

NATURAL PHILOSOPHY
OF
CAUSE AND CHANCE

**NATURAL PHILOSOPHY
OF
CAUSE AND CHANCE**

**BY
MAX BORN**

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PREFACE

A DRAFT of these lectures, written before they were delivered, contained considerably more technicalities and mathematics than the present text. Facing a large audience in which physicists and mathematicians were presumably a minority, I had to change my plans and to improvise a simplified presentation. Though this did not seem difficult on the platform of the Hall of Magdalen College, Oxford, the final formulation for publication was not an easy task. I did not like replacing rigorous mathematical reasoning by that mixture of literary style, authority, and mystery which is often used by popularizing and philosophizing scientists. Thus, the idea occurred to me to preserve the mathematics by removing it to a detailed appendix which could also contain references to the literature. The vast extension of the latter, however, compelled me to restrict quotations to recent publications which are not in the text-books. Some of these supplements contain unpublished investigations of my school, mainly by my collaborator Dr. H. S. Green. In the text itself I have given up the original division into seven lectures and replaced it by a more natural arrangement into ten chapters.

I have to thank Dr. Green for his untiring help in reading, criticizing, and correcting my script, working out drafts of the appendix, and reading proofs. I am also indebted to Mr. Lewis Elton not only for proof-reading but for carefully preparing the index. I have further to thank Albert Einstein for permission to publish sections of two of his letters.

My most sincere gratitude is due to the President and the Fellows of Magdalen College who gave me the opportunity to plan these lectures, and the leisure to write them down for publication.

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M. B.

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