ESTIMATION OF PRODUCTION FUNCTIONS FOR INDIAN INDUSTRY

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INTRODUCTION

The concept of the production function plays an important role both at the micro and macro economics. At the macro level it is a major tool in theories of growth and distribution. At the micro level it deals with problems such as profit maximization of the firms, factor substitutability, returns to scale.

The concept of the production function, when associated with a particular technological process is supposed to represent the output of one commodity as a function of quantities of various factors of production combined according to a given technology (we rule out joint production).

There are various approaches to the theory of production. Sometimes substitutability of factors of production is ruled out as in the Leontief framework. On the other hand, we have substitutability in the traditional theory. The traditional theory assumes that the factors of production are continuously divisible and productive processes are continuously variable. Thus, it arrives at a smooth and continuous function.

The theory of production, in the set theoretic framework, incorporates the above as special cases. It is a more general theory which embraces the various approaches to the theory of production. So, in the first chapter, we start with the set theoretic framework.

But to obtain the empirical estimates of the parameters

of the production function, we should have framework in which we can specifically define production functions. So to obtain estimates of the production function, we have to go by the traditional theory. In the traditional theory based on the assumption of differentiability, we mainly deal with three forms of production function: the Cobb Douglas production function which has the built in property of unitary elasticity of substitution; the Constant Elasticity of Substitution production function which enables us to obtain an estimate of the elasticity of substitution; the Variable Elasticity of Substitution production function in which the elasticity of substitution is a function of the capital/labor ratio.

In Chapter II, we discuss the methods of estimating these production functions. There are single equation and multi-equation models to estimate these production functions. The various models are based on the assumptions of constant returns to scale, perfect competition, profit maximization.

Chapter III studies the empirical estimations done for Indian manufacturing, both for the whole manufacturing sector and industrywise. These studies provide estimates of the elasticity of substitution, returns to scale and technological progress which vary substantially with the level of aggregation and with assumptions underlying the model. Looking at the empirical evidence, one does realise the need to test for the validity of constant returns to scale, elasticity of substitution and perfect competition.

CHAPTER I

PRODUCTION FUNCTIONS

This chapter sets out some preliminary theoretical results on production functions, that are used in estimation procedures. Section 1 outlines the modern notion of the production possibility set. Section 2 gives the traditional notion of a production function. Section 3 discusses the notions of elasticity of substitution, returns to scale, and technological progress. In Section 4, we discuss and derive some properties of three specific production functions - the CD, the CES, the VES. In the last section we discuss profit maximization, an underlying assumption in most of the estimations, and its implications.

Section 1: THE PRODUCTION POSSIBILITY SET:

There are a number of production processes available to a firm. Let us suppose that we have n goods and s processes. We denote the vector of net output by y with y_j as its j^{th} component. If $y_j > 0$, then the j^{th} good is an output. If $y_j < 0$, then the j^{th} good is an input. If $y_j = 0$, then the j^{th} good is irrelevant for the production process. There are a such processes available to an economic unit. We represent the j^{th} process by

$$q^j = \begin{bmatrix} q_{1j} \\ q_{2j} \\ \vdots \\ q_{nj} \end{bmatrix} \qquad j = 1, 2, \ldots s \qquad (1.1)$$

and its level of activity by $x_j \geq 0$.

If all the processes are simultaneously employed, the total output produced is given by

$$y = \sum_{j=1}^{s} x_j a^j \qquad x_j \geq 0 \quad j = 1, 2, \ldots, s \qquad (1.2)$$

We define the (n, s) matrix,

$$A = [a^1, a^2, \ldots, a^s] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & & & \vdots \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{ns} \end{bmatrix} \qquad (1.3)$$

If we denote the vector of activity levels by x with x_j as its j^{th} component, (1.2) becomes,

$$y = Ax \quad , \quad x \geq 0 \qquad (1.4)$$

We call matrix A, which represents technological processes, a technology matrix. All points that can be expressed in the form (1.4) are a collection of production possibilities available to an economic unit. A set of all such points, denoted by Y, is called the production possibility set.

A production possibility set is assumed to satisfy, (1) additivity, (2) divisibility, (3) convexity.

In addition to above properties, Koopmans has made two more specifications about the production possibility set.

(a) Irreversibility of Production Processes:

The postulate says that, "it is impossible to find a set of positive amounts of some or all activities of which the joint effect is a zero net output for all commodities."[1]

The postulate implies that if we produce positive outputs from certain inputs of other commodities, there exists no combination of producers' choices that reverses the outcome by producing inputs from these given outputs, i.e.

$$Y \cap \{-Y\} = \{0\}$$

(b) The Impossibility of the Land of Cockaigne:

"It is impossible to find a set of positive amounts of some or all activities of which the joint production consists of a positive net output for at least one commodity without causing a negative net output for at least one other commodity".[2]

This postulate implies that the production possibility set does not have in common any point other than the origin with the non-negative orthant.

Section 2:

In our discussion so far, we have not made any assumptions about differentiability. But the traditional approach proceeds on the assumption of differentiability. The set theoretic

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1. T.C. Koopmans (1957) Allocation.

2. Ibid.

approach is more general in the sense that it takes into account both, differentiable and non-differentiable production processes. Such assumptions of differentiability are unnecessary in the set theoretic framework.

Nevertheless, the traditional approach based on differentiability and profit maximization is extensively used for empirical estimations of production functions. Notions like elasticity of substitution and technological progress are either impossible or difficult to interpret in the set - theoretic framework. These parameters have important theoretical and policy implications.

If $y_1, y_2, \ldots, y_m$ are factors of production which produce $x_{m+1}, x_{m+2}, \ldots, x_n$ outputs, we can write a production function in its implicit form as[3],

$$F(y_1, y_2, \ldots y_m, x_{m+1} \ldots, x_n) = 0 \qquad (1.5)$$

The function (1.5) should satisfy

$$f(0, y_1) = f(0, y_2) = \cdots = f(0, y_m) = 0 \qquad (1.6a)$$

$$\partial f / \partial y_i \geq 0 \; ; \; \partial^2 f / \partial y_i^2 \leq 0 \qquad (1.6b)$$

The first condition (1.6a) and the postulate (b) of the preceding section have the same implications i.e., all the factors of production are indispensable for production. The second condition (1.6b) says that marginal products of all

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3. Hicks (1946).

factors of production are non-negative and that the production function is concave, which corresponds to the convexity of the production possibility set assumed in the previous section.

The traditional theory also assumes that the factors of production are continuously divisible and that the production processes are continuously variable. The implications of these assumptions are similar to the implications of divisibility and additivity for a production possibility set. The partitioning of factors of production and output also implicitly assumes the irreversibility of the processes of the preceding section.

Section 3:

Usually, a production function is estimated in order to test hypotheses with respect to, (1) elasticity of substitution; (2) returns to scale; (3) technological progress; and their implications at the macro and micro levels.

3.1 The Elasticity of Substitution:

"The elasticity of substitution is measure of the ease with which the varying factors can be substituted for each other."[4]

The elasticity of substitution (σ) is defined as the ratio of a proportionate change in the ratio of factor inputs to the proportionate change in the ratio of marginal products (i.e. marginal rate of substitution).

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4. Hicks (1963).

Let us rule out joint production so that the production function can be written in explicit form as $x = x(y_1, y_2, \ldots y_m)$

More specifically, let us have one output Y and two inputs capital (K) and labor (L) so that

$Y = f(L,K)$.

With two factors of production, labor and capital, we can obtain σ as,

$$\sigma = \frac{d(K/L)}{(K/L)} \Big/ \frac{d(MRS_{LK})}{MRS_{LK}} = \frac{d \ln (K/L)}{d \ln (MRS_{LK})} \qquad (1.7)$$

MRS_{LK} is the marginal rate of substitution between labor and capital. Positive marginal products ensure the non-negativity of the elasticity of substitution.

3.2 Returns to Scale:

In the long run, the expansion of output may be achieved by varying all the factors of production. The output may be increased by changing all factors of production by the same proportion or by different proportions. Given a production possibility set Y_j (1) non-decreasing returns to scale prevail if for any given y_j one can arbitrarily increase the scale of operations, (2) non-increasing returns to scale prevail if for any given y_j , one can arbitrarily decrease the scale of operations, (3) constant returns to scale prevail if for any given y_j one can arbitrarily change the scale of operations.[5]

For a given production function, returns to scale are

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5. Gerard Debreu (1959).

determined by the degree of homogeneity of the production function. For production functions assumed to be homogeneous,

$$Y = f(L, K) ;$$

if we increase both the factors of production by the same proportion λ, we get,

$$Y = f(\lambda L, \lambda K)$$
$$= \lambda^h f(L,K)$$
$$= \lambda^h Y$$

The degree of homogeneity h determines the returns to scale. If $h = 1$, the production function exhibits constant returns to scale. If $h < 1$, the production function exhibits decreasing returns to scale. The output increases less than proportionately with the increase in inputs. If $h > 1$, we have increasing returns to scale. The output increases more than proportionately with the increase in inputs.

3.3 Technological Progress:

We define a production function as being the known set of efficient production techniques. The production function is therefore susceptible to change as our knowledge of production techniques changes. Thus technological change causes a shift in the production function.

One type of technological change is, when the production function shifts over time, reflecting greater efficiency in combining the inputs. It is called "disembodied technological progress".[6] It can be represented by the production function,

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6. Hahn & Mathews (1964).

$$Y = f(L,K,t) \qquad (1.8)$$

or $Y(t) = f(L(t),K(t),t)$

where t stands for time.

Change in the output over time is given by

$$\frac{dY}{dt} = \frac{\partial f}{\partial L}\cdot\frac{\partial L}{\partial t} + \frac{\partial f}{\partial K}\cdot\frac{\partial K}{\partial t} + \frac{\partial f}{\partial t} \qquad (1.9)$$

The first two terms indicate the change in the output due to a movement along the production function. The last term, $(\partial f/\partial t)$ gives the change in the output due to disembodied technological progress, i.e. a shift in the production function. This type of change is called disembodied because it is not embodied in the factor inputs. It involves a reorganization of inputs.

On the other hand, embodied technological progress involves an augmentation in the effectiveness of factor inputs due to various possible improvements in their quality and efficiency over time.

Technological change shifts the entire production function. To decide which point on the old production function is to be compared with which point on the new production function, some criterion is needed. A number of criteria have led to the formulation of alternative definitions of neutral technological progress.

3.3.1 Hicks-neutral Technological Progress:

Hicks' definition of neutral technological progress is that the marginal rate of substitution should be independent

of time for each fixed capital/labor ratio. The points on two production functions are compared for a given L/K ratio.

Treating technological progress as an exogenous function of time t, the production function can be written as

$$Y = f(L,K,t)$$

For technological progress to be Hicks-neutral the production function should take the form

$$Y = A(t)f(L,K) \qquad (1.10)$$

where A(t) is the index of technological progress.[7] A(t) is generally assumed to take the form $e^{\lambda t}$

3.3.2 Harrod-neutral Technological Progress:[8]

Here we compare those points on the production functions at different points of time where the marginal product of capital (assumed to be equal to the rate of profit) is constant. Harrod's definition is framed particularly with the idea in mind that capital is a produced factor of production.

With the capital/labor ratio remaining constant, technological progress normally increases the marginal productivity of capital. For the marginal product of capital to remain constant in the face of technological progress, the K/L ratio should increase. Technological progress is neutral in the Harrod sense if the level of K/L is such as

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7. Hahn & Matthews (1964).

8. Hahn & Matthews (1964).

to cause the K/Y ratio to remain constant. Harrod-neutral technological progress is represented by the production function,

$$Y = f(K, A(t)L) \qquad (1.11)$$

With the production function (1.11), assuming constant returns to scale, an equal proportionate rise in capital and A(t)L must lead to an equal proportionate rise in the output. Harrod-neutral technological progress increases the labor force measured in efficiency units A(t)L.

3.3.4 Solow-neutral Technological Progress:

A third type of neutral technological progress is capital-augmenting technological progress. The production function can be written as,

$$Y = f(A(t)K, L) \qquad (1.12)$$

Assuming constant returns to scale, an equal proportionate rise in A(t)K and labor leads to an equal proportionate rise in output.

Section 4: FORMS OF PRODUCTION FUNCTIONS AND THEIR PROPERTIES:

Algebraic representation of production functions can take many forms. The production function, whichever form it takes, should satisfy (1.6). We consider here mainly three forms of production function.

(1) Cobb-Douglas, (2) Constant Elasticity of Substitution, (3) Variable Elasticity of Substitution production function. There are various other types of production functions, but these are the three most important

ones from the point of view of estimation. Various other production functions will be mentioned, as and when necessary, while discussing the estimations.

4.1 The Cobb-Douglas (CD) Production Function:

Douglas made the first attempt to specify and empirically quantity an aggregate production function.[9] He specified the production function as

$$Y = AL^{\alpha}K^{\beta} \qquad (1.13)$$

where,

$Y \geq 0$ $\quad \alpha \geq 0$

$K > 0$ $\quad \beta \geq 0$

$L > 0$ $\quad A > 0$

(a) α , β are elasticities of labor and capital with respect to output respectively.

$$\frac{L}{Y}\frac{\partial Y}{\partial L} = \alpha \; ; \quad \frac{K}{Y}\frac{\partial Y}{\partial K} = \beta \qquad (1.14)$$

(b) The function is homogenous of degree $\alpha+\beta$

$Y = AL^{\alpha}K^{\beta}$. We increase the factors of production by a proportion λ, $\lambda > 0$.

$$Y_1 = A(\lambda L)^{\alpha}(\lambda K)^{\beta}$$

$$= \lambda^{\alpha+\beta} AL^{\alpha}K^{\beta}$$

$$= \lambda^{\alpha+\beta} Y \qquad (1.15)$$

Output increases by the proportion $\lambda^{\alpha+\beta}$. The degree of

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9. P.H. Douglas (1934).

homogeneity ($\alpha+\beta$) determines the nature of returns to scale. If $\alpha+\beta=1$, the production function is linearly homogenous, exhibiting constant returns to scale. If $\alpha+\beta>1$, the production function exhibits increasing returns to scale. If $\alpha+\beta<1$, the production function exhibits decreasing returns to scale.

(c) For the CD function the elasticity of substitution is always unity

$$\sigma = d\ln(K/L) / d\ln(MRS_{LK})$$

$$MRS_{LK} = \frac{MP_L}{MP_K} = \frac{\alpha}{\beta} \cdot \frac{K}{L}$$

$$\ln(MRS_{LK}) = \ln(\alpha/\beta) + \ln(K/L)$$

$$d\ln(MRS_{LK}) = d\ln(K/L)$$

$$\sigma = \frac{d\ln(K/L)}{d\ln(MRS_{LK})} = \frac{d\ln(K/L)}{d\ln(K/L)} = 1 \qquad (1.16)$$

That the elasticity of substitution of a CD function is always unity, is a property of the CD function itself. It is not dependent on market conditions and the behavior of the firm.

4.2 <u>The Constant Elasticity of Substitution (CES) Production Function</u>:

One of the implications of a CD function is that factor rewards remain in fixed proportions. The empirical observations that the share of national income going to labor was not fixed but varied as the wage rate varied, prompted

Arrow, Chenery, Minhas, Solow to test this proposition. They introduced a production function with constant elasticity of substitution.[10]

The CES function takes the form,

$$Y = \gamma \left[\delta K^{-\rho} + (1-\delta) L^{-\rho} \right]^{-1/\rho} \qquad (1.17)$$

where,

γ - efficiency parameter

δ - distribution parameter, $0 \leq \delta \leq 1$, which determines the distribution of factor incomes.

ρ - substitution parameter, a function of the elasticity of substitution, $\sigma = \frac{1}{1+\rho}$

(a) The CES function is homogenous of degree one.

$$Y = \gamma \left[\delta K^{-\rho} + (1-\delta) L^{-\rho} \right]^{-1/\rho}$$

We increase capital and labor by the same proportion $\lambda, \lambda > 1$.

$$Y_1 = \gamma \left[\delta (\lambda K)^{-\rho} + (1-\delta)(\lambda L)^{-\rho} \right]^{-1/\rho}$$

$$= \lambda \gamma \left[\delta (K)^{-\rho} + (1-\delta)(L)^{-\rho} \right]^{-1/\rho}$$

$$= \lambda Y \qquad (1.18)$$

The output also increases by the same proportion λ . The CES function can be extended to the case of non-constant returns to scale as well.

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10. Arrow, Chenery, Minhas, Solow (1961).

$$Y = \gamma\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-h/\rho} \qquad (1.19)$$

$$Y_1 = \gamma\left[\delta(\lambda K)^{-\rho} + (1-\delta)(\lambda L)^{-\rho}\right]^{-h/\rho}$$

$$= \lambda^h Y \qquad (1.20)$$

h determines the degree of homogeneity.

If $h<1$, we have decreasing returns to scale

If $h=1$, we have constant returns to scale

If $h>1$, we have increasing returns to scale.

The CES production function can also be generalised to the case of more than two inputs.

(b) The elasticity of substitution depends on the value of ρ, the substitution parameter.

$$Y = \gamma\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho}$$

$$MP_L = \frac{\partial Y}{\partial L} = \gamma^{-\rho} Y^{1+\rho}(1-\delta)L^{-(1+\rho)}$$

$$MP_K = \frac{\partial Y}{\partial K} = \gamma^{-\rho} Y^{1+\rho}(\delta)K^{-(1+\rho)}$$

$$MRS_{LK} = \frac{MP_L}{MP_K} = \frac{(1-\delta)}{\delta}\left(K/L\right)^{+(1+\rho)}$$

$$d\ln(MRS_{LK}) = (1+\rho)\, d\ln(K/L)$$

$$\sigma = \frac{d\ln(K/L)}{d\ln(MRS_{LK})} = \frac{d\ln(K/L)}{(1+\rho)\, d\ln(K/L)} = \frac{1}{1+\rho} \qquad (1.21)$$

To ensure non-negativity of σ, we have $\rho \geq -1$.

We have three special cases.

(1) $\rho = -1$; $\sigma = \infty$

(2) $\rho = 0$; $\sigma = 1$

(3) $\rho = \infty$; $\sigma = 0$

The first case is the case of perfect substitutability. In this case, we get linear isoquants with the slope $-\delta/(1-\delta)$. In the third case, the function approaches the Leontief input-output framework where the substitution between the factors is totally ruled out.

In the second case, the CES production function reduces to the CD function.

$$Y = \gamma \left[\delta L^{-\rho} + (1-\delta) K^{-\rho}\right]^{-1/\rho}$$

$$\ln Y = \ln \gamma - \frac{1}{\rho} \ln \left[\delta L^{-\rho} + (1-\delta) K^{-\rho}\right] \qquad (1.22)$$

When $\lim \rho \to 0$, the expression inside the brackets takes an indeterminate form. Therefore, we apply L'Hospital's rule. Let

$$V = \frac{1}{\rho} \ln \left[\delta L^{-\rho} + (1-\delta) K^{-\rho}\right]$$

Differentiating with respect to ρ,

$$\frac{\partial V}{\partial \rho} = \frac{-L^{-\rho} \delta \ln L - (1-\delta) K^{-\rho} \ln K}{\delta L^{-\rho} + (1-\delta) K^{-\rho}}$$

$$\lim_{\rho \to 0} \frac{\partial V}{\partial \rho} = -\frac{\delta \ln L - (1-\delta) \ln K}{\delta + 1 - \delta}$$

Substituting in (1.22) we get

$$\ln Y = \ln \gamma + \delta \ln L + (1-\delta) \ln K. \qquad (1.23)$$

or

$$Y = \gamma L^{\delta} K^{(1-\delta)}$$

which is a CD production function.[11]

The CES production function is thus a more general form of the production function and includes the CD function as a special case.

4.3 The Variable Elasticity of Substitution (VES) Production Function:

Though the CES function is less restrictive than a CD function in that it allows the elasticity of substitution to take a value other than unity, it does not allow the elasticity of substitution to vary with factor proportions or with output. A function which allows the elasticity of substitution to vary with output was developed by Lu and Fletcher.[12]

$$Y = \gamma \left[\delta K^{-\rho} + (1-\delta) L^{-\rho} (K/L)^{-c(1+\rho)} \right]^{-1/\rho} \qquad (1.24)$$

The function has the properties of (1) positive marginal products, (2) downward sloping marginal productivity curves, (3) homogeneity of degree one, (4) variable elasticity of substitution. Moving along the isoquant, the elasticity of substitution varies with the capital/labor ratio:

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11. Kmenta's approximation for a CES production function can also be used to prove the above result. This uses Taylor's expansion and is discussed in the chapter on estimation.

12. Lu & Fletcher (1968).

From $\frac{WL}{Y} = (1-\delta)^{b} \gamma^{-\rho b} W^{1-b} X^{-c}$ $\quad \because X = K/L$

they obtained

$$\sigma = \frac{b}{1-c\left(1+\frac{1}{(1+\beta/2(X^{A}-B)}\right)} \qquad (1.25)$$

We can also show that the VES is a generalised form of the CES production function.

If C takes the value zero, the multiplier (K/L) vanishes and the function takes the CES form of production function.

Section 5: PROFIT MAXIMIZATION:

Whatever the form of the production function, the firm is very often assumed to operate on the basis of the profit maximization principle. It chooses to operate at that point on the production function where it maximises its profit.

It is assumed in addition that the firm is atomistic, so that it is a price taker in both, the product and the factor markets. Let p be the price of the product Y, w the price of the input L and r the price of the input K. We denote profit by Π. The firm then maximizes the profits

$$\Pi = pY - wL - rK \qquad (1.26)$$

The firm maximizes the profits subject to the production function

$$Y = f(L,K) \qquad (1.27)$$

We set up a Lagrangean,

$$H = pY - wL - rK - \lambda[Y - f(L,K)] \quad (1.28)$$

We get the first order conditions for a maximum

$$\partial H/\partial K = -r + \lambda(\partial Y/\partial K) = 0;$$

$$\partial H/\partial L = -w + \lambda(\partial Y/\partial L) = 0;$$

$$\partial H/\partial \lambda = Y - f(L,K) = 0.$$

Assuming that the second order conditions in terms of the bordered Hessian are satisfied, we eliminate λ and obtain,

$$\partial Y/\partial K = r/p, \quad \partial Y/\partial L = w/p \quad (1.29)$$

Each factor is thus utilized upto the point where the marginal product is equal to its real price, i.e., its price relative to the price of the product.

Solving (1.28) and (1.27) we can obtain the profit maximizing values of Y, L, K. For instance if (1.27) takes the form of a CD production function, we would have

$$r/p = \beta Y/K \quad \& \quad w/p = \alpha Y/L \quad \text{or}$$

$$\alpha = wL/pY \quad \& \quad \beta = rK/pY$$

$$\alpha + \beta = (wL + rK)/pY$$

Thus α is the factor share of labor, the amount paid to the labor as a proportion of the total value of output. β is the factor share of capital. When $\alpha + \beta = 1$, we have constant returns to scale, the value of the total product is exactly

exhausted in payments to the factors.

We can repeat the same exercise for CES and VES production functions. More about profit maximization and its implications will be discussed in the chapter on estimation.

CHAPTER II

ESTIMATING PRODUCTION FUNCTIONS

So far in our analysis, K and L have stood for capital and labour services. For the empirical estimation of production functions we have to measure these services as well as the output (Y).

Output (Y) is usually measured as value added per year (deflated for price changes in case of time series data). It is also measured as the gross value of the output per year. If the firms carry on with the production as far as gross profits are positive (non-negativity of quasi-rent) output should be measured gross of depreciation charges. As far as choice between value added and gross output as measure of output is concerned, we see that at economy level, gross output suffers from double counting because one firm's output can be used as another firm's input. This can be avoided by measuring output as value added. In principle, even at the micro level, one should use value added. But value added data are not always available.

Labour (L) is measured in terms of man hours. It is also measured in terms of wages and salaries paid to the workers. There are various types of labor - male, female, skilled, unskilled, and of different age groups. One should construct an index to convert all these various types of labor into one homogenous commodity, but this is also difficult in

practice.

The measurement of capital is a complex problem. It can be measured as net capital stock (net of depreciation) or in terms of machine hours. When we measure the capital in terms of machine hours, it gives us an estimate of capital services, going into production which might be different from the capital services available for production. Specially this is important because firms are reluctant to discard or sell unused machinery and plant. In that case, if we measure capital as net capital stock, we might not get a true estimate of capital, infact we are likely to overestimate it.

There are many other problems also involved in the measurement of capital, e.g. the vintage of capital. Old and new machines are not likely to provide the same services because of deterioration and obsolescence.

These variables are measured at firm level, industry level, for the entire manufacturing sector, depending on at what level of aggregation we want to estimate the production function. The measurement of these variables almost invariably involves the aggregation of heterogenous quantities. This leads us to a brief discussion of the aggregation problem.

Section 1: THE AGGREGATION PROBLEM:

The aggregation problem, as far as production functions are concerned is given a lot of importance because of the uses of aggregate production function. It is one of the basic constituents of growth models representing the technical

possibilities in an economy. It is also estimated for the economy or industry in order to draw empirical inferences about technological progress, returns to scale, elasticity of substitution, which might be useful for forecasting purposes.

Under what conditions can one meaningfully aggregate the variables labor and capital to arrive at an aggregate production function?

If labor and capital are homogenous, all firms are equally efficient, we can have an aggregate production function without any problems. However, the variables are not homogenous across the firms. Since all the firms do not have equal access to the best technology available (also, there are lags in adoption of new techniques) capital is heterogenous across the firms. The techniques are embodied in capital goods so firms differ in efficiency also. Nevertheless, Fisher (1969) while analysing the conditions for existence of aggregate production function assumes constant returns to scale, which implies that, small and big, all firms are equally efficient.

Given that output and labor are homogenous, aggregate production functions will exist if there exists a capital aggregate. Assuming diminishing returns to labor, the necessary and sufficient condition for a capital aggregate to exist is additive separability i.e. all the micro production functions should be additively separable. Moreover, the extent to which heterogeneity of capital is

very limited. A capital aggregate will exist under these conditions only if the firms differ from each other by capital augmenting differences.

If we make our analysis a little less restrictive by allowing more than one kind of capital good, the conditions for the existence of a capital aggregate become more stringent. Change in one kind of capital good will affect the marginal product of other capital goods and at the same time there will be a reassignment of labor.

Fisher gives an illustration with two kinds of capital goods, K1 and K2. Suppose we want to aggregate over K1 only and leave out K2. Then the marginal rate of substitution between K1 for different industries must be independent of K2. If K2 is also to be included in the capital aggregate, the marginal rate of substitution between K1 and K2 must be independent of labor (L). We have constant returns to scale with K1, K2, L. But this does not mean that we have constant returns to scale for K1 and L or K2 and L. The above conditions for aggregation hold only under the assumption of constant returns to scale. So, we cannot have an aggregate of only K1 or K2 because we cannot have constant returns to scale condition with K1 and L or K2 and L. We can obtain an aggregate of capital K = K1 + K2 only if the firms differ by capital augmenting differences.

Sato (1975) takes a different approach to solve the problem of heterogenous capital and obtain a capital aggregate.

He defines a capital aggregate as the total productive capacity of the industry. To arrive at an aggregate production function, we must have a link between the micro functions and the aggregate production function. Sato links the two via productive efficiency, unlike Fisher, he recognizes that it differs from firm to firm.

We define a production function for i^{th} firm as

$$Yi = f(diki, cili) \qquad (2.1)$$

where d_i and c_i are efficiency co-efficients of capital and labor and are different for different firms because of heterogeneity of capital. He defines a capacity density function for the given level of efficiency of labor $\phi(c)$ which is a function of efficiency co-efficients of labor and capital.

For each level of labor efficiency we have a capacity density function $\phi(c)$. Suppose that the efficiency of labor varies between 0 and Co, and Co is the highest level of efficiency of labor. So, integrating over the range (0, Co) the various capacity density functions, we can get the total productive capacity of the industry which is defined as capital aggregate (J).

$$J = \int_0^{Co} \phi(c)\, dc \quad - \quad (2.2)$$

This function, which gives the distribution of capital among firms, is a crucial link between macro and micro functions. The capital aggregate we obtained in (2.2) is

for that given efficiency distribution. Solow (1964) and Fisher (1969), in their analysis, go across the firms to obtain capital aggregate irrespective of what the efficiency distribution is and arrive at the existence conditions. But this analysis will hold only if the production function is invariant between alternative distributions of capital. To avoid this comparison which, is not valid, Sato divided the aggregation problem into two parts (1) existence problem (2) invariance problem.

1.1 The Existence Conditions of an Aggregate Production Function:

The production function for a firm is given by (2.1). The amount of labor each firm employs is a function of real wage rate (w) and the efficiency of labor for the given capital. (2.2) gives the capital aggregate. For the given capital aggregate J, different firms can operate at different levels of labor efficiency. Over this range of labor efficiencies, we should get the industry employment by integrating over the labor employment functions for the firms. We denote this integral function, which gives the industry employment, by H. We obtain H for the given total productive capacity of the industry. Then, we can get an aggregate production function which is a function of labor aggregate for the given total productive capacity J. The conditions for the aggregate function to exist are the conditions for the integral function H which gives us the labor aggregate. If conditions for existence of the integral function H are satisfied, then

there exists an aggregate production function that can be uniquely determined from H and to which corresponds a unique capacity density function ϕ.

1.2 Invariance Conditions:

Over time, techniques change and consequently the distribution of capital among firms also changes. That will change our capital aggregate J which we defined as total productive capacity of the industry. Corresponding to this new capital aggregate, there will be another aggregate production function. To get an aggregate production function which will hold over time, irrespective of changes in the capacity density function is the invariance problem. The same aggregate function will hold for the new efficiency distribution only if the efficiency distribution exhibits very particular and systematic shifts. These particular shifts in the efficiency distribution will give us the invariance conditions.

We replace $\phi(c)$ by $\phi(c,t)$ and Co by Co(t).

We have two cases:

The new distribution is simply scaled up proportionately to give us a distribution at t_1. So we get,

$$\phi(c,t1) + a\,\phi(c,to)$$

$$Co(t) = Co(t1) = Co.$$

The capital aggregate will be $J(t1) = aJ(to)$. This is nothing but capital augmenting technological change. The aggregate production function F is invariant between $\phi(c,to)$ and $\phi(c,t1)$.

Another case is J(t1) = J(to) but the distribution of J is different. The distribution stretches out proportionately. We have,

$$Co(t1)\ \phi(c,t1) = Co(to)\ \phi\left\{\frac{Co(to)}{Co(t1)}\ C,\ to\right\}$$

$$Co(t1) = bCo(to) \qquad C' = C/b$$

This is the case of labor augmenting technological change.

Combining the two cases we can obtain,

$$\phi(c,t_1) = (a/b)\,\phi(c/b,\ t_0) \qquad \because\ b = C_0(t_1)/C_0(t_0),\quad a = J(t_1)/J(t_0)$$

This kind of an efficiency distribution will account for labor and capital augmenting technological change.

Thus, the aggregate production function is invariant if the shifts in the efficiency distribution are of the type described above. This implies that the aggregate production function is invariant if and only if technical change is purely factor augmenting. Any other kind of shift will lead to changes in the form of the macro production function.

So far in our analysis we have discussed only the short run aggregate production function (we obtain F for the given efficiency distribution which means techniques are fixed) and shifts in the short run aggregate production function (shifts in the efficiency distribution). So the capital-labor substitution will refer only to the short run. This function cannot be used in growth analysis and for forecasting purposes.

These purposes are served by the aggregate production functions of exante nature. Can we estimate such an aggregate production function from the short run aggregate production function?

In the short run macro production function, the capital aggregate represents the existing capital (K). We define a capital aggregate which measures capital in use (Ku). The production function that corresponds to Ku, is called full capacity production function. If the efficiency distribution remains stable when it shifts, there will be a stable relationship between exante and full capacity production functions. So we try and relate full capacity and short run aggregate production functions.

For a stable efficiency distribution, given the degree of utilization, Ku/K should be a constant fraction. This will cause an equiproportionate shift from the exante production function. And the locus of such points will give us the full capacity production function as the efficiency distribution moves along the exante function. However, the full capacity production function does not satisfy the marginal productivity conditions.[1] So we have to consider a long run production function which will envelope the SR functions and thereby satisfy the marginal productivity conditions.

But this causes econometric problems. The marginal

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1. The function CD is an exception.

productivity conditions hold for both long run and short run aggregate production functions. So we have an identification problem. Unless there are other identifying conditions the two functions cannot be identified.

There are other aspects of aggregation problem such aggregation of variables, aggregation under conditions of imperfect competition etc.

Section 2: METHODS OF ESTIMATING PRODUCTION FUNCTIONS:

Mostly three forms of production function are estimated - CD, CES, VES. We can broadly classify the methods of estimating production functions into single equation models and multiequation models.

If we choose to estimate a single equation model, we have to face problems if the Standard Ordinary least Squares assumption are not followed. Violation of these assumptions will lead to the problems of heteroscedasticity, multicollinearity, autocorrelation, simultaneous equations bias. If we choose to estimate a system of simultaneous equations, identification problems arise. If the production equation is not identified, we might end up estimating some other relationship like a marginal productivity equation. With only endogenous variables in the system the problem of identification becomes all the more difficult.[2]

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2. Nerlove (1959) suggested a better way of estimating the system. Since in our empirical studies we do not come across such models, the dynamic model will not be dealt with here.

A number of estimation methods are based on the assumptions of perfect competition, profit maximization and constant returns to scale.[3]

These assumptions have led to estimation of a number of relationships which equate the marginal product of labor (capital) to the wages (rental) to the wages (rental). As we shall see in the next chapter, whatever the form of the production function, the relationships based on the marginal productivity conditions have been estimated very often and they provide a very good fit also. These estimates, at aggregate level, predict labor's share in the total output fairly well.

However, Simon (1979) has pointed out that just the fact that the production relations based on the marginal productivity assumptions fit well to the aggregate data does not prove the existence of aggregate production function. It does not imply that the marginal productivity theory has been proved empirically. These production function estimates predict the labor's share in total output fairly well and very often the exponents of labor and capital add up to one. But for the CD as well as the CES production function (homogeneous of degree one), Simon has shown that these results could follow from the identity that cost of labor and

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3. Dhrymes (1965) constructed a model in which constant returns and perfect competition are not assumed but become testable hypothesis.

cost of capital added together give value added, without making any marginalist assumptions.

In the following sections, we discuss various methods of estimating the production functions.

2.1 Estimation of the CD function:

The CD function is loglinear.

$$\ln Y_i = \ln A + \alpha \ln L_i + \beta \ln K_i + U_i \qquad (2.3)$$

U is the stochastic disturbance term which includes variations in productive and technical capabilities of the firms. One way to estimate the production function is to estimate (2.3) directly given the data on Y, L, K. The OLS regression will give the estimates of α and β. Here we are estimating the production function without putting any restrictions. We call it unrestricted Cobb-Douglas production function (URCD).

But though we estimate (2.3) without any restrictions, it leads to a number of econometric problems. L & K are endogenous variables jointly determined with Y. They are not independent of the stochastic disturbance term. This leads to a problem of simultaneous equations bias. They also tend to be dependent on each other leading to a problem of multicollinearity.

The occurance of multicollinearity leads to very high standard errors of the estimates. That will make the confidence intervals for the relevant population parameters large, and consequently, the probability of accepting a false

hypothesis increases.

Moreover, as the stochastic disturbance term accounts for the variations in technical and productive capabilities it might lead to a problem of heteroscedasticity. That means the estimates do not satisfy the minimum variance conditions, they are no longer efficient. So, though we do not make any assumptions about perfect competition; profit maximization, returns to scale, the reliability of the estimates is doubtful.

Another way of estimating a CD function is based on the assumptions of perfect competition, profit maximization. These assumptions imply that marginal product of the factor equals real price per unit of that factor. In that case α and β are the shares of labor and capital respectively in the total value of the output. Under constant returns to scale, the value of the total output gets exhausted by the payments to the factors of production.

Under the assumption of constant returns to scale, (2.3) takes the form

$$\ln(Y_i/L_i) = \ln A + (1-\alpha)\ln(K_i/L_i) + U_i \qquad (2.4)$$

This relates output per unit of labor to the capital labor ratio. Estimating (2.4) yields an estimate of $(1-\alpha)$. The use of ratios Y/L and K/L reduces the problem of heteroscedasticity to some extent. But the only disadvantage of estimating (2.4) is that we cannot test the returns to scale hypothesis because we are assuming constant returns to scale.

Under the same set of assumptions i.e., perfect competition, profit maximization, constant returns to scale, we can also arrive at a relationship.

$$\ln(Y_i/L_i) = \ln(w/p) - \ln\alpha + U_i \qquad (2.5)$$

This gives output per unit of labor as a function of real wages. By estimating (2.5) we can reduce the problems of simultaneity, multicollinearity, heteroscedasticity. But these problems are reduced at the cost of restrictive assumptions by which we arrive at (2.5). (2.4) and (2.5) are estimated on the assumption of $\alpha + \beta = 1$. So we call these Restricted Cobb Douglas production function (RCD).

All these are single equation estimations. Another approach is to estimate a system of simultaneous equations. The system consists of the production function and the first order conditions of profit maximization.

$$Y = AL^{\alpha}K^{\beta}e^{U_0}$$

$$\frac{\partial Y}{\partial L} = \frac{Y}{L}\cdot\alpha = \frac{w}{p}e^{U_1}$$

$$\frac{\partial Y}{\partial K} = \frac{Y}{K}\cdot\beta = \frac{r}{p}e^{U_2} \qquad (2.6)$$

U_0 is the disturbance term affecting the efficiency of the production process and U_1, U_2 are the economic disturbance terms affecting the attainment of profit maximization. Taking logarithms we obtain a linear system,

$$\ln Y = \ln A + \alpha \ln L + \beta \ln K + u_0$$

$$\ln Y = -\ln \alpha + \ln L + \ln w/p + u_1$$

$$\ln Y = -\ln \beta + \ln K + \ln r/p + u_2 \qquad (2.7)$$

$\ln Y, \ln K, \ln L$ are endogenous variables whereas w/p and r/p are exogenous variables. Here we face the identification problem. We write (2.7) as,

$$\begin{bmatrix} 1 & -\alpha & -\beta \\ 1 & -\beta & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \ln Y \\ \ln L \\ \ln K \end{bmatrix} = \begin{bmatrix} \ln A \\ \ln \frac{w}{p} - \ln \alpha \\ \ln \frac{r}{p} - \ln \beta \end{bmatrix} \qquad (2.8)$$

Can we obtain the reduced form? We can see that for $\alpha + \beta = 1$, the matrix is singular and there is no solution i.e. the output is indeterminate in the constant returns to scale case. Given the non-negativity of K, L, in order that the marginal productivity conditions give us profit maximizing solution, there must be decreasing returns to scale. So the production equation is identified only if $\alpha + \beta < 1$. If prices are fixed, equal proportional increases in K and L away from the optimum must, in order to yield less profit lead to a less than proportionate increase in Y. This identification condition $\alpha + \beta < 1$ i.e. decreasing returns to scale,

amounts to the condition $f_{KK} \cdot f_{LL} - (f_{LK})^2 > 0$ for a production function. Thus constant or increasing returns to scale are incompatible with a determinate solution in this model based on perfect competition and profit-maximization. But a priori we cannot rule out the possibility of increasing or constant returns to scale.

We go back to (2.7). Even after using the condition $\alpha + \beta \neq 1$, we can see that the production function is still not identified. This follows from rank and order conditions. From order conditions we get identification if we add an exogenous variable to one of the other equations, making sure that it does not appear in the production function.

Klein (1953) estimated the production function from the factor shares using one of the marginal productivity conditions. Taking marginal productivity of labor,

$$\alpha = \frac{Y}{L} = \frac{w}{p} e^{u_1} \qquad (2.9)$$

which can alternatively be written as,

$$\ln \alpha = \ln \frac{wL_i}{PY_i} + u_{1i} \qquad (2.10)$$

$$\widehat{\ln \alpha} = \frac{1}{N} \sum_{i=1}^{N} \ln \left(\frac{wL_i}{PY_i} \right) \qquad (2.11)$$

$\widehat{\log \alpha}$ is an unbiased and consistent estimator of $\log \alpha$.

But $\hat{\alpha} = \sqrt[N]{\prod_{i=1}^{N} (wL_i / PY_i)}$

is biased and consistent.[4]

2.2 Estimation of the CES Production Function:

Like the CD function, the CES function can also be estimated using conditions of profit maximization and perfect competition. The marginal productivity conditions give,

$$Y = A\left[\delta L^{-\rho} + (1-\delta)K^{-\rho}\right]^{-1/\rho}$$

$$\partial Y/\partial L = A'(Y/L)^{1+\rho} \qquad (2.12)$$

Setting real wage = marginal product of labor we get,

$$A'(Y/L)^{1+\rho} = w/p \qquad (2.13)$$

Solving for output per unit of labor we get,

$$\frac{Y}{L} = A''\left(\frac{w}{p}\right)^{1/1+\rho} \qquad (2.14)$$

Its loglinear form is,

$$\ln(Y/L) = \ln A'' + \left(\frac{1}{1+\rho}\right)\ln\left(\frac{w}{p}\right) = a + \sigma \ln\left(\frac{w}{p}\right) \qquad (2.15)$$

This equation relates output per worker to real wage and yields an estimate of elasticity of substitution.

Alternatively we can also estimate,

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4. Dhrymes (1962) derived an alternative estimate of α which he showed was unbiased, efficient, sufficient, consistent.

$$\ln\left(\frac{w}{p}\right) = a' + (1+\rho)\ln(Y/L) \qquad (2.16)$$

Let us assume that real wage is exogenous and all economic units in the cross-section are governed by the same production function. Then we can estimate(2.16)using cross section data on $Y, L, w/p$. This approach was used by Arrow (1961) et.al. for their cross section analysis of 19 countries.

But these two methods do not account for the case of non-constant returns to scale cases. The original formulation of the CES production function was restricted to the case of constant returns to scale and the function was linear in the logarithms of the variables. Therefore using marginal productivity conditions one could obtain an estimate of σ. If the CES function is extended to the case of non-constant returns to scale, this method of estimation is no more feasible.

The CES function incorporating non-constant returns to scale is given by,

$$Y = \gamma\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-h/\rho} \qquad (2.17)$$

Taking logarithms,

$$\ln Y = \ln\gamma - \frac{h}{\rho}\ln\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right] \qquad (2.18)$$

(2.18) is a non-linear relationship. Kmenta (1967) used Taylor's series to linearise this relationship and linearised it about $\rho = 0$. That gives,

$$\ln Y = \ln\gamma + h\delta \ln L - h(1-\delta)\ln K$$

$$-\frac{1}{2} h\rho\delta(1-\delta)[\ln L - \ln K]^2 \qquad (2.19)$$

We can split (2.19) into two parts. At $\rho = 0$, the last term vanishes and the equation reduces to a CD function. We may take the last term as representing a correction factor due to the departure of ρ from zero. OLS regression on (2.19) gives us the estimates of the parameters of the CES production function.

Kmenta (1967) obtained the estimates of the parameters of the CES production function using a simultaneous equations method also. He considered two cases:

(1) estimation with uniform prices

(2) estimation with non-uniform prices.

<u>Estimates with Uniform Prices</u>:

In this model, the firms operate under perfect competition and obtain their inputs at fixed prices in the same market.

Then we get,

$$\ln Y_i = \ln\gamma - \frac{h}{\rho}\ln[\delta K_i^{-\rho} + (1-\delta)L_i^{-\rho}] + u_{0i} \qquad (2.20)$$

$$\left(\frac{\rho}{h}+1\right)\ln Y_i - (\rho+1)\ln K_i = \ln[\gamma^{(\rho/h)}(ph\delta)^{-1}]R_1 + u_{1i} \qquad (2.21)$$

$$\left(\frac{\rho}{h}+1\right)\ln Y_i - (\rho+1)\ln L_i = \ln[w\,\gamma^{(\rho/h)}(ph)^{-1}(1-\delta)^{-1}]R_2 + u_{2i} \qquad (2.22)$$

Where p is the price of the product, w is wage rate, r is price of the capital input. R1 and R2 are introduced to allow for the possibility of systematic deviations from profit maximization due to restrictions on firm behavior. If there are no restrictions then R1 = R2 = 1.

This model is underidentified in the sense that the conditions for minimising generalised residual variance do not contain enough information to solve for all the unknown parameters to be estimated. This difficulty can be overcome by imposing restrictions on the variance-covariance matrix of the disturbances. We assume this variance-covariance matrix of disturbances to be diagonal. One way to estimate this model is by non-linear full information method. Alternatively we replace the production function in the system by its linear approximation.

$$\ln Y_i - h\delta \ln K_i - h(1-\delta)\ln L_i + \frac{1}{2}\rho h\delta(1-\delta)[\ln K_i - \ln L_i]^2 = k_0 + u_{0i} \qquad (2.23)$$

$$\left(\frac{\rho}{h}+1\right)\ln Y_i - (\rho+1)\ln K_i = k_1 + u_{1i} \qquad (2.24)$$

$$\left(\frac{\rho}{h}+1\right)\ln Y_i - (\rho+1)\ln L_i = k_2 + u_{2i} \qquad (2.25)$$

k_0, k_1, k_2 are constants. u_0, u_1, u_2 are disturbances and they are assumed to be normally distributed, and cross-sectionally independent. The maximum likelihood estimates can be obtained using indirect least squares method.

Let $F = \left(\frac{\rho}{h}+1\right)/(\rho+1)$

Then,

$$F\ln Y_i - \ln K_i = \gamma_{1i}$$

$$F\ln Y_i - \ln L_i = \gamma_{2i}$$

$$[\ln K_i - \ln L_i]^2 = \gamma_{3i}$$

Now we get a regression equation

$$\ln Y_i = a_0 + a_1\gamma_{1i} + a_2\gamma_{2i} + a_3\gamma_{3i} + e_i \qquad (2.26)$$

The co-efficients of (2.26) can be identified with those of production function.

$$\ln \gamma = \frac{a_0}{(1 - Fa_1 - Fa_2)} \qquad (2.27)$$

$$h\delta = \frac{-a_1}{(1 - Fa_1 - Fa_2)} \qquad (2.28)$$

$$h(1-\delta) = \frac{-a_2}{(1 - Fa_1 - Fa_2)} \qquad (2.29)$$

$$-\frac{1}{2}\rho h\delta = \frac{a_3}{(1 - Fa_1 - Fa_2)} \qquad (2.30)$$

(2.28) and (2.29) give together with F,

$$a_3 = -\frac{1}{2}(F-1)\,a_1 a_2 \qquad (2.31)$$

The least squares estimates of (2.26) subject to (2.31) will be consistent. Substituting the values of the estimates of a_0, a_1, a_2, a_3 in (2.27) through (2.30) we obtain the estimates of the parameters of the CES function - $\log \gamma, h, \delta$ and ρ.

Estimates with Non-uniform Prices:

In this model we allow the prices of inputs and outputs to vary over the sample period. So, our model consists of,

$$\ln Y_i = \ln\gamma - h\delta(\ln L_i - \ln K_i) + h\ln L_i - \tfrac{1}{2}h\rho\delta(1-\delta)(\ln L_i - \ln K_i)^2 + u_{0i} \qquad (2.32)$$

$$\left(\tfrac{\rho}{h}+1\right)\ln Y_i - (\rho+1)\ln K_i = \ln\left(r_2/p\right)_i + \ln \gamma^{(\rho/h)-1}(h\delta)R_1 + u_{1i} \qquad (2.33)$$

$$\left(\tfrac{\rho}{h}+1\right)\ln Y_i - (\rho+1)\ln L_i = \ln\left(\frac{w}{p}\right)_i + \ln \gamma^{(\rho/h)-1} h^{-1}(1-\delta)^{-1}R_2 + u_{2i} \qquad (2.34)$$

(2.34) - (2.33) gives,

$$\ln L_i - \ln K_i = \left(\frac{1}{1+\rho}\right)\ln\left(\frac{1-\delta}{\delta}\right)\frac{R_1}{R_2} - \left(\frac{1}{1+\rho}\right)\ln\left(\frac{w}{r_2}\right)_i + \left(\frac{1}{1+\rho}\right)(u_{2i}-u_{1i}) \qquad (2.35)$$

Regression of In (L/K) on $(w/r)_i$ gives best linear unbiased estimates of $1/(1+\rho)$

For the estimation of the other parameters, we obtain a reduced form equation for $\ln L_i$ by solving (2.32), (2.34), (2.35) (2.35).

For $\ln L_i$ and we get,

$$\ln L_i = b_0 + b_1 \ln\left(\frac{r}{p}\right)_i + b_2 \ln\left(\frac{r}{w}\right)_i$$

$$+ b_3\left[\ln\left(\frac{r}{w}\right)_i\right]^2 + b_4 V_{1i}^2 + V_{2i} \quad (2.36)$$

$$V_{1i} = (\rho+1)^{-1}(u_{1i} - u_{2i}). \quad V_{2i}$$

is the linear combination of all three disturbances. We can replace V_{1i}^2 . by the squares of the residuals of (2.35). If $\widehat{\ln L_i} - \widehat{\ln K_i}$ are OLS estimates from (2.35) and $\widehat{\ln L_i}$ are obtained from (2.36) with residuals $\hat{V}_{1i}$ and $\hat{V}_{2i}$ respectively,

then,

$$\ln L_i - \ln K_i = (\widehat{\ln L_i} - \widehat{\ln K_i}) + \hat{V}_{1i}$$

$$\ln L_i = \widehat{\ln L_i} + \hat{V}_{2i}$$

$$(\ln L_i - \ln K_i)^2 = [\widehat{\ln L_i} - \widehat{\ln K_i}]^2 + \hat{V}_{1i}^2 + 2\hat{V}_{1i}(\widehat{\ln L_i} - \widehat{\ln K_i})$$

We replace the values of the explanatory variables by their reduced form fitted values and estimate the relationship,

$$\ln Y_i = \ln \gamma - h\delta(\widehat{\ln L_i} - \widehat{\ln K_i}) + h \ln L_i$$

$$- \frac{1}{2}\rho h \delta(1-\delta)\left[[\widehat{\ln L_i} - \widehat{\ln K_i}]^2 + \hat{V}_{1i}^2\right] + V_{0i} \quad (2.37)$$

We can substitute $\hat{\rho}$ for ρ in (2.37) and obtain least squares estimates of the CES parameters.

Estimation of the VES Function:

The ACMS formulation is based on the assumption of "the existence of a relationship between Y/L and W independent of the stock of capital."[5] If this assumption does not hold, we get distorted estimates of σ. The estimation gives biased estimates of σ. So Lu and Fletcher (1968) obtained estimates of σ from the generalised CES production function. The estimation of a VES production function is also based on the assumption of perfect competition, profit maximization. Lu and Fletcher have done the estimation in 2 stages. They start with the relationship,

$$\ln(Y/L) = \ln a + b\ln W + c\ln(K/L) + e \qquad (2.38)$$

The assumptions of perfect competition and profit maximization give

$$P\frac{\partial Y}{\partial L} = W$$

Given these assumptions and the relationship (2.38) they arrived at

$$\frac{WL}{Y} = (1-\delta)^{b}\gamma^{-\rho b}W^{1-b}X^{-c} \qquad (2.39)$$

where $X = K/L$.

Assuming neutral technological progress $\gamma(t) = \gamma_0 e^{\lambda t}$

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5. Arrow et al (1961).

one obtains,

$$\ln(WL/Y) = \beta_0 + \beta_1 \ln W + \beta_2 t + \beta_3 \ln(K/L) \qquad (2.40)$$

where

$$\beta_0 = b\ln(1-\delta) + (b-1)\ln\gamma_0$$

$$\beta_1 = (1-b)$$

$$\beta_2 = \lambda(b-1)$$

$$\beta_3 = -c$$

Given the data on $\frac{WL}{Y}$, the wage rate and K/L we can estimate the relationship (2.40) to obtain OLS estimates of β's and consequently of b, λ and c.

From the estimates of b, λ, c, and δ we can calculate σ using the relationship

$$\sigma = \frac{b}{1 - c\left(1 + \frac{1}{\beta/\alpha\,(x^A - 1\beta)}\right)}$$

where $A = \frac{b+c-1}{b}$ $\qquad \beta = \frac{c}{b+c-1}$ $\qquad \alpha = (1-\delta)\gamma^{(1/b-1)}$

We can see here that σ is an explicit function of the capital/labor ratio.

We discussed both single equation and multiequation models to estimate production functions. But as we shall see in the next chapter, most of the estimations done for the Indian data are based on single equation models.

CHAPTER III

EMPIRICAL ESTIMATIONS FOR INDIAN MANUFACTURING

The empirical estimation of production functions is mainly done to obtain estimates of the parameters - elasticity of substitution, returns to scale, factor shares; and to measure technological change.

Section 1: SOURCES OF DATA:

For the estimation the data could be obtained from CMI (Census of Manufacturers in India), SSMI (Sample Survey of Manufacturing Industries), ASI (Annual Survey of Industries).

The CMI gave data on 29 industries for the period 1946-58. It covered factories registered as per the Factories Act of 1934 (i.e. firms using 20 or more workers with power). As per this definition it covered only 6500 to 7000 factories.

However, CMI was replaced by the ASI in 1959. The ASI coverage is much broader than the CMI. The ASI covers the factories registered under the Factories Act of 1948 (i.e. it covers factories employing 50 or more workers with power and 100 or more workers without power). The remaining factories, which are not covered by Factory Act of 1948, were covered by sampling methods. The ASI data are available for both sample and census sectors.

The ASI and CMI classifications are different. The differences in coverage and classification lead to the

problem of comparability of the data in case of time series analysis. Because of this problem of comparability of the data, we find in the empirical studies a tendency either to restrict the analysis upto 1958 or not to cover the CMI period at all. In that case, the later studies as pointed out by Mehta (1980), leave out an important period of Indian industrialization. To cover both ASI as well CMI periods in the analysis, one has to make adjustments so as to make the data comparable. But it was found that not many studies have made an effort to make the data comparable.

Mehta (1980) obtained a comparable CMI - ASI time-series for the period 1953-65. The ASI gives data on 63 groups of industries as against the CMI which gives data for 29 industries. Mehta found that 27 out of 29 groups were identified with the ASI groups. The two CMI groups which could not be identified with the ASI groups were the 22nd and 28th group.

He made two types of adjustments to make CMI-ASI series comparable; one with regard to the industrial classification and second with regard to the units covered. As far as the units covered are concerned, to make ASI and CMI data comparable, Mehta suggested two alternatives. One is to add the data of factories using 20 to 49 workers with power to the ASI series. The second alternative was to remove the non-ASI data from the CMI series and make the whole series comparable with the ASI base. He ruled out the first

alternative because of lack of appropriate data. So, he constructed a continuous time-series for the period 1953-65 by removing the non-ASI data from the CMI.

Section 2: MEASUREMENT OF VARIABLES:

Output:

Output is often measured as gross value added or gross value of the output. GVA is preferred to net value added because depreciation figures need not always reflect the actual capital consumption. The CMI depreciation is calculated on the basis of rates allowed for income-tax purposes. In such cases, the depreciation figures, instead of reflecting the capital consumption, depend on the prevailing law. Therefore, GVA is preferred to NVA. Murty and Sastry (1957) have drawn data from company balance sheets. They measured output as net value of the output from the profit and loss account of the firms.[1]

Labor:

In the case of labor, CMI and ASI give data on three categories of labor: workers directly employed, persons other than workers and workers employed through contractors. Wages and salaries data are given separately for these three categories. However, labor is also measured in terms of man hours. But the use of man hours as a flow of labor services involves a strong assumption. "It assumes that the inherent

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1. Net value of the output = Sales + closing stock - opening stock - manufacturing expenses - depreciation.

average physical and mental capacity of persons employed in each occupation is constant over time."[2] However, Banergi thinks that one can assume the average capacity to produce to remain roughly the same over a "reasonable" time-period. Denison (1961) observed that output per man varies less than output per man hour. So, he preferred employment data as a measure of effective labor input.

Capital:

Capital is very often measured as gross fixed capital. The data on capital poses a number of problems. It is measured as fixed capital by Sankar (1970), Mehta (1975), Rajalakshmi (1985). Sankar (1970) divided the fixed capital items into structures and plants and machinery. He used separate deflators for the two categories - index number of building costs and index number of machinery prices respectively. In his analysis, as he himself has mentioned, he did not take into account the age structure of capital.

Mehta (1975) measured fixed capital as the aggregate of the book value of land, buildings, plants and machinery. He deflated the time series by the index of machinery and transport equipments.

The use of gross fixed capital can be justified "particularly in case of under developed countries, on the ground that capital stock are probably more often used at

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2. Banergi (1975).

approximately constant levels of efficiency for periods far beyond the accounting life measured by the normal depreciation." (Banergi (1971). Banergi (1971) also pointed out that since large amount of expenditure is incurred in order to keep the asset in more or less similar productive capacity and keep the equipment more or less intact, there is no need to subtract depreciation from the gross value of the capital stock.

There are many more problems in the measurement of capital arising from heterogeneity, age structure, price deflators. The age structure of capital affects the gross-net adjustments. Hashim and Dadi (1971) made the gross-net and price adjustments to obtain an adjusted continuous capital series of Indian Manufacturing.

The data on age-structure of capital cannot be obtained. So, they calculated the average gross-net ratio for each industry for a particular year and converted the book value into gross value. Suppose we obtain the gross-net ratio (r) for the i^{th} year, then we can build a capital series using annual gross additions to the stock of capital as,

$$G^{i+2} = N^{i}_{(r)} + A^{i+1} + A^{i+2}$$

where G - gross value

r - gross-net ratio

N - Net value (depreciated book value)

A - gross additions to the capital stock during a year.

Thus, they obtained gross capital series at purchase price of fixed assets for 28 industries which are comparable to CMI industries.

These complexities in the measurement of capital led to the use of capital surrogates such as horse power ratings of electric motors and electricity consumption. These proxies do not raise any index number problems. Also, to take account of substitution of one fuel for another, one can construct a composite measure. Sastry (1981) constructed a composite measure of total energy consumption for the cotton industry. He took into account 16 fuel components and converted them to a common base (in terms of coal replacement). The conversion factors he used were obtained from the Ptroleum Information Service, U.N. World Energy Supplies.

What led him to the use of capital surrogates was the fact that in most of the studies, where capital is measured traditionally, the elasticity of output with respect to capital is very small or even negative and statistically insignificant. With the use of the capital surrogate he obtained significant estimates of elasticity of output with respect to capital.

Section 3: EMPIRICAL ESTIMATIONS:

3.1 Estimations for the whole Manufacturing Sector:

The CD and the CES are the most widely used forms of production function for empirical estimations. Though the VES production function permits the capital/labor ratio to

be an explanatory variable in determining the elasticity of substitution, not many estimations are based on the VES function.

The empirical estimates of the parameters of the production function are not only of theoretical interest, they may also provide useful guidelines for economic policy.

The elasticity of substitution has a number of implications. It has been shown by Solow (1956) that the unstable balance of the Harrod-Domar growth model depends critically on the elasticity of substitution. Lucas (1970), Sargent and Wallace (1975) have shown that the elasticity of substitution has implications for the cyclical behavious of the labor's share. The elasticity of substitution can also provide guidelines for resource allocation.

The capital and labor co-efficients indicate their relative importance in the industry concerned. If the production function is linearly homogeneous, the sum of the co-efficients will determine the nature of returns to scale.

Technological change has been identified as a dominant factor of economic growth. Technological change can be measured either by productivity indices or by shifts in the production function. A time series of total factor productivity indices would show the intertemporal changes in the capacity of the production system to generate output. There are three indices of Total Factor Productivity (TFP) often used for the purpose of estimating technological

progress.

(1) Solow Index A(t)

(2) Kendrick Index K(t)

(3) Domar Index D(t).

Solow used the term technological change as a "short hand expression for any kind of shift in the production function."[3] He assumed technological change to be neutral in the sense that the marginal rate of substitution remains the same with the change in output.

Yt = A(t) F(Lt, Kt) - (3.1)

A(t) measured the effect of cummulative shifts over time. Differenting (1) with respect to time and dividing by Y we obtain,

$$\dot{Y}/Y = \dot{A}/A + A(\partial F/\partial K)(\dot{K}/K) + A(\partial F/\partial L)(\dot{L}/L) \qquad (3.2)$$

or,

$$\dot{Y}/Y = \dot{A}/A + W_K(\dot{K}/K) + W_L(\dot{L}/L) \qquad (3.3)$$

Dots denote the partial derivatives with respect to time.

Solow derived a whole series of technological progress from the identity

$$A(t+1) = A(t)\left(1 + \frac{\Delta A(t)}{A(t)}\right) \quad \because A(0) = 1 \qquad (3.4)$$

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3. Solow (1957).

The Kendrick index (Kt) is based on the assumptions of perfect competition, perfect substitutability, profit maximisation.

$$K_t = \frac{Y_t}{\alpha L_t + \beta K_t} \qquad (3.5)$$

α & β are labor and capital co-efficients respectively.

Domar constructed an index of TFP for $Y = A L^{\alpha} K^{\beta}$ function. It is a geometric index.

$$\ln Y = \ln A + \alpha \ln L + \beta \ln K$$

$$(1/Y)(dy/dt) = (1/P)(dP/dt) + \alpha(1/L)(dL/dt) + \beta(1/K)(dK/dt)$$

$$\dot{P}/P = \dot{Y}/Y - [\alpha(\dot{L}/L) + \beta(\dot{K}/K)]$$

or

$$= \frac{\Delta Y}{Y} - [\alpha(\Delta L/L) + \beta(\Delta K/K)] \qquad (3.6)$$

3.2 Estimation of Elasticity of Substitution:

Table 1 summarises the results of the estimations of the production functions for the whole Indian manufacturing sector. The estimations are done for both time-series as well as cross-section data.[4] The studies are restricted to the CMI period. Even the studies in early 70's do not extend beyond the CMI period - Banergi (1971), Narsimham and Fabrycy

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4. Cross section: Dutta (1955), Murty & Sastry (1957)
Time series: Dutta Majumdar (1967), Banergi (1971), Narsimham & Fabrycy (1974).

(1973). However, Dutta Mujumdar (1967) has estimated the production function for the period 1951-61.

As for the cross-section examination of CES production function, Gujarati (1966) estimated the CES production function for manufacturing sector consisting of 28 industries for the year 1958. He obtained direct estimate of elasticity of substitution based on a relationship between value added per unit of labor and wage rate. His findings rejected the hypothesis of

The two major cross-section studies by Dutta and Murty and Sastry were carried out before the notion of the CES production function was introduced.

In case of time-series estimations, Banergi (1971) and Narsimham, Fabrycy (1974) have estimated CD as well as CES for the period 1946-58. So, these studies have tested the hypothesis of $\sigma=1$ as against the CD function with a built-in property of $\sigma=1$.

To estimate the elasticity of substitution, Banergi (1971) has estimated a number of relationships based on CES production function including a lagged model. He estimated the following relationships.

$$\ln(Y/L) = \sigma \ln \frac{\gamma^{\rho}}{1-\delta} + \sigma \ln W \qquad (3.7)$$

This gives a relationship between average labor productivity and the wage rate. One advantage of estimating this relationship is that we avoid the use of data on capital.

However, this relationship does not give us separate estimates of the efficiency and the distribution parameters.

Given the equilibrium relationship

$$\ln(Y/L)^{*}_{t} = \alpha + \sigma \ln W_t + u_t \qquad (3.8)$$

and allowing for partial adjustments towards equilibrium, one can write

$$\ln(Y/L)_t - \ln(Y/L)_{t-1} = \lambda\left[\ln(Y/L)^{*}_{t} - \ln(Y/L)_{t-1}\right], \quad 0<\lambda<1.$$

or

$$\ln(Y/L)^{*}_{t} = 1/\lambda \ln(Y/L)_t + (1-(1/\lambda))\ln(Y/L)_{t-1} \qquad (3.9)$$

Substituting in (3.8) we obtain

$$\ln(Y/L)_t = \alpha\lambda + \sigma\lambda \ln W_t + (1-\lambda)\ln(Y/L)_{t-1} + \lambda u_t \qquad (3.10)$$

He also estimated a serial correlation model with a lagged wage rate term to take account of serial correlation arising out of misspecifications in the error terms. But the co-efficient of the lagged wage rate term was found to be statistically insignificant.

But, these relationships do not allow for non-constant returns to scale. A CES production function with non-constant returns to scale, together with profit maximizing conditions gives,

$$\ln(Y/L)_t = -h/(h+\rho)\ln h^{\delta} + (h/h+\rho)\ln W_t + \frac{\rho(h-1)}{(h+\rho)}\ln L_t + u_t \quad (3.11)$$

The relationships (3.7), (3.10), (3.11) gave very good estimates with R^2 of .78 and above. But none of these relationships gave an estimate of elasticity of substitution significantly different from one.

On the other hand, Narsimham and Fabrycy (1974) found that the substitution parameter is sensitive to specification changes. They obtained the estimates of the parameters of the CES function using Hartley's (1961) iterative method. They estimated the CES function with and without the time trend. With the inclusion of a time trend, the estimate of elasticity of substitution was reduced substantially. The estimate of elasticity of substitution without the time trend was .7821. With the introduction of the time variable, they obtained an estimate of elasticity of substitution as low as .3534. They also estimated the relationship (3.7) which appeared to be a weak relationship with an R^2 = .2855 and gave a still lower estimate of elasticity of substitution of .2373.

Thus, the estimates of elasticity of substitution, as

we have seen, are sensitive to specification changes and also depend on the period under consideration. So it would not be very reasonable to assume unitary elasticity of substitution and go on to estimate a CD function. We should obtain the estimates of the elasticity of substitution and then only determine the form of the production function.

3.3 Estimation of Returns to Scale:

The sum of labor and capital co-efficients of the URCD gives an estimate of returns to scale. Dutta (1955) undertook a cross-section study of Indian Manufacturing for the years 1946 and 1947. He estimated an unrestricted CD function for Indian manufacturing. For both the years he obtained very good fits with R^2 of .968 and .971. His estimates of returns to scale conform to the hypothesis of constant returns to scale for both years. But over the years, the estimate of the elasticity of labor with respect to output has gone down substantially and that of capital has gone up.

On the other hand, as seen from the estimates of Murty and Sastry (1957), who carried out a cross-section analysis for the years 1951, 1952, even though the elasticities move in the same direction the magnitude is only marginal. Their estimates also favored the hypothesis of constant returns to scale.

All these estimations by Dutta, Murty and Sastry gave very good fits with R^2 of .95 and above which implies that

more than 95% of the variations in the output are explained by the factors of production.

As for the time-series estimations, Dutta Majumder (1966) chose to estimate restricted CD function for the period 1951-61. So, under the assumptions of unitary elasticity of substitution and constant returns to scale, all that he obtained were estimates of elasticity of labor and capital with respect to output (.46 and .54 respectively).

On the contrary, in his extensive study, Banergi (1971) estimated four relationships based on URCD.

$$\ln Y = A + \alpha \ln L + \beta \ln K + \epsilon_1 \qquad (3.12)$$

$$\ln Y = A + \alpha \ln L + \beta \ln K + \gamma \ln U + \epsilon_2 \qquad (3.13)$$

$$\ln Y = A + \alpha \ln L + \beta \ln K + \lambda t + \epsilon_3 \qquad (3.14)$$

$$\ln Y = A + \alpha \ln L + \beta \ln K + \lambda t + \gamma \ln U + \epsilon_4 \qquad (3.15)$$

Out of these four relationships only (3.12) yielded significant estimates of both capital and labor co-efficients with R^2 of the order of .82. The sum of the co-efficients was greater than one at 10% level of significance.

In (3.13), he included a capacity utilization variable into the relationship. However, nowhere in his analysis has he mentioned how he has measured capacity utilization. It can be seen from Table 1 that (3.13) gave a higher R^2. But

we cannot really say that this gives a better fit unless we compute $\bar{R}^2$, especially when we are estimating the relationships with so many variables. To Banergi, (3.13) estimates seemed more 'plausible'. But not knowing how capacity utilization has been measured, we cannot say anything about the 'plausibility' of the estimates. Though we do not want to go into the details of the capacity utilization problems, it should be mentioned here that if capacity utilization is treated as synonymous to capital utilization, capital is being measured twice in relationship (3.13), which might lead to problems of collinearity/serial correlation.

Krishnamurty (1961) gives a better way of dealing with the utilization problem. He considered two production functions; the full capacity production function and the production function relating labor and capital utilised to output. The later relationship can expressed as

$$Y = AL^{\alpha}(KU)^{\beta}$$

where K is the capital stock and U is capacity utilization rate.[5] (KU) thus gives the capital stock corrected for utilization.

(3.13) gave him an estimate of returns to scale not

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5. There are various measures of capacity utilization like Maximum output, minimum capital output ratio measure. But we do not go into the problem of measurement of capacity utilization.

significantly different from one.

Another relationship Banergi (1971) estimated to obtain a measure of returns to scale (3.14) was URCD with the introduction of a time trend. The co-efficients of time variable and capital were statistically insignificant. The sum of the capital and labor co-efficients was not significantly different from one. The estimates obtained with the introduction of both time and utilization variable (3.15) were as Banergi stated 'implausible and unstable'.

(3.11), the CES production function with non-constant returns to scale, also gave evidence in favour of constant returns to scale.

So, on the whole, the evidence was more in favour of constant returns to scale. Moreover, (3.12) gave an indication of increasing returns to scale only at 10% significance level. So, (3.12) also does not give a strong indication of increasing returns to scale.

Narsimham, Fabrycy (1974) estimated RCD and CES function with constant returns to scale for the period 1946-58. However, looking at the evidence in favor of constant returns to scale at aggregate level, their assumption of constant returns to scale appears to be reasonable.

3.4 Technological Change:

To measure technological change, Banergi (1971) and Narsimham, Fabrycy (1974) have tried to find out to what extent technological change has contributed to a rise in

output.

Banergi studied the productivity trends separately over the CMI (1946-58) and ASI (1959-64) periods because of the problem of the comparability of the data. He found that the Kendrick and the Solow index move a very closely together. So, he concentrated only on the Solow Index. For the period 1959-64, A(t), the Solow Index showed a downward trend. Labor productivity showed an increasing trend. He could find the presence of capital deepening but not of technological change.

On the contrary, Narsimham, Fabrycy (1974) could trace the presence of technological change. When they estimated the CD production function with a time trend, assuming constant rate of technological change, the presence of technological change was not significant. But rate of technological change need not be constant and may vary over the period. So, they used the analysis of covariance. Using this method, technological changes were found to be highly significant and were found to occur in spurts. Over time, they estimated technological progress to be approximately at a rate of 2% per year, but with zero advance in 1950 and 1956 and around 6% in 1954 and '55.

4.1: Estimations at a Disaggregated Level:

In the above section, we reviewed the findings with regard to elasticity of substitution, returns to scale and technological change for the Indian manufacturing at an

aggregate level. However, estimations at a disaggregated level may give a different picture. Also, for some of the policy issues like how much to invest between large and small scale industries, we have to analyse the problem at a disaggregated level, breaking up the industrial sector between different industries because aggregate estimates will not serve the purpose.

Diwan and Gujarati (1968) have discussed this particular problem in connection with the policy objectives of employment and productivity. These two objectives are conflicting. To maximise long run output, one should go in for a capital intensive technology. On the other hand, to maximise employment in the short run, labor intensive techniques would prove superior.

The output is determined by the market as well as the technique chosen for the production. So, the relationships they estimated are of the nature of demand functions for employment. Recognising the need to test the assumption of unitary elasticity of substitution, these relationships are based on the CES production function.

In the first model, they make allowances for market imperfections. The assumption of profit maximization gives the relationship,

$$\frac{\partial L}{\partial Y} = aw \qquad (3.16)$$

W is real wage rate. 'a' gives the degree of market

imperfections. Then a = 1 implies equality between the marginal productivity of labor and the real wage rate. 'a' is a function of the elasticity of demand for output and the elasticity of supply for labor. So, $a \gtrless 1$ will imply market imperfections.

They derived the marginal productivity relationship based on the CES production function and substituted in (3.12) to obtain a demand function for employment,

$$L_t = A' W_t^{-\sigma} Y_t^{\alpha} U_{1t} \qquad (3.17)$$

where,

$$\alpha = \frac{n+\rho}{n(1+\rho)}$$

(3.17) implies that output and wage rate are exogenously determined and these in turn determine employment.

In the second model, they assumed that there is unlimited supply of labor so that demand alone determines employment. But the existence of surplus labor does not ensure unlimited labor supply in all the subsectors. Specially in the industrial sector we need skilled laborers and the supply of skilled labor can be increased only with a time lag. Also, instead of making a simplifying assumption of homogeneous labor, they were more realistic and classified labor in two subsets - unemployed (U) and employed (E). Since in a technologically determined sector the supply can be adjusted only with a time lag, they arrived at a model which takes account of the dynamic adjustment process. The

adjustment process depends on the elasticity at which the supply adjusts to demand.

$$L_t = A' W_t^{\alpha_1} Y_t^{\alpha_2 \alpha_3} L_{t-1} U_{2t} \qquad (3.18)$$

where

$$\sigma = \frac{\alpha_1}{1-\alpha_3}$$

In the third model, they take account of the fact that changes in the market structure also influence the quantity of employment demanded and supplied. In this model, they take into consideration two factors - growth process in the output market and changes in the market.

For a continuous growth process, the planned output should always be higher than the actual or realised output. For simplicity, Diwan and Gujarati assumed the ratio of planned output to actual output to be constant.

They attributed growth in the wage rate to changes in the market structure and growth in the marginal product. The equation (3.16) can be modified to accommodate changes in the market structure. The ratio (a_{t-1}/a_t) will give the direction of changes in the market imperfections. On the hypothesis that market imperfections depend upon the wage rate, they postulated the relationship

$$a_t = \left(\frac{W_t}{W_{t-1}}\right)^{\mu} \qquad (3.19)$$

where μ is the elasticity of changes in the market imperfections with respect to rate of growth of wages.

(3.16) and (3.19) together give

$$\left(\frac{\partial Y}{\partial L}\right)_t = W_t^{(\mu+1)} W_{t-1}^{-\mu} = W_t^* \qquad (3.20)$$

-a relationship between changes in the market structure to the marginal product.

Incorporating these market imperfections in the first model and taking account of the fact that the demand for labor arises from planned output (Y_{tp}) and not actual output, they obtained the new demand function for employment

$$L_t = A' Y_{tp}^{\alpha} W_t^{*-\sigma} \qquad (3.21)$$

($Y_{tp} = (1+\lambda) Y_{t-1}$ where λ is planned growth rate)

Substituting for Y_{tp} and W_t^*, they obtained the reduced form

$$L_t = A^* W_t^{\beta_1} W_{t-1}^{\beta_2} Y_t^{\beta_3} Y_{t-1}^{\beta_4} U_{3t} \qquad (3.22)$$

and $\sigma = -(\beta_1 + \beta_2)$

They estimated (3.17), (3.18) and 3.22) for 28 industries for the period 1946-58. For the same period, Banergi (1971), as we have seen, obtained results in favour of unitary elasticity of substitution and constant returns to scale at an aggregate level. But the estimation of these three models by Diwan and Gujarati gives a different picture at a disaggregated level.

To say that one model is better than the other two, they set the criterion that it should give a value of $\bar{R}^2$

higher by at least .10 than the other two models. On this criterion, the first model holds for as many as 16 industries. However, in the case of a few industries, the second and third models do give an indication of improvement over the first model.

The estimation of these models generally yielded high $\bar{R}^2$ with the exception of vegetable oils (8), tanning (11), woollen textiles (19), Aluminium, copper and brass (22).

The estimates of elasticity of substitution obtained from these three models are summarised in Table II. Their findings with regard to the elasticity of substitution in the Indian industry were as follows:

Five industries, ceramics (14), Jute (20), chemical, drugs and pharmaceuticals (21), Aluminium (22), Iron and steel (23) gave the same estimates of the elasticity of substitution irrespective of the model used. For 12 industries - (1), (2), (3), (4), (5), (7), (8), (10), (15), (16), (27), (28), two out of the three models gave consistent estimates of elasticity of substitution. Out of these, 10 estimates were significantly different from zero. In the first case only the first two estimates are significant. For the remaining industries, the estimates of the elasticity of substitution differed from model to model. Out of these only 4 were significant.

The explanation they give for so many estimates of elasticity of substitution being insignificant is that "the

data relate to time series and since all variables are growing over time the estimates are liable to suffer from collinearity."

As for the magnitude of the elasticity of substitution, their evidence was not in favour of the hypothesis of unitary elasticity of substitution. Only starch and cement were found to have σ greater than one. All the remaining 26 industries showed an elasticity of substitution less than one. This should be noticed particularly because for the same period (1946-58), Banergi (1971) estimated four relationships based on the CES function at an aggregate level and rejected the hypothesis of $\sigma \neq 1$ for the Indian manufacturing. Out of these 26 industries, for four industries (1), (2), (16), (17) the value of σ was in the range .5 and 1. For 8 industries it was less than .5. In as many as 13 industries the estimates of σ were insignificant. So they came to the conclusion that σ in Indian industries was quite low.

Estimation of model II led them to the conclusion that the adjustment between demand and supply of labor does not take a very long time. They attributed this to the fact that the process of industrialization had just started and the employment level was not very high.

Rajalakshmi (1982) applied these models developed by Diwan and Gujarati (1968) to Rajasthan and All India for mineral and metal group industries for the period 1960-73.

She estimated the first two models i.e. (3.17) and (3.18) to determine the parameters - employment elasticity of output, elasticity of labor demand with respect to product wage and returns to scale for the above mentioned group of industries for All-India and Rajasthan.

As for the elasticity of labor demand with respect to product wage, it was not significant for 2 out of 6 industries. However, it was significant for Rajasthan in case of three industries - basic industrial chemicals, non-ferrous basic metals and electrical machinery. Employment elasticity of output was significant for Rajasthan as well as All-India in the case of electrical machinery. It was significant at the All-India level for metals except machinery and was insignificant for two industries. As far as the performance of the two models is concerned, the first model gave significant F values for 5 industries whereas the second model gave significant F values for only 2 industries. So one can say that the first model performed better than the second model.

However, even if Diwan and Gujarati found that elasticity of substitution was less than one for most of the industries, Banergi (1974) estimated a CD function for 5 industries namely cotton (1946-63), Jute (1946-63), Paper (1946-58), bicycle (1946-58), Sugar (1946-63). For the paper industry Diwan and Gujarati obtained an estimate of elasticity of substitution which was less than one for the

same period, 1946-58. Banergi on the other hand assumed it to be unity and went on to estimate a CD function. For Jute textiles, though Banergi's estimates refer to the period 1946-63 and those of Diwan and Gujarati to the period 1946-58, Diwan and Gujarati obtained an estimate of elasticity of substitution which was even less than .5 as opposed to the unitary elasticity in case of a CD function. For the remaining three industries the estimates obtained by Diwan and Gujarati were not even significant. Though Banergi's estimates relate to a longer period, it would have been reasonable to obtain the estimates of elasticity of substitution rather than take it to be unity.

Mehta (1980) estimated a number of relationships based on the CES function to obtain the estimates of elasticity of substitution. His estimations relate to the period 1953-65 for 27 industries.

The first relationship he estimated related value added per unit of labor to wage rate. Similarly, he estimated another relationship based on value added per unit of capital and rental to capital. In the third relationship, he equated the marginal rate of substitution between labor and capital to the wage rental ratio.

The first two relationships were estimated with and without the time trend. However, he came to the conclusion that dropping of the time trend does not generally affect the elasticity of substitution. The estimates of elasticity

of substitution obtained from these three relationships, $\sigma_1, \sigma_2, \sigma_3$ respectively are given in Table III.

It can be seen from the results summarised in Table III that the estimates of the elasticity of substitution are sensitive to specification changes. The regression of value added per unit of labor on wage rate yielded estimates of elasticity of substitution significantly different from zero for 17 industries. On the contrary, the regression of value added per unit of capital on rate of return on capital gave significant estimates of elasticity of substitution for all the industries. The R^2 also improved substantially in case of the second relationship. The explanation for this is perhaps that the assumption of perfect competition is more reasonable with regard to capital than labor. The third relationship gave significant estimates for 16 industries.

For 10 industries - (3), (4), (13), (14), (17), (18), (19), (20), (22), (25), all three models yielded significant estimates of elasticity of substitution. But as for the magnitude of the elasticity of substitution, it varied substantially with the model in a number of industries e.g. for glass and glassware $\sigma_1, \sigma_2, \sigma_3$ were 1.18, .43, .18 respectively and for sewing machine they were .87, .09 and .01 respectively.

Sankar (1970) in his study covered 15 industries which accounted for more than 95% of the value added in manufacturing. The industries were - cotton textile;

woollen textile; jute; iron and steel; aluminium, copper and brass; bicycles; general engineering; sugar; paper; soap; chemicals; paints and varnishes; cement, ceramics; glass and glassware.

He covered the period 1958-63. The estimates of the elasticity of substitution were obtained from the CES production function using the maximum likelihood method and Bayesian techniques.[6] He obtained the maximum likelihood estimates of elasticity of substitution ranging from .2727 for glass and glassware to 1.44 for bicycles and these estimates were significantly different from one in the case of 5 industries at a 5% level of significance.

The Bayesian estimates of σ led him to the conclusion that consumer oriented industries have higher elasticity of substitution than other industries.

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6. These two methods are not discussed in Chapter 2.

The estimations are based on the production function

$$Y_{it}^{*} = \exp(\Sigma_i \gamma_i Z_i + gt)\left[\delta K_{it}^{\frac{(\sigma-1)}{\sigma}} + (1-\delta) L_{it}^{\frac{(\sigma-1)}{\sigma}}\right]^{\frac{\lambda\sigma}{\sigma-1}} \exp U_{it}$$

The function is homogeneous of degree λ.
Z_i is the dummy variable which takes value 1 for the i^{th} region and zero for others. g in the rate of disembodied technical progress. He obtained the MLE estimate both with the dummy and time variables and without them also. However it was found that the introduction of dummy variables and time trend did not improve the estimates of σ in more than four industries.

Venkataswami (1975) also estimated a CES function for 29 industries for the period 1948-67. The relationship he estimated as based on value added per unit of labor and wage rate. The hypothesis of $\sigma \neq 1$ was rejected in the case of 21 industries. Estimation of the above relationship led him to the conclusion that 60% or more variations in labor productivity are explained by variations in wage rates alone.

However, all these estimates of elasticity of substitution were independent of the capital/labor ratio. Kazi (1980) and Rajalakshmi (1985) obtained estimates of elasticity of substitution which were a function of the ratio.

Kazi (1980) carried out the cross-section examination of the VES production function for the years 1973, 1974, 1975. He obtained the estimates for two-digit and three-digit classification covering 9 and 16 industries respectively.

Using Lu and Fletcher's formulation he estimated the equation

$$\ln Y = \ln A + b \ln W + c \ln (K/L) + u \qquad (3.23)$$

For two digit classification his results show that the co-efficient of capital/labor ratio is highly significant at 5% level in most of the cases. His results thus provide a justification for the proposition that the capital/labor ratio is an important explanatory variable in determining the elasticity of substitution.

In case of three digit classification also, the

co-efficient of capital/labor ratio was highly significant.

When he compared the estimates of elasticity of substitution without capital labor ratio (Table IV, W), it was found that the inclusion of the variable (K/L) in the production relation changes the estimates of elasticity of substitution significantly. For the year 1974, the estimation of a CES function gave estimates of elasticity of substitution not significantly different from one for 5 out of 9 industries. But with the inclusion of (K/L) ratio in the relationship, the same estimates were significantly less than unity at 5% level.

Similarly, for the year 1975, 7 out of 9 industries gave an estimate of elasticity of substitution not significantly different from one with the CES relationship. Whereas, all the estimates were significantly less than one when (K/L) was included in the relationship. The CES estimates of elasticity of substitution tend to have an upward bias. Most of the estimates of elasticity of substitution conform to the VES hypothesis that the (K/L) ratio is an important explanatory variable in determining the elasticity of substitution and σ is also variable among industries. Since the CES estimates might have an upward bias, one needs to test for the VES hypothesis of σ .

The findings of Diwan and Leonardson (1985) compare favourably with the above findings. Their analysis relating to 18 industries was done for the year 1971. They divided

the industries into small, medium and large on the basis of gross output. They estimated elasticity of substitution when technological change was neutral (σ_n) and also the elasticity of substitution when technological change was biased (σ_b). σ_b is a function of the share of capital in the output, which is variable. The estimates of σ_n were less than or equal to one for all the three groups but estimates of σ_b were considerably large (Table 6a). Thus they concluded that "there is a large amount of flexibility in the production structure to substitute capital for labor; given the nature of biased technical change."

They further classified the data into 5 groups according to the industry type. From the regression results of Kazi (1980), they estimated σ_n and σ_b for this industrywise classification (Table 6b). There was a close agreement in their estimates. Here again they found that σ_b is greater than one for 3 or the 5 groups - I, III, IV which reinforces their earlier conclusion.

Rajalakshmi (1985) estimated a VES function for the time-series 1960-75 for the purpose of productivity comparisons of All-India and Rajasthan. She has also resorted to Lu and Fletcher's formulation to obtain an estimate of elasticity of substitution. The function fitted very well with R^2 of .95 for Rajasthan and .84 for All-India. Also, the co-efficient of capital/labor ratio was significant for both the cases. On the basis of these regression results

she calculated elasticity of substitution for the years 1961, 1965, 1970 for All India and Rajasthan (Table 7). The elasticity of substitution was very high for the years 1961, 1965 and then slowly showed a declining trend. But even then she found that it was much higher than unity. She attributed the high values of elasticity of substitution to the massive investment programmes during the second and third five year plans which encouraged investment in fixed capital equipments. With the utilization of this installed capacity, σ showed a declining trend. It was noticed that for all the years $\sigma_R < \sigma_{AI}$. So, she concluded that there was large scope for substitution between labor and capital. The low value of elasticity of labor demand with respect to product wage indicates that in spite of the high substitution value, labor employment is insufficient. Thus higher level of employment could be generated in both the sectors (Rajasthan and All India).

Another such effort of productivity comparisons was made by Sastry (1981). He obtained the production function estimates for cotton textile industry (1946-70) for Maharashtra and Tamil Nadu and compared them with All India estimates. He estimated CES production function using the marginal productivity relationship and obtained estimates of elasticity of substitution not significantly different from one for Maharashtra, Tamil Nadu as well as All-India.

Thus, though in most of the studies at an aggregate

level, it was found that the elasticity of substitution is not significantly different from one, estimations at disaggregated level present a different picture as the elasticity of substitution varies substantially among industries.

4.2 Returns to Scale:

As we have seen, at aggregate level some studies assumed constant returns to scale[7], or, when the hypothesis of non-constant returns to scale was tested, it was rejected.[8]

However, in their major industrywise study, Diwan and Gujarati showed that there was no reason to proceed on the assumption of constant returns to scale without testing the hypothesis. They obtained the estimates of returns to scale based on all three models. They defined the measure of returns to scale (V) as

$$V = \frac{1-\sigma}{\lambda-\sigma}$$

For six industries (6), (8), (10), (19), (20), (22) (see Table II), they obtained a negative value for returns to scale. For the industries (5), (14), (15), (21), (25), (26), they obtained consistent estimates for all three models.

It can be seen from Table II that for a number of

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7. Gujarati (1966), Dutta Majumdar (1967), Narsimham and Fabryoy (1974).

8. Dutta (1955), Murty and Sastry (1957), Banergi (1971).

industries the value of V is quite high. They came to the conclusion that generally the value of V ranges between 2 and 3. The explanation they give for such a high value of V is that, in the period under consideration, the industrial sector was in a transitional stage. Government policy encouraged the building of capacity. Also, the technology itself was changing. New firms were going in for a more capital intensive technology. So capacity was growing faster than output, making it difficult to maintain the output at a capacity level. This might have resulted in over estimation of V. They pointed out that to transform these capacities into output becomes difficult due to bottlenecks in production like lack of raw materials or even spare parts depending upon imports which might result in stopping the production at less than capacity level. Thus in such a case "production is not only a function of labor and capital but also imports." However, the estimates of returns to scale obtained by Narsimham and Fabrycy (1974) for the period 1949-58 did not give as high estimates as obtained by Diwan and Gujarati.

Banergi (1974) estimated URCD for five industries to obtain an estimate of returns to scale. He estimated URCD with dummy variables for state and year. The use of dummy variables for state/year will allow each state/year to have its own intercept which will reduce the misspecification consequences of assuming the same slope for all states.

For cotton textile, the URCD estimates without state and time dummies gave an indication of increasing returns to scale. But with the introduction of state dummies, the estimates favored the hypothesis of constant returns to scale.

However, for the jute textile, he obtained estimates of constant returns to scale irrespective of state and time dummies.

For the sugar industry, URCD estimates gave evidence in favor of constant returns to scale. But with the introduction of state and time dummies, he obtained estimates significantly greater than one, implying increasing returns to scale. This gives an indication that the regional variations and cyclical fluctuations in this industry are significant. Other two industries - paper and bicycle, gave estimates of increasing returns, unanimously for all the models.

Sankar (1970) obtained maximum likelihood estimates (MLE) and Bayesian estimates of returns to scale from the CES production function. The two estimates are in close agreement for all 15 industries. Out of these 15 industries for three industries - cement, glass and glassware, bicycle, he obtained estimates of decreasing returns to scale for the period (1953-58). For the bicycle industry, Banergi (1974) obtained increasing returns to scale from a URCD estimation for the period 1946-58. For 5 industries both MLE and Bayesian estimates gave evidence for increasing returns to

scale for the period 1953-58.

Mehta (1980) estimated URCD for 27 industries to find that constant returns to scale prevail in most of the industries. When he applied the Tinter test to test the hypothesis of constant returns to scale, it was rejected in the case only 5 industries. However these estimations assume unitary elasticity of substitution. In case the hypothesis of unitary elasticity of substitution is not valid, to avoid the consequent misspecification, he estimated CES production function with non-constant returns to scale. These estimations also indicated presence of constant returns to scale in 17 industries and increasing returns to scale in only five industries. This led him to the overall conclusion that constant returns to scale prevail in most of the Indian industries.

Sastry (1981) also estimated CES function with non-constant returns for Maharashtra, Tamil Nadu, All-India for the cotton mill industry only to reject the hypothesis of non-constant returns to scale.

Rajalakshmi's estimates of returns to scale for metal and mineral group industries obtained from Diwan and Gujarati (1968) models, range from 1 to 4 in case of Model II giving an evidence of increasing returns to scale in this group of industries for the period 1960-73.

Similarly, she obtained evidence in favor of increasing returns to scale for the manufacturing sectors of All India

and Rajasthan for the period 1960-75, using the same relationships.

Thus we see that the empirical evidence on returns to scale varies as per the specification of the model and time period under consideration. The evidence so far at a disaggregated level does not strongly support any of the hypotheses of returns to scale.

4.3 Technological Change:

To measure technological change in Indian manufacturing Banergi (1971) computed the productivity index and came to the conclusion that over the years capital deepening was taking place and not technological progress. Narsimham and Fabrycy (1974) used the analysis of covariance to trace the presence of technological progress and found that it was occuring in spurts and not at a constant rate.

However, Sankar (1970) estimated technological progress using the production function estimates (Footnote 6 - g stands for Hicks-neutral technological progress in the CES function). In 10 out of 15 industries he found that the estimates of technological progress were positive. In 6 industries the estimates were significantly greater than zero at a 5% level of significance. However for cotton and cement industry the estimates were significantly below zero.

Venkataswamy (1975) traced the presence of technological progress using both CD and CES production function. The estimates obtained from CD function were a little lower than

those obtain from CES function. This is perhaps due to the reason, as mentioned earlier, that the co-efficients of a CD function carry the burden of expressing non-neutral technological progress as well as returns to scale. Venkataswamy found that the industries which experienced a high degree of technological change were - aluminium, copper and brass, electric fans, electric lamps, glass and glassware, bicycles, sewing machines, sugar, chemicals and cement over the period 1948-67. Though in industries like cotton and jute technological progress was negligible, on the whole he found that there was technological progress in the manufacturing sector.

Sakong and Narsimham (1974) analysed the nature and magnitude of technological progress in Indian manufacturing over the period 1949-58 using a different approach.

They applied Johansen's model which relates the rate of growth of labor productivity over a period to its income share in value added.[9]

They divided 28 industries into four groups - food and related industries; chemicals; metals and engineering; textile, ceramics, glass and wood.

The estimates of technological progress were further decomposed into inter-industry and intra-industry technological progress using Massell's model.[10] They found that over the period 1955-58. Also in the first period the decomposition of technological progress into intra and inter

9. Johansen (1961).
10. Massell (1960).

industry progress did not give a different picture which implied that there was no misallocation of resources. But in the second phase (1955-58) the intra industry technological progress was found to be greater than inter industry technological progress suggesting a misallocation of resources which they attributed to the Government policy during the second 5 year plan which emphasised investment in heavy industries.

The analysis at a disaggregated level shows that the estimates of the parameters of the production function vary with the form of the production function, with the level of aggregation. Not only this, the estimates may also vary with the particular assumptions made in the model under consideration.

CONCLUSION

In this study, we have made an attempt to review the literature on estimation of production functions and the estimations done for the Indian manufacturing sector. We were specifically interested in parameters - elasticity of substitution, returns to scale, and technological progress.

We started our analysis in the set-theoretic framework and defined the production possibility set. We also showed that this approach was superior to the traditional one. But, we also saw in the first chapter that to estimate the above mentioned parameters, which have important policy implications, one has to go back to the traditional framework. We mainly considered three forms of production function - CD, CES, VES.

The three variables, Y, L, K are measured at various levels of aggregation which very often implies adding up of heterogeneous quantities. The conditions under which they can be added up meaningfully, and for the aggregate production function to exist led to a brief discussion of the aggregation problem.

Solow and Fisher assumed that all the firms are equally efficient i.e. constant returns to scale. Then the aggregate production function exists only if the firms differ by capital augmenting differences. However, Sato (1975) allowed the firms to vary in productive efficiency. He showed that the existence of aggregate production is uniquely defined by the

efficiency distribution. Over time, the efficiency distribution changes. The aggregate production function should be invariant to the shifts in the efficiency distribution. It can be invariant to the shifts if and only if the technical change is factor augmenting. This means that the efficiency distribution should shift in a systematic manner. If the efficiency distribution stretches proportionately, it incorporates labor augmenting technologic progress. If it scales up proportionately, it accounts for the capital augmenting technological progress. Combining the two, we can also make a case for capital and labor augmenting technological progress. The aggregate production function is invariant between the efficiency distributions if the shifts are of the above mentioned type, i.e. factor augmenting technological progress.

There are many more aspects of the aggregation problem, e.g. aggregate production function serves as an important tool in growth analysis and also for forecasting purposes. In such cases we have to resort to aggregate production of exante nature. The aggregate production functions we discussed were of short run nature and shifts short run aggregate production functions. The relationship between the production function of short run nature and exante production function should be analysed. But it will form a part of more elaborate study.

As for the methods of estimating production function,

we discussed both, single equation and multiequation models. Estimation of single equation models leads to a number of problems if OLS assumptions are violated. One can also choose to estimate a system of simultaneous equations consisting of a production function and first order conditions of profit maximization. But here we have to face the identification problem. In such a case we have to add exogenous variable to the profit maximizing conditions. By estimating a system of simultaneous equations we eliminate simultaneity problem. Also, the complete system of equations, as pointed out by Intriligator (1978), "expresses the assumption that the data reflect both the behavior of the decision maker (the firm) and the technology, while the production function reflects only technology". Kmenta (1967) estimated a CES function using simultaneous equations. But most of the estimations done for the Indian economy are based on single equation models.

These methods of estimation are based on the assumptions of constant returns to scale, profit maximization and perfect competition. In some cases, constant returns to scale becomes a testable hypothesis.

A number of estimations have been done for the Indian manufacturing sector. We classified these studies into two broad categories: estimations done for the manufacturing sector as a whole; estimations done at a disaggregated level. The estimations are done using cross section as well

as time series data based on both CES and CD production functions. We reviewed the empirical findings about the elasticity of substitution, the returns to scale and technological progress.

If we look at the estimates of elasticity of substitution, while a number of studies have estimated a CD function, other studies do not unanimously provide evidence in favor of a particular hypothesis. Banergi (1971) estimated a number of relationships based on CES production function for the time-series 1946-58. But all of them rejected the hypothesis $\sigma \neq 1$. However Narsimham, Fabrycy (1974) found that the substitution parameter is sensitive to specification changes. Including time trend in the CES function they rejected the hypothesis $\sigma = 1$. On the contrary, from the cross-section examination of CES production function by Gujarati concluded that $\sigma = 1$. Similarly at a disaggregated level, the estimates of elasticity of substitution vary from industry to industry and according to the time span covered. So, the estimates of elasticity are sensitive to specification as well as to the level of aggregation. In the light of such a wide range of estimates of elasticity of substitution, and knowing its policy implications, the proposition $\sigma = 1$ should be made a testable hypothesis and not a built-in property of the production function.

The estimates of returns to scale mostly based on CD

function came to an aggreement in favor of constant returns to scale for time-series as well as cross section data at an aggregate level. Banergi's (1971) estimates of returns to scale based on the CES function with non-constant returns to scale also rejected the hypothesis of non-constant returns to scale. However, again at a disaggregated level, Gujarati and Diwan (1968) and other studies showed that the estimates of returns to scale also change from industry to industry. The estimates of returns to scale, at a disaggregated level also show a wide range. These findings make returns to scale a hypothesis that should be tested, rather assumption as seen in a number of estimation methods.

The findings about technological progress do not come to an unambiguous conclusion. Banergi (1971) and Narsimham, Fabrycy (1974) traced the presence of technological progress using different methods. Banergi (1971) estimated the Solow Index to come to the conclusion that over the years the process of capital deepening has set in and not technological progress. Narsimham, Fabrycy (1974) on the other hand, using analysis of covariance found technological change to be significant and it occurred in spurts. At a disaggregated level, Venkataswamy (1975), Mehta (1980) used the production function approach. They found that different industries experienced different magnitudes of technological progress.

Another aspect of this type of exercise i.e. estimation of a production function was brought by Rajalakshmi (1985)

where she made productivity comparisons of All India and Rajasthan. Other study bringing out this aspect was Sastry's (1981) estimates at All India level, for Tamil Nadu and Maharashtra for the cotton textile industry. Similarly Banergi (1974) incorporated dummy variables for the States to find that in case of some industries the estimates of production function change significantly with the introduction of dummies which implies that productivity varies from State to State.

Most of these estimations done for the Indian manufacturing are based on the assumptions of perfect competition, profit maximization, constant returns to scale. The empirical evidence does not conform to constant returns to scale, especially at a disaggregated level. We should therefore try to relax the assumption of constant returns to scale. As for the assumption of perfect competition, Dhrymes (1965) developed a model to test this proposition. So, we should test the validity of the assumption of perfect competition also.

Table 1: ESTIMATIONS FOR THE MANUFACTURING SECTOR AS A WHOLE

Author(s)	Nature of the function	Source of Data	Measurement of Capital	Measurement of Labor	Technological Progress	Labor Co-efficient	Capital Co-efficient	Elasticity of Substitution	R^2	No.of observations	Industry
A. Cross-Section Studies:											
1. Dutta (1955)	URCD 1946	CM1	Gross fixed Capital Assets	Average No. of Labor Employed	N.A.	.7725	.2326		.968	29	Indian Manufacturing
	URCD 1947	"	"	"	"	.5704	.4954		.971	29	"
2. Murty & Sastry (1957)	URCD 1951	Balance Sheets of firms	Total fixed assets net of depreciation + working capital	Total wages and salaries paid	"	.59	.40		.98	607	"
	URCD 1952					.53	.50		.95	320	
Gujarati (66)	CES 1958	CM1	-	Wages	-	-	-	.97	.66	28	"
B. Time-Series Studies:											
3. Dutta Majumdar (1967)	RCD 1951-61	CM1, AS1	"	"	N.A.	.46	.54				"
4. Banergee (1971)	a) URCD	CM1	Gross fixed capital assets	Total Employment	No	.992	.443		.82		"
	b) URCD with utilization variable	"	"	"	"	1.015	.199		.879		"
	c) URCD with time trend	"	"	"	"	1.329	.075		.829		"
	d) URCD with U & t	"	"	"	"	.223	.905		.900		"
	e) SMAC 1946-58	"	"	"	"			1.106	.785		"
	f) SMAC with time trend	"	"	"	"			0.607	.820		"
	g) SMAC distributed lag Model	"	"	"	"			.786	.839		"
	h) SMAC serial correlation	"	"	"	"			.576	.858		
5. Narsimham Fabry CY (1974)	RCD 1946-58	CM1	Fixed capital deflated by workers whole sale price index	No. of	Technological progress	.2271	.7729		.6252		28 Organised Large Scale Indian Industri
	CES				in spurts,	.6925	.3075	.7821	.9606		
	CES with time trend				not continuous process	.6973	.3027	.3534	.9132		

Table 2: Industrywise Estimates of Elasticity of Substitution, Returns to Scale

Industry	Diwan & Gujrati (1968) Estimates of σ Model I	II	III	Estimate of Returns to scale Model I	II	III	Nersimhem & Fabrycy (1974) Returns to scale
1. Fruits & vegetables	.9314	.6146	.6748	- .457	.7325	4.1532	1.10
2. Wheat flour	.8508	1.0419	.9618	- .4881	.0746	- .0751	3.05
3. Rice	.0556*	.1097	.1445	5.8009	6.2128	10.8565	1.70
4. Biscuits	- .0316*	1.3255	.3003	3.2832	.5874	3.8958	.89
5. Sugar	- .2438*	- .1552	.0725	3.2089	3.3119	3.2305	1.00
6. Distillaries and Breweries	.3254	.5993	.2057	-3.3495	-1.2459	-10.7192	.81
7. Starch	1.0045	1.1343	1.7338	.0133	.3325	1.4723	1.53
8. Vegetable oil	.2071*	.5081	.343	-5.0859	-1.2012	-1.6872	.22
9. Paints & Varnishes	- .1880*	- .1259	.0673	3.3474	3.6770	5.5254	.43
10. Soap	.2560	.2550	.0379	-3.4428	-3.5662	-4.8395	1.12
11. Tanning	- .0740*	.1412	- .0073	2.8352	5.5335	2.9020	.78
12. Cement	.9752*	1.3423	2.0782	- .0856	.5936	1.2178	1.54
13. Glass & Glassware	.0435*	.3025	.1382	7.6093	-24.3881	9.1680	.96
14. Ceramics	.2594	.2286	.2197	3.2886	3.1876	3.1249	1.37
15. Plywood & Tea Chest	.1813*	.1765	.0760	1.7254	1.7224	1.6491	1.50
16. Paper & Paper Board	.9396	1.2952	.8873	- .2003	.6435	- .4887	1.07
17. Matches	.3798*	1.1760	.5852	3.9730	.1577	1.4392	.91
18. Cotton Textiles	.0404*	.2454	.2242	4.7670	5.8450	3.2928	.87
19. Woollen Textiles	.0614*	.0324	.0168	-27.0489	-37.0727	-8.1458	2.06
20. Jute Textiles	.4430	.5051	.4201	-1.1119	-0.8785	-1.3676	.99
21. Chemical Drugs & Pharmaceuticals	.0580*	.0667	.0788	2.3656	2.3808	2.3035	1.20
22. Aluminium Copper & Brass	.0155*	.0670	.0112	-66.0738	-13.7205	-33.2929	.99
23. Iron & Steel	.1411*	.2020	.1396	7.3852	8.8174	6.3125	1.04
24. Bicycles	.0458*	- .4316	.0386	1.4503	.9874	1.4536	1.16
25. Sewing Machines	.0187*	.0979	- .0026	2.1421	2.0780	2.2132	1.70
26. Electric Lamps	-1.3306	-1.1214	-1.2750	1.8138	1.9757	1.8517	1.04
27. Electric Fans	.6611	.8512	.7373	-2.4086	9.9200	-1.7536	.94
28. General Electrical & Engineering	.3804	.4494	.3901	4.9135	6.7641	4.9304	1.76

* - refers to insignificent estimates.

Table 3 : Estimates of Elasticity of Substitution in Large-Scale Indian Industries (1953-65)

	Industry	Model I		Model II		Model III	
		σ	R^2	σ	R^2	σ	R^2
1.	Wheat flour	.56	.17	.77*	.96	.34*	.53
2.	Rice milling	1.99*	.72	.56*	.94	.003	.0002
3.	Biscuit making	1.22*	.53	.91*	.98	.71*	.81
4.	Fruit and vegetable processing	1.39*	.50	.76*	.88	.31	.16
5.	Sugar	0.54*	.67	.83*	.98	.70*	.95
6.	Distilleries & breweries	1.90*	.55	.69*	.94	.13	.06
7.	Starch	.99*	.54	.80*	.93	.22	.14
8.	Vegetable oils (excluding edible hydrogenated oils)	1.50	.27	.33*	.89	.0004	.0002
9.	Edible hydrogenated oil	.06	.16	.54*	.94	.075	.094
10.	Paints and varnishes	- .35	.05	.66*	.95	.18*	.36
11.	Soap	.05	.003	.74*	.96	.30*	.68
12.	Tanning	1.24*	.92	.41*	.82	.16	.03
13.	Cement	.59*	.27	.83*	.97	.40*	.69
14.	Glass and glassware	1.16*	.79	.43*	.93	.18*	.35
15.	Ceramics	1.38*	.89	.44*	.71	.18	.082
16.	Plywood and teachests	0.026	.0006	.46*	.56	.15	.12
17.	Paper and paper board	.31*	.64	.79*	.97	.60*	.90
18.	Matches	.94*	.49	.48*	.86	.22*	.29
19.	Cotton textiles	.70*	.65	.44*	.83	.43*	.70
20.	Jute textiles	1.30*	.75	.44*	.94	.14*	.31
21.	Woollen textiles	.71	.05	.58*	.82	.19	.16
22.	Chemicals (including drugs and pharma)	.90*	.86	.81*	.98	.76*	.88
23.	Iron and steel	2.65	.73	.81*	.98	.62*	.89
24.	Bicycles	.23	.022	.45*	.75	.18*	.26
25.	Sewing machines	.87*	.34	.09*	.66	.011*	.29
26.	Electric fans	.82*	.85	.91*	.84	.18	.07
27.	Electric lamps	1.27	.36	.70*	.94	.49*	.74

* - significantly different from zero.

Compiled from Mehta (1980).

Table 4: Estimates of Elasticity of Substitution by Kazi (1980) (Two-digit classification)

Industry	1974		1975	
	*σ_c	σ_1	σ_c	σ_1
1. Foodstuffs	1.3490^c (.1715)+	$.6275^a$ (.2287)	1.4771^c (.1416)	1.1766^b (.2465)
2. Tobacco & Textiles	$.5528^a$ (.1693)	$.3529^a$ (.1154)	1.1031^b (0.1127)	$.8373^a$ (.1628)
3. Furniture, Printing, Publishing, Footwear	$.5066^a$ (.1071)	$.4143^a$ (.1045)	1.1855^b (.1913)	$.8633^a$ (.1418)
4. Chemical and Chemical Products	$.8337^b$ (.4247)	$.3894^a$ (.4673)	$.9384^b$ (.2916)	$.5232^a$ (.3134)
5. Petroleum, Coal & Non-Metallic Mineral Products	1.4760^c (.1858)	1.6739^b (.4674)	1.3284^c (.1474)	1.5158^b (.3480)
6. Metal Industries	1.2451^b (.1987)	$.6037^a$ (.1899)	1.0605^b (.3425)	$.4258^a$ (.4444)
7. Machinery	$.8353^b$ (.2616)	$.7251^a$ (.2557)	1.0628^b (.2452)	$.9580^a$ (.2635)
8. Electric Machinery and Transportation	$.8555^b$ (.2966)	$.2703^a$ (.3046)	1.2879^b (.3640)	$.9507^a$ (.5100)
9. Miscellaneous Manufacturing Industries	$.9871^b$ (.1837)	$.5335^a$ (.2464)	1.0394^b (.1451)	$.6603^a$ (.1866)

Notes: * σ_c = Elasticity of substitution from CES production function.

σ_1 = Elasticity of substitution from CES production function with an additional explanatory variable (K/L).

a implies

b implies is not significantly different from unity.

c implies

The null hypotheses are tested at 5% level of significance.

Table 5 : Estimates of Elasticity of Substitution by Kazi (1980) (Three-digit classification)

	Industry (1975)	Omit	σ_0	σ_1
1.	Food products	1.1468	1.1468	.9496
2.	Tobacco and beverages	1.0798	1.0798	.6391
3.	Textiles	.9975	.9975	.9098
4.	Footwear and allied products	.5134	.5134	.4844
5.	Furniture and fixtures	.6751	.6751	.6013
6.	Paper and printed allied products	1.0728	1.0728	.8288
7.	Leather and rubber products	1.3462	1.3462	1.1192
8.	Chemicals and chemical products	1.8807	1.8807	.5674
9.	Petroleum and non-metallic mineral products	1.0354	1.0354	.8420
10.	Basic metal industries	.8658	.8658	.2768
11.	Metal products manufacturing	1.7780	1.7780	1.0040
12.	Manufacturing of machinery except electrical machinery	.7946	.7946	.7634
13.	Electrical machinery apparatus, appliances	1.1234	1.1234	1.1234
14.	Transport equipment	1.0727	1.0727	.8716
15.	Miscellaneous	1.0114	1.0114	.5936
16.	Electricity, gas, water and sanitary	.6676	.6676	1.1108

Table 6a : Estimates of Elasticity of Substitution (Diwan & Leonardson (1985)) (Groupwise)

Group Size	σ_n	σ_b
Large	.65	3.15
Medium	.53	1.84
Small	1.12	2.88

Table 6b : Estimates of Elasticity of Substitution (Diwan & Leonardson (1985)) (By Industry Type)

Industry Type	σ_n	σ_b
I	1.30	1.78
II	.33	- .35
III	.78	1.53
IV	.44	1.56
V	.49	.74

Table 7 : Elasticities of Substitution for Total Manufacturing Sectors of All-India and Rajasthan (Rajalakshmi (1985))

Year	All-India	Rajasthan
1961	8.6052	7.0052
1965	5.5536	2.4978
1970	2.4475	1.8983

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