THE THEORY OF INTERNATIONAL VALUES

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PREFACE

This work is in form a textbook but in fact an appeal for a thorough restatement of the theory of international values. Its purpose is three-fold. First, to integrate the 'pure' and 'monetary' aspects of international economic relations. Second, to formulate a multicountry multicommodity theory of trade. Third, to incorporate capital goods (and concomitantly the theories of growth and income distribution) into the fold of trade theory. By way of generalisation further subjects such as tariffs, transfers, customs unions, domestic distortions and fixed exchange rate systems have also been treated. Implications of the theory for global policymaking have been drawn out in the final chapter.

Although the treatment is 'advanced' it is nevertheless designed to be accessible to students with a 'subsistence knowledge' of matrix algebra. Prior familiarity with trade theory and/or general equilibrium economics and/or multicountry econometric models will undoubtedly help. However, youthful zeal coupled with a natural curiosity to understand "the ways of the world" will usually make up for deficiencies of technical expertise. At any rate all formalisms have been amply illustrated by numerical examples. These numericals may seem tiresome to the expert reader at first but will soon be found to be indispensible.

My greatest debts are to two pioneering monographs: Frank D. Graham's *Theory of International Values* and Piero Sraffa's *Production of Commodities by Means of Commodities*. The authors of these monographs spent three decades or more in composing their critiques. I hope that in placing reliance on these durable works I have not detracted attention from their true intents. In addition, a long, but necessarily partial list of debts appears in the bibliography. Lewis Carroll's *Through the Looking Glass* has provided the only relief from otherwise technical matter.

I am beholden to a number of individuals for generous donations. To B.G. Bapat, S.V. Bokil, V.M. Dandekar and P. Venkatramaiah for reviewing the manuscript. To Rajani Gupte for working with me throughout the project without losing hope or conviction. To Atul Desai, Deepak Havaldar and Vaishali Joshi for rendering computational assistance. To Pradeep Apte, N. Ashok Kumar and Shalini Bhargava for their comments in the final stages. To Catherine Dhanpal for typing and M.I. Thomas and his team at Wiley Eastern for producing the final script. All errors that remain are my own.

RKP

October, 1991.
Pune

CONTENTS

APPENDICES

1

EXCHANGE RATES

Such quantities of sand:
'If this were only cleared away',
They said, 'it would be grand',

1. Objects and Assumptions

The aim of this chapter is to provide a preliminary survey of the theory of exchange rate determination in a multicountry setting. We shall consider a world comprising several politically sovereign but economically interdependent nations. Political sovereignty implies the existence of distinct national currencies which are legal tender for transactions within each nation. International economic interdependence implies that these distinct national currencies must be exchanged against one another. We shall suppose that the national currencies are non-commodity or *fiat* moneys.

An important aspect of exchange rate determination that emerges from these assumptions must be noted at the very outset. Since the currencies are fiat, they will not be demanded for their own sake, their holding gives no utility. And since the currencies are legal tender, they will not be held outside the countries of their issue. It follows that the payments and receipts of each country must be exactly equal. For only this condition will ensure that the payments made by every country in terms of her own currency will be routed back through the foreign exchange market in the form of receipts to the country without being held abroad, where it cannot be used.

Our object in this chapter is to formulate a model of the process by which the foreign exchange market performs the function of balancing payments of all the countries and determines the exchange rates of the different currencies. Since the exchange rates are the unknowns to be determined, it is obvious that we shall be dealing with a regime of flexible exchange rates. In a later section of this chapter, and more particularly in Chapter 6, (section 12) below, we shall discuss the fixed exchange rate system in which exchange rates are predetermined and official, or accommodating, capital flows are the unknowns.

2. Two Currencies

Suppose at first that there are only two countries A and B whose economic relations with one another are confined to trading in commodities. Suppose that country A imports goods worth 40 million rupees from country B and country

B imports goods worth 2 million dollars from country *A* every 'year' Equilibrium in the balance of payments of both the countries requires that their payments (debits) should be exactly equal to their receipts (credits) in terms of their domestic currencies. Thus the exporters of country *A* must receive Rs. 40 million for the \$2 million worth of goods exported by them to country *B*. Likewise the exporters of country *B* must receive \$2 million for the Rs. 40 million worth of goods exported by them to country *A*. This can happen only if the rupee dollar exchange rate is Rs. 20 = \$1. Because balance of payments of country *A* requires that her exports of \$2 million, when converted into her own currency, rupees, must be equal to her import bill of Rs. 40 million, i.e. the exchange rate must satisfy the equation.

$$(\$2 \text{ million}) \times \text{Rupee-dollar rate} = \text{Rs. } 40 \text{ million}$$

At the same time the balance of payment equation of country B,

$$(\text{Rs } 40 \text{ million}) \times \text{Dollar-rupee rate} = \$2 \text{ million}$$

gives a dollar-rupee exchange rate \$0.05 = Re. 1 which is the reciprocal of the rupee dollar exchange rate Re 20 = \$1.

We may make further generalisations within the two currency context. Thus suppose that, in addition to importing from country *B*, citizens of country *A* invest Rs. 10 million every year in country *B*. The supply of rupees in the foreign exchange market increases to Rs.50 million whereas the supply of dollars remains at \$2 million. The balance of payments of country *A* requires the exchange rate to be such as to satisfy,

$$(\$2 \text{ million}) \times \text{Rupee-dollar rate} = \text{Rs. } 50 \text{ million}$$

which means a rupee-dollar rate of Rs. 25 = \$1 and a reciprocal dollar-rupee exchange rate of \$0.04 = Re1.

To sum up, in a two country setting the ratio of intercountry payments determines a unique exchange rate, which when adopted for foreign exchange transactions brings about equilibrium in the balance of payments of both countries and clears the supplies of currencies in the foreign exchange market.

3. Three or more Currencies

Matters become somewhat complicated when a third country *C*, with her own distinct currency, say yen, is introduced. Suppose now that the intercountry payments between *A*, *B* and *C* are as tabulated below.

	Payments to			
Payments by	*A*	*B*	*C*	Total
A	0	Rs.40 million	Rs.20 million	Rs.60 million
B	\$2 million	0	\$10 million	\$12 million
C	¥100 million	¥200 million	0	¥300 million

The entries in the rows of *international* or *multicountry payments matrix* show the payments made by each country to other countries. The total payments made by each country to the rest of the world are shown in the fourth column. The principal diagonal of the matrix contains only zeros because, by definition, no country can make *international* payments to herself. Every country's receipts from the other countries will be read from the columns of the matrix.

In equilibrium the total payments are equal to the total receipts of the countries. Our problem is to find the set of exchange rates that brings about equilibrium in the foreign exchange market. An obvious point may be noted immediately viz. the two currency rule of determining exchange rates by the ratio of intercountry payments is seen to break down in the three- country case. To see how, consider the results if we nevertheless apply the two-currency rule. Then the exchange rates suggested are as follows; the rupee-dollar exchange rate would be Rs.20 = \$1, the *A* -row, *B*-column element divided by the *B*-row, *A*-column element in the multicountry payments matrix, the yen-rupee exchange rate would be ¥5 = Re. 1, the *C*- row, *A*-column element divided by the *A*-row, *C*-column element and the yen-dollar exchange rate would be ¥20 = \$1, the *C*-row, *B*-column element divided by the *B*-row, *C*-column element. Observe, however, that these exchange rates are self contradictory. For instance, the rupee-dollar exchange rate, Rs.20 = \$1, and the yen- dollar exchange rate ¥20 = \$1 taken together, imply that ¥20 = Rs.20 = \$1, indicating a yen-rupee exchange rate of ¥1 = Re.1. But the yen-rupee exchange rate solved by the two-currency rule, ¥5 = Re.1 is widely different from the indirect yen-rupee exchange rate of ¥1 = Re.1. Hence the general inapplicability of the two-currency rule. The two currency rule is based on the fact that in a two-country world each country's payments must balance with respect to the other. This, when more than two countries are involved, is no longer necessary—it is quite possible for a country to have a deficit balance with one country and a surplus balance against another and yet have a balance in her overall payments.(1)

To find the correct set of exchange rates we must set up the balance of payments equations for all countries, i.e. we must equate the payments in domestic currency terms of each country to her receipts converted into her domestic currency. Thus we get the following three equations for *A*, *B* and *C* respectively,

$$(\$2\text{ mn}) \times \text{rupee-dollar rate} + (¥100\text{ mn}) \times \text{rupee-yen rate} = \text{Rs.}60\text{ mn.}$$

$$(\text{Rs.}40\text{ mn}) \times \text{dollar-rupee rate} + (¥200\text{mn}) \times \text{dollar-yen rate} = \$12\text{ mn.}$$

$$(\text{Rs.}20\text{ mn}) \times \text{yen-rupee rate} + (\$10\text{ mn}) \times \text{yen-dollar rate} = ¥300\text{ mn.}$$

In the three equations above the right -hand-sides show the world supplies of the currencies, viz. Rs.60 million, \$12 million and ¥300 million. The left-hand-sides show the demands for rupees, dollars and yen respectively. The exchange rates that will be determined by these equations will be such as to equate the demands for the currencies to their supplies. The difficulty is that there are three equations in six unknown exchange rates. The problem is nevertheless solvable since the exchange rates are interrelated with one another. Thus we know (or can always verify by looking at the daily foreign exchange quotations) that exchange rate relations of the following types always prevail in the market:

Rupee-dollar rate (Rs/\$) × Dollar-rupee rate (\$/Rs) = 1

Yen-dollar rate (¥/\$) × Dollar-yen rate (\$/¥) = 1, etc.

Rupee-Yen rate (Rs/¥) × Dollar-rupee rate (\$/Rs)
= Dollar-Yen rate(\$/¥)

Yen-rupee rate (¥/Rs) × Dollar-Yen rate (\$/¥)
= Dollar-rupee rate (\$/Rs) etc.

The meaning of the first two exchange rate relations is fairly obvious. If for any reason the exchange rates of any two currencies were to go out of line, people will simply buy a currency where it is cheap and and sell it where it is strong and make profit. This process, called *currency arbitrage*, will ensure that the rates come into line. Thus the first two equations ensure the absence of arbitrage in two currencies. The latter two equations ensure the absence of arbitrage in three currencies. The exchange rate relations, as we shall see, play a vital role in the determination of the equilibrium exchange rates.

We may now proceed with the solution. Express all the balance of payments equations in some one currency, say dollars[(2)]. To do so, multiply the first (*A*'s) balance of payments equation by the dollar-rupee rate and the third (*C*'s) by the dollar-yen rate keeping the second (*B*'s) as it is because it already is expressed in dollars. Then using the exchange rate relations above, the three balance of payments equations may be written as,

(\$2 m) + (¥100 m) × Dollar-yen rate = (Rs.60 m) × Dollar-rupee rate

(Rs.40 m) × Dollar-rupee rate + (¥200 m) × Dollar-yen rate = \$12 m.

(Rs.20 m) × Dollar-rupee rate + (\$10 m) = (¥300 m) × Dollar-yen rate.

We now have three equations in two unknown exchange rates, viz. the dollar rupee rate and the dollar-yen rate. We may use any two equations and obtain the solution : the dollar-rupee rate is \$0.1 = Re.1 and the dollar-yen rate is \$0.04 = ¥1. It can be verified immediately that these exchange rates always satisfy the third equation.

All the other exchange rates that we wish to ascertain can be found by using the exchange rate relations above. Thus the rupee-dollar rate and the yen-dollar rate are the reciprocals of the dollar-rupee rate and the dollar-yen rate solved above, their values are Rs.10 = \$1 and ¥25 = \$1. The rupee-yen rate can be found by multiplying the rupee-dollar rate and the dollar yen-rate, 10 × 0.4, giving Rs.0.4 = ¥1. And the yen-rupee rate will be ¥2.5 = Re.1

The exchange rate solutions corresponding to the multicountry payments matrix tabulated earlier is shown below.

There are three noteworthy features of the exchange rate table. Firstly, observe that the principal diagonal elements contain only 1's, since a unit of any currency can purchase only a unit of itself. Secondly, observe that the exchange rate lying below the principal diagonal are reciprocals of those lying above; e.g. the third-row first-column rupee-yen rate is the reciprocal of the first-row third-

		Currency Purchased		
		Rupee	Dollar	Yen
Currency	Rupee	1	0.1	2.5
sold	Dollar	10	11	25
		0.4	0.04	1

column yen-rupee rate, etc. Thirdly, observe that the exchange rates at the vertices of the triangles which form the star have a product equal to 1; e.g. the rupee-dollar rate, multiplied by the yen-rupee rate, 2.5, multiplied in turn by the dollar-yen rate, 0.04, gives 1, etc.

These features are nothing but the exchange rate relations which we used to solve for the exchange rates. These relations are called the *exchange neutrality conditions*, or simply the *neutrality conditions*. Considering their importance to the determination of exchange rates it will be worthwhile to understand the precise role they play. The reciprocity of exchange rates between any two currencies is called the two-point or two-currency neutrality condition. If A and B are any two currencies then,

$$E_{AB} E_{BA} = 1$$

where E denotes the exchange rate. The three-point, or three-currency neutrality condition for any three currencies A, B and C is,

$$E_{AB} E_{BC} E_{CA} = 1$$

And in general in the case of M currencies we must have,

$$E_{AB} E_{BC} \ldots E_{MA} = 1$$

The neutrality conditions as has been pointed out ensure that all possibilities of currency arbitrage are eliminated. Thus the two-point neutrality condition eliminates the possibilities of arbitrage in two currencies, i.e. nobody who has, say 1dollar, should be able, in equilibrium, to make a profit (or loss) by selling it for rupees, simultaneously selling the rupees for dollars to end up with more (or less) than 1 dollar. The three-point neutrality condition goes further to eliminate the possibility of arbitrage in three currencies–nobody with 1 dollar should be able to sell it for yen, sell the yen for rupees and simultaneously sell the rupees for dollars to end up with more (or less) than 1 dollar. The M-point neutrality condition does the some in the M-currency case. The neutrality conditions themselves have not been proved by us in the foregoing discussion. This, as well as further investigation of their properties, will be the task of the further analysis in which we shall allow ourselves more mathematics than we have so far.

4. The General Multilateral System

Consider a multicountry world consisting of Z countries $A, B, \ldots Z$. Let P_i ($i = A$, $B, \ldots Z$) be the total payments (debits) made by country i to the rest of the world and R_i ($i = A, B,...Z$) be the total receipts (credits) of country i from the rest of the world, both measured in the domestic currency of country i with reference to a stipulated time period of an hour, week, month or year. Let P_{ij} ($ij = A, B,..., Z$, $i \neq j$) be the payments made by country i to country j measured in the currency of country i. For obvious reasons $P_{ij} = 0$ for $i = j$. Equilibrium in the balance of payments requires that,

$$P_i = R_i \qquad \forall_i$$

$$\sum_j P_{ij} = R_i = \sum_j R_{ij} \qquad \forall_i \tag{1}$$

Since our problem is to determine the exchange rates from the balance of payments data in equation (1) we shall depict the intercountry payments and receipts in the following explicit form,

$$\begin{aligned}
0 + P_{AB} + P_{AC} + ...+ P_{AZ} &= R_A = P_A \\
P_{BA} + 0 + P_{BC} + ...+ P_{BZ} &= R_B = P_B \\
P_{CA} + P_{CB} + 0 + ...+ P_{CZ} &= R_C = P_C \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
P_{ZA} + P_{ZB} + P_{ZC} + ...+ 0 &= R_Z = P_Z
\end{aligned} \tag{2}$$

wherein both sides of each equation are expressed in the currencies of the respective countries. The equations in (2) above are mere identities in so far as they represent simply a country-wise break-up of the total payments of each country. The equations in this form will not directly help us to determine the exchange rates. But if we read the equations in (2) vertically it will at once be apparent that the entries in each column show the receipts of a country from the other countries, expressed in the currencies of the other countries; the first-column elements $P_{BA}, P_{CA}, ... P_{ZA}$ are the receipts of country A from countries $B, C,.. Z$ in the currencies of $B, C, ... Z$ respectively, the second-column elements $P_{AB}\, P_{CB} \,.....\, P_{ZB}$ are the receipts of country B from $A, C, ...Z$ in the currencies of $A, C, ..Z$ respectively, etc.

Now if the receipts in each column are multiplied by appropriate exchange rates so as to convert them into the currency of the receiving country and all such converted receipts are summed up then they must, under equilibrium conditions, be equal to the total payments P_i made by the respective countries to the rest of the world. Performing this operation for all the equations gives us the following system of balance of payments equations,

$$\begin{aligned}
0 + P_{BA}E_{AB} + P_{CA}E_{AC} +......+ P_{ZA}E_{AZ} &= P_A = R_A \\
P_{AB}E_{BA} + 0 + P_{CB}E_{BC} +......+ P_{ZB}E_{BZ} &= P_B = R_B \\
P_{AC}E_{CA} + P_{BC}E_{CB} + 0 +......+ P_{ZC}E_{CZ} &= P_C = R_C \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
P_{AZ}E_{ZA} + P_{BZ}E_{ZB} + P_{CZ}E_{ZC} +......+ 0 &= P_Z = R_Z
\end{aligned} \tag{3}$$

where E_{ij}($i, j = A, B,..., Z$ $i \neq j$) are the exchange rates denoting the quantity of currency i that must be given to purchase one unit of currency j. All the equations in (3), just as in (2) above, are expressed in terms of the own currencies of the countries.

It is important to understand the 'market' logic underlying the equations in (3). Our focus in on describing the market process by which the supplies of currencies are cleared against the demands for them. The world supply of each currency originates from the country to which that currency belongs and is accordingly equal to the total payments made by the country to the rest of the world during the period. Thus P_A is the world supply of currency A, P_B the world supply of currency B, ... and so on. Now a currency can be usefully possessed only in the country of its issue. Which means that an arrangement whereby the world supply of a country's currency can be rechanneled back to the country must be devised. This can be accomplished only if the receipts of the country in terms of her own currency are equal to her total payments. The foreign exchange market must therefore determine such prices of the foreign currencies as will ensure total payments = total receipts and supply = demand[(3)]

Each country's receipts, however, are the sum of the payments made by other countries and these naturally are in terms of the currencies of these other, in general $z-1$, countries. A country A's receipts, for example, are $P_{BA}, P_{CA}, ...P_{ZA}$ in currencies $B, C,...Z$ respectively. Their sum in the currency of country A, viz R_A is to be made equal to country A's total payments P_A. The payments made by other countries represent demands for the currency of A. These considerations form the basis of the equations in (3).

5. Underdeterminacy of the System

The system of equations (3) contains z equations in $z(z-1)$ unknown exchange rates $E_{ij}(i \neq j)$. Further note that of the z equations, any one must be linearly dependent on the remaining $z-1$ since the world payments must by definition be equal to the world receipts when both are evaluated in any one currency. After omitting the redundant equation the system of equations (3) is found to contain only $z-1$ independent equations in $z(z-1)$ unknown exchange rates. The system is $z(z-1)-(z-1)=(z-1)^2$ equations short of the required number so that at first sight the problem of determining the exchange rates seems to be hopelessly unsolvable.

6. Derivation of the Neutrality Conditions

We shall see, however, that it is not so. It will always be possible to derive exactly $(z-1)^2$ independent relations between the exchange rates so as to render the system exactly determinate. An interesting (though a surprisingly neglected) formulation makes this possible. The key to this formulation is the condition that all the balance of payments equations in (3) should balance not only in the domestic currencies of the countries but in *at least* one other currency as well.[(4)]

Accordingly, we subject system (3) to a transformation by placing the requirement that all its constituent equations should balance in terms of say currency A. To obtain the transformed system we keep the first equation intact because it already is in terms of currency A, we multiply the second equation by E_{AB}, the third equation by E_{AC},...., the z^{th} equation by E_{AZ}, and arrange the equations as follows,

$$
\begin{array}{l}
0 + P_{BA}E_{AB} + P_{CA}E_{AC} + \dots\dots\dots + P_{ZA}E_{AZ} = P_A \\
P_{AB}E_{AB}E_{BA} + 0 + P_{CB}E_{AB}E_{BC} + \dots\dots\dots + P_{ZB}E_{AB}E_{BZ} = P_B E_{AB} \\
P_{AC}E_{AC}E_{CA} + P_{BC}E_{AC}E_{CB} + 0 + \dots\dots\dots + P_{ZC}E_{AC}E_{CA} = P_C E_{AC} \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
P_{AZ}E_{AZ}E_{ZA} + P_{BZ}E_{AZ}E_{ZB} + P_{CZ}E_{AZ}E_{ZC} + \dots\dots\dots + 0 = P_Z E_{AZ}
\end{array}
\qquad (4)
$$

Now compare the column-sums to the corresponding row-sums. The elements in the first column are the payments made by country A to the rest of the world in her own currency. Their sum is, by definition, equal to the total payments in own currency terms P_A. Thus, we have

$$P_{AB}(E_{AB}E_{BA}) + P_{AC}(E_{AC}E_{CA}) + \dots\dots\dots\dots + P_{AZ}(E_{AZ}E_{ZA}) = P_A$$

At the same time the first equation in system (2) tells us that,

$$P_{AB} + P_{AC} + \dots\dots + P_{AZ} = P_A$$

For these two equations to hold, the exchange rate products $E_{AB}E_{BA}$, $E_{AC}E_{CA}$, ..., $E_{AZ}E_{ZA}$ must all be equal to unity: thus we obtain $z-1$ two-point neutrality conditions.

$$
\begin{array}{l}
E_{AB}E_{BA} = 1 \\
E_{AC}E_{CA} = 1 \\
\dots\dots\dots\dots\dots\dots \\
E_{AZ}E_{ZA} = 1
\end{array}
$$

or in short,

$$E_{Aj}E_{jA} = 1 \qquad j = B, C, \dots\dots Z \qquad (5)$$

The two-point neutrality conditions give us $z-1$ independent relations between the exchange rates.

Next, consider the equality of the second column-sum with the second row sum in (4),

$$P_{BA}E_{AB} + P_{BC}E_{AC}E_{CB} + \dots\dots + P_{BZ}E_{AZ}E_{ZB} = P_B E_{AB}$$

But the second equation in (2) tells us that,

$$P_{BA} + P_{BC} + \dots\dots + P_{BZ} = P_B$$

Comparing the two we infer that the following $z-2$ independent exchange rate relations must hold,

$$
\begin{array}{l}
E_{AC}E_{CB} = E_{AB} \\
E_{AD}E_{DB} = E_{AB} \\
\dots\dots\dots\dots\dots\dots\dots\dots \\
E_{AZ}E_{ZB} = E_{AB}
\end{array}
$$

The third column-sum and row-sum in (4) as compared with the third equation in (2) give further $z-2$ relations,

$$\begin{aligned} E_{AB}E_{BC} &= E_{AC} \\ E_{AC}E_{DC} &= E_{AC} \\ &\dots\dots \\ E_{AZ}E_{ZC} &= E_{AC} \end{aligned}$$

Each comparision from the second column onwards gives us $z-2$ independent three-point exchange-rate relations and there are $z-1$ comparisons to be made. Therefore we obtain $(z-1)(z-2)$ independent three-point equations of the type,

$$E_{Aj}E_{jk} = E_{Ak} \qquad j, k = B, C, ..., Z \quad j \neq k \tag{6}$$

or

$$E_{Aj}E_{jk}E_{kA} = 1$$

Thus the total number of independent exchange rate relationships that we have found are $(z-1)+(z-1)(z-2)=(z-1)^2$ the required number.

Our model of exchange rate determination contain $z-1$ independent balance of payments equations and $(z-1)^2$, independent exchange neutrality conditions, which is a total of $z(z-1)$ independent equations. These must determine $z(z-1)$ exchange rates E_{ij} $(i\,j = A, B, ..., Z\ i \neq j)$

7. Properties of the Neutrality Conditions

Equations (5) and (6), the two-point and three-point exchange neutrality conditions, deserve somewhat detailed consideration. Although written with reference to currency A *vis-a-vis* all the other currencies, we shall prove that they constitute a linearly independent set using which the neutrality conditions pertaining to all other currencies, between themselves, can be derived. Consider first the two-point neutrality condition. We know from equation (5) that for any currencies i and j $(i, j \neq A)$

$$\begin{aligned} E_{Ai}E_{iA} &= 1 \\ E_{Aj}E_{jA} &= 1 \end{aligned}$$

If we multiply these equations we get,

$$(E_{Ai}E_{iA})(E_{Aj}E_{jA}) = 1$$

i.e.

$$(E_{iA}E_{Aj})(E_{Ai}E_{jA}) = 1$$

We know from the three-point neutrality conditions (6) that,

$$\begin{aligned} E_{iA}E_{Aj} &= E_{ij} \\ E_{Ai}E_{jA} &= E_{ji} \end{aligned}$$

so that, we obtain,

$$E_{ij}E_{ji} = 1 \qquad \forall\, i, j, \quad i \neq j, \tag{7}$$

The two-currency neutrality conditions for all pairs of currencies can thus be directly derived from the two-currency and three-currency neutrality conditions for any specific currency. The former are therefore dependent on the latter. A similar argument holds also for the three-currency neutrality conditions. We know that for currency A,

$$E_{Aj} E_{jk} E_{kA} = 1 \qquad \forall j, k \quad j \neq k \neq A$$

From the three-point neutrality conditions (6) we also know that,

$$E_{Aj} = E_{Ai} E_{ij}$$
$$E_{kA} = E_{ki} E_{iA}$$

Substituting these above gives,

$$(E_{Ai} E_{ij})(E_{jk})(E_{ki} E_{iA}) = 1$$

which can be rearranged in the form,

$$(E_{Ai} E_{jA})(E_{ij} E_{jk} E_{ki}) = 1$$

Since $E_{Ai} E_{iA} = 1$ due to equation (5), the above reduces to

$$E_{ij} E_{jk} E_{ki} = 1 \qquad \forall i, j, k \quad i \neq j \neq k \tag{8}$$

Thus all the three-point neutrality conditions can be derived from the two-and three-point neutrality conditions for a specific currency.

It may be wondered why the set of independent neutrality conditions is confined to two-currency and three-currency arbitrage; why are there no independent relations amongst four, five or more currencies. This is because all these "higher order" neutrality conditions can be shown to be dependent on (7) and (8). The proof is simple. Consider a three-currency relation in (8),

$$E_{ij} E_{jk} E_{ki} = 1$$

and multiply either side by $E_{ip}(i \neq p)$ to get,

$$E_{ij} E_{jk} E_{ki} E_{ip} = E_{ip}$$

But we know from (8) that $E_{ki} E_{ip} = E_{kp}$ and from (7) that $E_{pi} = 1/E_{ip}$ so that the above equation reduces to

$$E_{ij} E_{jk} E_{kp} E_{pi} = 1 \qquad i \neq j \neq k \neq p$$

which is the general four-currency neutrally condition. In like manner all the higher-order conditions can be derived directly from (7) and (8).

The fact that all the higher-order neutrality conditions hold if the two-currency and three-currency neutrality conditions do is a remarkable theorem due to Chacholiades (1971) "If (2-point and) 3-point arbitrage is not profitable, then k-point ($k > 3$) is not profitable either." [Brackets mine. Also see Chacholiades (1978) and Deardorff (1979)]

It is important, however, to point out a difference in the methods of proof employed by these authors and that adopted in this text. The proof used in the text follows from the requirement that, in general equilibrium, the payments of every country must balance in terms of at least one currency other than her own. If this happens, it follows that the payments of all countries must balance in terms of all currencies. The exchange rates which meet this requirement conform, as has been shown, to the exchange neutrality conditions, i.e, they automatically prevent currency arbitrage. In the proof by Chacholiades, the neutrality conditions are derived directly from the requirement that the exchange rates should be such as to prevent currency arbitrage. Although both methods reach the same results, the differences in the methods of the proofs are revealing.

8. Solution of the Exchange Rates

After this brief, but necessary, digression on the neutrality conditions let us return to the question of exchange rate determination. We shall show how the system containing $z-1$ independent balance of payments equations and $(z-1)^2$ independent arbitrage, or neutrality, equations determine $z(z-1)$ exchange rates. Consider the system of equations (4). Substitute for all the exchange rate products in these equations, the appropriate exchange rates from the neutrality conditions (5) and (6); the second column exchange rate products $E_{AC}E_{CB}$, $E_{AC}E_{DB}$, ..., $E_{AZ}E_{ZB}$ will be replaced by E_{AB}, the third column exchange rate products $E_{AB}E_{BC}$,..., $E_{AZ}E_{ZC}$ will be replaced by E_{AC}, etc. Then we obtain the following system of equations in vector-matrix notation,

$$\begin{bmatrix} P_A & -P_{BA} & -P_{CA} & \cdots & -P_{ZA} \\ -P_{AB} & P_B & -P_{CB} & \cdots & -P_{ZB} \\ -P_{AC} & -P_{BC} & P_C & \cdots & -P_{ZC} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -P_{AZ} & -P_{BZ} & -P_{CZ} & \cdots & P_Z \end{bmatrix} \begin{bmatrix} 1 \\ E_{AB} \\ E_{AC} \\ \cdots \\ E_{AZ} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix} \tag{9}$$

The square matrix on the left-hand-side is the multicountry payments matrix (5). The sums of its column elements are equal to zero (see equation 2) because any one country's balance of payments equation is linearly dependent on the remaining equations. Therefore, the determinant of the matrix is zero. It follows that the system of equations will give a unique solution for $z-1$ exchange rates E_{AB} ......, E_{AZ}. Moreover the solution of the exchange rates will be positive because the matrix has a dominant principal diagonal; the diagonal elements being the total payments (receipts), will be greater than or equal to, the off-diagonal elements.

To obtain the solution explicitly, the following non-homogeneous system of equations can be used. Eliminate any one equation, say the zth from the system (9), and arrange the remaining $z-1$ equations as follows;

$$\begin{bmatrix} P_{BA} & -P_{CA} & \cdots & P_{YA} & P_{ZA} \\ -P_B & P_{CB} & \cdots & P_{YB} & P_{ZB} \\ P_{BC} & -P_C & \cdots & P_{YC} & P_{ZC} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{BY} & P_{CY} & \cdots & -P_Y & P_{ZY} \end{bmatrix} \begin{bmatrix} E_{AB} \\ E_{AC} \\ E_{AC} \\ \cdots \\ E_{AZ} \end{bmatrix} = \begin{bmatrix} P_A \\ -P_{AB} \\ -P_{AC} \\ \cdots \\ -P_{AY} \end{bmatrix} \tag{10}$$

where the square matrix on the left hand side is of the order $z-1 \times z-1$. Denoting the matrix by P, the vector of the exchange rates of currency A by E_a and the $z-1 \times 1$ vector of the payments of country A by P_a we can write (10) in concise notation as,

$$PE_a = P_a$$

and represent the solution of the exchange rates by,

$$E_a = (P)^{-1} P_a \tag{11}$$

Once these $z-1$ exchange rates are found from (11), all the remaining $(z-1)^2$ exchange rates can be determined from the neutrality conditions (5) and (6).

9. Properties of the Exchange Rate Solution

It remains now to point out some properties of the exchange rates solved. These will prove useful in later analysis of exchange rate movements.

Firstly, it is necessary to point out that the multicountry payments matrix P must be indecomposable, i.e. there must be at least one country which has international financial relations, directly or indirectly, with all other countries. Failing this condition, the world economy decomposes into sub-worlds, each with its own indecomposable multicountry payments matrix. These sub-worlds must be treated as independent economic systems in their own right.

Secondly, consider how the exchange rates move due to certain exogenous changes suppose the elements in the column vector of A's payments P_A, change uniformly by a factor μ. Then all the exchange rates $E_{Aj}(j \neq A)$ change by the same factor μ, because

$$\begin{aligned} E_a^* &= (P)^{-1} (\mu P_a) \\ &= \mu [(P)^{-1} P_a] \\ &= \mu E_A \end{aligned}$$

i.e. $$E_{Aj}^* = \mu E_{Aj} \qquad \forall j \neq A.$$

If any column in the matrix P changes by a factor μ, then the effect is to change the corresponding exchange rate by the factor $1/\mu$ with all other exchange rates remaining unchanged. This is best seen if we solve (11) by Cramer's rule,

$$E_{Aj} = \frac{det\,(P_j)}{det\,(P)}$$

where P_j is the matrix obtained by replacing the jth column ($j = B, C,... Z$) of P by the vector P_a. Denote the matrix obtained by multiplying the jth column of P by μ as P^*. Then,

$$det\,(P^*) = \mu\, det\,(P)$$

so that,

$$E_{Aj}^* = \frac{det\,(P_j)}{\mu\, det\,(P)} = \left[\frac{1}{\mu}\right] E_{Aj}$$

But every other exchange rate $E_{Ai}(i \neq j)$ remains what it was before the change because,

$$det\,(P_j^*) = \mu\, det\,(P_j)$$

so that

$$E_{Ai}^* = \frac{\mu\, det\,(P_j)}{\mu\, det\,(P)} = E_{Ai}$$

In general if each column of P changes by a factor μ_j ($j = B, C,..\, Z$) then the exchange rates of currencies change to,

$$E_a^* = \left[\frac{\prod\limits_{j=B}^{z}}{\prod\limits_{i \neq j}}\right] E_a \qquad\qquad j \neq A$$

where Π denotes the product operation.

Finally, if any row of the matrix P were to change, along with the corresponding row element of P_A, by a uniform factor μ then all the exchange rates remain unchanged.

10. The Multicountry Trade Matrix

In so far as the study of international trade alone is concerned, to the exclusion of other international transactions, the inter-country payments and receipts will contain only the intercountry import bills and the export earnings. Equilibrium in the balance of payments (rather, the balance of trade) will require that the export earnings of each country must be equal to her import bill in terms of all currencies.

All that we need to do to solve the exchange rates is to replace P_{ij} by M_{ij}, where M_{ij} is the import bill owed by country i to country j in the currency of country i, and P_i by M_i, where M_i the total import bill of country i will be equal to X_i, her total export earnings. The solution of the exchange rates will then be shown as,

$$\begin{bmatrix} M_{BA} & M_{CA} & \dots & M_{ZA} \\ -M_B & M_{CB} & \dots & M_{ZB} \\ \dots & \dots & \dots & \dots \\ M_{BY} & M_{CY} & \dots & M_{ZY} \end{bmatrix} \begin{bmatrix} E_{AB} \\ E_{AC} \\ \dots \\ E_{AZ} \end{bmatrix} = \begin{bmatrix} M_A \\ -M_{AB} \\ \dots \\ -M_{AY} \end{bmatrix}$$

11. Fixed Exchange Rate System

Thus far the discussion has been confined to the flexible exchange rate system in which the exchange rates can vary freely with every change in the intercountry payments. The analysis of fixed exchange rate systems can be developed as a special case. Formally speaking a fixed exchange rate system is one in which all the exchange rates are predetermined by the monetary authorities/governments of the countries. Presumably the predetermined exchange rates are such as to prevent currency arbitrage. Then, depending upon whether the countries have fixed the exchange rates of their currencies at higher (depreciated) levels or lower (appreciated) levels as compared to the exchange rates that would have prevailed under the flexible exchange rate systems, the countries will have surpluses or deficits in the *autonomous* transactions in their balance of payments.

The governments of the surplus countries would therefore be required to make *official* transfers to those of the deficit countries either directly, or indirectly through a financial institution (like the IMF), which the latter will use to *accommodate* their deficits. The magnitudes of the surpluses and deficits and the sizes of the official transfers become the unknowns when our system is applied to the case of fixed exchange rates.

Accordingly we may use the system of equations (9) to solve for the sizes of the surpluses and deficits of the different countries at predetermined exchange rates E^*_{ij}. There is, however, a point to be noted. Under the flexible exchange rate system the total autonomous payments of each country P_i were necessarily equal to her total autonomous receipts, R_i. This is no longer true. Thus we must use R_i instead of P_i. With this modification, the system (9) solves for the surpluses/deficits Q_i,

$$\begin{aligned}
Q_A &= R_A - P_{BA}E^*_{AB} - P_{CA}E^*_{AC} - \ldots - P_{ZA}E^*_{AZ} \\
Q_B &= -P_{AB} + R_B E^*_{AB} - P_{CB}E^*_{AC} - \ldots - P_{ZB}E^*_{AZ} \\
Q_C &= -P_{AC} - P_{BC}E^*_{AB} + R_C E^*_{AC} - \ldots - P_{ZC}E^*_{AZ} \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
Q_Z &= -P_{AZ} - P_{BZ}E^*_{AB} - P_{CZ}E^*_{AC} - \ldots + R_Z E^*_{AZ}
\end{aligned} \tag{13}$$

where all the Q_i $(i = A.......Z)$are measured in in the currency of country A. In vector-matrix noration we may write,

$$Q = RE^*_A$$

where Q_i is a $z \times 1$ vector of surplus/deficits, R is a $z \times z$ matrix with total receipts on its main diagonal and $-P_{ij}$ $(i \neq j)$ as its other elements and E^*_A is a $z \times 1$ vector of exchange rates with the first elements being 1.

It is clear from (13) that $\Sigma\, Q_i = 0$ because the sum of the columns on the right hand side are zero. Thus the clearing of foreign exchange transactions must be achieved by countries with positive Q's (surpluses) making official transfers to those with negative Q's (deficits).

12. The 'Mixed' exchange rate system

The mixed exchange rate system, or the managed float system, combines the characteristics of flexible and fixed exchange rate systems. In it both exchange rate variability and official transfers are allowed to play their parts in the process of clearing foreign exchange transactions. The decision-making is purely administrative. If foreign exchange reserves are dwindling and official transfers are not forthcoming, a deficit country allows her currency to depreciate; otherwise she may choose to maintain her currency at a high value using her reserves or official debts for the purpose. It is obvious that these are not the only dimensions of the exchange rate policy. Indeed, in a managed float system, the exchange rate policy is an aspect of overall foreign economic policy, in which are included policies regarding customs duties, exports subsidies, foreign investment policy, capital account convertibility, etc.

Notes

1. This is an aspect of exchange rate determination that is entirely neglected by the dominant theories of exchange rates such as the purchasing power parity theory or the partial equilibrium demand-supply theory. In the former the exchange rate is made to depend exclusively on the price levels in the two countries and, on that account, automatically excludes consideration of trade balance. In the latter, two countries are assumed so that balance payments for each country becomes axiomatic.
2. The system gives the same solution irrespective of the currency chosen. Thus we could as well convert all the balance of payments equations into rupees, solve the rupee-dollar and rupee-yen rates first, and then proceed to solve the remaining exchange rates.
3. A similar model, without the mathematics, will be found in Robinson (1947) who acknowledges Keynes (1928). And of course it will be found strewn throughout the literature under a 'contentless' title, "the balance-of-payments theory of exchange rates" which usually means the demand-supply diagram. It is implicit in the "Absorption Approach" to balance of payments. [See Alexander (1950)]
4. Since there is no restriction on the currency in which these equations should balance it follows that if the equations balance in any one currency other than the country's own, then they do so in terms of all.
5. There is considerable empirical work on the multicountry payments matrix, and particularly on the multicountry trade matrix. The pioneer effort is due to Folke Hilgerdt for the League of Nations in 1940. [See League of Nations (1942)] for an early application of the world trade matrix see Frisch (1947).] Annual world trade matrices are published by GATT. The IMF publishes data on multilateral financial flows, in addition to trade flows. These data have been used in a variety of macroeconometric models, e.g. Project LINK [IMF, See Ball (1973), Sawyer (1979)], Euro LINK [EEC], OECD linkage model [See OECD (1982)], COMET [Barten et.al. (1976)], DESMOS [Dramais (1974)], METEOR [Kooyman (1982)] and others. See Helliwell and Padmore (1985) for a survey of these models.

2

GAINS OF TRADE

"What is it you want to buy?" the Sheep said at last. "I don't quite know yet," Alice said very gently. "I should like to look round me first. If I might." "You may look in front of you, and on both sides if you like" said the Sheep; "but you can't look all round"

Introduction

To focus attention on international trade let us provisionally exclude all international transactions other than trading in commodities. [In Chapters V and VII we shall deal with international trade in the presence of other transactions]. Then, as we have seen in chapter 1 section 10, the exchange rate model can solve the exchange rates appropriate to a given pattern of intercountry import bills and export receipts. The exchange rate model, however, can do its work only after the international trade pattern, that is to say, the commodity-composition of the intercountry trade has been determined. We must therefore turn to inquire into the factors that determine the commodity-composition of world trade: which countries import which commodities, and from whom.

Countries trade because they gain by trade. An essential step towards determining the world trade pattern therefore is to determine the pattern of the gains of trade. Accordingly, the object of this chapter is to find the methods for identifying the intercountry gains of trade and to use them to find the countries of origin and the countries of destination of the various commodities. Taking advantage of the context, we shall also spell out the necessary and sufficient conditions under which a world trade pattern can constitute an international trade equilibrium.

1. The 'Natural' Rates of Currency Exchange

A concept of great significance to the identification of the pattern of the gains of trade will now be introduced. Let p_{ij} $(i = 1, 2, ..., n\ j = A, B, ...Z)$ be the money prices (in autarky) of commodity i in country j measured in the own currency of country j. These money prices may be tabulated as follows,

$$\begin{array}{cccc} p_{1A} & p_{1B} & p_{1C} \cdots\cdots & p_{1Z} \\ p_{2A} & p_{2B} & p_{2C} \cdots\cdots & p_{2Z} \\ p_{3A} & p_{3B} & p_{3C} \cdots\cdots & p_{3Z} \\ \cdots & \cdots & \cdots & \cdots \\ p_{nA} & p_{nB} & p_{nC} \cdots\cdots & p_{nZ} \end{array} \qquad (1)$$

Since our interest is to track down the gains of trade, it will be convenient to exclude for the present purposes all non-tradeable commodities. Thus all the commodities appearing in table (1) should be considered tradeables, i.e. commodities capable of being trade.

We shall define the *natural rates of currency exchange* or, in short, the *natural exchange rates* as the ratios of the autarkic money prices of commodities in different countries. Thus the natural exchange rates of currencies j and k are defined as,

$$E^i_{jk} = \frac{p_{ij}}{p_{ik}} \quad \forall i \quad j,k = A, B, ..., Z \quad j \neq k \tag{2}$$

There will be obviously be as many natural exchange rates between two currencies as there are commodities.

It should perhaps be immediately pointed out that the 'natural' exchange rates have been so called because they spring naturally from the autarkic money prices of the commodities in the various countries. Apart from this there is nothing natural about them. Competing terms include *transitional, threshold, cross-over, watershed, limbo, nodal* and *critical* exchange rates or the lengthier, *commodity rates of currency exchange.*

The idea is this: if the autarky prices of say wheat in countries A and B are Rs.10 and \$2 per kilo respectively, then the ratio of these prices suggests an exchange rate of currencies, Rs.5 = \$1, which we have called the natural exchange rate attributable to wheat. If trade develops and the actual exchange rate is Rs.5 = \$1 then neither country will gain by importing wheat from the other,. The commodity, wheat, will be in *limbo*, being indifferently a non traded good, an exportable of A or an exportable of B. If the actual exchange rate were to be lower than the natural rate, say Rs.2.50 = \$1, then citizens of A would import wheat from B because it would cost them Rs.5 per kilo in B which is lower than Rs.10 per kilo which they pay at home. On the other I and, if the actual exchange rate were Rs.10 = \$1, citizens of country B would import wheat from A because it would cost them \$1 per kilo to purchase wheat in A as compared to \$2 per kilo they are required to pay at home. It is in this sense that the wheat rate of 'rupee-dollar exchange', Rs.5 =\$1, is a *critical* or *threshold* value; if the actual exchange rate were to lie below it, wheat appears in A's import list and if the actual exchange rate were to lie above it, it appears in B's import list.

Corresponding to the table of money prices in (1) we can construct a table of natural exchange rates,

$$\begin{array}{l} E^1_{AB}\ E^1_{BC}\ E^1_{CD}\ \ldots.\ E^1_{ZA} \\ E^2_{AB}\ E^2_{BC}\ E^2_{CD}\ \ldots.\ E^2_{ZA} \\ E^3_{AB}\ E^3_{BC}\ E^3_{CD}\ \ldots.\ E^3_{ZA} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E^n_{AB}\ E^n_{BC}\ E^n_{CD}\ \ldots.\ E^n_{ZA} \end{array} \tag{3}$$

Three rather obvious properties of the natural exchange rates may be noted. Firstly, by definition, the natural exchange rate of currencies j and k must be the reciprocal of the natural exchange rate of currencies k and j,

$$E^i_{jk} = \frac{p_{ij}}{p_{ik}} = \frac{1}{(p_{ik}/p_{ij})} = \frac{1}{E^i_{kj}} \quad \forall i \tag{4}$$

Secondly, observe that the natural exchange rates in each row of table (3) have been derived from the ratios of the successive prices in the corresponding row in table (1), e.g. $E^1_{AB} = p_{1A}/p_{1B}$, $E^1_{BC} = p_{1B}/p_{1C}$, ..., $E^1_{ZA} = p_{1Z}/p_{1A}$. Since every price appears once in the numerator and once in the denominator, the product of the natural exchange rates in each row of (3) must be equal to 1:

$$E^i_{AB}\ E^i_{BC}\ E^i_{CD}\ \ldots\ E^i_{YZ}\ E^i_{ZA} = 1 \quad \forall i \tag{5}$$

because

$$\left[\frac{p_{iA}}{p_{iB}}\right]\left[\frac{p_{iB}}{p_{iC}}\right]\left[\frac{p_{iC}}{p_{iB}}\right]\ldots\left[\frac{p_{iY}}{p_{iZ}}\right]\left[\frac{p_{iZ}}{p_{iA}}\right] = 1$$

Thirdly, the natural exchange rates between any two currencies may either be obtained directly from the ratios of the prices in table (1) or indirectly from the natural exchange rates in table (3). For example.

$$E^i_{AD} = \frac{p_{iA}}{p_{iD}} \text{ or } E^i_{AD} = \left[\frac{p_{iA}}{p_{iB}}\right]\left[\frac{p_{iB}}{p_{iC}}\right]\left[\frac{p_{iC}}{p_{iB}}\right] = E^i_{AB}E^i_{BC}E^i_{CD}$$

2. Direction of Trade of a Commodity between Two Countries

A brief allusion was made to the role played by the natural exchange rate in determining the direction of trade in the example of wheat. We shall formalise it now.

Country A is said to gain by importing commodity 1 from B if the price of commodity 1 in country B converted to A's currency at the actual exchange rate of currencies A and B is lower then the price of commodity 1 country A, i.e. if

$$E_{AB}\,p_{1B} < p_{1A} \tag{6}$$

In the inequality p_{1B} is the price of commodity 1 in B's currency, say dollars, and p_{1A} is the price of commodity 1 in A, in rupees. To make the prices comparable to one another, p_{1B} the dollar price, is multiplied by the rupee-dollar exchange rate, so that $E_{AB}\,p_{1B}$ is rupee price that citizens of A will have to pay to purchase the commodity in country B. The condition above can be written in terms of the natural exchange rate. Dividing both sides of the inequality by p_{1B} gives,

$$E_{AB} < \frac{p_{1A}}{p_{1B}} = E^1_{AB} \tag{7}$$

On the other hand country B gains by importing commodity 1 from country A if the price of commodity 1 in A converted into dollars is lower than the price of commodity 1 in B, i.e., if,

$$E_{BA}p_{1A} < p_{1B}$$

or

$$E_{BA} < E^1_{BA} \tag{8}$$

Then provided the actual exchange rates of currencies E_{AB} and E_{BA} satisfy the two-point neutrality condition, viz $E_{AB}E_{BA} = 1$. we may write (8) as,

$$E_{AB} > E^1_{AB} \tag{9}$$

Finally if $E_{AB} = E^1_{AB}$ then neither country gains by trading commodity 1 with the other.

We have thus shown that the direction of the gains of trade in a commodity between any two countries is unambigously established by comparing the actual exchange rate with the natural exchange rate attributable to that commodity provided, however, that the actual exchange rate satisfies the two-point neutrality condition

3. Direction of Trade of a Commodity between Several Countries

Yet another proposition of great importance to the determination of the world trade pattern is as follows: if in a multicountry world, a country B gains by importing a commodity from a country A, then, so do all other countries, provided that the actual rates of currency exchange satisfy the two-point and three-point neutrality conditions.

In formal terms we are required to prove that if $E_{BA}p_{1A} < p_{1B}$ $(E_{AB} > E^1_{AB})$ then $E_{CA}p_{1A} < p_{1C}$ $(E_{AC} > E^1_{AC})$ for any other country C. The proof is simple. The fact that country B does not gain by importing the commodity from country C means either that (a) $p_{1B} < E_{BC}p_{1C}$ $(E_{BC} < E^1_{BC})$ meaning that the price of the commodity in B is lower than the price of the commodity in C converted into B's currency), or that (b) $E_{BA}p_{1A} < E_{BC}p_{1C}$ (i.e. the price of the commodity if imported from country A is lower than that if imported from country C). Whatever the circumstance, we must show that C would gain by importing the commodity only from country A.

If (a), then country C herself gains by importing the commodity from country B *vide* the results in section 3 above. But if, as we know, B finds it gainful to import the commodity from country A it is easy to guess that C should find it even more so. The formal proof is easy too. We know that,

$$E_{AB}p_{1A} < p_{1B}$$

and

$$p_{1B} < E_{BC}p_{1C}$$

Multiplying the inequalities and cancelling p_{1B} from both sides gives,

$$E_{BA}p_{1A} < E_{BC}p_{1C}$$

or $$\left[\frac{E_{BA}}{E_{BC}}\right] p_{1A} = E_{CB} E_{BA} p_{1A} = E_{CA} p_{1A} < p_{1C}$$

provided that the three-point neutrality condition $E_{CB} E_{BA} = E_{CA}$ is satisfied. If (b), then a slight readjustment of terms gives the result directly.

The propositions established above confer considerable advantages to the task of determining the international trade equilibrium. They demonstrate that for any set of consistent exchange rates there corresponds a unique pattern of the gains from trade and a corresponding trade pattern—an assignment of commodities to the countries that can result in gains for all the countries.

4. Gains of Trade and Terms of Trade

Given a set of currency exchange rates we can compute the quantities of different commodities that can be purchased in all the countries per unit of the currencies of these countries.

Thus a unit of currency A, for example, can purchase $1/p_{1A}$ units of commodity 1 or $1/p_{2A}$ units of commodity 2 or $1/p_{3A}$ units of commodity of 3 or.... $1/p_{nA}$ units of commodity n at home. Alternatively, the same unit of currency A can be converted into E_{kA} units of the currency of country k ($k = B$,...., Z) and used to purchase E_{kA}/p_{1k} units of commodity 1 or E_{kA}/p_{2k} units of commodity 2 or .... or E_{kA}/p_{nk} units of commodity n in country k. In general, a unit of currency A can purchase any one of the following quantities of different commodities in different countries,

$$\frac{E_{kA}}{p_{ik}} \quad i = 1,...,n \quad k = A,...,Z \quad E_{AA} = 1 \tag{10}$$

Similarly a unit of the currency of country B can purchase any one of the following quantities of different commodities,

$$\frac{E_{kB}}{p_{ik}} \quad i = 1,...,n \quad k = A,...,Z \quad E_{BB} = 1 \tag{11}$$

And so on for the currencies of other countries.

The quantities of commodities that a unit of a currency can purchase in the different countries can be arranged in the form of a table which we shall call the gain-from-trade table. Thus the quantities appearing in equation (10) for a unit of the currency of country A can be tabulated as follows.

A	A	B	C	....	Z
1	$1/p_{1A}$	$\boxed{E_{BA}/p_{1B}}$	E_{CA}/p_{1C}	....	E_{ZA}/p_{1Z}
2	$\boxed{1/p_{2A}}$	E_{BA}/p_{2A}	E_{CA}/p_{2C}	....	$\boxed{E_{ZA}/p_{2Z}}$
3	$1/p_{3A}$	E_{BA}/p_{3B}	$\boxed{E_{CA}/p_{3C}}$	....	$\boxed{E_{ZA}/p_{3Z}}$
⋮	⋮	⋮	⋮	⋮	
n	$\boxed{1/p_{nA}}$	E_{BA}/p_{nB}	E_{CA}/p_{nC}	....	E_{ZA}/p_{nZ}

This table forms the basis of country *A*'s decision to purchase different commodities from the various countries. From whom will country *A* decide to purchase the different commodities: Obviously from those countries from where the greatest quantities can be obtained per unit of *A*'s currency. Thus we can mark off the maximal elements in each row of the table. That fixes the commodities and countries from whom *A* will purchase them. If the boxed entries represent the row maxima the trade pattern indicated is *A*-2,*n* *B*-1, *C*-3,..., *Z*-2,3 i.e. *A* buys commodity 1 from *B*, commodity 2 from herself or from *Z*, commodity 3 from *C* and /or *Z*, etc.

We shall now prove a proposition of great importance: In the absence of trade impediments like transport costs and tariffs, the *ranking* of the elements of each row is identical for units of all (purchasing) currencies, provided the set of currency exchange rates E_{ij} $(i, j = A, \ldots, Z)$ is consistent. The proof is simple, it follows directly from the propositions established in sections (3) and (4) above. Consider the table of the quantities of different commodities that one unit of the currency of country *B* can purchase in the international markets.

B	*A*	*B*	*C*	....	*Z*
1	E_{AB}/p_{1A}	$1/p_{1B}$	E_{CB}/p_{1C}	....	E_{ZB}/p_{1Z}
2	E_{AB}/p_{2A}	$1/p_{2B}$	E_{CB}/p_{2C}	....	E_{ZB}/p_{2Z}
3	E_{AB}/p_{3A}	$1/p_{3B}$	E_{CB}/p_{3C}	....	E_{ZB}/p_{3Z}
⋮	⋮	⋮	⋮	⋮	
n	E_{AB}/p_{nA}	$1/p_{nB}$	E_{CB}/p_{nC}	....	E_{ZB}/p_{nZ}

We have to show that the ranking of the elements in rows of *B*'s table above is identical with that in *A*'s table.

Consider the row-1 column-*B* element in *A*'s table. By assumption it is the maximal element. Then we shall show that the row-1 column-*B* element in *B*'s table must be the maximal element in its row. By hypothesis we have, e.g.

$$\frac{E_{BA}}{p_{1B}} > \frac{1}{p_{1A}} \qquad \text{(in } A\text{'s table)}$$

which implies

$$\frac{1}{p_{1B}} > \frac{1}{E_{BA}p_{1A}}$$

so that

$$\frac{1}{p_{1B}} > \frac{E_{AB}}{p_{1A}} \qquad \text{(in } B\text{'s table)}$$

provided $E_{AB}E_{BA} = 1$. Consider a comparison with another element in the row. Thus by hypothesis,

$$\frac{E_{BA}}{p_{1B}} > \frac{E_{CA}}{p_{1C}} \qquad \text{(in } A\text{'s table)}$$

which implies

$$\frac{1}{p_{1B}} > \frac{E_{CA}}{E_{BA}p_{1C}}$$

so that

$$\frac{1}{p_{1B}} > \frac{E_{CA}E_{AB}}{p_{1C}}$$

only if $E_{AB}E_{BA} = 1$, and further that

$$\frac{1}{p_{1B}} > \frac{E_{CB}}{p_{1C}} \qquad \text{(in } B\text{'s table)}$$

provided $E_{CA}E_{AB} = E_{CB}$ and $E_{CB}E_{BC} = 1$. The same procedure applies not only to the rank of the maximal element, but any two elements of the table.

Thus in the absence of trade impediments the ranking of countries in terms of the quantities of commodities obtainable must be uniform from the point of view of *all* countries in the world economy. Apart from revealing important properties of multilateral trade, this result, as will be seen in the following chapter, saves a lot of tedious computation when determining the international trade equilibrium.

The ratio of the quantities of different commodities in each column of the gains-of -trade table shows the domestic rates of commodity exchange in each country. This ratio is usually referred to as the *domestic terms of trade*. The ratio of the *maximal* quantities of different commodities across the rows is the international ratio of commodity exchange or, in short, the *international terms of trade*. The international ratio of commodity exchange depends on the exchange rates of currencies. A depreciation of a country's currency implies that her exportables will in barter terms be offered on cheaper terms for her importables, and vice versa.

5. Two-Country Two-Commodity Trade

We have upto this point dealt with conditions determining the direction of the gains of trade of single commodities between two or more countries. Consider now the conditions for simultaneously determining the directions of trade of several commodities between several countries.

To proceed in gradual steps take the case of trade between two countries A and B in two commodities 1 and 2. The table of their money prices is,

$$\begin{matrix} p_{1A} & p_{1B} \\ p_{2A} & p_{2B} \end{matrix}$$

and the corresponding table of the natural exchange rates is,

$$\begin{matrix} E^1_{AB} & E^1_{BA} \\ E^2_{AB} & E^2_{BA} \end{matrix}$$

What conditions must be satisfied so that a pattern of trade, say A exports commodity 1 and B exports commodity 2, is gainful to the citizens of both countries? The result in section 2 shows that B gains by importing commodity 1 from A only if

$$E_{BA} p_{1A} < p_{1B}$$

or

$$E_{AB} > E^1_{AB} \tag{12}$$

Similarly A imports commodity 2 from country B only if

$$E_{AB} p_{2B} < p_{2A}$$

or

$$E_{AB} < E^2_{AB} \tag{13}$$

Combining (12) and (13), we obtain the necessary and sufficient condition for the trade assignment A-1 B-2 to be feasible,

$$E^1_{AB} < E_{AB} < E^2_{AB} \tag{14}$$

i.e. the natural exchange rate attributable to commodity 1 must be less than that attributable to commodity 2 and the actual exchange rate must lie between them. If the actual exchange rate were less than E^1_{AB} then A would find it profitable to import both commodities from country B. And if it were greater than E^2_{AB} then B would find it profitable to import both commodities from country A.

Should equality signs hold in (12), (13) or (14) then one or the other country will be indifferent as between purchasing one or the other commodity at home or abroad. Thus if

$$E^1_{AB} = E_{AB} < E^2_{AB} \tag{15}$$

the corresponding feasible assignment is A-1 B-1,2 because A gains by importing commodity 2 from B but B is indifferent to importing commodity 1 from A. On the other hand if

$$E^1_{AB} < E_{AB} = E^2_{AB} \tag{16}$$

the feasible trade assignment is A-1,2 B-2 whence B gains by importing commodity 1 from A but A is indifferent to importing commodity 2 from B. Finally if equalities hold at both ends so that,

$$E^1_{AB} = E_{AB} = E^2_{AB}$$

the situation may be characterised as a *null-trade* equilibrium where neither A nor B gains or loses by trading.

6. Comparative Advantage

Before we proceed to generalise the conditions determining the direction of the gains of trade in several commodities between several countries it is pertinent to pause and give some attention to the inequality (14) above. It states that for a trade assignment A-1 B-2 to be feasible it is necessary that,

$$E^1_{AB} < E^2_{AB} \tag{17}$$

This inequality is simply a statement of the bilateral comparative advantage condition. For $E^1_{AB} < E^2_{AB}$ means that,

$$\frac{p_{1A}}{p_{1B}} < \frac{p_{2A}}{p_{2B}}$$

which is the same as

$$\frac{p_{1A}}{p_{2A}} < \frac{p_{1B}}{p_{2B}} \tag{18}$$

in which form it was stated by Ricardo (1817). The statement of comparative advantage in terms of the natural exchange rates is thus identical with the statement in terms of the domestic autarky ratios of exchange in (18). There is, however, a third way in which the statement of comparative advantage can be made; it is the one which will prove useful in determining the feasible trade assignment in the general context of trade between several countries in several commodities. Write the inequality (17) as,

$$E^1_{AB} < \frac{1}{E^2_{BA}}$$

so that

$$E^1_{AB}E^2_{BA} < 1 \tag{19}$$

The statement (19) is of course the same as (17) and (18). But stated in this particular form it is amenable to the following interpretation: Country A is said to have a comparative advantage in the production of commodity 1 (and country B in the production of commodity 2) if the following sequence of international commodity arbitrage transactions is profitable—the purchase of commodity 1 in country A, its sale in country B and the simultaneous purchase of commodity 2 in country B and its sale in country A. To see what this means consider a trader having, say 1 rupee, the currency of country A. If he can purchase $(1/p_{1A})$ units of commodity 1 in country A and sell them for (p_{1B}/p_{1A}) dollars in country B, use the dollar proceeds to purchase $(1/p_{2B})(p_{1B}/p_{1A})$ units of commodity 2 in country B and sell them in country A for $(p_{2A}/p_{2B})(p_{1B}/p_{1A})$ rupees and make a profit, then A is said to have a comparative advantage in the production of commodity 1. Now, our trader makes a profit only if,

$$\left[\frac{p_{2A}}{p_{2B}}\right]\left[\frac{p_{1B}}{p_{1A}}\right] > 1 \text{ (rupee)}$$

or

$$E^2_{AB}E^1_{BA} > 1$$

leading to dollars in country B, use the dollar proceeds to purchase $(1/p_{2B})(p_{1B}$ $/p_{1B})$

$$E^1_{AB}E^2_{BA} < 1 \tag{20}$$

When this condition is satisfied by the autarky prices in the two countries we conclude that A-1 B-2 is a feasible trade assignment. Note, in addition, that when the *arbitrage sequence* in (20) holds and shows A-1 B-2 to be a feasible assignment it shows in the same breath that the opposite assignment, A-2 B-1, is infeasible. For the simple reason that $E^1_{AB}E^2_{BA} < 1$ implies $E^2_{AB}E^1_{BA} > 1$ meaning that the arbitrage sequence involving the purchases of commodity 1 in B and commodity 2 in A and their respective sales in countries A and B leads to losses.

The chief advantage of the arbitrage sequence conditions is that they give an indication of the feasible trade assignments directly from the autarky data, independently of the *actual* currency exchange rates. The benefit of the arbitrage sequence conditions will become apparent in the multicountry multicommodity case

7. Generalisation

To get a feel of the complications involved when several countries and commodities are considered, take the case of trade between three countries A, B, C in three commodities 1, 2 and 3. The autarkic money prices of the commodities in the three countries are,

$$\begin{matrix} p_{1A} & p_{1B} & p_{1C} \\ p_{2A} & p_{2B} & p_{2C} \\ p_{3A} & p_{3B} & p_{3C} \end{matrix}$$

and the natural exchange rates are,

$$\begin{matrix} E^1_{AB} & E^1_{BC} & E^1_{CA} \\ E^2_{AB} & E^2_{BC} & E^2_{CA} \\ E^3_{AB} & E^3_{BC} & E^3_{CA} \end{matrix}$$

Given any trade pattern, we can specify the necessary and sufficient conditions for that trade pattern to be a feasible trade pattern. Thus for a trade pattern say A-1,2 B-2 C-3 to be a feasible one, the following conditions must be satisfied;

$$\begin{array}{rclcrcll} E_{CA}p_{1A} & < & p_{1C} & & E_{CA} & < & E^1_{CA} & \text{(i)} \\ E_{BA}p_{1A} & < & p_{1B} & & E^1_{AB} & < & E_{AB} & \text{(ii)} \\ E_{AB}p_{2B} & = & p_{2A} & & E^2_{AB} & = & E_{AB} & \text{(iii)} \\ p_{2B} & < & E_{BC}p_{2C} & \text{implies} & E^2_{BC} & < & E_{BC} & \text{(21) (iv)} \\ E_{CA}p_{2A} & < & p_{2C} & & E_{CA} & < & E^2_{CA} & \text{(v)} \\ p_{3C} & < & E_{CA}p_{3A} & & E^3_{CA} & < & E_{CA} & \text{(vi)} \\ E_{BC}p_{3C} & < & p_{3B} & & E_{BC} & < & E^3_{BC} & \text{(vii)} \end{array}$$

These seven conditions can be arranged into composite conditions that must be satisfied by the actual currency exchange rates. Thus (21) (ii) and (21) (iii) can be combined into

$$E^1_{AB} < E^2_{AB} = E_{AB} \tag{22a}$$

Similarly 21 (iv) and 21(vii) give

$$E^2_{BC} < E_{BC} < E^3_{BC} \tag{22b}$$

And 21 (i), 21 (v) and 21 (vi) can be combined into,

$$E^3_{CA} < E_{CA} < E^2_{CA} \gtrless E^1_{CA} \tag{22c}$$

In the last inequality it makes no difference whether the natural exchange rate E^1_{CA} is greater than, equal to, or less than E^2_{CA}. All that is required is that *both* should be greater than the actual exchange rate E_{CA}.

Finally we must add to the conditions above, the two-point and three-point neutrality conditions to be satisfied by the actual exchange rates.

Conditions 22 (a), (b) and (c) are both necessary and sufficient for the pattern of the gains from trade to be consistent with the postulated pattern of trade A-1,2 B-2 C-3. The arbitrage sequences for the natural exchange rates are derived by considering the product of the actual exchange rates appearing in 22 (a), (b) and (c). Thus we know by the three-point neutrality condition that

$$E_{AB}E_{BC}E_{CA} = 1$$

Then multiplying the inequalities in (22) we observe that the products of the natural exchange rates appearing on the left hand sides of the actual exchange rates must satisfy

$$\begin{aligned} E^1_{AB}E^2_{BC}E^3_{CA} &< 1 \\ E^2_{AB}E^2_{BC}E^3_{CA} &< 1 \end{aligned} \tag{23}$$

and those on the right hand sides must satisfy

$$\begin{aligned} E^2_{AB}E^3_{BC}E^2_{CA} &> 1 \\ E^2_{AB}E^3_{BC}E^1_{CA} &> 1 \end{aligned} \tag{24}$$

The inequalities in (23) require that two arbitrage sequences should be profitable, (a) purchase of 1 in A, sale in B, purchase of 2 in B sale in C, purchase of 3 in C, sale in A and (b) purchase of 2 in A, sale in B, purchase of 2 in B, sale in C, purchase of 3 in C, sale in A. Obviously in the second sequence the arbitrage of commodity 2 in A and B will yield no profit because $E^2_{AB} = E_{AB}$, the profit well be made in the sale of 2 in C and the sale of 3 in A. The inequalities in (24) specify the unprofitable, indeed the loss-making, arbitrage sequences.

Another way of bringing out the correspondence between the feasible trade assignments and the profitable arbitrage sequences is as follows. The trade assignment A-1,2 B-2 C-3 can be conceived as being made up of two *elemental* *trade assignments*, A-1 B-2 C-3 and A-2 B-2 C-3. The overall trade assignment can be considered feasible only if all the elemental trade assignments of which it is composed are feasible. This is precisely what the arbitrage sequences in (23) state. Observe that superscripts of the natural exchange rates in (23) indicate the commodity, the first subscript indicates the country in which the commodity is purchased and the second subscript the country in which the commodity is sold in the course of arbitrage. Thus the first arbitrage sequence establishes the feasibility of the first elemental trade trade assignment A-1 B-2 C-3 where commodity 1 is purchased in A (sold in B), commodity 2 is purchased in B (sold in C) and commodity 3 is purchased in C (sold in A). Similarly the second arbitrage sequence establishes the feasibility of the second elemental trade assignment A-2 B-2 C-3.

The same analysis can be generalised to any number of countries and commodities and any trade pattern hypothesised.

8. McKenzie-Jones Conditions

The arbitrage conditions that we have used to determine the feasible trade assignments are a generalisation of the "product-of-prices" conditions due originally to McKenzie (1954) and Jones (1961). To see their relationship go back for a moment to the bilateral comparative advantage conditions (17) and write it as,

$$p_{1A}p_{2B} < p_{1B}p_{2A} \tag{25}$$

This is the McKenzie-Jones conditions as applied to the two-country two commodity case. It states that the assignment A-1 B-2 is *efficient* if the product of the prices, p_{1A} and p_{2B}, is less than the product of the prices p_{1B} and p_{2A}

In general with Z countries and Z commodities an assignment say A-1, B-2... Z-z is an efficient assignment only if the product of prices $p_{1A}, p_{2B}, \ldots p_{zZ}$ is less than the product of the prices for any other assignments which allocate a distinct commodity to each country such as A-2, B-1... Z-z, A-z B-1,..., Z-2, etc.

Two features of the McKenzie-Jones conditions limit their generality. First, they apply only to cases in which the number of countries are greater than, or equal to the number of commodities. Second, the conditions are designed to identify an *efficient* assignment wherein each country is specialised in the production of one, and only one, tradeable commodity.

In contrast our criteria, based on the products of the natural exchange rates, are designed to search feasible assignments and apply irrespective of the numbers of countries and commodities involved. Besides, although they are mathematically identical with the McKenzie-Jones conditions for feasible assignments, they are amenable to a lucid economic interpretation in terms of international commodity arbitrage which "the product of prices" criteria are not.

9. Importance of the Arbitrage Sequence Conditions

The importance of the arbitrage sequence conditions arises from their being the only clue as regards the feasible (i.e. gainful) trade patterns. The equilibrium trade pattern, if such exists, must either be a feasible trade pattern or some combination of the feasible trade patterns.

When we start the analysis the autarky equilibria of the different countries are all we know. But we are confronted with a frightfully large number of possible trade assignments. For instance with 2 countries and 2 commodities there are 7 possible trade patterns;

(i) *A*-1, *B*-2	(ii) *A*-1, *B*-1,2
(iii) *A*-1,2 *B*-1	(iv) *A*-1,2 *B*-2
(v) *A*-2, *B*-1	(vi) *A*-2, *B*-1,2
(vii) *A*-1,2 *B*-1,2	

[Interestingly, classical theory confined its attention to only the 2 patterns of complete specialisation *A*-1, *B*-2 and *A*-2, *B*-1 and neoclassical theory to 1 pattern of incomplete specialisation *A*-12, *B*-12]. With 2 countries and 3 commodities there will be 24 trade possibilities, ignoring the *null-trade assignment* *A*-1,2,3 *B*-1,2,3. The number rises rapidly as the dimensionality increases. With 4 countries and 5 commodities there will be 6,93,600 and in a 4-country, 7-commodity situation 167578320 possibilities. In general with z countries and n commodities the total number of trade assignments in which every commodity is allocated to at least one country and each country produces at least one commodity is given by the following recursive formula,

$$\begin{aligned} T_{n,z} = {} & (nC_1 + nC_2 + \dots + nC_n)^z \\ & - (nC_1 T_{1,z} + nC_2 T_{2,z} + \dots + nC_{n-1} T_{n-1,z}) - 1 \end{aligned} \tag{26}$$

where $T_{1z} = 1$ and $T_{2,z}, \dots, T_{n-1,z}$ are obtained from (26)

It is obviously impossible to try out every single trade pattern to determine which one is the equilibrium pattern. It is at this stage that the arbitrage sequence conditions come to the rescue by separating the feasible trade assignments from infeasible ones. All the infeasible assignments and their myriad combinations can be eliminated. This reduces considerably the computational effort required to determine the equilibrium trade assignment.

3

THE PROBLEMATIC

However this was anything like a regular bee: in fact, it was an elephant —as Alice soon found out, though the idea took her breath away at first. "And what enormous flowers they must be!" was her next idea.

1. 'Circular' Dependence in the System

It will be useful at this stage to pause, take stock, and spell out the problems that lie ahead. Chapter I has shown how the exchange rates of currencies are determined once the pattern of the intercountry import bills are know. It was then only natural to ask in Chapter II how the intercountry import bills were themselves determined. For in general, the import bill that one country owes another is composed of several bills on account of the several commodities imported. Therefore the aggregate import bill can be determined only after its constituent parts have been determined. Accordingly, in Chapter II, we were led to inquire into (1) which countries import which commodities and (2) from which countries. It is understood of course that (1) and (2) cannot be resolved independently of one another. For the citizens of a country do not first decide *whether* to import and then decide from *whom* to import the different commodities. Both decisions are arrived at simultaneously on the basis of a comparison between the costs of importing the commodities from each of the several countries and the cost of producing them domestically.

But the costs of importing commodities from the various countries cannot be ascertained until the exchange rates of currencies are known. Because both the direction as well as the magnitude of the gains from trade depends upon the exchange rates. Now the pattern of the intercountry gains from trade determines, as we have seen in Chapter II, the pattern of world trade and , therefore, the pattern of intercountry import bills. Observe that we come around in a complete circle—to know the exchange rates, the intercountry import bills must be known, to know the intercountry import bills the world trade pattern must be known, to know the world trade pattern, the pattern of the intercountry gains from trade must be known but, to know the pattern of the intercountry gains from trade the exchange rates must be known.

The task of trade theory is to resolve this 'circular' dependence amongst the unknowns of the system by constructing a wider system that can simultaneously determine the commodity-wise intercountry import bills and the exchange rates of currencies[1].

These circular dependencies are not confined to trade transactions alone—

they arise in other types of international transactions as well. Consider the example of intercountry tourist demand. The lower the value of a country's currency as compared to the foreign currencies the greater will be the tourist attraction for that country. On the other hand, the value of the country's currency itself depends on her tourist earnings. As another instance consider foreign direct investment. The more depreciated a country's currency the lower is the cost of non-tradeable capital items (land, construction), capital services (consultancy, legal and accounting fees, advertising) and labour and hence the greater is the profitability on foreign direct investment. On the other hand the value of the country's currency is itself determined by the size of the foreign direct investment flows that she attracts including factors attendant to it, viz. exports and imports by the investors, their lending and borrowing from abroad, their transfer-pricing practices and the repatriation of their earnings. Neither the volume of foreign direct investment nor the exchange rate can be ascertained without knowing the other.

2. Trade as an Aspect of International Equilibrium

It is evident that the object of determining the commodity composition of intercountry trade (as well as other transactions) cannot be achieved without considering the internal structures of the economic systems of the countries, that is to say, the endowments of primary resources, the technique of production and the tastes and preference of their citizens. For it is this data which determines the relative prices, the income levels and the income distribution in the countries.

The relative price structures of the countries determine the comparative advantages/disadvantages in the production of the different commodities in the countries which in turn determine which commodities will be imported and which produced and exported. On this depends, which industries will close down in the different countries and which will be expanded when trade is allowed to take place.

The tastes and preferences and the income levels determine the volumes of demands for the imported commodities and these along with the prices of commodities in the exporting countries determine the values of the import bills. At the same time the resource endowments and techniques of production determine the volumes and the values of the supplies of exported commodities. International equilibrium is achieved at the point where the *world* demands and the *world* supplies of commodities are equal in volumes and values. It is clear from the above that the values of the intercountry import bills (and export earnings) of individual countries, which we require for purposes of determining the exchange rates, can be determined only in the course of determining and matching the world demands and supplies of commodities. Our task is therefore, to find which industries will be operated in which countries, what their scales of output will be and how the domestic resources will be allocated between the industries operated to support the scales of outputs that are determined by the world demands for the commodities.

The upshot of all this is that the study of international trade becomes coextensive with the study of the "international allocation of economic activities" under specified assumptions such as the international immobility of labour and /or capital which, however, can be relaxed to relevant extents.[2]

Notes

1. It must be clarified that the term 'circular dependence' does not correspond rigorously with the mathematics of the situation. There is no such thing as circular dependence in mathematics. The term has been used to convey the idea that the system of equations that we h. ve to deal with is one that contains simultaneous *non-linear* equations. Thus there is a mapping f from the intercountry trade bills M_{ij}, to the exchange rates E_{ij}, $E_{ij} = f(M_{ij})$. This mapping describes the clearing of foreign exchange markets and ensures balance of payments of all the countries. At the same time there is a mapping from the exchange rates E_{ij} to the import bills M_{ij}, $M_{ij} = g(E_{ij})$. This mapping describes the gains of trade or, in other words, it is a mapping that admits only such M_{ij} as will satisfy the necessary and sufficient conditions for gains from trade for the i-j trade assignment. Thus $E_{ij} = f(M_{ij})$ and $M_{ij} = g(E_{ij})$ implies $E_{ij} = f[g(E_{ij})]$ or $E_{ij} = h(E_{ij})$ where $h = f[g]$ determines a fixed point for the mapping h from the space containing E_{ij} into itself.
2. The term "international allocation of economic activities" has been borrowed from a collection of the same title published by the Nobel Symposium (1977).

4

AUTARKY

Of course the first thing to do
was to make a grand survey of the
country she was going to go through.

1. Advantage of Studying the Classical Case

In this and the following two chapters we shall consider extremely simple societies which produce consumable commodities exclusively by means of labour. There is no capital and no profits. Natural resources claim no rents. (And there is no claim to realism either).

Although obviously simplistic, the study of this case offers two advantages. Firstly, it can serve as a model to discuss the propositions of the classical economists who usually excluded capital from their theories of international vales, in rather conspicuous contrast to their theories of domestic values in which capital occupied the central position. This they obviously did for the sake of simplicity, a procedure that we shall repeat[(1)]. Secondly, the simple case can serve as a vehicle for introducing a variety of concepts, techniques and results at an elementary level of the inquiry. This should facilitate the exposition at the more advanced level [Part 3. Chapters VII, VIII, IX]

Thus the discussion in the chapters of this part is intended to introduce the peculiarities of the *method* of international trade theory rather than to establish substantially realistic results.

2. Assumptions

Consider an economic system which produces several commodities by means of homogeneous labour. The money wage rate paid to labour will be assumed to be determined by contract and, consequently, changes in the money wage rates take place only by changes in the terms of contract. We shall suppose that each commodity is produced by a separate industry—there is no joint production. Since labour is homogeneous, and employs itself, it is only natural to suppose a state of continuous full employment[(2)]

And since there is no capital, there is no capital accumulation; all income is consumed[(3)].

It will be assumed that all industries have technologies that are linear homogeneous of degree one, i.e. constant-returns-to-scale, with the property that proportionate changes in the use of *all* inputs (in this case only labour) lead to proportionate results in the outputs produced. The production activities will be represented by input-output equations of the type used in activity analysis e.g. the Leontief-Sraffa equations[4]. The demand functions for all commodities are assumed to be homogeneous of degree zero in the prices and income, i.e., proportionate changes in all the prices and income have no effect on the quantities of commodities demanded. To represent the demand functions we shall particularly use the Engels' demand equations or the Stone-Geary linear expenditure systems.

3. Equilibrium with Engels' Demand Equations

Let $l_1, l_2, l_3,...$ be the quantities of labour required to produce unit outputs of the commodities $1, 2,..., n$ and let w be the uniform money wage rate paid to workers in all the industries[5]. Then the prices of the commodities, which must be equal to their unit costs of production, are

$$p_i = wl_i \qquad i = 1, 2, ... n \tag{1}$$

Suppose L to be the total labour in the economic system. Then the national income of the economy is simply equal to the wage bill, wL. Let $\alpha_1, \alpha_2, ..., \alpha_n$ be the proportions of the national income that the workers devote to the purchase of commodities $1,2,..., n$, respectively. These proportions, called the *Engels' coefficients* or *propensities to consume*, must be positive, less than unity with their sums adding to unity;

$$0 < \alpha_i < 1$$

$$\Sigma\alpha_i = 1$$

Accordingly the demand equations for the commodities may be written as,

$$p_i X_{di} = \alpha_i Y \qquad i = 1, 2, ..., n \tag{2}$$

where X_{di} stands for the quantity demanded of commodity i and Y, the national income is equal to wL, the wage bill. Substituting for the prices from equation (1) in equation (2), we get

$$X_{di} = \frac{\alpha_i Y}{p_i} = \frac{\alpha_i L}{l_i} \qquad i = 1, 2, ..., n \tag{3}$$

Equilibrium requires that the quantities of commodities produced and supplied by the different industries, X_{si}, must be such as to satisfy the demands for them,

$$X_{si} = X_{di} \qquad i = 1, 2,..., n \tag{4}$$

To produce the quantities demanded X_{di} requires that $L_i = l_i X_{di}$ units of labour should be employed in each industry i. And that means the production or supply equations must be,

$$wL_i = p_i X_{si} \qquad i = 1, 2, ..., n \tag{5}$$

But since $p_i X_{di} = \alpha_i wL$ by equation (2) we may write

$$wL_i = \alpha_i wL \qquad i = 1, 2, ..., n$$

which gives

$$L_i = \alpha_i L \qquad i = 1, 2, ..., n \tag{6}$$

What equation (6) means is that for the supplies of commodities to be equal to the demands for them, the labour employed in the industries which produce the commodities should be equal to the propensities to consume those commodities multiplied by the total labour in the system. Equation (6), therefore, spells out the rule for determining that allocation of the total labour between industries which equates the supplies of commodities to the demands. It is only when a quantity of labour $L_i = \alpha_i L$ is employed in the respective industries that the quantities supplied by them L_i/l_i are equal to their quantities demanded $\alpha_i L/l_i$.

We have, in the above, explained the economic equilibrium as if it took place by adjustment of quantities supplied to the quantities demanded while prices remained at the levels set by their costs of production. It must be mentioned, however, that the adjustment of supplies to demands can take place only if the price signals are appropriate. Thus it is necessary to note that if a disequilibrium of demands and supplies comes about, the demand prices of the commodities will diverge from their supply prices; rising above the supply prices in the case of others which are in excess supply. The industries whose products are in excess demand earn revenues greater than their costs of production and those whose products are in excess supply earn revenues lower than the costs borne by them. The former will of course use their additional revenues to expand outputs by hiring additional labour and the latter, who suffer losses, retrench their labour and reduce their outputs. This process continues until the retrenched labour from the excess supply industries is allocated to the expanding excess demand industries and the demand prices are equal to the supply prices.

That both full employment and market clearing are fulfilled at the same point of equilibrium can be established by Walras's Law. Denote the equilibrium allocation of labour by $L_i^* = \alpha_i L$, and let L_i be any other (disequilibrium) allocation of labour. Then the sum of the values of the excess demands and excess supplies for all the commodities is equal to zero;

$$\sum_i p_i(X_{di} - X_{si}) = \sum_i (p_{di} - p_i)X_{si} = 0 \tag{7}$$

Because both are equal respectively to,

$$\frac{\sum_i p_i(\alpha_i L - L_i)}{l_i} = \frac{\sum_i (\alpha_i wL - wL_i)}{l_i}$$

i.e. $$L\sum_i \alpha_i - \sum L_i = \left[\frac{w}{l_i}\right](L\sum_i \alpha_i - \sum_i L_i)$$

or $$L - L = \left[\frac{w}{l_i}\right](L - L) = 0$$

We have thus shown that every economic system which is described by the following data; the labour coefficients of production l_i, the Engels'coefficients of consumption α_i, the labour endowment L and the money wage rate w has a unique equilibrium of prices and outputs. If the data are positive and the Engels'coefficients add up to unity the equilibrium is positive. The state of the economic system,

$$wL_i^* = p_i X_i \quad i = 1, 2, ..., n \tag{8}$$

where

$$L_i^* = \alpha_i L \quad i = 1, 2, ..., n$$

will accordingly be called the *equilibrium state* of the economic system.

Consider an example. Suppose an economic system with an endowment of 15 units of labour, a money wage rate of \$2 per unit, labour coefficients of production of three commodities equal to 0.5, 0.25 and 0.125 and the propensities to consume equal to 0.2, 0.3, 0.5. Then from equation (1) we know that the equilibrium prices are \$1, \$0.5 and \$0.25 respectively. The equilibrium allocation of labour obtained by the formula in (6) is 3, 4.5 and 7.5 units of labour in the three industries respectively. And the outputs of the three commodities are obtained by dividing the labour allocated by the labour coefficients; these are 6, 18 and 60 units respectively. These outputs are exactly equal to the quantities demanded of the three commodities.

4. The Method of "Scale Multipliers"

In the above the equilibrium state of the economy has been found *ab initio* from the price and demand equations (1) and (2). The equilibrium state can also be determined by using the procedure of "scale multipliers." This procedure is used when the equilibrium state is to be determined by transforming a given disequilibrium state. To illustrate this procedure consider the economic system of the previous example. Suppose that the economic system stands in the following state,

$$\begin{aligned} 1.\ 5w &= 10p_1 \\ 2.\ 5w &= 20p_2 \\ 3.\ 5w &= 40p_3 \end{aligned} \tag{8}$$

That the state of system (8') is one of disequilibrium can be seen by calculating the quantities demanded. These are 6, 18 and 60 units of commodities 1, 2 and 3 respectively. The quantities supplied, on the other hand, are 10, 20 and 40 units. The same can be observed by comparing the demand prices of the commodities to their supply prices. For the quantities supplied as in (8'), the demand prices are \$0.6, \$0.45, and \$0.375 whereas the supply prices are \$1, \$0.5 and \$0.25 respectively. Clearly, there is excess supply of commodities 1 and 2 and an excess demand for commodity 3.

To bring about the equilibrium, the outputs of industries 1 and 2 will decrease and that of industry 3 will increase. This will be brought about by a change in the pattern of labour allocation. The equilibrium state, as we already know, is

$$\begin{aligned} &1. \quad 3w = 6\,p_1 \\ &2. \quad 4.5w = 18\,p_2 \\ &3. \quad 7.5w = 60\,p_3 \end{aligned} \tag{8''}$$

Now if we compare the states (8′) and (8″) we see that (8″) can be derived from (8′) by multiplying the first equation of (8′) by 0.6, the second equation by 0.9 and the third by 1.5. There are the scalar multipliers which transform the system (8′) to its equilibrium state in (8″). The general formula for these multipliers is

$$\lambda_i = \frac{L_i^*}{L_i} = \frac{\alpha_i L}{L_i} \quad i = 1, 2, \ldots, n \tag{9}$$

where L_i is the labour employed in the ith industry when the economic system is in a state of disequilibrium and L_i^* is the equilibrium allocation. It is clear that the multipliers for the excess supply industries will be lower than 1 and those for the excess demand industries will be greater than 1 because the former set of industries must shrink and the latter should expand in the course of attaining equilibrium.

The method of scale multipliers just discussed may seem somewhat artificial in the classical pure-labour case that is presently under discussion but it will prove invaluable in later parts when capital is introduced along with labour.

5. Equilibrium with a Linear-Expenditure System

Although the demand functions that have been considered so far are of the simple Engels' type, where the expenditure on each commodity is a constant fraction of income, the analysis can be suitably modified to accommodate more general and realistic demand patterns. Let us consider the properties of the equilibrium with a linear expenditure system which, of all the demand functions empirically estimated, has the greatest validity.

In a linear expenditure system the demand equations for the commodities are specified as follows:

$$p_i X_i = p_i \overline{X}_i + \alpha_i (Y - \Sigma\, p_i \overline{X}_i) \quad i = 1, 2, \ldots, n \tag{10}$$

where to use Stone's (1954) terminology $\overline{X}_i$ is the "committed" quantity of the ith commodity, $p_i \overline{X}_i$ is the "committed" expenditure of the consumers, Y is the total consumer income which in the present case is simply the wage bill wL, and $Y - \Sigma p_i \overline{X}_i$ is the "supernumerary" income. The "committed" expenditure may more simply be referred to as the 'constant' or 'subsistence' expenditure and the "supernumerary" income as the 'variable' or 'luxury' income.

The quantities demanded of the different commodities derived from (10) are,

$$X_{di} = \overline{X}_i + \frac{\alpha_i(wL - \Sigma p_i \overline{X}_i)}{p_i} \quad i = 1, 2, ..., n \tag{11}$$

which since $p_i = wl_i$ reduce to,

$$X_{di} = \overline{X}_i + \frac{\alpha_i(L - \Sigma l_i \overline{X}_i)}{l_i}$$

Equilibrium requires that the quantities supplied should be equal to the quantities demanded, i.e.

$$X_{si} = \frac{L_i}{l_i} = \overline{X}_i + \frac{\alpha_i(L - \Sigma l_i \overline{X}_i)}{l_i}$$

which gives the equilibrium labour allocation,

$$L_i^* = l_i \overline{X}_i + \alpha_i(L - \Sigma l_i \overline{X}_i) \quad i = 1, 2, ..., n \tag{12}$$

In this case too it is possible to state the general conclusion that a unique positive equilibrium exists provided that the economic system conforms to the following conditions(7),

$$\begin{aligned} \Sigma\alpha_i &= 1 \\ \overline{X}_i &> 0 \\ L &> \Sigma\, l_i \overline{X}_i \end{aligned} \tag{13}$$

The last inequality requires the labour endowment to exceed the labour requirements of the 'subsistence' quantities of the commodities. If it falls short, the economic system is non-variable. If it is equal, then we are in the classical-von-Neumann subsistence world where consumers do not have the 'luxury' of choosing between commodities in accordance with their own tastes and preferences (i.e. α_i must be equal to zero). Such a world we shall not consider.

The scale multipliers that bring about equilibrium for the economic system that starts at an arbitrary state of disequilibrium are found from the formula,

$$\lambda_i = \frac{\overline{X}_i}{X_i} + \frac{\alpha_i(L - \Sigma l_i \overline{X}_i)}{L_i}$$

where L_i and X_i are respectively the labour employed in industry i and the output of industry i in the state of disequilibrium.

Finally, observe that Walras's law holds, for any (arbitrary) state of the economic system because,

$$\begin{aligned} \Sigma p_i (X_{di} - X_{si}) &= \Sigma[p_i \overline{X}_i + \alpha_i (wL - \Sigma p_i \overline{X}_i) - wL_i] \\ &= \Sigma p_i \overline{X}_i + wL\Sigma\, \alpha_i - (\Sigma p_i \overline{X}_i)\Sigma\alpha_i - w\Sigma L_i \\ &= 0 \quad \text{because } \Sigma\alpha_i = 1 \end{aligned}$$

To get a numerical idea let us extend the numerical example that we used in the earlier section. Suppose the (marginal) propensities to consume are 0.2, 0.3, 0.5 as before and let the 'subsistence' quantities $\overline{X}_1, \overline{X}_2, \overline{X}_3$ be 2, 6 and 20 units of the three commodities respectively . Then the quantities demanded are

$$X_{d1} = 2 + \frac{0.2 \times 20}{1} = 6 \text{ units}$$

$$X_{d2} = 6 + \frac{0.3 \times 20}{0.5} = 18 \text{ units}$$

$$X_{d3} = 20 + \frac{0.5 \times 20}{0.125} = 60 \text{ units}$$

The labour required to produce these commodities in their industries are 3, 4.5 and 7.5 units respectively which gives the equilibrium state of the economic system,

$$\begin{aligned} 3w &= 6p_1 \\ 4.5w &= 18p_2 \\ 7.5w &= 60p_3 \end{aligned} \tag{14}$$

The equilibrium state obtained with the linear expenditure system in (14) is identical with that obtained from the Engel's demand equations in (8″). This is purely coincidental. It happens only in the special case where $l_i X_i = \alpha_i \Sigma l_i X_i$ $\forall\, i$, which is true of our numerical example.

6. Behaviour of the System under Random Influences

The equilibrium of an economic system is continually disturbed by changes in the parameters (viz, the technical coefficients, the propensities to consume and the 'factor' endowments) on which the equilibrium is based. Broadly speaking such disturbances are of two types. Firstly, there are the systemic disturbances such as improvements in the techniques of production, the introduction of new products or new designs for existing products, changes in tastes and preferences, changes in population, discoveries of new natural resources, etc. Since all such changes are changes in the parameters of the economic system, their effect on the evolution of the economic system can be studied by simply extending the methods that have been used to determine the equilibrium. The sources as well as the effects of these progressive forces are a proper subject of economic inquiry. Secondly, there are the (more or less) random disturbances like earthquakes, floods, cyclones, weather changes, etc. whose effects are usually temporary. [The distinction is admittedly broad. For how do we classify wars, industrial unrest, changes in political relations, floods caused by irrigation policies, etc? But it will serve the purpose of this section]. Forces of the second type can, however, be studied without severe modifications of the deterministic methods illustrated above. All we need to do is replace the specification of the production equation for the output X_i of the industry i by

$$X_i = \hat{X}_i + U_i \tag{15}$$

where $\hat{X}_i = E(X_i)$ is the 'normal' or expected output corespondent to the labour used, L_i in industry i, and U_i are symmetrically distributed random variables with means equal to zero. The labour coefficients of production can therefore be written so as to accommodate the random influences.

$$l_i = \frac{L_i}{X_i} = \frac{L_i}{\hat{X}_i + U_i}$$

and can themselves be decomposed into average and random terms,

$$l_i = \hat{l}_i + u_i \tag{16}$$

where u_i is the random term,

$$u_i = -\frac{L_i U_i}{X_i \hat{X}_i} \tag{17}$$

Now consider a disturbance in 'weather' conditions. Then the industries whose outputs have risen above their equilibrium levels will find that their labour coefficients have fallen below their normal levels and the others, whose outputs have fallen below their equilibrium levels, will find that their labour coefficients have risen above their normal levels. As a consequence, *both* the supply prices and the demand prices of the former set of commodities will fall below, and those of the latter set of commodities will rise above, their normal levels. If the market expects that normal weather conditions will prevail in the following period, that is to say, it expects that the outputs of the former set of commodities will fall down to their normal levels and those of the latter will rise to their normal levels, then the forward prices of the former set of commodities will be greater than their spot (demand) prices while the forward prices of the latter set of commodities will stand lower than their spot prices. Accordingly the commodities whose outputs have risen above will stand at premium in the forward market and those whose outputs have fallen will stand at discount.

Consider now how the behaviour of the spot prices changes with changes in the demand equations. If the demand equations are of Engels' type, the supply prices and demand prices are always *equal* to one another however severe may be the disequilibrium in outputs, provided that the allocation of labour between industries is proportional to the Engel's consumption coefficients. To see how, take an example of an economy which pays a wage rate of ¥1, and is endowed with 15 units of labour, whose labour coefficients of production are 1 and 2 and whose Engels' coefficients of consumption ar 1/3 and 2/3 for the two commodities respectively. Then the normal equilibrium state of the economy is,

$$\begin{aligned} 5w &= 5p_1 \\ 10w &= 5p_2 \end{aligned} \tag{18}$$

with

$$p_1 = ¥1$$
$$p_2 = ¥2$$

Now suppose that, due to entirely random conditions, the outputs of the commodities 'harvested' change to 10 units of commodity 1 and 2 units of commodity 2 respectively. Since this amounts to changes in the realized labour coefficients of production from their normal level (1 and 2 respectively) to 0.5 and 5, the supply prices of the commodities change to ¥0.5 and ¥5 respectively. Note, however, that the demand prices of the two commodities also change to the same levels; because,

$$p_{1d} = \frac{\alpha_1 Y}{X_1} = \frac{1/3 \times (¥15)}{10} = ¥0.5$$

$$p_{2d} = \frac{\alpha_2 Y}{X_2} = \frac{2/3 \times (¥15)}{2} = ¥5$$

Consider on the other hand the behaviour of the spot prices when a linear expenditure system is applicable. Let $\bar{X}_1 = 2$, $\bar{X}_2 = 1$ and the marginal propensities to consume be 1/3 and 2/3, which are equal to the Engels' coefficients postulated above. Then we know from the argument of the previous section that the normal equilibrium state of the system is described by (18). In the disequilibrium, however, the demand prices of commodities with the linear expenditure system are,

$$p_{1d} = \frac{\alpha_1(Y - p_2\bar{X}_2)}{(X_1 - \bar{X}_1) + \alpha_1\bar{X}_1} = \frac{1/3(15-5)}{8 + 1/3 \times 2} = \frac{1}{2.6} = ¥0.3846$$

$$p_{2d} = \frac{\alpha_2(Y - p_2\bar{X}_2)}{(X_2 - \bar{X}_2) + \alpha_2\bar{X}_2} = \frac{2/3(15-1)}{1 + 2/3 \times 1} = ¥5.6$$

Observe that the spot (demand) prices are not equal to the supply prices of he commodities as was the case with the Engels" demand equations. The demand price of commodity 1 falls below the supply price and that of commodity 2 rises above the supply price. Commodity 1 as we know is in abnormal excess supply whereas commodity 2 is in abnormal excess demand.

It is only obvious to suppose that the restoration of equilibrium in an economic system whose demand behaviour is described by a linear expenditure system will be faster than one whose demand behaviour conforms to Engels' functions. Because any disequilibrium shows itself in a greater spread between demand and supply prices on the one hand and between the spot and forward prices on the other, creating losses here and profits there and quickening the process of restoring the equilibrium. Indeed, if Engels' demand equations are applicable, every disequilibrium arising from the side of outputs is simply an 'abnormal equilibrium.'

The same cannot, however, be said when the disequilibrium arises due to random influences from the sides of inputs (in this case labour), i.e. those, like temporary migrations, natural calamities, human diseases, or miscalculations in the demand for products, which disturb the equilibrium distribution of the labour force between the different industries. In that case, the outputs of industries who suffer sporadic immigration rises above their normal equilibrium values and those of the others fall below. The supply prices remain at their normal values but the demand prices of commodities whose outputs have risen fall below their equilibrium prices and those of the others rise. In the forward markets the former commodities stand at a premium and the latter at a discount.

Consider for instance the effects if the system were disarranged and stood in the following state:

$$\begin{aligned} &1.\quad 7.5w = 7.5p_1 \\ &2.\quad 2.5w = 1.25p_2 \end{aligned} \tag{19}$$

The supply prices of the commodities remain at p_1 = ¥1, p_2 =¥2. But the demand prices diverge to p_1 = ¥0.6666 and p_2 = ¥1.3333. The forward price of commodity 1 will stand above the spot price and that of commodity 2 will stand below if the dislocation of labour amongst the industries is expected to be corrected in the following period.

Notes

1. The notable exception is Ricardo (1817) . But the reference is somewhat oblique. Ricardo explicit stated that international trade will increase the rate of profit in a country if the subsistence real wage in that country included imported commodities. Since Ricardo usually considers the subsistence wage to be advanced to workers, the imported commodities may be considered to be intermediate capital goods. One can then infer that Ricardo recognised trade in capital goods. We shall, however, see that Ricardo's conclusion holds good even if the wage is supposed to be paid *post-factum* and does not form part of the capital stock.
2. The assumption of full employment has been relaxed in the Part 3, when capital is introduced, besides labour in the production of commodities.
3. This is in contrast with the standard Heckscher-Ohlin-Samuelson theory of international trade. A curious feature of the standard theory is that capital is assumed to exist but not capital accumulation.This leaves the question of how the capital came into being unanswered—capital is treated like "manna from heaven",
4. See Koopmans (1951), Leontief(1941), Sraffa (1960). A lucid treatment of activity analysis models is found in Dorfman, Samuelson and Solow (1958). These references are, however, more appropriate to the model explored in Part 3, which includes capital goods.
5. That the money wage rate is paid for some standard work unit in terms of magnitude and quality is implicit but obvious.
6. Demand patterns of the partial equilibrium type, $X = a+bP_1 + cP_2 + ... + yY + ...$ where X is the quantity demanded, P_i are the prices and Y is consumer income, have been widely estimated in case of a variety of agricultural and industrial products. But their form is totally inapplicable for the purposes of general equilibrium theory. Specifically note that, in general, such demand patterns do not have the property $\Sigma p_i X_i = Y$ which is vital to general equilibrium models.

7. This condition for a positive solution is a necessary condition. It is not sufficient. A more general condition that is both necessary and sufficient is,

$$L > \Sigma l_i X_i - \frac{l_i X_i}{a_i} \qquad \forall i$$

8. The assumption of a subsistence wage is starkly unrealistic outside a 'slave economy'. In a modern economy different categories of workers are paid different wage rates. Besides, workers do consume 'luxury' goods and also save and earn a share of the surplus. Thus the real wage rates are *unknowns* whose values must be determined by the system; they should not be assumed as predetermined.

5

TRADE

"I don't know what you're thinking about," said Tweedledum; "but it isn't so, nohow." "Contrariwise'" said Tweedledee, "if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic."

1. Two-country Two-commodity Trade

Let us begin with the celebrated case of trade between two countries in two commodities. Suppose the labour endowments in countries A and B, L_A and L_B, are 300 units and 400 units respectively. Suppose that the tastes and preferences of the citizens of the two countries are represented by the following Engels' coefficients,

	A	B
1	$\alpha_{1A} = 0.5$	$\alpha_{1B} = 0.15$
2	$\alpha_{2A} = 0.5$	$\alpha_{2B} = 0.85$

And further suppose that the labour coefficients of production of the two commodities in the two countries are,

	A	B
1	$l_{1A} = 5$	$l_{1B} = 16$
2	$l_{2A} = 20$	$l_{2B} = 4$

Finally suppose the money wage rates w_A and w_B to be Re.1 and \$1 per unit of labour in countries A and B respectively.
Following the procedure laid down in Chapter IV, we determine the equilibrium states of the countries under conditions of autarky. These are:

	A	B
1	$150w_A = 30\,p_{1A}$	$60w_B = 3.75\,p_{1B}$
2	$150w_A = 7.5p_{2A}$	$340w_B = 85p_{2B}$

The prices of the commodities under conditions of autarky are

$$p_{1A} = \text{Rs.}5 \qquad p_{1B} = \$16$$
$$p_{2A} = \text{Rs.}20 \qquad p_{2B} = \$4$$

and the corresponding natural exchange rates are,

$$E^1_{AB} = 0.3125 \qquad E^1_{BA} = 3.2$$
$$E^2_{AB} = 5 \qquad E^2_{BA} = 0.2$$

Since $E^1_{AB} < E^2_{AB}$ country A has the comparative advantage in the production of commodity 1 and country B in the production of commodity 2 . Or, in terms of the arbitrage sequence conditions introduced in Chapter II, since,

$$E^1_{AB}\,E^2_{BA} = (0.3125)(0.2) = 0.0625 < 1$$

the feasible trade assignment is *A-1 B-2* and its opposite *A-2 B-1* is infeasible.

Our task is to see whether the assignment *A-1 B-2* is an *equilibrium* assignment and, if so, what the specific properties of the equilibrium are. Thus suppose that *A-1 B-2* is the world trade pattern. Then the activities in the world economy would be represented as follows,

$$\begin{array}{lcc} & A & B \\ 1 & 300w_A = 60p_{1A} & \ldots\ldots\ldots\ldots \\ 2 & \ldots\ldots\ldots\ldots & 400w_B = 100p_{2B} \end{array} \qquad (1)$$

where country A specialises in commodity 1 and B in commodity 2, each devoting all her resources to her industry of specialisation.

Should these be the circumstances, the import bill of country A on account of her imports of commodity 2 would be,

$$M_{AB} = \alpha_{2A}\,w_A\,L_A = (0.5)(\text{Re.}1)(300)$$
$$= \text{Rs.}150$$

and that of country B on account of her imports of commodity 1 from A, would be

$$M_{BA} = \alpha_{1B}\,w_B\,L_B = (0.15)(\$1)(400)$$
$$= \$90$$

The exchange rate of currencies would therefore be determined by the ratio of the intercountry import bills,

$$E_{AB} = \frac{M_{AB}}{M_{BA}} = \frac{\text{Rs.}150}{\$60} = 2.5(\text{Rs}/\$) \qquad (2)$$

The question is whether the state of the world economy as described in (1) and the exchange rate implied by that state in (2) constitutes a world trade equilibrium. Now if a set of exchange rates are to qualify as equilibrium exchange rates, they must fulfil three requirements. Firstly, they should be such as to prevent currency arbitrage, i.e. they should clear the foreign exchange markets. Secondly, they must support the trade pattern from which they are derived, meaning, they all parties must stand to *gain* by trading at those exchange rates. And thirdly, the exchange rates must be such as to ensure that the world demands and supplies of *all* commodities are exactly equal.

Let us apply these tests to the exchange rate $E_{AB} = 2.5(\text{Rs}/\$)$. Its linearly dependent counterpart is

$$E_{BA} = \frac{M_{AB}}{M_{BA}} = 0.4(\$/\text{Re})$$

so that $E_{AB}E_{BA} = 2.5 \times 0.4 = 1$, thus satisfying the first requirement of preventing currency arbitrage. Next the exchange rate $E_{AB} = 2.5$(Rs/\$) lies between the limiting values of the natural exchange rates $E^1_{AB} = 0.3125$ and $E^2_{AB} = 5$. Accordingly we observe that the citizens of both countries gain by trade. A citizen of country A who could have purchased only 0.05 units of commodity 2 for 1 rupee if it were produced at home can, by trade, exchange the 1 rupee for 0.4\$ and obtain $(0.4)/4 = 0.1$ units of commodity 2 in country B, double of what he could get at home. At the same time a citizen of country B who could have obtained 0.0625 units of commodity 1 at home for \$1, can exchange \$1 for Rs. 2.5 and purchase $2.5/5 = 0.5$ units of commodity 1 in country A. Thus the exchange rates $E_{AB} = 2.5$ and $E_{BA} = 0.4$ also fulfil the second requirement in as much as they justify the trade assignment *A-1 B-2* by a supporting pattern of gains of trade.

Finally we check whether the world supplied, 60 units of commodity 1 and 100 units of commodity 2 (as given by (1)) are exactly cleared by the world demands for the commodities.

$$\begin{aligned}
\text{World supply of 1} = 60 &= A\text{'s demand for 1} + B\text{'s demand for 1} \\
&= \frac{\alpha_{1A} w_A L_A}{p_{1A}} + \frac{[\alpha_{1B} w_B L_B] E_{AB}}{p_{1A}} \\
&= \frac{(0.5)(1)(300)}{p_{1A}} + \frac{(0.15)(1)(400)(2.5)}{5} \\
&= 30 + 30 \\
&= 60
\end{aligned}$$

$$\begin{aligned}
\text{World Supply of 2} = 100 &= A\text{'s demand for 2} + B\text{'s demand for 2} \\
&= \frac{[\alpha_{2A} w_A L_A] E_{BA}}{p_{2B}} + \frac{\alpha_{2A} w_B L_B}{p_{2B}} \\
&= \frac{(0.5)(1)(300)(0.4)}{4} + \frac{(0.85)(1)(400)}{4} \\
&= 15 + 85 \\
&= 100
\end{aligned}$$

Thus, at the exchange rate $E_{AB} = 2.5$ (Rs./\$) and $E_{BA} = 0.4$ (\$/Re) the world supplies are exactly cleared by the world demands. We conclude, therefore, that the state of the world economy described by (1) and the exchange rate in (2) is a state of international trade 'equilibrium'.

It may be pertinent to observe, incidentally, that the *tradeable surpluses* (i.e. domestic production less domestic consumption) of the two commodities in the two countries which are 30 units of commodity 1 in A and 15 units of commodity 2 in B are exactly equal to the foreign demands, that is to say, the demand for commodity 1 in B and the demand for commodity 2 in A respectively, in equilibrium. This shows how neatly Adam Smith's 'vent-for-surplus doctrine' fits with Ricardo's principle of comparative advantage(1)

2. Properties of the Trade Equilibrium

Before proceeding to generalise it may be worthwhile to point out some properties of the trade equilibrium with regard to existence and uniqueness.

Observe at the outset that there is no guarantee for the existence of an international trade equilibrium despite differences in the comparative costs of production, because the market clearing exchange rate could well lie outside the interval $(E^1_{AB} E^2_{AB})$ and prevent gains of trade to one or the other countries. To see this consider what would happen if the size of country B, all other conditions remaining the same, were 4000 units of labour. The exchange rate would stand at $E_{AB} = 0.25 = $ (Rs/$) at which B would not produce anything at all! Thus the mere existence of comparative costs is not sufficient for the existence of a trade equilibrium.

A necessary condition for the existence of a trade equilibrium is when one of the countries simply cannot produce one of the commodities (i.e. produce it only at infinitely great cost) but the other country does so at finite cost. For example if $l_{2A} = l_{1B} = \infty$ the $E^1_{AB} = 0$ and $E^2_{AB} = \infty$ so that if *some* demand exists for both the commodities in both the countries, trade mast take place. Indeed in this circumstance it is the equilibrium under autarkic conditions that becomes inconceivable.

If, however, a trade equilibrium exists then we can definitely conclude that it will be unique. This follows from the proof of the uniqueness of exchange rates, that ensure clearing of markets for any given pattern of intercountry import bills, which was spelt out in Chapter I above.

We conclude that an international trade equilibrium may not exist; though, if it does exist, it will be unique. Although this conclusion about the non-existence has been established by the aid of the simple two country two-commodity case, it is easy to see that the simple case is a counter example which establishes the conclusion generally.[2]

3. Two-country Three-commodity Trade

In this section we shall generalise over the commodities. The later sections will generalise over the number of countries. Suppose, to begin with, that the two countries are producing three (tradeable commodities. Their equilibrium states in autarky are;

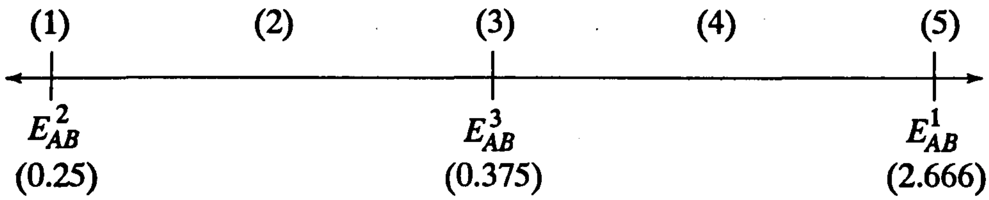
	A	B
1	$24w_A = 180p_{1A}$	$28w_B = 560p_{1B}$
2	$12w_A = 240p_{2A}$	$14w_B = 70p_{2B}$
3	$24w_A = 320p_{3A}$	$28w_B = 140p_{3B}$

Implicit in these states is the following data $\alpha_{1j} = 0.4$, $\alpha_{2j} = 0.2$, $\alpha_{3j} = 0.4$ ($j = A, B$) showing uniform tastes and preferences, the wage rates are $w_A =$ Re1 and $w_B =$ \$1, and the prices and the natural exchange rates are as tabulated below,

$$p_{1A} = 0.1333 \qquad\qquad p_{1B} = 0.05$$

$$p_{2A} = 0.05 \qquad p_{2B} = 0.2$$
$$p_{3A} = 0.075 \qquad p_{3B} = 0.2$$
$$E^1_{AB} = 2.666 \qquad E^1_{BA} = 0.375$$
$$E^2_{AB} = 0.25 \qquad E^2_{BA} = 4$$
$$E^3_{AB} = 0.375 \qquad E^3_{BA} = 2.666$$

The natural exchange rates E^i_{AB} can be depicted on a scale



which shows at once that A has the comparative advantage in the production commodity 2 and B in the production of commodity 1. Thus, if a trade equilibrium exists, we may confidently say that it will be one in which A exports commodity 2 and B exports commodity 1. What is not clear is to which country commodity 3 will be asigned, for that clearly depends upon the position of the actual exchange rate E_{AB}.

In fact there are five possible locations for the actual exchange rate (barring the possibility that it may lie outside the interval (E^2_{AB}, E^1_{AB}) in which case trade equilibrium does not exist) and corresponding to them are five feasible trade patterns. These are as follows.

(1) If $E_{AB} = E^2_{AB}$ the feasible trade pattren is A-2 B-1,2,3
(2) If $E^2_{AB} < E_{AB} < E^3_{AB}$ then A-2 B-1,3
(3) If $E_{AB} = E^3_{AB}$ then A-2,3 B-1,3
(4) If $E^3_{AB} < E_{AB} < E^1_{AB}$ then A-2,3 B-1
(5) If $E_{AB} = E^1_{AB}$ then A-1,2,3 B-1

Our task is to determine where the exchange rate would lie and which feasible trade pattern would be an equilibrium trade pattern. It is evident that the task can be accomplished only by a process of trial and error. Because, to know the exchange rate it is necessary to know beforehand the pattern of trade but, at the same time, the pattern of trade cannot be ascertained until the exchange rate is known. The circle has to be broken somewhere. To do so we shall follow the method of conducting hypothetical experiments in each of which we hypothesise a (feasible) trade pattern, find the exchange rate, check whether the pattern of the gains from trade is consistent with the trade pattern being tried out, if not try another feasible trade pattern, repeating the experiment until the equilibrium trade pattern is found.

Let us apply this method to the example above. Suppose we (arbitrarily) start with the feasible trade assignment A-2 B-1, 3 assuming implicitly that the exchange rate should lie between the natural exchange rates $E^2_{AB} = 0.25$ and $E^3_{AB} = 0.375$ we have then to work out the exchange rate to see whether it does in fact lie in the range hypothesised for it. There is, however, a hurdle to be cleared before this can be done. The difficulty is that of determining how the labour released from the industries which close down after trade is opened is allocated amongst those which are operated. It is upon the allocation of labour that the volumes, and therefore the values, of commodity production and exports depend. And further it is on the values of the intercountry trade bills that the exchange rate depends. Thus a method to simultaneously determine the allocation of labour, the levels of commodity outputs and the exchange rates is requires. Fortunately such a method can be obtained simply by extending the procedure of scale multipliers that was introduced in the context of autarky (Chapter IV section 4) to the case of international equilibrium of world supplies and world demands.

Accordingly let us set up the world supply-demand equations for the three commodities. Let X_{ij} denote the output of commodity i in country j. Let L_{ij} be the labour employed in industry i in country j under autarkic equilibrium conditions and let λ_{ij} be the scale multiplier applicable to industry i in country j in the post-trade situation. Then $\lambda_{ij} L_{ij}$ is the post-trade level of employment in industry i in country j and accordingly the output of commodity i in country j is,

$$X_{ij} = \frac{\lambda_{ij} L_{ij}}{l_{ij}} \quad \forall\ i, j \tag{3}$$

where l_{ij} is the labour coefficient of production. For an industry that closes down $\lambda_{ij} = 0$.

For the trade pattern A-2 B-1, 3 the world supply-demand equations are as follows;

$$\begin{aligned} X_{1B} &= \frac{\lambda_{1B} L_{1B}}{l_{1B}} = \frac{[\alpha_{1A} w_A L_A] E_{BA}}{p_{1B}} + \frac{\alpha_{1B} w_B L_B}{p_{1B}} \\ X_{2A} &= \frac{\lambda_{2A} L_{2A}}{l_{2A}} = \frac{\alpha_{2A} w_A L_A}{p_{2A}} + \frac{[\alpha_{2B} w_B L_B] E_{AB}}{p_{2A}} \\ X_{3B} &= \frac{\lambda_{3B} L_{3B}}{l_{3B}} = \frac{[\alpha_{3A} w_A L_A] E_{BA}}{p_{3B}} + \frac{\alpha_{3B} w_B L_B}{p_{3B}} \end{aligned} \tag{4}$$

We know that the total labour employed in countries A and B in the post-trade situation is the labour endowment L_A and L_B so that,

$$\begin{aligned} \lambda_{2A} L_{2A} &= L_A \\ \lambda_{1B} L_{1B} + \lambda_{3B} L_{3B} &= L_B \end{aligned} \tag{5}$$

And finally there is the two-point exchange neutrality condition

$$E_{AB} E_{BA} = 1 \tag{6}$$

The system of equations (4), (5) and (6) contains 6 equations in 5 unknowns, viz. λ_{1B}, λ_{2A}, λ_{3B}, E_{AB} and E_{BA}. By Walra's Law we know that any one of the three commodity balance equations (4) must be linearly dependent on the other two. Thus the system is exactly determinate—it contains 5 independent equations in 5 unknowns.

To obtain the solution fill up the data of the problem i.e. the values of α_{ij}, L_{ij}, L_j, w_j, p_{ij} and l_{ij} in equations (4) and (5). We then get,

$$X_{1B} = \left[\frac{28}{0.05}\right]\lambda_{1B} = \frac{(0.4)(1)(60)E_{BA}}{0.05} + \frac{(0.4)(1)(70)}{0.05}$$

$$X_{2A} = \left[\frac{12}{0.05}\right]\lambda_{2A} = \frac{(0.2)(1)(60)}{0.05} + \frac{(0.2)(1)(70)E_{AB}}{0.05}$$

$$X_{3B} = \left[\frac{28}{0.2}\right]\lambda_{3B} = \frac{(0.4)(1)(60)E_{BA}}{0.2} + \frac{(0.4)(1)(70)}{0.2}$$

which reduces to

$$\begin{aligned} 560\lambda_{1B} &= 480E_{BA} + 560 \\ 240\lambda_{2A} &= 240 + 280E_{AB} \\ 140\lambda_{3A} &= 120E_{BA} + 140 \\ 12\lambda_{2A} &= 60 \\ 28\lambda_{1b} + 28\lambda_{2B} &= 70 \end{aligned} \tag{4}$$

The corresponding solution is as follows,

$$\lambda_{1B} = 1.25 \quad \lambda_{2A} = 5 \quad \lambda_{3B} = 1.25$$
$$E_{AB} = 3.4285 \qquad E_{BA} = 0.29166$$

Observe, however, that the exchange rate $E_{AB} = 3.4285$ does not belong to the range (0.25, 0.375) that was hypothesised for it—the pattern of trade A-2, B-1,3 is feasible only if the actual exchange falls inside that range. That the exchange rate does not lie in the range is a contradiction of our hypothesis. We conclude that A-2 B-1,3 cannot be an equilibrium trade pattern because the market clearing exchange rates corresponding to this pattern do not give a supporting pattern of the gains of trade. To see this construct a gains from trade table and observe that, at the exchange rates in the solution, A would find it profitable to produce and export all the commodities.

	1	2	3
A	$\boxed{7.5}$	$\boxed{20}$	$\boxed{13.23}$
B	5.83	1.45	1.45

Now this situation has arisen because the hypothetical exchange rate $E_{AB} = 3.4285$ corresponds to so great a depreciation of A's currency as compared to B's that B finds it worthwhile to import all commodities from A. Thus if we are to bring the exchange rate within the feasible range (0.25, 2.666) we must move in the direction of raising the exports of A and/or reducing her imports from B, so that A's currency appreciates relative to that of B.

Accordingly let the trade pattern slide down to the right on the scale of exchange rate and stand at A-2,3 B-1,3 on the hypothesis that the exchange rate $E_{AB} = E^3_{AB} = 0.375$. Set up the corresponding world supply-demand equations and the labour-conservation equations.

$$X_{1B} = \frac{\lambda_{1B} L_{1B}}{l_{1B}} = \frac{[\alpha_{1A} w_A L_A] E_{AB}}{p_{1B}} + \frac{\alpha_{1B} w_B L_B}{p_{1B}}$$

$$X_{2A} = \frac{\lambda_{2A} L_{2A}}{l_{2A}} = \frac{[\alpha_{2A} w_A L_A]}{p_{2A}} + \frac{[\alpha_{2B} w_B L_B] E_{AB}}{p_{2A}} \tag{7}$$

$$X_{3A} + X_{3B} = \frac{\lambda_{3A} L_{3A}}{l_{3A}} + \frac{\lambda_{3B} L_{3B}}{l_{3B}} = \frac{\alpha_{3A} w_A L_A}{p_{3A}} + \frac{\alpha_{3B} w_B L_B}{p_{3B}}$$

$$\lambda_{2A} L_{2A} + \lambda_{3A} L_{3A} = L_A$$

$$\lambda_{1B} L_{1B} + \lambda_{3B} L_{3B} = L_B \tag{8}$$

Since the exchange rate is 'known', the system (7) and (8) contains 5 equations in 4 unknowns viz. $\lambda_{1B}, \lambda_{2A}, \lambda_{3A}, \lambda_{3B}$. Of course one of the equations in (7) is linearly dependent so that we have 4 independent equations in 4 unknowns[(4)]

Substituting the values of the data and the (hypothetical) value of the exchange rate $E_{AB} = E^2_{AB} = 0.375$ we get the solution,

$$\lambda_{2A} = 1.4375 \qquad \lambda_{1B} = 3.2857$$

$$\lambda_{3A} = 1.78125 \qquad \lambda_{3B} = -0.7857$$

This solution is not feasible because it requires country B to operate industry 3 in the *negative*, i.e. producing a negative output! A meaningless situation. It implies that country A herself has sufficient capacity to produce commodity 3 and that country B has to strain her resources beyond her labour endowment in order to supply the world with a sufficient quantity of commodity1. Surely a more reasonable allocation of resources is called for. We, therefore, knock out commodity 3 from country B and try the next trade pattern in A-2,3 B-1 with the hypothesis that the exchange rate should lie in the range (0.375, 2.666). The corresponding system of equations is,

$$X_{1B} = \frac{\lambda_{1B} L_{1B}}{l_{1B}} = \frac{[\alpha_{1A} w_A L_A] E_{BA}}{p_{1B}} + \frac{\alpha_{1B} w_B L_B}{p_{1A}}$$

$$X_{2A} = \frac{\lambda_{2A} L_{2A}}{l_{2A}} = \frac{\alpha_{2A} w_A L_A}{p_{2A}} + \frac{[\alpha_{2B} w_B L_B] E_{AB}}{p_{2B}} \tag{9}$$

$$X_{3A} = \frac{\lambda_{3A} L_{3A}}{l_{3A}} = \frac{\alpha_{3A} w_A L_A}{p_{3A}} + \frac{[\alpha_{3B} w_B L_B] E_{AB}}{p_{3A}}$$

$$\lambda_{2A} L_{2A} + \lambda_{3A} L_{3A} = L_A \tag{10}$$

$$\lambda_{1B} L_{1B} = L_B$$

$$E_{AB}E_{AB} = 1 \tag{11}$$

The system (9), (10) and (11) contains 5 independent equations in 5 unknowns viz. λ_{1B}, λ_{2A} λ_{3A}, E_{AB} and E_{BA}. In figures, the equations (9) and (10) are as follows,

$$\begin{aligned} 560\lambda_{1B} &= 280E_{BA} + 560 \\ 240\lambda_{2A} &= 240 + 280E_{AB} \\ 320\lambda_{3A} &= 320 + 373.33E_{AB} \end{aligned} \tag{9'}$$

$$\begin{aligned} 12\lambda_{2A} + 24\lambda_{3A} &= 70 \\ 28\lambda_{1B} &= 70 \end{aligned} \tag{10}$$

Using (11) to eliminate E_{BA} we obtain the solution,

$$\lambda_{1B} = 2.5 \quad \lambda_{2A} = 1.6666 \quad \lambda_{3A} = 1.6666$$
$$E_{AB} = 0.5714 \quad E_{BA} = 1.75$$

The exchange rate E_{AB} does fall in the range (0.375, 2.666) as required by the hypothesis of a trade assignment A-2,3 B-1; the pattern of the gains from trade supports the pattern of trade postulated as the boxed entries show.

	1	2	3
A	7.5	$\boxed{20}$	$\boxed{13.33}$
B	$\boxed{35}$	8.75	8.75

And *ex hypothesi* the world demands and supplies of commodities are equal to one another. We conclude that the trade equilibrium has been determined and depict the world economic activities in the equilibrium state;

	A	*B*
1	....	$70w_B = 1400p_{1B}$
2	$20w_A = 400p_{2A}$	....
3	$40w_A = 533.33p_{2A}$	....

where, the post-trade levels of employment in the industries in country A are $\lambda_{2A} L_{2A} = (1.666)(12) = 20$, $\lambda_{3A} L_{3A} = (1.666)(24) = 40$ and that in the industry in country B is $\lambda_{1B} L_{1B} = (2.5)(28) = 70$ units of labour.

We may also verify that the exchange rate(s) is equal to the ratio of the intercountry import bills: the import bill of country A is

$$\begin{aligned} M_{AB} = \alpha_{1A} w_A L_A &= (0.4)(\text{Re.}1)(60) \\ &= \text{Rs.}24 \end{aligned}$$

and that of country B is

$$\begin{aligned} M_{BA} = (\alpha_{2B} + \alpha_{3B})\, w_B L_B &= (0.2 + 0.4)(\$1)(70) \\ &= \$42 \end{aligned}$$

so that $$E_{AB} = \frac{M_{AB}}{M_{BA}} = \frac{\text{Rs.}24}{\$42} = 0.5714\ (\text{Rs}/\$)$$

and likewise for the reciprocal dollar-rupee exchange rate E_{BA}.

4. Two Country Several-commodity Trade

The method that we have used in the illustration above can be extended to deal with trade between two countries in several commodities. Briefly the procedure is as follows. Start at one end of the scale of natural exchange rates that is to say at the lowest (or highest) natural exchange rate and work the way step by step to the right (or left) trying successive trade patterns and rejecting all those which, (a) do not give a pattern of gains from trade consistent with the one hypothesised *and* (b) do not give positive scale multipliers for all the industries in all the countries. The pattern of trade that remains (if one remains) is the equilibrium trade pattern.

It may be noticed that the computational effort required by this procedure is excessive. A quicker procedure is as follows. Suppose that the natural exchange rates E^i_{AB} $(i = 1, ..., n)$ are such that,

$$E^1_{AB} < E^2_{AB} < E^3_{AB} < ... < E^n_{AB}$$

(If they are not, the commodities can always be renumbered to ensure it). More importantly, suppose for a moment that there are no commodities with the same natural exchange rate. Then we are required to try out $n-1$ possible trade situations where the exchange rate lies *between* two natural exchange rate and n possible situations where the exchange rate is equal to a natural exchange rate.

To ascertain whether the equilibrium trade pattern belongs to the former $n-1$ trade situations, the procedure is as follows, Suppose, to start with, that $E^1_{AB} < E_{AB} < E^2_{AB}$ assuming, in effect, that the trade pattern is A-1, B-2,3,...., n. Find the exchange rate,

$$E'_{AB} = \frac{M_{AB}}{M_{BA}} = \frac{[\alpha_{2A} + \alpha_{3A} + ... + \alpha_{nA}]\, w_A L_A}{\alpha_{1B}\, w_B L_B} \tag{12}$$

and check whether it belongs to the range hypothesised, (E^1_{AB}, E^2_{AB}). If it does, find the scale multipliers. If they are positive, the equilibrium is found. If not, suppose $E^2_{AB} < E_{AB} < E^3_{AB}$ assuming a trade pattern A-1, 2 B-3,..., n. Find the exchange rate

$$E''_{AB} = \frac{M_{AB}}{M_{BA}} = \frac{[\alpha_{3A} + \alpha_{4A} + ... + \alpha_{nA}]\, w_A L_A}{[\alpha_{1B} + \alpha_{2B}]\, w_B L_B} \tag{13}$$

If it belongs to the range (E^2_{AB}, E^3_{AB}) check if the scale multipliers are positive. If not move to the next situation $E^3_{AB} < E_{AB} < E^4_{AB}$ etc.

If the equilibrium does not belong to these $n-1$ trade patterns then turn to check whether it belongs to the n possible situations where the exchange rate is equal to one of the natural exchange rates, i.e., where the countries do not gain by trading some commodity. So suppose $E_{AB} = E^1_{AB}$ assuming A-1 B-1, 2,..., n and find the pattern of gains from trade at the exchange rate hypothesized. If it is consistent with the corresponding trade pattern, find the scale multipliers and check whether they are positive. If they are, equilibrium is found. If not try $E_{AB} = E_{AB}^2$ and repeat the procedure until the equilibrium (if it exists) is found.

The extension of this method to situations where two or more commodities correspond to the same natural exchange rates follows rather obviously by suitable modifications to the expressions for the intercountry import bills in equations (12), (13) etc., which determine the exchange rates for the different trading situations.

The procedure described above saves two sets of calculations; (a) the calculation of the scale multipliers when the exchange rates solved do not belong to the ranges hypothesised for then in the first $n-1$ trade patterns and (b) the calculation of the scale multipliers when the exchange rates, assumed to be equal to the natural exchanges rates in n possible trade patterns, do not yield patterns of gains of trade that are consistent with those being tried out. This reduces, to some extent, the computational effort required. [Compare the effort with that required to solve the example in section 3]. An even simpler method will be to go on calculating the gains of trade for every different configuration and calculate the scale multipliers only when a supporting pattern is found.

The procedure that we have so far used is in the spirit of that proposed by several classical economists like H.Von Mangoldt (1863), Edgeworth (1894), Bastable (1903) or Viner (1937) who constructed a scale of comparative cost ratios and conjectured that 'demand factors would break the chain' at some intermediate point on the scale. [See Graham (1948), Dornbusch, Fischer and Samuelson(1977) and Jones and Neary (1988), for the more modern versions of finding the trade equilibrium in Ricardian two-country situations].

At any rate, we shall see that when more than two countries are introduced, the quaint construction of the scale of natural exchange rates (or of comparative costs) becomes too cumbersome to be of any use. We will have to use the more powerful method of the arbitrage sequence conditions introduced in Chapter II. To get a feel of the power of these conditions, as well as to anticipate the methods that will be used heretofore, briefly reconsider the two-country three-commodity example of section 3 above. We find from the products of the natural exchange rates,

$$\begin{aligned} E_{AB}^2 E_{BA}^1 &= (0.25)(0.375) = 0.09375 \\ E_{AB}^3 E_{BA}^1 &= (0.375)(0.375) = 0.140625 \\ E_{AB}^2 E_{BA}^3 &= (0.25)(2.666) = 0.6666 \end{aligned} \tag{14}$$

that the elemental trade assignment in order of profitability are A-2 B-1, A-3 B-1 and A-2 B-3. No other trade assignment is profitable. Thus the equilibrium trade assignment (if it exists) is bound to be some combination of the three elemental trade assignments. To obtain the trial trade pattern we should, going by the order of profitability, assign commodity 2 to country A and commodity 1 to country B. What about commodity 3 ? Follow the logic and observe that the elemental assignment which gives 3 to A is more profitable than that which gives

3 to B. Thus the initial trade pattern will itself be set at A-2, 3 B-1. In this trade pattern all commodities are assigned to countries in accordance with the most profitable arbitrage sequences.

We know of course that A-2, 3 B-1 is actually the equilibrium trade assignment. What needs to be observed, however, is that by using the arbitrage sequence conditions we have arrived at the pattern much more quickly than we did by the traditional criterion of finding where 'demand factors break the chain'. To be sure, equilibrium is not determined by comparative advantage alone and if demand factors and/or the sizes of the countries were different the equilibrium pattern would not be A-2, 3 B-1. Even so, the arbitrage conditions take us directly to the trade pattern that is most profitable on the ground of comparative advantage alone. To achieve the object by using the chain of comparative costs' is, in comparison, cumbersome.

5. Three-country Three-commodity Trade

If the discussion of the arbitrage sequence conditions seemed unnecessarily elaborate in comparison with the inherently low degree of complexity of the two-country three-commodity example, the indispensability of these conditions will now be demonstrated in an example of trade between three countries in three commodities.

	A	B	C
1.	$40w_A = 200p_{1A}$	$800w_B = 160p_{1B}$	$400w_C = 400p_{1C}$
2.	$20w_A = 66.666p_{2A}$	$400w_B = 66.66p_{2B}$	$200w_C = 100p_{2C}$
3.	$40w_A = 400p_{3A}$	$800w_B = 8000p_{3B}$	$400w_C = 16000p_{3C}$
	w_A = Re.1	w_B = \$1	w_C = ¥1
	$p_{1A} = 0.2$	$p_{1B} = 5$	$p_{1C} = 1$
	$p_{2A} = 0.3$	$p_{2B} = 6$	$p_{2C} = 2$
	$p_{3A} = 0.1$	$p_{3B} = 0.1$	$p_{3C} = 0.025$

The natural exchange rates are

$E^1_{AB} = 0.04$	$E^1_{BC} = 5$	$E^1_{CA} = 5$
$E^2_{AB} = 0.05$	$E^2_{BC} = 3$	$E^2_{CA} = 6.666$
$E^3_{AB} = 1$	$E^3_{BC} = 4$	$E^3_{CA} = 0.25$

Just pause and imagine the mind boggling number of hypothetical trade patterns that can be conceived using combinations of the possible positions of exchange rates on the scales of the natural exchange rates. And imagine also the effort of trying them all out individually, finding the exchange rates and verifying whether they are consistent with the trading situation hypothesised, finding the scale multipliers and checking their feasibility.

But we shall see that the logic of the arbitrage sequence conditions will bring us to the equilibrium solution in only a couple of interactions. In general with z countries and n commodities there z^n arbitrage conditions of which n conditions pertaining to the n individual commodities must give a product equal to unity in all circumstances. Thus there are $z^n - n$ arbitrage sequences which assign different commodities to countries. In our problem there are $3^3 - 3 = 24$ such arbitrage sequences. Evaluating the products of the natural exchange rates corresponding to these sequences shows that 10 products have a value less than unity, 1 has a value equal to unity ($E^2_{AB} E^3_{BC} E^1_{CA} = (0.05)(4)(5) = 1$)and 13 have values greater than unity. thus 13 elemental trade assignments and *all* their combinations are out of question since they cannot offer gains from trade. The equilibrium trade assignment must therefore be some combination of the 10 profitable and 1 loss-preventing assignments. Our problem is to find that combination.

The natural exchange rates whose product is the lowest $E^1_{AB} E^2_{BC} E^3_{CA} = (0.04)(3)(0.025) = 0.003$, so that A-1 B-2 C-3 is the most profitable trade assignment. Since all commodities are covered i.e. assigned to at least one country, we may begin by hypothesising the trade pattern A-1 B-2 C-3. Then the world pattern of economic activities will look like this:

	A	B	C
1	$100w_A = 500p_{1A}$	......	.....
2	.....	$2000w_B = 333.333p_{2B}$	.....
3	.....	......	$1000w_C = 40{,}000p_{3C}$

All countries are in this state, specialised in the production of one commodity and devote all their resources to produce the commodities of their specialisation. In this state of the world system, the intercountry import bills are,

M_{AB} = Rs.20	M_{BA} = \$800	M_{CA} = ¥400
M_{AC} = Rs.40	M_{BC} = \$800	M_{CB} = ¥200
M_A = Rs.60	M_B = \$1600	M_C = ¥600

These can be used to solve for the exchange rates of currencies by constructing the system of intercountry balance of trace equations of chapter I.

$$M_A = M_{BA} E_{AB} + M_{CA} E_{AC}$$
$$M_B E_{AB} = M_{AB} + M_{CB} E_{AC}$$
$$M_C E_{AC} = M_{AC} + M_{BC} E_{AB}$$

Using, say, the first two equations (since any one must be linearly dependent on the others) we get,

$$60 = 800E_{AB} + 400E_{AC}$$
$$-20 = 1600E_{AB} + 200E_{AC}$$

which solve $E_{AB} = 0.025$, $E_{AC} = 0.1$. The neutrality conditions determine the other exchange rates, $E_{BA} = 40$, $E_{CA} = 10$, $E_{BA} = E_{BA} E_{AC} = (40)(0.1) = 4$ and $E_{CB} = 0.25$

Next we complete the table of the gains from trade to check whether it supports the trade pattern A-1 B-2 C-3.

	1	2	3
A	5	3.33	10
B	8	$\boxed{6.66}$	$\boxed{400}$
C	$\boxed{10}$	5	$\boxed{400}$

As the boxed entries in each column show, at the exchange rates just solved, C has the greatest advantage in the production of commodity 1, B in the production of commodity 2 and B and C have equal but greatest advantage in the production of commodity 3. It is clear that A-1 B-2 C-3 is not the equilibrium trade pattern.

To proceed to the new (trial) trade pattern two rules must be followed: (1) keep the initial trade pattern A-1 B-2 C-3 intact and (2) modify it to take account of the results revealed by the gains from trade table arrived at.

The first rule is important. It recognises that the trade pattern A-1 B-2 C-3 is firmly rooted in gains-of-trade considerations that underlie the arbitrage sequence conditions. Therefore until the assignment is *seen* to be infeasible it cannot be abandoned. The second rule, therefore, only adds commodities to the initial assignment. Thus we add commodity 1 to country C's portfolio and commodity 3 to country B's portfolio. with these additions the new (trial) trade pattern becomes A-1 B-2,3 C-1,3.

Verify immediately that all the elemental trade assignments which make up this trail assignment are feasible, i.e. $E_{AB}^{1} E_{BC}^{3} E_{CA}^{2} < 1$, $E_{AB}^{1} E_{BC}^{2} E_{CA}^{1} < 1$, etc., so that the trial assignment is itself feasible.

Now if the trial pattern A-1 B-2,3 C-1,3 is to be the actual trade pattern the exchange rates must be $E_{BC} = E_{BC}^{3} = 4$ and $E_{CA} = E_{CA}^{1} = 5$ because B and C produce commodity 3 in common and A and C produce commodity 1 in common. Also by the three-point exchange neutrality condition the exchange rate $E_{AB} = E_{AC} E_{CB} = (0.2)(0.25) = 0.05$. Thus we 'know' all the exchange rates corresponding to the trade pattern assumed. We may therefore immediately verify whether the assumed trade pattern is supported by the gains from trade evaluated from the corresponding exchange rates. The results are given below.

	1	2	3
A	$\boxed{5}$	$\boxed{3.33}$	10
B	4	$\boxed{3.33}$	$\boxed{200}$
C	$\boxed{5}$	2.5	$\boxed{200}$

We observe that the pattern of gains from trade indicates A should add commodity 2 to her portfolio. For the rest, the trade pattern is as assumed. Thus our trial trade pattern stands modified to A-1,2 B-2,3 C-1,3 with the corresponding exchange rate being $E_{AB} = E^2_{AB} = 0.05, E_{BC} = E^3_{BC} = 4, E_{CA} = E^1_{CA} = 5$. Fortunately, the values of the natural exchange rates, which are equal to the currency exchanges rates, meet the exchange neutrality conditions. Evaluating the gains from trade at the exchange rates arrived at gives a table identical with the one above showing, in effect that we have arrived at a consistent trade assignment. Whether it will be the equilibrium trade assignment can, however, be ascertained only after the allocation of labour is worked out.

To find the allocations of labour we set up the system of Walrasian supply demand equations and the labour conservation equations. Remembering that the world supply of commodity 1, comes from A and C, that of commodity 2 from A and B and that of commodity 3 from B and C, these equations are,

$$\begin{aligned}
\frac{\lambda_{1A} L_{1A}}{l_{1A}} + \frac{\lambda_{1C} L_{1C}}{l_{1C}} &= \frac{\alpha_{1A} w_A L_A}{p_{1A}} + \frac{(\alpha_{1B} w_B L_B) E_{AB}}{p_{1A}} + \frac{\alpha_{1C} w_C L_C}{p_{1C}} \\
\frac{\lambda_{2A} L_{2A}}{l_{2A}} + \frac{\lambda_{2B} L_{2B}}{l_{2B}} &= \frac{\alpha_{2A} w_A L_A}{p_{2A}} + \frac{\alpha_{2B} w_B L_B}{p_{2B}} + \frac{(\alpha_{2C} w_C L_C) E_{AC}}{p_{2A}} \qquad (15) \\
\frac{\lambda_{3B} L_{3B}}{l_{3B}} + \frac{\lambda_{3C} L_{3C}}{l_{3C}} &= \frac{(\alpha_{3A} w_A L_A) E_{CA}}{p_{3C}} + \frac{\alpha_{3B} w_B L_B}{p_{3B}} + \frac{\alpha_{3C} w_C L_C}{p_{3C}}
\end{aligned}$$

One might be tempted, but it would be misleading, to infer from the way in which the equations have been written out, that B imports commodity 1 from A alone, that C imports commodity 2 from B alone and that A imports commodity 3 from C alone. For, the precise quantities and values of the intercountry commodities exports/imports can only be ascertained *after* the labour allocations and levels of production have been determined. What the equations suggest is that B is indifferent to purchasing 1 from A and C, C is indifferent to purchasing 2 from A and B and A is indifferent to purchasing 3 from B and C.

To the supply-demand equations (15) we add to labour conservation equations,

$$\begin{aligned}
\lambda_{1A} L_{1A} + \lambda_{2A} L_{2A} &= L_A \\
\lambda_{2B} L_{2B} + \lambda_{3B} L_{3B} &= L_B \qquad (16) \\
\lambda_{1C} L_{1C} + \lambda_{3C} L_{3C} &= L_C
\end{aligned}$$

Since the exchange rates are known, the system of equations (15) and (16) contain 5 independent equations in 6 unknown scale multipliers. One scale multiplier must be given to determine the remaining five.

To see the implications of this, fill up the data including the exchange rates in (15) and (16). We get,

$$\begin{aligned}
200\lambda_{1A} + 400\lambda_{1C} &= 800 \\
66.66\lambda_{2A} + 66.66\lambda_{2B} &= 266.66 \qquad (15) \\
8000\lambda_{3B} + 16{,}000\lambda_{3C} &= 32{,}000
\end{aligned}$$

$$
\begin{aligned}
40\lambda_{1A} + 20\lambda_{2A} &= 100 \\
400\lambda_{2B} + 800\lambda_{3B} &= 2000 \\
400\lambda_{1C} + 400\lambda_{3C} &= 1000
\end{aligned}
\tag{16}
$$

Fix, say $\lambda_{1A} = 1$, to obtain $\lambda_{2A} = 3, \lambda_{2B} = 1, \lambda_{3B} = 2, \lambda_{1C} = 1.5$ and $\lambda_{3C} = 1$. Then the world economy looks like this;

	A	B	C
1	$40w_A = 200p_{1A}$	......	$600w_C = 600p_{1C}$
2	$60w_A = 200p_{2A}$	$400w_B = 66.66p_{2B}$	......
3	....	$1600w_B = 16{,}000p_{3B}$	$400w_C = 16{,}000p_{3C}$

In this scenario country A produces commodity 1 for herself and country C exports 200 units of commodity 1 valued at ¥200 to country B. Similarly country A exports 133.33 units of commodity 2 valued at Rs.40 to country C while country B produces just sufficient to meet her own requirements. And, finally, country B exports 8000 units of commodity 3 valued at \$800 to country A while C produces her own requirements. The import bills of countries A, B and C are Rs.40, \$800 and ¥200 respectively. We may simply verify from the intercountry trade data that the trade balancing exchange rates are $E_{AB} = 0.05, E_{BC} = 4, E_{CA} = 5$, as arrived at in the course of iterations.

The solution above holds for the (arbitrary) value of $\lambda_{1A} = 1$. We may obtain several solutions by fixing different values of λ_{1A}, or indeed any one of the multipliers, provided, however, that the value fixed exogenously does not give rise to negative multipliers. Thus, for example, if $\lambda_{2A} = 2$ then $\lambda_{1A} = 1.5, \lambda_{2B} = 2, \lambda_{3B} = 1.5, \lambda_{1C} = 1.25$ and $\lambda_{3C} = 1.25$. In this scenario the world activities are as follows:

	A	B	C
1	$60w_A = 300p_{1A}$	.....	$500w_C = 500p_{1C}$
2	$40w_A = 133.33p_{2A}$	$800w_B = 133.33p_{2B}$	....
3	.....	$1200w_B = 12{,}000p_{3B}$	$500w_C = 20{,}000p_{3C}$

Once again the intercountry trade bills will be found to be compatible with the same exchange rates $E_{AB} = 0.05, E_{BC} = 4$ and $E_{CA} = 5$, even though their values and composition are differert. In like manner we may generate any number of solutions.

The purpose of this section was to demonstrate the superiority of the algorithm based on the arbitrage sequence conditions over the traditional criterion of location the point at which 'demand factors break the chain of comparative costs'. We have incidentally used to context to demonstrate another result of considerable importance viz. that multiple allocations of labour and the different intercountry trade bills associated with them are compatible with a unique trade pattern and exchange rates.

6. Graham's Example

This section considers an example of trade between four countries in three commodities originally solved by Graham (1924, 1932, 1948). The reason for giving it a somewhat detailed consideration is threefold. Firstly, Graham's work represents the first systematic presentation of the theory of multicountry multicommodity trade. If the theory developed in the present monograph is to be regarded as valid then it is necessary (through by no means sufficient) that it should be compatible with Graham's results. Secondly, Graham used his example to criticise several propositions of the classical and neoclassical trade theories and to establish correct, or more general, propositions in their place. thus a discussion of these examples are vital to explain his critique of classical and neoclassical trade theory. Thirdly, Graham solved his example by a "tedious process of trial and error." We shall put to test the alogrithim developed and see to what extent its use mitigates the tedium[(4)].

Suppose the wage rates in the four countries A, B, C and D to be Re.1, \$1, ¥1 and DM1 respectively. Graham supposed the sizes of the countries, measured in terms of the output of industry 1 in autarky, to be in the ratio 1:2:3:4. And the supposed that 1.3 of the income in each country was spent on each commodity. If we suppose the labour endowments to be $L_A = 12, L_B = 18, L_C = 27$ and $L_D = 36$ we obtain the following autarky equilibrium states for the four countries,

	A	B	C	D
1	$4w_A = 10p_{1A}$	$6w_B = 20p_{1B}$	$9w_C = 30p_{1C}$	$12w_D = 40p_{1D}$
2	$4w_A = 19p_{2A}$	$6w_B = 40p_{2B}$	$9w_C = 45p_{2C}$	$12w_D = 112p_{2D}$
3	$4w_A = 42p_{3A}$	$6w_B = 48p_{3B}$	$9w_C = 90p_{3C}$	$12w_D = 160p_{3D}$

where the outputs are measured in thousands of units. Observe that the autarky outputs of commodity 1 in the four countries stand in the ratio 1:2:3:4 as required by Graham's example. The prices (per thousand units) are,

$$
\begin{array}{llll}
p_{1A} = 2/5 & p_{1B} = 3/10 & p_{1C} = 3/10 & p_{1D} = 3/10 \\
p_{2A} = 4/19 & p_{2B} = 3/20 & p_{2C} = 1/5 & p_{2D} = 3/28 \\
p_{3A} = 2/21 & p_{3B} = 1/8 & p_{3C} = 1/10 & p_{3D} = 3/40
\end{array}
$$

and the natural exchange rates are,

$$
\begin{array}{llll}
E^1_{AB} = 1.3333 & E^1_{BC} = 1.0000 & E^1_{CD} = 1.0000 & E^1_{DA} = 1.0000 \\
E^2_{AB} = 1.4035 & E^2_{BC} = 0.7500 & E^2_{CD} = 1.8666 & E^2_{DA} = 0.5089 \\
E^3_{AB} = 0.7619 & E^3_{BC} = 1.2500 & E^3_{CD} = 1.3333 & E^3_{DA} = 0.7875
\end{array}
$$

The international commodity arbitrage sequence that covers all commodities and yields the maximum profit is,

$$E^3_{AB} E^2_{BC} E^1_{CD} E^2_{DA} = 0.2908$$

Accordingly we may start the search for the trade equilibrium by supposing the trade pattern to be A-3, B-2 C-1 D-2, supposing alongside that $E_{BD} = E_{BA}E_{AD} = E_{BC}E_{CD} = E_{BD}^2 = 1.4$. The world supply-demand equations, after making the appropriate cancellations and substitutions are,

$$18 = 4E_{CA} + 6E_{CB} + 12E_{CD}$$

$$45.6 = 4E_{BA} + 9E_{BC}$$

$$8 = 6E_{AB} + 9E_{AC} + 12E_{AD}$$

Substituting, using the neutrality conditions, $E_{CA}E_{AB}$ for E_{CB}, $E_{CA}E_{AD}$ for E_{CD}, $E_{BA}E_{AC}$ for E_{BC} and multiplying the first and second equations by E_{AC} and E_{AB} respectively gives the following system of exchange rate determination,

$$\begin{bmatrix} 6 & -18 & 12 \\ 45.6 & -9 & 0 \\ 6 & 9 & 12 \end{bmatrix} \begin{bmatrix} E_{AB} \\ E_{AC} \\ E_{AD} \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 8 \end{bmatrix} \tag{17}$$

which solves the exchange rates $E_{AB} = 0.1754$, $E_{AC} = 0.4444$ and $E_{AD} = 0.2456$, the neutrality conditions solving the rest,. Observe that $E_{BA}E_{AD} = 1.4$, which is consistent with the assumed trade pattern in which both countries B and D produce commodity 2.

Our next step is to find the gains from trade, i.e. to find which countries have the maximum advantage in the production of which commodities at the exchange rates determined from the hypothetical trade pattern A-3 B-2 C-1 D-2. The table is given below.

	1	2	3
A	2.5	4.75	10.5
B	$\boxed{19.0}$	$\boxed{38.0}$	45.6
C	7.5	11.25	38
D	13.57	$\boxed{38.0}$	$\boxed{54.285}$

Observation of the maximal column elements shows that country B should add commodity 1 to her portfolio and country D should add commodity 3 to her's. Thus the first trade pattern stands modified to A-3 B-1,2 C-3 D-2,3. The fact that countries B and C produce commodity 1 in common, countries B and D produce commodity 2 in common and countries A and D produce commodity 3 in common, implies that the supporting exchange rates must be $E_{BC} = E_{BC}^1 = 1$, $E_{BD} = E_{BD}^2 = 1.4$ and $E_{DA} = E_{DA}^3 = 0.7875$.

Further, $E_{BC} = 1$ implies $E_{BA}E_{AD} = 1$ so that $E_{AB} = E_{AC}$. And $E_{BD} = 1.4$ implies $E_{BA}E_{AD} = 1.4$ or $E_{BA} = (1.4)\,E_{DA} = (1.4)(0.7875)$ leading to $E_{AB} = E_{AC} = 0.90703$. Similarly $E_{BD} = E_{BC}E_{CD} = 1.4$ gives $E_{CD} = 1.4$ In short, all the exchange rates corresponding to the trial trade pattern A-3, B-1,2 C-1, D-2,3 are known before hand. With all exchange rates known, the Walrasian equations need solve only the labour allocations. We have four independent equations in four scale multipliers λ_{1B}, λ_{2B}, λ_{2D} and λ_{3D}. The solution is $\lambda_{1B} = 1.535$ $\lambda_{2B} = 1.465$,

$\lambda_{2D} = 1.6321$ and $\lambda_{3D} = 1.3678$ which are all positive. Next we check whether the exchange rates support the assumed trade pattern. Observe from the table below that they do not specifically country C must add commodity 3 to her portfolio since her advantage in that line is the greatest at the trial exchange rates. The new trial trade pattern therefore becomes A-3 B-1,2 C-1,3 D-2,5.

	1	2	3
A	2.5	4.75	10.5
B	$\boxed{3.675}$	$\boxed{7.35}$	8.82
C	$\boxed{3.675}$	5.5125	$\boxed{11.025}$
D	2.625	$\boxed{7.35}$	10.5

But we have now reached a point where the trade pattern implies two mutually inconsistent sets of exchange rates. The first set, written in terms of the currency of country A, is the one which we have just tried out, $E_{AB} = 0.9070, E_{AC} = 0.9070$, $E_{AD} = 1.2698$. But now $E_{CD} = E^3_{CD} = 1.3333, E_{BC} = E^1_{BC} = 1, E_{BD} = E^2_{BD} = 1.4$ and $E_{AD} = E^3_{AD} = 1.2698$ which implies the set of exchange rates (in terms of A country's currency), $E_{AB} = 0.9523, E_{AC} = 0.9523, E_{AD} = 1.2698$.

As there is no going back we must take the bull by its horns. We set up the Walrasian supply-demand equations and the labour conservation equations for the trade pattern A-3 B-1,2 C-1,3 D-2,3 and determine the multipliers corresponding to both sets of exchange rates. Since A is completely specialised in commonly 3, $\lambda_{3A} = 3$, so the equations are :

$$\begin{aligned} 6\lambda_{1B} + 9\lambda_{1C} &= 4E_{BA} + 12E_{BD} + 15 \\ 40\lambda_{2B} + 112\lambda_{2D} &= 26.66\, E_{BA} + 60E_{BC} + 152 \\ 90\lambda_{3C} + 160\lambda_{3D} &= 226 \end{aligned} \tag{18}$$

$$\begin{aligned} 6\lambda_{1B} + 6\lambda_{2B} &= 18 \\ 9\lambda_{1C} + 9\lambda_{3C} &= 27 \\ 12\lambda_{2D} + 12\lambda_{3D} &= 36 \end{aligned} \tag{19}$$

Substituting in turn the two sets of exchange rates gives us two solutions for the multipliers. Corresponding to the former ($E_{AB} = 0.9070$, etc.) we get $\lambda_{1B} = 1.4914, \lambda_{2B} = -0.035, \lambda_{1C} = 2.333, \lambda_{3C} = 1, \lambda_{2D} = 2.1678$ and $\lambda_{3D} = 0.8321$. This solution is clearly unacceptable since it gives a negative multiplier for industry 2 in country B. To the latter set of exchange rates ($E_{AB} = 0.9523$, etc.) corresponds the following solution, $\lambda_{1B} = 2.6, \lambda_{2B} = 0.4, \lambda_{1C} = 2.266, \lambda_{3C} = 0.733, \lambda_{2D} = 2$ and $\lambda_{3D} = 1$. This solution is *prima facie* acceptable. So we turn to observe the pattern of the gains from trade at the latter set of exchange rates, $E_{AB} = 0.9523$, $E_{AC} = 0.9523$, $E_{AD} = 1.2698$. The table justifies the entire pattern of trade assumed, except B-2.

	1	2	3
A	2.5	4.75	$\boxed{10.5}$
B	$\boxed{3.5}$	7.0	8.4
C	$\boxed{3.5}$	5.25	$\boxed{10.5}$
D	2.625	$\boxed{7.35}$	$\boxed{10.5}$

The entire solution hinges upon the treatment that should be given to industry 2 in country *B*. At the former set of exchange rates, at which *B* had the advantage in the production of commodity 2, the multiplier for industry 2 in *B* was found to be negative, so the assignment *B*-2 had to discarded. At the latter set of exchange rates the multiplier is positive but *B*'s advantage in commodity 2 is inferior to that of country *D*. Either way the assignment of commodity 2 to country *B* is jinxed. So we drop *B*-2. Out next trial pattern becomes *A*-3 *B*-1 *C*-1,3 *D*-2,3 to which correspond the exchange rates $E_{AB} = 0.9523, E_{AC} = 0.9523$ and $E_{AD} = 1.2689$. We have just seen that the gains from trade fully justify the pattern of trade at those exchange rates. All that needs to be done, therefore, is to check whether the trade pattern gives feasible values for the scale multipliers. Indeed they do, $\lambda_{1A} = 3, \lambda_{1B} = 3, \lambda_{1C} = 1.911, \lambda_{3C} = 1.088, \lambda_{2D} = 2.2$ and $\lambda_{3D} = 0.8$ which are all positive.

The equilibrium world activities may be depicted as follows:

	A	*B*	*C*	*D*
1	....	$18w_B = 60p_{1B}$	$17.2w_C = 57.333p_{1C}$	......
2	......	....	....	$26.4w_D = 246.4p_{2D}$
3	$12w_A = 126p_{3A}$	....	$9.8w_C = 98p_{3C}$	$9.6w_D = 128p_{3D}$

The international commodity exchange ratio (or terms of trade) can be simply read off from the preceding (equilibrium) gains-from-trade table. It is 3.5 units of commodity 1 = 7.35 units of commodity 2 = 10.5 units of commodity 3, more conveniently expressed as 10 units of 1 = 21 units of 2 = 30 units of 3. [See Graham (1948), pp. 83-84 noting that he calls the commodities cloth, linen and corn respectively]. The world outputs of the commodities are 117.33, 246.4 and 352 (thousand) units respectively.

7. Effects of Changes in Demand

Graham then goes on to consider the effects of changes in tastes and preferences on the international trade equilibrium. He supposes demand to change to $\alpha_{ij} = 0.5, \alpha_{2j} = 0.3, \alpha_{3j} = 0.2$ ($j = A,.., D$) in the example above. (See Graham (1948) pp. 87-88). How does the trade equilibrium change?

For a ready reference we reproduce below the autarky equilibria at the new pattern of demand:

	A	B	C	D
1	$6w_A = 15p_{1A}$	$9w_B = 30p_{1B}$	$13.5w_C = 45p_{1C}$	$18w_D = 60p_{1D}$
2	$3.6w_A = 17.1p_{2A}$	$5.4w_B = 36p_{2B}$	$8.1w_C = 40.5p_{2C}$	$10.8w_D = 100.8p_{2D}$
3	$2.4w_A = 25.2p_{3A}$	$3.6w_B = 28.8p_{3B}$	$5.4w_C = 54p_{3C}$	$7.2w_D = 96p_{3D}$

To see whether the trade equilibrium does change we set up the Walrasian equations for the trade pattern A-3 B-1 C-1 D-2,3 and evaluate the new set of scale multipliers. The exchange rates are 'Known'; $E_{AB} = 0.9523, E_{AC} = 0.9523, E_{AD} = 1.2689$. The equations are,

$$\frac{9\lambda_{1B}}{0.3} + \frac{13.5\lambda_{1C}}{0.3} = \frac{6E_{BA}}{0.3} + \frac{9}{0.3} + \frac{13.5}{0.3} + \frac{18E_{BD}}{0.3}$$

$$\frac{10.8\lambda_{2D}}{0.1071} = \frac{3.6E_{DA}}{0.1071} + \frac{5.4E_{DB}}{0.1071} + \frac{8.1E_{DC}}{0.1071} + \frac{10.8}{0.1071}$$

$$\frac{5.4\lambda_{3C}}{0.1} + \frac{2.4\lambda_{3A}}{0.09523} + \frac{72\lambda_{3D}}{0.075} = \frac{2.4}{0.075} + \frac{3.6E_{DB}}{0.075} + \frac{5.4}{0.1} + \frac{7.2}{0.075}$$

In addition there are the labour conservation equations,

$$2.4\lambda_{3A} = 12$$
$$9\lambda_{1B} = 18$$
$$13.5\lambda_{1C} + 5.4\lambda_{3C} = 27$$
$$10.8\lambda_{2D} + 7.2\lambda_{3D} = 36$$

The solution obtained is $\lambda_{2A} = 5, \lambda_{1B} = 2, \lambda_{1C} = 2.666, \lambda_{3C} = -1.666, \lambda_{2D} = 2.182$ and $\lambda_{3D} = 1.726$. Thus industry 3 in country C closes down and the trade pattern A-3 B-1 C-1 D-2,3 presents itself for trial. Two exchange rates $E_{BC} = E^1_{BC} = 1$ and $E_{DA} = E^3_{DA} = 0.7875$ are implied by the trade pattern itself. Our task is to determine one exchange rate (the others will be determined by the neutrality conditions) and two multipliers for the allocation of labour in country D.

The new system of equations gives the solution $E_{AB} = 1.2825, E_{AD} = 1.2825$, with $\lambda_{2D} = 2.525$ and $\lambda_{3D} = 1.2125$. We know $E_{AD} = 1.2698$. the multipliers are feasible so we evaluate the gains of trade at these exchange rates and observe that commodity 1 must be added to country D's portfolio.

	1	2	3
A	2.5	4.75	$\boxed{10.5}$
B	2.599	5.918	6.237
C	2.599	5.898	7.79
D	$\boxed{2.625}$	$\boxed{7.35}$	$\boxed{10.5}$

Retaining the original assignment intact gives us our new trial pattern A-3 B-1 C-1 D-1,2,3 with the implied exchange rates $E_{BC} = E^1_{BC} = 1, E_{CD} = E^1_{CD} = 1$ $E_{BD} = E^1_{BD} = 1, E_{DA} = E^3_{DA} = 0.7875$. We obtain, $E_{AB} = 1.2698, E_{AC} = 1.2698$ and $E_{AD} = 1.2698$. the values of the multipliers solved from the supply demand and labour-conservation equations are $\lambda_{1D} = 0.012265$, $\lambda_{2D} = 2.5125$, $\lambda_{3D} = 1.2$. Observe the pattern of the gains from trade. It justifies the pattern of trade *A*-3 *B*-1 *C*-1 *D*-1,2,3. Besides, all the multipliers are positive. Hence the new trade equilibrium is determined.

	1	2	3
A	2.5	4.75	$\boxed{10.5}$
B	$\boxed{2.625}$	5.25	6.3
C	$\boxed{2.625}$	3.9375	7.875
D	$\boxed{2.625}$	$\boxed{7.35}$	$\boxed{10.5}$

As was only to be expected, a shift of demand in favour of commodity 1 (and away from commodities 2 and 3) has caused more countries to produce commodity 1 and fewer countries to produce commodity 3. Resources have shifted towards industries whose products are in greater demand. The output of commodity 1 has accordingly increased and that of commodity 3 decreased. This is seen from the table of world activities.

	A	*B*	*C*	*D*	Total
1	....	$18w_B = 60p_{1B}$	$27w_C = 90p_{1C}$	$0.22p_{1D} = 0.75p_{1D}$	150.75
2	......	......	......	$27.135w_D = 253.26p_{2D}$	253.26
3	$12w_A = 126p_{3A}$	.....	.....	$8.64w_D = 115.2p_{3D}$	241.2

The international ratio of commodity exchange is 2.625 units of commodity 1 = 7.35 units of commodity 2 = 10.5 units of commodity 3 or 10 units of 1 = 28 units of 2 = 40 units of 3. Observe that the shift of demand towards commodity 1 has raised its price relative to commodities 2 and 3 in the international market.

8. Criticisms of 2 × 2 Theory

Three conclusions of fundamental importance to trade theory emerge from the examples so far considered. The first conclusion is that in a multicountry multicommodity context, bilateral comparative advantage does *not* determine the direction of trade. Observe, for instance, that in the example of section 6 above, there are several pairs of countries and commodities which satisfy the bilateral comparative advantage conditions but the trade directions implied by them are nowhere to be found in the actual trade pattern; e.g. $p_{1A}/p_{2A} = 1.9$ is less than $p_{1D}/p_{2D} = 2.8$ but *A* does not export commodity 1 to country *D*, $p_{2B}/p_{3B} = 0.9$ is less than $p_{2C}/p_{3C} = 2$ but *B* does not export commodity 2 to country *C*, $p_{1A}/p_{2A} = 1.9$ is less than $p_{1B}/p_{2B} = 2$, but *A* does not export commodity 1 to country *B* (on the contrary she imports 1 from *B*!) nor does country *A* import commodity 2 from country *B*, $p_{1C}/p_{2C} = 1.5$ is less than $p_{1B}/p_{2B} = 2$ but both *B* and *C* export commodity 1 to the rest of the world. The upshot of this is that the existence of bilateral comparative advantage does not imply exports and the existence of bilateral comparative disadvantage does not imply imports. As a result, the classical claim 'that the same principles which determine the pattern of two-country two-commodity trade also apply to cases of complex 'trade' is, in general, false. [See Graham (1948), McKenzie (1954) and Jones (1961) for similar illustrations].

At the same time it is important to note that no trade assignment which does not satisfy (atleast the weak) bilateral comparative advantage condition can be present in the equilibrium trade assignment. That is to say the equilibrium trade pattern A-3 B-1 C-1 D-2,3 of the example is feasible because every bilateral comparison of which the trade pattern is made up conforms to the bilateral comparative advantage conditions., i.e. $p_{3A}/p_{2A} = 0.238$ is less than $p_{3B}/p_{1B} = 0.416$, $p_{3C}/p_{1C} = 0.333$ and $p_{3D}/p_{1D} = 0.25$; $p_{3A}/p_{2A} = 0.4523$ is less $p_{2D}/p_{2D} = 0.7$; $p_{1B}/p_{2B} = 2$ and $p_{1C}/p_{2C} = 1.5$ are both less than $p_{1D}/p_{3D} = 4$. [This is the ground which on Viner (1937) and Jones (1976) have defended the two-country two-commodity model against Graham's criticisms]. We conclude from this discussion that bilateral comparative advantage plays no role in the *determination* of the trade pattern in a multicountry multicommodity setting (unlike the two-country, two-commodity case) but the equilibrium trade pattern must nevertheless conform to bilateral comparative advantage.

Secondly we have seen that bilateral comparative disadvantage does not imply imports. The opposite is also true. imports do not imply bilateral comparative disadvantage. Thus, observe in the example of the preceding section, that country D produces 128 (thousand) units of commodity and, at the same time, imports 32 units from country A to satisfy her consumption requirements of 160 units. Graham (1948) has used this fact to refute Mill's (1848) claim that "no country would make anything for itself which it did not make for others" a claim that is generally valid only in the two-country, two-or-more-commodity case;

The third conclusion pertains to the general invalidity of conducting the exercise of determining the trade equilibrium in terms of "reciprocal demand". As Elliott (1950) has shown, the technique of reciprocal demand applies to the case of two countries trading in several commodities. In the general situation, the technique of reciprocal demand', by which is here meant 'demand for each other's goods' breaks down, its place is taken by world demands and world supplies of commodities. (See Graham (1948) chapter IX, and Metzler (1950)). This suggests that the genuine multicountry multicommodity problem cannot, in general, be reduced to a two-country two-commodity problem simply by rechristening countries as 'domestic' and 'foreign' and commodities as 'importables' and 'exportables'.(5) Any such attempt is spurious—it is not merely an approximation to reality, it is an attempt to reduce the reality to make it amenable for tools that have been forged to handle a two-country two-commodity setting.

It may be pertinent, in the light of what has been said, to make some final remarks on the controversy about the evaluation of the 'true' accomplishments of classical trade theory. Were the classical economists really guilty of spurious generalisation? Answers to questions like these frequently depend upon the way they are posed. From one standpoint we reason as follows: If the classical economists did believe that the twin principles of comparative advantage applied mechanically to multi-dimensional trade, they would have applied the *principles* mechanically and given us examples of multi-dimensional trade. The fact that they did not means that they imputed a wider meaning to these principles, and would readily have admitted that specific methods required modifications when dealing with the general situation. From the opposite standpoint, critics can point out that the classical economists drew conclusions from two-by-two exercises and stated them as if they applied generally, eg. that the terms of trade would 'usually' settle between comparative cost ratios, that a country will not import a commodity she produces, etc., so that the classical economists were, indeed, guilty of spurious generalisation.

Perhaps the best way to look at this matter is to decide the problem of international trade theory and then to evaluate the stages of accomplishment reached by different groups of economists. The task of international trade theory is to determine the commodity composition of international trade, the intercountry trade bills. The equilibrium exchange rates (and the terms of trade) and the allocation of resources in the countries. Then, taking benefit of a long perspective we may say that the classical theory, in the initial period from Ricardo through Mill to Marchall, perfected the two-country two-commodity model and, along with their followers like Nicholson (1897), Edgeworth (1894), Bastable (1903) amongst others, made deep penetrations into two-country several-commodity and several-country, two-commodity models. The next great stride was the extension to multicountry multicommodity trade, which was accomplished by Graham (1923, 1932, 1948) and perfected by McKenzie (1954, 1955), Jones (1961), and Dornbusch, Fischer and Samuelson (1977).

9. Critique of Graham's Critique

Graham used these, and similar examples, to criticise the classical theory of trade for giving exaggerated importance to the effects of changes in demand on the international ratio of commodity exchange. He worked out a series of examples to show that changes in the Engels' coefficients for the three commodities from 0.25 to 0.40 have no effect on the ratio of exchange. (See Graham (1948) pg. 87.88). And it is only when "catastrophic" changes in demand take place, as in the case of section 8 above, that the ratio of exchange undergoes a change. Graham used these results to argue the validity of the antithesis viz. Comparative costs, rather than demand, forms the correct basis of 'normal' international values. Both the classical thesis and Graham's antithesis deserve somewhat detailed consideration.

Observe to begin with that in a multicountry multicommodity setting the likelihood' is that some commodities will be produced in common between groups of countries. As a result the international ratio of commodity exchange will be determined by the points of commonality; it will form a chain in which the domestic cost ratios of the common commodities are the links (or in the terms employed in this text the exchanges rates will be equal to natural exchange rates). If established in this manner, changes in demand (over a definite range) will simply result in changes in the allocation of labour between the industries in the different countries while keeping the ratio of exchange unaltered. Graham argued that this feature of actual complex trade was completely ignored by the classical theory. The reason quite simply, is that classical trade theory was framed in terms of a two-country two-commodity trade in which the possibility that commodities will be produced in common is minimised and relegated to the extremes. In the classical model, therefore, the normal state of international affairs is one in which the ratio of exchange lies between the domestic cost ratios and the countries are completely specialised in one commodity. Every change in demand immediately affects the ratio of exchange–beyond that it has few, if any, effects. But this picture, Graham argued, is one sided. To which one may add that it would not, in itself, be a bad thing had not classical economists actually used it to make practical policy pronouncements on, for example, the affects of tariffs, etc.

Graham went further to state that all positions intermediate to the domestic cost ratios are in 'limbo'—they are 'unstable", "Precarious", "shifting" or "slippery", subject to fluctuations with "every change in the winds of desire", etc. On the contrary Graham argued that the international ratio of commodity exchange would usually be "firmly anchored" at the cost ratios of commodities produced in common and, therefore,that costs, rather than demand, fixed the "norms". This reflected Graham's opinion that costs are "stable", "fixed", "less prone to fluctuations" while demand is vagarious", "precarious", "unstable" and "shifting". Much of this criticism by Graham must sound somewhat quaint because we now know that both costs and demand have stable and random components and that the effects of both are fit subjects for economic study. But so strong is Graham's opinion on this matter that he even goes to the extent of letting the parameters, the sizes of countries and the demand coefficients, to change if a limbo situation were present. [See Graham (1948) pp. 70-74]. Instead of this extreme, it is better to suppose that equilibrium may well exist on the "flats", "ridges" or "corners", of the world production possibility surface. [See Whitin (1953). McKenzie (1954) and Jones (1976) for similar arguments][(6)].

But so attached was Graham to his presupposition that the international ratio of commodity exchange would be determined at domestic cost ratios of commodities produced in common that it led him to call all changes in demand which result in a change in that ratio, "catastrophic". (His example of a catastrophic change has been solved in section 8 above). However there is nothing really catastrophic—if the initial position were itself on the edge between a trade pattern and another, a 'small' change in demand could also bring about the catastrophe'

10. Complex Trade

We have not so far considered situations of complex trade. This typically involve a large-number of countries trading in an even larger number of commodities. Consider therefore an example of four countries trading in seven commodities, whose autarky equilibria are as follows:

	A	B	C	D
1.	$200w_A = 20p_{1A}$	$300w_B = 90p_{1B}$	$400w_C = 80p_{1C}$	$500w_D = 500p_{1D}$
2.	$200w_A = 50p_{2A}$	$300w_B = 60p_{2B}$	$400w_C = 16p_{2C}$	$500w_D = 25p_{2D}$
3.	$200w_A = 80p_{3A}$	$300w_B = 100p_{3B}$	$400w_C = 100p_{3C}$	$500w_D = 333.33p_{3D}$
4.	$200w_A = 200p_{4A}$	$300w_B = 30p_{4B}$	$400w_C = 120p_{4C}$	$500w_D = 75p_{4D}$
5.	$200w_A = 60p_{5A}$	$300w_B = 40p_{5B}$	$400w_C = 60p_{5C}$	$500w_D = 300p_{5D}$
6.	$200w_A = 100p_{6A}$	$300w_B = 180p_{6B}$	$400w_C = 40p_{6C}$	$500w_D = 100p_{6D}$
7.	$200w_A = 40p_{7A}$	$300w_B = 75p_{7B}$	$400w_C = 400p_{7C}$	$500w_D = 125p_{7D}$

Suppose the wage rates to be 1Re, 1\$, 1 yen and 1 mark respectively. The prices and the natural exchange rates are,

$p_{1A} = 10$	$p_{1B} = 3.333$	$p_{1C} = 5$	$p_{1D} = 1$
$p_{2A} = 4$	$p_{2B} = 5$	$p_{2C} = 2.5$	$p_{2d} = 20$
$p_{3A} = 2.5$	$p_{3B} = 3$	$p_{3C} = 4$	$p_{3D} = 1.5$
$p_{4A} = 1$	$p_{4B} = 10$	$p_{4C} = 3.333$	$p_{4D} = 6.666$
$p_{5A} = 3.333$	$p_{5B} = 7.5$	$p_{5C} = 6.666$	$p_{5D} = 1.666$
$p_{6A} = 2$	$p_{6B} = 1.666$	$p_{6C} = 10$	$p_{6D} = 5$
$p_{7A} = 5$	$p_{7B} = 4$	$p_{7C} = 1$	$p_{7D} = 4$
$E^1_{AB} = 3$	$E^1_{BC} = 0.666$	$E^1_{CD} = 5$	$E^1_{DA} = 0.1$
$E^2_{AB} = 0.8$	$E^2_{BC} = 2$	$E^2_{CD} = 0.125$	$E^2_{DA} = 5$
$E^3_{AB} = 0.8333$	$E^3_{BC} = 1.5$	$E^3_{CD} = 2.666$	$E^3_{DA} = 0.6$
$E^4_{AB} = 0.1$	$E^4_{BC} = 3$	$E^4_{CD} = 0.5$	$E^4_{DA} = 6.666$
$E^5_{AB} = 0.44$	$E^5_{BC} = 1.125$	$E^5_{CD} = 4$	$E^5_{DA} = 0.5$
$E^6_{AB} = 1.2$	$E^6_{BC} = 0.1666$	$E^6_{CD} = 2$	$E^6_{DA} = 2.5$
$E^7_{AB} = 1.25$	$E^7_{BC} = 4$	$E^7_{CD} = 0.25$	$E^7_{DA} = 0.8$

To set the initial trade pattern consider the most profitable arbitrage sequences which cover all commodities and countries. In all there are $n^z - n = 7^4 - 7 = 2394$ arbitrage sequences of which we shall consider the lowest six.

$$E^4_{AB}E^6_{BC}E^2_{CD}E^1_{DA} \cong 0.0002$$
$$E^4_{AB}E^6_{BC}E^7_{CD}E^1_{DA} \cong 0.0004$$
$$E^4_{AB}E^6_{BC}E^4_{CD}E^1_{DA} \cong 0.0008$$
$$E^4_{AB}E^1_{BC}E^2_{CD}E^1_{DA} \cong 0.0008$$
$$E^4_{AB}E^3_{BC}E^2_{CD}E^1_{DA} \cong 0.0009$$
$$E^5_{AB}E^6_{BC}E^2_{CD}E^1_{DA} \cong 0.0009$$

These indicate a trial trade pattern of *A*-4,5 *B*-1,3,6 *C*-2,7 *D*-1. The computations proceed as follows. In the first iteration there are 10 independent equations in 10 unknowns, λ_{1B}, λ_{1D}, λ_{2C}, λ_{3B}, λ_{4A}, λ_{5A}, λ_{6B}, λ_{7C}, and two exchange rate since $E_{BD} = E^1_{BD} = 3.333$. The solution gives a negative multiplier for λ_{1B}. At the same time gains from trade at the (trial) exchange rates suggest that *B* has the advantage in producing several commodities and *D* the advantage in producing commodity 5. But in view of the fact that *B* has a negative multiplier for industry 1 there is no reason to assign additional commodities to *B*. Keeping a status-quo in the original assignment our new trial trade pattern becomes *A*-4,5 *B*-3,6 *C*-2,7 *D*-1,5.

In the second iteration the multiplier for industry 5 in country *A* becomes negative and the pattern of gains from trade suggest a great advantage to *A* in the production of several commodities and to *D* in the production of commodity 3. Of course no additional commodities can be assigned to *A* so that the next trial pattern becomes *A*-4 *B*-3,6 *C*-2,7 *D*-1,3,5. In the third interation λ_{3B} turns negative but pattern of gains from trade supports exactly the trade pattern being tried out. This suggests that we are close to the equilibrium. Striking out commodity 3 from *B*'s portfolio, we come to the assignment *A*-4, *B*-6, *C*-2,7 *D*-1,3,5.

The solution obtained is $E_{AB} = 0.6666$, $E_{AC} = 1$, $E_{AD} = 1.2$, $\lambda_{1D} = 2.3333$, $\lambda_{2C} = 3.5$, $\lambda_{3D} = 2.3333$, $\lambda_{5D} = 2.3333$, $\lambda_{7C} = 3.5$ with $\lambda_{4A} = 7$ and $\lambda_{6B} = 7$. All the multipliers are positive and the exchange rates indicate the pattern of the gains from trade tabulated below. To avoid fractions the quantities reported are what Rs.10 can buy in the international market. As is readily seen the pattern of gains from trade exactly supports the trade pattern being tried out. Accordingly, we conclude that *A*-4 *B*-6 *C*-2,7 *D*-1,3,5 is the equilibrium trade pattern.

	A	*B*	*C*	*D*
1	1	4.5	2	$\boxed{8.33}$
2	2.5	3	$\boxed{4}$	0.416
3	4	5	2.5	5.55
4	$\boxed{10}$	1.5	3	1.25
5	3	2	1.5	$\boxed{5}$
6	5	$\boxed{9}$	1	1.66
7	2	3.75	$\boxed{10}$	2.083

The international ratio of commodity exchange is 8.33 units of 1 = 4 units of 2 = 5.55 units of 3 = 10 units of 4 = 5 units of 5 =9 units of 6 = 10 units of 7.

11. Non-tradeable Commodities

In this section we shall incorporate non-tradeable goods in the system of international general equilibrium. It is clear, of course, that non-tradeable goods must be produced by all the countries in the international equilibrium. Three features of the international trade problem with non-tradeable may be pointed out immediately. Firstly, there is no sense in calculating the natural exchange rates of the non-tradeable commodities for the simple reason that the natural exchange rates are required only to determine the feasible trade assignments, in which non-tradeables do not enter. Secondly, for the same reason, non-tradeable do not enter. Secondly, for the same reason, nontradeables will not enter the gains from trade calculations. Thirdly, irrespective of the trade pattern being tried out, the scale multipliers for the industries producing non-tradeables must be equal to 1 because, the demand-supply equations for them being purely domestic, there equilibrium sizes are already determined in the autarky equilibria.

These features are illustrated in an example of trade between three countries which produce three commodities of which one, say commodity 3, is non-tradeable. Suppose the autarky equilibria are:

	A	B	C
1.	$300w_A = 100p_{1A}$	$300w_A = 1000p_{1B}$	$500w_A = 100p_{1C}$
2.	$300w_A = 600p_{2A}$	$200w_A = 20p_{2B}$	$300w_A = 900p_{2C}$
3.	$400w_A = 100p_{3A}$	$500w_A = 100p_{3B}$	$200w_A = 200p_{3C}$

Supposing the wage rates to be Re.1, \$1, and ¥1, the natural exchange rates are,

$$E^1_{AB} = 10 \qquad E^1_{BC} = 0.06 \qquad E^1_{CA} = 1.666$$

$$E^2_{AB} = 0.5 \qquad E^2_{BC} = 30 \qquad E^2_{CA} = 0.666$$

The most profitable trade assignment clearly is A-2,3 B-1,3 C-2,3 because,

$$E^2_{AB}E^1_{BC}E^2_{CA} = (0.05)(0.06)(0.666) = 0.002$$

The Walrasian equations are,

$$300\lambda_{1B} = 300E_{BA} + 300 + 500E_{BC}$$

$$600\lambda_{2A} + 900\lambda_{2C} = 600 + 400E_{AB} + 900$$

$$400\lambda_{3A} = 400$$

$$500\lambda_{3A} = 500$$

$$200\lambda_{3C} = 200$$

and the labour-conservation equations are,

$$300\lambda_{1B} + 500\lambda_{3B} = 1000$$

$$300\lambda_{2A} + 400\lambda_{3A} = 1000$$

$$300\lambda_{2C} + 200\lambda_{3C} = 1000$$

There are thus 7 independent equations in 7 unknowns viz. λ_{1B}, λ_{2A}, λ_{2C}, λ_{3A}, λ_{3B}, λ_{3C}, and 1 exchange rate since $E_{CA} = E^2_{CA} = 0.666$. The solution is $\lambda_{1B} = 1.666$, $\lambda_{2A} = 2$, $\lambda_{2C} = 2.666$, $\lambda_{3A} = 1$, $\lambda_{3C} = 1$, $E_{AB} = 5.25$, $E_{AC} = 1.5$. The table of gains from trade shows that the trade pattern is perfectly justified.

	A	B	C
1	0.33	$\boxed{0.635}$	0.133
2	$\boxed{2}$	0.019	$\boxed{2}$

In computing the international ratio of commodity exchange too we may leave out the non-tradeables (or include them with their country names). Thus in the example the ratio is 0.635 units of commodity 1 = 2 units of commodity 2.

12. 'Inferior' and 'Luxury' Goods

We have until now described demand conditions exclusively by means of simple, linear Engels' equations in which the expenditures on the various commodities are fixed, positive proportions of he consumer's income. These proportions, the Engels' coefficients or the propensities to consume, are positive, less than unity but such as to add up to unity. The Engels' equations, therefore can only describe demand conditions for 'normal' goods i.e., those goods whose demand changes proportionately with changes in income. Inferior goods, whose demand falls as income increased, or luxury goods whose demand increases more than proportionately with changes in income cannot be covered by Engels' equations, for they would give rise to negative industries producing negative outputs. A more general demand specification will be required.

We shall therefore use the linear expenditure system to depict the demand conditions. The general demand equation, as we have seen in Chapter IV is,

$$p_i X_i = p_i X_i^* + \alpha_i (Y - \sum p_i X_i^*) \tag{20}$$

where X_i^* are the 'subsistence' quantities.

An inferior goods is one for which the *marginal* propensity to consume, α_i, is negative. Of course, since a positive output must be produced (unless the commodity is to become extinct) the *average* propensity to consume must be positive. A luxury good, on the other hand, is one for which the marginal propensity to consume is greater than the average propensity to consume, $\alpha_i > (p_i X_i / Y)$. And normal goods, which we been considering until now, will be those for which the marginal propensities to consume will be positive but less than, or equal to, their average propensities to consume.

The use of a linear expenditure system incidentally frees us from another restrictive assumption that the use of Engels' functions forced upon us; the assumption of unit price elasticities of demand. In general the price demand relationship implied by the linear expenditure system is unit price-elastic at all points on the demand curve. The elasticity coefficient is,

$$\left[\frac{dX_i}{dp_i}\right]\left[\frac{p_i}{X_i}\right] = \frac{-\alpha_i (Y - \sum p_j X_j^*)}{p_i X_i^* (1 - \alpha_i) + (Y - \sum p_j X_j^*)} \quad \forall\ i = j \tag{21}$$

For instance if $\alpha_i < 1$, the price elasticity of demand varies from 0 at an infinite price and rises gradually becoming equal to 1 at a zero price.

Consider an example of two countries trading in two commodities. The demand equations for the two commodities in country A are,

$$p_{1A}X_{1A} = 20p_{1A} + 0.5(40 - 20p_{1A} - 20p_{2A})$$
$$p_{2A}X_{2A} = 20p_{2A} + 0.5(40 - 20p_{1A} - 20p_{2A})$$

And the demand equations for the same commodities in country B are,

$$p_{1B}X_{1B} = 20p_{1B} - 0.2(10 - 20p_{1B} - 20p_{2B})$$
$$p_{2B}X_{2B} = 20p_{2B} + 1.2(10 - 20p_{1B} - 20p_{2B})$$

The labour endowments in the countries are $L_A = 40$ $L_B = 10$ and it is assumed that the wage rates are $w_A =$ Re.1 $w_B = \$1$. Accordingly the autarky equilibria in the two countries are,

	A	B
1	$18w_A = 90p_{1A}$	$8.4w_A = 21p_{1B}$
2	$22w_A = 55p_{2A}$	$1.6w_B = 8p_{2B}$

From the natural exchange rates,

$$E^1_{AB} = 0.5 \qquad E^1_{BA} = 2$$
$$E^2_{AB} = 2 \qquad E^2_{BA} = 0.5$$

we infer the trade pattern A-1, B-2. The trade equilibrium is found to be A-1,2 B-2,

	A	B
1	$22w_A = 110p_{1A}$	....
2	$18w_B = 45p_{2A}$	$10w_B = 50p_{2B}$

with an equilibrium exchange rate $E_{AB} = E^2_{AB} = 2$.

Since commodity 1 is an inferior good in country B, any increase in the size and hence income of country B will result in a decrease in country B's demand for commodity 1. Thus the production of commodity 1 in country A will fall but that of commodity 2 will increase. The exports of country A of commodity 2 will also increase. For example if $L_B = 12$ the new trade equilibrium is

	A	B
1	$21.2w_A = 106p_{1A}$	......
2	$18.8w_A = 47p_{2A}$	$12w_B = 60p_{2B}$

with the equilibrium exchange rate continuing to remain at $E_{AB} = E^2_{AB} = 2$.

13. Trade in 'Special' and Differentiated Commodities

Trade theory has usually confined itself to dealing with trade situations in which all commodities can be produced, and are consumed, in all countries.[7] Such trading situations surely have much importance for a rigorous demonstration of the principle of comparative advantage or the discussions of the theory of protection. But exclusive attention to such situations restricts the application of trade theory. Even the most casual observation shows that a substantial portion of international trade takes place in distinct commodities which can be produced in some countries but not in others. Several commodities require special natural resources and /or labour skills for their production. And countries are differentially endowed in these resources. The commodities therefore become unique to countries which posses these special factors of production. Also implicit in the very existence of a country is the notion that there must exist some 'special' commodity, eg. tourist demand, cultural exchange or diplomatic relations.

At the same time tastes and preferences differ across countries and cultures. Some commodities may not be consumed at all in some countries and cultures. Some product designs, of an otherwise similar commodity, may be more acceptable then others

Trade situations which reflect the variety in endowments, tastes and preferences must also be incorporated into the theory. Consider an illustration of three countries trading in four commodities. Suppose countries A and B have an exclusive possession of some special factors required to produce commodities 1 and 2. In other words if countries B and C, who do not have the special factors required to produce commodity 1 are nevertheless made to produce it, they must do so at an infinite cost. Similarly countries A and C, not having the special endowments to produce commodity 2, can produce it only at infinite cost. Suppose all countries can produce commodities 3 and 4 at finite cost. The prices of commodities are assumed to be as follows,

$$\begin{array}{lll} p_{1A} = 0.1 & p_{1B} = \infty & p_{1C} = \infty \\ p_{2A} = \infty & p_{2B} = 0.05 & p_{2C} = \infty \\ p_{3A} = 5 & p_{3B} = 4 & p_{3C} = 0.5 \\ p_{4A} = 6 & p_{4B} = 5 & p_{4C} = 1 \end{array}$$

Further suppose that the wage rates are w_A = Re.1, w_B = \$1, w_C = ¥1 and countries A, B, C each have 100 units of labour. Suppose the coefficients of consumption to be,

$$\begin{array}{lll} \alpha_{1A} = 0.2 & \alpha_{1B} = 0.4 & \alpha_{1C} = 0.25 \\ \alpha_{2A} = 0.2 & \alpha_{2B} = 0.3 & \alpha_{2C} = 0.25 \\ \alpha_{3A} = 0.3 & \alpha_{3B} = 0.3 & \alpha_{2C} = 0.25 \\ \alpha_{4A} = 0.3 & \alpha_{4B} = 0 & \alpha_{4C} = 0.25 \end{array}$$

Observe that we have not shown the autarky equilibria in this example. This is not impossible. (For example we can assume that under autarky all income perforce is spent on the commodities that can be produced and consumed in the countries and that once trade is opened, the real tastes as reflected in the post-trade Engels' coefficients above, are revealed and take effect. But it would be somewhat artificial. We, therefore, proceed directly to ascertaining the international trade equilibrium. The natural exchange rates are,

$$E_{AB}^1 = 0 \qquad E_{BC}^1 = \infty \qquad E_{CA}^1 = \infty$$
$$E_{AB}^2 = \infty \qquad E_{BC}^2 = 0 \qquad E_{CA}^2 = \infty$$
$$E_{AB}^3 = 1.25 \qquad E_{BC}^3 = 8 \qquad E_{CA}^3 = 0.1$$
$$E_{AB}^4 = 1.2 \qquad E_{BC}^4 = 5 \qquad E_{CA}^4 = 0.1666$$

The most efficient arbitrage sequences are,

$$\text{(i)} \qquad E_{AB}^1 E_{BC}^2 E_{CA}^3 = (0)(0)(0.1) = 0$$
$$\text{(ii)} \qquad E_{AB}^1 E_{BC}^2 E_{CA}^4 = (0)(0)(0.1666) = 0$$

[We have taken some liberty with the mathematics at two points. Firstly, in the calculation of the natural exchange rates eg. E_{BC}^1, E_{CA}^2, we have assumed $\infty/\infty = \infty$. This is to indicate that in the presence of a country which produces the commodity at finite cost, countries which do so at infinite cost simply will not features in the feasible trade assignment. Secondly, in the determination of the efficient arbitrage sequences we deem (i) to be more efficient than (ii), because, of the two zeros in (i) and (ii), the zero in (i) is "smaller" than that in (ii).]

The trade pattern indicated by (i) and (ii) is A-1 B-2, C-3,4. The demand-supply equations and the labour conservation equations are,

$$\frac{L_{1A}}{l_{1A}} = \frac{\alpha_{1A} w_A L_A}{p_{1A}} + \frac{(\alpha_{1B} w_B L_B)E_{AB}}{p_{1A}} + \frac{(\alpha_{1C} w_C L_C)E_{AC}}{p_{1A}}$$

$$\frac{L_{2B}}{l_{2B}} = \frac{(\alpha_{2A} w_A L_A)E_{BA}}{p_{2B}} + \frac{\alpha_{2B} w_B L_B}{p_{2B}} + \frac{(\alpha_{2C} w_C L_C)E_{BC}}{p_{2C}}$$

$$\frac{L_{3C}}{l_{3C}} = \frac{(\alpha_{3A} w_A L_A)E_{CA}}{p_{3C}} + \frac{(\alpha_{3B} w_B L_B)E_{CB}}{p_{3C}} + \frac{\alpha_{3C} w_C L_C}{p_{3C}}$$

$$\frac{L_{4C}}{l_{4C}} = \frac{(\alpha_{4A} w_A L_A)E_{CA}}{p_{4C}} + \frac{(\alpha_{4B} w_B L_B)E_{CB}}{p_{4C}} + \frac{\alpha_{4C} w_C L_C}{p_{4C}}$$

$$L_{1A} = L_A$$
$$L_{2B} = L_B$$
$$L_{3C} + L_{4C} = L_C$$

After filling in the data and making the elementary calculations we get,

$$100 = 20 + 40E_{AB} + 25E_{AC}$$
$$100 = 20E_{BA} + 30 + 25E_{BC}$$
$$L_{3C} = 30E_{CA} + 30E_{CB} + 25$$
$$L_{4C} = 30E_{CA} + 0 + 25$$
$$L_{3C} + L_{4C} = 100$$

The solution obtained is $E_{AB} = 0.90909$, $E_{AC} = 1.74545$, $L_{3C} = 57.8125$ and $L_{4C} = 42.1875$. As the table of the gains from trade shows, the trade pattern is an equilibrium one,

	A	B	C
1	$\boxed{10.0}$	0	0
2	0	$\boxed{22.0}$	0
3	0.2	0.275	$\boxed{1.145}$
4	0.166	0.22	$\boxed{0.572}$

The world pattern of economic activities is,

	A	B	C
1	$100w_A = 1000p_{1A}$	.....	......
2	......	$100w_B = 2000p_{2B}$	....
3	......	....	$57.8125w_C = 115.625p_{3C}$
4	.....	....	$42.1875w_C = 42.1875p_{4C}$.

It has already been shown that an international trade equilibrium need not exist when all commodities can be produced in all countries. But if each country produces at least one 'special' commodity that is demanded by at least one other country, a unique international trade equilibrium must exist. There are overwhelming reasons to believe that this should be so. The very existence of a country implies some unique feature or speciality due to which it is recognised by, and on whose basis, it comes into contact with, other countries; eg. diplomatic relations, cultural exchange or tourist attraction.

Besides, different countries have unique natural endowments, social cultures and traditional skills that leave an indelible mark on the products produced by them. these 'country' attributes of products cannot be exactly imitated or reproduced in other countries. The products therefore get the names of the countries that produce it. Examples abound in everyday life; Indian muslim, turkish carpets, German tools, Swiss watches, Italian wines, Australian cheese.

The analysis of special commodities have two consequences on the system of equations. Firstly, the concepts of commodity, industry and natural exchange rates become somewhat fuzzy. Thus our tables of natural exchange rats become studded with 0's and ∞'s that reflect the irreproducibility of quality attributes in different countries. Secondly, the demand equations originating from different countries will have to be disaggregated further for each special commodity produced in the different countries. Once these considerations are admitted into the system, several possibilities of *intra-industry trade* will be seen to arise, eg. countries exporting as well as importing commodities belonging to the same 'industry'.

14. Algorithm to Determine Equilibrium

The procedure to finding the international trade equilibrium can now be summarised. The steps are as follows:

(1) From the positions of the autarky equilibria find the natural exchange rates and consider the most profitable commodity arbitrage sequences (products of natural exchange rates) which (a) assign at least one tradeable commodity to each country and (b) assign each tradeable commodity to at least one country. This assignment serves as the initial trade pattern.

(2) Try this trade pattern—find the exchange rates, the scale multipliers for the industries and the pattern of gains from corresponding to the exchange rates determined. If the pattern of gains from trade supports the trade pattern hypothesised and the scale multipliers for the industries in all countries are positive, the trade equilibrium is found.

(3) If not, determine a new trade pattern. There are four rules which must be followed to determine it.

 (a)Strike out the commodities from a country's assignment if the scale multipliers appropriate to the industries producing them are negative,

 (b) Do not assign any additional commodity to any country which has even one negative multiplier, irrespective of the gains from trade.

 (c)Assign additional commodities to countries with all-positive multipliers in accordance with the pattern of the gains from trade.

 (d) For the rest, keep the original trade assignment intact, i.e. irrespective of the gains from trade retain the commodities assigned to a country, if it has all-positive multipliers.

(4) Try the new trade pattern by setting up the appropriate Walrasian supply-demand and full employment equations.

(5) If the results, i.e. the scale multipliers and the exchange rates support the trade pattern, equilibrium is found If not, repeat steps (3) and (4) until equilibrium is found.

(6) If a trade pattern which has been tried out and found to be inconsistent with the requirements of a trade equilibrium reappears in the course of the interactions, conclude that the world system postulated, does not possess an equilibrium and abandon further effort.

15. Critical Notes on Exchange Rate Theory

The theory of exchange rates that has been presented here is at sharp variance with the absolute version of the purchasing power parity theory which enjoys prominent place in the now-dominant "monetary Approach" to the balance of payments.(8)

The absolute version is open to two objections which are so severe as to knock down the theory once and for all. Firstly, let us see how the absolute version removes the very basis of international trade. We know that if the currency exchange rates are equal to the natural exchange rates of a commodity, no country can gain by trading in the commodity. Now the absolute version requires that the exchange rates of currencies should be equal to the ratio of the domestic price levels (or the reciprocal ratios of the domestic purchasing powers of the currencies). But with exchange rates so determined no country can gain by trading. Thus if the absolute version is to hold, there can be no gains from trade and hence no trade! The absolute version is inconsistent with the very fact of international trade!

One would not go very far in the attempt to rescue the theory by suggesting (a) that the purchasing powers referred to in the theory should be international instead of domestic for that would mean all exchange rates are equal to 1 or (b) that the price levels being averages, the exchange rate would settle at the ratio of the average price levels allowing individual commodities to be gainfully trade, because as is well-known the averages themselves are not unique; there will be as many exchange rates as there are averages.

The second objection to the absolute version is that it entirely subsumes the role of the demands and supplies of currencies and the role of foreign exchanges in coordinating the demands and supplies. Why, indeed should the foreign exchange market exist if exchange rates are to be predetermined by the price levels in the countries unless we are prepared to believe the far-fetched hypothesis that the price levels are determined by the foreign exchange market? Thus the absolute version calls into question the very existence of the foreign exchange market! Faced with the choice of accepting the facts of international trade and the existence of foreign exchanges on the one hand and the absolute version of the purchasing power parity theory on the other the verdict is obvious-abandon the absolute version completely.

In fact the purchasing power parity theorem is based on a rather elementary flow. It recognises the fact that the money of other countries is demanded not for its won sake but for the sake of the goods that it will buy. This gives the *raison d'etre* for demanding all money, not only those of other countries. But it hardly follows from this that *therefore* the exchange rates must be determined exclusively by the quantities of goods that the various moneys can buy for it amounts to calling the consequence the cause. What is blatantly neglected in such an inference is the *total* expenditure on goods of all countries. It is the *total* expenditures that determine the inter-country demands and supplies of currencies. And it is these that determine their values against one another. The prices of goods are one part of the total expenditures so they do play a role in the determining the exchange rates. But clearly they are not the only part.

They seem to be the only part when purchasing powers are compared. Because purchasing powers always mean purchasing powers of *units* of currencies. But currencies are not transacted *unit* by *unit* in the market. They cannot be. What are transacted are volumes of currencies and the exchange rates must be such as to clear the demands and supplies of the *volumes* of currencies transacted.

In contrast with the absolute version, the relative version of the purchasing power parity theorem is seen to possess unquestionable validity. Changes over time of the money wage rate in a country lead to proportionate changes in all prices and corresponding changes in the natural exchange rates, the value of the national income of the country and the import bills in domestic currency values.

The result is a proportionate depreciation of the domestic currency *vis-a-vis* all the other currencies. Neither the pattern of trade nor the gains from trade are affected by changes in the wage rates. Thus, in the absence of changes other than those of the countries' price levels, the exchange rates would respond proportionately to movements in the relative purchasing powers of the countries' currencies.

In fact a general multicountry formula for the relative purchasing power parity theorem can be derived to trace the movement over time of the exchange rates when the wage-price levels in different countries are changing. Let k_i ($i = A, B, \dots, Z$) be the rates of inflation in country i. Then every year the import bill M_{ij} changes by the factor $(1 + k_j)$. Go back to equation (12) of chapter I and attach the relevant inflation factors to the intercountry import bills. Then we get,

$$\begin{bmatrix} (1+k_B)M_{BA} & (1+k_C)M_{CA} & \dots & (1+k_Z)M_{ZA} \\ (1+k_B)M_B & (1+k_C)M_{CB} & \dots & (1+k_Z)M_{ZB} \\ \dots & \dots & \dots & \dots \\ (1+k_B)M_{BY} & (1+k_C)M_{CY} & \dots & (1+k_Z)M_{ZY} \end{bmatrix} \begin{bmatrix} E_{AB}^{(1)} \\ E_{AC}^{(1)} \\ \dots \\ E_{AZ}^{(1)} \end{bmatrix} = \begin{bmatrix} (1+k_A)M_A \\ -(1+k_A)M_{AB} \\ \dots \\ -(1+k_A)M_{AY} \end{bmatrix} \tag{22}$$

where $E_{Aj}^{(1)}$ ($j = B, \dots, Z$) represent the *current-period* exchange rates. Then solving by Cramer's rule we get,

$$E_{Aj}^{(1)} = \frac{\prod_{i \neq j}(1+k_j)\det M_j}{\prod_{j=B}(1+k)\det M} \quad j = B, \dots, Z \quad i \neq j \tag{23}$$

where M is the matrix of base-period intercountry import bills (equation 12 of chapter i), M_j is the matrix obtained by replacing the jth column ($j = B, \dots, Z$) of the matrix M by the column vector of country A's base-period import bill M_{Aj} and Π denotes the product of the inflation factors.

Since the base-period exchange rates are,

$$E_{Aj}^{(0)} = \frac{\det M_j}{\det M} \quad j = B, \dots, Z \tag{24}$$

We obtain on substitution,

$$E_{Aj}^{(1)} = KE_{Aj}^{(0)} \tag{25}$$

where K is the quotient of the products of the inflation factors appearing in equations (23) above. The remaining exchange rates can be worked out from the neutrality, conditions. We have thus shown that while *changes* in the exchange rates are determined by *changes* in prices, the levels of exchange rates are not determined by the *levels of prices*[9].

Finally, some comments are in order as regards why scholars seem to have generally accepted the absolute purchasing power parity theorem despite its drawbacks. First, the relative version, known to be supported by empirical evidence during some periods, seems logically to imply and to follow from the absolute version—if price level changes are important determinants of exchange rate changes it is indeed tempting to infer that price levels are important determinants of the exchange rate levels. The danger lies in this temptation.

Actually, the evidence about the relative version is mixed—sometimes exchange rates move in close conformity with the price levels and sometimes they do not. [See Genberg (1978), Isard (1978) and Ketseli-Papaefstratiou (1979) for comprehensive surveys of the empirical studies of the purchasing power parity theorem]. That the empirical evidence should be mixed is perfectly consistent with the theory presented in this book. For the sizes of countries, the tastes and preferences towards commodities, the techniques of production (and, as we shall see, intercountry investments transfers, tariffs, etc.) also determine the exchange rates. But the mixed evidence is inconsistent with the absolute version, for the absolute version requires that the price levels, and only the price levels, should determine the exchange rates.

Moreover consider this. How does one test for the validity of the absolute version independently of the relative version? The only way (which has not to my knowledge been tried out ever) is to conduct a cross-section exercise to check if individual exchange rates are equal to the quotients of the price levels, e.g.

$$E_{AB}^{t} = \frac{P_{At} / P_{A0}}{P_{Bt} / P_{B0}} \tag{26}$$

where 0 and t denote the base-period and current-period respectively. Even casual empiricism will reveal that wildly different answers will be obtained. [Perhaps it is on this account that econometricians have not published this exercise]. If a time series exercise is done, as has invariably been the case, only the validity of the relative version will be tested. Neither the validity nor the invalidity of the relative version *should* be imputed to the absolute version.

The second cause for the belief in the absolute version stands at a different level. In fact it establishes the validity of the absolute version in the circumstances assumed. However, the circumstances assumed are at great variance with international reality. Suppose that America and India had no economic relations whatsoever, direct or indirect. An American wants to visit India. He reckons that the basket of commodities he plans to consume while in India will cost him Rs.5000 whereas the same basket would cost him $200 in America. His exchange bank verifies these figures and accordingly sets the exchange rate (ignoring bank charges, commissions, etc.) at Rs.25 = $1. Why? Because the bank will have to buy Rs.5000. Nobody will buy $200 and sell Rs.5000 if an Indian brother does not want to make a reciprocal visit to America. So the bank must purchase a basket of commodities in America for $200 sell it in India for Rs.5000 and then make the rupees available to the American. (Let us not spoil the story by interrupting "what about non-tradeables?" "what about distinct goods?" etc.) Otherwise, the bank must borrow rupees. It will borrow the rupees only if it knows that it can clear the dollars against the rupees. It can do this only by using the dollars to purchase a basket of commodities in America and selling them in India for rupees and thus repaying the loan. The absolute version holds. But alas only in the artificial circumstances created for it to hold.

The purchasing power parity theorem also proves to be useful in a situation when nations commence economic relations after a period of prolonged hostilities, or wars, during which international payments are suspended. Black market transactions may exist but the market may be too thin to be considered representative. To commence economic relations *some* exchange rates need to be initiated. The best guess for the exchange rate under these circumstances, and in the absence of other information, is to multiply the pre-war rate by the quotient of the degree of inflation experienced by the countries during the transitional period, These exchange rates can be adopted, to start with. In due course the markets will bring out the correct exchange rates which reflects the structural changes that have taken place in addition to the changes in the relative purchasing powers of currencies during the transition. [See Cassel (1914)].

16. Fallacy in the Factor-price Equalisation Theorem

Another proposition which has received an undue share of attention in relation to its intrinsic vacuity, is the factor-price equalisation theorem developed in the early contributors by Heckscher (1919, 1949), Ohlin (1933), Lerner (1952), Samuelson (1948, 1949), Tinbergen (1949) McKenzie (1955) and on which by now a vast literature exists.

Let us try to find the key assumptions on which the equalisation of factor prices takes place. Suppose that in a two-country two-commodity trade situation the pattern of comparative advantage is A-1 B-2. Then the factor-prices are equalised only if both countries are incompletely specialised in the production of both commodities in the post-trade situation. For only then will the following relations hold;

$$\frac{w_A}{p_{1A}}=\frac{w_B}{p_{1B}}=\frac{w_A}{E_{AB}\,p_{1B}}=\frac{w_B}{E_{BA}\,p_{1A}}\quad\frac{r_A}{p_{1A}}=\frac{r_B}{p_{1B}}=\frac{r_A}{E_{AB}\,p_{1B}}=\frac{r_B}{E_{BA}\,p_{1A}}\tag{28}$$

$$\frac{w_A}{p_{2A}}=\frac{w_B}{p_{2B}}=\frac{w_A}{E_{AB}\,p_{2B}}=\frac{w_B}{E_{BA}\,p_{2A}}\quad\frac{r_A}{p_{2A}}=\frac{r_B}{p_{2B}}=\frac{r_A}{E_{AB}\,p_{2B}}=\frac{r_B}{E_{BA}\,p_{2A}}\tag{29}$$

where r_A and r_B are the rates of profit. It follows that in the post-trade equilibrium,

$$\frac{w_A}{w_B}=\frac{r_A}{r_B}=E^1_{AB}=E^2_{AB}=E_{AB}=\frac{M_{AB}}{M_{BA}}\tag{30}$$

It is evident that if the techniques of production are fixed, i.e. they do not change between the autarky and trade situation, the event posited in equations (30) can never come about unless it was the one which prevailed. But then it would contradict the assumption that A has the comparative advantage in commodity 1. A trade situation in which no country has any comparative advantage can always be called a trade equilibrium provided $E_{AB}^{1} = E_{AB}^{2} = E_{AB}$ but it is one in which no country gains from the trade. We may therefore call it a null-trade *equilibrium* in which factor-price equalisation holds trivially.

But if there are to be some gains from trade (say $E_{AB}^{1} < E_{AB}^{2}$) and yet factor price equalisation is to take place then it is mandatory that the techniques of production must be changed in both countries when they are faced with the trade situation. It is assumed that such techniques always exist. there is, however, a restriction placed by equation (30) on the new techniques of production that can be selected viz. that they should be such as to wipe off the comparative advantage that existed in the pre-trade situation. In other words in the pre-trade situation, the relative commodity price ratios and factor-price ratios must be unequal but in the post-trade situation such techniques will be chosen so as to equate both these ratios in both the countries. But this requires that in one country the rate of profit must actually *fall* to make the relative factor prices equal. Ask why should producers in that country select a technique which results in a fall in the rate of profit?

So we are in a dilemma. The factor-price equalisation theorem seems to be based on a criterion of the choice of technique that is different from the usual time-tested one viz. choose that technique which will maximise the *rate* of profit. We conclude that the factor-price equalisation theorem is valid on its own criterion of the choice of technique, whatever that criterion may be. There is a fallacy in the theorem only if he usual criterion is adopted.

Notes

1. The classical controversy on these doctrines was concerned with which of the two, viz the fact of a comparative advantage or the existence of a greater exportable surplus was the cause of trade. The answer is both. Because the existence of a greater exportable surplus is another way of stating the existence of comparative advantage provided tastes productive capacities are identical in both countries. If they are not, "vent-for-surplus" considerations must be stated separately. Observe in this context that if country A produces a greater output of commodity 1 than B on specialisation and country B produces a greater output of commodity 2 country A on specialisation then (a) $L_A/l_{1A} > L_B/l_{1B}$ and (b) $L_A/l_{2A} < L_B/l_{2B}$. Condition (a) implies that $l_{1A}/l_{1B} < L_A/L_B$ and (b) implies that $L_A/L_B < l_{2A}/l_{2B}$, combining which we get $l_{1A}/l_{1B} < L_A/L_B < l_{2A}/l_{2B}$, which contains the statement of bilateral comparative advantage.
2. A note of explanation is in order because McKenzie. (1954) by using a similar model seems to reach at the opposite conclusion. He proves that equilibrium always exists in a Graham model and , further, that the equilibrium is unique (upto a multiplicative constant). On a closer scrutiny, the disparity in conclusions will be seen to disappear because McKenzie does not introduce gains-of-trade considerations in his analysis. As a result, the possibility that the *autarky* situation itself may be an equilibrium is not excluded. A similar comment applies to the models of Mosak (1944) and Takayama (1972). The theorems as stated in this text, refer to the existence and uniqueness of *trade* equilibrium. Thus trade equilibrium may not exist, although if it does, it must be unique.
3. The scale of natural exchange rates is similar to Edgeworth's (1894) logarithmic scale of comparative advantages in function.

4. Graham did not indicate any method for his trial and error procedure except at one place where he suggested that greater numbers of commodities must be allotted to countries of greater sizes. See Graham (1948) pp. 92.93.
5. Marshall (1923) supposed that exportables and importables could be considered as fixed in composition and measured in terms of "representative bales". Edgeworth (1894)suggested that an "ideal" article typical of the *total* volume of a country's trade can be conceived for this purpose.
6. Within (1953) has formulated an ingenious linear programming version of Graham's model. In this version the value of the world output is maximised subject to technical coefficients, labour endowment and demand constraints of mixed type, i.e. using inequalities as well as equalities. His model is a faithful representation except at one place. His equation (6) in the Appendix, pp. 543 states that $\alpha_j Z = \lambda_j X_j$ where α_j is the proportion of the total value of *world* transactions devoted to product j. This amounts, as we know, to adding the labour in all countries—a procedure defensible only when discussing *interregional* as opposed to *international* equilibrium. The labour/income constraints must be drawn country-wise in a problem of international equilibrium. With this modification Whitin's model will give all the results of the standard Graham model.
7. This was certainly not the case with the early trade theories. Indeed the theories of Bastable (1903) and Cairnes (1874) gave a pride of place to special factors and special commodities. This is true of Ohlin (1932) as well. The Heckscher-Ohlin theorem and further propositions based on it, with their assumption of incomplete specialisation was largely responsible for restricting the scope of trade theory to the case of identical commodities. Smith (1776) and ricardo (1817) did assume identical commodities, but only to sharpen the principles. After the appearance of Leontiefs paradox [Leontief (1954), (1956)], however, a huge literature now exists which recognises the importance of human capital [Kravis (1956), Kenen(1965), Baldwin(1971)], natural resources, [Vanek(1963)], technology and innovation [Vernon(1966)], and tastes for specific product designs [Grubel and Lloyd(1975)].
8. The Monetary Approach was initiated by two collections-Frenkel and Johnson (1976) and Frankel and johnson(1978) and developed rapidly through the eighties to become the dominant approach in international macroeconomics. See Dornbusch (1980) for a lucid but substantial treatment of the method of this approach.
9. This is not a new criticism. It was made most forcefully by Keynes (1923) and by Samuelson (1964) amongst several others but has been conveniently ignored in the forward march of the "Monetary Approach".

6

GENERALISATIONS

"I declare its marked out like a chess board!"
Alice said at last. "It's a great huge game of chess that's
being played all over the world—
if this is the world at all, you know".

Introduction

It is now time to relax the several assumptions on which the model of trade in Chapter V was based. To begin with we shall study the effects arising from changes in the parameters/data of the model, i.e., the wage rates in different countries, the tastes and preferences of their citizens, the labour coefficients of production and the sizes of countries. Of these, we have already discussed the effects of changes in wage rates in section 13 and the consequences of changes in tastes and preferences and section 7 of Chapter V respectively. The remaining two: the effects of the changes in the sizes of countries on account of growth and the changes in the labour coefficients of production due to improvements in technology are studied in this chapter.

Subsequently we may relax the assumptions of the international immobility of labour and finance. (Dealt with in section 4 and in sections 10 and 11 respectively.) Finally, we shall also consider the effects of the instruments of international economic policy, viz. tariffs and quotas as well as the economics of fixed exchange rate systems. The theory of the customs unions has been considered as an extension of the theory of tariffs in section 8. And transport costs which, like tariffs, are trade deterrents have been considered in section 5.

There is no undue claim to realism here because the discussion will be conducted in the static classical realm—production exclusively by means of labour and no capital accumulation. Yet the discussion will not be without some interest from the theoretical standpoint. In the previous literature, many of these subjects have been treated in terms of the essentially static microeconomic and macroeconomic models, e.g. tariffs and quotas are discussed in the partial-equilibrium demand-supply model and intercountry transfers in the static Keynesian or Monetary Approach models.(1) One of the objects of this chapter will be to show why the division of international economics into micro and macro is unwarranted and how a theory that combines both aspects is possible.

1. Growth and Trade

Labour being the only factor of production, the only source of growth that we can presently admit is the growth in the labour force in different countries. Suppose each of the countries in the world economy to be growing at a 'natural' rate of growth determined by customs, tradition, tastes, preventive medicine, health-care and demographic characteristics.

An increase in the (relative) size of a country has two effects, both of which influence her trade pattern. Firstly, the country's capacity to produce goods also increases.

In so far as her propensities to import foreign goods are positive, an increase in the country's size results in an increase in her import bills, and if they are not offset by a similar increase in the demand for her goods, will lead to a depreciation of her currency. A continual depreciation of her currency has the effect of gradually discouraging imports and encouraging home production of the goods instead. This effect is further enhanced by the second effect i.e., an increase in the capacity to produce. In time, as the growth continues, the country may also become an exporter of the commodities which she was earlier importing.

If, however, the growing country has negative propensities to import from abroad the opposite effects would take place: her import bills will fall, her currency will appreciate and she will tend to specialise in production.

The effects just narrated depend crucially on the foreign countries' rates of growth as well as the foreign countries' intensity to import from the country which is growing. Of considerable importance is the behaviour of the trade pattern when, in the course of relative growth, the currency exchange rates change and hit one or more natural exchange rates. After this has happened the growth process, at least over a certain range, ceases to have any effect on the exchange rates, but only affects the intercountry interindustrial allocation of labour and the composition of outputs. This will be illustrated with a two-country example. A similar reasoning will be found to hold good in the multicountry case.

Consider countries A and B with the following autarky equilibria,

	A	B
1	$20w_A = 120p_{1A}$	$60w_B = 30p_{1B}$
2	$50w_A = 75p_{2A}$	$60w_B = 300p_{2B}$
3	$30w_A = 60p_{3A}$	$30w_B = 100p_{3B}$

With w_A = Re.1, w_B = \$1 the natural exchange rates are,

$$E^1_{AB} = 0.08333 \qquad E^1_{BA} = 12$$

$$E^2_{AB} = 3.333 \qquad E^2_{BA} = 0.3$$

$$E^3_{AB} = 1.6666 \qquad E^3_{BA} = 0.6$$

The international trade equilibrium is A-1, B-2,3. Country A's import bill is Rs.80 and her export earnings in foreign currency are \$60 so that the equilibrium exchange rate is $E_{AB} = 1.3333$. Now suppose growth to occur in country A so that her size increases to $L_A = 125$ and assume (for convenience) that country B remains stationary. In the new equilibrium country A's import bill rises to Rs.100 but her exports continue to fetch \$60 so that the equilibrium exchange rate moves to $E_{AB} = E^3_{AB} = 1.6666$. At this point country A gains nothing by importing commodity 3 from country B but nevertheless continues to do so. The international pattern of activities will show this;

	A	B
1	$125w_A = 750p_{1A}$	....
2	....	$97.5w_B = 487p_{2B}$
3	....	$52.5w_B = 175.0p_{3B}$

Next suppose country A grows further to $L_A = 137.5$. The exchange rate continues to remain at $E_{AB} = E^3_{AB} = 1.666$ but country A now allocates a greater quantity of labour to the production of commodity 3 which she partly produces and partly imports;

	A	B
1	$127.5w_A = 765p_{1A}$	....
2	....	$101.25w_B = 506.25p_{2B}$
3	$10w_A = 20p_{3A}$	$48.75w_B = 162.5p_{3B}$

Further (gradual) increases in the size of country A will only show an increasing production of commodity 3 in country A and a diminishing production in country B until country B becomes a net importer of commodity 3. This is shown by the example below. when the size of country A increases to 360, the equilibrium trade pattern shifts to A-1,3 B-2, with an equilibrium exchange rate $E_{AB} = 2$

	A	B
1	$192w_A = 1152p_{1A}$	....
2	....	$150w_B = 750p_{2B}$
3	$168w_A = 336p_{3A}$	....

The terms of trade of country A's exportables deteriorate further.

The connection of these results with the standard models of trade and growth due to Rybczynski (1955), Hicks (1953), Johnson (1958), Bhagwati (1958), Findlay (1973) and the 'North-South' models due to Prebisch (1950), Singer (1950), Lewis (1954) will be apparent at once if technical progress, which we have not so far dealt with, is left out and growth only of the factors of production is considered. To explore all manner of similarities and dissimilarities is obviously beyond the scope of this work. It must suffice to jot down a few indicative notes.

Firstly, observe that mere growth in one country has the effect of deteriorating her terms of trade with the smaller countries, and at the same time, a decreasing degree of her 'dependence' on the smaller countries in terms of the number of commodities she imports. Thus the share of growing countries in world exports tends to increase over time as they tend to produce more and more commodities which they hitherto imported. At first glance this looks like an increasing monopoly of the large countries in world trade. But the appearance is deceptive, because the large countries' gains from trade are deteriorating in this process while those accruing to the smaller countries are increasing.

Secondly, note that all (relative) growth is in fact immiserising in per capita terms if the growing countries' propensities to import are positive. Because growth induces these countries to move closer and closer to autarkic production and prevents them from availing the benefit of imports, unless they make transfers of income to the smaller countries (see sections 10 and 11 below) to prevent the depreciation of their own currencies; prevent, because the sum of the propensities to import of the growing and the small countries will tend to exceed unity for every appreciation of the growing countries' currency. In effect the large countries will at least, in part, be required to pay for their gains of trade.

2. Technical Progress

Labour being the only factor of production, the only kind of technical progress we can admit is a reduction in the labour coefficient of production. Technical progress of this kind has two effects. First, it effects the pattern of the intercountry comparative advantages as reflected in changes in the natural exchange rates. Second, it reduces the requirement of labour (per unit of output) in the industries enjoying the technical progress and releases labour that can be utilised in other industries. [This latter effect is absent when Engels' demand equations are used but becomes visible when the general linear expenditure system is used].

Generally speaking, technical progress, if it takes place in the industries producing the exportable commodities of a country will result in an increase in the overall advantage of trade and although it leads to a deterioration in the terms of trade of the exporting country, the levels of consumption in both countries improve as compared to the previous situation. 'Immiserising growth' cannot take place by an increase in the productivity of the exportable sector as has been suggested by Bhagwati (1958).

On the other hand if technical progress occurs in the industries producing the importables, it may beyond a certain point encourage domestic production of the importables and the currency of country *A* will appreciate leading to an improvement in her terms of trade. Even so, no country's welfare will be reduced by this kind of technical change either. The same remark applies also to a commodity that was previously non-traded between the countries on account of the equilibrium exchange rate being equal to the natural exchange rate attributable to the non-traded good.

Finally, if technical progress takes place in the industries producing non-tradeable commodities, and if demand conditions are adequately described by Engels' equations, there will be no effect on the international trade equilibrium, whereas if linear expenditure systems hold, the effects will be the same as those of an increase in the size of the country.

The remarks made above apply chiefly when technical progress is envisaged as progress in *process technology*. There is, besides, the other kind of technical progress involving improvements in product technology. This latter has complicated effects depending upon (a) whether the new products seek to partially substitute or altogether supplant the existing products, (b) whether it is an altogether new product and (c) if so, the manner in which the propensities to consume of the established products reduce to accommodate the new product, nationally and internationally.

3. Labour Migration

The consequences of the international migration of labour are manifold. The most obvious effect of migration of labour between countries is to change the relative sizes of the countries from which labour emigrates reduce in size and the others increase in size. Thus all the effects of relative growth on trade described in section 2 above follow upon the migration of labour, unless it be for temporary purposes like tourism.

But that is not all. The migrating workers may carry with them their long acquired habits of consumption as well as skills and knowledge of techniques of production. Thus both the pattern of tastes and preferences as well as the technological capabilities of the countries of immigration undergo a change. Moreover, specific non-tradeables or hitherto (tradeable but) non-traded goods that are a part of the basket of consumption of the immigrating population will either be produced or imported into the countries of immigration. Further, the immigrant labour may have dependents back home to whom they will send remittances, adding to effects of the factors above those of intercountry transfers. Finally, if there is economic discrimination of immigrant labour, the *average* money wage rate may rise or fall.

In short labour migration may result in a *complete* change in the data of the international trade problem.

4. Transport and distribution costs

The relatively greater incidence of transport costs has traditionally been considered a distinguishing feature of international, as opposed to interregional trade. To this may be added the relatively greater incidence of transaction costs in general; distributors' margins, commissions, brokerage, insurance, etc. As between the regions of a country so between the nations of the world, the effect of transport costs (a term which we shall use to include all types of transactions costs) is to reduce the extent of the gains from trade and offer incentives to local production. The manner of incorporating these costs in the international trade model and the effects of their incidence are illustrated below.

Let t^i_{jk} be the transport cost rate (ie. transport cost amount as a percentage of the price of commodity i in the country j) from country j to country k for commodity i. Since the transport costs amounts must be equal irrespective of whether commodity i is transported from country j to country k or vice versa, it follows that the intercountry transport costs rates (as percentages) must satisfy the condition,

$$t^i_{jk}\, p_{ij} = t_{kj}\, p_{ik}\, E_{jk} \tag{1}$$

where the term on the left hand side is the transport cost for commodity i from country j to country k in the currency of country j and that on the right hand side is the transport cost from country k to country j converted into j country's currency. From (1) we get,

$$t^i_{kj} = [E^i_{jk} / E_{jk}]\, t^i_{jk} \tag{2}$$

which means that our data need specify either t^i_{jk} or t^i_{kj} not both, since one determines the other.

Consider a two-country three-commodity illustration and suppose, for the sake of convenience, that the transport costs in domestic trade are zero. Suppose the autarky equilibria to be,

A	B
$120w_A = 1200p_{1A}$	$50w_B = 100p_{1B}$
$120w_A = 240p_{2A}$	$50w_B = 500p_{2B}$
$120w_A = 40p_{3A}$	$50w_B = 250p_{3B}$

where the wage rates are $w_A =$ Re.1 and $w_B =$ \$1. The prices in the two countries are

	A	B
1	0.1	0.5
2	0.5	0.1
3	3	0.2

The international trade equilibrium in the absence of transport costs is

A	B
$360w_A = 3600p_{1A}$	....
....	$75w_B = 750p_{2B}$
....	$75w_B = 375p_{2B}$

with the equilibrium exchange rate $E_{AB} = 4.8$

Now suppose the transport cost rate to be $t^i_{BA} = 0.5$ for all commodities i transported from country B to country A. Then by the formula in equation (2).

$$t^1_{AB} = 12 \quad t^2_{AB} = 0.48 \quad t^3_{AB} = 0.16$$

The question is whether the pattern of trade A-1 B-2,3 continues to hold after the introduction of transport costs. To find this, two gains-from-trade tables will need to be constructed. First, consider the gains from trade to country A's citizens who must exchange Re.1 for dollars and purchase commodities from country B at prices inclusive of transport cost, $p_{iB}(1 + t^i_{AB})$

	A	B
1	$\boxed{10}$	0.277
2	$\boxed{2}$	1.388
3	0.333	$\boxed{0.694}$

It shows that A will find it cheaper to produce commodity 2 rather than import it from country B. Next consider the position of country B which must exchange \$1 for (4.2) rupees and purchase from country A at prices $p_{iA}(1 + t^i_{AB})$ where t^i_{AB} $(i = 1, 2, 3)$ have been worked out above.

	A	B
1	$\boxed{3.692}$	2
2	6.486	$\boxed{10}$
3	1.379	$\boxed{5}$

B's gain-from-trade table shows that B would still find it profitable to import commodity 1 from country B. Accordingly suppose the trade pattern to be A-1,2 B-2,3 and proceed to set up the Walrasian equations. Before we do so, however, note that we shall suppose the transport services to be provided by the exporting industry and suppose that it is paid in terms of the commodities it exports. This assumption of the transport services being vertically integrated saves us the botheration of showing distinct transport industries in the countries whose turnover and employments will change with every change in the world trade pattern being tried out. The equation are,

$$\begin{aligned} 1200\lambda_{1A} &= 1200 + 500E_{AB} \\ 240\lambda_{2A} + 500\lambda_{2B} &= 2400 + 500 \\ 250\lambda_{3A} &= 600E_{AB} + 250 \\ 20\lambda_{1A} + 120\lambda_{2A} &= 360 \\ 50\lambda_{2B} + 50\lambda_{3B} &= 150 \end{aligned}$$

The terms $500E_{AB}$ and $600E_{BA}$ in equations (1) and (3) are inclusive of transport costs. The solution is $\lambda_{1A} = 2, \lambda_{2A} = 1, \lambda_{2B} = 2, E_{AB} = 2.4$. At this exchange rate the transport cost rates from country A to country B are,

$$t^1_{AB} = 6 \quad t^2_{AB} = 0.24 \quad t^3_{AB} = 0.08$$

We must now check whether the gains from trade at the new exchange rate and the new transport rates support the trade pattern A-1,2 B-2,3. The tables of country A and B respectively are as follows.

	A	B
1	$\boxed{10}$	0.555
2	$\boxed{2}$	1.777
3	0.333	$\boxed{1.388}$

	A	B
1	$\boxed{3.428}$	2
2	3.870	$\boxed{10}$
3	0.7407	$\boxed{5}$

They are found to support the trade pattern. Besides since the scale multipliers are positive we may conclude that the equilibrium is determined.

To find the trade equilibrium in the presence of transport costs, it is not always necessary to first find the trade equilibrium without transport costs and , then introduce transport costs to see how it is modified. We have done so because, in order to find the transport rates t^i_{AB} corresponding to the given rates t^i_{BA} requires, according to equation (1), that *some* exchange rates should be known. The closest approximation for the trial exchange rates are those obtained by finding the trade equilibrium without transport charges.

.One can, however, use the data to directly obtain the trade patterns in the presence of transport costs. This requires, firstly, that *all* the intercountry transport cost rates should be given as *data*. but how can we be sure that the rates to suppose that the transport cost rates assumed in the data are initial quotations that will be revised and made consistent after the exchange rates are known and/ or that there will be *transport arbitrage* among transporters who subcontract with those offering the lowest quotations. Either way, transport rates are bound to be revised until they satisfy equation(1)

Secondly, the arbitrage sequence conditions, which we use to determine the feasible trade assignment will have to be generalised to take account of the transport cost rates. Thus to determine whether an assignment, say A-1 B-2 is feasible we need to formulate the corresponding arbitrage sequence and check whether it is profitable. Thus a trader having Re.1 can purchase $(1/p_{1A})$ units of commodity 1 in country A, sell them in B for (p_{1B}/p_{1A}) dollars, obtain a net revenue of $(p_{1B}/p_{1A} - t^1_{AB}E_{BA})$ dollars after paying transport costs, use the net proceeds to purchase $(p_{1B}/p_{1A} - t^1_{AB}E_{BA})/p_{2B}$ units of commodity 2, sell them in country A at price of p_{2A}, pay transport costs at the rate $t^2_{BA}p_{2B}$ per unit of commodity 2 and make a profit only if,

$$\left[\frac{p_{1B}}{p_{1A}} - t^1_{AB}E_{BA}\right]\left[\frac{p_{2A}}{p_{2B}}\right] t^1_{AB}E_{BA} - t^2_{BA}\left[\frac{p_{1B}}{p_{1A}} - t^1_{AB}E_{AB}\right]E_{AB} > 1$$

Substituting to eliminate the currency exchange rates from equations (1) and (2) and rearranging terms gives the final conditions,

$$E^1_{AB}E^2_{BA} < 1 - t^1_{BA} - t^2_{AB} + t^1_{AB}t^2_{BA} \tag{3}$$

This condition is a general version of that obtained when transport costs are assumed away. It is a more stringent one because the term on the right hand side will be less than unity. The with–transport arbitrage sequence conditions can be generalized to several countries and several commodities by a similar procedure as that adopted above.

In so far as the alogrithm to find the trade equilibrium is concerned, the presence of transport costs, as we have seen, makes the computations more tedious; in each iteration as many gains-from-trade tables have to be computed as there are countries.

Essentially the same kind of results, that we have obtained in the two-country three-commodity example will appear in the general multicountry multicommodity context—increases in transport costs result in an overall reduction in the volume of foreign trade, although they may take away the advantage from some countries and confer it upon others.

5. Tariffs

Tariffs, or customs duties, on imported commodities have two effects. Firstly they raise the prices of imported commodities and hence reduce their demands. If the tariffs are sufficiently steep they may encourage domestic production of the commodities and result in a change of the world trade pattern Secondly , tariffs provide the government with revenue, which the government spends in some stipulated manner(2). To determine the consequences of tariffs it is necessary to consider both these effects.

Before we proceed an important limitation of the analysis must be mentioned. We are working in the classical framework. There is full employment in each country. Therefore, an important objective of tariffs viz., to protect employment, will be automatically ignored. Thus, we shall do no more than show how tariffs affect the trade patterns and real incomes in the various countries. The effects of tariffs in protecting employment, and on the real income distribution between countries, can be understood only in the more general framework of Chapter IX below.

Let us first consider the case of trade between two countries in two commodities. Suppose their autarky equilibrium to be,

	A	B
1	$50w_A = 100p_{1A}$	$50w_B = 50p_{1B}$
2	$50w_A = 50p_{2A}$	$50w_B = 100p_{2B}$

With wage rates w_A = Re.1 and w_B = \$1, the prices are

$$p_{1A} = 0.5 \qquad p_{1B} = 1$$
$$p_{2A} = 1 \qquad p_{2B} = 0.5$$

so that the international trade equilibrium is

	A	B
1	$100w_A = 200p_{1A}$	....
2	....	$100w_B = 200p_{2B}$

and the equilibrium exchange rate is $E_{AB} = 1$.

The levels of consumption in the post-trade situation of the two commodities in the two countries are tabulated below.

	A	*B*	*Total*
1	100	100	200
2	100	100	200

We wish to introduce tariffs in this setting. As a preliminary note that if the government of country *A* imposes a tariff at the rate t_{2A} on commodity 2 (charged on the foreign currency price of commodity 2), the consumers in country *A* will purchase the following quantity of commodity 2,

$$\frac{(\alpha_{2A} w_A L_A) E_{BA}}{p_{2B}(1 + t_{2A})} \tag{4}$$

Each unit of commodity 2 purchased is liable to tariff at the rate t_{2A} so that $t_{2A}\, p_{2B}\, E_{AB}$ is the tariff revenue per unit of commodity 2 in terms of the currency of country *A*. The total tariff revenue is the tariff revenue per unit multiplied by the number of units purchased of commodity 2, which works out to,

$$\left[\frac{(t_{2A})}{(1 + t_{2A})}\right] \alpha_{2A} w_A L_A \tag{5}$$

Thus if the rate of tariff $t_{2A} = 25$ per cent, the tariff revenue of the government of country *A* is (0.25/1.25)(Rs.50) = Rs.10. We shall suppose that the government spends this amount on commodity 1 alone. (Surely there is no sense to the tariff if the amount were to be entirely spent of commodity 2. Some proportion could be.)

To work out the equilibrium in the post-tariff situation we set up two Walrasian equations and two full-employment equations:

$$\begin{aligned}
100\lambda_{1A} &= \frac{60}{0.5} + \frac{50E_{AB}}{0.5} \\
100\lambda_{2B} &= \frac{50E_{BA}}{(0.5)(1.25)} + \frac{50}{0.5} \\
50\lambda_{1A} &= 100 \\
50\lambda_{2B} &= 100
\end{aligned}$$

where, in the first equation Rs.60 represents the *post-tariff* expenditure in country *A* on commodity 1, including Rs.50 by private consumers and Rs.10 by the government, The international trade equilibrium indicated is,

	A	*B*
1	$100w_A = 200p_{1A}$	....
2	....	$100w_B = 200p_{2B}$

which is identical, in terms of the outputs produced with that obtained in the pre-tariff situation, but with an exchange rate $E_{AB} = 0.8$. The levels of private consumption of the two commodities in the two countries are different too;

	A	*B*
1	100	80
2	100	100

Comparing this with the pre-tariff levels of consumption we observe that B's welfare has reduced on account of the depreciation of her currency and the consequent deterioration of her terms of trade. Country *A* has maintained the pre-tariff levels of consumption intact (and hence the pre-tariff level of welfare) because the appreciation of her currency and the resulting improvement of her terms of trade have exactly counter balanced the increase in the price of the commodity due to the tariff. Besides country *A* has a larger government activity than before which is effectively financed by the reduction in the welfare of country *B*. Thus although the world production levels have not reduced due to the tariff, the world welfare has got redistributed with country *B* losing in comparison with the pre-tariff position.

Every increase in the tariff rate will be increasingly harmful to country *B* until it reaches a point at which the exchange rate of currencies will be equal to the lower natural exchange rate, $E^1_{AB} = 0.5$, at which country *B* will prefer to produce commodity 1 at home, instead of importing it from country *A*.

In general in all cases of trade between two countries in two commodities, where the trade equilibrium is one of complete specialisation, the country on whose exports a tariff is imposed loses in the post-tariff situation whereas the country which levies the tariff gains. So long as the tariff rate is not large enough to change the trade pattern the world levels of production are unaltered. This proposition, however, requires that the government spend its tariff revenue on the exportable of that country. Then, in effect, the tariff will be extracted from the foreign country! It is no wonder that the foreign country would usually retaliate with a tariff in these circumstances, and a tariff war which simply reduces private welfare *ad infinitum* without a change either in the trade pattern or the levels of production is not altogether inconceivable.

A similar conclusion holds good in the more general case of trade between two countries in several commodities. So long as the government adopts the policy of spending the revenue on the exportables and of not charging very steep tariff rates, the terms of trade of the tariff imposing country must improve, to the detriment of the foreign country. In this general case, changes in world levels of production are not ruled out.

Should, however, the government spend even a part of the tariff revenue on imports the proposition is diluted. In this case, the private consumption of the imported commodities in the tariff imposing country must reduce to some extent.

To find the tariff rate at which a traded commodity becomes non-traded between two countries the following simple formula can be used. Suppose country A imposes a tariff of commodity i imported from country B at a rate t_{iA}. Then for commodity i to become non-traded t_{iA} must be such as to make the consumers of country A indifferent as between purchasing the commodity at home or importing it from B, i.e.

$$p_{iB}(1+t_{iA})E_{AB} = p_{iA}$$

$$t_{iA} = \frac{E^{i}_{AB}}{E_{AB}} \tag{6}$$

where E_{AB} is the free-trade equilibrium exchange rate. Thus if E^{i}_{AB} is close to E_{AB} in free-trade equilibrium, a small tariff will result in a change in the world trade pattern. [Elliott (1937) is one of the earliest results of this type.]

Next consider an example of three countries producing four commodities of which one, say commodity 4, is a non-tradeable. And suppose their autarky equilibria with wage rates Re.1, \$1 and ¥1 respectively are,

	A	B	C
1.	$20w_A = 120p_{1A}$	$60w_B = 150p_{1B}$	$60w_C = 25p_{1C}$
2.	$50w_A = 80p_{2A}$	$60w_B = 500p_{2B}$	$60w_C = 60p_{2C}$
3.	$30w_A = 60p_{3A}$	$30w_B = 100p_{3B}$	$80w_C = 800p_{3C}$
4.	$50w_A = 500p_{4A}$	$50w_B = 500p_{4B}$	$50w_C = 500p_{4C}$

The prices and the natural exchange rates are as follows,

$p_{1A} = 0.1666$	$p_{1B} = 0.4$	$p_{1C} = 0.8$
$p_{2A} = 0.625$	$p_{2B} = 0.12$	$p_{2C} = 1$
$p_{3A} = 0.5$	$p_{3B} = 0.3$	$p_{3C} = 0.1$
$p_{2A} = 0.1$	$p_{4B} = 0.1$	$p_{4C} = 0.1$
$E^1_{AB} = 0.4166$	$E^1_{BC} = 0.5$	$E^1_{CA} = 4.8$
$E^2_{AB} = 5.2083$	$E^2_{BC} = 0.12$	$E^2_{CA} = 1.6$
$E^3_{AB} = 1.6666$	$E^3_{BC} = 3$.	$E^3_{CA} = 0.2$

The equilibrium trade pattern is A-1,4 B-2,4 C-3,4 with a configuration of world activities given by $\lambda_{1A} = 5$, $\lambda_{4A} = 1$, $\lambda_{2B} = 2.5$, $\lambda_{4B} = 1$, $\lambda_{3C} = 2.5$, $\lambda_{4C} = 1$, and currency exchange rates $E_{AB} = 0.8666$, $E_{BC} = 0.5384$, $E_{CA} = 2.1428$ implying an international ratio of commodity exchange of 10 units of 1 = 16 units of 2 = 35.66 units of 3.

Suppose the government of country A impose a tariff on commodity 3 at the rate of t_{3A} per unit. The total tariff revenue is the tariff revenue per unit multiplied by the quantity of commodity 3 purchased in country A, which works out to

$$\left[\frac{(t_{3A})}{(1+t_{3A})}\right](\alpha_{3A}w_AL_A)$$

If the tariff rate is 50 per cent, the tariff revenue of the government of country A will be (0.5/1.5)(Rs.30) = Rs.10 for our example.

As has been mentioned before, the effect of the tariff also depends upon how the tariff revenue is disbursed by the government. Suppose, as one possible scenario, that the government spends the entire tariff revenue on the non-tradeable commodity 4. Then the equilibrium is $\lambda_{1A} = 4.5, \lambda_{4A} = 1.2, \lambda_{2B} = 2.5, \lambda_{4B} = 1, \lambda_{3C} = 2.5, \lambda_{4C} = 1, E_{AB} = 0.8, E_{BC} = 0.4583, E_{CA} = 2.7272$, and the world economy looks like this,

	A	B	C
1	$90w_A = 540p_{1A}$	....	....
2	....	$150w_B = 1250p_{2B}$	....
3	....	....	$250w_C = 2000p_{3C}$
4	$60w_A = 600p_{4A}$	$50w_B = 500p_{4B}$	$50w_C = 500p_{4C}$

An important point must be noted here. The tariff rate, t_{3A}, is on *commodity* 3, *not country C*. It appears as being imposed on country C because C happens to be the sole exporter of commodity 3. If it were levied on country C then commodity arbitrage would take place. A trader would buy a unit commodity 3 for ¥1, ship it to country B, and export it from country B, selling it for Rs.0.0366 and escape the tariff. In the direct sale from country C to country A, the price paid in A is Rs.0.055. Thus the tariff must be *general*, i.e. charged irrespective of the country of origin of a commodity(3). Or, if it is discriminatory, special arrangements must be made in country B to prevent re-export of the commodities taxed.

To see how the tariff affects the *private* real incomes in the three countries, observe the tables below. Table 1 show the free-trade levels of consumption of the tradeable commodities. Table 2 shows the post tariff levels of consumption.

Table 1: ***Pre-tariff consumption***

	A	*B*	*C*	*Total*
1	120.00	312.00	168.00	600
2	480.77	500.00	269.23	1250
3	642.85	557.15	800.00	2000

Table 2: ***Post-tariff consumption***

	A	*B*	*C*	*Total*
1	120.00	288.00	132.00	540
2	520.00	500.00	229.16	1250
3	545.45	654.54	800.00	2000

It is apparent that the levels of consumption in the three countries have changed rather unevenly due to movements in the exchange rates. As a result of appreciation of currency A relative to currency B, the consumption of commodity 1 in country B has decreased post-tariff as compared to the free-trade situation. On the other hand, due to an appreciation of country B's currency relative to country C's, the level of consumption of commodity 3 in country B has increased, while the consumption of commodity 2 in country C has fallen. Country C comes out as the net loser; her consumption of both importables has fallen after the tariff is imposed by country A.

The tariff has caused a reallocation of labour in country A and has resulted in a decline in the output of commodity 1. But the brunt of this decline is borne by countries B and C. The value of the world output of tradeables has fallen from Rs.323.33 to Rs.283.33. But this reduction in world 'welfare' has not been distributed evenly between the countries.

Welfare is known to be an elusive concept . We may nevertheless get some idea of how it is redistributed post-tariff by valuing the private consumptions of tradeable commodities at the autarky prices, in the countries, in the pre-and post-tariff situations. This method of measuring welfare has the advantage of being consistent with the concept of gains from trade which are also computed with reference to autarky prices. As the fourth column shows, 'welfare' in countries A and C has fallen in the post-tariff situation as compared to the

Welfare Changes (in own currencies)

	Autarky	Pre-tariff	Post-tariff	Changes
A	100	641.90	618.24	− 23.66
B	150	351.94	371.56	+ 19.62
C	200	483.64	414.76	− 68.88

pre-tariff situation whereas the welfare in country B has increased. Neither the tariff-imposer nor the tariff-imposer gain; it is country B which gains if A imposes a tariff on C. The analogy with the story of the cat and two monkeys would be exact, except that we have tacitly assumed that an increased government activity in country A does not increase the welfare in country A. This assumption makes the policy of imposing the tariff questionable in the first place. If, however, increased government activity was assumed to raise welfare the reduction in the value of private consumption on the regular commodities would be made good. At any rate the fact that the welfare in country B increases and that in country C diminishes is unambiguous[(4)]

By analogy with the two-country tariff example above, it is easy to guess that a similar end-result obtains if the tariff revenue is spent on the exportables, instead of the non-tradeables, by the tariff-imposing country. Whereas the expenditure of tariff revenue on non-tradeables effects the terms of trade by means of a reduction in the output of the exportables (due to a reallocation of labour towards the production of non-tradeables leading to a reduction in exportable surpluses) the expenditure of the tariff revenue on exportables affects the terms of trade by reducing the exportable surpluses directly.

The different manners of disbursing the tax revenue has different effects on the international economy. Thus if the government of country A were to spend 25 per cent of the revenue on each commodity, the equilibrium would be as follows,

	A	B	C
1	$97.5w_A = 585p_{1A}$	....	....
2	....	$150w_B = 1250p_{2B}$	....
3	....	....	$200w_C = 2000p_{3C}$
4	$52.5w_A = 525p_{4A}$	$50w_B = 500p_{4B}$	$50w_C = 500p_{4C}$

with the exchange rates $E_{AB} = 0.85, E_{BC} = 0.4705, E_{CA} = 2.5$ As is to be expected the appreciation of A's currency with respect to those of B and C is lower as compared to that when the tariff revenue is spent on the non-tradeables alone, because the import bills of country A are comparatively greater.

6. Retaliatory Tariff

We have seen how the welfare in a country, on whose exportables tariffs are imposed, declines. That was the position of country C in the example above. Now, suppose that the government of country C imposes a retaliatory tariff on commodity 1, and like its counterpart in country A, spends the tariff revenue on the non-tradeable commodity 4. If we suppose the tariff rate to be 50 per cent, the tariff revenue of the government of country C is (0.5/1.5) × ¥60 = ¥20. The equilibrium solution is $\lambda_{1A} = 4.5, \lambda_{2A} = 1.2, \lambda_{2B} = 2.5, \lambda_{4B} = 1, \lambda_{3C} = 2.25, \lambda_{4C} = 1.4, E_{AB} = 0.86111, E_{AB} = 2.18181$. the gains from trade evaluated at the post-tariff prices for all the three countries continues to support the trade pattern A-1,4 B-2,4 C-3,4. Observe, that, as compared to the situation in which country A alone imposed a tariff, the currency of country C has appreciated. The extent of appreciation, however, is less than the rate of tariff imposed by country C.

Consider now, the effects of the retaliatory tariff on the levels of consumption of the tradeable commodities in three countries. Country C gains in the consumption of commodity 2, the commodity not under tariff by either country,

Post-tariff consumption

	A	B	C	Total
1	120	310	110	540
2	483.87	500	266.13	1250
3	436.36	563.63	800	1800

as compared to country A. The unexpected result, however, is that country C loses her position in commodity 1 even further, despite the retaliatory tariff imposed by her. This is because the exchange rate appreciation effect is weaker than the effect of the tariff itself. The gains of the former effect accrue in terms of commodity 2 in country C. Surprisingly, the latter effect results in a gain to country *B* in which the level of consumption of commodity 1 rise from 288 units, when *A* alone imposes the tariff, to 310 units after *C* retaliates. (Refer to the table of post-tariff consumption in section 6.)

Finally observe the welfare changes after *C* imposes a tariff in the third column of the table below. The values of the consumption of the tradeables at autarky prices in the free-trade situation (first column) as well as the situation in which *A* alone imposes the tariff (second column) are reproduced for reference.

	Free-trade	A-tariff	A & C-tariff
A	641.90	618.24	540.59
B	351.94	371.56	353.09
C	483.64	414.76	434.13

A comparison of the second and third columns shows that country *C*'s retaliatory tariff has paid off—the welfare in country *C* improves and in countries *A* and *B*, it falls.

Three conclusion have been demonstrated above. Firstly, that the tariff imposing country may gain or lose, depending upon how we choose to evaluate the welfare effects of enlarged government activity. Secondly, the country on whose exports the tariff is imposed, undoubtedly loses. Finally, both the tariff imposer and the tariff imposee may lose while other countries with whom they are trading may gain.

Although these conclusions are more general than those arrived at in the standard theory (eg. Stolper-Samuelson theorem (1941)), they suffer from two limitations. Firstly, we have assumed in the examples unit-elastic demand curves for all commodities. This of course can be easily remedied. (Refer section 5 of chapter IV). Secondly, full-employment conditions have been assumed to the neglect of the employment-protecting effects of tariffs.

At any rate, the analysis above has given some idea of the kind of results that should be expected in the theory of distortions in a multicountry, multicommodity context. A more realistic analysis of tariffs, which can form the basis of the political economy of protectionism, will be found in Chapter IX below, where the employment-protecting role of tariffs becomes explicit.

7. Customs Unions

From the analysis of tariffs it is only natural to follow up and discuss the theory of customs unions, and to study in particular the consequences of the formation of the customs unions on the international economy. In so doing we shall find it beneficent to dispense with some of the traditional devices.

One traditional device is to start the analysis with a tariff-ridden world in which two or more countries decide to form a customs union and then remove the tariff walls selectively and study the 'trade-creating' and 'trade-diverting' influences of their removal[5] This device is obviously artificial. Besides, it is asymmetric to the theory of tariffs which is formulated with free-trade equilibrium as the backdrop. Thus even though our model is capable of using such a device we shall more usefully, start off with a free-trade situation. May it be noted that the traditional device is applicable when the objective is to study the consequences of a customs union for an actual tariff-ridden world, where tariff rates are so high as to prevent trade altogether.(6)

In like vein we shall dispense with the 'small union' assumption. There will be no restriction on the size of the union unless the object is to study a small union. Although not studied here, the analysis will be perfectly capable of determining how the sizes of the unions affect the world economy.

Finally we drop the customary assumption that the terms of trade between the partners in the union as well as those between the union and the rest of the world are fixed. In fact one of the objectives of the theory of customs unions should be to study how these respective terms of trade are altered. So to assume that they remains constant is like fixing the answer before asking the question.

To keep the discussion brief let us use the example of the preceding sections, in full awareness of the fact that the analysis can be generalised. Incidentally the example is similar, so far as dimensionality is concerned, to the models of Meade(1955), Vanek (1965), Kemp (1969), Lipsey (1970) and McMillan and McCann (1989) which too deal with a three-country three-commodity setup. The example we shall use contains an additional non-tradeable commodity.

Thus suppose countries A and B form a customs union and impose a 50 per cent tariff on commodity 3 which is exported by country C. Also suppose that the countries spend their tariff revenues, Rs.10 and \$10, on the non-tradeable commodity 4. From the corresponding system of supply-demand and full-employment equations we obtain the solution $\lambda_{1A} = 4.5, \lambda_{4A} = 1.2, \lambda_{2B} = 2.3333, \lambda_{4B} = 1.2, \lambda_{3C} = 2.5, \lambda_{4C} = 1, E_{AB} = 0.85714, E_{BC} = 0.36111, E_{CA} = 3.23076$(7). The ratio of commodity exchange which was 10 units of 1 = 16 units of 2 = 35.66 units of 3 in the free-trade situation changes to 10 units of 1 = 16.203 units of 2 = 53.846 units of 3. Thus whereas the terms of trade between countries A and B have not changed much (in the example used) those between A and B on the one hand and C on the other have altered substantially.

The effect of the changes in the terms of trade on the level of private consumption of the tradeable commodities in the three countries can be seen in the table below.

	A	*B*	*C*	Total
1	120.00	308.57	111.43	540.00
2	486.11	500.00	180.55	1166.66
3	646.15	553.85	800.00	2000.00

Now compare this with the free-trade levels of consumption. Countries A and B consume substantially similar quantities of all goods, after they form the customs union, as before. The outputs of their exportables decrease. But the brunt of these decreases is borne by country C. Country C's consumption of commodities 1 and 2, her importables, declines after the formation of the customs union. These are the consequences of the substantial deterioration of country C's terms of trade *vis-a-vis* the customs union.

At the same time, as we have seen, the intra-union terms of trade have not altered much. So in a way the customs union of our examples is an ideal one—both countries forming the union have substantially larger levels of government activities with practically unchanged levels of private consumption. In effect the tariff revenues earned by the union countries are paid by the reduction in consumption in country C.

Note, however, that this has come about because (1) the tariff rates imposed by the union countries are not 'high' enough and (2) the tariff revenues are spent by the governments on the non-tradeable commodity. If the tariff rates were 'high', the pattern of gains from trade would alter and autarkic production of one or the other importable would be encouraged in country C. In that case neither the improvement in the terms of trade of the union countries nor in their gains from trade with country C would be as large. Similarly, if the tariff revenues were spent on the importables the improvement in the terms of trade would be moderated. But were they spent in substantial measure on the exportables, a similar improvement of the terms of trade, as that observed when the revenues are spent on non-tradeables, would be observed.

The framework, it is easy to imagine, can be generalised to study the results of customs unions between any number of countries trading in any number of commodities, as well as to study 'optimal' combinations of customs unions.

To sum up, customs unions will result in raising the levels of consumption of the union countries at the cost of the non-union countries, provided that the common external tariffs are not so high as to change the basis of the gains from trade and provided that the tariff revenues are not dissipated in expenditures on importables. The stability of the customs union will, however, depend on the degree to which the intra-union terms of trade are affected—if that degree is large the union is unstable. The instability of a customs union can normally be removed only by permitting labour migration. This enables people of a member country, who intensively consume a commodity whose ratio has moved unfavorably, to migrate to another member country where that commodity is cheaper. Now this requires that there be no discrimination amongst citizens of member countries. It is partly on this account that customs unions give way gradually to economic unions.

In contrast to the positive effects of customs unions for member countries is their deleterious effect on the welfare of the non-union countries. The formation of customs unions is undoubtedly harmful to the non-union countries. The non-union countries are forced to take recourse to retaliatory tariffs which, of course, results in a loss of *world* welfare. The loss can be avoided only if the union members simultaneously make transfers to non-union members so as to mitigate their urge to retaliate with tariffs. But this brings into consideration the effects of intercountry transfers which are dealt with in section 11 below.

8. Import Quotas

The imposition of import quotas destroys the international trade equilibrium leading to excess supplies of those tradeable commodities on which the quotas are imposed and excess demands for those in the countries which impose the quotas. There is disequilibrium. If equilibrium is at all to be restored under these circumstances, there must be some arrangement in the countries on whose exportables the quotas are imposed, to absorb the excess labour in some non-market activities so that the supplies of commodities can be reduced to match the lower post-quota demands for them. Failing this, there is no option for these countries but to impose retaliatory quotas.

And in a world of intercountry quotas all trade and therefore exchange rates will be administered—market processes are replaced by administrative procedures. [See Bhagwati (1978) for a classic analysis of such regimes]

This view of import quotas stands in sharp contrast to the traditional analysis. For in traditional analysis quotas are almost exclusively studied by the aid of the partial demand-supply curves. (Tariffs are studied in both the partial and general equilibrium contexts). As a result, the general disequilibrium that arises when quotas are imposed, is ignored. It follows from the above that all attempts to compare tariffs and quotas are rather meaningless. Tariffs work through the market mechanism, quotas destroy the market mechanism.

9. Intercountry Transfers: Two Countries

Along with tariffs and devaluation the subject of intercountry transfers stands foremost among the applications of the theory of international trade. It is well-known that transactions on account of intercountry trade form a miniscule proportion of the total transactions in the foreign exchange market; the overwhelming portion consists of intercountry financial transactions. Indeed to call the theory of intercountry transfers an application of trade theory seems somewhat like wagging the dog by the tail. But of course it is not so—trade is essentially two-way, it creates the market, whereas transfers may go only one way. It follows that the problem of intercountry transfers must be studied against the backdrop of intercountry trade.

Consider, to begin with, a two-country two-commodity setup in a position of trade equilibrium with A exporting commodity 1 and B exporting commodity 2 and suppose that country A makes an 'annual' transfer of T_{AB} to country B. If the transfer is untied, i.e., the citizens of country B are free to spend the amount in accordance with their tastes and preferences, then the transfer amount will be spent on the importables and exportables in accordance with the Engels' coefficients. At the same time the demands for commodities in country A falls because the transfer amount is withdrawn from the citizens of country A[(8)]. Prior to the transfer, the equilibrium exchange rate was,

$$E_{AB} = \frac{\alpha_{2A} Y_A}{\alpha_{1B} Y_B} \tag{7}$$

where $Y_A = w_A L_A$, $Y_B = w_B L_B$ are the incomes in the two countries. After the transfer the import bill of country A to country B reduces by $\alpha_{2A} T_{AB}$ but that of country B to country A increases by $\alpha_{1B} T_{AB}$ in terms of the currency of country A. Thus the exchange rate moves to the ratio of the new levels of the intercountry debits,

$$E^{*}_{AB} = \frac{\alpha_{2A}(Y_A - T_{AB}) + T_{AB}}{\alpha_{1B}(Y_B + T_{AB} E^{*}_{BA})} \tag{8}$$

which, after rearranging the terms gives,

$$E^{*}_{AB} = E_{AB} + \frac{(1 - \alpha_{1B} - \alpha_{2A})T_{AB}}{\alpha_{1B} Y_B} \tag{9}$$

Equation (9) is a well-known result of international macroeconomics, although, it is never stated in this form. It shows that the movement of the exchange rate (terms of trade) due to the transfer depends upon the marginal propensities to import of the two countries. If the tastes in the two countries are identical, i.e., $\alpha_{1A} = \alpha_{2B}$, then as Keynes conjectured, no matter what the magnitude of the transfer may be, the transfer has no effect on the exchange rate and the terms of trade. If the sum of the marginal propensities to import is less than 1, the currency of country A depreciates as a result of the transfer. Interesting possibilities arise if one or the other importables is an inferior good with a negative marginal propensity to import. [See Keynes (1929), Ohlin (1929), Pigou (1932), Leontief (1936), Metzler (1942) for the earliest results. See Samuelson (1952) for a critical survey and generalisations.]

If the transfer were a 'tied' transfer, i.e., there were conditions attached to the manner of its disbursement by the citizens of country B, the effects would be different. For instance if it were mandatory that the transfer amount be spent by country B exclusively on the importable, the resulting exchange rate would be,

$$E^{*}_{AB} = E_{AB} - \frac{\alpha_{2A} T_{AB}}{\alpha_{1B} Y_B} \tag{10}$$

i.e. the currency of country A appreciates after the tied transfer if the propensities to import of both countries are positive.

All the standard results, however, have been derived on the assumption that transfers have no effect on the intercountry propensities to import, that is to say on the world trade pattern itself. The standard procedure, in keeping with the spirit of macroeconomic theorising, regards the intercountry propensities to import as constants. But this procedure is defensible only in the two-country two-commodity complete-specialisation environment. In fact, the intercountry propensities to import themselves change with the exchange rates and the exchange rates change as a result of intercountry transfers.

Consider an example of two countries trading in three commodities. The wage rates in the two countries A and B are Re.1 and \$1 respectively, and their autarky equilibria are,

	A	B
1	$24w_A = 144p_{1A}$	$60w_B = 30p_{1B}$
2	$60w_A = 96p_{2A}$	$60w_B = 300p_{2B}$
3	$36w_A = 72p_{3A}$	$30w_B = 100p_{3B}$

The international trade equilibrium, A-1, B-2,3 with an exchange rate $E_{AB} = 1.6$, is

	A	B
1	$129w_A = 720p_{1A}$	....
2	....	$97.5w_B = 487.5p_{2B}$
3	....	$52.5w_B = 175p_{3B}$

Now suppose country A makes a transfer of Rs.20 to country B. Since the tastes and preferences in the two countries are different, the post-transfer exchange rate will diverge from the pre-transfer rate $E_{AB} = 1.6$. Since the marginal propensity to import of country A is 0.8 ($\alpha_{2A} + \alpha_{3A}$) and that of country B is 0.4(α_{1B}), the sum of the marginal propensities to import of the two countries is greater than 1. As a result we will expect the currency of country A to appreciate. If we try the same trade pattern A-1 B-2,3 with the Walrasian equations modified for the transfer, we obtain the following international equilibrium,

	A	B
1	$120w_A = 720p_{1A}$	....
2	....	$97.82w_B = 489.13p_{2B}$
3	....	$52.17w_B = 173.91p_{3B}$

with an exchange rate $E_{AB} = 1.5333$. The results are as expected except that the composition of outputs varies as a result of the transfer[9].

Before we proceed to obtain further results let us pause to consider the effect of the transfer on the levels of consumption of the three commodities in countries A and B. This is tabulated below.

Transfer of Rs.20 from country A to country B.

	Pre-transfer			Post-transfer		
Commodity	A	B	Total	A	B	Total
1	144	576	720	120	600	720
2	187.5	300	487.5	163.04	326.08	489.13
3	75	100	175	65.21	108.69	173.91

The consumption of all commodities has increased in country B after the transfer and fallen in country A. But if we observe the increase in the *value* of consumption in country B in rupees, we find that it amounts only to Rs.16;

$$(600 - 576)p_{1A} + (326.08 - 300)p_{2B}E_{AB} + (108.69 - 100)p_{3B}E_{AB} = \text{Rs.}16$$

This amount is less than Rs.20 which country A transfers to country B. And the decrease in the value of consumption in country A is found to be Rs.16, which is the same as the increase in the value of consumption in country B;

$$(144 - 120)p_{1A} + (187.5 - 163.04)p_{2B}E_{AB} + (75 - 65.21)p_{3B}E_{AB} = \text{Rs.}16.$$

In other words the total value of the world economy is preserved in the course the transfer. But the transfer is 'undereffected', in real terms. This is obviously because the currency of country A appreciates due to the transfer, preventing the downslide in country A's levels of consumption. The reason for the currency of country A to appreciate due to the transfer is that the sum of the marginal propensities to import is greater than unity. Thus we may conclude that whenever the sum of the marginal propensities to import is greater (lower) than unity the transfer is under-(over-) effected. According we may distinguish between a *nominal* transfer which represents the sum paid out in accounting terms and an *effective* transfer that represents the change in the value of consumption in the transferor country.

Next consider a transfer of \$50 from country B to country A. We expect the currency of country B to appreciate and that of country A to depreciate. Since the pre-transfer equilibrium exchange rate is 1.6 and this is closest to the natural exchange of 1.666 rate as implied by the prices of commodity 3, let us suppose that the new exchange rate crosses this threshold and try the trade pattern A-1,3, B-2. However, the gains from trade do not support this trade pattern. What happens is that the depreciating effect on A's currency of the transfer of \$50, is washed out by the \$60 increase in country A's export of commodity 3 to country B resulting in an appreciation of A's currency overall.

If we try the trade pattern A-1,3 B-2,3 the equilibrium is found. The equations are,

$$\frac{\lambda_{1A}L_{1A}}{l_{1A}} = \frac{\alpha_{1A}(Y_A - T_{AB}) + (\alpha_{1B}Y_B)E_{AB} + \alpha_{1B}T_{AB}}{p_{1B}}$$

$$\frac{\lambda_{2B}L_{2B}}{l_{2B}} = \frac{\alpha_{2A}(Y_A - T_{AB})E_{BA} + \alpha_{2B}T_{AB}E_{BA} + \alpha_{2B}Y_B}{p_{2B}}$$

$$\frac{\lambda_{3A}L_{3A}}{l_{3A}} + \frac{\lambda_{3B}L_{3B}}{l_{3B}} = \frac{\alpha_{3A}(Y_A - T_{AB})E_{BA} + \alpha_{3B}T_{AB}E_{BA} + \alpha_{3B}Y_B}{p_{3B}}$$

$$\lambda_{1A}L_{1A} + \lambda_{3A}L_{3A} = L_A$$
$$\lambda_{2B}L_{2B} + \lambda_{3B}L_{3B} = L_B$$

After substituting the appropriate values in the equations, we obtain the equilibrium solution $E_{AB} = E^3_{AB} = 1.666$ and,

	A	B
1	$107.33w_A = 644p_{1A}$	....
2	....	$101w_B = 505p_{2B}$
3	$12.66w_A = 25.33p_{3A}$	$49w_B = 163.33p_{3B}$

Now consider the following result. Suppose country B transferred only \$20 to country A. The international equilibrium would show the same trade pattern A-1,3 B-2,3 and the exchange rate $E_{AB} = E^3_{AB} = 1.666$ but with different inter-industry allocation of labour,

	A	B
1	$117.33w_A = 704p_{1A}$	....
2	....	$98w_B = 490p_{2B}$
3	$2.66w_A = 5.33p_{3A}$	$52w_B = 173.33p_{3B}$

The important conclusion that may be drawn from this illustration is as follows: that transfers may have no effect on the exchange rate (terms of trade) if the exchange rate is equal to one of the natural exchange rates, even when tastes and preferences as between the transferor and transferee countries are different. The illustration is a counter example to the standard macroeconomic result of equation (9) which predicts that transfers (and changes in the transfer amounts) must *always* result in a change in the exchange rate in a determinate direction provided only that the tastes are different in the countries.

There is, to be sure, a certain range of values of the transfer amounts over which this happen. As was seen above, as the transfer from country B to country A changes from \$20 to \$50, there is an increase in the production of commodity 3 in country A and a simultaneous reduction in its production in country B.

If the transfer amount is increased beyond a certain threshold, the production of commodity 3 shifts entirely into country A and the world trade pattern changes to A-1,3 B-2. There are, in effect, then two threshold levels of transfer, one when the trade pattern shifts from A-1 B-2,3 to A-1,3 B-2,3 and second when the trade pattern shifts from A-1,3 B-2,3 to A-1,3 B-2. As the transfer amount increases from one threshold level to the other, it causes a gradual shift in the internal allocation of labour, raising the production of the commodity produced in common (commodity 3) in country A, the transferee, and simultaneously lowering it in country B, the transferor, until the next threshold is reached, at which point the production of commodity 3 shifts entirely to the transferee country. Changes in the transfer amount between these thresholds has no effect on the exchange rates, there are only inter-industry reallocations of labour and consequent changes in the composition of the countries outputs.

Thus when the trade position lies in between successive natural exchange rates the exchange rate responds smoothly to changes in the transfer amount. At the position of a natural exchange rate the response is nil. Thus over the entire range of feasible exchange rates, the exchange rate exhibits a spasmodic response to continuous changes in the transfer amount.

In sum, the existence of differences in tastes and preferences between countries is neither a necessary nor a sufficient condition for transfers to affect the exchange rates and terms of trade. Similarly, the conditions of the sum of the marginal propensities to import (whether less than, equal to or greater than 1), which determine the direction of exchange rate movements consequent upon a transfer, do not guarantee that exchange rates will in fact move as posited—exchange rates may not move at all; though it is certain that they will not move in directions opposite of those indicated by the conditions.

At this point it will be worthwhile to check on the comparison of the nominal vis-a-vis the effective transfer when transfer amounts are varying between the thresholds. It is obvious that the two will be equal for the simple reason that there is no movement of the exchange rate, which continues to be equal to the natural exchange rate implied by the commodity produced in common. This may be verified from the table of the levels of consumption below.

Transfer from country B to country A.

	Transfer $20			Transfer $50		
	A	B	Total	A	B	Total
1	184	520	704	244	400	644
2	230	260	490	305	200	505
3	92	86.66	178.66	122	66.66	188.66

The value of the increase in country A's consumption (decrease in country B's consumption) when the transfer amount increases from $20 to $50 is,

$$(184 - 144)p_{1A}E_{BA} + (305 - 230)p_{2B} + (122 - 92)p_{3B} = \$30$$

i.e. the transfer is exactly effected irrespective of the marginal propensities to import.

Upto this point the intercountry transfer has been assumed to be of the nature of an international tribute, indemnity or gift that gives no recompense to the transferor country. Let us now suppose that the transfer is a loan or an investment that must be serviced with interest payments and use our example to work out the elementary dynamics of the world economy over time. Suppose country A transfers Rs.20 every year as a loan to country B, charging 100 per cent rate of interest. Since the dynamic path of the world economy depends upon the repayment schedule let us assume, as one simple scenario, that country B does not repay the principal at any stage but only pays the interest amount on the accumulated debt. As before we shall suppose that both the transfer and the interest payments are made smoothly throughout the year so that they are matched with the regular trade payments.

During Year 1, when the loan is first initiated, the international equilibrium position is reproduced below.

	A	B
1	$120w_A = 720p_{1A}$	....
2	....	$97.82w_B = 489.13p_{2B}$
3	....	$52.17w_B = 173.91p_{3B}$

The exchange rates in Year 1, $E_{AB} = 1.5333$. In the following Year 2, country B will be required to transfer Rs.20 on account of interest at the same time as country A makes out a fresh loan of Rs.20. Thus in Year 2 the transfers are self-cancelling so that the pre-transfer international equilibrium is re-attained,

	A	B
1	$120w_A = 720p_{1A}$	....
2	....	$97.5w_B = 487.5p_{2B}$
3	....	$52.5w_B = 175p_{3B}$

with an equilibrium exchange rate $E_{AB} = 1.6$. In Year 3, country B will pay Rs.40 on her accumulated debt of Rs.40 to country A and receive the fresh loan of Rs.20. There is thus a net transfer of Rs.20 from country B to country A and the international equilibrium changes to,

	A	B
1	$120w_A = 720p_{1A}$	....
2	....	$97.2w_B = 486p_{2B}$
3	....	$52.8w_B = 176p_{3B}$

with an equilibrium exchange rate $E_{AB} = 1.666 = E^3_{AB}$. The lower threshold has been reached. In Year 4 country B will pay Rs.60 on her accumulated debt and receive a fresh loan of Rs.20. The net transfer from country B to country A is Rs.40 in year 4 and the international equilibrium will show an increase in the production of commodity 3 in country A and a reduction in country B. This process continues until the exchange rate touches $E^2_{AB} = 3.125$ whereafter trade relations between the countries will be snapped at a point where country B is heavily indebted to country A but cannot make good even a part of the debt by means of exports.

In the example above the sum of marginal propensities to import was greater than 1. As a result, the transferor country's currency appreciated with every increases in the transfer amount and weakened her export position. If the sum of the marginal propensities to import is less than 1, the transferor country's currency depreciates and strengthens the export position. Over a period of time as the transferor country is transformed into a net transferee, her currency appreciates while that of the other country depreciates. Thus the accumulated debt can be made good eventually by an improvement in the export position.

10. Intercountry Transfers: Three-or-more Countries

As before the complexity of the problem of intercountry transfers increases rapidly when more than two countries are considered. Suppose a world economy consisting of three countries A, B and C. Let μ_{ij} be the marginal propensity of country i to import from country j. Then,

$$\mu_{ij} = \sum_k \alpha^k_{ij} \qquad i, j = A, B, C \quad i \neq j \tag{11}$$

where α_{ij}^{k} denotes country's propensity to consume commodity k which she imports from country j. It should be immediately noted that μ_{ij} simply cannot be defined when country i imports commodity k from two, or more, countries, that is to say when the exchange rate of the currencies of those countries j is equal to natural exchange rate implied by commodity k.
Further, the propensity to import of country i from the rest of the world will be,

$$\mu_i = \sum_j \mu_{ij} = \sum_j \sum_k \alpha_{ij}^{k} \qquad i, j = A, B, C \tag{12}$$

Adhering to the same notation and assumptions as those of the previous section, let us first formulate the purely macroeconomic conditions to determine the magnitudes and directions of exchange rate movements following an intercountry transfer.

Suppose country A transfers an amount T_{AB} to country B. After the transfer the intercountry import bills attain the following values,

$$\begin{array}{lll} M_A = \mu_A (Y_A - T_{AB}) & M_B = \mu_B (Y_B + T_{AB} E^{*}_{BA}) & M_C = \mu_C Y_C \\ M_{AB} = \mu_{AB} (Y_A - T_{AB}) & M_{BA} = \mu_{BA} (Y_B + T_{AB} E^{*}_{BA}) & M_{CA} = \mu_{CA} Y_C \\ M_{AC} = \mu_{AC} (Y_A - T_{AB}) & M_{BC} = \mu_{BC} (Y_B + T_{AB} E^{*}_{BA}) & M_{CA} = \mu_{CB} Y_C \end{array} \tag{13}$$

where E^{*}_{BA} is the post-transfer exchange rate. The balance of payments equations in terms of the currency of country A for the three countries are,

$$\begin{aligned} M_A + T_{AB} &= M_{BA} E^{*}_{AB} + M_{CA} E^{*}_{AC} \\ -(M_{AB} + T_{AB}) &= -M_B E^{*}_{AB} + M_{CB} E^{*}_{AC} \\ -(M_{AC}) &= M_{BC} E^{*}_{AB} - M_C E^{*}_{AC} \end{aligned} \tag{14}$$

We may use any two independent equations in (14) to determine the post transfer values of the exchange rates. Arranging, say, the first two equations in matrix system of Chapter I we have,

$$\begin{bmatrix} M_{BA} & M_{CA} \\ -M_B & M_{CB} \end{bmatrix} \begin{bmatrix} E^{*}_{AB} \\ E^{*}_{AC} \end{bmatrix} = \begin{bmatrix} M_A + T_{AB} \\ -M_{AB} + T_{AB} \end{bmatrix} \tag{15}$$

which solves for the exchange rates;

$$\begin{aligned} E^{*}_{AB} &= \frac{M_{CB}(M_A + T_{AB}) + M_{CA}(M_{AB} + T_{AB})}{M_B M_{CA} + M_{BA} M_{CB}} \\ E^{*}_{AC} &= \frac{M_B(M_A + T_{AB}) - M_{BA}(M_{AB} + T_{AB})}{M_B M_{CA} + M_{BA} M_{CB}} \end{aligned} \tag{16}$$

Substituting (13) into (16) and isolating the pre-transfer value of the exchange rate, E_{AB}, we get,

$$E_{AB}^{*} = E_{AB} + \frac{(1-\mu_A-\mu_{BA})\mu_{CB}+(1-\mu_{AB}-\mu_B)\mu_{CA}\,T_{AB}}{(\mu_{BA}\mu_{CB}+\mu_B\mu_{CA})\,Y_B} \tag{17a}$$

$$E_{AC}^{*\prime} =$$

$$\frac{(\mu_A\mu_B-\mu_{AB}\mu_{BA})Y_A+[\mu_B(1-\mu_A)-\mu_{BA}(1-\mu_A)](Y_B+T_{AB}E_{BA}^{*})T_{AB}-\mu_{BC}(Y_A+T_{AB})}{(\mu_B\mu_{CA}+\mu_{BA}\mu_{CB})Y_BY_C+(\mu_B\mu_{CA}+\mu_{BA}\mu_{CB})T_{AB}E_{BA}^{*}Y_C} \tag{17b}$$

It is apparent from equation 17(a) that the exchange rate and terms of trade between the transferor and transferee countries depend not only on their propensities to import from one another and from country C, but also on the propensities of country C to import from them. The direction of the change in the exchange rate is determined by the sign of the parameter,

$$(1-\mu_A-\mu_{BA})\mu_{CB}+(1-\mu_{AB}-\mu_B)\,\mu_{CA} \tag{18}$$

provided all the intercountry propensities to import are positive. Substituting $\mu_{AB}+\mu_{AC}=\mu_A$, $\mu_{BA}+\mu_{BC}=\mu_B$ and $\mu_{CA}+\mu_{CB}=\mu_C$ in (18) and rearranging terms we get the condition for the currency of country A to depreciate against that of country B as,

$$\mu_{AB}+\mu_{BA} < 1-\left\{\mu_{AC}\left[\frac{\mu_{CB}}{\mu_C}\right]+\mu_{BC}\left[\frac{\mu_{CA}}{\mu_C}\right]\right\} \tag{19}$$

Observe that (19) is a generalisation of the two - country condition in equation (9) of the previous section. The special two-country condition follows if $\mu_{AC} = \mu_{BC} = 0$ and/or $\mu_{CA} = \mu_{CB} = 0$. In the three-country version, the sum of the marginal propensities to import from one another of the transferor and transferee countries should be less than 1 minus the *weighted* sum of their propensities to import from country C, for the currency of the transferor country is to depreciate. This is clearly a more stringent condition than that obtained in the case of two countries. It is also obvious that the stringency of the conditions will increase as more and more countries are introduced.

By a similar procedure we may also derive the conditions for the movement of the other exchange rates, E_{AC}^{*} and E_{BC}^{*}. Of course, as has already been indicated, the conditions determining the exchange rate movements hold only when countries produce no commodities in common. If the exchange rates are tied down to some natural exchange rate(s) they may not move at all after the transfer. Similar remarks also apply to the relationship of the nominal and effective transfer amounts between countries

To obtain a fuller appreciation of these, and more general results, consider an example of three countries trading in four commodities. Their autarky equilibria are,

	A	B	C
1.	$30w_A = 120p_{1A}$	$100w_B = 400p_{1B}$	$90w_C = 180p_{1C}$
2.	$20w_A = 200p_{1A}$	$100w_B = 300p_{2B}$	$60w_C = 160p_{2C}$
3.	$20w_A = 60p_{3A}$	$100w_B = 600p_{3B}$	$60w_C = 60p_{3C}$
4.	$30w_A = 240p_{4A}$	$100w_B = 800p_{4B}$	$90w_C = 210p_{4C}$

With wage rates in the three countries being $w_A =$ Re.1, $w_B = \$1$, $w_C =$ ¥1 the international trade equilibrium is as follows,

	A	B	C
1	....	$43.75w_B = 175p_{1B}$	$300w_C = 600p_{1C}$
2	$100w_A = 1000p_{2A}$	....	....
3	....	$162.5w_B = 875p_{3B}$	....
4	....	$193.75w_B = 1550p_{4B}$	....

with exchange rates $E_{AB} = 0.61538, E_{BC} = E^1_{BC} = 0.5, E_{CA} = 3.25$. The ratio of commodity exchange is 6.5units of 1 = 10 units of 2 = 9.75 units of 3 = 13 units of 4.

Now suppose that countries A and C transfer respectively Rs. 20 and ¥50 to country B. While the disposable incomes in countries A and C fall by the amount of their respective transfers, the disposable income of country B increases. The post-transfer international equilibrium obtained is as follows,

	A	B	C
1	....	$41.93w_B = 167.72p_{1B}$	$300w_C = 600p_{1C}$
2	$100w_A = 1000p_{2A}$	....	....
3	....	$166.14w_B = 996.84p_{3B}$	....
4	....	$191.93w_B = 1535.44p_{4B}$	....

with the exchange rates $E_{AB} = 0.6019, E_{BC} = E^1_{BC} = 0.5, E_{CA} = 3.32278$. The new international ratio of commodity exchange changes to 6.64 units of 1 = 10 units of 2 = 9.968 units of 3 = 13.29 units of 4. In the new equilibrium, the "terms of trade" between A and B have moved in favour of country A for she can obtain more of B–exportables for a unit of her own exportable commodity 2. But because the exchange rate of countries B and C has been tied to the natural exchange rate E^1_{BC}, A's terms of trade against country C also improve. It will follow, from a comparison of the pre-and post-transfer levels of consumption, that the transfer from country A to country B is undereffected but that from country C to country B is exactly effected.

Two points need to be mentioned to complete our discussion of intercountry transfers. Firstly, it is perfectly possible to allow the intercountry transfer to be collected by a central agency, like the government, which spends the amount in stipulated proportions. Secondly, it is also possible to incorporate non-tradeables into the analysis. The presence of non-tradeable commodities would broadly have the effect of suppressing the magnitudes of intercountry propensities to import. As a result, the weighted sums of the intercountry propensities to import are likely to be less than 1, so that the transfer is usually overeffected.

11. Fixed Exchange Rates

In a regime of universally fixed exchange rates a ranking of countries in the production of different commodities emerges automatically and determines the world trade pattern. Since, by presumption, the exchange rates must be such as to prevent currency arbitrage, they ensure that this ranking is uniform from the viewpoint of all countries (refer Chapter II).

It is immediately obvious that if the predetermined exchange rates are those which do not balance the *autonomous*(or private) receipts and payments of the different countries, some countries will be in a deficit position and others in a surplus position. Consistency of the exchange rates ensures that the value of the sum of the deficits will be exactly equal to the value of the sum of the surpluses, when these are evaluated in terms of any one currency. The question is, how are the accounts of the individual countries squared in these circumstances?

There are three adjustment mechanisms in a fixed exchange rate system, each of which has some historical significance. The first is the celebrated classical specie-flow mechanism formulated by Hume (1752) under gold standard conditions. In this mechanism deficit countries have to settle their deficits by paying gold to surplus countries. As a result the money supply, wages and prices fall in terms of gold in the deficit countries and rise in the surplus countries. In our terminology, the *natural* exchange rates of the deficit countries' currencies appreciate and those of the surplus countries' currencies depreciate. This happens until the trade balance is restored at the fixed currency exchange rate. The final effect of the price-specie flow mechanism is to bring about the same result as would be brought about under flexible exchange rates. How and why, we shall see presently.

It is important to note here that Hume's adjustment mechanism works just as well even if gold movements in particular are omitted, that is to say even if currencies are based on the fiat money principle(10). But this requires that the money wage rates in all the countries should be flexible. That is a matter of overall economic policy, legislation pertaining to the employment contracts and the nature of wage bargains. If the wage rates are flexible, then those countries faced with a deficit will be forced to reduce their wage rates and the prices of their commodities while the countries with surpluses will be forced to raise their wages and prices, until trade balance is restored at the fixed currency exchange rate.

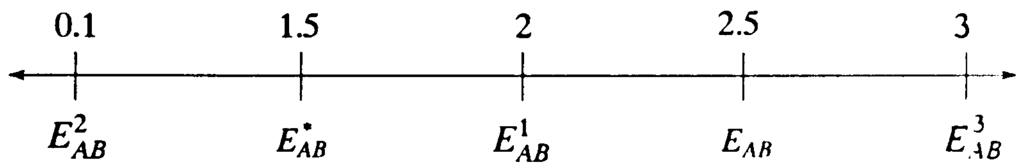
To see how this second adjustment mechanism works consider an example of two countries trading in three commodities. Their autarky equilibria are,

	A	*B*
1	$60w_A = 37.5p_{1A}$	$60w_B = 75p_{1B}$
2.	$60w_A = 300p_{2A}$	$60w_B = 30p_{2B}$
3	$30w_A = 100p_{3A}$	$80w_B = 800p_{3B}$

Suppose the wage rates to begin with are w_A = Re.1 and w_B = 1. Then the prices and natural exchange rates are

$$p_{1A} = 1.6 \qquad p_{1B} = 0.8$$
$$p_{2A} = 0.2 \qquad p_{2B} = 2$$
$$p_{3A} = 0.3 \qquad p_{3B} = 0.1$$
$$E^1_{AB} = 2 \qquad E^1_{BA} = 0.5$$
$$E^2_{AB} = 0.1 \qquad E^2_{BA} = 10$$
$$E^3_{AB} = 3 \qquad E^3_{BA} = 0.333$$

If the countries were to trade under conditions of flexible exchange rates (with fixed wage rates) the international trade equilibrium would be A -2 B-1,3 with an equilibrium exchange rate E^*_{AB} = 1.5. In terms of the scale of natural exchange rates this position would be as follows,



Suppose, however, that the countries decide to trade at a fixed exchange rate E_{AB} = 2.5. The gains from trade at this exchange rate would indicate a world trade pattern A-1,2 B-3. But with this trade pattern country A has a surplus and country B has a deficit, for the import bill of country A falls to Rs.30 but that of country B increases to \$120. The balance of payments accounts of the two countries evaluated in their domestic currencies at the fixed exchange rate E_{AB} = 2.5 are shown below.

A		*B*	
Dr.	Cr.	Dr.	Cr.
Imports Rs.30	Exports Rs.300	Imports \$120	Exports \$12
Surplus Rs.270	....	....	Deficit \$108
Total Rs.300	Total Rs.300	Total \$120	Total \$120

In the absence of other adjustment mechanisms, such as inter-country transfers, the wage rate in the surplus country A must rise and that in the deficit country B must fall, in order to avert this situation of disequilibrium. The extent to which wage rates must change depends on the deviation of the fixed exchange rates E_{AB} and the equilibrium exchange rate E^*_{AB}. In fact the ratio of the new wage rates will be determined by the ratio of the fixed exchange rate to the equilibrium exchange rate, i.e.

$$\frac{w_A}{w_B} = \frac{E_{AB}}{E^*_{AB}} = \frac{2.5}{1.5} = 1.6666 \tag{2}$$

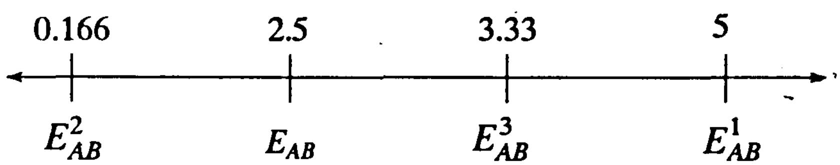
Any pair of values of w_A and w_B whose ratio is 1.6666 will give essentially the same result. Thus it is not correct to say, as we have, that the wage rate in the surplus country rises *and* the wage rate in the deficit country falls. Strictly speaking it is the ratio of the money wage rate of the surplus country to the deficit country that must rise, e.g. w_A = Rs.0.5 and w_B = \$0.3, would give the same result. This point is made clear in the example below.

Suppose, that the wage rate is country A rises to w_A = Rs.1.5 and that the wage rate in country B falls to w_B = \$0.9. The prices of commodities and the natural exchange rates will stand modified to,

$$
\begin{array}{llll}
p_{1A} & = 2.4 & p_{1B} & = 0.72 \\
p_{2A} & = 0.3 & p_{2B} & = 1.8 \\
p_{3A} & = 0.45 & p_{3B} & = 0.09 \\
E^1_{AB} & = 3.3333 & E^1_{BA} & = 0.3 \\
E^2_{AB} & = 0.1666 & E^2_{BA} & = 6 \\
E^3_{AB} & = 5 & E^3_{BA} & = 0.2
\end{array}
$$

Note that the values of the natural exchange rates will be the same, whatever may be the wage rates, for these values depend upon the relative wage rates. So long as the ratio is 1.6666 the natural exchange rates will be the same.

Now with the currency exchange rate fixed at E_{AB} = 2.5 observe the pattern of the gains of trade. It will show a trade pattern A-2 B-1,3 to be feasible. And indeed it is the equilibrium trade pattern.



For the import bill of country A is,

$$
\begin{aligned}
M_{AB} = (\alpha_{1A} + \alpha_{2A})\, w_A L_A &= (0.4 + 0.2)(1.5)(150) \\
&= \text{Rs.}135
\end{aligned}
$$

and that of country B is,

$$
\begin{aligned}
M_{BA} = \alpha_{2B}\, w_B L_B &= (0.3)(0.9)(200) \\
&= \$54
\end{aligned}
$$

The ratio of the intercountry import bills is 2.5 which is precisely the fixed exchange rate E_{AB}. Further the levels of consumption in both the countries are exactly what they would have been in equilibrium under a regime of flexible exchange rates. In fact, observe that the fixed rate E_{AB} is in real terms , equal to the equilibrium exchange E^*_{AB}.

$$
\frac{(\text{Rs.}2.5)(\$1)}{(1.5)(0.9)} = 1.5(\text{Rs}/\$)
$$

where the fixed exchange rate has been deflated by the wage rates in the two countries.

The same principle holds, as can be easily verified, in the multicountry case. If E^*_{ij} are the equilibrium exchange rates under the flexible regime and E_{ij} are the fixed exchange rates then the ratios of the money wage rates which ensure that the fixed exchange rates become equilibrium rates. are,

$$\frac{w_i}{w_j} = \frac{E_{ij}}{E^*_{ij}} \quad \forall i, j \;\; i \neq j \tag{21}$$

So long as E_{ij} conform to the neutrality conditions the solution of the relative wage rates is unique. Thus, with exchange rates and wage rates fixed for Z countries, the $Z-1$ independent balance of payments equations solve for $Z-1$ surpluses/deficits. With exchange rates fixed but wage rates flexible, the same equations can be used to solve $Z-1$ (relative) wage rates that ensure payments equilibrium.

In summing up the classical adjustment mechanism we note that the international trade equilibrium with fixed exchange rates and flexible wage rates is identical to the international trade equilibrium with flexible exchange rates and fixed wage rates in all respects except in the nominal values of the wage rates, national incomes and intercountry trade bills. In the former case the scales of the natural exchange rates slide up and down until trade equilibrium is achieved at the predetermined exchange rates. In the latter the currency exchange rates slide along fixed scales of natural exchange rates to ensure trade equilibrium.(11)

Form the standpoint of practical economic policy, however, the superiority of the flexible exchange rate system over the fixed system is quite obvious. Because, whereas the flexible exchange rate system is compatible with both the fixed and flexible wage rate conditions, the fixed exchange rate system works only if the wage rates are flexible.

The flexible exchange rate system provides an automatic mechanism to ensure equilibrium even if wage rates and other parameters are changing from time to time. In the fixed system on the other hand it is the wage rates which must adjust, whatever may be the cause of disequilibrium. Thus even through, theoretically, the fixed system can achieve the same result by revisions in the exchange rates, it can do so only after protracted domestic and multilateral negotiations.

Besides a flexible wage policy has great disadvantages. It introduces instability in the process of making agreements and contracts by shrouding them in cloaks of suspicion. Imagine for instance what will happen to the employment contract if a flexible wage policy is followed. Fluctuating wage rates will destroy the basis of the stability of the contract. The same applies to all types of commercial contracts if prices keep fluctuating all the time. Even international negotiations are not as free of suspicion under the fixed exchange rate system as one might except. A deficit country which is unable to make the necessary domestic adjustments and expresses a wish to devalue her currency is liable to be misinterpreted for stealthily attempting to improve her competitive advantage by offering domestic inability as an excuse. On a balance of considerations it makes much sense to maintain the stability of the terms on which the overwhelming proportion of domestic contracts are made and sacrifice, to some extent, the stability of the foreign exchanges[12] which perform a smaller proportion of the transactions. To a considerable extent the exchange rate uncertainty entailed by the flexible exchange rate system can be hedged by recourse to the forward exchange markets.

Another method of adjustment under the fixed exchange rate system is the method of accommodating transfers, which involves quite simply the requisite transfer of funds from surplus to deficit countries either directly, or indirectly through an institution like the International Monetary Fund. It is obvious that this cannot be a permanent arrangement. It may entail indefinite charity on the part of some countries

The above remarks about the relative superiority of the flexible exchange rate system are valid only when international economic relations are largely confined to trade transactions. They weaken substantially when the greater part of international relations are of the nature of debts and investment.

When large amounts of intercountry debts have been committed any change in the exchange rates lead to massive changes in the current values of the debts (or investments) and in turn lead to massive redistributions of income between the countries. These unforeseen redistributions of income arise purely on account of the nature of revaluations—there are no economic or technological grounds for them. Depreciation of currencies impose burdens on the indebted countries, appreciation of currencies on the creditor countries—burdens that are unforeseen and do not arise from the actions of the debtors or creditors themselves. This creates frustration and hopelessness in the minds of the people.[13]

Besides this, there are three other disadvantages of flexible exchange rates. (A) Long-term investment projects may suffer windfall losses and profits only on account of exchange rate movements, and quite independently of technological or managerial capabilities. [These risks cannot be hedged by forward exchange contracts because such contracts are seldom available for more than a year.] In short the process of capital formation and employment creation can be vitiated . (B) The flexible exchange rate system creates the atmosphere for "hot money" flows and encourages speculation. An undue dominance of speculative transactions that shift values of currencies which form the basis of real investment and trade flows will always be suspect. (C) A large element of instability enters the process of making international contracts, the number of clauses and covenants in the agreements multiply whose interpretation and enforcement becomes ambiguous and time-consuming.

Thus, whereas the flexible exchange rate system is well-suited to situations in which international trade transactions are preponderant, the fixed exchange rate system is appropriate where international debt and investment transactions dominate. It is an irony of international history that the Bretton Woods fixed exchange rate system was adopted during a time in which trade transactions dominated while in the present times of unprecedented growth in intercountry debts and investments we are limping along with a flexible exchange rate system. It is obvious that this cannot be a permanent arrangement.

The Bretton Woods System on which the world monetary system operated from 1948-1971-3 was essentially an attempt to combine some features of the three adjustment mechanisms under fixed exchange rates discussed above, as well as the flexible exchange rate mechanism; (a) fixed par values of currencies against one another with changes permitted only through multilateral negotiations (b) a band of variability around the par values permitted to let flexibility of exchange rates iron out day-to-day fluctuations in demands and supplies of currencies, (c) gold recognised as a means of settling the balances of international payments, and gold used to impart confidence to international money in the form of a gold-dollar conversion promise and (d) accommodating transfers (drawing) from International Monetary Fund to tide over balance of payments difficulties. The weakest links were of course (c) and (d). Countries having surpluses with America did not accommodate America (presumably on the ground that the dollar as *numeraire* currency was itself weakening on account of the American deficit vis-a-vis their own currencies so that the par values were becoming unrealistic) and insisted on settling the balances in gold. And America could not go on, directly or through the IMF, accommodating the deficits of other countries. Of course, matters would not have reached the stage of demise if appropriate changes in the par values of currencies had been more frequent for surplus and deficit countries alike. The American dollar was the *numeraire* currency and there was little point in the insistence on its depreciation. Nobody wants less money because its purchasing power falls—on the contrary people demand more of it. Likewise, if the demand for the American dollar had been *raised*, by revaluing the currencies against the dollar, international equilibrium could have been restored, there would have been no dollar glut. The glut should rightly have been ploughed back into the U.S.A. in the form of deposits/investments.

12. Public Finance

Finally let us turn to study the effects of governement's domestic fiscal policies on world trade. To begin with consider a country whose autarky equilibrium is as follows

$$\begin{array}{lrcl} & & \text{A} & \\ 1. & 400w_A & = & 2000p_{1A} \\ 2. & 200w_A & = & 666.66p_{2A} \\ 3. & 400w_A & = & 4000p_{3A} \end{array} \tag{22}$$

Suppose the wage rate is Re.1. Next. suppose the government imposes an income tax at a flat rate of 20% on the income earners. The government will collect Rs. 200. To get a complete idea of the effects of the tax we must make some assumption in regard to the manner in which the government spends this revenue.

If the government spends the revenue in the same proportions as the private consumers would (viz. with co-efficient 0.4, 0.2 and 0.4) there is no change; the equilibrium state remains as in (22). If the proportions differ the equilibrium will be modified. For example, if the government of A spent the entire revenue of Rs. 200 on commodity 1, then the total expenditure on commodity 1 will be (0.4) (Private Disposable Income) + (Government Spending) = (0.4) (800) + 200 = Rs. 520. On commodities 2 and 3 the expenditure will be (0.2) (800) = Rs. 160 and (0.4) (800) = Rs. 320. The new equilibrium would stand at

$$\begin{array}{lrcl} & & \text{A} & \\ 1. & 520w_A & = & 2600p_{1A} \\ 2. & 160w_A & = & 533.33p_{2A} \\ 3. & 320w_A & = & 3200p_{3A} \end{array} \qquad (23)$$

The prices of the commodities remain unchanged whatever may be the allocation of government revenue.

If however, the government spends its revenue to finance a new activity, e.g. a public good, say defence, then the prices will be found to increase by a mark-up factor $(1/1 - t_A)$ where t_A is the income tax rate. In our example the mark-up factor will be $(1/1 - 0.2) = (1/0.8) = 1.25$.
Accordingly, the equilibrium state will be

$$\begin{array}{lrcl} & & \text{A} & \\ 1. & 320w_A & = & 1600p^*_{1A} \\ 2. & 1600w_A & = & 533.33p^*_{2A} \\ 3. & 320w_A & = & 3200p^*_{3A} \\ 4. & 200w_A & = & D \end{array} \qquad (24)$$

where $p^*_{iA} = (1.25)p_{iA}$ and D denotes the supply of the public good.

Next consider the effects if, instead of an income tax, the governement employed an excise tax to secure the revenue. Thus if the government imposed an excise tax at 20% and used the proceeds to finance the supply of a public good it would collect Rs. 200 in revenue and the equilibrium would be as in (24) above with prices marked up by the factor $(1/1 - t_{iA})$ where $t_{iA} = 0.2$ is the rate of tax.

If the Rs. 200 were created by deficit financing then the money supply would rise by Rs. 200, i.e. by 20%. However, the prices of the commodities would rise by 25% i.e. by a mark up factor of (1/1 – (D/M)) where D/M is the ratio of the deficit to initial money supply. The new equilibrium state will be the same as in (24) above.

Leaving the reader to work out the effects of various combinations of taxes, tax rates, deficits and the manners of government spending we turn to tracing the effect of these domestic distortions on international trade.

Consider a two-country three-commodity example. We reproduce below the autarky and trade equilibria for ready reference;

Autarky

	A	B
1.	$400w_A = 2000p_{1A}$	$800w_B = 160p_{1B}$
2.	$200w_A = 666.66p_{2A}$	$400w_B = 66.666p_{2B}$
3.	$400w_A = 4000p_{3A}$	$800w_B = 8000p_{3B}$

Trade

	A	B
1.	$666.67w_A = 3333.33p_{1A}$	....
2.	$333.33w_A = 1111.11p_{2A}$	....
3.	....	$2000w_B = 20000p_{3B}$

The wage rates are Re.1 and \$1 in countries A and B and the equilibrium exchange rate is $E_{AB} = 0.333$. Now suppose the government of country A imposes an income tax at the rate of 20%. If it spends the revenue in the same manner as the private citizens of A there is no change whatsoever.

Suppose it spends the proceeds to purchase commodity 1. Since there is no change in the prices, the natural exchange rates and the pattern of comparative advantages will remain exactly as before. The new trade equilibrium is as follows:

	A	B
1.	$766.66w_A = 3833.33p_{1A}$	....
2.	$233.33w_A = 777.77p_{2A}$	....
3.	....	$2000w_B = 20000p_{3B}$

with the exchange rate $E_{AB} = 0.2666$. The currency of country A has appreciated on account of reduction in her import bill and secured favourable terms of trade for her citizens. Obviously the policy cannot be taken too far. Every extension of such a policy will result in appreciation of A's currency and reduction in her capacity to supply the exportables until B may find it profitable to produce commodity 1. But until then the government of country A does manage to extract a part of the tax from citizens of country B.

Next suppose the government of country A spends tax revenue on a public good. As remarked earlier the prices of both commodities in country A will increase by 25%. However, the comparative advantages will be unaffected.

To find the trade equilibrium in country A note that the labour conservation equation in A will be $400\,\lambda_{1A} + 200\lambda_{2A} = 800$ since the government has taxed away Rs. 200 which suffices to purchase 200 units of labour required for the public activity. The trade equilibrium under this scenario is,

$$
\begin{array}{llll}
 & \text{A} & & \text{B} \\
1. & 533.33w_A = 266.663p^{*}_{1A} & \ldots\ldots\ldots\ldots\ldots & \\
2. & 266.66w_A = 888.88p^{*}_{2A} & \ldots\ldots\ldots\ldots\ldots & \\
3. & \ldots\ldots\ldots\ldots\ldots & 2000w_B = 20000p_{3B} & \\
4. & 200w_A = D & \ldots\ldots\ldots\ldots\ldots &
\end{array}
\qquad (25)
$$

where $p^{*}_{1A} = 1.25p_{1A}$, $p^{*}_{2A} = 1.25\ p_{2A}$. The equilibrium exchange rate is $E_{AB} = 0.3333$. The curious thing is that even though country A has faced price-inflation of 25% due to the taxation policy there is no depreciation of its currency. The tax is passed on not only to consumers in country A but also to those in country B. Even if the government of country had financed a deficit of Rs. 200 to create the public activity, the position remains that in (25).

In sum the general equilibrium approach to the international values can turn out conclusions that the intuition, trained on usual macroeconomic concepts, cannot easily apprehend.

Notes

1. Not always. Tariffs have been discussed in offer-curve terms, i.e. as in the general equilibrium framework. It is only when tariffs have been compared to quotas that the partial equilibrium analysis has been employed. Quotas have not been treated in terms of offer-curves and for good reason, as section 9 will show. Similarly intercountry transfers have also been dealt with using general equilibrium theory. See Leontief (1936) or Samuelson (1951, 1952). But the larger part of the literature on the transfer problem uses macroeconomic theory.
2. The secondary effects of the tariff viz. the effect of raising revenue for the government and the manner of the disbursement of that revenue are usually assumed away. Writers who do not assume them away typically suppose that the revenue is redistributed between the citizens of the country in proportion to their incomes. This procedure is unduly artificial. The natural alternative, that of letting the government spend the tariff revenue itself in proportions determined by it has been followed here.
3. This incidentally is the chief advantage of tariffs over the increasingly - popular non-tariff barriers (NTB's). NTB's give rise to several opportunities for commodity arbitrage because they are specified on a country-to-country basis on vague and ambiguous criteria. The advantage of NTB's is political—they are not expressible in figures.
4. Unambiguous, despite the fact that in country B the consumption of commodity 1 has reduced and that of commodity 3 has increased. But this clearly is not a movement along an indifference curve, it is a movement of the curve as the budget valuation shows.
5. The procedure was introduced by Viner(1950) and later followed almost generally.
6. This should straighten the record so far as the traditional theory goes. The theory was formulated in a setting of protectionism from which customs unions were expected to emerge in Europe.
7. The exchange rates are such as to ensure absence of commodity arbitrage, i.e., rupee price of say commodity 3 in country A is equal to the dollar price of commodity 3 in country B converted into rupees. Failing this the customs union will break up.
8. It may be assumed that the transfer amount is drawn from country A by means of a

proportional income tax and distributed in country *B* as a proportional income subsidy so that the average consumption patterns in the two countries are not affected.

9. This is a point of considerable interest. Taussig (1927) was puzzled by the fact he observed while studying British trade, of "the unmistakably close [and rapid] connection between international payments and the movements of commodity imports and exports." [Brackets mine]. He found "the explanation of the speedy adjustment" in "the sensitiveness of the British monetary system" but admitted that the same speediness existed in other countries with less sensitive monetary systems. Actually it is the sensitiveness of the system of general economic allocation, not just of the monetary systems, that accounts for Taussig's observation.
10. See Viner (1939) pp. 155, 185.
11. This is not a proof of the purchasing power parity theorem as the Monetary Approach would have us believe. The classical adjustment mechanism shows how the fixed exchange rate can be justified if *balance of payments* is to be ensured. That the adjustment takes place through wage-price changes does not mean that, therefore, wages and prices are the determinants of the exchange rate.
12. See Keynes (1936) pp. 269-271 on further reasons for following a fixed wage-policy.
13. The comparison of the redistributing effects of currency depreciation with those of domestic inflation is revealing. Inflation redistributes wealth from lenders of funds to borrowers of funds. Currency depreciation (of which domestic inflation incidentally is a cause) has the opposite effect. It redistributes wealth from borrowers of international funds to lenders of international funds. Just as indexation of debt principals protect lenders of funds from inflation, there is a case for "non-indexation" to protect borrower countries from currency depreciation. Non-indexation means that international debts must be settled at the exchange rates prevailing at the time they were contracted. Only this principle will prevent unwarranted losses in wealth for borrowers of funds when their currencies depreciate and lenders of funds when their currencies appreciate.

7

AUTARKY

And so she did: wandering up and down, and trying turn after turn, but always coming back to the house, do what she would.... "Oh, it's so bad!" she cried. "I never saw such a house for getting in the way! Never!"

1. Assumptions

This chapter formulates a model of economic equilibrium in the general tradition of Walras (1874), Von-Neumann (1947), Leontief (1941) and Sraffa (1960). For a brief survey of these models see Appendix 3. The assumptions on which the model is based are as follows.

We shall assume homogenous single-quality labour and single-product industries which produce "commodities by means of commodities". All industries produce by means of working capital–there are no durable capital goods or natural resources. Every industry has only one technique of production. Of course we allow techniques of production to change with growth in the technical knowledge. All industries produce under conditions of constant returns to scale. The demand functions are homogenous of degree zero in the prices and income. For the sake of simplicity the demand functions are described by Engels' equations. But the entire discussion can easily be adapted to include more general demand equations, eg. linear expenditure systems. Finally, the money wage rate is assumed to be fixed by contract. It can, of course, change with revisions of the contract. The wage is assumed to be paid *post-factum*. This too can easily be generalised as the need arises. (See Appendix 2)

2. Two Commodities: All Income Consumed

In the first three sections of this chapter we shall consider the peculiar properties of an economic system which produces only two commodities. If the two commodities are such that they can be used as both consumption and capital goods, their price equations can be written as follows,

$$\begin{aligned}(A_{11}p_1 + A_{21}p_2)(1+r) + wL_1 &= p_1X_1\\ (A_{12}p_1 + A_{22}p_2)(1+r) + wL_2 &= p_2X_2 \qquad (1)\end{aligned}$$

where A_{11} and A_{21} are the quantities of commodities 1 and 2 required to produce an output X_1 of commodity 1, A_{21} and A_{22} are the quantities of 1 and 2 required to produce an output X_2 of commodity 2, L_1 and L_2 are the labour requirements, w is the money wage rate and r is the rate of profit. We shall assume that the technical coefficients of production are fixed.

$$\begin{aligned} a_{11} &= A_{11}/X_1 & a_{21} &= A_{21}/X_1 \\ a_{12} &= A_{12}/X_2 & a_{22} &= A_{22}/X_2 \\ l_1 &= L_1/X_1 & l_2 &= L_2/X_2 \end{aligned}$$

The equations in (1) state that under competitive conditions, the sales revenues of both industries must be just sufficient to recover the cost of the raw materials (which is assumed to have been invested for one period) with profit at a uniform rate as well as the wage bill which is assumed to be paid *post-factum*.

We shall suppose the money wage rate w to be given. Then the system (1) contains 2 equations in 3 unknowns, viz. The prices of the two commodities and the rate of profit. The system of price equations (1) is by itself underdeterminate. To obtain a determinate system we must augment it to include the demand equations of the commodities.

Let us suppose, as one instance, that the economy spends its entire income, both profits and wages, on the two commodities. The profit income is $rK = r[(A_{11} + A_{12})p_1 + (A_{21} + A_{22})p_2]$ where K is the capital employed in the two industries. The wage income is $wL = w(L_1 + L_2)$. The definitional identity of factor incomes to the net national product is,

$$\begin{aligned} rK + wL &= p_1(X_1 - A_{11} - A_{12}) + p_2(X_2 - A_{21} - A_{22}) \\ &= p_1F_1 + p_2F_2 \end{aligned}$$

where F_1 and F_2 are the physical surpluses of commodities 1 and 2. Now suppose that the demand equation for commodity 1 is,

$$\alpha_1 wL + \beta_1 rK = p_1F_1 \qquad (2)$$

where α_1 is the proportion of wage income and β_1 is the proportion of profits that are spent on commodity 1. Since, by assumption, our economy consumes the whole income $\alpha_1 + \alpha_2 = 1$ and $\beta_1 + \beta_2 = 1$ where α_2 and β_2 are proportions of wages and profits spent on commodity 2. Thus

$$\alpha_2 wL + \beta_2 rK = p_2F_2 \qquad (3)$$

Equations (1) and (2), or the equivalent (1) and (3), form a determinate system with 3 equations in 3 unknowns. Writing K as $(A_{11} + A_{12})p_2 + (A_{21} + A_{22})p_2$ in (2) the system can be represented as follows:

$$\begin{bmatrix} X_1 - A_{11}(1+r) & -A_{21}(1+r) & -L_1 \\ -A_{12}(1+r) & X_2 - A_{22}(1+r) & -L_2 \\ F_1 - \beta_1(A_{11} + A_{12})r & -\beta_1(A_{21} + A_{22}) & -\alpha_1 L \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (4)$$

where w is the known money wage rate. The equations in (4) form a homogenous system of *non-linear* equations(1). A unique non-trivial solution for the prices exists if and only the determinant of the square matrix on the left-hand-side is equal to zero.

Setting the determinant equal to zero gives a quadratic polynomial in r whose (lower) positive root is the equilibrium competitive rate of profit. Substituting this value of r in the price-equations (1) along with the known money wage rate gives the corresponding solution for the prices of commodities,

$$\begin{bmatrix} X_1 - A_{11}(1+r) & -A_{21}(1+r) \\ -A_{12}(1+r) & X_2 - A_{22}(1+r) \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} wL_1 \\ wL_2 \end{bmatrix} \tag{5}$$

The solution for the rate of profit and the prices is positive only if the economic system produces physical surpluses of both commodities, i.e., $F_1, F_2 > 0$ and the propensities to consume α_i and β_i are positive, less than one and unequal to one another[(2)]. Then the rate of profit determined by (4) will be lower than the maximum rate of profit R corresponding to a zero wage[(3)].

The maximum rate of profit R is the lowest positive root of the quadratic polynomial obtained by setting the determinant of the matrix in (5) equal to zero. It is the rate of profit that corresponds to a zero wage rate. Thus R is a parameter whose value is determined entirely by the technology of production: the more efficient the technology, that is to say, the lower the technical coefficients of production $a_{ij} = A_{ij}/X_j$, the greater the value of R. On the other hand as is clear from (4), the equilibrium, or actual, rate of profit depends on the production coefficients as well as tastes and preferences.

If the rate of profit r is less than the maximum rate R, the matrix in (5) has a dominant diagonal, i.e. , $X_1 - A_{11}\ (1 + r) > A_{12}\ (1 + r)$ and $X_2 - A_{22}\ (1+ r) > A_{21}\ (1 + r)$ provided at the same time $R > 0$. The equations (5) will accordingly give a solution with positive prices for both commodities[(4)].

Some of the features of the model outlined above can be brought out from the example given below. Suppose the price equations for the economic system to be,

$$\begin{aligned} (3p_1 + 5p_2)(1 + r) + 6w &= 30p_1 \\ (2p_1 + 4p_2)(1 + r) + 10w &= 40p_2 \end{aligned} \tag{6}$$

Suppose the wage rate to be $w = £1$ and suppose that the wage earners spend $\alpha_1 = 0.6$ and the profit earners spend $\beta_1 = 0.4$, of their incomes on commodity 1. Then the demand equations for commodity 1 is

$$(0.6)(w)(16) + (0.4)(5p_1 + 6p_2)r = 25p_1 \tag{7}$$

where $5p_1 + 6p_2$ is the capital in the economy and $25 = 30 - 3 - 2$ is the physical surplus or net supply of commodity 1 available for final consumption. Equation (7) specifies the equality of demand with supply. Equations (6) and (7) can be arranged in vector-matrix notation as follows,

$$\begin{bmatrix} 30 - 3v & -5v & -6 \\ -2v & 40 - 4v & -10 \\ 27 - 2v & 3.6 - 3.6v & -9.6 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

where $v = 1 + r$. Setting the determinant of the matrix equal to zero gives a quadratic polynomial,

$$6.4v^2 - 1381.2v + 3960 = 0 \tag{9}$$

whose roots are $v_1 = 2.9062$ and $v_2 = 212.9062$. The higher value may be discarded since it will correspond to negative prices and negative real payments to workers. Indeed it is greater than the maximum rate of profit R which is obtained from (6) by putting $w = 0$,

$$\begin{bmatrix} 30-3v & -5v \\ -2v & 40-4v \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{10}$$

Setting the determinant of the matrix equal to zero gives the polynomial

$$2v^2 - 240v + 1200 = 0 \tag{11}$$

whose lower positive root, $V_1 = 5.227$, i.e. $R = 4.227$ gives the maximum rate of profit. Since the actual rate of profit $r = 1.9062$ is less than the maximum rate $R = 4.227$, there is some room for positive real wages to be paid to wage earners.

The prices of commodities corresponding to $r = 1.9062$ can be obtained by substituting it in (6) to get;

$$\begin{bmatrix} 21.281 & -14.531 \\ -5.821 & 28.375 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix} \tag{12}$$

solving which we get $p_1 = 0.4768$ $p_2 = 0.6075$. Observe that in the matrix in (12), the principal diagonal elements dominate the off-diagonal elements and yield positive solutions for the prices

Other derived variables can be determined from the solution of the rate of profit and the prices. thus the real wage rate in terms of the two commodities is.

$$w_1 = w/p_1 = 2.097$$
$$w_2 = w/p_2 = 1.646$$

Similarly the national income is $rK + wL = p_1F_1 + p_2F_2$ is £30.96 and the share of wages in national income is 51.66 percent.

The growth rate is zero since no part of income is saved and invested. The capital-output ratio is 0.2535. And so on.

Finally, observe that even if we had used the consumption equation of commodity 2 viz,

$$(0.4)(w)(16) + (0.6)(5p_1 + 6p_2)r = 31p_2$$

along with the price equations (6), the solution obtained would be identical. This happens due to the simple reason that the consumption equation of commodity 2 is linearly dependent on the equation of commodity 1, because of our assumption that all income is consumed.

3. Two Commodities: Saving and Growth

Let us now allow a system in which profit-earners and wage-earners save a part of their incomes and consume the other. Let the propensity to consume of wage earners be $\alpha = \alpha_1 + \alpha_2$ and the propensity to save $1 - \alpha$. Similarly suppose the propensity to consume of the profit earners is $\beta = \beta_1 + \beta_2$ and $1 - \beta$ is their propensity to save. Further suppose that the wage-earners own a proportion τ of the capital stock in the economy and the profit earners own $1 - \tau$. Then, of the total profit income of any period rK, τrK is distributed to wage-earners and $(1 - \tau)\, rK$ is distributed to profit earners.

With these considerations in mind let us proceed to find the demand equations for the commodities. Since both commodities are dual-purpose goods, i.e., they are used both as consumption and capital goods we must find the demands for them in both capacities.

The incomes of the workers and capitalists are,

$$\begin{aligned} Y_w &= wL + \tau rK \\ Y_c &= (1-\tau)\, rK \end{aligned} \tag{13}$$

Of these incomes, each class spends proportions α_1 and β_1 respectively on commodity 1. Thus the consumption expenditure on commodity 1 is,

$$\alpha_1 Y_w + \beta_1 Y_c = \alpha_1(wL + \tau rK) + \beta_1 (1-\tau)\, rK \tag{14}$$

To obtain the investment demand for commodity 1 we reason as follows. The saving in the economy is the sum of the saving by workers and capitalists,

$$\begin{aligned} S &= (1-\alpha)\, Y_w + (1-\beta)\, Y_c \\ &= (1-\alpha)\,(wL + \tau rK) + (1-\beta)(1-\tau)\, rK \end{aligned} \tag{15}$$

The saving represents the addition to capital stock in the economy as a whole. But how is it to be apportioned between the industries and, further, between the commodities which make up each industry's capital? It will be clear that on account of our assumption of constant returns to scale production conditions, the allocation of saving must be made proportionately. Thus if K_1 and K_2 represent the industry capital stocks, i.e. $K_1 = A_{11}p_1 + A_{21}p_2$, $K_2 = A_{12}p_1 + A_{22}p_2$ and $K_1 + K_2 = K$, the saving will be allocated in proportion to their values. Thus the additions to the industry capital stocks will be $(K_1/K)S$ and $(K_2/K)S$ respectively.

We must further allocate these sums between the commodity constituents of each industry's capital. This allocation too must be made in proportion to the *values* of the commodities in the capital of each industry. For only this method of allocation is consistent with our assumption of constant returns to scale and fixed technical coefficients of production. Thus the investment expenditures on commodity 1 in the two industries are,

$$\left[\frac{A_{11}p_1}{K_1}\right]\left[\frac{K_1}{K}\right]S = \left[\frac{A_{11}p_1}{K}\right]S \tag{16}$$

$$\left[\frac{A_{12}p_1}{K_2}\right]\left[\frac{K_2}{K}\right]S = \left[\frac{A_{12}p_1}{K}\right]S$$

The total expenditure on commodity 1 is the sum of consumption expenditure (14) plus the investment expenditures (16). The total expenditure must be equal to the value of the net supply (after providing for replacement) of commodity 1, viz p_1F_1. Thus, with reference to (15) we write the demand-supply equation for commodity 1 as,

$$\alpha_1(wL + \tau rK) + \beta_1(1-\tau)rK + \left[\frac{(A_{11}+A_{12})p_1}{K}\right]$$
$$[(1-\alpha)(wL+\tau rK) + (1-\beta)(1-\tau)rK] = p_1F_1 \tag{17}$$

Equation (17), or its linearly dependent counterpart equation for commodity 2, gives us the required independent equation which, along with the price equations for the two commodities [equation (1)], will enable us to determine the prices and the rate of profit.

There are, however, two hurdles to be cleared before this can be done. Observe that in (17) the capital stock K appears in the denominator. But since K itself contains terms in the prices p_1 and p_2, it is impossible to obtain a polynomial equation in the rate of profit r alone; the polynomial will contain, besides r, the prices of commodities which themselves are unknowns. Moreover (17) contains an additional unknown τ which represents the proportion of capital stock owned by workers. Fortunately both the hurdles can be cleared in one stride. Consider the shares of ownership. Clearly, the relative shares of ownership must be determined by relative proportions of saving by the two classes. Thus, we must have

$$\frac{\tau}{1-\tau} = \frac{(1-\alpha)(wL+\tau rK)}{(1-\beta)(1-\tau)rK} \tag{18}$$

whence we get,

$$\tau = \frac{(1-\alpha)wL}{(\alpha-\beta)rK} \tag{19}$$

and

$$\frac{wL}{K} = \frac{\tau(\alpha-\beta)r}{(1-\alpha)} \tag{20}$$

Substituting (19) and (20) into (17) and eliminating τ from (17) we get an equation that is in directly usable form,

$$\begin{aligned}[F_1(\alpha-\beta) - (\alpha-\beta)(1-\beta_2)(A_{11}+A_{12})r]p_1 \\ -[\beta_1(\alpha-\beta)(A_{21}+A_{22})r]p_2 \\ -[\alpha_1(1-\beta)-\beta_1(1-\alpha)]wL = 0\end{aligned} \tag{21}$$

Letting

$$\begin{aligned}\phi_1 &= (\alpha-\beta)(1-\beta_2)(A_{11}+A_{12}) \\ \phi_2 &= \beta_1(\alpha-\beta)(A_{21}+A_{22}) \\ \phi_3 &= [\alpha_1(1-\beta)-\beta_1(1-\alpha)]\end{aligned}$$

we can write (21) as,

$$[F_1(\alpha-\beta)-\phi_1 r]\,p_1-(\phi_2 r)p_2-(\phi_3 L)w \;=\; 0 \qquad (22)$$

The price equations (1) and the demand-supply equality (22) gives the final system,

$$\begin{bmatrix} X_1 - A_{11}(1+r) & -A_{21}(1+r) & -L_1 \\ -A_{12}(1+r) & X_2 - A_{22}(1+r) & -L_2 \\ F_1(\alpha-\beta)-\phi_1 r & -\phi_2 r & -\phi_3 L \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (23)$$

The remaining steps of the solution are identical to those followed in section(2).

There are two conditions for a viable solution for the prices, the rate of profit and the workers' share of ownership, in addition to those mentioned in section 2. Firstly, for the share of workers to be greater than, or equal to zero, it is necessary that the propensity to consume of the workers must exceed that of the capitalists: with reference to (19) we see that,

$$\tau \geq 0 \text{ only if } \alpha > \beta$$

Secondly, for the share of workers' ownership to be less than 1 by equation(20),

$$\frac{(1-\alpha)wL}{(\alpha-\beta)\pi K} < 1$$

i.e.,
$$wL < \frac{(\alpha-\beta)\pi K}{(1-\alpha)}$$

In what follows economic systems which do not satisfy these conditions for $0 \leq \tau < 1$, will not be considered.

Some features of this generalised system are noteworthy. The new investment expenditures on the commodity 1 are obtained from equations (15) and (16). A similar equation can also be derived for commodity 2. The investment expenditures on commodities 1 and 2 are

$$I_1 = \left[\frac{(A_{11}+A_{12})p_1}{K}\right]\left[(1-\alpha)(wL+\tau rK)+(1-\beta)(1-\tau)rK\right] \qquad (24)$$

Similarly the investment expenditure on commodity 2 will be,

$$I_2 = \left[\frac{(A_{21}+A_{22})p_2}{K}\right]\left[(1-\alpha)(wL+\tau rK)+(1-\beta)(1-\tau)rK\right] \qquad (25)$$

The ratios of the new investment expenditures to the original capital stocks of both commodities will give the growth rates. These will, of course be the same for both commodities because technical coefficients are fixed and saving has been allocated proportionately.

If we make the classical assumption that profits are wholly saved and wages wholly consumed, $\tau = 0$, $\alpha = 1$, $\beta = 0$ then the investment expenditures in (24) and (25) reduce simply to $(A_{11}+A_{12})p_1 r$ and $(A_{21}+A_{22})p_2 r$ and the growth rate of the economy is,

$$g = \frac{(A_{11}+A_{12})p_1 r}{(A_{11}+A_{12})p_1} = \frac{(A_{21}+A_{22})p_2 r}{(A_{21}+A_{22})p_2} = r \tag{26}$$

If profits are partly saved and partly consumed but wages are wholly consumed, $\tau = 0$, $\alpha = 1$, then (24) and (25) give

$$I_1 = (A_{11}+A_{12})p_1(1-\beta)r$$
$$I_2 = (A_{21}+A_{22})p_2(1-\beta)r$$

So that,

$$g = r(1-\beta) \tag{27}$$

Even in the general case of part–saving by workers and part-consumption by capitalists, the relationship between the growth rate and the rate of profit will continue to remain as it is in (27). This can be easily seen by substituting the value of wL/K as given in equation (20), into (24) and (25). Thus the *relation* of the rate of growth and the rate of profit stands irrespective of the workers' propensity to save or the workers' share of ownership although the particular magnitudes of the rates of growth and profit are bound to be affected by workers' saving.

The results just derived(equation 27) are well-known as the "Cambridge growth and distribution equation" developed in the course of a series of contributions by Keynes (1930), Kalecki (1942, 1953), Robinson (1956), Kaldor(1956), Champernowne (1958), Kahn (1959) and Pasinetti (1962). The chief difference is that in the original macroeconomic versions the stock of capital is assumed to be given exogenously whereas in the model described above the stock of capital will be found along with the rates of growth and profit.

The growth rate itself must be equal to the ratio of the propensity to save of *national income* and the capital-output ratio. This Harrod Domar equality, however, is brought about by the *prices* of commodities—the prices will be such as to simultaneously determine values of the aggregate saving propensity and the capital-output ratio which satisfy the Harrod-Domar equality(5).

How does the system behave if there is a discrepancy between the rates of growth 'warranted' by capital formation and the 'natural' rate of population growth? Neoclassical growth theory presupposes that techniques will usually exist to accommodate the discrepancy. But if our reasoning is valid, it will not be necessary to go that far with the assumptions; *relative* price flexibility is sufficient in cases where there is unutilised labour. If there is none, and the warranted growth rate exceeds the 'natural' growth rate then a closed economy will perforce be tied down to the lower natural growth rate. The sizes of the consumption goods industries will increase and the prices of all commodities will fall. If the economy is open, however, capital will be exported and/or labour will be imported.

To get a numerical idea of the solution explained above consider an example. Suppose the price equations are,

$$(2p_1 + 3p_2)(1 + r) + 10w = 20p_1$$
$$(3p_1 + 1p_2)(1 + r) + 5w = 25p_2$$

Suppose the propensities of the workers to consume commodities 1 and 2 are $\alpha_1 = 0.4$, $\alpha_2 = 0.4$ and those of the capitalists and $\beta_1 = 0.1$, $\beta_2 = 0.1$. Then $\alpha = \alpha_1 + \alpha_2 = 0.8$ and $\beta = \beta_1 + \beta_2 = 0.2$. The demand-supply equality for commodity1 (equation (21)) is,

$$(9 - 2.7r)\,p_1 - (0.24r)p_2 - 4.5w = 0$$

If the wage rate is $w = £1$ and denoting $1 + r$ by v, the above three equations in matrix notation are,

$$\begin{bmatrix} 20 - 2v & -3v & -10 \\ -3v & 25 - 1v & -5 \\ 11.7 - 2.7v & 0.24 - 0.24v & -4.5 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Setting the determinant of the coefficients on the left-hand-side equal to zero gives us the polynomial,

$$1.32v^2 - 32.07v + 69.9 = 0$$

The profit factor $v = 2.42082$ so that the rate of profit is $r = 1.42082$. Substituting the profit factor in the price-equations we get the solution for the prices, $p_1 = 0.51263\,p_2 = 0.90530$ so that the capital stock $K = 5p_1 + 4p_2 = 6.5770$. Finally, equation (19) solves for the shares of ownership, $\tau = 0.53505$, $(1 - \tau) = 0.46495$, of the workers and capitalists respectively, and equation (27) solves for the rate of growth, $g = r\,(1 - \beta) = 1.13665$.

4. Two Commodities: Decomposable System

In the preceding two sections both commodities were supposed to play the dual role of capital as well as consumption goods. If, however, one of the commodities say commodity 1, is purely a capital good whilst the other, commodity 2, is purely a consumption good, the system becomes simpler to handle.

Since commodity 2 is not used as an input in the production of either commodity, $A_{21} = A_{22} = 0$. All investment expenditure will be concentrated on commodity 1 and all consumption expenditure on commodity 2. Thus the saving of the economy, whatever it may be, must be used up in the purchase of net physical surplus of commodity 1. As a result, the growth of the economy can be ascertained independently of the prices simply by taking the ratio of the physical surplus of commodity 1 to the quantity of the capital good that forms the capital in both industries,

$$g = \frac{S}{K} = \frac{p_1 F_1}{p_1(A_{11} + A_{12})} = \frac{F_1}{(A_{11} + A_{12})} \tag{28}$$

With the growth rate thus known the Cambridge equation (27) solves for the rate of profit with reference to the given propensity to save of capitalists. The price equations determine the prices at the given wage rate so that the capital stock, $K = (A_{11} + A_{12})p_1$, can be ascertained. Then equation (19) solves for the shares of ownership τ and $1 - \tau$.

Thus suppose $w = £1, \alpha = \alpha_2 = 0.8, \beta = \beta_2 = 0.4$ and the price equations to be,

$$5p_1 (1 + r) + 2w = 16p_1$$
$$5p_1 (1 + r) + 1w = 30p_2$$

By the method just described the growth rate is found by the formula,

$$g = \frac{16 - 10}{10} = \frac{6}{10} = 0.6$$

and the rate of profit is,

$$r = \frac{g}{1 - \beta} = \frac{0.6}{0.6} = 1$$

The prices therefore are $p_1 = 0.3333, p_2 = 0.1444$ and the capital stock

$$K = 10p_1 = £3.3333$$

Finally . the share of ownership in capital stock of the workers is,

$$\tau = \frac{(1 - \alpha)wL}{(\alpha - \beta)rK} = \frac{(0.2)(1)(3)}{(0.4)(1)(3.333)} = 0.45$$

It can be verified that the same solution will be obtained even if we follow the matrix method the price-equations and the demand-supply equality for commodity 2.

5. Difficulty of Generalising from 2 to *N*

The method devised for the determination of prices and income distribution in the two-commodity case ceases to apply when more than two commodities are considered. This is best seen if we attempt to generalise the logic of the two-commodity case to a situation of several commodities. Recall that in the two-commodity case the levels of output of both the commodities are given. This leaves us with 3 unknowns—the two prices and the rate of profits which are determined by three equations—two price equations and one independent demand equations. With n commodities, therefore, there will be n price equations and $n - 1$ independent demand equations to determine n prices and the rate of profit, i.e. $n + 1$ unknowns. The number of equations will be greater than the number of unknowns and the system becomes overdeterminate for $n > 2$.

The fault, however, as we shall see, lies not in the system but in the attempt to generalise. In fact, the additional equations will allow us to determine the outputs of commodities which are regarded as given in the two commodity case. In what follows, therefore, the commodity outputs will be regarded as unknowns along with the prices and income distribution.

6. The System of Equations: Decomposable Economy

Consider a competitive economic system which produces n commodities in n separate industries. We will suppose, to start with, that of the n commodities, m are purely capital goods whilst the rest $n-m$ are purely final consumption goods. These latter do not enter either directly or indirectly into the production of any commodity. The price equations are,

$$
\begin{aligned}
(A_{11}p_1 + A_{21}p_2 + \dots\dots\dots + A_{m1}p_m)(1+r) + wL_1 &= p_1X_1 \\
(A_{12}p_1 + A_{22}p_2 + \dots\dots\dots + A_{m2}p_m)(1+r) + wL_2 &= p_2X_2 \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
(A_{1n}p_1 + A_{2n}p_2 + \dots\dots\dots + A_{mn}p_m)(1+r) + wL_n &= p_nX_n
\end{aligned}
\tag{29}
$$

in which, it will be observed that $A_{ji} = 0$ for all consumption goods $j = m+1,.. n$. Further there are the demand-supply equations for the consumption goods. Assume, for the sake of simplicity, that wages are entirely spent while profits are partly spent and partly saved. Then the demand equations for the consumption goods are,

$$\alpha_i wL + \beta_i rK = p_i X_i \qquad i = m+1, \dots, n \tag{30}$$

where

$$
\begin{aligned}
\Sigma\,\alpha_i &= \alpha = 1 \\
\Sigma\,\beta_i &= \beta < 1
\end{aligned}
$$

A portion $(1-\beta)$ of the profits will be saved every year. Thus the total amount saved will be,

$$S = (1-\beta)\,rK$$

It follows from the saving -investment identity that the value of the physical surpluses of the capital goods must be equal to the amount saved. Thus,

$$S = (1-\beta)rK = \sum_{j=1}^{m} p_j F_j$$

where

$$K = \sum_{j}^{m}\sum_{i}^{n} A_{ji}p_j$$

$$F_j = X_j - \sum_{j}^{m} A_{ji}$$

The saving-investment identity will not of course help to determine anything. We need to analyse how the saving will be allocated between industries and between the capital goods that make up the capital stocks of the different industries. Thus to find the demand-supply equations of the capital goods we must consider them individually.

The annual saving S will be allocated in proportion to the capital stocks of the different industries. Thus each industry's allocation will be,

$$\left[\frac{K_i}{K}\right]S = \left[\frac{K_i}{K}\right][(1-\beta)rK] = (1-\beta)rK_i \qquad i = 1, ..., n$$

This amount must be further reapportioned to purchase the capital goods required by the different industries. Thus the amount allocated towards the purchase of the individual capital goods will be

$$\left[\frac{A_{ji}p_j}{K_i}\right](1-\beta)rK_i = (1-\beta)rA_{ji}p_j \qquad j = 1, ... m, \; i = 1, ..., n$$

Our aim is to find the demand for each capital good by all industries put together. In the expression above, the demand for the jth capital good by the ith industry has been given. We therefore sum up over the industries to obtain the demand for capital goods j in the economy as a whole,

$$\sum_{i=1}^{n}(1-\beta)rA_{ji}p_j = (1-\beta)r\cdot\sum_{i=1}^{n}A_{ji}p_j \qquad j = 1, ..., m$$

Since F_j is the physical surplus of capital good j that is available for new investment, the demand-supply equality of the capital goods will take the form.

$$(1-\beta)r\sum_{i=1}^{n}A_{ji}p_j = p_jF_j \qquad j = 1, ..., m$$

Cancelling p_j on both sides the equation becomes,

$$(1-\beta)r\sum_{i=1}^{n}A_{ji} = F_j \qquad j = 1,..., m \tag{31}$$

where ΣA_{ji} is total use made of capital good j in the economic system.

7. Procedure to Find the Equilibrium

It is evident that the equations in (30) and (31) will not hold for arbitrary levels of outputs of the commodities. We have to find the specific levels of outputs, X_i, for which the demands and supplies are equal. The methods for finding them are given below.

As a preliminary note that since F_j is the new addition to the stock of capital good j, ΣA_{ji}, their ratio denotes the growth in the use of commodity j,

$$g_j = \frac{F_j}{\sum_i A_{ji}} = r_j(1-\beta) \qquad i = 1, ..., n \;\; j = 1, ..., m \tag{32}$$

Since by hypothesis, the techniques of production are constant returns to scale, the growth rates of the different capital goods used as inputs must all be equal to one another. If they are not equal, there will be excess supplies of some capital goods and excess demands for others. The imbalance can only be avoided if the outputs of the capital goods are such that the growth in the stocks of all individual capital goods is uniform, that is to say, there is a balanced rate of growth,

$$g_i = g_j = g \qquad \forall \; i, j = 1, \dots m. \tag{33}$$

The outputs of the capital goods must be such as to satisfy the replacement demands and provide surpluses that will result in a uniform rate of growth. To determine these outputs we shall use a variant of the *standard system* of equations due to Sraffa (1960). We are in search of scale multipliers $\lambda_1, \lambda_2, \dots \lambda_m$ which, when used to multiply the initial (arbitrary) levels of production of the capital goods industries X_i in (29) will result in outputs $\lambda_i X_i$ that are consistent with a uniform rate of growth. The equations which determine the λ's are,

$$\begin{aligned}
(A_{11}\lambda_1 + A_{12}\lambda_2 + A_{13}\lambda_3 + \dots + A_{1m}\lambda_m + A_1)(1+g) &= \lambda_1 X_1 \\
(A_{21}\lambda_1 + A_{22}\lambda_2 + A_{23}\lambda_3 + \dots + A_{2m}\lambda_m + A_2)(1+g) &= \lambda_2 X_2 \\
(A_{31}\lambda_1 + A_{32}\lambda_2 + A_{33}\lambda_3 + \dots + A_{3m}\lambda_m + A_3)(1+g) &= \lambda_3 X_3 \qquad 34(a) \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
(A_{m1}\lambda_1 + A_{m2}\lambda_2 + A_{m3}\lambda_3 + \dots + A_{mm}\lambda_m + A_m)(1+g) &= \lambda_m X_m
\end{aligned}$$

where $A_1, A_2, \dots, A_m$ are the quantities of the capital goods 1, 2, ..., m required by the consumption goods industries, of course, at their initial (arbitrary) levels of production. Thus,

$$A_i = \sum_j A_{ij} \qquad i = 1, \dots, m \quad j = m+1, \dots, n$$

The right hand side of each equation in 34(a) shows the supply of the capital goods. The left-hand side shows the total demand for the capital goods consisting of the replacement demand in the capital goods industries, $\sum_j A_{ij}\lambda_i$ $(i, j = 1, \dots m)$, the replacement demand in the consumption goods industries A_i $(i = 1, \dots m)$ and new investment demand at a uniform rate of growth g. Equations 34 (a) contain m equations in $m + 1$ unknowns, $\lambda_1, \dots \lambda_m$ and g. We need one more equation. This equation, which we shall call the conservation-of-labour equation, is,

$$\lambda_1 L_1 + \lambda_2 L_2 + \dots + \lambda_m L_m = L_1 + L_2 + \dots + L_m = L_k \tag{34(b)}$$

where L_k is the total labour employed in the capital goods industries.

The system of the $m + 1$ equations 34(a) and (b) can be written in matrix notation,

$$\begin{bmatrix}
X_1 - A_{11}(1+g) & -A_{12}(1+g) & \dots & -A_{1m}(1+g) & -A_1(1+g) \\
-A_{21}(1+g) & X_2 - A_{22}(1+g) & \dots & -A_{2m}(1+g) & -A_2(1+g) \\
\dots & \dots & \dots & \dots & \dots \\
-A_{m1}(1+g) & -A_{m2}(1+g) & \dots & X_m - A_{mm}(1+g) & -A_m(1+g) \\
-L_1 & -L_2 & \dots & -L_m & L_k
\end{bmatrix}
\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \lambda_m \\ 1 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{bmatrix} \tag{34}$$

Setting the determinant of the matrix of the left hand side of (34) equal to zero gives a polynomial in g whose lowest positive root, say g^0 is the rate of growth. If the system is productive and obeys the Hawkins-Simon conditions (1949):

$$\sum_{j}^{n} A_{ij} \leq X_i \,, \quad \text{i.e.,}\ F_i \geq 0 \qquad i = 1,\ldots,m$$

or

$$\sum_{j=1}^{n} a_{ij} \leq 1 \qquad \forall\ i = 1,\ldots,m$$

with strict inequality holding for some i, then a positive rate of growth exists. As long as there is some demand for consumption goods ($\alpha_i > 0$, $\beta_i > 0$ for $i = m + 1,\ldots, n$) this rate of growth will be lower than the maximum rate of growth/ profit, R, corresponding to zero consumption.

Once the rate of growth is known, equations 34(a) solve for the scale multipliers λ_i,

$$\begin{bmatrix} X_1 - A_{11}(1+g^{(0)}) & -A_{12}(1+g^{(0)}) & \ldots & -A_{1m}(1+g^{(0)}) \\ -A_{21}(1+g^{(0)}) & X_2 - A_{22}(1+g^{(0)}) & \ldots & -A_{2m}(1+g^{(0)}) \\ \ldots & \ldots & \ldots & \ldots \\ -A_{m1}(1+g^{(0)}) & -A_{m2}(1+g^{(0)}) & \ldots & X_m - A_{mm}(1+g^{(0)}) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \ldots \\ \lambda_m \end{bmatrix}$$

$$= \begin{bmatrix} A_1(1+g^{(0)}) \\ A_2(1+g^{(0)}) \\ \ldots \\ A_m(1+g^{(0)}) \end{bmatrix} \tag{35}$$

If the matrix on the left hand side has a dominant diagonal, i.e,

$$X_i > \sum_{i}^{m} A_{ij}(1+g^0) \qquad \forall\ i = 1,\ldots,m \tag{36}$$

then its inverse will be positive and so will the scale multipliers. That the matrix will possess a dominant diagonal if the system obeys the Hawkins-Simon conditions is obvious because $g^0 < R$.

The λ's obtained from (35) $\lambda_1^{(0)}, \lambda_2^{(0)}, \ldots, \lambda_m^{(0)}$ will be used to multiply the price equations of the m capital goods in (29). After the multiplication, the equations are brought into a state of a uniform rate of growth at the given, arbitrary, levels of production of the consumption goods industries. The scale multiplier procedure also equates the demands and supplies of the capital goods. The levels of input, labour and outputs which ensure the equality of the demands and supplies of the capital goods are,

$$A_{ji} = A_{ji}^{(1)} \qquad\qquad i, j = 1,\ldots,m$$

$$L_j = L_j^{(1)} \qquad j = 1, ..., m \tag{37}$$

$$X_j = X_j^{(1)} \qquad j = 1, ..., m$$

where the terms on the right hand side denote the new terms in the price equations of the capital goods industries.

Next we turn to the sphere of the consumption goods. As has been pointed out, the outputs of the consumption goods are arbitrary—their equilibrium levels have to be determined. To determine these we must find the demands for consumption goods and then change the supplies in such a way that they satisfy the demands. To find the demands for the consumption goods we need to know all the prices and the rate of profit. The rate of profit as is clear from (32) and (33) will be,

$$r^{(0)} = \frac{g^{(0)}}{(1-\beta)} \tag{38}$$

where $g^{(0)}$ is growth rate obtained from(34). Substituting the value of $r^{(0)}$ into the m price equations of the capital goods (whether multiplied by $\lambda_j^{(0)}$ or not makes no difference) gives a solution for the capital goods prices. The system of equations is,

$$\begin{bmatrix} X_1 - A_{11}(1+r^{(0)}) & -A_{12}(1+r^{(0)}) & \dots & -A_{1m}(1+r^{(0)}) \\ -A_{22}(1+r^{(0)}) & X_2 - A_{22}(1+r^{(0)}) & \dots & -A_{2m}(1+r^{(0)}) \\ \dots & \dots & \dots & \dots \\ -A_{1m}(1+r^{(0)}) & -A_{2m}(1+r^{(0)}) & \dots & X_m - A_{mm}(1+r^{(0)}) \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \dots \\ p_m \end{bmatrix}$$

$$= \begin{bmatrix} wL_1 \\ wL_2 \\ \dots \\ wL_m \end{bmatrix} \tag{39}$$

The prices obtained from (39) will be positive if the matrix has a dominant diagonal, which will be true only if $r^{(0)} < R$. We shall assume $r^{(0)} < R$, which means we shall admit only such values of β as will ensure it.

From the prices of the capital goods, denoted by $p_1^{(1)}, p_2^{(1)}, ..., p_m^{(1)}$, we can firstly ascertain the value of the capital stock,

$$K^{(1)} = \sum_j^m \sum_i^n A_{ji}^{(1)} p_j^{(1)}$$

and secondly, using the price equations of the consumption goods find the n–m supply prices of consumption goods, $p_{m+1}^{(1)}, ..., p_n^{(1)}$. Then from the equations in (30) we find the quantities demanded of the consumption goods,

$$X_{di} = \frac{\alpha_i wL + \beta_i rK^{(1)}}{p_i} \qquad i = m+1, ..., n \tag{40}$$

Generally X_{di} will not be equal to the arbitrary outputs of the consumption goods $X_i^{(0)}$ although the sum of the values of excess demands must, by Walras's Law, be equal to zero,

$$\sum_{i=m+1}^{n} p_i(X_{di} - X_i^{(0)}) = 0$$

Our aim, however, is to find those outputs of the consumption goods for which the excess demand/supply of every consumption good is zero.

Thus if the demands and supplies are unequal we must change the outputs of the consumption goods in the direction of their demands and check whether at the new output levels, the demand-supply discrepancies have been wiped out. The immediate problem is to set the new supply levels. For this purpose we shall use Newton's method of successive approximations. Suppose the new supply levels to be,

$$X_i^{(1)} = X_i^{(0)} + \frac{(X_{id} - X_i^{(0)})}{2} \qquad i = m+1, \ldots, n \tag{41}$$

i.e. we shall try the averages of the demands and initial supplies as the possible candidates for the equilibrium consumption goods outputs.

This is the beginning of another trial. Firstly observe that we cannot suppose the growth rate to remain $g^{(0)}$ because $g^{(0)}$ was calculated at $X_i^{(0)}$ outputs of the consumption goods industries (in equation (34). With output levels at $X_i^{(1)}$ the inputs and labour required will be,

$$\begin{aligned} A_{ji}^{(1)} &= a_{ji}X_i^{(1)} \qquad j = 1, \ldots\ldots, m \qquad i = m+1, \ldots, n \\ L_i^{(1)} &= l_i X_i^{(1)} \qquad\qquad\qquad\qquad i = m+1, \ldots, n \end{aligned} \tag{42}$$

where a_{ji} and l_i are technical coefficients of production of consumption goods i. The capital goods requirements of the consumption goods industries will therefore change to,

$$A_j^{(1)} = \sum_i A_{ji}^{(1)} \qquad i = m+1, \ldots, n \quad \forall j \tag{43}$$

The capital goods outputs $X_j^{(1)}$ which ensured balance between their demands and supplies at the old consumption goods levels $X_i^{(0)}$, will no longer continue to do so with consumption goods outputs $X_i^{(1)}$. A fresh round of calculations will be required. Accordingly we once again set up a system of equations to ensure balance of the capital goods' demands and supplies and a uniform rate of growth. This system is simply a revised version of (34).

$$\left[\begin{array}{c|c|c|c} X_1^{(1)} - A_{11}^{(1)}(1+g) & -A_{12}^{(1)}(1+g) & -A_{1m}^{(1)}(1+g) & -A_1^{(1)}(1+g) \\ -A_{21}^{(1)}(1+g) & X_2^{(1)} - A_{22}^{(1)}(1+g) & -A_{2m}^{(1)}(1+g) & -A_2^{(1)}(1+g) \\ \cdots & \cdots & \cdots & \cdots \\ -A_{m1}^{(1)}(1+g) & -A_{m2}^{(1)}(1+g) & X_m^{(1)} - A_{mm}^{(1)}(1+g) & -A_m^{(1)}(1+g) \\ -L_1^{(1)} & -L_2^{(1)} & -L_m^{(1)} & L_K \end{array}\right]$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \lambda_m \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{bmatrix} \tag{34}$$

where $X_j^{(1)}, A_{ji}^{(1)}, L_j^{(1)}, (j = 1, ... m)$ are obtained from (37) and $A_j^{(1)}$ are obtained from (42) and (43).

From (34) we find the growth rate $g^{(1)}$. The scale multipliers λ_j are found from the system

$$\begin{bmatrix} X_1^{(1)} - A_{11}^{(1)}(1+g^{(1)}) & -A_{12}(1+g^{(1)}) & \dots & -A_{1m}^{(1)}(1+g^{(1)}) \\ -A_{21}^{(1)}(1+g^{(1)}) & X_2 - A_{22}(1+g^{(1)}) & \dots & -A_{2m}^{(1)}(1+g^{(1)}) \\ \dots & \dots & \dots & \dots \\ -A_{m1}^{(1)}(1+g^{(1)}) & -A_{m2}^{(1)}(1+g^{(1)}) & \dots & X_m^{(1)} - A_{mm}^{(1)}(1+g^{(1)}) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \lambda_m \end{bmatrix}$$

$$= \begin{bmatrix} A_1^{(1)}(1+g^{(1)}) \\ A_2^{(1)}(1+g^{(1)}) \\ \dots \\ A_m^{(1)}(1+g^{(1)}) \end{bmatrix}$$

Denote the solution obtained from (35) by $\lambda_1^{(1)}, \lambda_2^{(1)}, ..., \lambda_m^{(1)}$. The input-output levels of the capital goods industries will then stand revised to,

$$\begin{aligned} A_{ji}^{(2)} &= \lambda_j^{(1)} A_{ji}^{(1)} & i, j &= 1, ..., m \\ L_j^{(2)} &= \lambda_j^{(1)} L_j^{(1)} & j &= 1, ..., m \\ X_j^{(2)} &= \lambda_j^{(1)} X_j^{(1)} & j &= 1, ..., m \end{aligned} \tag{37}$$

At the same time find the revised value of the rate of profit $r^{(1)}$ corresponding to the growth rate $g^{(1)}$ from (38) , find the prices of the capital goods $p_j^{(2)}$ $(j = 1, ..., m)$ using (39) and then the supply prices of the consumption goods $p_i^{(2)}$ $(i = m + 1, ..., n)$. Next find the quantities demanded from (40), remembering that the level of employment will be $L^{(1)} = \sum_i^n L_i^{(1)}$ and the capital stock will be $K^{(1)}$, and set the new trial supply levels of consumption goods at,

$$X_i^{(2)} = X_i^{(1)} = \frac{X_{di} - X_i^{(1)}}{2} \qquad i = m+1, ..., n \tag{41}$$

(42) and (43) give the input requirements of these supply levels, which will be fed in, along with the revised scales of the capital goods industries from (37) into (35) .... and so on until at the end of t iterations the demand and supplies of the consumption goods are equal: $X_{di} = X_i^{(t)}$. By the very logic of the procedure, it

will follow that in the final stage $\lambda_j^{(t)} = 1 (j = 1, ..., m)$ and $g^{(t)} = g$, showing the capital goods sphere to be in equilibrium as well. The final solution of the prices $p_i^{(t)}$ $(i = 1, ..., n)$, outputs $X_i^{(t)}$ $(i = 1, ..., n)$ the rate of growth $g^{(t)}$ and the rate of profit $r^{(t)}$ and the level of employment $L^{(t)}$ emerge in the final iteration.

There are six conditions which ensure the existence of the unique positive equilibrium for any economic system of the type that has been described.

(1) The economy conforms to the assumptions mentioned in the first section.

(2) The technology satisfies the Hawkins-Simon conditions for viability,

$$\sum_{j}^{n} a_{ij} < 1 \qquad \forall\ i = 1, ..., m,$$

(3) The propensity to consume for each consumption good is positive,

$$\alpha_i, \text{ or } \beta_i > 0 \qquad \forall\ i = m + 1, ..., n$$

(4) The aggregate propensity to consume of workers is greater than that of the capitalists,

$$\alpha > \beta$$

(5) The capitalists propensity to consume is less than the ratio of the rate of the consumed surplus to the maximum rate of surplus

$$\beta < \frac{R - g}{R}$$

(6) The share of ownership of the workers is less than hundred per cent, for which,

$$L < \frac{(\alpha - \beta) rK}{(1 - \alpha) w}$$

Conditions (2) and (3) ensure a positive rate of growth that is less than the maximum R and thereby ensure that the standard system for the determination of the outputs of capital goods has a dominant diagonal and therefore, gives a positive solution. Condition (4) ensures that the share of workers in the ownership is non-negative, i.e., zero in case $\alpha = 1$, or positive in case $\alpha > 1$. Condition (5) ensures that the rate of profit is lower than the maximum R and thereby ensures that the system of equations which determine the prices of capital goods has a dominant diagonal and therefore yield positive prices. Finally, condition (6) ensures that the workers share of ownership is less than 1.

8. Examples

For those who have found the discussion of section 6 rather abstract, some examples should help to break the tedium. Two examples have been presented in this section. First consider the example of an economy that produces two capital goods and one consumption good. Suppose the wage rate is $w = £1$. For the sake of simplicity assume that all wages are spent on the consumption good while all profits are accumulated. Let the initial state of the economy be as follows,

$$\begin{aligned}(2p_1 + 3p_2)(1 + r) + 5w &= 20p_1\\(4p_1 + 5p_2)(1 + r) + 5w &= 30p_2\\(3p_1 + 3p_2)(1 + r) + 5w &= 30p_3\end{aligned} \tag{44}$$

The consumption equation is,

$$15w = 30p_3 \tag{45}$$

In this state the economy is in disequilibrium because the physical surpluses of capital goods are $F_1 = 20 - 9 = 11$, $F_2 = 30 - 11 = 19$ and their ratios to the quantities of capital goods 1 and 2 used are unequal,

$$\frac{F_1}{A_{11} + A_{12} + A_{13}} = \frac{11}{9} \neq \frac{19}{11} = \frac{F_2}{A_{21} + A_{22} + A_{23}}$$

showing a relative shortage of capital good 1 and a relative excess of capital good 2.

Accordingly, our first step is to ensure a balance between the demands and supplies of the capital goods. To this end we use the variant of Sraffa's standard system (equation (34) above),

$$\begin{aligned}(2\lambda_1 + 4\lambda_2 + 3)(1 + g) &= 20\lambda_1\\(3\lambda_1 + 5\lambda_2 + 3)(1 + g) &= 30\lambda_2\\5\lambda_1 + 5\lambda_2 &= 10\end{aligned} \tag{46}$$

with λ_1 and λ_2 being the scale multipliers and $L_k = 10$ is the labour employed in the capital goods' industries. We write the capital goods' equations above in the form,

$$\begin{bmatrix} 20 - 2v & -4v & -3v \\ -3v & 30 - 5v & -3v \\ -5 & -5 & 10 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $v = 1 + g$. The polynomial obtained by setting determinant of the matrix equal to zero is,

$$20v^2 + 2350v - 6000 = 0$$

whose relevant root is $v = 2.5$ giving the growth rate $g = 1.5$. Substituting this value of the growth rate in the first two equations in (47) above gives the solution $\lambda_1 = 1.1$, $\lambda_2 = 0.9$. Using these scale multipliers, we modify the arbitrary initial state i.e., we multiply the price equations of the capital goods' industries by λ_1 and λ_2 respectively and get,

$$\begin{aligned}(2.2p_1 + 3.3p_2)(1 + r) + 5.5w &= 22p_1\\(3.6p_1 + 4.5p_2)(1 + r) + 4.5w &= 27p_2\\(3p_1 + 3p_2)(1 + r) + 5w &= 30p_3\end{aligned} \tag{46$'$}$$

It will be noticed now that

$$\frac{F_1}{A_{11}+A_{12}+A_{13}} = \frac{13.2}{8.8} = 1.5 = \frac{16.2}{10.8} = \frac{F_2}{A_{21}+A_{22}+A_{23}}$$

Substituting the value of $r = 1.5$ in the price equations gives $p_1 = 2/3$, $p_2 = 2/3$ which, when substituted in the price equation for the consumption good 3, gives its supply price $p_3 = 0.5$. The quantity demanded of commodity 3 at the supply price is,

$$\frac{15w}{0.5} = \frac{15}{0.5} = 30$$

which is equal to the quantity supplied. Alternatively we may find the demand price at the initial quantity supplied,

$$\frac{15w}{30} = \frac{15}{30} = 0.5$$

and verify that it is equal to the supply price. We conclude that (46) is an equilibrium state with $X_1 = 22, X_2 = 27, X_3 = 30, p_1 = 2/3, p_2 = 2/3, p_3 = 1/2$ and $r = g = 1.5$.

Next let us consider an economic system in which two capital goods and two consumption goods are produced. Suppose the state of the system is,

$$\begin{aligned}
(3p_1 + 4p_2)(1+r) + 10w &= 15p_1 \\
(4p_1 + 5p_2)(1+r) + 20w &= 18p_2 \\
(2p_1 + 1p_2)(1+r) + 30w &= 40p_3 \\
(1p_1 + 2p_2)(1+r) + 40w &= 35p_4
\end{aligned} \tag{47}$$

Further suppose that the wage rate is $w = £1$, all profits are accumulated and all wage incomes are spent on the two consumption goods. And suppose the coefficients of consumption to be $\alpha_3 = 0.8$ and $\alpha_4 = 0.2$.

The state of the system as it appears in (47) seems to be one of equilibrium. For if we compare the own rates of growth (i.e. the ratios of the physical surpluses of the capital goods to their total use),

$$\frac{F_1}{\Sigma A_{1j}} = \frac{5}{10} = 0.5$$

$$\frac{F_2}{\Sigma A_{2j}} = \frac{6}{12} = 0.5$$

we see that they are equal. Yet the system is not in equilibrium. This can be seen by calculating the prices p_1 and p_2 from the first two equations, (at the rate of profit $r = 0.5$) substituting them in the next two to find p_3 and p_4 and then finding the quantities demanded of commodities 3 and 4. They work out to

$$X_{3d} = \frac{\alpha_3 wL}{p_3} = (0.8)(1)(100) = 71.8367$$

$$X_{4d} = \frac{\alpha_4 wL}{p_4} = (0.2)(1)(100) = 12.6229$$

Thus commodity 3 is in excess demand and commodity 4 in excess supply,

$$X_{3d} - X_3^{(0)} = 31.8367$$
$$X_{4d} - X_4^{(0)} = -22.3770$$

where $X_3^{(0)} = 40$ and $X_4^{(0)} = 30$ are the initial supplies in (47).

A series of iterations will be required to eliminate these. We fix revised supply levels by Newton's method.

$$\begin{aligned} X_3^{(1)} &= X_3^{(0)} + \frac{X_{3d} - X_3^{(0)}}{2} \\ &= 40 + \frac{31.8367}{2} \\ &= 55.91835 \end{aligned}$$

$$\begin{aligned} X_4^{(1)} &= X_4^{(0)} + \frac{X_{4d} - X_4^{(0)}}{2} \\ &= 30 - \frac{22.3770}{2} \\ &= 23.8115 \end{aligned}$$

We then calculate the inputs of capital goods 1 and 2 and the labour required to produce these supplies. For commodity 3,

$$\begin{aligned} A_{13}^{(1)} &= \left[\frac{A_{13}}{X_3}\right] X_3^{(1)} = \left[\frac{2}{40}\right](55.91835) \\ &= 2.7959 \end{aligned}$$

$$\begin{aligned} A_{23}^{(1)} &= \left[\frac{A_{23}}{X_3}\right] X_3^{(1)} = \left[\frac{1}{40}\right](55.91835) \\ &= 1.39795 \end{aligned}$$

$$\begin{aligned} L_3^{(1)} &= \left[\frac{L_3}{X_3}\right] X_3^{(1)} = \left[\frac{30}{40}\right](55.91835) \\ &= 41.9387 \end{aligned}$$

For commodity 4,

$$\begin{aligned} A_{14}^{(1)} &= \left[\frac{A_{14}}{X_4}\right] X_4^{(1)} = \left[\frac{1}{35}\right](23.8115) \\ &= 0.68032 \end{aligned}$$

$$\begin{aligned} A_{24}^{(1)} &= \left[\frac{A_{24}}{X_4}\right] X_4^{(1)} = \left[\frac{2}{35}\right](23.8115) \\ &= 1.36065 \end{aligned}$$

$$L_4^{(1)} = \left[\frac{L_4}{X_4}\right] X_4^{(1)} = \left[\frac{40}{35}\right](23.8115)$$
$$= 27.21314$$

From the standard system of the capital goods equations, corresponding to the state of the system (47), viz,

$$(3\lambda_1 + 4\lambda_2 + 3)(1 + g) = 15$$
$$(4\lambda_1 + 5\lambda_2 + 3)(1 + g) = 18$$
$$10\lambda_1 + 20\lambda_2 = 30$$

we get $g = 0.5$, $\lambda_1 = 1$, $\lambda_2 = 1$. Multiply the price equations of the capital goods 1 and 2 by the scale multipliers λ_1 and λ_2 respectively. We then obtain a revised state,

$$\begin{aligned} (3p_1 + 4p_2)(1 + r) + 10w &= 15p_1 \\ (4p_1 + 5p_2)(1 + r) + 20w &= 18p_2 \\ (2.79p_1 + 1.39p_2)(1 + r) + 41.93w &= 55.91p_3 \qquad (47) \\ (0.68p_1 + 1.36p_2)(1 + r) + 27.21w &= 23.81p_4 \end{aligned}$$

where, for convenience, we drop the decimal points beyond the second.

In this state, however, the own-rates of growth are unequal,

$$\frac{F_1}{\Sigma A_{1j}} = \frac{4.53}{10.47} = 0.4326$$

$$\frac{F_2}{\Sigma A_{2j}} = \frac{6.25}{11.75} = 0.5319$$

So we carry on. First to balance the demands and supplies of the capital goods, that is to make the own rates equal we formulate the revised standard system,

$$(3\lambda_1 + 4\lambda_2 + 3.47)(1 + g^{(1)}) = 15$$
$$(4\lambda_1 + 5\lambda_2 + 2.75)(1 + g^{(1)}) = 18$$
$$10\lambda_1 + 20\lambda_2 = 30$$

From these we find $g^{(1)} = 0.48867$, $\lambda_1 = 1.044089$, $\lambda_2 = 0.97955$. From $g^{(1)}$ we get the rate of profit $r^{(1)}$, find the prices $p_1^{(1)}, p_2^{(1)}, p_3^{(1)}, p_4^{(1)}$ and determine the quantities demanded

$$X_{3d} = \frac{\alpha_3 w L^{(1)}}{p_3^{(1)}}$$

$$X_{4d} = \frac{\alpha_4 w L^{(1)}}{p_4^{(1)}}$$

where $L^{(1)} = L_1 + L_2 + L_3^{(1)} + L_4^{(1)}$ and comparing these to the revised supplies $X_3^{(1)}$ and $X_4^{(1)}$, ascertain the excess demands. These work out to,

$$X_{3d} - X_3^{(1)} = 15.9604$$

$$X_{4d} - X_4^{(1)} = 11.2007$$

We use these to fix the revised supply levels,

$$X_3^{(2)} = X_3^{(1)} + \frac{X_{3d}^{(1)} - X_3^{(1)}}{2}$$

$$= 55.91835 + \frac{15.9604}{2}$$

$$= 63.89855$$

$$X_4^{(2)} = X_4^{(1)} + \frac{X_{4d} - X_4^{(1)}}{2}$$

$$= 23.8115 - \frac{11.2007}{2}$$

$$= 18.21115$$

Find $A_{13}^{(1)}, A_{23}^{(2)}, A_3^{(2)}, A_{14}^{(2)}, A_{24}^{(2)}, L_4^{(2)}$ and multiply the price equations of capital goods 1 and 2 by λ_1 and λ_2 to get the re-revised state,

$$\begin{aligned}
(3.132p_1 + 4.176p_2)(1 + r) + 10.440w &= 15.66p_1 \\
(3.918p_1 + 4.897p_2)(1 + r) + 19.591w &= 17.63p_2 \qquad (47)'' \\
(3.195p_1 + 1.597p_2)(1 + r) + 47.924w &= 63.898p_3 \\
(0.520p_1 + 1.040p_2)(1 + r) + 20.812w &= 18.211p_4
\end{aligned}$$

And so on. At the end of 22 iterations, the results of which are partially tabulated below, we succeed in reducing the excess demands to magnitudes less than 1×10^{-5}. But even greater accuracies are possible[(6)]

Iteration Number	Variable				
	$X_{3d} - X_3$	$X_{4d} - X_4$	λ_1	λ_2	$1 + g$
0	31.836739	−23.377048	1.000000	1.000000	1.500000
2	7.995903	− 5.607128	1.020924	0.988831	1.483051
6	0.502693	− 0.352267	1.001253	0.999290	1.477797
12	0.007919	− 0.005548	1.000020	0.999989	1.477451
18	0.000130	− 0.000087	1.00000	1.00000	1.477446
22	0.000008	− 0.000005	1.00000	1.00000	1.477446

The equilibrium rates of profit and growth are

$$r_A = g_A = 0.477446$$

and the equilibrium prices and outputs obtained after the 22nd iteration are as follows.

Commodity	Price	Employment	Outputs
1	2.90474	10.87689	16.31533
2	3.50204	19.12310	17.21079
3	1.09391	53.93008	71.90678
4	1.56113	14.39632	12.59678

This state of the system is the approximate equilibrium state. Once this is ascertained the other derived unknowns like the level of employment, the national income, the relative shares of wages and profits, the capital-output ratio, etc. can be determined.

If the consumption coefficients are $\alpha_3 = 0.5$ and $\alpha_4 = 0.5$, i.e., the demand-intensity of commodity 3 is stronger as compared to the previous situation, a different equilibrium is obtained. The equilibrium rates of profit and growth are $r = g = 0.496473$. And the prices, employments and outputs (to 3 digits) are as follows.

Commodity	Price	Employment	Outputs
1	3.010	10.136	15.205
2	3.614	19.863	17.877
3	1.110	33.681	44.909
4	1.580	36.057	31.509

To consider yet another situation suppose the consumption coefficients are $\alpha_3 = 0.2$, $\alpha_4 = 0.8$ showing even lower demand-intensity for commodity 3, then the equilibrium rates of profit and growth obtained are $r = g = 0.516119$ and the prices, employments and outputs are as given below.

Commodity	Price	Employment	Outputs
1	3.126	9.377	14.066
2	3.739	20.622	18.560
3	1.128	13.447	17.929
4	1.602	57.741	50.524

Observe that as the intensity of demand for commodity 3 goes down its output falls, which is all right, but its price *increases*. To all of us who have been trained on the demand-supply diagram this observation is bound to have an unfamiliar ring. But such are the results when interdependence is taken into account. In fact the price of commodity 3 increases simply because of the rise in the rate of profits. Of course relative commodity prices will go in all sorts of ways depending upon the capital-labour ratios in the different industries. The commodity produced by industries which have higher capital-labour ratios will rise in price relative to those which have lower capital-labour ratios with every increase in the rate of profit. (For a complete analysis see Sraffa (1960), Chapter III or the literature on the Sraffa system cited in Appendix 3.) In fact it can also be shown that the relative prices do not vary *monotonically* with changes in the rate of profit i.e. the relative prices may fall and rise with a continuous increase in the rate of profit. [See Garegnani(1970) or Steedman(1970), Chapter V].

9. Adjustments in Disequilibrium

The procedure of finding the equilibrium has to some extent concealed the actual adjustment processes that take place when the equilibrium of the economic system is disturbed. This has been described in what follows.

Thus suppose an economy, which is initially in equilibrium, is disarranged by a temporary random shock so that the outputs of some commodities rise above their equilibrium levels (rather growth path) whilst those of others fall below their equilibrium levels. In so far as the capital goods are concerned, a disturbance of this type means that the ratios of their physical surpluses to their quantities used in the capital stock of the economy become unequal to one another, rising in the case of those capital goods whose outputs have risen above the equilibrium levels and falling in the case of others. There will thus emerge a relative excess supply of the former set of capital goods and a relative shortage of the latter. The spot prices of the former set of commodities will fall below their equilibrium prices and those of the others will rise above.

Since the shock is temporary, i.e. it is not a result of a change in the parameters of the system viz. tastes and technology, the markets will expect the disequilibrium to be corrected in future. Thus the markets will expect the supplies of the commodities whose outputs have risen above the normal equilibrium levels to *fall* in future and at the same expect the supplies of the other commodities to *rise* in future. As a result the forward prices of the former set of commodities will *rise* above their equilibrium levels and the forward prices of the latter will *fall* below. Commodities which are currently in excess supply will stand at a *premium* and commodities which are currently in excess demand will stand at a *discount* in the forward markets. This much is clear. But how do we know which capital good is in excess demand and which is in excess supply in a given state of the economic system?

For this purpose we shall use the concept of the own rate of profit. The *own rate of profit* on a commodity is defined as the sum of the equilibrium rate of profit and the forward premium (discount) on that commodity[7].

$$r_i = r + \frac{p_{Fi} - p_{si}}{p_{si}} \tag{48}$$

where p_{Fi} is the forward price and p_{si} is the spot price. The difference between the forward and spot price expressed as a proportion of the spot price is the premium on commodity i (if positive) or the discount on commodity i (if negative) in the forward market for the commodity. In a disequilibrium situation the own rates of profit r_i are, in general, different for different commodities. Or, in other words, the forward and spot prices for the commodities diverge from one other. Only in equilibrium are the two prices equal for all the commodities so that all the own rates become equal to one another and in turn equal to the equilibrium rate of profit.

The magnitudes of the own rates of profit in any given state are found from our basic system of price and demand equations,

$$r_i = \frac{g_i}{1-\beta} = \left[\frac{1}{1-\beta}\right]\frac{F_i}{\sum\limits_j A_{ij}} \qquad i = 1, \ldots, m \tag{49}$$

Since in any given state the own rates of growth g_i can be readily ascertained, so can the new rates of profit. Thus equation(49) determines the magnitudes of the own rates of profit which can be substituted into (48) to obtain the forward premia/discounts in the various commodities. Their signs will determine which capital goods are currently in excess supply or demand—commodities that stand at a forward premium are in excess supply and those that stand at forward discount are in excess demand in the current period.

Upto this point the sizes of the forward premia/discounts have been determined. How do we determine the spot and forward prices individually? The spot price of a capital good depends upon the demand and supply in the present period. Thus if the economy was initially in equilibrium then the value of the total demand for capital good i will be,

$$\sum A_{ij}(1+r)p_i^*$$

where p_i^* is the equilibrium price. The value of the total supply is $X_i p_{si}$ where X_i is the output of capital good i in the disequilibrium state. Then equating the two and solving for p_{si} gives,

$$p_{si} = \frac{(1+r)p_i^* \sum\limits_j A_{ij}}{X_i} \tag{50}$$

With the spot price ascertained from (50) and the forward premia and discount from (48), we can find the forward prices of commodities.

$$\begin{aligned} p_{Fi} &= p_{si}(1+f) \\ &= [(1+f)(1+r)p_i^*]\left[\frac{\sum\limits_j A_{ij}}{X_i}\right] \end{aligned} \tag{51}$$

where f is the forward premium/discount.

Let us apply these ideas to the first numerical example of section (7). Equations (45) show a disequilibrium state of the economy because,

$$\frac{F_1}{\sum\limits_j A_{1j}} = \frac{11}{9} \neq \frac{19}{11} = \frac{F_2}{\sum\limits_j A_{2j}}$$

Since $\beta = 0$, in that example the own rates of profit are,

$$r_1 = \frac{11}{9} = 1.22222$$

$$r_2 = \frac{19}{11} = 1.72727$$

We know (and the markets know) that the equilibrium rate of profit is 1.5. From equation (47) we may therefore ascertain the forward premia on the two capital goods,

$$f_1 = r_1 - r = -0.27777$$
$$f_2 = r_2 - r = 0.22727$$

which shows that commodity 1 is in excess demand and commodity 2 is in excess supply.

The spot prices of the two commodities are found as follows. The total quantity of commodity 1 that constitutes the capital of the economy is 9 and that of commodity 2 is 11. Their equilibrium prices are known to be $p_1^* = 2/3$, $p_2^* = 2/3$ respectively and the disequilibrium outputs are 20 & 30. Using equations (50) we get,

$$p_{s1} = \frac{9(1+1.5)(2/3)}{20} = 0.75$$

$$p_{s2} = \frac{11(1+1.5)(2/3)}{30} = 0.611111$$

As is to be expected since capital good 1 is in excess demand its spot price *exceeds* its equilibrium price while capital good 2 being in excess supply, it spot price falls *below* its equilibrium price. Finally, we may find the forward prices,

$$p_{F1} = p_{s1}(1+f_1) = (0.75)(1-0.27777)$$
$$= 0.541666$$
$$p_{F2} = p_{s2}(1+f_2) = (0.61111)(1+0.22727)$$
$$= 0.75$$

The forward price for commodity 1 falls *below* the equilibrium price while that of the commodity 2 *rises* above.

In the transition towards the equilibrium, the outputs of those commodities whose own-rates of profit are low and which, on that account, stand at discounts in the forward markets, increase whilst those of the others fall. Thus the scale multipliers applicable to the former commodities will be greater than 1 and those applicable to the latter are lower than 1. In our example for instance the multipliers were found to be $\lambda_1 = 1.1$, $\lambda_2 = 0.9$. Accordingly their effect is to increase the output of commodity 1 from 20 units to 22 units, whose own-rate $r_1 = 1.2222$ is relatively lower, and to simultaneously decrease the output of commodity 2 from 30 units to 27 units, whose own-rate $r_2 = 1.7272$ is relatively higher.

It must at all times be remembered that the own-rate of profit is *not* the realised rate of profit. On the contrary what will invariably be observed is that the industries which produce commodities with low own-rates will realise higher rates of profit than industries which produce commodities with the relatively greater own rates. The reason is obvious–commodities with low own-rates are in excess demand and command spot prices greater than their costs of production evaluated at the equilibrium prices as compared to commodities with high own-rates. In our example the rates of profit realised by the industries in the period of disequilibrium are,

$$r_1' = \frac{(p_{s1}X_1 - wL_1)}{(A_{11}p_1^* + A_{21}p_2^*)} - 1 = \frac{(0.75)(20) - 5}{[(2)(2/3) + 3(2/3)]} - 1 = 2$$

$$r_2' = \frac{(p_{s2}X_2 - wL_2)}{(A_{12}p_1^* + A_{22}p_2^*)} - 1 = \frac{(0.6111)(30) - 5}{[(4)(2/3) + (5)(2/3)]} - 1 = 1.2222$$

which respectively are greater and lower than the equilibrium rate $r = 1.5$.

Finally as regards the own-rates of profit of the consumption goods. Consider the system of equations consisting of the m price equations of the capital goods and the price equation of the consumption good whose own-rate is to be determined. There will be $n - m$ such systems corresponding to $n - m$ consumption goods. In each system there are $m + 1$ equations in $m + 2$ unknowns viz. m prices of capital goods, the price of the consumption good and the rate of profit. As such each system is underdeterminate. However, the value of the expenditure on each consumption good in terms of the prices of the capital goods and the wage payments is known. Therefore we may write,

$$p_{si}X_i = \alpha_i wL + \beta_i rK = (\sum_j^m A_{ji}P_j)(1 + r) + wL_i \tag{52}$$

where K itself is determined by the prices of the capital goods p_j ($j = 1, ..., m$). With this modification one unknown viz. the price of the consumption good is eliminated. Thus each system will contain $m + 1$ equations in $m + 1$ unknowns: m prices of capital goods and the rate of profit. The solution for the rate of profit so obtained (of course at the disequilibrium output levels) is the own-rate of profit on the consumption good whose equation has been used. There will of course be $n - m$ such own-rates.

The spot prices of the consumption goods are quite simply their demand prices,

$$p_{si} = \frac{\alpha_i wL + \beta_i rK}{X_i} \qquad i = m + 1, ..., n \tag{53}$$

where r is the equilibrium rate of profit and K is valued at the equilibrium prices. The own-rates, the equilibrium rate and the spot prices determine, by equation (48), the forward prices of the consumption goods.

The process of adjustment from disequilibrium to equilibrium that has been described in this section holds only in the case of those commodities for which forward markets exist and whose spot prices move freely in response to excess supply/demand situations. In the case of commodities whose prices are "sticky" over a wide range of demand situations ("fix-prices" as Hicks(1974) has called them) the adjustment will take place exclusively by variations in the output while their prices remain constant.

A notable property of an economy in disequilibrium, which was obliquely referred to earlier in this chapter, is the existence of multiple solutions. If we consider the system of equations consisting of n price equations and any one demand equation at a time, there will be n such systems, each containing $n + 1$ equations in $n + 1$ unknowns viz., n prices and the rate of profit. Each system will in general give a different solution for the prices and the rate of profit. This incidentally reveals an important property of the equilibrium state. The equilibrium state is one in which no matter which one of the n possible systems one may consider, the same solution for the prices and the rate of profit will be found to hold.

10. Dual-purpose Goods Incorporated

If commodities serve in a dual capacity as capital as well as consumption goods, the system of equations for the determination of outputs will require to be modified. A dual-purpose good's equation as part of the standard system of equations will be specified as follows,

$$(A_{j1}\lambda_1 + A_{j2}\lambda_2 + \dots + A_{jm}\lambda_m + A_j)(1+g) + C_j = X_j \tag{54}$$

where A_j is the quantity of the commodity used in the production of the consumption goods and C_j is its quantity used for final consumption. The final demand equation of the dual-purpose good will, accordingly, be,

$$\alpha_j wL + \beta_j rK = p_j C_j \tag{55}$$

In the process of finding the equilibrium a trial quantity $C_j^{(1)}$ will have to put in arbitrarily to ascertain the initial values of the rate of profit and prices. It will be successively corrected by finding the quantity of commodity j demanded for final consumption from equation (55).

When there are many such commodities the standard system of equations will be as follows,

$$\begin{bmatrix} X_1 - A_{11}(1+g) & -A_{12}(1+g) & \dots & -A_{1m}(1+g) & -(A_1(1+g)+C_1) \\ -A_{21}(1+g) & X_2 - A_{22}(1+g) & \dots & -A_{2m}(1+g) & -(A_2(1+g)+C_2) \\ \dots & \dots & \dots & \dots & \dots \\ -A_{m1}(1+g) & -A_{m2}(1+g) & \dots & X_m - A_{mm}(1+g) & -(A_m(1+g)+C_m) \\ -L_1 & -L_2 & \dots & -L_m & L_k \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \lambda_m \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{bmatrix} \tag{56}$$

System (56) solves for the rate of growth which when substituted into the first m equations gives the corresponding solution for the scale multipliers $\lambda_1, \ldots, \lambda_m$.

Rather indiscriminately we have all along, referred to the system of equations that determine the capital goods' outputs as the standard system of equations. There are, however, some important differences between this system and the one conceived by Sraffa (1960).

Firstly, in Sraffa's standard system the non-basic commodities, i.e., commodities which do not enter directly or indirectly into the production of *all* commodities, play no role. The standard system is composed only of the basic goods. In the standard system used here *all* capital goods are included. In addition the capital goods requirements of the consumption goods industries (which may be non-basics or basics as in (57) appear in the system to determine the outputs of the capital goods.

Secondly, because all non-basic goods are eliminated from Sraffa's standard system as also the basic good requirements of the non-basic industries, the rate of profit can only be the maximum rate, R which Sraffa accordingly has called the *standard ratio*. On the other hand, the quantities of the capital goods used outside the sphere of the capital goods industries constrain the rate of growth to a value lower than the maximum R, in our system.

These differences seem to suggest that the distinction between intermediate and final goods supersede, in some respects, that between basic and non-basic goods, even though, in the variant of the standard system that has been used by us we assume the existence of at least one basic good. While the former distinction is relevant to the determination of the equilibrium in a realistic economic system which does produce non-basics, the relevance of the latter seems to be restricted to the definition of the "standard commodity".

11. Choice of Techniques

We have so far assumed that every commodity is produced by one technique only. Suppose instead that to produce one of the commodities there were two alternative techniques. Which technique would be chosen? The answer obviously is that the technique which gives a higher *rate of profit* will be chosen. Thus we must find the rates of profit on the two techniques.

Now except in the most simplistic instance e.g. where technique I uses less of some input(s) than technique II but no more of others for the same level of output, the answer is not straightforward. Usually, each technique has its own peculiarity, it uses more of some inputs and less of others and/or uses inputs not used at all in the other, etc. As a result, each technique is interconnected with the rest of the economic system in its own peculiar way, so that to find the rates of profit which the techniques will earn, we must solve the systems of equations to which the techniques respectively belong.

Thus we formulate two systems of equations, alike in all respects except one viz. System I contains the price equation appropriate to technique I and system II that of technique II, for the commodity under consideration. Next, we ascertain the equilibrium of the two systems and choose the system (along with its technique) that yields the higher rate of profit.

The same logic applies when several techniques are available to produce one or more of the commodities. With several commodities and several techniques available to produce each of the commodities, the number of computations required to ascertain the equilibrium increases considerably. Denote the number of techniques available to produce commodity i by T_i. Our problem will be to find which techniques are most suitable to the different commodities. Several combinations exist and all must be tried out. There will be a total of $T_1 \times T_2 \times T_3 \times \ldots \times T_n$ economic systems that will require to be solved and the system with the highest rate of profit will be selected.

12. Secular Dynamics

It might be thought that the model of general economic equilibrium that has been investigated in this chapter applies exclusively to an economic system whose industries grow at a uniform rate over time. This is not so. Dynamic changes such as the introduction of new techniques, changes in tastes and preferences, changes in the product and product designs can be suitably studied by adopting the analytical framework appropriately. Dynamic changes of the types mentioned result in an alteration of the basic data, or the givens of the model. As the data changes, the equilibrium of the economic system is modified over time.

Notes

(1) The same solution can be obtained by arranging the equations as a system of non-homogeneous equations,

$$\begin{bmatrix} K_1 & -X_1 & 0 \\ K_2 & 0 & -X_2 \\ \beta_1 K & -F_1 & 0 \end{bmatrix} \begin{bmatrix} r \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} -(K_1 + wL_1) \\ -(K_2 + wL_2) \\ -\alpha_1 wL \end{bmatrix}$$

where $K_1 = A_{11}p_1 + A_{21}p_2$, $K_2 = A_{12}p_1 + A_{22}p_2$, are the capitals employed in industries 1 and 2 and $K = K_1 + K_2$. Next solve for p_1 and p_2 by Cramer's rule to obtain two polynomials in the two prices. Eliminate one of the prices by substitution and find the lowest positive root of the other.

(2) This condition is essential. If $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_2$ a solution for relative prices emerge automatically which will not, in general, be consistent with the price equations themselves. Because,

$$\alpha_1 wL + \beta_1 rK' = p_1 F_1$$
$$\alpha_2 wL + \beta_2 rK = p_2 F_2$$

If $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_2$, then

$$\frac{\alpha_1 F_2}{\alpha_2 F_1} = \frac{p_1}{p_2}$$

3. The maximum rate of profit is that which corresponds to zero wage. The concept is due to Sraffa (1960). In the two-commodity case above if we put $w = 0$ we get R as the lowest

positive root of the determinantal equation,

$$\begin{bmatrix} X_1 - A_{11}(1+r) & -A_{21}(1+r) \\ -A_{12}(1+r) & X_2 - A_{22}(1+r) \end{bmatrix} = 0$$

The maximum rate R, also called the *standard ratio*, is determined exclusively by the technology. The lower the coefficients A_{ij} is in relation to the outputs X_j, the greater is R.

4. For the proof and further extensions of the role of dominant-diagonal matrices and their importance in activity analysis of economic systems see McKenzie (1959). Also of interest is Debreu G and Herstein I.N. (1953).
5. Meade (1963), Samuelson and Modigliani (1966) and Solow (1956) have proved the Harrod-Domar formula by assuming technology with flexible 'real' capital to 'real' output ratio. This is quite unnecessary. Prices which ensure demand-supply equilibrium are sufficient. Besides they avoid the difficulties of 'real' capital/output.
6. For methods to compute fixed points in the Arrow-Debreu type of general equilibrium models see, Smale (1976) and Scarf(1987). Scarf's article surveys alternative methods of fixed-point computation. Of course there is no rate of profit in the Arrow-Debreu class of models.
7. The concept of the *own* rate of profit and interest, also called the *natural rate* of profit is of great importance to dynamic general equilibrium theory. See Sraffa (1932), Keynes (1936), Sen (1962), and Morishima (1958). Under equilibrium conditions the own rates for all commodities must be equal to one another and in turn equal to the rate of profit. And the outputs and prices which ensure this as well as the equilibrium rate of profit must be determined by the general equilibrium system. This last aspect has usually been neglected in Arrow-Debreu general equilibrium theory. See Debreu (1959) and Hahn (1982). In their theory the own rates of interest are different for different commodities even under competitive equilibrium conditions. Also see Sraffa (1926, 1932), Hayek's (1932) reply to Sraffa (1932a) and Sraffa's (1932b) rejoinder in the same issue.

8

TRADE

'The time has come,' the Walrus said,
'To talk of many things:
Of shoes — and ships — and sealing wax -
of Cabbages — and kings.

1. The Complications

This chapter proposes to deal with the complications that arise when international trade in capital goods is recognised (1). To this end we shall extend the closed-economy model of the preceding chapter in much the same way that was followed in Part 2. Despite the enlarged context, the assumptions, it will be noticed, are not substantially modified. We continue to assume, as in the pure-labour case, the international immobility of labour and (finance) capital. This assumption can be relaxed in the course of generalising the theory. Moreover, the general assumptions on which the closed-economy model is based viz. constant returns to scale production functions in all industries, homogenous demand functions of the Engels' or Stone-Geary type, the absence of joint products, fixed capital, human capital, natural resources and public goods, will continue to hold throughout the discussion (2). In terms of the scope of the system, the only additional considerations are the determination of the levels of employment in the trading countries, their savings and investments and their rates of growth and profit.

Thus the source of complications that arise when capital goods trade is recognised is not so much in the generality of the assumptions as it is in the number of details to be co-ordinate—(tentative) relocations of labour between the surviving industries in different countries so as to equate the world supplies and demands for commodities lead to changes in the compositions of outputs which lead to changes in the rates of growth and profit in the countries which, in turn, cause changes in the prices of commodities, the distributions of incomes and therefore the world demands for the various commodities. Moreover, once capital goods are trade there is yet another circularity to be resolved—until the exchange rates are known it is impossible to ascertain the domestic currency prices of the commodities in different countries but, until the domestic currency prices of commodities are known the intercountry trade bills cannot be ascertained and, consequently, the exchange rates cannot be ascertained. The price systems of the countries get interlinked with one another, in fact the individual countries' price systems stand replaced by a wider, international, price system.

In the circumstances, international equilibrium can be ascertained only after a series of rather strenuous calculations. It will, therefore, be convenient to proceed example by example and to generalise only after getting a grip on the several adjustments required.

2. Two-country: One Capital Good, One Consumption Good Trade

Consider, to start with, the simple case of trade between two countries in two commodities of which commodity 1 is a capital good and commodity 2 is a consumption god. We shall suppose that all wage incomes are spent on the consumption good and all profits are saved for accumulation in both countries [(3)]. Suppose the autarky equilibria in the two countries are as follows.

	A	B
1	$5p_{1A}(1+r_A)+4w_A = 15p_{1A}$	$5p_{1B}(1+r_B)+3w_B = 15p_{1B}$
2	$5p_{1A}(1+r_A)+6w_A = 20p_{2A}$	$5p_{1B}(1+r_B)+2w_B = 20p_{2B}$
	$10w_A = 20p_{2A}$	$5w_B = 20p_{2B}$

Assuming that the wage rates are $w_A = \text{Re. } 1$, $w_B = \$1$, the autarkic real wage rates in terms of the consumption good 2, w_A^* and w_B^*, the rates of profit r_A and r_B and the prices of commodities are as follows.

$$w_A^* = (w_A / p_{2A}) = 2 \qquad w_B^* = (w_B / p_{2B}) = 4$$

$$1 + r_A = 1.5 \qquad 1 + r_B = 1.5$$

$$p_{1A} = 0.5333 \qquad p_{1B} = 0.4$$

$$p_{2A} = 0.5 \qquad p_{2B} = 0.25$$

The natural exchange rates

$$E_{AB}^1 = 1.3333 \qquad E_{BA}^1 = 0.75$$

$$E_{AB}^2 = 2 \qquad E_{BA}^2 = 0.5$$

indicate a (trial) trade pattern *A–1 B–2*. Only for the purposes of this section suppose that the levels of pre-trade employment are maintained in both countries. This is only to facilitate comparisons of our results with those in the literature on trade in two-sector growth models which has invariably assumed a condition of full-employment both pre-and post-trade. But it should be clear that the levels of pre-trade employment simply cannot be maintained after one industry in each country closes down. For when industries close down there capitals are destroyed, their workers are retrenched and without profits or wage incomes being generated, the funds to purchase capital goods and consumption goods respectively are also destroyed. To be sure the level of pre-trade employment may be attained, but that is only after the surviving industries have grown to that level, which happens over periods of time that need not be the same for both countries. Thus only for the purposes of this section suppose that the post-trade levels of employment are $L_A = 10\,L_B = 5$ in countries A and B,

	A	B
1	$12.5p_{1A}(1+r_A)+10w_A = 37.5p_{1A}$	....
2	....	$2.5p_{1A}E_{BA}(1+r_B)+5w_B = 50p_{2B}$

The supply equations of both countries are expressed in the own currencies of the countries.

Our task is to find whether the trade pattern *A-1*, *B-2* is an international trade equilibrium and if so, what are the post-trade values of the outputs of commodities, the post-trade prices, the rates of profit in the two countries, the rates of growth and the equilibrium exchange rate. There are thus 9 unknowns in the system viz. the post-trade values of $p_{1A}, p_{2B}, r_A, r_B, g_A, g_B, X_{1A}, X_{2B}$ and E_{AB}. The equations, we shall soon see, are fewer in number.

Firstly, our assumption that the pre-trade levels of employment in both countries are maintained in the post-trade situation determines directly two output levels, $X_{1A} = 37.5$, $X_{2B} = 50$, in the two countries. Secondly, the assumption that profits are not consumed and wages are not saved implies that the rates of profit in the two countries will be equal to their rates of growth, ie. $r_A = g_A$, $r_B = g_B$. Thirdly, we have two prices equations for commodities 1 and 2 in countries A and B respectively which help to eliminate two unknown prices p_{1A} and p_{2B}. Thus the effective unknowns reduce from 9 to 3, that is r_A, r_B, and E_{AB}, and our task is to see whether there are 3 independent equationts which will determine these unknowns. A moment's reflection will show that there aren't. There are only 2 Walrasian demand-supply equations for the two commodities of which only 1 is independent. These equations are,

$$X_{1A} = A_{11A}(1+r_A) + A_{12B}(1+r_B) \tag{1}$$

$$p_{2B}X_{2B} = w_A L_A E_{BA} + w_B L_B \tag{2}$$

Equations (1) states that the post-trade output of commodity 1 in country A must be sufficient to meet the post-trade replacement requrements of commodity 1 in both countries and besides, provide a surplus for capital accumulation at the post-trade growth (i.e. profit) rates. Equation (2) states that the post-trade output of commodity 2 in country B must be sufficient to meet the demands of the wage-earners in both countries.

Finally, we have the identity of the wage bill in the capital goods industries and the capital plug-profits in the consumption goods industries,

$$A_{12B}p_{1A}(1+r_B) = w_A L_A \tag{3}$$

which, although it happens to be the trade balance equation in our problem, cannot be used to determine any unknown. It is linearly dependent on the price equation of commodity 1 in country A and equation (1) above.

This, then, is our first (somewhat depressing) conclusion viz. that the trade situation involving two countries and two commodities of which one of the commodities is purely a capital good, is indeterminate. The cause of the indeterminacy is not hard to find. In a two-commodity world, equilibrium can be determined only when the outputs are given exogenously. (See Chapter VII). So long as we are concerned with individual countries there is no problem because the equilibrium rate of growth is obtained simply by dividing the net output of the capital good by the total use made of it as an input, both of which are data,. But when two countries are involved, with one country specialised in the capital good and the other in the consumption good, we are required to resolve two rates of profit from just one international demand-supply equation for the capital good. This accounts for one part of the indeterminacy. The other part arises due to the fact that, when wages are entirely consumed and profits entirely accumulated, the *inter-industrial exchange identity* (viz. wage bill in the capital goods industry equal to the sum of capital and profits in the consumption goods industry) prevails. But this identity happens also to be the *intercountry* trade equation when the 2 countries are specialised respectively in the production of the capital and consumption goods. Thus a unique exchange rate cannot be found—several exchange rates exist which satisfy the trade balance equation. Therefore some of the unknowns must be fixed exogenously to obtain a solution. Which unknowns? Another property is revealed here. It will not help to fix the rates of profit r_A and r_B in the hope that some solution of the exchange rate will be obtained. Along with one rate of profit, either r_A or r_B, one of the two, the price of commodity2, p_{2B}, or the exchange rate E_{AB} must also be fixed. There is thus a dichotomy even in the exogenous fixation of the unknowns.

Let us follow the strategy that is intuitively most comfortable. We shall fix the rate of profit in country A, r_A, and the exchange rate E_{AB} exogenously. Equations (1) and (2) will then solve the corresponding values of country B's rate of profit r_B and price of commodity 2, p_{2B}. And price equation for commodity 1 will solve for p_{1A}. Substituting the post-trade values of the outputs and wage bills in (1) and (2) we get,

$$1 + r_B = \frac{37.5 - 12.5(1 + r_A)}{12.5} \tag{1}$$

$$p_{2B} = \frac{10\,E_{BA} + 5}{10} \tag{2}$$

along with

$$p_{1A} = \frac{10}{37.5 - 12.5(1 + r_A)}$$

If we fix the post-trade rate of profit in country A at its autarky value then, for different exogenously specified exchange rates in the feasible range, we obtain the results tabulated below.

	$(1+r_A)'$	p_{1A}	$1+r_B$	$(E_{AB})'$	p_{2B}	$\overset{\bullet}{w_A}$	$\overset{\bullet}{w_B}$
1	1.5	0.5333	1.5	1.3333	0.25	3	4.0
2	1.5	0.5333	1.5	1.4285	0.24	2.9166	4.166
3	1.5	0.5333	1.5	1.5384	0.23	2.826	4.347
4	1.5	0.5333	1.5	1.6666	0.22	2.7272	4.545
5	1.5	0.5333	1.5	1.8181	0.21	2.619	4.761
6	1.5	0.5333	1.5	2	0.20	2.5	5.0

• denotes the unknowns whose values are fixed exogenously.

In this table we have considered several values of the exchange rate lying in the feasible range for gainful trade as determined by the natural exchange rates $E^1_{AB} = 1.3333$ to $E^2_{AB} = 2$. There are two noteworthy features of the results that follow , which are in contrast with the pure-labour case. Firstly, the gain to country A by importing commodity 2 does not disappear when the exchange rate touches the upper limiting value $E^2_{AB} = 2$; even at that exchange rate the real wage rate in country A, 2.5, is greater than that under autarky, that is, 2. The reason for this is simple. The import of the capital good by country B at a low price reduces the cost of production of commodity 2 in country B and therefore lowers the price of commodity 2. This increases the real wage rates in both countries, not only in the country importing commodity 2. It also raises the natural exchange rate attributable to commodity 2 and makes the feasible range of natural exchange rates wider than the one calculated at the autarky prices with capital goods produced domestically in both countries. In fact if we carry the calculation further, it is found that the real wage rate in country A reaches the autarky level when the exchange rate rises up to $E_{AB} = 3$. Thus the range of feasible exchange rates itself widens to become $E^1_{AB} = 1.3333$ to $E^2_{AB} = 3$ after trade. In general, then, once trade in capital goods is allowed to natural exchange rates themselves become variable and must be determined in the course of finding the equilibrium. There will be two sets of natural exchange rates, pre-trade and post-trade, so that the gains from trade must be ascertained with reference to both sets when determining the trade equilibrium.

Secondly, observe that the real wage rates in terms of the consumable commodity ($\overset{\bullet}{w_A}$, $\overset{\bullet}{w_B}$) in both countries are greater than those under autarky, even though the rates of profit in the post-trade situation are equal to the autarky rates in both countries. This means that the strictly inverse relationship of the real wage rate and the rate of profit, due to Ricardo(1817), Marx (1867), Samualson (1965) and Sraffa (1960), is applicable only to a closed economy; in the open economy this relationship is modified and depends on the exchange rate(4). Later, in more general contexts, we shall show that international trade raises both the rates of profit and the real wage rates in both countries by shifting outwards the wage-profit frontiers of both countries(5).

In the above we fixed the rates of profit post-trade tat the same levels as the autarky rates. Different results will obtain if these are fixed at different levels. thus for values of r_A greater (lower) than the autarky rate of 0.5, the rate of profit in country B will be correspondingly lower (greater). The post-trade values of the natural exchange rates and therefore the range of the feasible exchange rates would also change. But the price of commodity 2 and real wage rates corresponding to the different values of the (exogenously fixed) exchange rates would remain invariant. Some results are tabulated below.

$(1 + r_A)'$	p_{1A}	$(1 + r_B)$	E'_{AB}	p_{2B}	w_A	w_B
1.666	0.6	1.333	1.333	0.25	3.0	4.0
1.666	0.6	1.333	1.666	0.22	2.7272	4.545
1.666	0.6	1.333	2.0	0.2	2.5	5.0

' denotes the variable fixed exogenously.

There is, however, sense in disregarding every solution with post-trade rates of profit lower than the autarky rates as is the case with country B in the table above. For international trade can be considered gainful only if it raises, or at least does not lower, the rates of growth and profit in the trading countries. Accordingly, we may narrow down the solution space to consider only those solutions as being admissible in which the post-trade rate of profit is greater than, or equal to, the autarky rate for all the trading countries.

Of course even after adding this restriction the indeterminacy of two-country two-commodity trade situation (in which one commodity is a capital good) does not disappear. Several solutions exist which satisfy the requirements of trade equilibrium.

The indeterminacy encountered in the case under discussion is in such striking contrast with the standard 2 × 2 theory that it deserves some explanatory remarks. In the Heckscher-Ohlin theorem for example, a determinate result is obtained. The difference in results is naturally due to the difference in the assumptions. In the Heckscher-Ohlin theorem capital is treated as an endowment which is neither produced nor traded between countries. On the other hand in the present model, the capital good is traded amongst countries. Therefore, using one post-trade demand-supply equation for it, two rates of growth/profit, have to be resolved. In the Heckscher-Ohlin theorem the demand and (given)supply of the capital endowment along with a change of technique in the post-trade situation determines the rate of profit. There seems to be no way of reconciling the diametrically opposite conclusions until a proper critique is made of the assumptions, their validity and the precise definitions of the terms involved.

A proper critique also is required of the fact that in the standard theory the (entirely national) act of autarkic countries opening themselves to trade *necessarily* results in changes of the techniques of production used in both the countries. There is something awkward and doubtful about this. For two reasons. First, the initial assumptions of the autarkic situation, is meant to be a device for identifying which way the comparative advantages go and to understand the magnitude of the gain from trade. It is surprising to note that trade compels a change of techniques of production especially when, as in the standard theory. Trade in capital goods does not take place. Second, in the standard theory, opening to trade means an opening of market opportunities—only the size and the composition of demand changes from the viewpoint of each nation. Now according to the non-substitution theorems, changes in the demand should have no effect whatever on the techniques of production

3. Heckscher-Ohlin Theorem: a "Counter Example"

As an illustration of how widely the results may diverge when the same terms are used in different senses, let us demonstrate a counter-example to the Heckscher-Ohlin theorem. As a preliminary, note that if the technical coefficients of production are the same in all countries and so also the tastes and preferences, the relative prices of the commodities in both countries will be identical. This ensured by the assumption of constant returns to scale. If relative prices are identical no country has any comparative advantage. Thus it must be recognised at the outset that if gainful trade is to occur the specific techniques used under autarky must be different. This is recognised by the Heckscher-Ohlin theorem and we shall adhere to it in the example below. Consider two countries whose autarky equilibria are as follows

	A	B
1	$5p_{1A}(1 + r_A) + 4w_A = 15p_{1A}$	$6p_{1B}(1 + r_B) + 2w_B = 16.5p_{1B}$
2	$5p_{1A}(1 + r_A) + 6w_A = 20p_{2A}$	$5p_{1B}(1 + r_B) + 2w_B = 30p_{2B}$
	$10w_A = 20p_{2A}$	$4w_B = 30p_{2B}$

All profits are accumulated and wages spent on consumption in both countries. Suppose w_A = Re.1, w_B = \$1. Then the autarky solutions are.

$$1 + r_A = 1.5 \qquad 1 + r_B = 1.5$$
$$p_{1A} = 0.5333 \qquad p_{1B} = 0.2666$$
$$p_{2A} = 0.5 \qquad p_{2B} = 0.1333$$

Comparative advantage clearly indicates a trade pattern *A-1 B-2*. This can also be seen from the natural exchange rates,

$$E^1_{AB} = 2 \qquad E^1_{BA} = 0.5$$
$$E^2_{AB} = 3.75 \qquad E^2_{BA} = 0.2666$$

Consider the endowments of capital and labour in the two countries. In country a there are 10 units of commodity 1 in the capital stock and 10 units of labour. In country B there are 11 units of commodity 1 in the capital stock but only 4 units of labour. Evidently, even in *physical* terms, A is endowed with relatively more labour and B with relatively more capital:

$$\frac{K_{pA}}{L_A} = \frac{10}{10} = 1 < 2.75 = \frac{11}{4} = \frac{K_{pB}}{L_B}$$

where K_p is capital in physical terms. (Capital can be measured in physical terms only when there is one capital good in the countries.) But consider the physical capital-intensities of production of the two commodities in the two countries in the table below. The comparison show that

Country	Capital Intensity per unit output	
	Commodity 1	Commodity 2
A	$\frac{A_{11A}}{L_{1A}} = \frac{5}{4} = 1.25$	$\frac{A_{12A}}{L_{2A}} = \frac{5}{6} = 0.8333$
B	$\frac{A_{11B}}{L_{1B}} = \frac{6}{2} = 3$	$\frac{A_{12B}}{L_{2B}} = \frac{5}{2} = 2.5$

Commodity 1, the A-exportable, is produced capita-intensively in both countries as compared to commodity 2, the B-exportable:

$$\frac{A_{11A}}{L_{1A}} = 1.25 > 0.8333 = \frac{A_{12A}}{L_{2A}}$$

$$\frac{A_{11B}}{L_{1B}} = 3 > 2.5 \qquad = \frac{A_{12B}}{L_{2B}}$$

We have thus shown that the labour-abundant country A has a comparative advantage in the production of the capital-intensively produced commodity 1 and the capital-abundant country B has a comparative advantage in the labour-intensively produced commodity 2. This, even if capital-labor ratios can be compared physically, even when the specific techniques used in autarky differ, even when there are differences in factor-intensities in production in the two countries, and even in a two-country two-commodity situation with identical tastes and preferences. All that has been done by way of variation in the conditions of the original Heckscher-Ohlin theorem is to regard one of the commodities as being a produced capital good.

Things to do not improve even if we make the endowment comparisons in value terms. Suppose the exchange rate (of course given exogenously) is $E_{AB} = 3$, ie. it is such as to support the pattern of the gains from trade. Then the capital-labour endowment ratio of country A will be,

$$\frac{K_A}{L_A} = \frac{10p_{1A}}{10} = \frac{(10)(0.5333)}{10} = 0.5333$$

And the capital-labour endowment ratio of country B, measured in the currency of country A for purposes of comparison will be,

$$\frac{(K_B)E_{AB}}{L_B} = \frac{11p_{1B}E_{AB}}{4} = \frac{(11)(0.2666)(3)}{4} = 2.2$$

Comparing the two ratios we infer that in our example country A is endowed with less capital and more labour as compared to country B. But the value-capital intensities show commodity 1 to be produced capital-intensively in both countries as compared to commodity 2. Thus, even in value terms the results go opposite to those postulated by the Heckscher-Ohlin theorem.

	Value-Capital Intensities per unit output	
Country	Commodity 1	Commodity 2
A	$\frac{A_{11A}P_{1A}}{L_{1A}} = \frac{(5)(0.5333)}{4} = 0.666$	$\frac{A_{12A}P_{1A}}{L_{2A}} = \frac{(5)(0.5333)}{6} = 0.4444$
B	$\frac{A_{11B}P_{1B}E_{AB}}{L_{1B}} = \frac{(6)(0.2666)(3)}{2} = 2.35$	$\frac{A_{12B}P_{1B}E_{AB}}{L_{2B}} = \frac{(5)(0.2666)(3)}{2} = 2$

Of course it is also possible, within the same framework of assumptions used to develop the counterexample, to develop examples of trade that conform to the Heckscher-Ohlin theorem. And that precisely is the point being made. The purpose of the counter example is not to strike a nihilistic note and propose that "anything is possible in trade, the Heckscher-Ohlin theorem is wrong." Instead, the aim is to demonstrate that the terminology and the assumptions on on which the Heckscher-Ohlin theorem is based (as well as other propositions of the standard theory) are vague and imprecise—so imprecise that any attempt to give them some, quite usual, meanings may lead to results that refute the theorem itself.

The implication of this last point is entirely constructive. Thus it is possible to predict that empirical exercises to test the theorem as it is can lead to mixed results - they may support or refute the theorem because the exercise itself will inevitably be conducted on the basis of the *usual* meanings of terms whilst the theorem itself is based on different, if not unusual, meanings of the same terms[(6)]. Thus if the empirical studies do not support the theorem they need not be regarded as paradoxical.

All of us know that the Heckscher-Ohlin statement, viz that the trade pattern is determined by the pattern of endowments is correct. Middle East exports oil, India exports bauxite, Brazil exports coffee, Canada exports wheat, Bangladesh exports jute, etc. It is then all the more distressing to see the statement refuted only because the same meaning of the term endowment as it appears in these examples is not carried over when the term is used in the theorem. In the examples, endowment means natural endowment in the theorem endowment means the value of capital stock used, which are quite different things.

The remarks made above perhaps convey the urgency to evolve a common terminology. One suspects that if the different schools of economic thought were only to agree on the basic terminology, e.g. capital, cost of production, rate of profit, technique of production, endowment, etc. many of the differences that presently divide them, can be dissolved.

4. Two-country, One Capital Good, One Consumption Good, Trade: Unemployment

The reason for assuming in section 2 that the levels of pre-trade employment continue in the post-trade situation was, quite simply, deference to tradition.

The procedure is admittedly wrong. For once capital is involved in production, employment can grow only with the growth of capital stock. Capital stock can grow only as a result of capital accumulation and that takes time. Thus in going from autarky to trade we shall suppose that both capital as well as employment are simply destroyed in the industries which close down. In time, as growth takes place, the levels of employment in both countries will rise.

To fix the ideas let us briefly reconsider the trade situation in the example of section (2). The world activities, after industry 2 in country A and industry 1 in country B close down, are

	A	B
1	$5p_{1A}(1 + r_A) + 4w_A = 15p_{1A}$	....
2	....	$5p_{1A}E_{BA}(1 + r_B) + 2w_B = 20p_{2B}$

The solution with $r_A, r_B = 0.5$ is identical to that obtained in the earlier section except that both countries have larger levels of unemployment now than what they had in the autarky situation. Every year the capital stocks and employments in countries grow by 50 per cent so that after about 2.26 years the pre-trade levels of employment will be reached and thereafter exceeded.

So far as the example under discussion is concerned, the international trade pattern once established at an exogenously determined exchange rate, will be invariant to growth. The only effect of growth, since tastes are identical, will be a proportionate increase in the volumes of goods produced and traded. In the general case, however, differential growth, in conjunction with different tastes and preferences, can result in changes in the international trade pattern.

5. Two-country, One Capital Good, Two Consumption Goods Trade

In the approach to examples of complex trading situations it may be worthwhile to consider a situation of trade between two countries in three commodities, one of which is a non-tradeable capital good (e.g. an industrial workshop or an infrastructural service) and the other two are tradeable consumption goods. The two countries, we shall suppose, differ both in techniques as well as tastes. We shall assume, for the sake of simplicity, that wages are consumed and profits accumulated. Thus, suppose, that the Engels' coefficients of consumption out of wage incomes alone are $\alpha_{2A} = 0.8$, $\alpha_{3A} = 0.2$ in country A and $\alpha_{2B} = 0.375$ and $\alpha_{3B} = 0.625$ in country B. Let the wage rates be w_A = Re. 1 and w_B = \$1, and suppose the autarky equilibria to be,

	A	*B*
1	$5p_{1A}(1 + r_A) + 9w_A = 11p_{1A}$	$1p_{1B}(1 + r_B) + 1.2w_B = 11p_{1B}$
2	$4p_{1A}(1 + r_A) + 4.8w_A = 12p_{2A}$	$6p_{1B}(1 + r_B) + 2.95w_B = 60p_{2B}$
3	$1p_{1A}(1 + r_A) + 1.2w_A = 5p_{3A}$	$3p_{1B}(1 + r_B) + 5.85w_B = 2p_{3B}$

with

$$1 + r_A = 1.1 \qquad 1 + r_B = 1.1$$
$$p_{1A} = 1.6363 \qquad p_{1B} = 0.1212$$
$$p_{2A} = 1 \qquad p_{2B} = 0.0625$$
$$p_{2A} = 0.6 \qquad p_{3B} = 3.125$$

The natural exchanges rates are

$$E^2_{AB} = 16 \qquad E^2_{BA} = 0.0625$$
$$E^3_{AB} = 0.192 \qquad E^3_{BA} = 5.2083$$

They indicate *A-3 B-2* to be the only feasible trade pattern because $E^3_{AB}E^2_{BA} = 0.012 < 1$.

Trade opens, and, in keeping with the remarks of section 4 above, industry 2 in country A and industry 3 in country B close down so that the post-trade levels of employment in country A and country B reduce to $15 - 4.8 = 10.2$ and $10 - 5.85 = 4.15$, respectively. After the opening of trade the pattern of world economic activities is as follows:

	A	B
1	$5p_{1A}(1 + r_A) + 9w_A = 11p_{1A}$	$1p_{1B}(1 + r_B) + 1.2w_B = 11p_{1B}$
2	....	$6p_{1B}(1 + r_B) + 2.95w_B = 60p_{2B}$
3	$1p_{1A}(1 + r_A) + 1.2w_A = 5p_{3A}$	....

Since in each country there is one capital good and one consumption goods, we will not be required to determine the output levels. For as we have seen in the previous chapter (chapter 7) economic systems with two goods cannot be solved for outputs. The unknowns, therefor, are 7: two rates of profit, two prices in each country, i.e. 4 prices and the equilibrium exchange rate. The system contains exactly 7 independent equations to determine them; four price equations in the two countries, two domestic demand-supply equations for the capital goods in each country, to determine the rates of profit and two international demand-supply equations, of which only one is independent, to determine the exchange rate.

Consider firstly the determination of profit rates. The demand-supply equations for the non-tradeable capital good in country A is,

$$X_{1A} = (A_{11A} + A_{13A})(1 + r_A)$$

i.e.

$$11 = (5 + 1)(1 + r_A)$$

which gives,

$$1 + r_A = 1.8333$$

Similarly the equation for the non-tradeable capital good in country B is.

$$X_{1B} = (A_{11B} + A_{12B})(1 + r_B)$$

i.e.

$$11 = (1 + 6)(1 + r_B)$$

giving

$$1 + r_B = 1.5714$$

Substituting these into the price equations of the respective countries solves for the post-trade prices of the commodities produced,

$$p_{1A} = 4.9090 \qquad p_{1B} = 0.12737$$

$$p_{3A} = 2.04 \qquad p_{2B} = 0.069166$$

Next the prices of the traded consumption goods can be substituted in any one of the demand-supply equations for the consumption goods to solve for the exchange rate. Let us use the equation for commodity 2 produced in country B,

$$X_{2B} = \frac{\alpha_{2A} w_A L_A E_{BA}}{p_{2B}} + \frac{\alpha_{2B} w_B L_B}{p_{2B}}$$

where L_A and L_B will now stand for the total labour employed post-trade. Putting in the values we get,

$$60 = \frac{(0.8) \times 10.2) E_{BA} + (0.375 \times 4.5)}{0.0691566}$$

which gives $E_{AB} = 3.1460$. The international demand supply equations for commodity 3 produced and exported by country A,

$$X_{3A} = \frac{\alpha_{3A} w_A L_A}{p_{3B}} + \frac{(\alpha_{3B} w_B L_B) E_{AB}}{p_{3B}}$$

will confirm the exchange rate solution.

But to confirm whether *A-3 B-2* is an equilibrium trade pattern we must also check on the pattern of gains from trade. Thus the exchange rate will have to be compared to both the pre and post-trade natural exchange rates. So far as the pre-trade natural exchange rates are concerned, it is immediately seen that $E_{AB} = 3.1460$. Lies between $E^3_{AB} = 0.192$ and $E^2_{AB} = 16$, the trade pattern *A-3 B-2* is feasible.

To check whether the post-trade natural exchange rates also justify the same trade pattern we must find the post-trade natural exchange rates, for which we must compute the post-trade prices of all (tradeable) commodities. Of the commodities which are produced in the countries in the post-trade situation, the prices have already been computed. The prices of those which are not, i.e. the price of commodity 2 in A and the price of commodity 3 in B at their post-trade rates of profit, found. These can be worked out as,

$$p_{2A} = 3.4 \; p_{3A} = 3.225$$

Thus the post-trade natural exchange rates are

$$E^3_{AB} = 0.6322 \qquad E^2_{AB} = 49.156$$

since $E_{AB} = 3.1460$ lies between them there are gains from trade to both countries. This may be verified from the table below.

	A	*B*
2	0.294	$\boxed{4.595}$
3	$\boxed{0.490}$	0.0985

The general effects of trade on the economic systems of the trading countries are as follows. Firstly, the rates of profit in both countries *A* and *B* increase as a result of entering international trade. The reason is obvious. After opening to trade one consumption-good industry in each country closes down with the effect that the size of the surplus of capital goods increases in relation to quantities of the capital good used as inputs, which raises the rate of growth and hence the rate of profit. At the same time the shares of wages in national income, which ware about 90 percent in country *A* and 98.8 per cent in country *B* under autarky, are seen to reduce respectively to 29.35 per cent in *A* and 89 per cent in *B*. The downward moment of the wage shares holds up the rates of profit in both countries.

Secondly, observe the effect of international trade on the real wage rates in the two countries. These are tabulated below. Each row shows the real wage rates in terms of that commodity. And the columns show the position in pre-trade and post-trade terms. What come out rather clearly is that the real wage rates in terms of the two commodities move in different directions in both countries: the real

	A		*B*	
	Autarky	Trade	Autarky	Trade
2	1.0	4.595	16	14.45
3	1.666	0.490	0.32	1.54

wage rates in terms of the *importable* increase which is only to be expected, but the real wage rates in terms of the *exportable* actually decline as compared to the autarky situation. The latter effect, quite clearly, is the result of the increase in the rates of profit.

It is natural to ask whether the decline of the real wage rate in terms of the exportable does not imply that the autarky situation, is on that account alone, preferable. Would not the consumers of the exportable ask for protection from a trade situation that reduces the quantity which they can purchase? The answer is no. And the reason is entirely methodological. The gains of trade are compared by contraposing the autarky situation with the trade situation. This is a procedure that is appropriate to the theory, which seeks to explain trade, but not to the reality. In the corresponding reality, the countries would be trading all along; their autarky situation are purely hypothetical. Thus when the consumers make their calculations, obviously in the post-trade reality, they would do so at the prices prevailing at the *post-trade* rates of profit and would accordingly find that they can purchase just as great a quantity of the exportable when their nations are exporting as when they are not. But at the same time they will find that they can obtain less of the importable. They will not seek protection from trade. The only people who would do so are those seeking employment, not those who are employed.

The observations of the movements of the real wage rates in terms of different commodities incidentally have some bearing on the relationships between the real wage rates an the share of wages in national income and the interrelationships between the real wage rates in terms of different commodities with one another.-In a closed economy, if the real wage rate is cut in terms of any one commodity, it follows that it reduces in terms of *all* commodities[7]. That is to say, in a closed economy the real wage rates in terms of all commodities move in the same directions. But in an open economy they may move in different directions in our example, the real wage rates in terms of the importable have risen but in terms of the exportable they have fallen, in both countries. Similarly, whereas in a closed economy a reduction in the real wage rate in terms of any commodity (and therefore in terms of all commodities) necessarily implies a reduction of the share of wages in national income, it need not happen in the open economy—the real wage rates in our example have risen in terms of the importables yet the shares of wages in national income have fallen in both countries. It should made clear that these are the effects of "opening up" ; in the open economies as such the regular relation continue to hold.

6. Two-country, Two Capital-goods, One Consumption Good Trade

The example just worked out amply illustrates how the presence of capital goods, even if they do enter trade, nonetheless complicate the study of international trade. Let us now go a step further and study the observe case. Suppose there are two capital goods which are tradeables and one non-tradeable consumption good. Suppose the wage rates to be $w_A = \text{Re.}1, w_B = \$1$ and continue to assume that wages are entirely spent on the non-tradeable consumption good in both countries and profits are accumulated. The autarky equilibria in the two countries are as follows,

	A	*B*
1	$(2p_{1A} + 3p_{2A})(1 + r_A) + 1w_A = 27p_{1A}$	$(5p_{1B} + 13.166p_{2B})(1 + r_B) + 10w_B = 18p_{1B}$
2	$(9p_{1A} + 2p_{2A})(1 + r_A) + 8w_A = 21p_{2A}$	$(1p_{1B} + 1.5p_{2B})(1 + r_B) + 1w_B = 48p_{2B}$
3	$(7p_{1A} + 9p_{2A})(1 + r_A) + 1w_A = 20p_{3A}$	$(4.625p_{1B} + 12p_{2B})(1 + r_A) + 4w_B = 20p_{3B}$

The autarky rates of profit and prices of commodities are as follows,

$$1 + r_A = 1.5 \qquad 1 + r_B = 1.8$$
$$p_{1A} = 0.553535 \qquad p_{1B} = 1.30588$$
$$p_{2A} = 0.14505 \qquad p_{2B} = 0.07396$$
$$p_{3A} = 0.5 \qquad p_{3B} = 0.75$$

The natural exchange rates are $E^1_{AB} = 0.42387$ and $E^2_{AB} = 1.97375$, which indicates a world trade pattern *A-1 B-2* under which the world would look like this;

	A	*B*
1	$(2p_{1A} + 3p_{2B}E_{AB})(1 + r_A) + 1w_A = 27p_{1A}$	....
2	....	$(1p_{1A}E_{BA} + 1.5p_{2B})(1 + r_B) + 1w_B = 48p_{2B}$
3	$(7p_{1A} + 9p_{2B}E_{AB})(1 + r_A) + 1w_A = 20p_{3A}$	$(4.625p_{1A}E_{BA} + 12p_{2B})(1 + r_B) + 4w_B = 20p_{3B}$

Note that in the above, the supply equations of the industries have been expressed in terms of the own currencies of the countries. Thus the rupee-dollar exchange rate E_{AB} has been attached to the price of commodity 2 which is imported by country *A* and the dollar-rupee rate E_{AB} has been attached to the price of commodity 1 which is imported by country *B*.

To find whether the above configuration of activities is an international trade equilibrium we must set up the system of international equations. The unknowns are 7; the post-trade rates of growth and profit, the post-trade prices of the two capital goods, the post-trade prices of the non-tradeable consumption goods and the exchange rate. The equations are as follows. First there are the international demand-supply equations for the capital goods,

$$\begin{aligned} X_{1A} &= A_{1A}(1 + r_A) + A_{1B}(1 + r_B) \\ X_{2B} &= A_{2A}(1 + r_A) + A_{2B}(1 + r_B) \end{aligned} \tag{4}$$

where $r_A = g_A$, $r_B = g_B$, $A_{1A} = A_{11A} + A_{13A}$, $A_{1B} = A_{12B} + A_{13B}$, $A_{2A} = A_{21A} + A_{23A}$ and $A_{2B} = A_{22B} + A_{23B}$ are the replacement requirements of capital goods 1 and 2 in the industries which survive in countries *A* and *B* after trade. The equations in (4) will solve for the two post-trade rates of profit. Filling up the values we get,

$$\begin{aligned} 27 &= 9(1 + r_A) + 5.625(1 + r_B) \\ 48 &= 12(1 + r_A) + 13.5(1 + r_B) \end{aligned} \tag{4}$$

giving the solution $1 + r_A = 1.75$, $1 + r_B = 2$. Since both these rates of profit are greater than their respective autarky values, there is a *prima facie* case for proceeding with the solution.

Next the exchange rates E_{AB} and E_{BA} should be found because until these are known the post-trade prices of commodities cannot be ascertained. The exchange rate must be such as to clear the markets for foreign exchange, ie. it must be equal to the ratio of the intercountry trade bills,

$$E_{AB} = \frac{[A_{1B}(1 + r_B)]p_{1A}}{[A_{2B}(1 + r_B)]p_{2B}} \tag{5}$$

where $A_{1B}(1 + r_B)$ is the quantity of commodity 1 imported by country *B* and $A_{2A}(1 + r_A)$ is the quantity of commodity 2 imported by country *A*. From (5) we observe, however, that exchange rate E_{AB} cannot be determined until the post-trade prices of commodities 1 and 2 are determined. To resolve the prices the following strategy may be adopted. We know from (5) that

$$p_{2A}E_{AB} = \frac{A_{1B}(1+r_B)p_{1A}}{A_{2A}(1+r_A)}$$

which can be substituted in the price equation of commodity 1 in country A and these solve the post-trade price of commodity 1. Putting the appropriate values on the right-hand-side of the expression above we get,

$$p_{2B}E_{AB} = \frac{(5.625)(2)p_{1A}}{(12)(1.75)} = \left[\frac{11.25}{21}\right]p_{1A}$$

so that the price of commodity 1 can be obtained from the equation,

$$\left\{2p_{1A} + \left[\frac{3(11.25)}{21}\right]p_{1A}\right\}(1+r_A) + 1w_A = 27p_{1A}$$

Since $1 + r_A = 1.75$ we may also determine the price of commodity 2. From equation (5) we get,

$$p_{1A}E_{BA} = \frac{[A_{2B}(1+r_A)]p_{2B}}{A_{1B}(1+r_B)}$$

$$= \left[\frac{21}{11.25}\right]p_{2B}$$

Substituting for $p_{1A}E_{BA}$ in the post-trade price equation of commodity 2 in country B we get,

$$\left\{(1)\left[\frac{21}{11.25}\right]p_{2B} + 1.5p_{2B}\right\}(1+r_B) + 1w_B = 48p_{2B}$$

whence $p_{2B} = 0.0242325$. Substituting the values of the prices p_{1A} and p_{2B} in equation (5) gives the exchange rate $E_{AB} = 1.0686275$

The next step is to determine the prices of the non-tradeable consumption goods in the two countries. This can be done from the side of demand and/or cost of production. From the demand side we get

$$p_{3A} = \frac{2w_A}{20} = 0.1 \quad p_{3B} = \frac{5w_B}{20} = 0.25$$

These prices can be verified from the post-trade price equations for commodity 3 in the two countries.

The final step obviously is to check out on the gains from trade. To ascertain these we must find the prices of the commodities which are discarded in the post-trade situation at the post-trade exchange rate and rats of profit. These work out to $p_{2A} = 0.42152$ and $p_{1B} = 0.61614$. Thus the post-trade natural exchange rates are $E_{AB}^1 = 0.0748$ and $E_{AB}^2 = 17.394$. The equilibrium exchange rate lies between the values of both the pre-trade as well as the post-trade values of the natural exchange rats showing the trade to be gainful to both the countries.

Indeed it will be observed that the range of the post-trade natural exchange rates is wider than the range of the pre-trade natural exchange rates, showing a wider margin for gains from trade to both countries. But how can the gains from trade increase due to the fact of trade?

The answer clearly lies in the fact that gainful trade in capital goods leads to reductions in the prices of capital goods which in turn decrease prices, though not proportionately, of the goods in whose production they are used, The extent to which the prices decrease depends upon the exchange rate. The result is that the gap between the natural exchange rates increases. In short the post-trade natural exchange rates themselves depend upon the equilibrium exchange rate.

7. Section (6) in General Terms

Consider now some general properties of the example solved in section (6). The system of international equations (4) which determines the post-trade rates of profit in the trading countries can be written as follows,

$$\left[\frac{A_{1A}}{X_{1A}}\right](1+r_A) + \left[\frac{A_{1B}}{X_{1A}}\right](1+r_B) = 1$$

$$\left[\frac{A_{2A}}{X_{2B}}\right](1+r_A) + \left[\frac{A_{2B}}{X_{2B}}\right](1+r_B) = 1 \tag{6}$$

If a positive solution for the rates of profit r_A and r_B is to be obtained (i.e. $1 + r_A > 1$, $1 + r_B > 1$) then the coefficients of the profit factors must satisfy two sets of conditions. Firstly,

$$\frac{A_{1A}}{X_{1A}} + \frac{A_{1B}}{X_{1A}} < 1$$

$$\frac{A_{2A}}{X_{2B}} + \frac{A_{2B}}{X_{2B}} < 1$$

and secondly, the matrix of these coefficients must have a dominant diagonal, i.e.,

$$\frac{A_{1A}}{X_{1A}} > \frac{A_{1B}}{X_{1B}} \quad \text{and} \quad \frac{A_{2B}}{X_{2B}} > \frac{A_{2A}}{X_{2B}}$$

or

$$\frac{A_{1B}}{X_{1A}} > \frac{A_{1A}}{X_{1B}} \quad \text{and} \quad \frac{A_{2A}}{X_{2B}} > \frac{A_{2B}}{X_{2B}}$$

For our purposes it is not sufficient that the rates of profit be merely positive. Trade equilibrium requires that they be greater than the autarky rates of profit. This latter requires that

$$A_{1B} < A'_{1A}$$

$$A_{2A} < A'_{2B}$$

where A'_{1A} is the quantity requires of commodity 1 in the industry that closes down in country A(viz. industry 2) and A'_{2B} is the quantity of commodity 2 required in the industry that closes down in country B (i.e., industry 1). after trade begins.

If the matrix of coefficients appearing in (1) is symmetric and the elements on the principal diagonal are equal to one another, i.e.

$$\frac{A_{1B}}{X_{1A}} = \frac{A_{2A}}{X_{2B}}$$

$$\frac{A_{1A}}{X_{1A}} = \frac{A_{2B}}{X_{2B}}$$

or more generally even if they are unequal but,

$$\frac{A_{1A} + A_{1B}}{X_{1A}} = \frac{A_{2A} + A_{2B}}{X_{2B}}$$

then the rates of profit in both countries must be equal in the post-trade situation. The unlikelihood of the condition being satisfied does not require much explanation. Even in a grossly simplistic trade situation (one capital good and consumption good in each country) the equalisation of profit rates requires that the ratio of the *world* replacement requirements of the capital goods to their total production should be equal for all capital goods. This can happen only by fluke. Note, moreover, that the equalisation of profit rates foes not imply the equalisation of real wage rates. For the latter it is necessary that the ratio of post-trade wage bills should be equal to the ratios of the levels of production of the non-tradeable consumption good.

The same logic can be generalised to a situation where there are z countries producing z capital goods where each country is specialised in the production of one distinct capital good and a non-tradeable (or a tradeable but distinct consumption good). Then for the rates of profit to exceed the autarky rates of profit,

$$\sum_{j \neq A} A_{1j} < A'_{1A}$$

where A'_{1A} is the requirement of commodity 1 in all the industries that close down in country A (i.e. industries 2 to Z),

$$\sum_{j \neq B} A_{2j} < A'_{2B}$$

....

$$\sum_{j \neq Z} A_{zj} < A'_{zZ}$$

The international system of capital good's equations (6) can be written as,

$$CV = 1$$

where C is a $z \times z$ matrix of coefficients C_{ij} where,

$$C_{ij} = \frac{A_{ij}}{X_{ii}} \quad i, j = 1,\ldots,z \quad i,j = A,\ldots,Z$$

V is a $z \times 1$ vector of the profit factors $1 + r_j$ in the different countries and 1 is a $z \times 1$ vector containing the element 1. Then if C is symmetric, i.e. $C_{ij} = C_{ji}$ and $C_{ii} = C_{jj}$ for the rates of profit of all z countries are equalised. Alternatively if the row sums of the elements in the adjoint of C are equal, the rates of profit will be equalised.

Admittedly results of this type are trivial and would not have found a mention her had they not been accorded undue emphasis in the literature. Thus the only purpose, if any, of mentioning them is to show *how different* the condition for say profit rate equalisation becomes once the distinction of capital from capital goods is made explicit.

8. Two-country, Three Capital Goods, Two Consumption Goods Trade

In situations more general than the ones we have so far considered every country will be found to produce more than two goods in the post-trade situation. By analogy with the single-country case we may expect that a good deal of simplicity will be lost when more than two goods are produced. The search for the trade equilibrium will be much more arduous and will ordinarily require an elaborate iterative procedure.

This is illustrated by means of a relatively simple example of two countries trading in five commodities which include one non-tradeable capital good (commodity 1), two tradeable capital goods (commodities 2 and 3) and two tradeable consumption goods (commodities 4 and 5). Suppose wage rates to be w_A = Re.1, w_B = \$1, tastes to be uniform in both countries $\alpha_{4A} = \alpha_{4B} = 0.6$, $\alpha_{5A} = \alpha_{5B} = 0.4$ and the autarky equilibrium states to be, as in equation(7) below.

$$\begin{array}{lll}
 & A & B \\
1 & (5p_{1A} + 3p_{2A} + 1p_{3A})(1 + r_A) + 12w_A = 22.5p_{1A} & (3p_{1B} + 1p_{2B} + 5p_{3B})(1 + r_B) + 10w_B = 18p_{1B} \\
2 & (4p_{1A} + 3p_{2A} + 2p_{3A})(1 + r_A) + 5w_A = 22.5p_{2A} & (3p_{1B} + 4p_{2B} + 1p_{3B})(1 + r_B) + 20w_B = 12p_{2B} \\
3 & (1p_{1A} + 2p_{2A} + 4p_{3A})(1 + r_A) + 25w_A = 15p_{3A} & (5p_{1B} + 2p_{2B} + 4p_{3B})(1 + r_B) + 5w_B = 18p_{3B} \\
4 & (3p_{1A} + 4p_{2A} + 1p_{3A})(1 + r_A) + 36.56w_A = 300p_{4A} & (2p_{1B} + 2p_{2B} + 2p_{3B})(1 + r_B) + 41.25w_B = 15p_{4B} \\
5 & (2p_{1A} + 1p_{2A} + 2p_{3A})(1 + r_A) + 21.43w_A = 20p_{5A} & (2p_{1B} + 1p_{2B} + 3p_{3B})(1 + r_B) + 23.75w_B = 800p_{5B}
\end{array} \tag{7}$$

The rates of growth/ profit and the prices of commodities in autarky are as follows,

$$
\begin{array}{rclcrcl}
1+g_A = 1+r_A & = & 1.5 & \qquad & 1+g_A = 1+r_A & = & 1.2 \\
p_{1A} & = & 1.6839 & & p_{1B} & = & 1.8409 \\
p_{2A} & = & 1.7342 & & p_{2B} & = & 4.0227 \\
p_{3A} & = & 3.6365 & & p_{3B} & = & 1.9470 \\
p_{4A} & = & 0.2 & & p_{4B} & = & 4 \\
p_{5A} & = & 2 & & p_{5B} & = & 0.05
\end{array}
$$

The natural exchange rates are,

$$
\begin{array}{rclcrcl}
E^2_{AB} & = & 0.4311 & \qquad & E^2_{BA} & = & 2.3196 \\
E^3_{AB} & = & 1.8677 & & E^3_{BA} & = & 0.5354 \\
E^4_{AB} & = & 0.05 & & E^4_{BA} & = & 20 \\
E^5_{AB} & = & 40 & & E^5_{BA} & = & 0.025
\end{array}
$$

The most profitable arbitrage sequences which assign all tradeable commodities to the countries are ,

$$
\begin{aligned}
E^4_{AB}E^5_{AB} &= 0.00125 \\
E^2_{AB}E^5_{BA} &= 0.0107 \\
E^4_{AB}E^3_{BA} &= 0.0267
\end{aligned}
$$

Since commodity 1 is a non-tradeable capital good it will be produced in both countries so that the trade pattern which presents itself for a first trial is *A-1,2,4 B-1,3,5*. The number of unknown t be determined are 15 which include 2 rates of profit/growth in the two countries, 6 prices of the commodities produced in the post-trade situation in the two countries, the 6 outputs of these commodities and the exchange rate. There will be 15 independent equations to determine then, viz 3 standard-system equations for each country to determine the rates of growth/profit along with the levels of output of the capital goods, i.e. 6 equations in all, 6 price equations, international demand–supply equations for the consumption goods and 1 trade balance (or Walras' Low) equations.

After trade begins the price equation in the two countries in terms of the own currencies are as shown in equation (8) below:

	A	*B*
1	$(5p_{1A} + 3p_{2A} + 1p_{3B}E_{AB})(1 + r_A) + 12w_A = 22.5p_{1A}$	$(3p_{1B} + 1p_{2A}E_{BA} + 5p_{3B})(1 + r_B) + 10w_B = 18p_{1B}$
2	$(4p_{1A} + 5p_{2A} + 2p_{3B}E_{AB})(1 + r_A) + 5w_A = 22.5p_{2A}$	....
3	....	$(5p_{1B} + 2p_{2A}E_{BA} + 4p_{3B})(1 + r_B) + 5w_B = 18p_{3B}$

$$4 \quad (3p_{1A} + 4p_{2A} + 1p_{3B}E_{AB})(1 + r_A) + 36.56w_A = 300p_{4A} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$5 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (2p_{1B} + 1p_{2A}E_{BA} + 4p_{3B})(1 + r_B) + 23.75w_B = 800p_{5B} \tag{8}$$

The equations as they are shown here are not, however, in their equilibrium proportions. To find the equilibrium state we proceed in two stages.

The first stage is to ensure balance between the demands and supplies of the capital goods in both the countries and to obtain a first, tentative, solution for the rates of growth profit. The equations which do so are the equations of the standard system [Chapter VII section (7)]. There is, however, a difference. Since country *A* exports capital good 2 in our example, the total production of commodity 2 in country *A* must be sufficient not only to meet her own replacement and growth requirements but also those of country *B*. Similarly since country *B* exports capital good 3 the supply of commodity 3 in country *B* must be sufficient to meet *A*'s replacement and growth demand in addition to *B*'s own. Now the demand for commodity 2 in county *B* is,

$$(A_{21B} + A_{23B} + A_{25B})(1 + g_B) = A_{2B} \tag{9(a)}$$

and the demand for commodity 3 in country *A* is,

$$(A_{31A} + A_{32A} + A_{34A})(1 + g_A) = A_{3A} \tag{9(b)}$$

because industries 1, 3 and 5 are operated in country *B* while industries 1, 2 and 4 are operated in country *A* in the post-trade situation.

Thus the capital goods demand-supply equations including the intercountry demand for capital goods will be as follows,

$$\begin{aligned}
X_{1A} &= A_{1A}(1 + g_A) \\
&= (A_{11A} + A_{12A} + A_{14A})(1 + g_A) \\
X_{1B} &= A_{1B}(1 + g_B) \\
&= (A_{11B} + A_{13B} + A_{15B})(1 + g_B) \\
X_{2A} &= A_{2A} + A_{2B} \\
&= (A_{21A} + A_{22A} + A_{24A})(1 + g_A) + (A_{21B} + A_{23B} + A_{25B})(1 + g_B) \\
X_{3A} &= A_{3A} + A_{3B} \\
&= (A_{31A} + A_{32A} + A_{34A})(1 + g_A) + (A_{31B} + A_{33B} + A_{35B})(1 + g_B)
\end{aligned}$$

Since both, the levels of output and inputs, are now unknowns along with the growth rates, we shall write these equations in the (10) below,

$$\begin{array}{ll}
\qquad\qquad A & \qquad\qquad B \\
1(A_{11A}\lambda_{1A} + A_{12A}\lambda_{2A} + A_{14A})(1 + g_A) + 0 = X_{1A}\lambda_{1A} & (A_{11B}\lambda_{1B} + A_{13B}\lambda_{3B} + A_{15B})(1 + g_B) + 0 = X_{1B}\lambda_{1B} \\
2(A_{12A}\lambda_{1A} + A_{22A}\lambda_{2A} + A_{24A})(1 + g_A) + A_{2B} = X_{2A}\lambda_{2A} & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{array} \tag{10}$$

	A	B
3	....	$(A_{13B}\lambda_{1B} + A_{33B}\lambda_{3B} + A_{25B})(1 + g_B) + A_{3A} = X_{3B}\lambda_{3B}$
L	$L_{1A}\lambda_{1A} + L_{2A}\lambda_{2A} = L_{AK}$	$L_{1B}\lambda_{1B} + L_{3B}\lambda_{3B} = L_{BK}$

The zeros in the first equations in both countries are put in to convey the idea that commodity 1 is not traded between the countries, it is neither exported not imported. The second equations in each country include the foreign demands for the tradeable capital goods along with the exporting country's own demand. These equations show the interlinkage between the growth rates in the two countries because, as is apparent from equations 9 (a), (b) above, the terms A_{2B} and A_{3A} include the requirements of growth as well. At the same time these equations indicate the difficulty of simultaneously resolving the growth rates of the two countries—until we know g_B we do not know A_{2B} and hence cannot find g_A and until we know g_A we do not know A_{2A} and cannot find g_B.

To resolve this problem a procedure based on Newton's method of successive approximations must be used. After filling in the data into (10) we get,

	A	B
1	$(5\lambda_{1A} + 4\lambda_{2A} + 3)(1 + g_A) + 0 = 22.5\lambda_{1A}$	$(3\lambda_{1B} + 5\lambda_{3B} + 2)(1 + g_B) + 0 = 18\lambda_{1B}$
2	$(3\lambda_{1A} + 5\lambda_{2A} + 4)(1 + g_A) + A_{2B} = 22.5\lambda_{2A}$	....
3	....	$(1\lambda_{1B} + 2\lambda_{3B} + 1)(1 + g_B) + A_{3A} = 18\lambda_{3B}$
L	$12\lambda_{1A} + 5\lambda_{2A} = 17$	$10\lambda_{1B} + 5\lambda_{3B} = 15$ (10)′

It is clear that if the values of A_{2B} and A_{3A} are known, each country has three independent equations in three unknowns, viz., the scale multipliers for the capital goods produced in the country and the country's rate of growth. The trouble is that A_{2B} and A_{3A}, the quantities of capital goods 2 and 3 imported by countries B and A, are not known. These quantities must be determined along with the multipliers and the rates of growth.

Thus put in some value of A_{2B}, say $A_{2B}^{(1)}$, exogenously. A convenient starting point is to consider the autarky equilibrium values of $A_{21B}, A_{23B}, A_{25B}$ and g_B so that $A_{2B}^{(1)}$, for our example is 4(1.2) = 4.8. Supposing $A_{2B} = A_{2B}^{(1)}$, find the corresponding values of $\lambda_{1A}^{(1)}, \lambda_{2A}^{(1)}$ and $g_A^{(1)}$ from the equations for country A in (10′). Multiply the price equations of the capital goods in country A respectively by $\lambda_{1A}^{(1)}$ and $\lambda_{2A}^{(1)}$ and find the quantity of capital good 3 required in country A,

$$A_{3A}^{(1)} = (A_{31A}^{(1)}\lambda_{1A}^{(1)} + A_{32A}^{(1)}\lambda_{2A}^{(1)} + A_{34A})(1 + g_A^{(1)})$$

which can be substituted into the equations of country B. Then corresponding to $A_{3A} = A_{3A}^{(1)}$ we obtain the solution for the scale multipliers and the growth rate in country B, $\lambda_{1B}^{(1)}, \lambda_{3B}^{(1)}, g_B^{(1)}$, which in general, will be different from the autarky growth rate. Multiplying the price equations for the capital goods in country B by $\lambda_{1B}^{(1)}$ and $\lambda_{3B}^{(1)}$ gives us country B's demand for capital good 2,

$$A_{2B}^{(2)} = (A_{21B}\lambda_{1B}^{(1)} + A_{23B}\lambda_{3B}^{(1)} + A_{25B})(1 + g_B^{(1)})$$

If $A_{2B}^{(2)} = A_{2B}^{(1)}$ it is plain luck and we may proceed to tackle the demands and supplies of consumption goods. If not, the first stage is still incomplete.

Accordingly the next step must be taken in the direction of finding the right quantities of intercountry demands for capital goods. Firstly we consider a transformed system of price equations in which all the initial data are multiplied by the scale multipliers obtained in the first iteration, ie. $A_{11A}^{(1)} = A_{11A}\lambda_{1A}^{(1)}$, $A_{12A}^{(1)} = A_{12A}\lambda_{2A}^{(1)}$, $L_{1A}^{(1)} = L_{1A}\lambda_{1A}^{(1)}$, etc. and $A_{11B}^{(1)} = A_{11B}\lambda_{1B}^{(1)}$, i.e. $A_{13B}^{(1)} = A_{13B}\lambda_{3B}^{(1)}$, $L_{1B}^{(1)} = L_{1B}\lambda_{1B}^{(1)}$, etc. The capital goods' equations in (10′) will contain these new values. Next, we must put in an improvised value of A_{2B} into the equations of country A. The improvised value to be put in is obtained from the formula,

$$A_{2B}^{(3)} = A_{2B}^{(1)} + \frac{(A_{2B}^{(2)} - A_{2B}^{(1)})}{2}$$

Putting in $A_{2B}^{(3)}$ gives us the improvised value of $\lambda_{1A}^{(2)}, \lambda_{2A}^{(2)}$ and $g_A^{(2)}$ from the equations of country A and hence an improvised value of $A_{3A}^{(2)}$ which, when put into the equations of country B give the improvised values of $\lambda_{1B}^{(2)}, \lambda_{3B}^{(2)}$ and $g_B^{(2)}$. Thus,

$$A_{3A}^{(2)} = (A_{31A}^{(1)}\lambda_{1A}^{(2)} + A_{32A}^{(1)}\lambda_{2A}^{(2)} + A_{34A})(1 + g_A^{(2)})$$

$$A_{2B}^{(4)} = (A_{21B}^{(1)}\lambda_{1B}^{(2)} + A_{23B}^{(1)}\lambda_{3B}^{(2)} + A_{25B})(1 + g_B^{(2)})$$

If $A_{2B}^{(4)} \neq A_{2B}^{(3)}$ we must repeat the process. Accordingly the new value of A_{2B} to be put into A's capital goods equations (in which the data are once again transformed by the multipliers $\lambda_{iA}^{(2)}$) is

$$A_{2B}^{(5)} = A_{2B}^{(3)} + \frac{(A_{2B}^{(4)} - A_{2B}^{(3)})}{2}$$

And so on. The iterations will continue until the $A_{2B}^{n} \cong A_{2B}^{n-1}$, that is to say the difference $A_{2B}^{n} - A_{2B}^{n-1}$ is made small enough to obtain the desired accuracy for the growth rates. At this stage the balance between the demands and deliveries of capital goods, both tradeable and non-tradeable, has been ensured at *given* levels of activity in the consumption goods industries. In our example, at the end of this first stage, the outputs of the capital goods in the two countries and the growth rates are,

$$X_{1A} = 20.5333 \qquad X_{1B} = 15.0104$$

$$X_{2A} = 27.219191 \qquad X_{3B} = 23.9790$$

$$1 + g_A = 1.655675 \qquad 1 + g_B = 1.344711$$

We now move to the second stage in which we must equate the demands and supplies of the consumption goods. The first step is to check the quantities demanded of the consumption goods and compare them to the supplies. To find the quantities demanded we must know the post-trade prices of the consumption goods at the post-trade rates of profit, which since they are equal to the growth rates on the basis of our assumptions, are respectively $r_A = 0.655675$, and $r_B = 0.344711$. These rates of profit must be substituted into the price equations (5) along with the exchange rates to find the post-trade prices. Naturally, we first solve the post-trade prices of the capital goods and then use them to find the post-trade prices of the consumption goods. In so doing we encounter the difficulty mentioned earlier—until the exchange rate is known we cannot determine the prices of commodities but until the prices are known the values of the intercountry import bills and the exchange rate cannot be determined. In other words, the system of the price equations of the capital goods produced in both countries is indeterminate. We have four equations in four unknown prices, p_{1A}, p_{2A}, p_{1B}, and p_{3B} and one unknown exchange rate E_{AB}. (The other exchange rate E_{BA} is determined from $E_{AB}E_{BA} = 1$). The remedy thus lies in adding the trade balance equation which determines the exchange rate. We will then have five equations in five unknowns.

Having eliminated one of the exchange rates by the neutrality condition we must write all the four price equations in (8) in terms of the currency of any one country, say A. Then, in formal notation, we obtain the international price system (11),

$$
\begin{aligned}
(A_{11A}p_{1A} + A_{21A}p_{2A} + A_{31A}p_{3B}E_{AB})(1 + r_A) + w_A L_{1A} &= p_{1A}X_{1A} \\
(A_{12A}p_{1A} + A_{22A}p_{2A} + A_{32A}p_{3B}E_{AB})(1 + r_A) + w_A L_{2A} &= p_{2A}X_{2A} \\
(A_{11B}p_{1B}E_{AB} + A_{21B}p_{2A} + A_{31B}p_{3B}E_{AB})(1 + r_B) + w_B L_{1B}E_{AB} &= (p_{1B}E_{AB})X_{1B} \\
(A_{13B}p_{1B}E_{AB} + A_{23B}p_{2A} + A_{33B}p_{3B}E_{AB})(1 + r_B) + w_B L_{3B}E_{AB} &= (p_{3B}E_{AB})X_{3B}
\end{aligned}
$$

In which all equations, including the third and fourth which pertain to the production of commodities in country B, are expressed in currency A. To the four equations above, we add the following trade balance equation. The import bill of country A in terms of her own currency is,

$$M_{AB} = \alpha_{5A} w_A L_A + A_{3A} p_{3B} E_{AB}$$

and the import bill of country B in her own currency is,

$$M_{BA} = \alpha_{4B} w_B L_B + A_{3B} p_{2A} E_{BA}$$

Thus the exchange rate E_{AB} is,

$$E_{AB} = \frac{M_{AB}}{M_{BA}} = \frac{\alpha_{5A} w_A L_A + A_{3A} p_{3B} E_{AB}}{\alpha_{4B} w_B L_B + A_{2B} p_{2A} E_{BA}}$$

where L_A and L_B are the *post-trade* levels of employment in A and B and A_{3A} and A_{2B} are the quantities of intercountry capital goods imports obtained at the end of the first stage of iterations.

Readjusting terms we may write the above equation as,

$$p_{3B}E_{AB} = \frac{(\alpha_{4B} w_B L_B)E_{AB} + A_{2B} p_{2A} - \alpha_{5A} w_A L_A}{A_{3A}} \qquad (12)$$

Equation (11) and (12) from a determinate system. Substituting for $p_{3B}E_{AB}$ from equation (12) into the equations in (11) gives a 'hybrid' system in which one of the unknown prices p_{3B} is eliminated. The system has been called 'hybrid' because it is a combination of two seemingly distinct and unconnected systems of equations; the price equations of capital goods ad the overall trade balance equations. The hybrid system can be arranged as follows,

$$\begin{bmatrix} H_{11} & H_{12} & 0 & H_{14} \\ H_{21} & H_{22} & 0 & H_{24} \\ 0 & H_{32} & H_{33} & H_{34} \\ 0 & H_{42} & H_{43} & H_{44} \end{bmatrix} \begin{bmatrix} p_{1A} \\ p_{2A} \\ p_{1B}E_{AB} \\ E_{AB} \end{bmatrix} = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} \tag{13}$$

where

$H_{11} = [X_{1A} - A_{11A}(1 + r_A)]A_{3A}$

$H_{21} = -(A_{21A}A_{3A})(1 + r_A)$

$H_{12} = -[A_{21A}A_{3A} + A_{31A}A_{2B}](1 + r_A)$

$H_{22} = X_{2A}A_{3A} - (A_{22A}A_{3A} + A_{32A}A_{2B})$
$(1 + \mathrm{rA})$

$H_{14} = -(A_{31A}\alpha_{4B}w_BL_B)(1 + r_A)$

$H_{24} = -(A_{32A}\alpha_{4B}w_BL_B)(1 + r_A)$

$H_{32} = -(A_{21B}A_{3A} + A_{31B}A_{2B})(1 + r_B)$

$H_{42} = [X_{3B} - A_{33B}(1 + r_B)]A_{2B} - A_{23B}$
$(1 + \mathrm{rB})$

$H_{33} = -[X_{1B} - A_{11B}(1 + r_B)]A_{3A}$

$H_{43} = -A_{13B}A_{3A}(1 + r_B)$

$H_{34} = -[A_{31A}\alpha_{4B}w_BL_B(1 + r_B)$
$+ w_BL_{1B}A_{3A}]$

$H_{44} = [X_{3B} - A_{33B}(1 + r_B)](\alpha_{4B}w_BL_B)$
$- w_BL_{3B}A_{3A}$

$J_1 = w_AL_{1A}A_{3A} - A_{31A}\alpha_{5A}w_AL_A$
$(1 + r_A)$

$J_2 = w_AL_{2A}A_{3A} - A_{32A}\alpha_{5A}w_AL_A$
$(1 + r_A)$

$J_3 = -[A_{31B}\alpha_{5A}w_AL_A(1 + r_B)]$

$J_4 = [X_{3B} - A_{33B}(1 + r_B)](\alpha_{5A}w_AL_A)$

In system (13) the first two rows belong to the capital goods produced in country A, viz. commodities 1 and 2 and the next two rows to the capital goods produced in country B, viz. Commodities 1 and 3. Accordingly, the zeros in the first two rows of the H matrix show that the capital good 1, which is produced in country B, is not required in the production of the capital goods in country A. Similarly, the zeros in the next two row show that capital good 1 produced in country A is not required in the production of capital goods produced in country B.

The hybrid system will not seem so remote to the intuition if it is constantly kept in mind that the elements in matrix H and the vector J are the usual coefficients of the capital goods' price equation but modified by two effects, (a) the coefficients in the balance of trade equations and (b) the elimination from the system of one of the capital goods' prices, viz. p_{3B}. But even in the somewhat awkward and counterintuitive form in which the hybrid system presents itself the features that stands out prominently is that the exchange rate is solved along with the prices of capital goods in the trading countries. This conveys the idea that the domestic price systems of the countries become inextricably intertwined, and the twine which binds them is the exchange rate.

At the rates of profit $r_A = 0.655675$, $r_B = 0.344711$ we solve equations (13) to obtain the post-trade prices of the capital goods p_{1A}, p_{2A} produced in country A and $p_{1B}E_{AB}$ and E_{AB}. Once E_{AB} is known we can immediately extricate p_{1B} and p_{3B}, the prices of capital goods produced in country B in terms of B's currency. Next we compute the prices of the consumption goods in the own currencies of countries using the corresponding equations in (8) and are ready to find the quantities demanded of the commodities.

In our example, the solution of system (10) obtained is,

$$p_{1A} = 1.53214 \qquad p_{1B} = 1.58886$$
$$p_{2A} = 1.43759 \qquad p_{3B} = 1.53682$$
$$E_{AB} = 1.04107$$

And the corresponding prices of the consumption goods are,

$$p_{4A} = 0.187808 \qquad p_{5B} = 0.045099$$

We may thus compute the quantities demanded of the commodities 4 and 5. These are,

$$X_{4d} = \frac{\alpha_{4A} w_A L_A}{p_{4A}} + \frac{(\alpha_{4B} w_B L_B) E_{AB}}{p_{4A}}$$
$$= \frac{(0.6)(1)(53.5625) + (0.6)(1)(38.75)(1.04107)}{0.187808}$$
$$= 300$$

$$X_{5d} = \frac{(\alpha_{5A} w_A L_A) E_{BA}}{p_{5B}} + \frac{\alpha_{5B} w_B L_B}{p_{5B}}$$
$$= \frac{(0.4)(1)(53.5625)(0.960549) + (0.4)(1)(38.75)}{0.045099}$$
$$= 800$$

The quantities supplied of the two commodities are 300 and 800, as the post-trade equations (8) show, so that one of the requirements of an international trade equilibrium is fulfilled. As regards the other requirements, it can be verified that the pattern of the gains from trade supports the trade pattern. Besides, the rates of profit in both countries post-trade are greater than the respective autarky rates. We conclude that *A-1, 2, 4 B-1, 3, 5* is an equilibrium trade pattern.

The levels of industrial activity in the two countries in trade equilibrium are as follows:

	A		B	
	Labour	Output	Labour	Output
1	10.9513	20.5336	8.3391	15.0104
2	6.0487	27.2191	—	—
3	—	—	6.6608	23.9790
4	36.5625	300	—	—
5	—	—	23.75	800

The levels of working inputs A_{ij} corresponding to these levels of activities can be found by using the technical coefficients.

As compared with the autarky situation, the scale of activities in the non-tradeable capital good have reduced in either country. Instead, labour has been reallocated in favour of the tradeable capital goods in both countries. The scales of the consumption goods industries remains unchanged. This last will be recognised as a peculiarity of any system in which there is only one consumption goods [(See Chapter VII section (4)). We shall present in the following section an example in which each country exports more than one consumption good; the equilibrium scales of activity in the consumption goods sphere will not in general be equal to the autarky scales.]

The system converges to the international equilibrium solution in 19 iterations for an accuracy less than 1×10^{-5}. To get an idea of the speed of convergence the results of some select iterations have been reproduced below. The initial trial value of A_{2B} is 5. The number of iterations required to get the right value of A_{2B} (= $A_{2B}^{(n)}$) do not change appreciably with the size of the initial trial value.

Iteration No.	$(A_{2B}^{(n)} - A_{2B}^{(n-1)})$	λ_{1A}	λ_{2A}	λ_{1B}	λ_{3B}	z_A	z_B	$A_{2B}^{(n)}$
1	0.44227	0.95759	1.17377	0.84034	1.31930	1.69226	1.36056	5.42274
4	0.21310	0.99663	1.00620	0.99992	1.00008	1.66305	1.34497	6.049371
9	0.01323	0.99989	1.00018	0.99999	1.00000	1.65590	1.34471	6.04886
14	0.000102	0.99999	1.00000	1.00000	1.00000	1.655678	1.34471	6.04885
19	0.00000	1.00000	1.00000	1.00000	1.00000	1.65567	1.34471	6.04885

It is a peculiarity of the trade pattern being tried (one consumption good exported by each country) that no further iterations are required to ensure demand-supply balance of the consumption goods. We shall see in more general cases, (where countries produce more than one consumption good) that further iterations will be required.

9. Two-country, Two Capital Goods, Four Consumption Good Trade

In the example just presented there were only two consumption goods. Let us now allow some more. Consider two countries producing two non-tradeable capital goods and four consumption goods, Suppose their autarky equilibria to be,

	A	*B*
1	$(3p_{1A} + 4p_{2A})(1 + r_A) + 10w_A = 16.5p_{1A}$	$(3p_{1B} + 4p_{2B})(1 + r_B) + 30w_B = 21.6p_{1B}$
2	$(4p_{1A} + 5p_{2A})(1 + r_A) + 20w_A = 16.5p_{2A}$	$(4p_{1B} + 5p_{2B})(1 + r_B) + 25w_B = 19.2p_{2B}$

3 $(2p_{1A} + 2p_{2A})(1 + r_A) + 13.85w_A = 200p_{3A}$ $(2p_{1B} + 1p_{2B})(1 + r_B) + 20.9w_B = 30p_{3B}$

4 $(1p_{1A} + 2p_{2A})(1 + r_A) + 2.88w_A = 500p_{4A}$ $(5p_{1B} + 2p_{2B})(1 + r_B) + 18.93w_B = 10p_{4B}$

5 $(3p_{1A} + 1p_{2A})(1 + r_A) + 32.12w_A = 20p_{5A}$ $(1p_{1B} + 2p_{2B})(1 + r_B) + 0.47w_B = 100p_{5B}$

6 $(2p_{1A} + 2p_{2A})(1 + r_A) + 21.15w_A = 30p_{6A}$ $(3p_{1B} + 2p_{2B})(1 + r_B) + 4.7w_B = 500p_{6B}$

where the coefficients of consumption (from the wage income alone) are,

$$\alpha_{3A} = 0.2 \qquad \alpha_{3B} = 0.3$$
$$\alpha_{4A} = 0.1 \qquad \alpha_{4B} = 0.4$$
$$\alpha_{5A} = 0.4 \qquad \alpha_{5B} = 0.1$$
$$\alpha_{6A} = 0.3 \qquad \alpha_{6B} = 0.2$$

The wage rates are w_A = Re.1 w_B = \$1, so that the prices and the rates of profit in the equilibrium are,

$$1 + r_A = 1.1 \qquad 1 + r_B = 1.2$$
$$p_{1A} = 1.5734 \qquad p_{1B} = 2.4049$$
$$p_{2A} = 2.4476 \qquad p_{2B} = 2.7685$$
$$p_{3A} = 0.1 \qquad p_{3B} = 1$$
$$p_{4A} = 0.02 \qquad p_{4B} = 4$$
$$p_{5A} = 2 \qquad p_{5B} = 0.1$$
$$p_{6A} = 1 \qquad p_{6B} = 0.04$$

The natural exchange rates are,

$$E^3_{AB} = 0.1 \qquad E^3_{BA} = 10$$
$$E^4_{AB} = 0.005 \qquad E^4_{BA} = 200$$
$$E^5_{AB} = 20 \qquad E^5_{BA} = 0.05$$
$$E^6_{AB} = 25 \qquad E^6_{BA} = 0.04$$

showing A-3,4 B-5,6 to be feasible trade pattern.
Open the countries to trade so that their post-trade position stand at,

A *B*

1 $(3p_{1A} + 4p_{2A})(1 + r_A) + 10w_A = 16.5p_{1A}$ $(3p_{1B} + 4p_{2B})(1 + r_B) + 30w_B = 21.6p_{1B}$

2 $(4p_{1A} + 5p_{2A})(1 + r_A) + 20w_A = 16.5p_{2A}$ $(4p_{1B} + 5p_{2B})(1 + r_B) + 25w_B = 19.2p_{2B}$

3 $(2p_{1A} + 1p_{2A})(1 + r_A) + 13.85w_A = 200p_{3A}$ ....

4	$(1p_{1A} + 2p_{2A})(1 + r_A) + 2.88w_A = 500p_{4A}$	....
5	....	$(1p_{1B} + 2p_{2B})(1 + r_B) + 0.47w_B = 100p_{5B}$
6	....	$(3p_{1B} + 2p_{2B})(1 + r_B) + 4.70w_B = 500p_{6B}$

where the post-trade levels of employment are $L_A = 46.73$ and $L_B = 60.17$. That the state of the system, as portrayed above, is not an equilibrium one is obvious because in neither country are the own-rates of profit on commodities 1 and 2 equal to one another.

The steps to find the equilibrium are as follows. The first step is to ensure the balance between the demands and supplies of capital goods. Since the capital goods are non-tradeable the balances must be brought about separately in the two countries. thus we have two sets of equations which we shall write in formal terms to facilitate explaining the algorithm;

	A	B
1	$(A_{11A}\lambda_{1A} + A_{12A}\lambda_{2A} + A_{1A})(1 + g_A) = X_{1A}\lambda_{1A}$	$(A_{11B}\lambda_{1B} + A_{12B}\lambda_{2B} + A_{1B})(1 + g_A) = X_{1B}\lambda_{1B}$
2	$(A_{21A}\lambda_{1A} + A_{22A}\lambda_{2A} + A_{2A})(1 + g_A) = X_{2A}\lambda_{2A}$	$(A_{21B}\lambda_{1B} + A_{22B}\lambda_{2B} + A_{2B})(1 + g_B) = X_{2B}\lambda_{2B}$
L	$L_{1A}\lambda_{1A} + L_{2A}\lambda_{2A} = L_{AK}$	$L_{1B}\lambda_{1B} + L_{2B}\lambda_{2B} = L_{BK}$

where L_{AK} and L_{BK} denote the labour employed in the capital goods industries in countries A and B and $A_{1A} = A_{13A} + A_{14A}, A_{2A} = A_{23A} + A_{24A}, A_{1B} = A_{15B} + A_{16B}$ and $A_{2B} = A_{25B} + A_{26B}$.

Solving these two sets of equations at the given levels of activities in the consumption goods industries. i.e. $A_{1A} = 3, A_{2A} = 3, A_{1B} = 4, A_{2B} = 4$ we obtain values of the scale multipliers $\lambda_{1A}^{(1)}, \lambda_{2A}^{(1)}$ and $g_A^{(1)}$ which ensure balance of demands and supplies of the capital goods in country A and $\lambda_{1B}^{(1)}, \lambda_{2B}^{(1)}$ and $g_B^{(1)}$ which do the same for country B. Accordingly, we multiply the price equations of the capital goods industries by the scale multipliers. At the same time we obtain initial solutions for the rates of profit $r_A^{(1)}$ and $r_B^{(1)}$ using which we can find the prices of the consumption goods in the two countries.

Having done this we turn to the consumption goods equations. Since the trade pattern under trial is one of complete specialisation the exchange rate can be worked out directly,

$$E_{AB} = \frac{(\alpha_{5A} + \alpha_{6A})w_A L_A}{(\alpha_{3B} + \alpha_{4B})w_B L_B}$$

$$= \frac{(0.4 + 0.3)(1)(46.73)}{(0.3 + 0.4)(1)(60.17)}$$

$$= 0.776633$$

However, at this exchange rate and the prices of consumption goods already calculated the world demands for the consumption goods do not match the supplies; commodities 4 and 5 are in excess demand whilst commodities 3 and 6 are in excess supply. Denote the quantities demanded as $X_{id}^{(1)}$.

Our next step is in the direction of eliminating the excess demands and supplies of e consumption goods. Accordingly we change the levels of outputs in the consumption goods industries in the direction of their demands using Newton's method for that purpose. Thus the new trial outputs are,

$$X_i^{(1)} = X_i + \frac{X_{id} - X_i}{2} \quad i = 3, \ldots, 6$$

From the technical coefficients calculate the new values of $A_{1A}^{(1)}, A_{2A}^{(1)}, A_{1B}^{(1)}, A_{2B}^{(1)}$, in order to find the scale multipliers and rates of growth that will ensure balance of capital goods demands and supplies corresponding to the new levels of outputs in the consumption goods industries. Denote these values by $\lambda_{1A}^{(2)}, \lambda_{2A}^{(2)}, g_B^{(2)}, \lambda_{1B}^{(2)}, \lambda_{2B}^{(2)}, g_B^{(2)}$. and multiply the price equations of the first round by the scale multipliers. Ascertain the prices of consumption goods in both countries and once again find the quantities demanded of commodities $X_{id}^{(2)}$. If these are not equal to the quantities supplied $X_i^{(1)}$, determine new levels of output.

$$X_i^{(2)} = X_i^{(1)} + \frac{X_{id}^{(2)} - X_i^{(1)}}{2} \quad i = 3, \ldots, 6$$

And so on until at some stage $X_i^n = X_i^{n-1}$ and at this stage the equilibrium states of the countries are as follows,

	A		B	
	Labour	Output	Labour	Output
1	08.606	014.201	25.473	018.341
2	21.393	017.649	29.526	022.676
3	12.597	181.917	—	—
4	04.132	717.402	—	—
5	—	—	00.788	167.812
6	—	—	04.318	466.079

Observe that the volumes of employment in the post-trade equilibrium in the two countries are equal to the sums of the autarky employments in the industries operated in the post-trade situation ie. $L_A = L_{1A} + L_{2A} + L_{3A} + L_{4A} = 46.73$ and $L_B = L_{1B} + L_{2B} + L_{5B} + L_{6B} = 60.17$. All that has happened is that these volumes of employment have got redistributed between the industries operated in the post-trade situations.

This is of course a peculiarity of the trade situation under consideration and does always carry over to the general case where there are several tradeable and non-tradeable capital and consumption goods.

The effect of trade on the compositions or output is worth observing. Firstly. we find the outputs of commodity 2 rising in both countries at the cost of commodity 1. This is due to the fact that commodity 2 is used more intensively in the production of all consumption goods as compared to commodity 1 in both countries. Secondly, we find that the post-trade outputs of commodity 3 in country A and commodity 6 in country B fall while those of commodity 4 and commodity 5 increase. These output effects depend upon a variety of factors like the coefficients of consumption, the exchange rate, the volumes of post-trade employments and the post-trade prices of commodities.

Finally it may be verified that the pattern of gains from trade evaluated at the exchange rate $E_{AB} = 0.776633$ and the prices of commodities at the post-trade rates of profit $r_A = 0.40405$ and $r_B = 0.56146$ does indeed support the trade pattern being tried out.

10. Three-country, Two Capital Goods, Three Consumption Goods Trade

The preceding sections have developed the methods for handling trade in consumption as well as capital goods. Without exception, however, all the examples deal with trade between only two countries. As a step on the direction of developing the general theory of multicountry multicommodity trade it may be useful to get a grip on the complications that arise when trade between more than two countries s involved. Let us consider, therefore, an example of trade between three countries in five commodities. Of the five commodities suppose two, say commodities 1 and 2 are capital goods and the rest are tradeable consumption goods. Of the two capital goods suppose commodity 1 to be non-tradeable.

The autarky equilibria of the three countries are as follows,

$$
\begin{array}{ccc}
A & B & C \\
(2p_{1A} + 3p_{2A})V_A + 10w_A = 12p_{1A} & (8p_{1B} + 2p_{2B})V_B + 40w_B = 24p_{1B} & (1p_{1C} + 1p_{2C})V_C + 10w_C = 14.4p_{1C} \\
(3p_{1A} + 4p_{2A})V_A + 15w_A = 18p_{2A} & (2p_{1B} + 2p_{2B})V_B + 10w_B = 24p_{2B} & (6p_{1C} + 7p_{2C})V_C + 20w_C = 18p_{2C} \\
(1p_{1A} + 0.5p_{2A})V_A + 17.1w_A = 200p_{3A} & (4p_{1B} + 10p_{2B})V_B + 76.6w_B = 100p_{3B} & (2p_{1C} + 1p_{2C})V_C + 24.15w_C = 15p_{3C} \\
(0.5p_{1A} + 1.5p_{2A})V_A + 36.18w_A = 80p_{4A} & (4p_{1B} + 2p_{2B})V_B + 44.1w_B = 30p_{4B} & (1p_{1C} + 3p_{2C})V_C + 8.54w_C = 500p_{4C} \\
(3.5p_{1A} + 6p_{2A})V_A + 21.72w_A = 40p_{5A} & (2p_{1B} + 4p_{2B})V_B + 29.24w_B = 600p_{5B} & (2p_{1C} + 3p_{2C})V_C + 37.31w_C = 100p_{5C}
\end{array}
$$

where $V_A = 1 + r_A = 1.2$ $V_B = 1 + r_B = 1.2$ $V_C = 1 + r_C = 1.2$

The coefficients of consumption (from wage incomes alone) on the commodities 3, 4 and 5 in the three countries are as follows,

$$
\begin{array}{lll}
\alpha_{3A} = 0.2 & \alpha_{3B} = 0.5 & \alpha_{3C} = 0.3 \\
\alpha_{4A} = 0.4 & \alpha_{4B} = 0.3 & \alpha_{4C} = 0.2 \\
\alpha_{5A} = 0.4 & \alpha_{5B} = 0.2 & \alpha_{5C} = 0.5
\end{array}
$$

With wage rates $w_A =$ Re.1, $w_B = \$1$, $w_C = ¥1$ the autarky prices and the natural exchange rates are as follows,

$p_{1A} = 1.635$	$p_{1B} = 2.908$	$p_{1C} = 1.016$
$p_{2A} = 1.582$	$p_{2B} = 0.786$	$p_{2C} = 2.845$
$p_{3A} = 0.1$	$p_{3B} = 1$	$p_{3C} = 2$
$p_{4A} = 0.5$	$p_{4B} = 2$	$p_{4C} = 0.04$
$p_{5A} = 1$	$p_{5B} = 0.066$	$p_{5C} = 0.5$
$E^2_{AB} = 2.012$	$E^2_{BC} = 0.276$	$E^2_{CA} = 1.798$
$E^3_{AB} = 0.1$	$E^3_{BC} = 0.5$	$E^3_{CA} = 20$
$E^4_{AB} = 0.25$	$E^4_{BC} = 50$	$E^4_{CA} = 0.08$
$E^5_{AB} = 15$	$E^5_{BC} = 0.1333$	$E^5_{CA} = 0.5$

The most profitable trade assignments which cover all commodities and countries are

$$E^3_{AB} E^5_{BC} E^4_{CA} \cong 0.001$$

$$E^3_{AB} E^2_{BC} E^4_{CA} \cong 0.002$$

Accordingly the trade pattern is *A-1,3 B-1, 2, 5 C-1, 4* which the world economy with each country's accounts in her own currency looks like this

	A	B	C
1	$(2p_{1A} + 3p_{2B}E_{AB})V_A p + 10w_A = 12p_{1A}$	$(8p_{1B} + 2p_{2B})V_B + 40w_B = 24p_{1B}$	$(1p_{1C} + 1p_{2B}E_{CB})V_C + 10w_C = 14.4p_{1C}$
2	....	$(2p_{1B} + 2p_{2B})V_B + 10w_B = 24p_{1B}$	....
3	$(1p_{1A} + 0.5p_{2B}E_{AB})V_A + 17.1w_A = 200p_{3A}$	....	....
4	....	....	$(1p_{1C} + 3p_{2C}E_{CB})V_C + 8.54w_C = 500p_{4C}$
5	....	$(2p_{1B} + 4p_{2B})V_B + 29.24w_B = 600p_{5B}$	....

The international trade equilibrium must be found in the following steps. First, we set up the systems to determine the growth rates and post-trade outputs of the capital goods in the three countries. Since in the trade pattern being tried out countries A and C produce just one capital good the scale multiplier for the capital goods industry is 1 both of then. In other words the rates of growth are determined by the equations

	A	C
1	$(2\lambda_{1A} + 1)(1 + g_A) = 12\lambda_{1A}$	$(1\lambda_{1C} + 1)(1 + g_C) = 14.4\lambda_{1C}$
	$10\lambda_{1A} = 10$	$10\lambda_{1C} = 10$

So that $1 + g_A = 4$ $\quad 1 + g_C = 7.2$

$\lambda_{1A} = 1$ $\quad \lambda_{1C} = 1$

For country *B* however the equations will be,

$$B$$

$$\begin{array}{ll} 1 & (8\lambda_{1B} + 2\lambda_{2B} + 2)(1 + g_B) + 0 = 24\lambda_{1B} \\ 2 & (2\lambda_{1B} + 2\lambda_{2B} + 4)(1 + g_B) + A_2 = 24\lambda_{2B} \\ 3 & 40\lambda_{1B} + 10\lambda_{2B} = 50 \end{array}$$

where
$$\begin{aligned} A_2 &= A_{2A} + A_{2C} \\ &= (A_{21A} + A_{23A})(1 + g_A) + (A_{21C} + A_{24C})(1 + g_C) \\ &= 14 + 28.8 \\ &= 42.8 \end{aligned}$$

The solution for country *B* is as follows

$$\begin{aligned} 1 + g_B &= 1.3308645 \\ \lambda_{1B} &= 0.6654322 \\ \lambda_{2B} &= 2.3382709 \end{aligned}$$

From the growth rates of the countries we obtain the rates of profit and we multiply country *B*'s price equations for the capital goods by the scale multipliers to bring the system of capital goods industries in a state of balance.

Since country *B* does not import any capital good in the trade pattern being tried out the prices of commodities in country *B* at the post-trade rates of profit can be determined independently of the exchange rates. There prices are,

$$\begin{aligned} p_{1B} &\doteq 3.167745 \\ p_{2B} &= 0.86785 \\ p_{3B} &= 0.07045 \end{aligned}$$

Thus the exchange rates can be ascertained without recourse to an international price system of the type that is required when capital goods are imported by some or all countries.

Any two balance of trade equations can be used to determine the exchange rates. Consider these for countries *A* and *B*,

$$(\alpha_{4A} + \alpha_{5A}) w_A L_A + A_{2A} p_{2B} E_{AB} = (\alpha_{3B} w_B L_B) E_{AB} + (\alpha_{3C} w_C L_C) E_{AC}$$

$$[(\alpha_{3B} + \alpha_{4B}) w_B L_B] E_{AB} = A_{3A} p_{2B} E_{AB} + \alpha_{5A} w_A L_A + (\alpha_{5C} w_C L_C) E_{AC}$$

where the left-hand-sides show the import bills of countries *A* and *B* and the right-hand-sides show their export earnings, all measured in the currency of country *A*. The balance of trade equations in matrix form are.

$$\begin{bmatrix} A_{2A} p_{2B} - \alpha_{3A} w_B L_B & -\alpha_{3C} w_C L_C \\ A_2 p_{2B} - (\alpha_{3B} + \alpha_{4B}) w_B L_B & \alpha_{5C} w_C L_C \end{bmatrix} \begin{bmatrix} E_{AB} \\ E_{AC} \end{bmatrix} = \begin{bmatrix} -(\alpha_{4A} + \alpha_{5A}) w_A L_A \\ \alpha_{5A} w_A L_A \end{bmatrix}$$

where $L_A = 27.1, L_B = 79.24, L_C = 18.54, A_{2A} = 14, A_2 = 42.8$. On filling in the other values we obtain the solution $E_{AB} = 0.649697$, $E_{AC} = 0.68245$.

It now becomes possible to determine the prices of the capital and consumption goods produced by countries A and C and to ascertain the demands and supplies of the consumption goods. As it happens in our example, in which every country produces just one consumable commodity, the world demands and supplies of the consumption goods are exactly equal at the exchange rates determined above. It remains only to check on the gains from trade. To do so calculate the post-trade prices of all commodities in all countries at the post-trade rats of profit and the exchange rates of currencies. These are,

$$\begin{array}{lll} p_{1A} = 4.1835 & p_{1B} = 3.16775 & p_{1C} = 2.1122 \\ p_{2A} = 4.1212 & p_{2B} = 0.86375 & p_{2C} = 8.7205 \\ p_{3A} = 0.1747 & p_{3B} = 1.0496 & p_{3C} = 4.1275 \\ p_{4A} = 0.5989 & p_{4B} = 2.1107 & p_{4C} = 0.8444 \\ p_{5A} = 2.3439 & p_{5B} = 0.07045 & p_{5C} = 0.8691 \end{array}$$

From these prices we ascertain the gains from trade, and verify that

	A	*B*	*C*
2	0.2426	$\boxed{1.7819}$	0.1680
3	$\boxed{5.7123}$	1.4664	0.3550
4	1.6696	0.7292	$\boxed{17.3520}$
5	0.4266	$\boxed{21.8478}$	1.6859

the trade pattern *A-1, 3 B-1, 2, 5 C-1, 4* is indeed consistent.

Thus, for example, the international trade equilibrium is as follows.

	A		*B*		*C*	
	Labour	Output	Labour	Output	Labour	Output
1	10	12	26.61	15.96	10	14.4
2	—	—	23.38	56.11	—	—
3	17.1	200	—	—	—	—
4	—	—	—	—	8.54	500
5	—	—	29.24	600	—	—

11. Three-country, Four Capital Goods, Seven Consumption Goods Trade: Formal Version

We have by now surveyed several examples of 'lower-dimensional' trade in which countries produce no more than two tradeable commodities in the post-trade situation. Obviously our interest in these low-dimensional situation was not so much due to their substantive merit as it was in their being preliminaries that would lead us to the study of the general situation of multicountry multicommodity trade. In what follows we shall turn all attention to the general situation.

On the basis of casual empirical knowledge we know that the general case must have three features: (1) the existence of several countries, (2) the production of several capital goods, both tradeable and non-tradeable in these countries and (3) the production of several consumptions goods, both tradeable and non-tradeable. Since it is the general situation that the trade theorist must in practice deal with, it will be useful to outline the methods that he is most likely to require in trying out different trade patterns. Thus in this section we shall describe in purely formal terms a simple trade situation which incorporates to some extent all of the three features of actual trade situations that are most likely to be encountered in practice.

Consider a situation of trade between 3 countries in 11 commodities. Of the 11 commodities 4 are capital goods and 7 are consumption goods. Of the 4 capital goods suppose commodity 1 to be non-tradeable and commodities 2,3 and 4 to be tradeable capital goods. Of the 7 consumption goods suppose commodities 5, 6, 7, 8, 9, and 10 to be tradeable and commodity 11 to non-tradeable. Finally suppose the trade pattern indicated by the autarky equilibrium natural exchange rates in *A-1, 2, 5, 6, 11 B-1, 3, 7, 8, 11 C-1, 4, 9, 10, 11*. Observe that since in the post-trade situation each country produces two capital goods and (more than) two consumption goods. The system does not have the properties that are peculiar to low-dimensions, mentioned in Chapter VII. Thus even though the trade pattern itself is far from being even a representative of the general case, it is nevertheless detailed enough to serve the purpose of explaining the rules that must be followed when dealing with the complex situation. The first step towards finding the post-trade solution of the trade pattern shown above is to formulate the equations that ensure balance of the demands and supplies of capital goods and determine the growth rates at given levels of activities in the sphere of the consumption goods industries. The next step is to ensure balance of demand s and supplies in the consumption goods industries, But, as we know, there are several trials to be undergone between the two steps.

The capital goods equation's in the three countries are as follows,

$$A$$

$$\begin{aligned}
(A_{11A}\lambda_{1A} + A_{12A}\lambda_{2A} + A_{1A})(1+g_A) + 0 &= X_{1A}\lambda_{1A} \\
(A_{21A}\lambda_{1A} + A_{22A}\lambda_{2A} + A_{2A})(1+g_A) + A_{2e} &= X_{2A}\lambda_{2A} \qquad (14)\\
L_{1A}\lambda_{1A} + L_{2A}\lambda_{2A} &= L_{1A} + L_{2A}
\end{aligned}$$

where
$$A_{1A} = A_{15A} + A_{16A} + A_{1,11A}$$
$$A_{2A} = A_{25A} + A_{26A} + A_{2,11A}$$

are the quantities of capital goods 1 and 2 required in the consumption goods industries country A and,

$$A_{2e} = A_{2B}(1+g_B) + A_{2C}(1+g_C)$$

where $$A_{2B} = A_{21B} + A_{23B} + A_{27B} + A_{28B} + A_{2,11B}$$

$$A_{2C} = A_{21C} + A_{24C} + A_{29C} + A_{2,10C} + A_{2,11C}$$

are the quantities of capital good 2 required for replacement purposes alone in countries B and C. Clearly A_{2e} shows the total, ie. replacement plus growth requirements of commodity 2 in countries B and C and accordingly it represents the export of commodity 2 by country A.

$$B$$

$$\begin{aligned}
(A_{11B}\lambda_{1B} + A_{13B}\lambda_{3B} + A_{1B})(1+g_B) + 0 &= X_{1B}\lambda_{1B} \\
(A_{31B}\lambda_{1B} + A_{33B}\lambda_{3B} + A_{3B})(1+g_B) + A_{3e} &= X_{3B}\lambda_{3B} \qquad (15) \\
L_{1B}\lambda_{1B} + L_{3B}\lambda_{3B} &= L_{1B} + L_{3B}
\end{aligned}$$

where

$$A_{1B} = A_{17B} + A_{18B} + A_{1,11B}$$

$$A_{3B} = A_{27B} + A_{28B} + A_{2,11B}$$

are the quantities of capital goods 1 and 3 required in the consumption goods industries in country B and

$$A_{3e} = A_{3A}(1+g_A) + A_{3C}(1+g_C)$$

where $$A_{3A} = A_{31A} + A_{32A} + A_{35A} + A_{36A} + A_{3,11A}$$

$$A_{3C} = A_{31C} + A_{34C} + A_{39C} + A_{3,10C} + A_{3,11C}$$

are the quantities of capital good 3 required for replacement in countries A and C. A_{3e} is therefore the total (replacement plus growth) quantity exported by country B. Finally,

$$C$$

$$\begin{aligned}
(A_{11C}\lambda_{1C} + A_{14C}\lambda_{4C} + A_{1C})(1+g_C) + 0 &= X_{1C}\lambda_{1C} \\
(A_{41C}\lambda_{1C} + A_{44C}\lambda_{4C} + A_{4C})(1+g_C) + A_{4e} &= X_{4C}\lambda_{4C} \qquad (16) \\
L_{1C}\lambda_{1C} + L_{4C}\lambda_{4C} &= L_{1C} + L_{4C}
\end{aligned}$$

where $A_{1C}, A_{4C}, A_{4e} = A_{4A}(1+g_A) + A_{4B}(1+g_B)$ have meanings that follow from the notation.

It is clear from (14), (15) and (16) that the growth rats of all countries are interlinked so that none can be resolved right away, a process of trial and error will have to be resorted to. Thus put in any two values, say $A_{3e}^{(0)}$ and $A_{4e}^{(0)}$ arbitrarity into 2 and 3 For instance we may initially put in from the autarky equilibrium data. Then since all the remaining parameters viz. A_{ijk}'s and the L_{ik}'s ($i, j = 1, 2, 3, 4, k = A, B, C$) are known we an find a trial solution from (15) and (16) for the scale multipliers and the rates of growth in countries B and C. Denote these by $\lambda_{1B}^{(1)}, \lambda_{3B}^{(1)}, g_B^{(1)}$ and $\lambda_{1C}^{(1)}, \lambda_{4C}^{(1)}, g_C^{(1)}$ and multiply the original coefficients A_{ijk} by the appropriate multipliers to get the new (trial) equations of the capital goods produced in B and C. At the same time evaluate

$$A_{2e} = A_{2B}(1+g_B^{(1)}) + A_{2C}(1+g_C^{(1)})$$

where

$$A_{2B} = A_{21B}\lambda_{1B}^{(1)} + A_{23B}\lambda_{3B}^{(1)} + A_{27B} + A_{28B} + A_{2,11B}$$

$$A_{2C} = A_{21C}\lambda_{1C}^{(1)} + A_{24C}\lambda_{4C}^{(1)} + A_{29B} + A_{2,10B} + A_{2,11B}$$

Put the value of A_{2e} in equations (14) and solve for $\lambda_{1A}^{(1)}, \lambda_{2A}^{(1)}, g_A^{(1)}$. Multiply the original coefficients A_{ijA} by these multipliers to obtain the new (trial) equations for capital goods 1 and 2 in country A.

The coefficients of the capital good's equations are thus revised to the following vales,

$$\begin{array}{lll}
A_{11A}\lambda_{1A}^{(1)} = A_{11A}^{(1)} & A_{12A}\lambda_{2A}^{(1)} = A_{12A}^{(1)} & X_{1A}\lambda_{1A}^{(1)} = X_{1A}^{(1)} \\
A_{21A}\lambda_{1A}^{(1)} = A_{21A}^{(1)} & A_{22A}\lambda_{2A}^{(1)} = A_{22A}^{(1)} & X_{2A}\lambda_{2A}^{(1)} = X_{2A}^{(1)} \\
L_{1A}\lambda_{1A}^{(1)} = L_{1A}^{(1)} & L_{2A}\lambda_{2A}^{(1)} = L_{2A}^{(1)} & \\
A_{11B}\lambda_{1B}^{(1)} = A_{11B}^{(1)} & A_{13B}\lambda_{3B}^{(1)} = A_{13B}^{(1)} & X_{1B}\lambda_{1B}^{(1)} = X_{1B}^{(1)} \\
A_{31B}\lambda_{1B}^{(1)} = A_{31B}^{(1)} & A_{33B}\lambda_{3B}^{(1)} = A_{33B}^{(1)} & X_{3B}\lambda_{3B}^{(1)} = X_{3B}^{(1)} \\
L_{1B}\lambda_{1B}^{(1)} = L_{1B}^{(1)} & L_{3B}\lambda_{3B}^{(1)} = L_{3B}^{(1)} & \\
A_{11C}\lambda_{1C}^{(1)} = A_{11C}^{(1)} & A_{14C}\lambda_{4C}^{(1)} = A_{14C}^{(1)} & X_{1C}\lambda_{1C}^{(1)} = X_{1C}^{(1)} \\
A_{41C}\lambda_{1C}^{(1)} = A_{41C}^{(1)} & A_{44C}\lambda_{4C}^{(1)} = A_{44C}^{(1)} & X_{4C}\lambda_{4C}^{(1)} = X_{4C}^{(1)} \\
L_{1C}\lambda_{1C}^{(1)} = L_{1C}^{(1)} & L_{4C}\lambda_{4C}^{(1)} = L_{4C}^{(1)} &
\end{array}$$

Also find the quantities of capital goods 3 and 4 demanded at the new trial state by the countries importing these goods,

$$A_{3e}^{(1)} = A_{3A}^{(1)}(1+g_A^{(1)}) + A_{3C}^{(1)}(1+g_C^{(1)})$$

where $$A_{3A}^{(1)} = A_{31A}\lambda_{1A}^{(1)} + A_{32A}\lambda_{2A}^{(1)} + A_{35A} + A_{36A} + A_{3,11A}$$

$$A_{3C}^{(1)} = A_{31C}\lambda_{1C}^{(1)} + A_{34C}\lambda_{4C}^{(1)} + A_{39C} + A_{3,10C} + A_{3,11C}$$

$$A_{4e}^{(1)} = A_{4A}^{(1)}(1+g_A^{(1)}) + A_{4B}^{(1)}(1+g_B^{(1)})$$

where $$A_{4A}^{(1)} = A_{41A}\lambda_{1A}^{(1)} + A_{42A}\lambda_{2A}^{(1)} + A_{45A} + A_{46A} + A_{4,11A}$$

$$A_{4B}^{(1)} = A_{41B}\lambda_{1B}^{(1)} + A_{43B}\lambda_{3B}^{(1)} + A_{47B} + A_{48B} + A_{4,11B}$$

The revised values of A_{3e} and A_{4e} to be put into the new trial capital good's equations are now obtained internally

$$A_{3e}^{(2)} = \frac{A_{3e}^{(0)} + A_{3e}^{(1)}}{2}$$

$$A_{4e}^{(2)} = \frac{A_{4e}^{(0)} + A_{4e}^{(1)}}{2}$$

The capital goods equations in the three countries for the new trial are as follows,

$$A$$

$$\begin{aligned}
(A^{(1)}_{11A}\lambda_{1A} + A^{(1)}_{12A}\lambda_{2A} + A_{1A})(1+g_A)+0 &= X^{(1)}_{1A}\lambda_{1A} \\
(A^{(1)}_{21A}\lambda_{1A} + A^{(1)}_{22A}\lambda_{2A} + A_{2A})(1+g_A)+A_{2e} &= X^{(1)}_{2A}\lambda_{2A} \qquad (14)\\
L^{(1)}_{1A}\lambda_{1A} + L^{(1)}_{2A}\lambda_{2A} &= L^{(1)}_{1A} + L^{(1)}_{2A}
\end{aligned}$$

$$B$$

$$\begin{aligned}
(A^{(1)}_{11B}\lambda_{1B} + A^{(1)}_{12B}\lambda_{3B} + A_{1B})(1+g_B)+0 &= X^{(1)}_{1B}\lambda_{1B} \\
(A^{(1)}_{31B}\lambda_{1B} + A^{(1)}_{33B}\lambda_{3B} + A_{3B})(1+g_B)+A^{(2)}_{3e} &= X^{(1)}_{3B}\lambda_{3B} \qquad (15)\\
L^{(1)}_{1B}\lambda_{1B} + L^{(1)}_{3B}\lambda_{3B} &= L^{(1)}_{1B} + L^{(1)}_{3B}
\end{aligned}$$

$$C$$

$$\begin{aligned}
(A^{(1)}_{11C}\lambda_{1C} + A^{(1)}_{14C}\lambda_{4C} + A_{1C})(1+g_C)+0 &= X^{(1)}_{1C}\lambda_{1C} \\
(A^{(1)}_{41C}\lambda_{1C} + A^{(1)}_{44C}\lambda_{4C} + A_{4C})(1+g_C)+A^{(2)}_{4e} &= X^{(1)}_{4C}\lambda_{4C} \qquad (16)\\
L^{(1)}_{1C}\lambda_{1C} + L^{(1)}_{4C}\lambda_{4C} &= L^{(1)}_{1C} + L^{(1)}_{4C}
\end{aligned}$$

Repeat the process, i.e. solve for $\lambda^{(2)}_{1B}, \lambda^{(2)}_{3B}, g^{(2)}_B$ and $\lambda^{(2)}_{1C}, \lambda^{(2)}_{4C}, g^{(2)}_C$, evaluate the new value of A2e and solve for $\lambda^{(2)}_{1A}, \lambda^{(2)}_{2A}, g^{(2)}_A$. Change the coefficients of all three equations above by means of the new multipliers,

$$\begin{aligned}
A^{(2)}_{ijk} &= A^{(1)}_{ijk}\lambda^{(2)}_{jk} \\
X^{(2)}_{jk} &= X^{(1)}_{jk}\lambda^{(2)}_{jk} \qquad i,j = 1,\ldots,4 \quad k = A, B, C\\
L^{(2)}_{jk} &= L^{(1)}_{jk}\lambda^{(2)}_{jk}
\end{aligned}$$

to obtain the new trial coefficients of the capital goods equations. Also find $A^{(3)}_{3e}$ and $A^{(3)}_{4e}$, the requirement for capital goods 3 and 4 by the countries importing them,

$$\begin{aligned}
A^{(3)}_{3e} &= A^{(2)}_{3A}(1+g^{(2)}_A) + A^{(2)}_{3C}(1+g^{(2)}_C) \\
A^{(3)}_{4e} &= A^{(2)}_{4A}(1+g^{(2)}_A) + A^{(2)}_{4B}(1+g^{(2)}_B)
\end{aligned}$$

and calculate the revised values,

$$A^{(4)}_{3e} = \frac{A^{(2)}_{3e} + A^{(3)}_{3e}}{2}$$

$$A^{(4)}_{4e} = \frac{A^{(2)}_{4e} + A^{(3)}_{4e}}{2}$$

to be put into the revised capital goods equations or the second trial. And so on until $A^{(n)}_{3e} = A^{(n-1)}_{3e}$ and $A^{(n)}_{4e} = A^{(n-1)}_{4e}$ at which point the exportable quantity of commodity 2 in country A must also be equal to the quantity of commodity 2 demanded by countries B and C. At these points it will also be observed that $\lambda_{jk}^{(n)} = \lambda_{jk}^{(n-1)} = 1\ (\forall\, j, k)$ showing that the balance of capital goods demands and supplies shave been achieved.

All these iterations have, however, been performed at given levels of activity in the consumption goods industries. We must therefore move in the direction of eliminating the excess demands and supplies of the consumptions. To evaluate these excess demands/supplies it is necessary to find the prices of the consumption goods, to find which it is necessary to find the price of the capital goods and the exchange rates of currencies simultaneously. The price equations for the capital goods in any one currency, say currency, are as follows,

$$
\begin{aligned}
&(A_{11A}p_{1A} + A_{21A}p_{2A} + A_{31A}p_{3B}E_{AB} + A_{41A}p_{4C}E_{AC})(1 + r_A) + w_A L_{1A} \\
&\qquad = p_{1A}X_{1A} \\
&(A_{12A}p_{1A} + A_{22A}p_{2A} + A_{32A}p_{3B}E_{AB} + A_{42A}p_{4C}E_{AC})(1 + r_A) + w_A L_{2A} \\
&\qquad = p_{2A}X_{2A} \\
&(A_{11B}p_{1B}E_{AB} + A_{21B}p_{2A} + A_{31B}p_{3B}E_{AB} + A_{41B}p_{4C}E_{AC})(1 + r_B) \\
&\qquad + (w_B E_{AB})L_{1B} = (p_{1B}E_{AB})X_{1B} \\
&(A_{13B}p_{1B}E_{AB} + A_{23B}p_{2A} + A_{33B}p_{3B}E_{AB} + A_{43B}p_{4C}E_{AC})(1 + r_B) \\
&\qquad + (w_B E_{AB})L_{3B} = (p_{3B}E_{AB})X_{3B} \\
&(A_{11C}p_{1C}E_{AC} + A_{21C}p_{2A} + A_{31C}p_{3B}E_{AB} + A_{41C}p_{4C}E_{AC})(1 + r_C) \\
&\qquad + (w_C E_{AC})L_{1C} = (p_{1C}E_{AC})X_{1C} \qquad (17) \\
&(A_{14C}p_{1C}E_{AC} + A_{24C}p_{2A} + A_{34C}p_{3B}E_{AB} + A_{44C}p_{4C}E_{AC})(1 + r_C) \\
&\qquad + (w_C E_{AC})L_{4C} = (p_{4C}E_{AC})X_{4C}
\end{aligned}
$$

From the growth rates arrived at we can find the rates of profit r_A, r_B, r_C, (In the general case in which profit earners consume and workers save, the rates of profit are derived from the equations $r_k = g_k/1 - \beta_k\,(k = A, B, C)$ where β is the propensity to consume of the profit earnest). The system of equations (17) contains 6 equations in 8 unknowns viz. $p_{1A}, p_{1B}, p_{1C}, p_{2A}, p_{3B}, p_{4C}, E_{AB}, E_{AC}$. Two independent equations are required. Any two trade balance equations can be used to fill in this deficit. Consider those of countries A and B. There import bills and export earnings in terms of the currency of country A are,

$$
\begin{aligned}
M_A &= A'_{3A}p_{3B}E_{AB} + A'_{4A}p_{4C}E_{AC} + (\alpha_{7A} + \alpha_{8A} + \alpha_{9A} + \alpha_{10A})w_A L_A \\
X_A &= A'_{2e}p_{2A} + (\alpha_{4B} + \alpha_{5B})w_B L_B E_{AB} + (\alpha_{4C} + \alpha_{5C})w_C L_C E_{AC} \qquad (18) \\
M_B &= A'_{2B}p_{2A} + A'_{4B}p_{4C}E_{AC} + (\alpha_{4B} + \alpha_{5B} + \alpha_{9B} + \alpha_{10B})w_B L_B E_{AB} \\
X_B &= A'_{3e}p_{3B}E_{AB} + (\alpha_{7A} + \alpha_{8A})w_A L_A + (\alpha_{7C} + \alpha_{8C})w_C L_C E_{AC}
\end{aligned}
$$

(In the general case of consumption by capitalists and saving by workers the import bills and export earnings will contain additional items.) Arranging the equations in (18) in vector-matrix notation we get,

$$\begin{bmatrix} A'_{3A} & A'_{4A} \\ A'_{3e} & -A'_{4B} \end{bmatrix} \begin{bmatrix} p_{3B}E_{AB} \\ p_{4C}E_{AC} \end{bmatrix}$$

$$= \begin{bmatrix} A'_{2e}p_{2A} + (\alpha_{4B}+\alpha_{5B})w_B L_B E_{AB} + (\alpha_{4C}+\alpha_{5C})w_C L_C E_{AC} \\ \quad -(\alpha_{7A}+\alpha_{8A}+\alpha_{9A}+\alpha_{10A})w_A L_A \\ A'_{2B}p_{2A} + (\alpha_{4B}+\alpha_{5B}+\alpha_{9B}+\alpha_{10B})w_B L_B E_{AB} \\ -(\alpha_{7A}+\alpha_{8A})w_A L_A - (\alpha_{7C}+\alpha_{8C})w_C L_C E_{AC} \end{bmatrix}$$

Find the solutions for $p_{3B}E_{AB}$ and $p_{4C}E_{AC}$ in terms of p_{2A}, E_{AB} and E_{AC} and substitute them in equations (17). Then we obtain the hybrid system,

$$\begin{bmatrix} H_{11} & H_{12} & 0 & H_{14} & 0 & H_{16} \\ H_{21} & H_{22} & 0 & H_{24} & 0 & H_{26} \\ 0 & H_{32} & H_{33} & H_{34} & 0 & H_{36} \\ 0 & H_{42} & H_{43} & H_{44} & 0 & H_{46} \\ 0 & H_{52} & 0 & H_{54} & H_{55} & H_{56} \\ 0 & H_{62} & 0 & H_{64} & H_{65} & H_{66} \end{bmatrix} \begin{bmatrix} p_{1A} \\ p_{2A} \\ p_{1B}E_{AB} \\ E_{AB} \\ p_{1C}E_{AC} \\ E_{AC} \end{bmatrix} = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} \tag{19}$$

for which we obtain the first tentative solution for the exchange rates and the prices of the capital goods. Equation (18) then gives the tentative values of p_{3B} and p_{4C}. Once these are known we may find the prices of the consumption goods produced in each country and evaluate the excess demands and supplies.

Denote the quantities demanded of the consumption goods by $X^{(1)}_{jk}$ ($j = 5,\dots 11$, $k = A, B, C$) and their quantities supplied in autarky to be X_{jk}. Then the quantities demanded of the tradeable consumption goods 5, to 10 are

$$X^{(1)}_{jk} = \frac{\alpha_{jA}w_A L_A E_{kA} + \alpha_{jB}w_B L_B E_{kB} + \alpha_{jC}w_C L_C E_{kC}}{p_{jk}} \quad E_{kk} = 1, Vk \tag{20}$$

and the quantities demanded of the non-tradeable commodity 11 are,

$$X^{(1)}_{11,k} = \frac{\alpha_{11,k}w_k L_k}{p_{11,k}} \quad k = A, B, C \tag{21}$$

Should these be different from the quantities supplied X_{jk} we proceed to put in revised values of the outputs of consumption goods,

$$X^{(2)}_{jk} = \frac{X^{(0)}_{jk} + X^{(1)}_{jk}}{2} \quad j = 5,\dots 10 \;\; k = A, B, C \tag{22}$$

Next calculate the quantities of the capital goods that are required to produce these outputs as well as the labour required country-wise. Thus we get new values $A^{(1)}_{1A}, A^{(1)}_{2A}, A^{(1)}_{1B}, A^{(1)}_{3B}, A^{(1)}_{1C}, A^{(1)}_{4C}$ to be put into equations (1), (2) and (3) as well as new values of the levels of employment $L^{(1)}_A, L^{(1)}_B, L^{(1)}_C$ that will be required to calculate the new levels of excess demands in the second iteration.

Repeat the process of calculations in (14), (15), (16), (17), (18) and (19) and calculate the new levels of the quantities demanded $X_{jk}^{(3)}$ of the consumption goods and put in revised values

$$X_{jk}^{(4)} = \frac{X_{jk}^{(2)} + X_{jk}^{(3)}}{2} \tag{23}$$

If the process converges to definite levels of the rates of growth and profit that are grater than, or equal to, the autarky rates and to definite levels of employment, proceed to evaluate the prices of all tradeable commodities in all the countries at the post-trade rates of profit, prices of capital goods and exchange rats. Evaluate the gains from trade. If these are found to support the trade pattern then equilibrium is found. If not the trade pattern must be changed in accordance with the pattern of gains from trade revealed.

If during the process of the convergence any of the following situations arise, (a) the rates of growth and profit in a country begin falling without limit, (b) the volume of employment in some country diverges and rises without limit or (c) some scale multipliers become negative, the trade pattern being tried is not the equilibrium trade pattern it may be abandoned and replaced by a new trade pattern.

If (a) and (c) happen it means that some capital goods must be allotted to another (larger) country as well, provided, of course, that the assignment forms a part of the arbitrage sequence conditions. If (b) happens, it means that the country cannot by herself satisfy the demands for the consumption goods she exports so that another country must in addition be assigned the consumption goods.

12. General Algorithm

We may now generalise. Suppose there are Z countries producing n commodities of which m are capital goods and $n-m$ are consumption goods. Suppose that of the n commodities t are tradeable goods and $n-t$ are non-tradeables. The steps to find the trade equilibrium, if it exists, are as follows:

1. Determine the autarky equailibria of the Z countries by the methods described in Chapter VII above. Find the pre-trade prices and natural exchange rates.
2. Find the $Z^t - t$ arbitrage sequences and identify all those which give products less than unity.
3. To set the initial trial trade pattern choose the set of the most profitable arbitrage sequences that assign all tradeable commodities to some country and all countries to some tradeable commodity.
4. Try the trade pattern; that is to say,
   (a) Ensure balance in the capital goods sphere in all countries and determine the outputs of the capital goods and the growth rates at given levels of activities in the consumption goods industries.

(b) From the growth rates determine the rates of profit and formulate the system of the price equations of all capital goods in all countries in terms of any one currency. These will be m equations in unknown prices an $z - 1$ unknown exchange rates, i.e. $m + z - 1$ unknowns.

(c) Use $z - 1$ independent trade balance equations to eliminate $z - 1$ unknown prices (or exchange rates) and obtain the 'hybrid system of equations.

(d) Solve the hybrid system of the prices of capital goods and exchange rates and use them to find the prices of consumption goods from the relevant price equations.

(e) Using the demand equations for the consumption goods, determine the quantities demanded of the consumption goods.

(f) Move the supplies in the direction of the demands, calculate the capital goods requirements and labour requirements of the new supply levels and got back to (a), repeating (a) to (f) until the solution converges.

5. If the solution converges find the prices of all commodities in all countries at the post-trade rates of profit, post-trade prices of capital goods and the exchange rates.
6. Find the pattern of the gains from trade. If it is not consistent with the trade pattern being tried out, determine the new trial trade pattern, i.e. keeping status-quo on the previous trade pattern add to the portfolio of different countries those commodities with which they yield the greatest gain from trade.
7. Go back to step (2) and try the new trade pattern, repeating (2) to (6) until the trade equilibrium is found.
8. If in step (4), (i) the rats of growth an profit in some country begins to fall without limit, and her scale multiplier becomes negative stop, and assign one of her exportable commodities to another country and go back to step (2). If (ii) the size of labour employed begins to rise without limit, assign the exportable consumption good which is persistently in excess demand to another country which conforms to the arbitrage sequence conditions and go back to step (2).

Notes

1. Literature on this subject is scarce by all of it is significant: Notable contributions include McKenzie (1955), Vanek (1963) Schweinberger (1975), Steedman, Metcalfe, Mainwaring and Parinello in Steedman (1979a), Steedman (1979b), Sanyal and Jones (1982), Smith (1982).
2. Readers interest in pursuing the implications of fixed capital, human capital, natural resources and public expenditure for international trade may find Parchure (1989) useful.
3. Generalisation of the results to include consumption by profit-earners and saving by wage-earners can be made on the lines suggested in Chapter VII.
4. The reason for emphasising this rather obvious point is that Ricardo was opt to miss it. See Chipman (1965). Also see Morishima (1989). Morishima has shown that the inverse

relationship of the rate of profit and the real wage rate does not hold in an open economy.

5. This proposition is due to Ricardo(1817) who made it with specific reference to the import of a subsistence wage good. It clearly applies to intermediate capital goods as well. See Steedman (1979) for a proof.
6. The reference clearly is to Leontief's pathbreaking (1953, 1956) studies, which found that the capital-abundant U.S.A.. exports labour-intensively produced goods and imports capital-intensively produced goods. Similar results were found by Baldwin (1971). But Stern and Maskus found the paradox to be "reversed" in the case of U.S.A. trade in 1972. Such mixed evidence is consistent with the theory presented here.
7. Our conclusion on two-sector models is diametrically opposed to that obtained in the literature. Two-sector models in the tradition of Johnson (1971), Vanek (1971), Deardorff (1973) usually assume the international relative price as given exongenously. The fact is that it is indeterminate and must, therefore, be given exongenously. Our conclusion is also opposed to models in Heckscher-Ohlin-Samuelson tradition as also those of Oniki and Uzaws (1965), Bardhan (1965), Stiglitz (1970) Takayama (1970) all of whom find a unique trade equilibrium.
8. Obviously in an economy in equilibrium and under conditions of no technical change. See Sraffa (1960), Chp VI. for the proof.

9

GENERALISATIONS

"That wouldn't be very nice, I am afraid —"
"Not very nice alone", he interrupted, quite eagerly: "but you've no idea what a difference it makes, mixing it with other things — like gunpowder and sealing-wax."

1. Transfers: Preliminaries

We shall in this chapter consider generalisations of the dynamic model of capital and trade explained in the previous chapter. To keep the size of this chapter within decent limits and to avoid repetition, we shall consider only two applications, as illustrations—intercountry transfers and tariffs. For the rest, the methods explained in Chapter VI may be followed with suitable modifications.

Consider first the problem of autonomous intercountry transfers in an environment of flexible exchange rates[1]. When we dealt with this problem under the simple conditions of Chapter VI, our task was much less severe, for the transfer was withdrawn from wage incomes in one country and distributed to wage-earners in another, proportionately. But in the general context that we are presently working in, the proportions in which the transfer amount is drawn from wages and profits in one country and distributed between the wages and profits in the other, enter the analysis as additional desiderata. For, as we shall soon see, the effects of the transfer change depending upon the manner of its financing as well as the manner of its disbursement.

To the extent that the transfer is financed from wage incomes and distributed to wage-earners the price equations in the different countries are not affected. (Of course, their specific solution will change). But should the transfer amount be drawn exclusively out of profits and distributed between profits, the price equations will be modified: in the transferor country the transfer amount will be added on the cost-of-production side while in the transferee country the same amount (of course adjusted for the appropriate currency) will be subtracted.

Thus if the price equation of the jth commodity in country A is

$$\sum_i (A_{ijA} p_{iA})(1+r_A) + w_A L_{jA} = p_{jA} X_{jA} \qquad i = 1, \ldots, m \quad j = 1, \ldots, n$$

and T_{AB} is the amount of the transfer from country A to country B the price equations will be modified as follows,

$$\sum_i (A_{ijA} p_{iA})(1+r_A) + w_A L_{jA} + \left[\frac{K_{jA}}{K_A}\right] T_{AB} = p_{jA} X_{jA} \quad i = 1, \ldots, m \quad j = 1, \ldots, n \quad (1)$$

where (K_{jA}/K_A) is the proportion of the capital stock in industry j in country A. The same equation can be written as,

$$\sum_i (A_{ijA}p_{iA})(1+r_{At}) + w_A L_{jA} = p_{jA}X_{jA} \qquad i=1,\ldots,m \quad j=1,\ldots,n \qquad (1')$$

where r_{At} is the post-transfer rate of profit.

On the other hand the price equations of the jth commodity in the transferee country B will be,

$$\sum_i (A_{ijB}p_{iB}) + w_B L_{jB} - \left[\frac{K_{jB}}{K_B}\right](T_{AB}E_{BA}) = p_{jB}X_{jB} \quad i=1,\ldots,m \quad j=1,\ldots n \quad (2)$$

or,

$$\sum_i (A_{ijB}p_{iB})(1+r_{Bt}) + w_B L_{jB} = p_{jB}X_{jB} \quad i=1,\ldots m, \; j=1,\ldots n \quad (2')$$

Note, however, that in (1), (1'), (2), (2') only those industries j operated in the *post-trade* equilibrium need appear, not the ones operating in autarky.

If a proportion Ω_A of the transfer amount is drawn from wages W_A and $1-\Omega_A$ is drawn from profits in the transferor country A and the disbursement is such that a proportion Ω_B is distributed to wage-earners and $1-\Omega_B$ to the profit earners in the transferee country, then equations (1'), (1) and (2), (2') respectively, will contain only the amounts drawn from, and disbursed to, profits in the two countries, viz. $(1-\Omega_A)T_{AB}$ and $(1-\Omega_B)T_{AB}E_{BA}$. Accordingly the post transfer rates of profit will be altered only to the extent of the transfer from and to profit incomes.

Transfers, to the extent that they are drawn out of, and disbursed to, profits, have a direct effect on the relationship of the rate of growth and the rate of profit in both countries. In the transferor country the post-transfer rate of profit is the rate of growth *plus* the transfer amount as a proportion of the capital stock,

$$r_{At} = g_A + \frac{(1-\Omega_A)T_{AB}}{K_A} \qquad (3)$$

In the transferee country the post-transfer rate of profit will be the rate of growth *minus* the transfer (converted to foreign currency) as a proportion of the capital stock, in the transferee country,

$$r_{Bt} = g_B - \frac{(1-\Omega_B)T_{AB}E_{BA}}{K_B} \qquad (4)$$

In other words the rate of growth in the transferor country tends to decline relative to the rate of profit while that in the transferee country tends to increase relative to the rate of profit. Of course (3) and (4) are based on the assumption that profits in both countries are entirely saved and wages entirely consumed. But it is easy to guess that similar effects will be observed even in the general case.

2. Two Countries, Three Commodities

With this background let us consider some examples. To start at a simple level consider first the effects of transfers in the two-country three-commodity example of section 5 of Chapter VIII. Of the three commodities, one is a non-tradeable capital good and the other two are tradeable consumption goods. The trade equilibrium we have found is as follows,

	A	B
1	$5p_{1A}(1+r_A)+9w_A = 11p_{1A}$	$1p_{1B}(1+r_B)+1.2w_B = 11p_{1B}$
2	....	$6p_{1B}(1+r_B)+2.95w_B = 60p_{2B}$
3	$1p_{1A}(1+r_A)+1.2w_A = 5p_{3A}$	....

with a trade pattern *A-3 B-2* and the equilibrium exchange rate $E_{AB} = 3.1460$. Recall that we have assumed all wages to be consumed ($\alpha_{2A} = 0.8$, $\alpha_{3A} = 0.2$, $\alpha_{2B} = 0.375$, $\alpha_{3B} = 0.625$) and all profits to be saved so that the rates of profit are $r_A = 0.8333$, $r_B = 0.5714$.

Now suppose country A transfers Rs. 5 to country B. And to consider alternative scenarios of the manner of financing and disbursement of the transfer suppose, firstly, that the transfer amount is drawn entirely out of wage incomes in A and disbursed entirely to wage-earners in B. The effects of the transfer are easily traced. The import bills of countries A and B change to,

$$M_{AB} = (0.8)(\text{Rs.}10.2-\text{Rs.}5) = \text{Rs.}4.16$$
$$M_{BA} = (0.625)(\$4.15+\text{Rs.}5) = \$2.59375+\text{Rs.}3.125$$

The balance of payments equation of any one country can solve for the post-transfer exchange rate. Using the equation of country A we write,

$$M_{AB}+T_{AB} = X_A$$

i.e.,
$$\text{Rs.}4.16+\text{Rs.}5 = (\$2.59375)E_{AB}+\text{Rs.}3.125$$

so that $E_{AB} = 2.3267$. The currency of country A appreciates after the transfer, because the sum of the two countries' propensities to import exceeds unity, Besides this, and the associated redistribution of the level of consumption in the two countries, there are no immediate important effects.

Consider next the other extreme. Suppose the transfer amount is collected entirely from profits and disbursed entirely to profits. The price equations in the two countries will be as follows,

	A	B
1	$5p_{1A}(1+r_A)+9w_A+(5/6)T_{AB} = 11p_{1A}$	$1p_{1B}(1+r_B)+1.2w_B-(1/7)T_{AB}E_{BA} = 11p_{1B}$
2	....	$6p_{1B}(1+r_B)+2.95w_B-(6/7)T_{AB}E_{BA} = 60p_{2B}$
3	$1p_{1A}(1+r_A)+1.2w_A+(1/6)T_{AB} = 5p_{3A}$	....

In this scenario, the intercountry import bills remain as they were in the trade equilibrium but the exchange rate moves to.

$$E_{AB} = \frac{M_{AB} + T_{AB}}{M_{BA}}$$

$$= \frac{(0.8)(10.2) + 5}{(0.625)(4.15)}$$

$$= 5.0737$$

Thus, irrespective of the "sum-of-the-marginal-propensities" condition the currency of the transferer county depreciates. The usual macroeconomic condition is seen to weaken perceptibly outside the static pure-labour context of Part II above.

To find the general effects of the transfer we begin by noting that, in our simple example, the growth rates in the two countries remain at their post-trade values,

$$g_A = 0.8333$$
$$g_B = 0.5714$$

But the post-transfer profit rates will diverge from the growth rates. To find them, however, it will be first necessary to find the capital stocks and the post-transfer prices of the capital good in the two countries. But these prices cannot be ascertained until the post-transfer rates of profit are known.

Let us cut through the maze by using the international demand-supply equations to determine the prices of the consumption goods. The equation for commodity 2 is

$$60 = \frac{(0.8)(10.2)E_{BA} + (0.375)(4.15)}{p_{2B}}$$

which on substituting for E_{BA} gives $p_{2B} = 0.0527423$. Similarly the equation for commodity 3,

$$5 = \frac{(0.2)(10.2) + (0.625)(4.15)E_{AB}}{p_{3A}}$$

gives $p_{3A} = 3.04$.

In the post-transfer forms (equations (1)′, (2)′) the price equations in A and B can be written as follows,

	A	B
1	$5p_{1A}(1 + r_{At}) + 9w_A = 11p_{1A}$	$1p_{1B}(1 + r_{Bt}) + 1.2w_B = 11p_{1B}$
2	....	$6p_{1B}(1 + r_{Bt}) + 2.95w_B = 60p_{2B}$
3	$1p_{1A}(1 + r_{At}) + 1.2w_A = 5p_{2A}$	....

Substituting the values of p_{2A} and p_{2B} in the price equations above gives the value $p_{1A}(1 + r_{At}) = 14$ and $p_{1B}(1 + r_{Bt}) = 0.0357563$. Substituting these in the price equations for commodity 1 in the two countries gives $p_{1A} = 7.181818$ and

$p_{1B} = 0.11234$. Finally, we find from the above the post-transfer profit factors $1 + r_{At} = 1.949367$, $1 + r_{Bt} = 0.318219$. The post-transfer rate of profit in country A rises relative to the growth rate and that in country B falls. Of course the rates of disposable profit remain at the values set by the growth rates. Which incidentally means that a negative r_B obtained in our results need not surprise.

The post-transfer prices of the commodities in country A rise while those in country B fall. However, it may be verified that they are such as to justify the trade pattern *A-3 B-2*.

A variety of intermediate situations can also be considered. Consider for example a situation in which workers and capitalists finance a share of the transfer proportionate to their shares in the transferer country's national income and workers and capitalists in the transferee country receive shares of the transfer in proportion to their shares in their national income. The shares of wages m countries A and B in the post-trade equilibrium are 0.2935 and 0.8907 respectively and the shares of profits are 0.7965 and 0.1093 respectively. Since these are to be equal to the shares in the transfer, the withdrawals from wages and profits in country A are,

$$0.2935 \times 5 = \text{Rs.}1.4675$$
$$0.7065 \times 5 = \text{Rs.}3.5325$$

And the disbursements to wages and profits in country B in rupees are,

$$0.8097 \times 5 = \text{Rs.}4.4535$$
$$0.1093 \times 5 = \text{Rs.}0.5465$$

On the basis of the wage distributions we can work out the import bills and using the balance of payments equation of any one country solves the equilibrium exchange rate $E_{AB} = 3.5478$. Using this rate we can translate the transfer amount into country B's currency.

Next apportion the transfer amount withdrawn from, and distributed to profits in proportion to the capital stocks and write out the price equations in both countries in post-transfer profit rate terms.

Following the steps explained earlier we find,

$$\begin{aligned} p_{3A} &= 2.7464 & p_{2B} &= 0.0666 \\ p_{1A} &= 6.5146 & p_{1B} &= 0.12494 \\ 1 + r_{At} &= 1.9237 & 1 + r_{Bt} &= 1.3954 \end{aligned}$$

These results are intermediate to those obtained under the two extreme scenarios of transfer exclusively from wage to wage income and from profit to profit incomes.

To get a flavor of the kind of results that should be expected it will be instructive to find the condition under which the transferor country's currency depreciates (i.e. the terms of trade move against her). Continuing our assumption that wages are consumed and profits are saved, the reduction in the disposable wage income in the transferor country A and the increase in the disposable wage income in the transferee country B are,

$$TW_A = \Omega_A T_{AB}$$
$$TW_B = \Omega_B T_{AB} E_{BA}$$

where Ω_A is the proportion of the transfer drawn from wages in A and Ω_B is the proportion of the transfer disbursed to wages in B. The new import bills, therefore, are,

$$M_{AB} = \mu_{AB}(W_A - TW_A) = \mu_{AB}(W_A - \Omega_A T_{AB})$$
$$M_{BA} = \mu_{BA}(W_B + TW_B) = \mu_{BA}(W_B + \Omega_B T_{AB} E_{BA})$$

where μ_{AB} and μ_{BA} are the propensities to import.

The balance payments equation of any one country solves for the exchange rate,

$$E^*_{AB} = \frac{\mu_{AB}W_A + (1 - \mu_{AB}\Omega_A - \mu_{BA}\Omega_B)T_{AB}}{\mu_{BA}W_B}$$

In the absence of the transfer the equilibrium exchange rate is,

$$E_{AB} = \frac{\mu_{AB}W_A}{\mu_{BA}W_B}$$

Thus for the currency of country A to depreciate,

$$\mu_{AB}W_A + \mu_{BA}W_B < 1$$

If, for instance transfers are not drawn from wages or distributed to wages at all, i.e., $\Omega_A = \Omega_B = 0$ then the currency of country A must depreciate no matter what the propensities to import may be. If the transfer is drawn entirely from the wage incomes, i.e., $\Omega_A = \Omega_B = 1$, then the usual macro condition obtains provided, of course, that no part of profits is spent on consumption goods. Finally, if the initial trade equilibrium is at some natural exchange rate then the effect of the transfer will simply be to change the composition of the outputs in the two countries without any change in the exchange rate. (See Chapter VI sections 10 & 11)

3. Two Countries, Five Commodities

The same principles can be extended to the general case. Consider for instance the transfer problem in the setting of the example solved in section 8 of Chapter VIII. The equilibrium trade pattern in this example is A-*1,2,4* B-*1,3,5* where commodity 1 is a non-tradeable capital good, commodities 2 and 3 are tradeable capital goods and commodities 4 and 5 are tradeable consumption goods.

Suppose country A makes a transfer of T_{AB} of country B. then the following changes will be made in the system of equations. Firstly, as we have seen above, the growth-profit relationship will be modified,

$$r_A = g_A + \frac{TP_A}{K_A} = g_A + \frac{(1-\Omega_A)T_{AB}}{K_A} \qquad (5)$$

$$r_B = g_B - \frac{TP_B}{K_B} = g_B - \frac{(1-\Omega_B)T_{AB}E_{BA}}{K_B} \tag{6}$$

where TP_A is the portion of the transfer amount financed from profits in country A and TP_B is the portion of the transfer received by profit earners in country B. Unlike in the example of section 2 above, TP_B will be ascertained only after the post-transfer exchange rate E_{AB} has been ascertained, that is to say when the post-transfer equilibrium is found.

Secondly, since the profit factors are modified so will the price equations in both the countries and the balance of payments equations be modified. Since capital goods are being traded, the hybrid system, which is a combination of the capital goods' price equations and the balance of payments equation will also be modified. The hybrid system is,

$$\begin{bmatrix} H_{11} & H_{12} & 0 & H'_{14} \\ H_{21} & H_{22} & 0 & H'_{24} \\ 0 & H_{32} & H_{33} & H'_{34} \\ 0 & H_{42} & H_{43} & H'_{44} \end{bmatrix} \begin{bmatrix} p_{1A} \\ p_{2A} \\ p_{1B}E_{AB} \\ E_{AB} \end{bmatrix} = \begin{bmatrix} J'_1 \\ J'_2 \\ J'_3 \\ J'_4 \end{bmatrix} \tag{7}$$

The effect of the transfer is to change the coefficients in the fourth column on the left hand side, H'_{14} and the J_i coefficients on the right hand side. Their post-transfer values are,

$$H'_{14} = H_{14} - A_{31A}\,\alpha_{4B}\,(TW_B)(1+r_A)$$

$$H'_{24} = H_{24} - A_{32A}\,\alpha_{4B}\,(TW_B)(1+r_A)$$

$$H'_{34} = H_{34} - A_{31B}\,\alpha_{4B}\,(TW_B)(1+r_B)$$

$$H'_{44} = H_{44} + (X_3 - A_{33B}(1+r_B))\alpha_{4B}\,(TW_B)$$

$$J'_1 = J_1 + A_{31A}\,(\alpha_{5A}\,(TW_A) - T_{AB})(1+r_A)$$

$$J'_2 = J_2 + A_{32A}\,(\alpha_{5A}\,(TW_A) - T_{AB})(1+r_A)$$

$$J'_3 = J_3 + A_{31B}\,(\alpha_{5A}\,(TW_A) - T_{AB})(1+r_B)$$

$$J'_4 = J_4 - (X_{3B} - A_{33B}\,(1+r_B)(\alpha_{5A}\,(TW_A) - T_{AB})$$

where H_{ij} and J_i are as given in the example of section 8 of chapter VIII above.

Finally, the international equations for the consumption goods will be modified; in country A the disposable wage income will reduce by TW_A while in country B it will rise by TW_B. Thus, depending upon the tastes and preference in the two countries for the two commodities the relative demands for the commodities will change.

In the actual computational procedure one important modification will be required. Note that the rates of profit r_A and r_B in (5) & (6) cannot be determined until the capital stocks K_A and K_B and the exchange rate E_{AB} are known. But the values of these can be found out only after the post-transfer equilibrium has been ascertained. To get the process started any arbitrary (but positive) initial values of K_A and K_B and E_{AB} may be put in. Of course the final values obtained are independent of the initial values because from the second round of computations onwards the system begins to feed back its own values. If, however, the pre-transfer equilibrium values of K_A, K_B and E_{AB} are known, a convenient method is to put them in.

Let us study the effect of a transfer from country A to country B by means of a numerical example. Consider a slightly modified version of example in section 8 of Chapter VIII. We will suppose the coefficients of consumption in country B to be $\alpha_{4B} = 0.4$, $\alpha_{5B} = 0.6$. As a result, the labour employed in industry 5 in country B will be 43.75 instead of 23.75. Then the pre-transfer equilibrium is,

	A		*B*	
	Labour	Output	Labour	Output
1	10.95	20.53	8.33	15.01
2	6.04	27.21	—	—
3	—	—	6.66	23.98
4	36.5625	300	—	—
5	—	—	43.75	800

with national incomes $Y_A = 82.48$, $Y_B = 73.68$, capital stocks $K_A = 44.10$ $K_B = 43.22$, profit and growth rates $r_A = 0.655$ $r_B = 0.344$ and the exchange rate $E_{AB} = 1.026$.

Suppose that a transfer $T_{AB} = 5$ is made from wage earners in country A to wage earners in country B. It will be found that there is no change in the composition of outputs in the two countries and no change in the rates of profit or growth because wages are assumed to be entirely spent on consumption goods. Thus a transfer from wages in A to wages in B does not make any difference to the sphere of capital goods when only consumption goods are produced in each country. The exchange rate, however, moves to $E_{AB} = 1.083$. The national income of country A increases, $Y_A = 83.00$, while that of country B falls to $Y_B = 73.47$. These relative movements are entirely due to the depreciation of the transferor country A's currency and the appreciation of the transferee country B's. In country A the depreciation causes a rise in the domestic currency price of the imported capital good 3 which in turns causes a rise in the prices of all goods produced in A. In country B the appreciation causes a fall in the domestic currency price of the imported capital good 2 which, in turn, causes a fall in the prices of all goods.

Next suppose a transfer $T_{AB} = 10$ is made from country A to country B. Suppose it is withdrawn in the ratio of 60:40 from wage earners and profit farmers. The equilibrium solution is as follows;

	A		B	
	Labour	Output	Labour	Output
1	10.95	20.535	8.33	15.00
2	6.04	27.215	—	—
3	—	—	6.66	23.985
4	36.42	298.85	—	—
5	—	—	44.21	808.59

The rates of profit are $r_A = 0.739$ $r_B = 0.305$, the rates of growth are $g_A = 0.657$ $g_B = 0.341$ and the exchange rate $E_{AB} = 1.18$. The transferor country's rate of growth is seen to increase while that of the transferee country is seen to fall. These growth rate effects are obviously due to the fact that there is a redistribution of income in both countries; in favour of profits, saving and growth in the transferor country and in favour wages and consumption in the transferee country. The national incomes change to $Y_A = 90.22$ and $Y_B = 71.88$. Even the disposable income of the transferor country $Y_{dA} = 80.22$ is greater than the pre-transfer income, Such are the effects of the adverse movement in the exchange rate and the rise in the rate of profit and the consequent increase in prices.

The prices pre-and post-transfer are tabulated below.

	A		B	
	Pre-transfer	Post-transfer	Pre-transfer	Post-transfer
1	1.527	1.711	1.592	1.512
2	1.431	1.65	—	—
3	—	—	1.542	1.450
4	0.1875	0.2	—	—
5	—	—	0.070	0.069

Finally consider the effects of a transfer $T_{AB} = 5$ drawn exclusively from profit earners in country A and distributed to profit earners in country B. The post-transfer equilibrium is,

	A		B	
	Labour	Output	Labour	Output
1	10.952	20.53	8.34	15.01
2	6.047	27.21	—	—
3	—	—	6.66	23.98
4	36.415	298.8	—	—
5	—	—	43.73	799.64

The exchange rate is $E_{AB} = 1.163$, the rates of profit $r_A = 0.761$ $r_B = 0.231$ and the rates of growth $g_A = 0.657$ $g_B = 0.344$. The outputs of both consumption goods show a decline because the transfer has moved the *international* distribution of income in favour of profit, saving and growth and away from consumption. Thus a slight increase in the growth rates of both countries will be observed.

4. Tariffs

Let us now consider the two-country five-commodity trade situation *A-1,2,4 B-1,3,5* in the presence of a tariff. Suppose country A imposes a tariff at a rate t_{AB}, *ad-valorem*, on the commodities she imports from country B. It is obvious since the tariff is charged on the selling price of the commodity that its rate t_{AB} must be less than 1(100 per cent). Otherwise, the entire selling price or more will be collected by the government, which is ridiculous.

If we suppose that the tariff is imposed on both the commodities imported by country A, viz. commodities 3 and 5, then the total tariff revenue of the government of country A is the sum of the tariff revenues on commodities 3 and 5. We must obtain a formal expression for these. Consider commodity 3. After the imposition of the tariff the selling price of commodity 3 in county A will rise to,

$$\frac{p_{3B}\,E_{AB}}{1-t_{AB}}$$

Each unit of commodity 3 purchased in country A in the post-tariff situation will fetch a revenue of

$$\left[\frac{t_{AB}}{1-t_{AB}}\right] p_{3B}\,E_{AB}$$

And since country A purchases a quantity $A_{3A}(1+g_A)$ of commodity 3, the tariff revenue on commodity 3 will be,

$$T_{3A} = \left[\frac{t_{AB}}{1-t_{AB}}\right](p_{3B}\,E_{AB})(A_{3A}) \tag{8}$$

There is, besides, the revenue earned on commodity 5, the consumption good. To find an expression for it we reason as follows. The expenditure on commodity 5 in country A is $\alpha_{5A}w_A L_A$. After the tariff is charged, the selling price of commodity 5 in country A rises to

$$\frac{p_{5B}\,E_{AB}}{1-t_{AB}}$$

On each unit of commodity 5 purchased the tariff amount is,

$$(t_{AB})\left[\frac{p_{5B}\,E_{AB}}{1-t_{AB}}\right]$$

so that the tariff revenue is the tariff per unit purchased multiplied by the quantity of commodity 5 purchased post-tariff,

$$T_{5A} = (t_{AB})\left[\frac{p_{5B}\,E_{AB}}{1-t_{AB}}\right]\left[\frac{\alpha_{5A}w_A L_A}{p_{5B}\,E_{AB}/1-t_{AB}}\right] = t_{AB}\alpha_{5A}w_A L_A \tag{9}$$

Thus the total tariff revenue of country A is the sum of the expression in (8) and (9),

$$T_A = T_{3A} + T_{5A} \tag{10}$$

Two additional considerations must be entered into the system; (a) the tariff revenue of country A, as given in equation (10), and (b) the manner in which it is disbursed by the government. As a result, three modifications will become necessary. Firstly, in the price equations of all the commodities produced in country A the post-tariff price of the capital good 3 will appear on the cost of production side. For example the price equations of the capital goods 1 and 2 produced in country A will be,

$$\left\{A_{11A}p_{1A} + A_{21A}p_{2A} + A_{31A}\left[\frac{p_{3B}E_{AB}}{1-t_{AB}}\right]\right\}(1+r_A) + w_A L_{1A} = p_{1A}X_{1A}$$

$$\left\{A_{12A}p_{1A} + A_{22A}p_{2A} + A_{32A}\left[\frac{p_{3B}E_{AB}}{1-t_{AB}}\right]\right\}(1+r_A) + w_A L_{2A} = p_{2A}X_{2A} \tag{11}$$

And likewise for commodity 4. Secondly, if we assume that the tariff revenue is not spent on any importable of country A, the balance of payments equations will also change. Specifically, the import bill of country A changes to,

$$M_{AB} = (p_{3B}E_{AB})(A_{3A}) + (1 + t_{AB})(\alpha_{5A} w_A L_A) \tag{12}$$

The effects of changes in (11) and (12) are automatically carried to the hybrid system of equations. The coefficients post-tariff will be as follows,

$$H'_{11} = H_{11}(1-t_{AB}) \qquad H'_{21} = H_{21}(1-t_{AB})$$

$$H'_{12} = H_{12} + t_{AB}A_{21A}A_{3A}(1+r_A) \qquad H'_{22} = H_{22} + A_{32A}A_{2B}(1+r_A)$$

$$H'_{13} = H_{13} = 0 \qquad H'_{23} = H_{23} = 0$$

$$H'_{14} = H_{14} \qquad H'_{24} = H_{24}$$

$$J'_1 = J_1(1-t_{AB}) \qquad J'_3 = J_3(1-t_{AB})$$

$$J'_2 = J_2(1-t_{AB}) \qquad J'_4 = J_4(1-t_{AB})$$

Thirdly, depending upon the manner of the government's spending of the tariff revenue there will be corresponding changes in the demand equations for the consumption goods. If we assume, for example, that the tariff revenue is spent entirely on commodity 4, country A's exportable, the demand equations will be,

$$X_{4d} = \frac{\alpha_{4A}w_A L_A + (\alpha_{4B}w_B L_B)E_{AB} + T_A}{p_{4A}} \tag{13}$$

$$X_{5d} = \frac{(1-t_{AB})(\alpha_{5A}w_A L_A)E_{BA} + \alpha_{5B}w_B L_B)}{p_{5B}}$$

Consider some numerical results in the two-country five-commodity trade situation if the government imposes t_{AB} at the rate of 10% on commodities 3 and 5. The post-tariff situation are shown below.

	A		*B*	
	Labour	Output	Labour	Output
1	10.95	20.53	8.33	15.01
2	6.04	27.21	—	—
3	—	—	6.66	23.98
4	36.5625	300.00	—	—
5	—	—	43.75	800.00

Obviously the levels of private consumption will differ; the consumption of commodity 5 falls in country *A* and increases in country *B*. The exchange rate, however,moves to $E_{AB} = 0.8906$, i.e., the currency of country *A* appreciates after the tariff is imposed and the terms of trade move in favour of country *A*. There would be no cause for wonder if country *B* retaliated by a tariff.

If the tariff rate is high enough it can result in a change in the pattern of the gains from trade. For example if a 70% tariff rate were imposed on commodity 5, consumers in country *A* would prefer to purchase commodity 5 at home. Thus the world production pattern would change to *A-1,2,4,5 B-1,3,5* with trade confided only to commodities *A-2,4 B-3*. Of course the post-trade employment in country *A* will be greater after the prohibitive tariff on commodity 5 is imposed.

5. Political Economy of Tariffs

The points just made serve to illuminate why tariffs are deemed desirable and can be easily justified by democratically elected governments. The imposition of tariffs discourages imports and result in an appreciation of the domestic currency and consequently an improvement in the terms of trade. Underdeveloped countries in particular are apt to charge high tariff rates on consumption goods' imports so as to obtain their capital goods' imports cheaper. Besides the favourable terms-of-trade effect, tariffs also serve as a source of government revenue provided of course that their rates are not so high as to prohibit imports.

In cases where the government is interested in protecting domestic income, it charges prohibitive tariffs. The revenue aspect is secondary. Prohibitive tariffs protect the domestic industrialists and workers employed in the industries which otherwise would be required to close down. The protection thus given to profits and employment is one of the major objectives of a democratic government. And appeals from other sections (those drawing their incomes from activities other than those that are protected) to reduce tariff rates can be *shown* to be based on narrow vested interests and being anti-welfare. Tariffs seem always to give a fairer option to potential importers then outright bans on imports—those who must have the imported goods can have them but must pay the tariffs; which provides the government with revenue.

6. Forward Exchanges

We have so far referred only to the spot markets for currencies and therefore, only to spot exchange rates. A brief look at the determination of forward exchange rates is now in order. In this respect we are under a somewhat serious handicap. For the theory has not said anything about the determination of interest rates in various countries. The theory has dealt exclusively with rates of profit. If, however, it is accepted that the rates of interest are closely linked to the rates of profit in all economic systems, then the determination of forward exchange rates becomes possible. For this purpose we need only to combine the spot exchange rates which are determined by multilateral clearing with Keynes' interest-parity condition which obtains due to covered interest arbitrage operations(2). According to this condition, the forward exchange rate F_{ij} corresponding to the spot exchange rate E_{ij} and the rates of interest r_i and r_j are obtained as follows,

$$F_{ij} = E_{ij} \frac{(1+r_i)}{(1+r_j)} \qquad i,j = A,\dots,Z \tag{14}$$

or in its more familiar approximate form,

$$F_{ij} - E_{ij} \cong r_i - r_j \qquad i,j = A,\dots,Z$$

The forward exchange rates for different maturities can be worked out by compounding the interest factors over the relevant maturities.

We know that in the case of regular commodities the spot and forward prices must coincide under equilibrium conditions. But this is not generally true of the spot and forward exchange rates of two currencies, even under conditions of international equilibrium the two rates will not be equal. Because as (14) shows, for the two rates to be equal for all currencies, the rates of interest/profit must be equal in all countries. And that requires complete international mobility of capital as well as labour. Thus the spread of the forward exchange rate from the spot exchange rate may serve as a measure of the degree of international capital market imperfection, not as a measure of the degree of market disequilibrium or market "inefficiency." The spread between forward and spot rates *minus* the spread between interest differentials (if such can be observed) does, however, give an idea of the degree of market disequilibrium.

Notes

1. The methods appropriate to accommodating transfers under fixed exchange rates belong to a distinct class and have been explained at length in Chapter VI.
2. There is a voluminous literature on forward exchanges and generalisation of the interest-parity condition. See Keynes (1923), Einzig (1961), Grubel (1966), Sohmen (1969).

10

THEORY AND REFORM

Alice laughed, "There's no use trying, she said: one can't believe impossible things." "I daresay you haven't had much practice," said the Queen." When I was your age, I always did it for half-an-hour a day. Why, sometimes I've believed as many as six impossible things before breakfast."

1. Aims

The leading theme of this essay has been to show that international economic relations are only an aspect, a surface manifestation, of the international allocation of economic activities; the two are related to one another as the superstructure is to the basis [Chapter III, V, VIII]. It follows at once that policy measures that seek to reform international economic relations alone, without addressing themselves to reforming the intercountry pattern of productive activities, are bound to prove futile in the long-run. Such policies will treat the symptom instead of the disease. To achieve durable international stability we must address ourselves directly to the question of a desirable pattern of the international division of labour and capital.

What are the principal aims of international economic policy? Unquestionably, they are (a) to maintain full employment conditions, (b) to ensure rising standards of living and (c) to utilise fully the resources and potentials of all the constituent nations of the world. The fulfillment of these aims is a task of global concern even though in some minds, conditioned by old classical thinking, it continues to be regarded as an issue of primarily national concern. It is only in the last twenty years that these quaint ideas have begun to be discarded. This is evidenced by three parallel developments in the field of international economic cooperation. By far the most important is the momentum that has been gained by the proposal for a New International Economic Order (NIEO). The NIEO has expressly recognised the need for international economic integration and the concomitant need to make, not only trade and commodity agreements, but also development financing and co-operation, poverty and ecology, issues of common global concern. [See U.N General Assembly (1974), Tinbergen(1977), Brandt Commission (1983), Singer, Hatti and Tandon (1988)]. Reforms traditionally considered to lie exclusively within the purview of national economic policy have already begun to be regarded as being solvable only through international effort. The second development is the radical change in the focus of the otherwise purely monetary institutions, the IMF and the World Bank. Both have now begun calling for far-reaching internal reforms on the production and investment fronts and efficient utilisation of aid in developing countries. The third development is the attempt being made to co-ordinate the functions between the trade and monetary regulations by GATT and the IMF. [See Viravan *et.al.* (1987) and GATT Agreements. Final text of the Uruguay Round, GATT (1994)]. The formation of the World Trade Organization in 1995 is a development of far-reaching significance.

In each of these developments there is a common thread viz. that the institutions involved have moved, and plan to go, considerably beyond their traditional charters. They are classic instances of the evolution of institutions towards ideas. The purpose of this chapter is to outline a framework for international reforms that are in the spirit of these developments which have begun to take shape at the initiation of the international institutions. The more one wishes to look into the future the more it is necessary to look into the past. So, before setting out the plan of reforms, it may be useful to quickly review classical thinking which has largely shaped international reforms in the past. Classical theory is still the dominant mode of thought on international matters. Until its strengths and limitations are not exposed the new ideas will not receive a critical appraisal.

2. Classical Theory and International Reform

The strategy of international reform in the twentieth century has been dominated by classical thought. Classical proposals for international reform are based on two aspects of their system of ideas. Firstly, the classical theorists believe that national welfare is a national responsibility, that national governments have all the necessary equipment to promote welfare and ensure domestic stability and that multicountry co-operation is necessary only in matters of genuine multicountry concern, which means ensuring unfettered trade and currency convertibility. Such a division of tasks between matters domestic and matters international, which seems so natural to the classical precepts, is in fact artificial. At first glance it does seem rather neat when we view the 'facts'. It is observed that most nations' international relations, especially trade relations, form a small portion of the gamut of their activities; in fact they seem to be of the nature of a small 'residual'. The substantial portion of their economic activities consists of their own-products for own-use. And in the domestic activities only the national government has exclusive control. Hence the classical inference that national governments by themselves can, and should, promote national welfare.

There are three shortcomings in this line of reasoning. (A) While trade may quantitatively appear to be a small part of nations' economic activities, it is a *vital* part. Imagine Japan without oil imports, classical England without cotton imports, India without capital goods' imports. Their miserable autarky equilibria would not even remotely resemble their post-trade equilibria[Chapter VIII]. Classical economists assumed that all countries were capable of producing all commodities [Chapter V, section 13]. They assumed away trade in capital goods. But once these assumptions are relaxed it becomes apparent that literally hundreds and thousands of activities come into being in a country due to the fact of international trade: activities which, without trade, would simply not have existed. (B) Once these 'vital' imports that change the economic structures of countries are recognised, it becomes necessary to recognise that the price and production systems of all the nations of the world are linked more or less rigidly so that a national government cannot tamper around with one part without creating an imbalance in some other nation [Chapters V, VI, section 12 and Chapters VIII & IX]. (C) Classical theory assumed full employment, or in more realistic moods, admitted unemployment, but assumed that the national governments were capable of ensuring full employment. By implication, the effects of the trade policies of nations on domestic employment were automatically ignored. Classical theorists remained content to analyse only the effects of various policy instruments on the terms of trade. Thus a tariff, for example, was not supposed to promote employment. And since a tariff, in the real world, begot a tariff leaving all worse off, classical theory failed to explain the inbuilt protectionist tendencies of national governments that wished to ensure full employment. It was content with denouncing protectionism. Due to these shortcomings classical theory advocated reforms that separated domestic and international economic policy; it never came to terms with the fact that the world is one economic system in which every nations' domestic and international policies affected the fortunes of all other nations [Chapter VI, section 12 and Chapter IX].

The second feature of the classical system is that it is made up of two mutually inconsistent divisions, the 'pure' and the 'monetary' divisions. The pure theory deals with the trade pattern, terms of trade, protection, etc. and assumes a fixed exchange rate, determined usually by some purchasing power parity condition. The monetary theory deals with exchange rates and the balance of payments and assumes trade patterns to be fixed. This division of the theory has had the most deleterious consequence for understanding actual world events and for framing the right policies. A few examples should suffice. A transfer from one country to another which results in a depreciation of the currency of the transferer country and promotes exports of hitherto non-traded or imported commodities is in reality an instrument of *protection* of the export industries [Chapter VI, sections 9 and 10]. Classical theory has not the means to perceive how this can happen. In fact, however, if the instruments of transfers is used along with the usual import-reducing measures (eg. tariffs), it is a potent combination for ensuring balance of trade surpluses. The effect of the import-reducing measures is to promote domestic employment but it also results in an appreciation of the domestic currency that tends to discourage exports. The effect of the transfer at the same time is to nullify the effect of currency appreciation and maintain the level of exports at a high level.

Or take the case of the breakdown of the Bretton Woods system in 1969-71. The American dollar was the *numeraire* currency, which in reality meant that the values of *all other* currencies were supposed to be adjusted, if any imbalance developed on the foreign exchanges. Instead, countries having surpluses with the U.S.A. argued that there was a dollar 'glut', i.e. the prices of American goods had risen with the result that Americans were buying too much abroad against dollars. Any commodity which is in excess supply must fall in price, so America should devalue. These countries insisted on purchasing American gold because, as they saw it, the relative price of American gold had risen. America, anxious to preserve its dwindling gold stocks, decided to remove convertibility and the system collapsed. That is history. But ask how could America possibly devalue the dollar? Only by importing more or lending more abroad, that is to say, by adding further to the alleged dollar glut! In fact if the dollar glut (whatever that may mean) was to be avoided the thing to do was to import goods (not gold) from America, or lend more in America [Chapter I]. But such is the nature of classical thinking that it could not protect a truly grand institution of its own creation.

Finally, consider the evolution of Bretton Woods institutions themselves. The separation of the 'pure' and the 'monetary' aspects in the classical system were given a natural institutional expression in the separation of the premier institutions of that system, the GATT on the one hand to deal with 'pure' matters like trade and tariff negotiations, and the IMF and the World Bank on the other hand to deal with exchange rate matters and monetary and financial co-operation. The division of functions along these lines was meant not merely to be administrative, it was substantive. Both institutions functioned in an exemplary manner. Individually they were successful. Together they were not, as history has shown. For instance in the sixties when countries of Western Europe began to convert their surplus dollar balances into gold, GATT should have stepped in to negotiate greater tariffs by U.S. on their imports and/or reduction in all protective action against U.S. exports. GATT should have realised the monetary danger. And the IMF and other financial institutions should have stepped in to lend to the U.S. and bring pressure against converting the dollars into gold. That did not happen.

It is since the eighties that we are witnessing the phenomenon of GATT pressing for financial support to countries that have deregulated imports. And that of the IMF asking countries to deregulate imports to be eligible for financial assistance. This is as it should be. But the confusion has by no means disappeared. Negotiations on the NIEO make appeals for greater real resource transfers to develcping countries. Yet the IMF and the World Bank insist on debt recovery at the same time as they ask deficit countries to devalue their currencies. Even now there is a lack of common focus, a lack of consensus on the objectives of international policy and the means for achieving the objectives.

The outstanding fault of the classical reformers was that they wanted their theory to become the reality. They assumed the international immobility of capital and labour, so they made no attempt to discourage it. They assumed that free trade was 'better' than protection. They, therefore, proposed arrangements to ensure free trade. They assumed that the real and monetary worlds were separate, so they advocated separate institutions to look after the real and monetary affairs. They assumed that national governments could by themselves promote national welfares. Therefore, they promoted arrangements whereby the governments would not be disturbed in the performance of their duties by alien governments.

The temptation to make their theories come true was not without good reason. The classicals, it must be remembered, were fighting the parochial protectionists, who, especially after the event of the Great Depression, had become stronger. In a world torn by war, tariffs, competitive devaluations, currency inconvertibility and sheltered bilateral negotiations, the classical appeal for institutional arrangements to ensure unfettered multilateral trade rang loud and clear. In this world the classical reformers had a *real* message. Parochial protectionists were squarely beaten by the impassioned yet reasonable arguments of the classicals.

The classicals, however, were content to beat back their opponents. They were content to show that free trade is better than no trade. But their cosmopolitan zeal invariably faltered when it came to the question of promoting the international mobility of capital, technology and labour. Finance, investment and technology, they insisted, were meant to be domestic. And labour would, in any case, be fully employed at home. They could not *imagine* a world in which capital, technology and labour could be mobile and what it could mean to world welfare. They could not relax their own assumptions to see how the world might look. And that too in the modern reality in which capital, technology and labour are, and wish to be increasingly mobile internationally than they were ever before. Policies for international reform that will be relevant in future will have to take cognizance of these new facts.

3. Beyond Classical Avenues

For all its faults the classical theory had the merit that it won. For that we are eternally grateful. Classical reformers believed that a trade equilibrium was better than a no-trade equilibrium. And that it was possible to create great international institutions that could ensure trade equilibrium, however strong might be the reasons to depart from it. The achievement of the classicals was to bring into practice reforms which practical men of affairs thought were unrealistic, fanciful, ambitious and utopian. Even greater was their achievement of sustaining the system for about three decades.

All creations have a method of embarrassing their creators. A history of the world can be written that has as its leading theme a description of a series of such tragic ironies. Thus, the orderly and neat fixed exchange rate system forged out on classical principles has, to a great extent, been the cause of a serious debt crisis. The objective of the fixed exchange rate system was to discourage frequent exchange rate changes because these disrupted normal (pure!) free trade relations. There was besides a fear of competitive devaluations. For adjustments, the system relied upon transfers from surplus to deficit countries [Chapter VI, section 11]. Naturally the system was based on unbounded charity, which is good as far as it can go, but hardly sufficient to base a more or less permanent system on.

The result of this history is that the initial conditions have been changed. The world is already faced with a serious debt crisis. The NIEO demands greater altruism among nations. Its success will surely depend upon how far the debt crisis is resolved. To meet the objectives of international economic policy a new set of strategies will have to be devised which can convert weaknesses into strengths and yet prove acceptable to all. Some broad and necessarily tentative suggestions are proposed below. One does not feel ashamed or guilty if the philosophy underlying them seems idealistic or utopian. That is as it must be. What needs to be ensured is that the suggestions are within our implementative imagination. As Myrdal (1956) has put it in a similar context, "The ideal is also a living force in our society and it is, therefore, part of the social reality we are studying.... The ideal works through people's valuations, their political attitudes. Whether we can hope for a gradual attainment of the ideal, and with what speed and to what degree, depends in great measure upon how strongly entrenched the ideal becomes in this sphere of human valuations."

4. Surpluses, Deficits and Suspicions

We begin with a home-truth, a bald and blunt accounting identity; the sums of the surpluses and deficits of all countries must be equal to one another [Chapter I, section 11]. This is a truth that is as frequently forgotten as it is well known. Forgetting it when conducting international negotiations is often the cause of considerable confusion. Many of the fears of deficit countries, their attitude to the surplus countries ('foreign hand', 'arm-twisters', 'big-bully', etc.) and the swagger of the surplus countries is largely because somehow the word surplus denotes 'good' and 'efficient' and the word deficit, 'bad' and 'inefficient'. In fact, however, surplus and deficit are two names for the same thing. As soon as this is realised, it follows that the balance of payments of one and all countries is a matter of collective responsibility of all countries. Thus the problem of a deficit in some countries is as much a problem of the surplus countries as it is of the deficit countries themselves.

It may be recalled that Keynes's (1933) Clearing Union proposal discussed in the Bretton Woods negotiations contained an important provision, viz. that *both* the creditor and debtor countries must be required to pay interest to the Clearing Union so that the burden of adjustment could be placed on all countries collectively. Unfortunately, the actual provisions worked out did not contain this proposal, largely because it violated established financial wisdom—creditors are supposed to receive interest, debtors are supposed to pay it. But as soon as the balance of payments identity is understood it becomes apparent how myopic and misplaced the established financial wisdom is.

Why then do countries strive for surpluses? In the olden days when men suffered from a Midas-mania, they found that by being surplus they could quench their thirst for gold. Hume and Smith showed how foolish it was. But what now? The only reason can be that the countries can lend their surpluses to deficit countries and in the process pile up a credit which gives them a claim to the future incomes of the deficit countries. Conversely, the disadvantage of being a deficit country is that she is required to mortgage a part of her future income to the creditor country. But note an important caveat. The future net incomes that the creditor countries expect to earn can be realised by them only if they have *deficits* in *their* balance of trade in the future periods. Not otherwise.

This has two implications for international negotiations. Firstly, there is no sense in being fetishistic about a surplus. A surplus today means a deficit tomorrow. Secondly, there is no sense in blaming a deficit country for being inefficient. For a deficit today means a surplus tomorrow. The first step towards international economic understanding follows from here. Surplus countries that have accumulated claims against deficit countries over the past several years have a claim on their real income which can be settled only by making the indebted countries bear trade surpluses [Chapter VI, sections 9 and 10].

5. Short-term Strategies

The question therefore is how this turnaround of surplus and deficit countries to deficit and surplus countries respectively is to be brought about. There are several ways. One way that has been suggested in several quarters is the method of mass debt default by the underdeveloped countries. Such a course of action is surely disastrous to the future of international economic relations. In contrast, its converse, mass debt relief by creditor countries has considerable merit. It links up rather well with the negotiations for a New International Economic Order and other proposals for internationalising aid. So the motto of the future must be, Debt relief, Yes! Debt default, No! Even then there are human limits to the zeal for this kind of action. Altruism is one of the fainter passions of humans. Its strength usually depends upon general well-being and stability. For the existence of altruism we are grateful. But it would be foolhardy to place all reliance on it.

Another way, which has caught the fancy of financial experts, is debt-equity conversion. It is surrounded by a host of legal and political difficulties; equity in what? how much? at what ratio? with what future prospects and liabilities? etc. And evidence, such as there is from the Latin American countries, in particular, indicates, at best, desperation on the part of the creditors.

A third way which seems natural to several lending agencies is for the indebted countries to simply make their currencies convertible and to begin repaying their loans. But the consequences, if the deficit countries are made to follow this course of action, are too terrifying to even imagine. Because this policy involves a steep depreciation of the debtor countries' currencies. With every depreciation the volume of debt to be repaid itself increases. With every depreciation the rate of inflation in these countries is stepped up, the consequence of which is to further weaken the currency. And sometime during the course of this immiserising process, the hitherto deficit countries are expected to have trade surpluses to repay their past debts. This policy seems natural to lending agencies because it is based on the present market rules. What is forgotten is that the debt itself was created in the course of a non-market history.

There is nothing natural about this policy. It is as natural as advising a man lying on his bed with a heart-attack to recuperate by starting vigorous exercises right away. The policy of rapid depreciation will prove to be equally disastrous to the creditor countries. If deficit countries are compelled to devalue rapidly it is clear that there will be severe import compression. This will immediately result in a severe recession in the export industries in the creditor countries leaving in its wake wastage of capital resources and unemployment.

Exchange rate adjustments must always be slow and gradual. Exchange rates are not like prices of onions. Exchange rate changes cause changes in the economic structures of nations.

6. New Vistas

It has been mentioned at the outset that any long-term and durable strategy of international reform must address itself directly to changing the international allocation of economic activities in ways appropriate to fulfilling the objective of world welfare. No amount of tinkering around with trade, tariffs, subsidies, exchange rates, bilateral agreements or shortsighted conditionalities can help except as measures of temporary relief. On the contrary such tinkering may well distort the international allocation of productive resources by creating market imperfections of all kinds. When an iceberg is to be moved it is the base, not the tip, that must be harnessed. The sooner it is understood that the world economic system is an *indecomposably* interconnected system, the faster will the attitude towards international reform to change. [Chapters V, VI, VIII, IX].

Following up the premises laid down in this thesis let us address ourselves to the following question of immense gravity to the future of international peace: What pattern of the allocation of economic activities between different countries can resolve the present world debt crisis, while at the same time giving a strong impetus to the growth of world welfare? The question is ambitious. And to find an adequate answer it is best to work our way from first principles.

Consider briefly the causes of the world debt crisis. Countries that are presently indebted are so because they have had trade deficits in the past. Countries that are creditors have had trade/current account surpluses which they have used directly, or through international institutions, to lend to the deficit countries. Why have the deficits and surpluses emerged and allowed to continue over such long periods of time? The answer, as we know, is that the deficit countries, by consent of the surplus countries, pegged their currencies at appreciated levels and supported them by means of borrowing. And the surplus countries, by lending to the deficit countries, pegged their currencies at depreciated levels and preferred an export surplus. The particular exchange rate policies followed by both the deficit and surplus countries had good intentions. For the deficit countries appreciated currencies meant low costs of vital imported inputs and therefore a higher rate of capital formation and higher standards of living. [Chapters VIII, IX] For the surplus countries depreciated currencies meant greater exports and therefore greater levels of employment and capacity utilisation. It is important to appreciate that the process of raising the world debt levels was *directly* beneficial for *both* the deficit and surplus countries and was accordingly adopted as a continuing policy for over four decades.

The conclusion that the extension of international debt actually raises the world product is not merely a fanciful inference of theory although it must necessarily seem so to the myopic "practical" man who reasons exclusively in terms of the two sides of the cash register. It is a matter of fact that has been proven by actual economic experiment. First, the fifties and early sixties was a period of sustained economic growth during which recessions were almost forgotten. This was the time of the Bretton Woods System in which international accommodation helped to revive war-torn economies and give a growth thrust to the developing countries. In contrast the seventies, having seen the breakdown of the Bretton Woods System, brought about world wide recession. This recession would have been all the more severe on account of the 1973 and 1979 oil shocks if the international institutions had not intervened by lending on liberal terms to the developing countries and sustaining the import demands. The decade of the eighties presents yet another contrast, it was the longest period of continuous and rapid economic growth during this century. It was brought about by great initiatives in debt extension: debt relief by IMF and the World Bank and even private banks during the early eighties and the Baker initiative in 1985, the 5-year debt offer program at the Paris G-7 meeting in 1989 and the Brady Plan of 1990's. The entire crisis of the seventies and eighties was averted by these progressive and determined initiatives.

But now that the time has come to resolve the debt crisis there is to be found an incoherent din and clamour of opinions, with suggestions ranging from wholesale debt default to wholesale currency depreciation by the underdeveloped countries. There is no need to go so far, if only it is understood that the world debt crisis can be resolved by mutual multilateral consent, in the same manner as it was created.

The obvious clue towards resolving the debt is to make arrangements, by mutual consent, to ensure that the debtor countries are brought into a trade surplus position and the creditor countries have trade deficits in the years to come. The matter of course is not as mechanical as it sounds. For, mutual multilateral consent on such a policy can be obtained only if the policy is beneficial all-round, i.e. it improves the welfare in all the countries. The process of raising the volume of world debt has spread prosperity. On no account should the opposite process of reducing the debt be allowed to reduce prosperity. Even debt default would be preferable to that.

It is evident that the policy of converting debtor countries into trade-surplus conditions and creditor countries into trade-deficit conditions calls for an international reallocation of economic activities. Specifically, the production of some commodities which are presently exportables of the creditors countries (i.e. importable of the debtor countries) will need to be relocated in the debtor countries and exported from there. Naturally, not all economic activities are eligible for such relocation. Non-tradeable commodities are excluded by definition. So are 'special' commodities that require specific natural/climatic/ cultural endowments. Thus only industrial activities like engineering, chemicals, pharmaceuticals, electronics, textiles, printing, etc. will qualify for reallocation between countries. If the arrangements can be made, the debtor countries will have a trade surplus which they will use to repay their accumulated debts and the creditor countries will have excess of imports which however, will be financed by debt retrievals on the invisible current and capital accounts. In effect, the creditor countries will not only obtain the goods but also the money to buy them! Such is the benefit of international as opposed to domestic debts.

All this is fine. The real question is: What set of incentives will permit the international reallocation of economic activities envisaged? The incentives must clearly work through the domestic and international market mechanisms. They must be such as to increase the real economic welfare of all the nations involved. And finally, they must be appropriate to both the creditor countries from which industrial activities will be released as well as the debtor countries which will receive them.

Let us first consider the incentives that can be devised for the release of industrial activities from the creditor countries. There are two sets of incentives that directly increase real welfare and yet reduce *industrial* activity. First, a reduction in the working hours in the industrial sphere. Second, an increase in the government (and private) expenditures on welfare-improving non-tradeable goods like education, housing, civic and urban amenities, community works, health, medicine and social security, research and development, environmental improvement and cultural activities like sports, the fine arts and creative entertainment. The mechanism that makes the scheme work is as follows. There is firstly a direct and immediate increase in welfare due to an increase in leisure and the quality of life provided there is no decrease in the quantum of material goods consumed. There need be no decrease because material goods consumption can simply be financed by increasing imports against settlement of past debts.

Secondly, the reduced capacity in industrial mass production gives an incentive for relocating these activities to countries will relatively abundant and cheap labour [Chapter VI, section 1]. The resulting increase in (a) the level of foreign investment and (b) the surplus for export in the debtor countries can be used to finance the material goods requirements of the creditor countries.

The scheme operates in the direction of satisfying the dominant aspirations of people in both the developed and underdeveloped countries. In the developed world large masses of people are still shackled to the drudgery of industrial work when their natural aspirations have moved towards needs like creative leisure, self-fulfillment and community leadership. In the underdeveloped world large masses of people languish in poverty outside the mainstreams of progressive economic life for want of capital and technology.

Some words of explanation may be in order for those who are apt to think of such ideas as being lofty, far-fetched or jejune. They will not seem so if two things are firmly kept in mind. Firstly, at high levels of income the preferences of people automatically shift towards leisure and various kinds of services. These are the natural psychological demands of the people, because these are the values of good living. Leisure does not mean idleness—it means the pursuit of emotionally satisfying work, it means an escape from the mire of economic strife, it means the final step to prosperity after material affluence. Only a political philosophy based on despotism repudiates and discourage the attainment of these cherished goals. Secondly, it must be understood that there is no sense in keeping permanent creditor status against other nations, i.e. lending more than receiving in the form of repayments. For this amounts merely to piling up book-entries, which, when they become unbearably large, can only result in war. The credits piled up by past labour must sometime be encashed in terms of real goods and leisure. Even the village money -lender knows this. But it is apt to be forgotten in the great affairs of international finance.

Needless to say that the policy for debt resolution that has here been outlined will prove to be of immense benefit to the debtor nations as well. They will benefit by a movement of capital and also by that of the technology that is required to service the demands of the high-quality products required by the developed countries whom they must pay in terms of these products, and which they will also begin to consume. The effects of this "development experience" will undoubtedly be of immense importance for the sustenance of future international growth. But more important, in immediate terms, is the benefit of reducing unemployment of the vast numbers of skilled and to-be-skilled people who are today in the agony of "forced leisure" in the developing countries, deprived of basic material amenities. The plan therefore is one that involves a reallocation of economic activities in which these vast masses work to produce material goods for themselves as well as for the people of the developed countries in an upward movement along the hierarchy of needs.

There is, to be sure, considerable restructuring required in the debtor countries who are to receive the new industries. For the greater part the impetus towards restructuring has already been provided by international financial institutions. It involves firstly, the creation of a liberal political, administrative and judicial atmosphere. Secondly, the provision of physical infrastructure, particularly in the area of transport and communications. And thirdly, the extension of education and skill-development among the people. In a great number of developing countries and the erstwhile socialist countries the process of economic restructuring has already begun and should soon be complete.

This book is a treatise on the theory. The myriad details that will have to be hammered out in the course of multilateral negotiations to formulate the concrete plans is not a part of its province. That will require work of different genre and the collaboration of minds of diverse capabilities. Suffice it to say that the reforms here envisaged, being long-term by their very nature, must be put through in an atmosphere of complete information and communication. The success of this, as of any plan, hinges upon whether it turns out to be *actually* profitable to the industries involved.

7. Multilateral Financial Arrangements

There can be no doubt that classical fixed exchange rates (with a band of variability to show the authorities where the markets are headed) is the system that is most ideally suited to an environment in which international transactions are rising faster than the world product; that is to say a world economy that is in the process of economic integration. The reasons are partly classical and partly go beyond. The classical reasons are two; (a) the fixed rate system reduces exchange risks and smoothens out and augments trade and investment flows and (b) it eliminates wasteful speculation most of the time, i.e. excepting situations when a change of parity becomes imminent. In addition there is a further advantage. The fixed exchange rate system eliminates the huge unintended windfall capital gains and losses that are imposed on nations every time the exchange rate moves. It is these "translation risks" that have been increasing in severity in the past few decades that have witnessed a rising proportion of purely financial transactions between nations.

The effects of translation risks are pernicious. Firstly, every change in the exchange rate results in gains and losses for people of all countries. Depreciations favour exporters and those who depend on the exportable industries at the cost of importers and those who depend on the exportables industries at the cost of importers and those who depend on them. Appreciations have the opposite effects. Secondly changes in exchange rates cause huge real income gains and losses to different nations. Depreciations lower the real incomes of the net debtors and increase the real incomes of the net creditors. Neither is healthy. The suffering of the net debtors is obvious. Less obviously, but equally, the creditor countries also suffer in two ways. First, the real demand for their exportables from the net debtors declines as the debts are discharged. Secondly, the creditor countries who make windfall capital gains will invariably spend them in frivolous or speculative avenues which lead in the long run to wastages in the flow of real production.

Translation risks are not completely eliminated under the fixed exchange rate system, they will arise every time the parity exchange rates are altered. An additional device, needs to be added to eliminate them. This device, which can be called *exchange rate indexation* or better, *non-indexation*, requires the use of historical exchange rates to settle past liabilities/credits; i.e. the use of the exchange rates at which the liabilities were incurred. This device, which must be operated through the foreign exchange dealers, will require the setting up of an *exchange neutralisation fund* whose function will be to collect all capital gains arising from exchange rate changes and meet all the capital looses. Since the sum of the capital gains must exactly equal the sum of the capital losses, (except for gaps in the buying and selling rates, and differences in the schedules of repayment on which depends the pattern of realised gains and losses) the device will automatically neutralise the windfalls. To make the device operative it will of course be necessary to report all lend-borrow transactions at historical exchange rates. This proposal which we may call "marking to history" has the same effect in international transactions that "marking to markets" has in financial transactions and "price-level accounting" in mercantile transactions [Chapter VI, 13n].

The fixed exchange rate system is appropriate when international finance and investment dominate world transactions. The flexible exchange rate system is appropriate when trade transactions dominate. It is ironical that the world adopted a fixed exchange rate system (1944 to 1971) when trade was dominant and moved to a flexible system (post 1971) when international finance and investment became dominant. The fixed exchange rate system needs to be re-established now, more than even before.

International economic policy has been increasingly based on the recognition of greater economic integration between nations in the recent past. It will continue to do so. It will retain the classical appeal for free multilateral trade and currency convertibility intact but will inevitably deny the classical principle of preventing the mobility of capital, technology and labour between nations of the world. The classical promise that free trade alone will equalize standards of living across the world even if factors of production stay put in the countries of their fortuitous origin has been falsified. World trade will take place under conditions of ever increasing mobility of capital and technology.

Global economic policies formulated by the international institutions have been directed towards giving producers, investors and consumers equal and equitable access to the markets and commodities in all countries. The day may not be far when they will have to think in terms of the only remaining adjustment mechanism, viz., ensuring equal and equitable access to workers in different countries.

Appendix 1*

NEUTRALITY CONDITIONS : ALTERNATIVE DERIVATION

The exchange neutrality conditions of Chapter I can be derived in several ways. The method followed in text required the condition that every country's international payments and receipts must balance in terms of at least one currency other than her own. The method used by Chacholiades (1971) is based on finding restrictions on the exchange rates which will prevent the presence of international currency arbitrage. In this appendix another method has been given.

Suppose the national incomes of the Z countries in the world economy are Y_A, Y_B. Y_C,....., Y_Z measured in the own currencies of the countries. Then the world income in terms of the various currencies are,

$$\begin{aligned}
W_A &= Y_A + E_{AB} Y_B + E_{AC} Y_C + \dots + E_{AZ} Y_Z \\
W_B &= E_{BA} Y_A + Y_B + E_{BC} Y_C + \dots + E_{BZ} Y_Z \\
W_C &= E_{CA} Y_A + E_{CB} Y_B + Y_C + \dots + E_{CZ} Y_Z \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
W_Z &= E_{ZA} Y_A + E_{AB} Y_B + E_{ZC} Y_C + \dots + Y_Z
\end{aligned} \tag{1}$$

The following equality must be obviously true,

$$W_A = E_{AB} W_B = E_{AC} W_C = \dots = E_{AZ} W_Z \tag{2}$$

Thus,

$$\begin{aligned}
W_A &= (E_{AB} E_{BA}) Y_A + E_{AB} Y_B + (E_{AB} E_{BC}) Y_C + \dots + (E_{AB} E_{BZ}) Y_Z \\
W_A &= (E_{AC} E_{CA}) Y_A + (E_{AC} E_{CB}) Y_B + E_{AC} Y_C + \dots + (E_{AC} E_{CZ}) Y_Z \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
W_A &= (E_{AZ} E_{ZA}) Y_A + (E_{AZ} E_{ZB}) Y_B + (E_{AZ} E_{ZC}) Y_C + \dots + E_{AZ} Y_Z
\end{aligned}$$

Compare each of these $z-1$ equations to the first equation in (1). The following relations must hold,

$$\begin{aligned}
E_{AB} E_{BA} &= 1 \\
E_{AC} E_{CA} &= 1 \\
&\dots\dots\dots \\
E_{AZ} E_{ZA} &= 1
\end{aligned}$$

These are $z-1$ two-point neutrality conditions. There are, besides, $z-1$ sets

---

* Refers to Chapter 1

containing $z - 2$ three-point neutrality conditions,

$$
\begin{array}{cccc}
1 & 2 & \cdots\cdots & (z-1) \\
E_{AC}E_{CB} = E_{AB} & E_{AB}E_{BC} = E_{AC} & \cdots\cdots & E_{AB}E_{BZ} = E_{AZ} \\
E_{AD}E_{DB} = E_{AB} & E_{AD}E_{DC} = E_{AC} & \cdots\cdots & E_{AC}E_{CZ} = E_{AZ} \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\
E_{AZ}E_{ZB} = E_{AB} & E_{AZ}E_{ZC} = E_{AC} & \cdots\cdots & E_{AY}E_{YZ} = E_{AZ}
\end{array}
$$

Thus we get a total of $z - 1$ two point conditions plus $(z - 1)(z - 2)$ three-point conditions giving a total of $(z - 1)^2$ conditions. These have been derived from the simple idea that "world income must be the same howsoever it may be measured."

Appendix 2*

SOLUTION BY MEANS OF A NON-HOMOGENOUS SYSTEM

In Chapter VII we assumed that wages are paid *post-factum*. If instead wages are paid *pre-factum* (i.e., are advanced along with the means of production to workers), the price equations will be as follows,

$$
\begin{aligned}
(A_{11}p_1 + A_{21}p_2 + wL_1)(1+r) &= p_1 X_1 \\
(A_{12}p_1 + A_{22}p_2 + wL_2)(1+r) &= p_2 X_2
\end{aligned}
\qquad (1)
$$

The capital stock will now be equal to,

$$K = (A_{11} + A_{12})p_1 + (A_{21} + A_{22})p_2 + wL$$

If we assume that profits and wages are consumed then the independent demand equation will be,

$$\alpha_1 wL + \beta_1 rK = p_1 F_1$$

i.e. $$(\beta_1 A_1 r)p_1 + (\beta_1 A_2 r)p_2 + \beta_1 wLr + \alpha_1 wL = p_1 F_1 \qquad (2)$$

Equations (1) and (2) can be arranged in vector–matrix notation to solve for the rate of profit and the prices of commodities,

$$
\begin{bmatrix}
X_1 - A_{11}(1+r) & -A_{21}(1+r) & -L_1(1+r) \\
-A_{12}(1+r) & X_2 - A_{22}(1+r) & -L_2(1+r) \\
F_1 - \beta_1 A_1 r & -\beta_1 A_2 r & -(\alpha_1 + \beta_1 r)L
\end{bmatrix}
\begin{bmatrix} p_1 \\ p_2 \\ w \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\qquad (3)
$$

and solved in the same manner as indicated in section 2. The same logic holds in the general case of several commodities.

* Accompanies chapter VII, sections 2 and 3.

Appendix 3*

THE LEONTIEF, VON-NEUMANN AND SRAFFA MODELS

This appendix provides brief, non-rigorous, summaries of the Leontief (1941), Von-Neumann (1937, 1945) and Sraffa (1960) models of general economic equilibrium. These models, as is well-known, are developments of the classical theories, particularly those of Smith (1976), Ricardo (1817) and Marx (1867) as well as the pioneer work of Walras (1874). The reader must be warned that the summaries given here are not exhaustive** ; they are only meant to give some motivation for the model that appears in Chapter VII.

Let us start with the Leontief model. Consider a closed capitalist economy which produces "commodities by means of commodities" and other primary factors of production. Every industry produces only one product by means of a fixed-coefficients constant-returns to-scale technique of production. Let a_{ij} be the quantity of commodity i required to produce unit output of commodity j. Then if X_j is the level of output produced, $A_{ij} = a_{ij}X_j$ is the quantity of commodity i required in the production of commodity j. Let F_j be the quantity of commodity j used for purposes of new investment and final consumption and assume that it is given exogenously i.e. independently of the prices of commodities and the income of the consumers.

The demand-supply equations for the n commodities are as follows,

$$\begin{array}{l} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \dots + a_{1n}X_n + F_1 = X_1 \\ a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \dots + a_{2n}X_n + F_2 = X_2 \\ a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + \dots + a_{3n}X_n + F_3 = X_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}X_1 + a_{n2}X_2 + a_{n3}X_3 + \dots + a_{nn}X_n + F_n = X_n \end{array} \qquad \text{L–1}$$

where the left hand side of each equation shows the total intermediate and new

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* Accompanies Chapter VII

** For exhaustive reading the reader may refer the following literature. For the Leontief model see Koopmans (1957) , Dorfman, Samuelson and Solow (1964), Mathur and Bharadwaj (1965) , Brody and Carter (1972). For the Von-Neumann model, Dorfman, Samuelson and Solow (1964), Los and Los (1974). For the Sraffa model and its bearing on economic theory see Mainwaring (1989), Roncaglia (1978), Abraham-Frois and Berrebi (1979) Steedman (1977), Brahmanand (1974), Garegnani (1970, 1990), Pasinetti (1978), Bharadwaj and Schefold (1991) and Hahn (1975). Finally for classical and early neoclassical models in sympathetic but rigorous terms see Morishima's trilogy on Marx, Ricardo and Walras (Morishima (1973), (1989), (1977))and Arrow (1978).

demands and the right hand side, the levels of gross supplies required to satisfy them. L–1 contains n equations in n unknowns, X_1, ..., X_n. Then if the coefficients a_{ij} conform to the Hawkins-Simon conditions, i.e.

$$a_{ii} < 1 \qquad \forall i$$

$$\sum_i a_{ij} < 1 \qquad \forall j$$

$$|I - A| > 0$$

where A is the matrix of coefficients (a_{ij}) then a unique positive solution for gross outputs corresponding to the given quantities of final demands F_j, is obtained. In matrix notation,

$$X = AX + F$$

$$X = (I - A)^{-1}F \qquad \text{L–2}$$

The Leontief price-system is dual to the output system L–1. Suppose the values-added per unit of output V_1, V_2,....V_n are given exogenously. Then the price of each commodity must be equal to the cost of raw materials plus the value-added per unit of output. Thus the price equations are,

$$\begin{aligned} a_{11}p_1 + a_{21}p_2 + ... + a_{n1}p_n + v_1 &= p_1 \\ a_{12}p_1 + a_{22}p_2 + ... + a_{n2}p_n + v_2 &= p_2 \\ a_{13}p_1 + a_{23}p_2 + ... + a_{n3}p_n + v_3 &= p_3 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{1n}p_1 + a_{2n}p_2 + ... + a_{nn}p_n + v_n &= p_n \end{aligned} \qquad \text{L–3}$$

In vector-matrix notation we write,

$$(A^T)P + V = P$$

where A^T is the transpose of the A matrix in L–2, P is the vector of prices and V is the vector of values-added per unit of output. Then given the v_i's, L–3 solves the absolute prices of the n commodities,

$$P = (I - A^T)^{-1}V \qquad \text{L–4}$$

Since both the F's and v's are given exogenously the consistency of the price and output systems must be ensured. This is ensured by two restrictions: first, the national income identity

$$\sum_i p_i F_i = \sum_i v_i X_i = \sum_k w_k N_k \qquad \text{L–5}$$

where w_k are the exogenously given rates of factor remuneration and N_k are quantities of factors k used to produce the gross outputs X_i. Second, there is the restriction on the feasibility of the production programme in L–1, and L–2 viz, that it must not require quantities of the primary factors N_k, that are greater than their available supplies, say N_k^*, i.e.,

$$N_k \leq N_k^* \qquad \forall k \qquad \text{L–6}$$

Thus only such values of F's, V's and w's are admissible which will satisfy restrictions L–5 and L–6. This is the simplest and the most rudimentary version of the Leontief-model. It is called a static open model. Static, because it does not study what happens to the economic system over a period of time as new investment takes place. Open, because the demands for consumption goods are assumed to be given independently of the factor incomes and the prices of commodities.

The static open model is incapable of determining the growth rate or the income-distribution in an economic system. Everything except the gross output levels and prices are assumed to be given. It is therefore mechanistic and leaves out several unknowns of theoretical interest. These inadequacies are partially remedied in Leontief's closed model (1941), Leontief's dynamic model (1955) and von-Neumann's (1945) model. The discussion of Leontief's more general models will be subsumed in what follows under that of the Von-Neumann model to which they are very similar.

The original von-Neumann model is very complex. It contains m production processes for n commodities and allows every process to produce a number of joint products. Secondly, it uses inequalities in the price and output systems, allowing some processes to remain inoperative and some goods to be free goods. We shall, however, think of an established economic system which produces commodities at positive prices and is arranged in single products industries. This greatly simplifies the mathematics. Thus consider a system of n processes, or industries, producing n commodities. And suppose the system to be 'indecomposable', i.e., all commodities are assumed to be used directly or indirectly, in the production of all commodities. Then it will be possible to write von-Neumann's system in terms of technical coefficients.

The output of each commodity must be such as to satisfy the requirements of all industries which use the commodity as an input and provide a surplus for balanced growth. Thus the output equations are,

$$
\begin{aligned}
(a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n)(1+g) &= X_1 \\
(a_{21}X_1 + a_{22}X_2 + \dots + a_{2n}X_n)(1+g) &= X_2 \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
(a_{n1}X_1 + a_{n2}X_2 + \dots + a_{nn}X_n)(1+g) &= X_n
\end{aligned}
\qquad \text{N–1}
$$

or

$$AX(1+g) = X$$

The system of equations, N–1, contain n equations in $n + 1$ unknowns, i.e. the rate of growth and n outputs $X_1, \dots\dots, X_n$. Clearly all the unknowns cannot be determined. The von-Neumann system determines only the relative outputs and the rate of growth. The rate of growth is the lowest positive root of the polynomial.

$$|I - A(1+g)| = 0 \qquad \text{N–2}$$

and X is the associated eigenvector.

The dual of the output system is the price system. The price of each commodity must be sufficient to cover the cost of production and a uniform rate of profit,

$$\begin{aligned}(a_{11}p_1 + a_{21}p_2 + \dots + a_{n1}p_n)(1+r) &= p_1 \\ (a_{12}p_1 + a_{22}p_2 + \dots + a_{n2}p_n)(1+r) &= p_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \\ (a_{1n}p_1 + a_{2n}p_2 + \dots + a_{nn}p_n)(1+r) &= p_n\end{aligned} \qquad \text{N–3}$$

or

$$A^T P(1+r) = P$$

From the determinantal equation,

$$|I - A^T(1+r)| = 0 \qquad \text{N–4}$$

the lowest positive root is the rate of profit and N–2 can be used to solve the relative prices. It is a well-known theorem in linear algebra that the roots of equations N–2 and N–4 must be identical. Hence in Von-Neumann's system $g = r$. The system of equations $N - 3$ determine the relative prices.

Income distribution and consumption do not appear explicitly in Von-Neumann's model. Even labour, if it is to be introduced into the system, must be regarded as one of the commodities produced by a "household industry" which uses as inputs, fixed quantities of the various goods. The relative price of labour, i.e. the cost of its subsistence in terms of the *numeraire* along with 'profit' to the owners of the household industry will then appear in the price system N–3. We can of course think of several grades of labour, produced by different industries with differing costs of production and prices. But that is as close as possible to reality that we can bring the von-Neumann model, no further.

In Sraffa's system the importance is given to the system of prices and income distribution. Sraffa assumes, along with the neoclassics, that income, especially wage income, is 'variable', i.e., it is free to be spent in accordance with the tastes and preferences of the income recipients, it is *not* tied down to subsistence, it may contain a share of surplus. (See Sraffa (1960), Chapter III section 8). Now it is well-known that the neoclassical revolution in economic theory was inaugurated by repudiating the classical assumption of a subsistence wage. (Jevons (1871), Walras (1974)). Sraffa too repudiates that notion. The result is that the wage costs are brought out of the system of production and prices, their physical content is unspecified. Sraffa also assumes for convenience, that the wage is paid post-factum so that the price system is as follows,

$$A^T P(1+r) + wL = P \qquad \text{S–1}$$

where L is the vector of labour coefficients and w is the wage rate. The price system S–1 is indeterminate, it contains n equations in $n + 2$ unknowns, i.e. the wage rate, the rate of profit and n prices. One of the unknowns can be eliminated either (a) by setting a price as the *numeraire* in the Ricardo-Walras tradition, on (b) setting the wage as *numeraire* in the Smith-Keynes tradition or (c) by the device of a 'standard commodity' which is what Sraffa does. Even then there will be n equations in $n + 1$ unknowns. Sraffa allowed his price system to remain indeterminate and investigated its properties.

There is no dual system for output determination. Curiously, it is in the course of devising the "standard commodity", a *numeraire* for prices and wage measurements that the *standard system* for the (hypothetical) determination of the outputs of basic goods appears *.

This system is similar to Von-Neumann's output system but shows labour as a primary factor rather than as a produced good,

$$AX(1+R) = X$$

$$\sum_i l_i X_i = \sum_i L_i$$

In the standard system S–2, R is the maximum rate of profit or the "standard ratio, "l_i are the labour coefficients of production and ΣL_i is the total labour employed in the basic goods' industries. The standard system's outputs are purely hypothetical, for they correspond to zero outputs of the non-basic industries.

Sraffa's equations are thus seen to contain two features that are more realistic than the classical/von-Neumann assumptions. First the notion of a subsistence wage is repudiated. Second, the notion of population growing in Malthusian manner, as part of the output system, is abandoned; labour is recognised as a primary factor which may grow or decline on its own.

The Sraffa system, however, is indeterminate. The approach that has been adopted in chapter VII to render it determinate is to add to the system demand equations. In this book the demand equations are described by Engels' equations or Linear expenditure systems. The latter are clearly more adequate than the former. They are, besides, closer to Sraffa's own position. (See Sraffa (1960), Chapter II, section 8).

Basic goods are goods which are used directly or indirectly, in the production of all commodities in the system.

Appendix 4

THE NEO-RICARDIAN CRITIQUE

Using the indeterminate Sraffa system (augmented, however, to allow several techniques of production) which has been briefly explained in Appendix 3, Steedman, Metcalfe, Mainwaring and Parinello launched a convincing critique of the standard Heckscher-Ohlin-Samuelson trade theory. Their contributions are fortunately collected in one volume, *Fundamental Issues in Trade Theory*, Steedman (1979). In fact a major reason for skirting some of the critical issues raised by Steedman *et.al.* in this book is that their contributions can be found in one place. It will, therefore, suffice to highlight their results and refer the interested reader to their volume.

The focus of the neo-Ricardian critique is the cavalier treatment of capital in the standard trade theory. Standard trade theory entirely neglects the crucial distinction between capital as the *value* of produced means of production and capital as a physical stock of heterogeneous capital goods. In the former role it forms the base for earning profits. In the latter role it performs specific technological functions in different industries. Besides neglecting this distinction, standard trade theory regards capital as an 'endowment' on the same footing as the natural endowments, land and labour. The neo-Ricardians argue that this manner of treating capital has naturally led to several inadequacies in the theory of international values and has cast doubt on the validity of several propositions of the standard theory. We enlist below some of these inadequacies.

(1) By far the most serious inadequacy entailed by treating capital as a homogeneous endowment is that the standard theory, right from the start, is rendered incapable of studying trade in capital goods, an everyday happening of world trade whose importance, moreover, is increasing day by day.

(2) Once heterogeneous capital goods are explicitly admitted in the system of equations there are two consequences; (a) relative commodity prices need not be monotonically related to the rate of profit(*), and (b) the capital-labour ratio of an industry may not be monotonically related to the rate of profit. But the validity of the factor-price equalisation theorem and the Heckscher-Ohlin theorem depends crucially on the monotonicities of

* Consequence (a) has another implication. When different techniques of production are allowed, "reswitching of techniques, "i.e. a phenomenon in which one technique is more profitable than another at two (or more) distinct rates of profit but unprofitable at the rates in between, cannot be ruled out. This destroys the basis of factor-price equalisation. For a thorough discussion of reswitching of techniques see Sraffa (1960) and Bharadwaj (1970).

these two relationships. Thus merely by admitting heterogeneous capital goods, two "core propositions" of the standard theory (as Ethier (1984) has called them) are unsettled.

(3) Once value capital is admitted, the equations that determine prices contain the rate of profit as a markup factor. As a result, the relative commodity prices diverge from the domestic (marginal) rates of transformation in production. In other words the profit markup appears as a "factor-market distortion". But, as is well-known, the equality of the price ratios to the rates of transformation is necessary for the post-trade gains due to specialisation to be positive. But if they diverge, the possibility of negative gains from trade, (where gains from trade are calculated in standard theoretic terms viz. as an increase in the value of total consumption post-trade) cannot be ruled out.

In effect the neo-Ricardian critique constructively demonstrates that the standard trade theory is not robust in relation to essential facts. Even a bare minimum of changes in the assumptions towards greater realism make many of its crucial results go awry.

It is obvious that almost all the properties of the neo-Ricardian model are present in the model investigated in Chapter VII and VIII because the price equations in both models are the Sraffa price equations. Differences are due to the extent of determinacy of the two models: the neo-Ricardian model is indeterminate while the model of Chapters VII and VIII is fully determinate. Thus, for example, since both the rate of profit and the rate of growth can be chosen at will in the neo-Ricardian model, one will find a discussion of cases in which one may be greater or lower than the other. But in our model the rate of growth can never exceed the rate of profit. (It can do so only in the event of intercountry transfers, etc.). Thus in terms of our model some of the neo-Ricardian discussion may seem hypothetical.

Appendix 5

THE METZLER-CHIPMAN MODEL: A CRITIQUE

The purpose of this note is to show that the Metzler-Chipman model [(Metzler (1950), Chipman (1950)]can serve the purpose of a multiregional model rather than a multicountry model. Its application to the multicountry problem may prove fruitful only if it is anymented by the system of exchange rate determination explained in Chapter I.

The Metzler-Chipman model as applied to the multicountry problem reads as follows. The national incomes are determined by the equations,

$$Y_i = A_i + C_i + X_i \qquad i = 1, \dots n \tag{1}$$

where A_i is autonomous home absorption, C_i is home-consumption of the home-product and X_i is the export earnings of country i. We specify,

$$C_i = c_i Y_i \qquad i = 1, \dots. n \tag{2}$$

and, letting μ_{ji} be country j's prospensity to import from country i, we have the trade balance equation,

$$X_i = \sum_j E_{ij}\mu_{ji}Y_j = \sum_j \mu_{ij}Y_i \qquad i=1,\dots,n \;\; i \neq j \tag{3}$$

Substituting from (2) and (3) into (1), we write

$$Y_i = A_i^* + \sum_j E_{ij}\mu_{ji}^* Y_j = A_i^* + \sum_j \mu_{ji}^* Y_i \qquad i=1,\dots,n \tag{4}$$

where

$$A_i^* = A_i/1-c_i \;\text{ and }\; \mu_{ji}^* = \mu_{ij}/1-c_i$$

In vector-matrix notation we may write equation (4) in the standard form.

$$Y = A^* + M^* Y \tag{5}$$

where M^* is the matrix (μ_{ji}^*)

The system of equations (3) contains n equations in n^2 unknowns i.e., $n(n-1)$ unknown exchange rates and n unknown values of the national incomes. We may eliminate $(n-1)^2$ exchange rates by imposing the neutrality conditions [Chapter II, equations (11), (12)] so that $2n-1$ unknowns remain. There are in addition $n-1$ trade balance equations (3) and n income determination equations (5) so that we have $2n-1$ equations in all.

The system of equations in the Metzler-Chipman version gives the solution only after considerable computational effort. Since the Y_i's must be solved from (5), it is first necessary to try out arbitrary exchange rates say E_{ij} (taking care that they do not violate the Hawkins-Simon stability conditions on μ^*) and compute the corresponding values of the national incomes say, Y_i. Next we compute the trade volumes and using the system (Chapter II, section 4) to find the corresponding exchange rates check whether they are identical with E_{ij}. If they coincide with the equilibrium values we are lucky; if not, we try another set E^2_{ij}, etc. until we find the equilibrium exchange rates.

There is, however, one instance in which the Metzler-Chipman model yields valid results viz. the case of rigidly fixed exchange rates with relative immobility of labour. A special case is that of a multiregional country with exchange rates fixed by definition at unity between all the regions, but in which due to social/cultural/political reasons labour movements are not substantial.

Appendix 6
INTERREGIONAL THEORY

The methods of general equilibrium theory explained in the text have immediate application to inter-regional economics. There are two peculiarities of inter-regional economic relations. Firstly, the currency is uniform so that exchange rates are predetermined at the ratio 1:1. Secondly, resources are much more mobile between regions. Two implications follow:

(a) Trade will take place by the principle of absolute advantage.

(b) Receipts and payments of regions will not generally balance - rather, capital and labour will emigrate from deficit regions towards surplus regions to bring about the balance. Eventually, deficit regions will be left with the production of non-tradeable goods and /or a population living on subsistence.

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AUTHOR INDEX

SUBJECT INDEX

Of Related Interest

READINGS IN INDIAN PUBLIC FINANCE (2/E)

D.N. DWIVEDI

On the subject of Indian Public Finance, only a few good books are available, and even these generally tend to be aridly academic. There are still fewer books which grapple with the current issues in the area, particularly in the context of India's declared objective, namely the creation of an egalitarian society. Of course, a number of brilliant papers on the subject have appeared in various scholarly and professional journals, but they remain scattered, and, hence inaccessible in a coherent volume to students, scholars, researchers, and general readers interested in economic problems.

This book, bringing together some most important contributions to the subject by India's top-ranking and internationally known economists, eminently fulfils the long-standing need. With relevant excerpts from the Report of the Indirect Taxation Enquiry Committee (1977), and the Seventh Finance Commission included in this volume, it becomes a standard work on the subject. Some of the major issues dealt with here are: the problems of agricultural taxation: buoyancy and elasticity of central and state taxes; reforming direct and indirect taxes: the nature, working and impact of black money; deficit financing and price level; and the question of equitability of devolution of central resources among states. These issues are as alive and urgent today as in the past, when the papers collected here were written, and are likely to remain so in the nineties. In that sense, the book is to serve as an invaluable guide to researchers, scholars, and policy-makers of the present decade.

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He is author of *Problems and Prospects of Agricultural Taxation in Uttar Pradesh* and *Economic Concentration and Poverty in India* and is also author of several textbooks. In addition, Dr. Dwivedi has contributed dozens of articles dealing with current economic issues facing the country in various scholarly and professional journals.

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On the subject of Indian Public Finance, only a few good books are available and even these generally tend to be hardly academic. There are still fewer books which grapple with the current issues in the area, particularly in the context of India's declared objective, namely, the creation of an egalitarian society. Of course, a number of brilliant papers on the subject have appeared in various scholarly and professional journals but they remain scattered and hence inaccessible in a coherent volume to students, scholars, researchers and general readers interested in economic problems.

This book, bringing together some most important contributions to the subject by India's top-ranking and internationally known economists, eminently fulfils the long-standing need. With relevant excerpts from the Report of the Indirect Taxation Enquiry Committee, 1977 and the Seventh Finance Commission included in this volume, it becomes a standard work on the subject. Some of the major issues dealt with here are the problems of agricultural taxation, buoyancy and elasticity of central and state taxes, reforming direct and indirect taxes, the nature, working and impact of black money, deficit financing and price level and the question of equitability of devolution of central resources among states. These issues are as alive and urgent today as in the past, when the papers selected here were written, and are likely to remain so in the nineties. In a sense, the book is to serve as an invaluable guide to researchers, scholars and policy makers of the present decade.

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