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**THEORY OF EXHAUSTIBLE RESOURCES
A SELECT SURVEY OF DETERMINISTIC
MODELS**

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THEORY OF EXHAUSTIBLE RESOURCES

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INTRODUCTION

Natural resources could refer to all the living and nonliving endowment of earth. These include agricultural land, forests, fisheries, mineral resources, water, nonmineral energy sources of solar, tidal, wind and geothermal systems, fresh air, scenic beauty of a woodland and a host of such natural endowments.

These natural resources are broadly classified into two groups :

- (i) exhaustible resources and
- (ii) renewable resources.

Exhaustible resources are those natural resources the stock of which declines with extraction. For example, oil, coal, natural gas etc. The total stock of these resources are given and cannot be increased. Improvement in technology can, of course, increase the usable stock. Again, discoveries of new deposits will increase the known stock. But the total (known plus unknown) stock cannot be augmented. In other words, these resources do not have a natural or biological regeneration process by which their stock size grows in a time-span relevant to human beings. Resources such as fossil fuels enjoy growth rates that operate only over geological time.

On the other hand, renewable resources are capable of being regenerated by some natural processes. This natural process may take the form of biological regeneration as in case of fish etc. or it may be a natural replenishment such as river water etc. Renewable resources include all resources having such a growth function, e.g., fish,

aquifers, forests, soil fertility.

An important point to be noted is that even renewable resources are exhaustible, if they are harvested at a rate much greater than their natural rate of regeneration. In such a case, their stock size will continuously decline. Thus renewability depends on appropriate non-destructive methods of management as with farm lands, fisheries etc., since some changes in natural resource systems are irreversible.

Natural resources have very important role to play in the functioning of an economic system. This role comes from the various uses which these resources have. Some resources have use in direct consumption, e.g., fish, water, fresh air etc. Some resources are used as inputs in intermediate processing, e.g., iron ore, copper ore etc., in smelting. Some resources have consumptive uses in intermediate processes such as fuels consumed in manufacturing and transport, while some other resources have in situ uses, e.g., free running rivers, wilderness etc.

This role of natural resources in the functioning of an economy was well recognised by economists. The classical economists were concerned about the impact of shortages of natural resources on human material well-being. We know about Malthus' doomsday forecast of increasing population pressure pressing on a fixed supply of agricultural land and resultant starvation as the ultimate population check. Ricardo, on the other hand pointed to the diminishing quality of all the natural resources as the cause of differential land rents and the ultimate check on the economic and population growth. Stanley Jevons in 1865

foresaw stagnation to Britain's industrial progress as he was convinced that coal, which was the major input in the then industrial activity was getting exhausted. On the other hand, J.S. Mill provided a more optimistic analysis of the natural resource situation holding out distinct hope that knowledge and technology could keep the Malthusian margin a good remove.

All these economists expressed concern over the effects of impending natural resource shortages, but surprisingly they did not go into the question of an appropriate rate of utilization of these resources. None had looked at the market mechanisms which determine the rate of depletion of these resources. Markets have historically played an important role in determining the exploration activity and rates of use. Besides, the analysis done by all the classical economists was aggregative. No effort was directed at looking at the impacts of exhaustible and the renewable resources separately.

It was only at the peak of the first conservation movement (1895-1927) that Lewis Cecil Gray attempted such an inquiry. The conservationists believed that the natural resources were being exploited too rapidly and their excessive cheapness had led to wasteful consumption. Hence they proposed that the production of these resources should be prohibited in certain regions or measures should be taken to hamper the production in general. In his two classic articles in 1913¹ and 1914², Gray investigated the question of the rate of extraction that a competitive profit maximizing mine owner will pursue. For him, it was necessary to know this profitable rate of extraction of a profit

maximizing competitive mine owner in order to inquire what are the conditions, which in case of the individual determine the profitability of a conservation policy. Once these conditions are known, one could see whether the market mechanism will lead to conservation or over exploitation of the resources.

Then came the seminal work of Harold Hotelling in 1931³. He was not so much concerned with the behaviour of an individual firm in a competitive mining industry, but was interested in the intertemporal price behaviour and the pace of extraction both by a competitive industry and a private monopoly, and whether market mechanism can ensure a socially optimal extraction of the resources. The contributions of Gray and Hotelling created the subject of the 'Economics of Exhaustible Resources'.

In this dissertation, we briefly review some select models of exhaustible resource depletion under various market structures. The chapter scheme is as follows.

In the first chapter, we discuss the problem of resource extraction by a profit maximizing competitive exhaustible resource firm as well as the time paths of price and output in a competitive exhaustible resource market.

In chapter two, we review some of the empirical studies which have attempted at testing the empirical validity of the 'Hotelling Rule' under competitive conditions.

Chapter three deals with monopoly in exhaustible resource markets. The role of elasticity demand in determining the monopoly outcomes is demonstrated.

In Chapter four, we discuss the intermediate market structure, viz, oligopoly in exhaustible resource markets. It is assumed that there exist a resource cartel and a competitive fringe. Price and output behaviour under both a Nash-Cournot and a Stackelberg dominant firm model is discussed. Besides, a few simulation models of the major world resource markets are discussed.

We conclude this select and brief review in chapter five where we mainly focuss on the main issues in exhaustible resource depletion.

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CHAPTER I

COMPETITIVE EXHAUSTIBLE RESOURCE MARKETS

In this chapter, we briefly review the literature which discusses the pattern of intertemporal resource allocation in competitive exhaustible resource markets. We concern ourselves here only with models which concentrate on pure depletion where if $R(t)$ represented the remaining reserves at any period t , and $q(t)$ production, $dR(t)/dt$ change in stock, then

$$dR(t)/dt = \dot{R}(t) = -q(t).$$

We shall be mainly discussing the two early models of Lewis Cecil Gray and Harold Hotelling and some of the extensions of these models.

Gray's Theory of a Competitive mine:^{1,2}

In Gray's problem, the competitive mine operator is faced with the problem of determining how rapidly to extract the resources such that the sum of the present value of each period's profits is maximised. The following assumptions are made.

(i) Price of the extracted resource (coal in Gray's example) is constant over the whole time horizon.

(ii) Quantity of resource is homogeneous.

(iii) Costs of extraction (both average cost and marginal cost) in any period are dependent upon the quantity of ore extracted in that period, and are independent of cumulative extraction costs.

(iv) Costs of extractions are time independent, i.e. cost curves are identical in each period.

(v) Convex cost conditions exist, implying that law of variable proportions holds.

(vi) The stock of ore in situ is fixed and is known.

(vii) No fixed capital is employed. The extraction of the ore is carried out only by employing "men with shovels" who could be 'hired or fired' as per need.

(viii) The resource is extracted and sold in the same period.

Suppose the interest rate is assumed to be zero. Then a dollar today is equivalent to a dollar tomorrow. Now, with U-shaped cost curves being postulated and cost curves being time independent, the competitive mine owner maximises his profit by extracting and selling a constant amount of the resource corresponding to the minimum average cost in each period. If he removes the resource at an average cost greater than the minimum level, his average return per ton extracted will be now less than what it would be if he postpones the extraction to a future period.

Suppose, now there is a positive rate of interest. Then a dollar today is worth more than a dollar tomorrow. The tendency to delay the removal of the ore which would otherwise have to be extracted today at a higher average cost per ton is checked by the fact that the value of return from future removal is lessened by the discount on future. The net return from each ton removed at present, even at increased average cost may be greater than the present value of the net return of the same ton removed at minimum expense in the future. Hence, we are required to compare, beyond the minimum average cost, the marginal net return from the extraction of the marginal ton of ore, and not the

average net return in each period. As long as the present value of marginal net return of postponing the extraction of a ton of ore is positive, it is an opportunity cost which should be added to the current marginal cost in determining the current level of extraction. Thus, the first condition of optimality for a competitive mining firm is the following:

$$\text{Price} = \text{Marginal Extraction Cost} + \text{Opportunity Cost.}$$

This opportunity cost is known by various names in the literature such as royalty, user cost, net price, in situ price, security rent etc.

The second condition of optimality required in choosing the optimum extraction path over time is that the discounted marginal net return should be the same for all the periods. Otherwise, profits could be increased by shifting the production from one period to another. Gray derives this condition on the basis of a simple arithmetical example in his 1914 article. To put the second condition in a different way, it means that the undiscounted royalty or marginal profit should rise at the rate of interest. This condition is widely known in the literature as "Hotelling Rule" since it was Hotelling who gave a definitive theoretical explanation of this result. (We shall discuss Hotelling's contributions shortly.)

With these two conditions, the next question is of determining the optimum time horizon for extraction. Gray (1914) derives the optimum horizon through the equality of maximum average discounted profit at the horizon with the marginal discounted profit at every period. It means that at the horizon, marginal profit is equal to average profit. So the life of the mine should be chosen in such a way that at

the horizon, extraction is carried out at minimum average cost.

The assumption of the constancy of resource price over time, coupled with the necessary and terminal conditions just discussed, gives a monotonically declining extraction path over time for the mining firm. Since price is constant over time, the only way in which the firm can have the discounted marginal profit the same in all periods is by moving its output down the increasing portion of the marginal cost curve until the minimum point of the average cost curve is eventually reached at the time of exhaustion.

The richness of Gray's analysis lies in the fact that he could solve the dynamic optimization problem of a profit maximizing competitive mining firm without any of the sophisticated mathematical tools but only with intuitive knowledge and simple arithmetic.

Hotelling (1931) :³

Chronologically, the work of Harold Hotelling which actually laid the foundation for the modern theory of exhaustible resources comes next. While Gray is concerned with the problem of a single competitive mine operator extracting a known stock of given resource, Hotelling is concerned with the price behaviour over time and the nature of extraction profiles for both a competitive industry and a private monopoly. All the assumptions which Gray has made in his analysis also hold in Hotelling's case, except that Hotelling does not assume the constancy of resource price over time.

One more thing to be noted is that Hotelling's analysis is carried out in terms of what he calls the 'net price'. In

a competitive mining industry, the net price stands for market price minus marginal extraction cost for a unit of the exhaustible resource. Market price and the net price become identical if extraction costs are zero. If constant costs are assumed, then net price simply refers to the profit margin. If the industry is monopolistic, it refers to the marginal revenue less marginal cost i.e., the marginal profit.

The Hotelling Rule :

Hotelling begins with the analysis of the decision making process of the owner of a mine in a competitive mining industry. The mine manager's objective is to maximise the present value of profits from the extraction and sale over time of his known stock of resource. Let $p(t)$ refer to Hotelling's 'net price'. Now Hotelling states that a mine owner in a competitive mine industry will be indifferent between receiving price, $p(0)$, now or a price, $p(0)e^{rt}$, after time t , where r is the rate of interest which is assumed to remain constant over time. To put in a different way, it means that for an optimum extraction of the resource over time, the net price of the resource should grow at the rate of interest. This is the famous "Hotelling Rule". If $\dot{p}(t)/p(t) \neq r$, the mine owner will find it profitable to shift production backward or forward in time, since this increases his profits. In other words there shall be scope for profitable intertemporal arbitrage. Thus the Hotelling Rule gives the condition of indifference in production between time periods, or this is the condition of dynamic equilibrium in competitive resource markets.

One point to be noted is that, according to the

Hotelling rule, in equilibrium, the net price rises exponentially at the rate of interest. This is not quite the same thing as the market price of the resources rising exponentially. The market price is the net price plus the extraction costs. The market price can fall or stay constant while net price is rising if extraction costs are falling through time, and if the net price is not a very large proportion of the market price. So the movement of market price depends on two factors, the movements of costs of production over time, and the proportion of scarcity rent in net price.

Nature of Depletion in Competitive Resource Industry

Having thus given the equilibrium condition derived from profit maximization of a competitive firm, Hotelling discusses the nature of resource depletion in a competitive exhaustible resource market.

The relative price at different times under pure competition is given by

$$p(t) = p(0)e^{rt} \quad 1.1$$

where $p(0)$ is the price at $t = 0$. The level of $p(0)$ will of course depend upon demand and upon the total initial reserves.

Hotelling assumes that under competition, extraction at any moment t will be determined according to the demand function,

$$q(t) = D[p(t)] \quad 1.2$$

It is assumed that the total reserves will be completely exhausted at some future time period, T .

$$\int_0^T q(t)dt = R_0 \quad 1.3$$

where R_0 is the initial reserves and the upper limit T is

the time of final exhaustion. Then at T , q will be zero,

Thus,

at $t = T$, $q(T) = 0$ and,

$$q(T) = D[p(0)e^{rT}] = 0 \quad 1.4$$

Thus, equations 1.2 to 1.4 determine $p(0)$, T and the time path of extraction, and the nature of solutions will depend upon the demand function $D[p(t)]$ which gives the value of q .

Now the question is whether complete exhaustion will take place in finite time or it will extend to infinity. This depends upon the nature of the demand function. If a finite value of p is required to make q vanish, then exhaustion will take place in finite time. If on the other hand, an infinite value of p is required for q to vanish, then the extraction of resource will continue forever, though gradually declining over time. If there is a finite p at which q becomes zero, it implies that the demand for resource is choked off at this price. The reasons for such an outcome may be many. One most plausible reason cited is that a substitute for this resource becomes viable at this price, which may have infinite supply elasticity at this price. This viable substitute is referred to as "backstop technology".

Social Optimality of the Competitive Extraction Path :

In order to assess whether the competitive extraction path is socially optimal, Hotelling defines first what he calls the "social value of the resource" and then compares the competitive and socially optimal outcomes.

Suppose the "social value of the resource" from production $q(t)$ is given by the area under the inverse demand curve, such that

$$U(q(t)) = \int_0^{q(t)} P(q(t)) dt \quad 1.5$$

The social planner would wish to maximize

$$\int_0^T U(q(t)) e^{-\delta t} dt \quad 1.6$$

$$\text{S.t. } \dot{R} = -q(t)$$

$$R(0) = R_0, \text{ given}$$

The current value Hamiltonian is

$$H = U(q(t)) - \mu(t)q(t). \quad 1.7$$

The necessary conditions are

$$U'(q(t)) - \mu(t) = 0 \quad 1.8$$

$$\dot{\mu} - \delta\mu(t) = 0 \quad 1.9$$

$$\dot{R} = -q(t). \quad 1.10$$

The end point conditions are

$$R(0) = R_0, \quad q(T) = 0.$$

Since $R(0)$ is fixed and equal to $\int_0^T q(t) dt$, the present value of marginal social value of the resource should be the same in all periods, This is given by

$$U'(q(t)) = P(q(t)) = p(t).$$

$$\text{Then } \mu(t) = p(t) \text{ and } \dot{\mu} / \mu(t) = \dot{p} / p(t) = \delta.$$

Thus, the welfare maximizing extraction path is identical to the competitive extraction path.

But this equality is dependent on certain strong assumptions. One such assumption is that the rate of discount assumed by the competitive producers and the social rate of discount are identical. We do not get into the debate over this issue since it is a part of a larger issue in welfare economics which is beyond the scope of this study.

A second assumption is that the total initial reserves which the society has are identical to the total initial reserves of the competitive industry.

Resource Extraction by a Single Competitive Firm

Till now, we have been discussing the price behaviour and the output profile in a competitive industry. But, we have not discussed in detail as to how a firm in the competitive industry decides on its extraction path, although we saw that the 'Hotelling Rule' gives the dynamic efficiency condition for an optimal intertemporal extraction by a firm. This analysis has been done by many writers after Hotelling such as Shulze(1974)⁴, Fisher (1981)⁵. We here follow the method given by Conrad & Clark (1987)⁶

Till now, we have been using Hotelling's concept of 'net price' in our discussion. But now we shall explicitly introduce extraction costs into the firm's optimization problem. So, $p(t)$ henceforth refers to the market price of the resource.

Suppose, the firm's extraction cost depends only on the rate of extraction at any time t . Let

$$C(t) = C(q(t)) \quad 1.11$$

be the cost of extraction at rate $q(t)$. The price $p(t)$ is exogenously given to the competitive firm and is known in advance. Let r be the rate of interest which is constant over time. The firm would wish to

$$\text{maximize}_{q(t) \geq 0} \int_0^T [p(t)q(t) - C(q(t))]e^{-rt} dt \quad 1.12$$

$$\text{subject to } \dot{R}(t) = -q(t)$$

$$R(0) \text{ given, } R(t) \geq 0, T \text{ is unconstrained.}$$

The current value Hamiltonian is

$$H(t) = p(t)q(t) - C(q(t)) - m(t)q(t). \quad 1.13$$

The first order necessary conditions imply

$$p(t) - C'(\cdot) - m(t) = 0 \quad 1.14$$

$$\dot{m}(t) = rm(t). \quad 1.15$$

If $C(\cdot)$ is convex, then the first order conditions are also sufficient. Thus, we assume that $C'(\cdot) > 0$, $C''(\cdot) > 0$. Equations 1.14 and 1.15 together give us the following

$$\{d/dt[p(t) - C'(q(t))]\}/[p(t) - C'(q(t))] = r. \quad 1.16$$

which implies that the price net of marginal costs rises at the rate of interest. This is the Hotelling Rule when we explicitly introduce costs of extraction.

Now let us look at the transversality conditions. For the free terminal time problem like this, the transversality conditions are

$$(TC1): H = 0 \text{ at } t = T. \quad 1.17$$

This implies from equation (1.13) that

$$m(T) = p(T) - [C(q(T))/q(T)]. \quad 1.18$$

On the other hand, equation (1.14) implies that

$$m(T) = p(T) - C'(q(T)) \quad 1.19$$

Equations 1.18 and 1.19 together give

$$C'(q(T)) = [C(q(T))/q(T)] \quad 1.20$$

Or, in other words, at the terminal time, marginal cost of extraction is equal to average cost. Let q^* be the output where $MC = AC$, then $q(T) = q^*$.

The value of terminal time T is given by the second transversality condition,

$$(TC2): R(T)m(T) = 0, \quad m(t) > 0. \quad 1.21$$

The two transversality conditions actually characterise an uneconomical mine and hence may be called "pit-closure" conditions. (TC2) implies that $R(T) = 0$ since $m(T) > 0$ at T .

Once we get T , we can determine $m(T)$ from 1.18 and from equation 1.15,

$$m(t) = e^{r(t-T)}m(T); \quad 0 \leq t \leq T. \quad 1.22$$

Now from equation (1.14) we have

$$C'q(t) = p(t) - m(t) \quad 1.23$$

from which $q(t)$ is determined for $0 \leq t \leq T$.

This completes the competitive firm problem.

Now let us look at the necessary conditions given by equations 1.14 and 1.15. These are the same two conditions which Gray has derived with his intuition and arithmetic. $m(t)$ in equation 1.14 is the opportunity cost, which is a wedge between the price and marginal cost in each period. For very large resource stocks, this is very small, and the solution becomes identical to one in static micro economic theory. Similarly, 1.15 traces the time path of $m(t)$ which corresponds to Gray's second condition that the discounted marginal net profit should grow at a rate equal to r . The only difference is that while for Gray $p = \bar{p}$; for Hotelling p varies with time.

Besides, Gray also stated the result implied in equation 1.20, i.e., at the horizon, marginal cost is equal to average cost.

Stock Effects:

In the preceding section, we assumed that the cost of extraction depends only on current rate of output, $q(t)$. But it may also depend upon the amount of cumulative production, or to look at in another way, on the stock remaining in the ground. As the mine goes deeper, the extraction costs increase and affect the mine-owner's profits. Hotelling

models this phenomenon by specifying that the 'net price' depends on cumulative production. Later, Gordon (1967),⁷ Cummings (1969)⁸, Schulze (1974)⁹, Levhari and Liviatan (1977)¹⁰ etc., have analysed this case. Here we do not go into the details of each of these studies. We shall discuss the impact of stock effects on the Hotelling rule. We follow Fisher(1981)¹¹ in deriving this.

Now, the cost function has to be modified. Let

$$C(t) = C[q(t), R(t)] \quad 1.24$$

be the cost of extracting at the rate, $q(t)$. $R(t)$ refers to the reserves at any period, t . $p(t)$ is exogenously given.

The mining firm would wish to

$$\underset{q(t) \geq 0}{\text{maximize}} \int_0^T \{p(t)q(t) - C[q(t), R(t)]\} e^{-rt} dt \quad 1.25$$

$$\text{subject to } \dot{R}(t) = -q(t)$$

$$R(0) = R_0 \text{ given, } R(T) = 0, T \text{ is unconstrained.}$$

The current value Hamiltonian is

$$H = p(t)q(t) - C[q(t), R(t)] - m(t)q(t). \quad 1.26$$

The First order necessary conditions imply

$$p(t) - C_q(t) = m(t) \quad 1.27$$

$$\dot{m}(t) = rm(t) + C_R \quad 1.28$$

$$\dot{R}(t) = -q(t). \quad 1.29$$

The transversality conditions are

$$(TC1): R(T)M(T) = 0, \quad m(t) > 0, \quad 1.30$$

$$(TC2): H(T) = p(T)q(T) - C[q(T), R(T)] - m(T)q(T) = 0, \quad 1.31$$

$$p(t), R(t), q(t) > 0.$$

As before, (TC1) and (TC2) characterise the terminal time, and conditions for closing the mine.

Equation 1.27 is the static efficiency condition which states that at any t , there is a wedge, $m(t)$, between price and marginal extraction cost, and it is the scarcity rent. This is the marginal value of the resource left in the mine.

Equation 1.28 is the dynamic efficiency condition. Now, a change in $m(t)$ is the sum of two factors, (i) rm , which represents the value of foregone interest that could have been earned by extracting the unit last period and investing the return in an alternative investment. (ii) The second term C_R represents the stock effect. $C_R < 0$; this indicates the reduction in future costs from leaving the resource in the ground. As a result, now scarcity rent will grow at a rate lesser than r . It can be seen that now the 'Hotelling rule' of royalty rising at the rate r does not hold. The rate of increase in royalty or scarcity rent must equal the opportunity cost of deferred extraction, i.e., forgone interest, r , minus the savings in future extraction costs.

One more point to be noted is that that the competitive mining firm takes price, $p(t)$, as given for all periods. But, how does the firm know what price prevails in each period t ? In the static analysis, it can be assumed that the firm is a price taker which knows the current market price.

Conrad and Clark(1987)¹² say that in a dynamic setting, the assumption of rational expectations on the part of the firm is to be made. This means that the firm can predict the entire price profile, $p(t)$, over time accurately. This assumption may be an extreme one to make but we can assume that the firm will be motivated to estimate $p(t)$ as correctly as possible, since any mistake in predicting $p(t)$ will lead to a sub-optimal extraction policy, and hence a loss of profit. And with discounting, the near term price predictions are very important in comparison with long-term predictions.

"Hotelling Rule" - A Condition of Asset Market Equilibrium:

Solow (1974)¹³ and Dasgupta and Heal(1979)¹⁴ deduce the 'Hotelling rule' as a condition of stock equilibrium in asset market. According to them, a resource deposit is like any other capital asset to its owner. The resource deposit has market value from the prospect of its extraction and sale. Therefore, it can earn a current return for its producer only by an appreciation in value. Equilibrium in asset market requires that the rate of return on all assets in a given risk class be identical; the return may be partly in the form of current dividend and partly as capital gain. The common rate of return on these assets is the rate of interest for that risk class. The resource underground does not yield any dividend. Thus, in equilibrium, the value of a resource deposit must be growing at the rate of interest. The value of deposit is the present value of future sales from it less the costs of extraction. Therefore, this net present value or the 'net price' should be increasing at a rate equal to the rate of interest for the resource owners to have it as a constituent part of their portfolio. Any deviation from this would give scope for profitable arbitrage.

We just said that resource deposits in the ground do not earn any dividend,. But Fisher (1981)¹⁵ has an interesting observation to make. Take the case where the cost of extraction is dependent on current extraction rate, $q(t)$, and the remaining stock of ore, $R(t)$. In such a case, we have seen (equation 1.28) that the net price does not grow at the rate of interest, but at a rate lower than that. But, there is an implicit dividend which the resource stock earns. Let us look at equation 1.28 once again.

$$\dot{m}(t) = rm(t) + C_R$$

Here, $rm(t)$ is the interest cost of investing in resources in the ground. $\dot{m}(t)$ is the capital gain. C_R represents the dividend -- in the sense that holding the resource in the ground and thus not reducing the stock has resulted in cost saving, because we know that a reduction in the stock has the effect of increasing subsequent costs. Thus noting that the sign of C_R is negative, we can rewrite 1.28 as

$$\dot{m}(t) - C_R = rm(t)$$

which means that the capital gains plus the dividend is equal to the interest cost of investing in the resource in the ground. Thus, in such a case the net price need not grow at a rate equal to r .

Conclusion:

In this chapter, we focussed on the fundamental aspects of competitive resource depletion. We derived the dynamic efficiency conditions for an optimal resource allocation over time for both a competitive industry and the firm. The basic condition was the 'Hotelling rule' around which all the discussion was concentrated. What Gray intuitively asserted has become the "fundamental principle" of the economics of exhaustible resources after Hotelling gave a rigorous and definitive mathematical proof. We also discussed the asset theoretic approach of Solow and Dasgupta and Heal to the same problem.

But this does not 'exhaust' all the extensions of the model which are carried out later. There are various other issues discussed such as the alternative equilibria that will emerge under various cost conditions, or introducing

time-varying discount rates, the effect of a substitute technology on the current resource allocation pattern etc., which are equally interesting and important. But our purpose here is to develop the basic theoretical framework as a first step towards discussing the monopolised and the partially cartelised resource markets in the following chapters.

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CHAPTER II

EMPIRICAL TESTS OF HOTELLING'S MODEL OF EXHAUSTIBLE RESOURCE EXTRACTION UNDER COMPETITIVE CONDITIONS

In this chapter, we shall present a summary review of various studies which attempted to test the empirical validity of the Hotelling type model of exhaustible resource extraction under competitive conditions.

Let us briefly outline the basic model for optimal rate of extraction from a known resource stock of an exhaustible resource by a competitive firm. As the model has been discussed in detail in the previous chapter, here we limit ourselves to derivation of basic efficiency conditions resulting from the firm's maximization. The empirical studies have concentrated their focus in some way or the other on these conditions while testing the consistency of the model with data.

The Model

The competitive firm has a known stock of an exhaustible resource. It is a price-taker both in the product and input markets. The firm's objective is to maximize the present value of profits from the extraction and sale of the resource. Extraction costs are dependent on both the rate of output and cumulative extraction.

The necessary conditions for efficiency are obtained by solving the firm's discounted profit maximizing problem.

The firm's objective function can be stated as follows.

$$\begin{aligned} & \underset{q(t) \geq 0}{\text{maximize}} && \int_0^T \{p(t)q(t) - C[q(t), R(t)]\} e^{-rt} dt \\ & \text{subject to} && \dot{R}(t) = -q(t) \\ & && R(0) = R_0 \text{ given, } R(T) = 0, \text{ } T \text{ is Unconstrained} \\ & && q(t), p(t), R(t) \geq 0. \end{aligned}$$

where,

R_0 = known resource stock

$p(t)$ = Price of the resource, exogenously given

r = rate of interest

$q(t)$ = rate of extraction

$R(t)$ = stock of unmined reserves at t

T = Terminal time period.

The necessary conditions for optimization are

$$m(t) = p(t) - C_q(t) \quad 2.1$$

$$\dot{m}(t) = rm(t) + C_R(t) \quad 2.2$$

$$\dot{R}(t) = -q(t) \quad 2.3$$

$$R(T)m(T) = 0; \quad m(t) \geq 0 \quad 2.4$$

$$\{p(T)q(T) - C[q(T), R(T)] - m(T)q(T)\} = 0 \quad 2.5$$

$$p(t), R(t), q(t) \geq 0.$$

The interpretation of these conditions has been done in detail in the previous chapter. Briefly, equation 2.1 gives the static optimality condition which states that at any time, t , there exists a difference, $m(t)$, between price and marginal extraction cost due to the scarcity of the resource. Equation 2.2 is the dynamic efficiency condition of the intertemporal extraction programme. It states that change in $m(t)$ is equal to the sum of two factors --(a) external opportunity cost of holding the resource in the ground; (b) stock effect or internal opportunity cost of extraction. Equations 2.3 and 2.4 are the transversality

conditions characterizing an "uneconomic pit" that will close down in finite time.

Classification of Empirical Studies:

We can classify the empirical studies into three broad groups:

- A. Studies which attempted to test the model by observing the long run or short term trends in prices of individual resources or resource commodity aggregates.
- B. Studies which introduced an arbitrage behaviour for the owners of capital assets, where an in situ resource is considered a capital asset.
- C. A third type of models which attempted to estimate $m(t)$ or the shadow price and then test whether its time-path satisfies equation 2.2

Under the first group, we have three important studies. They are in chronological order:

1. Barnett and Morse (1963)
2. V.K. Smith (1979)
3. Margaret E. Slade (1982)

Barnett and Morse (1963):¹:

Harold J. Barnett and Chandler Morse (hereafter referred to as B & M) pioneered the empirical study of the time path of resource prices. The objective of their study was to examine the hypothesis that increasing trend, over time, in the real price of any natural resource reflected the increasing scarcity of that resource. They did not make use of any complicated statistical techniques. Using data on resource prices for the period from 1870 to 1957 in the United States, they plotted on graph the relative prices

(relative to an index of output prices) of five natural commodity aggregates for this period. B & M found that the price trends of minerals and total extractive output (which are of interest to us) were "approximately horizontal". On the basis of this result, they reject the hypothesis that these resources were becoming increasingly scarce. The observed trend was explained by the following factors -- "(1) substitution of economically more plentiful resources to less plentiful ones; (2) increased discoveries and availability of domestic natural resources; (3) increased imports of selected metallic minerals; and (4) a marked increase in the acquisition of knowledge and socio-technical improvements."²

We are not concerned here whether the scarcity hypothesis is empirically valid or not. From the model of extraction we have, it can be deduced, that in the absence of stock effects, the real price of the resource would be increasing, ceteris paribus. B & M study empirically demonstrated that such a trend in resource prices was not observed in the long run.

Does it mean that the Hotelling type of model does not hold? If we look at the factors responsible for such an outcome, e.g., new discoveries, imports of minerals, development of cheaper substitutes, etc., it is evident that the model does not take them into account while explaining the optimal extraction process. These factors must be controlled for while testing for efficiency condition in the model from which the price behaviour has been deduced.

V.K.Smith (1979):³

V. Kerry Smith carried out a study to examine the scarcity hypothesis. His purpose was to carry out an exercise similar to that of B & M using advanced statistical techniques which allowed him to test for the stability and direction of movement of the price series over time. His focus was on the relative price movements of four commodity aggregates, viz., agriculture, minerals, forestry products and the total extractive sector, over the period from 1900 to 1973. In his model, the price index for the relevant commodity aggregate relative to the wholesale price index, (i.e., the price index of the relevant commodity aggregate deflated by the wholesale price index (base 1947-49) in order to get the real price series) was postulated to be a linear function of a time trend variable. He fitted the following time trend equation.

$$RP_{tj} = \alpha_{0j} + \alpha_{1j} + \epsilon_{jt} \quad 2.6$$

where,

RP_{tj} = relative price of the natural resource commodity aggregate j in period t , t = time variable.

Without going into the details of estimation procedure, let us briefly look at his results.

Smith found that the coefficient of the time-trend variable in the linear regression for the full period was positive, but not significant. This result was consistent with the B & M finding that there was no evidence of resource scarcity.

Smith estimated equation 2.6 a number of times from the time series, but taking a different end point in each case. For example, α_{0j} and α_{1j} can be estimated from data for the

period 1900-1910, then again for 1900-1911 and so on upto 1900-1972. He found that the estimated α_{1j} was large and negative over the early years and was approaching zero by 1972 in the mineral sector. This indicated that the counter scarcity effect was weakening in recent years.

Smith concluded that his results certainly did not support the B & M findings since the estimated α_{ij} were found to be unstable. He was quick enough to note that additional data on extraction costs and market structure was essential for doing any meaningful analysis of the scarcity hypothesis.

The purpose of both these studies was not to test the consistency of Hotelling model with data, but testing the empirical validity of the resource scarcity hypothesis. In the process, they estimated the long run time trends in real resource prices. But to arrive at conclusions on the validity of the 'Hotelling Rule' [equation 2.2] on the basis of the estimated long run time path of real resource prices is very absurd. The reason is that the market price of any resource has two components - the net price and the marginal extraction costs. In the absence of extraction costs, it is easy to obtain an explicit price trajectory. But as Solow puts it, "The market price can fall or stay constant while the net price is rising if extraction costs are falling through time, and if the net price or scarcity rent is not too large a proportion of market price."⁴ Hence, the modelling of the behaviour of market price over time should incorporate in itself the behaviour of marginal extraction cost. Marginal extraction cost may decline over a certain

period due to technological innovations in which case while the net price is actually rising (as Hotelling postulated), the market price may still show a downward trend in that period. Such an analysis was carried out by Margaret Slade which we shall go into shortly.

Secondly, taking an aggregate price index for several exhaustible resources is not proper in such an exercise. Various resources are distinct in terms of the known stock, extraction costs etc. Hence, the analysis needs to be done for each individual resource separately.

Slade (1982):⁵

Margaret E. Slade tried to reconcile empirical findings of B & M and Smith with the theoretical prediction based on the 'Hotelling Rule' of an increasing trend in real resource prices over time. Slade incorporated into her model of resource extraction by the competitive mining firm, exogenous technical change and endogeneous change in the grade of ore mined. Market price of the resource has two components -- marginal extraction cost and scarcity rent. Marginal extraction cost is dependent upon the grade of the ore mined and state of technology. The time rate of change of market price is equal to the rate of change of marginal cost due to changes in technology plus discount rate times the scarcity rent. The two equations she derived were the following:

$$p(t) = C_q + \hat{\lambda} \tag{2.7}$$

$$= h(g) + k(t) +$$

$$\dot{p} = \dot{k} + \rho \hat{\lambda} \tag{2.8}$$

where

$p(t)$ = market price of the resource at t .

C_q = marginal extraction cost.

$\hat{\lambda}$ = scarcity rent at t.

ρ = discount rate.

If $\dot{k} = 0$, then p will increase with time since $\hat{\lambda}$ is positive for all t and increasing at a constant rate ρ . . A technological change which can make \dot{k} a very large negative will make p to fall. If k falls with time, but at a decreasing rate while $\hat{\lambda}$ is increasing with time, the time path of market price p(t) will initially fall and then will rise upwards or in other words, it will be U - shaped.

To test this hypothesis of long run time-path of p(t), Slade fitted a quadratic price trend for all the eleven resource prices.⁶ She also fitted a linear trend. Her time series data on 11 resource prices pertained to the period between 1870 & 1978. Prices were deflated by the U.S. wholesale price index (1967=1). She estimated the following two equations.

$$P_{it} = a_{0i} + a_{1i}t + u_{it} \quad 2.9$$

and

$$P_{it} = b_{0i} + b_{1i}t + b_{2i}t^2 + v_{it} \quad 2.10$$

where,

P_{it} = deflated price of i^{th} commodity at t.

t = time

Slade's results showed that with the linear model, the trend coefficients were positive in some cases and negative in some other and were significant at 90% confidence level in only a little over half of the cases studied. Hence, no generalisation was possible of the linear trend.

In the quadratic model, the linear terms were negative and quadratic terms were positive, thus implying the

predicted U-shape. Besides, 11 of the twelve quadratic coefficients were statistically significant (excepting lead).

Slade compared her results with those obtained by Smith. In the last section, we have seen that in case of the mineral sector, the linear trend coefficients (α_{1j}) in Smith's study were initially negative, but increased in value with time finally to become zero by 1972. If the correct trend for price behaviour was non-linear, then the slope of a local linear approximation would depend upon the time period chosen. If the correct trend was U-shaped, then the trend coefficient would be initially negative and they would increase with time, which was what Smith found. Thus Slade found that her results were consistent with Smith's.

Mueller and Gorin (1985)⁷ criticised Slade's study on the ground that she did not take into account many real world phenomena like cartels, wars, depressions and discrete jumps in technology of mining etc. This neglect had resulted in a bias in her theoretical results and empirical findings. The U-shaped price paths which she got would disappear if her equations were corrected for these factors.

In her reply to Mueller and Gorin, Slade argued (Slade, 1985)⁸ that these influences were exogenous and there were many other such factors. But their effects were seen only in the short run, and in the long run, they might act as a random noise in the system. She found that a corrected equation (properly adjusted for exogeneous influences) still confirmed the persistence of a U - shaped price path.

Mueller and Gorin's criticism has relevance if short-run time-paths of resource prices are being examined. A quick look at the graphs given in Slade(1982), makes it

clear. The short-term trends in resource prices do show distinct breaks when the events cited by Mueller and Gorin have had their influence on resource prices. But they are of little significance when a study covers a very long period of time (100 years or more). Slade's results confirmed the predicted price behaviour by a Hotelling type model. As expected, such a behaviour persisted in the long-run.

Under the second group, we have the following studies:

- 1.Heal and Barrow (1980a)
- 2.Heal and Barrow (1980b)
- 3.V. Kerry Smith (1981)
- 4.T.D.Agbeyegbe (1989)

Let us quickly recapitulate the basic theory behind these models, which has been dealt in detail in the previous chapter.

When we consider the exhaustible resource underground as a capital asset, a necessary condition for its efficient allocation over time requires that the price, net of extraction costs, should rise at a rate equal to the rate of return on other assets (in the same risk class). In particular, if the resource owners regard it as a constituent part of their portfolio, they will hold it only if this resource yields a return not less than the return available on alternative assets. Equilibrium in the asset market will then imply the realization of the necessary condition stated earlier. An implication of this condition is that if the markets for exhaustible resources are functioning efficiently, then there will be a strong association between the rates of return on the other assets and the rate of change of resource prices. This implies an

arbitrage process in the resource markets and the return on holding this resource is a significant determinant in holding decisions.

Heal and Barrow (1980a):⁹

Geoffrey Heal and Michael Barrow (hereafter referred to as H & B) were the first to empirically test the dynamic behaviour of exhaustible resource markets with arbitrage behaviour built into them. Traders were free to switch funds from capital markets to resource markets and vice versa, and their supply and demand decisions for the resource in question were governed by portfolio management considerations, so that their demand and supply decisions were dependent on expectations of returns on the resource relative to the return on other assets. It was also assumed that price adjustments always took place in resource markets so that the resource supply was equated to resource demands.

H & B specified the demand function in the following manner.

$$D = p^{n(p)} y^{n(y)} \left[\frac{\tilde{p}/p}{\tilde{o}/o} \right]^{a_1} \quad 2.11$$

where $n(p)$ and $n(y)$ are the price and income elasticities
 p is the current price of the resource

o is the current price of the alternative asset.

\tilde{p} is the price expected to rule at some future date.

\tilde{o} is the price of alternative asset expected to rule at the same future date.

Thus, the demand for the resource is a function of its own price, income and a multiplicative term which is nothing but the ratio of expected rate of capital gain from this resource to the expected rate of capital gain attainable elsewhere. This term decides whether a resource is expected

to be a good investment or not in the near future and accordingly scales down or up the demand. If the price is expected to rise at a rate greater than those of other assets, then demand is increased and vice versa. Thus, this term introduces arbitrage in the model through the demand equation.

The supply function is specified as follows :

$$S(p', y') = p'^a + y'^n \quad (2.12)$$

The resource supply is dependent on two things - p' which is a weighted average of past prices of the resource, implying that supply responds to price changes with a lag, and y' , a weighted average of past incomes. The rationale for including y' is to capture the effects of increase in the level of economic activity on investment resulting in the extension of extractive and refining activity directly and not via output price change.

Market clearing requires that $S = D$.

With these demand and supply functions, together with a formulation of the relevant expectational process, H & B derived the reduced form equation in terms of the lagged values of rate of change in resources price and the current and lagged values of rate of interest (return on alternative asset). It also contained a growth rate variable, (a proxy for y) in its current and lagged values.

The reduced form of the model was the following.

$$r_c(t) = A_1 r_c(t-1) + A_2 r_c(t-2) + A_3 r(t) + A_4 r(t-1) + A_5 r(t-2) + A_6 g(t) + A_7 g(t-1) + A_8 g(t-2) + \epsilon_t \quad (2.13)$$

where,

$r_c = \dot{p}/p$, $r =$ interest rate, $g =$ growth rate variable.

H & B used monthly data on resource prices for the four metals studied (copper, tin, lead and zinc). The resource price used was the three-month forward price quoted in the London Metal Exchange. The rate of interest used was the return to maturity of a 91 day UK Treasury Bill. The growth rate variable was based on the O.E.C.D. Index of Industrial Production. The period of study was from July 1965 to June 1977. There were 144 monthly observations on each of the variables.

Without going into the elaborate estimation procedures and the various transformed forms of equation 2.13, let us briefly look into the results.

i) At least one of the lagged values of the dependent variable had a coefficient which was significantly different from zero.

ii) Interestingly, the results showed that the level of interest rate had no influence on metal price movements. An alternative form of equation 2.13 with rate of change of interest rate as the explanatory variable (with appropriate lag structure) was estimated. The coefficients turned out to be significant, suggesting that the relevant explanatory variable was not the level but the rate of change of interest rate. A constant interest rate implies a more or less constant resource price. But this kind of result contradicted the relationship between resource price movements and interest rate as suggested by the theory of a dynamic full information perfectly competitive resource market in equilibrium. An explanation of such a result was that investors were concerned more with the capital gains on their investment rather than the interest earnings that would accrue.

(iii) The growth rate variable was found to be not significant both in its current and lagged values.

The general conclusion seems to be that there was a quite strong relationship between the metal price movements and the returns to other assets, though it was not of the form that would be generated by efficient markets. The reason for such a difference might be the way expectations were formed.

There is one important point to be noted here in this study. The metal prices used here are prices of refined metals whereas the decision to deplete a resource or not is about unextracted and unrefined ones, and is therefore influenced by the rate of change of the ore price and not the metal price. But since the demand for ore is a derived demand, derived from the demand for metal, these two prices are interrelated. Dasgupta and Heal (1979) show that the relationship between interest rate and change of ore price over short periods will not be qualitatively different from that between interest rate and rate of change of metal price. (For a detailed proof, see Dasgupta P.S. and Heal G. 1979.)¹⁰

Heal and Barrow (1980b):¹¹

In this study, H & B extended their previous analysis of the relationship between interest rates and metal price movements to a longer time period. The period covered was from 1870 to 1978, and annual time series data on the prices of seven mineral products ¹² were used. Three different interest rates - short, medium and long term - were used. Since the study covered a period of over 100 years, H & B

are aware of the influence of many factors such as changes in extraction costs, new discoveries etc. on resource price movements, which could be ignored in the short run. In view of this, they tested a variety of arbitrage models where extraction costs were explicitly taken into account. Costs of extraction were assumed to depend on the level of output and cumulative output. We do not go into the details of cost estimation procedures etc. A brief outline of the results obtained is given below.

The results of this study did not essentially differ from the results of the earlier study by H & B. Still, some interesting points need to be noted.

(i) It appears that, as in the earlier study, the rate of capital gain on the resources was related to changes in rather than level of interest rate.

(ii) Cost considerations did play an important role. The cumulative output variable, acting as a proxy for cost charges, was found to be playing a significant role in determining price movements. It was found that changes in, rather than the level of costs, are important. A theoretical justification for this result is that changes in extraction cost at the margin affect the rate of return to a resource.

(iii) The current output variable performed very poorly relative to the cumulative output variable, indicating that resource prices were more sensitive to their exhaustibility than to capacity constraints in their extraction.

(iv) The rate of change of manufacturing output, unlike in the earlier case, showed a significant impact on long run price movements. Theoretically, one expects that growth of output should not affect price movements because changes in the level of output and consequent changes in

demand for extractive resources are expected to be fully foreseen. But the results showed that changes in the level of output were to some extent unanticipated, especially in the long run.

V.K.Smith (1981):¹³

Smith carried out another study on the lines of H & B, covering a wide range of minerals and examined their long-term price movements. Using data on the prices of twelve minerals over the period from 1900 to 1973, and rates of return on many alternative assets with various maturity periods, he tested an array of models ranging from the simplest form of Hotelling equilibrium model to the complex specifications of H & B.

Smith was quite aware of the constraints placed on the analysis by the available data. We know from theory that a multitude of factors such as extraction costs, new discoveries, changes in market structure etc. influence movements of resource prices. But information on such factors is generally very limited, and this paucity of information limits the robustness of the empirical results.

The superiority of Smith's analysis over that of H & B is due to

(i) consideration of a wide range of arbitrage models including some with multiple rates of return, and

(ii) use of forecasting properties outside the sample period to evaluate the forecasting performance of the models.

As in the H & B study, the simple Hotelling model with no extraction costs was rejected on basis of the empirical results obtained. Based on the criteria of the goodness of fit and the expected parametric restrictions on the model, the H & B specification, overall, was consistently among the 'best' models for the twelve minerals studies. (the paper). On the basis of the ex post forecasting performance also, H & B specification is the best of the arbitrage models studied. It exhibited the best forecasting performance in case of three of the twelve minerals.

T.D. Agbeyegbe (1989):¹⁴

Another study on the lines of H & B was done by Terence D. Agbeyegbe. He studied the prices of four different metals, viz., copper, zinc, lead and silver. The quarterly data on prices for the period from the second quarter of 1968 to the second quarter of 1982 were used. The difference between H & B model and Agbeyegbe's model was that H & B derived their reduced form equation from a model of dynamic behaviour of resource market in equilibrium using explicit supply and demand functions for a resource whereas Agbeyegbe incorporated expectational behaviour into the simple Hotelling model and derived his reduced form equation.

The results of this study once again confirmed the H & B finding that the rate of capital gain on exhaustible resources was positively related to the changes in the current returns available from holding other assets and not on their levels. The alternative returns used were 91 day UK Treasury Bill rates. It was also found that the rate of capital gain on the resource was positively related to its

own lagged value by one period.

All these studies have obtained one result in common, i.e., the rate of capital gain on a particular resource depended on changes in interest rates. Though this does not precisely conform to the predictions of theories of competitive resource markets with perfect information in equilibrium, it suggests the existence of a relationship between resource price movements and interest rates. As H&B put it, "However, this should not of itself be surprising: traders in markets studied clearly do not have access to perfect information about future, but have rather to base their decisions on expectations which must in essence be based on past observations ... in such situations, the relationships that will be established between resource price movements and the returns on other assets are more complex than those that emerge from the full-information equilibrium models of Hotelling and his successors....."¹⁵

Under the third group of studies, we have the following:

1. Miller and Upton (1985)
2. Farrow (1985)
3. Halvorsen and Smith (1991)

Miller and Upton (1985):¹⁶

Merton H. Miller and Charles W. Upton (hereafter referred to as M & U) adopted a new method to test the Hotelling model empirically. Instead of trying to estimate the time-path of market prices or the time-path of in situ prices and then to see whether this is consistent with the dynamic efficiency condition, equation 2.2, M & U derived

from a Hotelling type model, what they called the Hotelling Valuation Principle. The principle, in their own words, can be stated as follows - "In a world in which time-path of mineral prices, less marginal extraction costs, follows Hotelling's Principle, the value of the reserves in any currently operating, optimally managed mineral deposit depends on the current period prices and extraction costs, regardless of when the reserves are extracted."¹⁷ M & U tested this proposition by regressing the market value of a sample U.S. domestic oil and gas producing companies on their estimated Hotelling Values at several points in time during the years 1979 to 1981.

Let us derive the Hotelling Valuation Principle from the Hotelling type model given in the beginning of this chapter. We take the discrete time form of the firm's objective function. All variables except the cumulative output variable have the same interpretation as before.

The cumulative output variable here refers to the total quantity of resource extracted till now.

The present value of profits V_0 is to be maximized.

$$\begin{aligned} \text{maximize } V_0 &= \sum_{t=0}^T \frac{[p(t)q(t) - C(q(t), R(t))]}{(1+r)^t} & 2.14 \\ \text{subject to } \sum_{t=0}^T q(t) &\leq R_0; \quad R(t) = \sum_{j=0}^t q(j). \\ c(t) &= C_q(t) > 0; \quad C_R(t) > 0. \end{aligned}$$

For the sake of simplicity, let us assume that $C_R(t) = 0$, i.e., extraction costs are independent of cumulative output, then the first order condition for V_0 to be maximum now becomes

$$[p(t) - c(t)] \left[\frac{1}{(1+r)^t} \right] = \lambda, \quad \forall t. \quad 2.15$$

where λ is the Lagrangean multiplier associated with the constraint.

By solving 2.15, we get,

$$[p(0) - c(0)][1+r]^t = p(t) - c(t), \quad \forall t \quad 2.16$$

This is the Hotelling rule.

Now if we assume constant returns to scale, both in current as well as cumulative extraction, then the marginal and the average extraction costs are the same.

Now if we substitute $p(t)$ from 2.16 into 2.14 and rearrange the terms, we get the present value of total reserves

$$V_0 = [p(0) - c(0)] \sum_{t=0}^T q(t) = [p(0) - c(0)]R_0 \quad 2.17$$

i.e., if output prices net of extraction costs follow the Hotelling principle, then the present value of total reserves will always be proportional to the remaining reserves.

We can express 2.17 as

$$V_0/R_0 = p(0) - c(0) \quad 2.18$$

Since $p(0) - c(0) = p(t) - c(t)$ at the time of calculation, equation 2.18 means that the unit value of the resource remaining at any time is simply the unit operating profits at that time.

Equation 2.18 can be empirically tested. The estimating equation becomes

$$V_0^{it}/R_0^{it} = \alpha^{it} + \beta [p(0) - c(0)]^{it} + \epsilon_{it}$$

M & U call $V_0/R_0 = \text{VALUE}$ and $p(0) - c(0) = \text{HOTEL}$

Using these definitions, estimating equation can be expressed as

$$\text{VALUE}^{it} = \alpha^{it} + \beta (\text{HOTEL})^{it} + \epsilon_{it} \quad 2.19$$

Noting the influences which many factors like technological innovations in mining, uncertainty, price expectations, taxes etc., have on the Hotelling principle, (equation 2.2), M & U estimated equation 2.19. Data on

estimates of reserves, sales prices and operating costs of the 39 firms chosen as sample, which were in the business of oil and natural gas, was obtained from the 'Herolds'. Gas reserves were converted into oil reserves by using BTU equivalents. A composite $p(\emptyset) - c(\emptyset)$ was estimated.

$$p(\emptyset) - c(\emptyset) = \frac{R_{oil}}{R_o} [p_{oil} - c_{oil}] + \frac{R_{gas}}{R_o} [p_{gas} - c_{gas}]$$

where

$$R_{oil} + R_{gas} = R_o.$$

The market value of reserves for each firm in the sample was computed in the following way. Initially, the total value of the firm was calculated as sum of the market value of equity claims and debt outstanding. From this sum, the value of non-petroleum assets was deducted to arrive at the market value of reserves.

Using data thus compiled (94 observations), M & U estimated equation 2.19. The estimated equation was

$$\text{VALUE} = -2.24\emptyset + \emptyset.91\emptyset (\text{HOTEL})$$

$$(\emptyset.035) \quad (\emptyset.114)$$

$$R^2 = \emptyset.408; N = 94.$$

The figures in the brackets are the standard errors of the estimated parameters.

Both the coefficients were significant. The negative intercept was explained to reflect the market's anticipation of future development costs since 'Ro' measured 'proved' reserves rather than the reserves 'proved and developed'.

Thus the relationship between the market value of reserves and current net price appears to conform to the predicted one by the theory. This study thus provided empirical support for the Hotelling rule, by testing an implication of the rule, viz., the Hotelling Valuation

Principle.

Farrow (1985): 18

Scot Farrow (1985) attempted to test the consistency of the Hotelling model of the competitive mining firm's behaviour by using relevant data made available by a U.S. mining firm for the period from 1975 to 1981. The necessary condition for dynamic efficiency in a Hotelling type model is given by equation 2.2 giving us the time path of scarcity rent.

$$\dot{m}(t) = rm(t) - C_R(t)$$

Farrow's idea is very simple. The theoretically derived condition for dynamic efficiency of firm behaviour can be empirically tested, and if it is found to be consistent with data, then Hotelling's model provides the best description of firm behaviour. We know that an optimizing firm will adjust its time path of output, and therefore of marginal extraction costs, until its own path of net prices meets the Hotelling condition given in equation 2.2. Farrow's argument is in the reverse. If the actual time path of net price of the firm corresponds to Hotelling condition, then, Hotelling model gives the best description of firm behaviour. To empirically test this hypothesis, we have to estimate equation 2.2 and if the firm's time path of net prices follows the Hotelling condition, then we expect that estimate of r will be positive and the coefficient of $C_R(t)$ will be equal to 1.

A discrete time form of (2) can be expressed as follows (Fisher, 1981)¹⁹

$$\Delta m(t) = rm(t-1) + C_R(t) \quad 2.20$$

Farrow called this the basic model. Estimation of 2.20

required estimation of $m(t)$ which, in turn, required data on commodity price and marginal extraction costs. Using a translog cost function, Farrow estimated $C_q(t)$ i.e., the marginal extraction cost and $C_R(t)$ -- i.e., stock effects.

Besides equation 2.20, Farrow estimated three alternative specifications of 2.20, where he incorporated time varying discount rate, price expectations and capacity constraints on rate of production.

The empirical results he obtained rejected the basic Hotelling hypothesis as a description of firm behaviour. The estimated r , supposed to reveal the firm's discount rate, turned out to be negative in all the three specifications and the basic model. Farrow concluded on the basis of these results: "Hotelling model is insufficiently robust to be confidently applied to empirical problems. The rejection of the model as a description of a mining firm indicates the fruitlessness of increasing the effort devoted to the empirical examination of the extraction decisions"²⁰

Although Farrow's empirical test rejected "Hotelling rule" as a description of a competitive firm's behaviour, his conclusion is very strong. M&U, in the conclusion to their study we have just reviewed, say that "the role of Hotelling Principle or 'r-percent rule' ... is unlikely to be much affected by 'mere' empirical testing. No viable alternative paradigm exists"²¹ Therefore, as long as an alternative explanation of the firm behaviour is developed, we cannot jump to any conclusion as Farrow did.

Secondly, estimation of $m(t)$ plays a very important role in trying to test the empirical validity of the Hotelling rule. And it is obvious that in the estimation of

$m(t)$, the form of cost function adopted plays a crucial role. Hence, characterization of the costs of a mining firm is a key factor in deciding the final outcome.

Thirdly, Farrow is putting the cart before horse, by saying that if empirical results support the theoretical predictions, then the theory is correct. A theory is not an exact representation of reality. It abstracts from reality while logically deriving conditions, given the assumptions. An empirical rejection of the conditions derived from theory does not invalidate theory. It opens the scope for further investigation.

Fourthly, Hotelling rule is a long run phenomenon. The time period which Farrow considered is very short for such an analysis.

Halvorsen and Smith (1991):²²

The latest in the series of empirical tests of the Hotelling model is that of Robert Halvorsen and Tim R. Smith. They too tried to test empirically the Hotelling model by estimating the time path of $m(t)$ or the in situ price and then see whether this conformed to the time-path predicted by the model.

As we know, estimation of time path of $m(t)$ requires estimation of $m(t)$ itself which, in turn, requires the estimation of marginal extraction costs and the effect of cumulative output where extraction costs are influenced by cumulative output. The difficulty gets accentuated where resource extraction and refining activities are vertically integrated. Halvorsen and Smith derived an econometric model from the theoretical model of a competitive mining firm where the extraction and processing activities were

vertically integrated. A restricted cost function was used to estimate the in situ price, $m(t)$. Using data from the Canadian metal mining industry for the period from 1954 to 1974, they carried out the empirical test. The results obtained rejected the hypothesis that the in situ price of exhaustible resources grows at a rate equal to interest rate less stock effects. Halvorsen and Smith concluded that the theoretical model did not adequately characterise the privately optimal behaviour of the firms.

Conclusion:

We have gone through a wide range of empirical studies which attempted to test the consistency of a Hotelling type model of a competitive mining firm. Since each of the studies adopted a different approach in their studies, it is not easy to make a comparative analysis. The initial studies just tried to observe the time path of resource prices. A second group of studies tried to introduce arbitrage into the resource owners' behaviour who are also players in the capital market. A third group of studies attempted to estimate the net price directly and then observe its time path. A few of the studies found empirical support to the model, and others did not find any such support. But the fact that an alternative paradigm is not available underlines the important role of the model.

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CHAPTER III

MONOPOLY IN EXHAUSTIBLE RESOURCE MARKETS

In the previous chapter, we discussed the conditions for an intertemporal competitive equilibrium in the exhaustible resource markets. We may now turn our attention to the case of a resource market characterised by monopoly.

By monopoly, we mean an industry composed of a single producer producing a unique product, or in other words, pure monopoly. The monopolist in the exhaustible resource market is the sole owner of the resource. He extracts and sells it in the market where he faces a downward sloping demand curve. The total stock of the resource is known and is of uniform grade (or quality). Hotelling was the first to analyse the behaviour of monopoly in resource markets. As in the case of his analysis of a competitive resource market, he conducts his analysis in terms of the net price. As we noted earlier, this net price can be considered as average net profit or in case of zero extraction cost, it is the average profit.¹

The monopolist in the exhaustible resource market owns a known stock, R_0 , of a given resource. He is facing a downward sloping demand curve, $p(q)$, for his resource where p is the price of the resource at which the flow q is sold at period t . The monopolist's objective is to maximise the sum of the present value of the flow of profits derived from selling the resource over time. For the moment, let us assume the extraction costs to be zero. Then, the monopolist's problem is to choose q as a function of t such

that he maximises,

$$\int_0^{\infty} e^{-rt} p(q) dt \quad 3.1$$

subject to $\int_0^{\infty} q dt = R_0,$ 3.2

$$q \geq 0.$$

p is a continuous function of q with $p'(q) \leq 0$ and

$$\frac{\partial^2(pq)}{\partial q^2} < 0.$$

The upper limit of the integral in 3.1 can be taken to be infinity. If exhaustion is to take place in finite time T , $q = 0$ for $t > T$.

The monopolist's problem can be solved by using the Lagrangean multiplier approach. It requires that for every value of t , the following expression should be maximum

$$qp(q)e^{-rt} - \lambda q, \quad 3.3$$

where λ is Lagrangean multiplier associated with q . For q to be maximum at every t , the following conditions are required.

$$e^{-rt} \frac{dp(q)}{dq} - \lambda = 0 \quad 3.4$$

and

$$e^{-rt} \frac{d^2(pq)}{dq^2} < 0. \quad 3.5$$

With appropriate rearrangement, 3.4 becomes

$$\frac{d(pq)}{dq} = p + q \frac{dp}{dq} = \lambda e^{rt}. \quad 3.6$$

Equation 3.6 gives us the necessary condition for the monopolist's maximization problem. It states that, for intertemporal equilibrium in the monopoly market, the marginal revenue of the monopolist should grow at a rate equal to r (with the assumption of zero extraction costs). If p is interpreted as the net price, then $p + q \frac{dp}{dq}$ can be called as the marginal net revenue. This becomes very clear

if we extend the algebra by another two steps.

$$\text{Let } m(t) = p + q \frac{dp}{dq} = \lambda e^{rt}$$

$$\text{or } m(t)e^{-rt} = \lambda .$$

Differentiating logarithmically with respect to time, we get,

$$\frac{\dot{m}(t)}{m(t)} = r. \quad (\text{since } \frac{\dot{\lambda}}{\lambda} = 0.) \quad 3.6'$$

This is the 'Hotelling Rule' for the monopolist in the exhaustible resource market.

Let us have a closer look at equation 3.6. We now have an additional term, $q \frac{dp}{dq}$ which does not appear in the competitive case. This term takes into account the loss that would be incurred because of the slight reduction in price on the intramarginal units the monopolist sells. This is due to the fact that the monopolist faces a downward sloping demand curve, unlike the competitive producer. Since $\frac{dp}{dq} < 0$, the LHS of equation 3.6 is less than price.

We may rearrange 3.6 so that

$$\left[p + q \frac{dp}{dq} \right] e^{-rt} = \lambda . \quad 3.6''$$

which states that the present value of the marginal revenue should be the same in all periods. The term λ is the user cost for the monopolist of producing and selling the marginal unit at any period t , which should be the same in all periods for intertemporal equilibrium. If it is not the same in all periods, then some marginal reallocation of the extraction rate between time periods with different present value of marginal revenues would increase the present value of profits.

Following Solow (1974)² and Dasgupta and Heal (1979)³, we can use an alternative argument to obtain 3.6'. The

stock of resource underground can be viewed as a capital asset. Suppose the rate of return on the alternative asset of the same risk class is r . The rate of return on the resource stock is $\dot{m}(t)/m(t)$. Equilibrium in the asset market requires that the rates of return on various assets in the same risk class should be equal. Otherwise, it gives scope for profitable arbitrage which ensures this equality. This means that the resource should be so managed that it earns a rate of return just equal to r . Thus, equation 3.6' is both a flow and stock equilibrium condition.

Positive Extraction Costs:

Having derived the necessary conditions for an intertemporal equilibrium for a monopolist resource owner assuming zero extraction costs, we now introduce positive extraction costs. Costs are assumed to be a function of a current rate of extraction. Let $C(q(t))$ be the cost function with the property that $C'(q(t)) > 0$ and $C''(q(t)) > 0$.

The monopolist's objective function now is to maximise

$$\int_0^T [p(t)q(t) - C(q(t))]e^{-rt} dt \quad 3.7$$

subject to $\int_0^T q(t) dt = R_0 \quad 3.8$

$q(t) \geq 0$, $q(0) = q_0$, $q(T) = 0$, T is unconstrained.

This is the isoperimetric problem in the calculus of variations. We can use the Lagrangean multiplier technique. Appending 3.7 and 3.8 with a Lagrangean multiplier, λ , gives us

$$\begin{aligned} L &= \int_0^T [p(t)q(t) - C(q(t))]e^{-rt} dt - \lambda \left[\int_0^T q(t) dt - R_0 \right] \\ &= \int_0^T \{ e^{-rt} [p(t)q(t) - C(q(t))] - \lambda q(t) \} dt + \lambda R_0 \quad 3.9 \end{aligned}$$

A necessary condition for q to maximise the augmented

integrand 3.9 is that it satisfies the Euler Equation.

$$[p(t) + q(t)\frac{dp(t)}{dq(t)} - C'(q(t))]e^{-rt} = \lambda. \quad 3.10$$

The term inside the bracket on the LHS of 3.10 is nothing but $MR(t) - MEC(t)$ or marginal net profit. Equation 3.10 tells that in the presence of positive extraction costs, the present value of marginal net profit in each period should be the same in equilibrium; or, in other words, the current value of marginal net profit should grow at a rate equal to r to ensure dynamic efficiency.

If we rewrite 3.10 in a little different way, we get

$$p(t) + q(t)\frac{dp(t)}{dq(t)} = C'(q(t)) + \lambda e^{rt}. \quad 3.10'$$

Here, we can see the difference between the static efficiency condition in microeconomics, viz; $MR = MC$ and the static efficiency condition here i.e, the opportunity cost of extracting the last unit (λe^{rt}) should be added to $C'(q(t))$ in order to determine the level of output in each period.

Having derived the necessary condition for intertemporal monopoly equilibrium in exhaustible resource markets, we shall proceed to investigate the question of how does monopoly affect resource price and output levels. In other words, the question is whether a monopolist depletes the resource faster or slower when compared with competitive depletion. We revoke the assumption of zero extraction costs.

The price and output paths for the resource under monopoly will mainly depend upon the nature of demand function and its behaviour over time, since it is the nature

of the demand curve which determines the relationship between price and marginal revenue. This relationship is very important to analyse the outcome under monopoly. Under competition, a finite value of p as q approaches zero ensures finite exhaustion. But under monopoly, a finite exhaustion of the resource requires that $p + q \frac{dp}{dq}$ (=MR) be finite while q approaches zero. But it is quite possible to have a demand curve with the property that when p becomes finite, $q \frac{dp}{dq}$ becomes infinite. In such a case, the duration of exhaustion will be infinite under monopoly. Hotelling is quite aware of this possibility which he has discussed at length in section 7 of his article.⁴ On the basis of this, he concludes that there is a general tendency for production to be retarded under monopoly."⁵ But, as far as the time path of output is concerned, his view is that q will be a declining function of time, although there may be discontinuities depending upon the nature of the demand curve.

But retardation of production is only one possibility. Suppose we continue with our assumption of complete exhaustion of the resource. Then, one simple way of comparing monopoly and competitive depletion is to compare the rate of growth of price under both market structures. We assume that both industries face the same demand curve so that the comparison is meaningful. There will be a given extraction path associated with each price path. Now, we know that along an intertemporal competitive equilibrium, the resource price grows at a rate equal to r . We can compare the rate of growth of price under monopoly with r and see which depletes faster or slower under various

alternative assumptions about the nature of demand curve for this resource.⁶ Many modern theorists have analysed the effect of monopoly on resource price and output paths under varying assumptions about the nature of demand functions and/or their behaviour over time.

The Role of Elasticity of Demand:

Before we go into their analyses, let us derive a simple relationship between the rate of growth of price over time (\dot{p}/p) and the behaviour of elasticity of demand, n , which will help us in understanding various extensions of the theory.⁷

We know that marginal revenue, $m(t)$, and price are related in the following manner.

$$m(t) = p(t) \left[1 + \frac{1}{n} \right] \quad 3.11$$

where,

$$n = \frac{dq(t)}{dq(t)} \frac{p(t)}{q(t)}, \quad n \leq -1.$$

Taking logarithms on both sides and differentiating with respect to time, we get,

$$\frac{\dot{m}(t)}{m(t)} = \frac{\dot{p}(t)}{p(t)} + \frac{\dot{h}}{h} \quad 3.12$$

where,

$$h(t) = 1 + [1/n]$$

With slight rearrangement of 3.12, we get

$$\frac{\dot{p}(t)}{p(t)} = \frac{\dot{m}(t)}{m(t)} - \frac{\dot{h}(t)}{h(t)} \quad 3.13$$

We know from our earlier analysis that under monopoly, Now 3.13 can be written as

$$\frac{\dot{p}(t)}{p(t)} = r - \frac{\dot{h}(t)}{h(t)} \quad 3.14$$

Equation 3.14 states that under monopoly whether the market price more rapidly or less rapidly compared with

competitive conditions, depends upon the sign of $\dot{h}(t)/h(t)$

The sign of $\dot{h}(t)/h(t)$ is dependent on the nature of the demand curve. Various assumptions about the nature of the resource demand curves are made by theorists and the implications are analysed. We can broadly classify these into two groups.

(i) Models assuming an isoelastic demand curve in each period, but the elasticity changing over time.

(ii) Models assuming elasticity of demand to be a function of quantities only and not an explicit function of time.

In the first group, we have the models of Weinstein and Zeckhauser (1975)⁸ and Stiglitz (1976)⁹. They assume that the demand curve is isoelastic in each period, but the elasticity changes from one period to another. Stiglitz considers the case of demand curve shifting over time and becoming more elastic. Weinstein and Zeckhauser consider both cases of elasticity increasing over time and decreasing over time. They also assume that the total stock of resource is known and extraction costs are zero.

This type of formulation makes elasticity of demand, n , an explicit function of time, i.e., $n = n(t)$. We can now express

$$h(t) = 1 + [1/n(t)]$$
$$\dot{h}(t) = \frac{dh(t)}{dn(t)} \cdot \frac{dn(t)}{dt} = -\left[\frac{1}{n^2} \dot{n}(t)\right]. \quad 3.15$$

We know that $h(t)$ is always positive (since $n < -1$). The sign of $\dot{h}(t)$ is dependent on the sign of $\dot{n}(t)$. If the absolute value of n is increasing over time, then $\dot{n}(t) < 0$. If the absolute value of n is decreasing over time, then

$\dot{n}(t) > 0$. Therefore,

$$\text{Sign } \dot{h}(t) = - [\text{Sign } \dot{n}(t)]$$

Now, let us look at equation (3.14). Suppose $\dot{n}(t)=0$, i.e., elasticity is constant with respect to time. The same constant elasticity demand curve repeats in every t . Then $\dot{h}(t)/h(t)=0$. Then $\dot{p}(t)/p(t) = \dot{m}(t)/m(t) = r$. It is clear that in such a situation, the depletion policy followed by a monopolist is identical to the outcome realised along a path of intertemporal competitive equilibrium if the competitive industry faces the same demand function. The logic of such an outcome is very simple. When we assume zero extraction costs, while the resource price rises at the rate of interest along an intertemporal competitive equilibrium, and under pure monopoly, it is the marginal revenue which rises at this rate. With an isoelastic demand curve, price is proportional to marginal revenue. Hence, both the monopoly and the competitive solutions are identical in this case.

Now let us look into Stiglitz' case¹⁰. If $\dot{n}(t) < 0$, i.e., if the absolute value of elasticity is increasing with time, $\dot{h}(t)/h(t)$ is positive, and $\dot{p}(t)/p(t) < r$, i.e., price increases less swiftly than r . This means that initially prices are higher under monopoly compared with competitive prices. Consequently, monopoly sales are lesser in the initial periods when compared with the sales in the competitive market. But, since we have assumed complete exhaustion ($\lambda > 0$), monopolist will have to sell more in the later periods. His price path will be flatter when compared with the competitive time path of price. An explanation of this phenomenon is that the monopolist restricts his output

in the initial periods to take advantage of the relatively inelastic demand in the earlier periods.

Let us now look into the case considered by Weinstein and Zeekhauser. Here, $\dot{n}(t) > 0$, i.e., the absolute value of elasticity decreases with time. Now, $\dot{h}(t)/h(t)$ has a negative sign. From 3.14 we get $\dot{p}(t)/p(t) > r$, i.e., price increases faster than r . Hence, the initial monopoly price will be lower than competitive price leading to a faster depletion of the resource. The idea is that the monopolist will restrict his output in the later periods to take advantage of the relatively less elastic demand in the later periods.

But such an equilibrium may not be sustained since it creates an opportunity for profitable arbitrage. Secondly, the necessary condition for such an outcome is that $\dot{n}(t) > 0$. This is rather unlikely since a higher rate of growth of prices induces investments in the development of substitutes. Consequently, the resource will face more and more substitutes as time passes. This will increase the elasticity of demand over time.

One small question still remains. Will these price paths under monopoly and competitive markets intersect each other when they are not identical? Since we have put the condition of complete exhaustion under both the market structures, monopoly and competitive price paths will have to intersect once. The nature of intersection is dependent upon whether we assume $\dot{n}(t) > 0$ or < 0 .

The second group of studies considers a variety of

demand curves wherein the elasticity is different at each point on the demand curve. Here elasticity is not an explicit function of time. It is a function of the quantity sold. A change in the quantity sold in each period causes changes in elasticity. A detailed and rigorous mathematical derivation of the results can be seen in Lewis T.R. (1978)¹¹. Dasgupta and Heal (1979)¹² have also discussed at length the various outcomes. In our discussion, we follow the latter's method as it is relatively simpler.

Now, if elasticity n is a function of quantity, $q(t)$, i.e., $n = n(q(t))$, then

$$h(q(t)) = 1 + \frac{1}{n(q(t))}$$

Differentiating with respect to time, we have

$$\dot{h}(t) = \frac{dh(q(t))}{dq(t)} \dot{q}(t) = - \left[\frac{dn(\cdot)}{dq(t)} \right] \frac{\dot{q}(t)}{n^2}$$

$$\dot{q}(t) < 0.$$

Therefore,

$$\text{Sign } \dot{h}(t) = \text{Sign } \frac{dn}{dq(t)}$$

Taken together with 3.14, we get the following results:

1. If $dn/dq = 0$; $\dot{p}/p = r$. 3.16'
2. If $dn/dq > 0$; $\dot{p}/p < r$. 3.16''
3. If $dn/dq < 0$; $\dot{p}/p > r$. 3.16'''

In case (1), the monopolist is facing a constant elasticity demand schedule. We assume zero extraction costs as before. We see that the competitive and monopoly solutions are identical in this case, if the competitive industry faces the same demand function. The logic of this outcome is the same as the one behind an earlier case where we assumed $\dot{n}(t) = 0$.

Let us now introduce a variable elasticity of demand. We first consider the case where the absolute value of elasticity increases as output falls i.e., $dn/dq > 0$. In this case, $\dot{p}/p(t) < r$. Monopoly price path is flatter than the competitive price path. But they intersect once since the total quantity of the resources available is the same under both market structures and it has to be exhausted.

In this case, the monopoly outcome turns out to be more conservationist relative to competitive one. As price increases (supply falls), new substitutes come into the market. The demand for the resource thus becomes more elastic.

The third case is where $dn/dq < 0$, i.e., the absolute value of elasticity increases with increasing output. This requires the monopolist to lower the price for expanding his market. This brings him increasingly into competition with substitutes for his product which causes the absolute value of elasticity to rise. In this case, $\dot{p}/p > r$, giving a monopoly price path steeper than the competitive one. The two price paths (competitive and monopoly) intersect once. But the dynamic equilibrium generating this price path may be unstable since it provides opportunities for profitable arbitrage which may force the monopoly outcome to be identical with the competitive one.

A similar extension has been worked out by Lewis-Mathews and Burness (1979)¹³ where they introduce the existence of 'quasi-fixed costs' in production along with a stationary demand but elasticity increasing with output.

Again, they obtain the same result as Dasgupta and Heal. (For details, see Lewis T.R., Mathews S.A. and Burness H.S., 1979).

Stiglitz (1976)¹⁴ analyses a third situation introducing constant unit cost of extraction which declines over time. The assumption of constant elasticity demand schedules is retained. Besides, the assumption that $\dot{n}(t)=0$ is retained. Under these assumptions, Stiglitz shows that a monopolist depletes the resource slowly relative to depletion in a competitive market for resources. Besides, even when cumulative extraction costs are introduced, retaining the other assumptions, the same result follows (see Stiglitz, 1976 for a complete derivation of these results).

Empirical Studies of Monopoly Behaviour in Exhaustible Resource Markets :

Having discussed the various theoretical developments analysing monopoly behaviour in resource markets, let us very briefly look at one empirical study by John S. Soladay.¹⁵ His purpose was to develop "some quantitative evidence of the difference between competitive and monopolistic behaviour."¹⁶ This necessitated him to identify two different markets for a given resource of which one is competitive in nature, while the other, a monopoly. Soladay found that the U.S oil industry provided a real world example of such a situation. Crude oil producing states in the U.S. were classified by Soladay states into

three groups depending upon the type of oil production control adopted. These groups are outlined below. :

(i) Five "market demand states" - These were supposed to represent a monopoly situation as oil production in these states was regulated by the government. State agencies limited oil production by a system of uniform output prorationing among oil wells.

(ii) The competitive situation was represented by a second group of six states which had no direct controls on oil production.

(iii) There was an intermediate group called the maximum efficiency rate (MER) group in which oil production was controlled with the aim of preventing it from exceeding technological limits.

To show empirically the existence of difference between competitive and monopoly behaviour as suggested by theory, it is required to estimate the production profiles over time for both the monopoly as well as the competitive group and then compare them.

Soladay estimated the production profiles for all these groups for the period 1948-74. Without going into the details of his methodology of estimation procedures, we look at his results. His results showed that the monopoly type group exhibited a production profile which was initially lower, peaked later and eventually exceeded the profile estimated in the competitive group. This profile was thus tilted more towards future than a competitor's profile.

This study is broadly in supportive of the theory. But since this is the only study available (apart from the studies considering OPEC as a monopoly), we cannot say much about the empirical validity or otherwise of the model.

Conclusion :

We began this chapter by deriving conditions of intertemporal equilibrium in the exhaustible resource market characterised by monopoly. Later, we addressed ourselves to the question of whether monopoly depletes resources faster or slower than the competitive industry. We know that there are certain conditions under which monopoly depletes faster than competitive market but in many other cases a monopolist is found to be the "conservationist's best friend". The general impression is that there is a tendency for the monopoly to slow down depletion in a model where resource stock is of uniform quality and required to be exhausted in finite time. But Fisher (1981)¹⁷ and Tullock (1979)¹⁸ believe that this tendency will be strengthened if the requirement of complete exhaustion is relaxed. The factors responsible for such an outcome are perceived differently by of them. For Fisher, costs of extraction rise with cumulative depletion and high cost units remain in the ground indefinitely and cumulative production will not be identical under competitive and monopoly regimes. It may be lower for the monopolist who produces less in each period. Tullock's argument runs in terms of changing demand elasticities. If the elasticity is increasing with time, Tullock argues that in the initial years, the monopolist can realise larger profits by selling less at higher prices.

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Afterwards, he will simply sell exactly the same amount as in competitive market at the competitive price, thus incurring no loss. He would thus earn monopoly profits in the first period and incurs no loss in the second. This is more profitable than the depletion pattern he would follow when complete exhaustion is assumed. But this requires that a monopolist should not be bound to exhaust his resource.

Besides, there are many other issues to which we have not addressed ourselves in this chapter. The effects of various types of taxation on monopoly are equally important. Apart from this, the impact of development of a substitute, the effect of uncertainty etc., are equally vital issues which need to be considered, but are not discussed here due to constraints of time and space.

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5. Hotelling, Harold. op.cit., p. 152.
6. This is one simple method to compare the two outcomes. More rigorous derivations using the respective objective functions and maximization procedures will help one to determine the prices, output paths, the time period of exhaustion and the quantities at each time period etc., and then do the comparisons between the monopoly and competitive outcomes. This has been attempted by Lewis T.R. and others.
7. This is a slightly modified version of the equations used by Dasgupta and Heal. For original derivations, see Dasgupta and Heal. op.cit., pp 325-26.
8. Weinstein, M. and Zeckhauser, R.J. (1975), "The Optimal Consumption of Depletable Natural Resources." Quarterly Journal of Economics 89, pp. 371-92.

9. Stiglitz, J.E. (1976), "Monopoly and the Rate of Extraction of Exhaustible Resources." American Economic Review 66, pp. 655-61
10. Stiglitz derives his results in a more rigorous and complete manner using the objective function of the monopolist and deriving price and output paths from the monopolist's optimization problem. Here, we haven't used his method. For details, see Stiglitz, op.cit.,
11. Lewis, Tracy R. (1976), "Monopoly Exploitation of an Exhaustible Resource." Journal of Environmental Economics and Management 3, pp. 198-204.
12. Dasgupta, P and Heal G. op.cit., pp. 323-34.
13. Lewis T.R., Mathews, S.A. and Burness H.S. (1979), "Monopoly and the Rate of Extraction of Exhaustible Resources : Note." American Economic Review 69, pp. 227-30.
14. Stiglitz J.E. op.cit.
15. Soladay, John J. (1979), "Monopoly and Crude Oil Extraction." American Economic Review 69, pp.234-39.
16. Soladay, John J. Op.cit., p.234
17. Fisher, Anthony C. (1981), Resources and Environmental Economics. Cambridge Surveys of Economic Literature, Cambridge, p. 39.
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CHAPTER IV

PARTIALLY CARTELISTED EXHAUSTIBLE RESOURCE MARKETS

In the previous chapters, we considered the polar cases of pure competition and pure monopoly in exhaustible resource markets. But neither of these models is applicable to some very important real world markets for exhaustible resources, such as bauxite, tin, copper and above all, crude oil. These markets are mainly oligopolistic in nature. There are various models of oligopoly behaviour which are being applied to exhaustible resource markets. In this chapter, we restrict ourselves to a particular type of oligopolistic models which specifically take into account the characteristics of the real world exhaustible resource markets just mentioned.

The oligopolistic resource market can be characterised as partially cartelised in the sense that it consists of one group of producers who form a cartel and set the price for the resource. The second group comprises the competitive fringe which act competitively. The producers in the fringe are individually incapable of influencing the market price at any date, and have therefore to take the price as given in determining their supply plans.

We have a plethora of studies of partially cartelised exhaustible resource markets, both at the theoretical and the empirical level. Accordingly, they can be broadly classified into two categories.

(i) Studies or models which focus on theoretical issues, and

(ii) Simulation models (optimizing and non-optimizing) which are computation oriented, and frequently concerned

with forecasting the future price path of the resource in question (oil in many cases).

There are a number of aspects or building blocks which are common to all these models such as the specification of demand for the resource, cost functions and reserves for various producers and discount rates for the producers and the extant studies vary widely in assumptions made about these aspects.

The most important component of these models is the solution concept employed. There are two solution concepts which are most widely used. These are the Open-loop Nash - Cournot solution and Stackelberg solution.

The term Open-loop strategy has the following meaning in this context. The cartel announces a set of prices at all future dates for the resource. The fringe takes these prices as given and selects a production profile such that it maximises the present value of its profits. For the fringe to take the time path of prices announced by the cartel as given, it has to have the confidence that these prices will actually prevail at all the future dates. This necessitates the presence of a particular institutional framework, viz., a set of complete futures markets in which trading takes place now and then the markets are closed, or equivalently, there exists a set of costlessly enforceable long term contracts.

Meaning of the Two Solution Concepts:

As we noted before, the fringe is a follower in the market, i.e., it takes the price path set by the cartel as given and determines its supply plans so as to maximise its discounted future profits. The difference between the two

solution concepts lies in the characterisation of the cartel which is the dominant firm. In the Nash (or Nash-Cournot) equilibrium, the cartel takes the output path of the fringe as given while determining the price path for the resource so as to maximise its discounted profits. Equilibrium in the market occurs when the fringe's output path assumed by the cartel in making its decisions is the one which the fringe actually chooses, while the price-pattern assumed by the fringe firms in making their supply decisions should be the one actually selected by the cartel. Neither sector has any incentive to deviate from these choices. In Salant's words, "Any situation where each sector takes as given the optimal choice of the other and where neither can under that assumption, increase its profits by altering its own strategy is called a Nash-Cournot Equilibrium".² Besides, in each period, the price and output decisions must be market clearing.

In the Stackelberg equilibrium, the cartel plays a less passive role, and it recognises that the fringe's output path will be a function of the price path set by the cartel and takes this reaction function of the fringe into account while choosing its optimal price path. In other words, the cartel can calculate for any price path, how much will be demanded in each time period, how much the fringe will supply in each time period, and hence how much production will be required from its resources. The cartel will select a price path, which, while allowing market-clearing output decisions in each period, does not require the cartel's cumulative production to exceed its initial reserves, and at the same time maximises the present value of its profits.

I. Theoretical Models:

Open-Loop Nash and Stakelberg Equilibrium:²

Having stated the meaning of the two solution concepts, let us briefly review the models which are developed using these concepts. As noted before, we assume that there are two groups of producers -- a cartel and a fringe group. The producers or firms are identical within each group. The cartel acts as a cohesive group.

Let us define some notations at this stage. Let the reserves of the cartel, the fringe and the total market be denoted by $R_m(t)$, $R_f(t)$ and $R(t)$, respectively at time t , and their production rates will be $q_m(t)$, $q_f(t)$ and $Q(t)$, respectively, $R_m(0)$ and $R_f(0)$ are given. The costs of production of each group are independent of cumulative levels of production, but may depend on the current rate of production depending on the model being discussed. Let c^f denote the marginal extraction cost of the fringe, and c^m denote the marginal extraction cost of the cartel. (Of course if c^f and c^m are assumed to be constant, then we have constant cost functions for both the groups.)

The assumption is that time is continuous and the capital markets are perfect with constant interest rate 'r'. There is no uncertainty. A stationary demand function, $P = P(Q(t))$, is given and is known. It has the characteristic that $P(0) = \bar{P}$, i.e., at $P = \bar{P}$, demand for the resource falls to zero. \bar{P} thus is the choke-price. At any point in time, $Q(t)$ is the sum of $q_m(t)$ and $q_f(t)$. The absolute value of price elasticity of demand is allthrough increasing with price and the demand curve has a point of unitary elasticity.

Given these assumptions and using the notations defined, let us look at the strategies of the fringe. The set of open-loop strategies for the fringe is the set of all possible production paths that do not violate its initial reserves. Let F be this set where

$$F = \{q_f(t) : \forall t \geq 0; q_f(t) \geq 0, \dot{R}_f(t) = -q_f(t); R_f(0) \text{ given}\}$$

Similarly, the set of open-loop strategies for the cartel is the set of all possible price paths it can set. Let this be K where

$$K = \{P(t) : \forall t \geq 0; 0 < P(t) \leq \bar{P}\}$$

In both the equilibria, the fringe is a follower, and for any price path $P(t) \in K$, it will select an output path $q_f(t) \in F$, so as to maximise the present value of profits. We know that the profit maximizing path of output must satisfy the following dynamic equilibrium condition, i.e., if $q_f(\bar{t}) > 0$ then for any $t \geq 0$

$$(P(t) - c^f)e^{-rt} \leq (P(\bar{t}) - c^f)e^{-r\bar{t}}; q_f(t) \geq 0 \quad 4.1^3$$

where the inequalities hold with complementary slackness. (Recall that we assume the fringe firms act as competitive price takers, and hence if 4.1 is not satisfied, there will be scope for profitable intertemporal arbitrage)

Now let us first describe the equilibrium condition for the Nash-Cournot model.

Nash Equilibrium:

As noted before, the cartel takes the output path of the fringe as given so as to maximise its discounted profits, subject to the condition that the output it would be required to produce each period to clear the market is

non-negative ($q_m(t) \geq 0$), and cumulative output does not exceed initial reserves. Thus, in each period, it can deduct the sales(=production), $q_f(t)$, of the competitive fringe from the consumer demand curve to obtain a sequence of excess demand curves. The cartel now acts like a monopolist faced by these excess demand curves and will allocate production to cartel members so that the discounted value of marginal profits is constant in all time periods the cartel produces. Once it stops producing, the discounted value of marginal profits should not exceed the discounted marginal profits in any period of positive sales. Thus, if at time \hat{t} , $q_m(\hat{t}) > 0$, then the cartel's production decisions must satisfy for all $t \geq 0$,

$$[P(t) + \frac{dP(t)}{dq(t)}q_m(t) - c^m]e^{-rt} \leq [P(\hat{t}) + \frac{dP(\hat{t})}{dq(\hat{t})}q_m(\hat{t}) - c^m]e^{-r\hat{t}};$$

$$q_m(t) \geq 0 \quad 4.2^4$$

This is the dynamic equilibrium condition for a monopolist faced by the excess demand curves. "A Nash equilibrium is a price path and an output path such that the output path is the fringe's profit maximizing response to the price path while the price path is the cartel's profit maximizing response to the fringe's output path."⁵ The equilibrium, in general, consists of three phases, viz.,

- (i) One in which only the fringe produces, and the prices are given by the competitive rule of 4.1;
- (ii) One in which the cartel alone produces, in which the cartel's marginal revenue follows 4.2 and price path will be the one a pure monopolist would set; and

(iii) One in which there is simultaneous production in which case both 4.1 and 4.2 must be satisfied, and the price follows the competitive path. (This condition requires that elasticity should be increasing with price along the consumer demand curve.)

However, it is not necessary that all these phases occur in all cases of Nash equilibrium, nor is it necessary that they occur in the same sequence. It all depends on the cost functions which are used for both the fringe and the cartel. It is in regard to this aspect that the various theoretical models differ. Let us consider first the model developed by Salant.

Salant S.W.(1976):⁶

Salant's model has all the assumptions and parameters specified in the beginning of this chapter. The characteristics of the partially cartelised resource markets are also the same. He also employs the concept of Nash equilibrium in his model. Besides, he has one very restrictive assumption in his model. He assumes that all the firms in the market have identical cost functions and initial reserves of the resource (oil in his model). As a result, (i) the cartel owns larger reserves of the resource than any single firm in the market, (ii) in case of upward sloping marginal cost curves, the cartel can extract the resource at the same rate as any other single firm, but at a lower cost. These two characteristics give the cartel market power over all other firms.

Salant initially assumes that extraction costs are zero. The dynamic optimization conditions for the fringe as well as the cartel are the same as we discussed before in

the context of Nash equilibrium. Therefore, the two 'marginal' conditions of Nash-equilibrium as stated in 4.1 and 4.2 hold, but with a slight modification. Since zero extraction costs are assumed, the terms c^f and c^m will disappear. Condition 4.1 will now mean that as long as the sales of the fringe are positive, price should grow at the rate of interest. Condition 4.2 will mean that as long as cartel sales are positive, its marginal revenue derived from the excess demand curves should grow at the rate of interest.

Since the firms are assumed to be identical, both the fringe and the cartel will begin producing simultaneously. When the price reaches a certain level, the fringe exhausts reserves and the cartel continues to sell. It exhausts its output by the time price reaches \bar{P} . The reason why it is the cartel which completes extraction and sale last is the following. Suppose, the cartel drops out before the fringe exhausted its stock. Then, the price path would be rising at the rate of interest while both the sectors co-existed in the market. It would now continue to rise at the same rate after the cartel stopped selling. Now, let us compare some early period when cartel sales are positive to some later period when its sales are zero. When cartel's sales are positive, its marginal revenue will be less than price and will be equal to price when its sales are zero. Since price is still growing at the rate of interest, even after the cartel sales fall to zero, marginal revenue would have to grow at a higher rate than interest when compared with the marginal revenue in a period of positive sales. But this would give the cartel an incentive to alter its strategy. Therefore, in equilibrium, the competitors can't

be selling after the cartel drops out.

Thus, the 'Salant Equilibrium' has two phases - Phase I in which both the fringe and the cartel operate in the market and price as well as cartel's marginal revenue increase at the rate of interest. The fringe exhausts its stock by the end of the first phase. In phase II, cartel alone operates, price increases at a rate less than the interest rate while the marginal revenue still increases at the rate of interest. The cartel exhausts its resource stock when price reaches P.

If we denote by S the time at which the first phase ends and the second begins, then

Phase I : For $0 \leq t \leq S$, both $q_f(t)$ and $q_m(t)$ are positive. Both discounted price and discounted cartel marginal revenue are positive. .

$$P(t)e^{-rt} = P(0); \quad 0 \leq t \leq S$$

$$\left[P(t) + \frac{dP(t)}{dq_m(t)} q_m(t) \right] e^{-rt} = MR(0); \quad 0 \leq t \leq S$$

where,

$$MR(0) = P(0) + \frac{dP(0)}{dq_m(0)} q_m(0).$$

Both $q_f(t)$ and $R_f(t)$ decline to zero as $t \rightarrow S$, but $R_m(S)$ is positive.

Phase II: For $S \leq t < T$, $q_f(t)$ is zero and $q_m(t)$ is positive, cartel (and the market) discounted marginal revenue is constant, which implies that $P(t)$ is increasing and $q_m(t) = Q(t)$ is declining. Since elasticity of demand is increasing, $P(t)$ rises at less than the rate of interest everywhere.

$$\frac{d[P(t)e^{-rt}]}{dt} < 0.$$

$$S < t \leq T$$

$$\left[P(t) + \frac{dP(t)q_m(t)}{dq(t)} \right] e^{-rt} = MR(0); \quad S < t \leq T.$$

Both $q_m(t)$ and $R_m(t)$ approach zero as $t \rightarrow T$, so that $P(t)$ approaches P .

The equilibrium path of Salant's model will be as shown in fig 4.1.

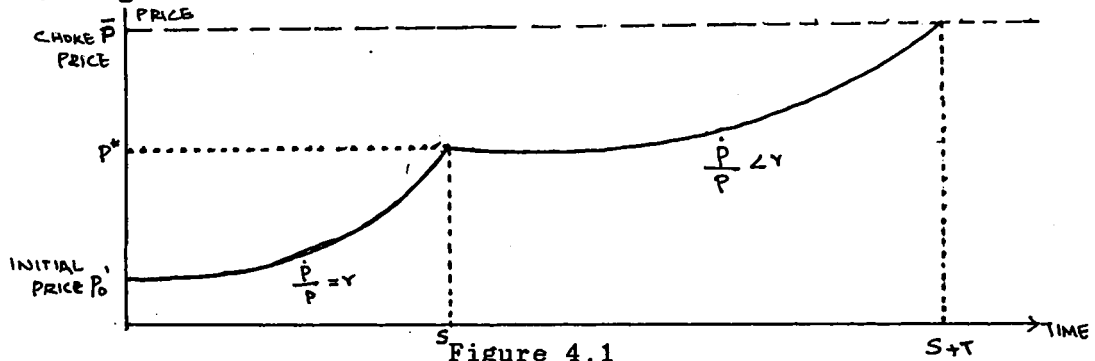


Figure 4.1

Equilibrium Time path of Price in Salant's Model

Retaining his assumption of identical nature of all firms in the markets, Salant shows that the characteristics of the equilibrium do not change when the assumption of zero extraction costs is relaxed and convex cost functions are introduced. We shall not go into the discussion of this case here since it is an extension of the basic model (For details, see Salant (1976), pp. 1088-1090)

As we noted just now, the important assumption in Salant's analysis is that all firms have identical reserves and cost functions. This is very restrictive and unrealistic.

Ulph A. and Folie M. (1980)⁷ and Ulph A. (1982):⁸

Ulph and Folie have a more general model than Salant's. They relax the assumption of identical cost functions and reserves for all firms in the market. They assume that cost

functions are linear, but they are not identical for the two groups. In their model, firms within each group are assumed to be identical. Since linear cost functions are assumed, both c^f and c^m are constants, but are not identical. Besides, they assume a stationary linear demand curve.

There are various cases that can occur in a Nash - Cournot equilibrium under these assumptions. But, here we will be concerned with two extreme cases, namely,

- (i) a case when the cartel has a 'substantial' cost advantage over the fringe, (i.e., $c^m < c^f$)
- (ii) when the fringe has a cost advantage over the cartel (i.e., $c^f < c^m$) and possesses 'significant' reserves.

In the first case, Ulph and Folie show that the Nash equilibrium will consist of three phases. In the first phase, the cartel alone produces, and the price path will be the one which a pure monopolist would set. Then, after some time the second phase begins and the competitive fringe commences production with both the cartel and the fringe firms producing together until the cartel exhausts its reserves. In this phase, the cartel would like to set the monopoly price, but it cannot do so, since the monopoly price path is undercut by the competitive price path. Finally, there will be a usual period during which the fringe alone produces until its resources are exhausted.

We can show the situation in the following figure (fig.4.2a). In this figure, time runs backwards from the final exhaustion date. The final exhaustion date is at the origin.

The first phase occurs between t_3 and t_2 and the cartel alone produces. The second phase of simultaneous production

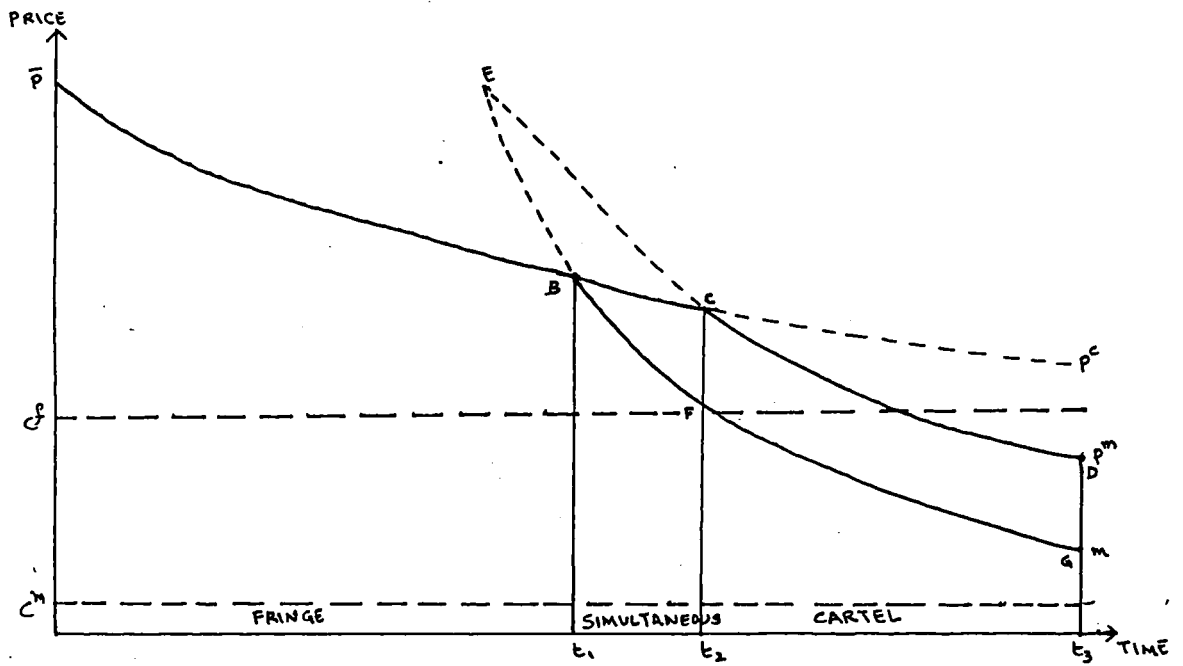


Figure 4.2a

Nash Equilibrium Price Path when $c^f > c^m$

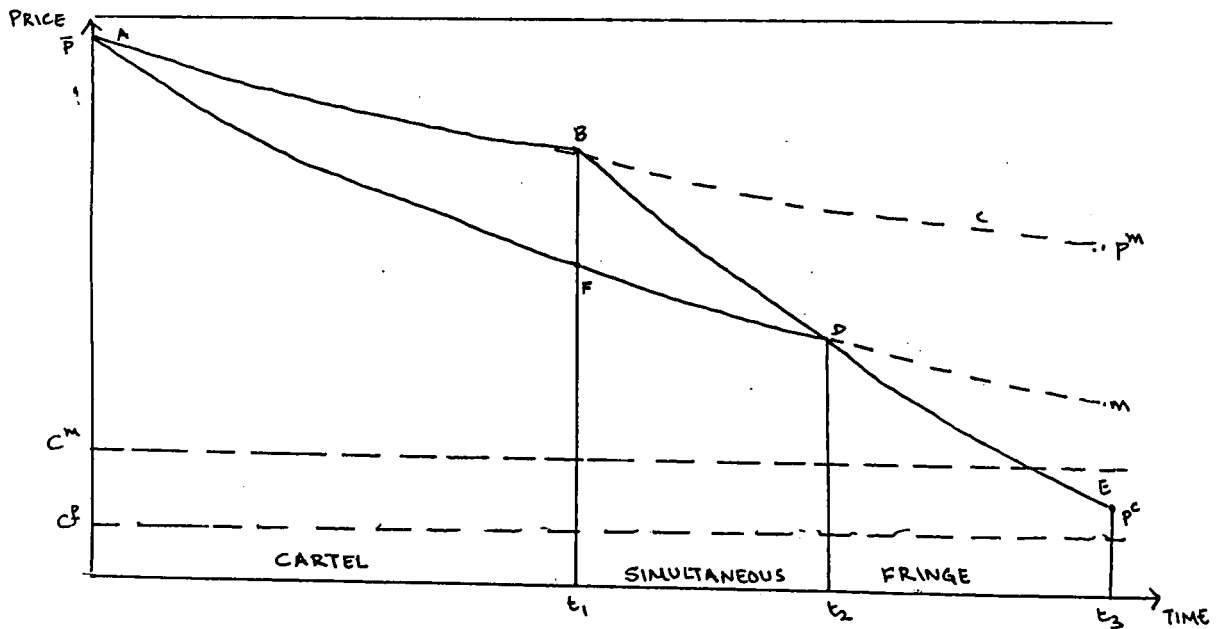


Figure 4.2b

Nash Equilibrium Price Path When $c^m > c^f$

is between t_2 and t_1 and price follows the competitive path. By the time, this phase ends t_2 , cartel exhausts its reserves and in the third phase only the fringe produces. At the end of this phase, i.e., the time of final exhaustion, fringe's reserves are zero and price rises to \bar{P} . The difference in steepness of the two price paths (competitive and monopoly) is due to the fact that $c^f > c^m$, and the competitive price path which is asymptotic to c^f will hence be less steeper than the monopoly price path.

In the second case, where the fringe has a cost advantage ($c^f < c^m$) and significant reserves, the Nash equilibrium consists of three phases, just as in the former case, but exactly in the reverse order. Initially, only the fringe firms produce and price follows the competitive price path. The second phase begins when the cartel also starts production, but it cannot charge the monopoly price in this period since the monopoly price path is undercut by the competitive path. This phase of simultaneous production ends when the fringe exhausts its stock and drops out leaving the market to the cartel. Later, cartel alone produces till its resources are exhausted when price reaches \bar{P} . This last phase has the price path set by the pure monopolist.

The situation is shown in fig 4.2b and the description remains the same as that of fig. 4.2a, except that the roles of the cartel and the fringe are interchanged.

Having briefly discussed a few cases of Nash equilibria, let us now consider the outcomes of using the concept of Stackelberg solution. Here we discuss the essentials of models of Ulph & Folie (1979),⁹ Ulph (1982),¹⁰ and Newbery (1981),¹¹ but not each of them

separately.

Stackelberg Equilibrium:

As mentioned before, even in the Stackelberg equilibrium, fringe is the follower and acts as a competitive price-taker. The cartel on the other hand, now takes the reaction function of the fringe into consideration while choosing its optimal price path. In the Nash-cournot model, the cartel acts like a monopolist faced by the excess demand curves but in the Stackelberg solution it would like to price like a monopolist in the resource market, faced by the total demand curve. But the presence of the fringe constrains the cartel since it knows the reaction function of the fringe to the price it sets. Hence, due to the presence of the fringe, the cartel may be constrained at time to set the price given by competitive price path which may lie below the pure monopolist's price path. In this situation, there can be the following three scenarios.

- (i) The competitive price may be ineffective as a constraint. The cartel then sets the monopoly price.
- (ii) The cartel may be constrained by the competitive price, but this lies above the cartel's marginal cost plus exhaustion rent. In this situation, the cartel will set the competitive price, and will still supply all the output the market demands.
- (iii) The competitive price can prove to a total constraint, in the sense that it does not allow the cartel to cover its marginal cost plus exhaustion rent. In such a situation, only the fringe will produce.

From this discussion, one very important point emerges

which is actually a crucial difference between the Nash and Stackelberg models; that is, there can be no simultaneous production by the cartel and the fringe in a Stackelberg equilibrium.

Let us now consider Stackelberg equilibria for the two extreme cases used for the Nash equilibrium, namely, (i) case I in which the cartel has a 'substantial' cost advantage over the fringe, and case II, where the fringe has a cost advantage over the cartel and possesses 'significant' reserves. These two cases are depicted in figures 4.3a and 4.3b, respectively.

In case I, since $c^f > c^m$, the cartel starts acting like a monopolist. In figure 4.3a, this is the time period between t_3 and t_2 . It sets the unconstrained monopoly price. But between t_2 and t_1 , the cartel is constrained to set price at the fringe's limit price, $P^C(t)$. Still the cartel is the sole producer at this stage. But, by t_1 , it exhausts its reserves. Between 0 and t_1 , the fringe alone produces until its resources are exhausted by the final exhaustion date, and price reaches \bar{P} . We can call the period between 0 and t_2 as the competitive phase and the period between t_2 and t_3 as the monopoly phase. The interesting point is that the cartel is the sole producer in the monopoly phase and in some part of the competitive phase.

It's now very obvious how the equilibrium will look like for case II. It is shown in figure 4.3b. Since there is no cost advantage, the cartel can act like a monopolist only in the last phase of production. There will be an intermediate phase between t_1 and t_2 when the cartel will be

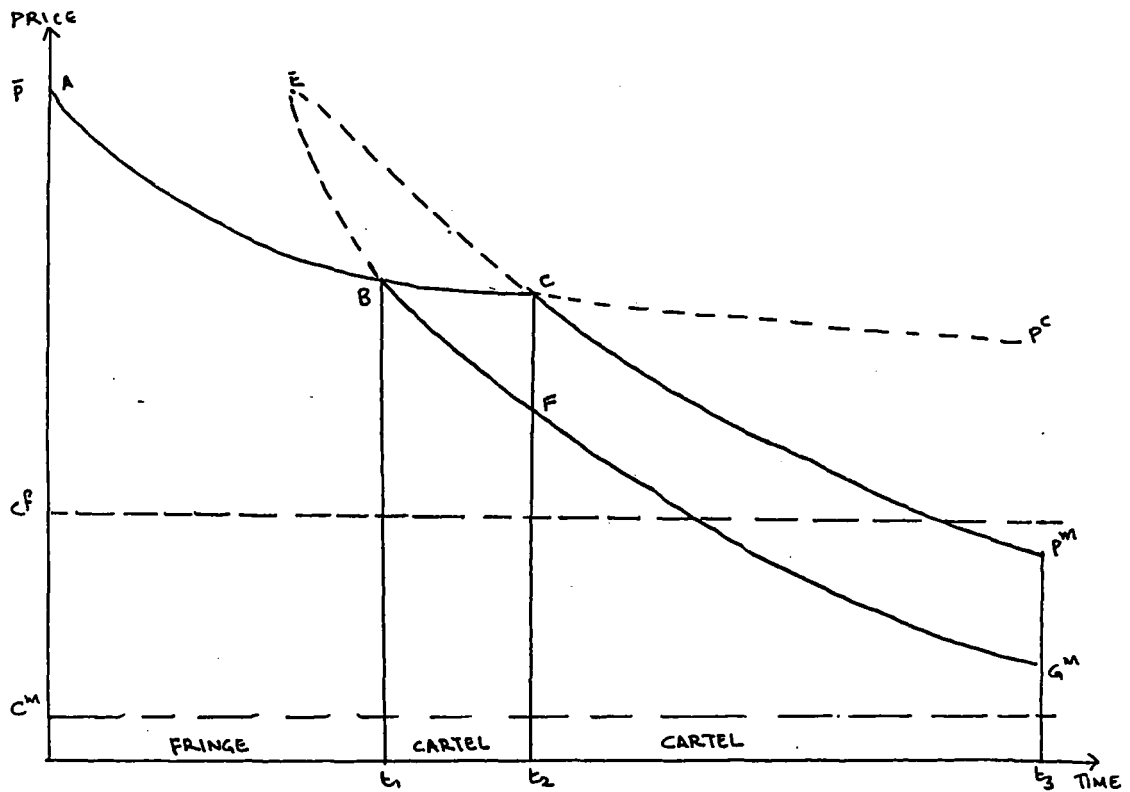


Figure 4.3a

Stackelberg Equilibrium Price Path when $c^f > c^m$

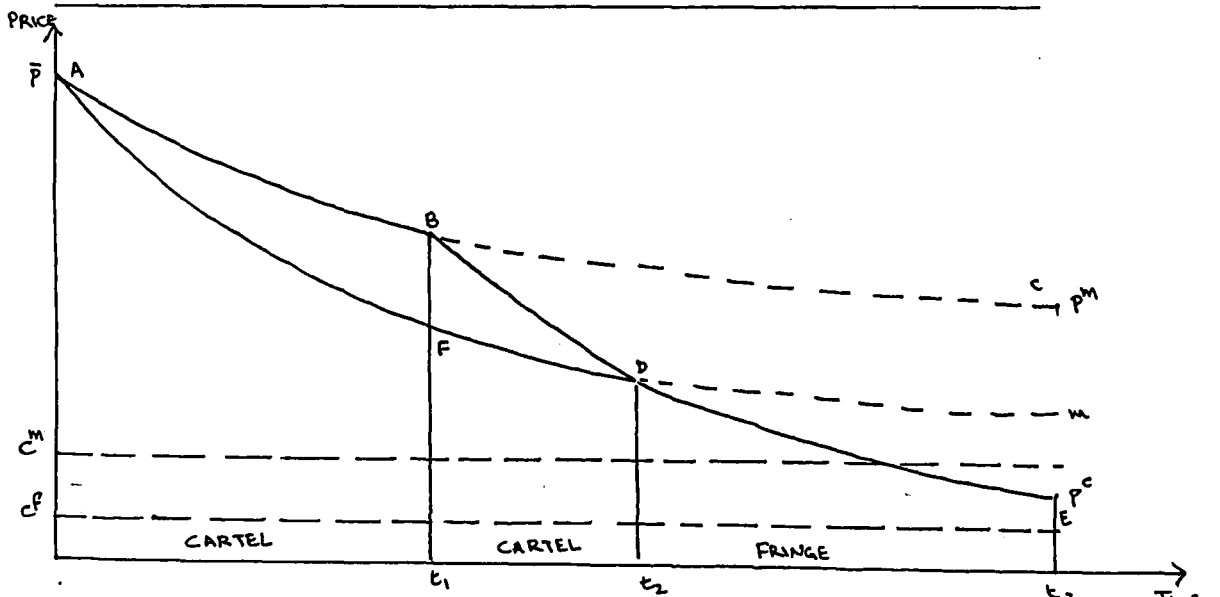


Figure 4.3b

Stackelberg Equilibrium Price Path when $c^f < c^m$

constrained to charge the competitive price while producing enough to clear the market. Of course, with its cost advantage, it is the fringe which starts off and competitive price prevails in the market, and the fringe produces in the beginning till its reserves are exhausted.

Dynamic Inconsistency:

We mentioned in the beginning of the chapter that in order for the open-loop strategies to be reasonable, it is necessary to assume the existence of a specific type of institutional framework which has either perfect future capital markets with no recontracting, or one of binding contracts. Suppose such an institutional framework is absent. Then there arises a question of discrepancy between ex-ante and ex-post equilibria, or dynamic inconsistency.

Let us consider the Stackelberg solution for case II. In this case, the competitive phase precedes the monopoly phase, and moreover, within the competitive phase, the fringe depletes its resources first. Suppose it takes \hat{t}_1 periods from the start at which fringe's resources are exhausted. Now the cartel will act competitively for a while after \hat{t}_1 . The reason for this is that if at \hat{t}_1 , the cartel acted as a monopolist there would be an immediate jump in price. If this was a part of an announced price path, the fringe would withhold production from earlier periods to make a capital gain at the jump point, as price would be increasing at a rate higher than the rate of interest at this point. This would continue until the competitive price path had risen sufficiently to meet the monopoly price path. But this would delay the start of the monopoly phase. The costs of this delay, are more than the gains of acting as a

monopolist throughout.

But \hat{t}_1 is reached in real time. The fringe resources are exhausted, and hence they are no longer a threat to the cartel and the cartel will immediately raise the price to the monopoly level. And thus, the ex-post equilibrium will differ from the ex-ante one.

However, we should note that all open-loop Stackelberg equilibria need not be dynamically inconsistent. For example, in case I equilibrium, the fringe does not deplete its reserves until the cartel has exhausted its resources, and therefore the problem of dynamic inconsistency does not arise. Dynamic inconsistency must occur, when $c^f < c^m$.

On the other hand, Open-loop Nash equilibria is dynamically consistent in both case I and case II. In case I, it is obvious since fringe produces and sells even after cartel exhausts. In case II equilibrium, though the fringe produces in the first period, there is a period of simultaneous production in which both the cartel and the fringe operate. Here, there is no risk of any price jump since the fringe holds reserves until the unconstrained monopoly trajectory is reached. Once the unconstrained monopoly path is reached, the cartel will not have any incentive to deviate from the predicted price path. In the period of simultaneous production, the cartel has no power to deviate from the competitive price path since its market share is given by the Nash equilibrium. Thus, open-loop Nash strategies are dynamically consistent.

When the open-loop Stackelberg strategies are dynamically inconsistent, another solution concept called 'feed back or closed loop' Stackelberg strategy is used. The computation of such strategies is highly complex and is

beyond the scope of our work. We just mention the idea behind this concept. In case of feed back strategies for our problem, we can think of the existence of a sequence of spot markets for the resource. The cartel sets the spot price for the resource in these markets, and the fringe sets its current output. The optimal decision depends upon two things -- (i) what happens in the spot market, and (ii) expectations of each group about the future. " The rational expectation is that at each moment of time, each group will act so as to maximize the present value of its profits from that moment onwards. An equilibrium set of 'feed back' strategies will be referred to then as a rational expectations equilibrium."¹²

The Phenomenon of Limit-Pricing:

Another type of Stackelberg models of pricing behaviour by a dominant firm in a market for an exhaustible resource are the limit-pricing models.

Gilbert (1978)¹³ has put forward one of the important models in this respect. The resource market in this model has all the characteristics noted in the beginning of this chapter. He analyses the dominant firm model. There is both a fringe and a competitively supplied backstop technology. He assumes an isoelastic demand function, with elasticity less than one throughout. The demand curve is also different from our initial specification in another respect, i.e., at $P = \bar{P}$, it is infinitely elastic since the substitute source of supply becomes active.

He assumes two different functional forms for the fringe costs of production. In the first case, marginal

production costs are independent of current rate of output, and in the second case, they are constant upto a capacity constraint.

In the first case, Gilbert shows that the optimal cartel strategy is independent of the cartel's production costs and discount rate, given the demand curve assumed in the model. Since fringe capacity is unconstrained, the cartel will set the price path such that it satisfies equation 4.1 and fringe reserves are exhausted by the time P reaches \bar{P} . After the fringe reserves are exhausted, the cartel maximizes profits by pricing marginally below the cost of a substitute source of supply (i.e., marginally below the limit price, \bar{P}).

In the second case, the fringe capacity is sufficiently small and its response to price is determined by the level of its capacity. In such a case, the cartel may find it in its interest to dominate the market at once. The smaller the fringe capacity, the greater is the monopoly power of the cartel. In such a case, the cartel may simply maximize the rate of profits, given the capacity-constrained response of the fringe. Eventually, the capacity constraint ceases to be binding; as depletion approaches, the optimal price will eventually rise to the level of price of the substitute.

The two optimal price trajectories may look as shown in figure 4.4.

Salant (1979)¹⁴ considers case of limit pricing phenomenon in exhaustible resource markets. Here, the cartel is the monopoly, and the competitive fringe supplies the backstop technology. Its supply is perfectly elastic at $P = \bar{P}$. So \bar{P} is the limit price for the cartel. The demand curve

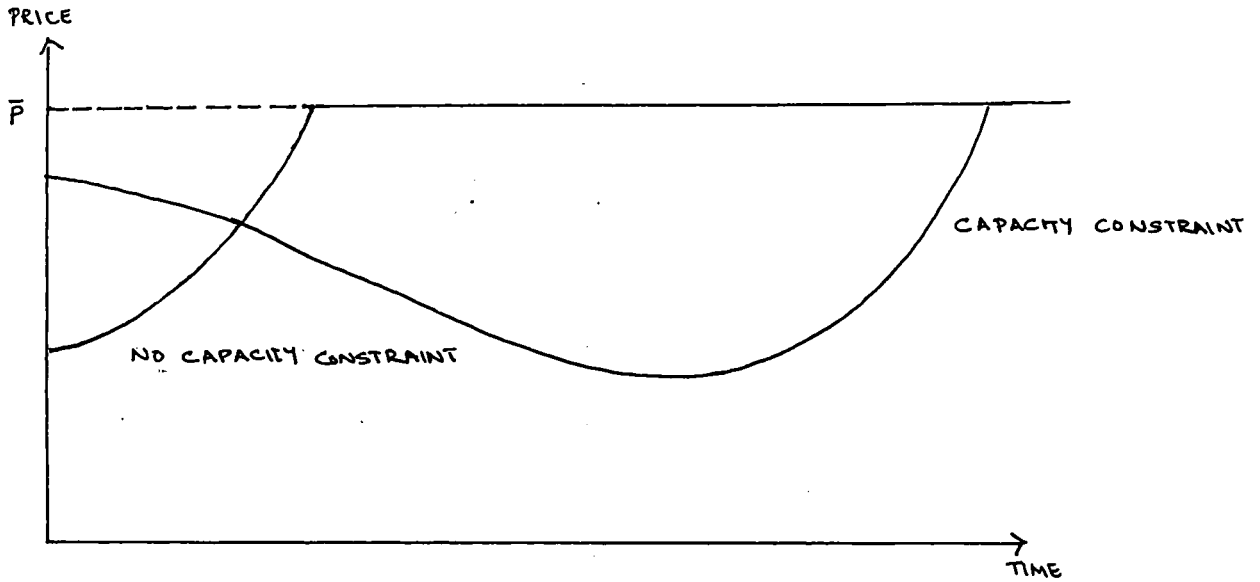


Figure 4.4

Cartel Limit-Pricing when there is both a fringe and a Competitively supplied backstop

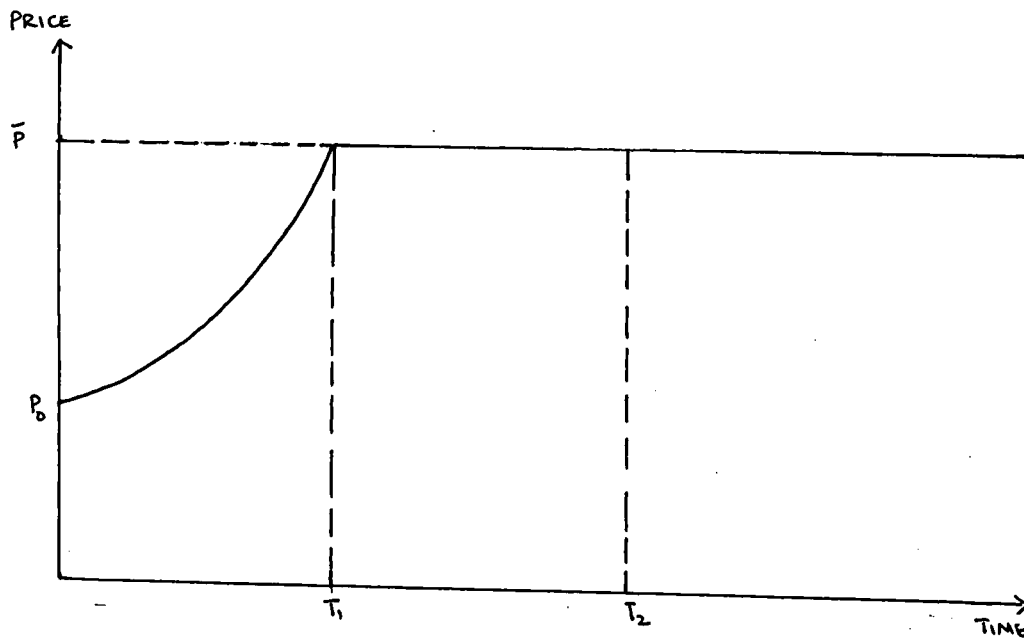


Figure 4.5

Cartel Limit-Pricing when Fringe supplies the backstop

is elastic throughout.

The optimal price strategy of the cartel will have two phases. In the first phase, the cartel's price will be such that its marginal profits increase at the rate of interest. In the second phase, the monopolist charges the limit price but prevents entry by supplying the entire market himself. The first phase exists if the cartel has substantial initial reserves.¹⁵ The second phase is necessary for the following reason. Suppose the fringe exhausts all its reserves by T_1 in Fig 4.5. This involves a moment of positive extraction immediately followed by a moment of zero extraction. This is not optimal since marginal profit would jump up between the two moments. This requires production to linger on for sometime after T_1 ; otherwise, it will make the monopolist to revise his production plans. The two phases are shown in figure 4.5.

Gilbert and Goldman (1978):¹⁶

We saw how a cartel responds if a competitively supplied substitute for its resource enters the market at a particular price. This particular price becomes the limit price for the cartel while choosing its optimal price policy.

Gilbert and Goldman examine the implications of potential competition in an exhaustible resource market initially controlled by a coordinated cartel (or monopoly) with a finite supply.

The entry behaviour of the competitors is assumed to be of two types.

(i) Quantity triggered entry: In this case at least one firm enters when the monopolist's reserves fall below a critical level. The decision to enter is independent of the

market price and is solely conditioned on the monopolist's remaining stock. The rationale of such an entry is that the potential competitor expects to have a substantial impact on the monopolist's market share.

(ii) Price triggered entry: In this case, potential entrants are myopic price-takers. A small entrant should be concerned only with the current price, since one small producer is not likely to cause a change in the monopolist's pricing policy. So, entry takes place when the current market price reaches a particular level.

In both the cases, it is assumed that potential entrants are high cost firms in comparison with the monopolist. The demand curve is assumed to be elastic throughout the relevant range.

Gilbert and Goldman show that in both cases, the threat of entry makes the monopolist to increase the initial price above the level that is optimal when entry is restricted, assuming that the monopolist's initial reserves are substantially large. Of course, competition may reduce prices in the long run, and thus offset the impact of the initial high prices. But, the net effect of potential competition may reduce welfare relative to a situation where entry is restricted. But, Gilbert and Goldman are not for protecting monopoly for this reason. The point is that the strategic monopoly response to entry may have unexpected consequences, and that threat of entry does not necessarily remove the damages of monopoly.

The foregoing discussion focusses on the theoretical issues that arise in the analysis of partially cartelised exhaustible resource markets. The basic purpose was to bring

out the ideas behind the alternative solution concepts and the resulting outcome, and not on the actual derivation of these results.

Let us now look at some of the models which have attempted to compute numerically the optimal price trajectories and thus to predict the likely nature of resource price paths.

II SIMULATION MODELS

The second group of studies of partially cartelised exhaustible resource markets are computation oriented. Basically, there are two groups of such models, namely, non-optimization simulation models and optimization simulation models.

In a non-optimization simulation model, a basic model is specified. Assuming plausible values for various parameters such as demand elasticities, demand growth rates, supply conditions for cartel and non-cartel firms, etc., a variety of price paths are generated. Of these paths, one or more is selected as the 'best' or 'most likely' according to same pre-specified criterion.

Alternatively, the optimization models (in principle at least) evaluate all the price paths possible. They use a plausible and quantifiable objective function, usually the present value of cartel profits, to select the best price path. In general, since the computation of optimal price trajectories is quite complex in optimal control problems, the specification of dynamic cartel models has been fairly simple, thereby limiting the generality of the conclusions on optimal price paths only to those functional forms.

Let us briefly discuss a few of these simulation models.

Blitzer - Meeraus - Stoutjesdijk (B-M-S) Model(1975):¹⁷

B-M-S attempt to evaluate the prospects for OPEC and the likely future path of world oil price using a dynamic simulation model. The problem facing the OPEC or any subgroup thereof, is fairly simple. There is an inelastic short-run demand curve for oil. This allows OPEC to obtain high prices and revenue today, but at the expense of future revenue, since OPEC market share is lost to other producers and to alternate fuels. Thus, OPEC's basic objective is to identify the price pattern over time which best satisfies the dual objectives of maintenance of the market share and high current revenue, as opposed to simply a 'maintainable' constant price, which implies the need for specifying some objective function. In this case, the criterion is the value of net foreign asset holdings in 1995, which is defined as the accumulated difference between the export revenues and import spending plus the value of oil still in the ground in 1995. That policy is the best which gives the highest value of this criterion function.

B-M-S evaluate the relative merits of various price trajectories considered under two assumed scenarios - (i) OPEC as a whole agrees to any necessary prorationing, and (ii) a hypothetical sub-OPEC cartel assumed to be price maintaining residual supplier, with other members maintaining production at currently projected capacity.

In the former model, the demand for OPEC oil is derived as the difference between global energy demand and non-OPEC

supply, or OPEC is treated as the residual supplier. In the sub-OPEC cartel model, it is the sub-OPEC cartel which takes the role of the residual supplier and the rest of the producers produce at projected capacity.

We do not go into the various alternative price policies evaluated in the model. The most frequently obtained result was that OPEC should reduce price from the 1974 level of \$10 per barrel. The main reason for the optimality of price reduction strategies was the necessity of discouraging investments in alternative fuels and preserving the OPEC's market share. The results of the sub-cartel model also confirm the results of the basic model.

Kalymon's Model (1975):¹⁸

Kalymon's model is explicitly optimizing in structure. His model essentially considers that OPEC is the residual supplier of oil, and thus faces the rest of the world's excess demand for oil. Optimal price trajectories for oil over time are calculated. A novel feature of Kalymon's model is that he has specified separate demand functions for export and domestic markets. This allows the examination of optimal price discrimination policies (in the two markets) as well, since it is essential to take into account the domestic markets which are expected to grow in importance in the future, imposing pressures on export oil pricing. Kalymon assumes constant marginal costs of extraction with respect to current output, but rising with cumulative output. Demand is linear and grows with time. He computes a price trajectory which maximises the present value of oil revenues less extraction costs plus the present value of

consumer surplus stemming from the domestic consumption of oil. The price trajectory thus obtained by an optimizing model reduces the possibility of a complex but optimal price strategy being overlooked.

The optimal price path generated by the optimization for a monolith OPEC required an immediate price reduction from its 1975 level of \$10 per barrel to \$8.68 per barrel, then a 1 percent increase per year until a price of \$15 per barrel is reached in the year 2027 and reserves exhausted. The strategy required a large measure of production restriction. The results obtained were quite sensitive to both the discount rate and the assumed substitution price. (Substitution price refers to the price at which demand for OPEC oil becomes zero.)

In addition to the optimization for the monolith OPEC, Kalymon carried out same exercise for the various sub-OPEC cartels, and examined the optimal price strategies when various subgroups act as residual suppliers. The results reported for the various coalitions were also similar to the ones reported for the monolith OPEC.

Thus, just as in the B-M-S model, Kalymon's results also showed that the 1975 price of oil (\$10 per barrel) was not optimal and needed to be reduced.

Cremer and Weitzman (1976):¹⁹

Cremer and Weitzman (C & W) have a true cartel model to simulate the long term world oil prices. It has all the characteristics of the resource market outlined in the beginning of this chapter with a price setting cartel and price following fringe, and the cartel is the Stackelberg dominant firm maximising the present value of its profits

while setting its price path taking into account the output response of the fringe to the price path it sets.

The fringe has an exogenously given capacity constraint which is assumed to be growing at a rate \dots . Extraction costs are assumed to be dependent on cumulative production. The world demand for oil is assumed to be linear and is assumed to grow at an annual rate g .

Based on the above model, C & W simulated the movement of oil price along a Stackelberg oligopoly equilibrium path.

Initially, C & W simulated a static model using the same cost and demand functions (which were used in the dynamic model) in their static form and assuming that the fringe produced at full capacity [11 billion barrels which was the non-OPEC oil output in 1975]. The cartel was assumed to maximise one period profits. This yielded a \$8.9 per barrel price of oil in any year for the above specification. OPEC's pure monopoly price in the static model would be \$18 per barrel where OPEC was the sole producer. So it was evidently clear that a greater fringe capacity lowered OPEC's monopoly power. This suggested that as the fringe capacity increases, OPEC's monopoly power would be eroded, and hence, this intertemporal consideration would lead the OPEC to raise initial (1975) price to a level above \$8.9 per barrel which was actually borne out by the dynamic simulation results.

In running the dynamic simulation, C & W measured time in intervals of ten years. In their 'preferred' specification, C & W assumed that both g and \dots equal 3 percent per year. As in the static model, fringe's capacity was taken to be 11 billion barrels per year. The long run

demand elasticity at \$10 per barrel was assumed to be -0.4 . The demand curve supposed that the world demand for oil would become zero at a price of \$35 per barrel (in 1975 dollars). The discount rate for the fringe was assumed to be 8 percent per year and for the OPEC, 5 percent per year.

The following are the results of the simulation with the above specification.

| | 1975-85 | 1985-95 | 1995-2005 | 2005-2015 |
|-------------------------------|---------|---------|-----------|-----------|
| 1. Price of oil per barrel | \$ 9.8 | \$ 10.3 | \$ 14.7 | \$ 20.8 |
| 2. Annual non-OPEC production | 13 | 17 | 12 | 4 |
| 3. Annual OPEC production | 5 | 6 | 14 | 21 |

All prices are constant prices in 1975 dollars and all the production figures are in billion barrels. A few crucial points need to be noted.

(i) The price of crude oil hardly rises in the first 20 years, whereas it rises very sharply in the next two decades to reach nearly \$21 per barrel in the period 2005-15.

(ii) A second point to note is that in the first 20 years, OPEC's market share is about a third, but then it starts increasing until it eventually holds a complete monopoly in the fourth period.

(iii) A third crucial point is that the price during the first 20 years is very close to the short run optimum price for OPEC which suggests that in the first 20 years, the fringe has a good deal of bite. One would in fact expect OPEC to set the initial price low so as to allow the fringe to deplete its resource faster and run up its extraction

cost before OPEC enters the market in a serious way. But the capacity constraint of the fringe does not allow this to happen.

In fact, this proposition has got the support of a simulation which is carried out on the specification as before except that now it is assumed that there is no capacity constraint for the fringe. This simulation shows that the prices in the first 20 years are much lower than the prices obtained in the previous simulation and OPEC does not produce anything in the initial years. It finds profitable to allow the fringe to dominate the market in the initial years. In the second period, OPEC controls nearly half the market. For the last two periods, both the simulations yield the same results. It is not surprising since during this period the fringe's capacity constraint does not have any impact.

Pindyck's Model (1978):²⁰

While C & W model of OPEC behaviour in the world oil market is a Stackelberg dominant firm equilibrium model, Pindyck's model is essentially a Nash-Cournot type of model where OPEC is the residual supplier who acts as a monopolist facing the 'net demand function' (i.e., total world demand minus fringe supply). The fringe, as usual, is a price-taker. Each sector maximises its present value of profits - given the optimal choices of the other sector.

Pindyck's purpose was to verify the widely held belief that the formation and the later success of OPEC as a cartel was due to the large gains to its members resulted

from cartelisation. This required measuring potential gains to producers from cartelisation of any exhaustible resource (which in this case was crude oil). Pindyck carried out similar studies to the cases of two other prominent resource cartels, namely, the International Bauxite Association (IBA) and the International Council of Copper Exporting Countries (CIPEC).

His approach to measure potential gains from cartelization was very simple. First, using the model of the cartelised resource market, the optimal price trajectory was calculated. Next, the optimal competitive price trajectory was calculated where it was assumed that the world resource market is perfectly competitive in the sense that even the members of the cartel in the previous exercise now behave as competitive price takers. In other words, the members of the cartel also behave as if the cartel was never formed. Once these two price trajectories were calculated, it was necessary to find the two profit figures under the two states of the world for the cartel members as a whole and then compare them.

Pindyck's model has some interesting features. He builds adjustment lags into his total demand functions and the fringe supply functions. A distinction is also made between the short run and long run elasticities of demand.

Essentially, the cartel selects a profit maximizing price sequence, subject to the excess demand to the commodity in question, since the fringe firms are assumed to supply all they wish at the prevailing market price.

Petroleum and Gains to OPEC:

Pindyck's simulated the world oil market with OPEC as the cartel and the other producers constituting the fringe. He initially applied 5 per cent discount rate for both OPEC and the fringe. His simulation yielded a 1975 oligopoly price of oil of \$13.25 per barrel. This falls to a little under \$10.00 per barrel during 1978-80. Then it starts rising gradually to a little over \$20.00 per barrel in the year 2010. Total world demand for oil stayed close to 17.5 billion barrels per year during these thirty five years. OPEC share of this market remained a little over 50 per cent until 1980 and then rose gradually until 2010 when it exceeded 70 per cent. The competitive price computed under the same specification rose steadily from \$4.62 per barrel in 1975 to about \$25.5 per barrel in 2010. Pindyck found that the cartelisation of oil market resulted in a 55 per cent increase in the present value of profits to members of OPEC -- indicating that the incentive for the cartel to remain cohesive was strong.

Bauxite and gains to IBA:

A specific feature of world bauxite demand schedule which Pindyck postulated was that the demand for bauxite is inelastic till a price of \$15.16 per metric ton (in 1973 prices) but beyond that it becomes almost infinitely elastic, since it is economical to produce alumina from sources other than bauxite at prices beyond this 'limit price'. Besides, the initial reserves of IBA were very large (i.e., 11,000 million metric tons). At 5 percent rate of discount, Pindyck's simulation runs yielded an

optimal price policy for the IBA wherein the prices per metric ton of bauxite lay between \$12.30 and \$13.00 in the period 1975-2000. Thereafter, it gradually increased to the 'limit price'. The optimal monopoly price trajectory had the typical characteristic of dropping initially for about 5 years and then rising slowly. The price fluctuated over a small range, but was always within a few dollars of the 'limit price'. The competitive price would have risen from \$6.40 to \$9.00 per ton during the same period. The total demand for bauxite increased from 62.9 million metric tons to 172.1 million metric tons by 2030. Throughout the period considered (1975-2045), IBA's market share remained between 70 and 80 per cent, which indicated that the rest of the producers did not matter. The cartelisation of bauxite market resulted in a 60 per cent increase in the sum of discounted profits for the IBA members. If the rate of discount was 10 per cent, then the same increased by a whopping 400 per cent.

Copper and Gains to CIPEC:

CIPEC accounted for 34 per cent of the non-communist world's copper production in 1974. An additional feature of the copper market is the secondary supply of copper from scrap supplied by the fringe besides its supply of new copper. The initial reserve level of CIPEC was also very low (135 million metric tons). Pindyck's simulation runs with 5 per cent rate of discount for both CIPEC and fringe yielded a 1975 price of copper at \$1.23 per pound (in 1974 U.S.dollars). The optimal monopoly price oscillated around \$1.00 per pound throughout till 2010. But an envelope of these prices showed the same pattern as the prices of

petroleum and copper. It fell in the initial years and later rose slowly as depletion neared. Total world demand for copper was stable around 7 million metric tons in the initial five years and then restarted rising slowly. CIPEC's share of this remained around 20 per cent throughout, which showed that the CIPEC did not have the monopoly power like OPEC or IBA. The relative long term gains to CIPEC from the cartelisation of the world copper market was also not too large. It resulted in only an 8 per cent increase in the sum of discounted profits to CIPEC members; which also required fluctuations in price (and hence in profits) which the member countries would like to avoid; and consuming countries might anticipate and counteract by stock piling. On the other hand, when the monopoly price trajectory was smoothed, the increase in profits reduced to only 3 per cent. Thus, in case of copper, the results indicated that there was little incentive for the members of CIPEC to have closely co-ordinated price and output policies.

Pindyck's simulations for the world oil, bauxite and copper markets bring forth the following issues.

In case of OPEC and IBA., the gains from cartelisation were substantial if optimal pricing policies were followed. But, this was not the case with CIPEC. The reasons for this are mainly two.

(i) In case of oil and bauxite, short term lag adjustments in demand and competitive supply are slow for price change, thus allowing for large short-term gains to the respective cartel, whereas secondary copper supply responds quickly to price changes.

(ii) The second crucial factor is the market share of the cartel in total non-communist world production. While

OPEC and IBA account for nearly two thirds of non-communist world's petroleum and bauxite production, respectively, CIPEC's share is only around one third in the total.

There are certain other issues which are ignored while estimating gains to cartel members, which Pindyck is not unaware of. For example, the supply response of the fringe producers will be different if they form expectations about future price. On the other hand, a situation of bilateral monopoly might emerge in the resource markets if the consuming countries form a buyers' cartel to weaken the producers' cartel in which case the price behaviour will be different.

Pindyck (1977):²¹

In the study just discussed, Pindyck was estimating the gains to the members from cartelisation of an exhaustible resource market as larger gains were supposed to indicate the presence of an incentive for the cartel to remain cohesive. He found that there were large potential gains to OPEC and IBA, if the world petroleum and bauxite markets were cartelised, respectively.

In this study, the author has extended the analysis with an additional question to answer - will the existence of total potential gains for the cartel as a whole from following the optimal price policy ensure that the cartel in question remains stable? It may not - since if one or more members of the cartel can earn higher revenues in the long run by undercutting the cartel price and expanding production .

Pindyck's objective in this paper was to study the stability of IBA as a cartel which was shown to have large

potential gains from the cartelisation of the world bauxite market.

In the IBA, in 1975, Australia was the largest producer of bauxite, having the largest proved resource base, and it had the largest growing production capacity. It had also moved to a position of competitive advantage by not increasing its tax which the other cartel members did. Besides, it was expanding its sales and relative market share at the expense of the Caribbean countries. These developments were taken as indications that Australia may be a weak link in the cohesiveness of the IBA.

If Australia was the weak link in the unified status of IBA, then an assessment of the stability of the IBA required estimation of benefits to Australia if it remained part of the IBA and its comparison with the benefits if it quit the IBA. If the former was greater, Australia would prefer to remain as a member of the IBA.

It is a matter of detail to discuss the method of estimating the gains to Australia under the two situations which we shall avoid. He estimated the gains to Australia under the following two specific situations --

- (i) Australia remains a part of the IBA and produces a constant share of the cartel output.
- (ii) Australia leaves the cartel and produces bauxite as a part of the competitive fringe.

He used the model of world bauxite market discussed in the previous section for this purpose. His simulation brought out two important points.

- (i) There was a strong incentive for Australia to leave the IBA since by doing so, it could nearly double its profits.

(ii) The other cartel members did have an incentive to retain Australia in the cartel. But their bargaining power was very limited; and any agreement over output shares acceptable to Australia would still leave the rest of the IBA with greatly reduced profits. Thus, Pindyck's simulation did not give any strong indication of the cohesiveness of the IBA.

Hnyilicza and Pindyck (1976):²²

In this paper, Hnyilicza and Pindyck (H & P) study a related problem in the stability of a cartel, where the cartel is viewed as being composed of members with different objectives and different degrees of bargaining power (as we have seen the case of Australia in the IBA). In such a case, the issue of allocation, among members, of the implied (by the cartel's optimal price strategy) output restrictions assumes relevance. Cartels are generally believed to be unstable because of the difficulties they face in trying to resolve this question.

This study by H & P showed that the dynamics in the exhaustible resource case suggested a more general solution, namely, rotate the cut-backs among the members. The cartel they studied was OPEC, and they classified the OPEC members into two groups:

- (i) Savers - with relatively low immediate cash needs and hence a low discount rate, and
- (ii) Spenders - with high current cash needs and a high discount rate.

In a numerical simulation of pricing and saver and spender output shares, discounted profits increased for both groups (over the amounts they would receive under

historically given output shares) by making the savers absorb the initial setbacks.

The results of the model simulation called for no production from the savers initially. This is very unrealistic and as H & P recognised, the temptation to cheat would be strong because savers would risk the breaking up of the cartel before they would even begin to deplete their reserves. Further, the model does not seem to take account of the costs of moving away from historical production levels, i.e., the spenders might not be able to expand their production as rapidly as they would need to in order to take up the slack caused by a complete shutdown by the savers. On the other hand, the savers, presumably, may not prefer a complete shutdown (though temporary), since substantial investments have already been made in producing, transporting and refining oil.

Conclusion

Thus, we have discussed in this chapter a variety of theoretical and simulation models of partially cartelised exhaustible resource markets. Various forms of market behaviour are discussed. These models give a deeper insight into the real world exhaustible resource markets than the models of perfectly competitive or monopoly resource markets give. The interesting part is that the analysis of an intermediate market structure of partially cartelised markets requires the results from the theory of exhaustible resources in the two extreme forms of market structures along with the theory of imperfect competition.

On the other hand, computational models attempt at quantifying the results of the theory and attempt to see whether there is any correspondence between the optimal

strategies suggested by the theory and the actual real world behaviour. Its interesting to see that the theoretical results which Gilbert (1978) obtained for the case of the fringe with a capacity constraint are borne out by the simulation results of Cremer & Weitzman (1976).

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CHAPTER V

SUMMARY AND CONCLUSION

As we noted in the introduction, the purpose of this dissertation is to briefly review some of the important models which deal with the question of the role of market mechanism in the intertemporal allocation of exhaustible natural resources. It was noted that the importance of market processes in natural resource depletion was first noted by Lewis Cecil Gray, and later by Harold Hotelling in their effort to find a link between the conservation ethic and economic theory. The model of Hotelling remained the foundation stone of modern theory of exhaustible resources.

In the first chapter, we discussed the pattern of resource depletion in competitive resource markets. We derived the static efficiency condition which says that the competitive (efficiency) price of an extracted resource is the sum of the marginal extraction cost and the price of the unextracted resource (or the royalty price). The dynamic efficiency condition states that if cost of extraction is independent of the remaining stock, the royalty price will grow at a compound rate equivalent to the rate of interest. This is popularly known as the "Hotelling Rule". We also showed how this condition can also be viewed as a necessary condition for equilibrium in an asset market if the resource underground is considered as a capital asset to its owner, forming a constituent part of his portfolio.

In the second chapter, we discussed the various empirical studies which attempted to test the empirical validity of the Hotelling's model. There were mainly three types of models: (i) models which attempted to see the

correspondence between the time path of resource prices as suggested by the theory and the long term or short term trends in actual resource price movements, (ii) studies which introduced an arbitrage behaviour for the owners of capital assets where the resource itself was considered as a capital asset, and (iii) a third group of models which attempted to estimate the royalty for the resource at the firm level and then see whether its time path corresponded to the one implied by the dynamic efficiency condition.

The results were divergent. While the results of some of these studies supported the empirical validity of the Hotelling type models, the results of some other studies did not. In this context, two important points are to be noted. First, though the models of competitive resource markets were very important from the point of view of theory and as a first step towards the analysis of the real world resource markets, they cannot be applied to reality since reality does not correspond to conditions of pure competition. Secondly, empirical testing the theory developed making an assumption of competitive resource markets using data generated in imperfect resource markets is not appropriate. Therefore, any conclusion about the empirical validity or otherwise of the Hotelling rule arrived on the basis of such studies should be taken with great caution.

In the third chapter, we concentrated on the polar opposite of pure competition, i.e., monopoly in exhaustible resource markets. The basic question that was discussed was the role of elasticity of demand in determining the optimal monopoly extraction paths. The main observation was that a monopolist initially conserves resources when compared with his counterpart in a purely competitive market and hence he

is the best friend of a conservationist. The question of monopolistic resource extraction is very relevant since the cases of large resource monopolies is not uncommon in real world.

In chapter four, we discussed a particular type of oligopolistic resource market which very closely captures the characteristics of many of the real world exhaustible resource markets, viz., partially cartelised exhaustible resource markets. The resource market was modelled to consist of a resource cartel which is the dominant firm and a group of firms forming a competitive fringe. We discussed the outcome of employing the open-loop Nash and Stackelberg solution concepts of interaction between the cartel and the fringe. A point to be noted is that neither of the solution concepts is satisfactory. Nash strategies make naive behavioural assumptions while for some parameter values, the open-loop Stackelberg strategy is dynamically inconsistent. When the latter occurs, a more appropriate solution concept is the closed-loop rational expectations Stackelberg equilibrium, which is very difficult to compute. Besides, we also reviewed a host of simulation studies of various major exhaustible resource cartels.

There are many important issues which are not discussed in this dissertation. It is not because of any doubts about their importance, but because of constraints of time and space which put a limit on the number of issues that can be discussed in a short essay.

One such important issue which is left out is the effect of various types of uncertainty on depletion. Uncertainty may take different forms. It may be uncertain

demand for the resource with uncertainty becoming more and more as long term predictions of demand are involved. This may result in a faster depletion of the resource.

Another kind of uncertainty which will affect the rate of extraction of the resource is the uncertainty about the date of invention of a substitute to this resource which will render the resource obsolescent. In the event of the development of such a substitute, the value of this resource will be destroyed. The owner faced with such an uncertainty will increase the pace of extraction and the resource will be depleted faster. Similar will be the outcome if there is uncertainty about the expropriation of the resource at some uncertain future date.

A third form of uncertainty is regarding the size of the resource stock itself. The outcome of an exploratory activity is largely uncertain. On the other hand, if the problem is simply one of optimally depleting a stock of unknown size, the resource owner will be worried about the possibility of running out of it unexpectedly. If the resource owner is risk averse, he will slow down depletion.

A second important question that is ignored here is the problem of intergenerational equity in exhaustible resource allocation. The basic question here is how much of the given stock of exhaustible is it fair for the current generation to use up and how much should be left for the generations to come. In other words, the problem is how to allocate the consumption of a given stock of resource over time in order to achieve a fair allocation between generations. Since the total stock of the resource is finite, the current generation owes the future generation a share of this resource and when the current generation depletes the

resource, it deprives the future generation from the use of it. So it is argued that the current generation should compensate the next generation for this.

Solow (1974b)¹ does not agree^{with} the view that the current generation owes to the future generation a share of the resource. On the other hand, he argues that the current generation at best owes to the future generation access to a certain standard of living. Intergenerational equity, according to Solow, requires that consumption per head be constant through time so that no generation is favoured over any other. Using Rawls' 'maximin' approach, Solow derives the best consumption path for the economy which is actually the largest sustainable level of consumption per head over generations subject to constraints on capital accumulation and availability of exhaustible resources.

The difficulty is that the maximin criterion requires a large initial capital stock. If it is small, the level of consumption will be small for ever because capital must not be accumulated by sacrificing the consumption of a generation that has little to begin with. But there is no reason why the initial capital stock should ever be large.

Phelps and Riley (1978)² take a more optimistic view under maximin approach to equity between generations. They allowed generations to overlap in order to mutually benefit from exchange. A generation that adds to capital stock has a claim to more retirement consumption provided by the labour of the next generation, which has in turn an obligation to put in more work in exchange for the added capital. Phelps and Riley did not explicitly consider exhaustible resources

but implicitly assumed the substitutability of capital, for resources. This gives rise to the question of substitutability of other factors for resources. Solow³ showed in a two-factor model that a constant level of consumption can be sustained indefinitely if any of the following conditions are satisfied.

(i) elasticity of substitution between resources and the other factor, capital, is greater than 1.

(ii) the elasticity is equal to one and the share of capital is greater than that of resources .

(iii) there is resource augmenting technical change.

Now, the question is how to achieve a constant consumption path suggested by Solow. Hartwick suggested a method for this, which is known as the 'Hartwick Rule' ⁴ in the literature. According to this rule, a specific investment policy should be followed. The rents from uses of natural resources should simply be invested in real capital, i.e., the current value of the exhaustible resources used up should be invested in new capital goods. The essence of Hartwick Rule seems to be to regard the resources as part of the capital stock and maintain the total capital stock over time.

Another important factor which affects the rate of resource depletion is taxation. Various types of taxes are imposed which affect resource rents and thus alter the pattern of depletion. Hotelling also has taken note of the effect of taxes on depletion in his work. Since this is a very vast topic by itself tracing the impact of a wide variety of taxes on resource prices and consequently on extraction, it could not be covered here.

All these issues are very crucial while analysing the question of optimal intertemporal allocation of resources. They have been dealt in great detail in the literature but could not be covered here, given the constraints of space and time.

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