STATISTICAL TESTING

OF

BUSINESS-CYCLE THEORIES

I

A METHOD

and its Application to

INVESTMENT ACTIVITY

BY

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PREFACE

About two years ago the League published a book of which Professor Gottfried VON HABERLER, now of Harvard University, was the author, under the title "Prosperity and Depression". The purpose of this book was to examine existing theories concerning the nature of what is currently termed the trade cycle, with a view to ascertaining what they had in common, the points at which differences of opinion arose and, in so far as possible, the causes of those differences. Its publication constituted the completion of the first stage of an enquiry into the nature and causes of the trade cycle that had been begun some years earlier. The second stage, as explained in the preface to Professor von Haberler's book, was to consist of an attempt "to confront these various theories with the historical facts—to subject them, in so far as those facts can be quantitatively expressed, to statistical analysis", and, in so far as they cannot be so expressed, to compare them with the recounted records of the past.

The present volume, entitled "Statistical Testing of Business-cycle Theories—A Method, and its Application to Investment Activity", is the first instalment of a brief series of pamphlets which it is proposed to issue in execution of one of the tasks involved by the second stage of the enquiry. It has been prepared by Professor J. TINBERGEN, who has been seconded for this purpose from the Central Statistical Bureau of the Netherlands. The primary object of this volume is to explain the statistical method which—subject to any suggestions that may be received—it is proposed to employ. With a view to illustrating this method—known as multiple correlation analysis—three examples of its application to economic phenomena have been given; these examples relate to fluctuations in total investment, residential building and net investment in railway rolling-stock.

The results obtained in the elaboration of these examples will, it is believed, prove of interest to students of the business cycle; but those results are in fact only incidental to the primary objects of this publication, which are, as I have stated, to explain the system of statistical analysis employed and, it is hoped, to arouse discussion concerning it that may prove of value to those in charge of the enquiry.

The manuscript of this volume has already been sent to a number of statisticians in different countries for comment, and two meetings of economists and statisticians have been held at which the assumptions made and methods adopted have been discussed. Thanks are due to all those who have helped by their criticisms and suggestions, and especially to Professor D. H. Robertson, who has ungrudgingly put his time at the disposal of the League for the purpose of consultation with Professor Tinbergen on the economic issues involved.

This introductory volume on method will be followed shortly by the first of the proposed analytical studies, which will be devoted to post-war business cycles in the United States of America. It is hoped that, before that study is completed, further comments and suggestions concerning the method here explained may be received either through Press reviews or directly from those who are interested and competent in this primary problem of methodology.

A. Loveday,

Director of the Financial Section and Economic Intelligence Service.

Geneva, January 1939.

CHAPTER I

INTRODUCTION

§ 1. Purpose of the Study

The purpose of this series of studies is to submit to statistical test some of the theories which have been put forward regarding the character and causes of cyclical fluctuation in business activity. Many of these theories, however, do not exist in a form immediately appropriate for statistical testing while most of them take account of the same body of economic phenomena—viz., the behaviour of investment, consumption, incomes, prices, etc. Accordingly, the method of procedure here adopted is, not to test the various theories one by one (a course which would involve much repetition), but to examine in succession, in the light of the various explanations which have been offered, the relation between certain groups of economic phenomena.

The enquiry is, by its nature, restricted to the examination of measurable phenomena. Non-measurable phenomena may, of course, at times exercise an important influence on the course of events; and the results of the present analysis must be supplemented by such information about the extent of that influence as can be obtained from other sources.

§ 2. Method employed

The method of study here employed, sometimes described as "econometric business cycle research," is a synthesis of *statistical business cycle research* and *quantitative economic theory.* A little may be said about each of these two elements.

(1) In the early phases of *statistical business cycle research*, attention was paid to somewhat superficial phenomena, such as the length of cycles, the degree of simple correlation between series and the relative amplitudes of their movements, the decomposition of series into trend, seasonal components, etc. Certainly all this work had its value, especially for the *negative* evidence it afforded on the validity of certain theories. For the purpose of applying more searching tests, however, it is necessary to dig deeper. An apparently simple relation, such as that between prices and production, is often not a direct causal relation at all, but a more or less complicated chain of many such relations. It is the object of analysis to identify and to test these direct causal relations: production, for instance, may be regarded as determined by the volume of orders; the volume of orders by the income of consumers and by prices; income by employment, wage rates and so on.

The part which the statistician can play in this process of analysis must not be misunderstood. The theories which he submits to examination are handed over to him by the economist, and with the economist the responsibility for them must remain; for no statistical test can prove a theory to be correct. It can, indeed, prove that theory to be incorrect, or at least incomplete, by showing that it does not cover a particular set of facts: but, even if one theory appears to be in accordance with the facts, it is still possible that there is another theory, also in accordance with the facts, which is the " true " one, as may be shown by new facts or further theoretical investigations. Thus the sense in which the statistician can provide " verification " of a theory is a limited one.

On the other hand, the rôle of the statistician is not confined to " verification ". As the above example illustrates, the direct causal relations of which we are in search are generally relations, not between two series only—one cause and one effect—but between one dependent series and several causes. And what we want to discover is, not merely what causes are operative, but also *with what strength each of them operates*: otherwise it is impossible to find out the nature of the combined effect of causes working in opposite directions. On this problem—the problem of " measurement ", as it may be called—the statistician can throw light by the use of the method called multiple correlation analysis. The details of this method are described in non-technical language in Chapter II, and in mathematical language in Appendix A.[1]

(2) *Economic theory*, to be capable of statistical test, must be expressed in quantitative—*i.e.*, in mathematical—form. What has usually been known, however, as mathematical economics deals chiefly with the conditions of an *equilibrium* which tends to be established in the long run, but is certainly not realised in the course of cyclical fluctuations. To be useful, therefore, for business cycle research, economic theory needs to be made " dynamic ". A " dynamic " theory, in the sense which is here attached to that ambiguous word, is one which deals with the short-term reactions of one variate upon others, but without neglecting the lapse of time between cause and effect. The equations in which it is expressed thus relate to non-simultaneous events, and take a form which Swedish economists have described as " sequence analysis ".

Take, for instance, the static concept of the functional relation between price and quantity supplied.[2] To convert this into a " reaction relation " or " direct causal relation " three things must be done. First, the relation must be exhibited in terms of cause and effect. Secondly, any time difference (lag) found to exist between change in price and change in quantity supplied should be mentioned explicitly—though in some cases, if the lag is very short (*i.e.*, if adaptation is almost instantaneous), it may legitimately be ignored. Thirdly, if quantity supplied varies to an important degree through causes other than changes in price (for instance, through changes in cost or in productive capacity), the influence of these other causes must be shown, and not left concealed in a *ceteris paribus* clause; though here again minor causes — *i.e.*, those whose combined effects are small — may legitimately be

[1] It is only in recent years that this method, developed especially by Mr. G. Udny Yule, and long known to mathematical statisticians, has been systematically applied in economic research, though some scattered applications to economic problems were made as long ago as 1906.

[2] This instance is taken for the sake of illustration only. In the study of cyclical fluctuations other " reaction relations ", such as those determining the movements in total outlay on investment or on consumption, appear to be of greater importance.

ignored, the formulation being confined to exhibiting the influence of major causes only. The necessary additions to static theory have, as a matter of fact, sometimes been found as a result of statistical research; in that sense, the statistician may supply theoretical suggestions to the economist.

Thus we find that the correlation analysis suggested by statistical technique and the sequence analysis dictated by " dynamicised " economic theory converge and are synthesised in the method employed in this study—the method, namely, of econometric business cycle research.

§ 3. Macro-economic Approach

There is one further feature of the method here employed which calls for remark. Economic analysis may be applied to the behaviour of individual persons or firms; or to the behaviour of " industries ", defined in some more or less arbitrary manner; or, again, to the behaviour of whole groups of industries, such as those producing consumption and investment goods respectively, and of whole categories of economic persons, such as those engaged in the credit market, or the labour market, as a whole. It is this last type of economic approach (sometimes spoken of as the " macro-economic " approach) which will be employed in this study. For it is this type of approach which seems most relevant to cyclical fluctuation, and which alone makes it possible to limit the number of variates considered to a figure which permits of their being effectively handled. It goes without saying that, in this approach, the coefficients found do not give any indications of the behaviour of individual entrepreneurs, consumers, etc., but only of the average reactions of many individuals.

CHAPTER II

ELEMENTARY OUTLINE OF THE METHOD OF CORRELATION ANALYSIS

§ 4. Simple Correlation

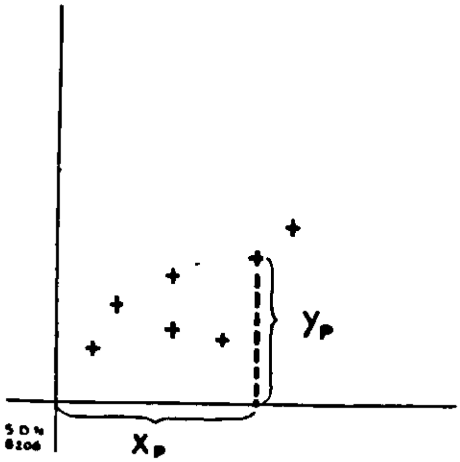
As has been pointed out in the previous chapter, the object of correlation analysis is twofold: (1) to test whether some expected relation between two or more variates exists (verification) and, (2) if so, to find the strength of the influences exerted by each causal phenomenon (measurement). The exact meaning of these terms and the consecutive steps in the analysis will now be discussed. It seems useful to begin with simple correlation.

Simple correlation.

Simple correlation is expected to exist if the fluctuations in any series Y are supposed to be caused (or chiefly caused) by the fluctuations in only one other series X. The simplest type of analysis that can be made in this case is to draw a scatter diagram. In such a

Graph II. 1.

Scatter Diagram.



Graph II. 2.

Perfect Correlation.

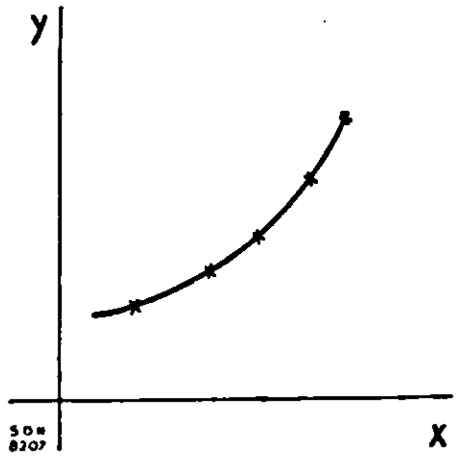


diagram (*cf.* Graph II. 1) each point, such as P, is so situated that, with a given scale, the length of its two "co-ordinates" X_P and Y_P are equal to corresponding values of X and Y. The collection of points obtained ("the scatter") may or may not lie approximately on one simple curve.

Perfect correlation.

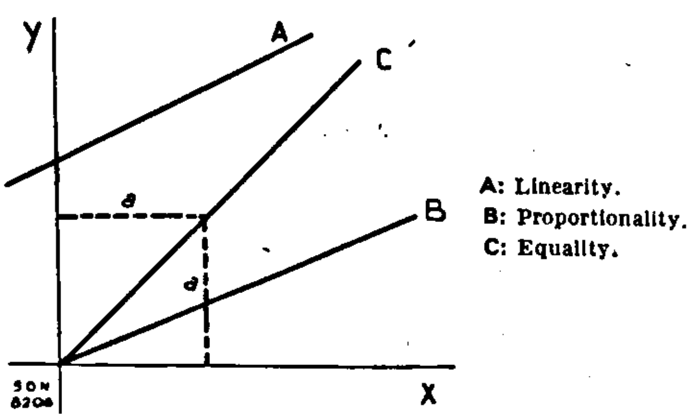
The extreme case of perfect correlation presents itself when all points lie exactly on one curve (Graph II. 2). In that case, the values of Y are exactly determined by those of the corresponding X's. X and Y are said to show a "functional relationship", and Y is a function of X, or X a function of Y. In other words, there is complete or perfect correlation between X and Y. Knowledge as to which is the cause can come only from outside.

Perfect linear correlation and perfect curvilinear correlation.

The curve may or may not be a straight line. If it is, the function or correlation is said to be linear; if not, it is called curvilinear. Linear relationship between X and Y does not necessarily mean proportionality; this occurs only if the straight line passes through the "origin" of the system of co-ordinates (*i.e.*, the point with co-ordinates zero-zero). A still more special case is that of equality between X and Y: then the line has to pass not only through (0,0) but also through every other point (*a*, *a*) with equal co-ordinates (Graph II. 3).

Graph II. 3.

TYPES OF LINEAR CORRELATION.



Regression curves and coefficients.

The curve through the points is called the *regression curve.* If it is a straight line, its slope is termed the regression slope and may be measured by a " *regression coefficient* ": this coefficient indicates the increase in Y which corresponds to a unit increase in X. In the table below, which indicates corresponding values for X and Y, arranged in ascending order, a unit increase in X clearly corresponds to an increase of 2 in Y. The regression coefficient is therefore 2. The relation between X and Y may also be described by the formula $Y = 10 + 2X$.

X	Y
10	30
11	32
12	34
13	36
14	38
15	40

Lags.

" Corresponding values " of X and Y will often be values for the same period. In some cases, however, the relation is between values of X and later values of Y. The time difference between corresponding values of X and Y is called the lag; Y lags behind X or X leads Y. It will be clear that if X is cause and Y effect, then X will lead Y. This fact may sometimes be used in order to find out which of two series is cause, which effect.[1]

The provisional determination of lags is best done with the help of an historical graph, showing the development in time of both series.

An example is to be found in Graph II.4, where two series have been drawn representing:

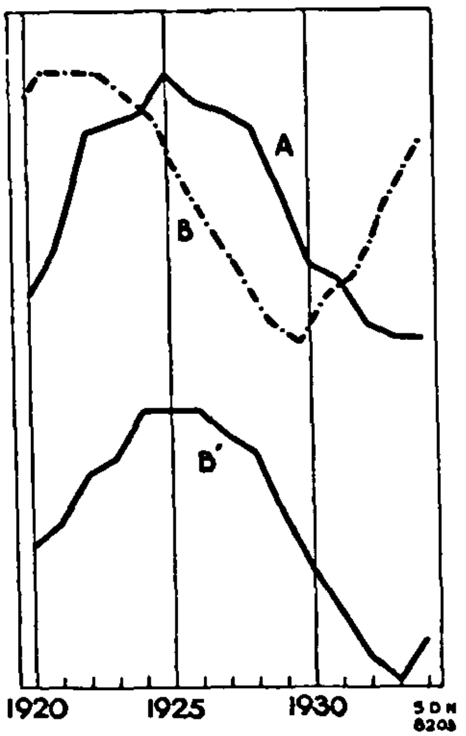
(A) Total volume of non-farm residential building in the United States, 1920-1935.

[1] One has, however, to be careful: it may happen, *e.g.*, that X leads Y, so that it would seem as if X were cause, Y effect. At the same time, however, $\dot{Y}$ (the rate of increase in Y) may lead X, and therefore $\dot{Y}$ may equally well be cause of X. Finally, it is possible that both causal connections exist: $\dot{Y}_t$ determining X_{t+1} and X_{t+1} determining Y_{t+2}.

(B) Total stock of houses, United States, deviations from trend (inverted).[1]

Graph II. 4.

Example of Lag.



It is at once clear that there is a lag of about three years and a-half between A and B. An immediate comparison of A with B′, where any value B′ equals the value of B three years and a-half earlier, confirms the existence of this lag.

Imperfect correlation.

As a rule, the scatter will not show perfect organisation. There may, however, still be a tendency for the points to group along a curve: then *imperfect correlation* is said to exist. That curve will now no longer be exactly determined. Various choices as to its type are possible, some of which will be discussed later. Once a choice has been made, the deviations between the actual points and the curve may be measured. Here also several methods of measurement may be chosen; but, after this second choice has been made, a measure for the degree of organisation can be given.

Correlation index; correlation coefficient.

The usual measure taken in the case of a general curve is the *correlation index*, which in the case of a straight line is reduced to a simpler measure called *correlation coefficient.* Both expressions have the property of being always less than or equal to unity; and they reach unity only if there is perfect correlation between the two variates (in the case of the correlation coefficient,

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[1] The trend of a series is a series indicating its general tendency. Details as to calculation of trend will be found in Appendix A.

if there is perfect linear correlation). Conversely, if they are equal to unity, there is perfect correlation (in the case of the correlation coefficient, perfect linear correlation).

Regression curves and coefficients.

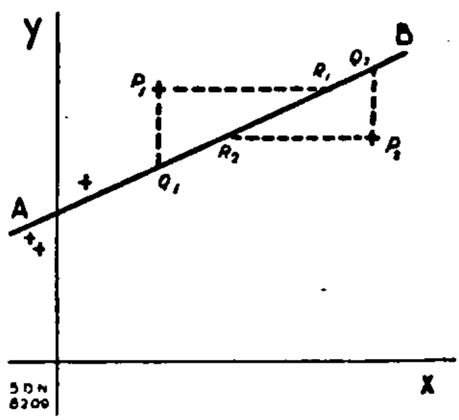
The notions of regression curve and regression coefficient, introduced above for the case of perfect correlation, are also used in cases of imperfect correlation; but they now depend on certain choices.

First, the type of curve has to be chosen. Usually a straight line is first tried. Secondly, a method of measuring deviations of the points from that line has to be devised. They may be measured in the direction of the Y-axis, in the direction of the X-axis or in other ways.

Graph II. 5 illustrates the procedure. The points representing the given observations are indicated by P_1, P_2, etc. As a regression curve, the line AB has been chosen. The deviations of P_1, P_2, etc., from AB, measured in the direction of the Y-axis, are indicated by P_1Q_1, P_2Q_2, etc. Those measured in the direction of the X-axis are indicated by P_1R_1, P_2R_2, etc.

Graph II. 5.

MEASUREMENT OF DEVIATIONS FROM A REGRESSION LINE.



The third step is to adopt some method for determining the curve in such a way that the deviations just defined will be as small as possible. Usually the " method of least squares " is taken: the sum of the squares of the deviations is made a minimum. In other words, that line is chosen as a regression line which shows the minimum sum of squares.

If deviations are measured in the Y direction—*i.e.*, in the direction of the dependent variate—the line obtained is called the *firs elementary regression line.* If deviations are measured in the X direction, the *second elementary regression line* is obtained. Each of the regression lines will be characterised by a regression slope and a regression coefficient.

In the case of perfect linear correlation, these two regression lines coincide, and no trouble arises as regards the choice indicated. When the correlation is not perfect, the difference between the two regression coefficients gives an idea of the degree of organisation of the scatter.

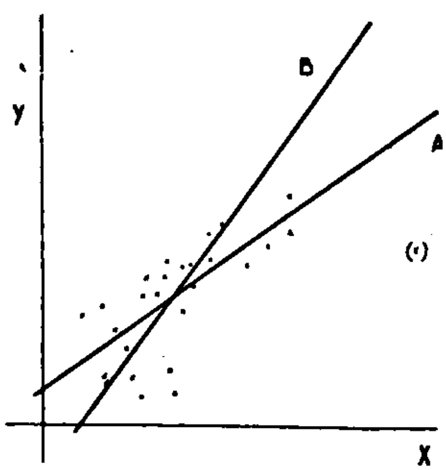
All that has been said applies as well to series x, y, etc., indicating the deviations which X, Y, etc., show from their average value $\overline{X}$, $\overline{Y}$, etc. over the period studied.

The correlation coefficient and the regression coefficients enable the two objects of the analysis to be substantially attained. The correlation coefficient tells whether or not the assumed relation between X and Y is exact, and therefore gives an answer to the verification problem. The regression coefficients indicate about how large a change in Y corresponds to a given change in X, and therefore answer the question of measurement. A first rough test of the economic significance of the coefficients is afforded by their signs, which may or may not be such as economic theory would lead one to expect.

Example. Graph II. 6 gives the scatter diagram between X, " value added " per ton of pig-iron, and Y, pig-iron production for Germany, 1881-1911. Value added per ton, which equals price minus raw-material cost, has been taken in order to eliminate the effect of the most important changes in production cost. Production has been measured in a somewhat unusual way, in order to eliminate influences of growth in productive capacity—viz., as the percentage deviation from trend. The relation is in its essence a supply relation, in which disturbing influences of cost and capacity changes have been eliminated by one of several possible methods. The scatter is moderately organised, and the only indication of curvilinearity is in the single point to the right, corresponding to the boom year 1900. Leaving aside this point, two elementary

Graph II. 6.

EXAMPLE OF SCATTER DIAGRAM.



linear regressions have been calculated and the corresponding lines drawn. The first elementary regression formula runs:

$$Y - \overline{Y} = 0.71\,(X - \overline{X})$$

in which $\overline{X}$ is the average value of all X's (except that for 1900) and $\overline{Y}$ is the average value of all Y's (except 1900). The meaning of the regression equation is that an increase of one point in X causes an increase of 0.71 points in Y. The second elementary regression would yield the figure 1.37 instead of 0.71 and shows that a rather high degree of uncertainty prevails here. The economic significance of these figures is closely connected with the elasticity of supply. In fact, it follows from the above definitions that an increase in prices by one unit, raw-material cost being supposed equal, would cause an increase in production of about 0.71% (the trend value of production being used as a basis). As the average price for the period was 59.7 Marks per ton, it may easily be deduced that the elasticity of supply was then 0.42. If the second elementary regression had been used, a figure of 0.82 would have been obtained.

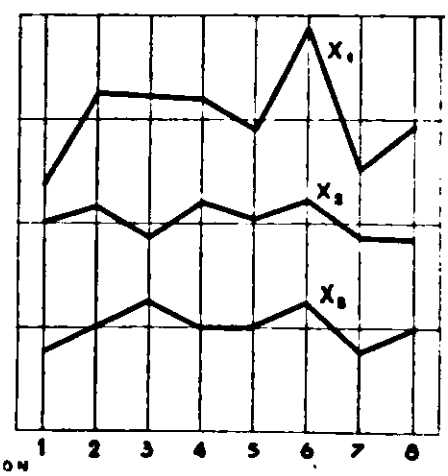
A first rough test of the economic trustworthiness of this figure is to see whether it has the right sign—*i.e.*, whether positive price changes are connected with positive changes in supply.

§ 5. Multiple Correlation

Multiple correlation.

As has already been said, in by far the greatest number of cases of economic importance, more than one cause is ordinarily assumed to have acted. Fluctuations in a series x_1 will have to be explained by the fluctuations in a number of other ("explanatory") series x_2, x_3, etc. To begin with, the nature of this problem may best be illustrated by an historical graph of all the series involved (*cf.* Graph II. 7). For

Graph II. 7.
Perfect Multiple Correlation.



the sake of simplicity, we may again start with a case of perfect linear relationship.

Perfect linear multiple correlation. The problem then is to find the figures b_2, b_3, etc., by which the series x_2, x_3, etc., have to be multiplied in order that the sum $b_2x_2 + b_3x_3 + ...$, calculated for each time-point, may equal the corresponding figure for x_1. In the theoretical example of Graph II.7, these numbers are 2 and 5. In fact, $2x_2 + 5x_3$ gives exactly x_1 for each observation. The graph shows some elementary features which are important for the carrying-out of the analysis and may therefore be stated. The decline in year 5 is caused entirely by series x_2, x_3 showing no decline at all in that year. On the other hand, the rise in year 8 can only be explained by x_3, as x_2 does not rise in that year. These two examples clearly show that only a combination of x_2 and x_3 can give the right result. Moreover, it is the combination with coefficients 2 and 5 which gives the best result, as is seen very clearly in year 3, where only that combination will produce the absence of change in x_1. These elementary remarks are intended to demonstrate that considerations of this kind may be helpful in the study of actual relationships, since they may show, after a mere visual inspection of the statistical material, whether or not success is to be expected.

Regression coefficients. As in the case of simple correlation, the coefficients 2 and 5 in the above example are called regression coefficients. As before, the regression coefficient indicates the increase (or decrease) in x_1 caused by a unit increase of x_2 or x_3 respectively; and, as before, a first rough test can be applied to this conclusion by enquiring whether the coefficient has the sign which economic theory would lead one to expect.

Influence of x_2. In the expression $2x_2 + 5x_3$, the term $2x_2$ (in general b_2x_2) may be called " the influence of x_2 " and $5x_3$ " the influence of x_3 ". In using these terms, one must, however, bear in mind that this expression is justified only so far as the economic theory which has prompted the calculation is accepted as valid. The special value of such a term in year t may be called " the influence exerted in that year "; whereas the strength of that influence in a given period may be characterised by, *e.g.*, the standard deviation of the term—*i.e.*, $2\sigma_{x_2}$, etc. All these expressions are independent of the units in which x_2 or x_3, etc., is measured.

Partial scatter diagrams. In multiple correlation analysis, the scatter diagram may still be used, but with a somewhat different function. Plotting three or more variates in a plane is not easy; but, instead, two or more partial scatter diagrams may be considered. The first uses as co-ordinates x_2 and $x_1 - 5x_3$ (in general, $x_1 - b_3x_3$): *i.e.*, x_1 " minus the influence of x_3 ", or " corrected for changes in x_3 ". The diagram so obtained illustrates the relation between x_2 and x_1 " other things being equal " or, more exactly, " other relevant things being equal ". A second diagram may be constructed comparing x_3 and $x_1 - 2x_2$ (in general, $x_1 - b_2x_2$).

Imperfect multiple correlation. The same technique [1] can be usefully employed in cases where no figures b_2, b_3, etc., can be found which make $b_2x_2 + b_3x_3 + \ldots$ exactly equal to x_1, for each time-point. This, in fact, is generally the case as long as the number n of series considered is smaller than the number N of time-points.[2] We must be satisfied if certain values for b_2, b_3, ... give a fairly good fit. As in the case of only two variates, such coefficients b_2, b_3, ... can be calculated after choosing the way in which deviations are to be measured and minimised. Again, b_2, b_3 are called regression coefficients, and the expression

$$x_1^* = b_2x_2 + b_3x_3 + \ldots$$

is called the *regression equation* of x_1 on x_2, x_3, etc.; x_1^* is often called the *calculated or theoretical value* of x_1. The differences $x_1 - x_1^*$ for each point of time are called *residuals*. If the line of best fit is chosen so as to make the sum of the squares of these residuals

[1] An example of this technique is found in Graphs III. 9-III. 11.

[2] If n equals N, then values b_2, b_3, etc., can always be found, as the number of unknowns b_2, b_3, etc., equals the number of relations which must be fulfilled.

as small as possible (*i.e.*, by application of the principle of least squares to the residuals), it is called the *first elementary regression.* The corresponding values for b_2, b_3, etc., will be written as

$$b_{12}, b_{13}, \dots$$

The deviations might, however, have been measured in other directions—*e.g.*, in that of x_2, by trying to find an expression $x_2^* = b_{21}x_1 + b_{23}x_3 + \dots$ which shows a minimal sum of the squares of $x_2 - x_2^*$. This is the second elementary regression. Of course, there are n such elementary regressions. In the calculations discussed in later chapters, the first elementary regression will generally be used; but information as to the other regressions will also be included.

The *total correlation coefficient* R between x_1 and x_1^* can be used as a measure of the degree of accordance between x_1 and x_1^*, and therefore, to some extent, as a measure of the success obtained.

The technique of partial scatter diagrams is again helpful to show whether or not the correlation obtained is satisfactory.

Multiple curvilinear correlation.

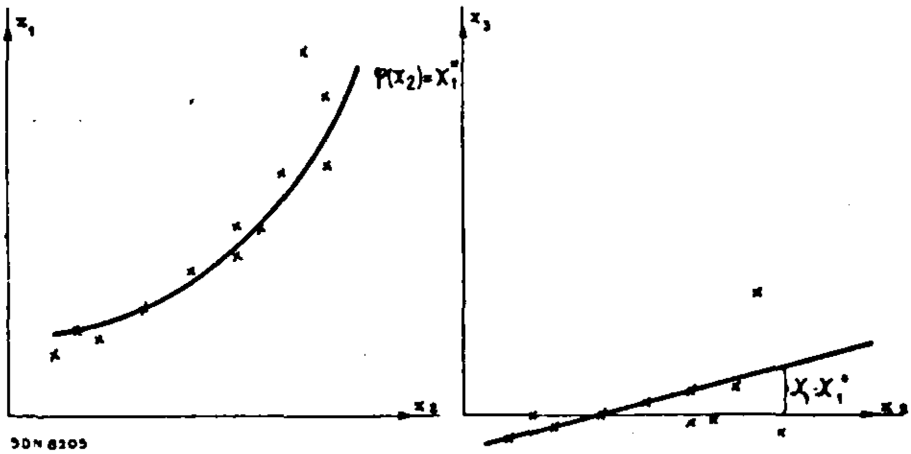
Partial scatter diagrams are especially helpful in order to test whether or not the assumption that the relation between x_1 and x_2, x_3 ... is linear, fits the facts.[1] If the partial scatters show curvilinearity, this assumption is no longer valid. Two ways are open for further attempts. First, more complicated algebraic formulæ can be tried and treated in a similar way to the linear ones; secondly, graphic methods can be used. These, however, can only start with a scatter between x_1 and one other variate (say, x_2), it being difficult to plot three or more variates in one chart. This scatter may show a tendency to a curvilinear relation, which may be drawn as a freehand curve through the cloud of dots. Let its ordinates (*cf.* Graph II. 8) be called $x_1^* = \varphi(x_2)$. Then for each point the difference between x_1 and the value x_1^* corresponding to its x_2 may be calculated, and this difference may be plotted again as x_3. If a close correlation—perhaps also curvilinear—is found, the curvilinear explanation may be more acceptable than the rectilinear one. Many alternatives are possible; to give details regarding them

[1] Graphs III. 9-III. 11 provide some examples.

and regarding the refinements of the method would, however, lead us too far.

Graph II. 8.

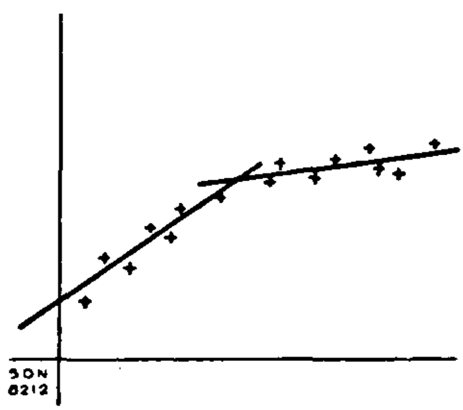
GRAPHIC ANALYSIS OF CURVILINEAR CORRELATION.



As a rule, curvilinear relations are considered in the following studies only in so far as strong evidence exists. A rough way of introducing the most important features of curvilinear relations is to use changing coefficients—for instance, one system of coefficients for the description of situations not far above normal and another for the description of extremely high levels. This amounts to approximating a curve by means of two straight lines (*cf.* Graph II. 9). Another way of introducing curvilinear relations is to take squares of variates, or still other functions, among the " explanatory series ".

Graph II. 9.

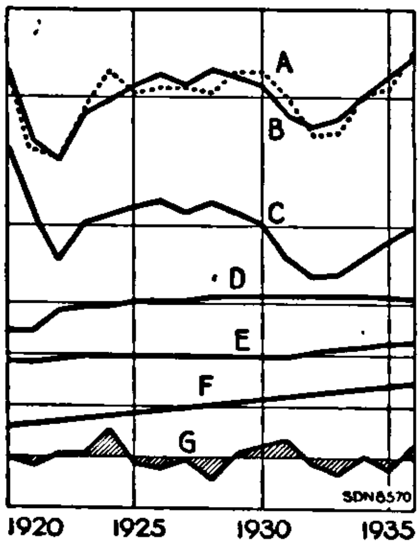
APPROXIMATION OF A CURVILINEAR RELATION BY TWO RECTILINEAR ONES.



Lags may also be introduced in multiple correlation analysis. The best lag, however, can no longer be determined by mere examination of historical graphs, since it depends on the relative influence of the various explanatory factors, and this relative influence varies in turn with the lag chosen. In principle, all possible lags must be tried and the regression coefficients calculated on each assumption. In practice, simplifications of procedure are possible if, for instance, one series only is of major influence and the others are secondary.

Graphic representation chosen.

As a rule, the results of multiple correlation calculations will be represented as in Graph II. 10. At the top, the actual series to be " explained " is indicated by dots, and on the same scale the theoretical values are indicated by a continuous line. Below the two lines, the various composing series b_2x_2, b_3x_3, etc., are drawn.



Graph II. 10.

Graphic Representation of a Correlation Calculation. " Explanation " of Iron and Steel Consumption. United Kingdom 1920-1936.

A = Actual iron and steel consumption.
B = Calculated iron and steel consumption.
C = Influence of profits one year before.
D = Influence of interest rate ½-year before.
E = Influence of price of iron ½-year before.
F = Influence of time.
G = Residuals, *i.e.*, A-B.

The ordinates of these lines are proportional to—not equal to—x_2, x_3 (and even proportional only in cases of linear formulæ). They represent what have been called " the influence of x_2 ", " the influence of x_3 ", etc. The advantages of this procedure are, first, that the scale of these series is comparable with that of the first series, and secondly, that it can be seen at once which of the series are important (*a*) in general or (*b*) for the explanation of any particular feature.

Example.

For example, Graph II. 10 is the result of a calculation aiming at " explaining " the fluctuations in iron and steel consumption in the United Kingdom 1920-1936; the explanatory series are profits of all industries one year before, bond yield and iron price half-a-year before, and time. The regression equation found is

$$x_1^* = 1.17x_2 - 0.08x_3 - 0.24x_4 + 2.39x_5$$

where x_1^* represents the calculated physical volume of iron and steel consumption in percentage deviations from average;

x_2, profits, all industries, percentage deviations from average;
x_3 bond yield, deviations from average in hundredths of 1%;
x_4 iron prices, percentage deviations from average;
x_5 time, years.

Obviously, this relation may be interpreted as a demand relation for iron where the series x_2, x_3 and x_5 have been taken as the other chief causes for changes in demand, and where a lag of one year for x_2 and of half-a-year for x_3 and x_4 has been assumed to exist. The movements of the three series x_2, x_3 and x_5 are responsible for the shifts in the demand curve during the period under review. The partial scatter diagram between $x_1 - 1.17x_2 + 0.08x_3 - 2.39x_5$ and x_4 would give the usual representation of the demand curve, shifts having been eliminated. As both x_1 and x_4 have been measured in percentage deviations from average, it will readily be seen that the elasticity of demand for iron would amount to -0.24 for prices and quantities near to their average values. Economically, the negative signs of the coefficients of x_3 and x_4 are as they should be. In addition, it may be mentioned that the " influence " of x_3 and x_4 is only small.

§ 6. Statistical Significance of Results

Reliability of results. The reliability of results may be judged by statistical as well as economic criteria. In general, the figures used are not exact. They are often derived from samples, or otherwise more or less inadequate for the problem under consideration. In addition, a number of minor explanatory causes are omitted; this seems to be the chief reason why observed and calculated values of x_1 in general do not coincide, and this lack of coincidence is responsible for a certain ambiguity in the results obtained. The question arises whether limits may be indicated for this uncertainty. As nothing is known about the factors omitted, it can be answered only if certain additional hypotheses are made.

Various methods of statistical testing have been worked out, using different hypotheses and leading, therefore, to different results. Some account of these methods will now be given. The non-mathematical reader should be warned that their comprehension will make somewhat greater demands on his attention than has the foregoing exposition of the method of multiple correlation analysis itself; and he may perhaps prefer to take the remainder of this chapter, together with Appendix A, on trust.

The classical method.

The classical method goes back to LAPLACE and GAUSS. It will be considered here in the final form that has been given to it by Professor R. A. FISHER.[1] According to this method, it is assumed that the unexplained parts—the residuals—are due to the circumstance that the " explained " variate, though essentially a linear function of the " explanatory " variates, contains an additional component representing the influence of neglected explanatory variates and may, moreover, be subject to errors of measurement. This so-called " *erratic component* " or " *disturbance* " in the explained variate not only gives rise to unexplained residuals, but also causes the regression coefficients calculated from the observations to differ from the coefficients of the true relation connecting the variates. The probable average magnitudes of these differences are derived from the assumption that the disturbances in subsequent time intervals are to be considered as " random drawings " from the " universe " of all possible values of these disturbances. In that " universe " there will be larger and smaller values of these disturbances, and these values are assumed to be normally distributed. This normal distribution means that the number of cases present in each class of magnitude will be determined by the so-called Gaussian law. In ordinary speech, small disturbances will be numerous and large disturbances will be few, their frequency obeying a simple law. The square root of the mean value of the squares of these disturbances is called their *standard deviation*, and is denoted by σ.

On certain further assumptions of a rather technical nature, it becomes possible to calculate what results with respect to the regression coefficients would have been obtained if another sample of disturbances had—by accident, so to say—been drawn. By comparing all possible results, one may say within what limits the results of the great majority of the possible cases will lie. These limits depend again on the choice one makes as to the " majority ". Often 99% or 95% is taken. If b_2 is one of the regression coefficients calculated, and σ_{b_2} the so-called standard error of b_2, about 95% of the cases lie between

[1] Cf. *Statistical Methods for Research Workers*, London and Edinburgh, 1936; " The goodness of fit of regression formulæ and the distribution of regression coefficients ", *Journ. Roy. Stat. Soc.*, **85**, 1922, p. 597; applications of " Student's " distribution, Metron, **5**, 3, 1926, p. 3.

$b_2 + 2\sigma_{b_2}$ and $b_2 - 2\sigma_{b_2}$, *i.e.*, in a range of width $4\sigma_{b_2}$ around b_2. About 99.7% lies between $b_2 \pm 3\sigma_{b_2}$.

This standard error σ_{b_2} is nothing else than the "standard deviation" of the differences between the calculated and the true regression coefficient in repeated samples. It depends—and with it the range of uncertainty in the calculated regression coefficients—on the following figures:

(1) The number (N) of observations containing mutually independent disturbances. The larger this number, the smaller σ_{b_2}. In economic problems, however, it is not always certain how large should be the time interval to which one observation refers in order to make successive values of the disturbances virtually independent.

(2) The number $(n - 1)$ of explanatory series. The larger this number, the larger σ_{b_2}. This will be understood if it is realised that, by $n = \mathrm{N}$ (*i.e.*, if the number of explanatory series is one less than the number of observations), a perfect correlation can be obtained by any set of mutually independent explanatory series, even if they do not bear at all on the subject.

(3) The total correlation coefficient (R). The nearer to 1 this number is, the smaller is σ_b; for $\mathrm{R} = 1$, σ_b becomes zero, except when there is perfect correlation between one of the explaining variates and a group of other explanatory variates.

(4) The correlations between two or more of the explanatory series. If at least one of these correlations is high, some of the regression coefficients show a larger σ_b (*i.e.*, are very uncertain). This, too, is easy to understand. In fact, in the extreme case, where two explanatory series were exactly parallel, it is clear that a substitution of one of them for the other would not change the correlation. The "best" fit could therefore be obtained with each of an infinite number of different combinations, in which one series would successively be substituted to a larger and larger extent for the other. The two regression coefficients of these two series would be entirely indeterminate; only some combination of them would be determinate.

Now even if the correlation between two explanatory series is not exact, small disturbances—which are always present—can change the result considerably, and therefore the various possible "samples" would show considerable differences. Hence σ_b will be large. The exact expression for σ_b and its computation are given in Appendix A, § 4.

Frisch's method.

Professor R. Frisch,[1] in his treatment of these problems, does not use the concept of some unknown "universe" from which a "sample" is drawn. He considers *every* variate as being built up of a systematic part and a disturbance. The relations assumed between the variates are supposed to hold good exactly between the systematic parts, and the regression coefficients in these relations

[1] Cf. *Statistical Confluence Analysis by Means of Complete Regression Systems.* Universitetets Økonomiske Institutt, Publ. Nr. 5, Oslo, 1934.

are called the true coefficients. The calculated coefficients may again show deviations from the true, and the object is to find these deviations or a limit to them.

On the further assumptions that there is no correlation (*i.e.*, that the correlation coefficient is zero) between: (i) the disturbances of different variates; (ii) the disturbances of one and the systematic part of another variate; and (iii) the disturbances and the systematic part of the same variate; it may be shown that, at least for problems of two variates the *true regression lies between the elementary regressions.*

Bunch-map analysis.

This is why Professor Frisch proposes to construct what he calls ***bunch maps.*** These indicate the regression slopes obtained for one pair of variates, if all possible elementary regression equations are solved. For a technical reason all variates are normalised—*i.e.*, expressed in their own standard deviation as units.

In order to explain the principle, a three-variate problem may be considered, where an endeavour is made to " explain " x_1 by x_2 and x_3. The first elementary regression equation provides an " explanation "

$$x_1' = b_{12\cdot3}\,x_2 + b_{13\cdot2}\,x_3 \tag{1}$$

with a regression coefficient $b_{12\cdot3}$ for x_2 and $b_{13\cdot2}$ for x_3. Taking the second elementary regression, we obtain an " explanation " of x_2

$$x_2' = b_{21\cdot3}\,x_1 + b_{23\cdot1}\,x_3,$$

which may, however, be transformed into an " explanation " of x_1 by putting $x_2' = x_2$ and solving for x_1:

$$x_1'' = \frac{1}{b_{21\cdot3}}\,x_2 - \frac{b_{23\cdot1}}{b_{21\cdot3}}\,x_3 \tag{2}$$

The two dashes have been added to indicate the second elementary regression as the origin of this estimate. Similarly, the third elementary regression

$$x_3' = b_{31\cdot2}\,x_1 + b_{32\cdot1}\,x_2$$

gives

$$x_1''' = -\frac{b_{32\cdot1}}{b_{31\cdot2}}\,x_2 + \frac{1}{b_{31\cdot2}}\,x_3 \tag{3}$$

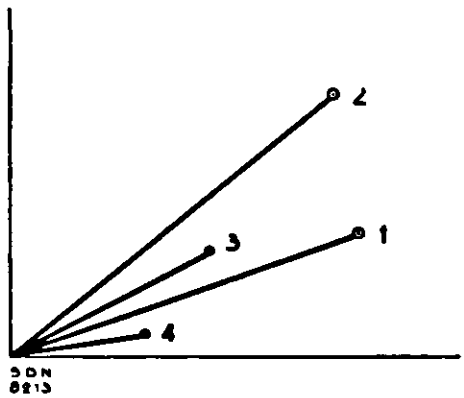
The equations (1), (2) and (3) are three estimates of the relation between the variates; two bunch maps are constructed to illustrate them. The first compares the three coefficients (in graphical representation, the slopes) obtained for the influence of x_2, viz. $b_{12\cdot3}$ from (1), $\frac{1}{b_{21\cdot3}}$ from (2) and $-\frac{b_{32\cdot1}}{b_{31\cdot2}}$ from (3). They are represented by three beams, numbered 1, 2 and 3 (being the numbers of the variates in whose direction the minimising has been performed). The beams 1 and 2 will be marked ⊙, indicating that the slopes are those between 1 and 2, 1 (the lower numbered variate) being considered as the variate to be " explained ". The second bunch map compares the three coefficients obtained for the influence of x_3 upon x_1 viz., $b_{13\cdot2}$, $-\frac{b_{23\cdot1}}{b_{21\cdot3}}$

and $\frac{1}{b_{31\cdot 2}}$. The beams are again numbered 1, 2 and 3, but here 1 and 3 are marked ⊙.

Similar bunch maps are made for all conceivable combinations of variates, starting with the simplest and ending with the " complete set " including all variates. The bunch maps for a two-set are of course extremely simple: they always consist of two beams only, which, by the choice of units referred to above, are necessarily situated symmetrically with respect to the two axes.

In general, each bunch map consists of a number of beams, two of which —the " leading beams "—have their ends marked ⊙ (*cf.* Graph II. 11). The numbers at the ends of these two beams indicate the variates, the regression between which is being studied. In the cases considered in Chapters III, IV and V, the variate with the lower number will always be taken as the one " to be explained ". Every other beam bears a number, and all the numbers together represent the group of variates used. The number attached to any beam represents the variate used as the left-hand variate in the regression equation before transformation. In other words, it indicates the direction in which deviations have been minimised in constructing the regression formula studied.[1]

Graph II. 11.
SPECIMEN OF BUNCH MAP.



A case of perfect relationship without any ambiguity is provided by bunch maps where all beams coincide; for it cannot make any difference in what direction we decide to measure deviations, if there are no deviations to be measured. But if in any case one of the explaining variates has been omitted, perfect correlation cannot exist, and no perfectly closed bunches appear.

Useful, superfluous and detrimental variates.

If, therefore, in a given case the bunch is not closed, the aim of further research, and in particular of including further variates in the analysis, is to close the bunch. Any economically significant variate which helps to close the bunch, or brings about a distinct change in the various slopes in the bunch without making it less closed, is called a *useful variate*. Any new variate which only slightly changes the bunch is called *superfluous*. There is, however, a third possibility: the bunch may " explode "—*i.e.*, show a larger spread after a new variate has been introduced. This happens if there is a high correlation between the new series and one or several of the previous explanatory factors. We are then faced with a situation which is called " *multicollinearity* ". It has been shown that,

[1] An explanation of differences in the length of the beams would lead us into too much technical detail.

in such a case, some of the regression coefficients become very uncertain; it is therefore possible that quite different results will be obtained if the deviations are measured in different directions. The new variate, or one of the older variates, is then called *detrimental*. This should be interpreted as meaning that, if all variates are included at the same time, no trustworthy measurement can be made. This does not mean that the variate in question may not be economically significant, but only that, owing to some circumstance (fortuitous or systematic), complete measurement is impossible. A less ambitious measurement may still be possible. Because of the great importance which attaches to these cases, the following simple example may be given.

Suppose one tries to determine the demand function for butter; x_1, the quantity of butter sold, has to be explained by

x_2 the price of butter,
x_3 the price of margarine,
x_4 the income of consumers.

Now we find that butter and margarine prices (at least their annual averages) are fairly highly correlated. Hence in the proposed demand equation

$$x_1' = b_2 x_2 + b_3 x_3 + b_4 x_4, \tag{1}$$

while b_4 may perhaps be readily determined, it will be impossible to find b_2 and b_3 separately with sufficient accuracy. One expedient, however, may be adopted. If x_3 is left out, the equation

$$x_1'' = b_2' x_2 + b_4' x_4 \tag{2}$$

may be tried; it will be possible to determine b_2' and b_4' provided the correlation coefficient found is not too bad (b_4' will be approximately equal to b_4). Equation (2) may be used instead of (1) if it is kept in mind that x_2 now stands for the combined influence of x_2 and x_3; it will be found that approximately $b_2' x_2 = b_2 x_2 + b_3 x_3$. This holds good only as long as the correlation between x_2 and x_3 persists. Equation (2) may therefore be used in all problems in which this correlation does not fail. For example, if the price of butter is raised by State regulation, but the price of margarine is raised as well so as to maintain the correlation between the two prices, then the consequences of the policy on the amount sold may be calculated. If, on the contrary, the regulation does not maintain the correlation, the formula becomes useless for this purpose.

A combination of the two methods.

Dr. T. Koopmans[1] has pointed out that the classical method and that of Frisch are complementary rather than alternative. Each of them deals with a part of the margin of uncertainty which must be assigned to calculated regression coefficients. That part of this margin which constitutes the object of Fisher's argument could be called the *error of sampling*. According to Fisher's hypotheses, it is due to the fact that the disturbances in the explained variate may affect the calculated regression coefficients to an unpredictable

[1] *Linear Regression Analysis of Economic Time Series*, Haarlem, 1936.

amount, which can be dealt with only by means of laws of probability. Since Fisher does not assume disturbances in the explanatory series, he thus rules out the type of uncertainty studied by Frisch. For this additional uncertainty arises from the circumstance that we usually do not know to what extent the disturbances found to be present in the whole set of data must be ascribed to this or that variate entering into the relation; or, in more technical terms, since we do not know exactly, in calculating the regression coefficients, what relative weights should be applied to express the relative accuracy of each of the several statistical series representing the variates, we incur, by any choice of weights whatever, the risk of introducing an *error of weighting* in the calculated coefficients. On the other hand, the error of sampling is excluded from Frisch's argument by his somewhat restrictive assumptions which have been indicated above.

Koopmans therefore combines the two theories into one method which deals simultaneously with the error of sampling and the error of weighting in the calculated coefficients. His procedure is as follows: For any set of relative weights of the variates that we may choose—*i.e.*, for any numerical guess we may make about the *relative* strength of the disturbances in the several variates,— mathematical deductions lead to:

(1) A set of " best estimates " for the regression coefficients, which takes the place of the first elementary regression in the classical method;

(2) A set of " standard errors " indicating the degree to which each of these estimates may be subject to errors of sampling; these standard errors correspond to those of the classical theory;

(3) A set of estimates of the standard deviations of the disturbances in each of the statistical series employed, which estimates measure the *absolute* strengths of these disturbances.

Where normally the correct relative weights are unknown, it appears that, under certain conditions including mutual independence of disturbances in different variates, the estimates of the regression coefficients mentioned under (1) remain within certain limits for all *a priori* possible weights. These limits correspond to those found by Frisch for the case of two variates, and are given by the two ultimate beams (not always the two " leading " beams) in the bunch map for the corresponding coefficient in the complete set of variates. They constitute ultimate limits to the error of weighting.

In a number of cases, however, narrower limits can be established with the help of the estimates mentioned under (3). It is often very improbable that the disturbances in any variate are of a size comparable to that of the variate itself. If such a result were arrived at from any presumed set of relative weights, such weights could be discarded as being inacceptable. Thus, frequently, the elementary regressions corresponding to variates that exercise only a secondary influence on the explained variate are excluded by this rule. Interpreting this proposition in terms of the bunch-map analysis, it might be said that, in these cases, the beams corresponding to such series should be disregarded, or at least be assigned less importance than the others, even if they are " leading " beams.

CHAPTER III

FLUCTUATIONS IN INVESTMENT

§ 7. The Relation tested

Problem chosen for testing.

In this and the two following chapters, a number of the results obtained in applying the method described above to one of the central relations in business-cycle theory will be discussed. The relation in question may be defined as that indicating the " proximate " objective causes of changes in investment activity, looked at from the demand side—*i.e.*, from the side of investing entrepreneurs and public authorities.

Calculations have been made for investment in general, as well as for residential building and railways as important special cases.

As emphasised in Chapter I, the principles underlying the procedure are that economic theory has to suggest the factors to be considered, while the statistical testing process shows the maximum degree of accordance obtainable and the relative strength of each factor required to obtain that degree of accordance.

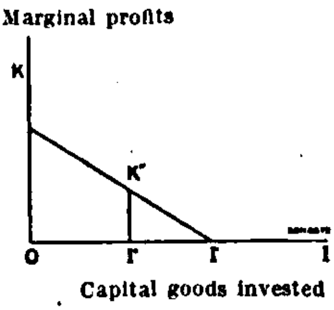
For the investigation of investment in general, the choice of the relevant factors has been based on the following considerations. Total investment activity is the sum of the investment activity of those individual entrepreneurs who decide to invest at all. The larger this *number*, the greater in general will the volume of investment be. Whether or not an entrepreneur decides to invest depends first of all on whether he expects to make profits or not. Therefore, the *number* of entrepreneurs planning investment will depend on profit expectations.[1]

[1] It is almost a tautology to say that investment is governed by profit expectations. It is not quite a tautology, however, since with the same profit expectations there may be different volumes of investment in different conditions, some of which will be mentioned later.

Next, the *extent* of the plans of those entrepreneurs who are planning investment will depend on a number of items entering into their calculations. These calculations, of course, partly reflect the profit expectations of the entrepreneur. There are, however, in them other elements of a more technical order. How much will be invested will also depend, *e.g.*, on the existing unused capacity to produce and on new technical possibilities.

Marginal profits and total profits.

Graph III. 1.



One aspect of these calculations may be considered in somewhat greater detail, with the help of Graph III. 1, which represents marginal profits (K) as a function of the number (I) of capital goods units in existence at a given moment. When this number is I″ at the moment of planning, the entrepreneur will *tend* to invest so many units that the point I′ with zero marginal profits [1] is reached. (If profits are understood to be taken before deducting interest payments, marginal profits minus interest payments are zero at point I′.) Provided that the curve K″I′, when it shifts upward and downward, does not change its slope—as is ordinarily assumed to be the case—I″K″, marginal profits, and the area OI″K″K, total profits on the existing plant, will show proportional changes. This parallelism will cease to be exact as soon as the curve K″I′ changes its slope too. Moreover, each addition to capital will, for the next time-unit, slightly change the base OI″ of the area, and so invalidate somewhat the proportionality between changes in total and changes in marginal profits in successive (instead of alternative) positions. Since, however, the effects of changes in marginal profits seem to be much greater than those due to this correction, in the statement that investment is determined by profit expectations the latter may with a fair degree of approximation be taken as expected total profits on existing plant.

[1] Evidently the argument of this paragraph also holds, *mutatis mutandis*, for *expected* marginal profits and *expected* total profits.

Finally, for many entrepreneurs—especially for public authorities planning to make investments—the *possibility of financing* investment activity will exert a considerable influence. From the theoretical point of view, it is perhaps superfluous to mention this aspect separately, since in profit calculations, in their widest sense, this financial possibility has somehow to be included. From the practical point of view it seems, however, useful to make the distinction, even if it cannot be maintained everywhere in the considerations which follow.

We shall now consider more closely the elements mentioned. Profit expectations themselves are, of course, hardly accessible to statistical measurement; but they will largely be determined by some objective criteria in the minds of most entrepreneurs. These objective criteria will in the first place be included as factors in our analysis.

Factors influencing profit expectations.

The factors which as a rule exert the greatest influence on *profit expectations* are by most authors assumed to be

(1) the magnitude of currently earned profits;
(2) the price of capital goods; and
(3) the rate of interest.

It might seem as if the rate of interest and the price of capital goods ought not to be included separately, since they enter through interest payments and depreciation into currently earned profits; but the interest rates and prices entering into these calculations are some sort of average over a long period and will therefore show almost no connection with the latest prices in both markets. Present investment will be governed by the rate of interest and the price of capital goods now prevailing, or at most will exhibit a fairly small and definite lag. For this reason, these factors are included separately; for most other cost items, this is not necessary.

Profit margin or total profit?

(4) Other authors have preferred to include as chief explanatory factor profit margins instead of total profits, profit margins representing the margin between average selling price and average prime cost. The chief reason for taking this course must lie in the hypothesis that the entrepreneur who plans to invest takes for granted the amount of additional output which he will

be able to sell as the result of his act of investment; in this case his expectation of profits will depend entirely on the margin between prices and prime cost. The possibility that his sales expectations will depend on the general business situation seems to have been neglected in this hypothesis. For entrepreneurs far from the margin, this may be justified to some extent; profit margins, as well as profits, have therefore been included as an explanatory variate. It may be asked whether it would not be preferable to include profit margin and volume of production separately as explanatory variates instead of total profits (which is about the same thing as the product of margin and volume of production) and profit margin. Since, however, statistics of total profits are, in general, more reliable than those of profit margins, and it may be expected that total profits as such have an important influence, it seems better to take total profits as one of the variates.

Profits and profit rates.

No distinction has so far been made between amount of profits and profit rate—*i.e.*, profit as a percentage of capital. Actually, this rate is commonly considered as the factor which has to be taken for our purpose. As will be seen below, in a number of cases figures are available and have been used. In other cases, however, only the amount of profits is available. From the statistical point of view the difference is very small. Generally the two series are very highly intercorrelated, as a consequence of the smooth movements of total capital stock. The results will therefore be very nearly the same—*i.e.*, the regression coefficient found for profits may be assumed to equal that which would be found for profit rate if the latter was used as explanatory series, provided that due corrections for changes in units were made.

(5) Another factor which has been mentioned is the rate of increase in prices. The underlying idea is that rising prices stimulate and falling prices curtail investment activity. Objects bought or constructed in times of rising prices show rising values and, therefore, rising possibilities of yielding a profit when sold; the reverse is true in periods of falling prices. The argument is of course more especially true for goods that are easily marketable, but applies also to some extent to capital goods, which may, *e.g.*, be constructed in advance when prices rise.

Financial possibilities. The factors determining the financial possibilities of investment are already included in the list just given. They are, first of all, currently earned profits, which are not only important for a great number of smaller enterprises, but also for public authorities. Public budgets will, with some lag, reflect private economic conditions and determine, in a high degree, the possibilities for public authorities to make large investments.

Indirectly, high current profits are also important for big enterprises, in so far as the raising of money by issuing new shares will be facilitated. The " easiness " of getting money in this way may also be inversely indicated by the share yield, which in some sense indicates the movements in the " interest rate " which the public expects to receive on new shares.

In addition, ordinary interest rates will be another indication. Long-term interest rates may play a rôle for those enterprises which usually finance their investments by bonds or mortgages (railways, building), or which base their calculations on those rates; short-term rates may influence financing by bank credits.

Technical circumstances. Apart from profit expectations, some technical circumstances also influence the volume of investment, as has already been observed. Apart from " autonomous " technical changes, to be discussed later, two factors seem outstanding.

Investment activity will be lower, the lower actual production is in proportion to existing productive capacity. As in general, and especially in the pre-war period, total capacity is a very smoothly moving series, the influence of that series may be neglected, since all our investigations deal only with the shorter fluctuations. As for the actual volume of production, it is so highly correlated with the volume of investment that it must not be included in our analysis as a separate series. Any calculation aiming at explaining investment activity will automatically take account of the circumstance discussed here.[1]

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[1] Indicating by v the volume of investment, by $u + v$ the total volume of production, and summarising in R all other factors in the " explanation " of investment activity, the relation to be tested will be: $v = a\ (u + v) + R$. Since $u + v$ is very near to bv (b constant), it follows that the equation may

(6) The second factor will be the rate of increase in the volume of production.[1] The larger this rate of increase, the larger the need for new capital goods. This is the tendency upon which the well-known " acceleration principle " is based. This connection is only a close one if no excess capacity is available; and this will be the case for only a small proportion of enterprises. In consequence, the strength of the force will be less than the pure theory of the simplest case would suggest.[2] Nevertheless it may have an influence.

Here, again, a mathematical difficulty presents itself under certain conditions—namely, when, as is the case in various periods and countries,[3] investment activity and consumers' goods production (and therefore also general production) are highly correlated. The rates of increase in the general production level and in investment activity are then also highly correlated. To include the series now proposed would therefore be almost the same as to include the rate of increase of the variate that is to be explained. It may be proved mathematically that this means introducing a small change in lag.[4] The new variate may therefore be given any importance (within some limits) one likes, if only the lag in the relation explaining investment activity be accordingly changed. Only if that lag may be known *a priori* will the problem be absolutely solvable. A further condition is, of course, that the lag, as well as the coefficient found for the rate of increase in total production, must be positive.

Series rejected.

The list of explanatory series omits two other series which have been emphasised by some authors and might possibly be considered as influencing investment fluctuations. These series have been rejected partly for *a priori* reasons, partly as a consequence of some provisional correlation calculations.

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also be written: $(1 - ab)\ \dot{v} = R$ or $v = cR$, meaning that $u + v$ could be left out as an explanatory variate.

[1] Not only consumers' goods production, since for the production of other goods machines, etc., are equally necessary.

[2] *Cf.* G. Haberler, *Prosperity and Depression*, page 84.

[3] *Cf.* discussion of results in § 10.

[4] *Cf.* end of § 8, p. 48.

(1) *"Weighted average of previous production."*

Some theorists have emphasised a new "explanatory" series by paying attention to what has been called the "echo effect". Suppose the duration of life of all capital goods were strictly equal to a given period: then production required for replacement of worn-out capital goods would be an exact repetition of total production of investment goods some time before. If, *e.g.*, that period were strictly seven years, then production in 1929 would be partly destined to replace capital goods produced in 1922, production in 1930 to replace those produced in 1923, etc. Now it is clear that (i) the life of various capital goods shows an enormous spread and (ii) even the life of, *e.g.*, one particular machine depends on business conditions in the year in which it is replaced. There will be a tendency to replace more in good than in bad years, even if the technical duration of life be the same. The first circumstance leads to the necessity of taking, for the explanation of 1929 production, not the year 1922 only, but a weighted average of a number of years which perhaps have their centre in 1922. Some experiments give the impression that the weighting flattens the curves so radically that practically no movements are left.[1] For this reason, as also on account of the uncertainty of the exact distribution of the duration of life, the variate "weighted average of previous production of capital goods" has been omitted. The second circumstance mentioned above—viz., the influence of the business position on replacement—is already taken into account by the inclusion of profits as one of the determining factors.

(2) *Autonomous changes in investment.*

Apart from the factors discussed, a number of extra-economic or autonomous factors will influence investment activity. Important inventions may do so; or political events which suddenly change expectations. These influences are considered, in this analysis, as non-systematic disturbances which act largely accidentally, in an irregular way, like lottery drawings. In general, such influences will exist whenever many mutually independent and small forces are acting, which will be the case in

[1] See also: P. de Wolff, "The Demand for Passenger Cars in the United States", in *Econometrica*, 6 (1938), page 113.

normal times. This is the approach to business-cycle problems which is known as the " shock theory of cycles ".[1] Some very exceptional events which do not obey these " laws " will be generally known, so that they may easily be eliminated before the analysis. This has been done, *e.g.*, with the English coalminers' strike in 1926, while for the American calculations the period since 1933 has been treated separately. With the exception of such events, the other autonomous influences are assumed to be included in the statistical residuals.

Two stages of investigation.

To sum up, there would be reason to include at least six explanatory series (indicated by the numbers 1-6), namely:

(1) current profits;
(2) price level of capital goods;
(3) interest rates (long, short or both);
(4) profit margin;
(5) the rate of increase of prices;
(6) the rate of increase in the volume of production.

To include this large number of series in all calculations would have meant such an amount of work that it seemed advisable to make the investigation in two consecutive stages. In the first stage, where a general orientation about the importance of each variate is the object, all series are included, but only three cases are considered (Germany and the United Kingdom before the war;

[1] *Cf.* R. Frisch : " Propagation Problems and Impulse Problems in Dynamic Economics " in *Economic Essays in Honour of Gustav Cassel*, London, 1933. The difference between this type of business-cycle theory and the other theories may be shortly characterised as follows: whereas most theories do not pay very much attention to external disturbances, and in most cases only start their argument with the assumption of an initial disturbance, the shock theory supposes that such disturbances work at very short intervals and are each of them of only little importance. From this simple fact, this theory concludes—by mathematical deductions—that the resulting movement may show, apart from the " endogenous " periods, apparent periods which are only the consequence of the cumulative effect of disturbances. The word " shock " has been chosen in analogy with physics, where such problems were already known.

the United States after the war), and the calculations are limited to regression formulæ and correlation coefficients. In the second stage, the calculations include the more important series only, but they are more detailed and cover six cases (adding to the above France and the United States before the war and the United Kingdom after the war). In addition, the significance of the latter calculations has been tested in various ways.

A rough and fairly easy first test of the significance of results consists in dividing up the period studied and repeating the calculations for the shorter periods. Similar results, with coefficients of the same order of magnitude, should be found: otherwise the significance of the results must be doubted. A number of calculations for the pre-war period have accordingly been repeated for two sub-periods—up to 1895 and from 1895 onwards. The year 1895 has been chosen as it is the turning-point of one of the so-called long waves.

In addition, another experiment has been made. As it has often been suggested that the "laws" governing upward phases and those governing downward phases of the cycle are different, the calculations have been repeated for (1) all years in upward phases and (2) all years in downward phases.

In addition, the more exact significance calculations mentioned in Chapter II have been made for some characteristic cases. They have not been repeated for all cases, as they are very laborious.

§ 8. The Statistical Material

The following details of the statistical series used in the calculations may now be given.

The countries and periods studied are:

Countries and periods.

Pre-war: Germany 1871-1912 (42 years);*
United Kingdom 1871-1910 (40 years);*
United States 1877-1913 (37 years);
France 1871-1908 (38 years).

Post-war: United Kingdom 1920-1936 (17 years);
United States 1919-1933 (15 years).*

Only the cases indicated with an asterisk have been included in the first stage.

The post-war figures for Germany are too much vitiated by autonomous events to afford a good basis for research. The period after 1933 for the United States has been left out, as the policy of the Government may have changed the relations investigated; the calculations have, however, been extrapolated (*cf.* Section 11).

Series used.

(a) *Description of series.* I. *Total investment* has as a rule been represented by estimates of the consumption of iron and steel (v_i). Alternatively, pig-iron production has been used. The main difference between the two series consists in exports of iron and steel (not included in consumption) and in scrap used in steel production (not included in production of pig-iron). The inclusion of exports may be interpreted as an attempt to take into consideration not only the home market, but also foreign markets. The exclusion of the second item is in most cases not serious, as there is a good parallelism between cycles in pig-iron and steel production. Only for the United Kingdom 1900-1910, where there was a marked divergency, must the consumption figures be preferred; it is interesting, however, to compare the results of the two attempts.

In the case of the United States, a more accurate estimate of the volume of investment (v) is available for the post-war years in the figures calculated by KUZNETS.[1] These figures, which distinguish between producers' durable goods, consumers' durable goods, and building, have also been used. Building has been excluded in all cases except one, where residential building (as estimated by WICKENS and FOSTER)[2] has been subtracted, and non-residential building retained.

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[1] S. KUZNETS, *Gross Capital Formation*, Bulletin 52, National Bureau of Economic Research, New York, 1934.

[2] D. L. WICKENS and R. R. FOSTER, *Non-farm Residential Construction 1920-1936*, Bulletin 65, National Bureau of Economic Research, New York, 1937.

The exact series used is indicated in each table. In general, the results obtained for the various series are not very different.

II. The explanatory series are taken from the following sources.

(1) *Profits earned* have been represented by the series indicated in the following table:

Country and period	Description of series	Source
United States, pre-war	(n) Share price index.	*Review of Economic Statistics.*
United Kingdom, pre-war	(E) Non-labour income.	BOWLEY, *Economic Journal*, 1904, completed with the help of data by STAMP, *British Incomes and Property* (based on Income Statistics).[a]
Germany, pre-war	(d) Dividends earned in % of capital.[b]	DONNER, *Die Kursbildung am Aktienmarkt, Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 36.
	(n) Share price index.	Same source.
France, pre-war	(n) Share price index.	Statistique générale de la France.
United States, post-war	(Z^c) Net income of corporations.	*Statistics of Income.*
	(n) Share price index.	Standard Statistics Co.
United Kingdom, post-war	(Z^c) Net profits earned in % of capital.[b]	*Economist.*

[a] Some experiments with statistical methods devised to find the annual figures out of the three-year-moving averages given by Income Statistics seemed to show that the cyclical movements of the latter and those of the former do not differ very much.
[b] Year of earning is taken to precede year of distribution.

(2) *Price of capital goods.* — The price of pig-iron (q_i) has been taken throughout, as it represents the most fluctuating item in the cost of capital goods. The sources are indicated in the following table:

Country and period	Description of series	Source
United States, pre-war	Price of No. 1 foundry pig-iron at Philadelphia.	*U.S.A. Statistical Abstract.*

Country and period	Description of series	Source
United Kingdom, pre-war	Sauerbeck's index of price of pig-iron.	S. Kuznets: *Secular movements in Production and Prices.*
Germany, pre-war	Average price of pig-iron produced.	*Statistisches Handbuch (1907); Statistische Jahrbücher.*
France, pre-war	Average price of pig-iron produced.	*Annuaire statistique de la France.*
United States, post-war	Price of Bessemer pig-iron at Pittsburgh.	*U.S.A. Statistical Abstract.*
United Kingdom, post-war	Price of Cleveland-Middlesbrough pig-iron.	Statist's index number of wholesale prices, *Journal of the Royal Statistical Society,* Part II.

(3) *Interest rates.* — In most cases, calculations have been made with (i) (m_s) market rate of discount or some other short-term interest rate and (ii) (m_{Lb}) bond yields.

In addition, some calculations have been made with (m_{Ls}) share yield, as representing a special category of " interest rates ", viz., the rate attributed to funds raised by share issues. The sources are summarised in the following table:

Country and period	Description of series	Source
United States, pre-war	(m_{Lb}) Long-term: 1890-1899: Yield on ten American railroad bonds.	*Review of Economic Statistics*, 1919.
	1900-1913: Yield on sixty bond issues combined.	Standard Statistics Co.
	(m_s) Short-term: market rate on 60-90 days paper.	I. Fisher, *The Theory of Interest.*
United Kingdom, pre-war	(m_{Lb}) Long-term: Yield on 2 ½% Consols.	,,
	(m_s) Short-term: market rate of discount.	,,
Germany, pre-war	(m_{Lb}) Long-term: Yield on fixed interest bearing securities.	Donner, see under (1).
	(m_s) Short-term: market rate of discount.	
France, pre-war	($^1/m_{Lb}$ Long-term: Index of price of 3% " rente ".	*Bulletin de la Statistique générale de la France,* 1919/20.

Country and period	Description of series	Source
United States, post-war	(m_{Lb}) Long-term: Bond yield, 60 issues combined.	Standard Statistics Co.
	(m_s) Short-term: market rate on 4-6 months commercial paper.	I. FISHER: *The Theory of Interest; League of Nations Monthly Bulletin of Statistics.*
	(m_{Ls}) Share yield: (i) Cash dividends of corporations in % of (ii) total capital stock.	(i) Statistics of Income. (ii) Statistics of Income; prior to 1925, estimates based on new security issues and index of share prices.
United Kingdom, post-war	(m_{Lb}) Long-term: Yield of 2 ½% Consols.	Statistical Abstract of the United Kingdom; Statistical Summary of the Bank of England.

(4) The calculation and sources of the figures for *profit margins* ($p-\frac{1}{2}\, l_w$) are given in this table:

Country and period	Description of series	Source
United States, pre-war	Index of cost of living — ½ index of hourly earnings.	National Industrial Conference Board.
United Kingdom, pre-war	(i) Index of prices of exported finished products — ½ (ii) index of wage rates.	(i) Calculation L.o.N. based on trade statistics. (ii) Index of Bowley and Wood, reproduced from Layton: *Introduction to the Study of Prices.*
Germany, pre-war	(i) General index of wholesale prices — ½ (ii) index of wages (both in % deviations from trend).	(i) JACOBS & RICHTER: Grosshandelspreise. *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft No. 37. (ii) J. KUCZYNSKI: *Löhne und Ernährungskosten in Deutschland.*

(5) The sources of the figures for (u) *production of consumers' goods* are:

Country and period	Description of series	Source
United Kingdom, pre-war	Index of production of consumers' goods.	HOFFMANN: *Weltwirtschaftliches Archiv*, vol. 40.
Germany, pre-war	Index of the Institut für Konjunkturforschung.	WAGENFÜHR, Die Industriewirtschaft. *Vierteljahreshefte zur Konjunkturforschung.* Sonderheft No. 31.

Country and period	Description of series	Source
United States, post-war	League of Nations estimates based on study by Warburton in:	*Journal of the American Statistical Association*, vol. 30.

(6) The figures for the *rate of increase in general price level* and their sources are:

Country and period	Description of series	Source
United Kingdom, pre-war	(Δq_i) Rate of increase in price of pig-iron. [a]	*Vide supra.*
Germany, pre-war	(Δq_i) *Idem.* [a]	*Idem.*
United States, post-war	(Δp) Rate of increase in cost-of-living index. [a]	National Industrial Conference Board.

[a] For pre-war times the general price level seemed to be well represented by that of investment goods. For post-war times iron prices are no more representative.

(b) *Trends.* — As the relation studied claims to represent only the causation of short-run movements in the volume of investment, deviations from trend have been taken throughout, except when otherwise stated. In general, trends have been calculated as nine-year moving averages for pre-war periods—which are long enough to allow of the first and last four years being omitted—and as rectilinear trends for post-war periods—which are too short to allow of omitting eight years.[1]

(c) *Other series included.* As has already been stated (Chapter III, § 7), an attempt has been made throughout to explain production of pig-iron or consumption of iron and steel (v_i) or total physical investment (v) by some of the six explanatory series mentioned above.

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[1] The trend chosen for the American figures (post-war period) may be somewhat biased by the fact that the period starts with a boom year and ends with a slump year.

In order to judge the importance of the trend, the first calculation of Table III.4 was made without correction for trend. The correlation coefficient was hardly affected, nor was the regression coefficient of profits (0.33 instead of 0.29). The combined influence of price and interest must therefore have been nearly the same; of the respective influence of these two factors, nothing can be said owing to a very high intercorrelation between the two variates (*cf.* Chapter II, § 6, page 29).

In some cases it has seemed useful to include still further series, sometimes in substitution for one of those mentioned already. In the case of the United States (pre-war), in order to obtain a fairly good explanation, it seemed necessary to include building volume (v_B) as a separate variate. This may be justified by the fact that the factors affecting the building volume are rather poorly represented in the four explanatory series: *e.g.*, building profits, which might play a rôle, are not reflected immediately in share-price indices. The same applies even more to other factors. The inclusion of building volume means, of course, only a postponement of the problem; the factors affecting building itself are to be studied afterwards (Chapter IV).

Lags considered.

(i) As a general starting-point, a lag of *half-a-year* was assumed to exist between the explaining series and investment activity.[1] In most cases, this seemed to be not far from reality.[2]

(ii) In the second stage, various experiments were made with *other lags*. They consisted either in assuming a lag of one year for all variates, or in introducing the same variate both with a lag of one year and without any lag and comparing the regression coefficients. For example, if the best explanation turns out to be: $0.8\, z + 0.4\, z_{-1}$, where z_{-1} stands for z one year earlier, this result represents a case of so-called " distributed lag " [3] with an average lag of 1/3 year. Similarly, lags of half-a-year may be introduced by simply taking as a variate $\frac{1}{2}\,(z + z_{-1})$ instead of z.

(iii) Here it should be remarked that an infinity of different interpretations can be given to the above formula, which, mathematically speaking, all come to the same. Instead, *e.g.*, of giving the above interpretation, one could read the formula:

$$1.2z - 0.4\,(z - z_{-1})$$

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[1] Except for the United Kingdom, where preliminary calculations showed a lag of one year for non-labour income to be preferable.

[2] A number of estimates available for the length of the production period would seem to indicate a few months for machinery, six months for house building, and one year for shipbuilding. *Cf.* " The Length of Certain Production Processes " (Dutch), *De Nederlandsche Conjunctuur*, August 1934, page 32.

[3] This term is due to Professor Irving Fisher. Distributed lags seem to be even more probable than simple lags.

saying that there are two influences, viz.:

(*a*) an influence of z, without lag and with strength 1.2;

(*b*) an influence of the rate of increase in z, with an (average) lag of half-a-year and a strength — 0.4.

§ 9. Chief Results

Before giving a detailed account of all the cases considered and results obtained, the conclusions drawn may be shortly summarised as follows.

(*a*) On the assumption that our estimate of iron and steel consumption (or the alternatives used) is a just index of investment activity, there is fairly good evidence that the fluctuations in investment activity are in the main determined by the fluctuations in profits earned in industry as a whole some months earlier.

(*b*) The influence of the other factors included is not considerable and is therefore, in many cases, numerically uncertain. This fact is reflected in

(i) the significance calculations in the ordinary sense, and

(ii) the fact that the " influence " of these other factors is sometimes positive and sometimes negative, and almost always small (*cf.* Chapter II, § 5).[1]

Nevertheless, for particular countries, fairly certain results are obtained (*cf.* next few sections).

(*c*) No systematic differences of a general character have been found to exist between upward and downward phases;

(*d*) As was to be expected, the difficulties arising from " multicollinearity "[2] increase with the number of variates and prevent the complete solution of a number of problems.

[1] The chief significance of this result is that in past cycles the rôle of interest rates and of prices of iron has been far less important than that played by profit changes. It does not follow for all cases—though it does for most of them—that interest rates might not have exerted a great influence if they had fluctuated more violently than in fact they have done.

[2] See Chapter II, § 6.

§ 10. Details of Results, First Stage.

The results of the calculations of the first stage are summarised in Tables III. 1, III. 2 and III. 3, giving the correlation and regression coefficients for the cases and variates indicated. The tables have been constructed in the following way. The chief explanatory variate, namely profits, has always been included; in the case of the United States two such variates were taken, namely profits and share yield. The selection of these variates has been based on their coefficient of simple correlation with investment activity v' and on the " influence " (*cf.* Chapter II) which they show in the more complete " explanations ".

The chief variate or the couple of chief variates has then been combined with each of the other variates, and with certain combinations of larger numbers of the latter. Variates showing regression coefficients with wrong signs have sometimes been excluded in order to reduce the number of possible combinations; *e.g.*, the variate profit margin, $(p_A - \frac{1}{2} l_w)_{-1/2}$ in the case of the United Kingdom in the four- and five-sets, and the variate rate of increase in price level (Δq) in the three-set. The same applies to variates showing a very small influence. This influence may be found by multiplying the standard deviation, given at the bottom of the tables, by the regression coefficients. In order to give an *a posteriori* test, this has been done for the highest regression coefficient in each column.

The tables have been used for three purposes. First, the increase in the correlation coefficient as the consequence of the inclusion of a new variate may be studied, and secondly, the stability of the regression obtained for one variate in various cases. Thirdly, conclusions as to the relevance of the variates may be drawn. These latter are based on:

(i) whether or not the variate in question increases the correlation coefficient to any considerable extent,

(ii) whether or not the sign of a fairly stable regression coefficient is right, and

(iii) whether or not the influence of that variate is perceptible.

Carrying out this programme for each of the variates, the following conclusions seem legitimate.

Table III. 1. "Explanation" of Investment Activity, First Stage.

United Kingdom, 1871-1910: Iron and Steel Consumption (v_i).

Case	Regression coefficients of: [1]							Correlation coefficients
	E_{-1}	$(q_i)_{-1/2}$	Δq_i	$(m_{Lb})_{-1/2}$	$(m_s)_{-1/2}$	Δu	$(p_A - 1/2\, l_w)_{-1/2}$	R
11	1.85							0.552
21	3.20	— 0.31						0.664
22	1.92		0.17					0.649
23	1.92			— 0.366				0.643
24	2.85				— 0.051			0.611
25	1.79					0.43		0.606
26	2.65						— 0.44	0.655
31	3.29	— 0.32	0.13					0.729
32	3.16	— 0.28		— 0.330				0.726
33	3.41	— 0.26			— 0.021			0.670
34	3.16	— 0.31				0.45		0.714
35	3.16	— 0.20					— 0.24	0.680
41	3.28	— 0.30	0.15	— 0.239				0.783
51	3.45	— 0.27	0.15	— 0.229	— 0.016			0.786
52	3.25	— 0.31	0.13	— 0.223		0.27		0.797

Standard deviations of variates: [1]

v_i								
7.48	2.23	13.25	14.90	6.74	57.87	4.38	7.26	
Maximum influence [2]	7.69	4.24	2.53	2.47	2.95	1.97	[3]	

[1] Units used: all series in percentage deviations from trend, except series m_{Lb} and m_s, which are in absolute deviations from trend, in units of 0.01 % and Δq_i and Δu, which are in percentages of the trend values of q_i and u respectively.

Meaning of symbols (for fuller explanation, see § 8).

v_i: consumption of iron and steel.
E: non-labour income.
q_i: price of iron.
Δq_i: rate of increase in $q_i = q_i - (q_i)_{-1}$.
m_{Lb}: bond yield
m_s: short-term interest rate.
Δu: rate of increase in consumers' goods production.
$p_A - 1/2\, l_w$ profit margin.

[2] Absolute value of product of largest regression coefficient with right sign and standard deviation.
[3] Wrong signs.

Table III. 2. " Explanation " of Investment Activity, First Stage.

Germany, 1871-1912: Iron and Steel Consumption (v_i).

Case	Regression coefficients of: [1]							Correlation coefficients
	$d_{-1/2}$	Δu	$(p-\frac{1}{2}l_w)_{-1/2}$	$(m_{LS})_{-1/2}$	$(q_i)_{-1/2}$	Δq_i	$(m_s)_{-1/2}$	R
11	5.37							0.853
21	5.37	0.23						0.857
22	5.18		0.10					0.854
23	5.63			— 0.013				0.854
24	5.39				— 0.006			0.853
25	5.51					— 0.05		0.854
26	4.96						0.027	0.865
31	4.98	0.32	0.21					0.859
32	5.70	0.25		— 0.017				0.857
33	5.19	0.28			0.038			0.857
34	5.44		0.09	— 0.013				0.854
35	5.32		0.28		— 0.101			0.856
36	5.78			— 0.015	— 0.020			0.854
41	5.80	0.36	— 0.13	— 0.024		— 0.07	0.034	0.875
Standard deviations of variates: [1]								
v_i 11.09	1.76	3.52	5.25	44.80	12.85	9.42	65.88	
Maximum influence [2]	10.21	1.27	1.47	1.08	1.30	[3]	[3]	

[1] Units used: all series in percentage deviations from trend except series d, m_{LS} and m_s, which are in absolute deviations from trend, in units of 1%, 0.01% and 0.01 % respectively. For Δu and Δq_i, see table III. 1.

Meaning of symbols (for fuller explanation, see § 8).

- v_i: consumption of iron and steel.
- d: dividends in % of capital.
- Δu: rate of increase in consumers' goods production.
- $p-\frac{1}{2}l_w$: profit margin.
- m_{LS}: share yield.
- q_i: price of pig-iron.
- Δq_i: rate of increase in price of pig-iron.
- m_s: short-term interest rate.

[2] Absolute value of product of largest regression coefficient with right sign and standard deviation.

[3] Wrong signs.

Table III. 3. “ Explanation ” of Investment Activity, First Stage.
United States, 1919-1933: Deliveries of Producers’ Durable Commodities + Non-residential Building (v').

Case	Regression coefficients of:[1]								Correlation coefficients
	$Z^c_{-½}$	$(m_{Ls})_{-½}$	$(q_i)_{-½}$	$(p-½l_w)_{-½}$	t	Δp	Δu	$(m_s)_{-½}$	R
11	0.19	—0.076							0.986
21	0.19	—0.074	—0.02						0.987
22	0 17	—0.083		0.10					0 987
23	0.18	—0.079			—0.13				0 986
24	0.20	—0.074				—0.29			0 990
25	0.19	—0.076					0.01		0.986
26	0.18	—0.078						0.004	0.986
31	0.16	—0.077	—0.01	0.27					0.989
32	0.18	—0.081	—0.07		—0.63				0.987
33	0.20	—0.071	—0.02			—0.29			0.991
34	0.17	—0.076		0.19	0.57				0.987
35	0.19	—0.076		0.02		—0.29			0.990
36	0.20	—0.072			0.10	—0.29			0.990
41	0.16	—0.075	—0.14	0.31	0.13				0.989
51	0.18	—0.071	—0.07	0.14	0.10	—0.23			0.991
52	0.16	—0.077	—0.17	0.31	—0.13		—0.06		0.989
53	0.17	—0.067	—0.18	0.24	0.27			0.012	0.990
	Standard deviations of variates:[2]								
v' 19.85	96.50	154.20	25.49	17.81	4.32	7.44	7.48	130.23	
Maximum influence[2]	19.30	12.80	4.54	5.52	2.72	[3]	0.07	[3]	

[1] Units used: All series in percentage deviations from the average, except series m_{Ls} and m_s, which are in absolute deviations from average, in units of 0.01 %. For Δp and Δu, see table III. 1 (Δq_i and Δu).

Meaning of symbols (for fuller explanation see § 8).

- v': deliveries of producers’ durable goods + non-residential building.
- Z^c: profits of corporations.
- m_{Ls}: share yield.
- q_i: price of pig-iron.
- $p-½l_w$: profit margin.
- t: trend.
- Δp: rate of increase in cost of living.
- Δu: rate of increase in consumers’ goods production.
- m_s: short-term interest rate.

[2] Absolute value of product of largest regression coefficient with right sign and standard deviation.
[3] Wrong signs.

The importance of profits (Z), and, in the case of the United States, of share yield (m_{Ls}), is confirmed: the increase in correlation and the influence of the variate are considerable; the signs are right.

Increase in correlation. The increase in correlation obtained by the inclusion of each variate may be taken from the following table:

Average Increase in Correlation Coefficient obtained by adding Each of the Following Variates.

Country	Profits	Share yield	Short-term interest rate	Price of iron	Rate of increase in		Profit margin
					production	prices	
United Kingdom 1871-1910	0.552	0.069 [a]	0.023	0.078	0.039	0.073	0.060 *
Germany 1871-1912	0.853	0.001	0.012 *	0.001	0.004	0.001 *	0.002
United States 1919-1933	0.795	0.191	0.001 *	0.001	0.000	0.003 *	0.001

[a] Bond yield.
* In this case the sign of the regression coefficient is wrong.

Signs. For the variates: short-term interest rate (m_s), rate of increase in price (Δq or Δp) and profit margin ($p - \frac{1}{2} l_w$), the results vary in different countries. All these variates show wrong signs in all cases for at least one country.

For the other variates and countries, the signs are right, but share-yield data for the United Kingdom are not available. The variate Δu, rate of increase in consumers' goods production, shows positive signs in at least one case for each country, but its influence is found to be small. In addition, as has been observed in the theoretical part of this chapter (§ 7), this influence may always be replaced by a shift in the lag assumed, especially in case of a high correlation between investment activity and consumers' goods production. Price of iron (q_i) shows negative signs in most cases for each country.

Influence. Taking the standard deviation of v_i equal to 100 in each country, the maximum influence found is the following:

Country	Profits	Share yield	Short-term interest rate	Price of iron	Rate of increase in: production	Rate of increase in: prices	Profit margin
United Kingdom	103	33[a]	39	57	26	34	*
Germany	92	10	*	12	11	*	13
United States	97	64	*	23	0	*	28

[a] Bond yield. * Wrong signs in all cases considered.

Conclusions. From the foregoing we conclude that, as stated already, the factors short-term interest rate, price of iron, rate of increase in production and in prices, and profit margin are, in the mean, far less important than profits and share yields. In particular cases some of them seem to be important, but a general indication is lacking. The most important variates that are to be considered in the second stage are therefore profits (and for the United States, post-war, also share yield), and in addition the price of iron and interest rates, since the theoretical considerations given in § 7 require that these should be considered at the same time. A second reason for including these two factors is the great importance which many economists attach to them as causal factors of investment.

§ 11. Details of Results, Second Stage.

A. *Examination of Regression Equations found.*

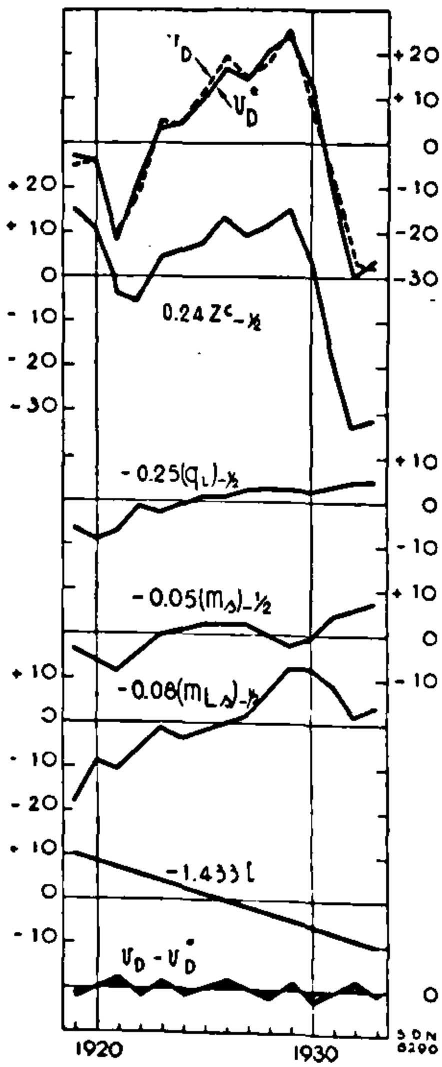
Correlation found. The correlation coefficients found (*cf.* Tables III. 4 to III. 9) are, in general, fairly high, their median value being about 0.80; and it is satisfactory that —in view of the better statistics available for post-war years—the post-war correlations are considerably higher than the pre-war ones for the same country. Especially interesting is the improvement obtained by taking such a careful estimate as Kuznets' index of investment activity instead of pig-iron production.

" Influence " of profits.

A feature common to all results is, as has already been observed, the important part which profits (or one of the series reflecting them) play in the " explanation ". The other factors play only a secondary part, as is easily seen from Graphs III. 2—III. 5, constructed in accordance with the rules laid down in Chapter II, § 5.

Graph III. 2.

"EXPLANATION" OF INVESTMENT ACTIVITY.
UNITED STATES OF AMERICA, 1919-1933.



Investment = Flow of producers' durable goods to enterprises, plus flow of consumers' durable goods to enterprises and households.

v_D : investment activity, actual.

v_D^* : investment activity, as explained by:

$Z^c_{-1/2}$: profits
$(q_i)_{-1/2}$: price of iron
$(m_s)_{-1/2}$: short-term interest rate
$(m_{Ls})_{-1/2}$: share yield

} lagged ½-year.

t : trend.

Graph III. 3.

"EXPLANATION" OF IRON AND STEEL CONSUMPTION.
UNITED KINGDOM, 1920-1936.

v_i: iron and steel consumption, actual.

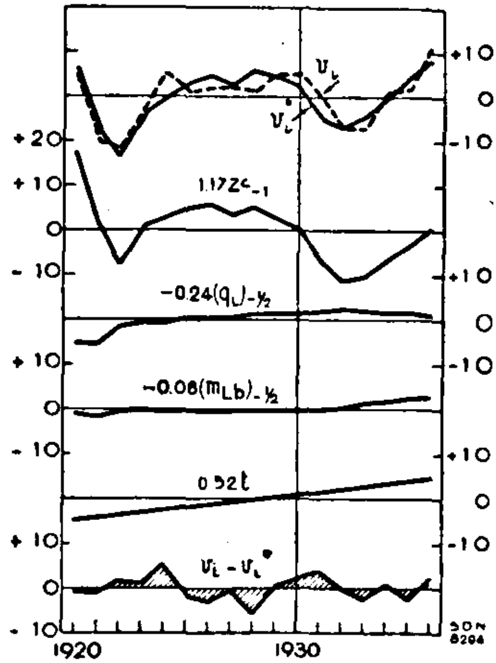
v_i^*: iron and steel consumption, as explained by:

z^c_{-1}: profits, lagged 1 year;

$(q_i)_{-1/2}$: price of iron } lagged ½-year;

$(m_{Lb})_{-1/2}$: bond yield } lagged ½-year;

t: trend.



Regression coefficients. The regression coefficients of profits vary according to the series taken to represent them. It is found in using such a comprehensive figure as corporation profits, for the United States, that when there is an increase in these profits of 1 % of their average level, there is a corresponding increase in investment activity of 0.3 % of its average level. The figure found for Germany (pre-war) is about twice as great, whereas the English figure (post-war) is nearly four times as great; but the latter is based on the *Economist* sample of profits, which seems to show relatively small percentage fluctuations.

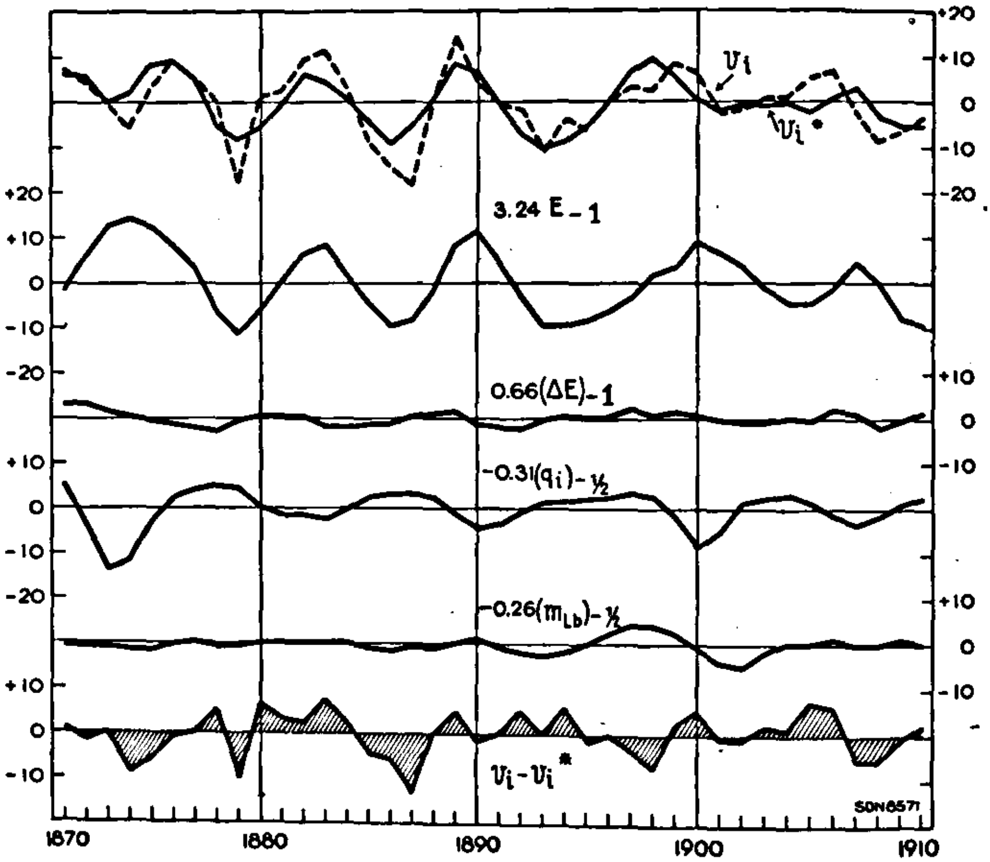
It is natural that share prices,[1] if used as representative of profits, obtain a larger coefficient, lying between 0.6 and 1.0.[2] This means that the change in investment caused by a 1 % change in share prices is also of the order of 1 %.

[1] For in pre-war times share prices in general showed smaller percentage fluctuations than dividends or profits. In the United States after the war the situation was different; the share yield was at a minimum in 1929 when prices were at a maximum.

[2] As far as the figures in Table III. 4 are concerned. Some of the exceptional figures in the other tables are explained below.

Graph III. 4.

"EXPLANATION" OF IRON AND STEEL CONSUMPTION.
UNITED KINGDOM, 1871-1910.



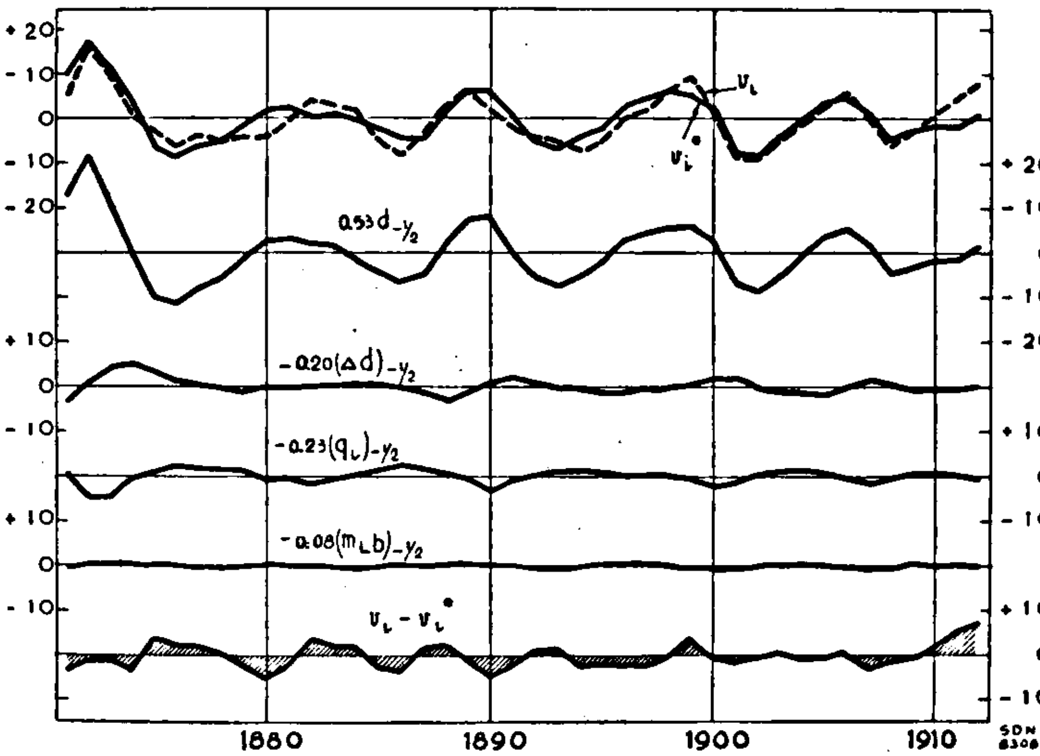
v_i:	iron and steel consumption, actual.	
v_i^*:	iron and steel consumption, as explained by:	
E_{-1}:	non-labour income	lagged 1 year.
$(\Delta E)_{-1}$:	rate of increase in non-labour income	
$(q_i)_{-1/2}$:	price of iron	lagged ½-year.
$(m_{Lb})_{-1/2}$:	bond yield	

Lag. In order to obtain an impression of the empirical evidence regarding the lag between profits and investment activity, Graph III. 6 has been constructed from all cases where a " free " lag was used in the calculations, *i.e.*, where both profits without lag and with one-year lag[1] were intro-

[1] Or, which comes to the same thing, profits and the rate of increase in profits. In Graphs III. 4 and III. 5, the latter's influence has been shown separately, whereas in the tables (except Table III. 10) it has been combined with the influence of profits.

Graph III. 5.

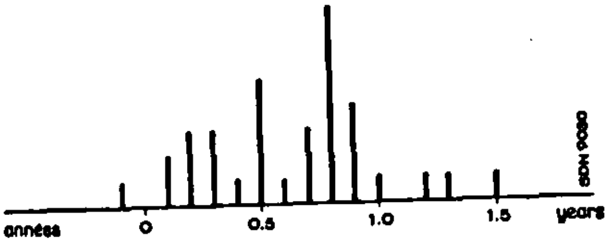
"EXPLANATION" OF IRON AND STEEL CONSUMPTION.
GERMANY, 1871-1912.



v_i:	iron and steel consumption, actual.	
$\overset{*}{v}_i$:	iron and steel consumption, as explained by:	
$d_{-1/2}$:	dividend in % of capital	lagged ½-year.
$(\Delta d)_{-1/2}$:	rate of increase in dividend in % of capital	lagged ½-year.
$(q_i)_{-1/2}$:	price of iron	lagged ½-year.
$(m_{Lb})_{-1/2}$:	bond yield	lagged ½-year.

duced and the real lag was assumed to be the average of these two, weighted by the regression coefficients corresponding to each. Graph III. 6 gives the frequency distribution of the lags found, the average length of which is about eight months. This figure supports fairly well the *a priori* reasoning in § 8.

Graph III. 6.
FREQUENCY DISTRIBUTION OF LAGS.



The two relations between profits and investment activity.

At this point, the validity of our general result, that profits have a large influence on investment activity, may be discussed in a more general framework. It is clear that, between these two variates, profits ($\overline{\overline{Z}}$) and investment activity ($\overline{\overline{v}}$), there exists another relation—viz., the definition (or, if one prefers it, the calculation) of profits. Writing

$\overline{\overline{u}}$ for consumption goods production

and

$\overline{\overline{z}}$ for the general profit margin,

profits will be defined (or calculated) by the equation [1]

$$\overline{\overline{Z}} = (\overline{\overline{u}} + \overline{\overline{v}})\,\overline{\overline{z}}\ .$$

Hence we would have two relations between profits and investment activity.

This fact might give rise to doubts as to the value of the preceding results in two respects:

(1) Is it not possible that only the second relation between profits and investment exists and that hitherto this has been wrongly taken as an *influence* of profits on investment ?

(2) Granted that there are two relations, is there not reason to fear that the coefficient found for the influence of profits is partly attributable to the other relation ?

It seems reasonable to answer both questions in the negative.

(1) As has been stated in the Introduction,[2] an essential element of our method is that the variates playing a rôle in each equation

[1] For the purpose of our argument, it is immaterial whether or not slightly different definitions (or methods of calculation) are adopted.
[2] *Cf.* p. 12.

must be known on *a priori* grounds. This also applies here. Even without any statistical evidence, few economists would deny that there is a causal influence of profits on investment.

But let us suppose, nevertheless, that evidence on this point is sought for in the statistics. A test should then be devised to prove that the apparent parallelism between profits and investment activity is not only due to the composition of profits but also to the causal connection between the two variates.

The obvious high correlation between Z and a combination of u, v and z does not imply that each of these three variates is closely correlated with Z. If the other relation between v and Z also exists, it is to be expected that the correlation between v and Z will be closer than that between u or z and Z. In most cases considered, however, u is closely correlated with v—as a consequence of what has been called the "multiplier" effect—which results in the fact that a high correlation between v and Z also causes a high correlation between u and Z and, hence, to some extent between the third variate z and Z. In these cases, therefore, the proposition cannot be tested along these lines. Only where u and v are not closely correlated will the test be possible.

Such a case is that of Germany before the war (*cf.* Graph III. 7, left-hand part). The upper pair of curves compares profits—represented here by dividends, d—with the *a priori* combination $u + v$, and z (for lack of better data, z has been approximately represented by the expression ($p - \frac{1}{2}l_w$) where p is the general price level, l_w the wage rate, $\frac{1}{2}$ being approximately the wage quota in prices). The three other pairs of curves compare d with each of the variates u, v and z separately.

The correlation between d and the combination is close. Of the three other correlations, that between d and v is the nearest. This is demonstrated by the following correlation coefficients:

Between	d	and	combination	:	0.80
"	d	"	$v_{+1/2}$	:	0.70
"	d	"	u	:	0.44
"	d	"	$p - \frac{1}{2} l_w$	:	0.64
"	u	"	v	:	0.26.

The result is therefore favourable to our thesis, though not strikingly so.

Another clear case is that of Germany in the years around 1929. The fluctuations in profits (Z) may be accounted for by the fluctuations in the factors $u + v$, volume of production, and p/l_w, the proportion between prices and wage-rates (following Donner,[1] who prefers the proportion to the margin). It is especially noticeable that the volume of total production is not correlated with home investment (v) so closely as profits are (*cf.* Graph III. 7, right-hand part). In this particular case, this must be ascribed to the fact that 1929 already showed a decline in the internal business-cycle position, whereas exports were even higher than in 1928. The satisfactory correlation between profits and investment activity —including, as a further variate, interest rates (m_s) with a negative sign—cannot therefore be a consequence of profits' depending chiefly on the volume of production and the latter's depending chiefly on the volume of investment; for the volume of production shows a lower correlation both with profits and with investment than these two series do with each other.

In most other cases for which data are available, the correlation between u and v is much closer. The only test possible in such cases is to ascertain whether the correlation between profits (Z) and v is higher than that between Z and z. This is found to be the case for the United Kingdom before the war—where, however, the figures for z are very unreliable—and for the United States after the war.

But, it must be repeated, the greatest importance must be attributed to the *a priori* argument to include profits in the explanation of investment.

(2) The problem of the reliability of the coefficients when two equations exist between two variates arises also in the statistical determination of supply and demand curves: the variates "price" and "quantity exchanged" occur both in the demand and in the supply function. Often a doubt is expressed as to whether in

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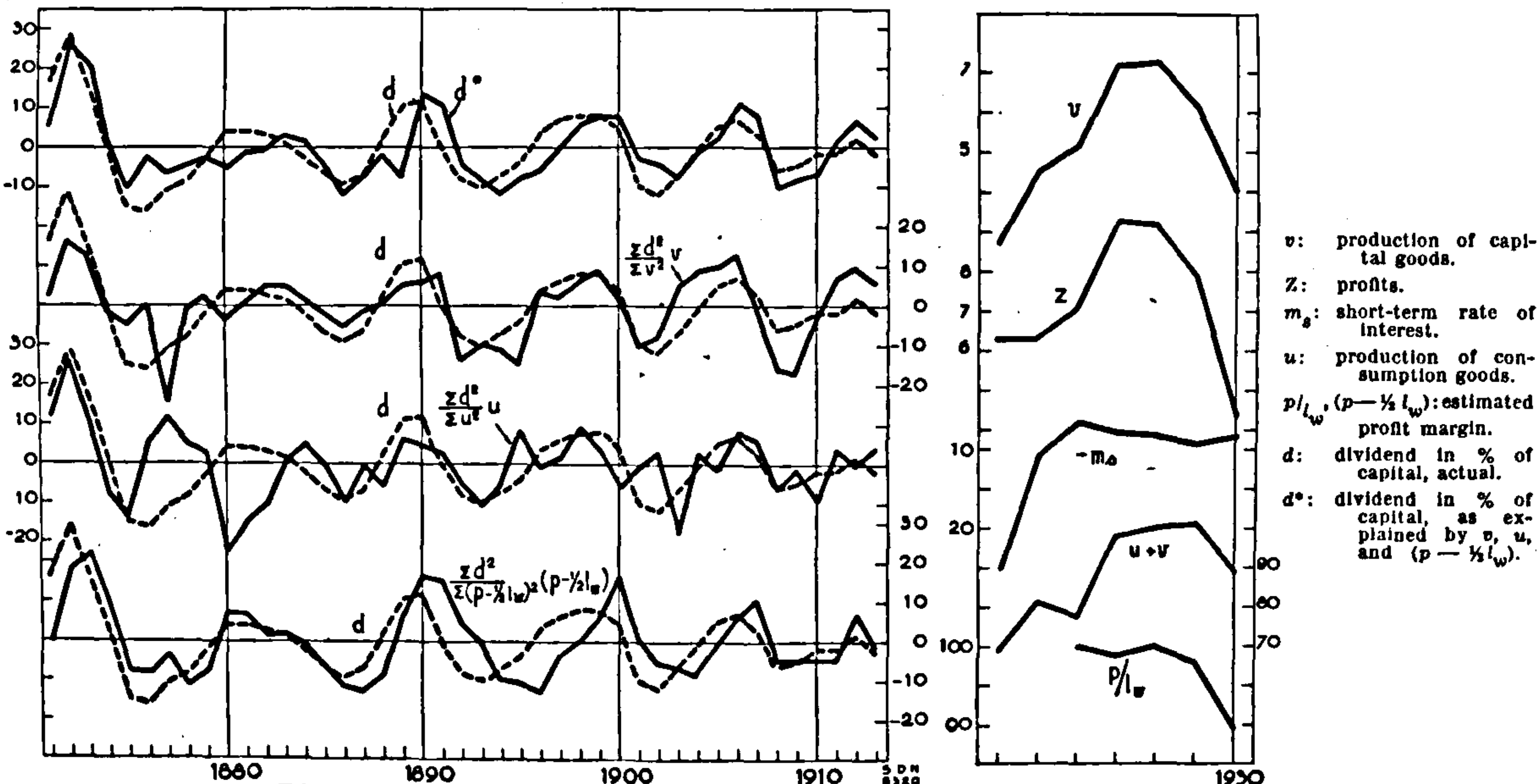
[1] Donner: "Die Kursbildung am Aktienmarkt", *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 36.

Graph III. 7.

THE TWO RELATIONS BETWEEN PROFITS AND INVESTMENT.
GERMANY 1924-1930 AND 1871-1911.



Multiplication by $\frac{\Sigma d^2}{\Sigma v^2}$ ..., etc., in order to equalise the standard deviations (amplitudes) of explained and explaining series.

such a situation both relations are statistically determinable. This doubt seems to be based on what happens in a special case—viz., when both relations are supposed to contain no, or no important, other variates, or when, for lack of statistical data, these other variates are left outside the calculations. In fact, in such a case, only one statistical equation will in general be found, and it would be difficult to maintain that this represents both economic relations.

In principle, this difficulty disappears, however, when, in at least one of the relations, other important variates play a part, provided, of course, that not exactly the same set of supplementary variates occurs in both equations. As soon as different variates occur in the two relations, statistical calculations will yield different results. A special case is the one where the lags are different in both relations. In the case of demand and supply relations for one market, consumers' income, or the price of a competitive commodity will, e.g., occur as complementary variate in the demand relation, whereas, in the supply relation, cost of production or productive capacity may come in. In some special cases, these variates may be of only minor importance, but the lag between price and quantity exchanged may be different for the two relations, as was assumed to be the case for sugar in Professor Schultz's investigations.[1]

The doubt referred to above is the less justified the more important the complementary variates are. And it is interesting to note that if both relations actually contain only the two variates, without differences in lag, these variates in general will no longer be variates but constants, since, in general, two equations are sufficient to determine the value of two unknowns.

In the case of investment activity and profits, it is clear that the relation representing the calculation of profits does contain other variates—viz., those indicated above: production of consumption goods and profit margin. There is therefore no particular reason for doubt as to the significance of our coefficients.

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[1] H. Schultz: *Statistical Laws of Demand and Supply*, Chicago, 1928.

Coefficient of price of iron.

The regression coefficients found for iron prices sometimes show positive signs, which contradict theoretical expectation. But these results are probably not significant, and simply illustrate the degree of indeterminateness of the figures. This makes it rather difficult to give figures for the elasticity of demand. The coefficients given in the tables are, owing to the method of measurement employed, estimates of that elasticity corresponding to trend values for the variates. None of the negative values is larger than unity; those for the three European countries are all between 0 and — 0,5, and in the case of France they are more often small positive than small negative numbers. The elasticity in those countries must have been small. The British figures are all between + 0.16 and — 0.31, and here it would appear that the elasticity is below one-half. The significance calculations (see page 80 and Table III. 10) indicate, however, that there is a wide range of uncertainty, though, even in the worst case, there is about a 95% chance that the elasticity is below 0.90. The American post-war figures are all between — 0.03 and — 0.36, which also suggests an elasticity of less than one-half; here also a rather large range of uncertainty exists.

The clearest historical example of the influence of iron prices on demand seems to be the upturn in demand in England in 1875, which took place in full depression, and followed the heaviest drop in prices among our observations. (See Graph III. 4 above.)

Coefficient of interest rates.

Even greater uncertainty seems to exist concerning the coefficient of interest rates. Here again, French and German figures are centred around zero, and American figures are not very far from it (*cf.* corresponding columns in Tables III. 4—III. 9). Only the English figures (relating to the influence of long-term interest rates) show a decidedly negative tendency, again with a large spread. Their median value is — 0.31 for the pre-war calculations; the post-war figure is considerably smaller: about — 0.1. The figure — 0.31 is in itself not a small figure: it means that a reduction of 1% in the long-term interest rate would cause a 31% increase in investment activity. It should not be forgotten, however, that the largest fall per annum in the long-term interest rate in any cycle before the war was 0.18% and that in most cases it was far less.

Not very different results are obtained in cases where share yield has been introduced as representing the interest rate, or where both share yield and the short-term interest rate have been used, except for the United States in the post-war period. Here share yields are found to have a considerable influence, possibly because they showed large fluctuations in a direction opposite to the usual fluctuations showed in pre-war times in Germany. This must be attributed chiefly to the exaggerated stock-exchange boom in 1928-29, which forced the yields down to very low levels, and to the confidence crisis in 1932, which resulted in very high share yields.

In addition, it must be noted that the fluctuations in share yield did not show any correlation with short-term interest rates; therefore the importance of share yield as an explanatory factor does not involve any proof of the importance of interest rates in the narrower sense.

Finally, the question may be put as to whether the influence found for a change of 1% in interest rates is the same as for a change of 1 % in profit rate, (*i.e.*, profits as a percentage of capital). If it is true that the difference between these two rates is the guide for investment activity, these two influences must be about equal. Unfortunately, the question can only be answered for the cases where profit rates are known, *i.e.*, for the United Kingdom after the war and Germany before the war. Here the regression coefficients calculated on this basis compare as follows:

Influence of 1% change in	Profit rate	Long-term interest rate
United Kingdom post-war	12	— 8
Germany pre-war	6	— 8

The result is not bad: the order of magnitude is the same, a deviation in one direction being counteracted by a deviation in the opposite direction. This would, in a sense, support the results obtained. For the United Kingdom before the war the coefficient of interest rates is about 3 times as great as that found for the post-war period, but roughly the same proportion seems to apply to the coefficient for profits.

Comparison between countries.

Table III. 4 (pages 68 and 69) enables us to compare the results obtained for the four countries studied. Most of the differences between these results are hardly significant, the most important being probably the difference in lags in the influence of profits. In particular, the lag of one year found for the United Kingdom (post-war) is decidedly longer than in most other cases. In addition, the influence of the price of iron and interest rates seems to be smaller in Germany and France than in England.

Comparison between production and consumption of pig-iron.

Table III. 5 (pages 70 and 71) shows an interesting difference in the lag with which profits enter into the English explanations of pig-iron production and consumption. Production reacts to profits much more quickly than does consumption. It follows that consumption probably lags behind production. One has the impression that the fluctuations in foreign demand for British iron must have had a leading influence on the business position in the United Kingdom. In the case of Germany, on the other hand, the difference in lag is much smaller. Apart from this difference, no essential feature emerges from Table III. 5. As was to be expected, the influence of profits is greater on consumption than on production, the difference being greater in Germany than in England.

Comparison between periods.

Table III. 6 (pages 70 and 71) shows comparable figures for three periods: (i) before 1895, the turning-point of the " long cycle ", (ii) between 1895 and the war, and (iii) after the war. If structural changes have occurred in the relation investigated, they must be reflected in changed coefficients. If, *e.g.*, it be true that investment activity at present reacts more violently to profit changes than it did before the war, this would be translated into a larger regression coefficient for profits. Unfortunately, it is almost impossible to obtain comparable figures for the pre-war and the post-war periods.

" EXPLANATION " OF INVESTME

Table III.

Note.— All series are in % deviations from trend, *except* series (9) and (10
indicated in brackets after each coefficient; when the lag has been determin

Country	Period	Series explained	Correlation coefficient R
(1)	(2)	(3)	(4)
United States	1919-1933	Production of pig-iron	0.94
,,	,,	Investment activity [a]	0.98
United Kingdom	1871-1910	Consumption of pig-iron	0.75
,,	1920-1936	,, ,, ,,	0.90
Germany	1871-1912	,, ,, ,,	0.87
United States	1895-1913	Production of pig-iron	0.77
Germany	1871-1912	Consumption of pig-iron	0.79
France	1871-1908	,, ,, ,,	0.81
United States	1895-1913	Production of pig-iron	0.76
Germany	1871-1912	Consumption of pig-iron	0.83

[a] Flow of producers' and consumers' durable commodities, building excluded.

Activity, Second Stage.

Comparison of Countries.

which are in absolute deviations from trend in units of 0.01%. The lag in years is "freely", the coefficient is in italics.

Regression coefficients and lags of:					
profits	non-labour income	share prices	price of pig-iron	short-term interest rate	long-term interest rate
(5)	(6)	(7)	(8)	(9)	(10)
† 0.29 (½)			— 0.03 (½)		† — 0.32 (½)
† 0.28 (½)			— 0.36 (½)		† — 0.05 (½)
	3.24 (0.8)		— 0.31 (½)		— 0.26 (½)
1.17 (1)			— 0.24 (½)		— 0.08 (½)
0.53 (0.9)			— 0.23 (½)		— 0.08 (½)
		0.85 (½)	— 0.51 (½)		— 0.27 (½)
		0.72 (½)	— 0.13 (½)		0.21 (½)
		0.94 (0.9)	0.10 (½)		— 0.08 (½)
		1.05 (0.4)	— 0.54 (½)	0.02 (½)	
		0.61 (0.3)	— 0.17 (½)	0.06 (½)	

† Intercorrelation between 0.75 and 0.80.

Table III. 5. Comparison of

Note. — See note to Table III. 4.

Country	Period	Series explained	Correlation coefficient R
(1)	(2)	(3)	(4)
United Kingdom	1871-1910	Production of pig-iron	0.79
,,	,,	Consumption ,, ,,	0.75
Germany	1871-1912	Production ,, ,,	0.75
,,	,,	Consumption ,, ,,	0.88
France	1871-1908	Production ,, ,,	0.69
,,	,,	Consumption ,, ,,	0.81

Table III. 6. Comparison

Note. — See note to Table III. 4.

Country	Period	Series explained	Correlation coefficient R
(1)	(2)	(3)	(4)
United States	1877-1895	Production of pig-iron	0.75
,, ,,	1895-1913	,, ,, ,,	0.68
,, ,,	1919-1924	,, ,, ,,	0.83
,, ,,	1919-1933	,, ,, ,,	0.61
United Kingdom	1871-1895	Consumption of pig-iron	0.77
,, ,,	1896-1910	,, ,, ,,	0.68
,, ,,	1920-1936	,, ,, ,,	0.90
Germany	1871-1895	Consumption of pig-iron	0.90
,,	1895-1912	,, ,, ,,	0.88

† Intercorrelation between 0.75 and 0.80.
* Because of the lack of correlation, the calculation of a lag has no point in this case.

ACTIVITY, SECOND STAGE.

Pig-iron Production and Consumption.

Regression coefficients and lags of:					
profits	non-labour income	share prices	price of pig-iron	short-term interest rate	long-term interest rate
(5)	(6)	(7)	(8)	(9)	(10)
	1.96 (0.1)		— 0.17 (½)		— 0.39 (½)
	3.24 (0.8)		— 0.31 (½)		— 0.26 (½)
0.23 (½)			0.08 (½)	— 0.01 (½)	
0.53 (0.8)			— 0.26 (½)	0.02 (½)	
		0.49 (0.1)	0.17 (½)		— 0.03 (½)
		0.94 (0.9)	0.10 (½)		— 0.08 (½)

of Various Periods.

Regression coefficients and lag of:					
profits	non-labour income	share prices	price of pig-iron	short-term interest rate	long-term interest rate
(5)	(6)	(7)	(8)	(9)	(10)
		0.94 (0.2)			
		0.74 (0.2)			
		2.71 (0.3)			
		0.18 [a]			
	3.29 (0.8)		— 0.33 (½)		— 0.56 (½)
	† *1.43* (lead 0.1)		† — 0.11 (½)		— 0.07 (½)
1.17 (1)			— 0.24 (½)		— 0.08 (½)
† 0.31 (½)			† 0.10 (½)	0.02 (½)	
0.90 (½)			0.13 (½)	— 0.03 (½)	

Profit figures for the United Kingdom and for the United States as used in post-war calculations are not available for pre-war years. Share prices, which in pre-war United States could probably be used as an indicator of profits, are no longer representative of profits in the entire post-war period, as they are quite out of line at the top of the 1929 boom. As a consequence, (i) the correlation obtained with share prices is no longer good and (ii) the regression coefficient obtained for the period 1919-1933 is quite different from that obtained for the period 1919-1924. The intensity with which investment activity reacts to share prices would seem to have decreased as compared with pre-war times according to the figure for the whole period, whereas it would seem to have increased very much according to the 1919-1924 figures. For the United Kingdom a post-war estimate for "non-labour income" by C. Clark,[1] which is about comparable to pre-war figures, leads to a regression coefficient considerably lower (1.36). There is therefore some evidence of a more intensive reaction after the war in the United States and a less intensive in the United Kingdom.

Comparison of the figures for the two pre-war periods shows that the coefficient for profits after 1895 is, in Germany, considerably higher than in the period before 1895, slightly lower in the United States and considerably lower in the United Kingdom. The American coefficients for the two sub-periods, as well as the 1871-1895 English coefficient, are in line with those for the entire pre-war period; the 1896-1910 English is quite out of line and, moreover, a lead instead of a lag has been found here between profits and investment. This lead may, however, be interpreted as an influence of the rate of increase in profits. In addition, the British business-cycle pattern was rather weak, partly perhaps because the Boer war counteracted the 1901 depression;[2] with less pronounced fluctuations, disturbing elements become more important and the results less reliable. Anyhow, the result for the period 1896-1910 is not satisfactory.

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[1] *National Income and Outlay*, London, 1937, page 60.

[2] *Cf.* Graph III. 4. It shows that pig-iron consumption fell only by about 10% from 1899 to 1901, whereas it fell by some 20 to 30% in the crises preceding 1901 and by 40% in Germany, *cf.* Graph III. 5, in 1901. The employment figures also showed the smallest fall ever seen in a crisis.

Extrapolation for the United States, 1934-37.

It has already been mentioned that one of the calculations for the United States has been extrapolated in order to cover the years 1934-37. In other words, the regression coefficients for the period 1919-33 have been applied to subsequent years (*cf.* Graph III. 8).

Graph III. 8.

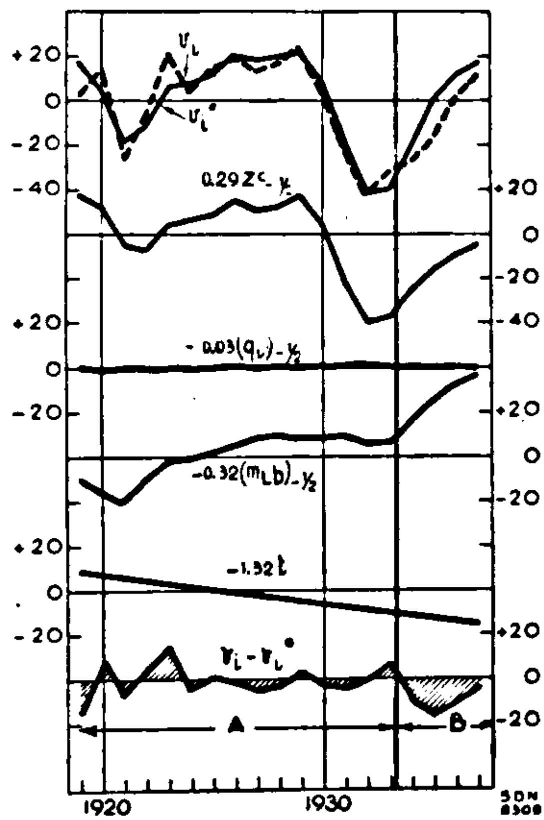
"EXPLANATION" OF PIG-IRON PRODUCTION.
UNITED STATES, 1919-1937.

v_i : pig-iron production, actual.

v_i^* : pig-iron production, as explained by:

$(Z^c)_{-½}$: profits
$(q_i)_{-½}$: price of iron
$(m_{Lb})_{-½}$: bond yield
(lagged ½-year.)

t : trend.



A — Calculated.
B — Extrapolated.

As Kuznets' estimates of investment activity are not available after 1933,[1] it was only possible to extrapolate the calculation for pig-iron production. One of the determining factors in the calculation being total profits, these had to be estimated for 1936 and 1937.

[1] It has not been possible to make use of the figures that have recently been published by KUZNETS in *National Income and Capital Formation in the United States, 1919-1935.*

Table III. 7. Comparison of

Note. — See note to Table III. 4. — Series (11) is in absolute deviations from

Country	Period		Series explained	Correlation coefficient
(1)	(2)		(3)	(4)
United States	1877-1913	Upward	Production of pig-iron	0.86
		Downward	,, ,, ,,	0.78
United Kingdom	1871-1910	Upward	Consumption of pig-iron	0.79
		Upward	,, ,, ,,	0.78
		Downward	,, ,, ,,	0.59
Germany	1871-1912	Upward	Consumption of pig-iron	0.90
		Downward	,, ,, ,,	0.89
France	1871-1908	Upward	Consumption of pig-iron	0.85
		Downward	,, ,, ,,	0.79

† Intercorrelation between 0.75 and 0.80. * Multiple correlation over 0.80.

Table III. 8. Comparison of

Country	Period		Series explained	Correlation coefficient
(1)	(2)		(3)	(4)
United States	1877-1913	Upward	Production of pig-iron	0.78
		Downward	,, ,, ,,	0.60
United Kingdom	1871-1910	Upward	Consumption of pig-iron	0.68
		Downward	,, ,, ,,	0.53
Germany	1871-1912	Upward	Consumption of pig-iron	0.84
		Downward	,, ,, ,,	0.84
,,	1871-1912	Upward	,, ,, ,,	0.72
		Downward	,, ,, ,,	0.85
France	1871-1908	Upward	Consumption of pig-iron	0.81
		Downward	,, ,, ,,	0.75

ACTIVITY, SECOND STAGE.

Upward and Downward Phases I.

trend, in units of 0.01%; series (12) is in % deviations from trend.

Regression coefficients and lags of:						
profits*	share prices	price of pig-iron	short-term interest rate	long-term interest rate	share yield	building volume
(5)	(7)	(8)	(9)	(10)	(11)	(12)
	* *0.26* (0.3)	* 0.42	—0.03 (½)			*0.27 (0)
	0.69 (0.9)	0.23	0.01 (½)			0.55 (0)
2.46 (0.8)		0.16 (½)		—0.27 (½)		
2.79 (0.8)				—0.29 (½)		
1.38 (0.8)				—0.31 (½)		
	0.36 (½)		0.04 (½)		0.13 (½)	
	† 0.54 (½)		0.04 (½)		†— 0.03 (½)	
	0.88 (1.3)	0.27 (½)		—0.29 (½)		
	† *3.15* (0.7)	—0.36†(½)		0.25 (½)		

* United Kingdom: non-labour income.

Upward and Downward Phases II.

Regression coefficients and lags of:		
profits	non-labour income	share prices
(5)	(6)	(7)
		0.73 (0.2)
		0.58 (1.2)
	2.24 (1.5)	
	1.41 (0.7)	
0.46 (½)		
0.41 (½)		
		0.57 (½)
		0.58 (½)
		1.92 (0.8)
		2.24 (0.7)

The estimate was based on two very high correlations found for the period 1919-1933, which held also for 1934 and 1935. The first " explains " profits by (1) total receipts of all corporations during the same year and (2) total receipts during the preceding year. The inclusion of lagged receipts is justified by the fact that " total deductions " (*i.e.*, the amounts to be deducted from total receipts in order to obtain profits of all corporations) are very closely correlated with receipts lagged over a few months, since they represent an " adaptation of costs to receipts " which takes some time. The second correlation is between " total receipts of all corporations " and

(1) Index of industrial production (Federal Reserve Board), multiplied by index of wholesale prices (Bureau of Labor Statistics); and
(2) Department-stores sales.

This latter correlation served to estimate total receipts for 1936 and 1937, which in turn determined the estimate of profits for 1936 and 1937. The result of the extrapolation is shown in Graph III. 8. The general direction of actual and calculated production of pig-iron is the same, but there seems to be a more or less systematic difference in level which might reflect the result of the changed structure.

Comparison between phases.

Tables III. 7 and III. 8 (pages 74 and 75) give comparable results for two sets of years; those showing a lower, and those showing a higher, investment figure (measured in deviations from trend) than the preceding year. The former set of years is called " downward phases ", the latter " upward phases " of the cycle. As the calculations for upward and downward phases are chiefly intended to be an illustration of the degree of uncertainty in the general results, it did not seem worth while to construct bunch maps, etc. As in Tables III. 4 to III. 6, only cases in which the correlation coefficients between the " explanatory " series do not exceed 0.80 have been included, in order to exclude—although admittedly only in a very rough way—cases of multicollinearity. Some of the cases given in Table III. 7 are probably still unreliable, however, in that respect, and Table III. 8 has therefore been added; in this table, only the chief variate—profits—or a representative series is employed, and multicollinearity is therefore impossible.

No systematic differences can be discovered. The regression coefficients of profits in Table III. 7, although rather divergent, do not differ from those for the whole period by more than three times the standard errors of the latter (*cf.* Table III. 10) in the case of the United Kingdom and Germany. For the United States and France, the results of either the upward or the downward phases are very uncertain because of a high intercorrelation between some of the explanatory variates. In Table III. 7, downward phases show a lower coefficient for the United Kingdom and a higher for Germany and the United States. These differences disappear, however, almost entirely in Table III. 8, except for the United Kingdom. The coefficients of the secondary factors sometimes show larger relative differences, but even these are not significant, as the coefficients are rather uncertain (*cf.* standard errors, Table III. 10), the only possible exception being the influence of building in the United States. It is therefore difficult to obtain evidence regarding the necessity of explaining downward phases by other relations than upward phases.

Inclusion of more variates.

Table III. 9 (pages 78 and 79) gives, for a few cases, the successive results obtained if more and more variates are included in the explanation. It is chiefly intended to show the great importance of profits as against the other variates in the " explanation ", and the relatively small improvements in correlation and the small change in regression consequent upon their inclusion. Nevertheless, these improvements will prove to be significant.

Partial scatter diagrams.

Graphs III. 9 to III. 11 represent partial scatter diagrams (*cf.* Chapter II, § 5) for three cases. As has already been observed, they enable us to test whether the hypothesis of rectilinear relationship is fulfilled or not. It may be seen from the graphs that there is no wide departure from rectilinearity, but that nevertheless a tendency to curvilinearity is present in a number of cases. The graphs containing the price of iron as the independent variate represent the demand curve for iron. (This is not quite correct in the case of the United States as the investment index includes other investment goods, but, owing to the rather large parallelism between the production of the various kinds of investment goods, the error cannot be important.)

Table III. 9. Influence of

Note. — See note to Table III. 4 and III. 7. Δ stands for "rate of increase in".

Country	Period	Series explained	Correlation coefficient R
(1)	(2)	(3)	(4)
United States	1919-33	Investment activity [b]	0.97
,, ,,	,,	,, ,,	0.98
,, ,,	,,	,, ,,	0.99
United Kingdom	1871-1910	Consumption of pig-iron	0.59
,, ,,	,,	,, ,, ,,	0.65
,, ,,	,,	,, ,, ,,	0.75
Germany	1871-1912	Consumption of pig-iron	0.79
,,	,,	,, ,, ,,	0.79
,,	,,	,, ,, ,,	0.83
,,	,,	,, ,, ,,	0.85

a For United Kingdom, non-labour income.
b Flow of producers' and consumers' durable commodities, building excluded.

Table III. 10. Results of Significance Calculations: Serial

Case	Corre-lation coeffi-cient	Serial correlation of residuals	Regression coefficients, with		
			Profits	Δ profits	Non-labour income
(1)	(2)	(3)	(4)	(5)	(6)
I	0.99	—0.46 ± 0.27	0.24 ±0.02 (½)		
II	0.77	0.01 ± 0.20			3.29 ±0.84 (1)
III	0.75	0.16 ± 0.16			3.24 ±0.58 (1)
IV	0.90	0.00 ± 0.25	1.17 ±0.17 (1)		
V	0.87	0.43 ± 0.16	0.53 ±0.09 (½)	—0.20 ±0.11 (½)	

I United States, Investment activity a. 1919-1933.
II, III, IV United Kingdom. Consumption of pig-iron. 1871-1895. 1871-1910. 1920-1936.
V Germany. Consumption of pig-iron, 1871-1912.

a Flow of producers' and consumers' durable commodities, building excluded.

TIVITY, SECOND STAGE.

nber of Variates included.

ies (13) is expressed in % of average level of price of pig-iron during period.

	Regression coefficients and lags of:					
rofits *	share prices	price of pig-iron	short-term interest rate	long-term interest rate	share yield	Δ price of pig-iron
(5)	(7)	(8)	(9)	(10)	(11)	(13)
31 (½)						
28 (½)					— 0.04 (½)	
24 (½)		— 0.25 (½)	— 0.05 (½)		— 0.08 (½)	
82 (0.6)						
90 (0.8)				— 0.33 (½)		
24 (0.8)		— 0.31 (½)		— 0.26 (½)		
	0.59 (½)					
	0.59 (½)	0.00 (½)				
	0.61 (0.3)	— 0.17 (½)	0.06 (½)			
	0.56 (0.9)	— 0.15 (½)	0.03 (½)			0.35 (½)

rrelation of Residuals and Standard Errors of Regression Coefficients.

ir standard errors, and lags of:				
Non-labour income	Price of pig-iron	Short-term interest rate	Long-term interest rate	Share yield
(7)	(8)	(9)	(10)	(11)
	—0.25 ±0.12(½)	—0.05 ±0.02(½)		—0.08 ±0.02(½)
.54 ±0.54 (1)	—0.33 ±0.13(½)		—0.56 ±0.49(½)	
.66 ±0.42 (1)	—0.31 ±0.10(½)		—0.26 ±0.14(½)	
	—0.24 ±0.14(½)		—0.08 ±0.10(½)	
	—0.23 ±0.17(½)		—0.08 ±0.14(½)	

B. *Significance calculations.*

Table III. 10 (pages 78 and 79) gives details regarding the significance calculations made. These have been restricted to five cases which seem representative and call for the following comments.

Serial correlation for residuals. First, the serial correlation for the series of the residuals has been calculated; *i.e.*, the correlation of that series with itself if lagged one year. This calculation serves to test the hypothesis at the basis of Fisher's theory—viz., that the residuals are to be considered as sample drawings from a "normally distributed universe". At the same time, it gives information as to whether the regression chosen satisfies the scheme of the shock theory (*cf.* § 7). In order to see whether the serial correlation differs from zero to any significant extent, the serial correlation coefficient has to be compared with its standard error, which equals $\frac{1}{\sqrt{N-1}}$.[1] It then appears that the greatest deviation from zero is found for Germany before the war, where the result is, however, still within a distance of three times the standard error.

Standard errors. The standard errors, calculated with Fisher's formula (*cf.* Appendix A, § 4) are such that all the regression coefficients for profits or the series replacing them are with a very high probability significantly positive: in all five cases, the regression coefficients are more than three times their standard error.

Only five out of ten coefficients tested for iron prices and interest rates are rigorously significant, *i.e.*, with a probability of over 95%. This fact is illustrated by Graph III. 12, where for each case the ranges within once the standard error on either side of the regression coefficient (*i.e.*, $b \pm \sigma_b$) have been indicated in black, those within twice the standard error (*i.e.*, $b \pm 2\sigma_b$) by shading. It will be seen that five of the latter areas go beyond the zero point. Nevertheless, there is more probability of the regressions, being negative. Moreover, it is satisfactory to the statistician that the range of error is generally smaller for post-war than for pre-war figures.

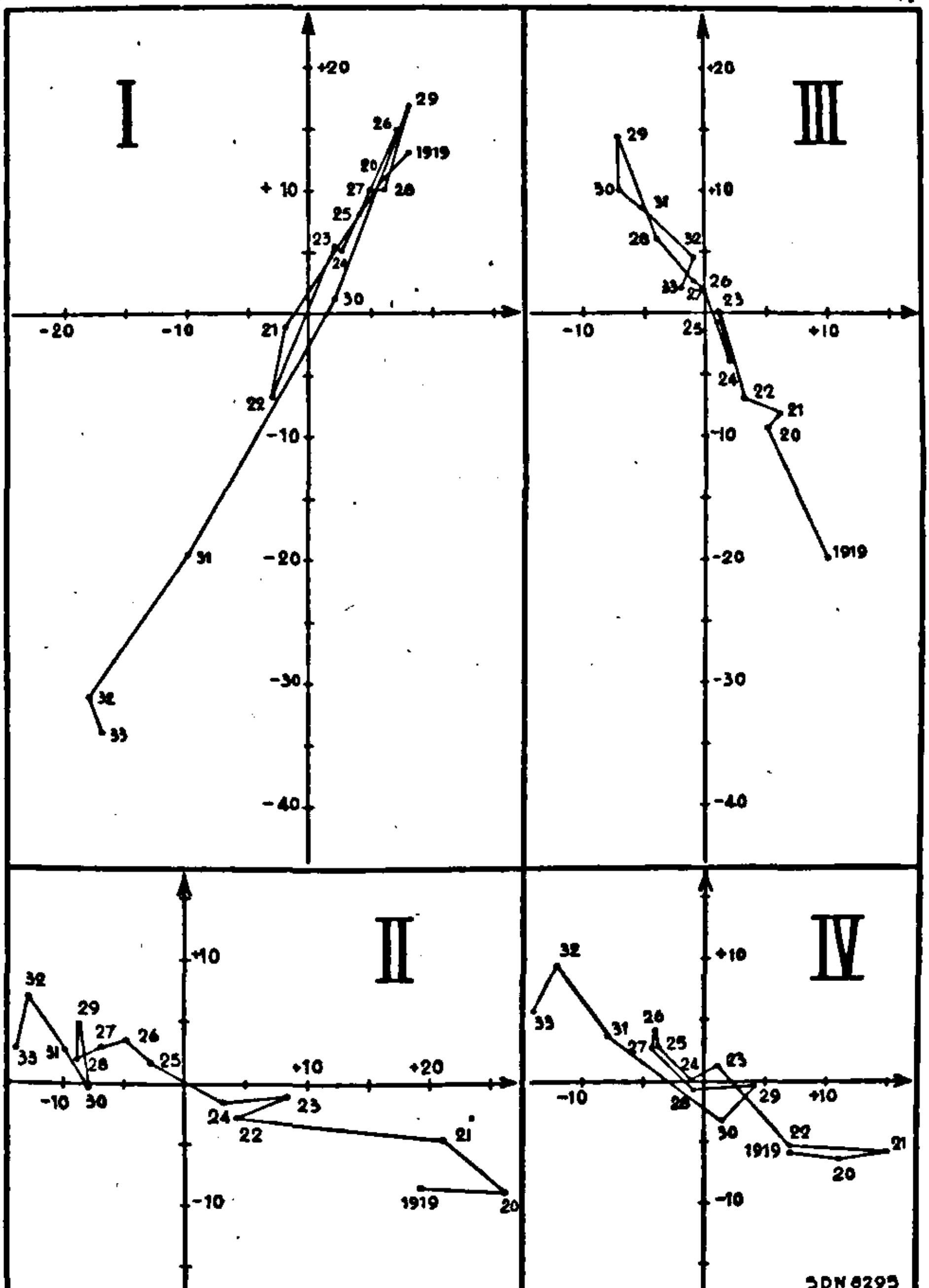
[1] *Cf.* M. S. Bartlett: "Some Aspects of the Time Correlation Problem", *Journal of the Royal Statistical Society*, 1935 (98), page 537, quoted in T. Koopmans, *loc. cit.*, page 129.

Graph III. 9.

Partial scatter diagrams.

INVESTMENT ACTIVITY: UNITED STATES, 1919-1933.

	Ordinates		*Abscissæ*
I II III IV	Investment activity (flow of durable producers' goods to enterprises, plus flow of durable consumers' goods to enterprises and households) corrected for influence of other "explanatory variates", i.e., *ceteris paribus*.	I II III IV	Profits $_{-1/2}$ Price of pig-iron $_{-1/2}$ Share yield $_{-1/2}$ Short-term interest rate $_{-1/2}$

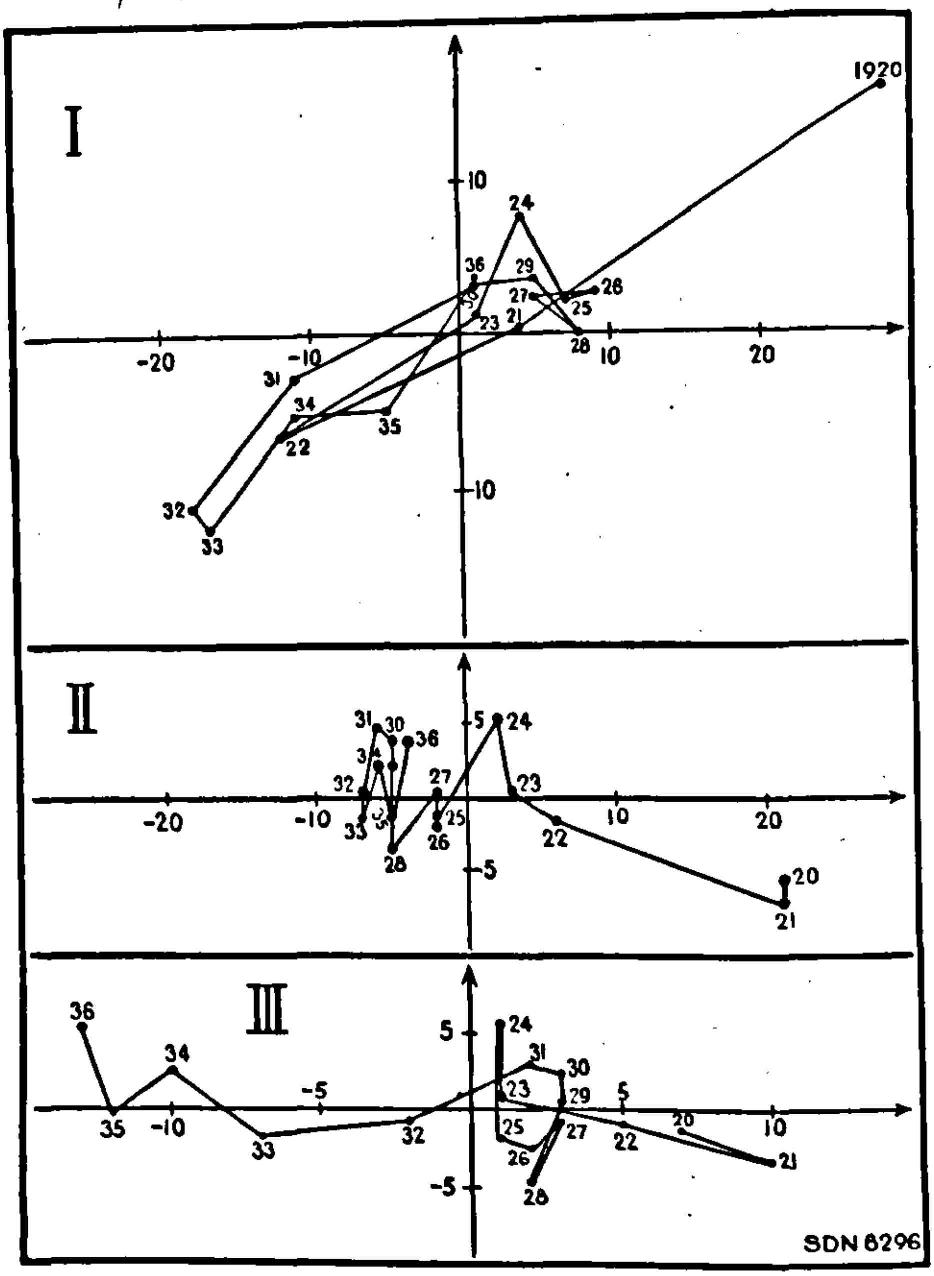


Graph III. 10.

Partial scatter diagrams.

PRODUCTION OF PIG-IRON: UNITED KINGDOM, 1920-1936.

Ordinates		*Abscissae*	
I, II, III	Production of pig-iron, corrected for influence of other "explanatory variates", *i.e., ceteris paribus.*	I	Profits $_{-1}$
		II	Price of pig-iron $_{-1/2}$
		III	Long-term interest rate $_{-1/2}$

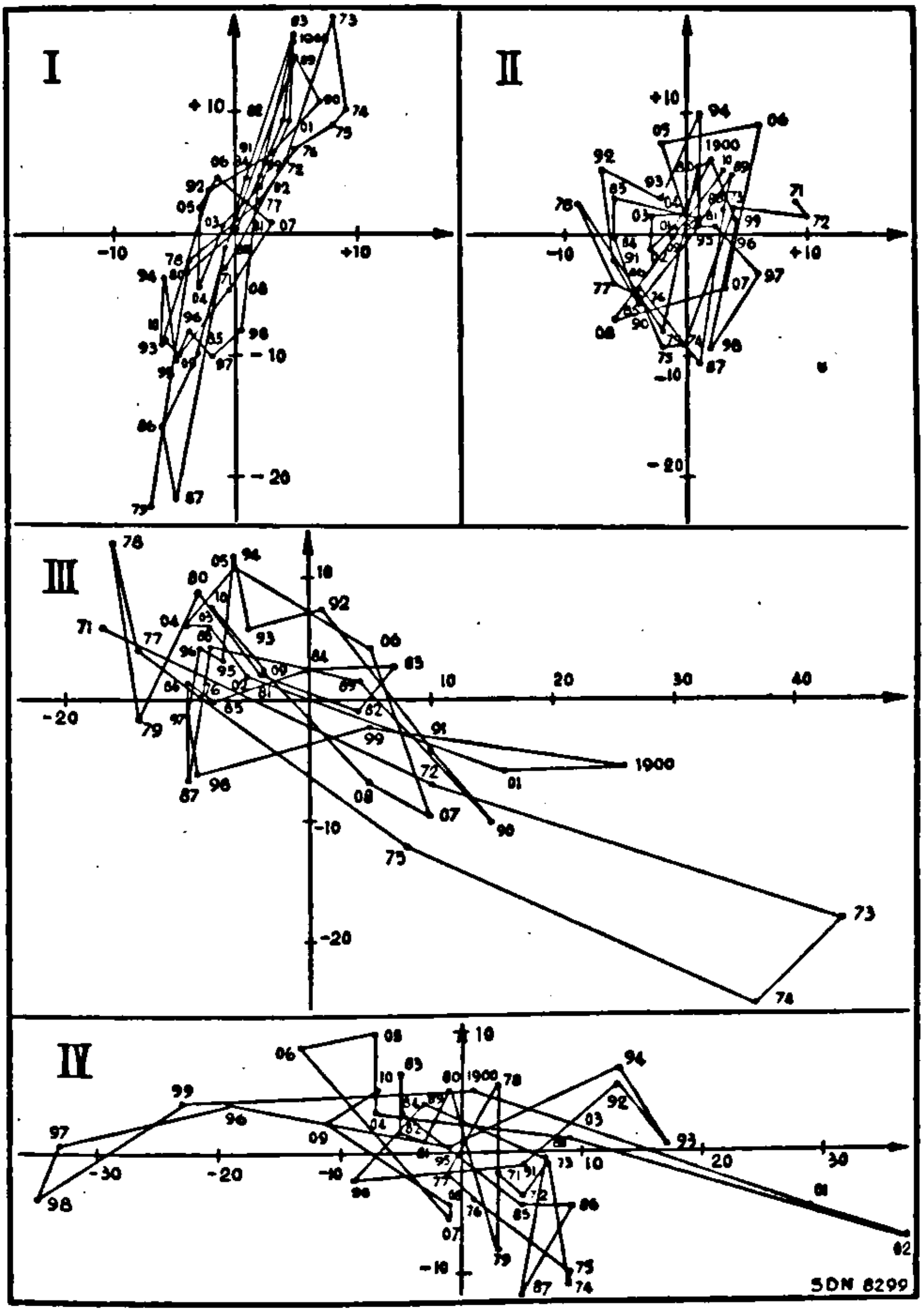


Graph III. 11.

Partial scatter diagrams.

CONSUMPTION OF IRON AND STEEL: UNITED KINGDOM, 1871-1910.

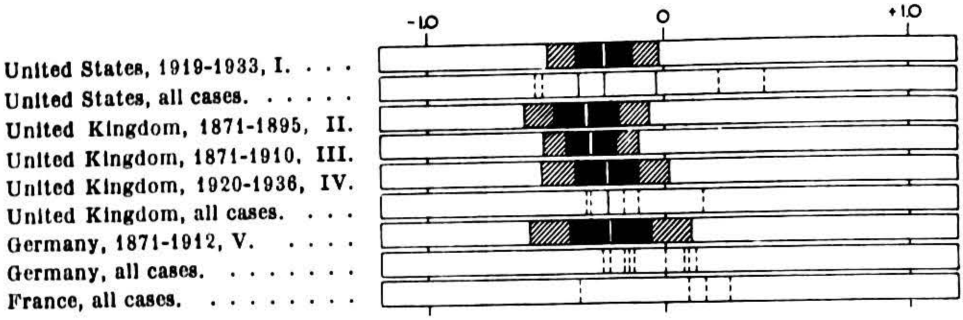
Ordinates		*Abscissæ*
I	Consumption of pig-iron, corrected for influence of other "explanatory variates", i. e.. *ceteris paribus*.	I Non-labour income $_{-1}$
II		II Δ non-labour income $_{-1}$
III		III Price of pig-iron $_{-1/2}$
IV		IV Long-term interest rate $_{-1/2}$



Graph. III. 12.

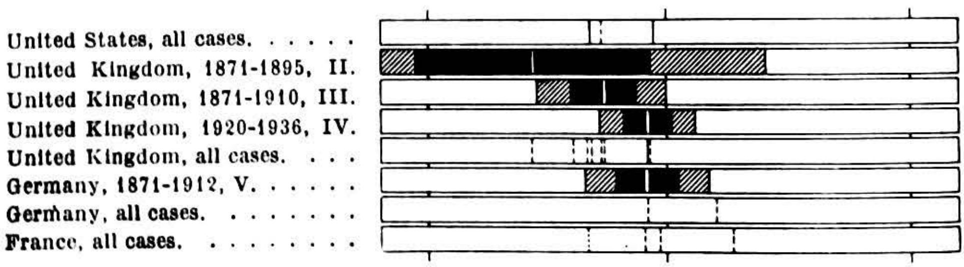
REGRESSION COEFFICIENTS AND RANGES OF INCERTITUDE.

Regression Coefficients of Iron Price.



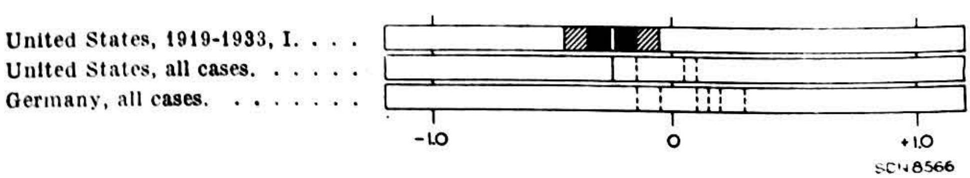
Regression Coefficients of Interest Rate.

A. Long-term, in 0.01 %.



Regression Coefficients of Interest Rate.

B. Short-term, in 0.05 %.



Explanation: Black range: range between b and $b \pm \sigma_b$ . . . . . ; Shaded range: range between b and $b \pm 2\sigma_b$ . . . . . — only for cases where σ has been calculated (cases I-V, table III. 10).

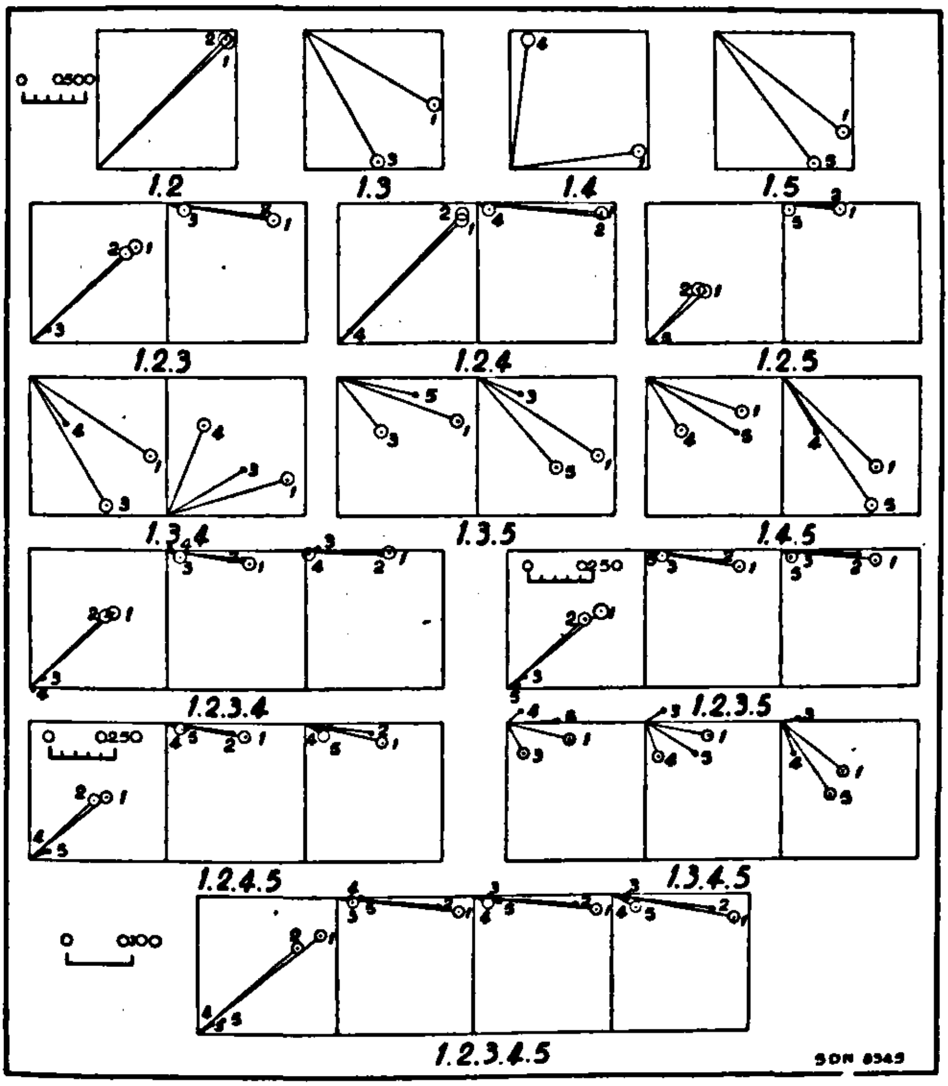
Solid line: post-war result; Broken line: pre-war result — for all cases.

Graph III. 13.

Bunch map.

INVESTMENT ACTIVITY: UNITED STATES 1919-1933

1 = Investment activity (flow of durable producers' goods to enterprises, plus flow of durable consumers' goods to enterprises and households).
2 = Profits$_{-1/2}$. 3 = Price of iron$_{-1/2}$. 4 = Short-term interest rate$_{-1/2}$.
5 = Share yield$_{-1/2}$.



In Graphs III. 13 to III. 16 a number of bunch maps are reproduced.

Bunch maps.

Graph III. 13 (page 85), relating to one of the United States post-war calculations, is very satisfactory, and even more so if—following Koopmans' argument—only the beams with numbers 1 and 2 are considered. In fact, in the 5-set (the case including all five variates) they coincide almost entirely, indicating a high degree of determinateness of all regression coefficients found. These findings are in close agreement with the findings on standard errors.

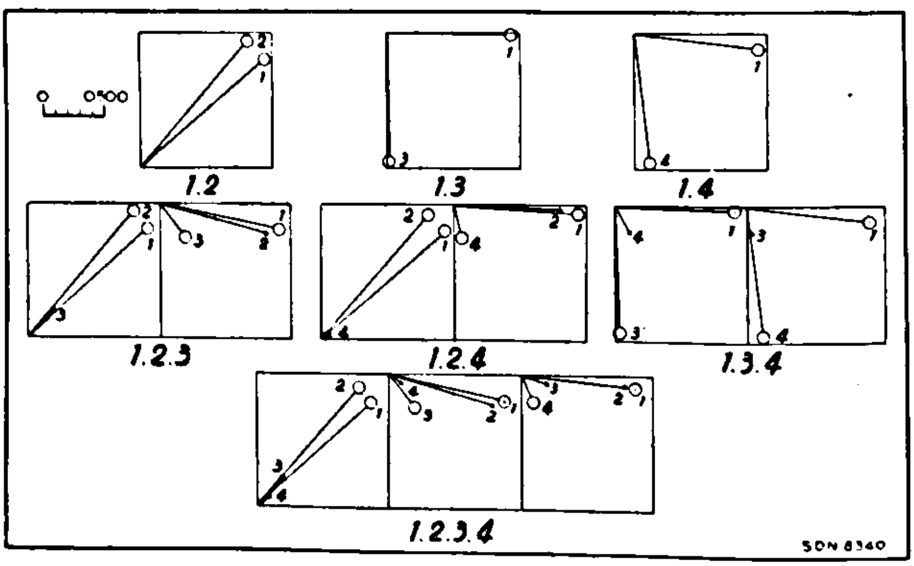
Graph III. 14, relating to the post-war calculation for the United Kingdom, is also very satisfactory, and the above observations apply. This case is particularly suitable for illustrating Koopmans' result regarding the minor importance of beams corresponding to variates which exercise only a secondary influence. Table III. 11 shows the coefficients of the four elementary regression equations, together with the probable limits to the true regression coefficients that take into account both the error of weighting

Graph III. 14.

Bunch map.

Consumption of Iron and Steel: United Kingdom, 1920-1936.

1 = Consumption of iron and steel. 2 = Profits$_{-1}$. 3 = Price of iron$_{-1/2}$
4 = Long-term interest rate$_{-1/2}$.



and the error of sampling in these coefficients. For the calculation of these limits, see Appendix A. 5. The assumption underlying the calculation of the limits to the error of weighting is that the standard deviation of the disturbances in each of the three explaining variates is not greater than a third of the standard deviation of the corresponding variate itself. Allowance for sampling errors of regression coefficients has been made by extending the limits to both sides by twice the standard error of sampling.

Table III. 11. Probable Limits to the True Regression Coefficients. Consumption of Iron and Steel: United Kingdom, 1920-1936.[a]

Explaining variate		2 Profits$_{-1}$	3 Price of iron$_{-\frac{1}{2}}$	4 Long-term interest rate$_{-\frac{1}{2}}$
1st	elementary regression coefficient	1.17	—.24	—.07
2nd	elementary regression coefficient	1.49	—.29	—.07
3rd	elementary regression coefficient	1.44	—1.31	—.21
4th	elementary regression coefficient	.11	—.07	—1.74
Limits to the error of weighting given by ultimate beams in bunch map	upper	1.49	—.07	—.07
	lower	.11	—1.31	—1.74
Maximum fraction of standard deviation admitted for disturbances		1/3	1/3	1/3
Narrower limits to the error of weighting:				
According to the rule of thumb	upper	1.34	—.07	—.07
	lower	.99	—.28	—.09
According to the strict rule	upper	1.28	—.24	—.07
	lower	1.17	—.28	—.09
Final limits including allowance for sampling errors	upper	1.62	.04	.12
	lower	.84	—.56	—.28

[a] For units, *cf.* Table III. 4.

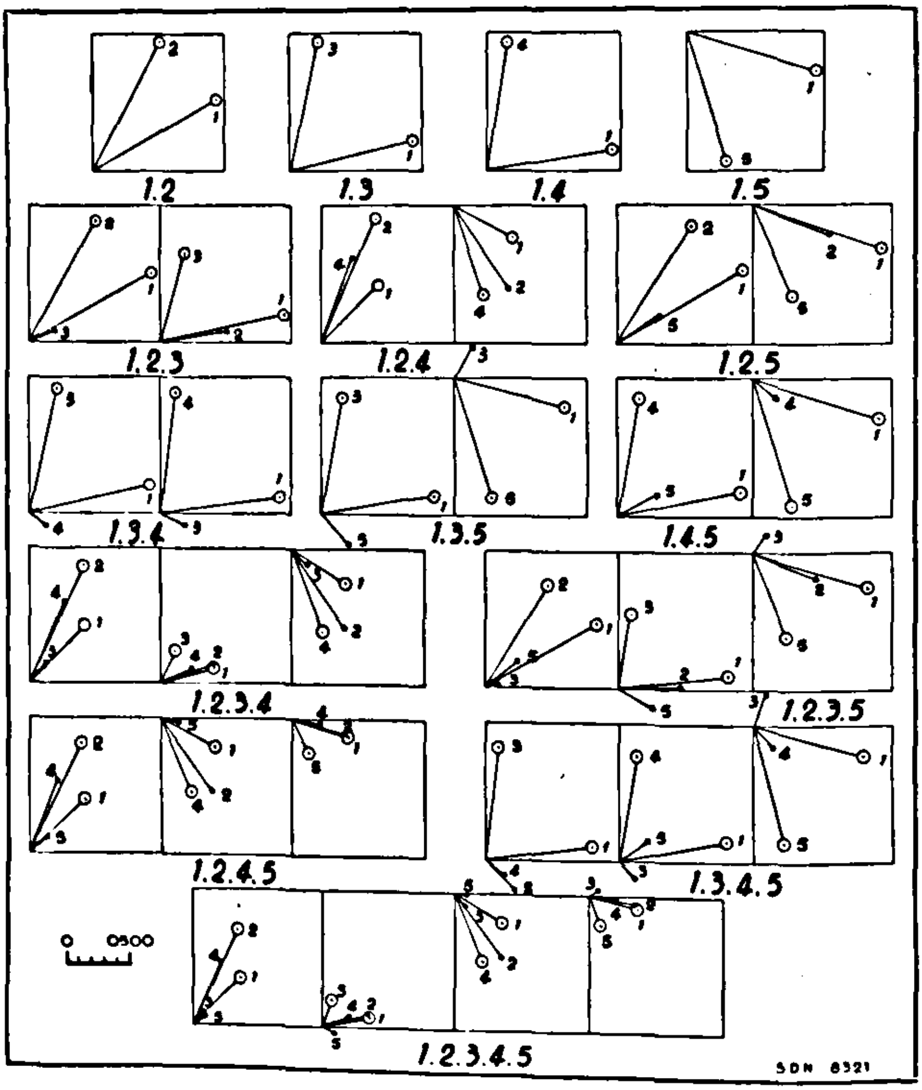
Graph III. 15 concerns one of the calculations for the United Kingdom before the war. It is certainly less good than the post-war cases: the beams spread much more widely. Nevertheless the result indicates the significance of all coefficients found. Here, it is interesting to see how the direction of the relation between variates 1 and 4 (consumption and price of iron) is changed by the inclusion of the other variates; even if only 2 is added (*cf.* set 124). The significance of variate 3 might be doubted, but if only the beams 1 and 2 are considered, the regression coefficient for 3 is again very well determined.

Graph III. 15.

Bunch map.

CONSUMPTION OF IRON AND STEEL: UNITED KINGDOM, 1871-1910.

1 = Consumption of iron and steel. 2 = Non-labour income$_{-1}$.
3 = Δ non-labour income$_{-1/2}$. 4 = Price of iron$_{-1}$. 5 = Bond yield$_{-1/2}$.

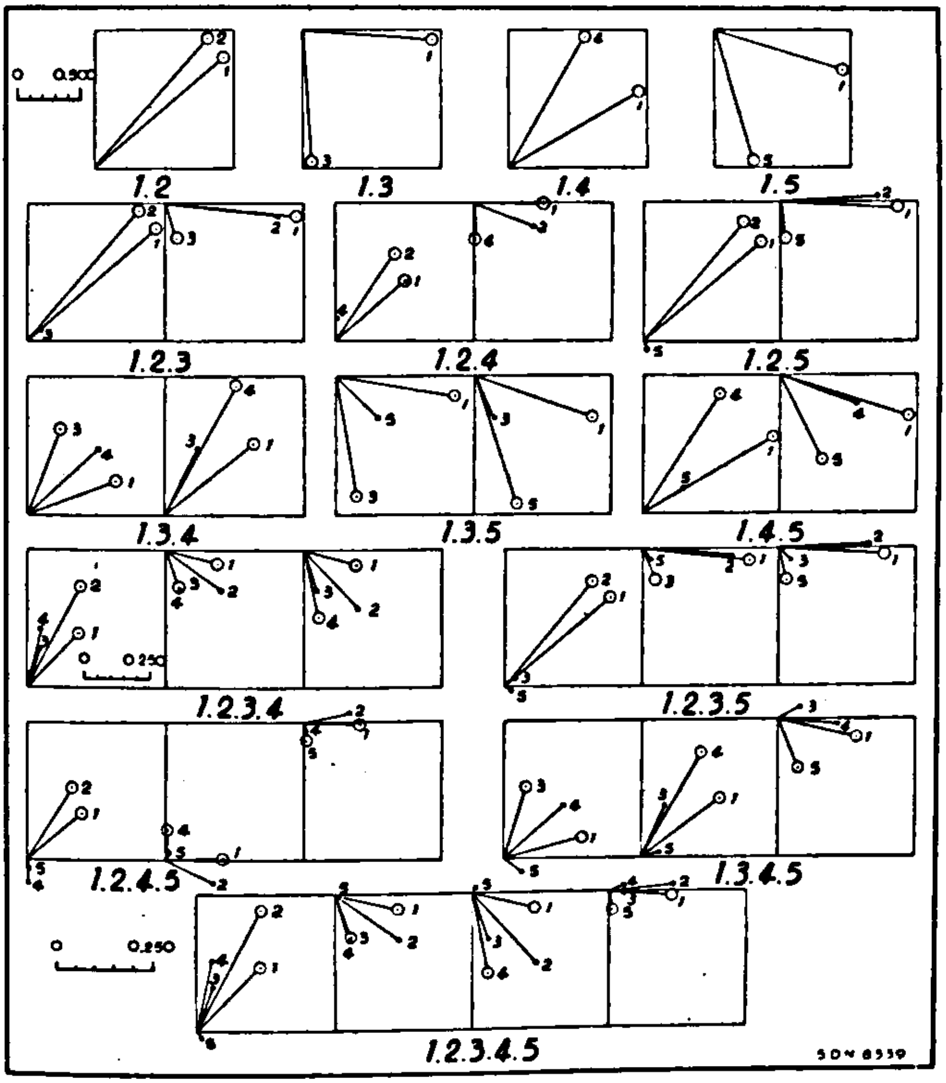


Graph III. 16, concerning one of the German (pre-war) calculations, is very similar to Graph III. 15. The spread of the beams is about the same; the relation between variates 1 and 4 is also reversed by the introduction of other variates and the regression coefficients of variates 3 and 5 are well determined if only beams 1 and 2 are considered. But the bunch map for Germany is less satisfactory in that the relation between variate 1 and variate 2 (profits) is not improved by the addition of any or all of the secondary variates.

Graph III. 16. Bunch map.

Consumption of Iron and Steel: Germany, 1871-1912.

1 = Consumption of iron and steel. 2 = Dividends$_{-1/2}$. 3 = Δ dividends$_{-1/2}$. 4 = Price of iron$_{-1/2}$. 5 = Long-term interest rate$_{-1/2}$.



CHAPTER IV

RESIDENTIAL BUILDING

§ 12. The Relation tested

Only private building studied.

The example to be considered next relates to a special type of investment activity—namely, the construction of dwelling-houses. So far as possible, the investigation has been confined to private activity in this field, as building by public authorities and societies may be governed by different considerations.[1]

Two groups of "explanatory" variates.

The "explanatory factors" included may be separated into two groups. The first group consists of some factors which roughly determine the profitability of owning houses. In a perfect market, this would be the most natural incentive to build. The second group forms, in a sense, a corrective to the first group, necessary because of the imperfection of the market.

First group of variates.

The profitability of owning houses depends chiefly on:

(*a*) The rent level;
(*b*) The cost of maintenance;
(*c*) Interest payments and
(*d*) Amortisation.

The amounts of interest payments and amortisation will first of all depend on the level of building costs; amortisation may be

[1] In pre-war time, building of dwellings by public authorities was non-existent or insignificant. For the post-war period, no detailed figures are available for the United States, but State intervention started only in 1933 and was rather indirect. For Sweden, the data for 296 cities cover all residential building, but those for Stockholm relate to private building only.

said to be a fixed percentage of the latter—which will be not far from 1%,[1] whereas interest will be the product of three factors, viz.:

(1) building costs;
(2) the percentage of building costs which on the average will be covered by mortgages; and
(3) the interest rate for mortgages.

It is rather difficult to get exact series on the cost of maintenance, but in general it will move about parallel to building cost, as it includes many elements also included in the latter. Its level may be roughly estimated to be about 1% of building cost per annum.

Series included.

It follows from the above that the following series should, first of all, be included in the " explanation ":

(1) The rent level;
(2) The cost of construction; and
(3) The rate of interest on mortgages.

Their relative " influence " may be deduced from the regression coefficients which will be calculated.

A priori *determination of relative influence.*

It may, however, also be determined by *a priori* considerations, based upon the structure of the profit account for holding houses.

Indicating the construction costs of a certain house by 100, this account will approximately show items of the following order of magnitude:

Receipts: rent	8
Deductions: amortisation and maintenance	2
interest on mortgage . . .	$0.7\overline{\overline{m}}_{Lb}$, where $\overline{\overline{m}}_{Lb}$ represents the interest rate.

If rents are measured by an index $\overline{\overline{m}}_R$, with average $\overline{m}_R = 100$, construction costs by an index $\overline{\overline{q}}_B$ with average $\overline{q}_B = 100$, and

[1] This figure may be somewhat too low in some cases; what matters, however, for the calculations is the total of amortisation and maintenance, which has been taken at 2% (see below).

interest rates $\overline{\overline{m}}_{Lb}$ in natural units (per cents), then profits, in per cents, from the holding of houses will be represented by

$$0.08\overline{\overline{m}}_R - 0.02\overline{\overline{q}}_B - 0.007\overline{\overline{q}}_B\overline{\overline{m}}_{Lb} = \overline{\overline{z}}$$

This expression may be also written as

$$0.08\,(\overline{m}_R + m_R) - 0.02\,(\overline{q}_B + q_B) - 0.007\,(\overline{q}_B + q_B)\,(\overline{m}_{Lb} + m_{Lb})$$

where the unbarred minuscules indicate deviations from average or from trend. These deviations will, in general, be small in comparison with the average values, and therefore their mutual products may be neglected. We then get:

$$\overline{\overline{z}} = (6 - 0.7\overline{m}_{Lb}) + 0.08m_R - (0.02 + 0.007\overline{m}_{Lb})q_B - 0.7m_{Lb}.$$

The first term in brackets is a constant, the average value of the expression: $\overline{z} = 6 - 0.7\overline{m}_{Lb}$. The deviations may as usually be indicated by z:

$$z = 0.08m_R - (0.02 + 0.007\overline{m}_{Lb})\,q_B - 0.7m_{Lb}\,.$$

The value of $\overline{m}_{Lb}$ will change from case to case, but usually it is of the order of magnitude of 5, which leads to

$$z = 0.08m_R - 0.055q_B - 0.7m_{Lb}.$$

Two sorts of calculation.

Two sorts of calculation have been made; calculations using m_R, q_B and m_{Lb} as separate variates (Table IV. 1), and calculations using z in their place (Table IV. 2).

Second group of variates.

Apart from these variates, a second group has been included. Their inclusion is due to the imperfection of some of the markets which play a rôle in our problem.

The variates of the second group are:

(4) The number of unoccupied houses (h') or the total number of houses present (h);

(5) Some income series (E).

Imperfection of market for housing services.

The reason for including the number of unoccupied houses is that it may directly discourage building, even if rents, building costs and interest rates are in a favourable relation to each other. In a perfect market for housing services, such a situation would not occur: rents would fall. The stickiness of rents, closely connected with the long duration of letting contracts and further imperfections in this market, prevents such a rapid adaptation and, consequently, the number of unoccupied houses is a largely independent factor which also influences building activity.

In one of the cases where no series for empty houses was available, the total number of houses could be included, after elimination of its trend. The trend elimination, together with the inclusion of an income series, forms a rough correction for the need for dwellings in that case.[1]

Income series are also included with another intention: they represent a demand factor, in so far as a number of houses are not built for letting at all, but by their future occupants. In the United States, about 50% of all inhabitants live in owned houses; it was estimated that about 75% of the new dwellings built during the last building boom in England were not for letting. Again this may be called an imperfection of the market for housing services.

Imperfection of credit markets.

In some investigations by other authors, explanatory series have been included which are connected immediately with the imperfection of credit markets. An extreme case is the one treated by Professor C. F. Roos,[2] concerning St. Louis, where, for the period studied, mortgage rates had not moved at all. Professor Roos includes instead the " foreclosure rate ", giving the number

[1] The number of family units is often used as an indication of the " need " for dwellings. As long, however, as family incomes are not taken into account, the number of family units reflects potential rather than actual demand; although the distribution of income over the various items of the budget is of course influenced by the number of families. For long-term investigations, it may be a useful guide; for an analysis of fluctuations it seems less important, as the number of family units usually develops smoothly. An exceptional growth of the number of family units is, however, regarded as one of the causes of the " building boom " in the United Kingdom from 1933 to 1936.

[2] *Dynamic Economics*, Bloomington, 1934, pages 69-110.

of foreclosures per 100,000 families. This rate he considers as a good inverse index of the willingness of banks to grant credits. A closer investigation shows that it is highly correlated with the number of unoccupied houses a short time before, which seems quite natural. In a sense, therefore, this factor is already included in our series h', provided we take the right lag.

A number of authors lay stress on the general state of confidence as a factor of importance, because of its close relation to the willingness to grant credits. This factor may be introduced in two different ways. As far as the fairly systematic changes in confidence during the business cycle are concerned, the income series will be a good index; and it has already been included. As far as acute and specific confidence crises occur, the years in which they have presented themselves may perhaps best be excluded. As a test, it may be investigated afterwards whether or not these years show, as compared to "calculated" building activity, an abnormally low level.

Profits of owning houses and of constructing houses.

Finally, it may be stated that the selection of the explanatory series, based as it is upon the profitability of owning houses, presupposes that the market for houses so nearly approaches perfection that the builder acts in the same way as the future owner of houses would have done: *i.e.*, it is assumed that his behaviour is not deflected by imperfect foresight with regard to the possibilities of selling the houses which he builds. This assumption cannot easily be avoided, as the statistical material available for prices of houses—and it is prices, not rents, which directly influence the mind of the builder—is very scanty.

Long and short waves. Trend elimination.

Several authors have pointed out the existence of a specific building cycle of fifteen to twenty years duration,[1] on which fluctuations of lesser duration would be superimposed. The present investigation has not been directed specially to the study of these long waves. For post-war years, the actual movement of building activity, without any correction for trend or long

[1] *Cf.*, *e.g.*, C. F. Roos, *loc. cit.*; J. R. Riggleman, "Building Cycles in the United States 1875-1932", *Journal of the American Statistical Association*, Vol. 28, pages 174-183.

cycle, has been explained. For pre-war, the secular trend has been eliminated, leaving both long and short waves to be studied. In the case of Sweden, however, satisfactory results were only obtained when the long cycle was in turn eliminated, by the use of moving averages of variable length. In this case, therefore, conclusions apply solely to short waves.

§ 13. The Statistical Material

The countries and periods studied are:

Countries and periods.

Pre-war: Germany (Hamburg) 1878-1913 (thirty-six years);
Sweden (Stockholm) 1884-1913 (thirty years).

Post-war: United Kingdom 1923-1935 (thirteen years);
United States 1915 or 1919-1935 (twenty-one or seventeen years);
Sweden 1924-1936, 1933 excluded * (twelve years).

In addition, extrapolations for 1936 and 1937 have been made for the United States and the United Kingdom.

Description of series.

The following table indicates the series which have been used to represent: (i) the volume of building and (ii) the explanatory factors mentioned above.

Volume of Building.

Country and period	Description of series	Source
Germany, pre-war	Net increase [a] in total number of "rooms" [b] in Hamburg.	Hunscha, *Die Dynamik des Baumarkts, Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 17.
Sweden, pre-war	Total number of newly built rooms or kitchens in Stockholm.	*Statistisk Årsbok för Stockholms Stad.*

* The year 1933 has been excluded in all calculations owing to big strikes in the building industry.

[a] Gross increase was only available for a shorter period, and referred to the number of dwellings without regard to their size.

[b] Lokalitäten.

Country and period	Description of series	Source
United Kingdom, post-war	Number of houses built by private enterprise without State assistance.	*Statistical Abstract for the United Kingdom.*
United States, post-war	(i) Estimated total value of non-farm residential construction in 1923/25 dollars. [a]	National Bureau of Economic Research, *Bulletin* No. 65, and *Statistical Abstract.*
	(ii) Contracts awarded, residential building, floor space of building.	*Statistical Abstract* (data from Dodge Co.).
Sweden, post-war	(i) Gross increase in number of rooms or kitchens in 296 cities.	*Sveriges Statistisk Årsbok.*
	(ii) Number of dwellings built in Stockholm by private enterprise.	*Stockholm Stadskollegiets utlåtanden och memorial.* Bihang No. 10 A, 1935.

Rent.

Country and period	Description of series	Source
Germany, pre-war	Average annual rent of occupied houses in Hamburg.	HUNSCHA: see under *Vol. of building.*
Sweden, pre-war	Average rent per room of houses to let in Stockholm.	MYRDAL, *The Cost of Living in Sweden.*
United Kingdom, post-war	Rent index of the Ministry of Labour cost-of-living index. [b]	*Abstract of Labour Statistics.*
United States, post-war	Rent index of the Bureau of Labor Statistics cost-of-living index.	*Statistical Abstract.*
Sweden, post-war	Rent index of the cost-of-living index.	*Sveriges Statistisk Årsbok.*

Construction Costs.

Country and period	Description of series	Source
Germany, pre-war	Prices of building materials. [c]	JACOBS und RICHTER, *Grosshandelspreise, Vierteljahreshefte zur Konjunkturforschung,* Sonderheft 37.
Sweden, pre-war	Index of building costs.	MYRDAL, *loc. cit.*

[a] *I.e.*, value at current prices deflated by index of construction costs, 1923-1925 = 100.

[b] Up to 1928, the index relates to controlled rents; from 1929 onwards, to controlled and uncontrolled rents combined. No better index is available.

[c] When reckoning the "profitability" of holding houses, allowance was made for wage costs: these were supposed to have been constant throughout the period and to account for 35 % of total construction costs.

Country and period	Description of series	Source
United Kingdom, post-war	Index of building costs.	COLIN CLARK, *Investment in Fixed Capital in Great Britain*, Special Memorandum No. 38, London & Cambridge Economic Service.
United States, post-war	Index of construction costs of *Engineering News Record*.	*Statistical Abstract.*
Sweden, post-war	(i) Index of building costs.[a]	*Svenska Handelsbanken:* " Index".
	(ii) Index of building costs in Stockholm.	*Statistisk Årsbok för Stockholms Stad.*
	Interest Rate.	
Germany, pre-war	Average rate on mortgage banks' new issues.	HUNSCHA: see under *Volume of building.*
Sweden, pre-war	Savings banks' rate.	LINDAHL, etc.: *The National Income of Sweden.*
United Kingdom, post-war	Yield on 2½% Consols.	*Statistical Abstract.*
United States, post-war	Yield on 60 bonds.	*Statistical Abstract* (from Standard Statistics).
Sweden, post-war	Savings banks' rate.	*Sveriges Statistisk Årsbok.*
	Index of Housing Needs.[b]	
Germany, pre-war	% of vacant dwellings in Hamburg.	HUNSCHA: see under *Volume of building.*
Sweden, pre-war	% of vacant dwellings in Stockholm.	*Statistisk Årsbok för Stockholms Stad.*
United States, post-war	Stock of houses,[c] deviations from trend.	*Statistical Abstract.*
	Profits or Income.	
Germany, pre-war	Dividends in % of capital.	DONNER, *Die Kursbildung am Aktienmarkt, Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 36.
Sweden, pre-war	Total real income.[d]	*Sveriges Statistisk Årsbok.*

[a] The figure for 1923 was obtained by combining the index of prices of building materials (*Statistisk Årsbok)* and the index of hourly wages in building (BAGGE: *Wages in Sweden*) with the same weights as in index for subsequent years— i.e., materials 60%, wages 40%.

[b] No data are shown for the United Kingdom. An attempt at computing the stock of houses, in deviations from its trend, led to unreliable results.

[c] Census data, interpolated on the basis of floor space of buildings for which contracts have been awarded.

[d] Total assessed income of following year deflated by cost-of-living index (MYRDAL: *The Cost of Living in Sweden*, 1830-1930).

Country and period	Description of series	Source
United Kingdom, post-war	Real income from wages and salaries. [a]	CLARK, *National Income and Outlay.*
United States, post-war	(i) Net income of corporations.	*Statistical Abstract.*
	(ii) Urban non-workers' income.	Estimates based on S. KUZNETS: *National Income.*
	(iii) Capital gains.	Estimates based on WARBURTON: *Journal of Political Economy,* Vol. 43.
Sweden, post-war	Total real income. [b]	*Sveriges Statistisk Årsbok.*

[a] Money income deflated by cost-of-living index (Ministry of Labour).
[b] Total assessed income of following year deflated by cost-of-living index.

Lags used. A uniform lag of one year has been assumed to exist between the series showing completed building and both the explanatory factors of the first group —rent, construction costs, interest rate—and the income series. In the case of the United States, where the series representing building is based either on contracts awarded or building permits delivered, the lag zero indicated in brackets after the regression coefficients corresponds to a real lag of probably not far from one year between the explanatory factors and the end of the building process.

As regards the total number of houses, or the number of vacancies, the lag was chosen between one-half and three and a-half years, according to the best result yielded.

§ 14. Results

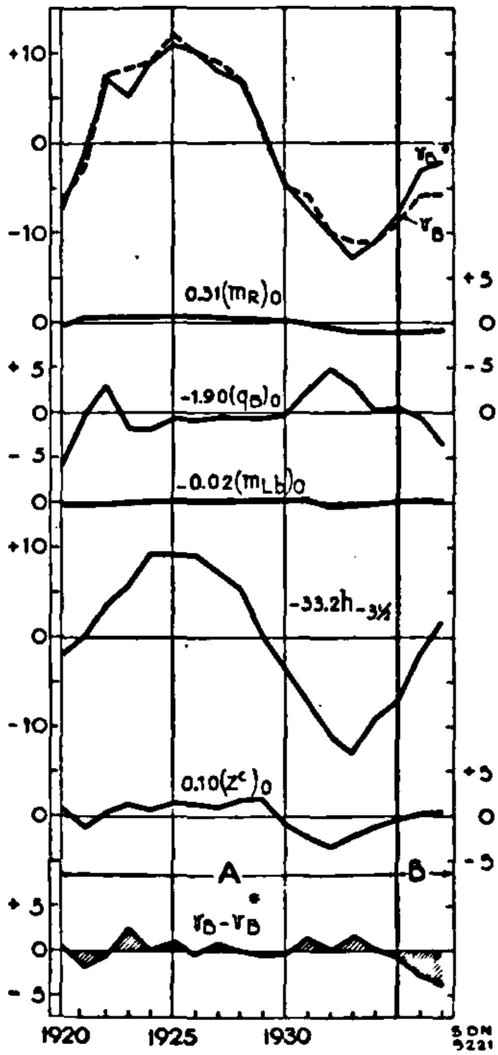
Chief results. The explanatory factors mentioned above permit, when rightly combined, of a good explanation of the movement of building activity, more especially after the war. But for each country, the respective influence of the various factors in the best combination varies greatly, as may be seen from the accompanying graphs. In the United States, the movement is dominated by the available stock of houses lagged over three and a-half years, while the influence both of the rate of interest and of income is almost negligible.

Graph IV. 1.

" EXPLANATION " OF BUILDING.
UNITED STATES 1920-1937.

(Free calculation.)

A = Calculated.
B = Extrapolated.

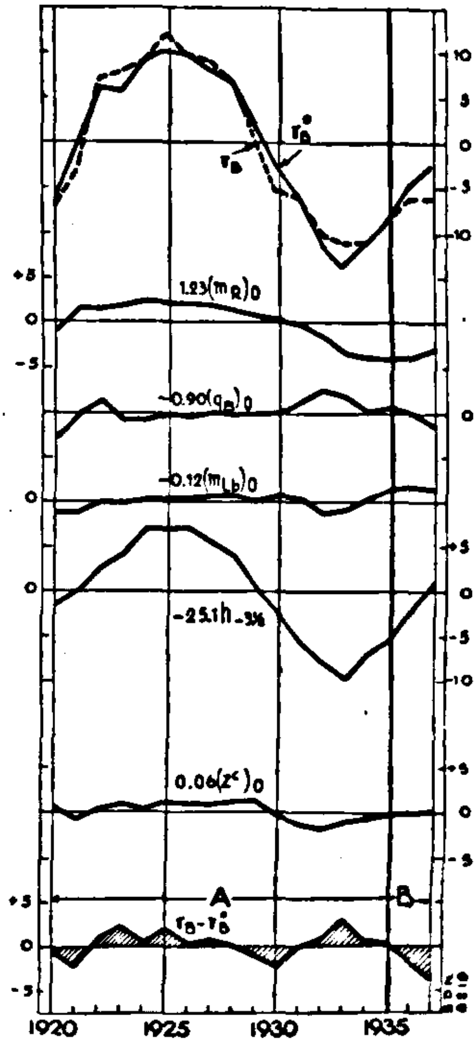


Graph IV. 2.

" EXPLANATION " OF BUILDING.
UNITED STATES 1920-1937.

(Rent, building costs and interest rate combined *a priori*.)

A = Calculated.
B = Extrapolated.



v_B :	building activity, actual.	
v^*_B :	building activity, as explained by:	
$(m_R)_0$:	rent	not lagged (see remark on p. 98);
$(q_B)_0$:	building costs	
$(m_{Lb})_0$:	bond yield	
$h_{-3½}$:	number of houses (in deviation from trend), lagged 3½ years.	
Z^c_0 :	profits, not lagged.	

Graph IV. 3.

" EXPLANATION " OF BUILDING.
UNITED KINGDOM 1923-1937.

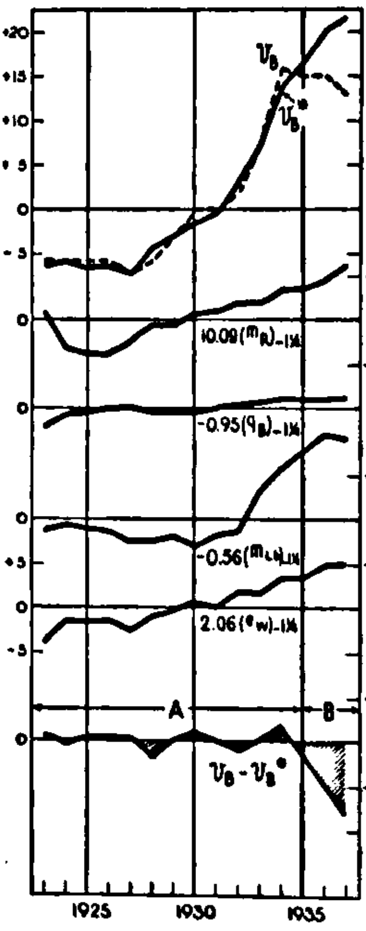
(Free calculation.)

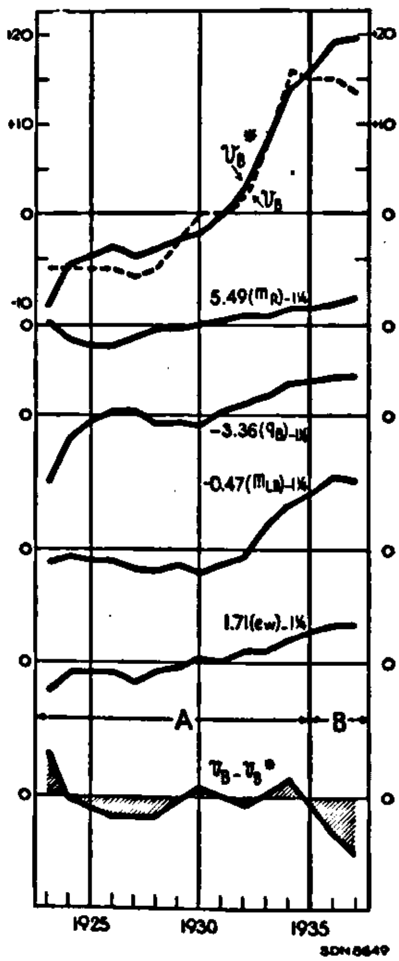
Graph IV. 4.

" EXPLANATION " OF BUILDING.
UNITED KINGDOM 1923-1937.

(Rent, building costs and interest rate combined *a priori*.)

A = Calculated. B = Extrapolated.





v_B:	building activity, actual.	
v_B^*:	building activity, as explained by:	
$(m_R)_{-1¼}$:	rent	lagged 1¼ years;
$(q_B)_{-1¼}$:	building costs	
$(m_{Lb})_{-1¼}$:	bond yield	
$(e_w)_{-1¼}$:	real labour income. lagged 1¼ years.	

Graph IV. 5.

" EXPLANATION " OF BUILDING.
SWEDEN 1924-1936.

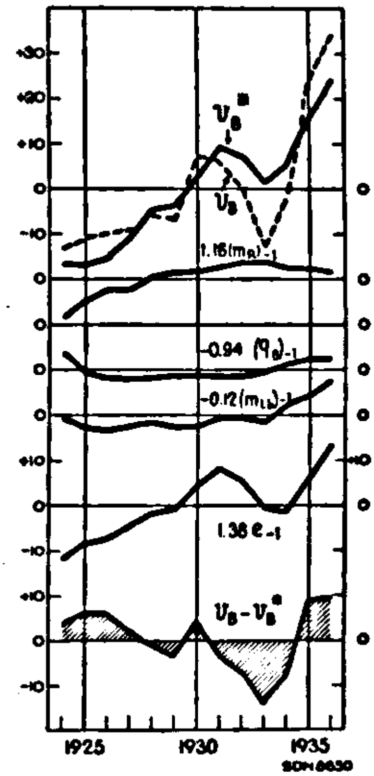
(Rent, building costs and interest rate combined *a priori*.)

v_B : building activity, actual.

v_B^* : building activity, as explained by:

$(m_R)_{-1}$: rent
$(q_B)_{-1}$: building costs
$(m_{Lb})_{-1}$: interest rate
e_{-1} : real income

} lagged 1 year.

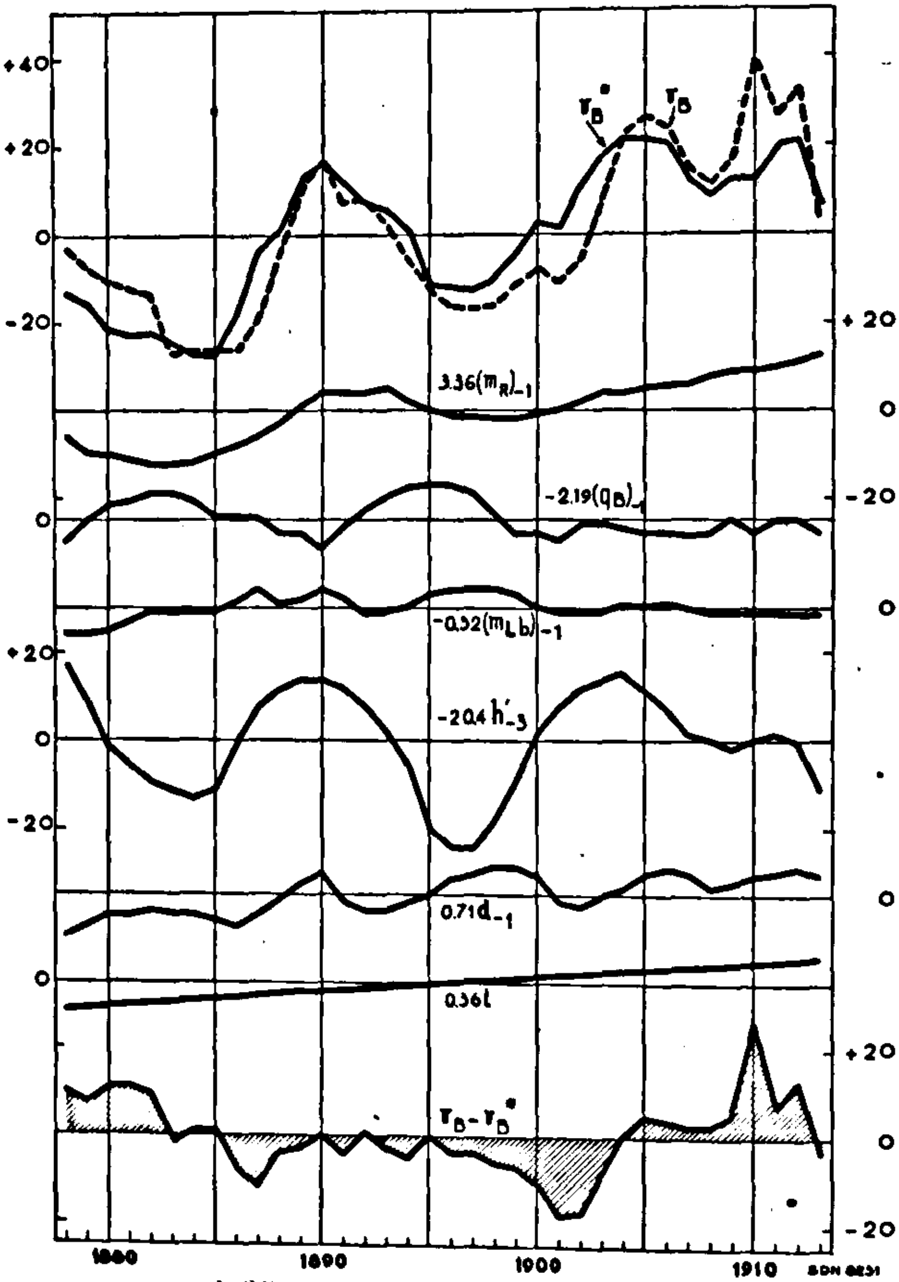


In the United Kingdom and Sweden, on the other hand, the greatest importance seems to attach to the interest rate and real income.

For before the war, the results are more doubtful and will be discussed in the next section.

Graph IV. 6.

"EXPLANATION" OF BUILDING. GERMANY (HAMBURG) 1878-1913.
(Rent, building costs and interest rate combined *a priori*.)



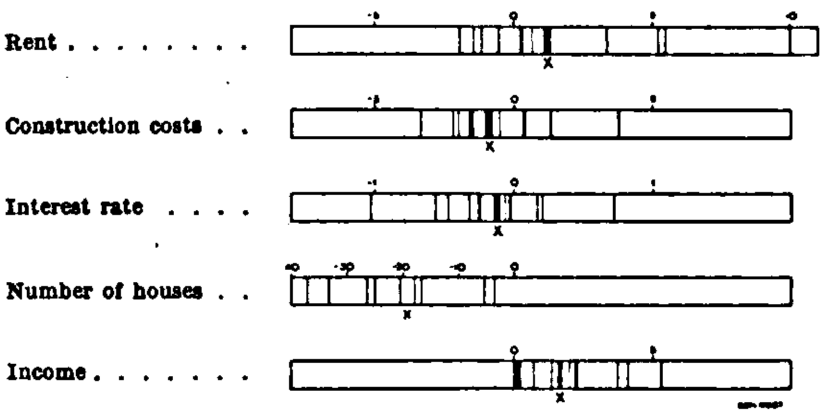
v_B :	building activity, actual.	
v_B^* :	building activity, as explained by:	
$(m_R)_{-1}$:	rent	lagged 1 year;
$(q_B)_{-1}$:	building costs	
$(m_{Lb})_{-1}$:	interest rate	
h_{-3} :	number of houses (in deviation from trend), lagged 3 years;	
d_{-1} :	dividends in % of capital, lagged 1 year;	
t :	trend.	

The following graph affords a comparison of all the regression coefficients found in the calculations comprised in the summary Tables IV. 1 and IV. 2. While the range of variation is rather wide, there is for all variates except the number of houses a well-defined mode which coincides with the median in three cases out of four.

Graph. IV. 7.

DISTRIBUTION OF REGRESSION COEFFICIENTS.

(Tables IV. 1 and IV. 2.)



(The x indicates the median.)

The values of the medians are roughly as follows: rent + 1; construction costs — 1; interest rate — 0.1; number of houses —20; income + 1½. Owing to the choice of the units, these figures represent the various elasticities, except in the case of interest rates. In this case, the figure indicates that an increase of 0.01% in the rate of interest will produce a decrease of 0.1% in the volume of building.

Extrapolations.

The equations found for the United States and the United Kingdom for the period up to 1935 have been applied to the data (or estimates) for 1936 and 1937.

For the United States, the volume of building thus calculated reflects the actual movement of building as shown by current statistics: both the free and the *a priori* calculations point

Building. Table IV. 1. Explanation

Note. — Unless otherwise stated, the units used are: for pre-war, % deviations from trend; interest rate, the deviations of which are expressed in units of 1 % and 0.01 % respectively. each series, see table on pages 95 *et seq.*

Country	Period	Series explained	Correlation coefficient R
Germany (Hamburg)	1878-1913	Net increase in number of rooms.	0.92
Sweden (Stockholm)	1884-1913	Number of new rooms built.	0.68
United States	1920-1935	Total volume of non-farm residential construction.	(i) 0.99 (ii) 0.99
United Kingdom	1923-1935	Number of houses built by private enterprise without State assistance.	0.99
Sweden	1924-1936*	Number of houses built in 296 cities.	(i) 0.97 (ii) 0.97
Sweden (Stockholm)	1924-1936*	Number of houses built without State assistance.	0.95

Building. Table IV. 2. Explanation of and Interest Rate

Note. — See note to Table IV. 1.

Country	Period	Series explained	Correlation coefficient R
Germany	1878-1913	Net increase in number of rooms.	0.87
Sweden	1884-1913	Number of new rooms built.	0.59
United States	(i) 1915-1935 (ii) } 1920- (iii) } 1935	Floor space of contracts awarded. Volume of non-farm residential construction.	{ 0.96 { 0.98 { 0.99
United Kingdom	1923-1935	Number of houses built by private enterprise without State assistance.	0.96
Sweden	1924-1936	Number of dwellings built in 296 cities.	(i) 0.89 (ii) 0.91

a % vacant houses.
b Net income of corporations.
c Total urban non-workers' income plus capital gains.
* 1933 excluded.

of Building: Free Calculations.

for post-war, % deviations from average, except for the " profitability of building " and the The lag, in years, is indicated in brackets after each coefficient. For a detailed description of

Regression coefficients and lags of:				
Rent	Construction costs	Interest rate	Number of houses	Income or profits earned
5.20 (1)	— 1.54 (1)	0.72 (1)	— 16.5[a] (3)	1.71 (1)
1.14 (1)	— 1.59 (1)	— 1.03 (1)	— 3.7[a] (1½)	3.75 (1)
0.31 (0) 0.23 (0)	— 1.90 (0) — 1.56 (0)	— 0.02 (0) — 0.03 (0)	— 33.2 (3½) — 37.0 (3½)	0.10[b] (0) 0.16[c] (0)
10.09 (1¼)	— 0.95 (1¼)	— 0.56 (1¼)	f	2.06 (1¼)
— 1.16 (1) — 1.43 (1)	0.34 (1) 0.39 (1)	— 0.24 (1) — 0.24 (1)	f f	2.22 (1) 2.25 (1)
— 0.52 (1)	3.74 (1)	0.21 (1)	f	4.12 (1)

Building: Rent, Construction Costs combined a priori.

Regression coefficients and lags of:					
Profitability of building				Number of houses	Income or profits earned
Total	Rent	Construction costs	Interest rate		
48.2 (1)	3.36 (1)	—2.19 (1)	—0.32 (1)	— 20.4[a] (3)	0.71 (1)
—19.0 (1)	—1.93 (1)	1.31 (1)	0.17 (1)	— 5.2[a] (1½)	5.34 (1)
13.3 (0) 12.3 (0) 11.9 (0)	1.21 (0) 1.23 (0) 1.19 (0)	—0.93 (0) —0.90 (0) —0.87 (0)	—0.12 (0) —0.12 (0) —0.11 (0)	— 17.8 (3½) — 25.1 (3½) — 26.5 (3½)	0.14[b] (0) 0.06[b] (0) 0.26[c] (0)
69.0 (1¼)	5.49 (1¼)	—3.36 (1¼)	—0.47 (1¼)	f	1.71 (1¼)
4.5 (1) 7.5 (1)	0.69 (1) 1.16 (1)	—0.52[d] (1) —0.94[e] (1)	—0.06 (1) —0.12 (1)	f f	1.66 (1) 1.38 (1)

[d] Index of construction costs of Svenska Handelsbanken.
[e] Index of construction costs of Stockholm Statistical Office.
[f] Series not included.

Building. Table IV. 3. Explanation of Building: Combination with Rent and

Note. — See note to Table IV. 1.

Country	Period	Series explained
Germany (Hamburg)	1878-1913	Net increase in number of rooms.
Sweden (Stockholm)	1884-1913	Number of new rooms built
United States	1920-1935	Total volume of residential construction
United Kingdom	1923-1935	Number of houses built
Sweden	1924-1936*	Number of houses built in 296 cities

Building. Table IV. 4. Influence of Number

Note. — See note to Table IV. 1.

Country	Period	Series explained
Germany	1878-1913	Net increase in number of rooms
	"	" " "
	"	" " "
United States	1920-1935	Volume of non-farm residential building
	"	" " "
	"	" " "
	"	" " "
	1920-1932	" " "
United Kingdom	1923-1935	Number of houses built without State assistance
	"	" " "
Sweden	1924-1936*	Number of rooms built in 296 cities
	"	" " "
	"	" " "

a Number of vacant houses, in %.
b Net income of corporations.
c Series not included.
* 1933 excluded.

Calculations using Interest Rate in a priori.
Construction Costs, and also freely.

Correlation coefficient R	Regression coefficients and lags of:			
	Profitability of building	Interest rate	Number of houses	Income or profits earned
0.93	76.8 (1)	1.04 (1)	—20.1[a] (3)	1.72 (0)
0.65	1.1 (1)	— 0.70 (1)	— 4.4[a] (1½)	4.29 (1)
0.98	12.5 (0)	0.02 (0)	—25.1 (3½)	0.06[b] (0)
0.97	20.5 (1¼)	— 0.56 (1¼)	[c]	3.11 (1¼)
0.97	— 5.3 (1)	— 0.35 (1)	[c]	2.02 (1)

Explanation of Building.
of Variates included.

Correlation coefficient R	Regression coefficients and lags of:			
	Profitability of building	Number of houses	Income or profits earned	Capital gains
0.66	3.8 (1)			
0.82		—15.5[a] (3)		
0.87	48.2 (1)	—20.4[a] (3)	0.71 (0)	
0.78	33.0 (0)			
0.97		—36.0 (3½)		
0.98	10.5 (0)	—29.8 (3½)		
0.98	12.3 (0)	—25.1 (3½)	0.06[b] (0)	
0.99	11.6 (0)	—22.5 (3½)	— 0.06[b] (0)	0.003 (0)
0.96	88.8 (1¼)			
0.96	69.0 (1¼)		1.71 (1¼)	
0.82	11.9 (1)			
0.87			2.36 (1)	
0.89	4.5 (1)		1.66 (1)	

to a slackening of residential building in 1937 as compared with 1936. For the United Kingdom, on the other hand, the extrapolations of both calculations point to a continuous rise through 1936 and, in a somewhat lesser degree, 1937, which does not agree with the actual stability in 1936 and decline in 1937. The explanation of this difference seems to lie in the special fact that the building boom, which consisted largely in the construction of small houses for the upper working-class and the lower middle class, had, round about 1935, attained a limit, which could only have been surpassed by making these owner-inhabited houses accessible to the earners of smaller incomes.

Details of results. Tables IV.1-IV.4.

In Tables IV. 1-IV. 4 the results of the principal calculations are set out. In Table IV. 1, all the explanatory factors enumerated above have been included separately in the correlation calculations; in Table IV. 2, the three factors of the first group have been combined *a priori* so as to reflect the profitability of building. In Table IV. 3, the interest rate has been added as a separate factor in addition to entering into the " profitability "; this is a way of introducing interest rates with a free coefficient—in order to find whether there is accordance with the *a priori* case—without increasing by two the number of variates, as is the case in Table IV. 1. Table IV. 4. shows the influence of the number of variates included.

Correlation coefficients.

With the exception of the case of Sweden, pre-war, all correlation coefficients in Tables IV. 1 and IV. 2 —the most important ones—vary between 0.87 and 0.99; for post-war, the median is even as high as 0.97.

Signs of regression coefficients.

All coefficients in Table IV. 1 have the right sign except the interest rate in the case of Germany, pre-war, which is positive instead of negative and all coefficients but two of the components of profitability for Sweden, post-war.

For Sweden, pre-war, the regression coefficient of profitability is negative in Table IV. 2, but it is reversed in Table IV. 3, which seems to point out that the influence of the rate of interest is much higher than assumed when calculating the " profitability ".[1] Table IV. 3 is otherwise not very satisfactory: two of the columns contain coefficients with a wrong sign.

In Table IV. 4, on the other hand, all signs are right.

Comparison between countries.

The main divergencies between countries as regards the order of magnitude of the regression coefficients of Tables IV. 1 and IV. 2—leaving aside those with a wrong sign[2]—may be briefly summarised:

Table IV. 1: In the United States, interest rate and rent have a much smaller coefficient than the average, while in the United Kingdom the coefficient of the latter is much greater. The coefficients of construction costs, in so far as they have the right sign, are rather close to each other; the coefficients of the number of houses, on the other hand, show a very wide spread.

In the United States, post-war, the coefficients for the income series are almost negligible.[3]

Table IV. 2: the coefficient of profitability is much larger than the average in Germany, pre-war, and the United Kingdom, post-war, and smaller in Sweden, post-war.

The coefficients for the number of houses are not very different from those in Table IV. 1. The coefficient of profits in the United States is, again, very small.

Comparison of Tables IV. 1 and IV. 2.

A comparison of the coefficients of rent, construction costs and interest rate in Tables IV. 1 and IV. 2 should make it possible to test the assumptions made when combining these factors into an index of profitability. To facilitate this comparison, the coefficients have been inserted in the following table. Unfortunately, the results show a rather wide range of variation.

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[1] This might be explained by the fact that the interest rate is, at the same time, a measure of the desirability of investing in bonds, and that this desirability influences the incentive to build.

[2] It may, in this connection, be noted that both the rent index and the construction costs index for Sweden, post-war, which obtain only wrong signs in Table IV. 1, are not very representative.

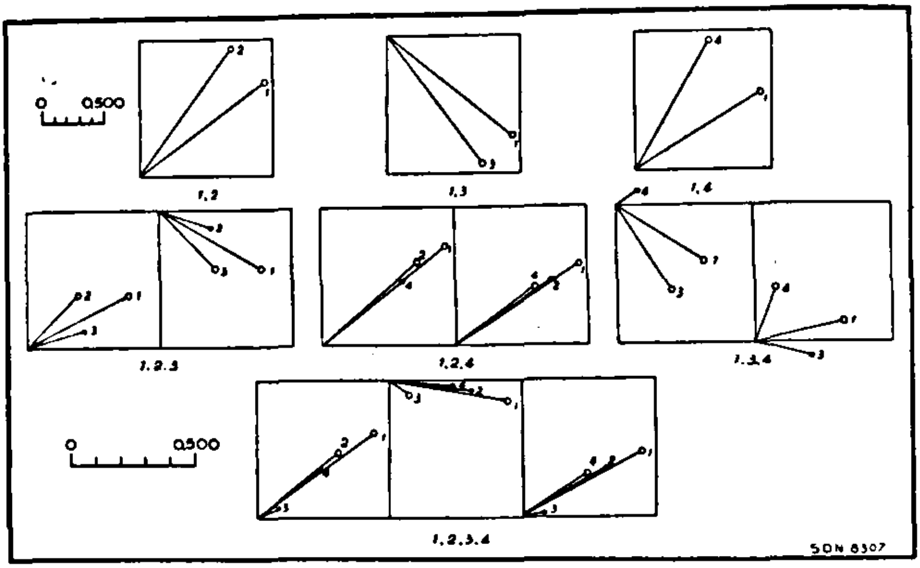
[3] These coefficients were somewhat raised when real instead of money income was used, but they still remained very low.

Graph IV. 8.

Bunch Map.

BUILDING: UNITED STATES 1915-1935.

1 = Building (contracts awarded). 2 = Profitability of building.
3 = Number of houses$_{-3^1/_2}$. 4 = Profits.

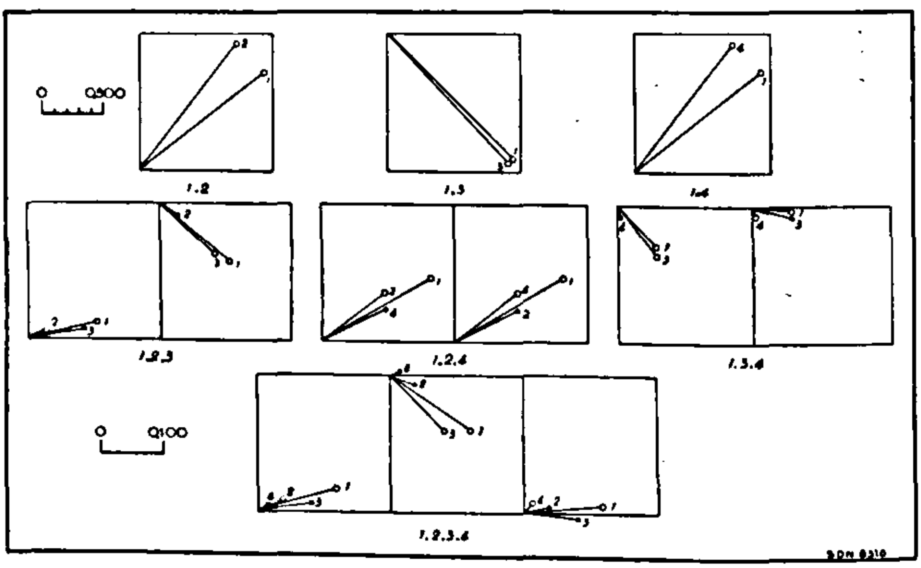


Graph IV. 9.

Bunch Map.

BUILDING: UNITED STATES 1920-1935.

1 = Building (total non-farm residential construction). 2 = Profitability of building.
3 = Number of houses$_{-3^1/_2}$. 4 = Profits.



In the German pre-war case, the coefficients of rent and construction costs are not too dissimilar. For after the war, the coefficient for rent is greater and that for construction costs smaller for the United Kingdom in the free calculation; but the reverse is true for the United States.

The two factors of the second group, on the other hand, have fairly stable coefficients.

Tables IV. 1 and IV. 2 combined.[a]

A = Free calculations. B = *A priori* calculations.

Country	Regression coefficients of:									
	Rent		Construction costs		Interest rate		Number of houses		Income	
	A	B	A	B	A	B	A	B	A	B
Germany 1878-1913	5.20	3.36	−1.54	−2.19	[b]	[b]	−16.5	−20.4	1.71	0.71
Sweden 1884-1913	[b]	[b]	[b]	[b]	[b]	[b]	− 3.7	− 5.2	3.75	5.34
United States 1920-1935	0.31	1.23	−1.90	−0.90	−0.02	−0.12	−33.2	−25.1	0.10	0.06
Utd. Kingdom 1923-1935	10.09	5.49	−0.95	−3.36	−0.56	−0.47	[c]	[c]	2.06	1.71
Sweden 1924-1936	[b]	[b]	[b]	[b]	−0.24	−0.06	[c]	[c]	2.22	1.66

[a] For the sake of clearness, the description of series, the indication of lags, and all footnotes have been omitted.
[b] Cases including wrong signs have not been included.
[c] Series not included.

Significance calculations.

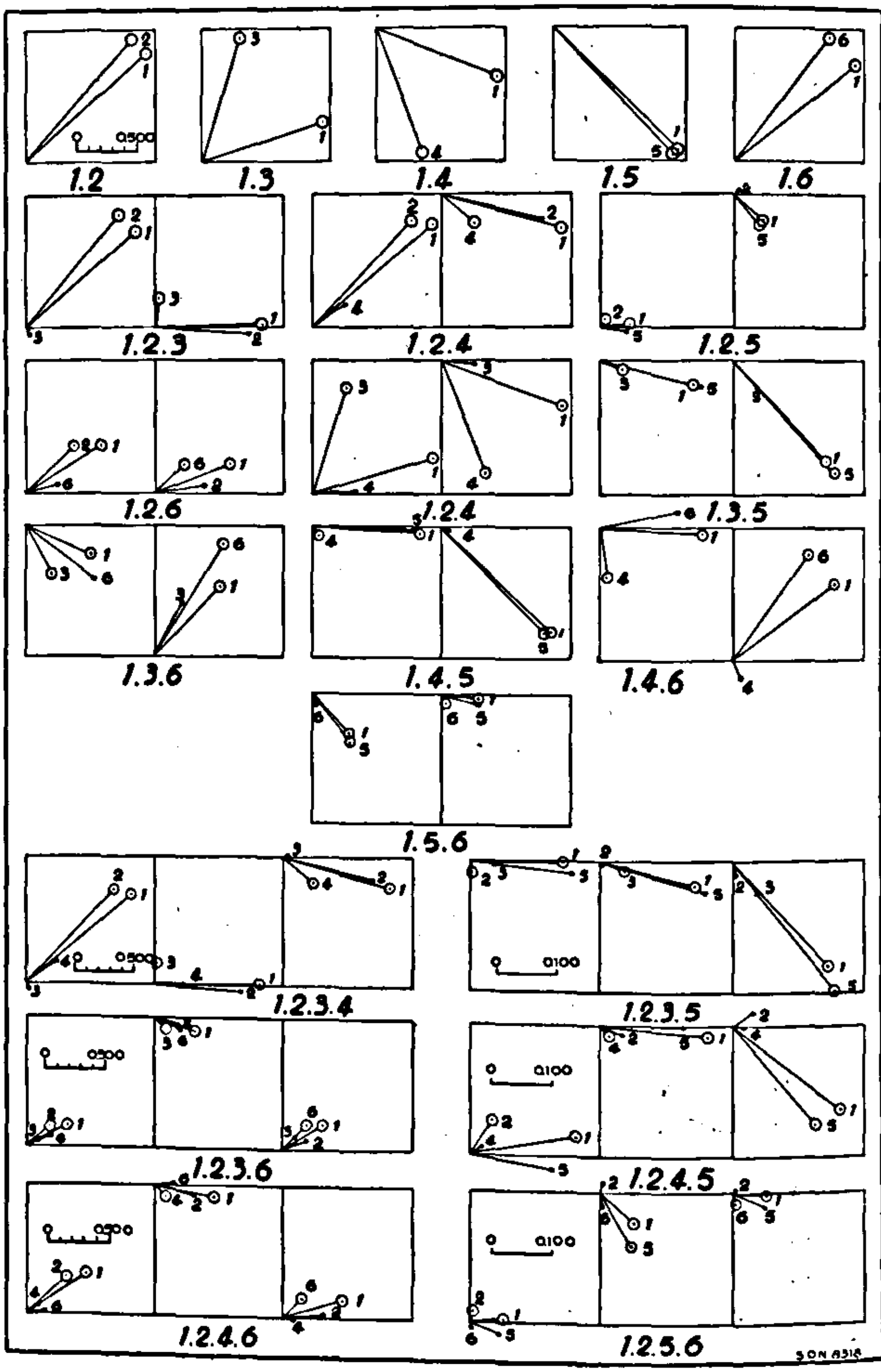
For the United States, three bunch maps have been drawn, two representing *a priori* calculations and the third the free calculation for 1920-1935.

As regards the first *a priori* calculation (Graph IV. 8, contracts awarded, 1915-1935), the final set (1234) is very satisfactory as to variate 2 (profitability). It is somewhat less satisfactory as to variates 3 (number of houses) and 4 (profits), the coefficients of which are not to be determined with the same degree of exactness, the angles between the beams being larger.

Bunch Map. *Graph*

BUILDING: UNITED STATES 1920-1935.

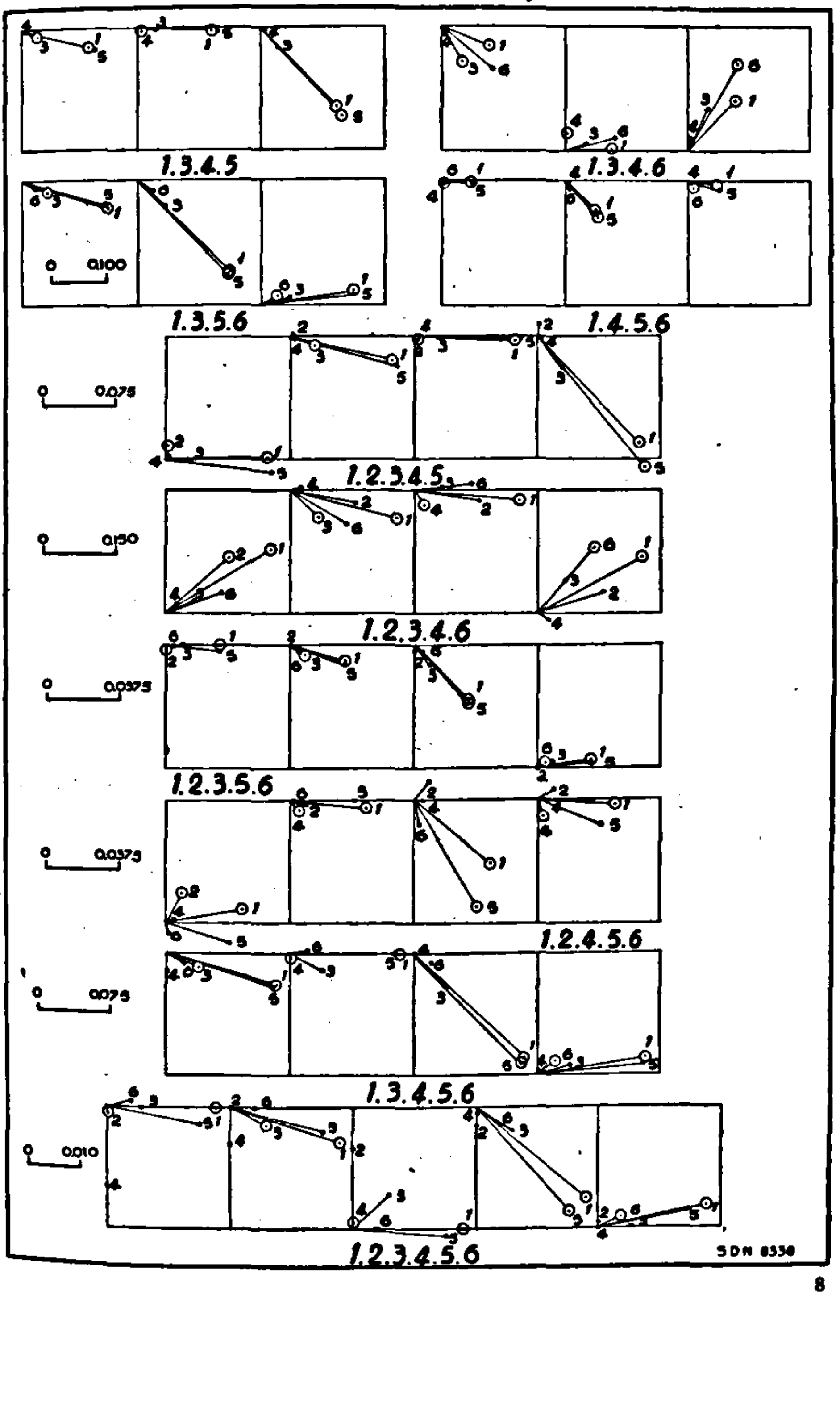
1 = Total non-farm residential construction. 2 = Rent. 3 = Construction



'V. 10. Bunch Map.

BUILDING: UNITED STATES 1920-1935.

osts.' 4 = Interest rate. 5 = Number of houses $_{-3^1/_2}$. 6 = Profits.



Taking account of the fact that, in this case, variates 2 and 4 appear to be the more important ones, one may, however, disregard beam 3 in the middle and right-hand parts of set 1234; the regression coefficients for 3 and 4 then become more certain, though that of 3 is small (small inclination of beams).

For the other *a priori* calculation (total non-farm residential construction, 1920-1935), the bunch map (Graph IV. 9) is less good; here the regression coefficient of 3 (the number of houses) is found to be both important and well determined if the two other variates, the coefficients of which are found to be small, are disregarded.

The bunch map (Graph IV. 10) for the free calculation over this period shows, in its final set (123456), a tendency to explosion, but nevertheless the variates 3 (construction cost), 5 (number of houses) and 6 (profits) are found to have well defined and important influences, especially 5. From the "best" sub-sets ("best" from the point of view of determinateness of regression coefficients)—viz., (135), (145), (1345), (12345)—similar conclusions are to be drawn.

CHAPTER V

NET INVESTMENT IN RAILWAY ROLLING-STOCK

§ 15. The Relation tested

Acceleration principle and profit principle.

In § 9, the result was obtained that, in the case of general investment activity, the acceleration principle yields an explanatory factor of only minor importance as compared with profits. For railways, it is possible to take a slightly different view. The two facts, (*a*) that railways usually are not permitted to refuse passengers or freight offered for transport and (*b*) that, generally, they are public enterprises or under some sort of control of public authority, both tend to replace pure profit considerations by more technical considerations as far as new investment is concerned. There is some reason to assume that profit considerations are in this case wholly or partially replaced by the considerations at the basis of the acceleration principle. For this reason, three types of calculation have been made. An attempt has been made to explain the net investment in railway rolling-stock v_R by the following primary factors:

(1) The rate of increase in traffic Δu_R only (" acceleration principle ");
(2) The profit rate Z_R only (" profit principle ");
(3) Both Δu_R and Z_R (" mixed principle ").

Secondary factors; lags.

As secondary factors the same factors have been chosen as in Chapter III, viz.,

The price of iron q_i;

The long-term rate of interest m_{Lb}.

About the probable lag, some information is available in the lags between orders of locomotives and of cars and the rate of increase in total stock of locomotives and of cars with the American railways. These data show a lag of about 1 year for cars and of about 1½ years for locomotives. As the lag between any incentive to invest and the actual increase in rolling-stock may be larger than the purely technical lag between orders and increase, it seemed a fair estimate to take 1½ years for all rolling-stock. To begin with, calculations with this lag were made. Inspection of the graphs showed that the lag seemed to be somewhat shorter for the United States, especially in the case of the profit principle; perhaps somewhat longer for France, and decidedly longer (2½ years) for Germany, if for these two countries the acceleration principle was accepted as the explanatory principle. Therefore, a lag of 2½ years for Germany has been taken, whereas for the other countries the lag of 1½ years was retained, with the exception of the profit principle for the United States, where a lag of 1 year was also considered. These lags may roughly be considered as the lags giving the highest correlation.

For the profit principle, somewhat more complicated calculations (indicated as calculations 2′) were made in addition: viz., calculations in which profits with two different lags are introduced as variates. This may give somewhat more accurate indications about lags, which will be discussed together with the results.

Significance calculations. Significance calculations have been made only for some of the most typical cases. As railway rolling-stock plays a decreasing rôle in total investment, it did not seem necessary for the ultimate objects of this enquiry to go into very much detail, the more so because the results were only moderately good.

§ 16. The Statistical Material

Countries and periods. The countries and periods studied are:

France, 1876-1908 (thirty-three years).
Germany, 1874-1908 (thirty-five years).
United Kingdom ,1873-1911 (thirty-nine years)
United States, 1896-1913 (eighteen years).

All necessary data on railways are taken from the Statistical

Year-books of these countries. For the secondary factors, the data referred to in Chapter III are taken.

Computation of indices.

Some preliminary work was involved in calculating the necessary indices.

Investment index.

An index v_R of net investment was calculated as a weighted arithmetic average of the percentage rates of increase in locomotives, freight cars and passenger cars. As weights, there were taken the products of the number of each type of rolling-stock present at the end of 1895 (for the United States 1905) by a weight factor which was taken as

20 for locomotives,
10 for passenger cars, and
1 for freight cars.

For the United Kingdom, where no separate data for both types of car were available, one weight factor 2 was used for all cars. The influence of the weights on the shape of the investment index is not large, as the rate of increase in locomotives and cars is usually highly correlated.

Profit series.

As profit series (Z_R), the following have been used:

United States: " Net operating income " as a percentage of " investment " (*i.e.*, capital invested);

United Kingdom: Ratio of net receipts to paid-up capital;

Germany: Profits as percentage of invested capital;

France: Net income per kilometre divided by cost of construction of one km.

Rate of increase in traffic.

An index for the rate of increase in traffic was calculated as a weighted arithmetic average of the percentage rates of increase in passenger traffic and freight traffic. The weights chosen are numbers roughly proportional to the total receipts for passenger traffic and freight traffic at about the middle of the period studied. They are indicated in the table below, together with the exact description of the traffic series used.

Traffic series and weights used.

Country	Passenger traffic		Freight traffic	
	Series used	Weight	Series used	Weight
United States	Passengers carried 1 mile	1	Freight carried 1 mile	3
United Kingdom	Total ordinary passenger journeys	4	Total tonnage of goods conveyed	5
Germany	Passengers carried 1 kilometre	1	Freight carried 1 kilometre (tons)	2
France	Passengers carried 1 kilometre	4	Freight carried 1 kilometre (tons)	5

For pig-iron prices and long-term interest rates, the same series have been used as described in Chapter III.

Trends. In order to eliminate trends, deviations from nine-year moving averages have been taken for all series except iron prices, where percentage deviations from nine-year moving averages were taken.

§ 17. Results

Details of the results obtained are presented in Tables V. 1 to V. 3 and Graphs V. 1 to V. 4. The following general features seem worth mentioning:

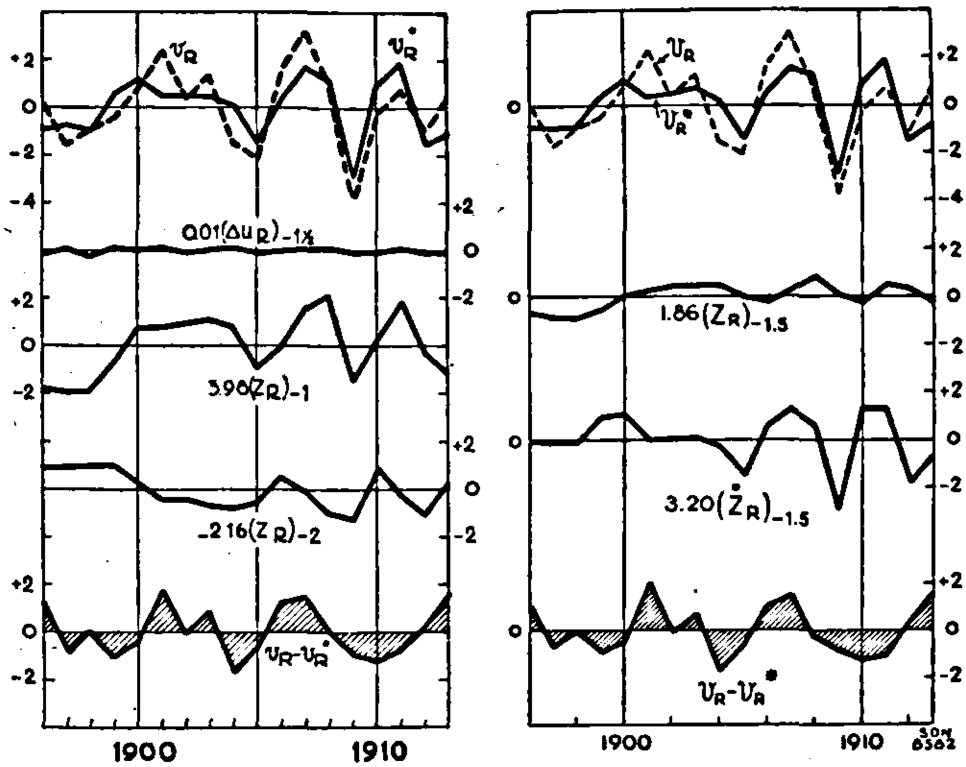
Results not better than for general investment activity. (i) Looking at the correlation coefficients obtained, one finds that the results are not, as might have been expected, better than those obtained for general investment activity. It therefore seems that the advantage of having more homogeneous material is counteracted by the larger influence of disturbances in a more restricted field of activity.

Lags. (ii) As has been said already, the lags chosen in the case of the acceleration principle are roughly those which give the best fit. They are 1½ years for the United States, the United Kingdom and France, and 2½ years for Germany. For the profit principle, these lags were tested by the calculations summarised in columns (7) to (9), Table V. 1. In the case of the United Kingdom and France, the regression coefficients obtained for profits with $2\frac{1}{2}$ years lag are small in comparison to those obtained for profits with $1\frac{1}{2}$ years lag. This means that the optimum lags are near to $1\frac{1}{2}$ years—somewhat more in France,

Graph V. 1.

"EXPLANATION" OF INVESTMENT IN RAILWAY ROLLING-STOCK. UNITED STATES 1896-1913.

Left-hand side: "Mixed principle" — Right-hand side: "Profit principle".



v_R :	Investment in railway rolling-stock, actual.
v_R^* :	Investment in railway rolling-stock, as explained by:
$(\Delta u_R)_{-1\frac{1}{2}}$:	increase in traffic, lagged $1\frac{1}{2}$ years;
$(Z_R)_{-1}$:	profits, lagged 1 year;
$(Z_R)_{-2}$:	profits, lagged 2 years;
$(Z_R)_{-1\frac{1}{2}}$:	profits } lagged $1\frac{1}{2}$ years.
$(\dot{Z}_R)_{-1\frac{1}{2}}$:	rate of increase in profits } lagged $1\frac{1}{2}$ years.

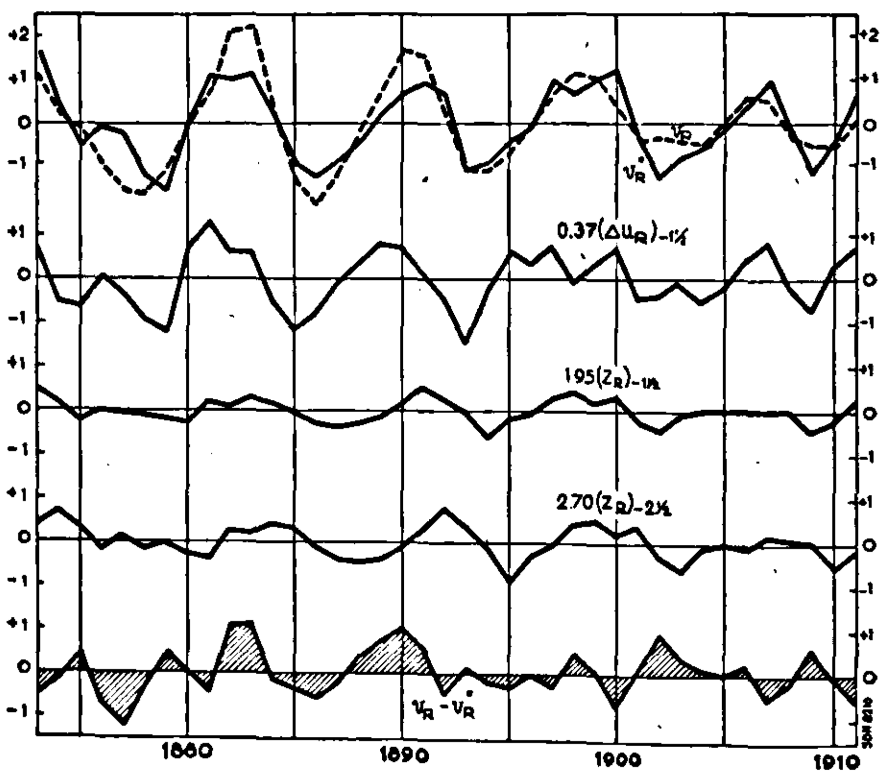
somewhat less in the United Kingdom. For Germany, both coefficients are equally important, pointing to an optimum lag of 2 years.

The regression coefficients obtained in the case of the United States indicate that a considerably smaller lag than even 1 year would be the optimum lag if profits were to be the only explanatory variate. This is, however, inacceptable, as delivery of rolling-

Graph V. 2.

" EXPLANATION " OF INVESTMENT IN RAILWAY ROLLING-STOCK.
UNITED KINGDOM 1873-1911.

" Mixed principle ".



v_R :	Investment in railway rolling-stock, actual.
v_R^* :	Investment in railway rolling-stock, as explained by:
$(\Delta u_R)_{-1\frac{1}{2}}$:	rate of increase in traffic, lagged $1\frac{1}{2}$ years;
$(Z_R)_{-1\frac{1}{2}}$:	profit rate, lagged $1\frac{1}{2}$ years;
$(Z_R)_{-2\frac{1}{2}}$:	profit rate, lagged $2\frac{1}{2}$ years.

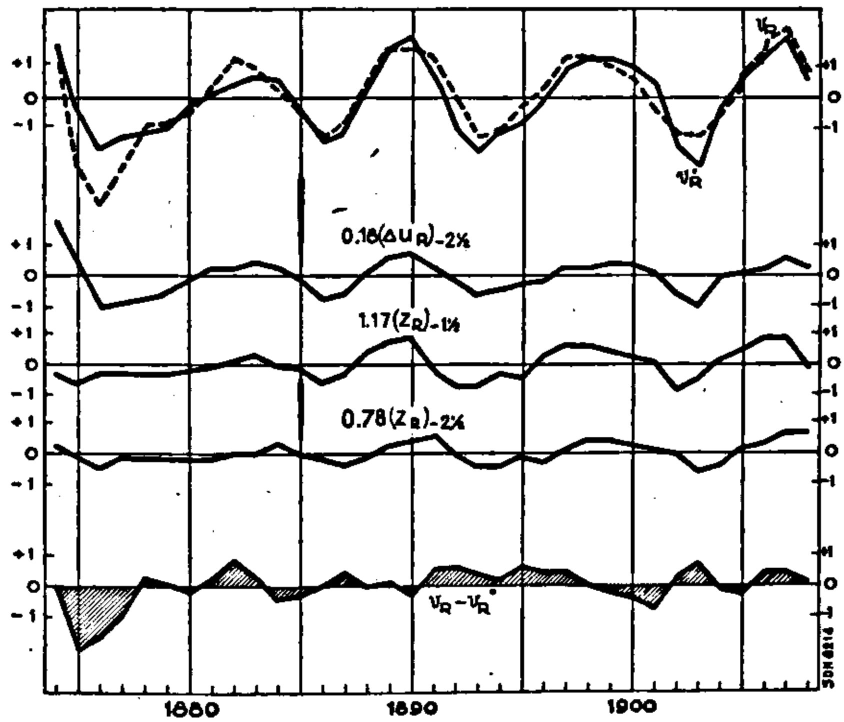
stock requires at least one year (see above). The profit principle in its simplest form—viz., that the amount of profits determines the volume of investment—is therefore inapplicable here; the regression equation yielded by this calculation could, however, be written in the form:

$$v_R = 0.93\,[(Z_R)_{-1} + (Z_R)_{-2}] + 3.20\,[(Z_R)_{-1} - (Z_R)_{-2}]$$

Graph V. 3.

"EXPLANATION" OF INVESTMENT IN RAILWAY ROLLING-STOCK. GERMANY 1874-1908.

"Mixed principle".



v_R : investment in railway rolling-stock, actual.

v_R^* : investment in railway rolling-stock, as explained by:

$(\Delta u_R)_{-2\frac{1}{2}}$: rate of increase in traffic, lagged $1\frac{1}{2}$ years;

$(Z_R)_{-1\frac{1}{2}}$: profit rate, lagged $1\frac{1}{2}$ years;

$(Z_R)_{-2\frac{1}{2}}$: profit rate, lagged $2\frac{1}{2}$ years.

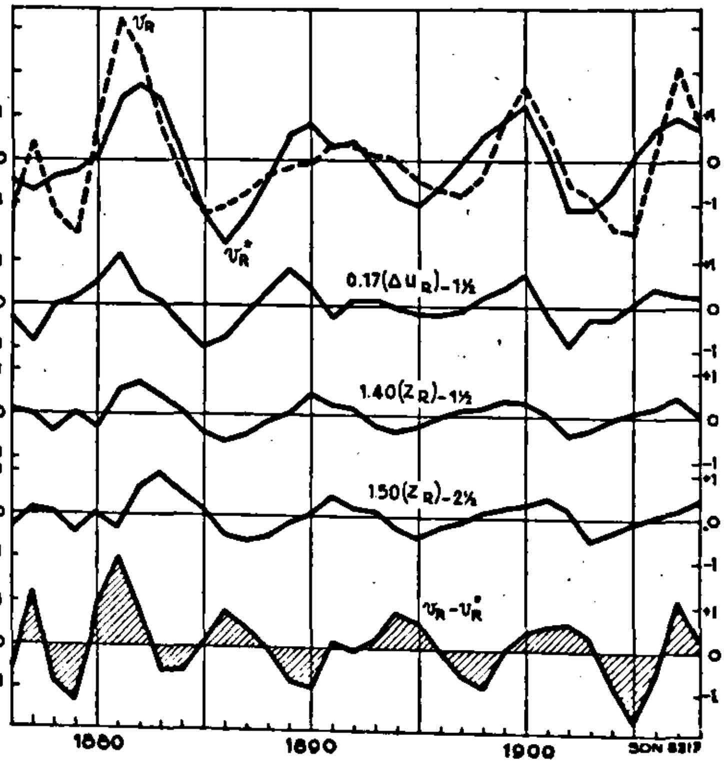
where the first expression in brackets is very near to twice profits with a lag of 1½ years and the second expression in brackets is the rate of increase in profits with a lag of 1½ years. Thus the rate of increase of profits, as well as profits themselves, is represented as exercising an influence on investment. Briefly, and very approximately, we get

$$v_R = 1.86\,(Z_R)_{-1.5} + 3.20\,(\dot{Z}_R)_{-1.5}\ (cf.\ \text{Graph V. 1}).$$

Graph V. 4.

" EXPLANATION " OF INVESTMENT IN RAILWAY ROLLING-STOCK.
FRANCE 1876-1908.

" Mixed principle ".



v_R : investment in railway rolling-stock, actual.
v_R^* : investment in railway rolling-stock, as explained by:
$(\Delta u_R)_{-1½}$: rate of increase in traffic, lagged 1½ years;
$(Z_R)_{-1½}$: profit rate, lagged 1½ years;
$(Z_R)_{-2½}$: profit rate, lagged 2½ years.

Acceleration principle and profit principle.

(iii) The correlation coefficients obtained with the calculations (1) and (2) mentioned above (§ 15) are not, on the average, very different (Table V. 1, columns (3) and (5)). So far as the differences are significant, it is remarkable that the acceleration principle gives a lower correlation than the profit principle for the United States and France, and about the same correlation as the profit principle for Germany and England.

Mixed principle.

Calculations (3) (Table V. 1, columns (10) to (13)), using both principles, show practically no influence of the rate of increase in traffic in the case of the United States; and the regression coefficients for profits are quite near to those found in columns (8) and (9).

To sum up, for the United Kingdom the correlation is considerably improved if the principles are combined; for France and Germany there is also some improvement, whereas for the United States the improvement is almost nil.

Calculations using secondary factors.

Calculations including "secondary factors" (*cf.* Tables V. 2 and V. 3) show considerable improvements in correlation if based upon the acceleration principle, and less improvement if based on the profit principle. The results obtained with the acceleration principle in table V. 2, with the exception of those for the United States, become somewhat better than those obtained with the profit principle, notwithstanding that the number of variates included is one less.

Regression coefficient for acceleration principle.

The regression coefficient obtained in case (1)— whether or not secondary factors are included makes no difference—is far lower than the acceleration principle in its simplest form [1] would suggest. In fact, it is often suggested that a given percentage increase in traffic would lead to an equal percentage increase in rolling-stock. Instead of unity, the coefficient found in Table V. 1, column (4), is, however, only one-sixth to one-third, or if the ratio between the standard deviations is taken, about

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[1] As given by Haberler: *Prosperity and Depression*, pages 84 and 85.

Table V. 1. " Explanation " of Investment

Units: Investment: Percentage increase in rolling-stock, deviations from 9 years moving average.
Δ Traffic: Percentage increase in traffic, deviations from 9 years moving average.
Profits: Percentage profits,[1] deviations from 9 years moving average.

Country	Period	(1) Acceleration principle		(2) Profit principle	
		Correlation coefficient	Regression coefficient and lag of Δ traffic	Correlation coefficient	Regression coefficient and lag of profits
(1)	(2)	(3)	(4)	(5)	(6)
United States	1896-1913	*0.54*	0.15 (1½)	*0.63*	3.20 (1)
United Kingdom	1873-1911	*0.63*	0.34 (1½)	*0.66*	4.80 (1½)
Germany	1874-1908	*0.79*	0.34 (2½)	*0.74*	2.41 (2½)
France	1876-1908	*0.57*	0.24 (1½)	*0.67*	3.42 (1½)

[1] Or the best approximation to it available.

Table V. 2. " Explanation " of Investment

Introduction of iron prices and long-term interest rates as

Country	Period	1. Acceleration principle			
		Correlation coefficient	Regression coefficients and lags of:		
			traffic	iron price	interest rate
(1)	(2)	(3)	(4)	(5)	(6)
United States	1896-1913	*0.78*	0.04 (1½)	—0.04 (1½)	—0.09 (1½)
United Kingdom	1873-1911	*0.75*	0.27 (1½)	0.02 (1½)	—0.04 (1½)
Germany	1874-1908	*0.88*	0.40 (2½)	—0.04 (2½)	0.01 (2½)
France	1876-1908	*0.83*	0.19 (1½)	0.06 (1½)	—0.05 (1½)

in Railway Rolling-stock.

Interest rates: deviations from 9 years moving average, in 0.01 %.
Iron prices: percentage deviations from 9 years moving average.
Lags: years.

(2') Profit principle (distr. lag)			(3) Mixed principle			
Correlation coefficient	Regression coefficients and lags of profits		Correlation	Regression coefficients and lags of:		
				Δ traffic	profits	
(7)	(8)	(9)	(10)	(11)	(12)	(13)
0.77	4.13 (1)	— 2.27 (2)	*0.77*	0.01 (1½)	3.98 (1)	— 2.16 (2)
0.66	5.10 (1½)	— 0.55 (2½)	*0.84*	0.37 (1½)	1.95 (1½)	2.70 (2½)
0.83	1.52 (1½)	1.53 (2½)	*0.88*	0.18 (2½)	1.17 (1½)	0.78 (2½)
0.68	3.00 (1½)	0.71 (2½)	*0.75*	0.17 (1½)	1.40 (1½)	1.50 (2½)

in Railway Rolling-stock (continued).

supplementary explanatory factors. Units: see Table V. 1.

2'. Profit principle				
Correlation coefficient	Regression coefficients and lags of:			
	profits		iron price	interest rate
(7)	(8)	(9)	(10)	(11)
0.87	2.55 (1)	— 1.80 (2)	— 0.02 (1½)	— 0.07 (1½)
0.70	3.80 (1½)	— 1.35 (2½)	0.02 (1½)	— 0.03 (1½)
0.84	2.14 (1½)	1.58 (2½)	0.01 (2½)	0.04 (2½)
0.86	1.89 (1½)	0.81 (2½)	0.03 (1½)	— 0.01 (1½)
0.79	2.15 (1½)	— 0.35 (1½)	0.04 (1½)	— 0.05 (1½)

Table V. 3. " Explanation " of Investment
Calculations using only interest rates

Country	Period	1. Acceleration principle		
		Correlation coefficient	Regression coefficient and lags of:	
			Δ traffic	Interest rate
(1)	(2)	(3)	(4)	(5)
United States	1896-1913	*0.69*	0.09 (1½)	—0.07 (1½)
United Kingdom	1873-1911	*0.67*	0.29 (1½)	—0.03 (1½)
Germany	1874-1908	*0.79*	0.34 (2½)	0.00 (2½)
France	1876-1908	*0.69*	0.19 (1½)	—0.06 (1½)

one-half,[1] which means a considerably smaller sensitivity of investment. After the introduction of the " secondary factors " and of the mixed principle, these coefficients grow less uniform, but in general still smaller, especially in the case of the United States. Nevertheless, the more general significance of the acceleration principle—viz., that percentage fluctuations in capital goods industries are larger than percentage fluctuations in consumers' goods industries—is not invalidated by these figures. The relatively low influence of the principle may be attributed to the fact that the technical necessity for its operation in its simplest form exists only if capacity is already being fully used. In all other circumstances, changes in capacity may be less than in proportion to changes in production.[2]

Influence of iron prices.

Not very much evidence is found of any influence of iron prices in the European countries. The regression coefficients found (Table V. 2, columns (5) and (10)) are positive and in general unimportant. Only in the United States do they seem to be clearly negative; the elasticity of demand at the point of the demand curve corresponding to trend values of prices and quantities (which, by

[1] This figure is obtained by dividing column (4) by column (3), and is therefore:

U.S.A.	U.K.	Germany	France
0.28	0.54	0.43	0.42

[2] In the case of the mixed principle for the U.K. and Germany, the correlation would improve if a continuous fall in the regression coefficients were assumed to exist (*cf.* Graph V. 2 and V. 3).

n Railway Rolling-stock (continued).
as supplementary factors. Units: see Table V. 1.

2'. Profit principle			
Correlation coefficient	Regression coefficient and lags of:		
	profits		interest rate
(6)	(7)	(8)	(9)
0.85	2.68 (1)	—2.65 (2)	—0.07 (1½)
0.67	4.62 (1½)	—0.56 (2½)	—0.02 (1½)
0.84	1.76 (1½)	1.71 (2½)	0.03 (2½)
0.83	1.37 (1½)	1.59 (2½)	—0.01 (1½)
0.76	2.27 (1½)	0.87 (2½)	—0.05 (1½)

the choice of units, is indicated by 30 × the regression coefficient) would be about unity.

Influence of interest rates.

On the other hand, the influence of interest rates seems to be quite clear (Table V. 2, columns (6) and (11), and Table V. 3, columns (5) and (9)). Here, as in other cases, the United States and Germany seem to represent two extremes between which France and the United Kingdom are situated, the influence of interest rates being largest in the United States. Owing to our figures, a fall of 0.1% (being ten times the unit used) in bond yields would, in the United States, lead to an increase in rolling-stock by 0.7 to 0.9% (ten times the regression coefficient found) more than normal, whereas the corresponding figures are 0.2, 0.3 and 0.4 for the United Kingdom, 0.5 and 0.6 for France, and 0.1 to —0.4 for Germany.

The decided importance of interest rates for investment activity in the field studied may find part of its explanation in the considerable length of life of railway rolling-stock and in the large part of this investment which, in the end, is financed through the capital market in the proper sense of that word. At the same time, the fact that in Chapter III, dealing with investment in general, a larger influence of interest rates on investment activity was found for pre-war times than for post-war times may now be explained, for investment in railway rolling-stock probably plays at present a less important rôle than it did before the war.

Significance calculations.

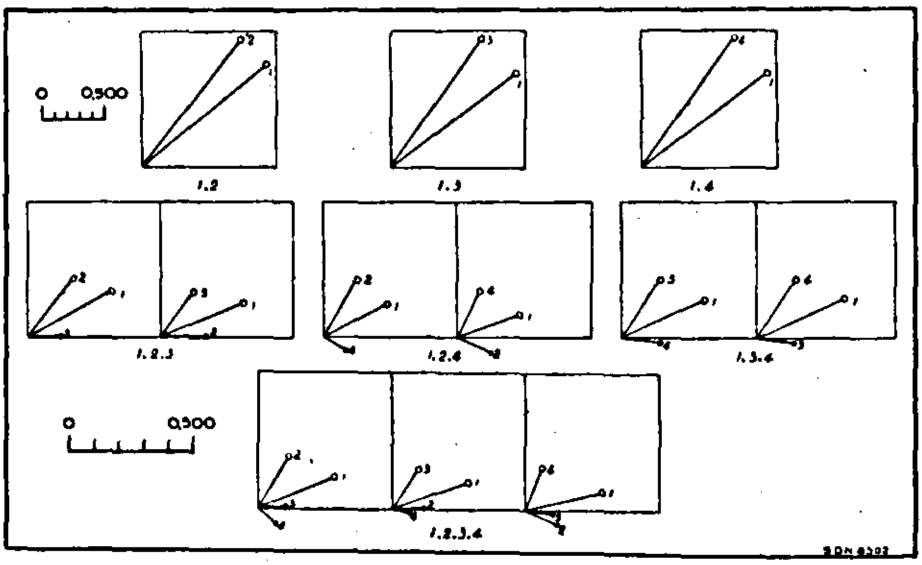
In addition to the information given in Tables V. 1 to V. 3, bunch maps have been calculated for four cases—viz., two for Germany and two for the United States—exhibiting the "mixed principle" without secondary factors and the acceleration principle with interest rates as a supplementary factor (*cf.* Graphs V. 5 to V. 8). These bunch

Graph V. 5.

Bunch Map.

RAILWAYS: GERMANY 1874-1908.

1 = Investment index. 2 = Δ traffic index$_{-2\frac{1}{2}}$. 3 = Profits$_{-1\frac{1}{2}}$. 4 = Profits$_{-2\frac{1}{2}}$.

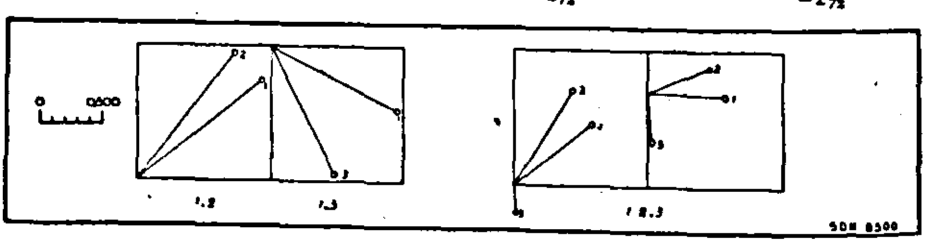


Graph V. 6.

Bunch Map.

RAILWAYS: GERMANY 1874-1908.

1 = Investment index. 2 = Δ traffic index$_{-2\frac{1}{2}}$. 3 = Interest rate$_{-2\frac{1}{2}}$.



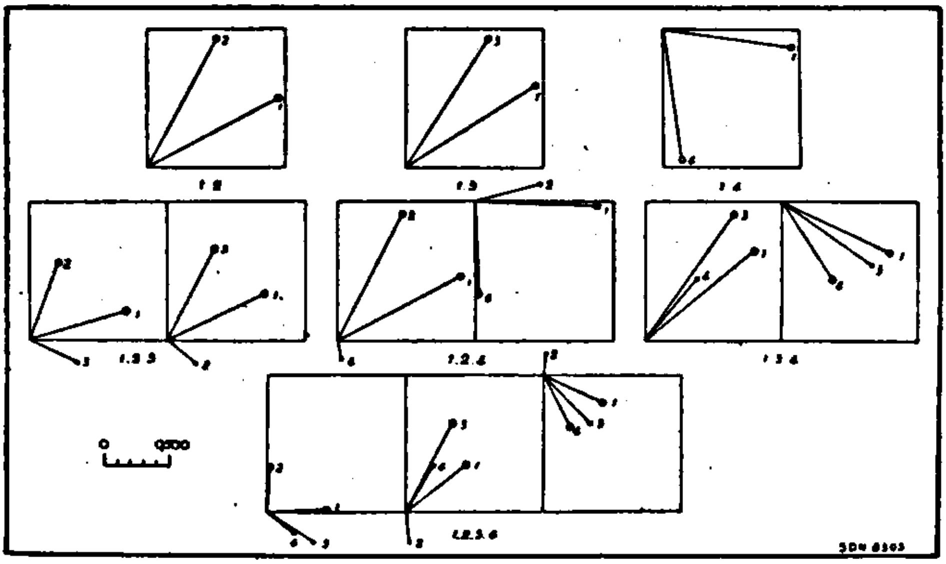
maps all seem to show that the figures obtained are very uncertain. Thus, Graph V. 6 gives a very wide spread for the beams in the right-hand part of set 123 which relates to the regression coefficient for 3 (interest rates). As 2 appears to be the most important explanatory variate in this set, beams 1 and 2 are the most important ones, which still supports our conclusion about a small

Graph V. 7.

Bunch Map.

RAILWAYS: UNITED STATES 1896-1913.

1 = Investment index. 2 = Δ traffic index_{-1}. 3 = Profits_{-1}. 4 = Profits_{-2}.

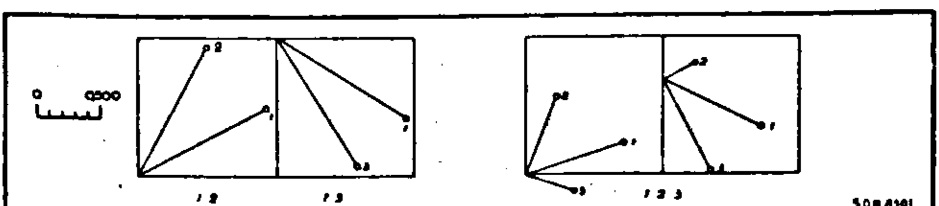


Graph V. 8.

Bunch Map.

RAILWAYS: UNITED STATES 1896-1913.

1 = Investment index. 2 = Δ traffic index_{-1}. 3 = Interest rate_{-1}.



influence of interest rates in Germany. In the case of the United States (Graph V. 8), beam 3 is, however, more important, supporting the view that a high influence of interest rates is present. Only if there are strong reasons for preferring the first elementary regression (which has been used in tables V. 1 to V. 3, as usually), can confidence be placed in the regression coefficients.

In this connection, it is of some interest that, among all the elementary regressions, only number 1 yields correct signs for all regression coefficients.

Explanation of differences between countries.

Most of the differences found to exist between the countries studied seem to point in the same direction. Investment in the United States reacts more quickly, and depends more on profits, interest rates and iron prices, and less on the purely technical acceleration principle, than it does in Europe, especially in Germany. This may be understood by realising that railways were, in the period investigated, more like free private enterprises in the United States and less so in the European countries; least of all in Germany, where already from 1878 onwards they were chiefly State enterprises.

Summary of findings.

To sum up, we have found that the correlations obtained for this branch of industry are on the average not higher than those obtained for general investment. The influence of interest rates seems to be rather high, except in Germany. The acceleration principle gives a somewhat better explanation than the profit principle, but the regression coefficients found are far below the theoretical values. Certain differences between the four countries included could be explained.

CHAPTER VI

APPLICATION OF RESULTS: FURTHER INVESTIGATIONS

The direct use of the results obtained is restricted. They indicate the relative strength of the various causes of fluctuations in investment activity discussed in Chapters III to V. In addition, something can be deduced about the chief proximate causes of turning-points in investment activity. Thus, the crisis of 1883 and the revival in 1887 in the United Kingdom may be ascribed to changes in profits (*cf.* graph III. 4), while the revival of 1875 would seem to be primarily explained by the fall in iron prices. To give another example, the proximate causes of the well-known building boom after 1933 in the United Kingdom would appear to be the fall in interest rates and building costs and the rise in real income (*cf.* graph IV. 4). In some cases, conclusions about policy may be drawn. The reduction of long-term interest rate necessary to raise the volume of investment by a given percentage may be estimated in a few of our cases with some certainty. Thus, it would seem that for the United States in the period 1919-1932 a reduction of this rate by 1% might have led, after about half a year or so, to an increase in investment activity of about 5% of the average level.

Many questions, however, still remain unanswered. This is partly due to the degree of uncertainty in a number of results found, which can be reduced only if better statistics and more precise theories are available.

But more important is the fact that, in this pamphlet—which is primarily intended to demonstrate a method—the argument ends with the influence of profits, interest rates, etc., on the volume of investment.

The economist and the statesman may be anxious to know what in turn influences profits, and how these influences have been changed, or could be changed, by policy. This problem can also be studied by means of the method described here, if that method is applied to a larger number of inter-relations between economic variates.

In addition to the equation explaining investment fluctuations, others explaining profits, prices of investment goods, interest rates and so on, will then have to be established. The total number of such equations should be equal to the number of variates necessary to describe adequately the business-cycle mechanism. The sum total of these relations may be called a complete system. Such a complete system is required to draw conclusions of the nature indicated above.

A first attempt in this direction, covering the United States after the war, will be published shortly as the second volume of this series. An explanation will there be given of profits as the difference between (*i*) total receipts of all enterprises, public authorities, etc., included, and (*ii*) total costs.

Taking the country as a whole, and regarding it as a " closed economy ", all costs that consist in payments from one enterprise to another cancel out. Total receipts may thus be taken as the total value of consumers' goods and services produced (U), plus the total value of investment goods and services sold by their producers (V); the cost items to be deducted are:

Wages L_w and salaries L_s;
Corporation managers' salaries L_c;
Rent payments K_r;
Interest payments K_i;
Depreciation allowances N.

Calling the amount of profits Z, we therefore get:

$$Z = U + V - (L_w + L_s + L_c + K_r + K_i + N),$$

and we may test this equation from the facts. On the basis of this relation, an observed fall in profits may now be found to be due to, *e.g.*, a fall in U, total consumption. The causation of the fall in this variate must then, in turn, be investigated, and so on.

It follows that full use of any relation can only be made after a complete system has been established. Some of the most important applications of the results of the present study can therefore only be stated when the work for the next volume has been completed.

APPENDIX A

DETAILS OF CALCULATIONS

1. *Trend Calculation.*

For pre-war periods, nine-year moving averages have been taken. In a few cases where seven-year moving averages were already available, the latter were used, as the differences were small. For post-war periods, which are shorter, rectilinear trends have been calculated. For short periods, the arbitrariness of this type of trend is less than for long periods, as the difference between a rectilinear trend and a moving average is in this case generally small. In addition, the rectilinear trend has the great advantage that no years need be left out of account, whereas, in the case of nine-year moving averages, four years are lost at each end of the series.

Let any series be represented by X_t, where t indicates the year (or any other unit period used) and assumes all values from 1 to N, N being the total number of years in the period considered. Let its rectilinear trend be $X_t^{(T)}$. The latter is then of the form

$$X_t^{(T)} = a + bt \qquad (1)$$

where a and b are constants, which should be chosen so as to obtain the " best " fit between X_t and $X_t^{(T)}$. This is usually done by applying the method of least squares, which prescribes that the expression

$$(X_1 - X_1^{(T)})^2 + (X_2 - X_2^{(T)})^2 + (X_3 - X_3^{(T)})^2 + \dots$$
$$\dots + (X_N - X_N^{(T)})^2 \quad \text{or} \quad \sum_1^N t (X_t - X_t^{(T)})^2 \qquad (2)$$

be made as small as possible by a suitable choice of a and b. This problem can be solved most simply if t, X_t and $X_t^{(T)}$ are measured as deviations from their mean values. If the mean values be indicated by $\bar{t}$, $\overline{\mathrm{X}}$ and $\overline{\mathrm{X}}^{(\mathrm{T})}$ respectively, and the deviations by t', x_t and $x_t^{(\mathrm{T})}$, we have:

$$\bar{t} = \frac{1}{\mathrm{N}} \sum_1^{\mathrm{N}} t\, t\,; \quad \overline{\mathrm{X}} = \frac{1}{\mathrm{N}} \sum_1^{\mathrm{N}} t\, \mathrm{X}_t\,; \quad \overline{\mathrm{X}}^{(\mathrm{T})} = \frac{1}{\mathrm{N}} \sum_1^{\mathrm{N}} t\, \mathrm{X}_t^{(\mathrm{T})} \qquad (3)$$

and

$$t' = t - \bar{t}\,; \quad x_t = \mathrm{X}_t - \overline{\mathrm{X}}\,; \quad x_t^{(\mathrm{T})} = \mathrm{X}_t^{(\mathrm{T})} - \overline{\mathrm{X}}^{(\mathrm{T})} \qquad (4).$$

It follows immediately that:

$$\Sigma t' = 0\,; \quad \Sigma x_t = 0\,; \quad \Sigma x_t^{(\mathrm{T})} = 0 \qquad (5).$$

Assuming now that the trend equation takes the form

$$x_t^{(\mathrm{T})} = a' + b' t' \qquad (1')$$

the method of least squares requires by analogy with (2):

$$\sum_1^{\mathrm{N}} t\, (x_t - x_t^{(\mathrm{T})})^2 \quad \text{minimum} \qquad (2').$$

Replacing $x_t^{(\mathrm{T})}$ by its value (1′), this becomes:

$$\sum_1^{\mathrm{N}} t\, (x_t - a' - b' t')^2 \quad \text{minimum.}$$

This expression depends on a' and b', since all other variates are given numbers drawn from observation. It is therefore a function $\mathrm{F}(a', b')$ of a' and b', and its minimum value must, according to a well-known statement in differential calculus, obey the two relations:

$$\frac{\partial \mathrm{F}(a', b')}{\partial a'} = 0 \qquad \frac{\partial \mathrm{F}(a', b')}{\partial b'} = 0 \text{ [1]} \qquad (6).$$

---

[1] It can be proved that these equations yield, in this case, a minimum and not a maximum value of F.

These relations come to: [1]

$$\Sigma x_t - \mathrm{N}a' - b'\Sigma t' = 0 \qquad\qquad \Sigma x_t t' - a'\Sigma t' - b'\Sigma t'^{2} = 0.$$

Applying relation (5), they are much simplified and become:

$$a' = 0 \qquad \Sigma x_t t' = b'\Sigma t'^2 \quad \text{or} \quad b' = \frac{\Sigma x_t t'}{\Sigma t'^2} \qquad (7).$$

The trend, if measured in deviations from its mean value, is therefore of the simple shape:

$$x_t^{(\mathrm{T})} = t'\frac{\Sigma x_t t'}{\Sigma t'^2} \qquad (8).$$

To obtain its actual level, the easiest way is to introduce t' into equation (1), as follows:

$$\mathrm{X}_t^{(\mathrm{T})} = a + b(t' + \overline{t}) = a + b\overline{t} + bt' \qquad (9).$$

As the average of $bt' = \frac{1}{\mathrm{N}}\Sigma bt' = \frac{b}{\mathrm{N}}\Sigma t' = 0$, it follows that $a + b\overline{t} = \overline{\mathrm{X}}^{(\mathrm{T})}$.

On the other hand, it may be deduced directly from (2), by applying the principle of least squares, that $\overline{\mathrm{X}}^{(\mathrm{T})} = \overline{\mathrm{X}}$ (this, in fact, is self-evident).

Finally, from (4) and (8), it will be seen that $b = b'$: hence the trend equation in natural units will be:

$$\mathrm{X}_t^{(\mathrm{T})} = \overline{\mathrm{X}} + t'\frac{\Sigma x_t t'}{\Sigma t'^2} \qquad (10).$$

2. *Multiple Correlation; Calculation of Regression and Correlation Coefficients.*

The technique used in this case is much the same as that used in the trend calculations just described. The problem is to find coefficients b_1, b_{12}, b_{13}, etc., which give the best fit between any

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[1] For simplicity, the suffixes to the sign Σ will in future be omitted.

given series X_{1t} and a linear function X'_{1t} of a number of " explanatory " series X_{2t}, X_{3t}, etc.[1]

$$X'_{1t} = b_1 + b_{12} X_{2t} + b_{13} X_{3t} + b_{14} X_{4t} \dots \qquad (11).$$

All series may again be measured in deviations from their averages. (The suffix t will in future be omitted in the formulæ, as no ambiguities can arise.) The deviations will be denoted by x_1, x'_1, x_2, etc., and, as before, it follows from their definitions that

$$\Sigma x_1 = \Sigma x'_1 = \Sigma x_2 = \Sigma x_3 = \dots \Sigma x_n = 0 \qquad (12).$$

By this choice, b_1 will become zero.

The problem is to find for b_{12}, b_{13} etc., values which make the expression

$$\Sigma x^2_{1.23\dots n} = \Sigma (x_1 - x'_1)^2 = \Sigma (x_1 - b_{12} x_2 - b_{13} x_3 \dots)^2 \quad (13)$$

a minimum. The quantities $x_{1.23\dots n}$ will be called the residuals.

They have to fulfil the following conditions known as " normal equations " and perfectly analogous to equations (6):

$$\left.\begin{array}{lll} \Sigma x_2 (x_1 - b_{12} x_2 - b_{13} x_3 \dots) = 0 & \text{or} & \Sigma x_2 x_{1.2\dots n} = 0 \\ \Sigma x_3 (x_1 - b_{12} x_2 - b_{13} x_3 \dots) = 0 & \text{or} & \Sigma x_3 x_{1.2\dots n} = 0 \\ \dots\dots\dots & & \end{array}\right\} \quad (14)$$

These may be written:

$$\left.\begin{array}{l} b_{12} \Sigma x_2^2 + b_{13} \Sigma x_2 x_3 + \dots + b_{1n} \Sigma x_2 x_n = \Sigma x_1 x_2 \\ b_{12} \Sigma x_2 x_3 + b_{13} \Sigma x_3^2 + \dots + b_{1n} \Sigma x_3 x_n = \Sigma x_1 x_3 \\ \dots\dots\dots\dots\dots\dots \\ b_{12} \Sigma x_2 x_n + b_{13} \Sigma x_3 x_n + \dots + b_{1n} \Sigma x_n^2 = \Sigma x_1 x_n \end{array}\right\} \quad (15)$$

where $n - 1$ is the number of "explanatory series". Equations (15) are usually the most convenient to use in numerical calculations.

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[1] In Chapters III-V, where only the first regression has been considered, X' and x' have been replaced by X^* and x^*.

All the "sum-expressions" Σx_2^2, $\Sigma x_2 x_3$, etc.... are known from observation.

A general definition of the correlation coefficient between two series is given by the formula:

$$r_{12} = \frac{\Sigma x_1 x_2}{\sqrt{\Sigma x_1^2 \, \Sigma x_2^2}} \qquad (16).$$

This definition may also be applied to the series x and x'_1, and then provides the "total correlation coefficient" $R_{1.23\ldots n}$. The result can be put in the form:

$$R_{1.23\ldots n} = \sqrt{\frac{b_{12}\Sigma x_1 x_2 + b_{13}\Sigma x_1 x_3 + \ldots + b_{1n}\Sigma x_1 x_n}{\Sigma x_1^2}}$$

which is convenient for numerical calculations.

If all series are measured in so-called normalised units—*i.e.*, in such units that their standard deviations become 1, then the normal equations may be given the form:

$$\left.\begin{array}{l} b_{12} + b_{13}r_{23} + \ldots + b_{1n}r_{2n} = r_{12} \\ b_{12}r_{23} + b_{13} + \ldots + b_{1n}r_{3n} = r_{13} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ b_{12}r_{2n} + b_{13}r_{3n} + \ldots + b_{1n} = r_{1n} \end{array}\right\} \qquad (17)$$

from which it will be seen that the regression coefficients in normalised units depend only on the system of correlation coefficients between all variates.

The coefficients b_{12}, b_{13}, etc., to be found from the normal equations (15) are the regression coefficients for the first elementary regression. If, for example, the second elementary regression is to be determined, the rôle of suffixes 1 and 2 should be interchanged and the coefficients in that regression —

$$x'_2 = b_{21}x_1 + b_{23}x_3 + b_{24}x_4 + \ldots b_{2n}x_n \qquad (18)$$

— have to be calculated from:

$$\left.\begin{array}{l} b_{21}\Sigma x_1^2 + b_{23}\Sigma x_1 x_3 + \dots b_{2n}\Sigma x_1 x_n = \Sigma x_2 x_1 \\ b_{21}\Sigma x_1 x_3 + b_{23}\Sigma x_3^2 + \dots b_{2n}\Sigma x_3 x_n = \Sigma x_2 x_3 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ b_{21}\Sigma x_1 x_n + b_{23}\Sigma x_3 x_n + \dots b_{2n}\Sigma x_n^2 = \Sigma x_2 x_n \end{array}\right\} \quad (19).$$

Equation (18) should be transformed into:

$$x_1'' = \frac{1}{b_{21}}(x_2 - b_{23}x_3 - b_{24}x_4 - \dots - b_{2n}x_n) \qquad (20)$$

by writing x_2 instead of x_2', and x_1'' instead of x_1 if it is to be used as a second estimate for the "explanatory equation" for x_1 (Frisch's method).

The regression used in this publication is the first elementary regression; for the most important cases, however, bunch maps (*cf.* 3 below) have been added.

3. *Construction of Bunch Maps.*

Here the technique outlined by Professor Frisch, who proposed the method, in his "Statistical Confluence Analysis by Means of Complete Regression Systems"[1] has been almost exactly followed. As has been pointed out in § 2, the regression coefficients in normalised units can be determined from the system of all correlation coefficients between the variates considered. Starting with these correlation coefficients, it is conceivable that every possible regression formula is calculated in the way indicated *sub* 2. Frisch's method is, however, far more efficient, as any repetition of operations is avoided. The general idea is that all symmetrical minors

[1] Publication No. 5 of the Economic Institute of the University of Oslo (Universitetets Økonomiske Institutt), Oslo, 1934.

of the determinant of the correlation coefficients:

$$\Delta_{12\ldots n} = \begin{vmatrix} r_{11} r_{12} \cdots r_{1n} \\ r_{21} r_{22} \cdots r_{2n} \\ \vdots \\ r_{n1} r_{n2} \cdots r_{nn} \end{vmatrix} \qquad (21)$$

should be calculated, and this is done in a systematic way.

As any minor of a two-rowed determinant is itself again an element of that determinant, no calculations are needed for such cases. We begin, therefore, with the minors of three-rowed determinants. Take, as an example, the determinant

$$\Delta_{123} = \begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{vmatrix} \qquad (22).$$

The minor $\hat{r}_{11}$ (this notation being provisional only) corresponding with the element r_{11} is found from $\hat{r}_{11} = r_{22} r_{33} - r_{23} r_{32} = 1 - r_{23}^2$. Similarly, $\hat{r}_{22} = 1 - r_{13}^2$ and $\hat{r}_{33} = 1 - r_{12}^2$. The minor $\hat{r}_{12}$ corresponding with r_{12} equals

$$- r_{21} r_{33} + r_{23} r_{31} = - r_{12} + r_{13} r_{23} \, .$$

Similarly,

$$\hat{r}_{13} = - r_{13} + r_{12} r_{23}$$
$$\hat{r}_{23} = - r_{23} + r_{12} r_{13} \, .$$

The figures for r_{ik} and for $\hat{r}_{ik}$ are written in a table showing them in the following way:

r_{ik}	1	2	3
1	r_{11}	r_{12}	r_{13}
2		r_{22}	r_{23}
3			r_{33}

$\hat{r}_{ik}$	1	2	3
1	$\hat{r}_{11}$	$\hat{r}_{12}$	$\hat{r}_{13}$
2	$\hat{r}_{12}$	$\hat{r}_{22}$	$\hat{r}_{23}$
3	$\hat{r}_{13}$	$\hat{r}_{23}$	$\hat{r}_{33}$

Because of the symmetry, the figures below the diagonal may be left out, as has been done in the left-hand table. The figures $\hat{r}_{ik}$ can now be easily checked, as there are three ways of calculating the value of Δ_{123}:

$$\begin{aligned}\Delta_{123} &= r_{11}\hat{r}_{11} + r_{12}\hat{r}_{12} + r_{13}\hat{r}_{13}\\ &= r_{12}\hat{r}_{12} + r_{22}\hat{r}_{22} + r_{23}\hat{r}_{23}\\ &= r_{13}\hat{r}_{13} + r_{23}\hat{r}_{23} + r_{33}\hat{r}_{33}\,,\end{aligned}$$

a well-known identity. This means simply that corresponding figures in the two above tables have to be multiplied and the results belonging to one and the same column to be added up. The same result must then be obtained, whatever column be taken.

The desired information concerning all possible regression coefficients is now obtainable from table $\hat{r}_{ik}$. The regression coefficient of variate 2 in the "explanation" of variate 1 is equal to $-\frac{\hat{r}_{12}}{\hat{r}_{11}}$ if the direction of minimising is that of variate 1; $-\frac{\hat{r}_{22}}{\hat{r}_{12}}$ if the direction of minimising is that of variate 2; and $-\frac{\hat{r}_{23}}{\hat{r}_{13}}$ if the direction of minimising is that of variate 3. Similarly, the regression coefficients of variate 3 in the explanation of variate 1 are respectively $-\frac{\hat{r}_{13}}{\hat{r}_{11}}$, $-\frac{\hat{r}_{23}}{\hat{r}_{12}}$ or $-\frac{\hat{r}_{33}}{\hat{r}_{13}}$ in the three cases mentioned. Each group of three coefficients supplies the material for the construction of one bunch map. Taking the first group, the number $-\frac{\hat{r}_{12}}{\hat{r}_{11}}$ indicates the slope of the beam to be indicated with ⊙ 1, $-\frac{\hat{r}_{22}}{\hat{r}_{12}}$ that of beam ⊙ 2 and $-\frac{\hat{r}_{23}}{\hat{r}_{13}}$ that of beam · 3; the symbol ⊙ being used for the "leading beams"—*i.e.*, for cases where minimising has taken place either in the direction of the variate to be "explained" or in that of the "explanatory" variate.

By the same technique, all "three-sets"—*i.e.*, groups of three variates—can be analysed.

Here it must be emphasised that the symbols used above, $\hat{r}_{13}$, etc., are, stricty speaking, incomplete and should be written $\hat{r}_{13(123)}$, indicating that it is the minor within Δ_{123} corresponding with the element r_{13}. Otherwise there would be no possibility of distinguishing it from, *e.g.*, $\hat{r}_{13(134)}$.

In order to make bunch maps for the next higher stage—*i.e.*, "four-sets"—the minors $\hat{r}_{12(1234)}$, etc., have to be calculated. Those relating to elements on the diagonal—viz., $\hat{r}_{11(1234)}$, $\hat{r}_{22(1234)}$, etc.—are, owing to their definition, simply equal to Δ_{234}, Δ_{134}, etc. Those not relating to an element on the diagonal have to be calculated with the formula:

$$\hat{r}_{ij(\alpha,\beta\ldots\gamma)} = -\Sigma_{k=\alpha,\beta\ldots)j(\ldots\gamma}\hat{r}_{ik(\alpha,\beta\ldots)j(\ldots\gamma)}r_{kj}$$
$$(i \neq j)\,.$$

The meaning of the suffixes is the following:

α, $\beta \ldots \gamma$ indicate the set studied.
i indicates the row, and
j the column of the element under consideration.
k is a "current suffix"—*i.e.*, it assumes all values indicated under the sum-sign.
$)j($ means that k must not assume the value j, whereas the $\hat{r}_{ik}$'s have to be taken as corresponding to the set without j.

As an example, $\hat{r}_{12(1234)}$ may be taken. Here $i = 1$, $j = 2$, α, $\beta \ldots \gamma = 1, 2, 3, 4$, and k evidently has to assume the values 1, 3, 4. We find:

$$\hat{r}_{12(1234)} = -\hat{r}_{11(134)}r_{12} - \hat{r}_{13(134)}r_{32} - \hat{r}_{14(134)}r_{42}\,.$$

No further new elements have to be introduced in the calculations.

4. *Calculation of Standard Error of Regression Coefficients* (Classical method).

Under the hypotheses formulated in Chapter II, the standard error $\sigma_{b_{1k}}$ for any regression coefficient b_{1k} must be calculated by means of the formula:

$$\sigma^2_{b_{1k}} = \frac{N}{N-n}\sigma^2_{1.23\ldots n}\frac{M_{11.kk}}{M_{11}} \qquad (23)$$

in which the symbols have the following meaning:

(*a*) $\sqrt{\frac{N}{N-n}}\sigma_{1.23\ldots n}$ is the standard deviation of the residuals $x_{1.23\ldots n}$, corrected for the number of coefficients in the regression formula (11) and therefore equal to:

$$\sqrt{\frac{1}{N-n}\Sigma x^2_{1.23\ldots n}} = \sqrt{\frac{1}{N-n}\Sigma(x'_1 - x_1)^2} \qquad (24).$$

The easiest way of calculating this standard deviation is given by:

$$\sigma^2_{1.23\ldots n} = \sigma^2_1(1 - R^2_{1.23\ldots n})$$

where σ_1 is the standard deviation of x_1.

(*b*) M_{11} is the determinant:

$$\begin{vmatrix} \Sigma x_2^2 & \Sigma x_2 x_3 \, \Sigma x_2 x_4 \ldots\ldots & \Sigma x_2 x_n \\ \Sigma x_2 x_3 & \Sigma x_3^2 & \vdots \\ \vdots & & \vdots \\ \Sigma x_2 x_n & \ldots\ldots\ldots\ldots\ldots & \Sigma x_n^2 \end{vmatrix}$$

i.e., the determinant formed by all moments of the " explanatory " series. In those cases in which calculations for bunch maps are already available, it can very easily be obtained from:

$$M_{11} = N^{n-1}\sigma_2^2\sigma_3^2 \ldots\ldots \sigma_n^2 \begin{vmatrix} 1 & r_{23} \, r_{24} \ldots\ldots & r_{2n} \\ r_{23} & 1 \ldots\ldots & \vdots \\ \vdots & \vdots & \vdots \\ r_{2n} & \ldots\ldots\ldots\ldots & 1 \end{vmatrix}$$

$$= N^{n-1}\sigma_2^2\sigma_3^2 \ldots \sigma_n^2 \Delta_{23\ldots n}$$

where $\sigma_2 \dots \sigma_n$ are the standard deviations of $x_2 \dots x_n$, N is the number of observations, and Δ is the symbol introduced in Appendix A, 3.

(*c*) M_{11kk} is the determinant remaining if the *k*th column and the *k*th row in M_{11} are dropped; *e.g.*:

$$M_{1133} = \begin{vmatrix} \Sigma x_2^2 & \Sigma x_2 x_4 \dots & \Sigma x_2 x_n \\ \Sigma x_2 x_4 & \Sigma x_4^2 & \Sigma x_4 x_n \\ \vdots & & \\ \Sigma x_2 x_n & \dots\dots\dots & \Sigma x_n^2 \end{vmatrix}$$

This, again, can be calculated by:

$$M_{1133} = N^{n-2} \sigma_2^2 \sigma_4^2 \dots \sigma_n^2 \begin{vmatrix} 1 & r_{24} \dots\dots & r_{2n} \\ r_{24} & 1 \dots\dots\dots & \vdots \\ \vdots & & \vdots \\ r_{2n} & \dots\dots\dots & 1 \end{vmatrix}$$

$$= N^{n-2} \sigma_2^2 \sigma_4^2 \dots \sigma_n^2 \Delta_{24\dots n}$$

5. *Calculation of Limits to the Error of Weighting in Regression Coefficients.*

The formulæ devised by Koopmans for computing limits to the error of weighting apply only when the signs of corresponding coefficients in all of the elementary regression equations are the same (after solving for the explained variate x_1). In terms of the bunch map, this means that in each of the maps referring to the complete set of variates, all beams should lie in one of the four right-angles formed by the axes.

If a possible regression equation is denoted by

$$x_1^* = b_{12} x_2 + b_{13} x_3 + b_{14} x_4$$

for the case of three explaining variates, where the variates are expressed in normalised units, a first set of limits to each of the b's is formed by the two ultimate beams in the corresponding bunch map involving the complete set of variates. Thus, b_{12} must lie between the largest and smallest of the four coefficients

$$b_{12}^{(1)} = -\frac{\hat{r}_{12}}{\hat{r}_{11}}, \quad b_{12}^{(2)} = -\frac{\hat{r}_{22}}{\hat{r}_{21}}, \quad b_{12}^{(3)} = -\frac{\hat{r}_{32}}{\hat{r}_{31}}, \quad b_{12}^{(4)} = -\frac{\hat{r}_{42}}{\hat{r}_{41}} \qquad (25).$$

Additional limits to the error of weighting in b_{12}, etc., can often be imposed from the following considerations. To the first elementary regression corresponds the assumption that disturbances occur only in the first variate. In this case, the standard deviation of these disturbances is estimated, according to (24), as

$$\sqrt{\frac{N}{N-n}}\,\sigma_{1.234} = \sqrt{\frac{N}{N-n}\,(1 - R_{1.234}^2)} \qquad (26),$$

the standard deviation σ of the variate x_1 itself being equal to unity. Similarly, if only the second variate is subject to disturbances, their standard deviation is estimated by

$$\sqrt{\frac{N}{N-n}}\,\sigma_{2.134} = \sqrt{\frac{N}{N-n}\,(1 - R_{2.134}^2)} \qquad (27),$$

and so on. The expressions containing the four multiple correlation coefficients R are easily computed from

$$1 - R_{1.234}^2 = \frac{\Delta_{1234}}{\Delta_{234}}, \quad 1 - R_{2.134}^2 = \frac{\Delta_{1234}}{\Delta_{134}}, \quad \text{etc.}$$

If any one of the standard deviations (26), (27), etc., of disturbances seems too large to be accepted, *a priori* considerations may lead one to adopt limits, say,

$$\rho_1', \quad \rho_2', \quad \rho_3', \quad \rho_4', \qquad (28)$$

that should not be exceeded by the estimated standard deviations of the disturbances in the corresponding variate.

Somewhat complicated calculations or diagrams are required to find exactly which values of b_{12}, etc., are still admitted by these limitations. For practical purposes, the uncertainty concerning the error of weighting may, however, already be considerably reduced, in a number of cases, by the following thumb rule,[1] which excludes a considerable part of those values of b_{12}, etc., that are incompatible with the limitations (28), but not always all of them. This rule imposes two additional limits to each of the regression coefficients b_{12} ..., the interval between these limits partly overlapping, or falling entirely within, the interval defined by the figures (25). Only values of b_{12} ... common to both intervals, then, have to be admitted.

The additional limits for b_{12} according to this rule are the largest and smallest of the quantities

$$j_{12}^{(1)} = -\frac{\hat{n}_{12}}{\hat{n}_{11}}, \quad j_{12}^{(2)} = -\frac{\hat{n}_{22}}{\hat{n}_{21}}, \quad j_{12}^{(3)} = -\frac{\hat{n}_{32}}{\hat{n}_{31}}, \quad j_{12}^{(4)} = -\frac{\hat{n}_{42}}{\hat{n}_{41}} \qquad (29).$$

Here, $\hat{n}_{12}$, etc., are the minors of the elements n_{12}, etc., in a determinant, obtained from the determinant Δ of the correlation coefficients by replacing the diagonal delements r_{11}, r_{22}, etc. (which equal unity), by

$$n_{11} = r_{11} - \rho_1, \quad n_{22} = r_{22} - \rho_2, \quad \text{etc.,}$$

where

$$\rho_1 = \frac{N - n}{N} \rho_1', \quad \rho_2 = \frac{N - n}{N} \rho_2', \quad \text{etc.}$$

Mutatis mutandis, similar limits for b_{13} and b_{14} are found, which may then be converted from normalised units to the units in which the variates were originally expressed.

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[1] This thumb rule, which has not yet been published, has been communicated to the author by Dr. KOOPMANS.

If greater precision is required, it may be useful to determine the full consequences of the limitations (28) on the coefficients b_{12}.... These can best be formulated in geometrical terms. A set of coefficients, b_{12}, b_{13}, b_{14}, may be represented by a point C in three-dimensional space having these coefficients as its rectangular co-ordinates. Then, instead of the simple limits (25) for b_{12}, and similar limits for b_{13} and b_{14}, the more restrictive proposition holds that the point C is confined to the tetrahedron formed by the four points $(b_{12}^{(1)}, b_{13}^{(1)}, b_{14}^{(1)})$, $(b_{12}^{(2)}, b_{13}^{(2)}, b_{14}^{(2)})$, etc. Further, as a consequence of the limitations (28), the point C must in addition be confined to the tetrahedron formed by the four points $(j_{12}^{(1)}, j_{13}^{(1)}, j_{14}^{(1)})$, $(j_{12}^{(2)}, j_{13}^{(2)}, j_{14}^{(2)})$, etc. Thus, the point C is confined to the common part of the two tetrahedra. In the case of four (or less) variates, the extension of this common part is most easily read from a geometrical figure, if necessary, using orthogonal projections.

In devising the maximum amounts ρ_1', ρ_2', etc., admitted for the standard deviations of the disturbances in each variate, it should be borne in mind that the disturbances in the dependent variate x_1 are also due to the omission of explaining variates of minor importance. The safest procedure, therefore, is to choose ρ_1' equal to, or at a round figure somewhat exceeding, the amount represented by (26). In this case, the limit ρ_1' does not contribute to a restriction of the possibilities left to the regression coefficients b_{12}, etc., by the remaining limitations, but only serves as an aid in computing the effect of the choice of ρ_2', ρ_3', ρ_4'.

For the computation of the errors of sampling corresponding to any of the regressions admitted by the limits to the error of weighting, reference may be made to the original publication.

APPENDIX B

STATISTICAL TABLES

UNITED KINGDOM

	Pig-iron				
Description of series	Pig-iron production	Consumption of iron and steel	Price of pig-iron	Production of consumers' goods	Profit margin
Units ..	000's tons	000's tons	1782 = 100	1913 = 100	1907 = 100
(1)	(2)	(3)	(4)	(5)	(6)
1870	5,964	2,736	44	53.5	168
1	6,627	3,372	47	58.3	164
2	6,742	3,441	74	57.9	188
3	6,566	3,396	85	61.0	190
4	5,991	3,274	66	62.7	170
5	6,365	3,680	52	60.9	156
6	6,556	3,969	46	60.5	136
7	6,608	3,927	42	61.5	136
8	6,381	3,812	36	58.3	134
9	5,995	3,224	37	54.5	122
80	7,749	4,020	42	64.6	130
1	8,144	4,057	37	62.9	118
2	8,586	4,281	39	66.7	120
3	8,529	4,339	37	68.0	112
4	7,812	4,131	32	69.0	104
5	7,415	3,782	31	64.6	94
6	7,009	3,614	30	65.2	86
7	7,559	3,486	31	67.8	82
8	7,999	4,415	31	71.6	84
9	8,323	5,064	38	74.9	84
90	7,904	4,829	39	75.4	92
1	7,406	4,772	36	78.2	90
2	6,709	5,014	33	73.0	82
3	6,977	4,766	32	72.3	80
4	7,427	5,356	32	75.3	72
5	7,703	5,458	32	79.5	66
6	8,660	6,151	34	81.5	68
7	8,796	6,619	34	79.1	66
8	8,610	6,889	36	83.6	64
9	9,421	7,574	47	86.0	78
1900	8,960	7,693	55	84.2	100
1	7,929	7,231	41	83.7	90
2	8,680	7,491	40	84.1	82
3	8,935	7,765	40	82.2	84
4	8,694	7,739	39	80.8	86
5	9,608	8,019	42	85.7	84
6	10,184	8,247	46	87.4	92
7	10,114	7,608	48	91.4	100
8	9,057	7,133	43	90.1	96
9	9,532	7,477	42	88.7	88
10	10,012	7,860	42	89.3	94
1	9,526	8,207	41	92.8	98
2	8,751	8,055	48	93.8	100
3	10,260	9,540	50	100.0	104

Notes and Sources.

(2) Production of pig-iron and ferro-alloys. British Iron and Steel Federation: *Statistics of the Iron and Steel Industries.*

(3) Home consumption of iron and steel. Unpublished estimate by Mr. A. Cairncross, Glasgow.

(4) Sauerbeck's price index. S. Kuznets: *Secular Movements in Production and Prices.*

(5) Index of production of consumers' goods. Hoffmann: *Weltwirtschaftliches Archiv*, Vol. 40.

(6) Profit margin = index of prices of exported finished products (Calculation L.o.N., based on trade statistics) — ½ index of wage rates (Index of Bowley and Wood, taken from Layton: *Introduction to the Study of Prices*).

PRE-WAR

Railways			Finance			
Railway investment index	Index of increase in traffic	Profit rate	Non-labour income	Long-term interest rate	Short-term interest rate	Description of series
%	%	%	million £	%	%	Units
(7)	(8)	(9)	(10)	(11)	(12)	(1)
			460	3.24	3.1	*1870*
8.51		4.69	490	3.23	2.7	*1*
5.33	9.06	4.78	525	3.24	3.8	*2*
6.49	7.19	4.62	545	3.24	4.5	*3*
4.70	1.56	4.40	555	3.24	3.5	*4*
3.82	6.16	4.47	560	3.20*	3.0	*5*
2.99	4.41	4.36	560	3.16	2.2	*6*
1.21	2.58	4.33	555	3.15	2.3	*7*
1.20	—0.13	4.24	540	3.15	3.5	*8*
0.77	1.41	4.18	545	3.08	1.8	*9*
2.49	9.45	4.41	560	3.05	2.2	*80*
2.89	3.75	4.33	575	3.00	2.9	*1*
3.74	4.78	4.36	590	2.98	3.4	*2*
5.55	4.18	4.32	585	2.97	3.0	*3*
4.17	—0.72	4.19	580	2.97	2.6	*4*
2.07	—0.25	4.04	580	3.02	2.0	*5*
1.10	1.32	4.01	580	2.98	2.1	*6*
1.26	3.60	4.02	595	2.95	2.4	*7*
2.15	3.20	4.08	615	3.02	2.4	*8*
3.12	5.01	4.23	640	2.87	2.7	*9*
3.85	3.52	4.41	640	2.85	3.7	*90*
4.70	2.82	4.31	635	2.87	2.5	*1*
3.62	0.89	4.17	625	2.85	1.5	*2*
2.16	—2.72	3.91	630	2.79	2.1	*3*
1.66	8.06	4.11	645	2.72	1.0	*4*
1.69	2.51	4.17	660	2.59	0.8	*5*
2.16	6.23	4.34	680	2.48	1.4	*6*
2.46	5.14	4.35	715	2.45	1.8	*7*
3.51	2.02	4.25	735	2.48	2.6	*8*
3.23	7.04	4.31	765	2.58	3.3	*9*
3.14	3.00	4.06	790	2.76	3.7	*1900*
1.36	0.05	3.89	800	2.92	3.2	*1*
1.29	3.34	4.06	805	2.91	3.0	*2*
1.01	1.13	4.07	810	2.83	3.4	*3*
0.55	0.89	4.02	825	2.83	2.7	*4*
0.25	1.42	4.05	835	2.78	2.6	*5*
0.57	4.92	4.09	875	2.83	4.0	*6*
1.59	3.77	4.10	905	2.97	4.5	*7*
0.37	—1.94	3.92	895	2.90	2.3	*8*
0.04	0.45	4.05	910	2.97	2.3	*9*
0.02	3.05	4.24	940	3.08	3.2	*10*
0.08	1.67	4.34	985	3.15	2.9	*1*
1.28	—1.40	4.18		3.28	3.6	*2*
				3.40	4.4	*3*

(7) % increase in number of locomotives and of carriages, wagons and trucks, weighted by 1 and 4 respectively. Original data from C. D. Campbell: " Cyclical fluctuations in the Railway Industry " in the *Transactions* of the Manchester Statistical Society, 1929/30 session.

(8) % increase in ordinary passenger journeys and in tonnage of goods conveyed, weighted by 4 and 5 respectively. Original data from C. D. Campbell, *loc. cit.*

(9) Ratio of net receipts to paid-up capital. C. D. Campbell, *loc. cit.*

(10) Bowley: *Economic Journal*, Vol. XIV, and J. Stamp: " *British Incomes and Property* ". Years ending June.

(11) Yield on British Consols. I. Fisher: *The Theory of Interest.*

(12) Market rate of discount. I. Fisher: *loc. cit.*

	Pig-iron					Finance				
Description of series	Pig-iron production	Consumption of iron and steel	Index of production of consumers' goods	Price of pig-iron	Profit margin	Dividends	Index of share prices	Long-term interest rate	Short-term interest rate	Share yield
Units	10^6 metric tons	10^6 metric tons	1913 = 100	Marks per ton	%	%	Nominal value = 100	%	%	%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1870	1.39	1.40	29.3	72.8	— 7.4	9.46	126.2	4.61	4.49	7.50
1	1.56	1.78	34.7	74.7	— 0.1	12.88	147.6	4.44	3.75	8.73
2	1.99	2.32	38.0	108.6	10.8	15.13	196.7	4.26	3.95	7.69
3	2.24	2.69	36.4	107.7	12.8	12.04	185.0	4.30	4.46	6.51
4	1.91	2.04	34.5	81.1	5.3	8.56	138.1	4.24	3.23	6.20
5	2.03	2.07	34.3	68.9	— 4.6	5.22	106.2	4.25	3.70	4.92
6	1.85	1.86	37.7	68.9	— 4.3	4.18	86.6	4.22	3.04	4.83
7	1.93	1.83	38.6	55.2	— 2.0	4.33	79.7	4.23	3.17	5.43
8	2.15	1.88	37.8	52.1	— 6.3	4.37	83.7	4.25	3.07	5.22
9	2.23	1.85	38.3	49.4	— 4.4	5.47	100.6	4.18	2.60	5.24
80	2.73	1.97	34.8	58.5	4.5	6.45	129.0	4.05	3.04	5.00
1	2.91	2.04	36.9	55.3	4.2	6.64	134.9	3.97	3.50	4.92
2	3.38	2.55	38.2	57.0	0.9	6.67	133.9	3.95	3.89	4.98
3	3.47	2.63	41.8	52.5	1.0	6.63	134.8	3.93	3.08	4.92
4	3.60	2.80	44.0	47.1	— 0.6	6.26	129.5	3.88	2.90	4.83
5	3.69	2.84	45.0	42.7	— 3.4	5.73	124.9	3.81	2.85	4.59
6	3.53	2.44	45.1	39.5	— 6.6	5.49	125.9	3.73	2.16	4.36
7	4.02	2.83	49.3	40.7	— 6.8	6.18	129.1	3.70	2.30	4.79
8	4.34	3.38	49.9	43.5	— 5.1	8.05	147.1	3.64	2.11	5.47
9	4.52	3.80	54.2	47.5	4.0	9.62	178.4	3.61	2.63	5.39
90	4.66	4.01	56.1	57.0	9.8	10.22	173.2	3.67	3.78	5.90
1	4.64	3.62	58.0	49.5	9.0	8.14	148.5	3.71	3.02	5.48
2	4.94	3.89	57.7	46.0	2.4	7.00	142.9	3.67	1.80	4.90
3	4.99	3.82	58.3	43.0	— 0.6	6.80	143.6	3.66	3.17	4.74
4	5.38	4.08	61.9	42.6	— 5.3	7.70	154.5	3.55	1.74	4.98
5	5.46	4.05	68.7	43.0	— 6.0	8.38	178.8	3.37	2.01	4.69
6	6.37	5.10	67.0	46.7	— 7.1	9.97	184.5	3.34	3.04	5.40
7	6.88	5.77	69.8	50.6	— 2.0	10.50	194.5	3.36	3.09	5.40
8	7.31	5.93	75.0	51.5	0.0	11.08	200.1	3.41	3.55	5.54
9	8.14	7.12	74.2	55.6	2.9	11.14	201.6	3.54	4.45	5.53
1900	8.52	7.60	72.2	64.3	9.8	10.23	184.5	3.68	4.41	5.54
1	7.88	5.73	75.7	62.2	0.5	7.74	163.4	3.65	3.06	4.74
2	8.53	5.25	79.3	53.2	— 2.8	7.31	164.3	3.52	2.19	4.45
3	10.02	6.45	72.8	52.1	— 3.3	8.29	170.2	3.53	3.01	4.87
4	10.06	7.22	82.2	51.6	— 4.6	9.09	182.5	3.57	3.14	4.98
5	10.88	7.47	82.3	53.0	— 0.6	10.16	199.4	3.57	2.85	5.10
6	12.29	8.76	88.0	58.2	3.8	10.90	198.5	3.63	4.04	5.49
7	12.88	9.63	89.2	64.0	6.3	10.40	182.7	3.75	5.12	5.69
8	11.81	8.03	87.0	60.6	— 2.3	8.92	178.1	3.80	3.52	5.01
9	12.64	8.40	90.6	55.7	— 2.4	9.44	189.3	3.70	2.87	4.99
10	14.79	9.88	89.1	55.7	— 2.2	10.09	200.8	3.76	3.54	5.02
1	15.57	10.13	97.1	57.4	— 2.5	10.40	201.9	3.79	3.54	5.15
2	17.85	11.82	96.3	60.6	4.1	11.06	200.9	3.91	4.22	5.51
3	19.31		100.0		— 1.0	10.47	192.9	4.09	4.98	5.43

Notes and sources:

(2) *Statistisches Handbuch für das Deutsche Reich* (1907) and British Iron and Steel Federation.

(3) Pig-iron production + net imports of iron and steel and their products + steel production — consumption of pig-iron for steel production.

(4) Institut für Konjunkturforschung, *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft No. 31.

(5) Average price of pig-iron produced. *Statistisches Handbuch für das Deutsche Reich (1907); Statistisches Jahrbuch.*

(6) General index of wholesale prices — ½ index of wages, both in % deviations from trend. Institut für Konjunkturforschung, *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 37, and J. Kuczynski: *Löhne und Ernährungskosten in Deutschland.*

(7) Yearly dividends in %. Institut für Konjunkturforschung, *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft No. 36.

(8) Index of share prices in % of nominal value. *Ibid.*

(9) Yield on fixed interest-bearing securities. *Ibid.*

(10) Market rate of discount. *Ibid.*

(11) Dividends paid in % of share prices. *Ibid.*

(12) Net increase in number of rooms (Lokalitäten) in Hamburg. Institut für Konjunkturforschung, *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 17.

(13) Number of rooms vacant as % of total. *Ibid.*

(14) Yearly average rent per room of occupied houses. *Ibid.*

PRE-WAR

Building						Railways			
Net increase in number of rooms in Hamburg	Vacancies	Rent of occupied houses	Index of price of building materials	Rate of interest of mortgage banks	Profitability of building	Investment index	Index of increase in traffic	Profit rate	Description of series
000's	%	Marks per room	1913 = 100	%	%	%	%	%	Units
(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(1)
						9.30	7.93	6.16	*1870*
						9.05	18.99	6.94	*1*
						17.01	17.33	5.98	*2*
						15.63	19.08	5.21	*3*
						12.87	2.14	4.71	*4*
						6.43	2.71	4.68	*5*
						3.38	3.07	4.45	*6*
		612	107	4.58	1.39	1.55	1.16	4.26	*7*
5.00	5.78	593	96	4.57	1.57	2.40	1.36	4.25	*8*
4.18	6.50	585	90	4.40	1.78	1.97	4.48	4.28	*9*
3.59	6.85	580	86	4.28	1.93	0.83	5.99	4.44	*80*
3.18	7.07	572	82	4.00	2.14	1.76	5.50	4.54	*1*
2.89	6.92	573	82	4.00	2.16	3.13	7.44	4.80	*2*
0.19	4.91	575	87	4.00	2.07	3.21	4.88	4.61	*3*
0.35	3.46	587	93	4.00	1.99	2.46	3.07	4.60	*4*
0.31	2.71	596	93	3.86	1.89	2.35	0.25	4.42	*5*
0.28	2.50	615	97	3.50	2.22	1.41	3.81	4.66	*6*
2.62	2.38	632	104	3.79	1.99	1.48	7.24	5.17	*7*
4.88	2.82	660	103	3.68	2.38	2.78	8.37	5.40	*8*
7.61	3.44	677	112	3.50	2.48	3.80	8.70	5.60	*9*
9.02	4.64	676	100	3.73	2.63	4.73	4.65	4.86	*90*
6.92	5.86	681	92	4.00	2.74	4.15	4.05	4.49	*1*
7.08	8.38	687	85	3.99	3.03	4.25	0.57	4.56	*2*
6.23	9.01	664	79	3.93	3.01	2.08	5.89	5.03	*3*
4.57	9.00	650	77	3.63	3.13	1.83	1.23	4.98	*4*
3.16	7.98	643	78	3.50	3.07	2.59	7.18	5.74	*5*
2.41	6.37	637	81	3.50	2.91	3.14	6.88	6.15	*6*
2.20	4.53	637	93	3.50	2.57	3.85	7.36	6.21	*7*
2.42	3.51	639	102	3.58	2.28	4.90	8.10	6.08	*8*
3.35	2.97	643	103	3.94	2.04	4.00	6.90	6.12	*9*
3.93	2.51	650	107	4.00	1.94	4.24	6.13	5.91	*1900*
2.41	2.15	665	98	4.00	2.38	3.06	− 2.01	5.14	*1*
4.29	2.72	676	98	3.98	2.51	2.42	3.46	5.40	*2*
7.40	3.54	681	101	3.78	2.61	1.91	7.31	5.95	*3*
10.20	4.38	686	104	3.87	2.52	2.60	4.66	6.00	*4*
10.72	4.80	689	102	3.84	2.63	3.37	8.04	6.29	*5*
10.38	5.32	694	106	3.94	2.48	4.75	8.28	6.35	*6*
8.60	4.65	705	103	4.02	2.64	6.42	6.44	5.60	*7*
8.03	4.56	714	97	4.02	2.92	5.82	− 0.33	4.51	*8*
8.85	4.82	709	104	4.00	2.67	4.06	6.77	5.09	*9*
13.39	6.87	717	97	4.00	2.97	3.17	6.20	5.74	*10*
10.85	7.21	724	97	4.00	3.04	3.74	8.94	6.41	*1*
12.16	7.16	737	104	4.03	2.94	4.05	6.37	6.29	*2*
6.18	6.02	747	100	4.11	3.12	5.22	2.58	5.70	*3*

(15) Index of price of building materials. Institut für Konjunkturforschung, *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 37.

(16) Average interest rate on new issues of German mortgage banks. Institut für Konjunkturforschung, *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 17.

(17) Profitability $= m_R - (0.65\, q_R + 0.35)(0.007\, m_{Lb} + 0.02)$

where m_R = rent index, in %, 1913 = 8.

q_R = index of cost of building materials, 1913 = 100.

m_{Lb} = interest on mortgage banks new issues.

(18) % increase in the number of locomotives, passenger cars and freight cars, with the respective weights 1, 1, 5. Original data from *Statistisches Handbuch (1907)* and *Statistisches Jahrbuch*.

(19) % increase in number of passengers and tonnage of goods carried one mile, with the respective weights 1 and 2. Original data from *Statistisches Handbuch (1907)* and *Statistisches Jahrbuch*.

(20) Net operating income in % of capital invested. *Statistisches Handbuch (1907)* and *Statistisches Jahrbuch*.

UNITED STATES OF AMERICA — PRE-WAR

	Pig-iron			Railways			Finance		
Description of series	Pig-iron production	Price of pig-iron	Building volume	Railway investment index	Index of increase in traffic	Profit rate	Stock prices	Long-term interest rate	Short-term interest rate
Units	000's tons	$ per ton	1913 = 100	%	%	%	$	%	%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1877	2,067	18.9	19				39.2		5.2
8	2,301	17.7	17				42.1		4.8
9	2,742	21.7	19				44.4		5.1
80	3,835	28.5	22				53.9		5.2
1	4,144	25.2	27				66.7		5.2
2	4,623	25.8	33				60.8		5.7
3	4,596	22.4	39				55.9		5.6
4	4,098	19.8	44				46.2		5.2
5	4,045	18.0	47				48.0		4.1
6	5,683	18.7	51				55.9		4.8
7	6,417	20.9	47				55.8		5.8
8	6,490	18.9	46				54.2		4.9
9	7,604	17.8	63				61.2		4.9
90	9,203	18.4	75			4.65	57.7	5.04	5.7
1	8,280	17.5	76		3.90	4.60	53.6	5.17	5.4
2	9,157	15.8	84		9.70	4.44	65.4	4.95	4.1
3	7,125	14.5	55		6.17	4.42	53.0	5.08	6.6
4	6,657	12.7	55		−10.52	3.80	49.4	4.90	3.0
5	9,446	13.1	73	−0.23	0.89	3.81	51.8	4.77	3.7
6	8,623	13.0	60	1.43	10.65	3.84	44.8	4.83	5.8
7	9,653	12.1	67	0.27	− 1.66	3.93	45.2	4.66	3.5
8	11,774	11.7	58	1.41	17.22	4.39	52.2	4.49	3.8
9	13,621	19.4	70	2.65	8.57	4.95	71.0	4.23	4.2
1900	13,789	20.0	46	4.23	13.36	5.13	61.2	4.15	4.4
1	15,878	15.9	66	6.06	4.95	5.43	69.6	4.07	4.3
2	17,821	22.2	69	4.76	8.58	5.63	65.1	4.06	4.9
3	18,009	19.9	71	6.31	9.16	5.72	55.8	4.24	5.5
4	16,497	15.6	80	3.74	1.76	5.38	54.2	4.23	4.2
5	22,992	17.9	106	2.70	7.28	5.51	79.5	4.06	4.4
6	25,307	21.0	109	6.02	13.28	5.90	93.9	4.18	5.7
7	25,781	23.9	98	7.41	9.73	5.95	76.2	4.51	6.4
8	15,936	17.7	89	4.74	− 4.55	4.91	74.7	4.55	4.3
9	25,795	17.8	125	−0.02	0.17	5.22	92.2	4.33	4.0
10	27,304	17.4	109	3.03	15.19	5.53	84.9	4.44	5.0
1	23,650	15.7	104	3.64	0.30	4.77	82.3	4.43	4.0
2	29,727	16.6	108	1.46	2.99	4.55	88.4	4.46	4.7
3	30,966	17.1	100	3.03	11.69	4.86	79.5	4.64	5.6

Notes and Sources.

(2) Production of pig-iron and ferro-alloys. British Iron and Steel Federation: *Statistics of the Iron and Steel Industries.*
(3) Price of No. 1 foundry pig-iron at Philadelphia. *U.S.A. Statistical Abstract.*
(4) W. H. Newman: *The Building Industry and Business Cycles.* Chicago, 1935.
(5) Per cent increase in the number of locomotives, passenger cars and freight cars, with the respective weights 2, 1 and 4. Original data from the *U.S.A. Statistical Abstract.*
(6) Per cent increase in number of passengers carried one mile and weight of freight carried one mile, with the respective weights 1 and 3. Original data from the *U.S.A. Statistical Abstract.*
(7) Net railway operating income in per cent of total railway investment. Original data on net traffic earnings, net revenue and income or net railway operating income and on railroad investment or capitalisation from the *U.S.A. Statistical Abstracts, 1904, 1910* and *1924.* The original series have been adjusted before being linked up.
(8) Average price of industrial stocks. *Review of Economic Statistics, 1919* and *1926.*
(9) 1890-1899: Yield on ten American Railroad bonds. *Review of Economic Statistics, 1919;* 1900-1913: Yield on sixty bond issues combined. Standard Statistics Co.
(10) Market rate on 60-90-day paper. I. Fisher: *The Theory of Interest.*

FRANCE — PRE-WAR

	Pig-iron			Railways			Finance	
Description of series	Production of pig-iron	Consumption of iron and steel	Price of pig-iron	Railway investment index	Index of increase in traffic	Profit rate	Index of share prices	Index of bond prices
Units	000's tons	000's tons	francs per ton	%	%	%	1901-10 = 100	1901-10 = 100
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1870	1,178	1,212	92	3.7	— 9.0	4.40	89	67
1	860	836	98	3.8	8.3	5.20	77	55
2	1,218	1,162	121	8.2	19.4	4.84	81	56
3	1,382	1,344	137	5.3	4.4	4.67	85	58
4	1,416	1,380	119	4.2	— 1.1	4.44	85	62
5	1,448	1,488	108	1.9	4.8	4.65	94	66
6	1,435	1,519	98	0.8	2.8	4.59	92	70
7	1,507	1,641	95	4.6	— 1.7	4.29	95	72
8	1,521	1,663	88	3.7	9.8	4.52	99	77
9	1,400	1,527	85	1.9	— 0.1	4.33	102	82
80	1,725	1,872	93	3.2	13.5	4.80	113	86
1	1,886	2,244	91	7.4	5.6	4.82	125	87
2	2,039	2,590	91	7.8	3.6	4.44	114	84
3	2,069	2,520	81	5.2	3.0	4.08	102	81
4	1,872	2,111	75	3.8	— 4.0	3.61	96	79
5	1,631	1,744	62	2.4	— 2.7	3 36	88	82
6	1,517	1,559	55	1.8	— 2.0	3.27	86	84
7	1,568	1,444	57	1.0	4.1	3 38	87	83
8	1,683	1,651	57	1.0	3.6	3.40	92	84
9	1,734	1,506	61	1.2	11.1	3.61	97	87
90	1,962	1,723	70	0.9	0	3.50	104	94
1	1,897	1,827	65	1.2	4.5	3.42	103	97
2	2,057	2,043	61	1.6	4.3	3.16	98	100
3	2,003	2,014	58	1.0	4.4	3.10	96	100
4	2,069	2,052	57	1.1	2.4	3.23	94	103
5	2,003	1,868	55	0.8	3.3	3.40	92	104
6	2,339	2,103	56	0.6	3.4	3.57	93	104
7	2,484	2,367	58	0.6	3.5	3.69	102	105
8	2,525	2,375	63	0.6	5.8	3.86	110	105
9	2,578	2,542	72	1.5	5.1	3.93	113	103
1900	2,714	2,908	82	3.1	9.1	3.86	109	103
1	2,389	2,362	70	3.0	— 5.1	3.52	96	102
2	2,405	2,067	68	1.1	1.2	3.67	89	103
3	2,841	2,489	64	1.0	1.8	3.82	89	100
4	2,974	2,561	65	1.1	1.1	3.93	90	99
5	3,077	2,637	68	— 0.1	5.3	4.03	101	101
6	3,314	2,983	80	0.7	4.5	4.16	105	100
7	3,590	3,075	87	3.6	5.8	3.97	105	97
8	3,401	2,925	82	4.9	4.1	3.86	102	98
9	3,574	3,152	82	1.9	2.8	3.89	108	100
10	4,038	3,715	78	1.8	3 2	3.84	116	100
1	4,470	4,278	76	3.6	5.1	3.72	121	98
2	4,939	4,649		3.6	5.3	3.82	130	94
3	5,207	4,817		4.5	5.3	3.91	127	89

Notes and Sources.

(2) *Annuaire statistique de la France.*

(3) Pig-iron production + net imports of pig-iron, iron and steel and manufactures thereof. Data from *Annuaire statistique de la France* and Institut de Recherches économiques de l'Université de Paris.

(4) Average price per ton of pig-iron produced. *Annuaire statistique de la France.*

(5) % increase in number of locomotives and of passenger and freight cars, weighted by 1 and 3 respectively. Original data from *Annuaire statistique de la France.*

(6) % increase in passengers carried one km. and goods carried one km., weighted by 4 and 5 respectively. Original data from *Annuaire statistique de la France.*

(7) Net receipts per km. of line divided by cost of construction of one km. *Annuaire statistique de la France.*

(8) Index of price of variable interest-bearing securities. *Bulletin de la Statistique générale de la France, 1919/20.*

(9) Index of price of 3 % "rente". *Bulletin de la Statistique générale de la France 1919/20.*

SWEDEN — PRE-WAR

	Building								
Description of series	Number of rooms built	Vacancies	Rent index	Cost of construction index	Long-term interest rate	Profitability of building	Total assessed income	Cost of living index	Real income earned
Units	000's	%	1861-70 = 100	1861-70 = 100	%	%	million kr.	1861-70 = 100	10^9 kr. 1861-70
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1880	5.3			107	5.8		286		
1	7.6			107	5.8		304		
2	9.4	3.0		108	5.7		318	106	.31
3	8.0	3.6	112	112	5.7	2.26	332	105	.31
4	10.5	4.2	116	112	5.7	2.58	332	101	.35
5	10.8	4.8	118	110	5.6	2.93	349	96	.36
6	6.8	5.4	112	108	5.6	2.56	345	92	.38
7	6.1	6.2	105	105	5.4	2.33	350	88	.41
8	6.3	5.1	100	109	5.1	1.92	355	91	.42
9	5.4	3.9	95	109	4.9	1.68	381	96	.43
90	7.6	6.5	95	112	4.7	1.60	411	98	.43
1	4.2	5.1	94	109	4.8	1.67	418	101	.43
2	1.7	5.0	93	107	4.9	1.62	425	99	.43
3	0.9	3.5	93	106	4.8	1.68	432	95	.47
4	2.5	1.6	95	100	4.8	2.24	448	90	.52
5	4.0	1.6	100	114	4.6	2.04	465	92	.54
6	6.3	0.8	107	117	4.5	2.54	495	91	.59
7	4.5	1.2	107	119	4.4	2.52	538	94	.64
8	6.2	0.3	120	124	4.4	3.30	601	98	.67
9	7.1	1.8	135	131	4.8	3.79	660	103	.70
1900	4.2	1.1	138	125	5.1	4.08	716	104	.73
1	4.3	1.6	138	115	5.3	4.47	764	101	.76
2	5.3	1.4	132	117	5.1	4.03	765	102	.81
3	9.9	2.8	135	136	4.9	3.46	831	104	.87
4	12.2	3.5	139	127	4.9	4.23	895	103	.91
5	8.6	2.3	141	138	4.9	3.79	944	105	.97
6	11.1	1.0	145	139	5.0	3.94	1,017	107	1.07
7	11.0	1.0	160	155	5.3	3.95	1,145	113	1.11
8	7.0	1.6	175	145	5.5	5.53	1,245	114	1.11
9	8.7	2.2	176	148	5.4	5.46	1,268	113	1.11
10	7.8	2.4	169	151	5.2	5.00	1,254	113	1.28
1	5.7	2.4	166	144	5.1	5.25	1,446	112	1.36
2	8.9	2.0	172	160	5.0	4.95	1,519	119	1.39
3	12.4	8.6	169	160	5.1	5.10	1,655	119	

Notes and Sources.

(2) Total number of newly built rooms or kitchens in Stockholm. *Statistical Abstract for Stockholm*. Since 1906, adjusted for change in scope.
(3) Number of rooms vacant on December 31st as a percentage of total. *Statistical Abstract for Stockholm*; 1883-7: estimated.
(4) and (5) Myrdal. *The Cost of Living in Sweden 1830-1930*.
(6) Interest rate of Swedish savings banks. Lindahl: *National Income of Sweden 1861-1930*.
(7) Profitability $= m_R - (0.007 q_B m_{Lb} + 0.02 q_B)$ where m_R = rent in % (1861-70 = 8); q_B = index of construction costs, 1861-70 = 100; m_{Lb} = long-term interest rate.
(8) Assessed income from capital and wages. *Sveriges Statistisk Årsbok*.
(9) Myrdal: *loc. cit*
(10) Assessed income of following year, divided by cost-of-living index.

SWEDEN — POST-WAR

	Building								
Description of series	Rooms built in 296 cities	Dwellings built in Stockholm	Rent index	Construction costs (i)	Construction costs (ii)	Savings banks interest rate	Profitability of building (i)	Profitability of building (ii)	Real income earned
Units	000's	000's	1.VII. 1914 = 100	1913 = 100	1914 = 100	%	%	%	10^9 Kr. 1914
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1923			169	211	210	5.2	1.6	1.7	2.65
4	40.0	2.8	180	216	230	5.5	1.8	0.9	2.78
5	42.3	4.2	186	219	235	5.6	2.0	1.0	2.82
6	43.5	6.1	188	215	235	5.5	2.4	1.2	2.96
7	43.9	6.6	196	213	232	5.4	3.3	2.3	3.07
8	46.7	6.0	199	210	232	5.5	3.6	2.4	3.10
9	46.4	5.6	200	209	232	5.5	3.7	2.5	3.34
30	59.5	8.4	204	208	232	5.2	4.5	3.3	3.48
1	59.3	7.7	206	201	232	5.2	5.2	3.5	3.36
2	52.7	6.9	206	194	228	5.3	5.4	3.4	3.12
3	40.2	3.7	203	192	221	4.7	6.2	4.6	3.10
4	51.2	3.0	202	191	217	4.4	6.6	5.2	3.39
5	76.7	5.8	199	193	217	3.9	6.7	5.7	3.70
6	86.9	6.9	197	193		(3.5)			(3.90)

Notes and sources.

(2) Gross increase in number of rooms or kitchens in 296 cities. LINDAHL: *The National Income of Sweden*, Part II—and *Sociala Meddelanden*, No. 7, 1937—Adjustments have been made because the number of cities varied slightly.

(3) Number of dwellings built by private enterprise only. *Stockholm Stadskollegiets utlåtanden och memorial*—Bihang, No. 10A, 1935. — *Sociala Meddelanden*, 1937.

(4) Index entering into the cost-of-living index. *Statistisk Årsbok för Sverige.*

(5) Index of construction costs. *Svenska Handelsbanken:* Index. See note (a) on page 97.

(6) Index of construction costs in Stockholm. *Statistisk Årsbok för Stockholms Stad.*

(7) Svenska sparbank föreningen. *Statistisk Årsbok för Sverige.*

(8) Profitability $= m_R - (0.007\, q_B m_{Lb} + 0.02\, q_B)$ where m_R = rent index in %, 1914 = 8; q_B = index of construction costs (i), 1913 = 100; m_{Lb} = long-term interest rate.

(9) As (8), but with index of construction costs (ii).

(10) Total assessed income of following year divided by cost-of-living index. *Statistisk Årsbok för Sverige.*

UNITED STATES

	PIG-IRON			BUILDING					
Description of series	Pig-iron production	Kuznets' investment index	Price of pig-iron	Residential construction (i)	Residential construction (ii)	Rent index	Building costs	Profitability of building	Stock of houses
Units	10^6 tons	10^9 $ 1929	$ per ton	10^6 $	10^6 sq. feet	1913 = 100	1923-25 = 100	%	000,000's
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1918	39.1		36.7			104.7	89.2	− 0.15	20.5
9	31.0	12.3	31.1		288	114.2	93.5	0.08	20.7
20	36.9	12.4	44.5	1122	164	134.9	118.5	− 1.28	20.9
1	16.7	9.2	25.3	1841	244	159.0	95.1	2.46	21.2
2	27.2	10.9	27.6	3115	370	160.9	82.3	4.40	21.6
3	40.4	14.2	29.0	3980	422	163.4	100.9	3.05	22.0
4	31.4	14.1	23.3	4244	442	168.0	101.6	3.46	22.5
5	36.7	15.4	22.3	4754	560	167.4	97.5	3.84	23.1
6	39.4	16.9	21.3	4314	521	165.4	98.1	3.78	23.7
7	36.6	16.2	20.4	4064	495	162.1	97.3	3.76	24.2
8	38.2	16.7	19.2	3813	568	157.6	97.5	3.45	24.8
9	42.6	18.4	20.5	2623	388	153.7	97.6	2.96	25.2
30	31.8	15.1	20.3	1456	230	149.6	95.7	3.02	25.4
1	18.4	11.3	18.7	1005	190	142.0	85.5	3.09	25.6
2	8.8	7.9	17.1	282	74	127.8	74.0	2.11	25.7
3	13.3	7.7	18.3	204	73	108.8	80.3	0.52	25.8
4	16.1		20.5	214	64	102.1	93.4	− 0.14	25.9
5	21.4		21.0	585	135	102.1	92.0	0.38	26.0
6	30.6		22.0	1202	222	104.6	97.3		
7	36.6		25.8	1278	236	110.0	109.5		

Notes and sources.

(2) Production of pig-iron and ferro-alloys. *Statistical Abstract of the United States and Survey of Current Business*, Department of Commerce.

(3) Flow of producers' durable commodities to enterprises + flow of consumers' durable commodities to households and enterprises.

(4) Price of Bessemer pig-iron at Pittsburgh. *Statistical Abstract.*

(5) Value of total new non-farm residential building. National Bureau of Economic Research *Bulletin* No. 65. To obtain an index of volume this series has to be deflated by construction costs.

(6) Construction contracts awarded, floor space of buildings. Data from DODGE Co., in *Statistical Abstract of the United States.*

(7) Index of the Bureau of Labor statistics. Figures for June of each year.

(8) Index of construction costs of the *Engineering News Record*. *Statistical Abstract.*

(9) Profitability $= m_R - (0.007 q_B m_{Lb} + 0.02 q_B)$

where m_R = rent index in %, average 1901-35 = 8.

q_B = index of construction costs, 1901-35 = 100.

m_{Lb} = long-term interest rate.

(10) Estimated number of houses at end of year based on census data and yearly volume of building.

ST-WAR

Production, Prices, Finance											
Pro-uction f con-umers' goods	Index of whole-sale prices	Cost-of-living index	Corpo-rations net income	Urban non-wor-kers' income	Capital gains	Profit margin	Index of stock prices	Long-term interest rate	Short-term interest rate	Share yield	Description of series
10^9 \$ 1929	1926 = 100	1923 = 100	10^9 \$	10^9 \$	10^9 \$	1929 = 100	1926 = 100	%	%	%	Units
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(1)
53	125.0	94.2	7.67				56.6	5.43	6.0	11.9	*1918*
51	134.4	102.3	8.42	14.1	1.2	126.4	72.6	5.49	5.6	9.0	*9*
50	157.7	118.2	5.87	15.9	1.1	140.2	66.1	6.12	7.5	9.1	*20*
45	114.0	102.3	0.46	15.0	0.5	116.4	51.6	5.97	6.8	9.6	*1*
53	91.4	97.4	4.77	16.4	1.3	110.6	64.7	5.10	4.7	6.9	*2*
59	102.0	100.0	6.31	17.9	1.4	107.8	66.6	5.12	5.0	8.1	*3*
59	99.6	101.3	5.36	18.7	2.0	106.4	69.6	5.00	3.9	7.7	*4*
61	102.9	103.7	7.62	19.9	4.6	111.2	88.4	4.88	4.0	7.3	*5*
65	103.2	104.3	7.50	20.2	3.6	110.4	100.0	4.73	4.2	7.3	*6*
66	96.5	102.0	6.51	20.9	4.4	105.8	118.5	4.57	4.0	6.4	*7*
70	96.4	100.6	8.23	22.1	7.8	103.0	154.3	4.55	4.9	5.1	*8*
74	95.9	100.1	8.74	23.7	6.2	100.0	189.4	4.73	5.8	4.6	*9*
66	92.5	96.7	1.55	22.4	− 3.5	93.2	140.6	4.55	3.9	5.2	*30*
61	78.2	87.2	−3.29	19.5	−10.4	78.2	87.4	4.58	2.6	6.1	*1*
56	67.3	77.9	−5.64	15.7	− 5.7	69.6	46.5	5.01	2.8	8.0	*2*
	61.0	74.9	−2.55	14.8		65.6	65.7	4.49	1.7	5.1	*3*
	72.2	79.4					81.1	4.00	1.0		*4*
	78.8	82.6					90.8	3.60	0.8		*5*
	80.6	84.8						3.24			*6*
	85.9	88.5						3.28			*7*

(11) Estimates based on data from Warburton, *Journal of the American Statistical Association*. Vol. 30.

(12) Index number of the Bureau of Labor Statistics. Figures for January of each year.

(13) National Industrial Conference Board.

(14) Corporations' net income minus deficit. *Statistical Abstract and Statistics of Income.*

(15) Estimates based on data from S. Kuznets: *National Income, 1919-1935*, National ureau of Economic Research, Bulletin No. 66.

(16) Estimates based on data from Warburton, *Journal of Political Economy*, vol. 43.

(17) Index of cost of living — ½ index of hourly earnings. National Industrial Conference oard.

(18) Index of Standard Statistics Co. for industrial stocks.

(19) Yield of sixty bonds combined, *Standard Statistics*. 1936 and 1937: estimates on the asis of forty-five bonds.

(20) Market rate, 4-6 months commercial paper. I. Fisher, *The Theory of Interest* and League Nations *Monthly Bulletin of Statistics*.

(21) Cash dividends in % of total capital stock. *Statistics of Income.* Up to 1925, capital ock has been estimated on the basis of new corporate issues and index of stock prices. But for slight difference of trend, the movements of the series of share yield thus calculated are highly orrelated with those of the series published since 1926 by the Standard Statistics Co., New York.

UNITED KINGDOM

	Pig-iron			Building	
Description of series	Pig-iron production	Pig-iron consumption	Price of pig-iron	Number of houses built	Rent index
Units	10^6 tons	10^6 tons	£ per ton	000's	VII.1914 = 100
(1)	(2)	(3)	(4)	(5)	(6)
1919	7.40	5.45	6.85		105
20	8.03	5.48	10.45		121
1	2.62	2.42	6.87		148
2	4.90	2.03	4.53		153
3	7.44	4.06	5.44	67.5	148
4	7.31	5.49	4.41	69.2	147
5	6.26	4.83	3.63	66.4	147
6	2.46	* 4.95	4.38	63.9	149
7	7.29	* 4.95	3.65	60.3	151
8	6.61	4.79	3.29	64.7	151
9	7.59	5.57	3.51	91.7	153
30	6.19	5.58	3.35	125.4	153
1	3.77	4.40	2.93	128.4	154
2	3.57	3.04	2.93	142.0	154
3	4.14	2.96	3.11	207.9	156
4	5.97	4.83	3.35	286.4	156
5	6.42	4.95	3.39	271.4	157
6	7.69	6.68	3.66	273.2	159
7				257.1	159

Notes and Sources.

(2) British Iron and Steel Federation: *Statistics of the Iron and Steel Industries.*

(3) Production minus net exports of iron and steel and their products. British Iron and Steel Federation and *Annual Statement of the Trade of the United Kingdom.* * Average of 1926 and 1927.

(4) Price of Cleveland-Middlesbrough pig-iron. Yearly article by the Editor of the *Statist*, *Journal of the Royal Statistical Society*, Part II.

(5) Number of houses built by private enterprise without State assistance; years ending following March; *Statistical Abstract for the United Kingdom* and *Annual Report, Ministry of Health.*

(6) Rent index of the cost-of-living index. Up to 1928, figures relate to controlled rents; since 1929, to controlled and uncontrolled rents combined. *20th Abstract of Labour Statistics* and *Monthly Labour Gazette.*

T-WAR

Building		Finance			
Index of onstruction costs	Profitability of building	Real income from wages and salaries	Profit rate	Long-term interest rate	Description of series
1929 = 100	%	10^6 £ 1914	%	%	Units
(7)	(8)	(9)	(10)	(11)	(1)
156.9	—2.64			4.62	*1919*
142.5	—0.28		15.2	5.32	*20*
115.6	2.14		10.3	5.21	*1*
103.6	2.55	12.5	7.0	4.42	*2*
99.9	2.64	13.6	9.8	4.31	*3*
97.1	2.75	13.6	10.3	4.39	*4*
97.1	2.79	13.7	10.9	4.44	*5*
100.0	2.73	13.0	11.3	4.55	*6*
100.0	2.79	14.1	10.5	4.56	*7*
100.0	2.78	14.2	11.1	4.47	*8*
96.8	3.05	14.8	10.5	4.60	*9*
95.2	3.25	14.7	9.8	4.48	*30*
93.2	3.78	15.2	7.2	4.39	*1*
89.5	4.27	16.4	5.8	3.74	*2*
88.9	4.47	16.2	6.1	3.39	*3*
87.9	4.70	16.7	7.2	3.10	*4*
88.1	4.77	17.2	8.5	2.89	*5*
92.7		17.4	9.7	2.94	*6*
					7

(7) Colin Clark, *Investment in Fixed Capital in Great Britain*, Special Memorandum No. 38, London & Cambridge Economic Service. This series has been continued on figures obtained from Mr. R. Stone.

(8) Profitability = $m_R - (0.007 q_B m_{Lb} + 0.02 q_B)$ where m_R = rent index, in %, 1929 = 8; q_B = index of construction costs, 1929 = 100; m_{Lb} = long-term interest rate.

(9) Income from wages and salaries according to C. Clark, *National Income and Outlay*, divided by index of cost of living.

(10) Ratio of profits of a sample of industrial corporations to their preferred and ordinary capital. *The Economist*.

(11) Yield of 2½% Consols. *Statistical Abstract for the United Kingdom* and *Statistical Summary of the Bank of England*.

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