

NEW FORMULA FOR DETERMINING DEPTH  
SPACING OF SUBSURFACE DRAINS IN IRREGULAR  
LANDS

By Lee D. Dumm

**NEW FORMULA FOR DETERMINING DEPTH AND SPACING  
OF SUBSURFACE DRAINS IN IRRIGATED LANDS**

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Errata Sheet

<u>Page</u>	<u>Correct Formula</u>	<u>Formula No.</u>
6	$y = y_0 \frac{4}{\pi} \sum_{n=1, 3, 5, \text{ etc.}}^{n=\infty} \frac{e^{-\frac{\alpha n^2 \pi^2 t}{L^2}}}{n} \sin \frac{n \pi}{L} x$	2
7	$y_{L/2} = y_0 \frac{4}{\pi} e^{-\frac{\alpha \pi^2 t}{L^2}}$	3
7	$\log_e \frac{\pi y_{L/2}}{4 y_0} = -\frac{\alpha \pi t}{L^2}$	4
7	$L = \pi \sqrt{\frac{K D_a t}{v \log_e \left( \frac{4 y_0}{\pi y_{L/2}} \right)}}$	5
8	$\frac{\partial}{\partial x} KZ \frac{\partial Z}{\partial x} = v \frac{\partial Z}{\partial t}$	6

## NEW FORMULA FOR DETERMINING DEPTH AND SPACING OF SUBSURFACE DRAINS IN IRRIGATED LANDS

Practically every area where irrigation has been carried on for any length of time has land that has become waterlogged or salted or lands on which crop production has been affected by high watertables. Therefore the provision of adequate drainage is an essential part of the planning, construction, and operation of an irrigation development.

Drainage problems in humid areas are usually evident from visual inspection of an area, since they are the result of natural conditions which have prevailed for many years. This is not true, however, of a proposed irrigation development. Here the drainage problems are generally the result of man-made conditions brought about by irrigation and may not be apparent until several years after the start of irrigation.

The Bureau of Reclamation in planning new irrigation developments and determining the feasibility of projects is required to forecast or predict the extent of future drainage problems in areas where none now exist. And, since the feasibility of a project depends upon the benefits derived versus the costs involved, it is essential that the cost estimates be sufficiently adequate to include ultimately required drainage facilities.

Most irrigated areas developed in the past have had favorable natural substrata drainage characteristics which did not require

the use of closely spaced drains to control high watertables. In general, most of these have been in arid lands underlain by sand or gravel strata into which excess water can percolate vertically and then move laterally through the permeable sand or gravel to natural outlets or constructed drains. Over the years a vast amount of experience has been gained in determining drainage requirements of such lands.

Since the better lands have been developed, however, we now find that many proposed developments do not have these favorable natural drainage characteristics and therefore closely spaced drains are usually required. Experience in subsurface drainage of such lands is rather meager. This lack of experience has led to the development of various drain-spacing formulae to meet the needs of planning and developing such areas.

The typical profile often encountered and for which the formulae have been developed is one in which a relatively permeable soil overlies a very slowly permeable strata or one in which the profile becomes less permeable with depth. Under these conditions relatively close spacing of drains is required, since the excess water must be moved laterally through the profile to the drains.

Drain spacing formulae, to date, have been developed on the basis of steady-state flow conditions, which do not entirely satisfy the conditions of irrigation. Other formulae have been developed to meet the requirements of a particular area.

The Bureau of Reclamation is planning, developing, and operating many irrigation projects in the western states. These projects embrace a wide diversity of soils and groundwater conditions. The need for a drain-spacing formula which would be applicable to these widely varying soil and groundwater conditions has become increasingly evident. Such a formula should include as many factors pertaining to irrigation drainage as possible. To meet this need, Mr. R. E. Glover, Engineer of the Bureau, has developed a theoretical mathematical formula which we feel incorporates most of these factors.

Our purpose in presenting the formula at this meeting is to point out the factors which we think have a bearing on irrigation drainage and to explain the methods we have used in developing the formula. Although we have not yet been able to conduct extensive laboratory experiments and field trials to verify the use of the formula we find it more compatible with our experiences over a variety of conditions than any other. Until we can further substantiate the results, we would welcome comments or research checks from anyone interested in drainage.

The development and use of the formula are predicated upon the following conditions and assumptions:

1. Normal irrigations at regular intervals.
2. Uniform or stratified soils overlying a slowly permeable stratum or barrier layer.

3. Permeability of the profile equal to the weighted average permeability of the soil or soils above the barrier layer.
4. The drainage requirement, or excess water to be disposed of, is the difference between the net input of water and the available moisture storage in that portion of the profile which must be kept clear of gravity or noncapillary water in order to provide for root growth and salinity control.
5. Excess water must be removed between irrigations in order to maintain the watertable below the root zone. This would require that sufficient storage space for gravity water be provided below the root zone for the inflow of excess water from the following irrigation.
6. Water moving through the soils is in a transitory state rather than a steady state, i. e., the head available to push the water toward the drains varies with time.
7. Movement of water through a soil is proportional to the groundwater gradient and to the depth of saturated thickness between the watertable and the slowly permeable layer or barrier.

Many of the results obtained from the theory of heat conduction in solids also apply to the movement of water through soils. Let us first assume a condition represented by Figure 1 below.

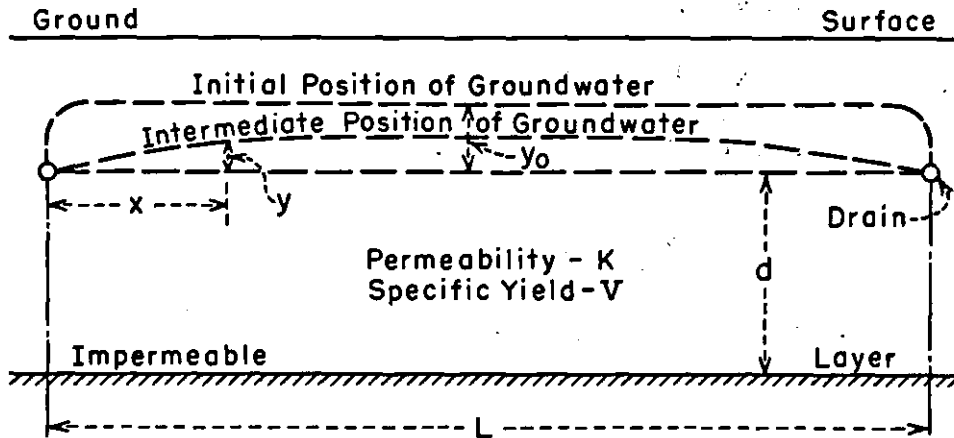


FIGURE 1

In this case, an increment of groundwater of an initial uniform drainable depth  $y_0$  flows toward horizontal drains spaced at a distance  $L$  apart. The aquifer, of permeability  $K$ , extends below the level of the drains to a depth  $d$ . If  $y$  represents the depth of the groundwater above the elevation of the drains at some horizontal distance  $x$  measured from one line of the drains toward the other, as shown on Figure 1, then the condition of continuity is, approximately, if  $y_0$  is small compared to  $d$ :

$$\frac{\partial y}{\partial t} = \frac{KD_d}{V} \frac{\partial^2 y}{\partial x^2} \quad (1)$$



Where  $D_a = d+y_0/2$  represents an average thickness of aquifer available for carrying water to the drains, a solution of this equation subject to the conditions:

$$y = 0 \text{ when } x = 0$$

$$y = 0 \text{ when } x = L$$

$$y = y_0 \text{ when } t = 0 \text{ for } 0 < x < L$$

and with

$$\alpha = \frac{K D_a}{V}$$

is:

$$y = y_0 \frac{4}{\pi} \sum_{n=1,3,5,\text{etc.}}^{\infty} \frac{\epsilon}{n} \frac{-\alpha n^2 \pi^2 t}{L^2} \sin \frac{n\pi}{L} x \quad (2)$$

A simplified formula useful for estimating drain spacings may be obtained from the above expression on the following basis: the most difficult point to drain is the point midway between the drains, or the point at which  $x = L/2$ . For this value of  $x$  the sine terms take on the successive values of + 1 and - 1. After the lapse of sufficient time the water level midway between the drains begins to lower, and at about this time all the terms in the series, except the first term, have also become quite small. When the parameter  $\frac{\alpha t}{L^2}$  reaches 0.025, for example, the first term has the value 0.7788 while the second term has the value -0.0362. Then we can say that,

for significant amounts of lowering of the watertable at  $x = L/2$ , we can consider the first term only and, to a sufficient approximation,

$$y_{L/2} = y_0 \frac{4}{\pi} e^{-\frac{\alpha \pi^2 t}{L^2}} \quad (3)$$

We can write this relation in the form

$$\log_e \frac{\pi y_{L/2}}{4 y_0} = -\frac{\alpha \pi^2 t}{L^2} \quad (4)$$

The drain spacing formula obtained by solving this expression for  $L$  when  $\alpha = \frac{K D_a}{V}$  is:

$$L = \pi \sqrt{\frac{K D_a t}{V \log_e \left( \frac{4}{\pi} \frac{y_0}{y} \right)}} \quad (5)$$

The successive groundwater profiles, as given by (2) are shown in Figure 2. By use of this chart the pattern of movement of water to the drains may be traced. The drain spacing required to remove groundwater to a specified depth may also be found by use of the chart by finding the value of  $\alpha t/L^2$  required to produce the necessary lowering and solving this relation for  $L$ .

Although the above formulas are derived under the assumption that  $y_0$  is small compared to  $d$  we shall see later that if we use  $D_a = d + (y_0/2)$  as the effective thickness of the aquifer, they do give

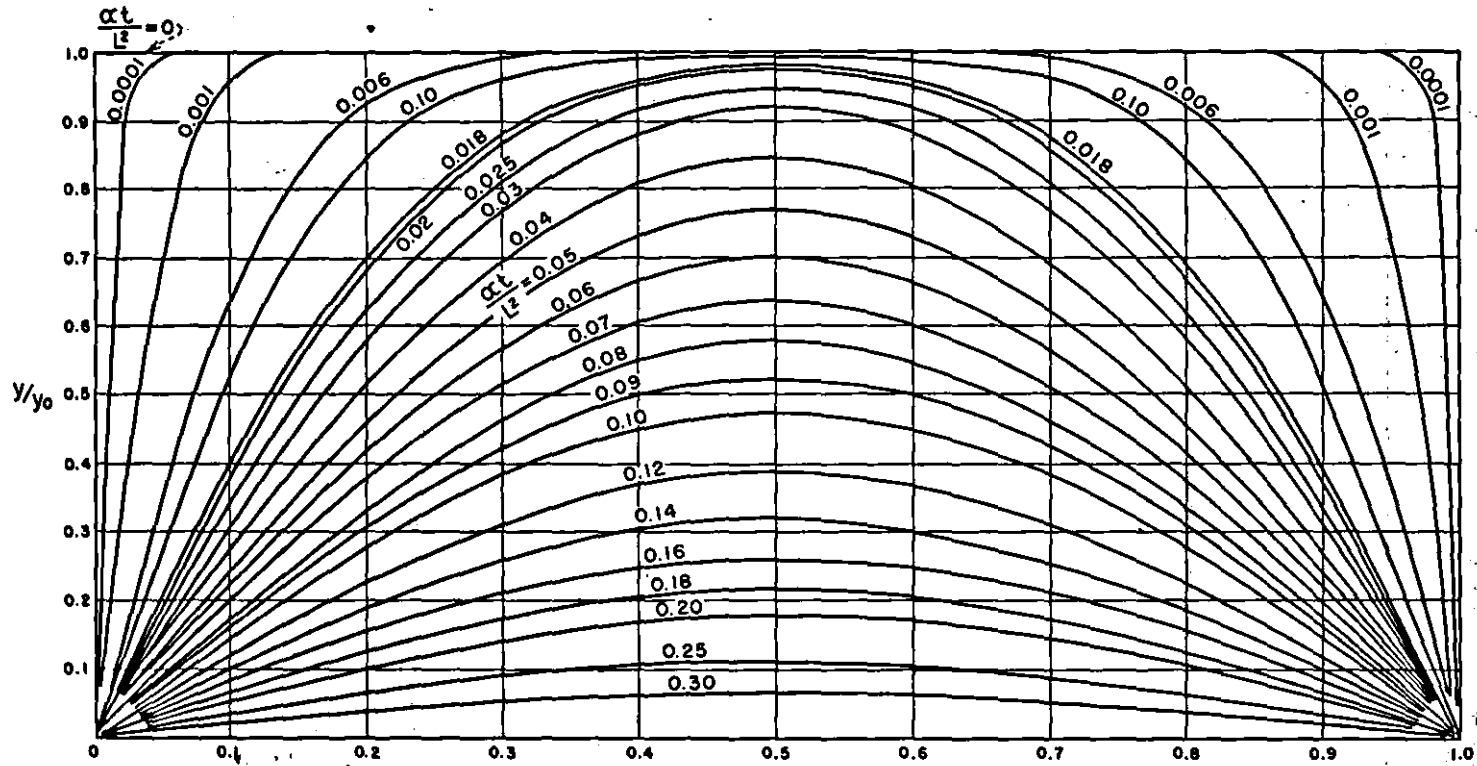
a fairly satisfactory approximation even though the drains are placed at the bottom of the aquifer.

We next give consideration to a case where the drains are placed at the bottom of the aquifer and in which we take account of the saturated thickness of the aquifer explicitly. This development provides a means of comparison for the extreme case and permits an evaluation of the error introduced by using the methods outlined above when  $y_0$  is not small compared to  $d$ .

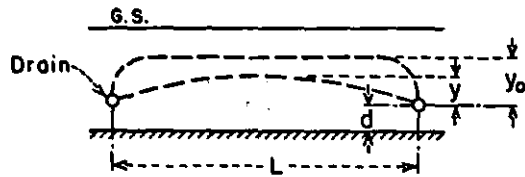
Let us assume a groundwater profile, terminating at drains spaced at distance  $L$  apart at the bottom of the aquifer, of permeability  $K$ , and an initial saturated depth  $H$  at a point midway between the drains. If  $Z$  represents the saturated thickness of the aquifer at the distance  $x$ , measured horizontally from one line of drains, as shown on Figure 3, then the condition of continuity is:

$$\frac{\partial}{\partial x} K Z \frac{\partial t}{\partial x} = V \frac{\partial Z}{\partial t}$$

⑥



EXAMPLE:



when:  
 $K = 0.20"/\text{Hr.}$   
 $t = 336 \text{ Hr.}$   
 $V = 0.05$   
 $y_0 = 48"$   
 $y = 18.6"$   
 $D_0 = d + y_0/2 = 72"$

$$\alpha = \frac{KD_0}{V} = \frac{0.2 \times 72}{.05} = 288$$

when  $y/y_0 = \frac{18.6}{48} = 0.392$ ;  $\frac{\alpha t}{L^2} = 0.119$

$$L^2 = \frac{\alpha t}{0.119} = \frac{288 \times 336}{0.119} = 813,000$$

$$L = 903" \text{ or } 75'$$

FIGURE 2

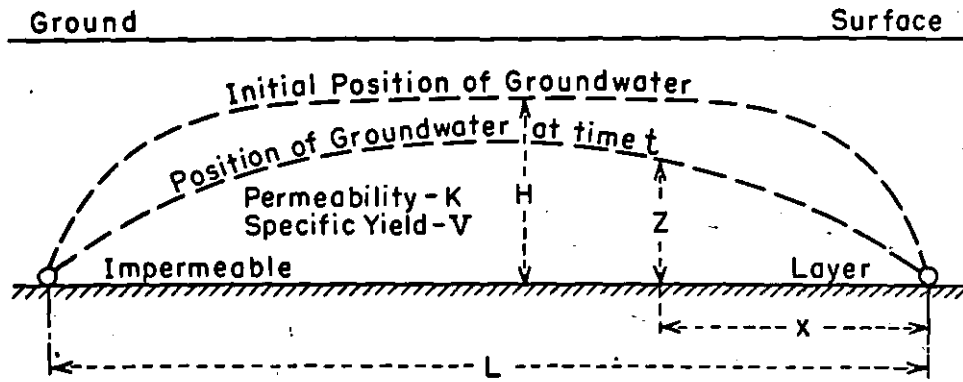


FIGURE 3

Let

$$u = \frac{z}{H}$$

$$\xi = \frac{x}{L}$$

$$\tau = \frac{KH}{VL^2} t$$

Then the above differential equation takes the form:

$$\frac{\partial}{\partial \xi} \left( u \frac{\partial u}{\partial \xi} \right) = \frac{\partial u}{\partial \tau} \quad (7)$$

A solution of the equation is:

$$U = \bar{W} \bar{Y}$$

Where  $\bar{W}$  is determined from the relation:

$$\int_0^{\bar{W}} \frac{\bar{W} D \bar{W}}{\sqrt{1 - \bar{W}^2}} = \sqrt{3} \xi \quad (8)$$

And  $\bar{Y}$  is given by

$$\bar{Y} = \frac{1}{\frac{9}{2} \left( \frac{\alpha t}{L^2} \right) + 1}$$

Where in this case:

$$\alpha = \frac{KH}{V}$$

If  $Z_c$  represents the value of  $Z$  at  $x = L/2$  at the time  $t$ , then, since  $\bar{Y} = Z_c/H$ , a rearrangement of the expression for  $\bar{Y}$  yields

$$t = \frac{2L^2}{9\alpha} \left( \frac{H}{Z_c} - 1 \right) \quad (9)$$

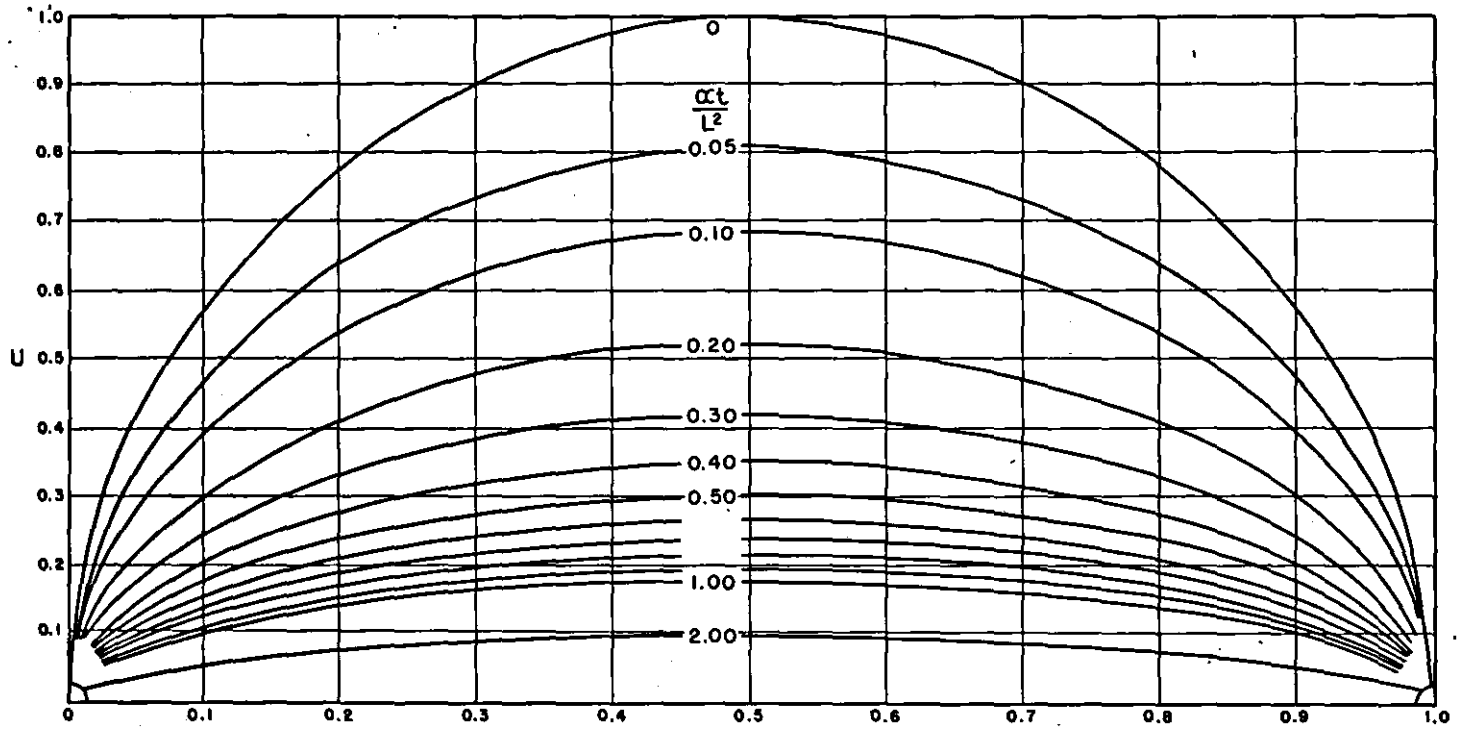
If this expression is solved for  $L$ , we get a formula for the drain spacing when  $\alpha = \frac{KH}{V}$  which is:

$$L = \sqrt{\frac{9KHt}{2V\left(\frac{H}{Z}-1\right)}} \quad (10)$$

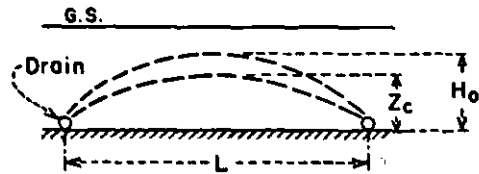
The groundwater surface profiles implied by this solution are shown on Figure 4. It will be noted that a uniform depth of groundwater is not assumed to exist at time zero, as in the first case. The substitution of a reasonable initial profile is done as a concession to the mathematical difficulties since the nonlinear differential equation that applies to this case is very difficult to solve when a uniform initial distribution is assumed. It is believed that no appreciable error is introduced by making this concession.

With these two cases we now possess treatments of drainage rates where the drains are placed at shallow depths in the aquifer and where the drains are placed at the bottom of the aquifer. The intermediate case remains without adequate treatment, and we will, therefore, try to provide this by making a choice of  $D_a$  so that the first case may be used for drain placement near the middle of the aquifer also. We will make the choice of  $D_a$  by comparing a linear and a nonlinear steady state case. In these cases we will assume that the water flowing to the drains is provided by a steady infiltration  $i$ . In the steady state case the continuity condition will take the form:

$$KD_a \frac{dy^2}{dx^2} = -i \quad (11)$$



EXAMPLE:



when:

$K = 0.20''/\text{Hr.}$   
 $t = 336 \text{ Hr.}$   
 $V = 0.05$   
 $H_0 = 96''$   
 $Z_c = 66.6''$   
 $U = Z_c/H_0$

$$\alpha = \frac{KH_0}{V} = \frac{0.2 \times 96}{.05} = 384$$

$$\text{when } Z_c/H_0 = \frac{66.6}{96} = 0.694; \frac{\alpha t}{L^2} = 0.10$$

$$L^2 = \frac{\alpha t}{0.10} = \frac{384 \times 336}{0.10} = 1,290,240$$

$$L = 1135'' \text{ or } 95'$$

FIGURE 4



and the solution, subject to the requirements that

$$y = 0 \text{ when } x = 0$$

$$y = 0 \text{ when } x = L$$

is:

$$y = \frac{i}{2KD_0} (Lx - x^2) \quad (12)$$

For the nonlinear case the continuity condition is

$$\frac{d}{dx} \left( KZ \frac{dZ}{dx} \right) = -i \quad (13)$$

And its solution subject to the requirements

$$Z = d \text{ when } x = 0$$

$$Z = d \text{ when } x = L$$

is:

$$\frac{Z^2}{2} - \frac{d^2}{2} = \frac{i}{2K} (Lx - x^2) \quad (14)$$

If the infiltration  $i$  is the same in both cases and we make  $Y = H - d$  and  $Z = H$  at  $X = L/2$ , then by comparison of Equations 12 and 14 we can write

$$D_o(H-d) = \frac{1}{2}(H^2 - d^2)$$

from which we obtain

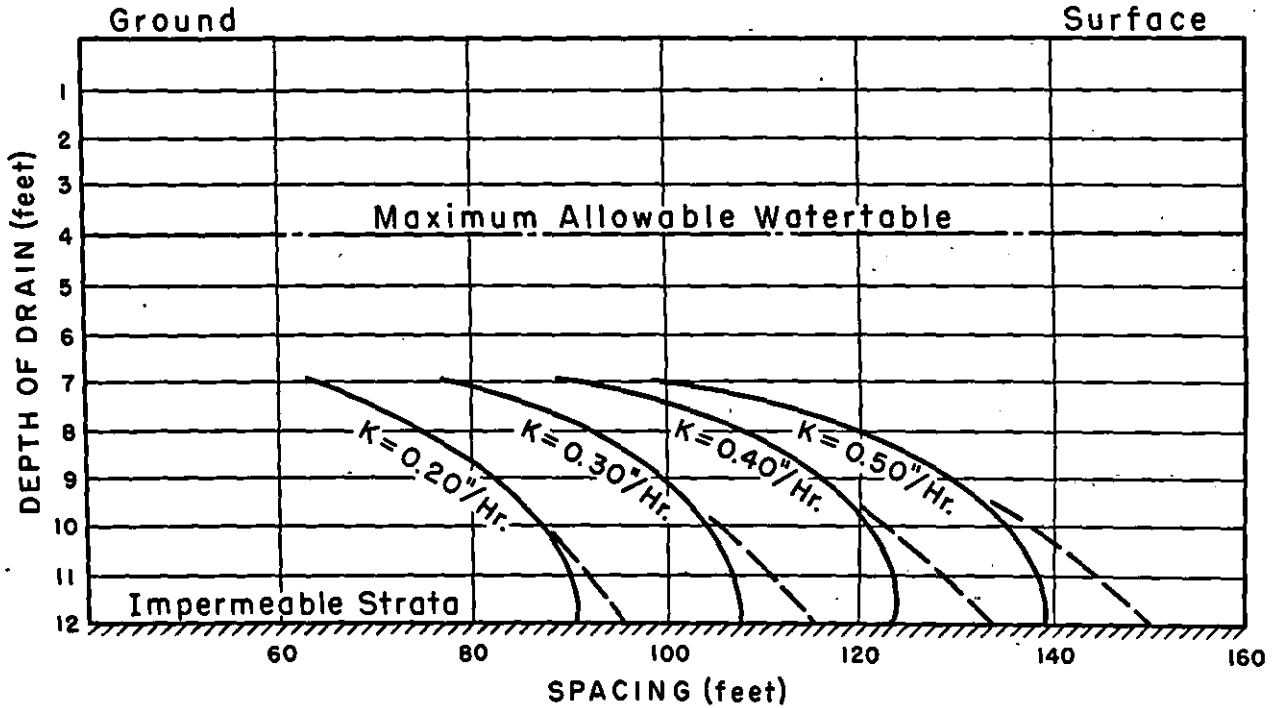
$$D_o = \frac{H+d}{2}$$

In the notation used with the original case this is equivalent to:

$$D_o = d + \frac{y_o}{2}$$

This result indicates that our best choice of  $D_a$  is obtained as the arithmetic mean of  $H$  and  $d$ . A comparison of the first case with this value of  $D_a$  with the second case, on the basis of the drain spacing required for a moderate lowering of the watertable will show that the first case provides a reasonably accurate treatment of the extreme case where the drains are placed at the bottom of the aquifer. Figure 5 shows that the use of this value of  $D_a$  in Formula (5) for the first case gives values within 10 percent of those obtained for the extreme case by the use of Formula (10). We may conclude, therefore, that the intermediate case can be solved approximately by this method also.

The use of the above value of  $D_a$  succeeds in removing the restrictions on the use of the first case for the intermediate drain



Comparison of the drain spacing formula (5) using the value of  $D_a = d + y_0/2$  for the extreme case where the drain is placed at the impermeable strata (shown by solid lines) versus the use of formula (10) (shown by dashed lines).

Note:

Curves computed for same conditions, except for drain depths and permeabilities, as outlined in example.

FIGURE 5

placement, but it does not remove restrictions of another sort. It still remains essential to restrict the use of the first case to those situations where the amount of lowering of the watertable is moderate. It will be readily apparent that if we were to try to estimate the time required to drain away, say  $3/4$  of the original saturated depth, that the initially chosen value of  $D_a$  would become inappropriate to represent the saturated thickness of the aquifer which would prevail toward the end of the drainage period. It is thought that in actual practice lowering of the watertable by moderate amounts will be the rule, but if for any reason it should be desirable to apply the first case to situations having relatively large amounts of drawdown, it is suggested that the computation be made in steps with a new value of  $D_a$  selected for each step. Restrictions on the amount of drawdown are not necessary when using the second case.

An example of the use of the formula is given in the following paragraphs.

Assume a clay-loam soil with a permeability of 0.2 inch per hour; a water-holding capacity of 2.15 inches per foot at wilting point and 3.46 inches per foot at field capacity; and a specific yield of 5 percent. Twelve feet of this material overlies a very slowly permeable stratum. Assume a 6-inch irrigation application every 14 days, a 10 percent surface loss, and a maximum allowable watertable of 4 feet below the ground surface. The problem is to determine the spacing of drains when the tile line is placed at 8-foot depth.

If a 10 percent surface loss occurs, only 90 percent or 5.4 inches of water actually goes into the soil. If the top 4 feet of the soil profile is to be kept free of gravity water, then this 4-foot column of soil will store only that amount of water used by the plants from this column between irrigations. Research information indicates that during a normal irrigation period of about 14 days the consumptive use of the plants is approximately the difference between field capacity and wilting point in the top 2 feet of the soil profile. In the next 2 feet the consumptive use is approximately one-half this difference. Therefore, the amount of capillary water that can be stored from the 6-inch irrigation in the top 4 feet is  $2(3.46 - 2.15) + \frac{2(3.46 - 2.15)}{2} = 3.93$  inches. The remaining amount of water,

$5.4 - 3.93 = 1.47$  inches, entering the soil must, therefore, be temporarily stored below the 4-foot zone and drained out before the next irrigation if the watertable is not allowed to rise above 4 feet below the ground surface. Since the specific yield of the soil is 5 percent, it will take  $1.47 \times 20 = 29.4$  inches of soil to store the drainage requirement of 1.47 inches.

The following chart, Figure 6, illustrates the conditions under this assumption.

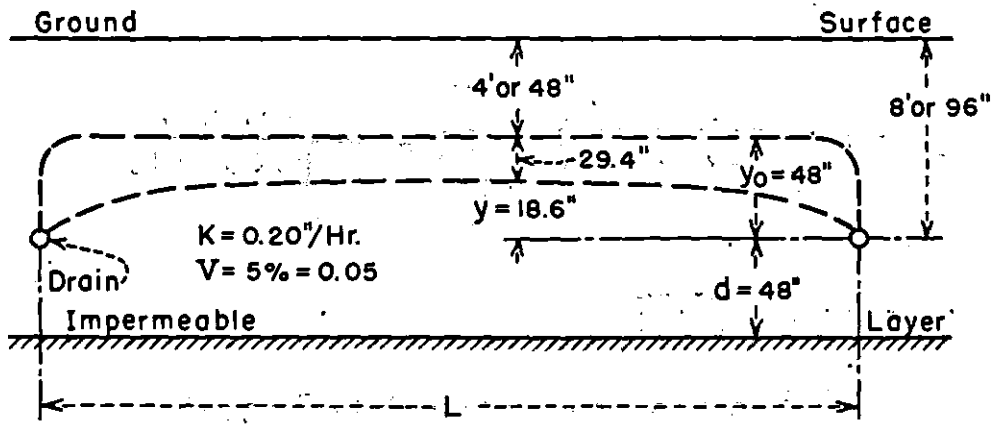


FIGURE 6

The drain spacing formula is in the form of

$$L = \pi \sqrt{\frac{KD_a t}{V \log_e \left( \frac{4}{\pi} \frac{y_0}{y} \right)}}$$

where:

$L$  = spacing, in inches

$K$  = permeability, in inches per hour

$t$  = time between irrigations, in hours

$V$  = specific yield of the soil

$y_0$  = distance between tile and maximum allowable groundwater, in inches

$y$  = distance between tile and watertable at end of period between irrigations, in inches

$d$  = distance between tile and impermeable layer, in inches

$D_a$  = average depth of drainable section or  $d + y_0/2$ , in inches

Then:

$$L = \pi \sqrt{\frac{0.2 \times 72 \times 336}{0.05 \log_e \left(1.27 \times \frac{48}{18.6}\right)}} = \pi \sqrt{\frac{4838.4}{0.05 \times 1.18}}$$

$$L = \pi \sqrt{82,200}$$

$$L = 905 \text{ inches or approximately } 75 \text{ feet}$$

Check by chart--Figure 2

$$\alpha = \frac{KD_0}{V} = \frac{0.2 \times 72}{0.05} = 288$$

$$\text{when } y/y_0 = \frac{18.6}{48} = 0.388 \quad ; \quad \frac{\alpha t}{L^2} = 0.119$$

then:

$$L^2 = \frac{\alpha t}{0.119} = \frac{288 \times 336}{0.119} = 813,000$$

$$L = 903 \text{ inches or approximately } 75 \text{ feet}$$

If in the above example we had assumed a 4.8-inch irrigation application we would have a drainage requirement of 10 percent of the net input. Using this value as the amount of water to be removed, and with all other conditions the same, we would find that the formula gives a drain spacing of 126 feet rather than the 75 feet in the example. It can be seen, therefore, that the amount of water to be removed by the drains is an important consideration in the use of the formula.

It might be asked, at this point, why we do not apply only enough irrigation water to provide the plants with their actual requirements and thereby do away with the necessity of providing drains to dispose of excess groundwater. There are several answers to this such as the requirement of a certain amount of deep percolation for salinity control; the physical impossibility of attaining 100 percent efficiency in irrigation; and the practical limits imposed by the managerial ability and equipment of the average irrigator.

In conclusion, it should be kept in mind that while a proper mathematical equation may be written to satisfy the conditions set up for a particular problem, it is practically impossible to satisfy completely these conditions on an area basis even if all of the factors involved could be measured with any great degree of accuracy. Permeabilities can be measured only in relative terms, soils are usually very nonuniform even over small areas, forecasts of irrigation practices can only be relative, and the amount of natural drainage that will occur is usually indefinite or unknown. Therefore, any answer



derived from a mathematical spacing formula must be used with proper judgment and should be checked with results obtained from experience before it is given widespread use as a basis of construction layout.

We find it necessary, however, to develop a reasonable estimate of future drainage requirements and costs for areas with the conditions described in order to determine feasibility of projects and to provide sufficient funds to meet the drainage problems when they arise. For these reasons we use drain-spacing formulas primarily as a means of predicting the approximate depth and spacing in areas where experience is lacking. We feel that the formula, when used with judgment, is satisfactory for this purpose.