

Money and General Disequilibrium: Economic Policies for the Integration of Monetary and Value Theories

Rajas Parchure

Gokhale Institute of Politics and Economics,
Pune 411 004, India
June 2011

Abstract

This paper is an inquiry into the question of the existence of equilibrium for a completely flexible monetary market economy from a theoretical standpoint that can be clumsily yet adequately labeled as 'Classical-Keynesian' or as 'neo-Ricardian post-Keynesian'. In parallel with the neo-Walrasian investigations of this question, this paper too concludes that equilibrium does not exist. But it goes one step further to quantify and compute the general disequilibrium in the form of a 'deflationary gap', and it also shows how disequilibria, both of the deflationary and inflationary types, can be corrected by a coordinated application of fiscal and monetary policies.

Does a completely flexible, competitive monetary economy have the inbuilt mechanisms to clear all markets and achieve a full employment equilibrium? The post 1965 literature has concluded that it does not [Hahn 1965, Clower 1967 Arrow and Hahn 1971, Grandmont 1983]. The idea that it does not was originally advanced by Keynes (1936). Its reaffirmation by modern writers, and that too by using the neo-Walrasian methodology which is so different from the Keynesian methodology, is surprising indeed. For in a subject like economics which is riddled with controversies that run deep into the very fundamentals of the subject, such a unanimity of conclusion is entirely unexpected: is it a fluke or is it evidence of the sheer robustness of the thesis that it should be thrown up irrespective of the methodology used to investigate it? To make further headway towards settling this question it may be worthwhile to conduct an inquiry into the thesis from a third perspective. If it is found that the thesis continues to be thrown up that will strengthen the grounds for considering it as valid. And economists of otherwise divergent schools of thought may have the satisfaction of agreeing upon one more result that is important from the viewpoint of practical policy even while agreeing to disagree about the theories that produce the result.

Accordingly, this paper investigates the question from the standpoint of classical capital and value theory as articulated by Sraffa (1960), whose methods are decidedly opposed to traditional neoclassical and to a large extent neo-Walrasian methods as well [Eatwell and Milgate 1998, Petri 1998]. This standpoint has been chosen because it is particularly well suited for the purpose of investigating the interconnections between money and the real variables viz. outputs, relative prices, investment employment and the like. The model to be presented in this paper has three interconnected components. First, the Sraffa system. This contains a depiction of how

capital consisting of diverse physical inputs is transformed by the application of labour into the gross and net national incomes consisting of diverse physical outputs. Considering that money is “a particular branch of the general stock of society” as Adam Smith [1776, Book II, Chapter II] taught us a long time ago, it finds a natural place as a constituent of capital in the Sraffa system and it is from here that it would partly exercise its influence on prices, distribution and outputs. Other real activities that will be influenced by the presence of money are consumption, saving and investment. Accordingly, the second component is a linear expenditure system [Stone 1954] that describes the manner in which the national income, whose formation and distribution is depicted in the Sraffa system, is utilized in the activities of consumption and saving. The linear expenditure system, besides its Keynesian flavour, has proved to be an empirically powerful description of actual consumption behaviour. The third component is a model of asset-liability matching in which banks determine the term structure of interest rates in the course of assigning their deposit supplies to loan demands subject to solvency/liquidity constraints. The loans advanced by banks to industries are used to finance a portion of their capital employed which appears in the Sraffa system with the interest costs and the remaining portion is financed by equity capital which earns profits. It will be shown that this somewhat sketchily outlined theoretical framework makes it possible to shed light on several questions that lie at the heart of the integration of monetary and value theories, viz. the relation of price theory and income analysis [Clower 1965], the relation between Walras’s Law and Keynesian economics [Clower 1965, Leijonhufvud 1968, Barro and Grossman 1976], the role of money in economic growth [Tobin 1965] and the eternal question of the neutrality of money.

The general plan of this paper is as follows. Section I gives the sectoral classification adopted, the model balance sheets of the sectors and their interdependences. Section II formulates the system of equations to determine the money wage and prices with monetary, financial and real elements appearing in the cost of production. Section III proposes an integrated model of the level and term structure of interest rates. Section IV contains the equations for the apportionment of income between the consumption goods in terms of the linear expenditure system and savings which are further apportioned between monetary and financial assets. Section V models the demand for capital goods; and Section VI formulates the procedure to determine the market-clearing outputs of capital goods and the rate of the growth that satisfy the demands of consumption goods industries and public sector’s demand. Sections VII and VIII outline an algorithm for solving all the component systems of equations simultaneously and give a numerical illustration that brings out the nature of the disequilibrium. Section IX contains a brief discussion on the causes and magnitude of the disequilibrium. Practical questions of the design and effective co-ordination of monetary and fiscal policies are deeply intertwined with the purely theoretical question of the existence of equilibrium in a monetary economy. Sections X and XI illustrate how the individual instruments of fiscal and monetary policies can be used to eliminate disequilibria of both deflationary and inflationary types. The controversy over the existence question has given rise to a subsidiary literature on the formal role of money and money-intermediaries in a general equilibrium system [Niehans 1969, Ostroy 1973, Grandmont and Younes 1972, Ostroy and Starr 1974]. Appendix I gives a *raison d’ etre* for money and money intermediaries in terms of their role in enhancing the productive efficiency of an economy. Appendix 1 shows how monetary exchange can improve the productive efficiency of an economy. Appendix II models

the impact of default risk on interest rates. Appendix III relaxes an assumption about the exogeneity of household demand for money and demonstrates that irrespective of whether the quantity theory is valid or invalid, a monetary economy has no equilibrium solution.

I Private and Social Wealth

Consider a closed competitive capitalist economy that uses currency and demand deposits as mediums of payment. The economy is divided into five sectors; worker households, capitalist households, industries, banks and government. Their balance sheets are as given in Table 1(a), Table 1(b), Table 1(c), Table 1(d) and Table 1(e).

Table 1(a): Workers' Balance Sheet

Workers	
Liabilities	Assets
	C_L Currency
Own Wealth W_L	D_L Deposits
	E_L Equity
Total =	Total

Table 1(b): Capitalists' Balance Sheet

Capitalists	
Liabilities	Assets
	C_K Currency
Own Wealth W_K	D_K Deposits
	E_K Equity
Total=	Total

Table 1(c): Banks' Balance Sheet

Banks	
Liabilities	Assets
Workers' deposits D_L	dK Loans
Capitalists' deposits D_K	
Industrial deposits D_I	
Total =	Total

Table 1(d): Industries Balance Sheet

Industries	
Liabilities	Assets
Loans dK	D_I Bank Balances
Equity $E = E_L + E_K = eK$	S Stocks
Total =K	Total = K

Table 1(e): Government's Balance Sheet

Government	
Liabilities	Assets
Currency $C = C_L + C_K$	Goodwill
Total =	Total

To keep the focus on essentials, it has been supposed in the above that households do not carry commodity stocks or avail consumer loans, that industries use only demand deposits and not currency to make payments, that the government (only at the preliminary stage) does not raise loans or make investments, that industries neither save nor manage financial portfolios and that banks are run as mutual companies without equity capital and with no intermediation costs but their profits accrue to the capitalist households. (Other possibilities e.g. proportionate distribution, do not affect the central conclusion). While the private wealth of this society is $W_L + W_K$, the net wealth is S , the stocks carried by industries.

II Prices

The system of price equations for a monetary economy needs to be formulated. Table 1(a) shows the assets and liabilities of the industrial sector. The liabilities side shows that both debt and equity have been used to finance the holding of assets. If e and d_q are the proportions of equity and debts of maturity q ($q = 1 \dots T$) with $e + \sum d_q = 1$ and if r and i_q represent the rate of profit and the interest rates on debts of maturity q , the weighted average cost of capital is $z = er + \sum d_q i_q$. Further, if the stocks $\sum A_{ji} P_j$ used in industry i are such that they are completely used up in the annual production cycle (Sraffa 1960) and if wL_i and $P_i B_i$ represent the wage bill and sales revenue of industry i , the price equations seen from the "liabilities side", are

$$eK_i r + \sum d_q K_i i_q + K_i + wL_i = P_i B_i \quad \forall i \quad \dots (1a)$$

where $K_i = D_{oi} + \sum A_{ji} P_j$ is the capital employed in industry i includes D_{oi} , the transactions balances of industry i . The same equations seen from the "assets side" are

$$(D_{oi})z + (\sum A_{ji} P_j) (1+z) + wL_i = P_i B_i \quad \forall i \quad \dots (1b)$$

In equations (1a), (1b), which represent the Sraffian version, all industries exactly use up the beginning-of-period stocks A_{ji} during the period. In other words, every item of stock turns over exactly once during the production period within each industry and in all the industries. A less restrictive characterization of the production processes, which nevertheless retains all the essential properties of Sraffa's system is as follows,

$$e (D_{oi} + \sum S_{ji} P_j) r + \sum d_q (D_{oi} + \sum S_{ji} P_j) i_q + \sum A_{ji} P_j + wL_i = P_i B_i \quad \forall i \quad \dots (1c)$$

where S_{ji} are the continuing stock requirements. Production processes as described in equation (2) recognize that industrial production takes place continuously; firms are buying inputs and selling outputs every day and they are enabled to do this by means of the holding of stocks of raw materials and semi-finished and finished goods.^{(1),(2)} The stock-turnover ratio within an industry differs across items of stocks, e.g. fast and

slow moving inventories. Also, the overall stock-turnover ratios greatly differ across industries, e.g. the retail industry rotates its stocks very rapidly as compared say to the aircraft manufacturing industry. It should be immediately clarified that none of the conclusions of this paper are in any way affected whether the price system is described by (1b) or (1c) or even a mix of (1b) and (1c) side by side, e.g., equations 1(b) to describe seasonal agricultural activities and equations (1c) to describe manufacturing activities. Two observations about the price systems (1b) and (1c) are in order. Firstly, both systems satisfy the national income identity as can be seen by adding the equations across the industries. Secondly, other than the obvious difference that (1b) and (1c) include money inventories along with commodity stocks, both systems satisfy the properties of the Sraffa's (1960) price system. Specifically, both systems move with one degree of freedom; z stands in an inverse relationship with the real wage rate given the outputs and if, besides the outputs, the interest rates too are given, there is an inverse relationship between the real wage rate and the rate of profit r . Besides, it can be readily verified that system (1c), like system (1b), is prone to reversals in the direction of relative price movements and reswitching of techniques consequent upon uniform changes in the distributive variables.^{(3), (4)}

The capital that is 'locked up' in the money stocks D_{oi} carried by the industries for transactions purposes needs to be given a concrete meaning in terms of values. This can be done by supposing that the amount of money held by an industry is some (optimal) fraction / multiple of its sales revenue,

$$D_{oi} = m_i P_i B_{is} \quad \dots (2)$$

Substituting (2) into (1) gives a set of N price equations involving $2N+T+2$ unknowns, viz. N outputs, N money prices, T interest rates, the money wage rate w , and the rate of profit r . Of these, the rates of interest will be determined by equations governing the banking activities of deposit mobilisation and lending to be spelt out in the next section. Even if the outputs and the rate of profit are provisionally taken as known, it still leaves $N+1$ unknowns (N money prices and 1 money wage rate) with only N equations. The appropriate equation to add is the demand-supply equation for *total* capital

$$\left[C + \sum m_i P_i B_{is} + \sum S_{ji} P_j \right] (1 + g) = C + D + E + V \quad \dots (3)$$

where g denotes the rate of growth and V denotes the saving.

It may be objected that symmetrical treatment has not been accorded particularly to the currency and demand deposit balances of households in terms of expressing them as fractions of household incomes as has been done for industrial cash balances in equations (2) and (3). This is easily done and demonstrated in Appendix 3. It may also surprise the reader to notice that the closing equation to determine the absolute money wage and prices, equation (3), is not the quantity theory of money which has traditionally been accorded that role. Appendix 3 demonstrates why the quantity theory fails to give acceptable solutions. It will suffice to note here that equalizing the demand and supply of money alone, which is only one component of society's capital, to the neglect of the other components will lead to unacceptable solutions.

In equation (3) the left-hand-side shows the total demand for capital, i.e., replacement demand plus new investment demand at a uniform growth rate (which is the natural outcome of assuming a fixed-coefficients constant returns to scale technology) and the right hand side is the total supply of capital including existing capital $C+D+E$ and the supply of new capital in the form of savings, V which includes dC , dD and dE , being the desired additions to currency, deposits and equity capital respectively. The rate of growth g is an additional unknown so that the total number of unknowns becomes $2N + T + 3$.

It is of utmost importance to note a methodological point. Neoclassical models require that “value capital” be given exogenously for the determination of relative prices [Petri 1999]. It might be thought that specifying the capital stock magnitudes in historically given balance sheets in Table 1 is tantamount to supposing that capital stock is given exogenously in the present model as well. This is not true. Firstly, because the unknowns in the present case are not relative prices, they are the absolute levels of the money wage and money prices. Secondly, the demand-supply equation (3) for capital contains unknown prices, saving, rate of growth and outputs. Also, the cash balances D_{oi} held by industries depend on the unknown prices and outputs. Thus, the capital stock will not be determined until all the unknowns of the system are determined [Refer to illustration in section 8 below].

Equations (1) and (3) may be combined into a system of equations to determine the money wage and prices. Let,

$$H_{ii} = (er + \sum_d i_q)(S_{ii} + m_i B_i) + A_{ii} \quad \dots (4a)$$

$$H_{ji} = (er + \sum_d i_q)(S_{ji}) + A_{ji} \quad \dots (4b)$$

$$H_i = S_i + m_i B_i \quad \dots (4c)$$

where $S_i = \sum_j S_{ji}$ denotes the total use made of commodity j in the capital stock of the economy, $m_i B_i$ is the physical (finished goods) representation of the transaction balances D_{oi} held by the industries and the term in the first bracket on the right hand sides of equation (4a) and (4b) is the weighted average cost of capital z .⁽³⁾ The solution for the money wage and prices is obtained from the system,

$$\begin{bmatrix} B_{1s} - H_{11} & -H_{21} & \cdots & -H_{n1} & -L_1 \\ -H_{12} & B_{2s} - H_{22} & \cdots & -H_{n2} & -L_2 \\ \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots \\ -H_{1n} & -H_{2n} & \cdots & B_{ns} - H_{nn} & -L_n \\ H_1(1+g) & H_2(1+g) & \cdots & H_n(1+g) & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_1 \\ \vdots \\ \vdots \\ P_n \\ w \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ -gC + D + E = V \end{bmatrix} \quad \dots (4)$$

If the magnitudes of the provisionally exogenous unknowns B_{is}, i_q, r, g and V are appropriately chosen a unique positive solution for the wage and prices is obtained.

By appropriate is meant that the magnitudes should satisfy the Hawkins-Simon conditions, $(I - A)^{-1} > 0$ and $g, r < R$ (the standard ratio), etc.

We may now proceed to the determination of the other unknowns whose magnitudes have been provisionally fixed in (4), viz. the interest rates, the saving, the growth rate, the commodity outputs and the rate of profit.

III Interest Rates

Interest rates are determined by banks in the course of matching the supplies of deposits to the demands for loans. Deposits supplied by households depend on their desired allocations of wealth and deposits supplied by the industries depend on their desired transaction balances. Loan demands from industries depend on their desired debt-equity ratios and by the risk appetites of banks that lend to them. In the course of matching deposit supplies to loan demand banks seek to maximize profits subject to a liquidity/solvency constraint. If we assume away default risk for the moment (see Appendix II), the main source of risk for banks is the possibility of deposit outflows. Established banks having a fairly diversified customer base and having sound estimates of the withdrawal behaviour of their depositors will be required to provide reserves to meet these withdrawals. Evidently, the best point on the liquidity–profitability trade-off in a competitive banking scenario is the one at which banks have arranged their loan portfolio in such a way that the inflows by way of loan repayment and interest receipts are just sufficient to meet the deposit outflows by way of withdrawals and interest payments. Now, in principle, loans of all maturities can be financed by deposits of all maturities. Banks will seek that particular assignment which will minimize the reserves and maximize the interest earning assets.

To find the optimal assignment of deposits to loans the following procedure can be followed. Let $p_u(u = 0 \dots T, v = 1 \dots T)$ be the probability of net withdrawal from a deposit of maturity u in the period v . The matrix of withdrawal probabilities is as given in Table 2. The cumulative reserve that must be provided to make a loan of maturity v from a deposit of maturity u per dollar of deposit is

$$z_{u,v} = 1 - \prod_{u=0}^v (1 - p_{u,v}) \quad \dots (5)$$

because the dollar of deposit of maturity u depletes after cumulative withdrawals from it up to the maturity v to an amount equal to the second term on the right hand side of equation (5). The optimal assignment of deposits to loans D_{uv} is obtained as the solution of the following assignment problem,

$$\text{Minimize } Z = \sum \sum Z_{uv} = \sum \sum z_{uv} D_{uv} \quad \dots (6a)$$

$$\text{s.t. } \sum_v D_{uv} = D_u \quad v = 0 \dots T \quad \dots (6b)$$

$$\sum_u D_{uv} = d_v K \quad u = 1 \dots T \quad \dots (6c)$$

$$\sum_u \sum_v D_{uv} = \sum D_u = \sum d_v K \quad \dots (6d)$$

$$D_{uv} \geq 0 \quad \dots (6e)$$

In view of (8b) it is evident that the row sums of the matrix (I-D) are all less than or equal to 1 with at least one row sum being strictly less than one. The matrix therefore, satisfies Solow's (1952) sufficient condition for a strictly positive inverse. And since the column vector on the right hand side of (1) is strictly positive so is the solution for the interest factors $(1 + i_q)^q \quad q=1 \dots T$. Of course, we need to go one step further to show that the solution of the interest factors must be strictly greater than 1, i.e. the solution for the spot interest rates i_q is strictly positive. This too is easily done. Using Cramer's rule and an elementary property of

determinants the solution of 8(a) can be expressed as

$$(1+i_q)^q = \frac{\det(I-D)_{t/d_{oq}}}{\det(I-D)} + \frac{\det(I-D)_{t/z_q}}{\det(I-D)} \quad \forall t$$

where, the matrices $(I-D)_{t/d_{oq}}$ and $(I-D)_{t/z_q}$ ($q = 1 \dots T$) are the (I-D) matrix whose t^{th} column is replaced by d_{oq} and z_q respectively. The first term on the right hand side of (3) can be shown to be equal to 1 for all $t = 1 \dots T$ simply by multiplying all columns other than the t^{th} by -1 and adding them to the t^{th} column containing d_{oq} to obtain $\det(I-D)$ in the numerator. The second term on the right hand side of (3) can be written as

$$\frac{\sum_q Z_q D_{qt}}{\det(I-D)}$$

where D_{qt} are the cofactors of the t^{th} column elements and are all positive. Thus, the expression in (3) is one plus something positive so that $(1+i_q)^q > 1$ and $i_q > 0$.

It will usually be reasonable to suppose that the probabilities of withdrawal are decreasing functions of the maturity of deposits and increasing functions of the time spent in deposit, i.e., the columns of Table 2 are decreasing sequences and the rows are increasing sequences. If the probabilities of withdrawal rise at a rising rate with the time spent in deposit the north-west corner rule itself gives the optimal solution [Mitra and Mohan 1988] and the equations in (7) will be lower triangular, i.e., loan demands of each maturity will be financed by deposits of the same or lower maturities and an upward sloping solution of the term structure of interest rates would obtain. A declining term structure is obtained in extreme circumstances e.g. because the demand for long-term loans dries up or regulatory action causes a shortage of very short term deposits which forces banks to use long-term deposits to finance short-term loans and the matching becomes upper-triangular..

The similarity of this theory with Keynes' (1936, Chapter 17) liquidity preference theory becomes at once apparent if the probabilities of withdrawal are identified with the preference for liquidity; the greater these probabilities the more bankers have to pay by way of interest to induce depositors to part with liquidity and the more they have to charge borrowers. The probabilities of withdrawal determine the asking rates of loans. On the demand side are bid rates for loans of the borrowing industries which are determined primarily by the rate of profit on equity capital (Keynes' MEC). The term structure of the bid rates depends on the probabilities of future increases in the interest rate. The greater these probabilities the greater is the insurance premium (strictly the interest rate call option premium) that borrowers will be willing to pay the bankers to cover the risk of adverse increases in the interest rate as loans come up for

renewal.. And since the risk of future interest rate increases with the time period of exposure, the term structure of bid rates for loans will most often be upward sloping except in the extremely rare instance when the market consensus is a decline in the interest rate.⁽⁴⁾

IV Income, Consumption and Saving

It will be supposed that the government supplies public goods by acquiring goods $A_{1G}, A_{2G}, \dots, A_{nG}$ from the industries and finances the expenditure by levying an income tax at a uniform rate, t . The budget equation is

$$tY = A_{1G}P_1 + \dots + A_{nG}P_n + wL_G \quad \dots (9)$$

This introduces one more unknown in the system, viz. the tax rate t and also adds an equation.

The net national income of the economy is,

$$Y = r_eK + \sum i_q d_q K + wL$$

$$\text{where } L = \sum L_i + L_G$$

The disposable incomes of the capitalist and worker households are,

$$Y_K = (1-t) [\sum i_q D_{qK} + \sum i_q (d_q K - D_q) + rE_K] \quad \dots (10a)$$

$$Y_L = (1-t) [wL + \sum i_q D_{qL} + rE_L] \quad \dots (10b)$$

The consumption behaviour of the households may be described by Stone's (1954) linear expenditure system. Thus,

$$P_i B_{ik} = P_i B_{iko} + a_i (Y_K - \sum P_i B_{iko}) \quad \dots (11a)$$

$$P_i B_{iL} = P_i B_{iLo} + b_i (Y_L - \sum P_i B_{iLo}) \quad \dots (11b)$$

where a_i and b_i are the marginal propensities to consume commodity i and B_{io} denote the 'fixed' quantities of consumption. The overall propensities to consume are $a = \sum a_i, b = \sum b_i$ ($0 < a, b < 1$) and it will be supposed that $b > a$ as a necessary condition for macroeconomic stability [Kaldor 1956, Pasinetti 1962]. Since the demand equations (11) are homogenous of degree zero in income and prices money illusion is ruled out. From (10) and (11) the saving of each group is obtained,

$$V_k = (1-a)(Y_K - \sum P_i B_{iko}) \quad \dots (12a)$$

$$V_L = (1-b)(Y_L - \sum P_i B_{iLo}) \quad \dots (12b)$$

The saving of each group will be allocated among the assets available, viz. currency, deposits and equity capital, in accordance with the desires for them. Let

where L_K is the total labour employed in the capital goods industries⁵. Equation (15b) states that the output levels of capital goods that are equal to the demands for them must be ensured by reallocating the labour between those industries⁶.

VI Market Clearing

We now turn towards the determination of the outputs. The capital goods price equations in (1) will be multiplied by $x_1 \dots x_T$ respectively to ensure clearing of the markets for capital goods.

Next, the markets for N-I consumption goods must be cleared. This is done by resetting the supplies $B_{I+1} \dots B_N$ as given in equations (1) in line with the demands for them as determined in equations (11) and adding to them the government's demand $A_{I+1,G} \dots A_{NG}$. However, as soon as the supplies of the consumption goods industries are altered, their aggregate requirement for the capital goods S_{iC} and A_{iC} undergo a change so that the equations in (15) stand altered. In other words, the solution can only be found by an iterative procedure.

It will be noted that equation (15) stands in a relationship of duality with the price system (5). Therefore, an important relationship between scalars g and r that appear in the respective systems will need to be specified. Since

$$g = \frac{V}{C + D + E}$$

we get on substitution from (10) (11) and (12), the growth-profit relationship⁷,

$$r = \frac{g((C + D + E) + B_{K0} + B_{L0} - I_K - I_L - wL_s)}{(1-a)(1-t)E_K + (1-b)(1-t)E_L} \quad \dots (16)$$

where $B_{K0} = (1-a)\sum P_i B_{iK0}$, $B_{L0} = (1-b)\sum P_i B_{iL0}$

$$\begin{aligned} I_K &= (1-a)(1-t)(\sum i_q D_{qK} + I_B) & I_L &= (1-b)(1-t)(\sum i_q D_{qL}) \\ I_B &= \sum i_q d_q K - \sum i_q D_q \end{aligned}$$

Where I_K and I_L are the swings out of the interest earnings of capitalists and workers and I_B , the profits of the banking sector, has been supposed to be paid to capitalists.

Finally, the size of the full employment labour supply is also fixed,

$$L_s = \sum L_i + L_G \quad \dots (17)$$

The foregoing fully describes the model whose implications we wish to pursue. The unknowns are N money prices of commodities, N industrial outputs, T interest rates, 1 money wage rate, 1 tax rate, 1 rate of growth and 1 rate of profit; 2N+T+4 in all. To determine them there are N+1 wage-price equations in (4), T interest rate equations in (7), I+1 equations to determine capital goods outputs and the rate of growth in (15), N-I-1 independent demand equations [with government purchases of consumption

goods from (9) added in (11)], the budget equation (9), the growth profit relation (16) and the full employment equation (17); a total of $2N+T+4$ equations in all, which *prima facie* seem to be well-matched to the respective unknowns they are supposed to determine.

It turns out, however, that the equations are contradictory. They do not give an equilibrium solution. Before proceeding to illustrate this it will be necessary to adopt a convenient algorithm because the model contains interdependent blocks of simultaneous non-linear equations that can only be solved by an iterative process of trial and error.

VII Algorithm

Start with the balance sheet data, $W_K, W_L, W_K, E_L, C_L, C_K, D_{Kq}, D_{Lq}, D_q, L_q$ the probabilities of withdrawal $p_{u,v}$ and $S_{ji}, A_{ji}, L_i, B_{is}, A_{iG}, L_G, M_i$ in system (1) and $B_{iK0}, B_{iL0}, a_i, b_i$ in (11).

Step 1 – Using the p_{uv} find the z_{uv} (equation 5), match bank assets to liabilities (equation 6) and solve for the interest rates in (7) assuming a hypothetical value for D_{oi} , the transaction balances of the industries.

Step 2 - Take hypothetical values for r, g and V and solve equations (4) for the prices and wage rate (care should be taken in taking values within the domain of viability eg., $r, g < R, b > a$ etc.)

Step 3(a) - Take a hypothetical value for the tax rate ($0 < t < 1$) and using the solution for the prices and wage rate in step 2 ascertain the values of K_i, Y_K, Y_L, V and the quantities demanded of the consumption goods B_{id} at a wage income wL_s in equations 11(a) and (b) and add to them government purchases, if any. Bisect B_{id} and B_{is} and set them as the new levels of output of the consumption goods industries. Calculate the stock and flow requirement of capital goods at these output levels in each industry and ascertain the aggregate stock and flow requirements for each capital good S_{ic} and A_{ic} in the consumption goods industries.

Step 3(b) - Calculate the maturity wise loan demands, $d_v K$. Using the solution in 3(a), replace D_{oi} by $\sum m_i P_i B_{is}$ the demand deposits of banks at the current solution of prices and outputs and find a revised interest rate solution in Step 1.

Step (4a) - Substitute S_{ic} and A_{ic} in equation (15) and find $x_1 \dots x_m$ and g .

Step (4b) - Simultaneously ascertain the size of the government expenditure at the wage-price solution in (2) and solve for the tax rate in equation (9).

Step 5 - Using values of g and t in step (4) and other data find the revised solution for the rate of profit in equation (16).

Step 6 - Multiply the capital goods price equations in (1) by $x_1 \dots x_7$ and use the value of r in step (5) and the interest rates in step (3) to find the revised solution for prices and wages. From this point onwards none of the unknowns will need to be specified exogenously; they will all be generated by the equations.

Step 7 - Repeat Steps 2 to 6 until the successively revised solution converges to the point where the demands and supplies of commodities are equalized.

A word of caution is necessary for implementing this algorithm on the computer. Considering that all of the unknowns bear definite relationships with one another as governed by the equations, when values of the unknowns are initially fed exogenously care should be taken to ensure that they are not widely off the mark from one another.

It is observed that the system converges at positive prices, interest rates, outputs, etc. and both commodity and financial markets are cleared but a glaring gap remains – the net national product at factor cost exceeds the net national product at market prices! A curious situation in which if one tries to satisfy the equations an identity is violated! The solution itself, even though it represents a disequilibrium, is unique; that is to say even if the initial exogenous values are chosen differently (subject to the word of caution mentioned above) the system converges to the same disequilibrium.

VIII Illustration

Consider an example in which the wealth of the capitalist households and worker households are $W_K = \$500$ $W_L = \$250$ respectively. Suppose the proportions in which they desire to hold assets are $h_{KC} = h_{KD0} = h_{KD1} = h_{KD2} = h_{KD3} = 0.05$ and $h_{LC} = h_{LD0} = h_{LD1} = h_{LD2} = h_{LD3} = 0.15$ for capitalists and workers respectively. Thus $h_{Ke} = 0.75$ and $h_{Le} = 0.25$ are the proportions of wealth invested in equity capital. If we suppose that industries demand loans of three maturities in equal proportions, i.e., $d_1 = d_2 = d_3 = 1/3$ the balance sheet of the banking sector including the transaction balances held by industries worth a hypothetical figure of \$10 in demand deposits will be as given in Table 3.

Table 3: Bank Balance Sheet

Liabilities		Assets	
D ₀	72.50	-	
D ₁	62.50	86.66	d _{1K}
D ₂	62.50	86.66	d _{2K}
D ₃	62.50	86.66	d _{3K}
Total	260	Total	260

Suppose the probabilities of withdrawal and the cumulative reserve factors $p_{u,v}$ and $z_{u,v}$ are as tabulated in Table 4. The initial levels of inputs and outputs in the industries and cash balances are as given in Table 5.

Table 4: Withdrawal Probabilities and Cumulative Reserve Factors

	$P_{u,v}$			$z_{u,v}$			
	1	2	3	1	2	3	
0	0.05	0.06	0.07	0	0.05	0.107	0.1695
1	0.03	0.05	0.06	1	0.03	0.0785	0.1338
2	0.02	0.03	0.05	2	0.02	0.0494	0.0969
3	0.01	0.02	0.03	3	0.01	0.0298	0.0589

Table 5: Input-Output Table

$m_i B_{is}$	S_{ji}					A_{ji}					L_i	B_i	
1	13	12	25	0	0	2	5	3	0	0	5	→	25
2	22	25	13	0	0	5	7	5	0	0	5	→	35
2.5	12	23	25	0	0	4	1	4	0	0	10	→	35
3	23	15	16	0	0	3	2	5	0	0	10	→	60
2.5	22	23	15	0	0	2	5	7	0	0	10	→	50
	92	88	94	0	0	16	20	24	0	0	40		

It is supposed that the government purchases one unit each of the five commodities and employs one unit of labour to make public services available to the citizens; $A_{iG} = 1, L_G = 1$. Thus the total labour supply is $L_s = 41$. And suppose the consumption of capitalists and workers is described by the following parameters,

$$\begin{array}{llll}
 B_{4K0} = 1.5 & a_4 = 0.05 & B_{4L0} = 0.5 & b_4 = 0.4 \\
 B_{5K0} = 1 & a_5 = 0.05 & B_{5L0} = 0.25 & b_5 = 0.5
 \end{array}$$

Let the marginal propensities to invest the annual saving in the different assets be equal to the average propensities purely for the sake of simplicity. With this data we may proceed to compute the system. Plug in hypothetical values $r=0.25, g=0.25, e=0.5, d_q=0.1666$ and $V = 10$, and compute the prices and wage rate. Follow the algorithm of Section 7. The tables 6(a) to 6(g) illustrate the convergence to the solution.

Table 6(a): Convergence of Profit and Growth

Iteration No.	r	g	x_1	x_2	x_3
1	0.1571	0.1189	1.0832	0.8791	1.0188
3	0.2170	0.1233	1.0024	1.0013	0.9980
10	0.2297	0.1281	1.0001	1.0000	0.9999
50	0.2331	0.1293	1.0000	1.0000	1.0000

Table 6(b): Convergence of Prices and Wage Rate

Iteration No.	w	P_1	P_2	P_3	P_4	P_5
1	2.5552	2.0318	1.8552	1.912	1.0964	15062
3	3.6250	2.5993	2.3714	2.4592	1.4293	1.9554
10	3.8864	2.7611	2.5188	2.6165	1.5209	2.0799
50	3.8502	2.7757	2.5324	2.6239	1.5247	2.0864

Table 6(c) : Convergence of Outputs and Excess Supplies

Iteration No.	B_1	B_2	B_3	B_4	B_5	$B_{4s} - B_{4d}$	$B_{5s} - B_{5d}$
1	27.0801	30.7696	35.6591	56.5322	48.4852	3.4677	1.5147
3	27.1175	30.8490	35.5932	53.6417	47.5926	2.0208	1.5165
10	27.1844	30.8964	35.5235	51.4583	46.0020	0.0803	0.0764
50	27.2034	30.9021	35.5065	50.9998	45.546	0.0001	0.0001

Table 6(d): Convergence of Interest Rates, Deposits and Loans

Iteration No.	i_1	i_2	i_3	D	$\Sigma d_v K$
1	0.0578	0.0973	0.1438	260.000	260.000
3	0.0531	0.0906	0.0826	273.250	277.334
10	0.0526	0.0883	0.1506	272.785	272.677
50	0.0526	0.0884	0.1508	272.786	272.786

Table 6(e): Convergence of Capital Stock, Income, Saving, Tax Revenue

Iteration No.	Y_K	Y_L	V_K	V_L	t	$T = X_G$	K
1	108.449	131.602	90.322	12.468	0.0455	10.936	545.50
3	103.257	172.252	84.371	16.202	0.0524	14.440	686.64
10	100.449	183.853	81.586	17.262	0.0541	15.383	710.14
50	101.943	182.626	82.850	17.146	0.0540	15.393	710.28

Table 6(f): Convergence of Leverage Ratios and Income Velocities*

Iteration No.	e	d	v_1	v_2
1	0.6272	0.3727	1.5263	0.6389
3	0.6120	0.3879	1.7760	0.7842
10	0.6160	0.3839	1.8647	0.8219
50	0.6159	0.3840	1.8632	0.8212

* v_1 = Narrow Money Velocity v_2 = Broad Money Velocity

Table 6(g): Convergence of Industrial Employment

Iteration No.	L_1	L_2	L_3	L_4	L_5	Total
1	5.4160	4.3956	10.1883	9.4220	9.6970	39.1190
3	5.4235	4.4070	10.1694	8.9402	9.5183	38.4588
10	5.4368	4.4135	10.1495	8.5763	9.2004	37.7768
50	5.4406	4.4145	10.1447	8.4999	9.1093	37.6093

Attention is called to the presence of a *deflationary gap* even after all the commodity and financial markets have been cleared. It will be recalled that the labour supply has been supposed to be $L_s = 41$ including 40 in the private industries and $L_G = 1$ in the public goods activity. However, as the table above shows the total employment offered is only 38.6093 (i.e. 37.6093 by the private industries and one for the public goods provision), showing a gap of 2.3907 units of labour which in value terms is,

$$\begin{aligned}
 \text{Deflationary Gap} &= w(L_s - L_d) \\
 &= 3.8502 (41 - 38.6093) \\
 &= 9.2051 \qquad \dots (18a)
 \end{aligned}$$

It will be noticed that the deflationary gap shows itself as the difference between net national product at factor cost and the net national product at market prices;

$$\begin{aligned}
 \text{Deflationary Gap} &= NNP_{fc} - NNP_{mp} \\
 &= 284.5709 - 275.3658 \\
 &= 9.2051 \qquad \dots (18b)
 \end{aligned}$$

Equivalently, it is also the gap between financial saving and the value of physical investment;

$$\begin{aligned}
 \text{Deflationary Gap} &= dC + dD + dE - \Sigma p_i F_i \\
 &= 6.7144 + 26.8578 + 66.4241 - 90.795 \\
 &= 9.2051 \qquad \dots (18c)
 \end{aligned}$$

where $F_i = B_i - \Sigma A_{ji}$ are the physical surpluses of the capital goods which constitute the net investment ΔS_i in the economy.

IX Deflationary Gap

The particular characterizations of the deflationary gap in equation (18) should not detract attention from the general conclusion that they represent, which is that a monetary economy has no equilibrium. These characterizations are simply the result of the algorithm that has been employed in which the attempt is directed towards finding a full-employment equilibrium solution in which the commodity markets and the financial market are cleared. But, for the reason that a full-employment equilibrium solution does not exist, a gap is found elsewhere in the system which can either be interpreted as an employment gap [equation (18a)], as a supply-demand gap (equation (18b)) or as a saving-investment gap (equation (18c)). With reference to Walras' law, the gap in (18c) may be regarded as the excess demand for financial assets over the value of investment and that in (18a) as the corresponding excess supply of labour, and the sum of these gaps is zero. It may be verified that the deflationary gap shows up in all circumstances, e.g. even if only currency money is used or only deposit money is used.⁽⁸⁾ In the example of Section VIII above, suppose $h_{KC} = h_{LC} = 0$ and $h_{kd0} = h_{kd1} = h_{KD2} = h_{KD3} = 0.0625$ and $h_{LDO} = h_{LD1} = h_{LD2} = h_{LD3} = 0.1875$ and $h_{KE} = 0.75$, $h_{LE} = 0.25$, $a_4 = 0.25$, $a_5 = 0.25$, $b_4 = 0.5$, $b_5 = 0.5$, i.e., there is no currency. Yet the solution $r = 0.4942$, $L_s = 41$, $L_d = 39.6687$, $NNP_{fc} = \$ 280.588$, $NNP_{mp} = \$279.500$ etc. shows a deflationary gap of \$1.088.

Some remarks on the interpretation of the deflationary gap are in order. The existence of the deflationary gap is directly attributable to the *kind* of money that is used viz. an inconvertible fiat money that is not produced under economic conditions but is nevertheless demanded for the unique functions that it performs. This implies that the regular mechanism of demand and supply which operates to clear other markets is inapplicable to money itself. Nor for that matter is the price of money itself free to vary in response to demand-supply imbalances because it is definitionally fixed at one. Currency, deposits and equity are all produced at negligible physical cost. In effect the rate of return on the production of new money itself, if it were privately produced, is infinite and the singularity that it represents is a formal equivalent of the

non-existence of equilibrium. The essential point can be phrased differently. While there are effective market forces based on the profit motive that will seek to clear the commodity markets and the financial market, there are no forces that would operate to clear the capital market, i.e., to ensure that the value of financial assets demanded (saving) must be equated to the value of physical investment undertaken or for that matter, to ensure that the labour market be cleared.⁽⁹⁾ As Keynes put it, "... men cannot be employed when the object of desire is something which cannot be produced and the demand for which cannot be readily choked off" [Keynes 1936, Chapter 17]. The deflationary gap, if interpreted in terms of equation (18a), gives the unmistakable impression that the economy is perpetrating some kind of a fraud in which L_s workers are paid wages but only L_d workers seem to be actually employed and the fraud is revealed as a violation of the accounting identity between the net national incomes measured respectively at factor cost and at market prices in (18b). On the other hand, if the requirement is placed that all L_s workers that are paid the wages should actually be employed, the result will be an excess supply of commodities in which case the deflationary gap would represent a deficiency of demand for currently produced output [Yeager 1968].

This conclusion has of course been established in the post-Keynesian literature for quite some time. Clower (1965) obtained essentially the same result when he found that in a situation in which realized factor income falls short of the aggregate money value of planned demand for factor inputs excess demand for commodities may fail to appear in the system under less than full employment conditions. Davidson (1978) explained it in terms of a structural deficiency of effective demand in a monetary economy that prevents labour market adjustments from fully realizing themselves. Chick (1983) and Rogers (1989) found the chief cause of unemployment to lie in the fact that the money rate of interest (presumed to be equal to the rate of profit) was too high in a monetary equilibrium which occurs at a point short of the full employment real sector equilibrium.

It is easy to guess that the size of the deflationary gap is directly proportional to the propensities to hold currency and/or deposits as opposed to equity capital. For instance, if $h_{KC} = h_{KDO} = h_{KDI} = h_{KD2} = h_{KD3} = 0.1$ and $h_{LC} = h_{LDO} = h_{LD1} = h_{LD2} = h_{LD3} = 0.18$ for the capitalists and workers respectively so that $h_{Ke} = 0.5$ and $h_{Le} = 0.1$ then, all else remaining the same, the disequilibrium solution obtained is $g = 0.1520$, $r = 0.3955$, $t = 0.0538$, $i_1 = 0.0559$, $i_2 = 0.0941$, $i_3 = 0.1599$, $e = 0.4065$, $d = 0.5934$, $v_1 = 1.2073$, $v_2 = 0.5142$ (v_1, v_2 are the velocities of narrow and broad money), $K = 676.4232$, $D = dK = 401.43$, $T = X_G = 14.540$, $w = 2.9393$, $L_s = 41$, $L_d = 35.7226$, $NNP_{fc} = 270.7909$, $NP_{mp} = 255.2655$ showing a deflationary gap of \$ 15.5256 and prices, outputs and employments as tabulated in Table 7.

Table 7: Effect of Shifts to Safe Assets

Industry	Price	Output	Employment
1	2.8618	27.5001	5.5000
2	2.6181	31.0156	4,4308
3	2.5899	35.2420	10.0691
4	1.4952	43,0593	7.1765
5	2.0696	37.7305	7.5461
			35.7226

X Fiscal Policy

The general deficiency of demand can be remedied by an expansionary fiscal policy financed either by issuing money or by issuing public debt with differing consequences as well as with alternative mixes of the public expenditure. Thus suppose as one instance that the government decides to finance a deficit by issuing money and purchasing additional quantities ΔA_{iG} of the 5 commodities in equal quantities. Then additional purchases of $\Delta A_{iG} = 0.6855$ entailing a total deficit financing of \$7.5855 exactly fills up the gap of \$9.2051 shown in equation (18) to give overall general economic equilibrium with $g=0.1052$, $r=0.1679$, $t=0.0548$, $i_1 = 0.0525$, $i_2 = 0.0882$, $i_3 = 0.1505$, $e=0.6156$, $d=0.6156$, $v_1 = 1.9094$, $v_2 = 0.8426$, $w=4.5034$, $L_s = L_d = 41$, $K=710.6146$, $D=dK= 273.114$, $NNP_{fc} = NNP_{mp} = 282.813$ and prices, outputs and industrial employment as shown in Table 8(a).

Table 8(a): Equilibrium with Deficit Financing

Industry	Price	Output	Employment
1	2.6034	26.9291	5.3858
2	2.3723	30.8231	4.4033
3	2.5614	35.7380	10.2108
4	1.4978	57.4641	9.5573
5	2.0298	52.1135	10.4227
			40.0000

If the government chooses an alternative expenditure-mix by buying additional quantities of only commodities 4 and 5 then purchases of $\Delta A_{4G} = \Delta A_{5G} = 2.317$ entailing a deficit financing of \$8.1078 suffices to give the equilibrium $g = 0.1125$, $r = 0.1882$, $t = 0.0543$, $i_1 = 0.0525$, $i_2 = 0.0883$, $i_3 = 0.1506$, $e = 0.6157$, $d = 0.3842$, $v_1 = 1.8981$, $v_2 = 0.8373$, $w=4.24$, $L_s = L_d = 41$, $K=710.5125$, $D=dK=273.012$ and $NNP_{mp} = NNP_{fc} = 280.942$. Or, to give one more example, if the government chooses only to employ additional labour “to dig holes and fill them up” then an additional employment of $\Delta L_G = 1.8942$ gives the equilibrium with $g=0.1126$, $r=0.1850$, $t=0.0529$, $i_1 = 0.0525$, $i_2 = 0.0883$, $i_3 = 0.1506$, $e=0.6157$, $d=0.3842$, $v_1 = 1.9547$, $v_2 = 0.8623$, $w=4.2819$, $L_s = L_d = 42.8942$, $K = 710.528$, $D= dK = 273.028$, $NNP_{fc} = NNP_{mp} = 289$ and prices, outputs and industrial employment shown in Table 8(b).

Table 8(b): Equilibrium with Deficit Financing

Industry	Price	Output	Employment
1	2.6133	26.9185	5.3837
2	2.3818	30.7835	4.3976
3	2.5445	35.7652	10.2186
4	1.4852	57.5882	9.5980
5	2.0178	52.0099	10.4019
			40.0000

If the government resorts to issuing public debt to finance the deficit the system of equations would have to be modified in two ways. Firstly, the size of the public debt M_G would have to be subtracted from the right hand side of equation (3) considering that the proceeds will be a diminution of capital funds available to the private economy. Secondly, if we suppose that the public debt is sold exclusively to banks, the maturity-wise amounts purchased by them will be added on the right hand sides of the asset-liability equations (7). (We shall suppose that the same interest rates hold for public and private debt. See Appendix III for the procedure to relax this assumption). Thirdly, in the budget equation (9) the public debt issued will be added on the left hand side as a receipt and debt-servicing expenditure according to the predetermined schedule will be added to the right hand side. Thus if it is supposed that the government issues straight debt with equal sizes of the three maturity buckets, the budget equation will be,

$$tY + M_G = \sum A_{iG} P_i + wL_G + M_{1G}(1 + i_1) + i_2 M_{2G} + i_3 M_{3G} \quad \dots (19)$$

with $M_{qG} = 1/3 M_G$. A debt financed deficit of \$8.0950 is found to bring about the following equilibrium; $g=0.1126$, $r=0.1732$, $t=0.0295$, $i_1 = 0.0526$, $i_2 = 0.0885$, $i_3 = 0.1506$, $e=0.6267$, $d=0.3732$, $v_1=1.8829$, $v_2=0.8295$, $w= 4.2850$, $K = 698.016$, $D=dK=272.658$, $NNP_{fc} = NNP_{mp} = 278.052$ and prices, outputs and employment as tabulated in Table 8(c).

Table 8 (c): Equilibrium with Debt Financed Deficit

Industry	Price	Output	Employment
1	2.5637	26.9174	5.3834
2	2.3365	30.7846	4.3978
3	2.5056	35.7654	10.2187
4	1.4635	57.5628	9.5938
5	1.9865	52.0319	10.4063
			40.0000

The choice of alternative public finance and expenditure strategies that is adopted by the government to close the deflationary gap and remove the disequilibrium belongs to the subject of political economy.

XI Monetary Policy

It is evident that the initial deflationary gap that is found in the free market context cannot be removed by the operation of the customary tools of monetary policy simply because the gap is obtained in the absence of active monetary policy. To bring out the stabilizing role of monetary policy consider the third example of Section (9) above in which government employs additional labour by deficit financing and suppose initially that the government overplays its role by hiring $\Delta L_G = 2$ (instead of $\Delta L_G = 1.894$), units of excess labour and indulges in deficit spending of \$8.5682 (instead of \$8.1078). There will now be an inflationary gap of size¹⁰

$$w(L_s - L_d) = (4.3023)(41 - 41.1278) \\ = -0.5104$$

The solution in disequilibrium stands as follows; $g=0.1118$, $r=0.1800$, $t=0.0528$, $i_1 = 0.0525$, $i_2 = 0.0883$, $i_3 = 0.1506$, $NNP_{fc} = 289.4715$, $NNP_{mp} = 289.9856$ etc. In what follows the operation of two instruments of monetary policy viz. the Cash Reserve Ratio (CRR) and Open Market Operations (OMO) to eliminate the inflationary gap are illustrated.

When banks are subjected to a CRR the system of equations will need 3 modifications. Firstly, the sizes of the effective lendable deposits are reduced by the CRR, but the reserves required to meet withdrawals will continue to be provided on total deposits held. Secondly, banks will recover the interest amounts that they pay on the CRR funds by charging them on the loans, i.e., they will add the terms $cD_q(1+i_q)^q - 1$ to the left hand sides of equation (7). Thirdly, equation (3) will now include only the effective deposits $(1-c)D$ and $(1-c)dD$ in place of D and dD .

A CRR of 8.7 per cent is found to eliminate the inflationary gap and give the equilibrium solution $g = 0.1126$, $r = 0.1760$, $t = 0.0526$, $i_1 = 0.0585$, $i_2 = 0.0885$, $i_3 = 0.1939$, $e = 0.6389$, $d = 0.3610$, $L_s = L_d = 43$, $w = 4.0349$, $v_1 = 1.9918$, $v_2 = 0.8953$, $D = 272.1550$, $dK = 247.1699$, $NNP_{fc} = NNP_{mp} = 278.4385$ and prices, output and employment as tabulated in Table 9.

Table 9: Equilibrium with CRR

Industry	Price	Output	Employment
1	2.5222	26.9197	5.3839
2	2.2991	30.7821	4.3974
3	2.4446	35.7651	10.2186
4	1.4260	57.6210	9.6035
5	1.9395	51,9830	10.3966
			40.0001

Alternatively, the gap can be eliminated by the central bank selling (previously issued) government securities to the banks. For the sake of illustration, suppose that only the required size has been previously issued by the government and is being held by the central bank. OMO requires that the system of the equations be modified in three ways. Firstly, the outstanding public debt will be subtracted from the right hand side of equation 3. Secondly, the maturity wise debts will be added on the asset side of the interest rate system (7). Thirdly, the debt servicing expenditure will appear in the budget equation (9). The inflationary gap is eliminated by the central bank selling securities worth \$1.2968 to commercial banks with one-third in each maturity bucket to give the equilibrium, $g=0.1126$, $r=0.1851$, $t=0.0547$, $i_1 = 0.0525$, $i_2 = 0.0883$, $i_3 = 0.1306$, $e=0.6169$, $d=0.3830$, $w=4.2706$, $L_s = L_d = 43$, $D=dK=272.983$, $NNP_{fc} = NNP_{mp} = 289.349$ etc. The budget equation must now be read as,

Taxes + Deficit Finance = Net Revenue Expenditure

$$15.8419 + 8.1091 = (15.8419 + 8.5413 - 0.4323)$$

in which $w\Delta L_G = 8.5413$ and $0.4323 = (1/3)(1.2968)$ = repayment of debt principal during the year. The outstanding debt itself makes no appearance on the left hand side because it is supposed that it has been previously issued and therefore, is not being used to support the current year's budget.

It should be clarified immediately that the foregoing manner of presentation should not give the impression that fiscal policy is meant for clearing deflationary gaps and monetary policy for clearing inflationary gaps : obviously both can be used for both purposes. Thus, if government expenditure slackens for say some administrative reasons but revenue collections are brisk, the resulting deflationary gap may be covered by an expansionary monetary policy provided of course that these developments take place in an initial situation in which a restrictive monetary policy is in force. More generally, both types of policies can be closely coordinated to achieve the equilibrium.

XII Concluding Remarks

Using the classical economic framework of capital and value theory as presented by Sraffa, Keynes's thesis that a monetary economy does not possess an equilibrium, has been conclusively demonstrated. Apart from that the paper can offer some clarifications on some post-Keynesian doctrinal controversies in economic theory. Firstly, it emerges quite clearly that the non-existence of equilibrium in a monetary economy can neither be attributed to any rigidity in the wage, prices or interest rates (all of these are unknowns of the system and so are perfectly variable) nor to the absence of a Walrasian auctioneer [Clower 1965, Leijonhuvud 1967, 1968] nor for that matter to the existence of uncertainty in its more or less quantifiable connotations [Keynes 1937]. The non-existence of equilibrium is attributable to the existence of an inconvertible money and a financial system based on it. To be sure, uncertainty and the resulting pessimism, because they cause asset preferences to shift towards safe financial assets (currency and deposits) causes the disequilibrium to become more severe as illustrated in Section IX, vindicating Keynes' idea. Secondly, the non-existence of equilibrium in a monetary economy is a conclusion that stands irrespective of the validity or otherwise of the quantity theory of money. Even if a world with pure outside money is considered and the quantity theory is used in place of equation (3) the deflationary gap continues to appear. And with inside money included it gives wild results (*See* Appendix III). Thirdly, the Cambridge equation relating the growth rate to the rate of profit [Kaldor 1956, Pasinetti 1962] stands generalized in a monetary/financial economic system to include the rates of interest and the financial leverage ratio. Fourthly, in the general context of both inside and outside money, any change in the values of nominal wealth W_L and W_K (trivially) leaves the real solution unaffected but any changes in the desired *composition* of nominal wealth are not neutral.

For the sake of brevity, a number of specific conclusions about the effects of changes in the data which include the production technology, consumption and saving behaviour, the monetary technology of payments and banking habits, and the fiscal

and monetary instruments etc. on the individual economic variables such as the rates of growth, profit, interest or prices, outputs and income distribution will not be reported here. Instead it should suffice to point out a drawback and a strength each of the theoretical framework adopted in this paper. The drawback is the absence of any discussion about the wealth effects on consumption demand, an issue that has featured prominently in the controversies on this subject particularly and generally in the field of monetary economics [Patinkin (1955), Friedman (1956), Hahn (1962)]. The reason for this omission is that disaggregate empirical consumption demand equations that incorporate wealth variables containing outside money and yet possess the theoretically desirable properties of homogeneity, additivity and symmetry (of the substitution matrix) are not available. Since one of the purposes of this paper is to present a theory based on empirical equations containing inter-industrial, financial and behavioural data this omission is inevitable. On the other hand, the strength of the theory lies in the use of empirical equations with numerically specific mathematical forms which can prove useful for the quantification of disequilibria and for measuring the impact of the alternative tools of fiscal and monetary policies on the economic variates.

Appendix I

Money and Economic Efficiency

The productive efficiency of an economic system has a measure that is independent of prices. It is Sraffa's standard ratio R which represents both the maximum rate of profit corresponding to a zero wage and the maximum rate of growth corresponding to zero outputs of non-basic industries. If S and A represent the stock and flow coefficients then the solution for prices and the rate of profit in the standard system is obtained from

$$SPR + AP = P$$

$$\text{i.e. } P = (I - A)^{-1}SPR$$

Letting $\lambda = 1/R$ gives the detrimental equation

$$[\lambda I - (I - A)^{-1}S]P = 0$$

whose dominant value must be positive if $(I - A)^{-1}S$ is non-negative. If the matrix $I - A$ satisfies the Hawkins – Simon conditions, $(I - A)^{-1}$ is positive and since S is strictly non-negative so is $(I - A)^{-1}S$ so that $\lambda_d > 0$ and also its reciprocal R . R is a decreasing function of the elements of A and S . [Kurz and Salvadori (1995)].

In a barter economy all industries would be required, without loss of generality, to hold stocks of all the commodities to perform transactions with one another, i.e., $S_{ijB} \geq 0 \forall i, j$. With the use of a general medium of exchange they would not be required to do so. They would instead be required to hold $M_i / P_i = m_i B_i$ worth of money stocks measured in terms of their own outputs. Then, provided

$$m_i B_i P_i < \sum S_{ijB} P_i \quad \forall i$$

the standard ratio R_m for the monetary economy must be greater than that for the barter economy R_B , i.e. the productive efficiency of the economy improves with the introduction of money.

Another source of efficiency is the saving in information processing and dissemination costs. In the absence of a numeraire, each industry would be required to give B-1 quotes for its products, i.e., B industries would give B(B-1) quotes. With a general *numeraire* there is a need for only B quotes. If the costs of information processing are considered as being included in A_{ji} , $A_{jm} < A_{jB}$ so that $R_m > R_B$. Yet another source of efficiency is the saving of decentralized inventory keeping costs and transport costs. If one commodity is generally accepted as numeraire and medium of exchange each industry must carry a separate hoard of it and incur storage, security transport costs for making or collecting payments from B-1 other industries, i.e. B(B-1) trips have to be made. If, instead, all money hoards are kept in a bank the number of hoards is reduced from B to 1 and the number of trips from B(B-1) to B as

payments are effected by bank transfers. If storage, security and transport costs are reckoned to be part of the A matrix, once again $A_{ijm} < A_{ijB}$ and $R_m > R_B$. The issue of currency against the commodity hoard and further, the creation of chequable deposits in place of paper currency, give even more economies to improve efficiency. This gives the *raison-d'etre* for the existence of money in a real economy.

Appendix II

Default Risk

It would be particularly inexcusable to ignore default risk after the world shaking bankruptcies experienced in 2007-2008. Let us therefore, generalize the interest rate equations (8) to include the risk of default. Let f_{yq} be the net default probability (i.e., net of loan recovery probability) of a loan of quality y . If $y = 0, 1, \dots$ indexes loan quality from the highest (zero default probability) to the lowest, then f_{yq} is an increasing function of y . Also it is reasonable to suppose f_{yq} rises with the maturity of loan, or at least does not decline. Then the default probability and default provisioning is as given in Table 10.

Table 10: Provisioning for default risk

Period	Loan	Default Probability	Reserve	Cumulative Reserve
0	1	f_{y_1}	f_{y_1}	f_{y_1}
1	$(1 - f_{y_1})$	f_{y_2}	$f_{y_2}(1 - f_{y_1})$	$f_{y_1} + f_{y_2}(1 - f_{y_1})$
2				
T	$\prod_{q=1}^T (1 - f_{yq})$	f_{y_1}	$f_{y_1} \prod (1 - f_{yq})$	$1 - \prod_{q=1}^T (1 - f_{yq})$

The right hand sides of the interest rate equations (7) would be segregated into loans of various qualities y and the cash flows from these with default risk loadings will be

$$L_{yq} (1 + i_{yq})^q = L_{yq} (1 + i_q)^q \left[\frac{1}{\prod_{q=1}^T (1 - f_{yq})} \right] = L_{yq} (1 + i_q)^q F_y$$

where $i_{yq} = i_q$ for $y=0$, i.e. sovereign debt and $i_{yq} = (1 + i_q) (F_y)^{1/q} - 1$.

Underestimation of default probabilities on mortgage loans (due to federal guarantees and inflation in collateral prices), overestimation of loan recovery probabilities (for the same reasons) and the resulting underpricing of default risk and/or underinsurance against default risk and weakening of the loan recovery process due to securitization on the one hand and on the other a tightening of federal funds rate from one per cent in June 2003 to 5.25 per cent from June 2006 to June 2007 brought down the present values of existing loans as well as collateral values thus precipitating default, the scale of which was exacerbated by the practice of linking the loan size to collateral value instead of the repayment capacities of borrowers.

Appendix III

The Quantity Theory of Money

Suppose we decide to use the quantity theory of money in the price system in place of equation 3 to determine the absolute wage and prices. A conceptual difficulty lurks in the background. To the extent that deposits are money, irrespective of whether we choose to call only demand deposits as money or all deposits as money their supplies are identically equal to their demands, so there is no independent equation. Another difficulty pertains to the static nature of the quantity theory; it supposes fixed outputs. But the economic system being considered is one that produces a surplus and grows. There is no equation available in the literature that represents the quantity theory for a growing economy.

In its absence let the following equations represent the money market clearing condition in its narrow and broad forms in a growing economy,

$$[C + D_0 + \Sigma m_i P_i B_i](1 + g) = C + D_0 + dC + dD_0 \quad \dots \text{(i)}$$

$$[C + D + \Sigma m_i P_i B_i](1 + g) = C + D + dC + dD \quad \dots \text{(ii)}$$

If these are used in place of equation (3) the solutions obtained for the example of Section VIII are $g=0.0924$, $r=17339347$, with negative prices, national income, etc. and $g=0.07$, $r = -1.2747e^{-20}$ and negative prices respectively. It does not matter if C and D_0 or C and D are made as functions of the incomes of workers and capitalists.

If money is considered to be purely outside money and if it is supposed that the demand for money by the capitalists and workers is given by the fractions 0.05 and 0.025 of their incomes then the following quantity theory equation can be added in the place of equation (3)

$$(0.05)Y_K + (0.025)Y_L + \Sigma m_i P_i B_i = C = \$2500(\text{say})$$

Then supposing for simplicity that there are no taxes and public expenditure, the solution obtained is $g = 0.1494$, $r = 0.1791$, $w = 24.3269$, $L_S = 40$, $L_d = 32.2946$, i.e., a deflationary gap of \$ 187.45. This can, of course, be eliminated by deficit financing in the usual way. Thus, if the government chooses to buy 1.6393 units of each of the 5 commodities, a deficit expenditure of \$172.6479 gives the equilibrium solution $g = 0.0882$, $r = 0.1089$, $w = 41.0046$, $NNP_{fc} = NNP_{mp} = 2629.755$ and prices, output and employment as given in Table 11. In equilibrium the quantities of currency demanded by the capitalists, workers and industries are \$ 26.0468, \$ 63.2643 and \$ 2583.338 respectively whose sum exactly equals the existing supply of \$2500 plus the new currency supplied by the deficit financing route worth \$ 172.647. In other words, even though the quantity theory of money gives a viable solution in a world with outside money and the money market is cleared it fails to give an equilibrium solution for the economic system as a whole. It may be observed in passing that while the size the currency supply makes no difference to relative prices, the manner in which the new money is injected i.e., the composition of expenditure chosen certainly affects the relative prices.

Table 11: Equilibrium with currency printing

Industry	Price	Output	Employment
1	23.6332	25.7340	5.1468
2	22.3211	29.4111	4.2015
3	25.8938	37.2806	10.6116
4	13.6359	65.4097	10.9016
5	19.8290	45.4926	9.0985
			40.000

Endnotes

1. A slight generalization of the original Sraffa system has been employed to capture the continuous nature of industrial production;

$$\left(\sum S_{ji} P_j\right)r + \sum A_{ji} P_j + wL_i = P_i B_{is} \quad \forall i$$

where S_{ji} represents the stocks/inventories of commodity j and A_{ji} and L_i are the flow inputs that are matched to the flow outputs.

2. When the system in note one above is extended to a monetary economy the money stocks carried by the industries are added to the stocks of inventories, i.e., the capital stock employed in each industry $K_i = D_{oi} + \sum S_{ji} P_j$, which in turn is financed by a mix of debt and equity. This forms the basis of the price equations (1) in the text. Incidentally the way equation (1) has been written may give an impression that the repayment of debt principal has been omitted from the cash outgoes on the left-hand-sides. That is not so. Debt principal is indeed repaid but it is concealed by the simultaneous loan renewals to replenish the stocks that serve as the security for the loan.

3. It may be noted that Sraffa's idea of the maximum rate of profits, i.e., standard ratio, will now stand generalized. The price system (1) for unit output levels can be written in vector-matrix notation as

$$eZPr + \sum d_q ZP i_q + AP + wL = P$$

where $z_{ii} = (m_i B_i + S_{ii}) / B_i$ and $z_{ji} = S_{ji} / B_i$. If $w = 0$, then the reciprocal of the dominant eigenvalue of the matrix $(I-A)^{-1}Z$ gives a maximum rate that corresponds to Sraffa's standard ratio (provided $(I-A)^{-1}Z$ is non-negative) which we may call the maximum weighted average cost of capital. If we let $w = 0$ and $i_q = 0$, the maximum rate of return on equity capital is obtained as the reciprocal of the dominant

eigenvalue of the matrix $(I-A)^{-1}(eZ)$. Denoting the former by R and the latter by R^* , we obtain $R^* = \frac{R}{e}$

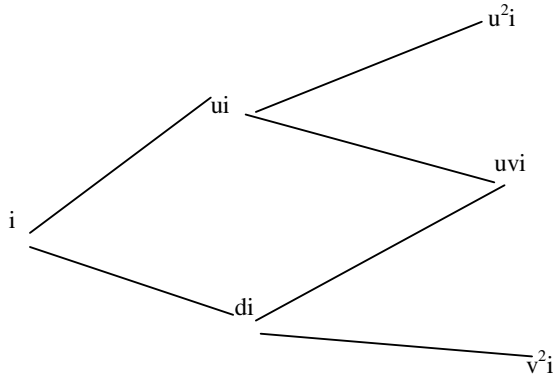
$= R + R(d/e)$ in which the second term shows the effect of leverage.

4. In reality banks co-ordinate four sets of interest rates; the asking rates for deposits, banks' bid rates for deposits, banks' asking rates for loans and borrowers' bid rates for loans. Depositors' asking rates for deposits of various maturities depends on the probabilities of requiring cash at different points of time in future and borrowers' bid rates for loans of different maturities depend on the rate of profit and the probability of adverse increases in the future interest rates. Both the sets of interest rates are likely to contain subjective probability estimates while bankers' bid and ask rates are likely to be based more on objective probabilities.

To derive the term structure of bid rates a simple binomial formula can be used. The rate of profit on equity can be expressed as

$$r = z + d(z - i)$$

where z is the average cost of capital and d is the debt to equity ratio. It is also equal to bid rate for the 1-period ban. Now suppose the future course of the interest is as shown below:



All outcomes over horizon n for which $u^n i > i$ are undesirable for the borrower. To hedge the risk he must buy d interest rate call options, or be prepared to pay the premium to the banker for the facility of locking in the spot rate for longer maturities. Clearly, the call option premium is an increasing function of the period n , which will cause the bid rates for loans to increase with maturity.

5. The similarity of the system of equations (15) with Sraffa's standard system must be noted. Specifically, the standard system represents an extreme special case of (15) in which the levels of output of all non-basic intermediate and consumption goods are equal to zero. It should be noted though that Sraffa's standard system applies to a point-input point-output technology whereas the standard system that is embedded in equation (15),

$$(SX)R + AX = \hat{B} X$$

$$LX = \Sigma L$$

as applicable for continuous-input, continuous-output technologies.

6. If a commodity j serves in a dual capacity both as a capital good and as a consumption good, its consumption demand equation will appear in equation (11) and the investment demand in equation (15). The quantity demanded for final consumption B_{jF} will be added in the j^{th} equation in (15) to A_{jC} and A_{jG} .

7. The very notion of the rate of economic growth stands modified in the context of a monetary economy. When there is no money the growth rate of the economy is the rate of *physical* addition to the stock of capital. In a multicommodity situation the rate of balanced industrial growth is $g = g_i = F_i/S_i$, (where F_i is the net output of capital good i and S_i its existing stock) which simultaneously represents the rate of growth of private wealth, national wealth and national product. A monetary economy, in contrast, is characterized by a divergence between private and national wealth. Thus, the growth rate appearing in equations (5), (15) and (16) is the growth rate of private wealth, i.e., currency, deposits and equity. Any attempt to retain the real economy notion of growth e.g. by excluding industrial cash balances $m_i B_i$ from equation (15), violates the duality between the price system (5) and the output system (15) and gives nonsensical solutions. Nor is confining the scope of growth to include only industrial wealth, i.e., loans plus equity, satisfactory because it precludes the possibility of understanding the manner in which the propensities to hold currency, deposits and equity affect the system.

8. This case corresponds to Keynes' definition of money in the *General Theory*, "As a rule, I shall, as in my *Treatise on Money*, assume that money is co-extensive with bank deposits" [Keynes 1936, Chapter 13, n1]. Keynes offered no justification for omitting currency from money. This is particularly surprising because sharp increases in the currency-deposits ratio during 1930-1933 (increase from 0.2 to 0.32 in the U.S.) was a major cause of financial collapse during the Great Depression. Nevertheless it does not affect the central conclusion. Incidentally the crisis of 2008 presents an opposite anomaly in the behaviour of the currency-deposit ratio; it fell steeply from 2.49 in August 2008 to 1.77 in December 2008. However, closer examination of the data will reveal that currency holdings were rising rapidly prior to and during that period they rose by seven per cent. There was a loss of confidence in banks and Fed responded by injecting a massive dose of deposits into banks raising them from \$311 bn in August 2008 to \$459 bn in December 2008; an increase of 50 per cent in four months! causing the

currency-deposit ratio to decline. But even after that deposits dipped down to \$390 bn in March 2009 (See Federal Reserve Board website).

9. Incidentally, the deflationary gap may also emerge when saving from current income gets diverted towards the purchase of existing assets, e.g. real estate.

10. These remarks have some bearing on the question of an appropriate foundation for money. In the post-war period proposals were vigorously advanced to base money on commodity reserves [Graham 1942, Graham 1943]. The subject was debated and the general consensus seemed to be that the plan was not feasible for a variety of reasons including the non-storability and/or high storage costs of the commodities and the heavy transaction costs. [Friedman 1951]. However, post Sraffa, it may be a good idea to ponder over the feasibility of introducing a money founded upon and convertible into the standard commodity. The standard commodity consists only of those goods that are directly or indirectly used to produce all the commodities in the economy and would therefore, consist of a more manageable set of commodities which, besides, would serve as a more elastic foundation for money than say gold. Moreover, the prices of all commodities would have a more or less definite relation with the prices of commodities that constitute the standard commodity.

11. The inflationary gap does not emerge only on account of government's action. It can arise if declines in private preferences for currency and deposits cause an excess demand for currently produced outputs of capital and consumption goods industries. This will produce an inflationary boom in the economy which, however, will last only for the duration over which the declines take place where after the deflationary gap would once again prevail.

References

- Arrow, K.J. and F.H. Hahn (1971), *General Competitive Analysis*, San Francisco: Holden-Day Inc.
- Barro, R.J. and H.I. Grossman (1976), *Money, Employment and Inflation*, Cambridge: Cambridge University Press.
- Chick, V. (1983), *Macroeconomics After Keynes*, London: Philip Allan.
- Clower, R.W. (1965), The Keynesian Counterrevolution: A Theoretical Appraisal, in *The Theory of Interest Rates*, F.H. Hahn and F.P.R. Brechling (Eds.), pp. 103-125, London : Macmillan
- Davidson, P. (1978), *Money and the Real World*, 2nd edn., London: Macmillan.
- Deleplace, G. and E.J. Nell (1996), *Money in Motion*.
- Eatwell, J. and M. Milgate (1998), Some Deficiencies of Walrasian Intertemporal general equilibrium in Mongiovi G. and F. Petri (Eds.) *Value, Distribution and Capital: Essays in Honour of Pierangelo Garegnani*, London and New York: Routledge.
- Friedman, M. (1951), Commodity Reserve Currency, *Journal of Political Economy*, 59(3): 203-217, January.
- (1956), The Quantity Theory of Money – A Re-statement in Friedman M. (Ed.), *Studies in the Quantity Theory of Money*, pp. 3-21, Chicago: Chicago University Press
- Graham, B. (1943), The Critique of Commodity Reserve Currency: A Point by Point Reply, *Journal of Political Economy*, 11(1): 66-69, February.
- Graham, F.D. (1942), Social Goals and Economic Institutions, Princeton: Princeton University Press, Chapter V.
- Grandmont, M. (1983), *Money and Value: A Reconsideration of Classical and Neoclassical Monetary Theories*, Cambridge: Cambridge University Press.
- Hahn, F.H. (1962), Real Balances and Consumption, *Oxford Economic Papers*, 14: 117-123, June.

- Hahn, F.H. (1965), On Some Problems of Proving the Existence of An Equilibrium In A Monetary Economy, in *The Theory of Interest Rates*, (Eds.), F.H. Hahn and F.P.R. Brechling, pp. 126-135, London: Macmillan
- (1972), Keynesian Economics and General Equilibrium in Harcourt G.C. (Ed.), *The Microfoundations of Macroeconomics*, London: Macmillan.
- Hayek, F.A. (1931), *Prices and Production*, London: George Routledge and Sons.
- Kaldor, N. (1956), Alternative Theories of Distribution, *Review of Economic Studies*, 23: 83-100.
- (1982), *The Scourge of Monetarism*, Oxford: Oxford University Press.
- Keynes, J.M. (1936), *The General Theory of Employment Interest and Money*, London: Macmillan.
- (1937), The General Theory of Employment, *Quarterly Journal of Economics*, 51: 209-223.
- Kurz, H. and N. Salvadori (1995), *Theory of Production: A Long Period Analysis*, New York: Cambridge University Press.
- Leijonhufvud, A. (1968), *On Keynesian Economics and The Economic of Keynes*, London: Oxford University Press
- Lavoie, M. (1992), *Foundations of Post-Keynesian Economic Analysis*, Hants: Edward Elgar.
- Mitra, S.K. and S.R. Mohan (1988), *On the Optimality of North West Corner Solution in Some Applications of the Transportation Theory*, Technical Report No. 8801, Indian Statistical Institute, Delhi.
- Moore, B.J. (1979), The Endogenous Money Stock, *Journal of Post-Keynesian Economics*, 2: 49-70.
- Niehans, J. (1969), Money in a Static Theory of Optional Payment Arrangements, *Journal of Money, Credit and Banking*, 1: 706-726.
- Ostroy, J.M. (1973), The Informational Efficiency of Monetary Exchange, *American Economic Review*, 63: 597-561
- Ostroy, J.M. and R.M. Starr (1974), Money and the Decentralisation of Monetary Exchange, *Econometrica*, 42: 1093-1113.
- Panico, C. and N. Salvadori (Eds.) (1992), *Post-Keynesian Theory of Growth and Distribution*, Cheltenham: Edward Elgar.
- Pasinetti, L. (1962), Rate of Profit and Income Distribution In Relation to the Rate of Economic Growth, *Review of Economic Studies*, 29: 267-279.
- Patinkin, D. (1955), *Money, Interest and Prices: An Integration of Monetary and Value Theory*, New York: Harper and Sons.
- Petri, F. (1999), Professor Hahn on the ‘neo-Ricardian’ criticism of neoclassical economics in Mongiovi G. and F. Petri eds. Value, Distribution and Capital: Essays in *Honour of Pierangelo Garegnani*, London and New York: Routledge.
- Rogers, C. (1989), *Money, Interest and Capital: A Study in the Form of Monetary Theory*, Cambridge: Cambridge University Press
- Smith, A. (1776), *The Wealth of Nations*, Dent: London & Toronto.
- Sraffa, P. (1960), *Production of Commodities by Means of Commodities*, Cambridge: Cambridge University Press.
- Stone, J.R.N. (1954), Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand, *Economic Journal*, 64: 511-527.
- Tobin, J. (1965), Money and Economic Growth, *Econometrica*, 33: 671-684, October.
- Tsiang, S.C. (1966), Walras’s Law, Say’s Law and Liquidity Preference in General Equilibrium Analysis, *International Economic Review*, 7: 329-345.

- Wicksell, K. (1898), *Interest and Prices*, translated by R.F. Kahn London: Macmillan (1936).
- Wicksell, K. (1901), *Lectures In Political Economy*, Vol. II, translation by Claasen E. and L Robbins, London: Routledge (1935).
- Yeager, E. (1968), Essential Properties of the Medium of Exchange, *Kyklos*, 21: 45-68.