A New Model of the Interaction between Industries and Households

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I Introductory

Closed input-output models incorporate household consumption in 2 ways, (a) by including the consumption vector in the input coefficients matrix and/or (b) by including some of the consumption coefficients in the input matrix and including the consumption-income relationship of others in the final demand vector. In this paper we propose a model in which household consumption demand is kept entirely outside the input matrix. In effect households are neither wholly nor even partially treated as industries so that all interactions between industries and households can only take the form of independent market forces of demand and supply; the forces of demand being conditioned by consumer habits, conventions, tastes and preferences and the forces of supply, by technology and profits. It is then shown that this model can fruitfully be extended/applied to the analysis of public goods provision by government and to the subject of international trade.

The paper is divided into seven sections. The basic model is presented in the second section which shows the existence of an economically meaningful equilibrium solution. The market processes by which the economy effects the transition from disequilibrium to equilibrium is spelt out in the third section. The fourth section incorporates public goods in the basic model. The fifth section presents an application of the model to the theory of international trade in both intermediate capital and consumption goods, and the sixth section sums up the conclusions.

II Basic Model

Consider a static self-replacing economy that produces n commodities by means of commodities and homogenous labour. The economy has a fixed endowment of labour L*. We may suppose that of the n commodities some are purely capital goods, some are dual purpose goods that are used for intermediate as well as final consumption purposes and others are purely consumption goods. However, we shall suppose that there is in the system at least 1 basic good [Sraffa 1960] which is used directly or indirectly in the production of all commodities. We also make the assumption of a zero rate of excess profit which is customary in static general equilibrium models.

The system of price equations is,

$$P^T A + wL^T = P^T \qquad \ldots (1)$$

where A and L are the matrix of input coefficients and the vector of labour coefficients, P is the vector of prices and w is the wage rate. 1 of the n prices in (1) can be set as the numeraire so that equation (1) determines a unique, positive set of

relative prices and the real wage rate provided the economy is viable, i.e., I-A obeys the Hawkins-Simon conditions.

We shall confront the price system (1) by Stone's (1954) linear expenditure system,

$$P_iC_i = P_iC_{i0} + \alpha_i (Y - \Sigma P_iC_{io}) \quad \ldots (2)$$

where the national income $Y = wL^* = \Sigma P_iC_i$ and $\Sigma \alpha_i = 1$. It is well-known that this system of demand equations, besides being empirically proven, satisfies the properties of additivity, homogeneity and symmetry which are considered desirable from the standpoint of consumer theory. Substituting the (p, w) solution from (1) into (2) gives the solution for the quantities of consumption goods demanded C_{id}. This done, the solution for the corresponding gross outputs X_i is obtained from

$$AX + C = X \quad \ldots (3)$$

It can be readily shown that the solution of (1), (2) and (3) is a unique positive equilibrium solution for 2n variables; n-1 relative prices, n absolute industrial outputs and the real wage rate. All commodity markets have been made to clear because the solution of the quantities supplied (equation 3) is such as to meet the direct and indirect requirements for final consumption demands (equation 2) calculated at the price-wage solution of equation 1. It remains only to show that this price output solution clears the labour market as well. To see this post-multiply the price system (1) by the output solution of (3), i.e., $X = (I - A)^{-1}C$ to obtain,

$$P^TAX + WL^T (I - A)^{-1}C = P^TX$$

Now $P^T = wL^T(I-A)^{-1}$ is the solution of (1) so $P^TC = wL^TX$. Since P^T is a 1xn vector of prices and C is an n x 1 vector whose elements are $C_i = C_{io} + [\alpha_i (Y - \Sigma P_iC_{io}] / P_i$,

$P^TC = wL^TX = \Sigma P_iC_{io} + (wL^* - \Sigma P_iC_{io}) (\Sigma \alpha_i) = wL^*$

so that $L^TX = L^*$; i.e., the aggregate labour requirement of the n industries to satisfy market demand is exactly satisfied by the available labour supply. In other words, the labour endowment L* has been allocated across industries in such a way that commodity demands are equal to supplies.

III Adjustments in Disequilibrium

It is not enough merely to prove the existence of an equilibrium unless it can also be shown that an economic system, starting from a state of disequilibrium, will inevitably develop forces within itself that will impel to move to the position of equilibrium. This will be demonstrated with the aid of a numerical example. Thus consider an economy that produces 3 goods of which commodity 1 is a purely capital good, commodity 2 serves both as a capital good and as a consumption good and commodity 3 is a pure consumption good. Suppose that it has a labour endowment L* = 12000 units. The technology described by A and L is

$$A = \begin{bmatrix} 0.2 & 0.1 & 0.15 \\ 0 & 0.25 & 0.30 \\ 0 & 0 & 0 \end{bmatrix} \quad L = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$$

The demand equations are:

$P_2C_2 = 30P_2 + 0.4\,[12000W - (30p_2 + 50P_3)]$
$P_3C_3 = 50P_3 + 0.6\,[12000W - (30P_2 + 50P_3)]$

The equilibrium solution for this economy is w = 0.4, $P_2 = 2.2666$ and $P_3 = 2.03$, Y = wL* = 4800, $C_{2d} = 847.147$, $C_{3d} = 1418.62$, $X_1 = 478.113$, $X_2 = 1696.977$, $X_3 = 1418.62$. Thus the economy in its state of equilibrium looks as follows:

$$95.622 + 0P_2 + 956.226w = 478.113$$
$$169.697 + 424.244P_2 + 6787.908w = 1696.977P_2$$
$$212.793 + 425.586P_2 + 4255.86w = 1418.620P_3$$

Suppose that due to some disturbance/dislocation the economy stands in a state of disequilibrium,

$$80 + 0P_2 + 800w = 400$$
$$200 + 500P_2 + 8000w = 2000P_2$$
$$160 + 320P_2 + 3200w = 1066.66P_3$$

Then the market prices of commodities 2 and 3 will stand at $P_{2d} = 0.9806$ and $P_{3d} = 2.70133$, i.e., in the first case the price will be less than the cost of production and in the latter greater than the cost of production. Industry 2 makes losses and Industry 3 makes excess profits of 1376.788 and 751.43 respectively while industry 1 is at breakeven. Consequently industry 2 will contract and industry 3 will expand as they plough back the profits in the next round of production. The cutback in the levels of input use in the second industry consistent with its technology will be as follows:

$$\left(\frac{200}{C_2}\right)(-1376.78) = -60.7406$$
$$\left(\frac{500P_2}{C_2}\right)\left(\frac{-1376.788}{P_2}\right) = -151.8516$$
$$\left(\frac{8000w}{C_2}\right)\left(\frac{-1376.788}{w}\right) = -2429.625$$

where $C_2 = 200 + 500P_2 + 8000w = 4533.33$. Industry 3 on the other hand will expand by

$$\left(\frac{160}{C_3}\right)(751.432) = 55.5246$$
$$\left(\frac{320P_2}{C_3}\right)\left(\frac{751.432}{P_2}\right) = 111.049$$

$$\left(\frac{3200w}{C_3}\right)\left(\frac{751.432}{w}\right) = 1110.492$$

where $C_3 = 160 + 320P_2 + 3200w = 2165.333$. Industries 2 and 3 will now employ 9880.864 units of labour and the remaining 2119.136 will be employed in industry 1. The new disequilibrium state is as follows:

$211.913 + 0P_2 + 2119.136w = 1059.568$
$139/259 + 348.148P_2 + 5570.372w = 1392.593P_2$
$215.524 + 431.049P_2 + 4310.492w = 1436.83P_3$

The markets prices of commodities 2 and 3 now are $P_{2d} = 3.006$ and $P_{3d} = 1.9945$ at which industry 2 makes an excess profit of 1029.980 and industry 2 incurs a loss of 50.9874. The results of repeating the steps above are recorded in the graphs which show the convergence to equilibrium. Graphs 1 and 2 shows the convergence of prices to their cost of production, Graphs 3, 4 and 5 of the outputs to their equilibrium levels and graphs 6 and 7 of the excess profit/loss show their convergence to zero.

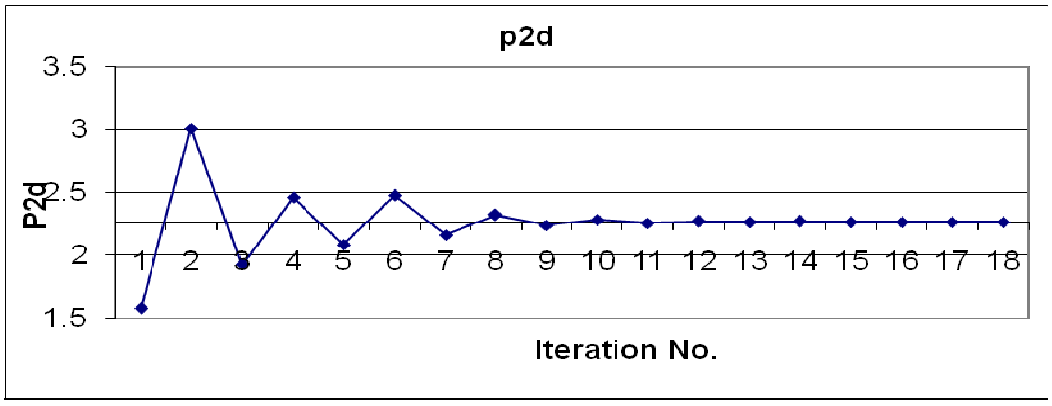
It may be pointed out in passing that we have nowhere been required to invoke a "Walrasian auctioneer" or "recontract mechanisms" to effect the transition from disequilibrium to equilibrium.

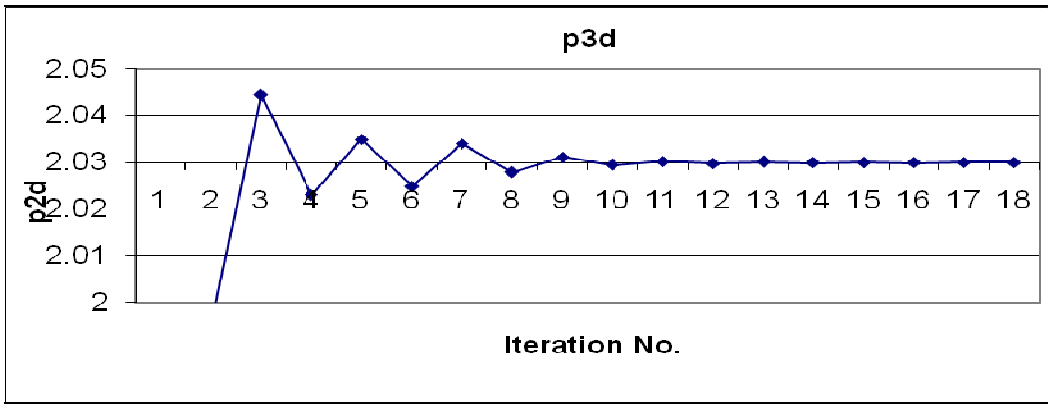
IV Public Goods

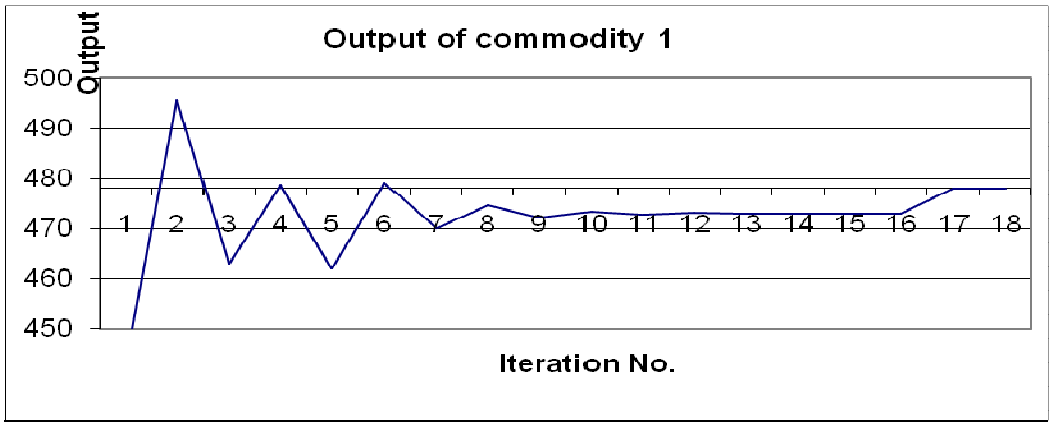
The model can easily be extended to include the provision of public goods. Since public goods are collectively consumed, neither their price nor their quantity considered individually and separately has any definite meaning. Thus suppose the government acquires $G_1, G_2 \ldots G_n$ units of the n commodities and employs L_G units of labour to provide the public goods and finances their acquisition by means of an income tax. Then the budget equation to solve for the tax rate is,

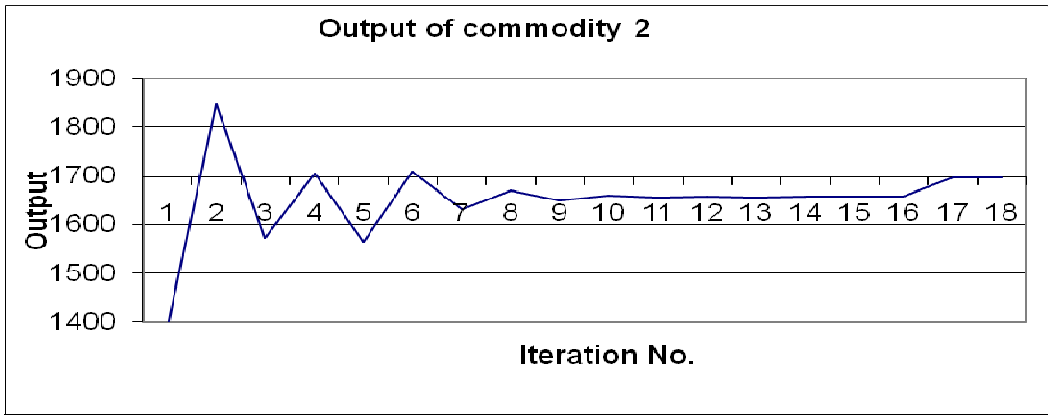
$$tw(L_P + L_G) = twL^* = \Sigma G_i P_i + wL_G \qquad \ldots (4)$$

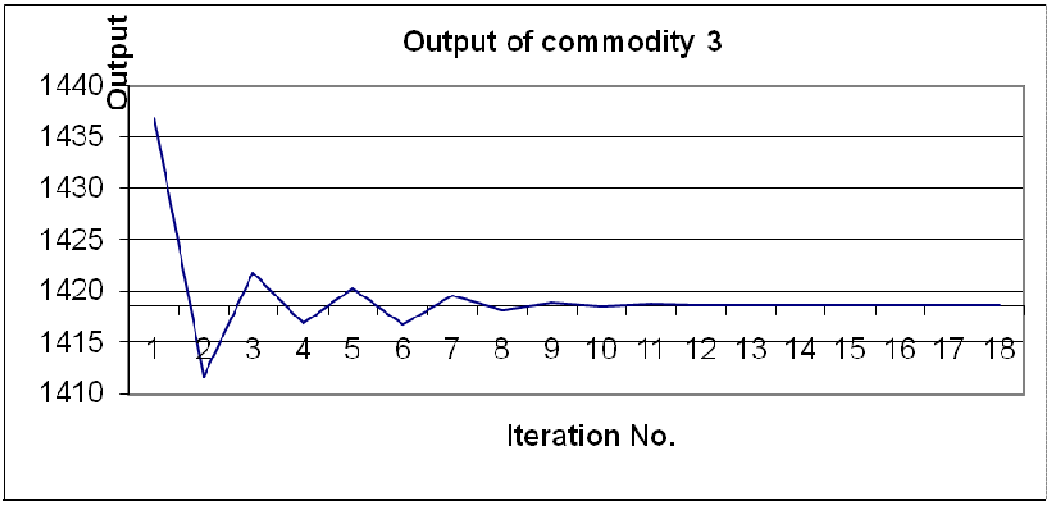
The disposable income (1-t)wL* will be spent on consumption goods in the manner shown in equation (2). Adding to them the government's demand, the quantities of consumption goods demanded will now be:

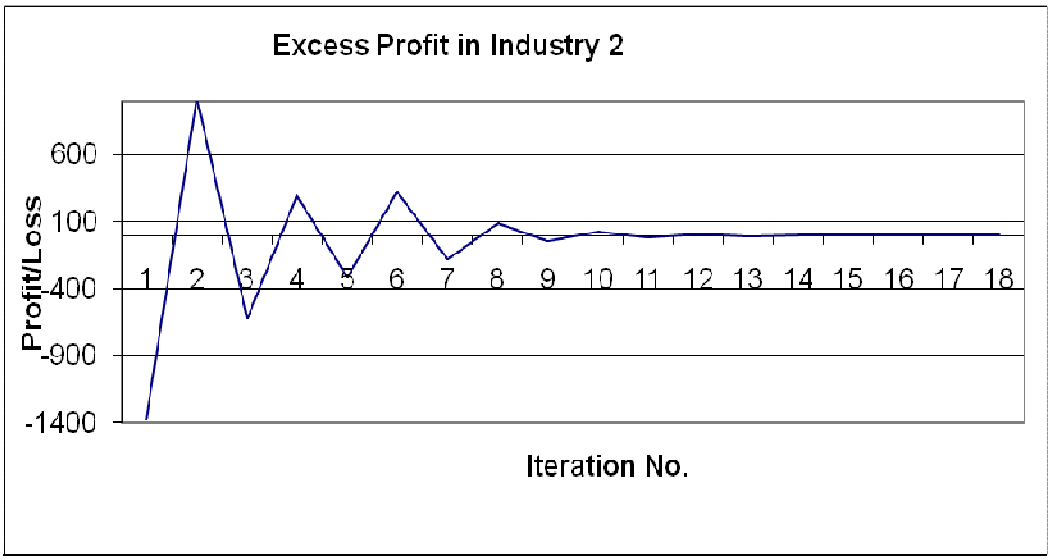


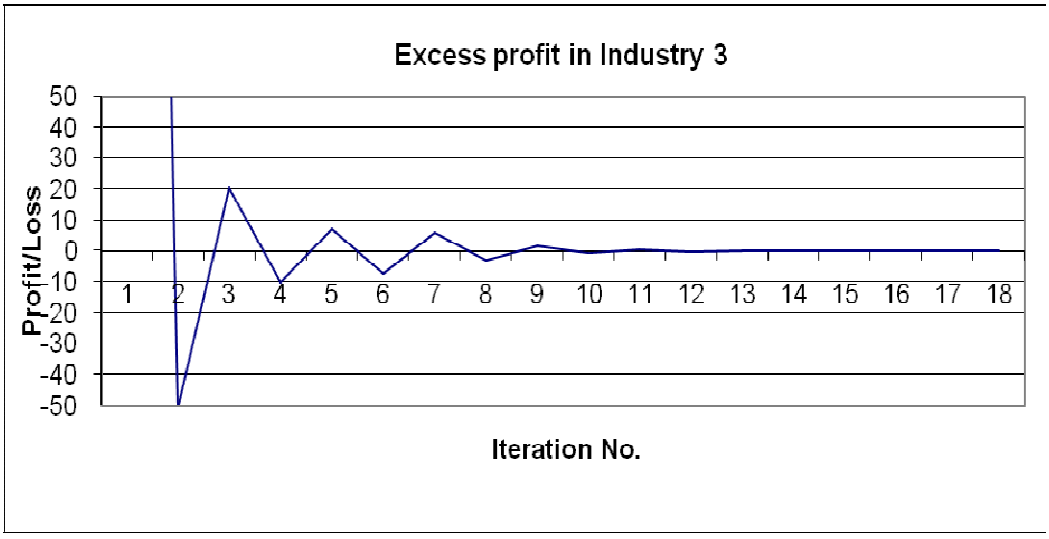

p3d
2.05
2.04
2.03
2.02
2.01
2
p2d
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Iteration No.


Output of commodity 1
Output
500
490
480
470
460
450
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Iteration No.


Output of commodity 2
1900
1800
1700
1600
1500
1400
Output
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Iteration No.


Output of commodity 3
Output
1440
1435
1430
1425
1420
1415
1410
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Iteration No.


Excess Profit in Industry 2
Profit/Loss
600
100
-400
-900
-1400
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Iteration No.


Excess profit in Industry 3
Profit/Loss
50
40
30
20
10
0
-10
-20
-30
-40
-50
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Iteration No.

$$C_i + G_i = C_{io} + G_i + \frac{\alpha_i[(1-t)wL^* - \Sigma P_i C_{io}]}{P_i}$$

The vector of final demands F will contain elements $C_i + G_i$ for the consumption goods and G_i for the others and the solution for gross output is obtained from:

$$AX + F = X \qquad \ldots (5)$$

As before, multiplying (1) by $X = (I-A)^{-1}F$ gives

$$P^T A X + wL^T(I - A)^{-1} F = P^T X$$

In view of $P^T = wL^T (I-A)^{-1}$ and $P^TF = wL^TX + wL_G$.
$wL_G + wL^TX = wL_G + P^TF = wL_G + \Sigma P_i(C_{io} + G_i)\ [(1 - t)wL^* - \Sigma P_i C_{io}]$
$= wL_G + \Sigma G_i P_i + wL^* - twL^*$
i.e. $w(L_G + L_P) = wL^*$ … (6)

If an excise tax were imposed instead of an income tax the prices of goods would rise to $P_i/(1-t)$ and $t\Sigma X_i P_i = \Sigma G_i P_i$. The real wage rate would decline to (1-t)w and post-tax consumption would be lowered by the extent of tax/government expenditure

The ease with which public goods have been incorporated in the model may be difficult to replicate in standard neo-classical general equilibrium theory; public goods have no definable quantities or prices which makes it impossible to find a place for them in the utility functions and budget constraints respectively of the households. Even if the quantity of a public good could be somehow proxied by the vector (G_1, …,G_n, L_G) that the government purchases to provide the public good, there would still be problems, e.g. the government could be buying goods like ICBM's, H. bombs and WMD's to provide say defence, but these goods are not likely to find a comfortable place in the utility functions and budgets of households.

V International Trade

The purpose of this section is to indicate in a very preliminary way how the model can be applied to international trade. We shall confine the analysis to the ubiquitous case of 2 countries and 2 commodities in this paper chiefly to highlight the starkly contrasting results produced by our model as compared to the propositions of standard neoclassical theory.

Thus consider 2 countries A and B, say US and Japan, each of which produce 2 commodities; commodity 1 is a pure capital good and commodity 2 is a pure consumption good. We shall make only 1 small modification to the model of Section II viz. we will suppose w to be numeraire instead of a price and set it equal to a unit of currency in each country. For obvious reasons this does not make the slightest difference to the conclusions. The autarky equilibria of the 2 countries are as follows:

A	B
$25P_{1A} + 10W_A = 30P_{1A}$	$15P_{1B} + 10W_B = 20P_{1B}$
$5P_{1A} + 10W_A = 50P_{2A}$	$5P_{1B} + 20W_B = 30P_{2B}$

The equilibrium prices are:

A (USD)	B(JPY)
$P_{1A} = 2$	$P_{1B} = 2$
$P_{2A} = 0.4$	$P_{2B} = 1$

Now observe that the endowments in the 2 countries A and B are such that A is relatively capital abundant as compared to B both in physical terms (30/20 > 20/30) as well as in value terms (60/20 > 40/30). Also observe that commodity 1 is capital intensively produced in both countries as compared to commodity 2; per unit output of commodities 1 and 2 the capital to labour ratios in country A in physical terms are 2.5 and 0.5 and in value terms five and 1 whereas in B they stand at 1.5 and 0.25 and 3 and 0.5 respectively. Yet the pattern of comparative advantage shows that $P_{1B}/P_{2B} = 2 < 5 = P_{2A}/P_{2B}$, i.e. country B, the labour abundant country, has the comparative advantage in the capital intensive commodity 1. The Leontief paradox [Leontief 1953] is not sounding as paradoxical as standard theory claims it to be!

Next open the 2 countries to trade in accordance with their comparative advantages. Then each country will employ its entire labour force in the industry that it operates so that the production activity in the 2 countries will be as follows;

	A	B
1		$45P_{1B} + 30W_B = 60P_{1B}$
2	$10P_{1B}E_{AB} + 20W_A = 100P_{2A}$	

where E_{AB} is the USD/JPY exchange rate. To find the exchange rate equate the value of country A's imports (of commodity 1) to its exports (of commodity 2),

$$10P_{1B}E_{AB} = 30\ W_B \qquad \ldots (7)$$

and obtain the solution $E_{AB} = 30/20 = 1.5$(USD/JPY). If we ascertain the gains from trade it is found that country A loses because at an exchange rate of 1.5; it buys commodity 1 from country B for 2 JPY x 1.5 USD/JPY = 3USD per unit but it could have produced commodity 1 at 2USD per unit at home. But country B gains because it can buy 1.666 units of commodity 2 from country for 1 JPY as against 1 unit at home. So we may try the trade pattern A - 1, 2 B - 1 to prevent A from losing but yet enabling B to gain. But in that case A is indifferent as between producing the commodity at home and buying it from B which can happen only if the exchange rate $E_{AB} = P_{1A}/P_{1B} = 1$ (USD/JPY).

However since A is now incompletely specialised we must find its allocation of labour between industries 1 and 2. Denote it by $L_{1A}X_{1A}$ and $L_{2A}X_{2A}$ with $L_{1A}X_{1A} + L_{2A}X_{2A} = L_A$ where L_{1A}, L_{2A} denote the autarky allocation. Now the pattern of production activity will be as follows:

	A	B
1	$25X_{1A}P_{1A} + 10X_{1A}W_A = 30X_{1A}P_{1A}$	$45P_{1B} + 30W_B = 60P_{1B}$
2	$5X_{2A}P_{1A} + 10X_{2A}W_A = 50X_{2A}P_{2A}$	

Since $P_{1A} = 2$, the first equation of country A is $50X_{1A} + 10X_{1A} = 60X_{1A}$ so that $X_{1A} = 1$. And since $10X_{1A} + 10X_{2A} = 20$, $X_{2A} = 1$ which means that country A operates the 2 industries just as it would in autarky in a self-sufficient manner, in which case it does not require to trade. Naturally B would do so too and we are back to square 1. A meaningful international trade equilibrium need not exist when 1 commodity is a pure capital good and the other is a pure consumption good!

Finally, consider an example of 2 countries producing 2 commodities both of which serve the dual purpose of being means of production as well as consumption. Suppose that in each country half the income is spent each commodity for final consumption (Only for the sake of simplicity the intercept terms of the consumption demand equations are zero). The autarky equilibria are as follows:

1. $3.052P_{1A} + 4.069P_{2A} + 9.156W_A = 20.347P_{1A}$ $\quad 5.0966P_{1B} + 4.988P_{2B} + 9.977W_B = 19.955P_{1B}$
2. $3.943P_{1A} + 4.928P_{2A} + 10.843W_A = 10.715P_{2A}$ $\quad 5.0138P_{1B} + 4.0111P_{2B} + 8.0222W_B = 20.055P_{2B}$

$\underline{\quad\quad}$ $\quad\quad$ $\underline{\quad\quad}$

20 $\quad\quad$ 18

$\underline{\quad\quad}$ $\quad\quad$ $\underline{\quad\quad}$

The prices are $P_{1A} = 0.7489$, $P_{2A} = 0.9330$ (USD) and $P_{1B} = 1.0050$ and $P_{2B} = 0.8140$ (JPY) indicating that country A has the comparative advantage in commodity 1 and B in commodity 2. In the post-trade situation each country employs its entire labour force in the industry of its specialization so that the production activity looks as follows

	A	B
1.	$6.666P_{1A} + 8.888P_{2B}E_{AB} + 20W_A = 44.444P_{1A}$	_____
2.	_____	$11.25P_{1A}E_{BA} + 9P_{2B} + 18W_B = 45P_{2B}$

Country A imports 8.88 units of commodity 2 for <u>intermediate</u> use and another $10W_A$ worth for final consumption from country A. Likewise country B imports 11.25 units of commodity 1 from country A for intermediate use and another $9W_B$ worth for final consumption. The trade balance equation of country A is

$$8.8888\ P_{2B}E_{AB} + 10 = 11.25P_{1A} + 9E_{AB}$$

in which both sides are expressed in the domestic currency of country A (USD). The exchange rate which equalizes exports to imports is

$$E_{AB} = \frac{10 - 11.25P_{1A}}{9 - 8.888P_{2B}} \qquad \ldots (8)$$

It is evident from (8) that E_{AB} cannot be determined until P_{1A} and P_{2B} are determined. But the post-trade price equations show that P_{1A} and P_{2B} cannot be determined until E_{AB} is determined. To resolve this circularity we must proceed iteratively. Take a tentative initial value of $E_{AB} = E_{AB}^{0}$. For example, by substituting the autarky prices in equation (8) we obtain $E_{AB}^{0} = 0.8925$. Substituting this in the price equations will solve for P_{1A} and P_{2B} which in turn can be used to make a revised guess E_{AB}^{1} in (8) and repeat the process until the results converge to the required level of accuracy. For our example the solution converges in 15 iterations (to an accuracy of the 7^{th} place of decimals) to $E_{AB} = 0.9160$, $P_{1A} = 0.6877$, $P_{2B} = 0.7346$. Observe that both the prices

are lower than their autarky levels showing the gains from trade. The post-trade real wage rates in terms of commodities 1 and 2 are $W_A/P_{1A} = 1.4541$ and $W_A/P_{2B}E_{AB} = 1.4861$ in country A and $W_B/E_{BA}P_{1A} = 1.3319$ and $W_B/P_{2B} = 1.3612$ in country B. Although, our trade model is 2 x 2 x 2 (static model with 2 countries, 2 factors, capital and labour and 2 commodities) just like the standard Heckscher-Ohlin Samuelson model it does not lead to equalisation of real wage rates in the post-trade situation.

VI Concluding Remarks

The model of general equilibrium investigated in this paper is founded entirely on 2 models that are rich in their empirical content viz. the input-output model of interindustrial technology and the linear expenditure system for household consumption. Although the model is based on the same assumptions as neo-classical general equilibrium theory, i.e., homogenous of degree 1 production functions and homogenous of degree zero demand equations its results diverge from those of the standard theory. Some of the model's properties are as follows:

Firstly, the paper demonstrates that it is not at all necessary to close I-0 models by endogenising final consumption into the input coefficients matrix either wholly or partially. The consumption block and the production block can be kept entirely separate and the market processes that are involved in the interaction of the 2 blocks can be appropriately described. Secondly, the model can be used to give a reasonable description of the behaviour of an economy in disequilibrium states and also of the process by which the economy attains it equilibrium state. Thirdly, the model is versatile enough to incorporate government's provision of public goods and determine the tax rate. Fourthly, the model can be fruitfully applied to the study of international trade. Apart from the fact that the model can handle trade in capital goods (a subject that has proved intractable for the methods of neo-classical theory) it produces results that diverge from the neo-classical theory of trade in 3 respects, (i) a trade equilibrium need not exist even if there are comparative cost differences, (ii) the pattern of comparative advantage may be quite contrary to the pattern of comparative factor endowments and (iii) equalisation of real wage rates by trade need not take place. Finally, because the model is based on the input-output matrix and the linear expenditure system, it is suitable for empirical implementation.

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