
An Introduction to
LOGIC
and
SCIENTIFIC
METHOD

BY

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PREFACE

Though formal logic has in recent times been the object of radical and spirited attacks from many and diverse quarters, it continues, and will probably long continue, to be one of the most frequently given courses in colleges and universities here and abroad. Nor need this be surprising when we reflect that the most serious of the charges against formal logic, those against the syllogism, are as old as Aristotle, who seems to have been fully aware of them. But while the realm of logic seems perfectly safe against the attacks from without, there is a good deal of unhappy confusion within. Though the content of almost all logic books follows (even in many of the illustrations) the standard set by Aristotle's *Organon*—terms, propositions, syllogisms and allied forms of inference, scientific method, probability and fallacies—there is a bewildering Babel of tongues as to what logic is about. The different schools, the traditional, the linguistic, the psychological, the epistemological, and the mathematical, speak different languages, and each regards the other as not really dealing with logic at all.

No task is perhaps so thankless, or invites so much abuse from all quarters, as that of the mediator between hostile points of view. Nor is the traditional distrust of the peacemaker in the intellectual realm difficult to appreciate, since he so often substitutes an unclear and inconsistent amalgam for points of view which at least have the merit of a certain clarity. And yet no task is so essential, especially for the beginner, when it is undertaken with the objective of adjusting and supplementing the claims of the contending parties, and when it is accompanied by a refusal to sacrifice clarity and rigor in thought.

IN so far as an elementary text permits such a thing, the present is intended to bring some order into the confusion of tongues concerning the subject matter of logic. But the resolution of the confusion between various schools which it effects appears in the selection and presentation of material, and not in extensive polemics against any school. The book has been written with the conviction

that logic is the autonomous science of the objective though formal conditions of valid inference. At the same time, its authors believe that the aridity which is (not always unjustly) attributed to the study of logic testifies to the unimaginative way logical principles have been taught and misused. The present text aims to combine sound logical doctrine with sound pedagogy, and to provide illustrative material suggestive of the rôle of logic in every department of thought. A text that would find a place for the realistic formalism of Aristotle, the scientific penetration of Peirce, the pedagogical soundness of Dewey, and the mathematical rigor of Russell—this was the ideal constantly present to the authors of this book.

However inadequately this ideal is embodied in the present text, the embodiment is not devoid of positive doctrine, so presented that at least partial justice is done to supplementary approaches to logic.

1. The traditional view of logic as the science of valid inference has been consistently maintained, against all attempts to confuse logic with psychology, where by the latter is meant the systematic study of how the mind works. Logic, as the science of the weight of evidence in all fields, cannot be identified with the special science of psychology. For such a special science can establish its results only by using criteria of validity employed in other fields as well. And it is clear that questions of validity are not questions of how we happen to think, but of whether that which is asserted is or is not in conformity with certain objective states of fact.

2. On the other hand, the pedagogical applications of psychological logics have not been ignored. We have aimed to present the subject in such a manner that discussion of doctrines new to the student is made continuous with his presumed knowledge at the outset. We have therefore avoided as far as possible the synthetic method of exposition: the method which begins with highly abstract elements and constructs a science out of them. Instead, we have followed what seems to us psychologically a more appropriate method. Illustrations with which a college student may reasonably be supposed to be familiar are usually taken as the text for discussion, and abstract, formal elements are gradually revealed as abstract phases of the subject matter. In this way, we trust, we have removed many of the difficulties which face the young student, and at the same time have indicated to him the important rôle played by logic in all of man's activities.

3. Again, while we have tried to present the significant results of symbolic or mathematical logic to those who have no previous

knowledge of the subject, we have not tried to develop the technique of symbolic manipulation for its own sake. In our opinion, such a technique, while very valuable, belongs properly to mathematics developed as an organon of science, and not to an elementary book on logic. Nor do we share the rather hostile attitude towards the Aristotelian logic expressed by some of the more zealous workers in the newer fields. We have not been sparing in indicating the limitations of the traditional presentation of our subject. But we think that the newer achievements in exact logic have served to extend as well as to correct the Aristotelian logic. We have thus given a great deal of attention to traditional views that might well be left out in a systematic presentation of our present knowledge. For we think that the discussion and correction of the limitations of the traditional views has many pedagogical advantages in making our final ideas clear.

4. We do not believe that there is any non-Aristotelian logic in the sense in which there is a non-Euclidean geometry, that is, a system of logic in which the contraries of the Aristotelian principles of contradiction and excluded middle are assumed to be true, and valid inferences are drawn from them. What have recently been claimed to be alternative systems of logic are different systems of notation or symbolization for the same logical facts. We have drawn freely on the natural sciences for illustrations of logical principles, precisely because the logical structure of these sciences is clearly more than linguistic. We have therefore frankly indicated the metaphysical significance of logical principles, and have not failed to note that the structure of language is itself often a clue to something other than linguistic fact. While maintaining that logic as an autonomous science must be formal, we have insisted that its principles are not therefore without significant content; on the contrary, we have taken the position that they are inherently applicable because they are concerned with ontological traits of utmost generality. We think that the category of objective possibility is essential to logical discussion.

In the main, therefore, we view the history of logic as that of a series of contributions of diverse value by the various schools. If our point of view is consequently somewhat eclectic, seeking to give the student a liberal rather than a narrow view of the subject, we have nevertheless striven hard to maintain clear distinctions as to fundamentals. Florence Nightingale transformed modern hospital practise by the motto: Whatever hospitals do, they should not spread disease. Similarly, logic should not infect students with fallacies and

confusions as to the fundamental nature of valid or scientific reasoning.

Different instructors will naturally attach more value to different parts of the book. Not all of it can be presented in a one-semester course, and enough material has been included to occupy the student's attention for a full year. In a one-semester course, the authors have found that the substance of Book II, with the inclusion of Chapters III, IV, and VIII of Book I, gives the most satisfactory results. Those not interested in mathematics may omit Chapter VII. Books are tools which wise men use to suit their own ends. One of the authors, who has given courses in elementary logic for over twenty years, has generally treated the contents of Book II (Applied Logic and Scientific Method) before the formal logic of Book I. There are, to be sure, some topics in Book II which presuppose the solutions of Book I. But experience shows that such difficulties are readily surmountable. It is especially the hope of the authors that general readers as well as students of the natural and social sciences will find this book helpful towards an understanding of scientific method.

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E. N.

The continued demand for this book, which has exhausted three printings of it, has given us a chance to correct certain errors and to revise some statements in the interest of greater clarity.

M. R. C.

E. N.

January 7, 1936

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