MONEY AND GENERAL DISEQUILIBRIUM, PART I: STATIC ECONOMY

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ABSTRACT

This paper explores the existence of monetary general equilibrium in the context of a classical model of general equilibrium theory. It begins by constructing the model of an ideal economy in which money is inessential This model is the counterpart of the Arrow-Hahn model that has no place for money. It then proceeds to articulate a more realistic model of a barter economy in which money plays an essential role in reducing transaction costs and materially improving the economic outcome. Techno-institutional arrangements of currency and credit moneys have been discussed and the conditions under which a unique positive general equilibrium exists have been obtained. General disequilibrium is shown to prevail if a part of the proceeds or income generated by currently produced outputs are devoted to accumulating money balances, i.e. saving exceeds investment. Fiscal solutions for restoring full employment general equilibrium have also been demonstrated.

1. INTRODUCTORY

Modern discussions of money in general equilibrium theory have their origins in the controversies about Keynes' (1936) thesis that monetary economies do not possess automatic inbuilt mechanisms to clear all markets including the labour market and consequently that the government will need to intervene by means of an expansionary fiscal policy to ensure full employment. Pigou (1943) and later Patinkin (1965) were of the opinion that Keynes' thesis was not theoretically sustainable, that he neglected the operation of the real balance effect which would serve as an automatic stabiliser of aggregate demand in the event of deflation and bring it line with aggregate supply. Investigations by Clower (1965) and Hahn (1965) revealed however that in Patinkin's model money and goods were indistinguishable from one another so that "goods were allowed to buy other goods". And further investigations by Hahn (1965) and Arrow and Hahn (1972) which attempted to give a satisfactory treatment of money in which "money bought goods and goods bought money but goods did not directly buy goods"

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came up with the completely nugatory conclusion that modern general equilibrium theory has no place for money, that money has no essential role to play in that theory. By the early seventies Ostroy (1973) summed up the dilemma of integrating monetary and value theories by asking, "How to make money appear without making standard theory disappear?" and Hahn (1973) concluded that, "There is nothing we can say about the equilibrium of an economy with 'money' that one cannot say about the equilibrium of a non-monetary economy". About a decade later Hahn (1982) noted that, "The most serious challenge that the existence of money poses to the theorist is this: the best developed model of the economy cannot find room for it".

During the last three decades the chief concern of monetary general equilibrium theory has been directed towards finding substantive reasons for the existence of an "essential" money and demonstrating the conditions under which general equilibrium can prevail in the presence of money. A variety of strategies have been brought to bear on this which nevertheless have one feature in common, viz. all of them alter the initial on this question which nevertheless have one feature in common. Viz. all of them alter the initial assumptions of general equilibrium theory itself to make room for a medium of exchange which has a positive exchange value even though it is by itself worthless. This has meant introducing constraints, restrictions, frictions, imperfections, inefficiencies, uncertainties, non-convexities, etc. into the general equilibrium model. Some of the 'successful' strategies have been the cash-in-advance constraint [Clower, (1967), Shapley and Shubik (1971), Sargent (1987)] infinite agent infinite horizon models [Bewley (1980), Gale and Hellwig (1984)], overlapping generations models [Brock 1974, Wallace 2001], sequence economies [Hahn (1971), (1973), Lucas and Stokey (1987)], credit verification costs [Woodford, 1986], search and random matching models [Trejor and Wright, 1995], positive bid-ask spreads [Duffie, 1990], separate budget constraints for individual transactions [Starr, 2002], incomplete markets [Magill and Quinzi (1992), Cass (2006)] uncertainty [Bewley (1980)], utility of holding money [Brock (1974)], acceptability of money in payment of taxes [Starrett (1973)], restriction that Pareto-optional allocations require trade [Duffie, 1990], etc. Reviewing these developments Gale (2010) in his entry on "money and general equilibrium" in *The New Palgrave* quoted the conclusion of Ostroy (1987), "We shall argue that the incorporation of monetary exchange tests the limits of general equilibrium theory……." and himself concluded, "That comment is just as true today as it

was then, and remains a great challenge for economists who want to develop more satisfactory models of the process of monetary exchange at the level of the economy as a whole".

While these developments have been apace the original Keynesian concerns which was their original source, "Does the working of a monetary economy differ in a fundamental way from the working of a real economy?" "Is the use of money compatible with the existence of full employment general equilibrium?" "Is money neutral?" etc. seem to have faded into the background at least in the literature on money in the context of general equilibrium theory. The question of the essentiality of money has occupied far more attention in that literature as compared to the question of the neutrality of money[1].

This paper addresses both the questions of essentiality and neutrality of money from the standpoint of a *classical* model of general equilibrium. In parallel with the neoclassical literature it attempts to find a minimal set of conditions that can account for the existence of an essential money. It then proceeds to investigate the conditions in which money, though essential, might also be non-neutral. The paper is divided into six sections. The second section articulates a classical model of general equilibrium of a barter economy that has no place for money. This model may be regarded as the classical counterpart of the Arrow-Hahn (1972) neoclassical model that has no place for money. The conditions for the existence of an essential money have been investigated in a "less than ideal" model of a barter economy in the third section. The fourth and fifth sections deal with currency money and credit money respectively. In these sections the conditions under which monetary disequilibria can arise and the policy actions by which the disequilibria can be corrected have been investigated. The sixth section makes some concluding observations.

2. GENERAL EQUILIBRIUM: IDEAL BARTER ECONOMY

The production side of the classical model will be described by a static open Leontief model that produces gross outputs are just sufficient to meet the replacement requirements of all industries and the final consumption demands of the workers'

households[2]. Net saving and net investment are zero. Final consumption demands are described by fixed share Engels' coefficients, these being the simplest possible representation of consumer demand as homogenous of degree zero functions of income and prices[3]. Money illusion is absent. It can be shown that this economic system has a unique positive equilibrium at full employment. The price system is ,

$$\mathrm{PA} + \mathrm{wL} = \mathrm{P} \qquad (1)$$

Where A is the matrix of I-O coefficient satisfying the conditions for economic viability, L and P are row vectors of labour coefficients and prices and w is the wage rate. One of the prices can be set as numeraire and a unique positive solution of the relative prices and the real wage rate can be found from equation (1). Let L^* be the total labour employed. The real net national income is $Y = wL^*$. Accordingly, the quantities demanded for final consumption goods of the n goods will be

$$C_i = \frac{\propto_i wL^*}{p_i} \qquad \sum \propto_i = 1 \qquad (2)$$

The gross outputs required to satisfy these final consumption demands and the resulting inter-industrial demands for capital goods are obtained as

$$X = (I - A)^{-1}C \qquad (3)$$

Where X and C are column vectors. The markets for the goods are cleared. It remains to show that the solution (P,w,X) must be such as to ensure full employment. This is easily done. The quantity of labour required to produce the output vector X is

$$LX = L(I - A)^{-1}C$$

From (1) we know $P=wL\ (I-A)^{-1}$ so that

$$\mathrm{wLX} = \mathrm{wL}\ (\mathrm{I\text{-}A})^{-1}\ \mathrm{C} = \mathrm{PC} = \sum \frac{p_i \propto_i wL^*}{p_i} = wL^*$$

and $LX = L^*$ implying that the demand for labour required to produce gross outputs X equals the labour available L^*. In effect there are n price and n output equations to solve for n-1 relative prices, n outputs and 1 real wage rate.

There is no place whatsoever for money to exist in this economic system. All the commodities and labour are costlessly exchanged for one another. Every agent,

whether the F firms working in N industries or the H households that supply labour transact in all commodities costlessly. A quantity theory equation of the type

$$\mathrm{M_d} = \sum m_i p_i X_i + m_H w L^* = M^* \qquad (4)$$

where m_i, m_H are proportions of industry sales and household incomes and M^* is the supply of money can be added to determine n money prices, 1 money wage rate and n outputs but the solution does not differ in any way from that obtained with (1), (2) and (3). Money is not held by anybody – it does not appear either in price equations of the industries nor in the demand equations of the households. The strict classical dichotomy of real and monetary sectors prevails. The existence of money makes no difference whatsoever, it is as if money does not exist. Besides being completely inessential money is also completely neutral. As Sraffa (1932, p. 42) put it, "……. a state of things in which money is "neutral" is identical with a state of things in which there is no money at all …….".

3. WHY MONEY?

What is needed to explain the emergence of essential money is a less than the ideal world of (1), (2) and (3). The harsh features of the wretched economic life that agents lead in a barter economy must be reflected in the model of the economy if the essentiality of money is to be brought into sharp focus. After all money is a techno-institutional arrangement that is designed to reduce the costs of making transactions and therefore must consist of (a) a commodity (or commodities) having a peculiar set of properties, viz. durability, divisibility, portability, etc. that enable its use to reduce transactions costs and (b) of monetary institutions that administer monetary payments. So let us first visualize a barter economy in which the commodities themselves are used to perform the monetary function. In the absence of double coincidence of wants every agent would be required in general to hold a stock of each of the goods S_{ik} (k=1---F+H) separately as it were, only for the purposes of making transactions[4]. Let the annual costs of storage, security, transport, deterioration, decay, damage, pilferage, theft and wastage associated with holding of each commodity stock for transactions purposes in terms of the commodity itself be t_{ik} (k=1 --- F+H). Then the sum $\sum t_{ik}S_{ik}$ for firms k belonging to industry j when divided by industry j's output will be a coefficient t_{ij} that

represents the annual cost of carrying stock i in industry j for transactions purposes. The gross I-O coefficient matrix for the barter economy will be

$$A_B = A + T = a_{ij} + t_{ij} \qquad i,j=1\text{---}n$$

The net wage income of the households who too carry transaction stocks and incur the associated costs will stand reduced to

$$w_N L^* = wL^* - \sum_i \sum_k t_{ik} p_i \qquad k=1....H$$

Thus a more realistic description of the barter economy than (1), (2), (3) is obtained as follows,

$$P_B = P_B A_B + wL \qquad (1)'$$

$$C_{iB} = \frac{\alpha_i w_N L^*}{P_{iB}} \qquad \sum \alpha_i = 1 \qquad (2)'$$

$$X_B = (I\text{-}A_B)^{-1} C_B \qquad (3)'$$

This system too will give a unique positive equilibrium if $I\text{-}A_B$ fulfils the viability conditions but it will be one which results in higher prices, lower outputs and lower real net income than the system (1), (2), (3) because $A_B > A$ and $w_N < w$ imply $P_B/w > P/w$, $C_B < C$ and $X_B < X$.

If one commodity (or a small set of commodities) exists for which $t_{mk} < t_{ik}$ $(i \neq m)$ then the number of commodity stocks required to be held for transactions throughout the economy would reduce from N (F+H) to F + H, a drastic reduction from a power of 2 to a power of 1 and this commodity could serve as the medium of exchange. Thus the condition for the existence of an essential commodity money can be stated as follows,

$$t_{mk} p_m < \sum\sum t_{ik} p_i \qquad k=1,....F+H$$

In the single commodity money economy the I-O matrix will be $A_M = A +$ column (t_{mi}) and $w_n L^* = wL^* - \sum t_{mk} p_m$. If we reasonably suppose that $A_B > A_M > A$ then we get $C_B < C_M < C$, $P_B/w > P_M/w > P/w$ and $X_B < X_M < X$; the single commodity money economy is more efficient than the realistic barter economy (1)', (2)' and (3)', but less efficient than the ideal barter economy (1), (2), (3) for which $t_{ik} = 0$.

The efficiency of the commodity money economy in which say gold serves as money can be further improved upon. The F+H decentralised hoards that are required to be held for transactions purposes can be centralized into a single hoard in a bank to achieve further economies in the expenses towards storage, security and transport. But even that is not quite efficient because each agent would have to make say two trips to the bank each day, one to withdraw gold at the start of the day for making purchases during the day and another one at the end of the day to deposit his collections, i.e. a total of 2 (F+H) trips, which would entail transport and in-transit security expenses. If the bank issues bearer currency notes against the gold deposits (and undertakes to convert them when required) these expenses are further economized. Even this can be improved upon by issuing cheque books, installing ATM's and by e-banking. At the same time the costs of interregional payments can be economized by individual banks centralizing their hoards into a single central bank which can issue notes of a uniform quality and set up clearing facilities[5]. By this point the monetary technology attains such a high level of efficiency that people don't mind even if government nationalizes the central bank to appropriate its gold reserves and suspends the convertibility of currency notes into gold and itself issues notes. It is this monetary system whose behavior is the principal concern of monetary economics.

4. CURRENCY MONEY

We shall suppose at this stage that banks only administer the system of payments in currency notes (credit money will be the subject of the next section). In performing its function banks will incur expenses which they recover by charging a fee per dollar of deposit for services of safekeeping, withdrawals of cash over the counter or by ATM, issue of cheque books, clearing services, replacement of worn-out notes and accounting. The introduction of a non-commodity money into the price system needs to be carefully done. Money is used but no part of it is used up in production. As Adam Smith taught us a long time ago, "Money is a branch of the general stock of society" but is peculiar in that, "it is neither a material to work upon nor a tool to work with". [Smith (1976), Book II, Chapter II]. Thus only the direct cost associated with holding of

monetary stocks can enter the price equations. The price equations for the economy are as follows,

$$p_c m_i p_i X_i + \sum A_{ji} p_j + wL_i = p_i X_i \quad (5)$$

$$\sum A_{jc} p_j + wL_c = p_c M^* \quad (6)$$

$$\sum m_i p_i X_i + m_H wL^* = M^* \quad (7)$$

In other words the matrix of I-O coefficients in the price system should now be read as $A_C = A + \text{diag}\,(p_c m_i p_i)$ and it should be presumed that $t_{ck}=0$ (notes and deposits entail no physical wastages), $p_c m_i p_i < t_{mi} p_m$ for each industry i and $p_c m_H wL^* < \sum t_{mk} p_m$ k=1---H. Equations (5), (6) require respectively that the sales revenues of industries and banks must cover their costs and equation (7) requires that the market for currency be cleared. The total labour is now $L^*=\sum L_i + L_C$. As regards the demand equations for the goods three specifications suggest themselves. The first is to net out bank service charges from the gross income from wages and then apply Engels' coefficients to their net income

$$F_i = \frac{\propto_i (wL^* - p_c m_H wL^*)}{p_i} + A_{ic} \qquad \sum \propto_i = 1 \quad 8(a)$$

$$X = (I\text{-}A)^{-1} F \quad 8(b)$$

It should be noted that the cost coefficients of banking services appears in the price equations but will not appear in the output equations 8(b). Of course the effect of bank service charges that households pay and which lower their income as well as the commodity input requirements of the banks will affect the solution of X in 8(b). Also the price equations (5) become non-linear since p_c and p_i, both unknowns appear as a product. But this does not pose a problem because an iterative solution is always possible. The system of equations (5) to (8) give an equilibrium solution for n money prices of commodities, n outputs, the bank fee p_c and the money wage rate w provided the terms $p_c m_i$ appearing on the main diagonal of the price system are low enough to ensure overall economic viability. Letting d = diag $(p_c m_i)$ the price equations in (5) may be written in matrix notation as

$$P\,[A+d] + wL = P$$

$$wL = P\,(I - A - d)$$

Multiplying both sides by the market clearing gross output vector from 8(b) gives

$$wLX = P\,[I-A-d]\,(I-A)^{-1}\,F$$
$$= PF - PdX \qquad (9)$$

From 8(a) we know that

$$PF = wL^{*} - p_c m_H wL^{*} + PA^{T}_{C}$$

Multiplying (7) by p_c on both sides and substituting for the second term on the right hand side gives $p_cM^{*} - p_c \sum m_i p_i X_i = p_cM^{*} - PdX$. Further, from equation (6) we know $p_cM^{*} = PA^{T}_{c} + wL_c$ so that

$$PF = wL^{*} - wL_c + PdX.$$

So in view of equation (9) we must have

$$wLX = wL^{*} - wL_c$$
$$\text{i.e. } LX + L_c = L^{*}$$

In short a full employment market clearing equilibrium with positive prices and outputs exists.

Observe also that the price and output systems are not exactly duals of one another as they were in equations (1) and (3) and (1)' and (3)'. Indeed it can be said that the exact duality of the price and output systems is a key feature of barter economies whether ideal or less-than-ideal; monetary economies are distinguished by the fact that their price and output systems are not exact duals of one another. The costs of using money and/or monetary institutions are loaded into the cost-price system but they do not appear in the output system as they do in the case of the barter economies of section 3. It means that the use of the monetary technology for transactions has enabled the economic system to operate its production technology at full efficiency. We may now suppose that $A_B > A_m > A_c = A$ so that $X_B < X_m < X_c < X$, $C_B > C_m > C_c > C$ and $\frac{P_B}{w} > \frac{P_m}{w} > \frac{P_C}{w} > \frac{P}{w}$. The currency money economy delivers greater outputs than the barter economy and the single commodity monetary economy. This conclusion stands in sharp contradiction with neoclassical findings that the introduction of essential

money having positive exchange value results in allocations that are not generally Pareto efficient. [See Starr (2010)]. Neo-Walrasian theory fails to capture these great advantages of monetary exchange over barter because it lacks the framework of cost-based prices and production-based incomes.

A second way to specify the consumer demand equations is to suppose that consumers make the bank service itself an object of conscious consumer choice, that is to say

$$F_i = \frac{\propto_i (wL^* - p_c m_H wL^*)}{p_i} + A_{iC} \quad \text{where i=1.... n, c} \qquad \sum \propto_i = 1$$

8(c)

This does not in general give an equilibrium solution except in the special case $\alpha_c = p_c m_H$.

The third specification that presents itself for consideration is to make currency balances itself an object of conscious consumer choice, i.e. $\alpha_c \, wL^* = \Delta C$ (whether real or monetary balances does not matter). This requires some justification considering that households are already holding balances of $m_H wL^*$ for performing transactions. Therefore the additional balances demanded out of net income must be justified on "precautionary" or "speculative" grounds. So far as a static economy is considered it must be supposed that the grounds are provided by something external say the prospect of a war, of political and industrial unrest or of a drought etc. because internal sources of potential risk such as stock, bond or real estate market collapses are absent. Whatever the reason, if it happens that a fraction of the net income is devoted to an addition to currency balances there will be disequilibrium. At least one of the markets for the commodities or for labour will fail to clear.

There is very little point in arguing that the hoarding cash out of receipts or incomes earned from currently produced outputs amounts to "irrational" behavior and for that reason will never occur. For if this is what people actually do under some circumstances then the reasons for that behavior must be sought(7).

Consider a numerical example that illustrates monetary general equilibrium and disequilibrium in turn. Consider the following data;

$$A = \begin{pmatrix} 0.05 & 0.1 & 0.1 & 0.15 \\ 0.25 & 0.30 & 0.15 & 0.25 \\ 0.2 & 0.25 & 0 & 0.15 \\ 0.1 & 0.05 & 0.075 & 0.1 \end{pmatrix} \quad L=(0.5, 2, 1, 4) \quad m_I=(0.05, 0.01, 0.025, 0.04) \quad m_H = 0.3$$

$A_{ic} = (0.5, 0.1, 0.15, 0.25) \quad L_c = 5$

$L^* = 100 \quad M^* = 100 \quad \alpha_i = 0.25 \ (i=1...4)$

If consumer demand is specified as per equation 8(a) then the equilibrium solution for equations 5-8(a), 8(b) is shown in Table No. 1

Table No. 1

Commodities	Prices	Outputs	
1	8.3898	13.6759	
2	12.93471	18.8187	$LX=L^*=100$
3	6.9724	17.9489	$Y=wL^*=275.46$
4	18.6025	8.1439	$p_c m_H w L^*=7.4874$
5	0.2495	--	
6	2.7546	--	

If demand is specified according to specification 8(c) in which a fraction of household income is devoted to holding additional money balances so that $\alpha_i=0.2$ (i=1....4) and $\alpha_C=0.2$, the disequilibrium solution obtained is shown in Table No. 2. The commodity markets are cleared but there is an excess supply of labour.

Table No. 2

	Price	Output	
p_1	8.6876	11.0482	
p_2	13.3869	15.1403	LX=81.5498=Demand for labour
p_3	7.2178	14.4174	
p_4	19.2576	6.5819	
p_c	0.2583	--	
w	2.8500	--	

This deflationary gap measured in terms of labour is 100-81.5498=18.4502 and in value terms it is \$2.85 x 18.4502 = \$52.5830 represents the excess demand for money and the excess supply of labour. If, on the other hand, we force employment to remain at 100 across industries then there will be an excess supply of one or all of the commodities. It is obviously more reasonable to suppose that the commodity markets are cleared and the labour market fails to clear for the simple reason that suppliers of the commodities would like to cut down their losses and maintain a breakeven by supplying only as much as is demanded, a mechanism that is inapplicable to the labour market. "The first characteristic which tends towards the above conclusion is the fact that money has, both in the long and the short period, a zero, or at any rate a very small, elasticity of production …….. Money, that is to say, cannot be readily produced; - labour cannot be turned at will by entrepreneurs to produce money in increasing quantities as its price rises in terms of the wage unit. In the case of inconvertible managed currency this condition is strictly satisfied". [Chapter 17 p. 230]

The only way in which the disequilibrium can be removed would be for the government to finance a deficit and purchase commodities and/or labour by printing notes. As an example suppose government decides to buy commodities from the four industries. Then government purchases of G_i=0.735 units of each of the four commodities (these are added to the final demand vector F) entailing an expenditure of $\sum p_i G_i$=\$35.6861 brings about the full employment equilibrium shown in table No. 3.

Table No. 3

	Price	Output	
p_1	8.3972	13.3373	
p_2	12.9454	18.7542	
p_3	6.9786	17.3962	$LX+L_c$=100.0077
p_4	18.6189	8.3585	
p_c	0.2498	--	
w	2.7570	--	

It may be observed that the physical multipliers (i.e. increment in industrial output due to additional final demand of 0.735 units due to government purchases) are all greater than 1.

Several alternative mixes of deficit spending may be employed to remove the deflationary gap. For example the government might choose to concentrate the deficit spend on say commodity . In that case the purchase of $G1 = 4.1352$ units requiring deficit spending of \$76.1155 would restore overall equilibrium that is shown in Table No. 4.

Table No. 4

Price	Output	
8.3007	16.8437	
12.7987	18.7835	
6.8991	17.3064	$LX+L_c=100.0077$
18.4067	7.9262	
0.2469	--	
2.7261	--	

5. CREDIT MONEY

In the course of administering the payments mechanism of the society banks discover that the net withdrawals during a period are only a fraction of the total moneys in deposit with them so that if banks can hold a fraction of the deposits as reserves to meet the periodic withdrawals they can lend out the remaining amounts as loans and earn interest income. The balance sheet of the banks is shown in Table No. 5.

Table No. 5
Banks' Balance Sheet

Liabilities		Assets	
Industrial Deposits	M_I	qM^*	Reserves
Household Deposits	M_H	$(1-q)M^*$	Advances
Total Liabilities	$M_I+M_H=M^*$	M^*	Total Assets

As before $M_I = \sum m_i p_i X_i$ and $M_H = m_H w L^*$. If we suppose that only firms take loans the price equations of the economy are

$$\delta k(m_i p_i X_i + \sum A_{ji} p_j + wL_i) + \sum A_{ji} p_j + wL_i = p_i X_i \quad (9)$$

$$\sum A_{jB} p_j + wL_B = k(1-q)M^* \quad (10)$$

$$\delta(\sum m_i p_i X_i + \sum\sum A_{ji} p_{ji} + w\sum L_i) = (1-q)M^* \quad (11)$$

$$\sum m_i p_i X_i + m_H w L^* = M^* \quad (12)$$

$$F_i = \frac{\propto_i w L^*}{p_i} + A_{iB} \quad (13)$$

$$X = (I-A)^{-1} F \quad (14)$$

As before $\delta k m_i$ should be small enough to ensure overall economic viability.

It is supposed that the banks because they compete for deposits do not now levy services charges on depositors but recover their costs from the interest payments of borrowers[6]. Also since the deposits are demand deposits no interest is paid for them. Equations 9-14 contain 2n+3 independent equations to determine as many unknowns, i.e. n money prices of commodities, n outputs, 1 money wage rate, 1 interest rate (k) and 1 debt-equity ratio (δ). Equations (9) require that sales revenues of industries should cover costs, equation (19) requires the interest revenue of banks to cover their costs, equation (11) clears the deposit and loan markets, equation (12) is the quantity equation that clears the money market and equations (13) and (14) clear the commodity markets. To prove the existence of a full employment market clearing equilibrium we proceed as follows. Let $d = \text{diag}(\delta k m_i)$. Then the price equations (9) in matrix notation are

$$Pd + (1+\delta k)\, PA + (1+\delta k) wL = P$$

so that

$$(1+\delta k) wLX = P[I-A-\delta kA - d]\,(I-A)^{-1} F$$

$$= PF - \delta kPAX - PdX \quad (15)$$

From equation (13)

$$PF = wL^* + PA^T_B$$

$$= wL^* + k(1-q)M^* - wL_B$$

$$= wL^* - wL_B + \delta k[\sum m_i p_i X_i + PAX + wLX]$$
$$= wL^* - wL_B + PdX + \delta kPAX + \delta kwLX$$

In view of (15) however

$$(1+\delta k)\, wLX = wL^* - wL_B + \delta kwLX$$

so that

$$LX + L_B = L^*$$

If we let $\alpha_i = 0.25 \quad i=1...4$, for the numerical example above (all data remain the same except that $M^*=100$ now stands for deposits and $q=0.1$ is the fraction of bank deposits held as reserves) an equilibrium is obtained. This is shown in Table No. 6.

Table No. 6

	Price	Output	
p_1	9.0609	13.5735	
p_2	13.8437	18.7658	$LX = 100$
p_3	7.4430	17.9130	
p_4	19.5019	8.1921	
w	2.7176	--	
k	0.2832	--	
δ	0.1345	--	

If, however $\alpha_i = 0.2 \; \forall i=1...4$, and $\alpha_D = 0.2$ (a fraction of household income is devoted to holding additional deposits) the resulting disequilibrium is shown in Table No. 7.

Table No. 7

	Price	Output	
p_1	9.6792	10.7168	
p_2	14.7023	14.7825	LX=80
p_3	7.9092	14.0856	
p_4	20.6279	6.4977	
w	2.8164	--	
k	0.2970	--	
δ	0.1620	--	

In parallel with the earlier case of currency money Walras's law holds – the excess demand for money balances $\alpha_D wL^*$ equals the excess supply of labour $w(L^* - L_d)$ so that $\alpha_D L^* = L^* - L_d = 20$. The original insight of Keynes (1936) bears repetition, "Unemployment develops, that is to say, because people want the moon; men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off". [Chapter 17 p. 235]

This deflationary gap can be eliminated by deficit financing of \$54.44 to purchase G_i=1.09 units each of the four commodities and restore the full employment equilibrium shown in Table No. 8. It will be observed that in this case too the physical multipliers, that is to say, increments in gross outputs due to additional purchases of 1.09 units of the commodities, are all greater than 1.09 units.

Table No. 8

	Price	Output	
p_1	9.0810	13.0905	
p_2	13.8719	18.6800	LX=100.009
p_3	7.4583	17.1000	
p_4	19.5391	8.5009	
w	2.7210	--	
k	0.2837	--	
δ	0.1354	--	

Even though in the foregoing the disequilibrium consequences of an excess demand for money have been illustrated with reference to households it is important to note that this will be true of excess demand for money from industries too. This excess demand must be necessarily shown in terms of additional money balances $m_{ie}p_iX_i$ on the left hand sides of the price equations with no corresponding term on the right hand side. The resulting net national income equation will be

$$\sum m_{ie}p_i X_i + wL^* = \sum p_i X_i - \sum\sum A_{ji}p_j - \sum A_{jB}p_j + p_B w_H wL^* \qquad (15)$$

In equation (15) the firms are using a part of their sales revenue to hoard cash. In this case even if there is no households' excess demand for money there will be a deflationary gap. Alternatively, the deflationary gap may occur because firms disinvest

their commodity stocks say on fears of political or industrial unrest and hold the proceeds as deposits but refrain from taking additional loans so that the proceeds are carried by banks as excess reserves.

A notable feature of the systems of equations (5) to (8) for currency money and (9) to (14) for credit money is that they conform to Walras' Law and explicitly include the quantity theory of money as an assumption (equations 7 and 12). That has not prevented the systems from generating the disequilibria; indeed, the disequilibria are such that the excess demand for money equals the excess supply of labour with all other markets cleared.

A disequilibrium with an inflationary gap is seen to arise if previously hoarded cash is brought into play into the market for currently produced outputs whether by industries or households. In that case $LX > L^*$ and the disequilibrium can be corrected by means of a surplus budget.

6. CONCLUDING REMARKS

The principal objectives of this paper have, I think, been achieved. Firstly, it has been shown that the use of money promotes economic efficiency which allows the economic system to deliver larger outputs at lower real prices which is what makes money essential. Secondly, monetary general equilibrium has been shown to exist if and only if no part of the proceeds or income from the sale of currently produced output is hoarded in the form of money. Failing this condition, a general disequilibrium will prevail which no amount of flexibility in prices, wages, interest rates and outputs (in this paper we have supposed all of them to be fully flexible) would be able to correct. Fiscal solutions for correcting the disequilibrium have been illustrated. As the discussion and the numerical illustrations clearly demonstrate money is non-neutral.

Of course there is absolutely nothing in all this that can be called new. The subject of the essentiality of money has been talked about over and over again in elementary economics classrooms in the context of the disadvantages of barter exchange for centuries and the subject of its non-neutrality has been talked about in the context of the Keynesian revolution during the last eight decades. And of course it was

all there in Keynes right from the beginning. A quote from Keynes (1936) would be appropriate,

"…….. money is a bottomless sink for purchasing power, since there is no value for it at which demand is diverted from it so as to slop over into a demand for other things".

[Chapter 17 p. 231]

This is exactly what lies at the heart of the disequilibria illustrated in sections 4 and 5 of this paper. In closing this paper it may be mentioned that the setting of a static economy in which monetary theory is usually written is a somewhat restrictive setting in the sense that the activities of saving and investment can be considered at best as sporadic incidents. The appropriate setting ought to be a growing economy in which saving and investment are continuing activities. It is in this setting that the divergence between saving which primarily takes the form of additions to financial assets including money and investment which consists of additions to real assets can be properly investigated. Also instead of a uniform single rate the entire term structure of interest rates must find a place in a more complete monetary-financial model. Only then can other instruments of fiscal policy such as public debt as well as the operation of monetary policy instruments can be brought into play. It will also be worthwhile to examine the issue of whether an excess demand for existing assets (land, bonds, stocks, gold, houses, etc.) financed from receipts of currently produced outputs will also result in deflationary gaps. Some or all of these questions this shall be hopefully attempted in Part II of this paper.

NOTES

1. The post Keynesian literature however, has continued its study of the neutrality questions and has insisted on demonstrating the non-neutrality of money. See Chick (1973, 1978), Davidson (1978), Lavoie (1984), Kohn (1986), Rogers (1989) among several others for some post Keynesian viewpoints.

2. Attempts to treat money in the context of I-O models are by no means new; Leontief and Brody (1993), Brody (2000), Tsujimura and Mizoshita (2003) among others have applied I-O techniques to monetary and financial flows. However this literature has not concerned itself with the existence of monetary general equilibrium.

3. None of the conclusions of the paper are materially affected if use is made of the theoretically more general and empirically more effective demand systems such as the Linear Expenditure System or the Almost Ideal Demand System. Purely qualitative demand functions of the type $Q_i = f(p_i, w_i)$ that are customary in neoclassical theory have been avoided for two reasons, (a) that demand functions should be "income-constrained" for any Keynesian type of investigation [Clower (1967)] and (b) that the customary purely qualitative functions are not amenable to a numerical understanding of disequilibria.

4. Of course perishables and/or services will not qualify to serve as means of payment. Only say D out of N goods which are durable will qualify. But this is no way affects the generality of the argument.

5. I am not strictly adhering to the actual chronology of the development of money and monetary institutions.

6. Alas, modern bankers faced with serious NPA's have reverted to the olden days and have begun charging their depositors for every service they offer! And for some time now there has also been talk of charging *negative* interest rates on deposits!

7. Keynes (1937), p. 216) argued that uncertainty about the future was the main cause for holding money. "……. partly on reasonable and partly on instinctive grounds our desire to hold money as a store of wealth is a barometer of the degree of our distrust of our own calculations and conventions concerning the future….. The possession of actual money lulls our disquietude ……..".

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