

INTERNATIONAL TRADE IN CAPITAL GOODS

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DECLARATION

I, Ujwala Sakharam Kamble hereby declare that this thesis entitled, “*International Trade in Capital Goods*” is submitted for the award of Degree of Doctor of Philosophy in Economics to the Gokhale Institute of Politics and Economics, Pune 411004. It is an original contribution and has been completed during my tenure as a research scholar at Gokhale Institute of Politics and Economics, Pune. This thesis has not been submitted by me elsewhere for the award of any degree or diploma-part or full. The information gathered by me elsewhere for the thesis is original, true and factual. Such material as has been obtain from other source has been duly acknowledged in the thesis. I hereby request, to consider the thesis for the award of the degree of ‘Doctor of Philosophy’.

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Abstract

A central concern of this thesis is to take seriously the implication of the capital controversies for trade theory pointed out by Steedman et. al. (1979). The trade models developed in this thesis are of a “classical” nature. It has kept strictly intact the distinction between value capital and heterogeneous capital goods and allowing the latter to be internationally traded. The trade models developed in this thesis themselves may be regarded as *complete* in the sense that they determine all variables of interest. This thesis has developed multi-country multi-commodity models of international trade in static and dynamic settings. Accordingly, the thesis contains two main chapters.

The third chapter, “Leontief Trade Models” applies the static open Leontief model to study the international trade. Traditionally the Leontief input-output model has been used as methodological apparatus for testing alternative theories of trade. We have shown in this section how international trade theory itself can be written in terms of the Leontief model. The Leontief trade model then makes a claim to greater generality by accommodating trade between several countries in several commodities which has been very difficult for standard trade theories to do. The Leontief trade model determines the comparative advantages, trade patterns, terms of trade and currency exchange rate simultaneously. The price system of the Leontief static open model is the unified price system for all countries and can be referred to as world price system which determines a solution for prices. The Leontief trade model has the ability to synthesize the ‘pure’ and the ‘monetary’ aspects of international trade.

The fourth chapter, “Dynamic Trade Models’ goes further to apply the Sraffa (1960) system to study international trade. This chapter considers the problem of determining the international trade equilibrium for growing economic systems. In the dynamic model, if and when trade equilibrium exist, then it determines;

- (a) the pattern of specialization in production viz. which countries produce which commodities
- (b) the post-trade prices of commodities
- (c) the post-trade outputs of commodities
- (d) the pattern of gains from trade i. e. which country gains how much by importing which commodity from which country
- (e) the commodity wise volumes of inter-country exports and imports
- (f) the equilibrium exchange rates and the international terms of trade

- (g) all value magnitudes such as the gross national products, net national incomes and capital stocks in the countries
- (h) the post-trade real wage rates in the countries
- (i) the post-trade rates of growth of the countries
- (j) the post-trade function of distribution of incomes in the countries

In this chapter the effect of mutually gainful trade in capital goods on the prices and outputs of commodities as well as on the distributional variables and growth rates have been illustrated. Specifically, it is shown that international trade in capital goods has the effect of lowering the costs of production in the trading countries and consequently has the effect of increasing the rates of profit and growth.

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List of Abbreviations

A : The technical coefficients matrix

A^T : The transpose of matrix A

A^{-1} : The inverse of matrix A

B : The diagonal matrix of supplies

a_{ijK} : The technical coefficient in country k .

E_{ij} : The exchange rate between country i and j .

E_{ij}^k : The natural exchange rate

L_j : The labour endowment in country j

l_{ij} : The labour coefficient of industry i in country j

p_{ij} : The price of commodity i in country j

P : The price vector

X_{di} : The quantity demanded of commodity i

X_{si} : The quantity supplied of commodity i

x_i : The scale multipliers

W_j : The wage vector of country j

w_j : The money wage rate in country j

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Chapter 1

Introduction and Review of Literature

1.1: Introduction

International trade theory has been dominated for many decades by models that either explicitly, or implicitly evade altogether the questions of the heterogeneity of capital goods, and the issue of distribution as a consequence of international trade. The capital controversies of the 1960's have had inevitable implications for neoclassical trade theories that were based on the notion of capital as non-produced endowments whose quantity is defined independently of the prices of good. Four kinds of developments have taken place in the literature since then.

- (1) Steedman¹ and his co-authors wrote a series of papers to show that if the distinction between capital as the value of heterogeneous capital goods and the capital goods themselves is made, as it ought to be made, many of the standard propositions and theorems of modern trade theory would either fail to hold or would, at the very least, have to be heavily qualified.
- (2) Dornbusch, Fischer & Samuelson², Shiozawa³ and others, on the other hand, developed the Ricardian model and extended it to several commodities including trade in intermediate goods. Specifically, they showed that with international trade in intermediate goods it is no longer possible to solve the price system of any country in isolation from the price systems of its trading partners; the price system becomes international in nature.
- (3) Eaton and Kortum⁴ developed a model of multi-country multi-commodity trade which allowed trade in intermediates but assumed quite restrictively that the pattern of absolute and comparative advantages across countries and commodities

¹ Steedman, I. (ed.1977 a), *Fundamental Issues in Trade Theory*, Macmillan, London.

² Dornbusch, R. S. Fisher and P. A. Samuelson (1977) "Comparative Advantage and Payments in a Ricardian Model with a Continuum of Goods", *American Economic Review*, Vol. **67**, December.

³ Shiozawa, Yoshinori (2007) "A New Construction of Ricardian Trade Theory—A Many-country, Many-commodity Case with Intermediate Goods and Choice of Production Techniques", *Evolutionary and Institutional Economics Review*, Vol.3 (2).

⁴ Eaton and Kortum (2002) "Technology, Geography, and Trade", *Econometrica*, Vol. 70, No. 5, pp. 1741-1779.

can be represented by index numbers and countries can be monotonically ranked by those indexes.

- (4) New trade theory explained trade which focuses on the role of increasing returns to scale, intra-industry trade and the concentration of industries in locations where all the industries can have the maximal advantage in production and trade. New trade theorists relaxed the assumption of constant returns to scale, and some build up a huge industrial base in certain industries which will then allow those sectors to dominate the world.

While the Ricardian trade model of Dornbusch, Fisher and Samuelson (DFS) (1977) supposes that production is carried out by labour alone the Shiozawa (2007) models allow intermediate goods and labour to be agents of production. There is another difference in dimensionality between the models; the DFS model consider trade between countries in a (infinite) continuum of goods whereas the Shiozawa allows several country trade in discrete goods. Their principal results also differ. The DFS model determines the equilibrium exchange at the point where it is equal to the ratio of the wage rates of the countries whereas the Shiozawa model does not have much to say about the determination of the exchange rates or the international terms of trade.

The Eaton-Kortum model on the other hand is based on the DFS model but it's aggregative in nature. It is not designed to ask questions such as the cause of trade, the determination of comparative advantage in a multi-country settings and the determination of the international terms of trade. Instead its purpose is to incorporate the gravity model, and barriers to trade and derive expressions relating bilateral trade volumes to deviations from purchasing power parity and to technology and geographical barriers.

The development of "new trade theories" e. g. Krugman⁵(1981) is motivated entirely differently from the DFS or Shiozawa model. Its purpose is to explain that portion of trade (which of course has been growing in importance) that is not attributable to comparative advantage but to tastes and preferences for product varieties. Consequently it concentrates on differentiated products produced in monopolistically competitive industries in which firms produce under condition of increasing returns to scale. New trade theory has been successful in explaining intra-industry trade and the concentration

⁵ Krugman, P. R. (1981), "Intra-industry specialization and the gains from trade," *Journal of Political Economy*, 89, 959–973.

of industries in locations where all the industries can reap the maximal advantage in production and trade.

In the latest development in international trade theory the attention has been given to “trade between firms”. This development has been motivated by the almost universal observation that not all firms in an industry are involved in international trade. In fact that proportion is quite small. This has prompted inquiries into the factors that account for the comparative advantages that firms enjoy instead of entire industries which allows them but not their industry peers to export.

The trade models developed in this thesis are of a “classical” nature. They are not much concerned either with intra-industry trade or with trade between firms; they share stronger kinship with the theories of Graham (1948) and Shiozawa (2002). At the same time a central concern is to take seriously the implication of the capital controversies for trade theory pointed out by Steedman et. al. (1979). Accordingly, the thesis contain two main chapters. Chapter III applies the static open Leontief model to study the international trade and chapter IV goes further to apply the Sraffa (1960) system to study international trade.

The present thesis has three distinct features. Firstly, it formulates a model of international trade in capital as well as consumption goods. Secondly, it does not compartmentalise the ‘pure’ and ‘monetary’ aspects of trade. In other words, comparative advantage, trade patterns and exchange rates are determined together simultaneously. Thirdly, it does not confine trade to two countries and two commodities. Instead it formulates a method to determine international trade between several countries in several commodities. A constructive algorithm is proposed for numerical solutions and the international trade equilibrium is found in which the size and composition of international trade, the industrial outputs in the countries, the distributive variables, the prices of commodities, the exchange rate and the international terms of trade are fully determined.

1.2: Classical and Neoclassical Value & Trade Theories

The theory being advanced in this thesis is based on the classical theory of value as revived by Leontief (1953) and Sraffa (1960) as opposed to the neoclassical theory of value. Very naturally it differs from the neoclassical theory both in its approach as well

as the methodology. Trade theory is usually an extension of value theory. The following table shows the differences in approach and therefore in the results in classical and neoclassical value.

Table 1.1: Classical and Neoclassical Value Theory

Value Theory	Classical	Neoclassical
Given Data	Land, Labour, Technical coefficients and Consumer preferences.	Land, Labour, Capital, Production function and Consumer preferences.
Variables determined	Outputs of various goods, Relative prices, Level of consumption of goods, National incomes of economies and Capital stock.	Levels of outputs, Levels of consumptions, Relative prices

In the neoclassical value theory, for given endowment of labour and capital, resources and technology, goods are produced considering the standard production functions. The produced goods consumed by consumers according to their taste and preferences. The neo-classical value theory determines the relative prices of goods and their production and consumption at the equilibrium where production is always equal to the consumption at full employment.

In the classical value theory, for given endowment of labour goods are produced using specific technology. These goods then consumed by consumers according to their preferences. The classical value theory determines output of various goods, relative prices of commodities, level of consumption of consumption goods, national incomes of economies and the capital stocks and where applicable the rates of growth and distribution of income between profits and wage.

The following table shows the differences in approach and therefore in the results in classical and neoclassical trade theory.

Table 1.2: Classical and Neoclassical Trade Theory

Trade Theory	Classical	Neoclassical
Determination	Levels of post trade productions, trade pattern, volumes of export and imports, levels of consumption, prices, rates of growth, terms of trade, exchange rate, post-trade capital stocks, national incomes, wage rates.	Levels of outputs, post-trade production= post-trade consumption, volumes of exports and imports, terms of trade, equalised factor prices.
Special Features	Capital goods are traded, Non-tradable commodities are included in the trade.	Capital goods are not traded. Non-tradable goods are not considered in trade.

With usual assumption of immobility of factors of production neoclassical trade theory determines levels of outputs, post-trade production which is always equal to post-trade consumption, volumes of exports and imports, terms of trade and equalised factor prices. The neoclassical trade theory supposes that all goods are consumption goods.

In fact, there are some goods that cannot be consumed but they are traded internationally. The trade in these types of goods is not considered in neoclassical trade theory. Secondly, in international trade of neoclassical type, the production possibility frontiers are not affected, only the consumption possibility frontiers are affected. But what is generally seen in the international trade that, production possibilities are also undergo a change.

In the classical trade theory, for given endowments in capital, labours, technology and consumer preferences at international trade equilibrium, levels of post trade productions, trade pattern, volumes of export and imports, levels of consumption where applicable, prices, rates of growth, terms of trade, exchange rate, post-trade capital stocks, national incomes and wage rates are determined in static framework.

In our trade models we have shown that production possibilities changes after countries open to trade chiefly because capital goods are allowed to be traded so that if they are more cheaply available the production possibility frontiers of countries are

enlarged. The trade models that are developed considers consumption as well as capital goods. To construct dynamic trade models we have considered savings, investments and growth. In both static and dynamic context the trade models determine all variables of interest such as volume of world trade, terms of trade, direction of trade, national incomes, relative prices, exchange rates and capital stock. In addition to this dynamic model determines rates of growth and profit.

1.3: Chapter Scheme

This thesis consists of five chapters. The chapter scheme is as follows:

Chapter I reviews the literature on the subject of international trade theories and elaborates the research questions and objectives of the thesis.

Chapter II gives the basic theoretical framework, preliminaries of trade between countries and related methodology.

Chapter III elaborates the trade equilibrium in static framework with various illustrations.

Chapter IV illustrates the trade equilibrium in dynamic framework.

Finally, Chapter V consists of brief summary of findings and concluding remarks. This chapter also discusses the limitations of the thesis.

1.4: Review of Literature

The capital controversies of the 1960's raised a theoretical debate among economists of two schools of economic analysis concerning the nature and role of capital goods or means of production and led to a critique of the dominant neoclassical vision of aggregate production and distribution. These controversies have not yet been wholly resolved by economists. `

In fact, capital consists of a heterogeneous collection of several capital goods. Neo-classical trade theories have typically assumed that capital and labour are immobile between countries and are fully employed at home. However, this assumption is

unsustainable because individual capital goods that constitute the capital stocks of the countries are in fact commodities that are traded between countries. International trade and growth are closely related. Over the years there has been unprecedented growth in capitalist economies. As the growth in international trade has taken place, its composition has changed. Relative importance of intermediate goods and machinery that lead to capital accumulation and role of produced means of production both in domestic and world trade; has increased to that of final consumption goods trade. Accordingly, the subject of international trade in capital goods has been gaining empirical importance.

The dominant theory, established by Heckscher-Ohlin-Samuelson or as we shall refer to it henceforth, the H-O-S⁶ theory, has placed relatively little emphasis on such issues because the H-O-S model of trade is ultimately rooted in the general model of exchange economy. The simple H-O-S model presents production as being carried out by the unassisted primary inputs, land and labour with no role being played by produced means of production, either in the form of intermediates or in that of fixed capital goods. The H-O-S theory has said rather little about the continuous expansion of the output of both consumption goods and means of production. There has been development of H-O-S inspired theories of growth and trade, e.g. Johnson⁷ and Stiglitz⁸ who were concerned with the transition to long-run equilibrium, starting from an arbitrary 'capital-labour' ratio, in models assuming a single capital good, combined with labour following a neo-classical production function. But such analysis does not give much insight into the long-run equilibrium properties of multi-capital good models which must be developed to deal adequately with real world problems of trade and accumulation.

⁶ There are four basic theorems of the Heckscher-Ohlin-Samuelson (HOS) model:

(i) The Factor Price Equalization theorem, (ii) the Stolper Samuelson theorem, (iii) the Rybczynski theorem, and (iv) the Heckscher-Ohlin theorem. The factor price equalization theorem gives conditions under which trade in commodities is a perfect substitute for the international mobility of factors. The Stolper-Samuelson theorem gives conditions under which a change in relative commodity prices has an unambiguous effect on real factor returns. The Rybczynski theorem shows how a change in factor supply alters production, holding fixed all prices. Finally, the Heckscher-Ohlin theorem shows the relation between relative factor endowments and comparative advantage.

⁷ Johnson, H. G. (1957) "The Transfer Problem and Exchange Stability", *Journal of Political Economy*, Vol. **64**

⁸ Stiglitz, J. E. (1970) "Factor Prize Equalization in Dynamic Economy," *Journal of Political Economy*, Vol. **78**

Chapter 2: Preliminaries

The purpose of this chapter is to introduce some preliminary concepts that will be required for the development of the models articulated in chapter III and IV. Firstly, we shall develop a model to determine market clearing exchange rates in a multi-country world. Secondly, we shall develop a method to identify the pattern of comparative advantage direction of trade, gains of international trade and the method of scale multipliers in a multi-country multi-commodity setting.

2.1: The System of Foreign Exchange Rates

Consider a world that comprises of several politically sovereign and economically interdependent countries. They have distinct national currencies which are used for transactions within them. Economic interdependency among them implies, however, that these distinct currencies must be exchanged against one another. We shall further assume that currencies are legal tender and fiat moneys. The former implies that they will not be held outside the countries of their issue and latter implies that they will not be demanded for their own sake and their holdings gives no utilities. By assuming these two conditions we ensure that once payment in own currency made by a country will be routed back as a receipt through foreign exchange market.

We shall now begin to determine the unknown exchange rate by formulating the model based on foreign exchange market function of balancing the payments of all countries. Here the point needs to be noted that we are in the regime of flexible exchange rates. Suppose in the simplest case that the world comprises of only two countries A (India) and B (USA). Their currencies are INR and USD respectively. Suppose country A imports INR 40 million worth of goods from country B and country B imports USD 20 million worth of goods from country A. In order to determine the exchange rate we will consider trade balance equation of either of these two countries. Consider the trade balance equation of country A as follows,

$$\text{Import bill of country A} = \text{Export earnings of country B}$$

Therefore, $\text{INR } 40 \text{ million} \times \text{dollar-rupee rate} = \text{USD } 20 \text{ million}$

$\text{INR } 2 \times \text{dollar-rupee rate} = \text{USD } 1$

Thus the dollar-rupee rate is 0.5. The reciprocal of dollar-rupee exchange rate gives the rupee-dollar exchange rate = 2. In case of three countries A (India), B (USA) and C (Japan). Their currencies are INR and USD and JPY respectively. The inter-country payment between them are as shown in table 2.1.

Table 2.1: Inter-country Payments

(million)	Payments to			Total
	A	B	C	
Payments by				
Country A (RS.)	0	40	20	60
Country B (\$)	2	0	10	12
Country C(JPY)	100	200	0	300

In the equilibrium, the total payments must equal to the total receipts of the countries. We therefore need to find the set of exchange rates that clear exchange rate market. That is the payments in the domestic currency of each country to its receipts converted into domestic currency must equal. Thus we set up the balance of payments equations for all countries A, B, and C as follows.

$$(\text{USD}2 * \text{Rs} / \text{USD}) + (100\text{JPY} * \text{Rs} / \text{JPY}) = \text{Rs } 60$$

$$(\text{Rs}40 * \text{USD} / \text{Rs}) + (200\text{JPY} * \text{USD} / \text{JPY}) = \text{USD } 12$$

$$(\text{Rs}20 * \text{JPY} / \text{Rs}) + (10\text{USD} * \text{JPY} / \text{USD}) = \text{JPY}300$$

There should not be currency arbitrage opportunities, which is to say that the exchange rate must follow the following conditions.

$$(\text{Rs} / \text{USD}) \times (\text{USD} / \text{Rs}) = 1$$

$$(\text{JPY} / \text{USD}) \times (\text{USD} / \text{JPY}) = 1$$

Which automatically implies that $(\text{Rs} / \text{JPY}) \times (\text{USD} / \text{Rs}) = \text{USD} / \text{JPY}$. We can now convert all the balance of payments equations in one currency say, USD by multiplying first equation by USD / Rs and third equation by USD / JPY . Second equation is balance of equation of country B which is already in its own currency USD. Therefore, the equation will be as follows,

$$(USD2) + (100JPY * USD / JPY) = Rs\ 60 * (USD / Rs)$$

$$(Rs40 * USD / Rs) + (200JPY * USD / JPY) = USD\ 12$$

$$(Rs20 * USD / Rs) + (10USD) = JPY300 * (USD / JPY)$$

Now, we are left with 3 equations in 2 unknown exchange rates namely, the USD / Rs and USD / JPY . Solving any 2 equations simultaneously we get $USD / Rs = 10$ i. e. $USD1 = Rs. 10$ and $USD / JPY = 25$ i. e. $USD1 = JPY25$. These exchange rate will satisfy the third equation automatically. Hence if E_{ij} is the exchange rate between country i and j, then we can write in general,

$$E_{AB} E_{BA} = 1$$

$$E_{AB} E_{BC} = E_{AC}$$

The remaining all exchange rates are given in the following table.

Table 2.2: Exchange Rates

Currency Sold	Currency Purchased		
	A (RS.)	B(\$)	C(JPY)
Country A (RS.)	1	0.1	2.5
Country B(\$)	10	1	25
Country C(JPY)	0.4	0.04	1

Now consider a multi-country world consisting of n countries. Let P_i and R_i be the total payments made by country i to the rest of the world and total receipts of country i from the rest of the world in its own currency.

P_{ij} $i \neq j$ be the payments made by country i to country j and R_{ij} be the receipts of country i from country j. In equilibrium,

$$P_i = R_i$$

$$P_i = \sum_j P_{ij} = R_i = \sum_j R_{ij} \quad \forall i, j$$

The above form of equation can be shown as follows,

$$0 + P_{BA} E_{AB} + P_{CA} E_{AC} + \dots + P_{ZA} E_{AZ} = P_A = R_A$$

$$P_{AB} E_{BA} + 0 + P_{CB} E_{BC} + \dots + P_{ZB} E_{BZ} = P_B = R_B$$

.....

$$P_{AZ} E_{ZA} + P_{BZ} E_{ZB} + P_{CZ} E_{ZC} + \dots + 0 = P_Z = R_Z$$

Where E_{AB}^1 is the natural exchange rate (commodity price based exchange rate).

Similarly, country B gains by importing commodity 1 from A if following condition gets satisfy,

$$E_{BA} p_{1A} < p_{1B}$$

$$E_{BA} < \frac{p_{1B}}{p_{1A}} (= E_{BA}^1)$$

by applying neutrality condition $E_{AB} E_{BA} = 1$

$$E_{AB} > E_{AB}^1$$

If $E_{AB} = E_{AB}^1$ then neither of A nor B will gain by trading commodity 1.

Now suppose A gains by importing commodity 2 from country B,

$$\text{i.e. } E_{AB} < E_{AB}^2$$

Country B gains by importing commodity 1 from country A,

$$E_{AB} > E_{AB}^1$$

In other words A exports commodity 1 and B exports commodity 2. This trade assignments denoted by A-1 B-2. The necessary and sufficient condition for the trade assignment A-1 B-2 to be feasible if we combine above two equations,

$$E_{AB}^1 < E_{AB} < E_{AB}^2$$

The trade assignment A-2 B-1 will be feasible if,

$$E_{AB}^2 < E_{AB} < E_{AB}^1$$

Or in term of arbitrage sequence condition $E_{AB}^2 * \frac{1}{E_{AB}^1} < 1$ or $E_{AB}^2 * E_{BA}^1 < 1$

Finally, if $E_{AB}^1 = E_{AB} = E_{AB}^2$ holds then neither country A nor B gains or losses by trading.

In multi-country set up, if country B gains by importing commodity 1 from country A, then all other countries also gain by importing commodity 1 from country A if and only if the currency exchange satisfy two and three point neutrality condition. In notation form

currency of exchange satisfy $E_{AB}E_{BA} = 1$ and $E_{AB}E_{BC}E_{CA} = 1$ and if $E_{AB} > E_{AB}^1$ then for any other country C $E_{AC} > E_{AC}^1$

2.3: Gains of International Trade

For the set of currency exchange given in earlier section, now we can compute the quantities of different commodities that can be purchased in all the countries per unit of the currencies of their own. That is to say a unit of currency of country A can purchase $1/p_{iA}$ units of commodity i for $i=1, 2, \dots, n$ at home. Alternately, the same unit of currency A can be converted into E_{kA} units of the currency of country k and used to purchase E_{kA}/p_{ik} units of commodity i.

In general, a unit of the currency of country j say A can purchase quantities of different commodities from the different countries can be tabulated. We shall call this table as the gain from trade table.

Table 2.3: Country A's Gains of Trade

A	A	B	C	Z
1	$1/p_{1A}$	E_{BA}/p_{1B}	E_{CA}/p_{1C}	E_{ZA}/p_{1Z}
2	$1/p_{2A}$	E_{BA}/p_{2B}	E_{CA}/p_{2C}	E_{ZA}/p_{2Z}
3	$1/p_{3A}$	E_{BA}/p_{3B}	E_{CA}/p_{3C}	E_{ZA}/p_{3Z}
⋮	⋮	⋮	⋮		⋮
⋮	⋮	⋮	⋮		⋮
n	$1/p_{nA}$	E_{BA}/p_{nB}	E_{CA}/p_{nC}	E_{ZA}/p_{nZ}

On the basis of above gains from trade table, country A will decide to purchase different commodities from different countries. Suppose that boxed quantities are the maximum quantities of commodities that can be obtained per unit of A's currency. Therefore country A will buy those commodities from various countries. This gives the trade pattern as A-2, B-1,2, C-3, ..., Z-n. The ranking of the elements in rows of country B's gains of table

above is identical with A's table 2.3. To analyse this property consider country B's gains of trade table below.

Table 2.4: Country B's Gains of Trade

B	A	B	C	Z
1	E_{AB} / p_{1A}	$1 / p_{1B}$	E_{CB} / p_{1C}	E_{ZB} / p_{1Z}
2	E_{AB} / p_{2A}	$1 / p_{2B}$	E_{CB} / p_{2C}	E_{ZB} / p_{2Z}
3	E_{AB} / p_{3A}	$1 / p_{3B}$	E_{CB} / p_{3C}	E_{ZB} / p_{3Z}
⋮	⋮	⋮	⋮		⋮
⋮	⋮	⋮	⋮		⋮
⋮	⋮	⋮	⋮		⋮
n	E_{AB} / p_{nA}	$1 / p_{nB}$	E_{CB} / p_{nC}	E_{ZB} / p_{nZ}

The ranking of the elements of each row is identical for units of all currencies, provided the set of currency exchange rates E_{ij} is consistent. This can be seen by considering the table of quantities of different commodities that one unit of the currency of country B can purchase in the international markets. For example, consider the maximal element of row-1 column B in A's table 2.3. By assumption, it is the maximum element. Then in A's gains of trade table;

$$\frac{E_{BA}}{p_{1B}} > \frac{1}{p_{1A}}$$

$$\text{or } \frac{1}{p_{1B}} > \frac{1}{E_{BA} p_{1A}}$$

Which is same as $\frac{1}{p_{1B}} > \frac{E_{AB}}{p_{1A}}$ ($\because E_{AB} E_{BA} = 1$) i.e. the maximum element

of row 1 column B in country B's gains of trade table 2.4

Thus we have shown that gains of trade tables are invariant of currencies under consideration, that is to say rankings in A's and B's gains of trade tables are identical.

2.4: Comparative Advantage

We have seen the necessary and sufficient condition for the trade assignment A-1 B-2 to be feasible which is,

$$E_{AB}^1 < E_{AB} < E_{AB}^2$$

This inequality is nothing but the bilateral comparative advantage. The above equation implies

$$\begin{aligned} & E_{AB}^1 < E_{AB}^2 \\ \text{i.e.} \quad & \frac{P_{1A}}{P_{1B}} < \frac{P_{2A}}{P_{2B}} \quad \dots (1) \end{aligned}$$

$$\text{Which is same as} \quad \frac{P_{1A}}{P_{2A}} < \frac{P_{1B}}{P_{2B}} \quad \dots (2)$$

The inequality (1) is the statement of comparative advantage in terms of the natural exchange rate which is identical with inequality (2), the statement in terms of domestic ratios of exchange.

In case of several countries and several commodities, consider the case of trade between three countries A, B, C in say three commodities 1,2 and 3. We shall suppose that a trade pattern A-1, B-2, C-3 is feasible trade pattern. The necessary and sufficient conditions for the feasible trade assignment are as follows,

$$E_{AB}^2 = E_{AB}^2 = E_{AB} \quad \text{A-2, B-2}$$

$$E_{AB}^1 < E_{AB}^2 = E_{AB} \quad \text{A-1, B-2}$$

i.e. $\frac{P_{1A}}{P_{2A}} < \frac{P_{1B}}{P_{2B}}$ Which is the statement of comparative advantage of country A and B in producing commodities 1 and 2 respectively.

$$E_{BC}^2 < E_{BC}^3 \quad \text{C-3}$$

i.e. $\frac{P_{2B}}{P_{3B}} < \frac{P_{2C}}{P_{3CB}}$ Which is the statement of comparative advantage of country C in producing commodity 3. Thus the similar exercise can be done in m-country n-commodity framework.

2.5: Engel's Demand Equations

The demand functions considered in this thesis are homogeneous of degree zero in the prices and income. That is to say the proportionate change in all the prices and income have no effect on the quantities of commodities demanded. To represent the demand functions we shall particularly use Engel's demand equations. Suppose there are n industries which produce consumption commodities $1, 2, \dots, n$. Let l_1, l_2, \dots, l_n be the quantities of labour required to produce one unit of outputs of commodities $1, 2, \dots, n$. Suppose L to be the total labour in the economy and w be the uniform wage rate paid to workers in all industries. Then the wage bill (wL) is the national income (Y) and the price of the commodities must be equal to their unit costs of production, are

$$p_i = wl_i \quad i = 1, 2, \dots, n$$

Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be the proportional of national income that the wage earners spend to purchase commodities $1, 2, \dots, n$. These proportions are called and Engel's coefficients or propensities to consume. They are positive less than one, their sum is unity.

$$0 < \alpha_i < 1 \\ \sum \alpha_i = 1$$

Let X_{di} be the quantity demanded of commodity i . Then demand equations for the commodities can be written as,

$$p_i X_{di} = \alpha_i Y \quad i = 1, 2, \dots, n \\ X_{di} = \frac{\alpha_i Y}{p_i} = \frac{\alpha_i L}{l_i} \quad i = 1, 2, \dots, n$$

The quantity supplied by different industries denoted by X_{si} is,

$$X_{si} = \frac{wL_i}{p_i} \quad i = 1, 2, \dots, n$$

At equilibrium quantity demanded will be equal to quantity supplied which imply,

$$wL_i = \alpha_i Y = \alpha_i wL \quad i = 1, 2, \dots, n \\ L_i = \alpha_i L \quad i = 1, 2, \dots, n$$

From the above equations we can see that the supplies of commodities to be equal to the demands, the labour employed in the industries must be equal to their consumption coefficients multiplied by the total labour in the economic system. This is therefore become the rule for determining the labour allocation in industries which equates the demands and supplies of commodities.

2.6: The Method of Scale Multipliers

In section 2.5 the state of equilibrium of the economy has found using the price and demand equations of commodities. A given disequilibrium state can be transformed into the equilibrium state of the economy by using the method of scale multipliers. Suppose an economic system is in disequilibrium state i.e. the quantities demanded are not equal to supplies. To arrive at equilibrium, the outputs of industries needs to increase/decrease according to their demand. This could be done by a change in the pattern of labour allocation. There are scale multipliers which transform the disequilibrium state to its equilibrium state. The general formula for these scale multipliers is;

$$x_i = \frac{L_i^*}{L_i} = \frac{\alpha_i L}{L_i} \quad i = 1, 2, \dots, n$$

Where L_i is the labour employed in the i^{th} industry when the economic system is in a disequilibrium state, L_i^* is the equilibrium allocation, α_i is the consumption coefficient of i^{th} commodity and L is total labour in an economy.

Chapter 3: Leontief Trade model

3.1: Introduction

From its very beginnings international trade theory has been formulated in a general equilibrium perspective. The standard protocol has been to first work out the general equilibrium of autarkic economies on the assumption of given endowments, technologies and tastes and preferences. The autarkic equilibrium contains solutions of domestic relative prices and determines the pattern of comparative advantages. Next the international trade equilibrium, if it exists, is worked out to find the international terms of trade and the post-trade outputs of commodities. Finally, the results like the welfare gains etc. are extracted.

We shall follow the standard protocol of international trade theory. However, there will be some important departures so far as the assumptions are concerned. Firstly, capital in its value connotation will not be considered as an “endowment” in the Leontief-trade model; in other words, the distinction between physical capital and value capital will be strictly adhered to. Thus, we will speak only of the physical endowments of individual capital goods in relation to labour ratios not value capital to labour ratios. Secondly, since capital goods will be treated as produced commodities the assumption that the same physical size of each capital good will be employed both pre- and post-trade becomes unsustainable. We shall however suppose full employment of labour in the pre- and post-trade situations.

Thus we shall suppose to begin with that there are two static competitive economies producing two commodities by means of constant returns production functions and consuming them by means of homogenous of degree zero functions that are identical in both countries. This will facilitate a comparison between the results of the Leontief trade model with those of the standard theory. Remaining part of this chapter consists of five sub-sections. Section (3.2) gives the basic theoretical model to find out the equilibrium in the state of autarky. Section (3.3) deals with the two-country two-commodity trade models. Section (3.4) discusses two-country trade in several commodities. The last section (3.5) contains a more general model of trade between several countries producing several commodities. Finally, section (3.6) contains some important remarks on the current chapter.

3.2: Equilibrium in Autarky

In this section we will develop the basic theoretical model to find out the equilibrium in the state of autarky using only the notations which we will further use in the various sections of this chapter. Consider country A which produces two commodities that can be used as consumption as well as capital goods. Their price system of equations can be written as follows,

$$\begin{aligned} a_{11A}p_{1A} + a_{21A}p_{2A} + w_A l_{1A} &= p_{1A} \\ a_{12A}p_{1A} + a_{22A}p_{2A} + w_A l_{2A} &= p_{2A} \end{aligned} \quad \dots (1)$$

Where,

a_{ijA} : the quantity of commodity (i) required to produce one unit of output of commodity (j) in country A. ... i=1, 2. & j=1, 2.

l_{jA} : the requirement of labour in jth industry in country A ... j=1, 2.

w_A : the money wage rate in country A.

p_{jA} : the price of commodity j in country A.

The demand of each of these commodities is comprises of intermediate demand and final consumption demand. Thus,

Total quantity demanded of commodity 1 =Intermediate demand
+ Final consumption demand

$$\therefore (a_{11A} + a_{12A}) + F_{1A} \quad \dots (2)$$

F_{1A} : the final consumption demand for commodity 1 in country A.

Total quantity demanded of commodity 2 =Intermediate demand
+ Final consumption demand

$$\therefore (a_{21A} + a_{22A}) + F_{2A} \quad \dots (3)$$

Final consumption for our purpose can be described by Engel's equations. It should be made clear that this seemingly unduly restrictive assumption has been made only for simplicity and computational convenience. None of the substantial conclusions of this thesis are affected by it. All the general methods of this thesis holds even if it were substituted by more realistic demand systems like the linear expenditure system or almost ideal demand systems. All that is strictly required is that the demand functions should be

homogeneous of degree zero in income and price. Thus suppose proportions α_{1A} and α_{2A} ($\alpha_{1A} + \alpha_{2A} = 1$) of the wage income are spent on commodities 1 and 2. Then the quantities demanded for final consumption are

$$F_{iA} = \frac{\alpha_{iA} w_A l_A}{P_{iA}} \quad (i = 1, 2) \quad \dots (4)$$

System (1) can be written in the matrix form and solved simultaneously for prices as follows,

$$A_A^T P_A + w_A L_A = P_A$$

$$P_A = (I - A_A^T)^{-1} w_A L_A$$

Also, substituting the quantities demanded for final consumption at these prices from (4) into the Leontief output system, we obtain the equilibrium gross outputs (B_A) that are required to the quantities demanded for final consumption as'

$$B_A = (I - A_A)^{-1} F_A$$

3.3: Two Country Two Commodity Trade

In this section we will develop two-country two-commodity trade models. This section has five sub-sections. Section (3.3.1) identifies the factors that are responsible for non-existence of trade equilibrium with an illustration of two-country two-commodity trade. Section (3.3.2) discusses the conditions under which the trade equilibrium exists for the example of previous section (3.3.1). In the next two sections (3.3.3) and (3.3.4) we will construct the theory that can generate situations in which the trade pattern is consistent with the Heckscher-Ohlin prediction as well the Leontief paradox prediction. Section (3.3.5) deals with trade in heterogeneous capital goods and several consumption goods.

3.3.1: Non-Existence of Trade Equilibrium: An Illustration

Before we proceed to construct trade models, it may be worthwhile to point out some properties of the trade equilibrium with regard to existence and uniqueness. In international trade, it is not necessary to have a trade equilibrium always. It may or may

not exist depending on various factors. In this section we will examine the factors which are responsible for non-existence of trade equilibrium. Let us begin with the case of trade between two countries in two commodities. Suppose country A and B are producing two tradable commodities that are used as consumption as well as intermediate capital goods. Their labour endowments are 20 units and 25 units respectively. Each country spends half of its net national income on each of the commodities. That is to say Engel's coefficients for each commodity in all countries is (1/2). The wage rates are $w_A = USD 1$ in country A and $w_B = JPY 1$ in country B. Then their economic activities at autarky equilibrium are found out using the method discussed in section 3.2 as follows;

$$\begin{array}{r}
 \text{Country A} \\
 1. \quad 3.052 p_{1A} + 4.069 p_{2A} + 9.156 w_A = 20.347 p_{1A} \\
 2. \quad 3.943 p_{1A} + 4.928 p_{2A} + 10.843 w_A = 19.715 p_{2A} \quad \dots (1) \\
 \text{-----} \\
 \qquad \qquad \qquad 20 \\
 \text{Country B} \\
 1. \quad 5.749 p_{1B} + 4.791 p_{2B} + 16.290 w_B = 19.165 p_{1B} \\
 \quad \quad 5.443 p_{1B} + 4.354 p_{2B} + 8.709 w_B = 21.772 p_{2B} \quad \dots (2) \\
 \text{-----} \\
 \qquad \qquad \qquad 25
 \end{array}$$

The prices of commodities under conditions of autarky are,

$$\begin{array}{ll}
 p_{1A} = 0.7489 & p_{1B} = 1.5678 \\
 p_{2A} = 0.9330 & p_{2B} = 0.9899
 \end{array}$$

The corresponding natural exchange rates are,

$$\begin{array}{ll}
 E_{AB}^1 = \frac{p_{1A}}{p_{1B}} = 0.4776 & \therefore E_{BA}^1 = \frac{1}{E_{AB}^1} = 2.0933 \\
 E_{AB}^2 = \frac{p_{2A}}{p_{2B}} = 0.9425 & \therefore E_{BA}^2 = \frac{1}{E_{AB}^2} = 1.0609
 \end{array}$$

Since $E_{AB}^1 < E_{AB}^2$, country A has the comparative advantage in the production of commodity 1 and country B in the production of commodity 2. Or, in terms of the arbitrage sequence conditions, since the lowest product of the above exchange rates is,

$$E_{AB}^1 * E_{BA}^2 = 0.4503$$

The trade assignment is A-1, B-2. Our task is to see whether this trade assignment is an equilibrium assignment and, if not, what the responsible factors are. Suppose that A-1, B-2 is the world trade pattern where country A specialises in commodity 1 and country B in commodity 2. Each country uses all of its resources in its industry of specialisation. Then the international price system of countries in the post trade situation would be represented as follows,

Country A

$$1. \quad 6.6667p_{1A} + 8.8881p_{2B}E_{AB} + 20w_A = 44.4451p_{1A} \quad \dots (3)$$

.....Industry 2 is closed.....

Country B

.....Industry 1 is closed.....

$$2. \quad 15.6228p_{1A}E_{BA} + 12.4971p_{2B} + 25w_B = 62.4913p_{2B} \quad \dots (4)$$

We shall convert country B's price system (4) in A's currency by multiplying it by E_{AB} ,

$$15.6228p_{1A} + 12.4971p_{2B}E_{AB} + 25w_BE_{AB} = 62.4913p_{2B}E_{AB} \quad \dots (5)$$

Observe that the price equations (3) and (4) of the industries have been expressed in terms of the own currencies of the countries. Thus the dollar-yen exchange rate E_{AB} has been attached to the price of commodity 2 which is imported by country A and yen-dollar exchange rate E_{BA} has been attached to the price of commodity 1 which is imported by country B. However, the exchange rate E_{AB} and E_{BA} need to qualify as the equilibrium exchange rates by fulfilling following three requirements. (1) They should be such as to prevent currency arbitrage i.e. $E_{AB}E_{BA} = 1$. (2) Country A and B must gain from trade by trading at these exchange rates. (3) World demands must equate supplies at these exchange rates.

To find whether the configuration of activities given by price equations (3) and (5) is an international trade equilibrium, we set up the world demand- supply equations for the

commodities; for the two tradable commodities the sources of demands are domestic and foreign. Therefore the market clearing equation for commodity 1 is as follows,

$$\frac{L_A}{l_{1A}} = \left(\frac{\alpha_{1A} w_A L_A + \alpha_{1B} w_B L_B E_{AB}}{p_{1A}} \right) + \left(\frac{a_{11A} L_A}{l_{1A}} + \frac{a_{12B} L_B}{l_{2B}} \right)$$

Where left hand side (LHS) of the above equation is supply and right hand side (RHS) is demand for commodity 1 which again has two components;

Total demand= Final consumption demand+ Intermediate demand

$$= \left(\frac{\alpha_{1A} w_A L_A}{p_{1A}} + \frac{\alpha_{1B} w_B L_B E_{AB}}{p_{1A}} \right) + \left(\frac{a_{11A} L_A}{l_{1A}} + \frac{a_{12B} L_B}{l_{2B}} \right)$$

Where, $\left(\frac{\alpha_{1A} w_A L_A}{p_{1A}} + \frac{\alpha_{1B} w_B L_B E_{AB}}{p_{1A}} \right)$

= (Domestic final consumption demand +International final consumption demand)

And $\left(\frac{a_{11A} L_A}{l_{1A}} + \frac{a_{12B} L_B}{l_{2B}} \right)$

= (Domestic intermediate demand +International intermediate demand)

The size of the intermediate demand depends directly on the post-trade outputs

produced in them; country A produces $\left(\frac{L_A}{l_{1A}} \right)$ units of commodity 1 post-trade which

requires $\left(\frac{a_{11A} L_A}{l_{1A}} \right)$ units of commodity 1 as input. Country B produces $\left(\frac{L_B}{l_{2B}} \right)$ units of

commodity 2 post-trade which requires $\left(\frac{a_{12B} L_B}{l_{2B}} \right)$ units of commodity 1 as input.

Similarly, commodity 2's demand supply equation is as follows.

$$\frac{L_B}{l_{2B}} = \left(\frac{\alpha_{2A} w_A L_A E_{BA} + \alpha_{2B} w_B L_B}{p_{2B}} \right) + \left(\frac{a_{21A} L_A}{l_{1A}} + \frac{a_{22B} L_B}{l_{2B}} \right)$$

Plugging the numerical values in above market clearing equations,

$$44.4451 = \left(\frac{0.5 \times 1 \times 20 + 0.5 \times 1 \times 25 E_{AB}}{p_{1A}} \right) + (6.6667 + 15.6228) \quad \dots (6)$$

$$62.4913 = \left(\frac{0.5 \times 1 \times 20E_{BA} + 0.5 \times 1 \times 25}{P_{2B}} \right) + (8.8881 + 12.4971) \quad \dots (7)$$

The equations (6) and (7) are linearly dependent, therefore one of them can be used to solve the exchange rate E_{AB} . However notice that there is a circularity. The prices p_{1A} and p_{2B} in the post-trade situation cannot be determined until the exchange rate E_{AB} is determined. On the other hand E_{AB} cannot be determined until at least one of p_{1A} and p_{2B} known. So this circularity has to be resolved by proceeding iteratively. Thus substituting initially the autarky price p_{1A} in the equation (6) the initial solution for the exchange rate is obtained as $E_{AB}^{(1)} = 0.4076$. This exchange rate can be substituted in equations (4) and (5) to get first approximate prices for $p_{1A}^{(1)}$ and $p_{2B}^{(1)}$ by solving them simultaneously. This value of $p_{1A}^{(1)}$ can be used in equation (6) to get second approximate solution for exchange rate $E_{AB}^{(2)}$ and this $E_{AB}^{(2)}$ can be used in equations (4) and (5) to get second approximate solution for prices $p_{1A}^{(2)}$ and $p_{2B}^{(2)}$ & so on.

After 28 such iterations the converged solutions for exchange rate and prices are obtained up to 6 decimal places of accuracy as $E_{AB} = 0.2744$, $p_{1A} = 0.6062$, $p_{2B} = 1.1902$. To calculate gains from trade we need price of commodity 2 in country A (p_{2A}) and price of commodity 1 in country B (p_{1B}). These prices can be calculated using post trade prices p_{1A} , p_{2B} and the exchange rate E_{AB} . In country A, if commodity 2 was produced then its price would have been $p_{2A} = 0.7529$ and the price of commodity 1 in country B would have been $p_{1B} = 1.1810$. Next we complete the gains from trade table:

Table 3.1: Gains from Trade in Non-Supporting Trade Pattern

Commodity	Country A	Country B
1	1.649	2.012*
2	1.328	3.060*

* The maximum quantity

The gains from trade table does not support the trade pattern A-1, B-2. Therefore we conclude that it cannot be an equilibrium trade pattern because the market clearing exchange rates corresponding to this trade pattern do not give a supporting pattern of the

gains of trade.

From the gains from trade table (3.1) we observe that country B has comparative advantage in both the commodities. Country B would find profitable to produce and export commodity 1 and 2 to country A. Accordingly let us set the next trade pattern as B-1, 2. But since both of A's industries are closed there would not be any intermediate demand for production. Also the entire labour will become unemployed therefore there would not be any income generation hence there will not be any consumption demand for both the commodities. So the trial pattern B-1, 2 is not workable and the process of finding international trade equilibrium terminates here itself.

From first trial trade pattern A-1, B-2 and the gains from trade table 3.1 it is clear that country A definitely does not have comparative advantage in producing commodity 2 but it had comparative advantage in producing commodity 1 in first trade pattern. Taking into account the limitations of last trial trade pattern, the next trial trade pattern can be set as A-1, B-1, 2. The world price system now will look as follows,

Country A

$$1. \quad 6.6667p_{1A} + 8.8881p_{2B}E_{AB} + 20w_A = 44.4451p_{1A} \quad \dots (8)$$

.....Industry 2 is closed.....

Country B

$$1. \quad 5.749p_{1B} + 4.791p_{2B} + 16.290w_B = 19.165p_{1B} \quad \dots (9)$$

$$2. \quad 5.443p_{1B} + 4.354p_{2B} + 8.709w_B = 21.772p_{2B} \quad \dots (10)$$

The international and domestic demands of both the countries for commodity 1 shall be satisfied by both the countries A and B. Let x_{1B} and x_{2B} be the scale multipliers which when multiply with price equations for industry 1 and 2 in country B respectively, will give rise to levels of output that are required to internationally clear the markets for commodity 1 and 2. Now there will be two market clearing equations and one full employment equation for country B.

$$\frac{L_A}{l_{1A}} + \frac{L_{1B}x_{1B}}{l_{1B}} = \frac{M_1}{p_{1A}} + \left[\frac{a_{11A}L_A}{l_{1A}} + \frac{a_{11B}L_{1B}x_{1B}}{l_{1B}} + \frac{a_{12B}L_{2B}x_{2B}}{l_{2B}} \right]$$

$$\left(\frac{L_{2B}x_{2B}}{l_{2B}} \right) E_{AB}^1 = \frac{M_2}{p_{2B}} + \left[\frac{a_{21A}L_A}{l_{1A}} + \frac{a_{21B}L_{1B}x_{1B}}{l_{1B}} + \frac{a_{22B}L_{2B}x_{2B}}{l_{2B}} \right] E_{AB}^1$$

where $M_i = \alpha_{iA} w_A L_A + \alpha_{iB} w_B L_B E_{AB}$

and

$$L_{1B}x_{1B} + L_{2B}x_{2B} = L_B$$

The exchange rate $E_{AB} = E_{AB}^1$ because both A and B produce commodity 1 and there is no gain to either by trading in it. The LHS of first two equations are supplies and RHS are demands. Commodity 1 is produced in common in A and B, so their demand for commodity 1 will be satisfied by both the countries. Also, the exchange rate is fixed based on commodity 1's relative price i.e. $E_{AB} = E_{AB}^1 = p_{1A} / p_{1B}$. After putting numerical values in the above market clearing equations we get,

$$44.4451 + 19.165x_{1B} = \frac{0.5 \times 1 \times 20 + 0.5 \times 1 \times 25 \times 0.4776}{p_{1A}} + [6.6667 + 5.749x_{1B} + 5.443x_{2B}] \dots (11)$$

$$(21.772x_{2B}) \times E_{AB}^1 = \frac{0.5 \times 1 \times 20 + 0.5 \times 1 \times 25 \times E_{AB}^1}{p_{2B}} + [8.8881 + 4.791x_{1B} + 4.354x_{2B}] \times E_{AB}^1 \dots (12)$$

$$16.29x_{1B} + 8.71x_{2B} = 25 \dots (13)$$

In equations (11) and (12) the unknown prices and scale multipliers appeared as the multiples of one another. To solve these equations the method of iteration is used. For the first iteration the autarky prices are plugged into equations (11) and (12) and then solved them for two scale multipliers to get first approximate solutions $x_{1B}^{(1)}$ and $x_{2B}^{(1)}$. This solution of scale multipliers then used in equations (11) and (12) to get the first approximate solution for prices $p_{1A}^{(1)}$ and $p_{2B}^{(1)}$. The first approximate solution for the price of commodity 1 in country B, $p_{1B}^{(1)}$ can be easily calculated by plugging values of $p_{2B}^{(1)}$ and $p_{1A}^{(1)}$ in equation (8). Then the first approximate solution for exchange rate becomes $E_{AB} = E_{AB}^{(1)} = p_{1A}^{(1)} / p_{1B}^{(1)}$. We repeat this procedure until all solutions converged.

After 16 such iterations the solution obtained up to the accuracy of 6 decimal places is $E_{AB} = 0.3965$, $x_{1B} = 0.4352$, $x_{2B} = 3.5937$, $p_{1A} = 0.6217$, $p_{2A} = 0.7725$, $p_{1B} = 1.5678$, $p_{2B} = 0.9899$. With this values of x_{1B} and x_{2B} equation (13) may be read as $(16.2908)(0.4352) + (8.709)(3.5937) = 37.91 \neq 25$, which is inconsistent. Also, if one

of the equations (11), (12) would have solved simultaneously with equation (13) after plugging autarky equilibrium prices then the solution would have been $E_{AB} = 0.2744, x_{1B} = 0, x_{2B} = 2.8705$ indicating that country B should closed down industry 1. If commodity 1 is removed from B's trade pattern then the trade pattern will be revert to A-1, B-2 which was not an equilibrium trade pattern as found earlier.

To conclude, the trade equilibrium may not exist despite having differences in the comparative costs of production. In this particular example the non-existence of trade equilibrium is chiefly due to the relative sizes of labour endowments of both trading countries. That is to say, the labour in country B remained underemployed whereas country A requires more than existing labour to meet international demand for commodities post-trade. However, if a trade equilibrium exists then it will be unique. This follows from the conditions for uniqueness of exchange rates which were spelt out at the beginning of this section.

3.3.2: Two Country Trade Equilibrium

In this section we shall work out the trade equilibrium for the same example of section 3.3.1. The trade patterns in previous section A-1, B-2 and A-1, B-1, 2 were not feasible because of various reasons discussed in the previous section. All other conditions remaining the same the trade pattern A-1, B-2 would have been feasible trade pattern if,

1. Labour endowment in country A was more than existing 20 units.
2. Labour endowment in country B was less than existing 25 units.

Following cases will illustrate these two situations.

Case 1) Suppose country B's labour is constant at 25 units. We set country A's labour at 28 units more than its existing 20 units. Then autarky equilibrium looks as follows,

Country A

$$\begin{aligned}
 1. \quad & 4.2728p_{1A} + 5.6966p_{2A} + 12.8184w_A = 28.4858p_{1A} \\
 2. \quad & 5.5202p_{1A} + 6.8992p_{2A} + 15.1816w_A = 27.601p_{2A} \quad \dots (1)
 \end{aligned}$$

Country B

$$\begin{aligned} 1. \quad & 5.7491p_{1B} + 4.7911p_{2B} + 16.2901w_B = 19.1652p_{1B} \\ 2. \quad & 5.4431p_{1B} + 4.3541p_{2B} + 8.7091w_B = 21.7721p_{2B} \end{aligned} \quad \dots (2)$$

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The autarky prices are,

$$\begin{aligned} p_{1A} &= 0.7489 & p_{1B} &= 1.5678 \\ p_{2A} &= 0.9330 & p_{2B} &= 0.9899 \end{aligned}$$

The corresponding natural exchange rates are,

$$\begin{aligned} E_{AB}^1 &= \frac{p_{1A}}{p_{1B}} = 0.4776 & \therefore E_{BA}^1 &= \frac{1}{E_{AB}^1} = 2.0933 \\ E_{AB}^2 &= \frac{p_{2A}}{p_{2B}} = 0.9425 & \therefore E_{BA}^2 &= \frac{1}{E_{AB}^2} = 1.0609 \end{aligned}$$

Since, $E_{AB}^1 = \frac{p_{1A}}{p_{1B}} = 0.4776 < 0.9425 = \frac{p_{2A}}{p_{2B}} = E_{AB}^2$ the trade pattern is A-1, B-2.

The post-trade economic activities are,

Country A

$$1. \quad 9.3333p_{1A} + 12.4434p_{2B}E_{AB} + 28w_A = 62.2232p_{1A} \quad \dots (3)$$

.....Industry 2 is closed.....

Country B

.....Industry 1 is closed.....

$$2. \quad 15.6228p_{1A}E_{BA} + 12.4971p_{2B} + 25w_B = 62.4913p_{2B} \quad \dots (4)$$

The market clearing equation for commodity 1 is as follows,

$$\frac{L_A}{l_{1A}} = \left(\frac{\alpha_{1A}w_A L_A + \alpha_{1B}w_B L_B E_{AB}}{p_{1A}} \right) + \left(\frac{a_{11A}L_A}{l_{1A}} + \frac{a_{12B}L_B}{l_{2B}} \right)$$

Similarly, commodity 2's demand supply equation is as follows.

$$\frac{L_B}{l_{2B}} = \left(\frac{\alpha_{2A} w_A L_A E_{BA} + \alpha_{2B} w_B L_B}{p_{2B}} \right) + \left(\frac{a_{21A} L_A}{l_{1A}} + \frac{a_{22B} L_B}{l_{2B}} \right)$$

Plugging the numerical values in above market clearing equations,

$$62.2232 = \left(\frac{0.5 \times 1 \times 28 + 0.5 \times 1 \times 25 E_{AB}}{p_{1A}} \right) + (9.3333 + 15.6228) \quad \dots (6)$$

$$62.4913 = \left(\frac{0.5 \times 1 \times 28 E_{BA} + 0.5 \times 1 \times 25}{p_{2B}} \right) + (12.4434 + 12.4971) \quad \dots (7)$$

The equations (6) and (7) are linearly dependent, therefore equation (6) can be used to solve the exchange rate E_{AB} . The prices p_{1A} and p_{2B} in the post-trade situation cannot be determined until the exchange rate E_{AB} is determined. On the other hand E_{AB} cannot be determined until at least one of p_{1A} and p_{2B} known. So this circularity has to be resolved by proceeding iteratively. Thus substituting initially the autarky price p_{1A} in the equation (6) the initial solution for the exchange rate is obtained as $E_{AB}^{(1)}$. This exchange rate can be substituted in equations (4) and (5) to get first approximate prices for $p_{1A}^{(1)}$ and $p_{2B}^{(1)}$ by solving the equations simultaneously. This value of $p_{1A}^{(1)}$ can be used in equation (6) to get second approximate solution for exchange rate $E_{AB}^{(2)}$ and this $E_{AB}^{(2)}$ can be used in equations (4) and (5) to get second approximate solution for prices $p_{1A}^{(2)}$ and $p_{2B}^{(2)}$ and so on until all solutions converged.

After 8 such iterations the converged solutions for exchange rate and prices are obtained up to 6 decimal places of accuracy as $E_{AB} = 0.9391$, $p_{1A} = 0.6907$, $p_{2B} = 0.7299$. To calculate gains from trade we need price of commodity 2 in country A (p_{2A}) and price of commodity 1 in country B (p_{1B}). These prices can be calculated using post trade prices p_{1A} , p_{2B} and the exchange rate E_{AB} . In country A, if commodity 2 was produced then its price would have been $p_{2A} = 0.9175$ and the price of commodity 1 in country B would have been $p_{1B} = 1.4749$. Next we complete the gains from trade table.

Table 3.2: Gains from Trade in Two Country Trade Equilibrium
(Country B's labour constant)

Commodity	Country A	Country B
1	1.4479*	0.7220
2	1.0899	1.4589*

* The maximum quantity

Thus, the above gains from trade table supports the trade pattern A-1, B-2. Therefore, we conclude that the world economy described by equations (3) and (4) is a state of international trade equilibrium.

Some observations from a comparison of the autarky and post-trade situations may be summarized as follows,

Table 3.3: Pre and Post Trade Prices, Output and Real Wage Rates
(Country B's labour constant).

	Country A		Country B	
	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	0.7489	0.6907	1.5678	0.7355
p_2	0.933	0.6854	0.9899	0.7299
B_1	28.4858	62.2232	19.165	-----
B_2	27.601	-----	21.772	62.4913
w/p_1	1.3353	1.4478	0.6378	1.3596
w/p_2	1.0718	1.4589	1.0102	1.3701

Table 3.3 shows that, the world outputs of all commodities have increased, the post-trade prices of all commodities have declined in both the countries and consequently real wage rates have increased in both countries. Hence, both countries gain by trading the commodities according to their comparative advantages.

Case2) Suppose now country A's labour is constant at 20 units. If country B's labour is decreased to 18 units from its existing 25 units, then autarky equilibrium is,

Country A

$$1. \quad 3.052p_{1A} + 4.069p_{2A} + 9.156w_A = 20.347p_{1A}$$

$$2. \quad 3.943p_{1A} + 4.928p_{2A} + 10.843w_A = 19.715p_{2A} \quad \dots (1)$$

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Country B

$$1. \quad 4.1393p_{1B} + 3.4495p_{2B} + 11.7288w_B = 13.7988p_{1B}$$

$$2. \quad 3.9190p_{1B} + 3.1349p_{2B} + 6.2712w_B = 15.6758p_{2B} \quad \dots (2)$$

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Since we kept labour coefficients, technical coefficients, consumption coefficients, wage rate same as they were in previous example (1), the autarky prices remained same as in case (1). Hence the trade pattern is A-1, B-2, it can be seen from,

$$E_{AB}^1 = \frac{P_{1A}}{P_{1B}} = 0.4776 < 0.9425 = \frac{P_{2A}}{P_{2B}} = E_{AB}^2$$

The post-trade economic activities are,

Country A

$$1. \quad 6.6667p_{1A} + 8.8881p_{2B}E_{AB} + 20w_A = 44.4451p_{1A} \quad \dots (3)$$

.....Industry 2 is closed.....

Country B

.....Industry 1 is closed.....

$$2. \quad 11.2486p_{1A}E_{BA} + 8.9979p_{2B} + 18w_B = 44.9938p_{2B} \quad \dots (4)$$

The market clearing equation for commodity 1 is as follows,

$$\frac{L_A}{l_{1A}} = \left(\frac{\alpha_{1A}w_A L_A + \alpha_{1B}w_B L_B E_{AB}}{P_{1A}} \right) + \left(\frac{a_{11A}L_A}{l_{1A}} + \frac{a_{12B}L_B}{l_{2B}} \right)$$

Similarly, commodity 2's demand supply equation is as follows.

$$\frac{L_B}{l_{2B}} = \left(\frac{\alpha_{2A} w_A L_A E_{BA} + \alpha_{2B} w_B L_B}{p_{2B}} \right) + \left(\frac{a_{21A} L_A}{l_{1A}} + \frac{a_{22B} L_B}{l_{2B}} \right)$$

Plugging the numerical values in above market clearing equations,

$$44.4451 = \left(\frac{0.5 \times 1 \times 20 + 0.5 \times 1 \times 18 E_{AB}}{p_{1A}} \right) + (6.6667 + 11.2486) \quad \dots (6)$$

$$44.9938 = \left(\frac{0.5 \times 1 \times 20 E_{BA} + 0.5 \times 1 \times 18}{p_{2B}} \right) + (8.8881 + 8.8979) \quad \dots (7)$$

The equations (6) and (7) are linearly dependent, therefore equation (6) can be used to solve the exchange rate E_{AB} . By proceeding iteratively. After 10 such iterations the converged solutions for exchange rate and prices are obtained up to 6 decimal places of accuracy as $E_{AB} = 0.9163$, $p_{1A} = 0.6731$, $p_{2B} = 0.7346$. To calculate gains from trade we need price of commodity 2 in country A (p_{2A}) and price of commodity 1 in country B (p_{1B}). These prices can be calculated using post trade prices p_{1A} , p_{2B} and the exchange rate E_{AB} . In country A, if commodity 2 was produced then its price would have been $p_{2A} = 0.9167$ and the price of commodity 1 in country B would have been $p_{1B} = 1.4766$. Next we complete the gains from trade table:

Table 3.4: Gains from Trade for Two Country Trade Equilibrium
(Country A's labour constant).

Commodity	Country A	Country B
1	1.4540*	0.7391
2	1.0908	1.4856*

*The maximum amount

Thus, the above gains from trade table supports the trade pattern A-1, B-2. Therefore, international trade equilibrium is found.

As shown in table 3.6 the world outputs of all commodities have increased, the post-trade prices of all commodities have declined in both the countries and consequently real wage rates have increased in both countries. Both countries gain by trading.

Table 3.5: Pre and Post Trade Prices, Output and Real Wage Rates
(Country A's labour constant).

	Country A		Country B	
	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	0.7489	0.6878	1.5678	0.7506
p_2	0.933	0.6731	0.9899	0.7346
B_1	20.3470	44.4451	13.7988	-----
B_2	19.7150	-----	15.6758	44.9938
w/p_1	1.3353	1.4539	0.6378	1.3323
w/p_2	1.0718	1.4856	1.0102	1.3613

The conclusion of above two case is that the international trade equilibrium may not exist. If it exist then it will be unique as shown in case (1) and (2). The equilibrium found in the previous examples was entirely due to their special character. In order not to leave a doubt regarding the question of the determinacy of the international trade equilibrium, we will construct the theory that can generate situations in which the trade pattern is consistent with the Heckscher-Ohlin prediction as well the Leontief paradox prediction in the next two sections (3.3.3) and (3.3.4).

3.3.3: Leontief Paradox

Consider the following numerical example of two countries producing two goods both of which serve as means of production and for final consumption in Leontief trade model. Their autarky equilibria are as follows;

$$\begin{array}{rcl}
 \text{Country A} & & \text{Country B} \\
 A_{11A} p_{1A} + L_{1A} w_A = X_{1A} p_{1A} & & A_{11B} p_{1B} + L_{1B} w_B = X_{1B} p_{1B} \\
 A_{12A} p_{1A} + L_{2A} w_A = X_{2A} p_{2A} & & A_{12B} p_{1B} + L_{2B} w_B = X_{2B} p_{2B} \\
 \underline{X_{1A}} & \quad \quad & \underline{X_{1B}} \quad \quad \underline{L_B} \\
 \underline{L_A} & & \\
 & & \dots (1)
 \end{array}$$

The conditions that must be fulfilled for the Leontief paradox to exist can now be stated. We need under autarky the following conditions to be met, i.e.

- i) Country A is relatively capital abundant.

$$\frac{X_{1A}}{L_A} > \frac{X_{1B}}{L_B}$$

ii) Commodity 1 is capital intensively produced in both countries.

$$\frac{a_{11A}}{l_{1A}} > \frac{a_{12A}}{l_{2A}}, \quad \frac{a_{11B}}{l_{1B}} > \frac{a_{12B}}{l_{2B}}$$

iii) Yet, the capital abundant country A has the comparative advantage in commodity 2.

$$\frac{p_{2A}}{p_{1A}} < \frac{p_{2B}}{p_{1B}}$$

For (iii) to hold

$$\frac{l_{1A}}{a_{12A}l_{1A} + (1 - a_{11A})l_{2A}} > \frac{l_{1B}}{a_{12B}l_{1B} + (1 - a_{11B})l_{2B}}$$

In the post-trade situation the position of the countries will be as follows

<i>Country A</i>	<i>Country B</i>
(1). <u>Industry 1 is closed</u>	$\left(a_{11B} \frac{L_B}{l_{1B}} \right) p_{1B} + L_B w_B = \left(\frac{L_B}{l_{1B}} \right) p_{1B}$
(2) $\left(a_{12A} \frac{L_A}{l_{2A}} \right) p_{1B} E_{AB} + L_A w_A = \left(\frac{L_A}{l_{2A}} \right) p_{2A}$	<u>Industry 2 is closed</u>

Where E_{AB} is the exchange rate (units of currency A per unit of currency B).

(iv) The post-trade endowments should show A to be relatively capital abundant.

$$\left[\left(a_{12A} \frac{L_A}{l_{2A}} \right) / L_A \right] > \left[\left(a_{11B} \frac{L_B}{l_{1B}} \right) / L_B \right]$$

i. e. $\frac{a_{12A}}{l_{2A}} > \frac{a_{11B}}{l_{1B}}$

(v) Next, neither country must lose by trading with at least one of them positively gaining from trade. This requires;

$$\frac{p_{1B}}{p_{2B}} \leq T_{12} \leq \frac{p_{1A}}{p_{2A}} \quad \text{in commodity terms}$$

$$\frac{P_{2A}}{P_{2B}} \leq E_{AB} \leq \frac{P_{1A}}{P_{1B}} \quad \text{in exchange rate terms}$$

Where T_{12} is the terms of trade, i.e. the international ratio of commodity exchange.

(vi) Finally, A's requirement of commodity 1 post trade must be equal to or less than B's exportable surplus of that commodity,

$$a_{12A} \frac{L_A}{l_{2A}} \leq (1 - a_{11B}) \frac{L_B}{l_{1B}}$$

Which means that

$$\frac{L_A}{L_B} \leq \left(\frac{1 - a_{11B}}{a_{12A}} \right) \left(\frac{l_{2A}}{l_{1B}} \right)$$

The following illustration will give clearer picture of Leontief paradox.

Now consider two countries A and B whose autarkic equilibria are as follows;

Country A

$$(1) \quad 34.0909p_{1A} + 24.5454w_A = 40.9090p_{1A}$$

$$(2) \quad \underline{6.8181}p_{1A} + \underline{5.4545}w_A = 10.9090p_{2A}$$

$$\underline{40.9090} \qquad \underline{30.00}$$

Country B

$$(1) \quad 3.589p_{1B} + 6.666w_B = 10.255p_{1B}$$

$$(2) \quad \underline{6.666}p_{1B} + \underline{33.333}w_A = 26.666p_{2A}$$

$$\underline{10.255} \qquad \underline{40}$$

The wage rates and autarky prices are as follows.

$$w_A = \$1$$

$$w_B = £1$$

$$p_{1A} = \$3.6$$

$$p_{1B} = £1$$

$$p_{2A} = \$2.75$$

$$p_{2B} = £1.5$$

Although country A is relatively better endowed in capital relative to labour as compared to B and commodity 1 is relatively capital intensively produced in both countries than commodity 2, it is the labour abundant country B which has the comparative advantage in the production of commodity 1, which can be depicted in commodity terms as,

$$\frac{P_{1B}}{P_{2B}} = 0.6666 < \frac{P_{1A}}{P_{2A}} = 1.3090$$

or equivalently in exchange rate terms as,

$$E_{AB}^2 = \frac{P_{2A}}{P_{2B}} = 1.8333 < 3.6 = \frac{P_{1A}}{P_{1B}} = E_{AB}^1$$

Open the countries to trade and let A and B specialize in commodities 2 and 1 respectively and employ all their labour in their production. The situation will be as follows,

<i>Country A</i>	<i>Country B</i>	
(1) <i>Industry 1 is closed</i>	$21.538p_{1B} + 40w_B = 61.238p_{1B}$... (3)
(2) $37.5p_{1B}E_{AB} + 30w_A = 60p_{2A}$	<i>Industry 2 is closed</i>	

The world outputs of both commodities have increased due to specialization. Although the exchange rate (international terms of trade) is indeterminate, if it lies in the interval [1.833, 3.6] the gains to both countries are positive. For example if we choose an exchange rate of 2(\$/£) country A gets access to commodity 1 at \$2 instead of \$3.6 that it would cost at home. As a result the price of commodity 2 in country A reduces to \$1.75 so that consumers in A will enjoy 17.1428 units and 42.8571 units will be exported to B and consumed there; both quantities are greater than the autarky consumption levels, both countries gain by trade. The international terms of trade is 1 unit of commodity 1 = 1.1428 units of commodity 2. The example clearly shows that comparative advantage is not determined by factor endowments- it may well be opposite.

3.3.4: Leontief Non-Paradox

The Leontief trade model also yields results of the Heckscher-Ohlin type. The conditions for that happen are as follows

- (i) Country A is relatively capital abundant and commodity 1 is capital intensively produced in both countries

$$\frac{A_{1A}}{L_A} > \frac{A_{1B}}{L_B} \quad \text{and} \quad \frac{a_{11A}}{l_{1A}} > \frac{a_{12A}}{l_{2A}}, \quad \frac{a_{11B}}{l_{1B}} > \frac{a_{12B}}{l_{2B}}$$

- (ii) Country A has comparative advantage in commodity 1

$$\frac{P_{1A}}{P_{2A}} < \frac{P_{1B}}{P_{2B}}$$

(iii) Country A is capital abundant in the post trade situation as well

$$\frac{a_{11A} \left(\frac{L_A}{l_{1A}} \right)}{L_A} > \frac{a_{12B} \left(\frac{L_B}{l_{2B}} \right)}{L_B}$$

$$\frac{a_{11A}}{l_{1A}} > \frac{a_{12B}}{l_{2B}}$$

(iv) Both countries gain from trade

$$\frac{p_{1B}}{p_{2B}} \geq T_{12} \geq \frac{p_{1A}}{p_{2A}} \quad \text{in commodity terms}$$

$$\text{or } \frac{p_{2A}}{p_{2B}} \geq E_{AB} \geq \frac{p_{1A}}{p_{1B}} \quad \text{in exchange rate terms}$$

(v) Country B's post trade requirement of commodity 1 must be equal to or less than country A's exportable surplus of commodity 1,

$$a_{12B} \frac{L_B}{l_{2B}} \leq (1 - a_{11A}) \frac{L_A}{l_{1A}}$$

If we combine (i) and (iii) the conditions for observing the Heckscher- Ohlin trade pattern are

$$\frac{a_{11A}}{l_{1A}} > \frac{a_{12A}}{l_{2A}}, \quad \frac{a_{11B}}{l_{1B}} > \frac{a_{12B}}{l_{2B}} \quad \text{and} \quad \frac{a_{11A}}{l_{1A}} > \frac{a_{12B}}{l_{2B}}$$

$$\text{But } \frac{a_{12A}}{l_{2A}} > \frac{a_{12B}}{l_{2B}} \quad \text{or} \quad \frac{a_{12A}}{l_{2A}} < \frac{a_{12B}}{l_{2B}} .$$

But combining the corresponding conditions for the Leontief paradox gives a sharp hierarchy of factor intensities viz.

$$\frac{a_{11A}}{l_{1A}} > \frac{a_{12A}}{l_{2A}} > \frac{a_{11B}}{l_{1B}} > \frac{a_{12B}}{l_{2B}}$$

Clearly, trade patterns conforming to the Heckscher- Ohlin theorem will be more numerous as compared the Leontief paradox because the conditions required for the latter are far more restrictive and may explain why the paradoxical pattern has been observed less frequently than the Heckscher- Ohlin pattern in the empirical literature.

The following example shows how the Heckscher- Ohlin pattern can be obtained from the Leontief trade model.

<i>Country A</i>	<i>Country B</i>
(1) $15p_{1A} + 10w_A = 25p_{1A}$	$10p_{1B} + 10w_B = 15p_{1B}$
(2) $10p_{1A} + 20w_A = 10p_{2A}$	$5p_{1B} + 10w_B = 30p_{2B}$... (1)
$\begin{array}{r} \underline{25} \quad \underline{30} \end{array}$	$\begin{array}{r} \underline{15} \quad \underline{20} \end{array}$

$w_A = \$1$	$w_B = £1$
$p_{1A} = \$1$	$p_{1B} = £2$
$p_{2A} = \$3$	$p_{2B} = £0.6666$

Country A is relatively better endowed in capital than B and commodity 1 is more capital intensively produced than commodity 2 in both countries. The comparative advantage of producing 1 is in favour of A because $\frac{p_{1A}}{p_{2A}} < \frac{p_{1B}}{p_{2B}}$. When the countries are opened to trade the picture is as follows,

<i>Country A</i>	<i>Country B</i>
(1) $45p_{1A} + 30w_A = 75p_{1A}$	<hr style="width: 100%; border: 0.5px solid black;"/> <i>Industry 1 is closed</i> .
(2) <i>Industry 2 is closed</i>	$10p_{1A}E_{BA} + 20w_B = 60p_{2B}$... (2)

Here too the exchange rate (international terms of trade) is indeterminate. However an exchange rate between 0.5(\$/£) and 4.5(\$/£) will give positive gains to both countries. For instance if $E_{AB} = 2$ (\$/£) country B would obtain for £ 1 an amount equal to 2 units of commodity 1 as against 1 unit which it should have obtained at home. This reduces the price of commodity 2 in country B to £ 0.4166 which is lower than the autarky price. As a result consumers in country B will enjoy 48 units of commodity 2 and consumers in country A will obtain 12 units of commodity 2 both of which are more than what they consumed under autarky. Alas, the individual markets do not clear in the simplified trade situation considered above; in the first example 2.5 units of commodity 1, i.e. the exportable surplus of commodity 1 in country B minus A's; requirements are wasted and there is an unfulfilled demand for 2.8571 units of commodity 2 in country B; excess supply of commodity 1 worth \$ 5 of exactly equals the excess demand for commodity 2 which is \$ 5. In the second example the excess supply of commodity 1 is 20 units worth \$ 20 and the excess demand of commodity 2 is 24 units also worth \$ 20. Although both countries will definitely gain from trade but a market clearing exchange rate cannot be found.

In the static open Leontief model with one capital good and one consumption good, in autarky $w_{1A}L_{1A}$ must equal $A_{11A}p_{1A}$ (imports of industry 1 must be equal to its exports to industry 2) and both commodity markets must clear. This, when specialization takes place internationally, will not be true except in the very special case,

$$(1 - a_{11A}) \frac{L_A}{l_{1A}} = a_{12B} \frac{L_B}{l_{2B}}$$

Failing this condition, the trade balance equation $w_A L_A = A_{12B} p_{1A}$ becomes inconsistent and is unable to determine a unique exchange rate which clears the markets. It may however be noted that in the one capital-good one consumption-good case the markets do not clear irrespective of whether the Leontief paradox holds or the Heckscher-Ohlin theorem does.

3.3.5: Trade in Heterogeneous Capital Goods

In this section we will develop a model of two-country two-commodity trade whose endowment ratios and factor intensities are such that the Heckscher-Ohlin model would fail to make any prediction but for which the Leontief trade model readily determines the international trade equilibrium. Consider the two commodities both of which serve as means of production as well as consumption. The following procedure determines their autarky equilibria.

- (i) Determine the prices from the Leontief price system for some given money wage.

$$P = (I - A^T)^{-1} wL$$

- (ii) Determine the quantities demanded for final consumption

$$F_{id} = \frac{wL_k}{p_{ik}} \quad k = A, B$$

- (iii) Determine the equilibrium gross outputs required to supply quantities F_{id} for final consumption

$$X = (I - A)^{-1} F$$

These give autarkic equilibrium outputs and prices which can be used to determine the comparative advantage. Suppose the comparative advantages shows,

$$\frac{p_{1A}}{p_{1B}} < \frac{p_{1B}}{p_{2B}}$$

Then the indicated trade pattern is A-1 B-2. Accordingly, set up the post-trade production equations with full employment of the labour endowments being employed in the industries of specialization.

<i>Country A</i>	<i>Country B</i>
(1) $A_{11A}p_{1A}^* + A_{21A}p_{2B}^*E_{AB} + w_A L_A = X_{1A}p_{1A}^*$	<i>Industry 1 is closed</i>
(2) <u><i>Industry 2 is closed</i></u> .	$A_{12B}p_{1A}^*E_{BA} + A_{22B}p_{2B}^* + w_B L_B = X_{2B}p_{2B}^*$

... (1)

These are two equations in three unknowns, viz. post-trade prices p_{1A}^*, p_{2B}^* and the equilibrium exchange rate E_{AB} . $p_{2B}^*E_{AB}$ is the price of commodity 2 in the currency of country A and $p_{1A}^*E_{BA}$ is the price of commodity 1 in the currency of country B. The third equation is obviously the trade balance equation of one of the countries, say country A in the currency of country A.

$$A_{21A}p_{2B}^*E_{AB} + \alpha_{2A}w_A L_A = A_{12B}p_{1A}^* + \alpha_{1B}w_B L_B E_{AB} \quad \dots (2)$$

The left hand side shows the total imports of country A of commodity 2 and the right hand side its exports of commodity 1. The three equations form a set of simultaneous non-linear equations on account of the presence of the products of two unknowns like $p_{2B}^*E_{AB}$ and $p_{1A}^*E_{BA}$. Nevertheless they can be solved by linear methods in the following manner. Express the price equation of commodity 2 in country B in the currency of country A by multiplying it by E_{AB} (obviously $E_{AB}E_{BA} = 1$) and arrange the three equations in vector- matrix notation as a non-homogeneous system;

$$\begin{bmatrix} X_{1A} - A_{11A} & -A_{21A} & 0 \\ -A_{12B} & X_{2B} - A_{22B} & -w_B L_B \\ A_{12B} & -A_{21A} & \alpha_{1B}w_B L_B \end{bmatrix} \begin{bmatrix} p_{1A}^* \\ p_{2B}^*E_{AB} \\ E_{AB} \end{bmatrix} = \begin{bmatrix} w_A L_A \\ 0 \\ \alpha_{2A}w_A L_A \end{bmatrix} \quad \dots (3)$$

Inverting the matrix on the left hand side and multiplying it to the vector on the right hand side gives a unique positive solution for p_{1A}^* , p_{2B}^* and E_{AB} where $p_{2B}^* = p_{2B}^* E_{AB} / E_{AB}$.

A numerical example will illustrate the workings of the Leontief trade model. Suppose the matrices of the I-O coefficients and the vectors of labour coefficients are

$$A_a = \begin{bmatrix} 0.55 & 0.45 \\ 0.10 & 0.25 \end{bmatrix} \quad L_a = \begin{bmatrix} 0.60 \\ 0.45 \end{bmatrix}$$

$$A_b = \begin{bmatrix} 0.85 & 0.05 \\ 0.7 & 0.10 \end{bmatrix} \quad L_b = \begin{bmatrix} 1 \\ 0.3 \end{bmatrix}$$

Also suppose $w_a = \$1$, $w_b = £1$, $L_A = 20$, $L_B = 18$ and $\alpha_{ik} = 0.5$ the coefficients of consumption. Then the prices in two countries are

$$P_a = [I - A_a^T]^{-1} w_a L_a \quad P_b = [I - A_b^T]^{-1} w_b L_b$$

$$\begin{bmatrix} p_{1A} \\ p_{2A} \end{bmatrix} = \begin{bmatrix} 1.6923 \\ 1.6154 \end{bmatrix} \quad \begin{bmatrix} p_{1B} \\ p_{2B} \end{bmatrix} = \begin{bmatrix} 11.1000 \\ 0.9500 \end{bmatrix}$$

The quantities demanded for final consumption are $\frac{\alpha_{ik} w_k L_k}{p_{ik}}$ ($k = A, B$)

$$F_a = \begin{bmatrix} 5.9091 \\ 6.1905 \end{bmatrix} \quad F_b = \begin{bmatrix} 0.8108 \\ 9.4737 \end{bmatrix}$$

The equilibrium gross outputs therefore are;

$$X_a = [I - A_a]^{-1} F_a \quad X_b = [I - A_b]^{-1} F_b$$

$$\begin{bmatrix} X_{1A} \\ X_{2A} \end{bmatrix} = \begin{bmatrix} 24.6753 \\ 11.5440 \end{bmatrix} \quad \begin{bmatrix} X_{1B} \\ X_{2B} \end{bmatrix} = \begin{bmatrix} 12.0341 \\ 19.8862 \end{bmatrix}$$

The autarkic equilibria may then be depicted as follows.

Country A

$$(1) 13.5714 p_{1A} + 2.467 p_{2A} + 14.8052 w_A = 24.6753 p_{1A}$$

$$(2) \frac{5.1948}{18.7662} p_{1A} + \frac{2.8860}{5.3530} p_{2A} + \frac{5.1948}{20} w_A = 11.5440 p_{2A}$$

Country B

$$(1) 10.2290 p_{1B} + 8.4239 p_{2B} + 12.0341 w_B = 12.0341 p_{1B}$$

$$(2) \frac{0.9943}{11.2233} p_{1B} + \frac{1.9886}{10.4125} p_{2B} + \frac{5.9658}{18} w_B = 19.8862 p_{2B}$$

... (4)

The endowment ratios of the individual capital goods to labour show A to be better endowed in good 1 but poorly endowed in good 2 as compared to B.

$$\frac{A_{1A}}{L_A} = 0.9833 > \frac{A_{1B}}{L_B} = 0.6235$$

$$\frac{A_{2A}}{L_A} = 0.2677 < \frac{A_{2B}}{L_B} = 0.5785$$

And the factor intensities in terms of individual capital goods are as follows;

$$\frac{A_{11A}}{L_{1A}} = 0.9166 < \frac{A_{12A}}{L_{2A}} = 1, \quad \frac{A_{21A}}{L_{1A}} = 0.9166 < \frac{A_{22A}}{L_{2A}} = 1$$

$$\frac{A_{11B}}{L_{1B}} = 0.85 > \frac{A_{12B}}{L_{2B}} = 0.166, \quad \frac{A_{21B}}{L_{1B}} = 0.7 > \frac{A_{22B}}{L_{2B}} = 0.33$$

Commodity 1 is labour intensively produced in country A as compared to commodity 2 but capital intensively produced in country B. It is hard to see what the Heckscher-Ohlin theorem would do in this setup. But the Leontief trade model can proceed without a hiccup. From the price solution it is clear that

$$\frac{P_{1A}}{P_{2A}} (= 1.0470) < \frac{P_{1B}}{P_{2B}} (= 11.6800) \text{ so A-1 B-2 is the pattern of post-trade specialisation;}$$

Country A

$$(1) 18.3333 p_{1A}^* + 3.3333 p_{2B}^* E_{AB} + 20 w_A = 33.3333 p_{1A}^*$$

$$(2) \underline{\text{Industry 2 is closed}}$$

Country B

(1) Industry 1 is closed

$$(2) 3.00p_{1A}^*E_{BA} + 6.00p_{2B}^* + 18w_B = 60.00p_{2B}^*$$

... (5)

The trade balance equation is;

$$3.3333p_{2B}E_{AB} + 10 = 3.00p_{1A} + 9E_{AB} \quad (\because \alpha_{2A}w_A L_A = 10, \alpha_{1B}w_B L_B = 9) \quad \dots (6)$$

The solution is $E_{AB} = 0.7654$, $p_{1A}^* = 1.4074$ and $p_{2B}^* = 0.4354$. The international terms of trade are 1 unit of commodity 1 = 4.2232 units of commodity 2 which lie between the domestic terms of trade. Since both p_{1A}^* and p_{2B}^* are lower than their pre-trade values p_{1A} (1.6923) and p_{2B} (0.9500) respectively, both countries gain from trade and the international trade equilibrium is found. All markets are cleared. For example the total production of commodity 1 is 33.3333 units of which 21.3333 units are absorbed in countries A and B for investment purposes leaving 12 units for final consumption. With half of the income of each country being spent on commodity 1 the total consumption demand is $\frac{10w_A + 9w_B E_{AB}}{0.4354} = 7.1053 + 4.8906 = 12$ units.

We have seen that the Leontief trade model is found to apply in the case of 2-country two capital goods situations in which a country is better endowed in capital relative to labour in terms of one capital good but poorly endowed in terms of another. In this kind of a situation the Heckscher-Ohlin apparatus would be helpless; it would be unable to give any definite answer.

3.4: Two-Country Trade in Several Commodities

In the previous sections we have investigated properties related to trade model in a 2x2 setting. We now consider a 2xn model of two countries trading in several commodities.

3.4.1: Trade in Several Capital Goods

Consider two countries A and B that produce four tradable commodities. Their labour endowments are 60 and 120 units respectively. All four commodities have consumption

as well as intermediate demand. The consumption coefficient in each country is assumed to be uniform $(1/4)^{\text{th}}$ for each commodity. The autarkic equilibria for the countries are as follows,

Country A

$$\begin{aligned}
 1) & 4.7848p_{1A} + 7.1772p_{2A} + 4.7848p_{3A} + 9.5696p_{4A} + 11.9620w_A = 239.23967p_{1A} \\
 2) & 1.4111p_{1A} + 11.2889p_{2A} + 8.4667p_{3A} + 12.7000p_{4A} + 14.1111w_A = 141.1116p_{2A} \\
 3) & 11.2507p_{1A} + 13.1259p_{2A} + 9.3756p_{3A} + 13.1259p_{4A} + 14.0634w_A = 93.7562p_{3A} \\
 4) & 14.8976p_{1A} + 11.9181p_{2A} + 9.9317p_{3A} + 11.9182p_{4A} + 19.8634w_A = 99.3172p_{4A} \\
 & \text{-----} \\
 & \qquad \qquad \qquad 60 \qquad \qquad \qquad \dots (1)
 \end{aligned}$$

Country B

$$\begin{aligned}
 1) & 21.8284p_{1B} + 17.4627p_{2B} + 15.2799p_{3B} + 26.1941p_{4B} + 32.7426w_B = 218.2838p_{1B} \\
 2) & 18.5629p_{1B} + 13.9221p_{2B} + 18.5629p_{3B} + 9.2814p_{4B} + 23.2036w_B = 464.0712p_{2B} \\
 3) & 22.4959p_{1B} + 26.2453p_{2B} + 18.7466p_{3B} + 26.2453p_{4B} + 28.1199w_B = 187.4661p_{3B} \\
 4) & 26.9505p_{1B} + 21.5604p_{2B} + 17.9670p_{3B} + 17.9670p_{4B} + 35.9340w_B = 179.6698p_{4B} \\
 & \text{-----} \\
 & \qquad \qquad \qquad 120 \qquad \qquad \qquad \dots (2)
 \end{aligned}$$

The autarky equilibrium prices are,

$$\begin{array}{ll}
 p_{1A} = 0.0725 & p_{1B} = 0.2336 \\
 p_{2A} = 0.1537 & p_{2B} = 0.0779 \\
 p_{3A} = 0.2451 & p_{2B} = 0.2566 \\
 p_{4A} = 0.2884 & p_{2B} = 0.3000
 \end{array}$$

Since the prices are known at autarky equilibrium, the natural exchange rates can be found out as follows,

$$\begin{array}{ll}
 E_{AB}^1 = \frac{p_{1A}}{p_{1B}} = 0.3104 & \therefore E_{BA}^1 = \frac{1}{E_{AB}^1} = 3.2215 \\
 E_{AB}^2 = \frac{p_{2A}}{p_{2B}} = 1.9717 & \therefore E_{BA}^2 = \frac{1}{E_{AB}^2} = 0.5072 \\
 E_{AB}^3 = \frac{p_{3A}}{p_{3B}} = 0.9552 & \therefore E_{BA}^3 = \frac{1}{E_{AB}^3} = 1.0469 \\
 E_{AB}^4 = \frac{p_{4A}}{p_{4B}} = 0.9613 & \therefore E_{BA}^4 = \frac{1}{E_{AB}^4} = 1.0403
 \end{array}$$

From the above exchange rates we can find the lowest products as follows,

$$E_{AB}^1 E_{BA}^2 = 0.1574$$

$$E_{AB}^1 E_{BA}^4 = 0.3229$$

$$E_{AB}^1 E_{BA}^3 = 0.3249$$

Hence we set up the first trial trade pattern as A-1, B-2, 3, 4. Country A is now fully specialized in producing commodity 1 and all its resources will be devoted to industry 1. In case of country B, the difficulty of labour allocation in post-trade situation can be resolved by adopting the procedure of scale multipliers which simultaneously determine allocation of labour and the levels of commodity outputs. The labour released from industry 1 which is closed down, after trade is opened is allocated amongst operating industries 2, 3 and 4. The volumes, and therefore values of commodity production, exports and further the values of intercountry trade bills and exchange rates are depended on this allocation of labour. Let x_{2B} , x_{3B} and x_{4B} are the scale multipliers, they will appear in post-trade price equations of commodity 2, 3 and 4 respectively.

The post trade price systems expressed in A's currency is,

Country A

$$1. \quad 24 p_{1A} + 36 p_{2B} E_{AB} + 24 p_{3B} E_{AB} + 48 p_{4B} E_{AB} + 60 w_A = 1200 p_{1A} \quad \dots (3)$$

Industry 2, 3, 4 are closed.

Country B

Industry 1 is closed

$$2. \quad x_{2B} (18.5629 p_{1A} + 13.9221 p_{2B} E_{AB} + 18.5629 p_{3B} E_{AB} + 9.2814 p_{4B} E_{AB} + 23.2036 w_B E_{AB}) = 464.0712 p_{2B} E_{AB}$$

$$3. \quad x_{3B} (22.4959 p_{1A} + 26.2453 p_{2B} E_{AB} + 18.7466 p_{3B} E_{AB} + 26.2453 p_{4B} E_{AB} + 28.1199 w_B E_{AB}) = 187.4661 p_{3B} E_{AB}$$

$$4. \quad x_{4B} (26.9505 p_{1A} + 21.5604 p_{2B} E_{AB} + 17.9670 p_{3B} E_{AB} + 17.9670 p_{4B} E_{AB} + 35.9340 w_B E_{AB}) = 179.6698 p_{4B} E_{AB}$$

... (4)

Observe that the scale multipliers are multiplied to both RHS and LHS of respective price equations. Since, the prices are invariant of these scale multipliers, we first find out the post-trade prices of all commodities, i.e. 4 prices and one exchange rate.

For that we will take the trade balance equation as our 5th equation along with 4 post-trade price equations. The trade balance equation is,

$$36p_{2B}E_{AB} + 24p_{3B}E_{AB} + 48p_{4B}E_{AB} + \frac{3}{4} \times Rs.1 \times 60$$

$$= (18.5629x_{2B} + 22.4959x_{3B} + 26.9505x_{4B})p_{1A} + \frac{1}{4} \times \$1 \times 120E_{AB}$$

Solving four equations and one trade balance equation simultaneously we get,

$$p_{1A} = 0.1167$$

$$p_{2B} = 0.0673$$

$$p_{3B} = 0.2218$$

$$p_{4B} = 0.2609$$

$$E_{AB} = 3.8105$$

Next, we will set up the demand supply equations for all 4 commodities and full employment equation for country B as follows,

$$1) 1200p_{1A} =$$

$$24p_{1A} + 15w_A + 18.5629x_{2B}p_{1A} + 22.4959x_{3B}p_{1A} + 26.9505x_{4B}p_{1A} + 30w_B E_{AB}$$

$$2) 464.0712x_{2B}p_{2B} =$$

$$36p_{2B} + 15w_A E_{BA} + 13.9221x_{2B}p_{2B} + 26.2452x_{3B}p_{2B} + 21.5603x_{4B}p_{2B} + 30w_B$$

$$3) 187.4661x_{3B}p_{3B} =$$

$$24p_{3B} + 15w_A E_{BA} + 18.5628x_{2B}p_{3B} + 18.7466x_{3B}p_{3B} + 17.9669x_{4B}p_{3B} + 30w_B$$

$$4) 179.6698x_{4B}p_{4B} =$$

$$48p_{4B} + 15w_A E_{BA} + 9.2814x_{2B}p_{4B} + 26.2452x_{3B}p_{4B} + 17.9669x_{4B}p_{4B} + 30w_B$$

$$5) 23.2305x_{2B} + 28.1199x_{3B} + 35.9339x_{4B} = 120$$

We have 5 equations in 3 unknowns scale multipliers x_{2B} , x_{3B} and x_{4B} . Out of these 5 equations 2 are linearly dependent. We can use any two equations along with the last full-employment equation to get the unknown scale multipliers. Substituting post-trade prices and exchange rate which are already determined, in above equations and solving equations 2, 3 and 5, we get $x_{2B} = 1.6948$, $x_{3B} = 1.3611$ and $x_{4B} = 1.1799$. On the other hand, solving equations 3, 4 and 5 we get $x_{2B} = 0.8405$, $x_{3B} = 1.7436$ and $x_{4B} = 1.4323$. That is to say the scale multipliers do not have unique solution. In addition to this the gains from trade table 3.4 given below shows that both the countries have comparative advantage in producing commodity 3.

Table 3.6: Gains from Trade for First Trial Trade Pattern in Several Capital Goods

Gains From Trade Table		
Commodity	Country A	Country B
1	8.9039*	1.3702
2	1.9749	4.1770*
3	1.2680*	1.2680*
4	1.0568	1.0779*

*The maximum quantity

Therefore, we now set the trade pattern where both the countries produce commodity 3. Hence the next trial trade pattern would be A-1, 3 and B-2, 3, 4. Since commodity 3 is produced in common, the exchange rate is fixed based on commodity 3's relative price i.e. $E_{AB} = E_{AB}^3 = p_{3A} / p_{3B}$. We can replace p_{3A} by $p_{3B}E_{AB}$ which will eliminate 1 separate unknown p_{3A} . In both countries we will require to allocate the labour using the method of scale multipliers. Accordingly the post-trade price system will be represented as follows,

Country A

$$x_{1A}(4.7848p_{1A} + 7.1772p_{2B}E_{AB} + 4.7848p_{3B}E_{AB} + 9.5696p_{4B}E_{AB} + 11.9620w_A = 239.23967p_{1A})$$

$$x_{3B}(11.2507p_{1A} + 13.1259p_{2B}E_{AB} + 9.3756p_{3B}E_{AB} + 13.1259p_{4B}E_{AB} + 14.0634w_A = 93.7562p_{3B}E_{AB})$$

Industry 2 and 4 are closed

... (5)

Country B

Industry 1 is closed

$$x_{2B}(18.5629p_{1A} + 13.9221p_{2B}E_{AB} + 18.5629p_{3B}E_{AB} + 9.2814p_{4B}E_{AB} + 23.2036w_B E_{AB} = 464.0712p_{2B}E_{AB})$$

$$x_{3B}(22.4959p_{1A} + 26.2453p_{2B}E_{AB} + 18.7466p_{3B}E_{AB} + 26.2453p_{4B}E_{AB} + 28.1199w_B E_{AB} = 187.4661p_{3B}E_{AB})$$

$$x_{4B}(26.9505p_{1A} + 21.5604p_{2B}E_{AB} + 17.9670p_{3B}E_{AB} + 17.9670p_{4B}E_{AB} + 35.9340w_B E_{AB} = 179.6698p_{4B}E_{AB})$$

... (6)

Solving price system (5) and (6) simultaneously for prices and exchange rate we get,

$$\begin{aligned}
 p_{1A} &= 0.0688 \\
 p_{2B} &= 0.0693 \\
 p_{3A} &= p_{3B} = 0.2284 \\
 p_{4B} &= 0.2683 \\
 E_{AB}^3 &= E_{AB} = 1
 \end{aligned}$$

Next step is to set up the demand supply equations for all 4 commodities and full employment equation for country B. They are as follows,

1. $239.2396x_{1A}p_{1A} = 4.7847x_{1A}p_{1A} + 11.2507x_{3A}p_{1A} + 15w_A + 18.5629x_{2B}p_{1A} + 22.4959x_{3B}p_{1A} + 26.9505x_{4B}p_{1A} + 30w_B E_{AB}$
2. $464.0712x_{2B}p_{2B}E_{AB} = 7.1171x_{1A}p_{2B}E_{AB} + 13.1258x_{3A}p_{2B}E_{AB} + 15w_A + 13.9221x_{2B}p_{2B}E_{AB} + 26.2452x_{3B}p_{3B}E_{AB} + 21.5603x_{4B}p_{4B}E_{AB} + 30w_B E_{AB}$
3. $(93.7561x_{3A} + 187.4661x_{3B})p_{3B}E_{AB} = 4.7847x_{1A}p_{3B}E_{AB} + 9.3756x_{3A}p_{3B}E_{AB} + 15w_A + 18.5628x_{2B}p_{3B}E_{AB} + 18.7466x_{3B}p_{3B}E_{AB} + 17.9669x_{4B}p_{3B}E_{AB} + 30w_B E_{AB}$
4. $179.6698x_{4B}p_{4B}E_{AB} = 9.5695x_{1A}p_{4B}E_{AB} + 13.1258x_{3A}p_{4B}E_{AB} + 15w_A + 9.2814x_{2B}p_{4B}E_{AB} + 26.2452x_{3B}p_{4B}E_{AB} + 17.9669x_{4B}p_{4B}E_{AB} + 30w_B E_{AB}$
5. $11.9619x_{1A} + 14.0634x_{2A} = 60$
6. $23.2305x_{2B} + 28.1199x_{3B} + 35.9339x_{4B} = 120$

We now substitute the post-trade prices and exchange rate in above demand-supply equations to get unknown scale multipliers. Note that the price equation of commodity 3 is linearly dependent. Therefore solving remaining 5 equations in 5 unknown scale multipliers we get,

$$\begin{aligned}
 x_{1A} &= 3.2605 & x_{2B} &= 1.6639 \\
 x_{3A} &= 1.4930 & x_{3B} &= 0.8658 \\
 & & x_{4B} &= 1.5874
 \end{aligned}$$

Substituting the scale multipliers in price equations (5) and (6) we get new levels of output and labour in operating industries post-trade as follows,

Country A

$$1) 15.6011p_{1A} + 23.4016p_{2B}E_{AB} + 15.6011p_{3A} + 31.2022p_{4B}E_{AB} + 39.0027w_A \\ = 780.0544p_{1A}$$

.....Industry 2 is closed.....

$$3) 16.7978p_{1A} + 19.5975p_{2B}E_{AB} + 13.9982p_{3A} + 19.5975p_{4B}E_{AB} + 20.9973w_A \\ = 139.9819p_{3A}$$

Industry 4 is closed

60

... (7)

Country B

Industry 1 is closed

$$2) 30.8873p_{1A}E_{BA} + 23.1655p_{2B} + 30.8873p_{3B} + 15.4437p_{4B} + 38.6092w_B \\ = 772.1832p_{2B}$$

$$3) 19.4785p_{1A}E_{BA} + 22.7249p_{2B} + 16.2321p_{3B} + 22.7249p_{4B} + 24.3481w_B \\ = 162.3208p_{3B}$$

$$4) 42.7820p_{1A}E_{BA} + 34.2256p_{2B} + 28.5214p_{3B} + 28.5214p_{4B} + 57.0427w_B \\ = 285.2136p_{4B}$$

120

... (8)

The state of world economy described by (7) and (8) is a state of international trade equilibrium if the gains from trade supports the trade pattern A-1, 3 and B-2, 3, 4. To construct gains from trade table we need to compute the post-trade prices of all commodities. The commodities which are produced in the countries in the post-trade situation, the prices have already been computed. The prices of commodities which are not produced post-trade are worked out as follows.

$$p_{2A} = 0.1441, p_{4A} = 0.2737 \& p_{1B} = 0.2106$$

Now the gains from trade table is constructed as follows,

Table 3.7: Gains of Trade for Trade Equilibrium in Several Capital Goods

Commodity	Country A	Country B
1	14.5446*	4.7483
2	6.9405	14.4237*
3	4.3792*	4.3792*
4	3.6541	3.7272*

*The maximum quantity

It can be seen that gains from trade table exactly supports the trade pattern A-1, 3 and B-2, 3, 4. Hence trade equilibrium is found. A comparison of the autarky and post-trade situations may be summarized as shown in table 3.8.

Table 3.8: Pre and Post Trade Prices, Output and Real Wage Rates in Trade Equilibrium in Several Capital Goods

	Country A		Country B	
	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	0.0725	0.0688	0.2336	0.0688
p_2	0.1537	0.0693	0.0779	0.0693
p_3	0.2451	0.2284	0.2566	0.2284
p_4	0.2884	0.2737	0.3000	0.2683
B_1	239.2397	780.0544	218.2838	-----
B_2	141.1116	-----	464.0712	772.1832
B_3	93.7562	139.9819	187.4661	162.3208
B_4	99.3172	-----	179.6698	285.2136
w/p_1	13.7931	14.5349	4.2808	14.5349
w/p_2	6.5062	14.4300	12.8370	14.4300
w/p_3	4.0780	4.3783	3.8971	4.3783
w/p_4	3.4674	3.7272	3.3333	3.7272

Table 3.8 shows that the world outputs of all commodities have increased, the post-trade prices of all commodities have declined in both the countries and therefore the real wage rates have increased in both countries.

In above illustration we have seen that the Leontief trade model can be applied under the conditions of 2 country n-commodity, and is capable of generating a greater variety of predictions even when the setting is generalized to the case of several capital goods.

3.4.2: Trade in Heterogeneous Capital Goods and Several Consumption Goods

Having investigated the properties of the Leontief trade model in a 2xn setting, it is worthwhile to go further and consider 2xn model with several heterogeneous capital goods and several consumption goods. By way of generalisation we shall also allow some commodities to be non-tradable. Thus consider an example where two countries produce two non-tradable capital goods, two tradable capital goods, two tradable consumption goods and one non-tradable consumption good. Their labour endowments are 380 and 400 units respectively. They spend $(1/3)^{rd}$ of their net national income on each of the consumption goods. Following are the states of autarkic equilibrium,

Country A

1. $9.0947p_{1A} + 7.6702p_{2A} + 5.8047p_{3A} + 4.1638p_{4A} + 21.7672w_A = 49.3086p_{1A}$
2. $7.8929p_{1A} + 11.3658p_{2A} + 6.8194p_{3A} + 4.0411p_{4A} + 37.6165w_A = 50.5147p_{2A}$
3. $5.4995p_{1A} + 4.2965p_{2A} + 6.3588p_{3A} + 3.0075p_{4A} + 38.3924w_A = 42.9652p_{3A}$
4. $3.5127p_{1A} + 5.1520p_{2A} + 3.3956p_{3A} + 1.9905p_{4A} + 23.2617w_A = 28.1022p_{4A}$
5. $7.8612p_{1A} + 6.5093p_{2A} + 6.5093p_{3A} + 3.7053p_{4A} + 90.1293w_A = 80.1149p_{5A}$
6. $7.4388p_{1A} + 8.5659p_{2A} + 8.3631p_{3A} + 3.6067p_{4A} + 84.5328w_A = 101.4394p_{6A}$
7. $8.0084p_{1A} + 6.9547p_{2A} + 5.7113p_{3A} + 7.5869p_{4A} + 84.2999w_A = 105.3749p_{7A}$

380

Country B

1. $6.5791p_{1B} + 5.4075p_{2B} + 4.7766p_{3B} + 4.3260p_{4B} + 22.5312w_B = 40.5562p_{1B}$
2. $5.6593p_{1B} + 8.6237p_{2B} + 4.7430p_{3B} + 4.5274p_{4B} + 32.3391w_B = 43.1188p_{2B}$
3. $3.8461p_{1B} + 3.5612p_{2B} + 4.5584p_{3B} + 3.2051p_{4B} + 24.9288w_B = 35.6126p_{3B}$
4. $4.3250p_{1B} + 6.7471p_{2B} + 4.1520p_{3B} + 3.8060p_{4B} + 34.6006w_B = 32.8705p_{4B}$
5. $7.3183p_{1B} + 5.8760p_{2B} + 5.8760p_{3B} + 5.0213p_{4B} + 96.1533w_B = 87.6064p_{5B}$
6. $6.1154p_{1B} + 7.2075p_{2B} + 7.0109p_{3B} + 5.6786p_{4B} + 92.8239w_B = 98.2842p_{6B}$

$$7. \quad 6.7127p_{1B} + 5.6959p_{2B} + 4.4955p_{3B} + 6.3059p_{4B} + 96.6227w_B = 122.0498p_{7B}$$

400

The autarky equilibrium prices are,

$$\begin{array}{ll} p_{1A} = 1.2449 & p_{1B} = 1.3586 \\ p_{2A} = 1.6472 & p_{2B} = 1.6041 \\ p_{3A} = 1.5594 & p_{3B} = 1.3571 \\ p_{4A} = 1.5861 & p_{4B} = 1.9589 \\ p_{5A} = 1.5810 & p_{5B} = 1.5219 \\ p_{6A} = 1.2486 & p_{6B} = 1.3566 \\ p_{7A} = 1.2020 & p_{7B} = 1.0924 \end{array}$$

Since commodities 1, 2 and 7 are non-tradable they will be produced post-trade in both countries. Consider therefore, the natural exchange rates for tradable commodities. They are,

$$\begin{array}{ll} E_{AB}^3 = 1.1490 & E_{BA}^3 = 0.8702 \\ E_{AB}^4 = 0.8097 & E_{BA}^4 = 1.2350 \\ E_{AB}^5 = 1.0388 & E_{BA}^5 = 0.9626 \\ E_{AB}^6 = 0.9204 & E_{BA}^6 = 1.0864 \end{array}$$

The lowest product of these exchange rates will suggest the trade pattern with the maximal comparative advantage. $E_{AB}^4 E_{BA}^3 = 0.7046$ and $E_{AB}^6 E_{BA}^5 = 0.8860$ are the lowest product, it suggests the trade pattern A-1,2,4,6,7 and B-1,2,3,5,7 with all tradable commodities assigned to one or the other country. According to trade pattern A-1, 2,4,6,7 and B-1, 2,3,5,7, countries now open to trade. Since commodity 7 is a non-tradable purely consumption good, demand would be domestic and it will be produced in both countries post-trade. Therefore at the beginning we keep the labour employed in industry 7 post trade at autarky level. For first approximation the labour from closed consumption and capital good industries are allocated to operating consumption and capital good industries post-trade respectively to meet their domestic as well as international demand. Further we shall use the method of scale multipliers to reallocate labour in capital goods industries to achieve trade equilibrium. Then the price systems of country A and country B in A's currency would be as follows,

Country A

$$\begin{aligned}
 x_{1A} (9.0947p_{1A} + 7.6702p_{2A} + 5.8047p_{3B}E_{AB} + 4.1638p_{4A} + 21.7672w_A &= 49.3086p_{1A}) \\
 x_{2A} (7.8929p_{1A} + 11.3658p_{2A} + 6.8194p_{3B}E_{AB} + 4.0411p_{4A} + 37.6165w_A &= 50.5147p_{2A}) \\
 \dots\dots\dots\text{Industry 3 is closed}\dots\dots\dots & \\
 x_{4A} (9.3104p_{1A} + 13.6553p_{2A} + 9.0001p_{3B}E_{AB} + 5.2759p_{4A} + 61.6541w_A &= 74.4836p_{4A}) \\
 \dots\dots\dots\text{Industry 5 is closed}\dots\dots\dots & \\
 15.3702p_{1A} + 17.6991p_{2A} + 17.2079p_{3B}E_{AB} + 7.4522p_{4A} + 174.6622w_A &= 209.5946p_{6A} \\
 8.0084p_{1A} + 6.9547p_{2A} + 5.7113p_{3B}E_{AB} + 7.5869p_{4A} + 84.2999w_A &= 105.3749p_{7A}
 \end{aligned}$$

Country B

$$\begin{aligned}
 x_{1B} (6.5791p_{1B}E_{AB} + 5.4075p_{2B}E_{AB} + 4.7766p_{3B}E_{AB} + 4.3260p_{4A} + 22.5312w_B E_{AB} &= 40.5562p_{1B}E_{AB}) \\
 x_{2B} (5.6593p_{1B}E_{AB} + 8.6237p_{2B}E_{AB} + 4.7430p_{3B}E_{AB} + 4.5274p_{4A} + 32.3391w_B E_{AB} &= 43.1188p_{2B}E_{AB}) \\
 x_{3B} (9.1845p_{1B}E_{AB} + 8.5042p_{2B}E_{AB} + 10.8853p_{3B}E_{AB} + 7.6537p_{4A} + 59.5294w_B E_{AB} &= 85.0421p_{3B}E_{AB}) \\
 \dots\dots\dots\text{Industry 4 is closed}\dots\dots\dots & \\
 14.3832p_{1B}E_{AB} + 11.5486p_{2B}E_{AB} + 11.5486p_{3B}E_{AB} + 9.8688p_{4A} + 188.9773w_B E_{AB} &= 172.1793p_{5B}E_{AB} \\
 \dots\dots\dots\text{Industry 6 is closed}\dots\dots\dots & \\
 6.7127p_{1B}E_{AB} + 5.6959p_{2B}E_{AB} + 4.4955p_{3B}E_{AB} + 6.3059p_{4A} + 96.6227w_B E_{AB} &= 122.0498p_{7B}E_{AB}
 \end{aligned}$$

We first set up the market clearing equations for capital and consumption goods for country A and B as follows,

Country A

Demand-supply equations for capital goods 1, 2 and 4

$$\begin{aligned}
 9.0947x_{1A} + 7.8929x_{2A} + 9.3104x_{4A} + (15.3702 + 8.0084) &= 49.3086x_{1A} \\
 7.6702x_{1A} + 11.3658x_{2A} + 13.6553x_{4A} + (17.6991 + 6.9547) &= 50.5147x_{2A} \\
 4.1638x_{1A} + 4.0411x_{2A} + 5.2759x_{4A} + (7.4522 + 7.5869) + BD &= 74.4836x_{4A}
 \end{aligned}
 \tag{3}$$

Where, BD represents country B's demand for capital good 4.

$$BD = 4.3260x_{1B} + 4.5274x_{2B} + 7.6537x_{3B} + (9.8688 + 6.3059)$$

The RHS of above equations are supplies and LHS are demands. The quantities in the parenthesis on LHS are consumption good industry's demand for respective commodities.

Demand-supply equations for consumption goods industry

i) For tradable commodity 6

$$209.5946p_{6A} = \left(\frac{1}{3} \times 1 \times 380 \right) + \left(\frac{1}{3} \times 1 \times 400E_{AB} \right) \quad \dots (4)$$

ii) For non-tradable commodity 7

$$105.3749p_{7A} = \frac{1}{3} \times 1 \times 380 \quad \dots (5)$$

LHS for above two equations is supply and RHS is demand. The first bracket in equation (4) is domestic demand and second bracket is international demand for commodity 6. For commodity 7 which is non-tradable the demand is domestic.

In the similar fashion the demand supply equations for country B are as follows,

Country B

Demand-supply equations for capital goods 1, 2 and 3

$$6.5791x_{1B} + 5.6593x_{2B} + 9.1845x_{3B} + (14.3832 + 6.7127) = 40.5562x_{1B}$$

$$5.4075x_{1B} + 8.6237x_{2B} + 8.5042x_{3B} + (11.5486 + 5.6956) = 43.1188x_{2B}$$

$$4.7766x_{1B} + 4.7430x_{2B} + 10.8853x_{3B} + (11.5486 + 4.4955) + AS = 85.0421x_{3B} \quad \dots (6)$$

Where AS is country A's demand for capital good 3.

$$AS = 4.1638x_{1A} + 4.0411x_{2A} + 5.2759x_{4A} + (7.4522 + 7.5869)$$

Demand-supply equations for consumption goods industry

i) For tradable commodity 5

$$172.1793p_{5B} = \left(\frac{1}{3} \times 1 \times 400 \right) + \left(\frac{1}{3} \times 1 \times 380E_{BA} \right) \quad \dots (7)$$

ii) For non-tradable commodity 7

$$122.0498p_{7B} = \frac{1}{3} \times 1 \times 400 \quad \dots (8)$$

Finally, we set up the trade balance equation;

$$\begin{aligned}
& (5.8047x_{1A} + 6.8194x_{2A} + 9.0001x_{4A} + 17.2079 + 5.7113)p_{3B}E_{AB} + \frac{1}{3} \times 1 \times 380 \\
& = (4.3260x_{1B} + 4.5274x_{2B} + 7.6537x_{3B} + 9.8688 + 6.3059)p_{4A} + \frac{1}{3} \times 1 \times 400 \times E_{AB} \\
& \dots (9)
\end{aligned}$$

Observe that in the system of equations (3) unless the scale multipliers x_{1B} , x_{2B} & x_{3B} which are appeared in BD are known x_{1A} , x_{2A} & x_{4A} cannot be depicted. Similarly, in equation (6) x_{1B} , x_{2B} & x_{3B} cannot be determined unless x_{1A} , x_{2A} & x_{4A} are known. This hurdle can be handled by proceeding iteratively. First we put the x_{1B} , x_{2B} & x_{3B} as 1 each in BD and solve equation (3) simultaneously to get first approximate solution $x_{1A}^{(1)}$, $x_{2A}^{(1)}$ & $x_{4A}^{(1)}$. Using this solution we can calculate first approximate values of $x_{1B}^{(1)}$, $x_{2B}^{(1)}$ & $x_{3B}^{(1)}$ by solving (6) simultaneously. The volumes, and therefore values of commodity production, exports and further the values of intercountry trade bills and exchange rates are depended on this allocation of labour. Next, we multiply the price equations of capital good industries in (1) and (2) by the first approximate scale multipliers. There are 6 unknown prices namely p_{1A} , p_{2A} , p_{4A} , p_{1B} , p_{2B} , p_{3B} and 1 unknown exchange rate E_{AB} . And there are 3 price equations each for two countries and one trade balance equation. Hence the number of unknowns matches with the number of equations and by solving these equations simultaneously the prices and exchange rate can be determined.

The prices of consumption goods are entirely dependent on capital goods prices and exchange rate. Therefore they can be calculated by substituting the known capital goods price into the consumption goods price equations. The price equation of consumption goods industry depicts the supply price and demand-supply equations for consumption goods industry depict the demand price. From these two prices quantity demanded and quantity supplied can be calculated. To equalize demand-supply the method of bisection is used. Again for the new quantities of capital goods demanded by consumption goods industries, the scale multipliers are calculated. This process continues till all markets cleared.

After 16 such iterations of the process the trade equilibrium is found. The exchange rate and equilibrium prices are,

	$p_{1A} = 1.1982$	$p_{1B} = 1.2822$
	$p_{2A} = 1.5966$	$p_{2B} = 1.5260$
$E_{AB} = 1.0113$	$p_{4A} = 1.5432$	$p_{3B} = 1.2940$
	$p_{6A} = 1.2188$	$p_{5B} = 1.4812$
	$p_{7A} = 1.1784$	$p_{7B} = 1.0599$

The post-trade equilibrium price system is given as follows,

Country A

$$9.1303p_{1A} + 7.7002p_{2A} + 5.8302p_{3B}E_{AB} + 4.1801p_{4A} + 22.0007w_A = 49.5015p_{1A}$$

$$8.8795p_{1A} + 12.7865p_{2A} + 7.6719p_{3B}E_{AB} + 4.54632p_{4A} + 42.6218w_A = 50.5147p_{2A}$$

.....Industry 3 is closed.....

$$7.5886p_{1A} + 11.1299p_{2A} + 7.3356p_{3B}E_{AB} + 4.3002p_{4A} + 50.5904w_A = 60.7084p_{4A}$$

.....Industry 5 is closed.....

$$15.7346p_{1A} + 18.1186p_{2A} + 17.6895p_{3B}E_{AB} + 7.6289p_{4A} + 178.8017w_A = 214.5621p_{6A}$$

$$8.1686p_{1A} + 7.0938p_{2A} + 5.8255p_{3B}E_{AB} + 7.7387p_{4A} + 85.9855w_A = 107.4818p_{7A}$$

380

Country B

$$6.8326p_{1B} + 5.6158p_{2B} + 4.9606p_{3B}E_{BA} + 4.4927p_{4A}E_{BA} + 23.3993w_B = 42.1187p_{1B}$$

$$5.1215p_{1B} + 7.8041p_{2B} + 4.923p_{3B} + 4.0972p_{4A}E_{BA} + 29.2655w_B = 39.0206p_{2B}$$

$$8.6632p_{1B} + 8.0215p_{2B} + 10.2676p_{3B} + 7.2194p_{4A}E_{BA} + 56.1507w_B = 80.2153p_{3B}$$

.....Industry 4 is closed.....

$$14.5826p_{1B} + 11.7087p_{2B} + 11.7087p_{3B} + 10.0056p_{4A}E_{BA} + 191.5960w_B$$

$$= 174.5653p_{5B}$$

..... Industry 6 is closed.....

$$6.9188p_{1B} + 5.8705p_{2B} + 4.6335p_{3B} + 6.4995p_{4A}E_{BA} + 99.5886w_B$$

$$= 125.7961p_{7B}$$

400

The next step is to calculate the price of commodities 3 and 5 if country A was to produce these commodities. Similar exercise needs to be carried out for country B for commodity 4 and 6. The solution is,

$$p_{3A} = 1.5083$$

$$p_{5A} = 1.5500$$

$$p_{4B} = 1.8747$$

$$p_{6B} = 1.3166$$

The gains from trade table can be calculated using all prices.

Table 3.9: Gains from Trade in Heterogeneous Capital Goods and Several Consumption Goods

Commodity	Country	
	A	B
3	0.6629	0.7641*
4	0.6479*	0.5274
5	0.6451	0.6675*
6	0.8204*	0.7510

*The maximum quantity

From above gains from trade table it can be seen that it exactly supports the trade pattern A-1, 2, 4, 6, 7 and B-1, 2, 3, 5, 7. Hence we conclude that the trade equilibrium is found. A comparison of the autarky and post-trade situations may be summarized as shown in table 3.10.

Table 3.10: Pre and Post Trade Prices, Output and Real Wage Rates in Trade in Heterogeneous Capital Goods and Several Consumption Goods.

	Country A		Country B	
	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	1.2449	1.1983	1.3586	1.2822
p_2	1.6472	1.5966	1.6041	1.5261
p_3	1.5595	1.3087	1.3572	1.2941
p_4	1.5861	1.5433	1.9589	1.5261
p_5	1.5811	1.4980	1.5220	1.4813
p_6	1.2487	1.2188	1.3566	1.2052
p_7	1.2021	1.1785	1.0925	1.0599
B_1	49.31	49.50	40.56	42.12
B_2	50.51	56.83	43.12	39.02
B_3	42.97	-----	35.61	80.22
B_4	28.10	60.71	32.87	-----
B_5	80.11	-----	87.61	174.57
B_6	101.44	214.56	98.28	-----
B_7	105.37	107.48	122.05	125.80
w/p_1	0.8033	0.8345	0.7360	0.7799
w/p_2	0.6071	0.6263	0.6234	0.6553
w/p_3	0.6412	0.7641	0.7368	0.7728
w/p_4	0.6305	0.6480	0.5105	0.6553
w/p_5	0.6325	0.6675	0.6570	0.6751
w/p_6	0.8008	0.8205	0.7371	0.8298
w/p_7	0.8319	0.8485	0.9154	0.9435

Above table shows that the world outputs of all commodities have increased, the post-trade prices of all commodities have declined in both the countries and consequently real wage rates have increased in both countries. The Leontief trade model is found to apply under conditions of 2xn model with several heterogeneous capital goods and several consumption goods allowing some commodities to be non-tradable.

3.5: Three or More Country Trade

It is been shown in the foregoing section that the Leontief trade model can be successfully applied to the two countries trading in m commodities with several capital goods, several consumption goods, and non-tradable goods. It is only natural to investigate further and explore the possibility of its being generalized to the $m \times n$ case of several countries producing several commodities.

3.5.1: Three Country Trade in Several Goods

Consider three countries A, B, C with labour endowments of 500, 600 and 800 respectively and each country spending $1/4^{\text{th}}$ of its net income on each of the commodities. The technical and labour coefficients are as follows:

$$\begin{array}{cccc}
 A_{ijA} & L_{iA} & A_{ijB} & L_{iB} \\
 \left[\begin{array}{cccc} 0.02 & 0.03 & 0.02 & 0.04 \\ 0.10 & 0.08 & 0.06 & 0.09 \\ 0.12 & 0.14 & 0.10 & 0.14 \\ 0.15 & 0.12 & 0.10 & 0.12 \end{array} \right] & \left[\begin{array}{c} 0.05 \\ 0.10 \\ 0.15 \\ 0.20 \end{array} \right] & \left[\begin{array}{cccc} 0.10 & 0.08 & 0.07 & 0.12 \\ 0.04 & 0.03 & 0.04 & 0.02 \\ 0.12 & 0.14 & 0.10 & 0.14 \\ 0.15 & 0.12 & 0.10 & 0.10 \end{array} \right] & \left[\begin{array}{c} 0.15 \\ 0.05 \\ 0.20 \\ 0.20 \end{array} \right] \\
 \\
 A_{ijC} & L_{iC} & & \\
 \left[\begin{array}{cccc} 0.10 & 0.12 & 0.09 & 0.10 \\ 0.12 & 0.16 & 0.14 & 0.10 \\ 0.04 & 0.03 & 0.04 & 0.02 \\ 0.02 & 0.03 & 0.05 & 0.03 \end{array} \right] & \left[\begin{array}{c} 0.25 \\ 0.20 \\ 0.05 \\ 0.05 \end{array} \right] & &
 \end{array}$$

If money wage rates are $w_A = \text{USD}1$, $w_B = \text{JPY}1$ and $w_C = \text{INR}1$ in the countries A, B, C respectively.

To get unique prices for each of the commodities unlike demand and supply prices in disequilibrium state the way out of autarky equilibrium has been adopted. The autarky equilibria price systems, prices and natural exchange rates are as follows,

Country A

1. $18.9834p_{1A} + 94.9170p_{2A} + 113.9004p_{3A} + 142.3755p_{4A} + 47.4585w_A = 949.1702p_{1A}$
2. $18.9834p_{1A} + 94.9170p_{2A} + 113.9004p_{3A} + 142.3755p_{4A} + 47.4585w_A = 949.1702p_{2A}$

$$\begin{aligned}
3. \quad & 20.5307p_{1A} + 61.5922p_{2A} + 102.6537p_{3A} + 102.6537p_{4A} + 153.9806w_A \\
& = 1026.5370p_{3A} \\
4. \quad & 35.0085p_{1A} + 78.7692p_{2A} + 122.5299p_{3A} + 105.0257p_{4A} + 175.0427w_A \\
& = 875.2137p_{4A}
\end{aligned}$$

500
... (1)

Country B

$$\begin{aligned}
1. \quad & 92.8233p_{1B} + 37.1293p_{2B} + 111.3880p_{3B} + 139.2350p_{4B} + 139.2350w_B \\
& = 928.23355p_{1B} \\
2. \quad & 86.3593p_{1B} + 32.3847p_{2B} + 151.1289p_{3B} + 129.5391p_{4B} + 53.9746w_B \\
& = 1079.492p_{2B} \\
3. \quad & 71.8256p_{1B} + 41.0432p_{2B} + 102.6080p_{3B} + 102.6080p_{4B} + 205.216w_B \\
& = 1026.08p_{3B} \\
4. \quad & 120.9446p_{1B} + 20.1574p_{2B} + 141.1021p_{3B} + 201.5744p_{4B} + 201.5744w_B \\
& = 1007.872p_{4B}
\end{aligned}$$

600
... (2)

Country C

$$\begin{aligned}
1. \quad & 123.6923p_{1c} + 148.4307p_{2c} + 49.4769p_{3c} + 247.3845p_{4c} + 309.2307w_c \\
& = 1236.923p_{1c} \\
2. \quad & 181.0036p_{1c} + 241.3381p_{2c} + 42.2508p_{3c} + 42.2508p_{4c} + 67.6100w_c \\
& = 1508.363p_{2c} \\
3. \quad & 152.1227p_{1c} + 236.6353p_{2c} + 67.6100p_{3c} + 84.5125p_{4c} + 84.5125w_c \\
& = 1690.252p_{3c} \\
4. \quad & 209.1682p_{1c} + 209.1682p_{2c} + 41.8336p_{3c} + 62.7504p_{4c} + 104.5841w_c \\
& = 2091.682p_{4c}
\end{aligned}$$

800
... (3)

$$\begin{array}{lll}
p_{1A} = 0.1470 & p_{1B} = 0.2696 & p_{1C} = 0.3503 \\
p_{2A} = 0.2393 & p_{2B} = 0.1580 & p_{2C} = 0.2972 \\
p_{3A} = 0.2184 & p_{3B} = 0.2885 & p_{3C} = 0.1345 \\
p_{4A} = 0.2931 & p_{4B} = 0.3448 & p_{4C} = 0.1210
\end{array}$$

$$\begin{array}{lll}
E_{AB}^1 = 0.5454 & E_{BC}^1 = 0.7697 & E_{CA}^1 = 2.3818 \\
E_{AB}^2 = 1.5137 & E_{BC}^2 = 0.5318 & E_{CA}^2 = 1.2421 \\
E_{AB}^3 = 0.7571 & E_{BC}^3 = 2.1439 & E_{CA}^3 = 0.6160 \\
E_{AB}^3 = 0.8500 & E_{BC}^3 = 2.8484 & E_{CA}^3 = 0.4129
\end{array}$$

The least products of above natural exchange rates are $E_{AB}^1 E_{BC}^2 E_{CA}^4 = 0.1197$ and $E_{AB}^1 E_{BC}^2 E_{CA}^3 = 0.1786$ therefore the first trial trade pattern is A-1, B-2, C-3,4. Country A and country B are fully specialized in commodity 1 and 2 respectively. While country C will produce commodity 3 and 4. The labour from the industries which are ceased to operate post-trade will reallocate into the operating industries. In case of country A and B this is a straight forward process but in case of country C the reallocation of labour into the operating industries will take place according to their proportion of labour endowments. After reallocation of labour the price system would look as follows,

Country A

$$200p_{1A} + 1000p_{2B}E_{AB} + 1200p_{3C}E_{AC} + 1500p_{4C}E_{AC} + 500w_A = 10000p_{1A} \quad \dots (4)$$

----- Industry 2, 3 and 4 are closed.....

Country B

$$960p_{1A}E_{BA} + 360p_{2B} + 1680p_{3C}E_{BC} + 1440p_{4C}E_{BC} + 600w_B = 12000p_{2B}$$

----- Industry 1, 3 and 4 are closed.....

Or in A's currency

$$\begin{aligned}
& 960p_{1A} + 360p_{2B}E_{AB} + 1680p_{3C}E_{AC} + 1440p_{4C}E_{AC} + 600w_B E_{AB} \\
& = 12000p_{2B}E_{AB} \quad \dots (5)
\end{aligned}$$

Country C

----- Industry 1 and 2 are closed.....

$$\begin{aligned}
& 643.5763p_{1A}E_{CA} + 1001.119p_{2B}E_{CB} + 286.0339p_{3c} + 357.5423p_{4c} + 357.5423w_c \\
& = 7150.847p_{3c}
\end{aligned}$$

Or in A's currency

$$\begin{aligned}
& 643.5763p_{1A} + 1001.119p_{2B}E_{AB} + 286.0339p_{3c}E_{AC} + 357.5423p_{4c}E_{AC} + 357.5423w_c E_{AC} \\
& = 7150.847p_{3c}E_{AC} \quad \dots (6)
\end{aligned}$$

$$884.9152p_{1A}E_{CA} + 884.9152p_{2B}E_{CB} + 176.9831p_{3C} + 265.4746p_{4C} + 442.4577w_c$$

$$= 8849.152p_{4C}$$

Or in A's currency

$$884.9152p_{1A} + 884.9152p_{2B}E_{AB} + 176.9831p_{3C}E_{AC} + 265.4746p_{4C}E_{AC} + 442.4577w_cE_{AC}$$

$$= 8849.152p_{4C}E_{AC}$$

... (7)

The market clearing equations for all commodities in A's currency and the full employment equation for country C for the trial trade pattern are as follows,

$$10000p_{1A} = (125 + 150E_{AB} + 200E_{AC}) + (200 + 960 + 643.5763x_{3C} + 884.9152x_{4C})$$

... (8)

$$12000p_{2B}E_{AB} = (125 + 150E_{AB} + 200E_{AC}) + (1000 + 360 + 1001.119x_{3C} + 884.9152x_{4C})$$

... (9)

$$7150.847p_{3C}E_{AC} = (125 + 150E_{AB} + 200E_{AC})$$

$$+ (1200 + 1680 + 286.0339x_{3C} + 176.9831x_{4C})p_{3C}E_{AC}$$

... (10)

$$8849.152p_{4C}E_{AC} = (125 + 150E_{AB} + 200E_{AC})$$

$$+ (1500 + 1440 + 357.5423x_{3C} + 265.4746x_{4C})p_{4C}E_{AC}$$

... (11)

$$357.5423x_{3C} + 442.4577x_{4C} = 800$$

... (12)

The LHS of each market clearing equation is quantity supplied and RHS is quantity demanded. The first parentheses of RHS of all market clearing equations is final consumption demand for that commodity while second parentheses contains the intermediate demand for the commodities. From the above set of market clearing equations it can be observed that the unknowns have appeared in multiples of each other. Therefore, to overcome the difficulty of non-linearity of the equations the process of iteration is used to solve the equations as follows. The trial values of scale multipliers x_{3C}, x_{4C} are chosen so that these should satisfy the equation (12). Substitute these values $x_{3C}^{(1)}, x_{4C}^{(1)}$ in equations (8), (9), (10) and (11). The price system and the system of market clearing equations left with unknowns viz. $p_{1A}, p_{2B}E_{AB}, p_{3C}E_{AC}, p_{4C}E_{AC}, E_{AB}, E_{AC}$. Note that equations (10) and (11) are linearly dependent and also the task after finding out the solution of unknowns pertains only to verify with equations (10) and (11). Finally the solution for scale multipliers are obtained which satisfy all the equations is $x_{3C} = 4.5947$ and $x_{4C} = 3.9364$. The trade equilibrium prices and exchange rates are as follows,

$$\begin{aligned}
p_{1A} &= 0.1083 \\
p_{2B} &= 0.1341 \\
p_{3C} &= 0.0634 \\
p_{4C} &= 0.0601 \\
E_{AB} &= 0.6769 \\
E_{AC} &= 2.8316
\end{aligned}$$

The remaining prices can be calculated using the autarky price equations and prices of imported inputs. They are as follows,

$$\begin{aligned}
p_{2A} = 0.2061 & & p_{1B} = 0.2409 & & p_{1C} = 0.2722 \\
p_{3A} = 0.1926 & & p_{3B} = 0.2682 & & p_{2C} = 0.2134 \\
p_{4A} = 0.2580 & & p_{4B} = 0.3093 & &
\end{aligned}$$

The next step is to construct gains from trade table.

Table 3.11: Gains from Trade for Three Country Trade in Several Goods

Commodity	Country		
	A	B	C
1	9.2293*	6.1310	1.2972
2	4.8519	11.0098*	1.6546
3	5.1918	5.5067	5.5633*
4	3.8745	4.7754	5.8754*

*The maximum quantity

From above gains from trade table it can be seen that it supports the trade pattern A-1, B-2, C-3, 4. Hence it is the equilibrium trade pattern. The equilibrium price system given as follows,

Country A

$$200p_{1A} + 1000p_{2B}E_{AB} + 1200p_{3C}E_{AC} + 1500p_{4C}E_{AC} + 500w_A = 10000p_{1A}$$

Country B

$$960p_{1A}E_{BA} + 360p_{2B} + 1680p_{3C}E_{BC} + 1440p_{4C}E_{BC} + 600w_B = 12000p_{2B}$$

Country C

$$698.9642p_{1A}E_{CA} + 1087.2780p_{2B}E_{CB} + 310.6507p_{3c} + 388.3134p_{4c} + 388.3134w_c$$

$$= 7766.269p_{3c}$$

$$823.3731p_{1A}E_{CA} + 823.3731p_{2B}E_{CB} + 164.6746p_{3c} + 24.0119p_{4c} + 411.6866w_c$$

$$= 8233.731p_{4c}$$

A comparison of the autarky and post-trade situations are summarized as shown in table 3.12,

Table 3.12: Pre and Post Trade Prices, Output and Real Wage Rates in Three Country Trade in Several Goods.

	Country A		Country B		Country C	
	Pre-trade	Post-trade	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	0.1471	0.1084	0.2696	0.1601	0.3503	0.0383
p_2	0.2393	0.0908	0.1581	0.1342	0.2973	0.0321
p_3	0.2185	0.1798	0.2885	0.2656	0.1346	0.0635
p_4	0.2932	0.1702	0.3449	0.2514	0.1211	0.0601
B_1	949.17	10000	928.23	-----	1236.92	-----
B_2	823.45	-----	1079.49	12000	1508.36	-----
B_3	1026.54	-----	1026.08	-----	1690.25	7766.27
B_4	875.21	-----	1007.87	-----	2091.68	8233.73
w/p_1	6.7996	9.2293	3.7085	6.2446	2.8547	26.1224
w/p_2	4.1784	11.0081	6.3252	7.4527	3.3640	31.1713
w/p_3	4.5774	5.5614	3.4657	3.7646	7.4304	15.7533
w/p_4	3.4108	5.8762	2.8994	3.9776	8.2589	16.6371

Above table shows that the world outputs of all commodities have increased, the post-trade prices of all commodities have declined in both the countries. The real wage rates in terms of consumption commodities have increased in both countries.

3.5.2: Three Country Trade in Capital and Consumption Goods

The method outlined above continues to hold even if capital goods and consumption goods are distinct. The only difference that arises in the nature of the international market clearing equations for the respective goods. This difference is illustrated in the example below.

Consider three countries A, B and C with labour endowments 457, 598 and 547 units respectively. They spend $(1/4)^{\text{th}}$ of their net national income on each of the consumption goods. They all are producing 2 non-tradable capital goods, 3 tradable capital goods, 3 tradable consumption goods and 1 non-tradable consumption good. Their initial conditions in terms of demand and supply are different from autarkic conditions. To get unique prices for each of the commodities unlike demand and supply prices in disequilibrium state the way out of autarky equilibrium has been adopted. Following are the autarkic price systems,

Country A

1. $0.6693p_{1A} + 1.5445p_{2A} + 1.6990p_{3A} + 0.5148p_{4A} + 2.9398p_{5A} + 5.1485w_A$
 $= 15.4457p_{1A}$
2. $0.8493p_{1A} + 1.3590p_{2A} + 2.3103p_{3A} + 1.0192p_{4A} + 1.0192p_{5A} + 16.9877w_A$
 $= 16.9877p_{2A}$
3. $0.4585p_{1A} + 0.3275p_{2A} + 0.7861p_{3A} + 0.7370p_{4A} + 1.1792p_{5A} + 6.8789w_A$
 $= 22.9299p_{3A}$
4. $0.7412p_{1A} + 1.3341p_{2A} + 2.0754p_{3A} + 0.8894p_{4A} + p_{4A} + 4.7438p_{5A} + 37.0610w_A$
 $= 14.8244p_{4A}$
5. $2.5935p_{1A} + 1.8976p_{2A} + 3.1628p_{3A} + 2.4037p_{4A} + 1.2651p_{5A} + w_{75.9078A}$
 $= 31.6282p_{5A}$
6. $1.8119p_{1A} + 1.1150p_{2A} + 3.0803p_{3A} + 0.8014p_{4A} + 5.9236p_{5A} + 85.7189w_A$
 $= 90.5972p_{6A}$
7. $2.9022p_{1A} + 3.6277p_{2A} + 3.2649p_{3A} + 2.6119p_{4A} + 2.9312p_{5A} + 81.2616w_A$
 $= 40.6308p_{7A}$
8. $3.9393p_{1A} + 3.3766p_{2A} + 2.1103p_{3A} + 2.7013p_{4A} + 3.0952p_{5A} + 80.1948w_A$
 $= 28.1385p_{8A}$
9. $1.4801p_{1A} + 2.4052p_{2A} + 4.4404p_{3A} + 3.1453p_{4A} + 7.6474p_{5A} + 67.8404w_A$
 $= 32.0700p_{9A}$

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Country B

1. $4.8166p_{1B} + 3.6125p_{2B} + 3.6125p_{3B} + 9.6333p_{4B} + 6.0208p_{5B} + 24.0833w_B$
 $= 36.1250p_{1B}$
2. $5.3577p_{1B} + 2.1430p_{2B} + 3.2146p_{3B} + 7.5007p_{4B} + 2.1430p_{5B} + 37.5039w_B$
 $= 37.5039p_{2B}$
3. $4.3656p_{1B} + 2.9104p_{2B} + 1.4552p_{3B} + 5.0932p_{4B} + 2.1828p_{5B} + 21.8280w_B$
 $= 29.1040p_{3B}$
4. $4.4929p_{1B} + 5.6161p_{2B} + 2.2464p_{3B} + 3.3697p_{4B} + 6.7394p_{5B} + 67.3940w_B$
 $= 50.5455p_{4B}$
5. $1.6639p_{1B} + 4.9917p_{2B} + 1.6639p_{3B} + 4.1597p_{4B} + 3.3278p_{5B} + 58.2365w_B$
 $= 41.5975p_{5B}$
6. $1.8498p_{1B} + 4.6245p_{2B} + 4.0881p_{3B} + 5.6882p_{4B} + 7.8617p_{5B} + 95.2664w_B$
 $= 33.2969p_{6B}$
7. $1.9395p_{1B} + 2.4243p_{2B} + 2.1819p_{3B} + 5.6245p_{4B} + 1.9589p_{5B} + 117.3402w_B$
 $= 124.1285p_{7B}$
8. $2.6812p_{1B} + 2.2982p_{2B} + 5.2667p_{3B} + 5.6689p_{4B} + 2.1067p_{5B} + 109.1656w_B$
 $= 28.7277p_{8B}$
9. $8.9575p_{1B} + 8.8829p_{2B} + 5.3745p_{3B} + 3.8069p_{4B} + 9.2561p_{5B} + 67.1818w_B$
 $= 38.8161p_{9B}$

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Country C

1. $2.0904p_{1C} + 2.7266p_{2C} + 3.9081p_{3C} + 4.5444p_{4C} + 2.7266p_{5C} + 45.4441w_C$
 $= 32.7198p_{1C}$
2. $6.1858p_{1C} + 4.1238p_{2C} + 3.0929p_{3C} + 5.1548p_{4C} + 3.0929p_{5C} + 61.8583w_C$
 $= 30.9291p_{2C}$
3. $2.9298p_{1C} + 3.9553p_{2C} + 2.1974p_{3C} + 0.8789p_{4C} + 1.4649p_{5C} + 32.9610w_C$
 $= 32.9610p_{3C}$
4. $5.8200p_{1C} + 3.6375p_{2C} + 3.4920p_{3C} + 9.0938p_{4C} + 3.7830p_{5C} + 29.1004w_C$
 $= 34.9205p_{4C}$
5. $2.6858p_{1C} + 3.4020p_{2C} + 2.3277p_{3C} + 4.7449p_{4C} + 2.2381p_{5C} + 87.7370w_C$
 $= 29.5441p_{5C}$
6. $2.5815p_{1C} + 4.1304p_{2C} + 3.0978p_{3C} + 1.4456p_{4C} + 2.4782p_{5C} + 85.7066w_C$
 $= 41.3043p_{6C}$
7. $1.6064p_{1C} + 2.9833p_{2C} + 8.0321p_{3C} + 1.7211p_{4C} + 2.2949p_{5C} + 82.6164w_C$
 $= 41.3082p_{7C}$

$$\begin{aligned}
8. \quad & 4.1245p_{1C} + 3.5353p_{2C} + 3.6826p_{3C} + 5.5976p_{4C} + 7.2916p_{5C} + 44.1919w_C \\
& = 58.9226p_{8C} \\
& 4.6951p_{1C} + 2.4345p_{2C} + 3.1301p_{3C} + 1.7389p_{4C} + 4.1735p_{5C} + 77.3838w_C \\
9. \quad & = 30.4318p_{9C}
\end{aligned}$$

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The autarky equilibrium prices are;

$$\begin{array}{lll}
p_{1A} = 1.3902 & p_{1B} = 2.4178 & p_{1C} = 3.1322 \\
p_{2A} = 1.9089 & p_{2B} = 2.2450 & p_{2C} = 4.5303 \\
p_{3A} = 0.6702 & p_{3B} = 2.0182 & p_{3C} = 2.2848 \\
p_{4A} = 4.0820 & p_{4B} = 2.3454 & p_{4C} = 3.4956 \\
p_{5A} = 3.1310 & p_{5B} = 2.2623 & p_{5C} = 4.8878 \\
p_{6A} = 1.2610 & p_{6B} = 4.4898 & p_{6C} = 3.3107 \\
p_{7A} = 2.8119 & p_{7B} = 1.2043 & p_{7C} = 3.3104 \\
p_{8A} = 4.0602 & p_{8B} = 5.2040 & p_{8C} = 2.3208 \\
p_{9A} = 3.5625 & p_{9B} = 3.8514 & p_{9C} = 4.4936
\end{array}$$

Since commodities 1, 2 and 9 are non-tradable, to find trial trade pattern, consider the natural exchange rates for tradable commodities. They are,

$$\begin{array}{lll}
E_{AB}^3 = 0.3321 & E_{BC}^3 = 0.8833 & E_{CA}^3 = 3.4087 \\
E_{AB}^4 = 1.7404 & E_{BC}^4 = 0.6709 & E_{CA}^4 = 0.8563 \\
E_{AB}^5 = 1.3839 & E_{BC}^5 = 0.4628 & E_{CA}^5 = 1.5610 \\
E_{AB}^6 = 0.2808 & E_{BC}^6 = 1.3561 & E_{CA}^6 = 2.6253 \\
E_{AB}^7 = 2.3347 & E_{BC}^7 = 0.3638 & E_{CA}^7 = 1.1773 \\
E_{AB}^8 = 0.7802 & E_{BC}^8 = 2.2423 & E_{CA}^8 = 0.5715
\end{array}$$

The least product of above natural rates will suggest trial trade pattern as given below,

$$E_{AB}^6 E_{BC}^7 E_{CA}^8 = 0.0584 \quad \text{A-6, B-7, C-8}$$

$$E_{AB}^3 E_{BC}^7 E_{CA}^4 = 0.0690 \quad \text{A-6, B-5, C-8}$$

$$E_{AB}^6 E_{BC}^5 E_{CA}^8 = 0.0743 \quad \text{A-3, B-7, C-4}$$

Trial trade pattern is A-1, 2, 3, 6, 9 B-1, 2, 5, 7, 9 C-1, 2, 4, 8, 9. The labour allocation will take place in tradable goods industries from the non-operating industries post trade. After opening all countries to trade the price system post trade would be as follows,

Country A

$$0.6693p_{1A} + 1.5445p_{2A} + 1.6990p_{3A} + 0.5148p_{4C}E_{AC} + 2.9398p_{5B}E_{AB} + 5.1485w_A = 15.4457p_{1A} \quad \dots (1)$$

$$0.8493p_{1A} + 1.3590p_{2A} + 2.3103p_{3A} + 1.0192p_{4C}E_{AC} + 1.0192p_{5B}E_{AB} + 16.9877w_A = 16.9877p_{2A} \quad \dots (2)$$

$$7.9898p_{1A} + 5.7070p_{2A} + 0.7861p_{3A} + 13.6969p_{4C}E_{AC} + 12.8408p_{5B}E_{AB} + 119.8478w_A = 399.4928p_{3A}$$

... (3)

.....Industry 4 is closed.....

.....Industry 5 is closed.....

$$5.2248p_{1A} + 3.2152p_{2A} + 8.8822p_{3A} + 2.3109p_{4C}E_{AC} + 17.0812p_{5B}E_{AB} + 247.1754w_A = 261.2423p_{6A} \quad \dots (4)$$

.....Industry 7 is closed.....

.....Industry 8 is closed.....

$$1.4801p_{1A} + 2.4052p_{2A} + 4.4404p_{3A} + 3.1453p_{4C}E_{AC} + 7.6474p_{5B}E_{AB} + 67.8404w_A = 32.0700p_{9A}$$

... (5)

Country B

$$4.8166p_{1B} + 3.6125p_{2B} + 3.6125p_{3A}E_{BA} + 9.6333p_{4C}E_{BC} + 6.0208p_{5B} + 24.0833w_B = 36.1250p_{1B} \quad \dots (6)$$

$$5.3577p_{1B} + 2.1430p_{2B} + 3.2146p_{3A}E_{BA} + 7.5007p_{4C}E_{BC} + 2.1430p_{5B} + 37.5039w_B = 37.5039p_{2B} \quad \dots (7)$$

.....Industry 3 is closed.....

.....Industry 4 is closed.....

$$4.2131p_{1B} + 12.6393p_{2B} + 4.2131p_{3A}E_{BA} + 10.5327p_{4C}E_{BC} + 8.4262p_{5B} + 147.4586w_B = 105.3276p_{5B} \quad \dots (8)$$

.....Industry 6 is closed.....

$$4.2131p_{1B} + 12.6393p_{2B} + 4.2131p_{3A}E_{BA} + 10.5327p_{4C}E_{BC} + 8.4262p_{5B} + 147.4586w_B = 340.3871p_{7B}$$

... (9)

.....Industry 8 is closed.....

$$8.9575p_{1B} + 8.8829p_{2B} + 5.3745p_{3A}E_{BA} + 3.8069p_{4C}E_{BC} + 9.2561p_{5B} + 67.1818w_B = 38.8161p_{9B}$$

... (10)

Country C

$$2.0904p_{1C} + 2.7266p_{2C} + 3.9081p_{3A}E_{CA} + 4.5444p_{4C} + 2.7266p_{5B}E_{CB} + 45.4441w_C = 32.7198p_{1C}$$

... (11)

$$6.1858p_{1C} + 4.1238p_{2C} + 3.0929p_{3A}E_{CA} + 5.1548p_{4C} + 3.0929p_{5B}E_{CB} + 61.8583w_C = 30.9291p_{2C}$$

... (12)

.....Industry 3 is closed.....

$$29.9597p_{1C} + 18.7248p_{2C} + 17.9758p_{3A}E_{CA} + 46.8120p_{4C} + 19.4738p_{5B}E_{CB} + 149.7986w_C = 179.7583p_{4C}$$

... (13)

.....Industry 5 is closed.....

.....Industry 6 is closed.....

.....Industry 7 is closed.....

$$19.8347p_{1C} + 17.0012p_{2C} + 17.7095p_{3A}E_{CA} + 26.9185p_{4C} + 35.0649p_{5B}E_{CB} + 211.5151w_C = 283.3534p_{8C}$$

... (14)

$$4.6951p_{1C} + 2.4345p_{2C} + 3.1301p_{3A}E_{CA} + 1.7389p_{4C} + 4.1735p_{5B}E_{CB} + 77.3838w_C = 30.4318p_{9C}$$

... (15)

The price systems of country B and C can be converted into A's currency by multiplying their price systems by E_{AB} and E_{AC} respectively. The market clearing equations for all commodities are as follows,

Country A

Demand-supply equations for capital goods industry

$$0.6693x_{1A} + 0.8493x_{2A} + 7.9898x_{3A} + (5.2248 + 1.4801) = 15.4457x_{1A} \quad \dots (16)$$

$$1.5445x_{1A} + 1.3590x_{2A} + 5.7070x_{3A} + (3.2152 + 2.4052) = 16.9877x_{2A} \quad \dots (17)$$

$$1.6990x_{1A} + 2.3103x_{2A} + 0.7861x_{3A} + (8.8822 + 4.4404) + BD3 + CD3 = 399.4928x_{3A}$$

... (18)

The RHS is supply and LHS is demand in each of the above equations. The x_{iA} 's are the scale multipliers. The quantity in the bracket on LHS is consumption good industry's demand for respective commodities. In equation (18) terms BD3 and CD3 on LHS represent the country B and C's demand for tradable capital good 3 respectively.

Demand-supply equations for consumption goods industry

i) For tradable commodity 6

$$209.5946p_{6A} = \left(\frac{1}{4} \times 1 \times 451\right) + \left(\frac{1}{4} \times 1 \times 598E_{AB}\right) + \left(\frac{1}{4} \times 1 \times 547E_{AC}\right) \quad \dots (19)$$

ii) For non-tradable commodity 9

$$32.0700p_{9A} = \frac{1}{4} \times 1 \times 451 \quad \dots (20)$$

Country B

Demand-supply equations for capital goods industry

$$4.8166x_{1B} + 5.3577x_{2B} + 4.2131x_{5B} + (4.2131 + 8.9575) = 36.1250x_{1B} \quad \dots (21)$$

$$3.6125x_{1B} + 2.1430x_{2B} + 12.6393x_{5B} + (12.6393 + 8.8829) = 37.5039x_{2B} \quad \dots (22)$$

$$6.0208x_{1B} + 2.1430x_{2B} + 8.4262x_{5B} + (8.4262 + 9.2561) + AD5 + CD5 = 105.3276x_{5B} \quad \dots (23)$$

The RHS is supply and LHS is demand in each of the above equations. The x_{iB} 's are the scale multipliers. The quantity in the bracket on LHS is consumption good industry's demand for respective commodities. In equation (23) terms AD5 and CD5 on LHS represents the country A's and C's demand respectively for tradable capital good 5.

Demand-supply equations for consumption goods industry

i) For tradable commodity 6

$$340.3871p_{7B} = \left(\frac{1}{4} \times 1 \times 451 \times E_{BA}\right) + \left(\frac{1}{4} \times 1 \times 598\right) + \left(\frac{1}{4} \times 1 \times 547E_{BC}\right) \quad \dots (24)$$

ii) For non-tradable commodity 9

$$38.8161p_{9B} = \frac{1}{4} \times 1 \times 598 \quad \dots (25)$$

Country C

Demand-supply equations for capital goods industry

$$2.0904x_{1C} + 6.1858x_{2C} + 29.9597x_{4C} + (19.8347 + 4.6951) = 32.7198x_{1C} \quad \dots (26)$$

$$2.7266x_{1C} + 4.1238x_{2C} + 18.7248x_{4C} + (17.7095 + 3.1301) = 30.9251x_{2C} \quad \dots (27)$$

$$4.5444x_{1C} + 5.1548x_{2C} + 46.8120x_{4C} + (26.9185 + 1.7389) + AD4 + CD4 = 179.7583x_{4C} \quad \dots (28)$$

The RHS is supply and LHS is demand in each of the above equations. The x_{iC} 's are the scale multipliers. The quantity in the bracket on LHS is consumption good industry's demand for respective commodities. In equation (28) term AD4 and CD4 on LHS represents the country A's and C's demand respectively for tradable capital good 4.

Country C

Demand-supply equations for consumption goods industry

i) For tradable commodity 6

$$283.3534p_{8C} = \left(\frac{1}{4} \times 1 \times 451 \times E_{CA} \right) + \left(\frac{1}{4} \times 1 \times 598 \times E_{CB} \right) + \left(\frac{1}{4} \times 1 \times 547 \right) \quad \dots (29)$$

ii) For non-tradable commodity 9

$$30.4318p_{9C} = \frac{1}{4} \times 1 \times 547 \quad \dots (30)$$

By using the demand-supply equations countries A, B and C the scale multipliers can be solved. Once they are calculated we multiply the price equations of capital goods by their respective scale multipliers to get new levels of outputs. Further, each country has one non-tradable consumption good and one tradable consumption good hence for these two commodities the respective price equation solve for supply prices and demand-supply equations solve the demand prices. Therefore by using the bisection method iteratively demand and supply of consumption goods can be equalized.

After 18 such iterations all prices are determined at the equilibrium;

$$\begin{array}{lll}
p_{1A} = 1.2302 & p_{1B} = 2.1561 & p_{1C} = 2.4633 \\
p_{2A} = 1.7448 & p_{2B} = 2.0316 & p_{2C} = 3.7517 \\
p_{3A} = 0.4587 & p_{5B} = 2.1430 & p_{4C} = 2.7415 \\
p_{6A} = 1.2128 & p_{7B} = 1.1760 & p_{8C} = 1.7551 \\
p_{9A} = 3.3052 & p_{9B} = 3.5076 & p_{9C} = 3.5939
\end{array}$$

The equilibrium exchange rates are;

$$E_{AB} = 1.2792 \text{ \& } E_{AC} = 1.0599$$

At these exchange rates the prices of commodities which are not traded are found to be;

$$\begin{array}{lll}
p_{4A} = 3.8343 & p_{3B} = 1.9079 & p_{3C} = 2.7415 \\
p_{5A} = 2.9819 & p_{4B} = 2.3048 & p_{5C} = 4.0700 \\
p_{7A} = 2.6651 & p_{6B} = 4.2549 & p_{6C} = 2.8877 \\
p_{8A} = 3.8465 & p_{8B} = 4.8970 & p_{7C} = 2.5726
\end{array}$$

The gains from trade is as follows,

Table 3.13: Gains from Trade in Three Country Trade in Capital and Consumption Goods

Gains from Trade			
Commodity	Country		
	A	B	C
3	2.1803*	0.4098	0.5002
4	0.2608	0.3392	0.3441*
5	0.3354	0.3648*	0.2318
6	0.8246*	0.1837	0.3267
7	0.3752	0.6648*	0.3667
8	0.2600	0.1596	0.5376*

*The maximum quantity

Hence the trade equilibrium is found. The predicted trade pattern holds true. The price system corresponding to equilibrium trade pattern is,

$$\begin{array}{c}
\text{Country A} \\
0.5036p_{1A} + 1.1621p_{2A} + 1.2783p_{3A} + 0.3873p_{4C}E_{AC} + 2.2119p_{5B}E_{AB} \\
+ 3.8738w_A = 11.6213p_{1A}
\end{array}$$

$$0.5056p_{1A} + 0.8089p_{2A} + 1.3752p_{3A} + 0.6067p_{4C}E_{AC} + 1.1325p_{5B}E_{AB} + 10.1114w_A = 10.1114p_{2A}$$

$$1.7619p_{1A} + 1.2585p_{2A} + 3.0204p_{3A} + 2.8316p_{4C}E_{AC} + 4.5306p_{5B}E_{AB} + 26.4285w_A = 88.0950p_{3A}$$

.....Industry 4 and 5 are closed.....

$$7.4283p_{1A} + 4.5712p_{2A} + 12.6280p_{3A} + 3.2856p_{4C}E_{AC} + 24.2847p_{5B}E_{AB} + 351.4137w_A = 371.4129p_{6A}$$

.....Industry 7 and 8 are closed.....

$$1.4219p_{1A} + 2.3107p_{2A} + 4.2658p_{3A} + 3.0216p_{4C}E_{AC} + 7.3467p_{5B}E_{AB} + 65.1726w_A = 30.8089p_{9A}$$

Country B

$$3.8961p_{1B} + 2.9221p_{2B} + 2.9221p_{3A}E_{BA} + 7.7921p_{4C}E_{BC} + 4.8700p_{5B} + 19.4803w_B = 29.2205p_{1B}$$

$$5.2230p_{1B} + 2.0920p_{2B} + 3.1380p_{3A}E_{BA} + 7.3220p_{4C}E_{BC} + 2.0920p_{5B} + 36.6098w_B = 36.6098p_{2B}$$

.....Industry 3 and 4 are closed.....

$$5.2083p_{1B} + 15.6250p_{2B} + 5.2083p_{3A}E_{BA} + 13.0208p_{4C}E_{BC} + 10.4166p_{5B} + 182.2913w_B = 130.208p_{5B}$$

.....Industry 6 is closed.....

$$4.6787p_{1B} + 5.8484p_{2B} + 5.2636p_{3A}E_{BA} + 13.5683p_{4C}E_{BC} + 4.7255p_{5B} + 283.0629w_B = 299.4385p_{7B}$$

.....Industry 8 is closed.....

$$10.2073p_{1B} + 10.1224p_{2B} + 6.1245p_{3A}E_{BA} + 4.3382p_{4C}E_{BC} + 10.5477p_{5B} + 76.5557w_B = 44.2322p_{9B}$$

Country C

$$3.5970p_{1C} + 4.6917p_{2C} + 6.7248p_{3A}E_{CA} + 7.8195p_{4C} + 4.6917p_{5B}E_{CB} + 78.1951w_C = 56.3005p_{1C}$$

$$8.2578p_{1C} + 5.5052p_{2C} + 4.1289p_{3A}E_{CA} + 6.8815p_{4C} + 4.1289p_{5B}E_{CB} + 82.5782w_C = 41.2891p_{2C}$$

.....Industry 3 is closed.....

$$21.6453p_{1C} + 13.5283p_{2C} + 12.9872p_{3A}E_{CA} + 33.8207p_{4C} + 14.0694p_{5B}E_{CB} + 108.2263w_C = 129.8715p_{4C}$$

.....Industry 5, 6 and 7 are closed.....

$$16.9552p_{1C} + 14.5330p_{2C} + 15.1386p_{3A}E_{CA} + 23.0107p_{4C} + 29.9744p_{5B}E_{CB} + 181.663w_C = 242.2173p_{8C}$$

$$5.8452p_{1C} + 3.0308p_{2C} + 3.8968p_{3A}E_{CA} + 2.1649p_{4C} + 5.1957p_{5B}E_{CB} + 96.3375w_C = 37.8855p_{9C}$$

A comparison of the autarky and post-trade situations may be summarized as shown in table 3.14 below.

Table 3.14: Pre and Post Trade Prices, Output and Real Wage Rates in Three Country Trade in Capital and Consumption Goods.

	Country A		Country B		Country C	
	Pre-trade	Post-trade	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	1.3902	1.2302	2.4179	2.1561	3.1323	2.4633
p_2	1.9090	1.7448	2.2451	2.0316	4.5304	3.7517
p_3	0.6703	0.4587	2.0182	0.3586	2.2848	0.4328
p_4	4.0820	2.9057	2.3454	2.2715	3.4956	2.7415
p_5	3.1310	2.7413	2.2624	2.1430	4.8879	2.5864
p_6	1.2611	1.2128	4.4899	0.9481	3.3108	1.1442
p_7	2.8119	1.5042	1.2044	1.1760	3.3105	1.4192
p_8	4.0603	1.8605	5.2040	1.4543	2.3208	1.7551
p_9	3.5625	3.3052	3.8515	3.5076	4.4936	3.5939
B_1	15.45	15.45	36.13	36.13	32.72	32.72
B_2	16.99	16.99	37.50	37.50	30.93	30.93
B_3	22.93	399.49	29.10	-----	32.96	-----
B_4	14.82	-----	50.55	-----	34.92	179.76
B_5	31.63	-----	41.60	105.33	29.54	-----
B_6	90.60	261.24	33.30	-----	41.30	-----
B_7	40.63	-----	124.13	340.39	41.31	-----
B_8	28.14	-----	28.73	-----	58.92	283.35
B_9	32.07	32.07	38.82	38.82	30.43	30.43
w/p_1	0.7193	0.8129	0.4136	0.4638	0.3193	0.4060
w/p_2	0.5238	0.5731	0.4454	2.7888	0.2207	0.2665
w/p_3	1.4919	2.1803	0.4955	2.7887	0.4377	2.3108
w/p_4	0.2450	0.3441	0.4264	0.4402	0.2861	0.3648
w/p_5	0.3194	0.3648	0.4420	0.4666	0.2046	0.3867
w/p_6	0.7930	0.8246	0.2227	1.0547	0.3020	0.8740
w/p_7	0.3556	0.6648	0.8303	0.8503	0.3021	0.7046
w/p_8	0.2463	0.5375	0.1922	0.6876	0.4309	0.5698
w/p_9	0.2807	0.3026	0.2596	0.2851	0.2225	0.2783

Above table shows that the world outputs of all commodities have increased, the post-trade prices of all commodities have declined in both the countries and consequently real wage rates have increased in both countries.

3.6: Remarks

Traditionally the Leontief input-output model has used as methodological apparatus for testing alternative theories of trade. We have shown in this section how international trade theory itself can be written in terms of the Leontief model. The Leontief trade model then makes a claim to greater generality to accommodate trade between several countries in several commodities which has been very difficult for standard trade theories to do. It may be noted that the price system of the Leontief static open model is the unified price system for all countries and can be referred to as world price system which determines a solution for prices. Because of factor immobility, in order to determine the volumes of outputs produced, consumed and traded, we need to introduce the sizes of employment and the consumption demand equations into the model and solve them separately. The Leontief trade model determines the comparative advantages, trade patterns, terms of trade and currency exchange rate simultaneously. It has the ability to synthesize the ‘pure’ and the ‘monetary’ aspects of international trade. In the current chapter we have established the Leontief models in static framework. In the next chapter we will deal with the trade models in dynamic framework.

Chapter 4: Dynamic Trade Models

4.1: Introduction

The Leontief trade model of the previous chapter pertained to trade between static economies whose rates of growth were zero. In this chapter we shall consider the problem of determining the international trade equilibrium for growing economic systems. Economies grow due to net saving and new investment. In the case of static economies investment activity is confined to replacing the capital stocks that are used in the various industries. We shall now be concerned with economies that not only replace the existing capital stocks but also add to them thus expanding productive capacity. Naturally, when such economies trade the world supplies of capital goods must be such as to satisfy both the replacement demands as well as the new investment demands for them considered at the different rates of growth of various countries that use them. In this way output systems of the trading countries are intertwined with one another.

Yet another complication will be encountered in the dynamic model. The dual of the output system containing the rate of growth will be the price system containing the rate of profit, both of which were zero in the previous chapter. The price systems of the various countries must now make provision to pay profits (besides costs of production) at the disparate rates in the various countries. And since capital goods are traded the cost of production and profitability of various countries are intertwined into an international price system. The result is that the dynamic trade model will determine the rates of growth and rates of profit (income distribution) in the various trading countries in the international trade equilibrium besides the outputs, the inputs, exports, domestic consumption and in terms of trade and exchange rates.

In case of assumptions, we will continue to assume, as in the preceding chapter, the international immobility of labour and capital, constant returns to scale production functions in all industries, homogenous demand functions, the absence of joint products, fixed capital, human capital, natural resources and public goods. For simplicity we will assume that wages are wholly consumed and profits are fully accumulated. Section 2 explains the procedure of finding the equilibrium in autarky. Section 3 contains a two-country two-commodity model of international trade in which the traded commodities are

capital goods. Section 4 shows how the world trade equilibrium in case of two country trade in consumption goods is determined. After getting grip of the trade in capital and purely consumption goods separately, section 5 deals with the two-country trade in capital as well as consumption goods considering some of them as non-tradable commodities. Lastly, section 6 contains the generalized model of three country trade in capital and consumption goods.

4.2: Equilibrium in Autarky

In this section we will develop theoretical model by considering an economy which produces three goods by means of a fixed coefficients constant returns to scale technology. Of the three commodities, two are purely capital goods and the third is purely a consumption good. It is supposed that profits are wholly accumulated and wages are wholly consumed. Also suppose that the money wage rate is exogenously given, say Re . 1. Then the following equations describe the price determination of the three goods and demand equations for the consumption good.

$$(a_{11}p_1 + a_{21}p_2)(1+r) + wL_1 = B_1p_1$$

$$(a_{12}p_1 + a_{22}p_2)(1+r) + wL_2 = B_2p_2$$

$$(a_{13}p_1 + a_{23}p_2)(1+r) + wL_3 = B_3p_3$$

$$wL = B_3p_3 \quad \dots (1)$$

The unknowns whose values in equilibrium are to be determined are the three prices, the rate of profit, the rate of growth and the outputs of the three commodities, i.e. 7 unknowns in all. Of course, in view of the classical assumption that wages are wholly consumed and profits are wholly saved, the equilibrium rate of growth will be equal to the equilibrium rate of profit. By equilibrium will be meant that demands and supplies of all the three goods and labour must be equal to one another. The demand-supply equation for the consumption good is the fourth equation of (1). The demand-supply equations for the capital goods appear in the dual of the system (1) which will be written as,

$$\begin{aligned}
(a_{11}x_1 + a_{12}x_2 + A_{1x})(1+g) &= B_1x_1 \\
(a_{21}x_1 + a_{22}x_2 + A_{2x})(1+g) &= B_2x_2 \\
L_1x_1 + L_2x_2 + L_3 &= L_1 + L_2 + L_3 = L_k
\end{aligned}
\tag{2}$$

The left hand sides of the first two equations are the demands for the two capital goods and the right hand sides are the supplies; the left hand side shows the replacement and new investment demands (at a uniform growth rate) for the capital goods from all the three industries in the economy which must be satisfied by the supplies of those goods shown on right hand side. The last equation of (2) is the full-employment condition which must be read as $L_1x_1 + L_2x_2 + L_3 = L_1 + L_2 + L_3$ implying $L_1x_1 + L_2x_2 = L_1 + L_2 = L_k$ where L_k is total employment in the two capital goods industries. The system (2) solves for the rate of growth (equal to the rate of profit) and the two scale intensities x_1 and x_2 . Multiplying the first two equations of (1) by x_1 and x_2 and writing $r = g$ the system of equation gives the equilibrium of the economic system.

$$\begin{aligned}
(a_{11}x_1p_1 + a_{21}x_1p_2)(1+r) + wL_1x_1 &= B_1x_1p_1 \\
(a_{12}x_2p_1 + a_{22}x_2p_2)(1+r) + wL_2x_2 &= B_2x_2p_2 \\
(a_{13}p_1 + a_{23}p_2)(1+r) + wL_3 &= B_3p_3
\end{aligned}
\tag{3}$$

When the solution of rate of profit is substituted in (3) it solves for the prices p_1 , p_2 and p_3 . It can be easily shown that the quantity demanded of commodity 3, B_{3d} at the price solution p_3 in equation (3) is exactly equal to the quantity supplied B_3 in equation (1). From the fourth equation of (1);

$$B_{3d} = \frac{wL}{p_3} = \frac{(a_{13}p_1 + a_{23}p_2)(1+r) + wL_3}{p_3} = B_3$$

$$\text{because } w(L_1 + L_2) = (a_{13}p_1 + a_{23}p_2)(1+r)$$

where, $w(L_1 + L_2)$ is value of exports of industry 3 to the capital goods industries and $(a_{13}p_1 + a_{23}p_2)(1+r)$ is the value of its imports (replacement plus new investment) from them. Thus the equality of the demand and supply of the consumption is attained at every arbitrary supply level of the consumption good B_3 . It is immediately pointed out that this

is not a general property; it is a property of an economic system that contains several capital goods but only one consumption good.

4.3: Two Country Trade in Capital Goods

In this section we will study the case of two country trade in capital goods. Consider two economies A and B which produce three commodities out of which two capital goods are tradable and one non-tradable consumption good. Suppose the wage rates for country A and B are $w_A = ¥1$ and $w_B = \$1$ respectively. We will assume that wages are spent on the non-tradable consumption good in both countries and profits are wholly accumulated. The labour endowments in country A and country B are $L_A = 12$ and $L_B = 14$ units respectively.

The autarky equilibria, ascertained by the method outlined in section 4.2 are shown below the system of equations (1).

Autarky Equilibria

	Country A	Country B
1	$(2.0p_{1A} + 3.0p_{2A})(1 + r_A)$ $+ 1.0w_A = 27.0p_{1A}$	$(3.375p_{1B} + 13.166p_{2B})(1 + r_B)$ $+ 9.0w_B = 16.2p_{1B}$
2	$(9.0p_{1A} + 5.5p_{2A})(1 + r_A)$ $+ 10.0w_A = 26.25p_{2A}$	$(1.0p_{1B} + 1.5p_{2B})(1 + r_B)$ $+ 1.0w_B = 48.0p_{2B}$
3	$(7.0p_{1A} + 9.0p_{2A})(1 + r_A)$ $+ 1.0w_A = 20.0p_{3A}$	$(4.625p_{1B} + 12p_{2B})(1 + r_B)$ $+ 4.0w_B = 20.0p_{3B}$
		... (1)

The rates of profits and prices in country A and country B are;

Country A	Country B
$r_A = 0.5$	$r_B = 0.8$
$p_{1A} = ¥0.1696$	$p_{1B} = \$1.0370$
$p_{2A} = ¥0.6828$	$p_{2B} = \$0.0632$
$p_{3A} = ¥0.6$	$p_{3B} = \$0.7$

The pattern of comparative advantages for the tradable goods 1 and 2 can be obtained from the prices;

$$\frac{P_{1A}}{P_{2A}} = 0.2485 < \frac{P_{1B}}{P_{2B}} = 16.3874$$

This condition may also be equivalently stated in exchange rate units,

$$\frac{P_{1A}}{P_{1B}} = 0.1636 < \frac{P_{2A}}{P_{2B}} = 10.7904$$

If we allow A to specialize in the production of commodity 1 and B to specialize in the production of commodity 2 then industries 1 and 2 in countries B and A respectively will cease to operate and the price equations in the two countries would be as below,

Post-Trade System of Equations

Country A

... (2)

1 $(2.0p_{1A} + 3.0p_{2B}E_{AB})(1 + r_A) + 1.0w_A = 27.0p_{1A}$

2Industry 2 is closed.....

Country B

1Industry 1 is closed.....

2 $(1.0p_{1B}E_{BA} + 1.5p_{2B})(1 + r_B) + 1.0w_B = 48.0p_{2B}$

3 $(4.625p_{1A}E_{BA} + 12.0p_{2B})(1 + r_B) + 4.0w_B = 20.0p_{3B}$

Observe that the price equations of the industries have been expressed in terms of the own currencies of the countries. Thus the yen-dollar exchange rate E_{BA} has been attached to the price of commodity 2 which is imported by country A and dollar-yen rate E_{AB} has been attached to the price of commodity 1 which is imported by country B.

To find whether the above configuration of activities is an international trade equilibrium, we set up the world demand-supply equations for the commodities; for the two tradable capital goods the sources of demands are domestic and foreign, whereas for the non-tradable consumption good the demand is only domestic. There are primarily 13 unknowns to be determined including 4 post-trade outputs, 4 post-trade prices, 2 post-trade rates of growth, 2 post-trade rates of profit and the exchange rate (or terms of trade). Of these, the post-trade outputs of the (non-tradable) consumption good will not need determination; they will be equal to the autarky levels. Also due to the classical

assumption that profits are accumulated and wages are consumed each country's growth rate will equal its profit rate. Thus, 4 unknowns are eliminated so that 9 remain to be determined. As regards the equations there will firstly be two world demand-supply equations for the capital goods which simply state that outputs of capital goods 1 and 2 from countries A and B must be such as to satisfy the replacement and new investment demands of all industries in both the countries. These are supplemented by the full-employment conditions in the two countries.

$$\frac{x_{1A}L_{1A}}{l_{1A}} = \left(\frac{a_{11A}x_{1A}L_{1A}}{l_{1A}} + \frac{a_{13A}x_{3A}L_{3A}}{l_{3A}}\right)(1 + g_A) + \left(\frac{a_{12B}x_{2B}L_{2B}}{l_{2B}} + \frac{a_{13B}x_{3B}L_{3B}}{l_{3B}}\right)(1 + g_B)$$

$$\frac{x_{2B}L_{2B}}{l_{2B}} = \left(\frac{a_{21A}x_{1A}L_{1A}}{l_{1A}} + \frac{a_{31A}x_{3A}L_{3A}}{l_{3A}}\right)(1 + g_A) + \left(\frac{a_{22B}x_{2B}L_{2B}}{l_{2B}} + \frac{a_{32B}x_{3B}L_{3B}}{l_{3B}}\right)(1 + g_B)$$

... (3)

$$x_{1A}L_{1A} + x_{3A}L_{3A} = L_A$$

$$x_{2B}L_{2B} + x_{3B}L_{3B} = L_B$$

... (4)

For reasons stated above $x_{3A} = x_{3B} = 1$ so that the systems (3) & (4) together will have 4 equations in 4 unknowns. Their solution is $x_{1A} = 11$ and $x_{2B} = 10$, $g_A = r_A = 4.92$, $g_B = r_B = 7.5688$.

At this stage we have determined the post-trade levels of outputs and the growth/profit rates in the two economies. We must now determine the prices of commodities and the exchange rate. To determine these there are two price equations in table 4.3, that for capital good 1 in country A and for capital good 2 in country B. Besides, there is one equation to determine the exchange rate. The equilibrium exchange rate E_{AB} must be such as to balance the trade. Considering that $E_{AB} = 1/E_{BA}$ we may write the trade balancing exchange rate as follows,

$$E_{AB} = \frac{x_{2B}p_{1A}(1 + r_B) + 4.625x_{3B}p_{1A}(1 + r_B)}{3x_{1A}p_{2B}(1 + r_A) + 9x_{3A}p_{2B}(1 + r_A)}$$

... (5)

Where, the numerator shows the imports of country B in terms of yen and the denominator shows the exports of country B in dollars.

Now observe the circularity; until the prices p_{1A} and p_{2B} are determined we cannot ascertain the exchange rate. But the prices p_{1A} and p_{2B} which are in yen and dollars respectively cannot be determined until the exchange rate is determined. However we have the required number of equations viz. the post-trade price equations for the two capital goods appearing in system of post-trade price equation (2) and the exchange rate equation (5) above. So the only way to resolve the circularity is to proceed iteratively. Thus substitute initially the autarky prices p_{1A} and p_{2B} in (5) and solve E_{AB} . Next substitute E_{AB} in the price equations of capital goods in (2) to find out p_{1A} and p_{2B} corresponding to it and substitute the solution in (5) to obtain a revised value of E_{AB} and so on until the result converge. Table 4.1 below shows the convergence to an accuracy of up to 4 places of decimals in 7 iterations.

Iteration No.	p_{1A}	p_{2B}	E_{AB}
1.	0.1696970	0.06328054	1.351616
2.	0.1600593	0.0553372	1.457851
3.	0.1609443	0.0551371	1.471232
4.	0.1610065	0.0551232	1.472171
5.	0.1610246	0.05511923	1.472443
6.	0.1610373	0.0551164	1.472468
7.	0.1610368	0.0551158	1.472477

This completes determination of 7 of the 9 unknowns. Finally given the solution of the capital goods prices and the exchange rate the prices of the non-tradable consumption good in each country can be determined from respective price equations in

system of equation (2). Of course since the demand prices of the goods are known in advance $\left(\frac{wL_A}{B_{3A}}, \frac{wL_B}{B_{3B}} \right)$ this last step only verifies that the cost of production of the post-trade capital goods' prices and rates of profit i.e. supply prices equal the demand prices. Since the exchange rate $E_{AB} = 1.4725$ lies between the limiting value of 0.163 and 10.790 both countries will gain from trade. In other words, the ratios of commodity exchange (terms of trade) viz. 6.2111 units of commodity 1=26.4360 units of commodity 2 or 4.2562 lies between the limiting values of 0.2485 and 6.387. If A and B were to produce capital goods 2 and 1 respectively in post-trade situations then their prices would worked out to be 0.8111 and 1.1365 respectively. Table 4.2 shows the quantities of commodity 1 and 2 that can be obtained for ¥1 at home and abroad at the equilibrium exchange rate;

Table 4.1: Gains from Trade in Two- Country Trade in Capital Goods.

	Country A	Country B
Capital good 1	6.21118*	0.5976
Capital good 2	1.2329	12.2584*

*The maximum quantity

Accordingly the world trade equilibrium may be written as shown below.

Equilibrium Price System

Country A

$$1 \quad (22p_{1A} + 33p_{2B}E_{AB})(1 + r_A) + 11w_A = 297p_{1A}$$

2..... Industry 2 is closed.....

$$3 \quad (7p_{1A} + 9p_{2B}E_{AB})(1 + r_A) + 1w_A = 20p_{3A}$$

Country B

1 Industry 1 is closed.....

$$2 \quad (10p_{1A}E_{BA} + 15p_{2B})(1 + r_B) + 10w_B = 480p_{2B}$$

$$3 (4.625p_{1A}E_{BA} + 12p_{2B})(1 + r_B) + 4w_B = 20p_{3B} \quad \dots (6)$$

Some observations that are apparent from a comparison of the autarky and post-trade situations may be summarized as follows.

Table 4.2: Pre-trade and Post-trade Prices, Outputs, Real Wage and Growth in Two- Country Two-Capital Goods Trade

	Country A		Country B	
	Pre-trade	Post-trade	Pre-trade	Post-trade
p_{1A}	0.1696	0.1610	1.037	0.1093
p_{2A}	0.6828	0.0816	0.0632	0.0554
p_{3A}	0.6	0.6	0.7	0.7
B_1	27	297	16.2
B_2	26.25	48	480
B_3	20	20	20	20
w/p_3	1.6667	1.6667	1.4287	1.4287
g	0.5	4.92	0.8	7.5683
r	0.5	4.92	0.8	7.5683

Table 4.2 shows at a glance the general consequences of trade in the (simplistic) situation of our example. The post-trade prices of the capital goods have declined in both countries, the world outputs of capital goods have increased, the rates of growth and profit have risen but the prices of the consumption good and consequently the real wage rate have remained unaltered. It should immediately be pointed out that this last observation is not a general conclusion; it holds exclusively for a model with a single consumption good. In general case the real wage rates too increase. To conclude,

- (i) Firstly, while handling trade in capital goods the distinction between value capital and capital goods has been strictly maintained with profits being charged on the value of capital invested and the capital goods having their individual market prices. Specifically, the rate of profit is not treated as a return on “physical

capital". It would appear that this treatment of capital goods, capital and profits exactly fulfils the requirements that Steedman et.al. would expect from their critique of trade theory in the context of the capital controversy.

- (ii) It should be pointed out that the assumptions of model do not guarantee the existence of a world trade equilibrium. If, for instance, if the labour endowments had been 10 and 15 in country A and country B respectively then the trade equilibrium would not exist.
- (iii) Gainful trade of capital goods between the countries is found to raise the rates of growth and the rates of profit. That the post-trade real wage rate remains unaltered should not be taken as a general conclusion; it is a property of a model which has only one consumption good. In the general case to be discussed below we should expect the real wage rates to rise. Access to international trade is akin to an access to a powerful technology that shifts outwards the wage-profit frontiers of both the countries.

4.4: Two Country Trade in Consumption Goods

In the previous section 4.3, we have seen that the trade equilibrium in case of two countries which were producing and trading two intermediate capital goods and one non-tradable consumption good. In this section, we will consider the case of trade between two countries in purely consumption goods and one non-tradable capital good (e.g. industrial infrastructure). Consider two economies with labour endowments 15 and 10 respectively. The wage rates for country A and B are $w_A = ¥1$ and $w_B = \$1$ respectively. We continue to assume that wages are entirely spent on the consumption goods and profits are wholly accumulated. We shall suppose that these countries differ in techniques of production and they have similar taste for consumption goods. Thus Engel's consumption coefficients are (1/2) for each consumption commodity in all countries. The autarky equilibria are as follows,

Country A	Country B
1 $(5p_{1A})(1+r_A) + 9w_A = 11p_{1A}$	$(6p_{1B})(1+r_B) + 5.85w_B = 11p_{1B}$
2 $(2.5p_{1A})(1+r_A) + 3w_A = 7.5p_{2A}$	$(1.08p_{1B})(1+r_B) + 3.19w_B = 21.6093p_{2B}$
3 $(2.5p_{1A})(1+r_A) + 3W_A = 12.5p_{3A}$	$(2.41p_{1B})(1+r_B) + 0.96w_B = 4.8132p_{3B}$

... (1)

The autarky rates of profit and prices of goods are as follows,

Country A	Country B
$r_A = 0.1$	$r_B = 0.1595$
$p_{1A} = ¥1.6363$	$p_{1B} = \$1.4469$
$p_{2A} = ¥1$	$p_{2B} = \$0.2314$
$p_{3A} = ¥0.6$	$p_{3B} = \$1.0388$

The natural exchange rates are;

Country A	Country B
$E_{AB}^2 = 4.3215$	$E_{BA}^2 = 0.2314$
$E_{AB}^3 = 0.5776$	$E_{AB}^3 = 1.7313$

Lowest product of above exchange rates $E_{AB}^3 E_{BA}^2 = 0.1337$ gives the most profitable trade pattern A- 1, 3 and B- 1, 2. After the opening of trade the pattern of world economic activities will look as follows,

A

$$1 \quad x_{1A} [(5p_{1A})(1+r_A) + 9w_A = 11p_{1A}]$$

$$2 \quad \dots\dots\dots \text{Industry 2 is closed} \dots\dots\dots$$

$$3 \quad [(1p_{1A})(1+r_A) + 1.2w_A = 5p_{3A}]$$

B

$$1 \quad x_{1B} [(6p_{1B})(1+r_B) + 5.85w_B = 11p_{1B}]$$

$$2 \quad [(1.08p_{1B})(1+r_B) + 3.19w_B = 21.6093p_{2B}]$$

$$3. \quad \dots\dots\dots \text{Industry 3 is closed} \dots\dots\dots$$

... (2)

Note that in above system of equation (2), x_{1A} and x_{1B} are the scale multipliers which is when multiply to price equation will give the new level of output. First, using the full employment equations we find out the values of these two scale multipliers

Using the full employment equation of country A we will determine x_{1A} as;

$$(9x_{1A} + 6) = 15;$$

$$x_{1A} = 1$$

Similarly, for country B the full employment equation will determine x_{1B} as follows,

$$(5.85x_{1B} + 4.15) = 10$$

$$x_{1B} = 1$$

After substituting the values of scale multipliers and reallocation of labour in the industries, the price systems would be as follows.

A	B
$1(5p_{1A})(1+r_A) + 9w_A = 11p_{1A}$	$(6p_{1B})(1+r_B) + 5.85w_B = 11p_{1B}$
$2 \text{ _____ Industry 2 is closed _____}$	$(1.40p_{1B})(1+r_B) + 4.15w_B = 28.11p_{2B}$
$3(5p_{1A})(1+r_A) + 6W_A = 25p_{3A}$	$\text{_____ Industry 3 is closed _____}$
	... (3)

Note that the values of scale multipliers are 1 each. The total labour in country A is 15 out of which 9 is employed in capital good's industry1 and remaining 6 unit is now employed in tradable consumption good's industry 3. Similarly, the total labour in country B is 10 out of which 5.85 unit is employed in capital good's industry1 and remaining 4.15 unit is now employed in tradable consumption good's industry 2.

Now there are 7 unknowns; viz. two rates of profit, two prices in each country i.e. 4 prices and an equilibrium exchange rate. The economic system contains 7 independent equations; 4 price equations in two countries, two demand-supply equations of the capital goods and one trade balance equation.

The rate of growth of country A can be solved from demand-supply equation for capital goods industry1,

$$(5 + 5)(1 + g_A) = 11 \quad \dots$$

(4)

$$g_A = 0.1$$

Similarly, the demand-supply equation for capital goods industry1 will determine the rate of growth of country B,

$$(6 + 1.4050)(1 + g_B) = 11 \quad \dots (5)$$

$$g_B = 0.4855$$

The trade balance equation of country A will depict the exchange rate,

$$\alpha_{2A} w_A L_A = \alpha_{3B} w_B L_B E_{AB} \quad \dots (6)$$

$$\therefore (0.5 \times 1 \times 15) = (0.5 \times 1 \times 10) E_{AB}$$

$$E_{AB} = 1.5$$

Finally we will find out the post-trade prices. Substituting the value of exchange rate, wage rates of country A and B and the consumption coefficients in the demand-supply equations for consumption goods industry2 of country B, we will get p_{2B} .

$$\alpha_{2A} w_A L_A E_{BA} + \alpha_{2B} w_B L_B = (21.6093) p_{2B}$$

$$\therefore (0.5 \times 1 \times 15 \times (1/1.5)) + (0.5 \times 1 \times 10) = (21.6093) p_{2B} \quad \dots (7)$$

$$p_{2B} = 0.5781$$

Using the demand-supply equations for consumption goods industry3 of country A, we will get p_{3A} .

$$\alpha_{3A} w_A L_A + \alpha_{3B} w_B L_B E_{AB} = 25 p_{3A}$$

$$\therefore (0.5 \times 1 \times 15) + (0.5 \times 1 \times 10 \times 1.5) = 25 p_{3A} \quad \dots (8)$$

$$p_{3A} = 0.6$$

The post-trade prices of capital goods can now be easily calculated by substituting growth rates and consumption goods prices for respective countries in above system of price equations (3). They are $p_{1A} = \text{¥}1.6363$ and $p_{1B} = \$2.8031$.

The next step is to construct the gains from trade table. For that we need the prices of commodities which are produced in the countries post-trade, they have already computed and the prices of commodities which are not produced post-trade, namely p_{2A} and p_{3B} . They can be calculated using the post-trade rates of growth and prices. All commodity prices, exchange rate and the rates of profits are;

Country A	$E_{AB} = 1.5$	Country B
$r_A = 0.1$		$r_B = 0.4855$
$p_{1A} = \text{¥}1.6363$		$p_{1B} = \$2.8031$
$p_{2A} = \text{¥}1$		$p_{2B} = \$0.5781$
$p_{3A} = \text{¥}0.6$		$p_{3B} = \$2.2844$

The gains from table shows the quantities that can be brought for yen 1 in both countries A and B.

Table 4.3: Gains from Trade in Two Country Trade in Consumption Goods

	Country A	Country B
Consumption good 2	1	2.5947*
Consumption good 3	1.6667*	0.6566

*The maximum quantity

The above gains from trade table supports the trade pattern A- 1, 3 and B- 1, 2 is feasible. Therefore, the trade equilibrium is found. The post-trade price system (3) is the price system at the equilibrium. Comparison between pre and post-trade prices, outputs, wage rates and rates of growth is follows;

Table 4.4: Pre and Post-Trade Prices, Outputs and Wage Rates in Two Country Trade in Consumption Goods

	Country A		Country B	
	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	1.6363	1.6363	1.4469	2.8031
p_2	1	0.8672	0.2314	0.5781
p_3	0.6	0.6	1.0388	0.4
B1	11	11	11	11
B2	7.5	-----	21.6093	28.11
B3	12.5	25	4.8132	-----
w/p_2	1.0000	1.1531	4.3215	1.7298
w/p_3	1.6667	1.6667	0.9626	2.5000
r	0.1	0.1	0.1595	0.4855
g	0.1	0.1	0.1595	0.4855

The price of commodities 1 and 3 remained unaltered whereas the price commodity 2 declined in country A post-trade. In country B, all commodities have become costlier as compared to their autarky prices.

The effects of the trade in purely consumption goods on trading countries are of peculiar to only the trade pattern discussed in this section, are as follows.

Firstly, the rate of profit in country B increases. This is because, after opening to trade the consumption good industry 3 in country B closes down with the effect that the size of the surplus capital good increases significantly in relation to quantity of the capital good used as input in consumption good industries 3 which raises the rate of growth (profit) in country B. On the other hand the rate of profit in country A remained unaltered.

The reason is obvious. Even though the consumption good industry 2 in country A has closed down after opening to trade, the effect of the surplus capital good is nullified as the quantities of the capital good used as input were same for both consumption good industries 2 and 3 in country A. Secondly, the real wage rate in country B declined which is the result of the increase in an rate of profit in country B. In country A, the real wage rate in terms of commodity2 is increased but in terms of commodity 3 remained same post-trade.

4.5: Two Country Trade in Capital and Consumption Goods

In the previous two sections we have seen that the countries were trading capital goods and purely consumption goods separately. In this section we will consider two countries trading both capital as well as consumption goods. Consider countries A and B which produce 7 goods out of which 2 are non- tradable capital goods, 2 are tradable capital goods, 2 are tradable consumption goods and one consumption good is non-tradable. Their labour endowments 380 and 400 respectively. The consumption coefficients are assumed to be equal (1/3) for all consumption goods in both the countries. Suppose that the wage rates to be $w_A = ¥1$ and $w_B = \$1$. Their autarky equilibrium price system and prices are as follows,

Country A

$$1 (10.1p_{1A} + 8.5p_{2A} + 6.4p_{3A} + 4.6p_{4A})(1 + r_A) + 24.3w_A = 54.6p_{1A}$$

$$2 (8.8p_{1A} + 12.7p_{2A} + 7.8p_{3A} + 4.5p_{4A})(1 + r_A) + 42.2w_A = 56.3p_{2A}$$

$$3 (6.1p_{1A} + 4.8p_{2A} + 7.0p_{3A} + 3.3p_{4A})(1 + r_A) + 42.8w_A = 47.5p_{3A}$$

$$4 (3.9p_{1A} + 5.7p_{2A} + 3.7p_{3A} + 2.2p_{4A})(1 + r_A) + 25.7w_A = 30.8p_{4A}$$

$$5 (7.5p_{1A} + 6.2p_{2A} + 6.2p_{3A} + 3.5p_{4A})(1 + r_A) + 85.7w_A = 76.2p_{5A}$$

$$6 (7.0p_{1A} + 8.1p_{2A} + 7.9p_{3A} + 3.4p_{4A})(1 + r_A) + 79.8w_A = 95.7p_{6A}$$

$$7 (7.6p_{1A} + 6.6p_{2A} + 5.4p_{3A} + 7.2p_{4A})(1 + r_A) + 79.5w_A = 99.4p_{7A}$$

Country B

$$\begin{aligned}
 1 & (7.5p_{1B} + 6.1p_{2B} + 5.4p_{3B} + 4.9p_{4B})(1 + r_B) + 25.5w_B = 46.0p_{1B} \\
 2 & (6.5p_{1B} + 10.0p_{2B} + 5.4p_{3B} + 5.2p_{4B})(1 + r_B) + 37.1w_B = 49.4p_{2B} \\
 3 & (4.4p_{1B} + 4.0p_{2B} + 5.p_{3B} + 3.6p_{4B})(1 + r_B) + 28.3w_B = 40.4p_{3B} \\
 4 & (4.9p_{1B} + 7.6p_{2B} + 4.7p_{3B} + 4.3p_{4B})(1 + r_B) + 39.1w_B = 40.4p_{4B} \\
 5 & (6.9p_{1B} + 5.6p_{2B} + 5.6p_{3B} + 4.8p_{4B})(1 + r_B) + 91.0w_B = 37.1p_{5B} \\
 6 & (5.8p_{1B} + 6.8p_{2B} + 6.6p_{3B} + 5.3p_{4B})(1 + r_B) + 87.4w_B = 82.9p_{6B} \\
 7 & (6.4p_{1B} + 5.4p_{2B} + 4.3p_{3B} + 6.0p_{4B})(1 + r_B) + 91.5w_B = 115.6p_{7B}
 \end{aligned}$$

... (1)

The autarky Prices & Rate of Profit are;

Country A

$$r_A = 0.074$$

$$p_{1A} = \text{¥}1.3937$$

$$p_{2A} = \text{¥}1.8153$$

$$p_{3A} = \text{¥}1.6843$$

$$p_{4A} = \text{¥}1.7280$$

$$p_{5A} = \text{¥}1.6632$$

$$p_{6A} = \text{¥}1.3229$$

$$p_{7A} = \text{¥}1.2742$$

Country B

$$r_B = 0.088$$

$$p_{1B} = \$1.5218$$

$$p_{2B} = \$1.7775$$

$$p_{3B} = \$1.4897$$

$$p_{4B} = \$2.1424$$

$$p_{5B} = \$1.6081$$

$$p_{6B} = \$1.4397$$

$$p_{7B} = \$1.1532$$

To find the trade pattern we shall consider natural exchange rates of traded commodities.

They are,

Country A

$$E_{AB}^3 = 1.1306$$

$$E_{AB}^4 = 0.8066$$

$$E_{AB}^5 = 1.0343$$

$$E_{AB}^6 = 0.989$$

Country B

$$E_{BA}^3 = 0.8845$$

$$E_{BA}^4 = 1.2398$$

$$E_{BA}^5 = 0.9669$$

$$E_{BA}^6 = 1.0883$$

Lowest products of above exchange rates are $E_{AB}^4 E_{BA}^3 = 0.7134$ and $E_{AB}^6 E_{BA}^5 = 0.8884$ therefore the most profitable trade pattern is A- 1, 2, 4, 6, 7 and B- 1, 2, 3, 5, 7. Country A will produce and export commodities 4 and 6 to country B and import commodities 3 and 5 from country B. Country B will produce and export commodities 3 and 5 to country A and import commodities 4 and 6 from country A. In country A industries 3 and 5 will cease to operate whereas industries 4 and 6 will cease to operate in country B. Commodities 1, 2 and 7 are not traded among the countries, hence both countries will produce them for their domestic demand.

The production of exportable commodities have to increase so that they meet world and domestic demand for the commodities. To reallocate the labour in each of the industries the method of scale multipliers is used. These scale multipliers can be solved using the demand-supply equations. After attaching the scale multipliers the price equations in two countries would look as follows;

Country A

$$1 [(10.1p_{1A} + 8.5p_{2A} + 6.4E_{AB}p_{3B} + 4.6p_{4A})(1 + r_A) + 24.3w_A] x_{1A} = 54.6p_{1A}x_{1A}$$

$$2 [(8.8p_{1A} + 12.7p_{2A} + 7.8E_{AB}p_{3B} + 4.5p_{4A})(1 + r_A) + 42.2w_A] x_{2A} = 56.3p_{2A}x_{2A}$$

3Industry 3 is closed.....

$$4 [(3.9p_{1A} + 5.7p_{2A} + 3.7E_{AB}p_{3B} + 2.2p_{4A})(1 + r_A) + 25.7w_A] x_{4A} = 30.8p_{4A}x_{4A}$$

5Industry 5 is closed

$$6 [(7.0p_{1A} + 8.1p_{2A} + 7.9E_{AB}p_{3B} + 3.4p_{4A})(1 + r_A) + 79.8w_A] = 95.7p_{6A}$$

$$7 [(7.6p_{1A} + 6.6p_{2A} + 5.4E_{AB}p_{3B} + 7.2p_{4A})(1 + r_A) + 79.5w_A] = 99.4p_{7A}$$

... (2)

Country B

$$1 [(7.5p_{1B} + 6.1p_{2B} + 5.4p_{3B} + 4.9p_{4A}E_{BA})(1 + r_B) + 25.5w_B] x_{1B} = 46.0p_{1B}x_{1B}$$

$$2 [(6.5p_{1B} + 10.0p_{2B} + 5.4p_{3B} + 5.2p_{4A}E_{BA})(1 + r_B) + 37.1w_B] x_{2B} = 49.4p_{2B}x_{2B}$$

$$3 [(4.4p_{1B} + 4.0p_{2B} + 5.2p_{3B} + 3.6p_{4A}E_{BA})(1 + r_B) + 28.3w_B] x_{3B} = 40.4p_{3B}x_{3B}$$

4 Industry 4 is closed

$$\begin{aligned}
5 & [(6.9p_{1B} + 5.6p_{2B} + 5.6p_{3B} + 4.8p_{4A}E_{BA})(1 + r_B) + 91.0w_B] = 37.1p_{5B} \\
6 & \dots\dots\dots \text{Industry 6 is closed} \dots\dots\dots \\
7 & [(6.4p_{1B} + 5.4p_{2B} + 4.3p_{3B} + 6.0p_{4A}E_{BA})(1 + r_B) + 91.5w_B] = 115.6p_{7B} \\
& \dots (3)
\end{aligned}$$

As shown in above system of equation (2) and (3), country A imports commodity 3 and country B imports commodity 4 therefore the exchange rate has been attached to price of these imported goods.

Now, we will set up the world demand-supply equation for each commodity. The demand-supply equations for capital goods are;

$$\begin{aligned}
& \text{Country A} \\
1 & [(10.1x_{1A} + 8.8x_{2A} + 3.9x_{4A} + 7.0 + 7.6)(1 + g_A)] = 54.6x_{1A} \\
2 & [(8.5x_{1A} + 12.7x_{2A} + 5.7x_{4A} + 8.1 + 6.6)(1 + g_A)] = 56.3x_{2A} \\
4 & [(4.6x_{1A} + 4.5x_{2A} + 2.2x_{4A} + 3.4 + 7.2)(1 + g_A)] \\
& + [(4.9x_{1B} + 5.2x_{2B} + 3.6x_{3B} + 4.8 + 6.0)(1 + g_B)] = 30.8x_{4A} \\
& \dots (4)
\end{aligned}$$

In contrast with capital goods 1 and 2 which are non-tradable note that country A exports tradable capital good 4 therefore, the supply should meet i) domestic replacement and growth demand and ii) country B's replacement and growth demand for capital good 4.

The demand-supply equations of consumption goods and the full-employment equation are as follows;

$$\begin{aligned}
& \text{Country A} \\
6 & [(380/3) + (400 * E_{AB} / 3)] = 95.7x_{6A}p_{6A} \\
7 & [(380/3)] = 99.4x_{7A}p_{7A} \\
& \text{Country A: Full employment equation} \\
& (24.3x_{1A} + 42.2x_{2A} + 25.7x_{4A}) = 135 \\
& \dots (5)
\end{aligned}$$

Similarly, for country B the systems of equations are as follows,

Country B's demand-supply equations of capital goods are;

Country B

$$\begin{aligned}
 1 & [(7.5x_{1B} + 6.5x_{2B} + 4.4x_{3B} + 6.9 + 6.4)(1 + g_B)] = 46.0x_{1B} \\
 2 & [(6.1x_{1B} + 10.0x_{2B} + 4.0x_{3B} + 5.6 + 5.4)(1 + g_B)] = 49.4x_{2B} \\
 3 & [(5.4x_{1B} + 5.4x_{2B} + 5.2x_{3B} + 5.6 + 4.3)(1 + g_B)] \\
 & + [(6.4x_{1A} + 7.8x_{2A} + 3.7x_{4A} + 7.9 + 5.4)(1 + g_A)] = 40.4x_{3B}
 \end{aligned}$$

... (6)

Note that country B exports tradable capital good 3 therefore, its supply should meet i) domestic replacement and growth demand and ii) country A's replacement and growth demand for capital good 3. Country B's demand-supply equations of consumption goods and the full employment equation are;

Country B

$$\begin{aligned}
 5 & [(380 * E_{BA} / 3) + (400 / 3)] = 37.1x_{5B} p_{5B} \\
 7 & [(400 / 3)] = 115.6x_{7B} p_{7B}
 \end{aligned}$$

Country B: Full employment equation

$$(25.5x_{1B} + 37.1x_{2B} + 28.3x_{3B}) = 130$$

... (7)

We solve above all post-trade equations to get post-trade prices, outputs, rate of profits, rates of growth and exchange rate. The number of equations and number of unknowns are counted as below;

No. of equations	No. of unknowns
10 price equations	10 post trade prices
1 trade balance equations	1 exchange rate
6 demand-supply equations for capital goods	2 rates of growth
full employment equations	6 scale multipliers

We have in all 19 equations in 19 unknowns. We shall solve these equations and find international trade equilibrium in following steps. All equations stated above are interdependent therefore proceed iteratively as follows.

First, determine the rates of growth and scale multipliers. The system of demand supply equation of capital goods is in the non-linear form and the full employment equation is in the linear form. Take autarky level of growth g_B assuming each of scale multipliers of country B as one. Then solve demand supply equation of capital goods and full employment equations of country A to get first approximate solutions of the scale multipliers x_{1A}, x_{2A}, x_{4A} and growth rate g_A . Next, multiply the capital goods equations by corresponding first approximate scale multipliers. While doing so the demand for capital good 3 will change which country A imports from country B. Plug this new replacement and growth demand of capital good 3 in B's demand supply equation of commodity 3. Solve the demand supply equations for capital goods along with full employment equation of country B to get first approximate solution of scale multipliers x_{1B}, x_{2B}, x_{3B} and growth rate g_B . Next, multiply the price equations of capital goods by these scale multipliers. Plug the growth rate in price systems of country A and B. Determine all post trade prices, by solving price systems of both the countries simultaneously along with trade balance equation. Make sure that demand of all consumption goods industries should meet by their respective supplies. Repeat this procedure until all solutions converged. After 8 such iterations our all solutions converge up to an accuracy 6 places of decimals. The state of equilibrium is as follows,

Country A

$$\begin{aligned}
 1 & (10.6p_{1A} + 8.9p_{2A} + 6.8E_{AB}p_{3B} + 4.9p_{4A})(1 + r_A) + 24.56w_A = 57.51p_{1A} \\
 2 & (10.4p_{1A} + 15.0p_{2A} + 9.0E_{AB}p_{3B} + 5.3p_{4A})(1 + r_A) + 50.01w_A = 66.68p_{2A} \\
 3 & \dots\dots\dots \text{Industry 3 is closed} \dots\dots\dots \\
 4 & (8.9p_{1A} + 13.1p_{2A} + 8.6E_{AB}p_{3B} + 5.1p_{4A})(1 + r_A) + 59.43w_A = 71.31p_{4A} \\
 5 & \dots\dots\dots \text{Industry 5 is closed} \dots\dots\dots \\
 6 & (14.6p_{1A} + 16.7p_{2A} + 16.4E_{AB}p_{3B} + 7.1p_{4A})(1 + r_A) + 165.68w_A = 198.81p_{6A} \\
 7 & (7.5p_{1A} + 6.5p_{2A} + 5.4E_{AB}p_{3B} + 7.1p_{4A})(1 + r_A) + 79.33w_A = 99.16p_{7A}
 \end{aligned}$$

Country B

$$\begin{aligned}
 1 & (8.2p_{1B} + 6.7p_{2B} + 6.0p_{3B} + 5.4p_{4A}E_{BA})(1 + r_B) + 28.08w_B = 50.55p_{1B} \\
 2 & (6.3p_{1B} + 9.5p_{2B} + 5.2p_{3B} + 5.0p_{4A}E_{BA})(1 + r_B) + 35.72w_B = 47.63p_{2B} \\
 3 & (10.2p_{1B} + 9.5p_{2B} + 12.1p_{3B} + 8.5p_{4A}E_{BA})(1 + r_B) + 66.20w_B = 94.57p_{3B} \\
 4 & \dots\dots\dots \text{Industry 4 is closed} \dots\dots\dots \\
 5 & (13.5p_{1B} + 10.8p_{2B} + 10.8p_{3B} + 9.3p_{4A}E_{BA})(1 + r_B) + 177.29w_B = 161.53p_{5B} \\
 6 & \dots\dots\dots \text{Industry 6 is closed} \dots\dots\dots \\
 7 & (6.4p_{1B} + 5.4p_{2B} + 4.3p_{3B} + 6.1p_{4A}E_{BA})(1 + r_B) + 92.71w_B] x_{7B} = 117.10p_{7B}
 \end{aligned}$$

... (8)

The rates of growth, equilibrium prices and exchange rate are;

$$\begin{aligned}
 r_A &= 0.1048 & r_B &= 0.1333 \\
 E_{AB} &= 1.0193 \\
 p_{1A} &= ¥1.3983 & p_{1B} &= \$1.4975 \\
 p_{2A} &= ¥1.8200 & p_{2B} &= \$1.7559 \\
 p_{4A} &= ¥1.7303 & p_{3B} &= \$1.4685 \\
 p_{6A} &= ¥1.3207 & p_{5B} &= \$1.5947 \\
 p_{7A} &= ¥1.2774 & p_{7B} &= \$1.1386
 \end{aligned}$$

Our next task is to construct gains from trade table, for that we worked out the prices of commodities which are not traded among the countries. They are $p_{3A} = ¥2.1793$, $p_{5A} = ¥1.7167$, $p_{4B} = \$2.1175$ and $p_{6B} = \$1.4259$.

From these prices we ascertain the gains from trade as follows,

Table 4.5: Gains from Trade in Two Country Trade in Capital and Consumption Goods

Commodity B	Country A	Country B
3	0.45	0.67*
4	0.58*	0.46
5	0.58	0.62*
6	0.76*	0.69

It can be verified from the gains from trade table that trade pattern A- 1, 2, 4, 6, 7 B- 1, 2, 3, 5, 7 is the consistent trade pattern. The state of price system (8) is at the equilibrium.

The following table 4.6 gives comparison between pre and post-trade variables.

Table 4.6: Pre and Post-Trade Prices, Outputs and Wage Rates in Two Country Trade in Capital and Consumption Goods

Commodity	Country A		Country B	
	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	1.3937	1.3984	1.5218	1.4975
p_2	1.8154	1.8200	1.7775	1.7559
p_3	1.6843	2.1793	1.4897	2.1380
p_4	1.7280	2.1584	2.1424	2.1175
p_5	1.6632	1.7167	1.6081	1.5947
p_6	1.3229	1.4534	1.4397	1.4259
p_7	1.2742	1.2774	1.1532	1.1386
B1	54.6	57.51	46	50.55
B2	56.3	66.68	49.4	47.63
B3	47.5	-----	40.4	94.57
B4	30.8	71.31	40.4	-----
B5	76.2	-----	37.5	161.53
B6	95.7	198.81	82.9	-----
B7	99.4	99.16	115.6	117.10
w/p_5	0.6013	0.5825	0.6219	0.6271
w/p_6	0.7559	0.6880	0.6946	0.7013
w/p_7	0.7848	0.7828	0.8672	0.8783
$g = r$	0.074	0.1048	0.088	0.1333

The general consequences of trade in the situation of our example can be seen from above table. The post trade growth and profit rates have risen in both countries. In country A the price of all commodities have increased post-trade. As a result real wage rates have declined in country A. Whereas in country B all commodities have become

cheaper post-trade. This is reflected in increase in wage rates. As compared to autarky level of outputs the world outputs for commodities have increased.

4.6: Three Country Trade in Capital and Consumption Goods

In the preceding sections we have developed the models which handled trade in consumption as well as capital goods in two-country setup. To develop the generalized model, we consider the trade between three countries. Let's consider an example of trade between three countries and five commodities. Of the five commodities, commodity 1 is non-tradable capital good, commodity 2 is tradable capital good and remaining 3 commodities are consumption goods. The coefficients of consumption on commodities 3, 4 and 5 in the three countries are equal to (1/3). The labour endowments in the countries A, B and C are 100, 200 and 300 units respectively.

The autarky equilibria of the three countries are as follows,

Country A

$$\begin{aligned}
 1 & (2.1p_{1A} + 3.1p_{2A})(1 + r_A) + 10.39w_A = 12.46p_{1A} \\
 2 & (2.9p_{1A} + 3.9p_{2A})(1 + r_A) + 14.61w_A = 17.54p_{2A} \\
 3 & (1.6p_{1A} + 0.82p_{2A})(1 + r_A) + 28.001w_A = 327.52p_{3A} \\
 4 & (0.41p_{1A} + 1.24p_{2A})(1 + r_A) + 29.81w_A = 24.72p_{4A} \\
 & (2.77p_{1A} + 4.75p_{2A})(1 + r_A) + 17.19w_A = 31.65p_{5A} \\
 5 & \text{-----} \\
 & \text{100}
 \end{aligned}$$

Country B

$$\begin{aligned}
 1 & (8.07p_{1B} + 2.02p_{2B})(1 + r_B) + 40.36w_B = 24.22p_{1B} \\
 2 & (1.93p_{1B} + 1.93p_{2B})(1 + r_B) + 9.64w_B = 23.14p_{2B} \\
 3 & (2.69p_{1B} + 6.71p_{2B})(1 + r_B) + 51.42w_B = 67.13p_{3B}
 \end{aligned}$$

$$\begin{aligned}
4 \quad & (4.48p_{1B} + 2.24p_{2B})(1 + r_B) + 49.42w_B = 33.62p_{4B} \\
& (3.36p_{1B} + 6.71p_{2B})(1 + r_B) + 49.15w_B = 1006.58p_{5B} \\
5 \quad & \text{-----} \\
& \qquad \qquad \qquad 200
\end{aligned}$$

Country C

$$\begin{aligned}
1 \quad & (0.97p_{1C} + 0.97p_{2C})(1 + r_C) + 9.65w_C = 13.90p_{1C} \\
2 \quad & (7.00p_{1C} + 7.40p_{2C})(1 + r_C) + 24.35w_C = 17.14p_{2C} \\
3 \quad & (2.61p_{1C} + 1.31p_{2C})(1 + r_C) + 26.30w_C = 19.58p_{3C} \\
4 \quad & (1.70p_{1C} + 5.10p_{2C})(1 + r_C) + 14.52w_C = 850.05p_{4C} \\
& (1.35p_{1C} + 2.02p_{2C})(1 + r_C) + 25.18w_C = 67.48p_{5C} \\
5 \quad & \text{-----} \\
& \qquad \qquad \qquad 100
\end{aligned}$$

... (1)

With wage rates $W_A = ¥1$, $W_B = \$1$ and $W_C = £1$ the autarky prices, rates of profit and the natural exchange rates are as follows,

Country A	Country B	Country C
$r_A = 0.27$	$r_B = 0.18$	$r_C = 0.02$
$p_{1A} = ¥1.7292$	$p_{1B} = \$2.8727$	$p_{1C} = £0.9978$
$p_{2A} = ¥1.6703$	$p_{2B} = \$0.7754$	$p_{2C} = £3.2826$
$p_{3A} = ¥0.1018$	$p_{3B} = \$0.9931$	$p_{3C} = £1.7024$
$p_{4A} = ¥1.3486$	$p_{4B} = \$1.9829$	$p_{4C} = £0.0392$
$p_{5A} = ¥1.0531$	$p_{5B} = \$0.0662$	$p_{5C} = £0.4939$
$E_{AB}^2 = 2.1542$	$E_{BC}^2 = 0.2362$	$E_{CA}^2 = 1.9653$
$E_{AB}^3 = 0.1025$	$E_{BC}^3 = 0.5833$	$E_{CA}^3 = 16.7270$
$E_{AB}^4 = 0.6801$	$E_{BC}^4 = 50.5676$	$E_{CA}^4 = 0.0291$
$E_{AB}^5 = 15.9000$	$E_{BC}^5 = 0.1341$	$E_{CA}^5 = 0.4691$

Lowest products of above exchange rates are $E_{AB}^3 E_{BC}^5 E_{CA}^4 = 0.0003$ and $E_{AB}^3 E_{BC}^2 E_{CA}^4 = 0.0031$. Therefore the most profitable trade assignment is A-1, 3; B- 1, 2, 5; C- 1, 4. To find the international trade equilibrium we proceed as follows. First, we allocate the labour from industries which are cease to operate post-trade into tradable goods industries to meet their international demands. In case of country A the labour from industries 2, 4 and 5 will be employed in tradable consumption good industry 3. Similarly, in country C, labour from industries 2, 3 and 5 will be employed in tradable consumption good industry 4. since country B is producing tradable capital good 2 as well as consumption goods, the demand for capital good 2 will not only domestic but also international. Therefore, labour from industries 3 and 4 can be allocated by using the method of scale multipliers.

Accordingly the world economy with each country's accounts in her own currency looks as follows,

Country A

$$\begin{aligned}
 &1 \quad (2.1p_{1A} + 3.1p_{2B} E_{AB})(1 + r_A) + 10.39w_A = 12.46p_{1A} \\
 &2 \quad \dots\dots\dots \text{Industry 2 is closed} \dots\dots\dots \\
 &3 \quad (5.2p_{1A} + 2.6p_{2B} E_{AB})(1 + r_A) + 89.61w_A = 1048.11p_{3A} \\
 &4 \quad \dots\dots\dots \text{Industry 4 is closed} \dots\dots\dots \\
 &5 \quad \dots\dots\dots \text{Industry 5 is closed} \dots\dots\dots
 \end{aligned}$$

Country B

$$\begin{aligned}
 &1 \quad x_{1A} ((8.07p_{1B} + 2.02p_{2B})(1 + r_B) + 40.36w_B = 24.22p_{1B}) \\
 &2 \quad x_{2A} ((1.93p_{1B} + 1.93p_{2B})(1 + r_B) + 9.64w_B = 23.14p_{2B}) \\
 &3 \quad \dots\dots\dots \text{Industry 3 is closed} \dots\dots\dots \\
 &4 \quad \dots\dots\dots \text{Industry 4 is closed} \dots\dots\dots \\
 &5 \quad x_{5A} ((3.36p_{1B} + 6.71p_{2B})(1 + r_B) + 49.15w_B = 1006.58p_{5B})
 \end{aligned}$$

Country C

$$1 (0.97p_{1C} + 0.97p_{2B}E_{BC})(1 + r_C) + 9.65w_C = 13.90p_{1C}$$

2 Industry 2 is closed

3 Industry 3 is closed

4

$$(10.58p_{1C} + 31.74p_{2B}E_{BC})(1 + r_C) + 90.35w_C = 5289.53p_{4C}$$

5 Industry 5 is closed

... (2)

We have in all 15 equations in 15 unknowns.

No. of equations	No. of unknowns
7 price equations	7 prices
2 trade balance equations	2 exchange rates
3 demand-supply equations for capital goods	3 rates of growth
3 full employment equations	3 scale multipliers

We solve these equations to determine unknowns in following steps.

1. Determine the growth rates and scale multipliers by solving the system of demand-supply equations.
2. Solve price system and balance of trade equation to get the post-trade prices and exchange rates.
3. Find out the prices of non-traded commodities at exchange rates depicted in step 2.
4. Construct gains from trade table and check whether the trade pattern supports the gains from trade for all countries
5. Finally if step 4 is true then the trade equilibrium is found. If not then set up the new trade pattern according to the gains from trade table and follow steps 1 to 5 till the equilibrium is found.

First, we set up the demand-supply equation of commodity 1 in country A and solve scale multiplier x_{1A} and rate of growth g_A .

Country A

$$1 \quad (2.1x_{1A} + 5.2)(1 + g_A) = 12.46x_{1A}$$

$$10.39x_{1A} = 10.39$$

$$x_{1A} = 1$$

$$\text{Therefore,} \quad (1 + g_A) = 1.7032$$

$$g_A = 0.7032$$

Similarly, from the demand-supply equation of commodity 1 in country C we get scale multiplier x_{1C} and rate of growth g_C .

Country C

$$1 \quad (0.97x_{1C} + 10.58)(1 + g_C) = 13.90x_{1C}$$

$$10.39x_{1C} = 10.39$$

$$x_{1C} = 1$$

$$\text{Therefore,} \quad (1 + g_C) = 1.2043$$

$$g_C = 0.2043$$

For country B, the demand-supply equations of capital goods 1 & 2 and the full-employment equation are;

Country B

$$1 \quad (8.07x_{1B} + 1.92x_{2B} + 3.36)(1 + g_B) = 24.22x_{1B}$$

$$2 \quad (2.02x_{1B} + 1.92x_{2B} + 6.72)(1 + g_B) + 49.15 = 23.14x_{1B}$$

$$40.36x_{1B} + 9.64x_{2B} = 150.85 \quad \dots (3)$$

Where, 49.15 is the sum of demands of country A and C for commodity 2.

The above system of equation can be solved by using the method of Eigen value determination.

The system of equations (3) can be written in the matrix form;

$$\begin{bmatrix} 8.07 & 1.92 & 3.36 \\ 2.02 & 1.92 & 6.72 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1B} \\ x_{2B} \\ 1 \end{bmatrix} (1 + g_B) = \begin{bmatrix} 24.22 & 0 & 0 \\ 0 & 23.14 & -49.15 \\ 40.36 & 9.64 & -150.85 \end{bmatrix} \begin{bmatrix} x_{1B} \\ x_{2B} \\ 1 \end{bmatrix}$$

$$AX(1 + g_B) = BX$$

Which is in the form:

$$AX(1 + g_B) = BX$$

$$B^{-1}AX = \frac{1}{(1 + g_B)} IX$$

Solving above equation we get

$$x_{1B} = 2.82, x_{2B} = 3.85 \text{ and } g_B = 1.0324$$

We multiply post-trade price equations of commodity 1 and 2 by x_{1B} and x_{2B} respectively. The price system will look as follows,

Country B

$$1 (22.75p_{1B} + 5.69p_{2B})(1 + r_B) + 113.75w_B = 68.25p_{1B}$$

$$2 (7.42p_{1B} + 7.42p_{2B})(1 + r_B) + 37.10w_B = 89.04p_{2B}$$

$$5 (3.36p_{1B} + 6.71p_{2B})(1 + r_B) + 49.15w_B = 1006.58p_{5B}$$

... (4)

Next step is to find post-trade prices of all commodities. First we solve price system (4) of country B simultaneously (since $r_B = g_B = 1.0324$) we get,

$$p_{1B} = \$6.0821$$

$$p_{2B} = \$1.7418$$

$$p_{3B} = \$1.6145$$

The exchange rates can be calculated using any two balance of trade equations, say for countries A and B all measured in the currency of country A.

Import bill = Export earnings

Balance of trade equation of country A

$$(\alpha_{4A} + \alpha_{5A})w_A L_A + A_{2A}p_{2B}E_{AB} = (\alpha_{3B}w_B L_B)E_{AB} + (\alpha_{3C}w_C L_C)E_{AC}$$

Balance of trade equation of country B

$$(\alpha_{3B} + \alpha_{4B})w_B L_B E_{AB} = A_2 p_{2B} E_{BA} + (\alpha_{5A} w_A L_A) + (\alpha_{5C} w_C L_C) E_{CA}$$

The balance of trade equations of country A and B can be written in matrix form as below;

$$\begin{bmatrix} A_{2A} p_{2B} - \alpha_{3B} w_B L_B & \alpha_{3C} w_C L_C \\ A_2 p_{2B} - (\alpha_{3B} + \alpha_{4B}) w_B L_B & -\alpha_{5C} w_C L_C \end{bmatrix} \begin{bmatrix} E_{AB} \\ E_{AC} \end{bmatrix} = \begin{bmatrix} (\alpha_{4A} + \alpha_{5A}) w_A L_A \\ (\alpha_{5A} w_A L_A) \end{bmatrix}$$

Substituting the respective values, we obtain the solution $E_{AB} = 1.0270$ & $E_{AC} = 0.4703$.

At this stage we know p_{2B} , E_{AB} and E_{AC} therefore we can find prices of remaining traded commodities.

Country A	Country B	Country C
$p_{1A} = \text{¥}2.2273$	$p_{1B} = \$6.0821$	$p_{1C} = \text{£}1.1050$
$p_{3A} = \text{¥}0.1121$	$p_{2B} = \$1.7418$	$p_{4C} = \text{£}0.0472$
	$p_{5B} = \$0.1136$	

To ascertain the gains from trade we need to find the prices of commodities which are not traded at E_{AB} and E_{AC} . They are,

Country A	Country B	Country C
$p_{2A} = \text{¥}2.1426$	$p_{3B} = \$1.6145$	$p_{2C} = \text{£}3.9422$
$p_{4A} = \text{¥}1.4216$	$p_{4B} = \$3.3542$	$p_{3C} = \text{£}1.8262$
$p_{5A} = \text{¥}1.3319$		$p_{5C} = \text{£}0.5371$

The gains from trade table is;

Table 4.7: Gains from Trade in Three Country Trade in Capital and Consumption Goods

	Country A	Country B	Country C
2	0.47	0.56*	0.54
3	8.92*	0.60	1.16
4	0.70	0.29	45.03*
5	0.75	8.57*	3.96

* The maximum amount.

It can be verified from the above table that the trade pattern A-1, 3; B- 1, 2, 5; C- 1, 4 is consistent.

Therefore following price system is in equilibrium.

Country A

- 1 $(2.1p_{1A} + 3.1p_{2B}E_{AB})(1 + r_A) + 10.39w_A = 12.46p_{1A}$
- 2 Industry 2 is closed
- 3 $(5.2p_{1A} + 2.6p_{2B}E_{AB})(1 + r_A) + 89.61w_A = 1048.11p_{3A}$
- 4 Industry 4 is closed
- 5Industry 5 is closed

Country B

- 1 $(22.75p_{1B} + 5.69p_{2B})(1 + r_B) + 113.75w_B = 68.25p_{1B}$
- 2 $(7.42p_{1B} + 7.42p_{2B})(1 + r_B) + 37.10w_B = 89.04p_{2B}$
- 3 Industry 3 is closed
- 4Industry 4 is closed
- 5 $(3.36p_{1B} + 6.71p_{2B})(1 + r_B) + 49.15w_B = 1006.58p_{5B}$

Country C

- 1 $(0.97p_{1C} + 0.97p_{2B}E_{BC})(1 + r_C) + 9.65w_C = 13.90p_{1C}$
- 2 Industry 2 is closed
- 3 Industry 3 is closed
- 4
- 5 $(10.58p_{1C} + 31.74p_{2B}E_{BC})(1 + r_C) + 90.35w_C = 5289.53p_{4C}$
- 5 Industry 5 is closed

... (5)

At this stage we determined all post-trade prices, exchange rates, equilibrium quantity of each commodity. We now construct a table which compares the these parameters at autarky and equilibrium state of countries as follows;

Table 4.8: Pre and Post-Trade Prices, Outputs and Wage Rates in Three Country Trade in Capital and Consumption Goods

	Country A		Country B		Country C	
	Pre-trade	Post-trade	Pre-trade	Post-trade	Pre-trade	Post-trade
p_1	1.7292	2.2273	2.8727	6.0821	0.9978	1.1050
p_2	1.6703	1.7889	0.7754	1.7418	3.2826	3.8040
p_3	0.1018	0.1121	0.9931	0.1092	1.7024	0.2384
p_4	1.3486	0.0222	1.9829	0.0216	0.0392	0.0472
p_5	1.0531	0.1167	0.0662	0.1136	0.4939	0.2481
B_1	12.4600	12.4600	24.2200	68.2500	13.9000	13.9000
B_2	17.5400	-----	23.1400	89.0400	17.1400	-----
B_3	327.5200	1048.11	67.1300	-----	19.5800	-----
B_4	24.7200	-----	33.6200	-----	850.0500	5289.53
B_5	31.6500	-----	1006.5800	1006.5800	67.4800	-----
w/p_3	9.8232	8.9206	1.0069	9.1617	0.5874	4.1950
w/p_4	0.7415	45.0525	0.5043	46.2699	25.5102	21.1864
w/p_5	0.9496	8.5712	15.1057	8.8028	2.0247	4.0307
$g = r$	0.27	0.7032	0.18	1.0324	0.02	0.2043

The effect of trade on composition of output is interesting to observe. Firstly, the outputs of non-tradable capital good 1 remained same whereas the outputs of tradable consumption good increased in country A and C. On the other hand the outputs of capital goods 1 and 2 increased but the output of tradable consumption good remained same in country C. However, the level of the world outputs for commodities have increased. The post-trade rates of growth increased significantly in all countries. All capital goods have become costlier.

Interestingly, the price of domestically produced consumption goods post-trade have increased i.e. price of commodity 3 in country A, commodity 5 in country B and commodity 4 in country C. The imported consumption goods became cheaper in all countries post-trade. This is reflected in the wage rates. The wage rate in term of commodity 3 in country A, commodity 5 in country B and commodity 4 has decreased but wage rates in terms of remaining consumption commodities have increased significantly. That is to say, the real wage rates in terms of the importable have increased but the real wage rates in terms of the exportable have declined as compared to the autarky situation. The latter effect is the result of the increase in the rates of profit. All these effects depend upon many factors such as the exchange rate, technical coefficients, and the volumes of post-trade employments and post-trade prices of commodities.

4.6: Remarks

In this chapter, the effect of mutually gainful trade in capital goods on the prices and outputs of commodities as well as on the distributional variables and growth rates have been illustrated. Specifically, it is shown that international trade in capital goods has the effect of lowering the costs of production in the trading countries and consequently has the effect of increasing the rates of profit and growth.

In the dynamic model we have seen that, if and when trade equilibrium exist, then the following variables are determined by the system of equations;

- (a) the pattern of specialization in production viz. which countries produce which commodities
- (b) the post-trade prices of commodities
- (c) the post-trade outputs of commodities
- (d) the pattern of gains from trade i. e. which country gains how much by importing which commodity from which country
- (e) the commodity wise volumes of inter-country exports and imports
- (f) the equilibrium exchange rates and the international terms of trade
- (g) all value magnitudes such as the gross national products, net national incomes and capital stocks in the countries
- (h) the post-trade real wage rates in the countries
- (i) the post-trade rates of growth of the countries
- (j) the post-trade function of distribution of incomes in the countries

Chapter 5

Concluding Remarks and Discussion

This thesis has been concerned with developing multi-country multi-commodity models of international trade in static and dynamic settings. As stated in the introductory, it has kept strictly intact the distribution between value capital and heterogeneous capital goods and allowing the latter to be internationally traded. Accordingly the value and composition of capital stocks and the national products of the countries have been traded as unknowns whose values must be determined by the model whether in static or dynamic contexts. The trade models themselves may be regarded as *complete* in the sense that they determine all variables of interest. Thus, in the static model of chapter III if trade equilibrium exists then the following variables are determined by the system of equations;

- (a) the pattern of specialization in production; which countries produce which commodities
- (b) the post-trade prices of commodities
- (c) the post-trade outputs of commodities
- (d) the pattern of gains from trade i. e. which country gains how much by importing which commodity from which country
- (e) the commodity wise volumes of inter-country exports and imports
- (f) the equilibrium exchange rates and the international terms of trade
- (g) all value magnitudes such as the gross national products, net national incomes, capital stocks, trade flows in the countries
- (h) the post trade real wage rates in the countries

In the dynamic trade models of chapter IV the model determines (a) to (h) and further determine

- (i) the post-trade rates of growth of the countries
- (j) the post-trade distribution of real incomes i.e. wages and profits in the countries

All the results are “constructive” in the sense that the theory has not been concerned with providing abstract results such as the existence or otherwise of an international trade equilibrium. On the contrary the thesis has emphasized on developing algorithms to determine the international trade equilibrium, if it exists.

In contrast with traditional trade theory, trade models developed in this thesis consider the internal structure of the economic systems of the countries i. e. the endowments of primary resources, the technique of production and tastes and preferences of consumers while determining the commodity composition of intercountry trade. The relative prices of commodities determine the comparative advantage in the production of the different commodities in the countries. Our trade models determine which commodities will be imported, which produced and exported and accordingly which industries will be close down in different countries and which will be expanded when trade takes place. The taste and preferences and the income levels of consumers determine the volumes of demand for the imported commodities. These along with the prices of commodities in the exporting countries determine the values of intercountry import bills of countries. The intercountry import bills determine the exchange rates. Simultaneously, the endowments and techniques of production determine the volumes and the values of the supplies of exported commodities. The international trade equilibrium is achieved at the exchange rate where the world supplies meets world demands in volumes and values.

The specific results of the theory of international trade presented in this thesis stand in sharp contrast to those of traditional trade theory. Attention is here drawn to critical results.

1. In section 3.2 of chapter 3 it has been shown that there is no guarantee for the existence of an international trade equilibrium despite differences in the comparative costs of production, because the sizes of labour endowment in trading countries are not sufficient to meet world demand for exportable commodities in post-trade situation.
2. If, however a trade equilibrium exists then it will be unique. This follows from the proof of the uniqueness of exchange rates that ensure clearing of markets for any given pattern of intercountry import bills, which was spelt out in section 3.3 of chapter3.
3. In section 3.4 of chapter 3 it has been shown that the theory can generate situations in which the trade pattern is consistent with the Heckscher-Ohlin prediction as well the Leontief paradox prediction.
4. The illustrations given in section 4.1 and 4.2 of chapter 3 have showed that the Leontief trade model can be applied to 2-country n-commodity, and it is capable of generating variety of results not only in the setting which is generalises to the

case of several capital goods but also in the setting when generalised to the case of several capital goods and several consumption goods along with some commodities to be non-tradable. Unlike the neo-classical trade theories which do not consider the non-tradable goods.

5. In sections 5.1 and 5.2 of chapter 3 it has been shown that Leontief trade model makes claim to greater generality to accommodate trade between several countries in several commodities which has been very difficult for standard trade theories to do.
6. In general, the international trade model of this thesis does not predict equalization of real factor rewards in all countries due to international trade.
7. Traditional trade theory is lacking in models of international trade in which economic growth is considered in an endogenous manner. The theory advanced in this thesis shows how this can be achieved and shows how trade is growth-enhancing chiefly due to access to cheaper capital goods.
8. In contrast with the conclusion of Dornbusch, Fisher and Samuelson (1977) who postulated a model of trade in a continuum of goods in which the exchange rate would be determined by the ratio of nominal wage rates in equilibrium, the present thesis concludes exchange rates would be determined at the point where trade of each country is balanced and foreign exchange markets are cleared.
9. In contrast with the assumption of Eaton and Kortum (2002) that each country imports a given commodity only from one country the model in this thesis allows imports from one or more countries and can therefore make a claim to greater generality.
10. The effects of international trade on the competitive rates of profit that prevail post-trade have been clearly brought out. The traditional trade model requires that the rate of profit in one country declines due to trade. Thus is an unsatisfactory result which is prevented from occurring in the trade model of this thesis.
11. The international trade model in this thesis is based on empirically observable inputs, viz. the input-output coefficients and consumption coefficients that can be obtained from linear expenditure systems or almost ideal demand systems so it is expected that it would be more amenable to empirical implementation.
12. While it is true that the 'pure' and the 'monetary' aspects of international trade have not been fully integrated in the theory, at least they have not been kept entirely separate. So it is, for example, perfectly possible to study the effects of

tariffs on exchange rate or the effects of intercountry transfers on the international trade equilibrium.

13. Traditional trade theory has never been satisfactorily extended to apply to multi-country trade problems. The trade model in this thesis has been shown to be capable of handling multi-country multi-commodity trade quite easily.

It might be appropriate in closing this thesis to point out the potential for generalization of the results. The generalization envisaged fall into two categories the first of which pertains to the assumptions of the model itself and the second pertains to the scope of modern economic circumstances that needs to be given consideration. Of these the first type of generalization are easier to remedy viz. the restrictive assumptions pertaining to consumption and saving behavior. Thus even though simple linear Engels' equations have been used to represent final consumption demand, more general and empirically relevant demand equations such as the Stones'(1954) linear expenditure system and/or Deaton and Muellbauer (1980) almost ideal demand systems can easily be substituted and will require minor modifications in the algorithm for finding the international trade equilibrium. Likewise the classical assumption that "profits are wholly accumulated and wages are wholly consumed" can be easily relaxed and replaced by a more general Kaldor-Pasinetti(1982) specification.

It is the second set of considerations viz. that of economic circumstances such as the facts of intra-industry trade, of increasing returns to scale in production and of trade being concentrated between firms in each industry which will pose real challenges. It is an open question whether these circumstances are at all amenable to being modelled using the input-output methodology. Indeed, the developers of new trade theory have themselves taken refuge in the simple Ricardian assumption of production by means of labour alone. And scholars who have developed the "trade between firms" literature have not yet developed any theoretical basis for the phenomenon, that they have studied but confined themselves to calling attention to empirical factors such as the sizes of firms, the extent of foreign ownership in them etc.

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