

CAPITAL BUDGETING : A SELECTED SURVEY OF LITERATURE

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CHAPTER I

INTRODUCTION

In the economic theory we are concerned with the allocation of scarce resources. The capital budgeting theory is concerned with the allocation of resources over a period of time, so as to maximize our objectives. Capital budgeting means investment decisions as : for the individual; purchase of a house, a car, or any other consumer durable good - for a firm; acquisition of a plant, a machine or expenditure on research and development - for a government; construction of a dam, a road, etc.

Although we shall concentrate on the investment decisions of a firm, the same is applicable to other entities mentioned above with some modifications. Every decision-maker is faced with a number of investment opportunities to be financed out of a limited amount of funds.

Generally, capital budgeting theory seeks to answer the following three questions :

- i) What particular projects to be accepted?
- ii) What should be the total amount of funds to be invested in these projects?
- iii) How should these investment expenditure be financed?

All the above three questions are closely interrelated. Decisions about the project choice and the method of

financing should be made simultaneously.

Similarly, project appraisal is impossible without considering the problem of the cost of funds to the firm, as this cost is itself influenced by the characteristics of the investment proposals.

However, it is beyond the scope of this study to go into the details of the problem of cost of funds, and the financial decisions of a firm. Throughout this study we shall assume the cost of capital to be given, and we shall not deal with the last of the above three questions, which is about the method of financing of the accepted projects.

There are several reasons for interest in capital budgeting. Developments in the past 30 to 40 years has stimulated the search for a systematic method of project appraisal. Firms with a limited amount of funds are, now, confronted with a greater number of investment opportunities because of the rapid economic growth.

Rapid economic development has caused the larger size of capital requirements and it has increased the rate of product and technological obsolescence. Therefore, correct and comprehensive project-appraisals are vital to modern industries. As most of the capital goods are made specific, they have very little value in alternative uses. Thus, mistakes are not easily corrected. Moreover, technological progress has made industry so capital-intensive that, once

an investment project is financed by a firm, it has simultaneously determined the future framework in which it will operate.

The coverage of this study is narrow as stated above. In the second chapter we will assume a world of certainty. Every future expected outcome will obtain without fail. Our estimations are the exact values which will happen in the future and so on. With this optimistic view in mind we shall study the relatively simpler methods of investment appraisal. In the third chapter we introduce the great cause of difficulty : "risk and uncertainty". Though the capital budgeting theory has not been able to solve this problem completely, a voluminous literature exists on the treatment of risk and uncertainty with regard to capital investment appraisal. We shall try to present a brief survey of this literature. The fourth chapter is devoted to the problem of firms which do not have adequate funds to finance all the projects which are worth more than they cost to the firm. These firms must "ration" their available capital among the various worthwhile investment proposals.

This dissertation ends with a brief summary of our results and conclusions.

CHAPTER II

INVESTMENT APPRAISAL UNDER CERTAINTY

2.1 Introduction

Corporate Management needs an objective means of measurement for selection of different projects, so that it will maximize the long-run benefits to the present owners of the firm. This implies selection and use of a formula for measurement of profitability of projects. This formula will reduce the attributes of an investment (like cost, returns, life, etc.) to a single number providing a comparative and absolute measure of the profitability of projects, i.e., the formula will help in comparing the available projects and in determining the total size of the budget.

There are different formulae or yardsticks or standards by which investment proposals can be evaluated under certainty. Basically we can divide these approaches into a) Non-discounting methods, and b) Discounted Cash Flow Methods.

2.2 Non-Discounting Methods

1) The Degree of Necessity¹: This method assigns a degree of urgency or the extent to which a proposal can not be

¹ Our discussion is based on a pioneering work by Dean J. (1954).

postponed. The investment proposals which can no more be postponed are taken first.

The serious defect of this method is that it does not take into account the productivity or profitability of capital, i.e., its effect on the company's earnings.

Moreover the degree of necessity is subjective and can not be measured. In this case, capital budgeting is likely to degenerate into a contest of managerial personalities.

ii) The Payback Period : The number of years in which the initial outlay is paid back by the project is the payback period. This method overweighs the initial receipts and ignores the distant earnings.

Therefore, it does not take into account the vital matter of the life pattern of earnings. If the earnings are to last longer, the profitability will be higher. Thus, it is not a good overall measure of a company's investment worth.

The pay off estimate¹ of the rate of profit is the reciprocal of the ratio of its cost to its annual earnings. If X denotes the annual earnings and K denotes the total cost of the project, then

$$R_p = \frac{X}{K} \quad \dots\dots\dots (1)$$

¹ Please see Gordon M.J. (1955) for further details.

where R_p represents the pay off estimated rate of return.

2.3 Discounted Cash Flow Methods

The mechanics of discounting a future sum of cash inflows or outflows is simple. It requires an understanding of the mathematics of compound interest. Basically there are two major variants of the DCF methods :

i) Net Present Value Approach : The gross present value of a project is the expected earnings of a project discounted back to the present at the market rate of interest. Generally, the gross present value of a project can be stated as :

$$G.P.V. = \sum_{t=1}^T \frac{X_t}{(1+r)^t} \dots\dots\dots (2)$$

where X_t is the expected net returns at the time t including depreciation allowances and interest charges, and r is the market rate of interest.

In order to secure returns from an investment programme it is usually necessary to invest money in one or more time periods. If Rs. K is the present value of all these outlays then G.P.V. less K may be called as the net present value (N.P.V.) of an investment. If this net present value (N.P.V.) is positive then it is profitable to invest in that project.

ii) The Internal Rate of Return Approach : The internal rate of return is defined as that discount rate which makes

the N.P.V. of an investment equal zero or equivalently that rate of discount which equates the present value of returns and the present value of total outlays. Generally it may be stated as :

$$K = \sum_{t=1}^T \frac{X_t}{(1+i)^t} \dots\dots\dots (3)$$

where X_{tS} are inclusive of depreciation allowances and interest charges. As before K represents the present value of total outlays, and i , here, is the internal rate of return.

If X_{tS} are net of depreciation, the R.H.S. in equation (3), the discounted value of returns, should be zero rather than the initial cost of the project. Because when depreciation is deducted, we have already accounted for the initial cost. Similarly, when X_{tS} are net of interest charges; then, i , should be compared with zero rather than r , the market rate of discount; to determine whether the project is profitable or not.

iii) Comparison Between N.P.V. and IRR Approaches¹: Under certain conditions² the approaches give the same result for accept or reject decisions. This is so because if

1 The discussion here is based on an article by Solomon E. (1956).

2 We shall see below how these approaches may give different answers.

the rate of return is greater than the company's cost of capital³ then for those projects the present value of receipts discounted at the company's cost of capital will be greater than present value of outlays again discounted at the company's cost of capital. For decisions involving more than a simple "accept or reject" answers the application of the two formulae sometimes gives ambiguous results. To illustrate, assume that we have to select one of the two mutually exclusive alternatives X and Y, where the company's cost of capital is given to be 10 per cent.

Project X requires Rs. 100 now and has an expected return of Rs. 120 after one year hence, i.e., at t_1 and nothing afterwards. Project Y requires Rs. 100 now and promises Rs. 174.9 after 4 years hence, i.e., at t_4 . The present value of X, at the company's cost of capital of 10 per cent, is Rs. 109.09. The present value of Y is Rs. 119.46. The internal rate of return of X is 20 per cent whereas that of Y is only 15 per cent. Thus, the rate of return approach chooses X over Y where P.V. method chooses Y over X. How can we reconcile these contradictory results?

The implicit assumption made by those who employ

3 The cost of capital may be defined as the minimum required rate of return or the opportunity cost of funds provided to the firm.

the IRR approach is that the funds can be reinvested at a rate of profit equal to that promised by the longer-lived of the two projects, in this case 15 per cent on project Y. On the other hand, the present value approach assumes that funds can be reemployed at the rate equal to the company's cost of capital. The two approaches can lead to a correct and consistent ranking of the projects only if the following factors are taken into account.

- 1) The possibility of reinvestment of funds should be taken into account while comparing investment proposals of different life and earning pattern. Thus, comparison should be made between the courses of action rather than the projects only. The ultimate criterion should be the total wealth that the investor will receive by the terminal date of the longer-lived project.
- 2) If the rate of return is to be used as the index of profitability, then we should consider the per annum rate of return for the courses of action to a common terminal date (usually the terminal date of the longer-lived project).
- 3) If the present value is to be used as an index of profitability, then the expected reinvestment rate or set of rates should be used. This rate does not necessarily have to be equal to the company's cost of capital.

- 4) And finally, when we compare projects requiring different outlays, it is necessary to find out the "present value per rupee of outlay", rather than the present values of the projects.

iv) The Problem of Multiple Rates of Return¹ : There are some projects which have a net cash outflow in the terminal section of their life. For example, there are some projects which have positive abandonment costs, such as old buildings which need to be demolished at the end of the project's life. The application of usual rate of return formula to these projects would yield more than one answer.

Consider two mutually exclusive investment opportunities : a) The existing equipment (e.g. a pump set for an oil well) earns Rs. 10,000 for 2 years, and, b) a new project (e.g. another pump set) which costs Rs. 1,600 and helps earnings increase upto Rs. 20,000 in the first year but no earnings for the next year.

The incremental cash flow of this new investment can be looked at as :

Time Period	t_0	t_1	t_2
Incremental cash flow due to investment	-1600	10,000	-10,000

¹ For further discussion please see Solomon, E. (1956).

The application of the IRR approach to this incremental cash flow would yield 25 per cent and 400 per cent. That is, at these two rates of interest the present value of the new project will equal the initial cost.

We can find out the true rate of profit of this type of projects, by first understanding what exactly the project is worth to the investor. Here, in our example, new investment will mean getting Rs. 10,000 a year earlier at the cost of Rs. 1,600. If the investor thinks that these earnings can be reinvested at a rate of x per cent, then the earlier returns are worth Rs. $100x$, i.e., if $x = 23\%$ then getting Rs. 10,000 a year earlier is worth Rs. 2300. Rs. 2300 is achieved at the end of the second year at a cost of Rs. 1600 now. This can be stated in terms of an "equivalent rate of return", which in this case would be 20 per cent. (Rs. 1600 at 20 per cent would amount to Rs. 2304 at the end of two years). This approach can be used to find out a meaningful and unique rate of return for any set of cash flows

v) Fisher's Rate of Return Over Cost¹: Fisher's Rate of Return over Cost was developed in order to rank investment proposals by the present value criterion. It can be defined with reference to at least two investment proposals.

¹ For further details please refer to Alchian A.A. (1955).

If X_t denotes the receipts stream and K_t represents the outlay stream then the net present worth of a project is defined as :

$$\text{N.P.V.} = \sum_{t=0}^T (X_t - K_t) (1+r)^{-t} \dots\dots\dots (4)$$

and

$$\sum_{t=0}^T [(X_{1t} - K_{1t}) - (X_{2t} - K_{2t})] (1+r)^{-t} \dots\dots\dots (5)$$

is the difference between the net present worths of the two proposals. Fisher's Rate of Return Over Cost is the rate r which sets this difference equal to zero. Fisher's rate of return over cost always involves a comparison of two investment proposals, rather than merely discounting the receipts and outlays of one project.

For illustration assume that there are two investment proposals each requiring an outlay of Rs. 25 immediately. One project, A, promises to yield Rs. 5 per year for 10 years and the other, B, promises to yield Re. 1 in the first year and Rs. 2 in the second year and so on for ten years. The IRR for project A is 17.5 per cent and for project B it is 12 per cent, where the Fisherian RRC of B compared with A is 6 per cent. That is, at 6 per cent both the projects have equal net present value. A will be preferred to B according to IRR criterion; yet B will have greater net present value for rates of interest below 6 per cent and for rates above 6 per cent, A will have

greater net present value. Therefore, according to Fisher, ranking of investment proposals depends on the market rate of interest. Fisher's RRC serves as a device to determine all the rates of interest at which investment 1 will be preferred to investment 2. It does not rank the investments once for all, for the simple reason that ranking changes as interest rate changes.

vi) Comparison of IRR and the Pay-off Estimate¹: Assume for simplicity that the investment's annual returns are constant throughout the life of the asset and it will have no scrap value after its useful life.

We have already seen above that IRR is that rate of discount for which the following equation holds.

$$K = \sum_{t=1}^n \frac{X}{(1+i)^t}$$

$$K = \frac{X}{i} - \frac{X}{i} \left(\frac{1}{1+i} \right)^n \quad (2) \quad \dots\dots\dots (6)$$

1 Our discussion is largely based on an article by Gordon, M.J. (1955).

2 We have

$$K = \sum_{t=1}^n \frac{X}{(1+i)^t}$$

which is

$$K = \frac{X}{(1+i)} + \frac{X}{(1+i)^2} + \dots\dots + \frac{X}{(1+i)^n}$$

where

$$K(1+i) = K + K_i = X + \frac{X}{(1+i)} + \dots\dots + \frac{X}{(1+i)^{n-1}}$$

Hence :

$$K_i = X - \frac{X}{(1+i)^n}$$

$$K = \frac{X}{i} - \frac{X}{i} \left(\frac{1}{1+i} \right)^n$$

The relationship between the rate of return and the reciprocal of pay-off can be seen by rewriting equation (6) as :

$$i = \frac{X}{K} - \frac{X}{K} \left(\frac{1}{1+i} \right)^n \dots\dots\dots (7)$$

An examination of equations (1) and (7) reveals that a project's true rate of return is equal to the reciprocal of the pay-off ratio minus the same quantity multiplied by some quantity $\left(\frac{1}{1+i}\right)^n$ which approaches zero as the life of an investment increases.

If an investment proposal will earn the same amount for ever, its rate of return is simply the reciprocal of its pay-off. And if the investment is finite the rate of profit is smaller than the pay-off reciprocal by the quantity $X/K \left(1/(1+i)\right)^n$. This margin falls as life of the investment increases. Consequently, subject to certain margin of error, a firm may take the reciprocal of its pay-off ratio as an estimate of its rate of profit.

CHAPTER III

CAPITAL BUDGETING UNDER UNCERTAINTY

3.1 Introduction

In this chapter we shall examine the ways in which uncertainty about future outcomes of individual projects can be handled. Assuming that the firms are wealth maximizers and the management is not interested in any other goal, these methods would be required to evaluate risky projects and recommend acceptance of those projects which would maximize the wealth of the investing firm.

It is necessary to note that all the methods to be considered here are, basically different variants of the DCF (Discounted Cash Flow) method, adjusted in some way or the other to incorporate the risk element. Thus, all these methods are based on the principle of discounting. Our task, here, is to explain these different methods and their relevance to the problem of capital budgeting under uncertainty.

3.2 Expected Net Present Value (ENPV)

If returns to a project are not known with certainty, but the probability distribution of future outcomes are known, then it is possible to find out the expected net present value of that project. This becomes more complicated

when returns in one period depend on returns in some other periods. We shall illustrate the concept of ENPV with the help of the following example¹.

Consider a project which requires Rs. 150 to initiate. There is a 0.7 probability that the project will return Rs. 130 after one year and a 0.3 probability that it will return only Rs. 90 after one year. If the project returns Rs. 130 in the first year, there is a 0.4 chance of having returns of Rs. 120 after two years and a 0.6 chance of Rs. 70 after two years. However, if the project returns Rs. 90 in the first year, there is equal probability of 0.5 each to get Rs. 130 and Rs. 100 after two years. This is shown more concisely in the following figure :

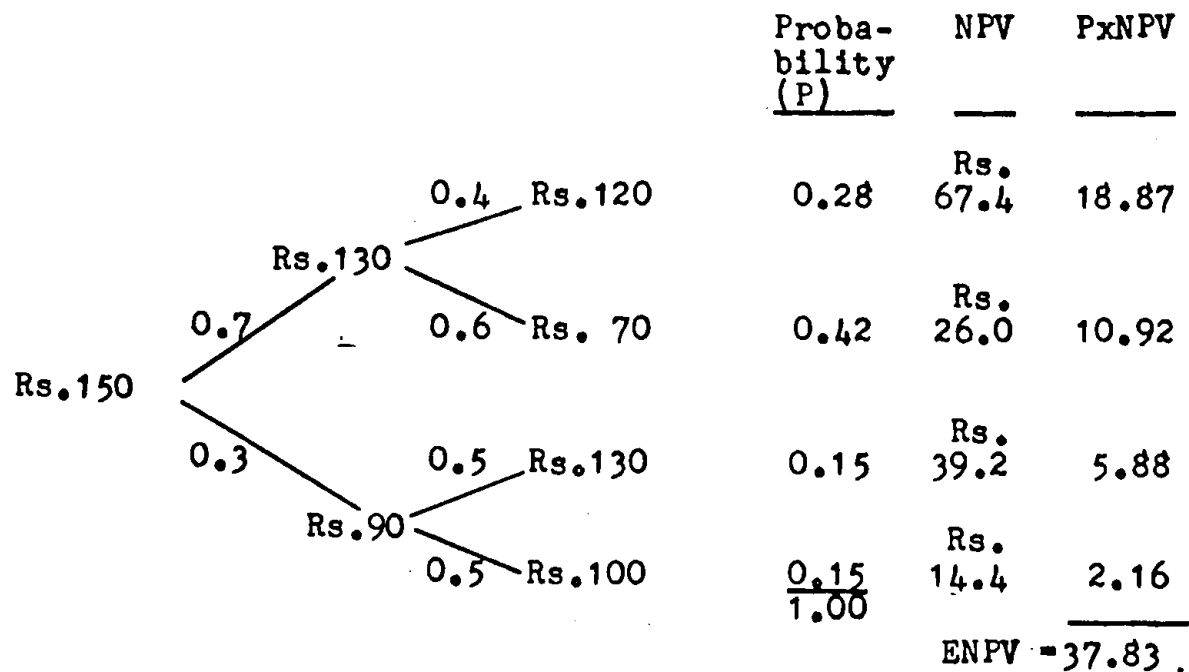


Figure 3.1

¹ This numerical example is reproduced from Wilkes (1977), pp. 21-25.

From the above figure it can be seen that there is a 0.28 chance of getting Rs. 130 after one year and Rs. 120 after two years. The net present value of this occurrence, assuming a 10 percent rate of discount, is Rs. $67.4 (= 130/(1.1) + 120/(1.1)^2 - 150)$. The other possible NPVs and their associated probabilities are shown in the figure. The expected net present value of this project is Rs. 37.83 which is the sum of all the possible net present values multiplied by their respective probabilities. The figure 37.83 is expected in the sense that if several trial runs of the project were made the average of the NPV outcomes would be very near to 37.83. A possible decision rule based on this method may be : accept those project which have positive ENPVs and reject those with negative ENPVs.

However, for the problems which involve more than a simple "accept or reject" decision, this rule needs some further clarifications. For instance, consider just two net present values : 200 with a probability 0.90 and -1000 with a 0.10 probability. The ENPV of this proposal is +80. There are good reasons for an investor not to prefer this ENPV to a +70 from possible NPVs of +80 (P=0.50) and +60(P=0.50). Investors are generally assumed to be "relative-risk-averse"¹; between two projects with equal ENPVs, that project which has the narrower spread of possible

¹ Please see Hirshleifer, J. (1966).

returns would be preferred. The variability of returns is taken to represent the riskiness of a project.

For a quantitative decision-making procedure, therefore, it is necessary to measure the degree of dispersion of possible outcomes. There are various statistical measures of dispersion, but the most widely used measures are standard deviation and variance of NPVs.¹

The formula for calculation of standard deviation (s.d.) is :

$$\text{s.d.} = \sqrt{\sum P_i (\text{NPV}_i - \text{ENPV})^2}$$

Variance, the alternative measure of dispersion, is just the square of s.d.

Thus, let us incorporate these considerations into the above rule.

From a pair of two projects with equal ENPV select that one which has the lower s.d. or, equivalently, from a pair of two projects with equal s.d. the one with higher ENPV will be preferred.

The rule applies, in principle, to an evaluation of an individual project. This exercise may be regarded as a comparison between the individual project and a null

¹ It may, quite rationally, be argued that only the deviations below the mean should be considered as risk.

project (i.e. doing nothing). It does not give an answer in all cases, a project with high returns may also have higher standard deviation.

If this rule is applied to all pairs of proposed projects, it will single out, what we may call as, the "mean-variance efficient" projects. A mean-variance efficient project has the highest ENPV for a given level of standard deviation and the lowest s.d. for a given level of ENPV.

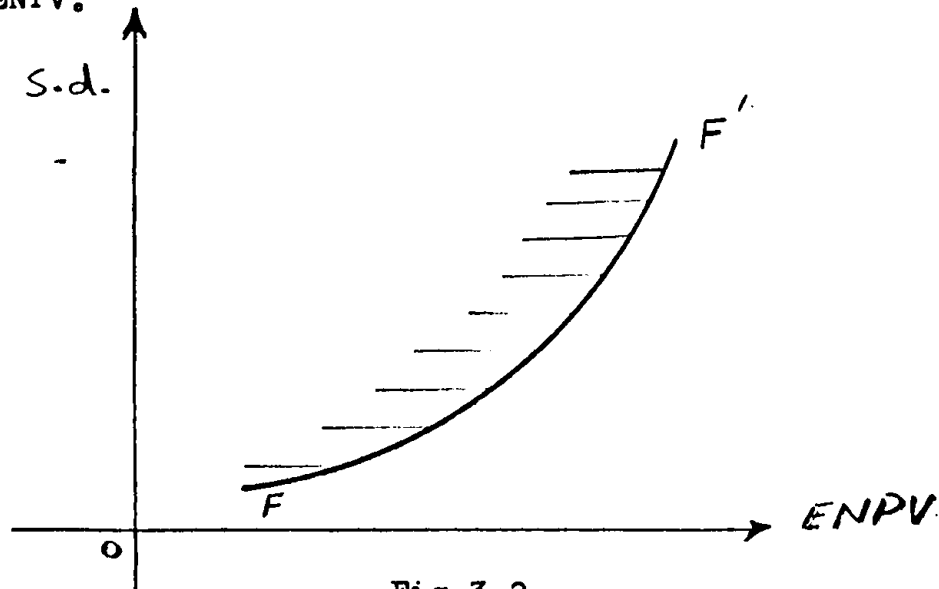


Fig.3.2

The line FF' in Fig. 3.2 is a hypothetical "frontier" of such "mean-variance" efficient projects. It is not necessary that the frontier should be smooth. But, the curvature of it may be explained as : eventually greater returns are achieved by acceptance of "increasingly" greater risk. Points to the left of the frontier are inefficient and points to the right are not achievable. Therefore, the investing firm's decision procedure is

reduced to a selection from the points on the frontier.

If indifference curves could be drawn reflecting the preferences of owner(s), it would be possible to find an optimum level of investment. Graphically, it implies selecting a point on the frontier that is on the highest (i.e., farthest to the right) indifference curve. However, for practical purposes, indifference curve analysis is somewhat of an idealized solution procedure. There are some alternative approaches, one of which argues that it is possible to summarize the risk-return combinations in a single variable, V , where

$$V = \frac{\text{s.d.}}{\text{ENPV}},$$

is called the coefficient of variation. A possible decision rule based on such a formula is : select a project which has the lowest coefficient of variation. The reason for the use of this rule, is quite plausible. For larger values of ENPV greater values of s.d. are acceptable.

3.3 Decision-Tree Approach

In the capital budgeting process, there are some investment opportunities which involve a sequence of decisions to be made over time. The decision-tree approach is an analytical tool which deals with this type of investment decisions. It studies various decisions in relation to their subsequent chance events. The value of decision-tree approach is in its laying out available information in a way that enables management to make systematic and better decisions. A decision-tree is not an answer in itself; rather, it helps the management determine which course of action would mean greatest gain to the present owners of the firm.

To illustrate, let us take a firm considering the introduction of a new product which it can distribute in the State of Maharashtra or throughout India. If it distributes in Maharashtra only, plant and machinery expenses would cost Rs. 50 lakhs, and the company can reevaluate the project at the end of 3 years to decide whether or not it should distribute at the national level, which will cost another Rs. 1 crore. To distribute nationally right from the beginning would cost Rs. 1.20 crores. Assume, that the risk-free after-tax cost of funds is 4 per cent per annum and the life of the project in either case is six years after which the plant will be worthless. In

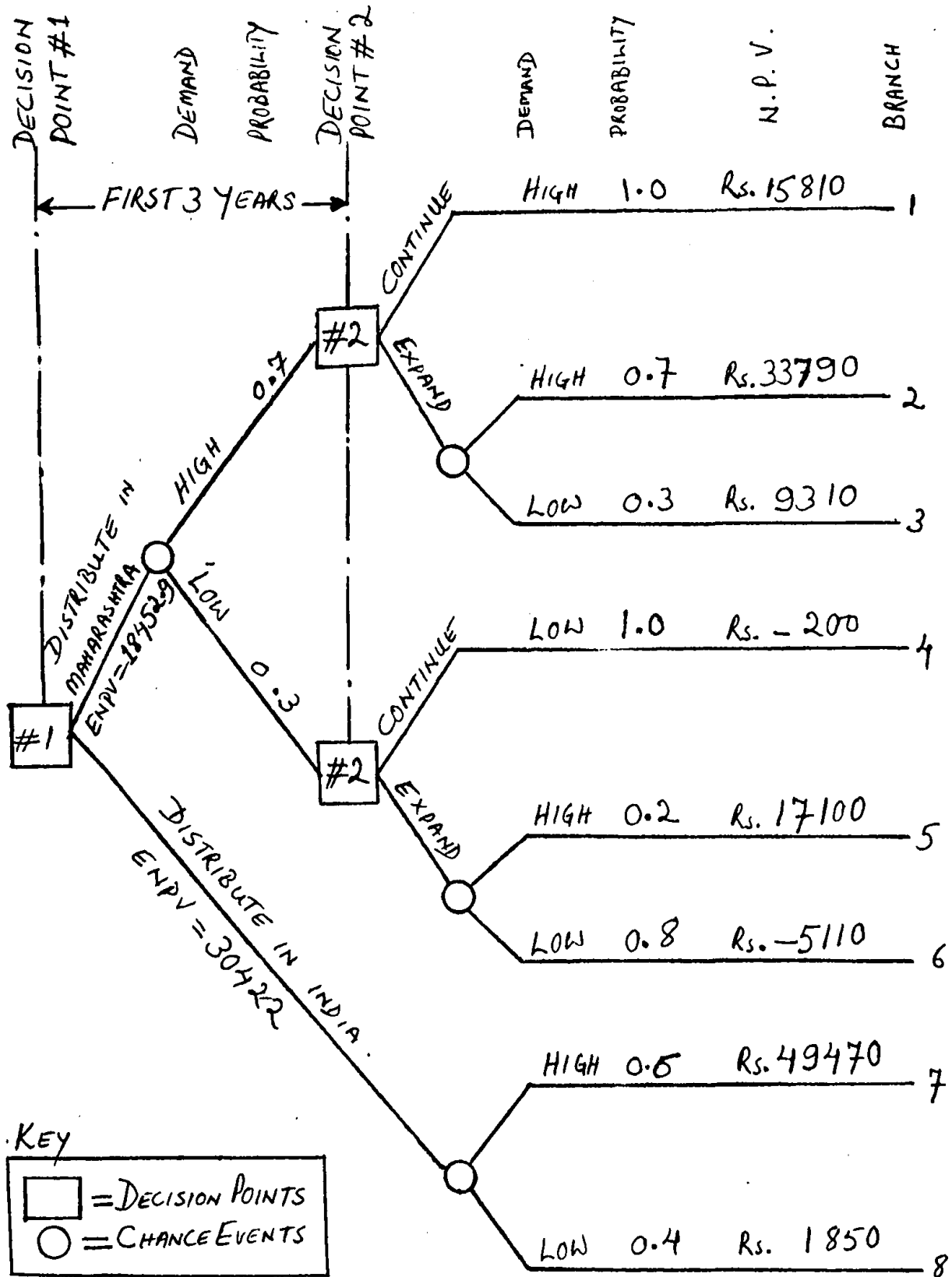


Fig - 3.3

Fig. 3.3 we have portrayed the decision points (rectangulars) and the subsequent chance events (circles) with their expected probabilities. Finally, there are eight courses of alternatives or branches illustrated by the tree. Of course, in decision-trees we do not lay out all the events that can happen, but only those which are considered to be important and have consequences which we would wish to compare. The expected cash flows for each branch, and the Net Present Value of these flows are shown in Table 3.1.

To come to decision # 1, i.e., whether to distribute the product in Maharashtra or all over India, we have to "roll back" the tree to find out the "position values" of decision point # 2 where demand for 3 years of distribution in Maharashtra has been low or high. If we build a small plant now, there is a 0.7 probability that initial demand in Maharashtra will be high and a 0.3 probability that it will turn out to be low. After selling for 3 years at a high level of demand, we have 2 options, either to continue statewide or expand our plant. If we continue statewide we are certain that demand will remain high, but if we expand our plant there is a 0.7 probability that demand will remain high and a 0.3 probability that it will prove to be low in India. The expected present value for expansion after 3 years where the statewide demand was high is

$$ENPV = 0.7 (33790) + 0.3 (9310) = \text{Rs. } 26447 \text{ (thousands)}$$

TABLE 3.1

Branches	Years							(Rs. '000)	
	0	1	2	3	4	5	6	GPV	NPV
1	-5,000	1,000	3,000	5,000	7,000	4,000	4,000	20,810	15,810
2	-5,000	1,000	3,000	-7,000	10,000	20,000	20,000	38,790	33,790
3	-5,000	1,000	3,000	-7,000	8,000	6,000	6,000	14,310	9,310
4	-5,000	200	400	1,000	2,000	1,000	1,000	4,800	-200
5	-5,000	200	400	-11,000	8,000	15,000	15,000	22,100	17,100
6	-5,000	200	400	-11,000	3,000	4,000	4,000	-110	-5,110
7	-12,000	3,000	10,000	15,000	20,000	12,000	12,000	61,470	49,470
8	-12,000	1,000	2,000	3,000	7,000	3,000	3,000	13,850	1,850

The position value of this decision point is Rs. 26447 (thousand) which implies that it is profitable to expand and distribute in the whole of India than to continue our distribution in Maharashtra only. Similarly, we can find out the position value of decision #2 where demand is low for initial 3 years in the state. The expected net present value for expansion is :

$$\text{ENPV} = 0.2(17100) + 0.8(-5110) = \text{Rs. } -668 \text{ (thousands).}$$

But this is not the position value, as the present value of the alternative, i.e., to continue distribution statewide, is higher. The position value of this decision point is Rs. -200(thousands). Therefore, it is profitable to continue statewide when initial demand is low.

Next, we must find out the expected present value of these "position values" which is nothing but the expected present value of distribution in the state initially and expansion after 3 years, if demand proves to be high in the initial years. This ENPV is equal to :

$$\text{ENPV} = 0.7(26447) + 0.3(-200) = \text{Rs. } 18452.9 \text{ (Rs. '000).}$$

The expected net-present value for distribution throughout India from the outset is :

$$\text{ENPV} = 0.6(49470) + 0.4(1850) = \text{Rs. } 30422 \text{ (Rs. '000).}$$

Therefore, at the decision point #1, we find that it is profitable to build a bigger plant in the beginning and produce and distribute throughout India for the entire life of the asset.

While concluding the note, we may enumerate the key steps in formulation and use of the decision trees.

Firstly, the problem and different alternatives must be identified. Secondly, we must lay out the decision-tree which is a formulation of the structure of alternatives. Thirdly, we obtain the data needed, especially the cash flows and the probabilities associated with each alternative and finally, we must evaluate each alternative course. A good evaluation will indicate which alternative is the most desirable in the light of standards used.

3.4 Sensitivity Analysis

Sensitivity analysis aids decision-making under circumstances of uncertainty. This method is highly practicable and is of the type of "what would happen if..." approach. For a given criterion, say N.P.V., we can do the calculations for different values of the discount rate. Different values of N.P.V., will show to what extent the project is sensitive to the discount rate employed. If the proposal is not sensitive, the answer is clear; otherwise, we have to study the cost of capital problem more closely. Sensitivity analysis can show the significance of variations in different parameters, like returns, probabilities (of sales, cost, price, etc.) and life time of the project.

To illustrate the use of sensitivity analysis consider the following situation, where an investment proposal

requires an immediate outlay of Rs. 35000 and would have an active life of five years. At the end of the fifth year, it becomes worthless and so there is no scrap value. It is estimated that the sales of the product, q , will be 1500 units in the first year, 2500, 4000, 2500, 1500 units in the second, third, fourth and the fifth year respectively. The unit selling price, p , is estimated to be Rs. 20 throughout the life of the asset and the unit cost, c , is estimated to be Rs. 15. The appropriate discount rate is thought to be 10 per cent. The expected returns, X , from the investment proposal in any year t is given by $X_t = (p-c)q_t$; thus, the anticipated cash flow is as given below.

X_{t0}	X_{t1}	X_{t2}	X_{t3}	X_{t4}	X_{t5}
-35000	7500	12500	20000	12500	7500

At the discount rate of 10 per cent, it is estimated that the Gross Present Value G.P.V. is Rs. 45,369.50 and the N.P.V. is Rs. 10,369.50. We want to analyse for what range of variations in each of the estimates, the N.P.V. of the project will remain non-negative.

Let us first consider the unit profit, $P-C$, which is estimated to be Rs. 5. With the other data unchanged, the N.P.V. of the project will not become negative unless the unit profit drops below $(35000/45369.5) (Rs.5) = Rs. 3.86$. Therefore, if unit cost is constant, the N.P.V. will not

become negative as long as the selling price is above Rs. 18.86 ($=15+3.86$). This implies that the price must not fall by more than 5.7 per cent. On the other hand if unit cost remains below Rs. 16.14, for the unit price of Rs. 20, then the N.P.V. will be non-negative. Thus, increase in unit cost must not exceed 7.6 per cent. If unit profits do not change but sales are 77.14 per cent ($= (3500/45369.5) (100)$ of the original estimates in each year (i.e. do not fall by more than 22.86 per cent), the project will remain worthwhile.

The N.P.V. will become zero for an initial outlay of Rs. 45,369.5, which means a 29.62 per cent increment, compared with the original figure. The internal rate of return or the yield of the project is approximately 20 per cent, so that a 100 per cent increase over the original discount rate is tolerable.

The above information is summarized in the following table :

Datum	% change
Price	5.70
Unit Cost	7.60
Sales Volume	22.85
Discount Rate	100.00

The figures in the percentage change column are unfavourable changes (decrease or increase as the case may be) which occurring individually will reduce the N.P.V. to zero. It is

clear from the table that the proposal's N.P.V. is highly sensitive to unit sale price and unit cost. What is implied by this analysis that the management should obtain more precise estimates of those figures, or if this is not done, it should be careful in controlling unfavourable changes in these variables. It is necessary to note that we have, for the sake of illustration, considered these unfavourable changes in only one variable at a time. However, this is an over-simplification. For instance consider changes in unit cost which will affect the unit sale price and the volume of sales.

All the variables are interrelated. Changes (unfavourable) in one will stimulate changes in the other variables and ultimately they will have a greater unfavourable impact on the N.P.V. of an investment

However, it is necessary to note that sensitivity analysis is not an answer in itself; it merely provides useful information to the management. The decision is yet to be taken.

3.5 The Risk Adjusted Discount Ratio Method (RAD)

This method is best illustrated notationally as

$$PV = \sum_{t=1}^H \frac{X_t}{(1+R_A)^t} \dots\dots\dots(1)$$

where,

PV = The present value of uncertain cash flows of the project,

X_t = Expected value of uncertain cash flows at time t.

R_A = The rate of discount which is greater than a risk-free rate of interest (R_F) by a margin of risk.

H = The economic life of the project.

The difference between R_A and R_F the risk-free rate of interest is referred to as the "risk-premium". This method does not specify the magnitude of risk-premium to be added to the risk-free rate of interest. Therefore, it is a subjective measure of risk adjustment applied to uncertain cash flows of a project.

This method has certain assumptions behind it.

- i) The probability density function of actual cash flows is known with certainty.
- ii) There exists a risk-free rate of interest which measures only the time value of money.
- iii) A premium has to be added to the risk-free rate of interest, as a price charged for the amount of risk involved in cash flows.

- iv) All the returns are equally risky, therefore, they are discounted at a single risk-adjusted discount rate.
- v) No distinction is made between the degree of uncertainties attached to the cash inflows and those of cash outflows, i.e., payments are as risky as receipts.

The term R_A is interpreted as the "required rate of return" which is appropriate to risky and uncertain characteristics of the cash flows of a project. This required rate of return must account for (a) the time value of money, and (b) the risk and uncertainty adjustment. The time value of money may be thought of as a risk-free rate of interest which may be known to every one. But as to the risk adjustment it is left to the decision maker to find out what is the appropriate risk-premium to be added to the market risk-free rate of interest.

In short, this method states that if cash flows are risky, then some risk-premium must be added to the risk-free rate of interest to obtain the required rate of return. But the basic problem is to obtain this premium.

3. The Certainty Equivalent Method (CEQ)

The CEQ method is an attempt to separate the effects of time and uncertainty on present value of future cash

flows. The rate of return on a certain receipt is a compensation for parting with money for a certain period of time. For example, if a certain receipt, \underline{X} , is to be received after one period of time, the present value of \underline{X} is $X/(1+R_F)$, where R_F is the risk-free rate of discount for that period. For the uncertainty effect, consider an uncertain receipt \tilde{X}_t to be received at period t . The smallest certain receipt X_t^* , also to be received at time t , which one may want to exchange with the uncertain receipt \tilde{X}_t , is the "certainty-equivalent" (CEQ) of \tilde{X}_t .

Let $\alpha_t = X_t^*/\tilde{X}_t$, then (CEQ) method can be stated as :

$$\begin{aligned} PV &= \sum_{t=1}^H \frac{X_t^*}{(1+R_F)^t} \\ &= \sum_{t=1}^H \frac{\alpha_t \tilde{X}_t}{(1+R_F)^t} \dots\dots\dots(2) \end{aligned}$$

The CEQ method converts every uncertain cash receipt (or cash payment), \tilde{X}_t , into its certainty equivalent, X_t^* . Then it finds the present value of these certainty equivalent receipts (or payments) at a risk-free rate of discount, R_F .

The simplifying assumptions of the CEQ are :

- i) There exists a risk-free rate of interest and this rate will remain constant over the economic life of the asset.

- ii) The coefficients α_t are determined according to the expectations of those who evaluate the project's cash flows. And to that extent it is subjective with no objective measure of risk given.

3.7 Risk Adjusted Discount Rate Vs. Certainty Equivalent Method

Robichek and Myers (1966) have compared these two methods in the manner outlined below.

As we have seen in the previous section RAD is defined as :

$$PV = \sum_{t=1}^H \frac{\tilde{X}_t}{(1+R_A)^t}$$

and CEQ method is :

$$PV = \sum_{t=1}^H \frac{\alpha_t \tilde{X}_t}{(1+R_F)^t}$$

If present value of a cash flow stream from the two methods is equal, then we must have :

$$\frac{X_t^*}{(1+R_F)^t} = \frac{\tilde{X}_t}{(1+R_A)^t} = \frac{1}{\alpha_t} \frac{X_t^*}{(1+R_A)^t} \dots \dots \dots (3)$$

where $X_t^* = CEQ(X_t)$ and $\alpha_t = \frac{X_t^*}{\tilde{X}_t}$

From (3) it follows¹ that $\alpha_t = \frac{(1 + R_F)^t}{(1 + R_A)^t} \dots\dots\dots (4)$

Similarly for the period $t + 1$ we have

$$\alpha_{t+1} = \frac{(1+R_F)^{t+1}}{(1+R_A)^{t+1}} = \frac{(1+R_F)}{(1+R_A)} \alpha_t \dots\dots\dots (5)$$

If $\alpha_0 = \frac{(1+R_F)}{(1+R_A)}$, is assumed to be unity,

then for that time period $R_F = R_A$. This seems reasonable because for the present, time zero, there is no interest rate and no risk at all. $R_F = R_A = 0$. Thus, equation (4) will be the general condition for the relationship between α_{ts} . The relation described above in equation (5) is assumed when a constant risk adjusted discount rate R_A is applied for discounting the cash flows. It means if the present value obtained by the RAD method is intended to be equal to that obtained by CEQ method, then it is implied that the ratios, $\alpha_t = X_t^*/\tilde{X}_t$ decreases over time at a constant rate.

$$= 1 - \frac{(1+R_F)}{(1+R_A)}$$

1 we have $\frac{X_t^*}{(1+R_F)^t} = \frac{1}{\alpha_t} \frac{X_t^*}{(1+R_A)^t}$

thus, $(1+R_F)^t / \alpha_t = (1+R_A)^t$

hence, $\alpha_t = \frac{(1+R_F)^t}{(1+R_A)^t}$

On the other hand if we assume that $\alpha_t = \alpha_{t+1}$, it implies that risk does not increase with time. If this is true, then equation (4) can not hold unless $R_F = R_A$, which means a zero risk-premium. However, for $R_A \neq R_F$, we have

$$\alpha_t = \frac{(1+R_F)^t}{(1+R_A)^t} \neq \frac{(1+R_F)^{t+1}}{(1+R_A)^{t+1}} = \alpha_{t+1}$$

Therefore, if we assume α_{t_s} to be equal, then a unique discount rate R_A can not evaluate the individual returns correctly. On the other hand if a unique rate of discount R_A is appropriate then α_{t_s} can not be equal, i.e., risk does not remain constant over time.

The above methods and the traditional discounting as well as non-discounting methods may be termed as "single stroke" valuation models. That is, the project evaluation, is done once for all. The intermediate circumstances, like change in prices which cause change in cash flows or revision of expectations during the time period are not considered.

When these considerations are taken into account, then one has to employ a sort of dynamic valuation model. The so called dynamic valuation models are based on single-period valuation models, with parameters adjusted from time to time. Next we shall consider these models in greater detail.

The recent developments in the valuation models¹ employ the famous Sharpe (1964)-Lintner (1965) model of capital asset pricing (CAPM).

We shall briefly state the essential features of the Sharpe-Lintner model and then examine how it is applied to find out the present value of a stream of cash flows of a project.

3.8 The Capital Asset Pricing Model (CAPM)

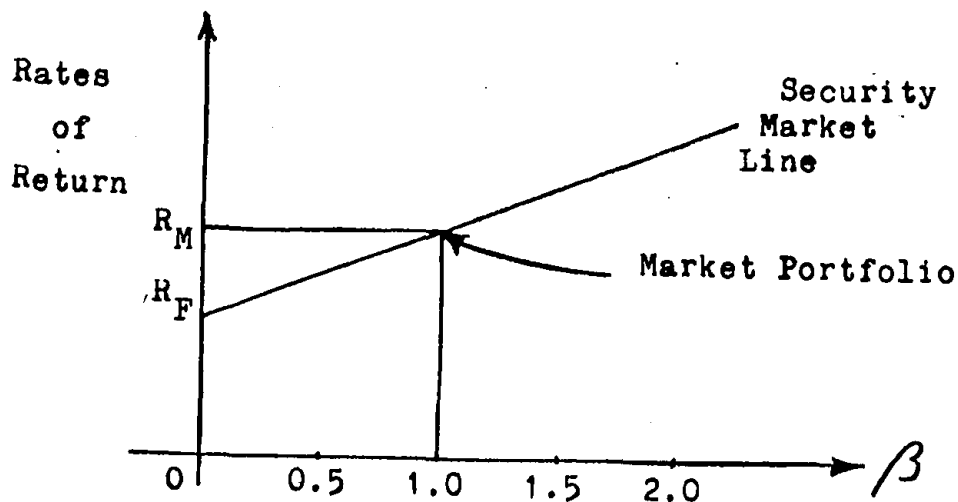
Consider a capital market with a large number of securities. The returns on these securities are not certain. Diversification can reduce the magnitude of this risk, because security prices are not perfectly correlated. However, diversification can not eliminate the variability risk altogether. The risk that potentially can be eliminated is called "unique risk". This is a risk which is particular to a firm; it stems from the perils that surround an individual firm. But there is another risk which can not be eliminated, no matter how much an investor diversifies. This is called the "market risk" and stems from the fact that there are some economy-wide perils which affect all the businesses and firms. To that extent, stocks have a

¹ We know that capital budgeting is essentially a problem of valuation. The exercise is to find assets which are worth more than what they cost.

tendency to move together and investors are, thus, exposed to "market uncertainties".

The risk of a well-diversified portfolio depends on the market risk of all the securities included in that portfolio. If we want to find out the contribution of an individual security to the market risk of a well diversified portfolio, it is no good thinking of that security held in isolation. We need to measure its market risk and this boils down to finding its sensitivity to market movements. This sensitivity measure is often called the beta (β) of that asset. The market risk of a well-diversified portfolio is an average of betas of individual securities included in it. A portfolio consisting of all the available risky securities in the market may be called as the "market portfolio". Investors in the market portfolio, however, well-diversified, would expect a rate of return which is greater than that offered by the most secured assets like Government bonds. This difference is referred to as the "market risk premium".

Illustrated graphically, we have :



The market portfolio has a risk beta of 1.0 and a risk premium of $R_M - R_F$. The CAPM suggests that risk premiums on portfolios other than market portfolio vary in direct proportion to their risk betas. This implies that in the above figure all investments must plot along the line known as security market line. Therefore, if a portfolio has a beta of 0.5 then the expected risk premium is half the market risk premium $R_M - R_F$. And a portfolio having a beta of 2.0 will require a risk premium which is two times the market risk premium.

The expected risk premium on a share is equal to its beta times market risk premium

$$R - R_F = \beta (R_M - R_F)$$

or

$$R = R_F + \beta (R_M - R_F) \dots\dots\dots(6)$$

where R is the required rate of return of an asset. When this rate is found, then, it can be used to discount the expected cash flows of that asset to find out its present worth. This method is similar to RAD method. But here the adjustment is not done subjectively and it does not depend on the expectations of the management of the firm. Here the discount rate is adjusted to the market risk premium and the historical sensitivity of the asset to market conditions.

We can also derive the CEQ method from the CAPM. To find out the present value of a return to be received after one period, we divide that return by $(1 + R)$, where R is the appropriate rate of discount for that project.

$$\begin{aligned} \text{i.e.} \quad PV &= \frac{\tilde{X}_1}{1 + R} \\ \text{or } 1 + R &= \frac{\tilde{X}_1}{PV} \dots \dots \dots (7) \end{aligned}$$

From CAPM we have

$$\begin{aligned} 1+R &= 1+R_F + \beta(R_M - R_F) \\ \frac{\tilde{X}_1}{PV} &= 1+R_F + \beta(R_M - R_F) \dots \dots \dots (8) \end{aligned}$$

$$\begin{aligned} \text{where, } \beta &= \frac{\text{COV}(R, R_M)}{\sigma^2_{R_M}} \\ &= \frac{\text{COV}[(\tilde{X}_1/PV) - 1, R_M]}{\sigma^2_{R_M}} \dots \dots \dots (9) \end{aligned}$$

$$\begin{aligned} &= \text{COV}\left[\frac{1}{PV}(\tilde{X}_1 - PV), R_M\right] / \sigma^2_{R_M} \\ &= \frac{1}{PV} \text{COV}[(\tilde{X}_1 - PV), R_M] / \sigma^2_{R_M} \\ &= \frac{1}{PV} \text{COV}(\tilde{X}_1, R_M) / \sigma^2_{R_M} \quad (1) \\ &= \text{COV}(\tilde{X}_1, R_M) / PV \sigma^2_{R_M} \dots \dots \dots (10) \end{aligned}$$

By substitution of (10) for β in (8), we get

$$\frac{\tilde{X}_1}{PV} = 1 + R_F + \left[\text{COV}(\tilde{X}_1, R_M) (R_M - R_F) \right] / PV \sigma^2_{R_M} \dots (11)$$

1 COV(X \pm a, Y) is the same as COV(X, Y).

The ratio $(R_M - R_F) / \sigma^2 R_M$ is known as the market price of risk and is generally denoted by λ . We can then write (11) as

$$\frac{\tilde{X}_1}{PV} = 1 + R_F + \frac{\lambda \text{COV}(\tilde{X}_1, R_M)}{PV} \dots\dots\dots(12)$$

multiplying through by PV we get,

$$\begin{aligned} \tilde{X}_1 &= PV (1 + R_F) + \lambda \text{COV} (\tilde{X}_1, R_M) \\ PV &= \frac{\tilde{X}_1 - \lambda \text{COV} (\tilde{X}_1, R_M)}{1 + R_F} \dots\dots\dots(13) \end{aligned}$$

Equation (13) is the CEQ form of CAPM. It implies that if the asset is risk-free, i.e., $\text{COV}(X, R_M) = 0$, then, the returns are simply discounted at a risk-free rate of interest. As we can see from equation (13), even in this form of CAPM, there are no subjective risk adjustments. The deductions from the uncertain return depend on the market price of risk, which is supposed to be given for an individual firm, in a perfectly competitive capital market, and the covariance between earnings and market rate of return.

3.9 Application of CAPM to Capital Budgeting of Risky Projects

The CAPM was applied to capital budgeting of risky projects by many writers. We shall summarize here the

works of i) Bouge M.C. and Roll R. (1974) (B & R hereafter),
 ii) Myers S.C. and Turnbull S. (1975) (M & T hereafter)
 and iii) Fama E. (1977).

The article by B & R is one of the pioneering works done in this area. M & T have extended their model and Fama's article is primarily a survey of literature, however, we shall outline the major developments brought about by him. The essentials of this approach can be found in B & R (1974) where they start with a single-period model of asset valuation using the famous Sharpe-Lintner model. They employ the following equivalent formulation of the CAPM :

$$V_0 = \frac{E[\tilde{V}_1] - \lambda_0 \text{COV}(\tilde{V}_1, \tilde{V}_{M1})}{1 + R_{F0}} \dots\dots\dots(14)$$

where,

V_0 = the present value of the firm.

\tilde{V}_1 = the end-of-period uncertain value of the firm
 (including any dividends paid over the period).

\tilde{V}_{M1} = the end-of-period uncertain value of all firms.

R_{F0} = the present risk-free rate of return.

$\lambda_0 = [E(\tilde{V}_{M1}) - (1-R_{F0}) V_{M0}] / \sigma^2(\tilde{V}_{M1})$ is the present
 market price per unit of risk.

If a project is expected to bring an incremental cash flow equal to \tilde{X}_1 , then the end-of-period value of the firm including the project will be :

$$V_0 + \Delta V_0 = \frac{E(\tilde{V}_1 + \tilde{X}_1) - \lambda_0 \text{COV}(\tilde{V}_1 + \tilde{X}_1, \tilde{V}_{M1})}{1 + R_{F0}} \dots\dots\dots(15)$$

This means

$$\Delta V_0 = \frac{E(\tilde{X}_1) - \lambda_0 \text{COV}(\tilde{X}_1, \tilde{V}_{M1})}{1 + R_{F0}} \dots\dots\dots(16)$$

Equation (16) is the present value of uncertain net cash flow to be received after one period. It is the addition to the value of the firm. If it is greater than the initial expenditure on the project, then the project, is desirable and its undertaking will increase the market price of the shares of the firm. On the other hand, if this amount is less than the initial expenditure, then it decreases the present value of the firm. Therefore, it is not to be undertaken.

Equation (14) is simply finding a CEQ for the expected value of the firm after one period, and then discounting it to the present at a risk-free rate of interest. This valuation as we have noted earlier, does not depend on the personal judgement of the management or any other individual preferences. It is the market assessment of a risky project. Because it depends on the market price of risk and the risk-free rate of interest, which is given for a firm, and the covariance of the value of that firm and the value of all other firms put together.

B & R in their model have assumed stochastic variations in R_F and λ , i.e., the value of these parameters are subject to variations over time.

This single period valuation model helps one to understand the problem and build a basis for a multiperiod valuation model which is needed for the valuation of projects which last more than one period. In the multi-period case the problem is to find V_0 , the changes in the present value of the firm due to uncertain cash flows, \tilde{X}_t ($t=1,2,\dots,T$), which will result in incremental changes, V_t . V_0 can be compared with the initial cash outlays to assess the desirability of the project.

The procedure used by B & R and later by M & T and Fama is an application of a single period valuation model to all the periods of the economic life of the asset. That is done by using equation (14) at the terminal point and working "backwards"!

To illustrate let us consider the last period. For that period, the uncertain changes in the value of the firm is trivially equal to the uncertain returns.

$$V_T = X_T$$

In the (T-1)the period, we have a one period valuation

1 To illustrate the backward working of this valuation model, consider an investment which requires an immediate outlay of Rs. 800, and promises with certainty to return Rs. 100, after one year, Rs. 1100 after two years and nothing afterwards. Assuming a 10% rate of discount, the present value of Rs. 1100 at the beginning of the second year is Rs. 1000. At the beginning of the first period, i.e., at present, we must find out the present value of Rs. 100 + Rs. 1000, to be received after one year, which is equivalent to Rs. 1000. With an initial outlay of Rs. 800, the NPV of this investment proposal works out to be Rs. 200.

problem. Applying one period valuation model we can find out the certainty equivalent to the final cash flow, and then deducting the value of final expenditure we can find out the incremental value at T-1. "For a t-period project, therefore, we have a t-period infinite state dynamic programming to solve. Each step in the solution is an application of the one-period valuation model, with the parameters depending on the state of the world at the beginning of that period."¹

The argument by M & T may be termed as an extension of the B & R model. Like B & R, they assumed that CAPM is true and will remain true in the future. They further assumed that expectations are based on information available at the beginning of the period and each time only one period ahead can be valued. They have assumed, unlike B & R, that the market price of risk, λ , and the risk-free rate of interest, R_F , will remain constant over time.

According to M & T, the problem with CAPM is that today's price cannot be calculated without knowing the probability distribution of tomorrow's price. They argue that to solve this problem, we must know how expectations are revised each period. Investors will forecast the future cash flows based on current information. The relation between the actual and expected cash flows can be written as :

$$\tilde{X}_t = E(\tilde{X}_t | \Phi_{t-1}) (1 - \tilde{\delta}_t) \dots \dots \dots (17)$$

¹ Bouge and Roll (1974).

where $\tilde{\delta}_t$ is a random disturbance term with mean equal to zero. This disturbance term is assumed to be a linear function of an index which measures unanticipated changes in the economy. Written in an equation form, the disturbance term is

$$\tilde{\delta}_t = \tilde{M}_t + b\tilde{I}_t$$

where \tilde{M}_t measures unique or firm-specific changes and b , assumed to be constant, is the measure of sensitivity of $\tilde{\delta}_t$ to unanticipated changes in the economy index. The forecasted value of the expected future cash flows are assumed to be generated by the process.

$$E(X_{t+1} | \phi_t) = a_1 X_t + a_2 X_{t-1} + \dots \quad \dots (18)$$

where $\sum_{i=1}^t a_i = 1$

If coefficients a_i decline geometrically over time, then it follows that expectations are revised by simple "adaptive expectations" model. Then it follows from (17) that

$$E(X_{t+1} | \phi_t) = E(X_t | \phi_{t-1}) (1 - \eta \tilde{\delta}_t) \dots \quad \dots (19)$$

where $\eta = a_1$, can be termed as the elasticity of expectations and will normally lie $0 \leq \eta \leq 1$. On the basis of these assumptions they build a one period valuation model which when applied to the period next to the last is¹:

¹ \tilde{M}_t is assumed to be uncorrelated to the market returns.

$$P_{T-1} = \frac{E(\tilde{X}_T | \phi_{T-1}) (1 - \lambda b \sigma_{IM})}{1 + R_F} \dots \dots \quad (20)$$

where $\sigma_{IM} = \text{COV}(\tilde{I}_T, \tilde{R}_{MT})$

P_{T-1} = the value of the project at the next to the last period.

At time T, the physical asset is assumed to be worn out and will sell at zero price, i.e., the scrap value of the asset is zero. Its addition to the value of the firm will be the CEQ of its earnings at the last period.

For valuation at period T-2 we must consider expected value of earnings at T-1 and the expected price of the asset at T-1 as of T-2. The present value of X_{T-1} at T-2 can be written as :

$$\frac{E(\tilde{X}_{T-1} | \phi_{T-2}) (1 - \lambda b \sigma_{IM})}{1 + R_F}$$

And the present value of the asset's price at T-1 will depend on how expectations are revised at T-1, after observing the actual value of earnings \tilde{X}_{T-1} and its expected value based on the information available at T-2. The present, T-2, value of the price of the asset at T-1 will then be

$$\frac{E(\tilde{P}_{T-1} | \phi_{T-2}) (1 - \lambda b \eta \sigma_{IM})}{1 + R_F} \dots \dots \dots (21)$$

$E(\tilde{P}_{T-1} | \Phi_{T-2})$ can be written in terms of $E(\tilde{X}_{T-1} | \Phi_{T-2})$ using equation (20).

$$\frac{E(\tilde{X}_{T-1} | \Phi_{T-2}) (1 - \lambda b \sigma_{IM}) (1 - \lambda \eta b \sigma_{IM})}{(1 + R_F)^2} \dots \quad (22)$$

Equation (22) is the present value of \tilde{P}_{T-1} at T-2 and it may be added to the present value of \tilde{X}_{T-1} , to arrive at the value of the asset at T-2. Therefore,

$$P_{T-2} = \left[\frac{E(\tilde{X}_{T-1} | \Phi_{T-2}) (1 - \lambda b \sigma_{IM})}{(1 + R_F)} + \frac{E(\tilde{X}_{T-1} | \Phi_{T-2}) (1 - \lambda b \sigma_{IM}) (1 - \lambda \eta b \sigma_{IM})}{(1 + R_F)^2} \right] \dots \quad (23)$$

In the same same way we can find P_{T-3} , P_{T-4} , etc. Eventually we get the current equilibrium value.

$$P_0 = E(\tilde{X}_1 | \Phi_0) q \sum_{t=0}^T z^t \dots \quad (24)$$

where $q = (1 - \lambda b \sigma_{IM}) / (1 + R_F)$ and
 $z = (1 - \lambda b \eta \sigma_{IM}) / (1 + R_F)$

The further extension of the model by M & T pertains to consideration of multiple cash flow stream and growth of earnings.

For illustration of the multiple cash flow, the authors give an example of an investment proposal's cash flow with two components, \tilde{X}_{1t} and \tilde{X}_{2t} . Components will be evaluated as if there were two separate projects, each with specific parameters, appropriate to the associated risk with the

cash flow component. In case of growth of earnings, a constant growth is assumed.

$$E(\tilde{X}_{t+1} | \phi_t) = E(\tilde{X}_t | \phi_{t-1}) (1+g) (1+\eta \tilde{\delta}_t)$$

where, g is the growth rate of earnings assumed to remain constant over time. It is argued that if there are some stochastic changes in the growth rate, it then can be introduced into the uncertain cash flows. The valuation formulae can be obtained by the same procedure used above.

The assumptions of Fama's model are similar to those adopted by M & T. With the background acquired from the above discussion of the approaches by B & R and M & T, we can cut short the discussion made by Fama on general aspects of CAPM. The essentials of Fama's model may be summarized as follows.

Consider a single uncertain cash flow \tilde{X}_t to be received at time period t . Applying the Sharpe-Lintner single period model, we get the value of that earning at $t-1$ as :

$$V_{t-1} = \left[E(\tilde{X}_t) - \phi_t \text{COV}(\tilde{X}_t, \tilde{R}_{Mt}) \right] / (1+R_{Ft})$$

or

$$V_{t-1} = E(\tilde{X}_t) \left[(1 - \phi_t \text{COV}(\tilde{X}_t, \tilde{R}_{Mt}) / E(\tilde{X}_t)) \right] / (1+R_{Ft}). \quad (25)$$

where

$$\phi_t = \frac{E(\tilde{R}_{Mt}) - R_{Ft}}{\sigma^2(\tilde{R}_{Mt})} = \lambda_t \tilde{V}_{Mt-1}$$

\tilde{R}_M = uncertain market rate of return.

R_F = the certain risk-free rate of interest.

λ_t = the market price of risk which is given by

$$\lambda_t = \frac{E(\tilde{V}_{Mt}) - (1+R_F) V_{M, t-1}}{\sigma^2(\tilde{V}_{Mt})}$$

where, \tilde{V}_{Mt} represents uncertain market value of all firms at t, and $V_{M, t-1}$ the known market value of all the firms at t-1.

We know that $E(\tilde{X}_t)$ could have been discounted back by $E(\tilde{R}_t)$, the risk adjusted rate of discount, which according to Sharpe-Lintner model is given by

$$E(\tilde{R}_t) = R_{Ft} + \beta(\tilde{R}_{Mt} - R_{Ft})$$

Thus, V_{t-1} could have been expressed as

$$V_{t-1} = [E(\tilde{X}_t) / (1+E(\tilde{R}_t))] \dots\dots\dots(26)$$

From equations (25) and (26), we have

$$\frac{1}{1+E(\tilde{R}_t)} = \frac{1 - \phi_t \text{COV}(\tilde{X}_t, \tilde{R}_{Mt}) / (E(\tilde{X}_t))}{1+R_{Ft}}$$

In this way we have found a risk adjusted discount rate for the last period. The value of this rate of discount will depend on $\text{COV}(\tilde{X}_t, \tilde{R}_{Mt})$, because λ_t and R_{Ft} are given for an individual firm.

If value of V_{t-1} is known at t-2, then we need to discount this certain return by the risk-free rate of interest. Thus,

$$V_{t-2} = \frac{V_{t-1}}{1+R_{Ft}}$$

And if there are no intermediate, uncertainties we can find out the current equilibrium value of V_{t-1} by

$$V_0 = \prod_{k=1}^{t-1} \left(\frac{1}{1+R_{Fk}} \right) V_{t-1} \dots\dots\dots(27)$$

From equation (26), the value of V_{t-1} is

$$V_{t-1} = \frac{E(\tilde{X}_t)}{1+E(\tilde{R}_t)}$$

Equation (27) can be rewritten as

$$V_0 = \prod_{k=1}^{t-1} \left(\frac{1}{1+R_{Fk}} \right) \left(\frac{1}{1+E(\tilde{R}_t)} \right) E(\tilde{X}_t) \dots\dots\dots(28)$$

However, if these are intermediate uncertainties, then for the intermediate periods we must apply appropriate rates of discount. Therefore, the current equilibrium value of a single earning, X_t , to be received at t can be written as

$$V_0 = \prod_{t=1}^T \left(\frac{1}{1+E(\tilde{R}_t)} \right) E_0(\tilde{X}_t) \dots \dots\dots(29)$$

If $E(\tilde{R}_t)$, are to be assumed constant, then equation (29) resembles the traditional models of capital budgeting.

$$V_0 = E_0(\tilde{X}_t) \left(\frac{1}{1+E(\tilde{R})} \right)^t$$

This procedure can be used to price $\tilde{X}_t - \tilde{I}_t$, when \tilde{I}_t represents uncertain cash outflows at t . The present value of $(\tilde{X}_t - \tilde{I}_t)$, a single cash inflow, can be written as

$$\begin{aligned}
& E_0(\tilde{X}_t - \tilde{I}_t) \left[\frac{1}{1+E(\tilde{R}_{t1})} \right] \left[\frac{1}{1+E(\tilde{R}_{t2})} \right] \dots \left[\frac{1}{1+E(\tilde{R}_{tt})} \right] \\
& = E_0(\tilde{X}_t - \tilde{I}_t) \prod_{\tau=1}^t \left[\frac{1}{1+E(\tilde{R}_{t\tau})} \right] \quad \dots(30)
\end{aligned}$$

Equation (30) is a valuation formula of a single cash flow to be received at t . The valuation method for a stream of cash flows can be written as

$$V_0 = \sum_{t=1}^T E_0(\tilde{X}_t - \tilde{I}_t) \prod_{\tau=1}^t \left[\frac{1}{1+E(\tilde{R}_{t\tau})} \right] \quad \dots(31)$$

Recall that the derivation of this method is based on the assumption that λt and R_{Ft} are market determined parameters that are given for any individual firm. And that $E(\tilde{R}_t)$ would depend on the covariance between $\tilde{X}_t/E_{t-1}(\tilde{X}_t)$ and the market rate of return, \tilde{R}_{Mt} .

Next we may review an interesting article which tackles a logical extension of the conventional models of capital budgeting.

Lewllen W.G. (1977) argues that it is not logical to assume: all the components of cash flows as equally risky. And application of a single rate of discount to both cash outflows and cash inflows is not appropriate when cash outflows are not as risky as cash inflows. In this case, it "seems logical" to discount cash outflows at a lower rate so that their present value will be higher, reducing the net present value of the relevant project.

The author furnishes a numerical example of two one period projects which are assumed (for simplicity) to have

certain cash inflows at the end of the period. But the cash outflows for the two projects are considered to be uncertain with equal expected values and different variances. Then assuming known probability distribution of future outcomes for the project and the market, he generates the numerical values for the betas of both the projects. Both the projects happen to have negative betas, i.e., they have inverse relation with market movements. According to CAPM, the project with lower beta is the more desired one. Here, both the betas are negative; therefore, the project with a higher negative beta is preferred to the other. When a project is preferred to some other project it must have greater net present value. The inflows are identical and at the same time certain; we must, therefore, discount the cash outflows of the desired or preferred project such that the ENPV of outflows is less at present, or equivalently the discounting must be such that the less desired project must have greater ENPV of its cash outflows. To do this we must discount the cash outflows of the more desired project at a higher discount rate. However, the more desired project here is the one with a larger negative beta, i.e., the one with more uncertain cash outflows. The conclusion based on his argument is that if a project has more uncertain cash outflows, we must apply higher rate of discount to find out the present value of outflows.

A possible explanation for his peculiar results may be that normally betas are generated on the basis of historical data. The assumption for the use of betas based on historical data is that the history will repeat itself. It is entirely a different issue if expectations are new to apply CAPM to cash outflows separately and at the same time, he tries to find out the "expected" sensitivity of the projects to market movements. His first argument is sensible, but we have reservations as to the acceptability of the second argument. If sensitivity to market movements is to be used as a measuring rod, it has to (or assumed to) be fixed. Here, the betas are regarded as the basis for the desirability of projects. If the betas themselves are exposed to uncertainties, then the assessments based on such betas will be uncertain.

CHAPTER IV

CAPITAL RATIONING

4.1 Introduction

In this chapter we shall review the literature¹ on "Capital Rationing". The explicit or implicit assumption made in this area is that only the supply of liquid capital in the sense of investible funds is fixed, i.e., there are some quantitative limits on investment expenditure that a firm can make.

There are basically two distinct approaches developed in this area : a) Fisherian or Isoquant analysis, and b) the mathematical programming approach.

This chapter is devoted to the presentation of these two approaches.

4.2 Fisherian Analysis

In this section we shall employ the Fisherian isoquant analysis to understand the two-period certainty model of capital rationing. The two-period consumption-investment decisions framework under certainty was developed by Irving Fisher (1930) and elaborated by J. Hirshleifer (1958) and is treated at length in some recent text books like

¹ A selected survey of literature which we do not claim to be complete.

J. Hirshleifer (1970) and Fama-Miller (1972).

With the help of this graphical analysis, we shall illustrate how different problems in inter-temporal allocation of resources can be handled.

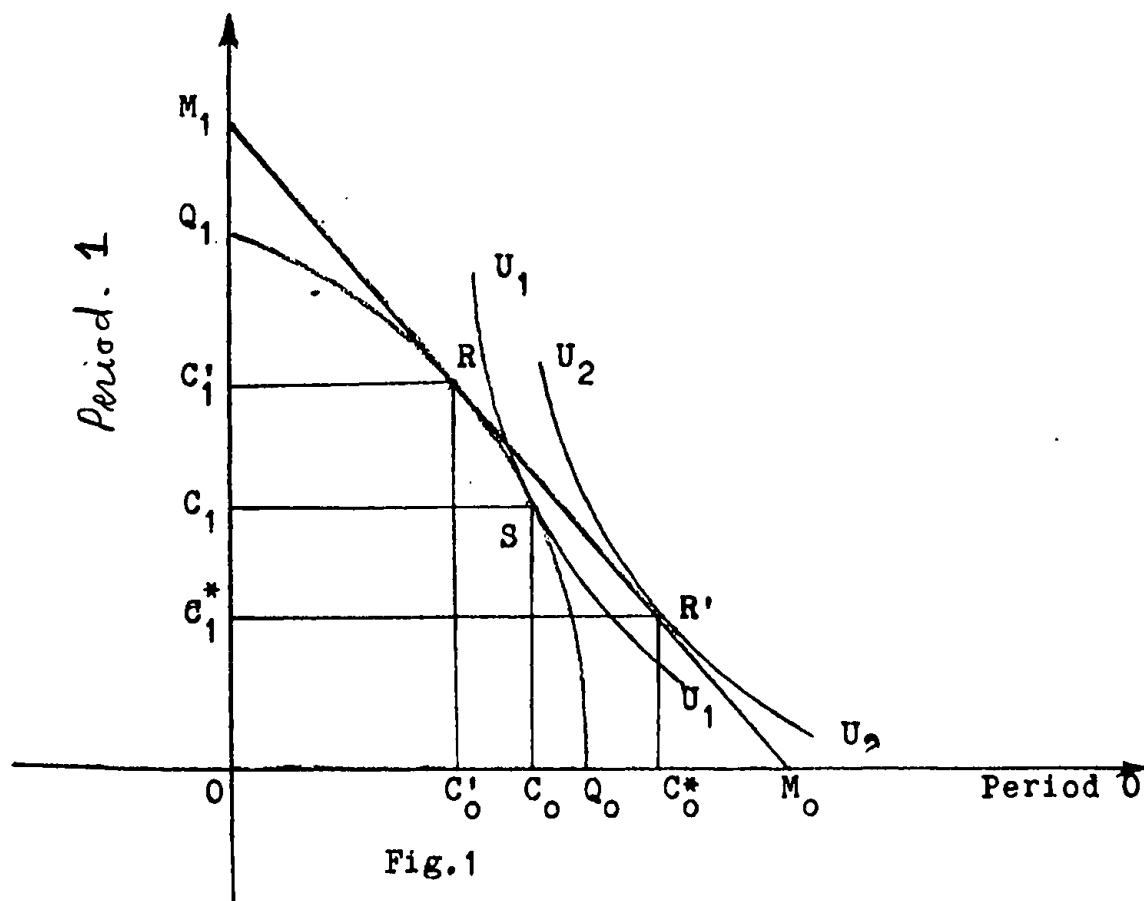


Fig.1

In the two-period world portrayed in Fig.1, the horizontal axis represents the amount of consumption and investment opportunities available to the owner-firm in the current period. The vertical axis shows the potential income available for consumption in the second period.¹ The

¹ i.e., no investment will take place in the second period.

resources available to the owner-firm, giving rise to consumption opportunities in period zero and/or one, are displayed along the Q_1RSQ_0 curve, here called as the production opportunities line (POL). In isolation, i.e. with no access to financial markets, the owner would choose an amount of investment so as to attain the most preferred level of consumption over time. The optimal level will be the point on the POL which reaches the highest indifference curve, which represents his preference for consumption in periods one and zero. In our diagram it is point S, which means OC_0 level of consumption in current period and OC_1 level of consumption in the next period.

When financial markets are introduced in which lendings and borrowings can take place, with no limits, at the market rate of interest, r , (This is the assumption of perfect capital market which means that lendings and borrowings can take place at the same rate of interest and no one (any individual or firm) is assumed to be able to affect the uniform market rate of interest by the magnitude of his borrowings and lendings), the consumption opportunities of the owner-consumer are extended to all the points on the line $M_1RR'M_0$, which is the "Financial Market Line" (FML). This line represents the amount of consumption achievable in the two periods in conjunction with the production opportunities either by borrowing or lending at the uniform

market rate of interest. With these additional consumption opportunities available to the owner-consumer, he chooses point R' which he prefers to point S . The optimum production level, R , is achieved by investing an amount $C'Q_0$ in productive assets which would yield an amount OC'_1 in the second period. By borrowing an amount $C'_0C^*_0$ and repaying an amount of $C^*_1C'_1$ in the second period, the owner-consumer attains his preferred consumption level shown by the point R' .

The marginal productivity of capital at the optimum level of production, R , which is located by the point of tangency between the POL and the FML , is equal to the market rate of interest. The superiority of this point to all other points on the POL is that it has the highest present value at the market rate of interest. The present value of point R is OM_0 . This is simply because the consumption in second period, OC^*_1 , has a present value equal to $C^*_0M_0$. This present value of consumption in second period plus the current consumption OC^*_0 is equal to OM_0 . On the other hand, point R has the greatest terminal value, which is equal to OM_1 . OM_1 is the terminal value for any point on the Financial Market line $M_1RR'M_0$. Similarly OM_0 is the present value of all the points on the line $M_1RR'M_0$. The most preferred consumption vector R' is characterized by the equality of the marginal rate of preferences of the owner-consumer and the market rate of interest (The equality

of the slopes of the FML and the indifferent curve of the owner-consumer). In this way, in terms of our diagram, the investment decision of the owner-firm depends on the slope and location of the POL and the market rate of interest and not on the indifference map of the owner-consumer. It is this argument that constitutes the essence of the "Separation Theorem" developed by Irving Fisher. In short, investment decisions can be made with no regard to the consumption preferences of the owner(s) of the firm.

4.2.1 The Case of Multiple Owners : If there is more than one individual owner in the firm under consideration and these individuals have all their wealth in the form of the shares of this firm only, then it does become necessary to distinguish between the firm and its owners.

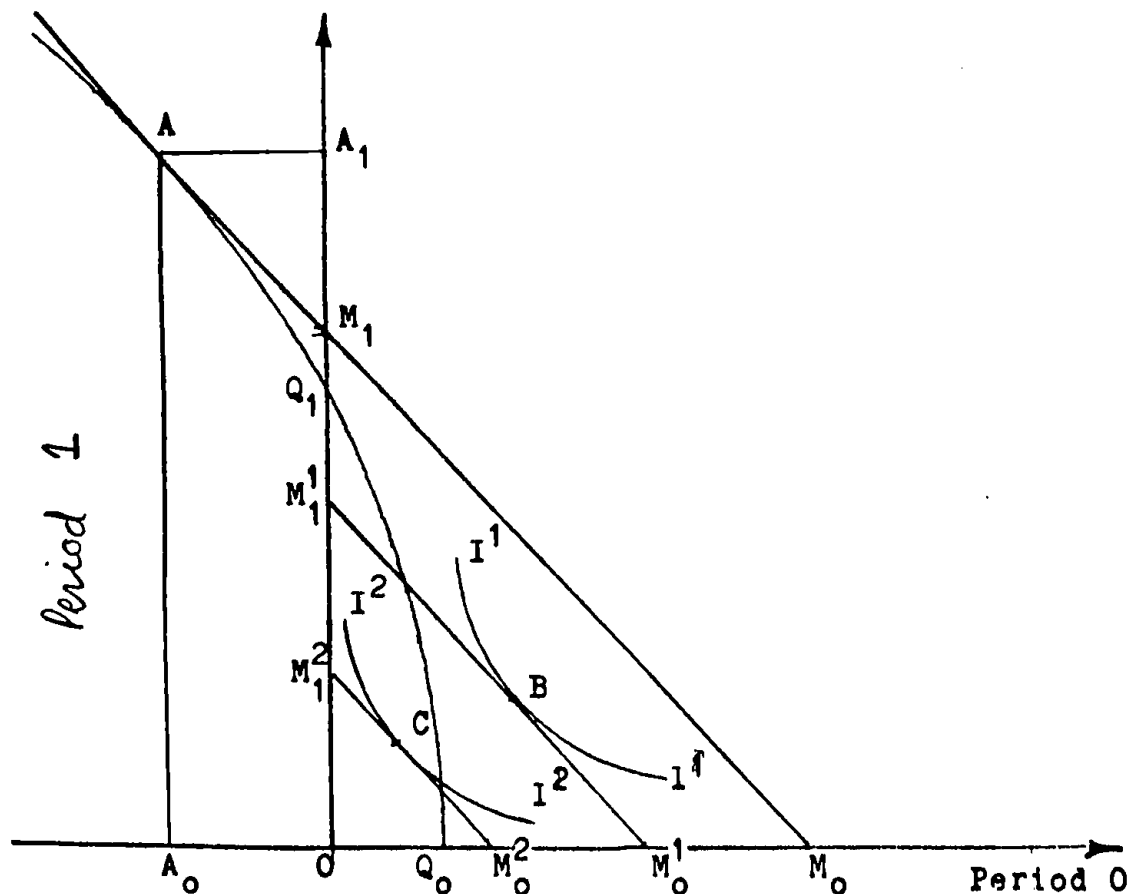


Fig.2

In the above figure, the earlier POL, Q_0Q_1 , is extended to the 4th quadrant. It is assumed that the firm is able to operate without taking additional external funds. In that case Q_0Q_1 would have been its maximum production in the next period. We could have drawn the POL starting from the origin. At the optimal level of output, point A, an amount of borrowing equal to OA_0 , is required. The amount invested may be divided into two parts : (a) amount OQ_0 on the private account (i.e. the shareholders' forgone income which could have been obtained by liquidating the firm now), and (b) the amount, OA_0 , on corporate account. The corporate borrowing has to be repaid in the next period. The amount to be repaid is $A_1M_1 = (1+r)OA_0$. The remaining amount, OM_1 , is the terminal value of the firm or M_0 is the present value of it at the market rate of interest. This amount could be distributed among the owners. In this figure we have assumed only two owners¹, one of them having $1/3$ of the shares and the other holding the remaining $2/3$ of the shares. This means $OM_0^1 + OM_0^2 = OM_0$, or equivalently $OM_1^1 + OM_1^2 = OM_1$.

Each of these owners arrives at his optimum point of consumption by reference only to the amounts available for

¹ These two owners, are assumed to have no other source of income. Otherwise, we can not determine their optimal level of consumption from this source of income.

his consumption and the market rate of interest. Points B and C are the optimal points for the individual owners. Thus, the investment decision can be made with no reference to consumption preferences of the owners of the firm.

4.2.2 Perfect Capital Market With Limits on Borrowing : In capital budgeting process, usually, limits are set on the investment expenditure of the firm. This situation may be illustrated with the help of Fig. 3, in which the owner-consumer has a limited borrowing facility.

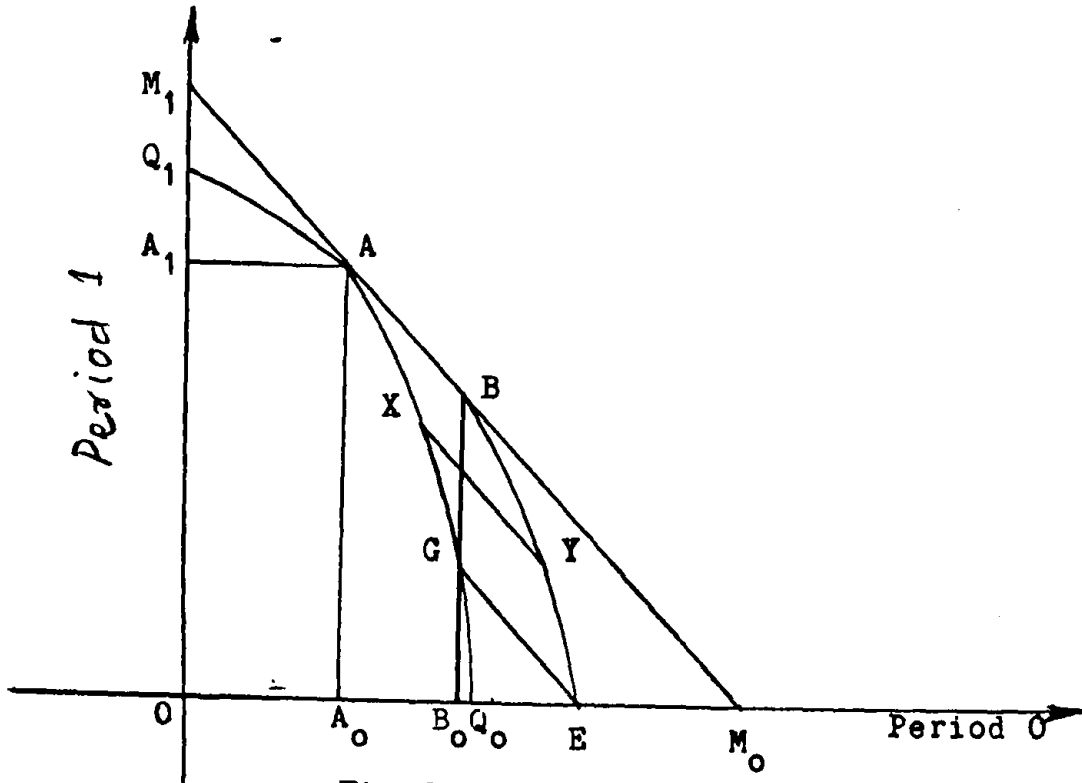


Fig.3

$A_0 - B_0$ is the fixed borrowing limit. An owner-firm will maximize the net present value by pushing the productive investment along the POL till point A, where the highest FML is achieved. At this point, the marginal rate of return on productive investment equals the market rate of

interest. Had there been no borrowing limit, the owner-consumer could have been on M_0BAM_1 consumption opportunities line. However, because of the borrowing limit equal to A_0B_0 , the segment BM_0 is unavailable to the owner-consumer. The new consumption opportunities line is formed in the following way. At any point on the POL below A, a maximum amount of borrowing at the market rate of interest can take place, i.e., he can be on any point on POL and utilize the borrowing facility to its limit, e.g., producing at X allows consumption at Y and the line GE in the above figure is another illustration of this feature. Thus, the total set of consumption opportunities is bound by M_1ABE .

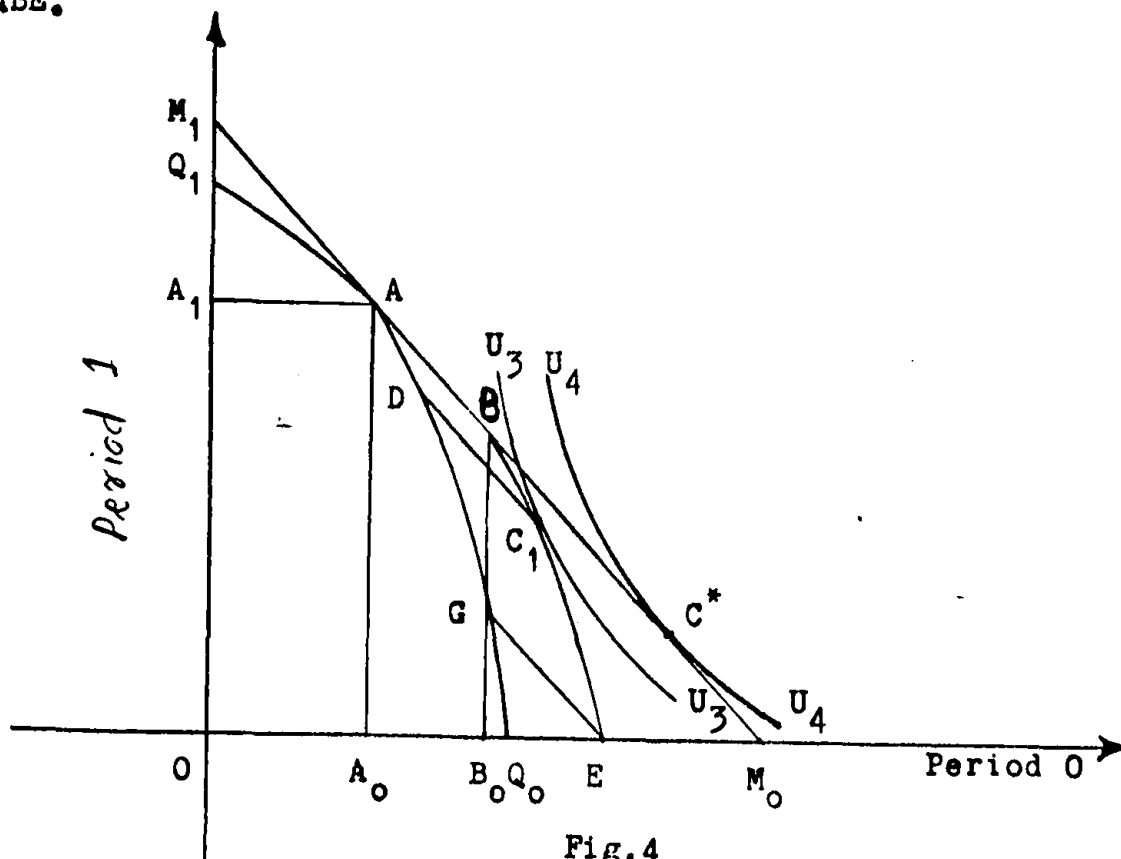


Fig. 4

In Fig. 4, we have superimposed the indifference curves of the owner-consumer on the consumption opportunities of Fig. 3. In the absence of limits on borrowing, the optimal point on the POL would be A, where the POL is tangent to the FML (M_1M_0). Producing at this point and utilizing the maximum level of borrowings, the owner-consumer could have been on point B. It may be observed from Fig. 4 that point B is not an optimum point for consumption decisions. It is clear that there may be many points on BE which are superior to the point B. C_1 is the optimum point in the sense that it permits the investor to achieve the highest possible utility curve. To reach that utility curve, he needs to produce at D on the POL and, by rearranging his income, he could obtain the consumption vector C_1 . In the case of a borrowing limit, therefore, investment decisions can not take place independently of consumption decisions. This leads to a fall of Separation Theorem developed by Irving Fisher.

When there is no limit on borrowing the marginal rate of return on productive investment is the market rate of interest, i.e., point A in Figures 3 and 4. This point also means the maximum present value of investment, at the market rate of interest. However, in the case of limit on borrowing the marginal rate of return on productive investment at the optimum level is higher than the market rate of interest.

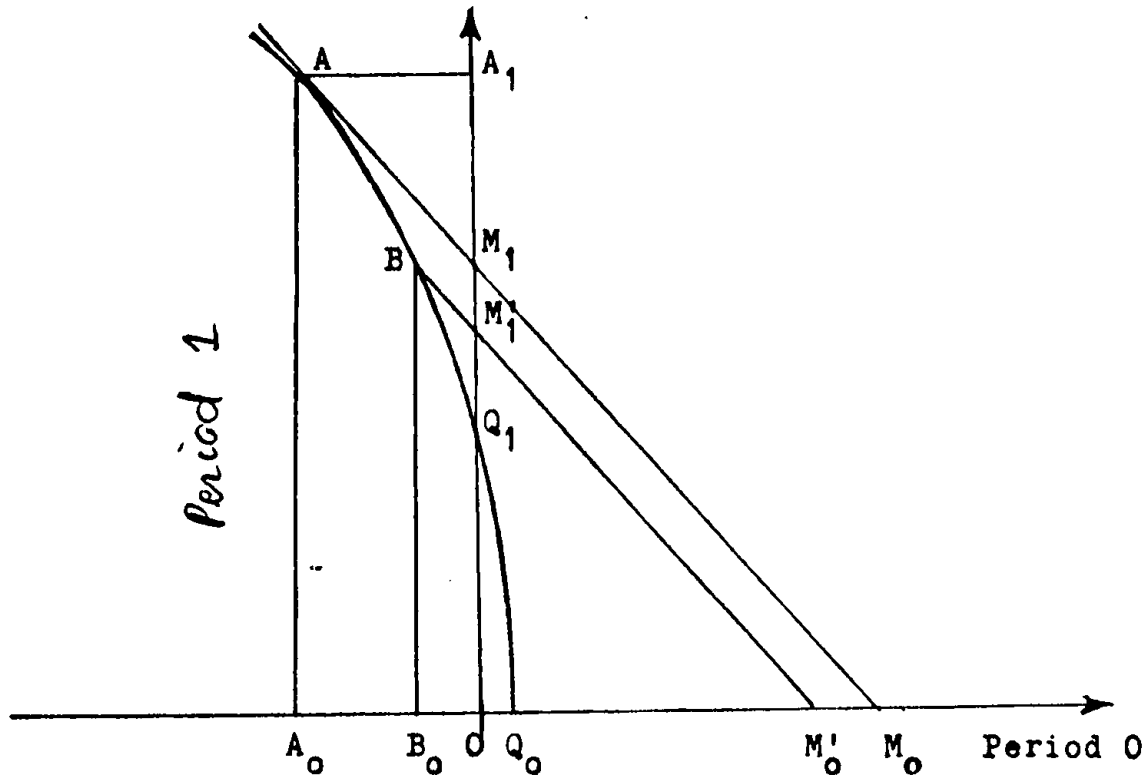
4.2.3 Borrowing Limit on Corporate Account

Fig.5

In the above figure, the firm, like in Fig.4, is assumed to be able to operate without additional funds. This time a maximum borrowing limit is imposed on the firm to the extent of OB_0 . Without any borrowing limit, the management would have pushed the productive investment till the POL becomes tangential to the FML, M_0M_1 at the point A. Because of the borrowing limit, this optimum level of output is no longer available. Now the firm maximizes its terminal value, subject to the borrowing limit OB_0 at the point B. After repayment of the loan at the market rate of interest, the income available for distribution among the owners is OM'_1 , which is less than the terminal value of the firm under perfect market situation. Assuming that the owners are able

to borrow and/or lend at the market rate of interest they will combine their income from this source and other sources and with reference to their inter-temporal consumption preferences and the market rate of interest, arrive at an optimum consumption vector over time.

A valid objection at this point is that if the owners are able to borrow without limit in the market, then they can borrow and themselves lend to the corporation so as to nullify the effect of capital rationing on the firm. By doing this, the firm can operate at the optimum level under perfect capital market situations and increase the terminal value to OM_1 . However, according to Weingartner (1977) it is unrealistic to assume that (a) owners but not others would be willing to lend to a rationed firm, because when a firm is rationed in the capital market, it may be because of the risky nature of the business in which the firm is involved, and (b) the owners of the rationed firm may not be able to borrow without limits on the basis of the shares of the rationed firm.

4.2.4 Different Corporate and Personal Borrowing Rates :

The interest rate on corporate borrowing, assuming absence of limit on borrowings, if different from the interest rate on personal borrowing, may be either lower or higher than the interest rate on personal borrowing.

Let us examine all these different rates and determine which one maximizes the terminal value to the owners of a Corporation which has to borrow at a rate of interest

different from the rate of interest on personal borrowings.

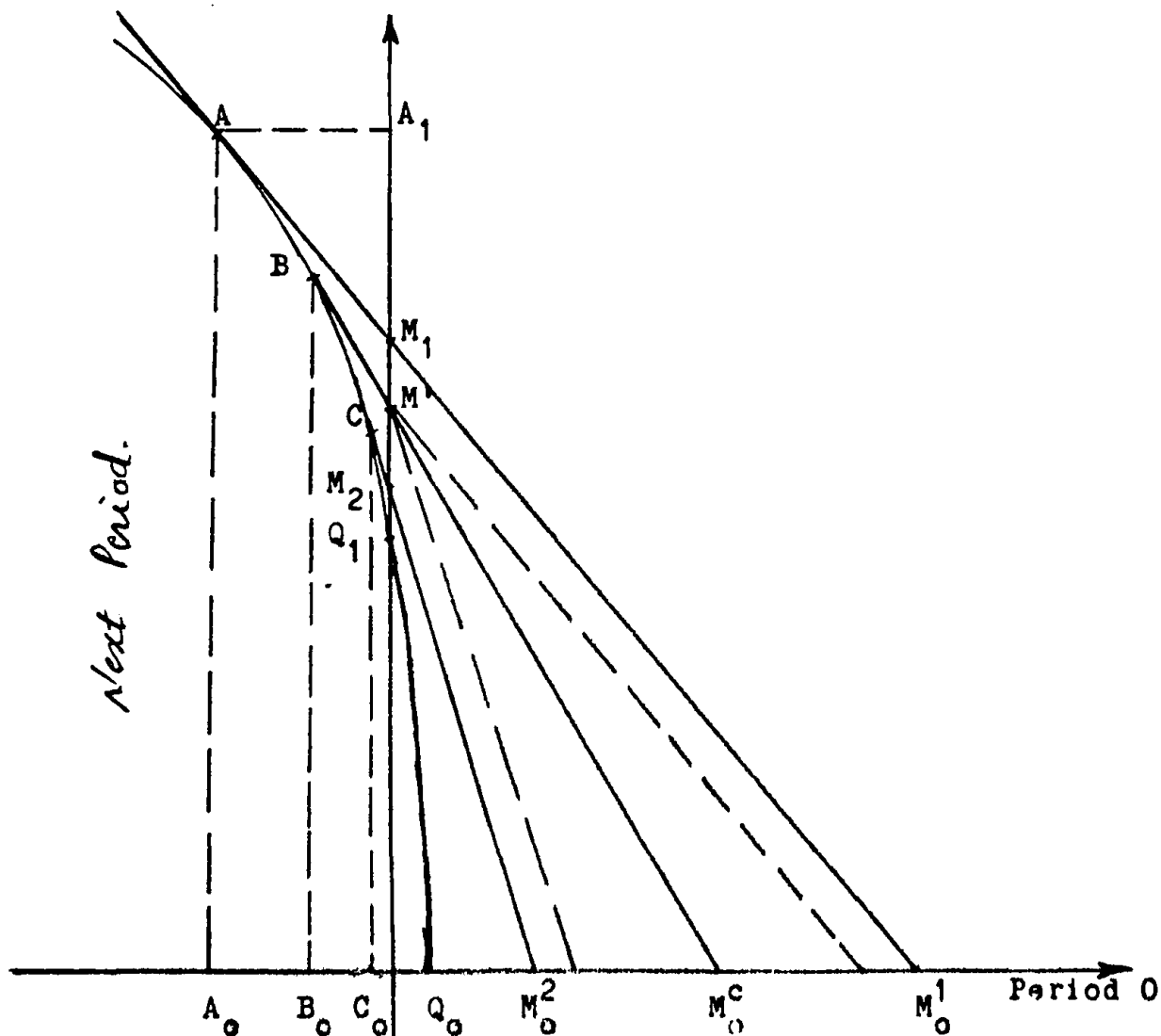


Fig.6

i) Corporate Rate r_c : In this case the management will invest until the marginal rate of return on productive investment equals the corporate rate, r_c . This is shown in Figure 6 by point B, where corporate FML is tangent to the POL of the firm. The terminal value of the firm, after repayment of loans, OB_0 , is OM^1 which is available for distribution among the owners of the firm. The owners will

combine this income with their other income and make their consumption decision taking into account the rate of interest on their personal borrowing or lending.

ii) Higher Rate of Interest on Personal Borrowing, r_2 :

This rate, r_2 , is indicated by a steeper FML. The marginal rate of return on productive investment is equal to this rate at point C. Producing at C and repaying the borrowings of the firm, OC_0 , at the rate of interest on corporate borrowing, r_c , will leave a terminal value which is less than OM^1 (terminal value achieved using the rate of interest on corporate borrowing as outlined above).

iii) Lower Rate of Interest on Personal Borrowing, r_1 :

Using this rate of interest for productive investment decision, the firm's optimum level of investment is shown by point A. After repayment of the loan, OA_0 , at the corporate rate, r_c , the terminal value available for distribution among the owners will be less than OM^1 ; because B is the point of tangency between the POL of the firm and the FML with the slope, $-(1+r_c)$, and a line parallel to this FML, $M_0^C M^1 B$, from point A will necessarily lie below M^1 .

Therefore, the conclusion is that those firms which have to borrow at a corporate rate of interest different from the rate of interest on personal borrowing are to maximize their owners' wealth by investing in productive assets as long as the marginal rate of return is greater than the rate of interest on corporate borrowing.

4.3 Capital Rationing - A Programming Approach

Mathematical programming is another approach towards the problem of capital rationing. The literature in this area is quite vast. What we present below is just a summary of a selected survey of the works of some of the most prominent writers on the subject. We shall start by reviewing a pioneering article by Lorie J. and L.J. Savage.¹

4.3.1 The Lorie-Savage Problem : Consider a firm which is confronted with several investment proposals and a fixed budget. The fixed budget of the firm is assumed to be independent of the investment proposals. This permits computation of the net present value for each investment proposal. The objective is to select from among the investment proposals the projects with positive net present values, which will maximize the present value of the firm.

In the single period case the solution offered by Lorie and Savage is that to rank the projects by the ratios of their net present values to their costs.²

Lorie and Savage have also pointed out that it is possible to have bigger proposals in size which may have higher ratios of present value to cost. However, acceptance of such proposals may preclude the adoption of other small proposals which together may prove to be more profitable.

1 Lorie, J. and L.J. Savage (1955).

2 This ratio is known as the profitability index.

For example, there are three projects with the present value to cost ratios of 1000/800, 620/500 and 620/500. Assume that the total budget is Rs. 1000 and the company's cost of capital is 10 per cent. If the selection is made according to the criterion suggested by Lorie-Savage, we would choose the first project which has a higher ratio of present value to cost. But had we chosen the other two smaller projects, they would together yield¹ more by fuller utilization of the budget. This is called the problem of indivisibilities to which they could not present a solution. Let Y_j represent the net present value of each investment proposal and c_{tj} be the present value of outlays for project j in time period t . Further assume C_t to be the present value of budget ceiling in period t .

In the single-period procedure we rank the investment proposals by Y_j / c_{1j} . Accept the projects from the top of this ranking till the budget is exhausted, i.e., the expression

$$\sum_{j=1}^K c_{1j} \leq C_1$$

ceases to hold.

For the chosen projects the quantity $Y_j - P_1 c_{1j}$ is positive or zero and for the rejected ones it is negative. Where, the quantity P_1 is required to determine the selected

¹ By "yield" we mean the rate of return rather than the magnitude of returns.

projects. The proper value of P_1 is found by trial and error method. The present value of the expenditures for the selected projects must not violate the budget constraint

$$\sum_{j=1}^K o_{1j} \leq C_1$$

First we must assume a positive trial value for P_1 to select the projects for which the expression $Y_j - P_1 c_{1j}$ is positive. Then we have to test whether -

$$C_1 - \sum_{j=1}^K o_{1j}$$

is positive or negative. If it is negative we must increase the value of P_1 and if positive, the value of P_1 is decreased. Finally, the trial and error ends when the budget C_1 is exhausted as fully as possible.

The solution procedure is the same if we use the ratios Y_j / c_{1j} or the expression $Y_j - P_1 c_{1j}$. However, it is the latter expression which can be extended to the multi-period case. For the multi-period case, the procedure must take into account budget limits in more than one period. In this case we must find, by trial and error, positive values of P_1, P_2, \dots, P_T , such that the quantity

$$Y_j - \sum_{t=1}^T P_t o_{tj}$$

is positive for the selected projects and negative for the rejected projects. T is the total number of time periods

for which the budget is fixed. The values of P_{ts} must be chosen such that the budget limitations are not violated

$$\sum_{j=1}^K c_{tj} \leq C_t$$

and are fully utilized.

With this method, they have solved a two period example of nine investment proposal in quite a satisfactory way. Their example is reproduced here.

Investments	1*	2	3*	4*	5	6*	7	8	9*
N.P.V. of Investments	14	17	17	15	40	12	14	10	12
P.V. of Outlays in Period 1	12	54	6	6	30	6	48	36	18
P.V. of Outlays in Period 2	3	7	6	2	35	6	4	3	3
Present value of Budget	$C_1 = 50, C_2 = 20$								

The desired values of P_1 and P_2 that were found by Lorie and Savage were 0.33 and 1 respectively. The quantity $Y_j - P_1 \cdot 1_j - P_2 \cdot 2_j$ is positive for the investments indicated by an *. For the selected project the present value of outlays is 48 in the first period and 20 in the second period.

The major shortcoming of this method is that, it gives no properly defined method of analysis. Trial and error is,

normally, not a sound analytical tool. Besides, when the number of proposals increases and becomes quite large, this method does not help much. Moreover, if the fixed budgets are binding in many years, it becomes practically impossible to find the auxiliary numbers, P_{tS} , which could be unique and maximizing the objective of greater present value!

4.3.2 The Weingartner Model : H.Martin Weingartner (1963) model of capital rationing is some times called the horizon model², because the objective is to maximize something (discounted cash flow here) over a given period of time. Weingartner's model tries to maximize the present value of cash inflows of the projects, under given constraints. His basic framework may be rewritten as follows³:

$$\text{Maximize} \quad \sum_{j=1}^J \sum_{t=1}^T \left[a_{jt} / (1+r)^t \right] x_j \quad \dots\dots\dots(1)$$

Subject to

$$- \sum_{j=1}^J b_{jt} x_j \leq M_t \quad \dots\dots\dots(2)$$

$$x_j \geq 0 \quad t = 0, 1, \dots, T$$

where,

r = a known discount rate

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- 1 For a critical analysis of the Lorie-Savage problem , please see Weingartner, H.M. (1963)
 - 2 For similar models, see, e.g., Charnes-Cooper & Miller (1959)
 - 3 We are changing his notations, just to make the model comparable to the other models.

a_{jt} = the net cash flow obtained from a unit of project during period t .

b_{jt} = the net amount of cash used up by a unit of project during period t .

x_j = the number of units of project j constructed.

M_t = the amount of cash made available from sources external to the project under consideration during period t .

The essential features of this model may be listed as under.

- i) It can have upper bounds on the number of units of projects to be constructed. For example, if only one unit of accepted proposal may be constructed then we have to specify in the constraints

as

$$x_j \leq 1$$

- ii) If the funds used, M_0, M_1, M_2, \dots , etc. are to be repaid at some future particular date, then this problem can easily be handled by setting the value of M_t of that repayment period, negative in the constraints. For example, if the amount, K , is to be repaid at some specific period h , then we set $M_h = -K$.
- iii) The problem of budget deferrals can also be easily handled in this type of solution to capital rationing of a firm. If the productive investment is expected to start at some future time, h , then we have to set the values of M equal to zero for

periods prior to h .

- iv) If some projects are indivisible in nature, i.e. they can not be undertaken in fractional magnitudes, then we have to employ the method of integer-linear programming. Otherwise the analysis is the same.
- v) The problem of interdependence¹ of the investment proposals can also be considered in the framework. If, for instance, there are two mutually exclusive projects \underline{a} and \underline{b} . We can take care of this problem by setting $x_a + x_b \leq 1$ in the constraint. Assuming indivisibilities of the projects, if x_a is accepted, i.e., $x_a = 1$, then it requires that x_b must be equal to zero, meaning that it cannot be undertaken.

There are some minor problems in the model, which, according to Baumol and Quandt (1965), can be taken care of. These are :

- a) Funds not required for investment in the firm may be invested outside the firm or may be allocated for personal consumption of owners. The model has made no allowance for this problem.
- b) Funds not used in one period may be retained and used in some other period(s). There is no provision for this possibility too in the original model. Basically Baumol and Quandt have cited

¹ We shall discuss this problem in detail later in this chapter.

three major problems with the Weingartner model.

- i) It is illogical to separate the cash inflows, a_{jt} , from the cash outflows, b_{jt} . There must be some relationship between these two, because a bigger inflow of funds can influence the budget limits or sources which are to finance b_{jt} . If R_{jt} and C_{jt} represent the revenue and cost of the j th project in period t , the net cash outflows are $b_{jt} = C_{jt} - R_{jt}$. However, we want to maximize the discounted value of net cash inflows $a_{jt} = R_{jt} - C_{jt}$. This implies $a_{jt} = -b_{jt}$.
- ii) The discount factor $(1+r)^t$ must, in solution, equal the marginal internal rate of return, ρ_t . Assuming capital rationing \underline{r} must itself be internally determined. If it is to be the true marginal opportunity rate, then we cannot use it in the objective function of the primal problem unless we know the values of dual evaluators, ρ_t . A dual problem cannot even be constructed without knowing the primal problem. Therefore, the whole system falls.
- iii) Baumol and Quandt argue that it is not correct to discount the earnings of a project. It would be better to discount the stream of dividends which are actually meant for consumption. Weingartner's

reaction (1966a) was that to discount the dividends we require, an identity between the firm and its owners or equivalently all the individual owners have the same preferences for present over future consumption.

According to Edwin Elton (1970), the discounting of earnings rather than dividends will amount to double counting. Earnings can add to the total satisfaction of owners when it is actually consumed. After all, investors ultimately want to increase their ability to consume. The discounting of total earnings, a part of which may be reinvested and not distributed for actual consumption, will amount to double counting.

The discounting of intermediate earnings, which are not consumed but used to augment the funds available for further investment, violate the economic logic of discounting in which the discount rate, r , represents a reward for postponed consumption.

4.3.3. The Baumol-Quandt Model : After criticising the Weingartner's model, Baumol and Quandt reformulate his model by stating that a utility approach will be more rational because the investors ultimately want to maximize their ability to consume. Assuming that utility is linear in money¹, the management will try to withdraw in the t th

¹ In terms of indifference curves, this implies a linear utility function, i.e., a constant slope.

period ($t = 0, 1, \dots, T$), W_{ts} , which are exclusively meant for consumption of the owners of the firm. The manager must try to maximize the utility of withdrawals (judged by himself) in periods zero, one, etc. The whole argument notationally is

$$\text{Maximize } \sum_{t=0}^T U_t W_t \dots\dots\dots (3)$$

Subject to

$$-\sum_{j=1}^J a_{jt} + W_t \leq M_t \dots\dots\dots (4)$$

$$x_j \geq 0, \quad W_t \geq 0 \quad \begin{array}{l} j=1, 2, \dots, J \\ t=0, 1, 2, \dots, T \end{array}$$

where,

U_t is the (fixed) constant utility of a unit of money in period t , or the marginal utility in t^{th} period.

W_t is the amount of withdrawals for the consumption of owners in period t .

$-a_{jt} = b_{jt}$ is the net cash outflows for project j in period t .

M_t as before is the budget ceiling for period t .

According to Carleton (1969), Baumol and Quandt must have presumed $U_0 \gg U_1 \gg U_2 \gg \dots \gg U_t \gg 0^1$.

1 Implies that the earlier returns have greater utility than the latter returns.

$$\text{Thus, } \frac{U_t}{U_{t-1}} = (1+r)^{-1}$$

where r is the required rate of discount.

Assuming a constant discount rate and setting the utility of money for every individual in period zero, U_0 , equal to unity, we can write :

$$\frac{U_t}{U_0} = U_t = (1+r)^t$$

$$\text{for } r_1 = r_2 = \dots = r_t = r.$$

The above primal linear programming problem has the dual :

$$\begin{aligned} &\text{Minimize } \sum_t \rho_t M_t \\ &\text{Subject to } - \sum_j a_{jt} \rho_t \geq 0 \\ &\rho_t \geq U_t \end{aligned}$$

ρ_t , as we have defined earlier, may be thought of as the productivity of an additional rupee of the fixed budget in the t^{th} period. And the ratio ρ_{t-1}/ρ_t can be considered as $1+R_t$, where R_t is the internal rate of return of an additional unit of money in period t . Or it may be termed as the marginal internal rate of return. In any period, if there are no withdrawals for consumption by the owners, it means it pays to invest rather than consume, i.e., for $W_t = 0$, $\rho_t > U_t$ and for $W_t > 0$, $\rho_t = U_t$.

Now, let us consider a case where $W_{t-1}=0$ and $W_t > 0$, then we have $\beta_{t-1} > U_{t-1}$ and $\beta_t = U_t$. Therefore,

$$\frac{\beta_{t-1}}{\beta_t} > \frac{U_{t-1}}{U_t}$$

or $1 + R_t > 1 + r_t$

or assuming r_{ts} to be constant, i.e., $r_1 = r_2 = \dots = r_t = r$

$$1 + R_t > 1 + r$$

$$R_t > r$$

Therefore, when marginal internal rate of return exceeds the required rate of return, it pays to invest rather than consume (the alternative for productive investment). That is exactly what Irving Fisher would have had us to expect.

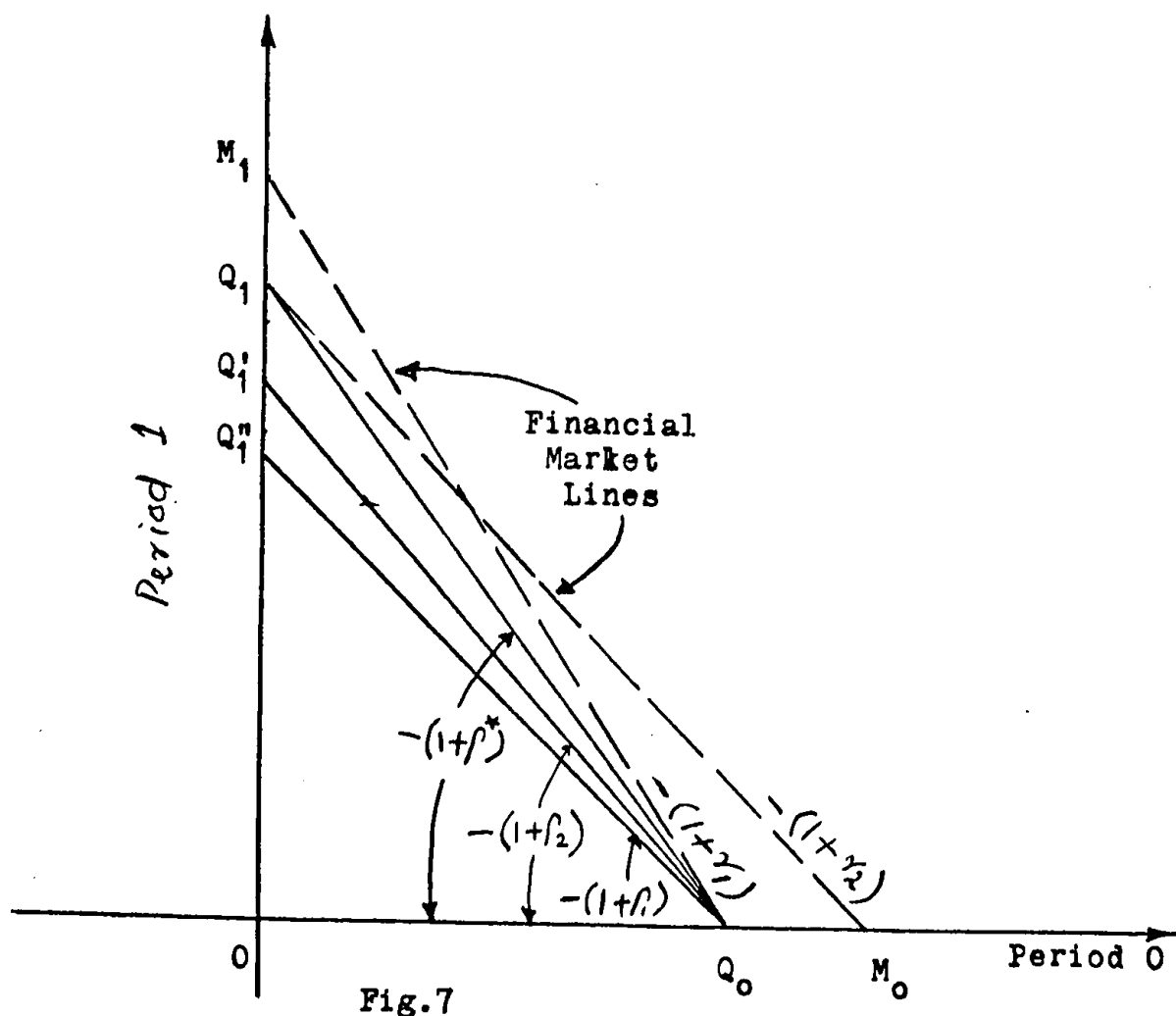
"The marriage between Fisher's analysis of the discount rate under capital rationing and the programming approach is thus complete".¹

Weingartner (1977) has criticized Baumol and Quandt on the ground that they dropped the notion of project and assumed instead constant returns to scale without any qualifications. This assumption might have been motivated by the result, provided by number of economists, that the long run cost curves are comparatively flat over a large range of plant sizes. As this assumption is also combined with the assumption

1 Baumol and Quandt (1965). p.327

of linear utility functions, it will be shown that, it leads to peculiar and unwarranted results for more general cases.

In terms of the two period Fisherian graphical analysis, which was outlined earlier, the investment proposals exhibiting constant returns to scale would be portrayed as straight lines with negative slopes all emanating from a point on the horizontal axis representing the owner's initial capital. In Figure 7, these are projected to the vertical axis which represents the income available for consumption in the next period.



Suppose there are three investment proposals each with constant rate of return ρ_1 , ρ_2 and ρ^* respectively. These proposals are indicated by three straight lines Q_0Q_1'' , Q_0Q_1' , and Q_0Q_1 , in Figure-7. As before, r is the rate of interest for purely financial transactions in the market. If this rate of interest indicated by r_2 , is less than the rate of return offered by the best investment proposal, ρ^* , then the investor will invest the maximum amount possible (which yield maximum present value for investment). The owner(s) can combine this income with other income(s), if any, and determine his (their) preferred level of consumption over time. In the other case, when market rate of interest is higher, which is indicated by r_1 , than the rate of return on the best investment proposal, ρ^* . Such a financial market line is indicated by Q_0M_1 . Here, the best investment decision is no investment at all. The owner(s) will consume some of his (their) present incomes and lend some to generate income for consumption in the next period. In this case the Separation Theorem holds and investment decisions are independent of consumption preferences of the owner(s).

Next, let us consider a case with explicit limit on borrowing by the owner-firm. In Figure-8, we consider only the best investment alternative, Q_0Q_1 , with a rate of return equal to ρ^* . The borrowing limit is $OA_0 = OB_0$. Assuming a market rate of interest, r , lower than the rate

of return offered by the best investment proposal, ρ^* , the POL extends to the 4th quadrant utilizing all available resources and maximum borrowing. The amount borrowed, OA_0 , has to be repaid at the market rate of interest. The payment equals $A_1M_1 = (1+r)OA_0$. With an explicit limit on borrowing by the owners of the firm, a new consumption possibilities line is formed parallel to the line of the best investment alternative. This consumption possibilities line is $Q_0'BM_1$. Its derivation is as follows. From every point on the POL, a FML segment, like Q_1B , is drawn.

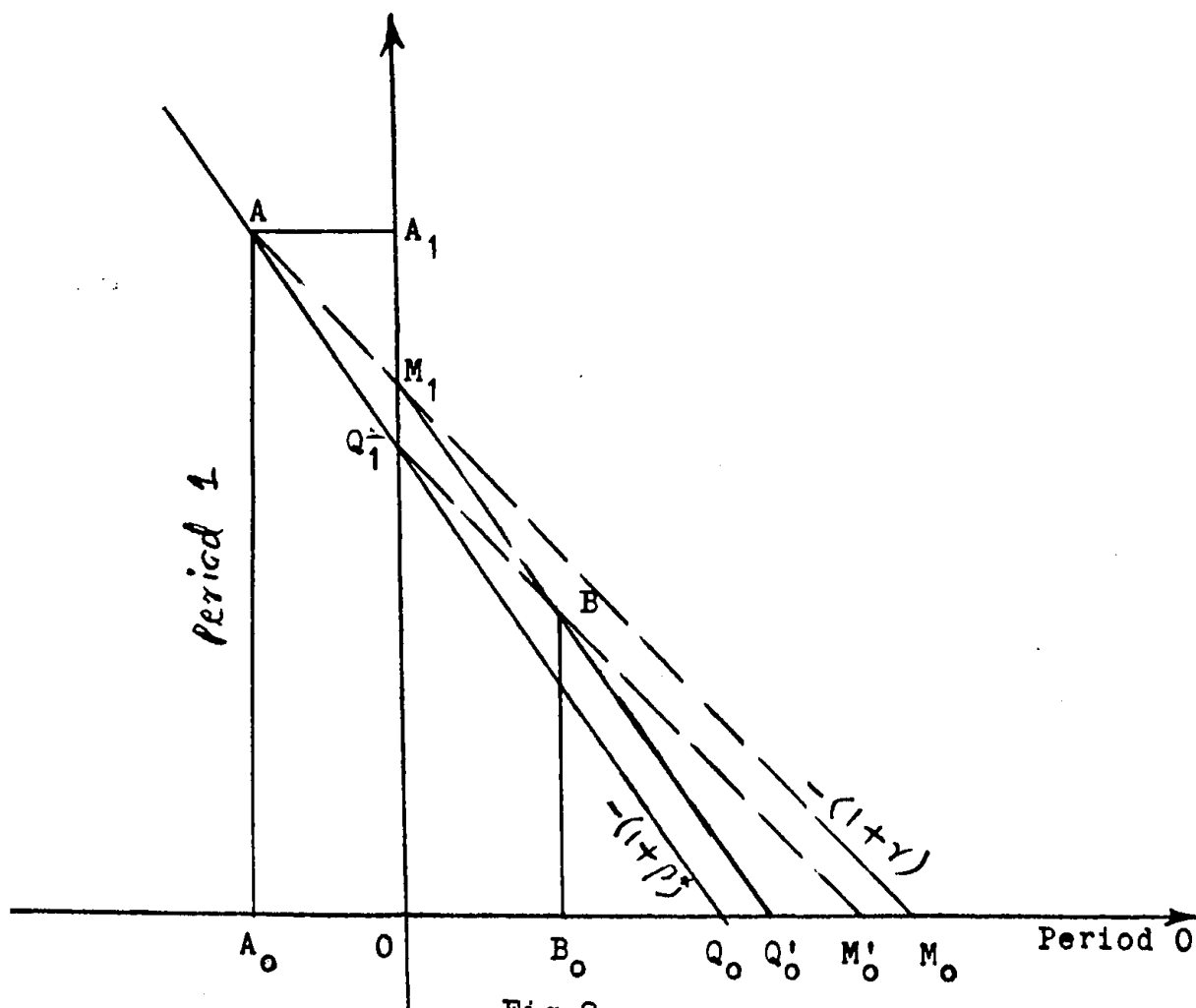


Fig.8

Since the borrowing limit is constant these segments will be of equal length and slope of $-(1+r)$.

The Separation Theorem cannot hold in this case, because now the consumption preferences of the owners do not have to be compared with the market rate of interest but with the new consumption possibilities line which is parallel to the POL of the best investment alternative. Therefore, investment decision cannot be taken independently of the consumption decision.

In case of $r > \rho^*$, no productive investment will take place and the borrowing limit is of no relevance.

Next, let us consider the assumption of constant returns to scale together with the assumption of linear utility functions. Baumol and Quandt and many others¹ have made both these assumptions.

Assuming a single owner we will draw indifference "curves" with a constant negative slope which is the ratio of (constant) marginal utility of money in period zero, U_0 , to the (constant) marginal utility of money in period one, U_1 , where the negative sign represents the trade off. This is portrayed in Figure-9, which is, except for the indifference curves, identical to Figure-8.

Three possible cases have to be examined.

1 Carleton (1969), Merville-Tavis (1973) and Myers (1972).

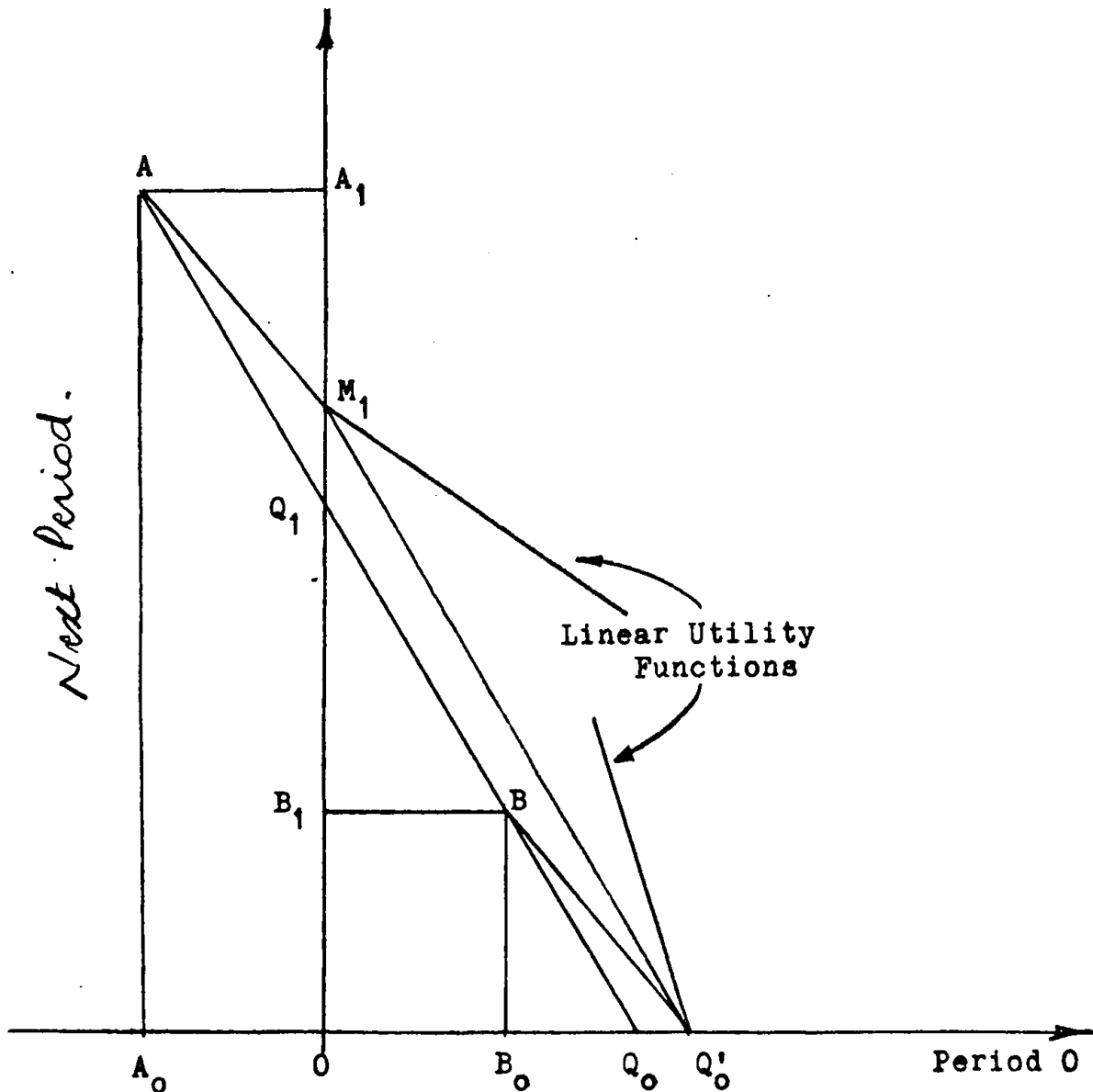


Fig.9

- i) When $\frac{U_0}{U_1}$ is greater than $(1+r^*)$: Then, the constant utility-maximizing investor will invest an amount equal to Q_0B_0 . Against the proceeds in the next period, OB_1 , he will borrow an amount equal to $B_0Q_0' - OA_0$ (the maximum borrowing limit) and consume OQ_0'

in the current period, leaving nothing for the next period.

ii) When $\frac{U_0}{U_1}$ is less than $(1+\rho^*)$, then the constant utility-maximizing investor will produce at A, using all available resource plus the maximum borrowing limit. This productive investment after repayment of loans leaves OM_1 in the second period. The investor will consume nothing in the current period, trying to consume the maximum possible OM_1 , in the next period.

iii) When $\frac{U_0}{U_1}$ is equal to $(1+\rho^*)$: only in this case we can expect positive amounts of consumption in the two periods. However, the situation $\frac{U_0}{U_1} = (1+\rho^*)$, where the slope of the linear utility function is equal to the slope of the consumption opportunities line, is rare. Thus, the assumption of linear utility functions together with the assumption of constant returns to scale would mean that consumption can take place in one or the other period, but not in both the periods. It is because of these reasons that Weingartner rejects both the assumptions of constant returns to scale and linear utility functions.

4.3.4 Reconciliation Between the Weingartner Model and the Baumol and Quandt Model : Many authors¹ have tried to bring about a resolution between these two models. Edwin

¹ Edwin Elton (1970), Carleton (1969), Merville-Tavis (1973) Myers (1974).

Elton (1970) argues that external discount rates are important for the investment decision-making of a firm under capital rationing. He tries to show his point by Fisherian analysis and mathematical programming. He does not provide a mathematical programming model of his own which incorporates the external discount rates. Elton states that as long as the objective functions are different, we are bound to get different results. What is needed is the derivation of a more appropriate objective function. However, again Elton does not provide the derivation of this "appropriate" objective function.

The model of Carleton (1969), even according to him, is a reinterpretation of Baumol and Quandt model. His major argument is that capital budgeting as an economic phenomenon, is a part of general financial planning of the firm. The funds not used in a specific capital budget may be reserved for other uses. In his objective function, he postulates maximization of the utility of withdrawals not for the consumption of the owners but as a release of funds from micro level (capital budget) to macro level (Financial Plan).

Merville-Tavis (1973) have formulated mathematically and illustrated graphically that the rate of return on productive investment may lie between the lending and borrowing rates of interest. This happens when there are neither lendings nor borrowings by the firm. When initial

funds available to a firm are limited, and if marginal rate of return on productive investment financed by the initial capital is still higher than market borrowing rate of interest, then borrowing will take place (for larger productive investment) until the marginal rate of return equals the market borrowing rate of interest.

On the other hand, when funds are large, the productive investment will stop just before its marginal rate of return becomes less than the market lending rate of interest. If funds are just sufficient to finance the investment proposal, the marginal rate of return on productive investment may lie between lending and borrowing rates of interest.

Merville-Tavis develop a model very similar to the Baumol and Quandt model into which are incorporated lendings and borrowings as well as lending and borrowing rates of interest. The objective function is exactly the same as the one in Baumol and Quandt model, that is, maximizing the constant utility of amount (withdrawals or dividends) consumed. They have developed this model, accepting the criticism, that additivity of utilities over time would mean either a single owner or a group of owners with a relatively homogeneous preferences for consumption over time.

The other very interesting resolution of the conflict is made by S. Myers (1972), where he tries to show that the Baumol and Quandt model is not actually different from the

original Weingartner model. Let us assume a world of certainty and perfect capital markets except for capital rationing. Now consider equation (4), i.e., the constraint in the Baumol and Quandt model. This will always be a strict equality rather than inequality, because in solution it would mean that W_t s can be increased, implying that the objective function can be improved.

Considering the constraint in equation (4) as a strict equality and solving for W_t and substituting it in the objective function, we can rewrite the Baumol and Quandt model as follows :

$$\text{Maximize } \sum_{t=1}^T U_t (M_t + \sum_{j=1}^J a_{jt} x_j) \dots \dots \dots (3a)$$

$$\text{Subject to } (\sum_{j=1}^J a_{jt} x_j) + W_t = M_t \dots \dots \dots (4a)$$

Mathematically the constant term $\sum_{t=1}^T U_t M_t$ may be dropped from the objective function.

The difference between Baumol and Quandt model and Weingartner model reduces to, (i) inclusion of "Slack" variables, W_t , in the constraint and (ii) the use of constant utilities, U_t , instead of $1/(1+r)^t$, the discount factor, in the objective function.

The inclusion of extra slack variables is not very important, but inclusion of U_t instead of the discount factors has to be examined. It is attempted below.

As we have already seen above in the discussion of

Baumol and Quandt model, according to Carelton (1969)

Baumol and Quandt presumed

$$U_0 \gg U_1 \gg U_2 \gg \dots \gg U_t \gg 0$$

Thus, $\frac{U_t}{U_{t-1}} = \frac{1}{1+r} \dots \dots \dots (5)$

where, r is the required rate of discount. Assuming r_t s to be constant over time and scaling every investor's utility function so that $U_0 = 1$, then in equilibrium, we have¹

$$\frac{U_t}{U_0} = U_t = \frac{1}{(1+r)^t} \dots \dots \dots (6)$$

Substituting equation (6) into equation (3a), the Baumol and Quandt model turns out to be exactly equal to the Weingartner model.

4.3.5 The Problem of Interdependent Projects² : The usual assumption in the capital budgeting literature is that all investment alternatives are independent, i.e., acceptance of any set of investment proposals would not affect the profitability or feasibility of any other different set of investment proposals. However, dependent or contingent

1 For further discussion of the classical theory of security valuation, see Hirshleifer (1970).
 2 Our discussion, here, draws heavily upon an article by Weingartner H.M. (1966a).

projects can also arise, for instance, acceptance of one project is dependent or contingent on acceptance of one or more projects. As an example consider the purchase of an extra-long boom for a crane which would be of little use if the crane itself were not purchased. However, the latter may be justified on its own, i.e., the crane can operate even without this additional investment. When dependent projects are combined with their independent "prerequisites" the combination may be called as a compound project.

The problem of interdependence of projects can be handled with the help of mathematical programming approach to capital budgeting. We must establish the relationship among the interrelated projects and then introduce these relationships into the constraint equations of the mathematical models.

Consider two interrelated projects r and s . Project r may be accepted only if project s is undertaken. Where, project s is an independent one. The relation between these two projects may be

$$x_r \leq x_s \dots\dots\dots(7a)$$

$$x_s \leq 1 \dots\dots\dots(7b)$$

where, x is a proportion of a project. Thus, in solution, if project s is accepted, the effective constraint

for project r would be $x_r \leq 1$. Otherwise, x_s would be zero and the constraint for project r would be $x_r \leq 0$. However, the non-negativity condition together with $x_r \leq 0$, will force x_r to equal zero (be rejected).

As another possibility consider two mutually-exclusive investment projects u and v . Project r is dependent on acceptance of either of these two projects. This type of interrelationship may be expressed as

$$x_u + x_v \leq 1 \dots\dots\dots(8a)$$

$$x_r \leq x_u + x_v \dots\dots\dots(8b)$$

Therefore, if any of the projects u or v is accepted the other must be rejected. This makes the constraint on project r as $x_r \leq 1$. But if both the projects are rejected then the constraint for dependent project becomes $x_r \leq 0$. With the non-negativity condition this implies that project r will be rejected.

Similarly, if we have two mutually exclusive projects r and s , where acceptance of either of these projects depends on the acceptance of either project u or project v , two mutually exclusive alternatives. This relationship may be expressed as :

$$x_u + x_v \leq 1 \dots\dots\dots(9a)$$

$$x_r + x_s \leq x_u + x_v \dots\dots\dots(9b)$$

Finally, we can build up the relationship for contingent

chains, where acceptance of a project, r depends on the acceptance of some other project s which in turn depends on the acceptance of project u .

$$x_u \leq 1 \quad \dots\dots\dots(10a)$$

$$x_s \leq x_u \quad \dots\dots\dots(10b)$$

$$x_r \leq x_s \quad \dots\dots\dots(10c)$$

The difficulty with the dependency equation (7a) may be seen by putting both the unknowns on the same side of the inequality (7a) as

$$x_r - x_s \leq 0 \quad \dots\dots\dots(11)$$

The zero on the R.H.S. presents no difficulty; However, the negative coefficient of x_s does. One possible way of handling this problem is to preorder the projects in such a way that the independent members of a set always precede the dependent projects.

4.4 Time-State Preference

Time-state preference approach to investment decisions under uncertainty is one of the most elegant and theoretically most appealing methods of investment valuation. It is important in the sense that it views the investment decision in a general equilibrium situation. In other approaches our considerations are partial and decisions are made in isolation i.e., taking only one firm at a time; we do not consider what happens to the other economic entities or the general environment. This method takes into account the states of the world that would occur in future. A rupee in one state of the world may have a greater or lower value than in some other state. An asset may return more or less (in money or real terms) in future depending on the state of the world then. To an individual, returns in different states are worth differently. If A is a bachelor now, he may be married then; if he has no or few children at present, he may have more children after ten years.

The state-preference model was initially developed by Arrow (1964) and Debreu (1959) and further extended by Hirshleifer (1965). Hirshleifer also discusses this approach and its applications in his later works¹.

1 See Hirshleifer (1970, 1966).

The simplified version of the theory, which follows, draws heavily upon Hirshleifer's (1965) pioneering work. For illustration, let us assume that

- i) there is only one present state which implies that there is certainty as to the present (time 0).
- ii) future is represented by a point in time (time 1) and there are only two alternative states: a and b. (either state a or state b must occur; two alternative states of the world might be thought of as war versus peace, for example, or depression versus prosperity).

In the two period certainty model, as developed in this chapter we have only two objects of choice, C_0 and C_1 , consumption in period zero and consumption in period 1, respectively. But here, in our example, the individual is confronted with three objects of choice :

C_0 = a certain level of consumption at the present (time 0);

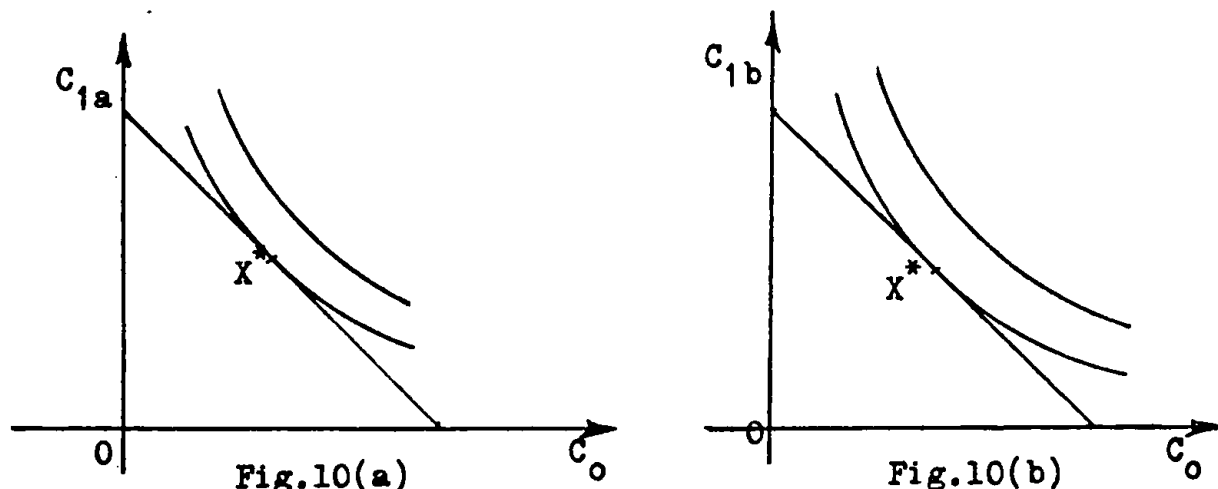
C_{1a} = an uncertain level of consumption in period 1, if state a obtains; and

C_{1b} = an uncertain level of consumption in period 1, if state b occurs.

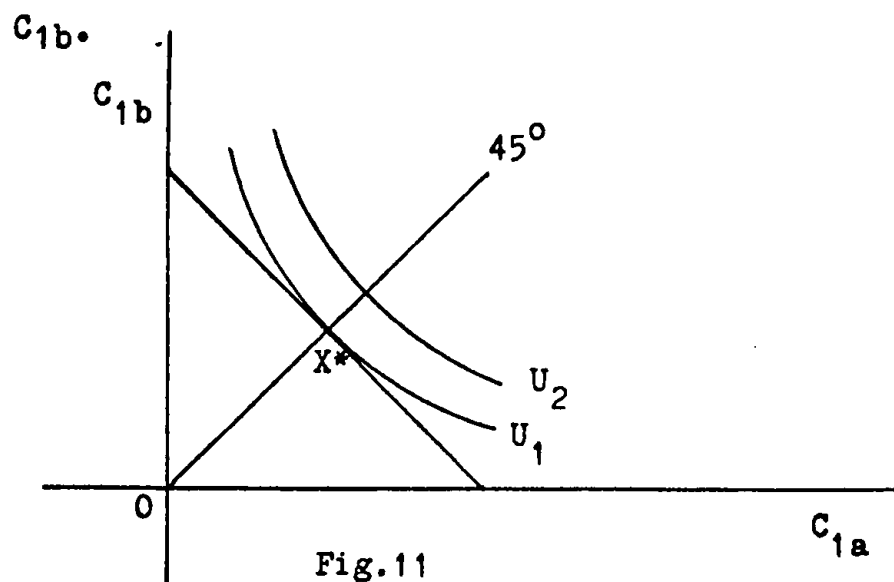
We can think of two types of choices :

- i) between a certain level of consumption and a future uncertain level of consumption; between C_0 and C_{1a} or between C_0 and C_{1b} . This is

illustrated in the following figure.



ii) between two contemporaneous risky claims C_{1a} and



We consider, here, only the situations of pure exchange leaving aside the problems introduced by production opportunities.

In Fig. 11, the amount of C_0 is implicitly fixed so that we can consider a simple exchange between two risky claims. We have drawn a 45° line through the origin which represents a set of points along with $C_{1a} = C_{1b}$. 45° line implies a certain level of claims whichever state obtains.

We call this line as the "certainty line". The figure shows the indifference curves of the individual with subjective probabilities of $\pi_a = \pi_b = \frac{1}{2}$, of the two states a and b. The slope of the indifference curves can be explained in several ways. One possible way is to observe the general behaviour that no one would be so imprudent as to prefer, if $\pi_a = \pi_b = \frac{1}{2}$, a combination $(C_{1a}, C_{1b}) = (1000, 0)$ to another combination $(C_{1a}, C_{1b}) = (500, 500)$, especially if receiving nothing in state b does not mean a tolerable loss of wealth but absolute impoverishment and starvation. Even a very mild degree of conservatism requires convex indifference curves. This statement implies risk aversion or "non-specialization" among alternative state of the world.

In the simplified model here, there are three prospects of consumption opportunities : C_0 , C_{1a} and C_{1b} . We may think of three "pure securities".¹ A pure security is regarded as one offering one unit of income in only one state of the world.

In our model there are only three possible pure securities.

- i) $(y, 0, 0) = (1, 0, 0)$
- ii) $(0, y_{1a}, 0) = (0, 1, 0)$
- iii) $(0, 0, y_{1b}) = (0, 0, 1)$

¹ Known also as Arrow-Debreu securities, because this concept was initially developed by K.J. Arrow (1964) & Debreu (1959) independently.

Let us denote a particular individual's initial endowment, W , as : (Y_0, Y_{1a}, Y_{1b}) . If this individual knows the prices of the above pure securities, then he can reallocate his initial endowment across states, should he so desire. Since we are assuming a pure exchange situation, where no production takes place, individuals can reallocate their consumption claims over time and state only by trading and in this trade they are constrained by their initial endowment which represents consumption claims over times and states.

$$W = P_0 C_0 + P_{1a} C_{1a} + P_{1b} C_{1b} \dots \dots \dots (12)$$

where P_0 , P_{1a} and P_{1b} are the prices of the above pure securities, and the constant W , initial wealth, is determined by

$$W = P_0 Y_0 + P_{1a} Y_{1a} + P_{1b} Y_{1b} \dots \dots \dots (13)$$

Since we have assumed certainty as to the present (time 0), we may set $P_0 = 1$. P_{1a} is the price of a rupee to be received in time 1, only if state 'a' occurs. When an individual values such a risky claim, he applies some rate of discount, which depends on his subjective probability about the occurrence of a state. This discount rate reflects the nature of individual's preference function which is itself influenced by the subjective probability of the occurrence of different states.

Let us further assume that total utility from

consumption does not depend on the state of the world. The prices of time-state claims can be defined in terms of discount rates :

$$P_{1a} = \frac{1}{1 + R_{1a}}, \text{ and } P_{1b} = \frac{1}{1 + R_{1b}}$$

where, R_{1a} and R_{1b} are the discount rates for both futurity and probability. It follows, therefore, that equations (12) and (13) can be reformulated in terms of present values of future wealth or consumption discounted at an appropriate rate for exchange between a present (certain) receipt and a future state-contingent claim.

$$W = Y_0 + \frac{Y_{1a}}{1+R_{1a}} + \frac{Y_{1b}}{1+R_{1b}} = C_0 + \frac{C_{1a}}{1+R_{1a}} + \frac{C_{1b}}{1+R_{1b}} \dots\dots(14)$$

Note that in a degenerate case of only one future state, the riskless rate of discount R_1 is defined as :

$$\frac{1}{1+R_1} = \frac{1}{1+R_{1a}} + \frac{1}{1+R_{1b}} \dots\dots\dots(15)$$

Consider a situation where the riskless rate of discount is 10%. An individual is offered Rs. 11 after one year with certainty. The present value of this offer would be Rs. 10. Now assume that there are two states, a and b, in time 1, with equal probabilities of $\frac{1}{2}$ for each state.

If the individual is offered Rs. 11 in either state, it implies that he is again certain to receive Rs. 11 in time 1. The value of Rs. 11, to be received either in State a or in State b, to the individual can not exceed Rs. 10. And since these offers are equally probable, the price of each one of them must not exceed Rs. 5. Therefore, in the special case of only two States a and b with probabilities $\pi_a = \pi_b = \frac{1}{2}$ and the riskless rate of interest $R_1 = 10\%$, the time state discount rate is 120%.

In this model, an individual, j, will be faced with a time and State preference function.

$$U = g^j (C_0^j, C_{1a}^j, C_{1b}^j, \pi_a^j, \pi_b^j). \quad \dots(16)$$

We have emphasized in this formulation of utility function that individual j tries to maximize his total utility which itself depends on his subjective probabilities, π_a^j, π_b^j . The total utility of consumption in these states can be expressed as :

$$U^j = V(C_0^j) + \pi_a^j V(C_{1a}^j) + \pi_b^j V(C_{1b}^j) \dots\dots\dots(17)$$

where $V(C_0^j)$ is the amount of total utility of consumption in period 0. Similarly $V(C_{1a}^j)$ and $V(C_{1b}^j)$ represent total utility of consumption in period 1 under state a and b respectively.

We have tried to demonstrate how to achieve a balance among contemporaneous risky claims. When these

claims are balanced, then we can apply a riskless rate of discount to find out the present value of such claims or to reallocate our initial endowment across time and states.

CHAPTER V

SUMMARY AND CONCLUSIONS

Under the assumption of certainty we have classified different methods of investment appraisal into two main groups of a) Non-discounting methods and b) Discounted Cash Flow (DCF) methods. With every model the assumption is that future is known with complete certainty. According to the Degree of Necessity approach, proposals which can no more be postponed are taken first. In this model the productivity or the profitability of capital is not taken into account. The payback period method is superior to the Degree of Necessity approach as it considers the number of years in which initial outlay is recouped from the project. However, it is criticised on the ground that it does not take into account the whole economic life of an asset and it only considers the early cash flows. Coming to DCF methods, we have studied the Net Present Value (NPV) approach which is defined as the discounted value of returns stream less the discounted value of outlays stream. Then we have studied the Internal Rate of Return (IRR) method of investment appraisal. In this method, a rate of discount is found which equates the present value of returns and the present value of outlays. We make a comparison between NPV and IRR

approaches to illustrate the possibilities where these models can lead to conflicting results. We have also considered the problem of multiple rates of return and tried to find out a meaningful and unique rate of return in such circumstances. We have elaborated the Fisher's Rate of Return over cost which serves as a device to determine all the rates of interest at which one investment will be preferred to another. Finally, we make a comparison between the IRR and the Payoff Estimate of the rate of return, where pay-off is the ratio of initial expenditure to annual earnings. We noted that if an investment will earn the same amount for ever, its rate of return is simply the reciprocal of its pay-off.

Though these DCF methods are superior to payback and other non-discounting methods (when the objective is to maximize the wealth of the present owners of a firm), they are based on certain simplifying assumptions. Therefore, the investment decisions based on the results obtained by these criteria will be unreliable.

Relaxing the assumption of complete certainty as to the future events, we dealt with more sophisticated methods of investment appraisal. First, we developed the concept of Expected Net Present Value (ENPV) to consider the problem of risk and variability in regard to future cash flows of a project. The problem with this method was its assumption of the known probability distribution of future outcomes

which may not be possible in real world. Next we have considered the Risk Adjusted Discount Rate (RAD) method, where we add a risk premium to the market rate of interest. This premium is appropriate to the risky nature of the project and it has to be decided subjectively. The Certainty Equivalent Method (CEQ) tries to find out a certainty equivalent for every cash inflow or cash outflow; then discounting at a risk-free rate of interest, it finds the present value of such certainty equivalent earnings. CEQ, too, offers no objective measure of risk and uncertainty. We have compared RAD and CEQ methods to understand their advantages and limitations.

We noted that all the above methods are single stroke valuation models. The project valuation is done once and for all, and the intermediate circumstances are not considered. The application of Capital Asset Pricing Model (CAPM) to capital budgeting decisions imparts greater adaptability to intermediate circumstances. It is a sort of step by step, time to time, dynamic valuation model. First, we developed the famous Sharpe-Lintner single period valuation model. Then we have presented how CAPM may be applied to capital budgeting decisions under uncertainty. Finally, we have also studied the concept of sensitivity analysis and decision-tree approach to investment appraisal under uncertainty. Both of these analytical tools are simple and practically useful as they can help management

of a firm in its investment decisions.

To consider the problem of capital rationing initially we employ a two period Fisherian isoquant analysis. With the help of this graphical analysis we have considered the optimal capital rationing of firms with single or multiple owners. In this way we have developed the essentials of the Separation Theorem and concluded that ownership may be separated from the management of a firm. Further, we have considered explicit limits on the amounts of borrowings by firms under capital rationing. We noted that the Separation Theorem can not hold in such cases. Finally, the possibility of the firms which have to borrow at a corporate rate different from the rate of interest on personal borrowings. As the loans are to be repaid at the corporate rate of interest the management need not consider other different rates of interest on personal borrowings.

Then we studied the Lorie-Savage problem and its extensions in mathematical programming approach to investment decisions. In this regard two famous models a) Weingartner (1963) and b) Baumol and Quandt (1965) are studied in greater detail. Subsequently, we considered a possible reconciliation between the two models. We have also studied the problem of capital budgeting of inter-related projects. It is shown that this problem could be handled with the help of mathematical programming approach to capital budgeting under constraints. In all these

models we have focused our attention only on the problem of "rationing" the available funds. Here, too, our assumption is "certainty" as to the future outcomes. As an extension of the Fisherian isoquant analysis we have studied the theoretically more appealing "time-state-preference" approach to investment decisions under uncertainty. Of course, there are some difficulties with the application of this model which we have noted.

Further, extensions and research in this area may be to consider capital rationing problem when we relax the assumption of certainty of future by specifically taking into account conditions of risk and uncertainty. This will lead to obtaining more realistic results. If uncertainties are taken into account, the ranking of investment proposals may not be the same as the one under the assumption of certainty. More sophisticated methods of investment appraisal under uncertainty may be used to rank the investment proposals in order of their desirabilities. The future availability of funds may also be subject to uncertainties leading to alteration in the ranking of the proposals, particularly in regard to investment proposals where outlay of funds in uncertain future periods need to be made. It will be a question of balancing uncertain stream of outlays with uncertain funds available in future time periods in order to maximize the present worth of investment in the light of standards used. However, it will make capital budgeting decisions more complicated and difficult which may prompt further research in this area.

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