

**AN INTRODUCTION TO THE  
THEORY OF STATISTICS.**

## OTHER BOOKS OF INTEREST

### BIOMATHEMATICS.

Principles of Mathematics for Students of Biological Science.

By **W. M. FELDMAN, M.D., B.S (Lond.), F.R.S.(Edin.), F.R.C.S.**

CONTENTS.—Introductory—Logarithms—A Few Points in Algebra—A Few Points in Elementary Trigonometry—A Few Points in Elementary Mensuration—Series—Simple and Compound Interests—Laws in Nature—Functions and Their Graphical Representation—Nomography—Differentials and Differential Coefficients—Maxima and Minima—Estimation of Errors of Observation—Successive Differentiation—Integral Calculus—Biochemical Applications of Integration—Thermodynamic Considerations and Their Biological Applications—Use of Integral Calculus in Animal Mechanics—Use of the Integral Calculus for Determining Lengths, Areas, and Volumes, also Centres of Gravity and Moments of Inertia—Special Methods of Integration—Differential Equations—Fourier's Series—Mathematical Analysis Applied to the Co-ordination of Experimental Results—Biometry—APPENDIX—INDEX.

**Second Edition.** Enlarged and Re-set. In Large Crown 8vo. Cloth, Pp. i-xviii+480. With many worked numerical examples, and 164 Diagrams 25s.

"An excellent introduction, and worthy of great praise."—*Edin. Med. Jour.*

### MEDICAL JURISPRUDENCE & TOXICOLOGY.

By **WILLIAM A. BREND, M.A.Cantab., M.D., B.Sc.(Lond.).**

CONTENTS.—Part I: Medical Jurisprudence—Introduction—Identification: of the Living; of the Dead—The Medico-Legal Relations of Death—Signs of Death—Death from Causes usually leading to Asphyxia—Death by Burning, Sunstroke, and Electricity—Death from Cold, and Death from Starvation—Wounds and Mechanical Injuries—Matters involving the Sexual Functions—Pregnancy and Legitimacy—Criminal Abortion—Birth: Infanticide—The Forms of Insanity—Legal Relationships of Insanity and Other Abnormal States of Mind—Medical Examinations for Miscellaneous Purposes—Medical Privileges and Obligations—Evidence and Procedure as regards the Medical Man. Part II: Toxicology—General Facts with Regard to Poisons—Corrosive Poisons—Irritant Poisons (Metals and Non-Metals)—Poisons of Animal Origin and Poisoning by Food—INDEX.

**Seventh Edition, Revised.** Pocket Size. Pp. i-xiii+325.

10s. 6d.

"A trustworthy work . . . especially suitable for students and practitioners."  
—*Lancet.*

**RADIO FREQUENCY MEASUREMENTS.** By Moullin. Second Edition. 487 pp. 289 Illustrations. 34s.

**STUDIES IN MOLECULAR FORCE.** By Chatley. 118 pp. 7s. 6d.

**THE CALCULUS FOR ENGINEERS AND PHYSICISTS.** By Smith. Second Edition. 207 pp. Diagrams and Plate. 9s.

**THE POLYNUCLEAR COUNT.** By Cooke and Ponder. 80 pp. Illus. 6s.

**ELEMENTARY HÆMATOLOGY.** By Cooke. 100 pp. 54 Illus. 7s. 6d.

**THE FINANCE OF LOCAL GOVERNMENT AUTHORITIES.** By Burton. 289 pp. 10s.

**MECHANISED ACCOUNTANCY.** By Curtis. 143 pp. 76 Illus. 15s.

*Prices net, Postage Extra*

**CHARLES GRIFFIN & CO., LTD.**

*Technical Publishers since 1820*

**2 DRURY LANE, LONDON, W.C.2**

# AN INTRODUCTION TO THE THEORY OF STATISTICS

BY

G. UDNY YULE, C.B.E., M.A., F.R.S.,

FELLOW OF ST JOHN'S COLLEGE, AND FORMERLY READER IN  
STATISTICS, CAMBRIDGE; HONORARY VICE-PRESIDENT  
OF THE ROYAL STATISTICAL SOCIETY

AND

M. G. KENDALL, M.A.,

FORMERLY MATHEMATICAL SCHOLAR OF ST JOHN'S COLLEGE, CAMBRIDGE;  
FELLOW OF THE ROYAL STATISTICAL SOCIETY.

With 55 Diagrams and 4 Folding Plates



ELEVENTH EDITION, REVISED THROUGHOUT AND RE-SET.

LONDON:  
CHARLES GRIFFIN & COMPANY, LIMITED,  
42 DRURY LANE, W.C. 2.

1937,

[All Rights Reserved.]

Printed in Great Britain by  
Nisbet & Co., Ltd., Edinburgh.

## ABRIDGED PREFACE TO THE ~~FIRST~~ EDITION.

---

THE following chapters are based on the courses of instruction given during my tenure of the Newmarch Lectureship in Statistics at University College, London, in the sessions 1902-1909. The variety of illustrations and examples has, however, been increased to render the book more suitable for the use of biologists and others besides those interested in economic and vital statistics, and some of the more difficult parts of the subject have been treated in greater detail than was possible in a sessional course of some thirty lectures. To enable the student to proceed further with the subject, fairly detailed lists of references to the original memoirs have been given: exercises have also been added for the benefit, more especially, of the student who is working without the assistance of a teacher.

The volume represents an attempt to work out a systematic introductory course on statistical methods—the methods available for discussing, as distinct from collecting, statistical data—suited to those who possess only a limited knowledge of mathematics: an acquaintance with algebra up to the binomial theorem, together with such elements of co-ordinate geometry as are now generally included therewith, is all that is assumed. I hope that it may prove of some service to the students of the diverse sciences in which statistical methods are now employed.

G. U. Y.

*December 1910.*

## PREFACE TO THE ELEVENTH EDITION.

---

THE "*Introduction to the Theory of Statistics*" having completed five-and-twenty years of life, it was decided that the time had come when a complete revision should be made. This, I felt, I could not personally undertake; it was clearly a task for a younger man, more in touch with recent literature and less affected by the prejudices of age in favour of the old and the familiar.

Mr Kendall undertook the task not merely with willingness but with enthusiasm. I read his typescript, but to him is primarily and almost solely due the credit for suggesting the general lines of the revision, and for carrying out the agreed suggestions: the only new chapter for which I am directly responsible is Chapter 24 on Interpolation and Graduation based on a few lectures sometimes included in former courses.

I hope that in its new form the book may long continue to be of service to further generations of students.

G. UDNY YULE.

CAMBRIDGE,  
July 1937.

---

IN the revision undertaken for this edition, apart from some substitution of new numerical illustrations for old, very little of the material appearing in earlier editions has been deleted. A few minor alterations have been made—the matter formerly included in supplements has been incorporated in the text, and there has been some rearrangement—but the major changes are almost entirely in the form of additions. Of these, the most important are several new chapters on Sampling, including an introductory chapter on Small Samples. Chapters have also been added on Moments and Measures of Skewness and Kurtosis, and on Simple Curve Fitting by the Method of Least Squares. Mr Yule has contributed a new chapter on Interpolation and Graduation. For the first time Tables of the various functions commonly required in statistical work have been assembled at the end of the book. Throughout the preparation of this new material I have had the benefit of Mr Yule's encouragement, criticism and advice.

The complete revision has presented the opportunity of issuing the book in new form, and it is hoped that the larger page and type will

found an improvement. A more distinctive system of paragraph numbering and paragraph headings has been introduced. Some further Exercises have also been added.

Notwithstanding the mathematical character of recent developments in statistical theory, an attempt has been made to keep within the limits laid down by Mr Yule for earlier editions of this book in regard to the knowledge of mathematics required by its readers. In one or two places it has been necessary to introduce the notation of the integral calculus, but this has been accompanied by explanations in terms of geometrical ideas.

It is a pleasure to record Mr Yule's and my indebtedness to "Student" and the proprietors of *Metron* for permission to reproduce a slightly condensed version of the former's tables of the  $t$ -integral; and to R. A. Fisher and Messrs Oliver & Boyd for permission to reproduce the tables of the significance points of the  $z$ -integral from Professor Fisher's *Statistical Methods for Research Workers*. The tables for the 0.1 per cent. level of  $z$  are due to W. E. Deming, Lola S. Deming and C. G. Colcord, who have also very generously given their consent to the reproduction.

I shall feel indebted to any reader who directs my attention to possible errors, omissions, ambiguities or obscurities.

M. G. K.

LONDON,  
July 1937.

# CONTENTS.

	PAGES
NOTES ON NOTATION AND ON TABLES FOR FACILITATING STATISTICAL WORK . . . . .	xi-xiii
INTRODUCTION . . . . .	PAGE 1
<b>CHAP.</b> 1. THEORY OF ATTRIBUTES—NOTATION AND TERMINOLOGY . . . . .	11
2. CONSISTENCE OF DATA . . . . .	25
3. ASSOCIATION OF ATTRIBUTES . . . . .	34
4. PARTIAL ASSOCIATION . . . . .	50
5. MANIFOLD CLASSIFICATION . . . . .	65
6. FREQUENCY-DISTRIBUTIONS . . . . .	82
7. AVERAGES AND OTHER MEASURES OF LOCATION . . . . .	112
8. MEASURES OF DISPERSION . . . . .	134
9. MOMENTS AND MEASURES OF SKEWNESS AND KURTOSIS . . . . .	154
10. THREE IMPORTANT THEORETICAL DISTRIBUTIONS—THE BINOMIAL, THE NORMAL AND THE POISSON . . . . .	169
11. CORRELATION . . . . .	196
12. NORMAL CORRELATION . . . . .	227
13. FURTHER THEORY OF CORRELATION . . . . .	241
14. PARTIAL CORRELATION . . . . .	261
15. CORRELATION: ILLUSTRATIONS AND PRACTICAL METHODS . . . . .	288
16. MISCELLANEOUS THEOREMS INVOLVING THE USE OF THE CORRELATION COEFFICIENT . . . . .	297
17. SIMPLE CURVE FITTING . . . . .	309
18. PRELIMINARY NOTIONS ON SAMPLING . . . . .	332
19. THE SAMPLING OF ATTRIBUTES—LARGE SAMPLES . . . . .	350
20. THE SAMPLING OF VARIABLES—LARGE SAMPLES . . . . .	373
21. THE SAMPLING OF VARIABLES—LARGE SAMPLES, continued . . . . .	394
22. THE $\chi^2$ DISTRIBUTION . . . . .	418
23. THE SAMPLING OF VARIABLES—SMALL SAMPLES . . . . .	434
24. INTERPOLATION AND GRADUATION . . . . .	462
REFERENCES . . . . .	495

## APPENDIX TABLES, ETC.

	PAGE
1. ORDINATES OF THE NORMAL CURVE FOR GIVEN VALUES OF THE DEVIATE . . . . .	531
2. AREAS OF THE NORMAL CURVE LYING TO THE LEFT OF THE ORDINATES AT GIVEN DEVIATES . . . . .	532



TABLE	PAGES
3. PROBABILITY THAT A NORMAL DEVIATE IS GREATER IN ABSOLUTE VALUE THAN A GIVEN VALUE . . . . .	533
4. VALUES OF THE $\chi^2$ INTEGRAL FOR ONE DEGREE OF FREEDOM—	
A, FOR VALUES OF $\chi^2$ FROM 0 TO 1 . . . . .	534
B, FOR VALUES OF $\chi^2$ FROM 1 TO 10 . . . . .	535
5. AREAS OF THE <i>t</i> -CURVES LYING TO THE LEFT OF THE ORDINATES AT GIVEN DEVIATES . . . . .	536-7
6. SIGNIFICANCE POINTS OF THE $z$ INTEGRAL—	
A, FOR THE 5 PER CENT. LEVEL . . . . .	538
B, FOR THE 1 PER CENT. LEVEL . . . . .	539
C, FOR THE 0.1 PER CENT. LEVEL . . . . .	540
DIAGRAM GIVING THE CONTOUR LINES OF THE SURFACE $P = F(v, \chi^2)$	<i>Facing</i> 540
ANSWERS TO EXERCISES . . . . .	541
INDEX . . . . .	553

LIST OF FOLDING PLATES.

Fig. 11.2. FREQUENCY-SURFACE FOR THE RATE OF DISCOUNT AND RATIO OF RESERVES TO DEPOSITS IN AMERICAN BANKS . . . . .	<i>Facing p.</i> 204
Fig. 11.3. FREQUENCY-SURFACE FOR STATURE OF FATHER AND STATURE OF SON . . . . .	,, 204
Table 11.9. CORRELATION BETWEEN LENGTH OF MOTHER-FROND AND LENGTH OF DAUGHTER-FROND IN <i>Lemna minor</i> . . . . .	,, 218
Fig. A1. CONTOUR LINES OF THE SURFACE $P = F(v, \chi^2)$ . . . . .	,, 540

## NOTES ON NOTATION AND ON TABLES FOR FACILITATING STATISTICAL WORK.

### A. Notation.

The reader is assumed to be familiar with the commoner mathematical signs, *e.g.* those for addition and multiplication. We shall also employ the following symbols, all of which are in general use:—

#### The Factorial Sign.

The symbol  $n!$ , read “factorial  $n$ ,” means the number

$$1 \times 2 \times 3 \times \dots \times (n-2) \times (n-1) \times n$$

Factorial  $n$  is by some writers expressed by the symbol  $[n]$ , but this notation appears to be falling out of use in favour of  $n!$ , probably owing to the greater ease with which the latter form can be printed and type-written.

#### The Combinatorial Sign.

The symbol  ${}^n C_r$  means the number of ways in which  $r$  things can be chosen from  $n$  things, *e.g.*  ${}^{52} C_{13}$  is the number of ways in which a hand of cards can be dealt from an ordinary pack of 52 cards.

In most text-books on Algebra it is shown that

$${}^n C_r = \frac{n!}{r!(n-r)!} = {}^n C_{(n-r)}$$

#### The Summation Sign.

The sum of  $n$  numbers  $x_1, x_2, \dots, x_n$  is written  $\sum_{r=1}^{r=n} (x_r)$ , read “sum  $x_r$  from one to  $n$ ,” *i.e.*

$$\sum_{r=1}^{r=n} (x_r) = x_1 + x_2 + x_3 + \dots + x_{(n-1)} + x_n$$

Where no ambiguity is likely to arise, the suffix  $r$  and the limits written above and below  $S$  are omitted, *e.g.* the above sum would be written simply  $S(x)$ , it being understood from the context that the summation extends over the  $n$  values.

Many writers use the Greek letter  $\Sigma$  instead of  $S$ .

#### The Greek Alphabet.

As the letters of the Greek alphabet will often be used as symbols, we give for convenience the names of those letters.

Small Letter.	Capital Letter.	Name.	Small Letter.	Capital Letter.	Name.
$\alpha$	A	alpha	$\nu$	N	nu
$\beta$	B	beta	$\xi$	$\Xi$	xi
$\gamma$	$\Gamma$	gamma	$\omicron$	O	omicron
$\delta$	$\Delta$	delta	$\pi$	$\Pi$	pi
$\epsilon$	E	epsilon	$\rho$	P	rho
$\zeta$	Z	zeta	$\sigma, \varsigma$	$\Sigma$	sigma
$\eta$	H	eta	$\tau$	T	tau
$\theta$	$\Theta$	theta	$\upsilon$	Y	upsilon
$\iota$	I	iota	$\phi$	$\Phi$	phi
$\kappa$	K	kappa	$\chi$	X	chi ( <i>pron. ki</i> )
$\lambda$	$\Lambda$	lambda	$\psi$	$\Psi$	psi
$\mu$	M	mu	$\omega$	$\Omega$	omega

### B. Calculating Tables.

For heavy arithmetical work a calculating machine is invaluable; but owing to their cost machines are, as a rule, beyond the reach of the student.

For a great deal of simple work, especially work not intended for publication, the student will find a slide rule exceedingly useful: particulars and prices will be found in any instrument-maker's catalogue. For greater exactness in multiplying or dividing, logarithms are almost essential.

If it is desired to avoid logarithms, use may be made of extended multiplication tables. There are a great many of these and some references to different forms are given in the list on pages 524-525.

In addition to general arithmetical tables of this kind, the student will derive invaluable aid from Barlow's "*Tables of Squares, Cubes, Square-roots, Cube-roots, and Reciprocals of All Integral Numbers up to 10,000*" (E. & F. N. Spon, London and New York, price 7s. 6d.), which are useful over a wide range of statistical work.

### C. Special Tables of Functions Useful in Statistical Work.

The tables and diagram at the end of this book will cover most of the student's ordinary requirements. Other tables appear in the works cited on page 525. The more advanced student will find it useful to have "*Tables for Statisticians and Biometricians*" (Cambridge University Press, Part 1, price 15s., and Part 2, price 30s.)—particularly Part 1. Research workers will wish to have the tables appended to R. A. Fisher's "*Statistical Methods for Research Workers*," 6th ed. (Oliver & Boyd, price 15s.).

### D. References to the Text.

Each section in the book is distinguished by a number in heavy type consisting of the number of the chapter in which the section occurs prefixed to the number of the section in that chapter and separated from it by a period; e.g. **7.13** means the Thirteenth Section of Chapter 7, and **10.1** refers to the First Section of Chapter 10. The Introduction,

which precedes Chapter 1, is for this purpose regarded as Chapter 0, *e.g.* 0.26 refers to the Twenty-sixth Section of the Introduction. References to sections are given simply by the number of the sections, *e.g.* "We saw in 8.3" means "We saw in the Third Section of Chapter 8."

Similarly, equations, tables, examples, exercises and diagrams are distinguished first of all with the number of the chapter in which they occur and then, separated by a period, with their serial number within the chapter, *e.g.* "Table 6.7" refers to the Seventh Table in Chapter 6, and "Equation (17.8)" refers to the Eighth Equation of Chapter 17. These figures are in ordinary type.

This simple notation saves a good deal of unnecessary wording. To facilitate quickness of reference we sometimes give pages as well.

A distinction is drawn between *examples*, which are given in the text for purposes of illustration, and *exercises*, which are set at the end of the chapter for the student to work out for himself.

## INDEX.

[The references are to pages. The subject-matter of the Exercises given at the ends of Chapters has been indexed only when such Exercises (or the Answers thereto) give constants for statistical tables in the text, or theoretical results of general interest; in all such cases the number of the Exercise cited is given. In the case of Authors' names, citations in the text are given first, followed by citations of the Authors' papers or books in the list of References. References to Greek letters follow the references under Roman letters.]

- ABILITY, General, refs., 513.  
 Absolute measures of dispersion, 149.  
 Accidents, Deaths from (Poisson distribution), 191.  
 —, Frequency-distributions, refs., 506, 508.  
 Achenwall, Gottfried, *Abriss der Staatswissenschaft*, footnote, 5.  
 Additive property of  $\chi^2$ , 426-427.  
 Adyanthaya, N. K., refs., Sampling, 523.  
 Ages at death from scarlet fever (Table 6.11), 100; (fig. 6.11), 101.  
 — of cows correlated with milk-yield; see Milk-yield.  
 — of husband and wife (Table 11.2), 198; constants, 220-221; correlation ratios (Ex. 13.2), 259.  
 Aggregate of classes, 14.  
 Agricultural labourers' earnings; see Earnings; minimum wage-rates, 137; calculation of mean and standard deviation, 136-138; of median and mean deviation, 145-146; of quartiles, 147.  
 Agricultural Market Report, data cited from (Table 11.7), 203.  
 Airy, Sir G. B., Use of term "error of mean square," 144.  
 Aitken, A. C., refs., Applications of generating functions to normal frequency, 505; fitting polynomials, 514, 515.  
 Allan, F. E., refs., Fitting polynomials, 515.  
 Ammon, O., Hair- and eye-colour data cited from (Table 5.2), 66.  
 Analysis of variance, 444-449; use in testing significance of correlation ratios, 453-455; of linearity of regression, 455-456; of multiple correlation coefficient, 456-458.  
 Analysis Situs, refs., Hotelling, 512.  
 Anderson, O., refs., *Einführung in die mathematische Statistik*, 496; *Korrelationsrechnung*, 512; correlation, 512.  
 Animal feeding-stuffs, Index numbers of prices of, correlated with price-index of home-grown oats (Table 11.7), 203; 215-218.  
 Annual value of estates in 1715 (Table 6.12), 105; (fig. 6.13), 103.  
 Approximations in the theory of large samples, 379-380.  
 Arithmetic mean; see Mean, Arithmetic.  
 Array, Def., 196; type of, 196; standard deviation of, 206, 214, 242, 266-268; homo- and hetero-scedasticity, footnote, 214; in normal correlation, 230, 232, 284.  
 Association—generally, 34-64; def., 37; degrees of, 38; testing by comparison of percentages, 39-43; constancy of difference from independence values for the second-order frequencies, 43; coefficients of, 44-45; illusory or misleading, 57-58; total possible number of associations for  $n$  attributes, 55-56; case of complete independence, 60-62; use of ordinary correlation coefficient as measure of association, 252-253; tetrachoric  $r$  as coefficient of association, 251-252, 253; refs., 499-500, 510.  
 Association, Partial—generally, 50-64; total and partial, def., 50-51; arithmetical treatment, 52-55; number of partial associations for  $n$  attributes, 55-56; testing, in ignorance of third-order frequencies, 58-60; refs., 500.  
 —, Examples: inoculation against cholera, 40, 42-43; deaths and occupations, 59-60; deaf-mutism and imbecility, 40-41; eye-colour of father and son, 41; eye-colour of grandparent, parent and offspring, 53-55, 60; colour and prickliness of *Datura* fruits, 44; defects in school-children, 52-53.  
 Asymmetrical frequency-distributions, 94-101; relative positions of mean, median and mode in, 125; diagram, 118; see also Frequency-distributions; Skewness.

- Attributes—theory of, generally, 11-81; def., 11; numerically defined, 77-78; notation, 12-14; positive and negative, 13; order and aggregate of classes, 14; ultimate classes, 15-16; positive classes, 17; consistence of class-frequencies, 26-31 (*see also* Consistence); association of, 34-49 (*see also* Association); sampling of, 350-372 (*see also* Sampling of attributes).
- Australian marriages, Distribution of, 96; (fig. 6.8), 97; calculation of mean and standard deviation, 140-141, 142; of third and fourth moments, 158-159, 160; of  $\beta_1$  and  $\beta_2$ , 161; median and quartiles given, 164; calculation of skewness, 164; of kurtosis, 165; standard error of mean (Ex. 20.7), 392; of median and quartiles (Exs. 20.6 and 20.8), 392; of standard deviation, 401; correlation between errors in mean and standard deviation (Ex. 21.5), 412.
- Averages—generally, 112-114; def., 112; desirable properties of, 113-114; forms of, 114; average in sense of arithmetic mean, 114; refs., 501-502. *See also* Mean, Median, Mode.
- Axes, Principal, in correlation, 231-232; in fitting straight lines to data, footnote, 314.
- BACHELIER, L., refs., *Calcul des probabilités*, 495; *Le jeu, la chance et le hasard*, 495.
- Baker, G. A., refs., Sampling of variables, 517, 521; of correlation coefficient, 521.
- Barlow, P., Tables of squares, etc., 71; refs., 524.
- Barometer heights (Table 6.10), 99; (fig. 6.10), 99; means, medians and modes of, 125; modes of, 488.
- Bartlett, M. S., refs., Sampling (*under* Wishart), 520.
- Bateman, H., refs., Poisson distribution (*under* Rutherford), 506.
- Baten, W. D., refs., Moments, 504, 509; frequency-distributions, 507.
- Bateson, W., Data cited from, 44.
- Bayes, T., refs., Doctrine of chances, 521.
- Becker, R., refs., *Anwendung der math. Statistik auf Probleme der Massenfabrikation*, 497.
- Beetles (*Chrysomelidae*), Sizes of genera (Table 6.13), 106.
- Benini, R., refs., *Principi di Statistica Metodologica*, 526.
- Bennett, T. L., refs., Cost of living, 503.
- Berkson, J., refs., Bayes' theorem, 521.
- Bernoulli, James, Binomial distribution, 169; refs., *Ars Conjectandi*, 505.
- Bertillon, J., refs., *Cours élémentaire de statistique*, 499.
- Bertrand, J. L. F., Quotation on chance, 339; refs., *Calcul des probabilités*, 495.
- "Best fit," of regression lines and polynomials, as given by method of least squares, 209-210, 262-264, 311, 313-314.
- Beta-function, 444; tables, refs., 525.
- Bias in sampling, 336, 337-339, 346-347; human bias, 337-339.
- in scale reading (Table 6.4), 86.
- Bielfeld, Baron J. F. von, Use of word "statistics," 4.
- Binomial distribution, 169-180; genesis of, in numbers of trials of events, 169-170; calculated series for certain values of  $p$  and  $n$  (Tables 10.1 and 10.2), 172; general form of, 171-173; mean and standard deviation of, 173-174; third and fourth moments of, 174;  $\beta$ -coefficients of, 174-175 (Tables 10.3-10.5); mechanical representation of, 175-176; deduction of normal curve from, 177-180; of Poisson distribution from, 187-189; in sampling of attributes, 351, *see* Sampling of attributes; refs., 505-508.
- Birge, R. T., refs., Fitting polynomials, 515.
- Birth-rate, Data on (Table 6.1), 83; standardisation of, 306; refs., 514.
- Bispham, J. W., refs., Sampling of partial correlations, 517.
- Bivariate distributions, 196; normal surface, 227-228.
- Blackman, V. H., quoting data of Ashby and Oxley on duckweed (Table 17.3), 317.
- Blakeman, J., refs., Tests for linearity of regression, 514, 517; probable error of contingency coefficient, 517.
- Boldrin, M., refs., Variation, 527; *Statistica*, 526.
- Boole, G., refs., *Laws of Thought*, 499.
- Booth, Charles, on pauperism, 289-290.
- Bortkiewicz, L. von, Data of deaths from kicks by a horse, as Poisson distribution, 191; refs., Poisson distribution, 506; sampling, 517.
- Bowley, A. L., refs., Cost of living, 503; index-numbers, 503; *Prices and Wages*, 503; sampling methods, 516; effect of errors on an average, 520; test of correspondence between statistical grouping and formulæ, 520; Edgeworth's contributions to mathematical statistics, 521.
- Bravais, A., refs., Correlation, 509.
- Breaking-up a group, in interpolation, 477-479.
- British Association, Data cited from, Stature (Table 6.7), 94; weight (Ex. 6.6), 110-111; *see* Stature; Weight; refs., Reports on Index-numbers, 503; mathematical tables, 525.
- Brown, J. W., refs., Index-correlations, 511, 513.

- Brown, W., refs., Effect of experimental errors on the correlation coefficient, 513; *The Essentials of Mental Measurement*, 496.
- Brownlee, J., refs., Frequency-curves (epidemiology and random migration), 508.
- Bruns, H., refs., *Wahrscheinlichkeitsrechnung und Kollektivmasslehre*, 495.
- Brunt, D., refs., *The Combination of Observations*, 496.
- Burnside, W., refs., *Theory of Probability*, 495.
- CAMBRIDGESHIRE, Mortality in, 468.
- Camp, B. H., refs., Normal hypothesis, 505; integrals for point binomial and hypergeometric series, 507; correlation, 511; Tchebycheff's inequality, 521; sampling, 521.
- Cantelli, F., refs., Interpolation, 526; probability, 527; variation, 527.
- Cards, Punched, for recording of data, 76-77; for sampling, 340.
- Carroll, Lewis (pseudonym), Ex. 1.10 cited from, 24.
- Carver, H. C., refs., Sampling, 518.
- Castellano, V., refs., Variation and concentration, 527.
- Cause and effect, 2-3.
- Cave, Beatrice M., refs., Correlation, 512.
- Cave-Browne-Cave, F. E., refs., Correlation, 512.
- Cells, in  $\chi^2$  test, 413-414.
- Census (England and Wales), Tabulation of infirmities in older, 22; data as to infirmities cited from, 40; classification of occupations, as example of a heterogeneous classification, 75; data as to deficiency in room space, quoted from Housing Report, 77; classification of ages, 86; data as to number of males cited from, 481; refs., 501.
- Chance, in sense of complex causation, 38; of success or failure of an event, 169-170, 350; in definition of "randomness," 336.
- Chances, Small, 191; see Poisson distribution.
- Charlier, C. V. L., Check, in calculation of moments, 156; alternative approach in sampling of attributes, 368-369; refs., Theory of frequency curves, resolution of a compound normal curve, 507.
- Chebycheff, Chebysheff, see Tchebycheff.
- Cheshire, L., refs., Sampling of correlation coefficient, 522.
- Chi-squared, see  $\chi^2$ .
- Childbirth, Deaths in, Application of theory of sampling (Table 19.1), 364, 363-365.
- Chokhate, J., see Shohat, J.
- Cholera and inoculations, Illustrations, 40, 42, 420, 426-427.
- Chotimsky, V., refs., Curve fitting, 515.
- Chrysomelide, Distribution of size of genus (Table 6.13), 106.
- Chuproff, Chuprow, see Tschuprow.
- Church, A. E. R., refs., Sampling from U-shaped population (under Holzinger), 518; sampling moments, 522.
- Class, in theory of attributes, 12; class symbol, 12; class-frequency, 13; positive and negative classes, 13-14; order of a class, 14; ultimate classes, 15-16.
- Class-interval, Def., 82-83; choice of magnitude and position, 85-86; desirability of equality of intervals, 82, 88-89; influence of magnitude on mean, 118, 119-120; on standard deviation, 141; on third and fourth moments, 160.
- Classification—generally, 11-12; by dichotomy, def., 12; manifold, 65-81; homogeneous and heterogeneous, 74-75; as a series of dichotomies, 75-76; of data on punched cards, 76-77; of a variable for frequency-distribution or correlation table, 82-88, 197-198.
- Closeness of fit, see Fit,  $\chi^2$ .
- Cloudiness at Greenwich (Table 6.14), 106; (fig. 6.15), 104.
- Coefficient of association, 44, 45, 55, (standard error) 410; of contingency (Pearson's), 68-69, (standard error) 410, (Tschuprow's) 70; of variation, 149-150, (standard error) 405-406; of rank correlation, 240-249, (standard error) 410; of correlation, partial correlation, multiple correlation, see Correlation.
- Colcord, C. G., see Deming.
- Colours, Naming a pair, Example of contingency (Ex. 22.5), 432.
- Complete beta-function, refs., Tables, 525.
- elliptic integrals, refs., Tables, 525.
- Complex frequency-distributions, 103, 105.
- Concentration, refs., 527.
- Condon, E., refs., Curve fitting, 515.
- Connor, R. L., refs., Tests of correspondence between statistical grouping and formulæ (under Bowley), 520.
- Consistency of class-frequencies—generally, 26-31; def., 26; conditions for, 27; conditions for, in the case of positive class-frequencies, 27-29; refs., 499.
- Consistence of correlation coefficients, 280-281.
- Constrained data, in Lexis' sense, 369.
- Constraints, in  $\chi^2$  distribution, 414-415; linear constraints, 415.
- Contingency, Coefficient of (Pearson's), 68-69; (Tschuprow's), 70; relationship with normal correlation, 239; standard error of, 410; refs., 500-501, 517-520.
- Contingency tables, Def., 65-66; treatment of, by elementary methods, 67;

- isotropy, 72-74, 237-239; degrees of freedom in, 415-416; testing of divergence from independence, 418-421.
- Contrary classes and frequencies, for attributes, 13; case of equality of contrary frequencies (Exs. 1.6 and 1.7), 23; (Ex. 2.8), 32; (Exs. 4.7, 4.8 and 4.9), 64.
- Correction of correlation coefficients for errors of observation, 298-299; for grouping, 221-222.
- of death-rates, etc., for age and sex distributions, 305-306; refs., 514.
  - of standard deviation, for grouping of observations, 141; of moments, 160, 399; comparison of corrections with sampling effects, 402; refs., 504-505.
- Correlation—generally, 196-308; construction of tables, 196-198; representation of bivariate frequency-distribution by surface and stereogram, 198-204, by scatter diagram, 205-206; treatment of table by coefficient of contingency, 206.
- Product-moment correlation coefficient, 209-213; def., 209; equations and lines of regression, 206-211; linear and curvilinear regression, 207, 242-243; coefficients of regression, 213; standard deviations of arrays, 214, 242; calculation of correlation coefficient for ungrouped data, 214-215, 215-218; for grouped data, 218-221; effect of fluctuations of sampling on, 221; correction for grouping, 221; elementary methods for cases of curvilinear regression, 242-243; rough methods for estimating coefficient, 241-242; correlation ratios, 243-246; effect of errors of observation on the coefficient, 298-299.
- Rank correlation coefficient, 246-251; relationship with product-moment coefficient, 249; grade correlation, 249-251; tetrachoric  $r$ , 251-252; coefficient for a fourfold table, direct, 252; intraclass correlation, 253-258; expression for coefficient, 256-258; limits to negative values of, 256-257; correlation between indices, 300-301; correlation due to heterogeneity of material, 301; effect of adding uncorrelated pairs to a given table, 301-302; application to theory of weighted mean, 302-303; correlation coefficient in theory of sampling, 407-408; small samples, 449-453; refs., 509-514, 517-524; for Illustrations, Normal, Partial, Ratio, see below.
- Correlation, Illustrations and Examples.
- Correlation between:
- Two diameters of a shell (*Pecten*),\* (Table 11.1), 197; constants (Ex. 11.3), 225.
  - Ages of husband and wife (Table 11.2), 198; constants, 220-221; correlation ratios (Ex. 13.2), 259.
  - Statures of father and son (Table 11.3), 199; (fig. 11.3), facing 204; (fig. 11.8), 211; constants (Ex. 11.8), 225; correlation ratios, 246; testing normality of table, 232-239; diagram of diagonal distribution (fig. 12.2), 234, of contour lines fitted with ellipses of normal surface (fig. 12.3), 236.
  - Age and yield of milk in cows (Table 11.4), 200; (fig. 11.9), 212; constant (Ex. 11.3), 225; correlation ratios (Ex. 13.1), 259.
  - Discount rates and percentage of reserves on deposit (Table 11.5), 201; (fig. 11.2), facing 204.
  - Sex-ratio and numbers of births in different districts (Table 11.6), 202; (fig. 11.10), 213; constants (Ex. 11.8), 225; correlation ratios, 246.
  - Monthly index-numbers of prices of animal feeding-stuffs and home-grown oats (Table 11.7), 203; scatter diagram (fig. 11.4), 205; constants, 215-218.
  - Length of mother- and daughter-froed in *Lemna minor*, 218-220.
  - Weather and crops, 291-292.
  - Movements of infantile and general mortality, 292-294.
  - Movements of marriage rate and foreign trade, 294-296.
  - Earnings of agricultural labourers, pauperism and out-relief (Ex. 11.2), 224; partial correlations, 270-272; geometrical representation (fig. 14.1), 276.
  - Changes in pauperism, out-relief, proportion of old and population, 288-291; partial correlations, 272-275.
- Correlation, Normal, 227-240, 282-286; deduction of expression for two variables, 227-229; homoscedasticity and linearity of regression, 229-231; contour lines, 230-231; normality of linear functions of normally distributed variates, 231; principal axes, 231-232; testing of correlation table for stature, 232-237; isotropy of normal correlation table, 237-239; relationship with contingency, 239; outline of theory for any number of variables, 282-286; coefficient for a normal distribution grouped to a fourfold form round the medians (Sheppard's theorem), (Ex. 12.4), 240; refs., 509-511.
- Correlation, Partial, 261-287; the problem, partial regressions and correlations, 261-262; notation and definitions, 263-264; normal equations, fundamental theorems on product-sums, 262-263, 265-266; meaning of generalised regressions and correlations, 266; reduction of standard deviation, 266-268,



- of regression, 268-269, of correlation, 269; arithmetical treatment, 269-275; representation by a model, 275-277; coefficient of multiple correlation, 277-279; expression of correlations and regressions in terms of those of higher order, 279-280; consistence of coefficients, 280-281; fallacies, 281-282; limitations in interpretation of the partial correlation coefficient, partial association and partial correlation, 282; partial correlation in the case of normal distribution of frequency, 284; refs., 511-512.
- Correlation ratios, 243-246; relation with measure of closeness of fit of simple curves, 329; standard error, 409; test of significance of, 453-455; partial, 282; refs., 510, 511, 514.
- Cosin, Values of estates in 1715 (Table 6.12), 105.
- Cost of living, refs., 503.
- per unit of electricity, *see* Electricity.
- Cotsworth, M. B., refs., Multiplication table, 524.
- Coutts, J. R. H., Data quoted from (Table 17.5), 322.
- Cows, Distribution according to age and milk-yield, *see* Milk-yield.
- Craig, C. C., refs., Seminvariants, 505, 518; sampling, 518, 522.
- Cramér, H., refs., Series used in mathematical statistics, 507.
- Crawford, G. E., refs., Proof that arithmetic mean exceeds geometric mean, 502.
- Crelle, A. L., refs., Multiplication tables, 525.
- Criminals, Relation between weight and mentality (Table 5.6), 78.
- Crops and weather, Correlation, 291-292.
- Cunningham, E., refs., *Omega*-functions, 507.
- Curve fitting, General, 309-331; the problem, 309-311; method of least squares, 311-313; equations for fitting polynomials, 312-313; equations for straight line, 313-314; calculation, 314-315; reduction of data to linear form, 316-320; fitting of more general polynomials, 320-324; case when independent variable proceeds by equal steps, 325-327; calculation of sum of squares of residuals, 327-328; measurement of closeness of fit, 328-329; relationship of measure with correlation ratio and multiple correlation coefficient, 329; general remarks, 324, 329; refs., 514-515.
- Curve fitting, Illustrations and Examples: Estimated distance and velocity of recession of extra-galactic nebulae (Table 17.1), 309-310; (fig. 17.1), 310; straight line fitted to, 315-316; measure of fit, 329.
- Growth of duckweed (Table 17.3), 317; (fig. 17.2), 317; logarithmic curve fitted to, 316-318.
- Working costs per unit and units sold per head of population in certain Electricity Undertakings (Table 17.4), 320; curve fitted logarithmically, 318-321; (figs. 17.3 and 17.4), 319 and 321.
- Temperature and loss in weight in soil (Table 17.5), 322; parabolas fitted to, 320-324; (fig. 17.5), 324; sum of squares of residuals, 327-328; closeness of fit, 329.
- Growth of population in England and Wales (Table 17.6), 326; parabola fitted to, 325; (fig. 17.6), 326.
- Curvilinear regression, *see* Regressions.
- Czuber, E., refs., *Wahrscheinlichkeitsrechnung*, 496, 505; *Die statistische Forschungsmethode*, 496.
- DARBISHIRE, A. D., Data cited from, 130, (Exs. 19.12 and 19.13), 372; refs., illustrations of correlation, 509, 516.
- Darmois, G., refs., Time series, 512; *Statistique mathématique*, 496.
- Data, Remarks on collection of, 6-7; on treatment of, 7; on summarisation of, 7-8; on analysis of, 8-9.
- Datura*, Association between colour and prickliness of fruit, 44, 432 (Ex. 22.6).
- Davenport, C. B., Data as to *Pecten* cited from (Table 11.1), 197.
- David, Census of Israelites, footnote, 2.
- Davis, H. T., refs., Curve fitting, 515; (Editor) *Tables of Higher Mathematical Functions*, 525.
- Day, E. E., refs., *Statistical Analysis*, 496.
- De Finetti, B., refs., Variation, 527.
- De Morgan, refs., *Formal Logic*, 499.
- De Vergottini, M., refs., Variation, 527.
- De Vries, H., Data cited from (Ex. 6.5 (d)), 110.
- Deaf-mutism, Association with imbecility, 40-41, 45; frequency among offspring of deaf-mutes (Ex. 6.5 (b)), 109.
- Deaths or death-rates, Association with occupation (partial correction for age-distribution), 59-60; from scarlet fever (Table 6.11), 100; (fig. 6.11), 101; infantile and general, correlation of movements, 292-294; standardisation of, for age- and sex-distribution, 59-60, 305-306, refs., 514; application of theory of sampling, deaths from accident, 359; deaths in childbirth, 363-365, (Table 19.1), 364; deaths from explosions in mines, 367-368; inapplicability of the theory of simple sampling to, 357-359; mortality in Cambridgeshire, 468.
- Deciles, 150-151; standard error of, 380-382.

- Defects, in school-children, association of, 16, 52-53, refs., 499.
- Degree of a fitted curve, 310.
- Degrees of freedom, in  $\chi^2$  test, 415-416; in estimates from small samples, 436-437.
- Deming, W. E., Lola S. Deming and C. G. Colcord, Tables of  $\alpha$ -integral, 444, and Appendix Table 6C.
- Demoivre, A., Discoverer of normal distribution, 169.
- Dependent variable, in curve fitting, 313-314.
- Design of statistical inquiries, in sampling, 335.
- Detlefsen, J. A., refs., Fluctuations of sampling in Mendelian population, 516.
- Deviation, Mean, 134; generally, 144-147; def., 144; is least round the median, 145; calculation of, 147, (Ex. 8.11), 153; comparison of magnitude with standard deviation, 146-147, 182; of normal curve, 182.
- , Quartile; see Quartiles.
- , Root-mean-square; see Deviation, Standard.
- , Standard, 134; def., 134-135; relation to root-mean-square deviation about any origin, 135-136; is least possible root-mean-square deviation, 136; little affected by small errors in the mean, 136; calculation from ungrouped data, 135-138; for grouped data, 138-141; influence of grouping, 141; range of six times the s.d. includes the bulk of the observations, 142; of a series compounded of others, 142-143; of  $N$  consecutive natural numbers, 143; of rectangular distribution, 143; of arrays in theory of correlation, 206, 214, 242; of generalised deviations (arrays), 264, 266-267; other names for, 144; of a sum or difference, 297-298; effect of errors of observation on, 298; of an index, 299-300; of binomial series, 174; of Poisson distribution, 189. For standard deviations of sampling, see Error, Standard.
- Dice, Records of throwing (Table 6.15 and fig. 6.16), 107, (Ex. 10.2), 193; testing for significance of divergence from theory, 351-353, 419-420, 423-424; refs., 516-517.
- Dickson, J. D. Hamilton, Normal correlation surface, 237; refs., normal correlation, 509.
- Difference method in correlation, 292-296, 477; refs., 512-513.
- Differences, in interpolation, 462-464; effect of errors in  $u$  on, 473-477; effect of subdividing an interval on, 477.
- Discounts and reserves in American banks (Table 11.5), 201; (fig. 11.2), facing 204.
- Dispersion, Measures of, 112, 134-153; absolute measures of, 149; range as a measure, 134; in Lexis' sense, normal, subnormal and supernormal, 369; refs., 503-505. See Deviation, Mean; Deviation, Standard; Quartiles.
- Distance-velocity relation in extra-galactic nebulae, 309-310, (Table 17.1), 309, (fig. 17.1), 810; straight line fitted to, 315-316.
- Distribution of frequency; see Frequency-distributions; sampling, see Sampling.
- Dodd, E. L., refs., Frequency-curves, 507; sampling, 518, 522.
- Doodson, A. T., refs., Mode, median and mean, 502.
- Duckweed, Correlation between mother and daughter-frond, 218-220; growth of, curve fitted to, 316-318.
- Duncker, G., Relation between geometric and arithmetic mean (Ex. 8.12), 153.
- Dunlap, H. F., refs., Sampling from rectangular populations, 518.
- EARNINGS of agricultural labourers, Correlation with pauperism and out-relief, data (Ex. 11.2), 224; partial correlations, 270-272; diagram of model (fig. 14.1), 276.
- Edgeworth, F. Y., Dice-throwing (Weldon), 107; refs., geometric mean, 502; index-numbers, 503; normal law and frequency-curves generally, 505, 506, 507, 508; dissection of normal curve, 508; correlation, 509-511; theory of sampling, probable errors, etc., 516-518; Edgeworth's contributions to mathematical statistics, see Bowley.
- Efficient estimates, 428.
- Elderton, E. M., refs., Variate difference correlation method (*under* Pearson), 513; sampling, (*under* Pearson), 523.
- Elderton, W. F., Tables of  $\chi^2$ , 425; refs., calculation of moments, 504; table of powers, 525; *Frequency Curves and Correlation*, 496, 504, 505.
- Electricity Commission, Data quoted from returns for 1933-34 (Table 17.4), 320.
- Electricity, Curve fitted to costs per unit and number of units sold per head of population for certain Undertakings, 318-320, (Table 17.4), 320, (figs. 17.3 and 17.4), 319, 321.
- Elliptic integrals, Tables of, refs., 525.
- Engineering, Applications of statistical method, refs., 497.
- Engledow, F. L., Data cited from, (Table 23.2), 446.
- Epidemiology, Applications of statistical method to, refs., 508.
- Error function, 183; see Normal distribution.
- Error, Law of, errors, curve of, see Normal distribution.

- Error, Mean, 144.  
 —, Mean square, 144.  
 — of mean square, 144.
- Error, Probable, in theory of sampling, 353-354. For general references, see Error, Standard.
- Error, Standard, def., 353, 380; of number or proportion of successes in  $n$  events, 351; when numbers in samples vary (Ex. 19.11), 372; when chance of success or failure is small, 356; of percentiles, median, quartiles, etc., 380-382; of semi-interquartile range, 385-386; of arithmetic mean, 386; of variance, 399; of standard deviation, 399-402; of coefficient of variation, 405-406; of moments about fixed point, 395-396; of moments about the mean, 397; of third and fourth moments about the mean, 403-404; of  $\beta_1$  and  $\beta_2$ , 406; of coefficients of correlation and regression, 407-409; approximate formula for correlation ratio and caution in case of multiple correlation coefficient, 409; of coefficient of association, 410; of coefficient of mean square contingency, 410; absence of, in certain cases for rank correlation coefficient, 410; refs., 516-520. See also Sampling, Theory of.
- Error, Theory of; see Sampling, Theory of.
- Estates, Value of, in 1715; see Value.
- Estimates, Precision of, 335; efficient, 428; in small samples, 434; of arithmetic mean, 434-435; of variance, 435-436; degrees of freedom of, 436-437.
- Estimation, Theory of, 334-335; of theoretical frequencies in the  $\chi^2$  test, 427-428; of position of maximum, 487-488.
- Exclusive and inclusive notations for statistics of attributes, 22.
- Existent universes, 333.
- Experiments on  $\chi^2$  test, 429-430.
- Explosions in coal mines, Deaths from, as illustrating theory of sampling, 367-368.
- Eye-colour, Association between father and son, 41, 45, 73-74; association between grandparent, parent and child, 58-55, 60; contingency with hair-colour, 66-67, 70-71; non-isotropy of contingency table for father and son, 73-74.
- Ezekiel, M., refs., Correlation, 511; sampling and curvilinear regression, 522; *Methods of Correlation Analysis*, 496.
- FALKNER, R. P., refs., Translation of Meitzen's *Theorie der Statistik*, 498.
- Fallacies in interpreting associations, Theorem on, 56-57, illustrations, 57-58, owing to changes of classification, actual or virtual, 75; in interpreting correlations, 281-282; "spurious" correlation between indices, 300-301, correlation due to heterogeneity of material, 301.
- Farm Economics Branch, School of Agriculture, Cambridge, data cited from records of, (Ex. 17.4), 330.
- Fay, E. A., Data cited from *Marriages of the Deaf in America*, (Ex. 6.5 (b)), 109.
- Fechner, G. T., refs., Frequency-distributions, averages, measures of dispersion, etc., 501, 503; *Kollektivmasslehre*, 501.
- Fecundity of brood-mares (Table 6.9), 98, (fig. 6.9), 98; mean, median and mode (Ex. 7.4), 132; inheritance, refs., 513.
- Fegiz, P. L., Data cited from, (Ex. 17.2), 330.
- Feldman, H. M., refs., Sampling, 518.
- Field experiments, refs., 497.
- Fieller, E. C., refs., Sampling distribution of an index, 518.
- Filon, L. N. G., refs., Probable errors, (under Pearson), 519.
- Finite and infinite universes, 332-333.
- Fisher, A., refs., *Mathematical Theory of Probabilities*, 496.
- Fisher, Irving, refs., Index-numbers, 503.
- Fisher, R. A., Criticism of use of standard error in test of linearity of regression, 409; tables of  $\chi^2$ , 418, 425; normality of  $\chi^2$  for large  $n$ , 422; tables of  $t$ , 439-440; data cited from, 442-443; application of  $t$ -distribution to regressions, 443; distribution of correlation coefficient, 449; transformation of, 451; refs., goodness of fit of regression lines, 510; curve fitting, 515; sampling of correlation coefficient, 518, 522; moments of sampling distributions, 518;  $\chi^2$  distribution, 520-521; tests of agreement between observation and hypothesis, 521; sampling theory, 522; extremes of sample, 522; statistical estimation, 522;  $t$ -distribution, 522; *Statistical Methods for Research Workers*, 496.
- Fisher's  $z$ -distribution, 443-444; Tables, 444, and Appendix Tables 6; use in analysis of variance, 448; in testing significance of correlation ratios, 453-455; significance of linearity of regression, 455-456; significance of multiple correlation coefficient, 456-458.
- Fit of simple curves to data; see Curve fitting; measure of closeness of fit, for simple curves, 328-329; "best" fit, "closest" fit, as given by method of least squares, 209-210, 262-264, 311-314; goodness of fit, see  $\chi^2$  distribution.
- Flux, Sir A. W., refs., Measurement of price-changes, 503.
- Food, Drink and Tobacco Trades, Data on size of firms in, (Ex. 6.5 (a)), 109.
- Footrule, Spearman's, footnote, 249.

- Forcher, H., refs., *Die statistische Methode als selbständige Wissenschaft*, 498.
- Fountain, Sir Henry, refs., Index-numbers of prices, 503.
- France, Anatole, Remark about the Chinese, 2.
- Freedom, Degrees of; *see* Degrees of freedom.
- Frequency of a class, 13, 83.
- Frequency-curve, Def., 92-93; ideal forms of, 93-104; refs., 501, 507-508; *see* Normal distribution.
- Frequency-distributions, 82-83; formation of, 85-89; graphic representation of, 90-92; ideal forms, symmetrical, 93-94, moderately asymmetrical, 94-98, extremely asymmetrical (J-shaped), 98-101, (U-shaped), 101-102; truncated distributions, 102-103; complex distributions, 103-104; pseudo-frequency distributions, 104, 108; reduction to absolute scale, 150; theoretical, 169; binomial distribution, 169, 169-180; normal distribution, 180-187; Poisson distribution, 187-191; refs., 501, 507-508. *See also* Binomial distribution; Normal distribution; Poisson distribution; Pearson curves; Correlation, Normal.
- Frequency - distributions, Illustrations: Birth-rates in England and Wales, 83; stigmatic rays on poppies, 84; lengths of screws, 84; final digits in measurements, 86; persons liable to sur- and super-tax in the United Kingdom, 89; head-breadths of Cambridge students, 90; statures of males in the United Kingdom, 94; Australian marriages, 96; fecundity of brood-mares, 98; barometer heights at Greenwich, 99; ages at death from scarlet fever, 100; annual value of estates in 1715, 105; degrees of cloudiness at Greenwich, 106; sizes of genera in *Chrysolmelider*, 106; dice-throwing, 107; male deaths in England and Wales, 107-108; size of firms in Food, Drink and Tobacco Trades (Ex. 6.5 (a)), 109; percentage of deaf-mutes in offspring of deaf-mutes (Ex. 6.5 (b)), 109; yield of grain (Ex. 6.5 (c)), 110; petals in the buttercup, *Ranunculus bubosus* (Ex. 6.5 (d)), 110; weights of males in the United Kingdom (Ex. 6.6), 110; wheat shoots (Table 18.1), 338. *See also* Correlation, Illustrations and examples.
- Frequency-polygon, Construction of, 90.
- Frequency-surface, Forms and examples of, 196-202; (figs. 11.1, 11.2, and 11.3), 204, and facing 204; *see* Correlation, Normal.
- Frisch, R., refs., Difference equations and frequency-distributions, 507; correlation, 509; time series, 512.
- Fry, T. C., refs., *Probability and its Engineering Uses*, 497.
- Fundamental sets, Specifying data, 17.
- GABAGLIO, A., refs., *Teoria generale della statistica*, 498.
- Galton, Sir Francis, Ogive curve, 150-151; binomial apparatus, 175-176; regression, 207; Galton's function (correlation coefficient), 242; normal correlation, 237; data cited from, 41, 53, 73; refs., geometric mean, 502; percentiles, 504; binomial machine, 506; correlation, 509; correlation between indices, 513; *Natural Inheritance*, 504, 506.
- Galvani, L., refs., Means, 527; variation and concentration, 527.
- Gamma-functions, refs., Tables, 525.
- Gauss, C. F., Normal distribution, 169; use of term "mean error," 144.
- Geary, R. C., refs., Frequency-distributions, 507.
- Geiger, H., refs., Poisson distribution (under Rutherford), 506.
- Geometric mean; *see* Mean, Geometric.
- Gibson, Winifred, refs., Tables for computing probable errors, 518.
- Gini, C., refs., Index-numbers, 503; curve fitting, 515; general, 526; interpolation, 526; means, 527; probability, 527; variability, 527; index-numbers, 528; statistical relations, 528; *Appunti di Statistica Metodologica*, 526; (Ed.) *Trattato Elementare di Statistica*, 526.
- Goodness of fit, 430; *see*  $\chi^2$  distribution.
- Grades, 150; grade correlation, 249-251; relationship with ranks and rank correlation, 249-251; *see* Ranks.
- Graduation, 480-485; *see* Interpolation.
- Gram, J. P., refs., Expression of functions in series by least squares, 515.
- Graphic method of representing frequency-distribution, 90-92; of interpolating for median and percentiles, 121-122, 150; of representing correlation between two variables, 205-206; of estimating correlation coefficient, 241-242; refs. (Italian), 526.
- Graunt, John, refs., *Observations on the Bills of Mortality*, (under Hull, C. H.), 498.
- Gray, John, Data cited from, 361.
- Greatest and least value of sample, refs., 522 (Dodds), 522 (Fisher and Tippett).
- Greenleaf, H. E. H., refs., Curve fitting, 515.
- Greenwood, M., Data cited from, 40, 42, (Table 10.3), 175; use of principal axis in curve fitting, footnote, 314; refs., inoculation statistics and association, 499; Poisson distribution, 506; multiple happenings, 508; index correlations (under Brown), 511, 513; errors of sampling, 516.

- Group, Breaking-up of, in interpolation, 477-478; formula for halving of, 479-480.
- Grouping of observations to form a frequency-distribution, Choice of class-interval, 82-83; influence of grouping on mean, 118, 119-120; influence on standard deviation, 141; influence on higher moments, 160.
- Growth of duckweed (Table 17.3), 317; curve fitted to data, 316-318; (fig. 17.2), 317; of population (Table 17.6), 326; curve fitted to data, 325; (fig. 17.6), 326.
- HAIR-COLOUR and eye-colour, Example of contingency, 66-67, 70-71; non-isotropy, 71-72; theory of sampling applied to certain data, 361-362.
- Hall, Sir A. D., Data cited from, (Ex. 6.5 (c)), 110.
- Hall, Philip, refs., Partial correlation, 511; distribution of means from rectangular universe, 522.
- Halving a group, in interpolation, 479-480.
- Harmonic mean; *see* Mean, Harmonic.
- Harris, J. A., refs., Short method of calculating coefficient of correlation, 514; intraclass coefficients, 514; correlation, miscellaneous, 512.
- Hart, B., refs., Effect of errors on correlation, 513.
- Head-breadths of Cambridge students (Table 6.6), 90; (figs. 6.1 and 6.2), 91.
- Height, Distribution of men according to; *see* Stature.
- distribution of wheat plants (Table 18.1), 338.
- Helguero, F. de, refs., Dissecting normal curves, 508.
- Hendricks, W. A., refs., Curve fitting, 515.
- Henry, A., refs., *Calculus and Probability*, 495.
- Heron, D., refs., Association (*under* Pearson), 499; relation between fertility and social status, 512; defective physique and intelligence, application of correction for age-distribution, 514; abae for giving probable errors of correlation coefficients, 518; probable error of partial correlation coefficient, 518.
- Heteroscedastic arrays, footnote, 214.
- Hilton, John, refs., Sampling inquiry, 516.
- Histogram, Construction of, 90-91.
- History of statistics generally, 4-5; refs., 498.
- Hojo, T., refs., Sampling distribution of medians, quartiles, etc., 518.
- Hollis, T., cited *re* Cosin's "Names of the Roman Catholics, etc.," 105.
- Holzinger, K. S., refs., Sampling from U-shaped universe, 518.
- Homoscedastic arrays, footnote, 214.
- Hooker, R. H., Correlation between weather and crops, 291-292; between movements of two variables, 294-296; refs., theory of partial correlation, 511; correlation between movements of two variables, 512; between weather and crops, 512; between marriage rate and trade, 512.
- Horst, P., refs., Evaluation of multiple regression coefficients, 511.
- Hotelling, H., refs., History, 498; limits to skewness, 505; analysis situs, 512; time series (*under* Working), 513; sampling of correlation ratio, 518; optimum statistics, 518; generalisation of "Student's" distribution, 522; sampling of rank correlation coefficient, 522.
- Houses, Inhabited and uninhabited, in rural and urban districts (Ex. 5.2), 80.
- Hubble, Edwin, Data cited from, (Table 17.1), 309.
- Hull, C. H., refs., *The Economic Writings of Sir William Petty, together with Observations on the Bills of Mortality more probably by Captain Graunt*, 498.
- Human bias, in sampling, 337-339.
- Humason, M. L., Data cited from, (Table 17.1), 309.
- Husbands and wives, Correlation between ages of (Table 11.2), 198; constants, 220-221; correlation ratios (Ex. 13.2), 259.
- Hypergeometric series, refs. (Karl Pearson), 506; (Camp) 507.
- Hypothetical universe, 338; sampling from, 345-346.
- ILLUSORY associations, 57-58.
- Imbecility, Association with deaf-mutism, 40-41, 45.
- Inclusive and exclusive notations for statistics of attributes, 22.
- Incomes liable to sur- and super-tax; *see* Sur- and super-tax.
- Incomplete beta-function, tables, refs., 525; gamma-function, tables, refs., 525; elliptic integrals, tables, refs., 525.
- Independence, Criterion of, for attributes, 34-35; case of complete, for attributes, 60-62; form of contingency or correlation table in case of, 74;  $\chi^2$  test for, 418-430.
- Independent variable in curve fitting, 813-814.
- Index-numbers of prices, 129-130; use of geometric mean for, 129-130; of animal feeding-stuffs and home-grown oats (Table 11.7), 203; correlation between, 215-218; refs., 502-503, 528.
- Indices, Correlation between, 800-801; refs., 513-514.
- Infinite and finite universes, 332-333; sampling from, 344-345.

- Inoculation against cholera, Examples, 40, 42-43, 420, 426-427.
- Inoculation against tuberculosis in cattle, Example, 425-426.
- Interclass correlation, 254; *see* Correlation.
- Intermediate observations in a frequency-distribution, Classification of, 85, 87-88; in correlation table, 197-198.
- Interpolation and graduation—generally, 462-493; simple interpolation, 462; differences, 462-464; Newton's formula, 464-468; interpolation of statistical series, 468-470; practical work, 470-473; number of differences to use, 470-471; choice of set of  $u$ 's, 472; possible forms of polynomials, 472-473; effect of errors on differences, 473-477; effect on differences of subdividing an interval, 477; breaking-up a group, 477-479; formula for halving a group, 479-480; graduation, 480-485; inverse interpolation, 485-487; estimation of the position of a maximum, 487-488; modifying central ordinates to equivalent areas, 489; refs., 524, (Italian) 526-527.
- Interval, Subdivision of, 477.
- Intraclass correlation, 253-258; coefficient of, 255-258; limits to negative values of coefficient, 256-257; in analysis of variance, 448.
- Inverse interpolation, 485-487.
- Irwin, J. O., refs., Recent advances, 495; sampling distribution of means, 518;  $\chi^2$  test, 521; analysis of variance, 522; frequency-distribution of means of samples, 522.
- Isotropy, Def., 72; generally, 71-74; of normal correlation table, 237-239; refs., 500.
- Isserlis, L., refs., Partial correlation ratios, 511; conditions for real significance of probable errors, 519; fitting polynomials (Tchebycheff), 515; probable error of mean, 522; small samples (under Greenwood), 522.
- JACOB, S. M., refs., Crops and rainfall, 512-513.
- Jeffery, G. B., refs., Sampling (under Pearson), 523.
- Jeffreys, H., refs., *Scientific Inference*, 495.
- Jensen, A., refs., Sampling methods, 516.
- Jevons, W. S., Use of geometric mean, 130; refs., system of numerically definite reasoning (theory of attributes), 499; *Pure Logic and other Minor Works*, 499; *Investigations in Currency and Finance*, 502.
- John, V., refs., *Der Name Statistik*, 498; *Geschichte der Statistik*, 498.
- Jordan, C., refs., Time series, 512; curve fitting, 515; *Statistique mathématique*, 496, 515.
- J-shaped frequency-distributions, 98-101.
- KAPTEYN, J. C., refs., *Skew Frequency-curves in Biology and Statistics*, 502, 507.
- Kelley, T. L., refs., Correlation, 511; tables to facilitate the computation of correlation coefficients, 525; *Statistical Method*, 496.
- Kelvin, Lord, Dictum on measurement and knowledge, 1.
- Keynes, J. M., refs., *A Treatise on Probability*, 495, 516.
- Khotimsky; *see* Chotimsky.
- Kick of a horse, Deaths from, following Poisson distribution, 191.
- King, George, Graduation of age statistics, 483-485.
- Kiser, C. V., refs., Bias in sampling, 516.
- Knibbs, Sir G. H., refs., Price index-numbers, 503; frequency-curves, 508.
- Kohlweiler, E., refs., *Statistik im Dienste der Technik*, 497.
- Kohn, S., refs., *Theory of Statistical Method*, 496.
- Kondo, T., refs., Standard error of mean square contingency, 519; of standard deviation, 519.
- Koren, J., refs., *History of Statistics*, 498.
- Kurtosis, Def., 165; calculation of, 165; of binomial series, 174; of Poisson distribution, 189-190; effect on standard error of standard deviation, 400.
- Labour Gazette*, Index-number, refs., 503.
- Labourers, Agricultural, Minimum wages-rates of; *see* Agricultural labourers' earnings; *see also* Earnings.
- Laplace, Pierre Simon, Marquis de, Normal distribution, 169; refs., *Théorie analytique des Probabilités*, 504, 519.
- Latshaw, V. V., refs., Curve fitting (under Davis), 515.
- Le Roux, J. M., refs., Sampling, 522.
- Leading term and leading differences, 463.
- Least squares, Method of, in fitting regression lines, 209-210, 262-263; in fitting curves generally, 309-331; equations, 312-313.
- Lee, Alice, Data cited from, (Table 6.9), 98, 125, (Table 11.3), 199; refs., generalised probable error in multiple correlation (under Pearson), 510; inheritance of fecundity and fertility (under Pearson), 513.
- Lemna minor*, Correlation between lengths of mother- and daughter-frond in, 218-221; rate of growth of, 316-318.
- Leptokurtic curves, 165.
- Lester, A. M., Unpublished data on screw measurements, (Table 6.3), 84.
- Levels of significance, in  $\chi^2$  test, 424-425; in  $t$ -test, 440; in  $z$ -test, 444.

- Levy, H., refs., *Elements of Probability*, 495.
- Lexis, W., Use of term "precision," 144; alternative approach in sampling of attributes, 368-369; refs., *Abhandlungen zur Theorie der Bevölkerungs- und Moralstatistik*, 496, 516; *Theorie der Massenerscheinungen*, 516.
- Linear constraints, 415.
- Linearity of regression, 207; tests for, 245, 409, 455-456.
- Lipps, G. F., refs., Measures of dependence (association, correlation, contingency, etc.), 499, 500; Fechner's *Kollektivmasslehre*, 501.
- Little, W., Data as to agricultural labourers' earnings cited from (Ex. 11.2), 224.
- Livi, L., refs., *Elementi di Statistica*, 526.
- Logarithmic increase in population, 127-129; in duckweed, 316-318.
- Loss in weight in soils, Percentage; see Percentage.
- Lottery sampling, 340-341.
- MACAULAY, F. G., refs., Smoothing time series, 512.
- Macdonell, W. R., Data cited from (Table 6.6), 90.
- Manifold classification; see Classification.
- March, L., refs., Index-numbers, 503; correlation, 512.
- Marriage rate and trade, Correlation of movements, 294-296.
- Marriages, Australian; see under Australian.
- Marshall, A., refs., *Money, Credit and Commerce*, 503.
- Martin, E. S., refs., Corrections to moments, 504.
- Maximum, Estimation of position of, 487-488.
- McAlister, Sir Donald, refs., Law of geometric mean, 502.
- McKay, A. T., refs., Sampling distribution of correlation coefficient, 519.
- McNemar, Q., refs., Partial correlation (under Kelley), 511.
- Mean, Arithmetic—generally, 114-120; def., 114; nature of, 114; calculation of, for a grouped distribution, 115-118; influence of grouping, 118, 119-120; position relatively to mode and median, 125; diagram (fig. 7.2), 118; sum of deviations from, is zero, 118; of series compounded of others, 119; of sum or difference, 119-120; comparison with median, 122-124, 387; summary comparison with median and mode, mean is best for all general purposes, 125-126; reciprocal character compared with harmonic mean, 130-131; of binomial distribution, 173; of Poisson distribution, 189; weighting of, 302-306; standard error of, 386-387, 388-391; means of two samples, 387-388, (small samples) 442-443; estimates of, 434-435; refs., 501-502, 517-520, 521-524, (Italian) 527.
- Mean deviation; see Deviation, Mean. — error, 144; see Error, Standard; Deviation, Standard.
- Mean, Geometric, 114; generally, 126-130; def., 126; calculation, 126; less than arithmetic mean, 126; difference from arithmetic mean in terms of dispersion, (Ex. 8.12), 153; of series compounded of others, 127; of series of ratios or products, 127; in estimating inter-censal populations, 127-129; convenience for index-numbers, 129-130; weighting of, 306.
- Mean, Harmonic, 114; generally, 130-131; def., 130; calculation, 130; is less than arithmetic and geometric means, 131; difference from arithmetic mean in terms of dispersion (Ex. 8.13), 153; reciprocal character compared with arithmetic mean, 130-131; in theory of sampling, when numbers in samples vary (Ex. 19.11), 372.
- Mean square error, 144.
- , Weighted, 302-306; def., 302; difference between weighted and unweighted means, 303-304; applications of weighting to corrections of death-rates, etc., for age- and sex-distribution, 305-306; refs., 514.
- Median, 114; generally, 120-124; def., 120; indeterminate in certain cases, 120; unsuited to discontinuous observations and small series, 120-121; calculation of, 121; graphical determination of, 121-122; comparison with arithmetic mean, 122-124, 387; advantages in special cases, 123-124; slight influence of outlying values on, 124; position relative to mean and mode, 125, (fig. 7.2), 118; weighting of, 306; standard error of, 380-385; refs., 517-520.
- Meidell, H. B., refs., Sampling, 519, 523.
- Meitzen, P. A., refs., *Geschichte, Theorie und Technik der Statistik*, 498.
- Mendelian breeding experiments as illustrations, 44, 130, 353; refs., fluctuations of sampling in, 516-517.
- Mentality, Relationship with weight in a selection of criminals (Table 5.6), 78.
- Mercer, W. B., Data cited from (Ex. 6.5 (c)), 110.
- Method of least squares; see Least squares.
- Methods, Statistical, Purport of, 3; def., 3.
- Mice, Numbers in litters, Harmonic mean, 130; proportions of albinos in litters, fluctuations compared with theory of sampling (Exs. 19.12 and 19.13), 372.
- Migration, Random, refs., 508.

- Milk-yield in cows, Correlation with age (Table 11.4), 200; (fig. 11.9), 212; constants (Ex. 11.3), 225; correlation ratios (Ex. 19.1), 259.
- Milton, John, Use of word "statist," 4.
- Miner, J. R., Tables for calculation of correlation coefficients, 525.
- Mises, R. von, refs., *Wahrscheinlichkeit, Statistik und Wahrheit*, 495; *Wahrscheinlichkeitsrechnung*, 496.
- Mixed sampling, 336, 347-348.
- Mode—generally, 124-125; def., 124; approximate determination from mean and median, 125; diagram showing position relative to mean and median (fig. 7.2), 118; weighting of, 306; refs., 502.
- Modifying central ordinates, 489.
- Modulus as measure of dispersion, 144; see Precision.
- Mogno, R., refs., Interpolation, 526.
- Mohl, R. von, refs., *Geschichte und Literatur der Staatswissenschaft*, 498.
- Moir, H., refs., Frequency-curves (mortality), 508.
- Molina, E. C., refs., Bayes' theorem, 523.
- Moments—first, def., 116; second, def., 135; general, def., 154; expression of moments about mean in terms of those round an arbitrary point, 155-156; calculation of, 156-159; Sheppard's corrections for, 160; of bivariate distribution, footnote, 214; standard errors of, 394-404; correlation between errors in, 394-404; refs., 505, 517-520.
- Moments, Examples of, Height distribution, 156-158, 160; marriage distribution, 158-159, 160; weight distribution (Ex. 9.1), 167; milk yield distribution (Ex. 9.5), 167-168.
- Montessus de Ballore, R. de, refs., *Probabilités et Statistiques*, 496.
- Moore, L. Bramley, Data cited from, (Table 6.9), 98; refs., inheritance of fertility and fecundity (under Pearson), 513.
- Morant, G., refs., Poisson distribution, 506.
- Mortality; see Death-rates.
- Mortara, G., refs., *Lexiconi di Statistica Metodologica*, 526.
- Movements, Correlation, in two variables, Methods, 292-296; refs., 512-513.
- Multiple correlation coefficient, 277-279; calculation of, 278; relation with measure of closeness of fit for simple curves, 329; use of standard error in judging significance of, 409; testing significance of, 456-458; see Correlation.
- NEGATIVE classes and attributes, 13.
- Newbold, Ethel M., Application of partial correlation methods to coefficients not determined by product-moment method, footnote, 270; refs., frequency-distributions, accidents, 506.
- Newsholme, Sir A., refs., Birth-rates, correction for age-distribution, 514; *Vital Statistics*, 497.
- Newton's formula, in Interpolation, 464-468; binomial coefficients in (Table 24.4), 470.
- Neyman, J., refs., Representative method in sampling, 516; use and interpretation of test criteria, 521, 523;  $\chi^2$  distribution, 521; small samples, 523.
- Niceforo, A., refs., *La Méthode statistique*, 496, (*Il Metodo Statistico*, 526); *La Misura della Vita*, 501.
- Nixon, J. W., refs., Experimental test of normal law, 506, 507.
- Normal dispersion, in Lexis' sense, 369.
- Normal distribution, 169; generally, 180-187; deduction from binomial distribution, 177-180; ordinates, 182-183; table of ordinates, Appendix Table 1; areas, 183-184; table of areas, Appendix Tables 2 and 3; standard deviation, 182; mean deviation, 182; moments, 182;  $\beta_1$  and  $\beta_2$ , 182; seminvariants, 182; fitted to a given distribution (fig. 10.3), 187; quartile deviation, 184-185; range  $\pm 3\sigma$  cuts off all but small fraction of whole, 185; as an error distribution, 185-186; occurrence of, in Nature, 186; place of, in theory, 186-187; numerical examples of use of tables, 183-184; normality of sampling distributions, 437-438; refs., general, 505-506; dissection of compound curve, 508. For normal correlation, normal surface, see Correlation, Normal.
- Norton, J. P., Data cited from (Table 11.5), 201; refs., *Statistical Studies in the New York Money Market*, 512.
- Numerical data, Statistics concerned with, 2.
- Nybelle, H. C., refs., *Theorie der Statistik*, 497.
- OATS, Home-grown, Index-number of prices of, Correlated with price index of animal feeding-stuffs (Table 11.7), 203, 215-218.
- Ogive curve, Galton's, 150-151.
- Oldis, E., refs., Sampling of correlation coefficient (under Cheshire), 522.
- Oppenheim, A., refs., Charlier's form of the frequency function (under Aitken), 515.
- Order of a class, 14; of generalised correlations, regressions, deviations, and standard deviations, 264; of multiple correlation coefficient, 278.
- Orthogonal polynomials, 324.
- Osculatory interpolation, 484.



- PABST, M., refs., Sampling of rank correlation coefficient (*under* Hotelling), 522.
- Paciello, U., refs., Variation, 527.
- Pairman, E., refs., Corrections to moments, 504.
- Palgrave, Sir R. H. I., *Dictionary of Political Economy*, 498.
- Parabolas, Fitting of, to data, 309-331; def., 310; degree of, 310.
- Parameters, Statistical, def., footnote, 373.
- Pareto, V., refs., *Cours d'économie politique*, 501.
- Parkes, A. S., refs., Sampling of attributes, 516.
- Partial association; *see* Association, Partial.
- correlation; *see* Correlation, Partial.
- Pauperism, Correlation with earnings and out-relief (Ex. 11.2), 224, 270-272; with out-relief, proportion of aged, etc., 272-275, 288-291.
- Pearl, R., refs., Probable errors, 519; *Introduction to Medical Biometry*, 497.
- Pearse, G. E., Data cited from, (Table 6.14), 106; refs., corrections to moments, 504.
- Pearson, E. S., refs., *The Application of Statistical Methods to Industrial Standardisation*, 496; tests for normality, 519; probable errors, 519; distribution of range, 519; polychoric coefficients, 500;  $\chi^2$  test, 521; use and interpretation of test criteria, 521, 523; sampling distribution of correlation coefficient, 521, 522, 523; small samples generally, 523.
- Pearson, Karl—contingency, 68-69; "correction" to coefficient of contingency, footnote, 69; coefficient of variation, 149; definition of  $\beta$ 's, footnote, 161; skewness, 162; binomial apparatus, 176; system of curves, 192; relationship between normal correlation and contingency, 239; sampling methods, 399; data cited from, 73, (Ex. 5.1), 79-80, 98, 125, 199; refs., historical notes, 498; biography of Galton, 498; obituary of Pearson by Yule, 498; correlation of characters not quantitatively measurable, 499; contingency, etc., 500, 501; mode, 502; standard deviation, 504; coefficient of variation, 504; correction to moments, 504; influence of broad categories on correlation, 504; frequency curves and correlation, 506-507; binomial distribution and machine, 507; hypergeometric series, 507, 517; dissection of compound normal curve, 508; general methods of curve fitting, 507; correlation and correlation ratio, 509, 510, 511, 512, 514; fitting of principal axes and planes, 510, 515; testing fit of regression and other curves, 510; inheritance of fertility, 513; correlation between indices, 514; weighted mean, reproductive selection, 514; curve fitting, 515; sampling of attributes, 516-517; probable errors, 519-520; sampling generally, 519, 523; tables of probability integrals for small samples, 519, 523;  $\chi^2$  distribution, 521; small samples, 523; (Editor) *Tracts for Computers*, 525; *Tables for Statisticians and Biometricians*, 525; *Tables of B-function*, 525; *Tables of Gamma-Function*, 525; *Tables of Elliptic Integrals*, 525.
- Pearson curves, 192.
- Peas, Applications of theory of sampling to experiments in crossing, 353.
- Pecten, Correlation between two diameters of shell, 197; constants (Ex. 11.3), 225.
- Pepper, J., refs., Sampling, 519, 520.
- Percentage loss in weight, Relation with temperature, for certain soils (Table 17.5), 322; curve fitted to data, 320-323; diagram (fig. 17.5), 324.
- Percentage, Standard error of, 351; when numbers in samples vary (Ex. 19.11), 372; *see also* Sampling of attributes.
- Percentiles, 150-151; def., 150; advantages and disadvantages, 151; use for unmeasured characteristics, 150-151; standard errors of, 380-382; correlation between errors of sampling in, 385; refs., 504, 517-520.
- Perozzo, L., refs., Applications of theory of probability to correlation of ages at marriage, 508.
- Persons, W. M., refs., *Index-numbers*, 503.
- Petals of *Ranunculus bulbosus*, Frequency of (Ex. 6.5 (d)), 110; unsuitability of median in case of such a distribution, 120.
- Peters, J., refs., Multiplication tables, 524.
- Petty, Sir William, refs. (*under* Hull), *Economic Writings*, 498.
- Pietra, G., refs., Interpolating plane curve, 515; *Statistica*, 526; interpolation, 526; variation, 528; statistical relations, 528.
- Platykurtic curves, 165.
- Plaut, H., refs., *Anwendungen der math. Statistik auf Probleme der Massenfabrication*, 497.
- Poincaré, H., refs., *Calcul des Probabilités*, 495, 516.
- Poisson, S. D., 169; refs., Sex-ratio, 517; *Recherches sur la Probabilité des Jugements*, 506.
- Poisson distribution, 169, 187-191; mean, standard deviation, third and fourth moments, 189-190; seminvariants, 190; frequency polygons (fig. 10.4), 190; illustrations, 191; ref. to tables of, 190.

- Polynomials, Fitting of, to data, 309-331; degree of, 310; shortcomings of, 329; orthogonal, 324; differences of, 464; possible forms of, in interpolation, 472-473; *see* Curve fitting; Interpolation.
- Poppies, Stigmatic rays on, Frequency (Table 6.2), 84; unsuitability of median in case of such a distribution, 120.
- Population, Estimation of, between censuses, 127-129; curve fitted to growth of, in England and Wales, 325-327; refs., 502.
- Positive classes and attributes, Def., 13; number of positive classes, 17; sufficiency of, for tabulation, 17; expression of other frequencies in terms of, 20-21.
- Precision, 144; def., 186; of estimates, 335; varies with square root of number of observations, 357.
- Pretorius, S. J., Data cited from, (Table 6.8), 96, (Table 6.10), 99; refs., skew frequency surfaces, 511.
- Prices, Index-numbers of, 129-130; use of geometric mean in, 129-130; refs., 502-503.
- Principal axes, in correlation, 231; in fitting straight lines, footnote, 314.
- Probability, and statistical inference, 9-10, 335; use of, in sampling distributions, 375-376; refs., 516, (Italian) 527. — integral, 183; *see* Normal distribution.
- Probable error; *see* Error, Standard.
- Pseudo frequency-distributions, 105, 108.
- Punched cards, Recording of information on, 76-77.
- Purposive sampling, 336, 346-348.
- QUARTILE deviation; *see* Quartiles.
- Quartiles, quartile deviation and semi-interquartile range, 147-148; generally, 147-149; defs., 147, 148; determination of, 147-148; ratio of q.d. to standard deviation, 148, 149; advantages of q.d. as measure of dispersion, 149; difference between deviations of quartiles from median as measure of skewness, 162; q.d. of normal-curve, 184-185; standard errors, 380-382, 385-386; refs., 504, 517-520.
- Quetelet, L. A. J., *Lettres sur la théorie des probabilités* (Ex. 19.2), 371.
- RANDOM sampling, 336-345; technique of, 339-345; numbers (Tippett's), 341-344; importance of, 345-346; *see* Sampling; Simple sampling.
- Range, as measure of dispersion, 134.
- Ranks, 150-151; rank correlation, 246-249; relationship with grades and grade correlation, 249-251; sampling of rank correlation coefficient, 410.
- Ranunculus bulbosus*, Frequency of petals (Ex. 6.5 (d)), 110; unsuitability of median for such distributions, 120.
- Reed, L. F., refs., Curve fitting, 515.
- Registrar-General: Correction or standardisation of death-rates, 305, refs., 514; estimates of population, refs., 502; data cited from Reports of, 40-41, 59-60, 83, 100, 198, 292-294, 294-295, 304, (Table 17.6), 326, (Table 19.1), 304, 364-365, 365-366, 468.
- Regressions—generally, 206-211; def., curves of, 207, coefficients of, 213; total and partial, 262-263; curvilinear, 207; test of curvilinearity, 245, 409; reduction to linear form in certain cases, 242-243; standard errors of coefficients, 408-409; test of significance of, 443; test of linearity of, 455-456; refs., 510-511, 514-515.
- Reserves and discounts in American banks, Correlation (Table 11.5), 201, (fig. 11.2), facing 204.
- Residuals, 311; sum of squares minimised by method of least squares, 311-312; calculation of sum of squares of, 327-328.
- Rhind, A., refs., Tables for computing probable errors, 520.
- Rhodes, E. C., refs., Law of error, 508; fitting polynomials, 515; sampling, 517, 520.
- Rider, P. R., Data cited from, 374; refs., recent advances, 495; small samples, 523.
- Rietz, H. L., refs., Frequency-distributions, 508; small samples, 523; *Mathematical Statistics*, 496; (Ed.) *Handbook of Mathematical Statistics*, 497.
- Ritchie-Scott, A., refs., Correlation of polychoric table, 500.
- Robinson, G., refs., *Calculus of Observations*, 496, 515, 524.
- Robinson, S., refs., Experiments on the  $\chi^2$  test, 521.
- Romanovsky, V., refs., Frequency-curves, 508; multiple regressions, 511; curve fitting, 515; sampling, 523, 524.
- Room space, Deficiency in, data from 1931 Census Housing Report (Table 5.5), 77.
- Ross, Sir R., refs., Frequency-curves (Epidemiology), 508.
- Roth, L., refs., *Elements of Probability*, 495.
- Royer, E. B., refs., Contingency, 500.
- Russell, W. T., refs., *Medical Statistics*, 497.
- Rutherford, Lord, refs., Poisson distribution, 506.
- SALISBURY, F. S., refs., Correlation, 511 (under Kelley).
- Salvemini, T., refs., Interpolation, 526.
- Salvosa, L. R., refs., Tables of Pearson's Type III Function, 525.
- Sampling, Theory of—introductory remarks, 9-10; preliminary notions,

- generally, 332-348; types of sampled universe, 332-334; estimation from samples, 334-335; precision of estimates, 335; types of sampling, 336; random sampling, 336-346; bias, 337-339; technique of random sampling, 339-340; lottery sampling, 340-341; Tippett's numbers, 341-344; sampling from infinite universes, 344-345; from hypothetical universes, 345; importance of random sampling, 345-346; purposive sampling, 336, 346-347; mixed sampling, 336, 344, 347-348; stratified sampling, 336, 347-348; simple sampling, 350; sampling distributions, 373-377; refs., 516.
- Sampling of attributes—conditions assumed in simple sampling, 350; standard deviation of number or proportion of successes in  $n$  events, 350-352; examples from artificial chance, 352-353; standard error, 353; probable error, 353-354; case when proportion of successes is estimated from the data, 354-355; examples, 355-356; case when chance of success or failure is small, 356; standard error independent of size of universe, 356-357; precision, 357; limitations of simple sampling, 357-358; comparing  $\bar{x}$  sample with theory, 359-360; comparing one sample with another independent thereof, 360-361; comparing one sample with another combined with it, 361-362; effect of removing conditions of simple sampling, 362-368; application to sex-ratio, 363-365; sampling from limited material, 367; alternative approach, 368-369; refs., 516-517. *See also* Binomial distribution; Normal distribution; Correlation, Normal.
- Sampling of variables, Large samples—generally, 373-412; sampling distributions, 373-375; use of, 375-377; simple sampling, 378-379; approximations in theory of large samples, 379-380; standard error, 380; for standard error of particular parameters, *see under* Error, Standard, or under the particular parameter; comparison of two samples, 387-388, 402-403; effect of breakdown of simple sampling conditions on standard error of mean, 388-391; general theorems on standard errors of moments, 394-398; effect of Sheppard's corrections on standard errors, 399; refs., 517-520.
- Sampling of variables, Small samples—generally, 434-461; estimates, 434; of arithmetic mean, 434-435; of variance, 435-436; degrees of freedom of estimates, 436-437; tests of significance, 437; assumption of normality, 437-438;  $t$ -distribution, 438-442; applied to two samples, 442-443; to significance of regression coefficients, 443;  $z$ -distribution, 443-444; analysis of variance, 444-449; significance of correlation coefficient, 449-453; Fisher's transformation for, 451-453;  $t$ -test for, 453; significance of correlation ratio in uncorrelated universe, 453-455; of measure of linearity of regression, 455-456; of multiple correlation coefficient, 456-458; refs., 521-522.
- Sanders, H. G., refs., *Field Experimentation*, 497.
- Saunders, Miss E. R., Data cited from, 44.
- Savorgnan, F., refs., *Variation*, 528.
- Scale reading, Bias in, 86-87.
- Scarlet fever, Ages at death from, (Table 6.11), 100; (fig. 6.11), 101; mean, 117; median, 121.
- Scatter diagram, 205-206; generalised, 275-277.
- Scheibner, W., Difference between arithmetic and geometric, arithmetic and harmonic means (Exs. 8.12 and 8.18), 153.
- Scottish Milk Records Association, 408.
- Screws, Measurements on (Table 6.3), 84.
- Semi-interquartile range; *see* Quartiles.
- Seminvariants, Def., 165; calculation of, 166; of normal distribution, 182; of Poisson distribution, 190; standard errors (Ex. 21.6), 412.
- Sex-ratio of births, Correlation with total births (Table 11.6), 202, 212, (fig. 11.10), 213, 245-246; constants (Ex. 11.3), 225; applications of theory of sampling to, 363-365; refs. (*under* Vigor), 517; standard error of ratio of male to female births (Ex. 19.8), 371.
- Shakespeare, W., Use of the word "statist," 4.
- Shea, J. D., refs., *Fitting polynomials*, (*under* Birgs), 515.
- Sheppard, W. F., Correction of standard deviation and higher moments for grouping, 160, 399; theorem on correlation of normal distribution grouped around medians (Ex. 12.4), 240; refs., calculation and correction of moments, 505; normal curve and correlation, 506; theory of sampling, 510, 520.
- Shewhart, W. A., refs., *Engineering Applications of Statistical Method*, 497; *Economic Control of Quality of Manufactured Product*, 497; small samples, 524.
- Shohat, J. (Chokhate, J.), refs., *Sampling*, 524.
- Significance, Levels of; *see* Levels of significance; tests of significance, 335-336, 437.
- Simple curve fitting; *see* Curve fitting.
- Simple interpolation, 462.
- Simple sampling of attributes, 350-353; limitations of, 357-359; applications of,

- 359-362; effect of removing limitations of, 362-368; simple sampling of variables, 378-379; effect on standard error of mean of removing limitations, 388-391.
- Sinclair, Sir John, Use of words "statistical," "statistics," 4-5.
- Sipos, A., refs., Time series, 513.
- Skew or asymmetrical frequency-distributions, 94-98; *see also* Frequency-distributions.
- Skewness, 96, 98; measures of, 162-164; standard error of Pearson's measure of, 407.
- Small chances, 191; *see* Poisson distribution.
- samples; *see* Sampling of variables, Small samples.
- Smith, H. B., refs., Time correlation, 513.
- Smith, C. D., refs., Tchebycheff inequalities, 524.
- Snedecor, G. W., refs., *Calculation and Interpretation of Analysis of Variance*, 524.
- Snow, E. C., refs., Estimates of population, 512; lines and planes of closest fit, 515.
- Soil, Relationship between temperature and percentage loss in weight; *see* Percentage loss in weight.
- Solomons, L. M., refs., Limits to a measure of skewness, 505.
- Soper, H. E., refs., Tables of Poisson Distribution, 506; *Frequency Arrays*, 508; probable error of correlation coefficient, 520; of bi-serial expression for correlation coefficient, 520; sampling, 520, 524.
- "Sophister" (pseudonym), refs., Small samples, 524.
- Southey, Robert, cited *re* Cosin's "*Names of the Roman Catholics, etc.*," 105.
- Spahlinger vaccine for tuberculosis in cattle, Example, 425-426.
- Spearman, C., "Foot-rule" coefficient of rank correlation, footnote, 249; effect of errors of observation on the standard deviation and correlation coefficient, 298-299; refs., effect of errors of observation, 513; rank method of correlation, 510, 513.
- Spurious correlation of indices, 300-301; refs., 513-514.
- Standard deviation; *see* Deviation, Standard.
- error; *see* Error, Standard; for standard error of a particular parameter, *see* under that parameter or under Error, Standard.
- Standardisation of death-rates, 305-306; refs., 514.
- "Statist," Occurrence of the word in Shakespeare and Milton, 4.
- "Statistic," Use of singular form, 3-4.
- Statistical, Introduction and development in meaning of the word, 4-5; *Statistical Account of Scotland*, 4; Royal Statistical Society, 5; scope of statistical methods, 2-10; design of statistical inquiries, 335.
- Statistical series, Interpolation of, 468-470.
- Statistics, Introduction and development in meaning of word, 4-6; def., 3; theory of, def., 3; sketch of field of, 6-10; popular attitude towards, 10.
- Stature, Correlation of, for father and son: (Table 11.3), 199; diagrams (fig. 11.3), facing 204, and (fig. 11.8), 211; constants (Ex. 11.3), 225; correlation ratios, 245; testing for normality, 232-237; for isotropy, 238-239; diagonal distribution (fig. 12.2), 234; contour lines (fig. 12.3), 236.
- Stature of males in the United Kingdom: (Table 6.7), 94, (fig. 6.6), 95; calculation of mean, 117, and of median, 121; of means and medians of individual countries (Ex. 7.1), 131; of standard deviation, 138-139; of percentiles, 151; of mean deviation, 146; of s.d., m.d. and quartiles of individual countries (Ex. 8.1), 152; of third and fourth moments, 156-158, 160; of  $\beta_1$  and  $\beta_2$ , 161; of skewness, 163-164; distribution fitted to normal curve (fig. 10.3), 187; standard errors of mean and median, 384; of first to ninth deciles, 385; of standard deviation, 400-401; of third and fourth moments, 404; correlation between errors in mean and s.d., (Ex. 21.5), 412.
- Stead, H. G., refs., Correlation coefficients, 513.
- Steffensen, J. F., refs., *Recent Researches*, 496, 524; interpolation, 524.
- Stevenson, T. H. C., refs., Birth-rates, correction of, for age distribution (*under* Newsholme), 514.
- Stigmatic rays on poppies, Frequency; *see* Poppies.
- Stirling, James, Expression for factorials of large numbers, 178.
- Stoessiger, B., refs., Probability integrals for small samples (*under* Pearson), 519, 523.
- Straight line fitted to data, 313; reduction of non-linear data to linear form, 316-320.
- Stratified sampling, 336, 347-348.
- "Student" (pseudonym), Mnemonic for platy- and leptokurtosis, 165; standard deviation of distribution of rank correlation coefficient, 410; refs., Poisson distribution, 506; elimination of spurious correlation due to position in time or space, 513; probable errors, 520; distribution of means of samples not drawn at random, 520; probable

- error of mean (*t*-distribution), 524; small samples, 524.
- "Student's" *t*-distribution, 438-443; form of, 439; tables of, 439-440, and Appendix Table 5; applications of, 440-442; comparison of two samples, 442; significance of regression coefficients, 443; significance of correlation coefficient, 453.
- Subdivision of intervals, in interpolation, 477.
- Subnormal dispersion, in Lexis' sense, 369.
- Sugar beet, Determination of sugar content, as illustration of sampling technique, 347-348.
- Supernormal dispersion, in Lexis' sense, 369.
- Sur- and super-tax, Data on incomes liable to, (Table 6.5), 89; median, upper quartile and ninth decile (Ex. 8.3), 153.
- t*-DISTRIBUTION; see "Student's" *t*-distribution.
- Tables of functions, etc., refs., 524-525; see also under subject headings.
- Tabulation of statistics of attributes, 14, 22; of a frequency-distribution, 88-89; of a correlation table, 197-198.
- Tangential interpolation, 484.
- Tappan, M., refs., Partial correlation, 511.
- Tschebycheff, refs., Fitting polynomials (see Isserlis), 515; means, 520; inequality (under Camp), 521, (under Smith, C. D.), 524.
- Tschouproff, Tschuprow, etc., see Tschuprow.
- Tedeschi, T., refs., Interpolation, 527.
- Temperature and percentage loss in weight of certain soils; see Percentage loss in weight.
- Tests of significance, 335-336; with  $\chi^2$ , 418-421; small samples, 437. See also Sampling of variables, Small samples.
- Tetrachoric *r*, 251-252; differs from product-moment correlation coefficient, 253; standard error of, 408.
- Thiele, T. N., refs., *The Theory of Observations*, 505.
- Thomson, G. H., refs., *The Essentials of Mental Measurement*, 496; computation of regression coefficient, etc., 511.
- Thorndike, E. L., refs., Methods of measuring correlation, 510.
- Ticket sampling, 340.
- Time-correlation problem, 292-296; refs., 512-513.
- Tippett, L. H. C., Sampling numbers, 341-344; sampling distributions obtained by use of, 374-375; refs., extremes of samples (under Fisher), 522; *The Methods of Statistics*, 497.
- Tocher, J. F., Data cited from, (Ex. 9.3), 167, 168; (Table 11.4), 200; correlation of milk-yield and butter fat, 408; refs., contingency (under Pearson), 500.
- Todhunter, I., refs., *History of the Mathematical Theory of Probability*, 498.
- Trachtenberg, M. I., refs., Property of the median, 504.
- Transvariazione, refs. (Italian), 527-528.
- Truncated frequency-distributions, 102-103.
- Tschebycheff, P. L.; see Tchebycheff.
- Tschuprow, A. A., Coefficient of contingency, 70-71; refs., *Korrelationstheorie*, 496; partial correlations, 511; mathematical expectations of moments, 520; distribution of means, 524.
- Tuberculosis in cattle, Vaccine for, Example, 425-426.
- Type of array, Def., 196.
- Types of universe, 332-334; of sampling, 336.
- ULTIMATE classes and frequencies, Def., 15-16; sufficiency of, for tabulation, 16.
- Undertakings, Electricity; see Electricity.
- Universe, Def., 25; specification of, 26; types of universe for sampling purposes, 332-334; finite and infinite universes, 332-333; universe of universes, 334.
- U-shaped frequency-distributions, 101-102, 104.
- VALUE of estates in 1715 (Table 6.12), 105, (fig. 6.13), 103.
- Variables, Theory of, Generally, 82-308; sampling of, generally, 373-461; see Sampling of variables.
- Variance, for square of standard deviation, 135; standard error of, 399; estimates of, 434-435; analysis of, see Analysis.
- Variate, Def., footnote, 82; see Variables.
- Variate-difference correlation method, 292-296, 477; refs., 512-513.
- Variation, Coefficient of, 149-150; standard error of, 405-406.
- Variation, refs. (Italian), 527.
- Velocity-distance relation among extragalactic nebulae, (Table 17.1), 309-310; straight line fitted to, (fig. 17.1), 310, 315-316.
- Venere, A., refs., Means, etc. (under Gini), 527.
- Venn, John, refs., *Logic of Chance*, 495, 516, 517.
- Veronese, G., refs., Interpolation, 527.
- Verschaffelt, E., refs., Measure of relative dispersion, 504.
- Vigor, H. D., Data cited from, (Table 11.6), 202; refs., sex-ratio, 517.
- Vinci, F., refs., Variation, 528.
- WAGES, Minimum rates for agricultural labourers, see Agricultural labourers; of agricultural labourers, correlated

- with out-relief, pauperism, etc., *see* Earnings.
- , Real, refs., 503.
- Walker, Helen M., refs., *History of Statistical Method*, 498.
- Warner, F., refs., Defects in school-children, notation for statistics of attributes, 499.
- Water analysis, Methods of, refs., 506.
- Waters, A. C., refs., Estimating intercensal populations, 502.
- Weather and crops, Correlation, 291-292; refs., 512.
- Weight of criminals, Relation with mentality (Table 5.6), 78.
- of males in the United Kingdom (Ex. 6.6), 111; mean, median and mode (Ex. 7.3), 132; standard deviation, mean deviation and quartiles (Ex. 8.2), 152; moments,  $\beta_1$ ,  $\beta_2$  and skewness (Exs. 9.1 and 9.2), 167; standard error of mean (Ex. 20.5), 392; of median and quartiles (Ex. 20.3), 392; of standard deviation (Ex. 21.1), 412.
- Weighted mean; *see* Mean, Arithmetic; *also* Mean, Geometric; Median; Mode.
- Weldon, W. F. R., Dice-throwing, (Table 6.15), 107, 351, 419, 423-424.
- Westergaard, H., refs., *Theorie der Statistik*, 497; *Contributions to the History of Statistics*, 498.
- Wheat-shoots, Distribution of (Table 18.1), 338.
- Whipple, G. C., refs., *Vital Statistics*, 497.
- Whitaker, Lucy, Data cited from, (Ex. 10.17), 194-195; refs., Poisson distribution, 507.
- Whiting, M. H., Data cited from, (Table 5.6), 78.
- Whittaker, E. T., refs., *Calculus of Observations*, 496, 515, 524.
- Wicksell, S. D., refs., Correlation, 513; in case of non-linear regression, 511.
- Wilks, S. S., refs., Analysis of variance, 524.
- Will, H. S., refs., Curve fitting, 513.
- Willcox, W. F., Citation of Bielfeld, 4.
- Willis, J. C., Data regarding *Chrysomelidae* (Table 6.13), 106.
- Wilson, G. S., and others, Use of coefficient of variation, 150; refs., *The Bacteriological Grading of Milk*, 505.
- Winters, F. W., refs., Small samples (*under* Shewhart), 524.
- Wishart, John, refs., *Field Experimentation*, 497; sampling distributions, 520, 524.
- Wolfenden, H. H., refs., Mortalities and death-rates, 514.
- Woo, T. L., Relationship between laterality of hand and laterality of eye (Ex. 5.10), 81; tables for testing significance of correlation ratio and multiple correlation coefficient, 455, and refs., 524.
- Woods, Frances, refs., Index-numbers, 503; index-correlations (*under* Brown), 511, 513.
- Woods, Hilda M., refs., *Medical Statistics*, 497.
- Working, H., refs., Time series, 513.
- Working classes, Cost of living, refs., 503.
- YATES, F., Data cited from, (Table 18.1), 338; refs., bias in sampling, 516.
- Yield of grain, Data on, (Ex. 6.5 (c)), 110, (Table 23.2), 446.
- of milk, Correlated with age in cows; *see* Milk-yield.
- Young, A. A., refs., Age statistics, 501.
- Yule, G. Udny, Problem of pauperism, 288-291; use of principal axis in curve fitting, footnote, 314; data cited from, 40, 42, 86, 106, (Table 11.6), 202, (Table 11.9), facing 218, 351-352, 446, 456; refs., history of words "statistics," "statistical," 498; obituary of Karl Pearson, 498; attributes, association, consistence, etc., 499, 500; isotropy, influence of bias in statistics of qualities, 500; determination of mode, 502; frequency curves, 506; application of Poisson distribution, 506; correlation, 509, 510, 511, 520; pauperism, 512, 513; birth-rates, 513, 514; time correlation problem, 513; correlation between indices, 514; sex-ratio, 517; fluctuation of sampling in Mendelian ratios, 517; probable errors, 520;  $\chi^2$  in case of association and contingency tables, 521.
- z-DISTRIBUTION, *see* Fisher's z-distribution.
- Zimmerman, E. A. W., Use of words "statistics," "statistical," in English, 4.
- Zimmerman, H., refs., Multiplication tables, 525.
- Zizek, F., refs., *Die statistischen Mittelwerthe* and translation, 502.
- $\beta$ -COEFFICIENTS, 161; standard errors of, 406.
- B-function, Tables of, refs., 525; use of, in z-test, 444.
- $\gamma$ -coefficients, 161.
- F-function, Tables of, refs., 525.
- $\chi^2$ —generally, 413-433; analogy with Lexis' Q, 369; def., 416-417; distribution, 417; tabulation of P for, 413, 425; cf. also Appendix Table 4 and diagram A1; use as test of significance, when cell frequencies are known *a priori*, 418-421; properties of the distribution, 422; normality for large  $\nu$ , 422; conditions on application of test, 422-423; effect of taking into account signs of deviations, 423-424; levels of significance, 424-425; additive property of, 426-427; estimation of theoretical frequencies from data, 427-429; experiments on, 429-430; goodness of fit, 430; refs., 520-521.