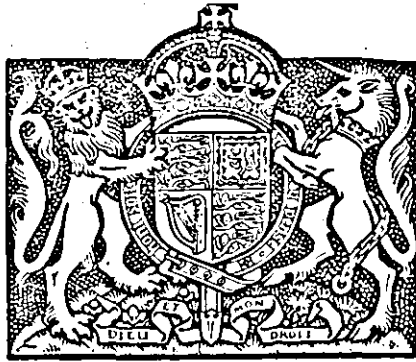


MEMORANDA ON COLONIAL
FERTILIZER EXPERIMENTS



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MEMORANDA ON COLONIAL FERTILIZER EXPERIMENTS

I

PLANNING AND CONDUCT OF
FERTILIZER EXPERIMENTS

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2

FERTILIZER EXPERIMENTS
IN COLONIAL AGRICULTURE

by E. M. Crowther, D.Sc., F.R.I.C.

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APPENDIX

1. Note on the statement of results of 2ⁿ experiments

In setting out the results of factorial experiments with all combinations of a number of treatments each at two levels, generally 0 and 1, it is convenient to give the general mean (G.M.), the "main effects" (single letters) and "interactions" (combinations of two or more letters). The "main effects" show the mean difference in yield or other data between plants with and those without a given treatment, averaging over all other conditions. Yields with and without a given treatment are obtained by adding to or subtracting from the general mean one-half of the main effect. Thus:—

$$\begin{aligned} \text{Mean yield without N} &= \text{GM} - \frac{1}{2} \text{N} \\ \text{Mean yield with N} &= \text{GM} + \frac{1}{2} \text{N} \\ \text{Difference} &= \text{main effect of N} = \text{N} \end{aligned}$$

The interaction between two treatments (*e.g.*, NP) shows how far the effect of one of the fertilizers is increased or decreased by the presence of the other. NP is defined as **one-half** of the difference between the effects of N on plots with and without P respectively. Thus:—

$$\begin{aligned} \text{Mean effect of N on plots without P} &= \text{N} - \text{NP} \\ \text{Mean effect of N on plots with P} &= \text{N} + \text{NP} \\ \text{Difference} &= 2 \text{NP} \end{aligned}$$

Alternatively the interaction may be regarded as **one-half** of the extra benefit from using two fertilizers together over the sum of their effects when used separately. This may be seen from the following statement of the way in which the yields for individual treatments are built up from main effects and interactions.

Mean yield for treatment with

$$\begin{aligned} \text{No N, no P} &= \frac{1}{8} (\text{GM} - \text{N} - \text{P} + \text{NP}) \\ \text{+ N, no P} &= \frac{1}{8} (\text{GM} + \text{N} - \text{P} - \text{NP}) \\ \text{No N, +P} &= \frac{1}{8} (\text{GM} - \text{N} + \text{P} - \text{NP}) \\ \text{+ N, +P} &= \frac{1}{8} (\text{GM} + \text{N} + \text{P} + \text{NP}) \end{aligned}$$

The above method of defining interactions has the advantage that all main effects and interactions are obtained by adding or subtracting all the individual plot yields in an experiment. They therefore have the same standard error. The calculation of main effects and interactions from the individual plot yields may be illustrated by the 8 treatment NPK experiment, using Fisher's conventions* by which small letters denote individual treatment or yields and (1) denotes the plots with none of the treatments considered. If each of the 8 treatments occurs 4 times (*i.e.*, 32 plots in all):—

$$\begin{aligned} \text{Main effect N} &= \left(\frac{1}{16} \text{npk} + \text{np} + \text{nk} + \text{n} - \text{pk} - \text{p} - \text{k} - (1) \right) \\ \text{Interaction NP} &= \left(\frac{1}{16} \text{npk} + \text{np} - \text{nk} - \text{n} - \text{pk} - \text{p} + \text{k} + (1) \right) \\ \text{Interaction NPK} &= \left(\frac{1}{16} \text{npk} - \text{np} - \text{nk} + \text{n} - \text{pk} + \text{p} + \text{k} - (1) \right) \end{aligned}$$

*Fisher's convention of using capital letters for effects and interactions, and small letters for treatments and plot yield has been followed here to facilitate reference to the books and papers of Fisher and Yates. For practical convenience in drawing up plans and in preparing labels and reports, the present writer much prefers to use the ordinary chemical symbols with any necessary suffixes for treatments and plot yields and, where necessary, to indicate effects and interactions by underlining the symbols in manuscript and typescript and by italics or heavy type capitals in print. Such treatment symbols as CaMg become very clumsy in small letters.

2. Note on the arrangement of 2^n experiments

Arrangements appropriate for 2^n experiments may be extended to cover four-level comparisons, *e.g.*, four blocks, four kinds of a given fertilizer, or three kinds and none of a given fertilizer, by the trick of writing the treatments pseudofactorially as (1) a, b, ab. Then, as usual

$$\begin{aligned} A &= \frac{1}{2} (ab + a - b - (1)) \\ B &= \frac{1}{2} (ab - a + b - (1)) \\ AB &= \frac{1}{2} (ab - a - b + (1)) \end{aligned}$$

The arrangement of 2^n factorial experiments in blocks with confounded interactions or fractional replication is facilitated by using a series of formal identities based on the special algebraic convention that the product of any letter by itself equals unity, *e.g.*, $A^2=1$, $ABC \times ABD = A^2B^2CD = CD$. By multiplying the identity $ABCD=1$ in turn by appropriate letters it will be seen that, if $ABCD=1$, then

$$\begin{aligned} A &= A^2BCD = BCD \\ AB &= A^2B^2CD = CD \\ ABC &= A^2B^2C^2D = D \end{aligned}$$

In other words, any letter or group of letters in such an identity may be regarded as being identical with the rest of the letters in the identity.

Confounding interactions with blocks means identifying certain treatment contrasts with certain block contrasts. Fractional replication means identifying certain treatment contrasts with certain other treatment contrasts. The identities show which identifications are to be made. Some may be chosen arbitrarily, but others then follow automatically. To write out the relevant identities provides a rapid and convenient means of seeing the implications of any restriction on complete replication.

It is convenient to denote treatment contrasts by early letters of the alphabet and block contrasts by late letters. Each independent identity written down halves the number of plots required to balance all treatments with the restriction covered by the identity. Thus in the 8 plot NPK test with two blocks—denoted by (1) and x—a full replication would require 16 plots but the number may be reduced to 8 by the condition $NPK.X=1$. Multiplying both sides by X gives $NPK=X$, which is equivalent to identifying the NPK interaction with the contrast between blocks. It will also be noted that any two-term interaction is identical with the interaction of the third treatment with blocks, *e.g.*, $NP=KX$.

When appropriate identities have been selected, the treatments within each of the blocks are chosen to satisfy the conditions that the letters representing each combination of treatments and blocks must have either no letters or an even number of letters in common with each of the identities. This may be expressed more concisely by saying that treatment and block letters must be "even" with the identities. Treatment and block letters must be considered together in choosing "even" combinations, but it is more convenient to tabulate them separately, setting out the block letters (*e.g.* (1), x, y, xy) at the heads of columns. The first block, denoted by (1), may be termed the principal block. It includes the plot (1) with none of the treatments. Two "even" combinations of treatment letters are then selected, together with their product. This gives four of the treatments in the principal block. Another set of four is obtained by picking another "even" combination and multiplying the first four by it. This process is continued until the principal block is completed. The treatments for the second block (x) are found by picking any "even" combination of X but not Y with treatment letters and using it to multiply all the treatment letters in the principal block, remembering the rule the $A^2=1$.

Thus, for the identity $NPK.X=1$, the principal block must contain the untreated plot (1), such a pair of combinations as np and nk and their product pk. The second block x must contain a pair of letters such as nx. Entering x as a heading for the block and multiplying the treatment letters of the principal block (1) by n, we have for block x the treatments n, p, k, npk.

Identity $NPK.X=1$

Block (1)	x
Treatments (1)	n
np	p
nk	k
pk	npk

The example given in paragraph 20 of arranging combinations of four treatments in 8 blocks of 2, confounding all two-factor interactions, requires 7 identities to cover the degrees of freedom for blocks.

$$\begin{aligned}
 AB.X &= 1 \\
 AC.Y &= 1 \\
 BC.XY &= 1 \\
 AD.Z &= 1 \\
 BD.XZ &= 1 \\
 CD.YZ &= 1 \\
 ABCD.XYZ &= 1
 \end{aligned}$$

Block (1)	x	y	xy	z	xz	yz	xyz
Treatments (1)	b	c	bc	d	bd	cd	bcd
abcd	acd	abd	ad	abc	ac	ab	a

A 2^5 experiment in four blocks of eight could be given by:—

$$\begin{aligned}
 ABC.X &= 1 \\
 ADE.Y &= 1 \\
 BCDE.XY &= 1
 \end{aligned}$$

Block (1)	x	y	xy
Treatments (1)	b	d	bd
bc	c	bcd	cd
de	bde	e	be
bcde	cde	bce	ce
abd	ad	ab	a
acd	abcd	ac	abc
abe	ae	abde	ade
ace	abce	acde	abcde

It will be found convenient to check the arrangement by setting out the treatments factorially and inserting the block letters, thus:—

	(1)	c	d	cd	e	ce	de	cde
(1)	(1)	x	y	xy	y	xy	(1)	x
a	xy	y	x	(1)	x	(1)	xy	y
b	x	(1)	xy	y	xy	y	x	(1)
ab	y	xy	(1)	x	(1)	x	y	xy

Half replicates are obtained by an identity involving only treatment letters, quarter replicates by two such identities and so on. Such identities may be used in conjunction with additional ones for blocks. It is convenient to write down all the identities or aliases involved and not merely the minimal number required to define the conditions. Thus a half replicate of a 2⁶ experiment in four blocks of eight would be given by :—

$$\begin{aligned} ABCDEF &= 1 \\ ABDE.X &= 1 = CF.X \\ ADF.Y &= 1 = BCE.Y \\ BEF.XY &= 1 = BEF.XY = ACD.XY \end{aligned}$$

Such designs have been used for Rothamsted experiments at many centres on three bulky organic manures at two rates and no organic manure, each tested with and without N, P, K.

The contrasts may be denoted : Blocks (1), x, y, xy.
 No organic manure (1) Rate 1 (1) No N (1) No P (1) No K (1)
 Sewage sludge a „ 2 d + N n + P p + K k
 Compost b
 Farmyard manure ab

The identities are :
 ABDNPK = 1
 ABNP.X = 1 = DK.X
 ANK.Y = 1 = BDP.Y
 BPK.XY = 1 = ADN.Y

Blocks	(1)	x	y	xy
Treatments	(1)	dp	np	dn
	adpk	ak	adnk	anpk
	dnpk	nk	dk	pk
	an	adnp	ap	ad
	bp	bd	bn	bdnp
	abdk	abpk	abdnpk	abnk
	bdnk	bnpk	bdpk	bk
	abnp	abdn	ab	abdp

The pattern of confounding may be seen and checked by writing the block letters (1), x, y, xy into a formal arrangement of the treatments.

Organic Manures		Fertilizers							
		(1)	n	p	np	k	nk	pk	npk
<i>Single rate</i>									
None	(1)	(1)	—	—	y	—	x	xy	—
Sludge	a	—	(1)	y	—	x	—	—	xy
Compost	b	—	y	(1)	—	xy	—	—	x
F.Y.M.	ab	y	—	—	(1)	—	xy	x	—
<i>Double rate</i>									
None	d	—	xy	x	—	y	—	—	(1)
Sludge	ad	xy	—	—	x	—	y	(1)	—
Compost	bd	x	—	—	xy	—	(1)	y	—
F.Y.M.	abd	—	x	xy	—	(1)	—	—	y

The confounding of the DK interaction with the X contrasts of blocks (*i.e.*, DK.X = 1) is seen by the circumstance that in the x and xy blocks the single rates of organic manures occur with potash and the double rates without potash.

It will be realized that very great care must be taken in the choice of interactions for confounding, especially with fractional replication. Where there are only one or two experiments it may pay to increase the size of the blocks to minimize confounding. In a series of closely related experiments different interactions may be confounded in different experiments.

In all these patterns the actual arrangement of plots within the blocks must, of course, be randomized, and it will generally be worth while randomizing the position of the blocks within the experiment.

For more detailed discussions of confounding and fractional replication in 2ⁿ and 3ⁿ experiments, reference should be made to:—

D. J. Finney, *Empire Journal of Experimental Agriculture*, 1947, Vol. 15, pp. 107–112, and O. Kempthorne, *Biometrika*, 1947 (in the Press).

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