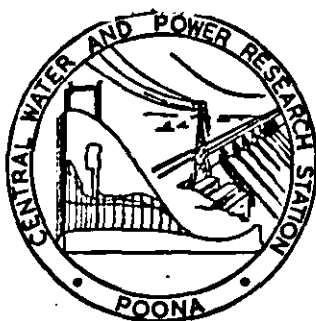


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with FOREWORD by  
**D V Joglekar**



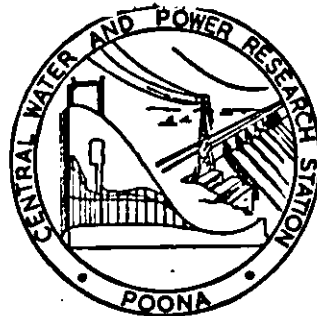
**Technical Memorandum HLO 2**  
**March 1962**

**GOVERNMENT OF INDIA**  
**CENTRAL WATER AND POWER RESEARCH STATION**  
**POONA**

# PEAK FLOW ESTIMATION BY METHOD OF MAXIMUM LIKELIHOOD

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**CENTRAL WATER AND POWER RESEARCH STATION**  
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## FOREWORD

It is sometime that Gumbel's Extreme Value formula has come to be almost universally accepted for deriving the most reliable estimate of the average peak flow magnitudes of long term return periods of occurrence. Pitfalls nonetheless obtain commonly in evaluating the parameters of the formula from the past observed data of the annual peak flow measures on record. Hypothetical definitions are postulated for the plotting positions representing the various observed flow magnitudes, while theoretically none exist. Gumbel has in fact clearly admitted this position in his latest published valuable book on Extreme Value Distributions(1958).

The maximum likelihood approach which involves "prohibitive" computational labour has been processed in the Technical Memorandum in a schematized form to be readily adopted by all design engineers who are equipped with the simple aid of only a routine calculating machine and ordinary tables of logarithms. The estimates resulting are shown to be precise as well as better consistent of magnitude (ie insusceptible to large alterations of value) to withstand effectively deletions of a few years doubtfully observed flow measures or limited additions of more recent data. The feature comprises a fundamental and a very significant contribution to the subject which has for long proved to be slippery.

Poona

March 17, 1962

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## APPENDIX I

### *Standard Error of Peak Flow Estimates by Maximum Likelihood*

Since from (5),  $x_T = u - \frac{1}{a} \log_e \log_e \frac{T}{T-1} \equiv u - \frac{1}{a} R$

$$\text{Variance } (x_T) = \text{Variance } (u) + R^2 \text{ Variance } \left(\frac{1}{a}\right) - 2R \text{ Covariance } \left(u, \frac{1}{a}\right) \dots (16)$$

or, denoting Variances and Covariances by Var and Cov

$$\text{Var}(x_T) = \text{Var}(u) + R^2 \text{Var} \left(\frac{1}{a}\right) - 2R \text{Cov} \left(u, \frac{1}{a}\right) \dots \dots (16)$$

Now the Variance-Covariance matrix of  $u$  and  $a$  viz

$$\begin{vmatrix} \text{Var}(u) & \text{Cov}(u, a) \\ \text{Cov}(u, a) & \text{Var}(a) \end{vmatrix} \dots (17)$$

is the reciprocal\* of the matrix

$$-n \begin{vmatrix} E\left(\frac{d^2 \log f}{du^2}\right) & E\left(\frac{d^2 \log f}{da du}\right) \\ E\left(\frac{d^2 \log f}{da du}\right) & E\left(\frac{d^2 \log f}{da^2}\right) \end{vmatrix} \dots (18)$$

where 'f' stands for the distribution function

$$\frac{-a(x-u)}{ae} - e^{-a(x-u) - e^{-a(x-u)}}$$

and  $E$  for the expected values of the second order partial derivatives of  $\log f$ ,

$$\text{e.g. } E\left(\frac{d^2 \log f}{du^2}\right) = \int_{-\infty}^{+\infty} \frac{d^2 \log f}{du^2} f dx$$

On evaluating and substituting the expected values, (18) becomes

$$n \begin{vmatrix} a^2 & (1-C) \\ (1-C) & \frac{1}{a^2} \left[ \frac{\pi^2}{6} + (1-C)^2 \right] \end{vmatrix} \dots (19)$$

( $C$  being Euler's Constant = 0.577216).

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\* M G Kendall; "The Advanced Theory of Statistics" Vol II, pp 36-42.

Reciprocal of (19) is

$$\frac{1}{n} \left\| \begin{array}{cc} \frac{1}{a^2} \left[ 1 + \frac{6(1-C)^2}{\pi^2} \right] & \frac{-6(1-C)}{\pi^2} \\ \frac{-6(1-C)}{\pi^2} & \frac{6a^2}{\pi^2} \end{array} \right\| \quad \dots \quad (20)$$

Comparison of (20) with (17) gives

$$\text{Var}(u) = \frac{1}{na^2} \left[ 1 + \frac{6(1-C)^2}{\pi^2} \right], \quad \text{Var}(a) = \frac{6a^2}{n\pi^2}$$

$$\text{Cov}(u, a) = \frac{-6(1-C)}{n\pi^2}$$

wherefrom we get\*

$$\text{Var}\left(\frac{1}{a}\right) = \left(-\frac{1}{a^2}\right)^2 \text{Var}(a) = \frac{6}{n\pi^2 a^2}$$

$$\text{and } \text{Cov}\left(u, \frac{1}{a}\right) = -\frac{1}{a^2} \text{Cov}(u, a) = \frac{6(1-C)}{n\pi^2 a^2}$$

substituting which in (16) gives

$$\begin{aligned} \text{Var}(x_T) &= \frac{1}{na^2} \left[ 1 + \frac{6(1-C)^2}{\pi^2} \right] + R^2 \frac{6}{n\pi^2 a^2} - 2R \frac{6(1-C)}{n\pi^2 a^2} \\ &= \frac{1}{na^2} \left\{ 1 + \frac{6}{\pi^2} \left[ (1-C)^2 + R^2 - 2R(1-C) \right] \right\} \\ &= \frac{1}{na^2} \left[ 1 + \frac{6}{\pi^2} (1-C-R)^2 \right] \end{aligned}$$

Therefore Standard Error  $(x_T) = \sqrt{\text{Var}(x_T)}$

$$= \frac{1}{\sqrt{n} a} \left[ 1 + \frac{6}{\pi^2} (1-C-R)^2 \right]^{\frac{1}{2}}$$

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\* M G Kendall, "The Advanced Theory of Statistics", Vol 1, pp 208, Charles Griffin & Co Ltd, London (1943)

APPENDIX II

Annual Peak Flows ( 000 cfs )

Year	Baitarni at Akhupada	Krishna at Vijayawada	Penner at Nellore	Godavari at Dowlaishwaram	Sutlej at Bhakra	Sone at Dehri	Hakmani at Himat Nagar	Mahanadi at Sambalpur	Mahanadi at Naraj	Yamuna at Tajewala	Ravi at Madhopur	Damodar at Rhondia	Sabarnati at Dharoi	Tapi at Kathore
1905	56	483	82	795										
06	172	463	48	1078										
07	275	495	144	1228										
08	101	572	105	1036										
09	122.5	653	473	901										
10	96	370	206	965										
1911	82	320	34	1144										
12	125	543	152	1186	237									
13	290	418	63	969	199									
14	92.5	949	29	1141	199									
15	83	578	98	1066	146									
16	65	955	382	915	75									
17	101	498	324	936	160									
18	85	254	72	718	70									
19	101	456	96	678	133									
20	263	392	69	528	145	674								
1921	65	558	142	998	136	580								
22	127	499	127	824	110	309	6.8							
23	140	624	91	555	93	1028	0.9							
24	52.5	634	151	629	237	965	5.9							
25	173	613	111	796	191	892	2.7							
26	125	552	30	933	96	871	34.9	841	1410					
27	325	560	158	845	141	600	29.7	698	1164	189.5				
28	172	461	17	725	85	NA	3.7	595	991	81				
29	125	458	67	845	162	638	7.1	840	1400	294	72			
30	102.5	370	169	972	150	NA	10.3	559	931	145	155			

Year	Baitarni	Krishna	Penner
1896	211	760	
97	50	619	
98	116	624	
99	72	405	
1900	228	719	
1901	114	513	
02	180	548	
03	96	1061	479
04	228	479	38

Year	Baitarni	Krishna
1874	87	
75	87	
1876	85	
77	183	
78	20	
79	95	
80	83	
81	250	
82	108	
83	110	
84	112	
85	54	
1886	80	
87	68	
88	112	
89	98	
90	72	
91	142	
92	112	
93	156	
94	195	578
95	75	496

1931	148	468	41	954	72	400	6.3	690	1150	82	.116			
32	146	512	6	933	135	953	2.9	566	943	213	157			
33	142	632	46	1020	93	580	15.5	847	1412	79	137	216		
34	140	384	69	1029	87	1209	31.4	NA	1185	89	77	106.5		
35	95	360	107	909	156	786	7.8	552	920	76	111	422.5	27	
36	91	350	95	1075	84	619.5	0.7	546	910	98	193	174.5	1	
37	127	393	51	785	83	614	60.1	825	1375	194	112	202	44	
38	47.5	370	143	1011	175	272	7.8	492	820	51	94	64	28	
39	117	402	158	896	83	681	7.8	816	1360	43	69	258.5	80	
40	217	354	199	1791	61	404	1.0	840	1400	70	115	266.5	32	243
1941	300	382	55	626	97	181	41.8	402	670	78	104	521.5	15	481
42	127	381	13	2108	177	959	10.8	656	1093	143	138	375.5	44	758
43	287	427	293	635	186	NA	7.7	714	1190	118	94	251	68	179
44	148	385	89	1794	81	959	45.6	793	1322	101	114.5	153	87	900
45	108	375	27	1164	109	279	20.6	589	981	97	114.5	121	66	722
46	192	475	420	1250	135	512	13.1	950	1445	129	103.5	314	16	339
47	45	443	132	1117	276	607	0.2	965	1495	563	566	260	20	291
48	64	369	15	671	83	561	0.1	948	1103	110	79	230	7	255
49	69	914	455	1119	107.5	443	9.0	701	634	126	133	230	17	662
50	134	565	33	1153	172	861		792	913	92	301	246	291	398
1951	83	360	42	846	325	421		450	394	215	71	347	36	162
52	84	406	294	526	83	356		848	1045	NA	101	168	2	115
53	108	478	55	2830	170	777		776	876	154	357	195		235
54	58	513	117	1084	199	206			450	224	152	243		
55	175	503		1364	130				NA	275	617	61		
56	142	830		1571					647	450	232	244		
57	62	592		1755					503	317	472			
58		740							931.5	199	128			
59										180				

NA denotes ' data not available. '

Year	Bairarni at Akhupada	Krishna at Vijayawada	Penner at Nellore	Godavari at Dowlaiswaram	Sutej at Bhakra	Sone at Dehri	Hatnani at Himat Nagar	Mahanadi at Sambalpur	Mahanadi at Naraj	Yamuna at Tajewala	Ravi at Madhopur	Dunodar at Rhondia	Subarnati at Dharoi	Tapl at Kathore
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## CWPRS TECHNICAL MEMORANDUMS

### SUBJECT CLASSIFICATION

Subject	Symbol
Water studies & Hydrology	HLO
Rainfall & Runoff	
Regeneration & Losses	
Quantity of Water	
Fluid Mechanics	FLM
Design of Hydraulic Structures	HYD
River Training & Flood Control	HYR
Irrigation, Drainage & Reclamation	IRD
Surface & subsurface irrigation	
Water requirements of crops	
Soil survey and crop pattern	
Drainage	
Reclamation	
Navigation	NAV
Ports, Harbours & Estuaries	
Inland Navigation	
Coastal Erosion	
Ships & Navigation Crafts	
Structural Design	STD
Soil Mechanics & Foundation Engineering	SFE
Investigations	
Stability, deformations & displacements of earth structures	
Foundation studies including geophysical	
Construction Materials & Practices	CMP
Concrete Technology	
Building & other Materials	
Instrumentation	IST
Model and Prototype comparisons	MPC
Other miscellaneous	MIS