

PROBIT ANALYSIS

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A STATISTICAL
TREATMENT OF THE SIGMOID
RESPONSE CURVE

BY

D. J. FINNEY, M.A., Sc.D.

*Lecturer in the Design and Analysis of Scientific Experiment,
University of Oxford*

WITH A FOREWORD BY

F. TATTERSFIELD, D.Sc., F.R.I.C.

*Head of the Department of Insecticides and Fungicides,
Rothamsted Experimental Station*

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FOREWORD

IN our work at Rothamsted on insecticides, their action and relative potency, we have been so dependent and have made so many calls on the skill and patience of the Statistical Department and in particular, in recent years, of Mr Finney, that it is a pleasure to learn that the statistical methods and techniques which he has placed so willingly at our disposal for the solution of our various problems are now to be expressed in a more permanent form and to be given to a wider field of workers. If this book receives its due, the investigators of toxicological problems throughout the world will find it a standby.

To many of us, engaged on the practical issues of devising the experimental material and methods to be employed in our laboratory tests, there will be much in these pages that has to be taken upon trust. They are meant to act primarily as an aid in computation, but there is a profound thread of reasoning running through them all, giving a coherence to the several chapters. One cannot but feel, therefore, that the more mathematical readers will find the book suggestive and stimulating.

Twenty-five or more years ago, when I entered the field of research from which Mr Finney takes so many examples for detailed computational study, the very whisper of the need for statistical analysis, falling upon the ears of the biological expert, was enough to bring down a storm of denial upon one's head. Although there may be some small residue of such a reaction still in existence, it now only persists in obscure nooks and crannies of the world of biological research. Much of this change is due to the school of statisticians founded by R. A. Fisher at Rothamsted, of which Mr Finney has been a distinguished member. They always showed a willingness to enter into one's experimental difficulties and an understanding of the limitations imposed by time and space upon the amount of work it was possible to do. The measure of the thought given to these matters can be gauged by the fact that some of the most important tables, given by Mr Finney in the following

pages, were in process of computation in actual anticipation of problems which we later carried to him. This brings up the now rather hackneyed, but none-the-less important, matter of the prior consultation with a competent statistician before designing experiments. Repeatedly is such a course justified by events, in that the plan can not only be often simplified, but so designed as to yield more information with little, if any, more labour.

Another issue, raised by such text-books as this, concerns the extent to which a training in statistics should enter into the curriculum of the biological and biochemical student. I have found, relatively late in life, how hard a task it is to pick up, during busy years, the requisite amount of basic statistical knowledge to follow the arguments set out in recent text-books written for one's own benefit. I therefore feel that more attention might be given to such a matter by the academic powers-that-be. The engineer may not need to know all there is to be scientifically known about the composition and manufacture of his tools and materials, but at least he should know enough to use them rightly. So too the quantitative biological investigator.

Mr Finney's text-book has a very long and dignified ancestry ranging back to early Egyptian and Mesopotamian times, when texts of mathematics were used for the training of scribes, the professional computers of those distant days. But this is a forward-looking book and is primarily meant for experimentalists. Enlarged editions may well follow this, the first, as the subject grows and new problems arise; and that there are many just round the corner anyone who has discussed these matters with the author is very well aware. Having had the benefit of his personal advice and help throughout the last six years I cannot but wish this book Godspeed on its helpful mission to others.

F. TATTERSFIELD

PREFACE TO FIRST EDITION

FROM the theory of probability, originally investigated in order to explain nothing more important than the results of games of chance, has developed the science of applied statistics. Over one hundred years ago Laplace wrote that

. . . la théorie des probabilités n'est au fond, que le bon sens réduit au calcul: elle fait apprécier avec exactitude, ce que les esprits justes sentent par une sorte d'instinct, sans qu'ils puissent souvent s'en rendre compte,

and these words might equally well be written of statistics to-day. In many fields of scientific research, and especially in the biological sciences, numerical studies are complicated by the inherent variability of the material under investigation, and conclusions must be based on averages derived from series of observations. The estimation of these averages and the assessment of their reliability are statistical operations, in the performance of which the experimenter inevitably employs a statistical technique even though he himself may not always recognize this fact. The operations may be simple or complex, depending upon the circumstances; if, however, they cease to be 'le bon sens réduit au calcul', they can no longer be expected to contribute to the understanding of the problem under investigation.

The recent rapid advances in the application of rigorous statistical methods to biological data began with the publication, in 1925, of R. A. Fisher's *Statistical Methods for Research Workers*. Not only did Fisher develop exact methods for the analysis of data from small samples to replace the older approximations from large-sample theory, but he also introduced new and powerful techniques for making the most efficient use of experimental results. Of equal importance to the growth of the present-day philosophy of experimentation was Fisher's suggestion that the statistician should be consulted during the planning of an experiment and not only when statistical analysis of the results is required, as his advice on experimental design

may greatly increase the value of the results eventually obtained. Since Fisher's book first appeared, the principles of experimental design and the methods of statistical analysis have been extended so rapidly as to make it increasingly difficult for any but the professional statistician to be familiar with the variety of methods needed in biological problems.

Many books since Fisher's have been written with the aim of surveying a wide field of biological statistics, but these can give only an outline of some important topics. There is to-day a need for books in which the specialized statistical methods appropriate to certain branches of science will be discussed in sufficient detail to enable biologists to appreciate them and apply them to their own problems.

One subject requiring fuller discussion than can reasonably be expected in any general text-book of statistics is the method of probit analysis, for the development of which J. H. Gaddum and C. I. Bliss are largely responsible. This method is widely used for the analysis of data from toxicity tests for the assay of insecticides and fungicides, and also of data from other types of assay dependent upon a quantal response. In this book I have tried to give a systematic account of the theory and practice of probit analysis, including as much as possible of the most recent extensions and refinements, in such a form that it may be understood by biologists, chemists, and others who have some knowledge of elementary statistical procedure; at the same time, I have endeavoured to satisfy the mathematical statistician by showing the theoretical background of the method. The less mathematically minded reader will no doubt be content to omit, or at most to read cursorily, Appendix II and other sections concerned with the mathematical basis of the technique. Full understanding and appreciation of statistical methods can be gained only by experience in their use, but careful study of the numerical examples should enable many who were previously unfamiliar with probit analysis to apply it satisfactorily to their own data.

This book has been written as a result of several years of close collaboration with members of the Insecticides Department

at Rothamsted Experimental Station, especially with Dr F. Tattersfield, Dr C. Potter, and, until he left Rothamsted, Dr J. T. Martin. I wish to express my gratitude to them for discussing with me a wide variety of their problems, for advising me on the experimental aspects of their results, and for the generosity with which they have permitted me to use their data both in this book and in earlier publications. I am also very grateful to my colleagues in the Statistical Department at Rothamsted for much helpful discussion, and particularly to Dr F. Yates for his detailed and constructive criticism in the preparation of my book. Others to whom my thanks are due include Miss G. M. Ellinger for assistance in German translation, Dr C. G. Butler for permission to use the numerical data of Ex. 33, Dr A. E. Dimond and Dr J. G. Horsfall for giving me very full information on the results discussed in § 41 and for permission to use their data, Professor G. H. Thomson for assistance in tracing the history of the probit method, the Editors of the *Annals of Applied Biology* for permission to reproduce the first half of Table II, Professor R. A. Fisher, Dr F. Yates, and Messrs Oliver and Boyd, Ltd. for permission to reproduce Tables I, VI and VII from their book *Statistical Tables for Biological, Agricultural and Medical Research*, and my father, Robt. G. S. Finney, for very considerable help in the correction of proofs.

D. J. FINNEY

ROTHAMSTED EXPERIMENTAL
STATION

August 1945

PREFACE TO SECOND EDITION

IN the four years that have elapsed since the first edition of this book was completed, the theory of the analysis of quantal response data has been extended and new applications of probit analysis and related methods have been made. To discuss these

in logical order would entail considerable rearrangement of the book, a course that was impracticable for this edition. Instead, I have adopted the compromise of correcting a small number of minor errors, of rewriting §§ 15, 28, parts of §§ 45 and 47, and Appendix II, and of adding a new chapter on Recent Developments. Table II has been extended to 90 % natural mortality, and additional information has been incorporated into Table V; the new Tables VIII and IX aid the analysis of problems discussed in Chapter 11.

I am aware of three major shortcomings that could not be removed without a more radical revision of the book. I have not discussed as fully as I should now choose to do alternative methods for the analysis of quantal response data, especially that based upon the angle transformation (§ 15) and approximations such as Kärber's (§§ 13, 49). My examples give the impression that probits are useful primarily for data from insecticide tests, whereas in fact they find application in many other branches of science. I may seem to imply that probit analysis is a statistical technique co-extensive with the analysis of biological assays; in fact, only one class of biological assays has any need of probits or related techniques, and probit analysis is used in the statistical examination of many experiments that would not generally be regarded as assays.

I am indebted to all who, privately and in reviews, have told me of errors in the first edition or have made suggestions which have helped the preparation of this edition. I am especially grateful to Mr P. S. Hewlett and Mr R. L. Plackett, for sending me a draft of a new paper and allowing me to comment on it in § 53, and to the Editors of *Biometrika* for permission to include Table VIII.

D. J. FINNEY

OXFORD

December 1949

Appendix I

THE COMPUTING OF PROBIT ANALYSES

THE chief hindrances to the more widespread adoption of the probit method for the statistical analysis of quantal response data are probably the apparent complexity of the mathematical theory and the apparent laboriousness of the computations. The more technical details of the theory (Appendix II) are admittedly difficult, especially for the many biologists who lack mathematical training. In Chapters 2 and 3 an attempt has been made to present a reasonably simple account of the aim and underlying principles of the method, in a form which the reader will be able to appreciate even though he has to take the theory on faith. For many routine purposes the computations can be largely replaced by the graphical analysis described in Chapter 3, relative potencies and other parameters being estimated by measurement on the diagrams. When the complexity of the data, or the desirability of using extensions of the probit method (such as have been described in later chapters), makes necessary arithmetical rather than graphical estimation, the labour can be much reduced and computational accuracy much increased by orderly arrangement and systematic working procedure. In this appendix is given an example of the computing required for the simplest type of analysis, the fitting of a single probit regression line; the arrangement is that used throughout this book, but the steps are set out in greater detail and recommendations for adequate checking are made. With modifications appropriate to the various circumstances, the same procedure may be used in computing any of the more complicated probit regression equations that have been described.

The computer is assumed to be working with a calculating machine; the same results can be obtained by pen-and-paper calculations, but almost any machine intended for general computing will materially improve both accuracy and speed. Before buying a machine it is well to take expert advice on the most suitable type, though to some extent personal preference must decide between the products of different manufacturers. If the machine is to be used frequently, an electrically operated model is desirable, and a 10-figure rather than an 8-figure keyboard is an advantage that usually outweighs the additional cost.

The reader should find little difficulty in becoming proficient at the arithmetical processes described in this book, but the only route to accuracy and efficiency is by way of experience. A machine is not a complete safeguard against arithmetical errors, and carelessness will lead to wrong answers just as certainly as in non-mechanized calculations. One of the most frequent sources of error lies in copying from paper to machine or from machine to paper; inversions of the order of two digits and similar mistakes are particularly easily made. Computations should therefore be planned so as to reduce to a minimum the necessity for copying on to paper figures that have later to be restored to the machine. The surest means of preventing errors is to have all work checked by another computer on another machine, but, by the application of full checks at every stage, one computer should be able to carry out the work satisfactorily. The chief danger is that any misreading of figures, wrong setting of the machine, or faulty working of the machine may be repeated in checking; in order to guard against this, where a section of the computations cannot be checked by an independent path and the steps have therefore to be duplicated, the order of setting the machine should be changed so as to avoid the complete repetition of every detail. For example, a column may be summed by starting from the bottom instead of from the top, or a product found by interchanging the roles of multiplicand and multiplier.

Inexperienced computers frequently carry a far greater number of digits in their computations than is warranted by the accuracy of the original data, and present their results to six places of decimals when three at most are justifiable. In machine calculations an increase in the number of digits does not increase the labour to the same extent as in pen-and-paper work, and undoubtedly it is sometimes easier to carry an additional decimal place than to decide how many can be justified. Nevertheless the saving of time through working with fewer digits may be considerable in a long series of analyses; it should further be remembered that copying errors are less common with four digit numbers than with seven. The general practice in the numerical examples of this book has been to cut out all unnecessary digits from the early stages of an analysis (i.e. in dosages, working probits, weights, etc.), to retain digits fairly fully at intermediate stages (sums of squares and products, elements of matrices, etc.) as an aid to checking, but to present results shorn of superfluous digits and free of any spurious appearance of accuracy. Unless doses have been measured to within 0.1% of their true values, two or three significant digits in the dosage, x , are sufficient. Percentage kills

based on batches of 200 individuals or less need not be expressed more precisely than to the nearest 1 %, thus enabling the tables of this book to be used without interpolation; there is little to be gained by calculating the kills to greater accuracy than the nearest 0.1 % unless the batches contain more than 2000 individuals.

Martin (1940, Table 3) has given data on the toxicity to *Aphis rumicis* of an ether extract of *Derris malaccensis*; the original observations of the experiment have been reconstructed from his figures in order that they may be used in an example of the typical probit analysis. Table 38 shows the greater portion of the computations for estimating a linear regression equation which will relate the probit mortality to the log concentration.

TABLE 38. Computations for the Fitting of a Probit Regression Equation

λ	x	n	r	p'	$p(C-2)$	Em- pirical probit	Y	nx	y	nxy	ny
520	2.72	49	49	100	100	∞	7.5	2.4	7.85	6.528	18.840
390	2.59	45	44	98	98	7.05	6.9	6.8	7.03	17.612	47.804
260	2.41	44	38	86	86	6.08	6.1	17.4	6.08	41.934	105.792
130	2.11	52	15	29	28	4.42	4.7	30.4	4.43	64.144	134.672
65	1.81	47	4	9	7	3.52	3.3	6.7	3.57	12.127	23.919
0	—	48	1	2	0	—	—	—	—	—	—
								63.7		142.345	331.027

$$1/Snw = 0.0156986, \quad \bar{x} = 2.2346, \quad \bar{y} = 5.1967.$$

$Snwx^2$	$Snwxy$	$Snwy^2$
321.72589	757.4672	1809.159
318.08633	739.7180	1720.234
<hr/>	<hr/>	<hr/>
3.63956	17.7492	88.925
		86.558
		$2.37 = \chi^2_{(3)}$

The various steps in building up Table 38 and in completing the estimation of the probit regression line are as follows:

1. In the column headed λ enter, in suitable units (here milligrams of dry root per litre of spray fluid), the doses tested, arranging these in descending order from the highest to the controls or zero concentration.

2. In the column headed x enter the logarithms of λ , to base 10, correct to two places of decimals. The doses may be multiplied or divided by a power of 10 throughout in order to make x take small

positive values, but a compensating adjustment must be made in later stages of the analysis if the results are to be expressed in the original units of dose.

3. In the columns headed n and r enter for each dose the number of insects tested and the number badly affected, moribund, or dead (Martin's $B + M + D$, which he takes as the 'kill').

4. Check that steps 1, 2 and 3 are correct, beginning each check from the bottom of the column.

5. Calculate the percentage kill, $p' = 100r/n$, to the nearest whole number; this step is most expeditiously performed on a slide rule. If n exceeds 200 for many of the doses, give the percentages to one decimal place.

6. The percentage kill among the controls is small ($c = 2$), and is estimated from about as many insects as the kill for other doses; the approximate analysis discussed in § 27 will therefore be sufficiently accurate. The population value, C , is taken as equal to c , and the adjusted kills calculated accordingly as

$$\begin{aligned} p &= \frac{p' - 2}{98} \times 100 \\ &= 1.020p' - 2.0; \end{aligned}$$

p also is taken correct to the nearest whole number (or to 1 decimal place if n is generally greater than 200). A slide rule can be used for the multiplication.

7. Check step 5, multiplying p' by n to give $100r$.

8. Check step 6 from the relation

$$p' = 0.98p + 2.$$

9. Enter the probits of p in the 'empirical probit' column, reading the values from Table I or Table 1, to 2 decimal places.

10. Plot the empirical probits against x , as shown in Fig. 23; draw a provisional straight line to fit the points, placing the line by eye and allowing for one point above the line at $x = 2.72$.

11. For each of the dosages used in the experiment read the value of the ordinate to the provisional line. These are the expected probits, Y , and are entered in the appropriate column of Table 38, correct to 1 decimal place; greater accuracy in Y is unnecessary unless n is very large.

12. Check steps 9, 10 and 11.

13. From the column $C = 2$ of Table II, read the weighting coefficient for each Y in Table 38, multiply by the corresponding n , and enter, to 1 place of decimals, in the column nw . Two or three significant digits in nw will generally be sufficient, and a good working rule is to use 2 places of decimals when n is less than 20, 1 place when n is between 20 and 200, and the nearest whole number for higher values

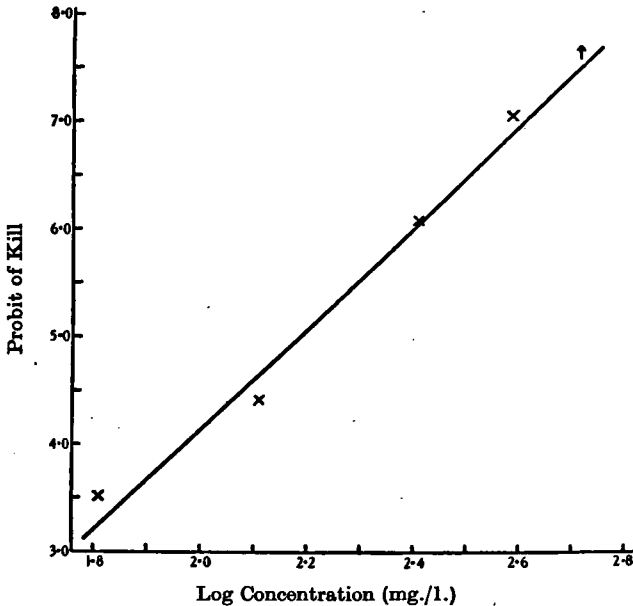


FIG. 23. Relationship between dosage of *D. malaccensis* and probit of kill of *A. rumicis*, showing probit regression line.

of n , modifying this rule so as to give the same number of decimal places for each dose throughout one analysis. In exceptional circumstances, such as the occurrence of a very wide discrepancy between the provisional line and an observation at either extreme of the range, a greater number of decimal places in nw may be desirable for the one dose. These recommendations assume that C is not very large; more decimal places may be required to give a reasonable number of significant digits when a high value of C reduces the values of w considerably.

14. From Table IV enter the working probit, y , corresponding to each p (not p') and Y . Thus from the last page of this table, the last

line of the column headed 7.5 gives $y = 7.85$ for $p = 100$; from the preceding page the column headed 6.9 gives $y = 7.03$ for $p = 98$, and so on. Use 2 decimal places if n is generally less than 200, otherwise 3. If Y is less than 2.0 or greater than 7.9, determine y from Table III as described in § 16.

15. Check steps 13 and 14; 13 may be checked satisfactorily with a slide rule. It will be observed that y usually, but not always, differs less from the empirical probit than does Y .

16. Place the first value of nw on the keyboard of the machine, multiply by the corresponding x , and enter the product in the column nwx ; leave the keyboard unaltered but clear the product from the machine, multiply by y , and enter in the column nwy . Clear the machine and repeat for each dosage, entering each result to its full number of decimal places. No entries are made in the line for $\lambda = 0$, as the controls are used only for estimating C .

17. Place the last value of x on the right-hand side of the keyboard, unity at the left-hand side, and multiply by nw ; without clearing the result, repeat with the next to the last value of x and so work to the top of the x column, thus accumulating the products at the right-hand side of the result register and the sum of nw at the left hand. Enter the totals

$$S_{nw} = 63.7,$$

$$S_{nwx} = 142.345,$$

in Table 38, and check the latter by addition of the nwx column from the top. Before S_{nwx} is cleared from the machine, divide it by S_{nw} to give \bar{x} to 4 places of decimals, and enter \bar{x} in Table 38.

18. Repeat these operations using y in place of x , thus checking the total S_{nw} and also obtaining

$$S_{nwy} = 331.027;$$

again check, by adding the nwy column from the top, and before clearing divide by 63.7 to give \bar{y} to 4 places of decimals. Enter \bar{y} in Table 38.

19. Find the reciprocal of S_{nw} , either from tables or by division, and enter at the bottom of Table 38 to at least 7 decimal places. Additional accuracy is needed here, as this quantity is to be multiplied by large numbers.

20. Set $1/S_{nw}$ on the keyboard and check that multiplication by S_{nw} gives unity; clear the result and multiply by S_{nwx} , so checking the values of \bar{x} ; clear the result and similarly check \bar{y} .

21. Square $Snwx$ and divide by Snw , multiply $Snwx$ and $Snwy$ and divide by Snw , square $Snwy$ and divide by Snw ; the answers 318.08633, 739.7180 and 1720.234, are to be entered in Table 38 as shown. Check by squaring the second, dividing by the first, and obtaining the third as the quotient; the first being still on the keyboard, multiply by 63.7 and divide by 142.345 to give 142.345 as a quotient. These checks may fail in the last digit as a result of rounding off. The parts played in the checks by the first and third of the three calculated quantities should be interchanged when the third has the greater number of significant digits.

22. Set the first entry for nwx on the keyboard, multiply by x , and repeat with successive lines of Table 38, accumulating the total so as to give

$$\begin{aligned} Snwx^2 &= 6.528 \times 2.72 + 17.612 \times 2.59 + \dots + 12.127 \times 1.81 \\ &= 321.72589. \end{aligned}$$

Enter this figure in the appropriate position in the lower part of the table. Multiply the last value in the nwx column, which is already on the keyboard, by y and repeat with successive entries *up* the table, to obtain

$$Snwxy = 757.4672;$$

enter this figure. Multiply nwy by x *down* the table, in order to check $Snwxy$. Before clearing the machine, subtract from $Snwxy$ the second of the three quantities calculated in step 21, without the machine, to give

$$S_{xy} = 17.7492;$$

enter in Table 38 and check the difference by means of the machine. Multiply nwy by y *up* the table to give $Snwy^2$, and enter in Table 38.

23. Check $Snwx^2$ by accumulating products of nwx and x *up* the table. Before clearing, subtract the first of the quantities calculated in step 21, without the machine, to give

$$S_{xx} = 3.63956;$$

enter in the table and check with the machine. Similarly check $Snwy^2$ *down* the table, derive

$$S_{yy} = 88.925,$$

enter in the table, and check.

24. Compute

$$\begin{aligned} b &= 17.7492 \div 3.63956 \\ &= 4.8767. \end{aligned}$$

25. Obtain the equation for the probit regression line as

$$Y = 5.1967 + 4.8767(x - 2.2346) \\ = -5.70 + 4.88x;$$

3 places of decimals are almost always sufficient for the coefficients of this equation, and consideration of the standard errors to which the parameters are subject often suggests, as here, that only 2 are justified.

26. Compute $(17.7492)^2 \div 3.63956$, and subtract the result from S_{yy} , giving

$$\chi^2_{(3)} = 2.37;$$

reference to Table VI shows this not to be indicative of significant heterogeneity, and variances may therefore be derived from true weights without any heterogeneity factor. When a significant χ^2 is obtained at this stage, modify subsequent steps as in Exs. 7 and 9.

27. To check b , set 3.63956 and multiply by 4.8767 to obtain 17.7492.

28. The variance of b is $1/S_{xx}$. Find the standard error of b as the square root of this reciprocal, and check by calculating the reciprocal of $\sqrt{S_{xx}}$; a slide rule or tables of square roots and reciprocals may be used. The result is

$$b = 4.88 \pm 0.52,$$

and the size of the standard error shows that there is no need to quote b to more than 2 decimal places.

29. Check step 25. Calculate values of Y for three values of x (say $x = 1.5, 2.0$ and 2.5), and plot Y against x on Fig. 23. The three points should be collinear and they define the probit regression line. This line is almost indistinguishable from the provisional line and has therefore not been drawn separately in the figure. Since the agreement is so good, there is no need to carry out a second cycle of computations.

30. Find the log LD50 as the value of x which gives $Y = 5$:

$$m = (5 + 5.70)/4.88 \\ = 2.193.$$

The antilogarithm of m is the estimated LD50, 156 mg./l.

31. Calculate

$$V(m) = \frac{1}{(4.877)^2} \left\{ 0.0157 + \frac{(2.193 - 2.235)^2}{3.640} \right\} \\ = (0.0157 + 0.0005)/23.79 \\ = 0.00068,$$

whence the standard error of m is ± 0.026 .

32. Calculate $g = t^2V(b)/b^2$ for the probability level at which fiducial limits are required; for the 5 % level $t = 1.96$ (Table VII), so that

$$g = (1.96)^2/3.640 \times 23.79 \\ = 0.044.$$

33. Since g is small, the fiducial limits to m may be taken at a distance 1.96×0.026 on either side of m , these therefore being 2.244 and 2.142. The corresponding concentrations are 175 and 139 mg./l. respectively.*

34. Check steps 30–33.

35. The conclusion from the analysis of these data is that the median lethal dose of Martin's ether extract of *Derris malaccensis* to *Aphis rumicis*, under the conditions of the experiment, has a maximum likelihood estimate of 156 mg./l.; with a fiducial probability of 95 %, the true value of the median lethal dose may be expected to lie between 175 and 139 mg./l.

The above is a detailed account of a systematic arrangement of the computations suitable for use when a single computer is responsible for the whole analysis. The plan need not be followed exactly and no doubt personal taste will suggest modifications. There is necessarily a conflict between the desirability of checking a result before too many further calculations have been based upon it and the desirability of delaying a check for some time so as to reduce the risk of unconscious repetition of mistakes. The computer's aim should be to carry out the original calculations correctly, and checking should be looked upon as a verification of correctness rather than as means of discovering errors. Where a result can be checked independently of its first calculation the check may be made immediately, but where the only check is to repeat the same processes a reasonable interval should be allowed to elapse first. In the arrangement of the computations that has just been described, notes on checks have been inserted, but generally the reader has been left to devise his own methods for these; they should not be made earlier than the points indicated, and preferably the original working should be so accurate that they can safely be left until much later. When a second computer is available to assist in the analysis, he should be responsible for all checks.

* Fiducial limits to m calculated from the exact equation (4.6) are 2.243 and 2.139, and the concentrations are 175 and 138 mg./l. respectively, thus confirming the statement that g is too small for the exact formula to be needed.

The reader who adopts either this or a similar arrangement of the work for the simple type of data should have no difficulty in adapting and extending it to the more complex analyses discussed in Chapters 5-10. For example, when relative potencies are being estimated for several poisons tested in a single experiment, the provisional lines (step 10) will be drawn parallel if there seems any possibility of data agreeing satisfactorily with the hypothesis that the probit regression lines have equal slopes. Thereafter the computations proceed as outlined above, for each poison separately, until the values of S_{xx} , S_{xy} and S_{yy} have been obtained for each. A test of parallelism and, if departures from parallelism are not significant, subsequent estimation of relative potencies based on a common regression coefficient follow as described in § 20. The computations are analogous to those in steps 24-34 above.

Appendix II

MATHEMATICAL BASIS OF THE PROBIT METHOD

MANY who are concerned with the use of the probit transformation in the analysis of numerical data will be content to accept the technique without explanation of the mathematical and statistical theory. In Chapters 1-11 of this book, the practical applications have been developed, and the computational processes illustrated, with a minimum of theoretical argument. For the benefit of the mathematically minded reader, this appendix gives a brief outline of the derivation of the equations of estimation which have been used elsewhere. The appendix is not intended as a complete account, but the whole theory of the probit method can be developed on the lines set out below; the mathematical statistician should have little difficulty in discovering for himself how the equations that follow lead to the computations described earlier, or in modifying these equations in order to obtain the analogous techniques for the alternative distributions of tolerance discussed in § 15. The essential features of the maximum likelihood estimation of the parameters in the quantal response problem were contained in a note by Fisher (1935), though the first adequate formal presentation appears to have been that of Garwood (1941); Finney (1949*d*) has given a more general result. As emphasized in § 49, the known optimal properties of maximum likelihood estimation relate to large samples, and some alternative may be superior in samples of finite size. No other method has yet been demonstrated to have an advantage, and only maximum likelihood will be considered here. Berkson's method of minimizing χ^2 (1949) is in many respects similar, and the construction of the iterative process may be of interest in connexion with problems other than those of quantal responses.

Suppose that a tolerance distribution is represented by equation (2.1), so that the probability of response to a dose λ_0 is P , as defined by equation (2.2). If a batch of n subjects is exposed to the stimulus at a dose λ_0 , and if the subjects react independently of one another, the probability of r responses is given by the binomial distribution (§ 7) as

$$P(r) = \frac{n!}{r!(n-r)!} P^r Q^{n-r}. \quad (\text{II, 1})$$

Suppose that a series of k doses is tested in an experiment; then the probability of a particular set of numbers killed in each group is proportional to e^L , where

$$L = Sr \log P + S(n-r) \log Q, \tag{II, 2}$$

and S denotes summation over all doses. The quantity e^L , or, more strictly, a quantity proportional to it but having a maximum value of unity, has been called by Fisher (1922) the *Likelihood* of the observations.

Now P, Q are functions of the dose which contain certain parameters, and the problem confronting the statistician is that of estimating the parameters from the experimental data. Fisher (1922, 1925) has shown that estimates of the parameters which maximize the likelihood are *asymptotically efficient* (Cramér, 1946, §33.3; Kendall, 1946, §17.27). The likelihood is a maximum when L is a maximum; hence, if θ is a parameter of the distribution of individual tolerances, the maximum likelihood estimate of θ must satisfy the equation

$$\begin{aligned} 0 = \frac{\partial L}{\partial \theta} &= S \frac{r}{P} \frac{\partial P}{\partial \theta} + S \frac{n-r}{Q} \frac{\partial Q}{\partial \theta} \\ &= S \frac{n(p-P)}{PQ} \frac{\partial P}{\partial \theta}, \end{aligned} \tag{II, 3}$$

where $p = r/n$ (II, 4)

is an empirical estimate of the value of P at a particular dose. If the tolerance distribution involves more than one unknown parameter, a set of equations of the form of (II, 3) must be satisfied simultaneously.

Direct solution of such equations is seldom possible, but iterative methods can be made to give successive approximations converging to the solutions. It will be sufficient to illustrate the procedure for two parameters, θ and ϕ . Suppose that θ_1, ϕ_1 are first approximations to the solutions of the equations (II, 3), obtained perhaps by rough graphical or arithmetical estimation. By the Taylor-Maclaurin expansion, to the first order of small quantities

$$\left. \begin{aligned} \frac{\partial L}{\partial \theta_1} + \delta \theta \frac{\partial^2 L}{\partial \theta_1^2} + \delta \phi \frac{\partial^2 L}{\partial \theta_1 \partial \phi_1} &= 0, \\ \frac{\partial L}{\partial \phi_1} + \delta \theta \frac{\partial^2 L}{\partial \theta_1 \partial \phi_1} + \delta \phi \frac{\partial^2 L}{\partial \phi_1^2} &= 0, \end{aligned} \right\} \tag{II, 5}$$

where the addition of the suffix to θ, ϕ indicates that the first approximations are to be substituted after differentiation. The second-order

differential coefficients $\frac{\partial^2 L}{\partial \theta_1^2}$, $\frac{\partial^2 L}{\partial \theta_1 \partial \phi_1}$ and $\frac{\partial^2 L}{\partial \phi_1^2}$ may be simplified by putting $p = P$ after differentiation, in order to give expected instead of empirical values (cf. Fisher, 1944, § 57.2). Equations (II, 5) then become

$$\left. \begin{aligned} \delta \theta S \frac{n}{P_1 Q_1} \left(\frac{\partial P_1}{\partial \theta_1} \right)^2 + \delta \phi S \frac{n}{P_1 Q_1} \left(\frac{\partial P_1}{\partial \theta_1} \frac{\partial P_1}{\partial \phi_1} \right) &= S \frac{n(p - P_1)}{P_1 Q_1} \frac{\partial P_1}{\partial \theta_1}, \\ \delta \theta S \frac{n}{P_1 Q_1} \left(\frac{\partial P_1}{\partial \theta_1} \frac{\partial P_1}{\partial \phi_1} \right) + \delta \phi S \frac{n}{P_1 Q_1} \left(\frac{\partial P_1}{\partial \phi_1} \right)^2 &= S \frac{n(p - P_1)}{P_1 Q_1} \frac{\partial P_1}{\partial \phi_1}. \end{aligned} \right\} \quad (\text{II, 6})$$

The solutions, $\delta \theta$, $\delta \phi$, are adjustments to θ_1 , ϕ_1 , which give improved approximations, $\theta_2 = \theta_1 + \delta \theta$, $\phi_2 = \phi_1 + \delta \phi$. Further adjustments are obtainable by recalculating the equations with θ_2 , ϕ_2 in place of θ_1 , ϕ_1 , and the process may be repeated until the latest set of adjustments is negligible. General theory of statistical estimation, indeed, states that if the first approximations are of non-zero efficiency, the first cycle of computation will yield fully efficient estimates. This, however, is strictly true only for large samples, and also is likely to be disturbed by the subjective element in determining θ_1 , ϕ_1 from the graphical process usual in probit analysis. The wisest course, therefore, is to iterate the calculations until the solutions of (II, 3) are approached; care and experience in the choice of first approximations will usually ensure that two cycles give a numerical accuracy sufficient for practical purposes.

If $\hat{\theta}$ is the estimate of a single parameter, obtained as a solution of the maximum likelihood equation (II, 3), its variance is (Cramér, 1946, § 33.3; Fisher, 1922; Kendall, 1946, § 17.25)

$$V(\hat{\theta}) = -1 / \frac{\partial^2 L}{\partial \hat{\theta}^2}, \quad (\text{II, 7})$$

in which expression $\hat{\theta}$ is to be substituted for θ after differentiation. When more than one parameter must be estimated, the variances and covariances of the estimates are the elements of the inverse matrix of second differential coefficients; thus, for two parameters,

$$V = \begin{pmatrix} -\frac{\partial^2 L}{\partial \hat{\theta}^2} & -\frac{\partial^2 L}{\partial \hat{\theta} \partial \hat{\phi}} \\ -\frac{\partial^2 L}{\partial \hat{\theta} \partial \hat{\phi}} & -\frac{\partial^2 L}{\partial \hat{\phi}^2} \end{pmatrix}^{-1} \quad (\text{II, 8})$$

This matrix, but with the previous set of approximations instead of $\hat{\theta}$, $\hat{\phi}$, is inverted as part of the last cycle of computations, since the

differential coefficients appear as the coefficients of $\delta\theta$, $\delta\phi$ in equations (II, 6). In practice, the variances and covariances may be taken as approximately equal to the elements of this inverse, so as to avoid the necessity of recalculation. A χ^2 test for the heterogeneity of the departures of the observations from the tolerance distribution specified by the parameters then follows.

The equations discussed above are of general applicability, whatever the form of P , and may easily be extended to the estimation of a greater number of parameters. Toxicity test data, and other quantal response problems, require their simplification to specialized forms suitable for computation. The most important form is that for the estimation of the parameters of the tolerance distribution given by equation (2.6), or

$$P = \frac{1}{\sigma\sqrt{(2\pi)}} \int_{-\infty}^x \exp \left[-\frac{1}{2\sigma^2} (x-\mu)^2 \right] dx, \quad \text{(II, 9)}$$

where x measures the dosage on a logarithmic or other normalizing scale. This distribution has been shown in §9 to be equivalent to a linear relationship between x and the probit of P , say

$$Y = \alpha + \beta x, \quad \text{(II, 10)}$$

in which the new parameters, α , β are related to the old μ , σ by

$$\left. \begin{aligned} \mu &= (5 - \alpha) / \beta, \\ \sigma &= 1 / \beta, \end{aligned} \right\} \quad \text{(II, 11)}$$

and Y is defined by equation (3.1), namely

$$P = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{Y-5} e^{-t^2} du.$$

Now
$$\frac{\partial P}{\partial Y} = \frac{1}{\sqrt{(2\pi)}} e^{-\frac{1}{2}(Y-5)^2} = Z,$$

the ordinate to the normal curve at the point whose abscissa is $(Y - 5)$, and therefore

$$\frac{\partial P}{\partial \alpha} = Z, \quad \frac{\partial P}{\partial \beta} = xZ.$$

Hence, if
$$Y = a_1 + b_1 x \quad \text{(II, 12)}$$

is a first approximation to the maximum likelihood estimate of

equation (II, 10), adjustments to a_1 and b_1 are given by equations (II, 6) or

$$\left. \begin{aligned} \delta a S \frac{nZ^2}{PQ} + \delta b S \frac{nZ^2}{PQ} x &= S \frac{nZ^2}{PQ} \left(\frac{p-P}{Z} \right), \\ \delta a S \frac{nZ^2}{PQ} x + \delta b S \frac{nZ^2}{PQ} x^2 &= S \frac{nZ^2}{PQ} x \left(\frac{p-P}{Z} \right), \end{aligned} \right\} \quad (\text{II, 13})$$

in which Z, P, Q are determined from equation (II, 12). If the *weighting coefficient*, w , is defined by

$$w = Z^2/PQ, \quad (\text{II, 14})$$

these last equations are seen to be equations for the estimation of the weighted linear regression of $(p-P)/Z$ on x , the weight nw being assigned to each value of $(p-P)/Z$. A convenient method of solution is to introduce the *working probit*

$$y = Y + \frac{p-P}{Z}, \quad (\text{II, 15})$$

and to add $SnwY$ to each side of the first, $SnwxY$ to each side of the second of equations (II, 13). The weighted linear regression equation of y on x is then calculated in the form

$$Y = \bar{y} + b(x - \bar{x}), \quad (\text{II, 16})$$

where

$$\left. \begin{aligned} \bar{x} &= Snwx/Snw, \\ \bar{y} &= Snwy/Snw, \end{aligned} \right\} \quad (\text{II, 17})$$

and

$$b = S_{xy}/S_{xx}; \quad (\text{II, 18})$$

equation (II, 16) may be seen to be a revised version of (II, 12), with

$$\begin{aligned} a_2 &= a_1 + \delta a = \bar{y} - b\bar{x}, \\ b_2 &= b_1 + \delta b = b. \end{aligned}$$

The iteration may continue with a new cycle of computations based on

$$Y = a_2 + b_2 x.$$

By equation (II, 8), which here is analogous to the ordinary formulae for weighted linear regression,

$$V(\bar{y}) = 1/Snw, \quad (\text{II, 19})$$

$$V(b) = 1/S_{xx}, \quad (\text{II, 20})$$

provided that the maximum likelihood estimates have been approached sufficiently closely. These results are, strictly, applicable only for

large samples, and research into the appropriate formulae for small samples is needed; the rapidity with which increase in the number of observations improves the approximation of a normal distribution to a binomial suggests that, when the total number of subjects tested is not very small, these variance formulae will be trustworthy.

Homogeneity of the experimental materials and mutual independence of the subjects have so far been assumed. If the n subjects in a batch do not react independently to the dose applied, but depart from the fitted equation (II, 16) more markedly than can be attributed to random variation, without showing any regularity such as would suggest the need for a different specification of the tolerance distribution, the weights of the observations must be decreased, and the variances correspondingly increased. This situation will in general be indicated by a significantly large value of the heterogeneity χ^2 , whose calculation is described in §§ 17, 18. Provided that no classes have had to be amalgamated (cf. Ex. 7), χ^2 will have $(k-2)$ degrees of freedom, two less than the number of batches tested; $\chi^2/(k-2)$ is then the factor by which the variances must be increased. In a more complex analysis, the heterogeneity factor remains the mean square derived from the heterogeneity χ^2 . When a heterogeneity factor has to be used, all variances and standard errors must be considered in relation to a t -distribution, not a normal, with degrees of freedom equal to those of χ^2 . This procedure is not peculiar to probit analysis but is the normal statistical practice of using an empirical variance instead of a theoretical when the data give evidence of the occurrence of variation other than that included in the theoretical value.

The problem of adjustment for a natural response rate (§ 28) which supplements the responses due to the stimulus provides an instructive example of the generalization of the method just described (Finney, 1949 *b, d*). When an average natural response rate, C , operates independently of the stimulus, the total response rate expected at any dose is as given in equation (6.1),

$$P' = C + P(1 - C).$$

The probabilities P' , Q' replace P , Q in (II, 1), and the prototype of the maximum likelihood equations is

$$0 = \frac{\partial L}{\partial \theta} = S \frac{n(p' - P')}{P'Q'} \frac{\partial P'}{\partial \theta}.$$

Now

$$\frac{\partial P'}{\partial C} = Q, \quad \frac{\partial P'}{\partial P} = (1 - C),$$

and also

$$p' - P' = (p - P)(1 - C).$$

In general, C also has to be estimated. If S is used for summation over all non-zero doses, and the n_c subjects tested at zero dose showed a response rate c , the equations corresponding to (II, 3) may be written

$$\frac{\partial L}{\partial x} = S \frac{nZ^2(1-C)}{Q\{C+P(1-C)\}} \left(\frac{p-P}{Z} \right) = 0,$$

$$\frac{\partial L}{\partial \beta} = S \frac{nZ^2x(1-C)}{Q\{C+P(1-C)\}} \left(\frac{p-P}{Z} \right) = 0,$$

$$\frac{\partial L}{\partial C} = \frac{n_c(c-C)}{C(1-C)} + S \frac{n(p-P)}{C+P(1-C)} = 0.$$

Differentiate again, and equate observed values with expected; if

$$w = \frac{Z^2}{Q\left\{P + \frac{C}{1-C}\right\}}$$

and

$$x' = Q/Z$$

be introduced, as in equations (6.3), (6.4), the coefficients required for equations (II, 5) may be written

$$\frac{\partial^2 L}{\partial \alpha^2} = -Snw,$$

$$\frac{\partial^2 L}{\partial \alpha \partial \beta} = -Snwx,$$

$$\frac{\partial^2 L}{\partial \beta^2} = -Snwx^2,$$

$$\frac{\partial^2 L}{\partial \alpha \partial C} = -\frac{1}{1-C} Snwx',$$

$$\frac{\partial^2 L}{\partial \beta \partial C} = -\frac{1}{1-C} Snwx'x',$$

$$\frac{\partial^2 L}{\partial C^2} = -\frac{n_c}{C(1-C)} - \frac{1}{(1-C)^2} Snwx'^2.$$

If the working probit is again introduced, the equations for a , b , δC may be reduced to the form most suitable for computing, equations (6.5), (6.6); these equations are formally the same as for a multiple linear regression of y on x , x' , and the analysis may proceed as illustrated in Ex. 17.

Estimates of the parameters for other expressions of the probability function, P , can be derived in a similar manner. Other examples in

which the basic distribution (II, 9) is retained but modified have been given in §§ 45, 47, 50, and the reader should have no difficulty in developing the equations required for these problems. The analogue of the probit method appropriate to any of the alternative tolerance distributions mentioned in § 15 can be found by alteration of (II, 9) to an integral based on the new distribution. This has been illustrated elsewhere (Finney, 1947*a*) for the logistic distribution, for which it leads to the maximum likelihood estimates in Berkson's (1944) logit method. The logistic sigmoid might be more appropriate than the normal for the problem of Ex. 33; it could be estimated without any change in the form of equations (10.1)–(10.6), except that P , Q , Z , w , x' , y , must be interpreted in terms of the logistic function:

$$P = e^Y / (1 + e^Y),$$

$$Z = e^Y / (1 + e^Y)^2,$$

with the other quantities related to P , Z by the usual formulae. Thus the method of § 47 can be used in the fitting of a logistic curve with unknown upper limit to observations with equal variances, and can easily be modified so as to take account of unequal variances. Since the logistic and normal sigmoids are so similar in shape over a wide range of values of P , a logistic analysis of the data of Ex. 33 would not in fact have given results appreciably different from those of the probit analysis.

It must not be forgotten that the probit transformation is only a convenient trick for representing the sigmoid (II, 9) by a straight line, and that probit analysis is only one convenient method for the solution of equations such as (II, 3) for the normal distribution of tolerances. Neither the transformation nor the method of analysis have any fundamental relationship to the problems in which they are used, beyond what is implied by this statement. The chief advantage of probit analysis over other methods of solving the same equations is that it enables the computational methods familiar in linear regression analysis to be applied in what is not really a linear regression problem. The wide range of tables now available reduces to a minimum the need for the calculation of special functions in any one problem; the procedure is a simple, easily checked routine that can be performed by workers who would be at a loss if merely ordered to solve equation (II, 3).

Nevertheless, methods other than those of this book are in some respects superior. Black (1950) suggests an alternative method of tabulating maximum and minimum working probits which, while

leading to what is mathematically the same analysis, may be more rapid in use than the usual, since the *new* column is built up directly. The gain is at the expense of the graphical representation, and the linear regression analogy is left a little less clear, but, at least for some types of data, it has advantages. Black's proposal, in fact, is equivalent to assigning a weight and a score to each subject, the score being a maximum or minimum function of the expected Y according as the subject does or does not respond; the analogy between probit analysis and linear scoring methods used in the estimation of genetic parameters (Finney, 1949*c*) is thus apparent. Garwood (1941) compared the use of expected values of the second differential coefficients in equations (II, 5) with the use of empirical values calculated from the complete expressions for these differential coefficients. The two methods must converge to the same estimates of the parameters, though not necessarily to identical values for the variances. Garwood found that the empirical coefficients gave a quicker convergence, in the sense that fewer cycles were needed in order to attain a specified closeness of approach to the limiting values; this itself is an empirical conclusion, based on trial of several sets of data, though it is likely to be generally true. The advantage of the empirical coefficients is counterbalanced by their greater algebraic complexity and the consequent greater labour of calculating them, so that the time required for computing one cycle becomes much greater than for the expected coefficients. This difficulty seemed to condemn the 'empirical' so long as the only tables available were of the type of those in this book for the 'expected'. Recent work by Cornfield and Mantel (1950) alters this situation by suggesting an improved computing scheme for the empirical coefficients and providing tables for use in it; the method is still more laborious, cycle for cycle, than that used in this book, but the advantage in speed of convergence may now be sufficient to outweigh this disadvantage. Extensive comparisons of the two will be necessary before a final decision is made as to which is best for routine use.

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TABLE I. Transformation of Percentages to Probits

%	0-0	0-1	0-2	0-3	0-4	0-5	0-6	0-7	0-8	0-9	1	2	3	4	5
0	—	1-9098	2-1218	2-2522	2-3479	2-4242	2-4879	2-5427	2-5911	2-6344	For more detail see values for 95-100				
1	2-6737	2-7096	2-7429	2-7738	2-8027	2-8299	2-8556	2-8799	2-9031	2-9251					
2	2-9463	2-9665	2-9859	3-0046	3-0226	3-0400	3-0569	3-0732	3-0890	3-1043					
3	3-1192	3-1337	3-1478	3-1616	3-1750	3-1881	3-2009	3-2134	3-2256	3-2376					
4	3-2493	3-2608	3-2721	3-2831	3-2940	3-3046	3-3151	3-3253	3-3354	3-3454	9	18	27	36	45
5	3-3551	3-3648	3-3742	3-3836	3-3928	3-4018	3-4107	3-4195	3-4282	3-4368	8	16	24	32	40
6	3-4452	3-4536	3-4618	3-4699	3-4780	3-4859	3-4937	3-5015	3-5091	3-5167	7	14	21	28	36
7	3-5242	3-5316	3-5389	3-5462	3-5534	3-5605	3-5675	3-5745	3-5813	3-5882	6	13	19	26	32
8	3-5949	3-6016	3-6083	3-6148	3-6213	3-6278	3-6342	3-6405	3-6468	3-6531	5	10	15	20	24
9	3-6592	3-6654	3-6715	3-6775	3-6835	3-6894	3-6953	3-7012	3-7070	3-7127	4	9	13	18	22
10	3-7184	3-7241	3-7298	3-7354	3-7409	3-7464	3-7519	3-7574	3-7628	3-7681	6	11	17	22	28
11	3-7735	3-7788	3-7840	3-7893	3-7945	3-7996	3-8048	3-8099	3-8150	3-8200	5	10	16	21	26
12	3-8250	3-8300	3-8350	3-8399	3-8448	3-8497	3-8545	3-8593	3-8641	3-8689	5	10	15	20	24
13	3-8736	3-8783	3-8830	3-8877	3-8923	3-8969	3-9015	3-9061	3-9107	3-9152	5	9	14	18	23
14	3-9197	3-9242	3-9286	3-9331	3-9375	3-9419	3-9463	3-9506	3-9550	3-9593	4	9	13	18	22
15	3-9636	3-9678	3-9721	3-9763	3-9806	3-9848	3-9890	3-9931	3-9973	4-0014	4	8	13	17	21
16	4-0055	4-0096	4-0137	4-0178	4-0218	4-0259	4-0299	4-0339	4-0379	4-0419	4	8	12	16	20
17	4-0458	4-0498	4-0537	4-0576	4-0615	4-0654	4-0693	4-0731	4-0770	4-0808	4	8	12	16	19
18	4-0846	4-0884	4-0922	4-0960	4-0998	4-1035	4-1073	4-1110	4-1147	4-1184	4	8	11	15	19
19	4-1221	4-1258	4-1295	4-1331	4-1367	4-1404	4-1440	4-1476	4-1512	4-1548	4	7	11	15	18
20	4-1584	4-1619	4-1655	4-1690	4-1726	4-1761	4-1796	4-1831	4-1866	4-1901	4	7	11	14	18
21	4-1936	4-1970	4-2005	4-2039	4-2074	4-2108	4-2142	4-2176	4-2210	4-2244	3	7	10	14	17
22	4-2278	4-2312	4-2345	4-2379	4-2412	4-2446	4-2479	4-2512	4-2546	4-2579	3	7	10	13	17
23	4-2612	4-2644	4-2677	4-2710	4-2743	4-2775	4-2808	4-2840	4-2872	4-2905	3	7	10	13	16
24	4-2937	4-2969	4-3001	4-3033	4-3065	4-3097	4-3129	4-3160	4-3192	4-3224	3	6	10	13	16
25	4-3255	4-3287	4-3318	4-3349	4-3380	4-3412	4-3443	4-3474	4-3505	4-3536	3	6	9	12	16
26	4-3567	4-3597	4-3628	4-3659	4-3689	4-3720	4-3750	4-3781	4-3811	4-3842	3	6	9	12	15
27	4-3872	4-3902	4-3932	4-3962	4-3992	4-4022	4-4052	4-4082	4-4112	4-4142	3	6	9	12	15
28	4-4172	4-4201	4-4231	4-4260	4-4290	4-4319	4-4349	4-4378	4-4408	4-4437	3	6	9	12	15
29	4-4406	4-4435	4-4464	4-4494	4-4523	4-4553	4-4582	4-4611	4-4640	4-4670	3	6	9	12	14

30	4-4756	4-4785	4-4813	4-4842	4-4871	4-4899	4-4928	4-4956	4-4985	4-5013	3	6	9	11	14
31	4-5041	4-5070	4-5098	4-5126	4-5155	4-5183	4-5211	4-5239	4-5267	4-5295	3	6	8	11	14
32	4-5323	4-5351	4-5379	4-5407	4-5435	4-5462	4-5490	4-5518	4-5546	4-5573	3	6	8	11	14
33	4-5601	4-5628	4-5656	4-5684	4-5711	4-5739	4-5766	4-5793	4-5821	4-5848	3	5	8	11	14
34	4-5875	4-5903	4-5930	4-5957	4-5984	4-6011	4-6039	4-6066	4-6093	4-6120	3	5	8	11	14
35	4-6147	4-6174	4-6201	4-6228	4-6255	4-6281	4-6308	4-6335	4-6362	4-6389	3	5	8	11	13
36	4-6415	4-6442	4-6469	4-6495	4-6522	4-6549	4-6575	4-6602	4-6628	4-6655	3	5	8	11	13
37	4-6681	4-6708	4-6734	4-6761	4-6787	4-6814	4-6840	4-6866	4-6893	4-6919	3	5	8	11	13
38	4-6945	4-6971	4-6998	4-7024	4-7050	4-7076	4-7102	4-7129	4-7155	4-7181	3	5	8	10	13
39	4-7207	4-7233	4-7259	4-7285	4-7311	4-7337	4-7363	4-7389	4-7415	4-7441	3	5	8	10	13
40	4-7467	4-7492	4-7518	4-7544	4-7570	4-7596	4-7622	4-7647	4-7673	4-7699	3	5	8	10	13
41	4-7725	4-7750	4-7776	4-7802	4-7827	4-7853	4-7879	4-7904	4-7930	4-7955	3	5	8	10	13
42	4-7981	4-8007	4-8032	4-8058	4-8083	4-8109	4-8134	4-8160	4-8185	4-8211	3	5	8	10	13
43	4-8236	4-8262	4-8287	4-8313	4-8338	4-8363	4-8389	4-8414	4-8440	4-8465	3	5	8	10	13
44	4-8490	4-8516	4-8541	4-8566	4-8592	4-8617	4-8642	4-8668	4-8693	4-8718	3	5	8	10	13
45	4-8743	4-8769	4-8794	4-8819	4-8844	4-8870	4-8895	4-8920	4-8945	4-8970	3	5	8	10	13
46	4-8996	4-9021	4-9046	4-9071	4-9096	4-9122	4-9147	4-9172	4-9197	4-9222	3	5	8	10	13
47	4-9247	4-9272	4-9298	4-9323	4-9348	4-9373	4-9398	4-9423	4-9448	4-9473	3	5	8	10	13
48	4-9498	4-9524	4-9549	4-9574	4-9599	4-9624	4-9649	4-9674	4-9699	4-9724	3	5	8	10	13
49	4-9749	4-9774	4-9799	4-9825	4-9850	4-9875	4-9900	4-9925	4-9950	4-9975	3	5	8	10	13
50	5-0000	5-0025	5-0050	5-0075	5-0100	5-0125	5-0150	5-0175	5-0201	5-0226	3	5	8	10	13
51	5-0251	5-0276	5-0301	5-0326	5-0351	5-0376	5-0401	5-0426	5-0451	5-0476	3	5	8	10	13
52	5-0502	5-0527	5-0552	5-0577	5-0602	5-0627	5-0652	5-0677	5-0702	5-0728	3	5	8	10	13
53	5-0753	5-0778	5-0803	5-0828	5-0853	5-0878	5-0904	5-0929	5-0954	5-0979	3	5	8	10	13
54	5-1004	5-1030	5-1055	5-1080	5-1105	5-1130	5-1156	5-1181	5-1206	5-1231	3	5	8	10	13
55	5-1257	5-1282	5-1307	5-1332	5-1358	5-1383	5-1408	5-1434	5-1459	5-1484	3	5	8	10	13
56	5-1510	5-1535	5-1560	5-1586	5-1611	5-1637	5-1662	5-1687	5-1713	5-1738	3	5	8	10	13
57	5-1764	5-1789	5-1815	5-1840	5-1866	5-1891	5-1917	5-1942	5-1968	5-1993	3	5	8	10	13
58	5-2019	5-2045	5-2070	5-2096	5-2121	5-2147	5-2173	5-2198	5-2224	5-2250	3	5	8	10	13
59	5-2275	5-2301	5-2327	5-2353	5-2378	5-2404	5-2430	5-2456	5-2482	5-2508	3	5	8	10	13

TABLE I (cont.)

%	0-0	0-1	0-2	0-3	0-4	0-5	0-6	0-7	0-8	0-9	1	2	3	4	5
60	5-2533	5-2559	5-2585	5-2611	5-2637	5-2663	5-2689	5-2715	5-2741	5-2767	3	5	8	10	13
61	5-2793	5-2819	5-2845	5-2871	5-2898	5-2924	5-2950	5-2976	5-3002	5-3029	3	5	8	10	13
62	5-3055	5-3081	5-3107	5-3134	5-3160	5-3186	5-3213	5-3239	5-3266	5-3292	3	5	8	11	13
63	5-3319	5-3345	5-3372	5-3398	5-3425	5-3451	5-3478	5-3505	5-3531	5-3558	3	5	8	11	13
64	5-3585	5-3611	5-3638	5-3665	5-3692	5-3719	5-3745	5-3772	5-3799	5-3826	3	5	8	11	13
65	5-3853	5-3880	5-3907	5-3934	5-3961	5-3989	5-4016	5-4043	5-4070	5-4097	3	5	8	11	14
66	5-4125	5-4152	5-4179	5-4207	5-4234	5-4261	5-4289	5-4316	5-4344	5-4372	3	5	8	11	14
67	5-4399	5-4427	5-4454	5-4482	5-4510	5-4538	5-4565	5-4593	5-4621	5-4649	3	6	8	11	14
68	5-4677	5-4705	5-4733	5-4761	5-4789	5-4817	5-4845	5-4874	5-4902	5-4930	3	6	8	11	14
69	5-4959	5-4987	5-5015	5-5044	5-5072	5-5101	5-5129	5-5158	5-5187	5-5215	3	6	9	11	14
70	5-5244	5-5273	5-5302	5-5330	5-5359	5-5388	5-5417	5-5446	5-5476	5-5505	3	6	9	12	14
71	5-5534	5-5563	5-5592	5-5622	5-5651	5-5681	5-5710	5-5740	5-5769	5-5799	3	6	9	12	15
72	5-5828	5-5858	5-5888	5-5918	5-5948	5-5978	5-6008	5-6038	5-6068	5-6098	3	6	9	12	15
73	5-6128	5-6158	5-6189	5-6219	5-6250	5-6280	5-6311	5-6341	5-6372	5-6403	3	6	9	12	15
74	5-6433	5-6464	5-6495	5-6526	5-6557	5-6588	5-6620	5-6651	5-6682	5-6713	3	6	9	12	16
75	5-6745	5-6776	5-6808	5-6840	5-6871	5-6903	5-6935	5-6967	5-6999	5-7031	3	6	10	13	16
76	5-7063	5-7095	5-7128	5-7160	5-7192	5-7225	5-7257	5-7290	5-7323	5-7356	3	7	10	13	16
77	5-7388	5-7421	5-7454	5-7488	5-7521	5-7554	5-7588	5-7621	5-7655	5-7688	3	7	10	13	17
78	5-7722	5-7756	5-7790	5-7824	5-7858	5-7892	5-7926	5-7961	5-7995	5-8030	3	7	10	14	17
79	5-8064	5-8099	5-8134	5-8169	5-8204	5-8239	5-8274	5-8310	5-8345	5-8381	4	7	11	14	18
80	5-8416	5-8452	5-8488	5-8524	5-8560	5-8596	5-8633	5-8669	5-8705	5-8742	4	7	11	14	18
81	5-8779	5-8816	5-8853	5-8890	5-8927	5-8965	5-9002	5-9040	5-9078	5-9116	4	7	11	15	19
82	5-9154	5-9192	5-9230	5-9269	5-9307	5-9346	5-9385	5-9424	5-9463	5-9502	4	8	12	15	19
83	5-9542	5-9581	5-9621	5-9661	5-9701	5-9741	5-9782	5-9822	5-9863	5-9904	4	8	12	16	20
84	5-9945	5-9986	6-0027	6-0069	6-0110	6-0152	6-0194	6-0237	6-0279	6-0322	4	8	13	17	21
85	6-0364	6-0407	6-0450	6-0494	6-0537	6-0581	6-0625	6-0669	6-0714	6-0758	4	9	13	18	22
86	6-0803	6-0848	6-0893	6-0939	6-0985	6-1031	6-1077	6-1123	6-1170	6-1217	5	9	14	18	23
87	6-1264	6-1311	6-1359	6-1407	6-1455	6-1503	6-1552	6-1601	6-1650	6-1700	5	10	15	19	24
88	6-1750	6-1800	6-1850	6-1901	6-1952	6-2004	6-2055	6-2107	6-2160	6-2212	5	10	15	21	26
89	6-2265	6-2319	6-2372	6-2426	6-2481	6-2536	6-2591	6-2646	6-2702	6-2759	5	11	16	22	27

90	6-2816	6-2873	6-2930	6-2988	6-3047	6-3106	6-3165	6-3225	6-3285	6-3346	6	12	18	24	29
91	6-3408	6-3469	6-3532	6-3595	6-3658	6-3722	6-3787	6-3852	6-3917	6-3984	6	13	19	26	32
92	6-4051	6-4118	6-4187	6-4255	6-4325	6-4395	6-4466	6-4538	6-4611	6-4684	7	14	21	28	35
93	6-4758	6-4833	6-4909	6-4985	6-5063	6-5141	6-5220	6-5301	6-5382	6-5464	8	16	24	31	39
94	6-5548	6-5632	6-5718	6-5805	6-5893	6-5982	6-6072	6-6164	6-6258	6-6352	9	18	27	36	45
95	6-6449	6-6546	6-6646	6-6747	6-6849	6-6954	6-7060	6-7169	6-7279	6-7392					
	97	100	101	102	105	106	109	110	113	115					
96	6-7507	6-7624	6-7744	6-7866	6-7991	6-8119	6-8250	6-8384	6-8522	6-8663					
	117	120	122	125	128	131	134	138	141	145					
97	6-8808	6-8957	6-9110	6-9268	6-9431	6-9600	6-9774	6-9954	7-0141	7-0335					
	149	153	158	163	169	174	180	187	194	202					
%	0-00	0-01	0-02	0-03	0-04	0-05	0-06	0-07	0-08	0-09	1	2	3	4	5
98-0	7-0537	7-0558	7-0579	7-0600	7-0621	7-0642	7-0663	7-0684	7-0706	7-0727	2	4	6	8	11
98-1	7-0749	7-0770	7-0792	7-0814	7-0836	7-0858	7-0880	7-0902	7-0924	7-0947	2	4	7	9	11
98-2	7-0969	7-0992	7-1015	7-1038	7-1061	7-1084	7-1107	7-1130	7-1154	7-1177	2	5	7	9	12
98-3	7-1201	7-1224	7-1248	7-1272	7-1297	7-1321	7-1345	7-1370	7-1394	7-1419	2	5	7	10	12
98-4	7-1444	7-1469	7-1494	7-1520	7-1545	7-1571	7-1596	7-1622	7-1648	7-1675	3	5	8	10	13
98-5	7-1701	7-1727	7-1754	7-1781	7-1808	7-1835	7-1862	7-1890	7-1917	7-1945	3	5	8	11	14
98-6	7-1973	7-2001	7-2029	7-2058	7-2086	7-2115	7-2144	7-2173	7-2203	7-2232	3	6	9	12	14
98-7	7-2262	7-2292	7-2322	7-2353	7-2383	7-2414	7-2445	7-2476	7-2508	7-2539	3	6	9	12	15
98-8	7-2571	7-2603	7-2636	7-2668	7-2701	7-2734	7-2768	7-2801	7-2835	7-2869	3	7	10	13	17
98-9	7-2904	7-2938	7-2973	7-3009	7-3044	7-3080	7-3116	7-3152	7-3189	7-3226	4	7	11	14	18
99-0	7-3263	7-3301	7-3339	7-3378	7-3416	7-3455	7-3495	7-3535	7-3575	7-3615	4	8	12	16	20
99-1	7-3656	7-3698	7-3739	7-3781	7-3824	7-3867	7-3911	7-3954	7-3999	7-4044	4	9	13	17	22
99-2	7-4089	7-4135	7-4181	7-4228	7-4276	7-4324	7-4372	7-4422	7-4471	7-4522	5	10	14	19	24
99-3	7-4573	7-4624	7-4677	7-4730	7-4783	7-4838	7-4893	7-4949	7-5006	7-5063	5	11	16	22	27
99-4	7-5121	7-5181	7-5241	7-5302	7-5364	7-5427	7-5491	7-5556	7-5622	7-5690	6	13	19	25	32
99-5	7-5758	7-5828	7-5899	7-5972	7-6045	7-6121	7-6197	7-6276	7-6356	7-6437					
99-6	7-6521	7-6606	7-6693	7-6783	7-6874	7-6968	7-7065	7-7164	7-7266	7-7370					
99-7	7-7478	7-7589	7-7703	7-7822	7-7944	7-8070	7-8202	7-8338	7-8480	7-8627					
99-8	7-8782	7-8943	7-9112	7-9290	7-9478	7-9677	7-9889	8-0115	8-0357	8-0618					
99-9	8-0902	8-1214	8-1559	8-1947	8-2389	8-2905	8-3528	8-4316	8-5401	8-7190					

I am indebted to Professor R. A. Fisher and Dr F. Yates, and also to Messrs Oliver and Boyd, Ltd. of Edinburgh, for permission to reprint Table I from Table IX of their book *Statistical Tables for Biological, Agricultural and Medical Research*.

TABLE II. The Weighting Coefficient and Q/Z

Y	Q/Z	Percentage natural mortality, C										
		0	1	2	3	4	5	6	7	8	9	10
1-1	5034	·00082	—	—	—	—	—	—	—	—	—	—
1-2	3425	·00118	·00001	—	—	—	—	—	—	—	—	—
1-3	2354	·00187	·00002	·00001	·00001	—	—	—	—	—	—	—
1-4	1634	·00235	·00004	·00002	·00001	·00001	·00001	·00001	—	—	—	—
1-5	1148	·00327	·00007	·00004	·00002	·00002	·00001	·00001	·00001	·00001	·00001	·00001
1-6	811·2	·00451	·00015	·00007	·00005	·00004	·00003	·00002	·00002	·00002	·00002	·00001
1-7	580·2	·00614	·00028	·00014	·00009	·00007	·00006	·00005	·00004	·00003	·00003	·00003
1-8	419·1	·00828	·00053	·00027	·00018	·00013	·00011	·00009	·00007	·00006	·00006	·00005
1-9	305·8	·01105	·00097	·00050	·00034	·00025	·00020	·00017	·00014	·00012	·00011	·00010
2-0	225·3	·01457	·00172	·00090	·00061	·00046	·00036	·00030	·00026	·00022	·00020	·00017
2-1	167·69	·01903	·00297	·00159	·00108	·00082	·00065	·00054	·00046	·00040	·00035	·00031
2-2	126·02	·02458	·00496	·00274	·00188	·00142	·00114	·00095	·00081	·00070	·00062	·00055
2-3	95·63	·03143	·00803	·00456	·00317	·00241	·00194	·00162	·00138	·00121	·00106	·00095
2-4	73·28	·03977	·01256	·00739	·00521	·00400	·00324	·00271	·00232	·00202	·00179	·00160
2-5	56·70	·04979	·01895	·01161	·00832	·00646	·00525	·00441	·00379	·00332	·00294	·00264
2-6	44·288	·06168	·02763	·01768	·01292	·01014	·00831	·00702	·00606	·00531	·00472	·00424
2-7	34·923	·07564	·03895	·02605	·01947	·01548	·01280	·01088	·00943	·00830	·00740	·00666
2-8	27·797	·09179	·05316	·03719	·02847	·02297	·01918	·01642	·01431	·01265	·01131	·01021
2-9	22·330	·11026	·07044	·05147	·04037	·03309	·02794	·02411	·02115	·01879	·01687	·01527
3-0	18·101	·13112	·09080	·06912	·05557	·04631	·03957	·03445	·03043	·02719	·02452	·02228
3-1	14·802	·15436	·11419	·09023	·07432	·06298	·05449	·04790	·04263	·03832	·03473	·03170
3-2	12·211	·17994	·14046	·11476	·09670	·08332	·07300	·06481	·05814	·05261	·04795	·04397
3-3	10·159	·20774	·16935	·14249	·12263	·10736	·09525	·08541	·07726	·07039	·06453	·05947
3-4	8·521	·23753	·20056	·17308	·15184	·13494	·12116	·10973	·10008	·09182	·08469	·07846
3-5	7·205	·26907	·23373	·20611	·18392	·16571	·15050	·13760	·12652	·11690	·10848	·10103

3-6	6-1394	·30199	·26842	·24107	·21836	·19921	·18283	·16867	·15631	·14541	·13575	·12711
3-7	5-2705	·33589	·30415	·27741	·25456	·23482	·21759	·20242	·18896	·17694	·16614	·15639
3-8	4-5571	·37031	·34043	·31453	·29186	·27187	·25409	·23819	·22387	·21092	·19915	·18840
3-9	3-9676	·40474	·37669	·35181	·32960	·30964	·29161	·27524	·26031	·24665	·23409	·22250
4-0	3-4770	·43863	·41237	·38864	·36707	·34739	·32937	·31279	·29749	·28334	·27020	·25797
4-1	3-0665	·47144	·44691	·42438	·40362	·38441	·36661	·35005	·33460	·32017	·30666	·29397
4-2	2-7206	·50260	·47973	·45844	·43858	·42000	·40259	·38623	·37085	·35634	·34264	·32969
4-3	2-4276	·53159	·51029	·49024	·47134	·45360	·43662	·42063	·40546	·39105	·37735	·36430
4-4	2-1780	·55788	·53806	·51924	·50134	·48430	·46805	·45255	·43774	·42357	·41002	·39702
4-5	1-9640	·58099	·56257	·54495	·52806	·51187	·49633	·48140	·46705	·45325	·43996	·42716
4-6	1-7797	·60052	·58341	·56694	·55106	·53574	·52095	·50666	·49286	·47951	·46659	·45409
4-7	1-6202	·61609	·60022	·58485	·56996	·55551	·54150	·52790	·51470	·50187	·48941	·47729
4-8	1-4814	·62742	·61271	·59840	·58446	·57089	·55766	·54478	·53221	·51996	·50801	·49635
4-9	1-3599	·63431	·62069	·60737	·59436	·58164	·56921	·55704	·54514	·53350	·52210	·51094
5-0	1-2533	·63662	·62401	·61165	·59963	·58765	·57599	·56455	·55332	·54230	·53149	·52087
5-1	1-1593	·63431	·62266	·61120	·59994	·58886	·57796	·56724	·55669	·54631	·53609	·52604
5-2	1-0759	·62742	·61667	·60607	·59562	·58532	·57516	·56515	·55527	·54553	·53592	·52644
5-3	1-0018	·61609	·60618	·59639	·58672	·57717	·56773	·55841	·54919	·54008	·53108	·52219
5-4	0-9357	·60052	·59140	·58238	·57346	·56462	·55588	·54722	·53866	·53018	·52178	·51347
5-5	0-8764	·58099	·57263	·56434	·55612	·54797	·53990	·53189	·52396	·51609	·50829	·50056
5-6	0-8230	·55788	·55022	·54262	·53507	·52759	·52015	·51278	·50545	·49818	·49097	·48380
5-7	0-7749	·53159	·52460	·51765	·51075	·50389	·49708	·49030	·48357	·47688	·47024	·46363
5-8	0-7313	·50260	·49624	·48992	·48363	·47737	·47114	·46495	·45879	·45266	·44657	·44050
5-9	0-6917	·47144	·46567	·45993	·45422	·44853	·44287	·43723	·43162	·42603	·42047	·41493
6-0	0-6557	·43863	·43343	·42824	·42308	·41793	·41281	·40770	·40261	·39754	·39249	·38746
6-1	0-6227	·40474	·40006	·39540	·39075	·38612	·38150	·37690	·37231	·36774	·36318	·35863
6-2	0-5926	·37031	·36613	·36196	·35781	·35366	·34952	·34540	·34128	·33718	·33308	·32900
6-3	0-5649	·33589	·33218	·32847	·32477	·32108	·31740	·31372	·31006	·30640	·30274	·29910
6-4	0-5394	·30199	·29871	·29543	·29216	·28890	·28564	·28238	·27913	·27589	·27266	·26942
6-5	0-5158	·26907	·26619	·26331	·26044	·25757	·25470	·25184	·24899	·24613	·24329	·24044

TABLE II (cont.)

Y	Q/Z	Percentage natural mortality, <i>C</i>										
		0	1	2	3	4	5	6	7	8	9	10
6-6	0.4940	·23753	·23502	·23251	·23001	·22751	·22501	·22251	·22001	·21752	·21503	·21255
6-7	0.4739	·20774	·20556	·20339	·20122	·19905	·19689	·19473	·19256	·19041	·18825	·18609
6-8	0.4551	·17994	·17808	·17621	·17435	·17249	·17083	·16877	·16691	·16506	·16320	·16135
6-9	0.4376	·15436	·15277	·15118	·14960	·14801	·14643	·14484	·14326	·14168	·14010	·13852
7-0	0.4214	·13112	·12977	·12843	·12709	·12575	·12442	·12308	·12174	·12040	·11907	·11773
7-1	0.4082	·11026	·10814	·10802	·10689	·10577	·10465	·10353	·10241	·10129	·10017	·09905
7-2	0.3919	·09179	·09086	·08993	·08900	·08807	·08714	·08621	·08528	·08435	·08342	·08249
7-3	0.3786	·07564	·07487	·07411	·07334	·07258	·07181	·07105	·07029	·06952	·06876	·06800
7-4	0.3661	·06168	·06106	·06044	·05982	·05920	·05858	·05795	·05733	·05671	·05609	·05547
7-5	0.3543	·04979	·04929	·04879	·04828	·04778	·04728	·04678	·04628	·04578	·04528	·04478
7-6	0.3432	·03977	·03937	·03897	·03857	·03817	·03777	·03737	·03697	·03657	·03617	·03577
7-7	0.3327	·03143	·03112	·03080	·03048	·03017	·02985	·02954	·02922	·02891	·02859	·02828
7-8	0.3228	·02458	·02434	·02409	·02385	·02360	·02335	·02311	·02286	·02261	·02237	·02212
7-9	0.3134	·01903	·01883	·01864	·01845	·01826	·01807	·01788	·01769	·01750	·01731	·01712
8-0	0.3046	·01457	·01442	·01428	·01413	·01399	·01384	·01369	·01355	·01340	·01326	·01311
8-1	0.2962	·01104	·01093	·01082	·01071	·01060	·01049	·01038	·01027	·01016	·01005	·00993
8-2	0.2882	·00828	·00819	·00811	·00803	·00795	·00786	·00778	·00770	·00762	·00753	·00745
8-3	0.2806	·00614	·00608	·00602	·00596	·00590	·00583	·00577	·00571	·00565	·00559	·00553
8-4	0.2734	·00451	·00446	·00442	·00437	·00433	·00428	·00424	·00419	·00415	·00410	·00406
8-5	0.2666	·00327	·00324	·00321	·00318	·00314	·00311	·00308	·00305	·00301	·00298	·00295
8-6	0.2600	·00235	·00233	·00231	·00228	·00226	·00224	·00221	·00219	·00217	·00214	·00212
8-7	0.2538	·00167	·00166	·00164	·00162	·00161	·00159	·00157	·00156	·00154	·00152	·00150
8-8	0.2478	·00118	·00117	·00116	·00114	·00113	·00112	·00111	·00110	·00108	·00107	·00106
8-9	0.2421	·00082	·00081	·00080	·00080	·00079	·00078	·00077	·00076	·00076	·00075	·00074
9-0	0.2367	·00056	·00056	·00055	·00055	·00054	·00054	·00053	·00053	·00052	·00051	·00051

Y	Q/Z	11	12	13	14	15	16	17	18	19	20
1-1	5034	—	—	—	—	—	—	—	—	—	—
1-2	3425	—	—	—	—	—	—	—	—	—	—
1-3	2354	—	—	—	—	—	—	—	—	—	—
1-4	1634	—	—	—	—	—	—	—	—	—	—
1-5	1146	·00001	·00001	·00001	—	—	—	—	—	—	—
1-6	811-2	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001
1-7	580-2	·00002	·00002	·00002	·00002	·00002	·00002	·00001	·00001	·00001	·00001
1-8	419-1	·00005	·00004	·00004	·00003	·00003	·00003	·00003	·00003	·00002	·00002
1-9	305-8	·00009	·00008	·00007	·00007	·00006	·00006	·00005	·00005	·00005	·00004
2-0	225-3	·00016	·00014	·00013	·00012	·00011	·00010	·00010	·00009	·00008	·00008
2-1	167-69	·00028	·00026	·00023	·00022	·00020	·00018	·00017	·00016	·00015	·00014
2-2	126-02	·00050	·00045	·00041	·00038	·00035	·00033	·00030	·00028	·00026	·00025
2-3	95-63	·00086	·00078	·00071	·00066	·00061	·00056	·00052	·00049	·00046	·00043
2-4	73-28	·00145	·00131	·00120	·00111	·00102	·00095	·00088	·00083	·00077	·00073
2-5	56-70	·00238	·00217	·00199	·00183	·00169	·00157	·00147	·00137	·00128	·00121
2-6	44-288	·00384	·00350	·00321	·00296	·00274	·00255	·00237	·00222	·00208	·00196
2-7	34-923	·00604	·00551	·00506	·00467	·00433	·00403	·00376	·00352	·00331	·00311
2-8	27-797	·00928	·00849	·00781	·00722	·00670	·00624	·00583	·00547	·00514	·00484
2-9	22-330	·01392	·01277	·01177	·01090	·01014	·00945	·00885	·00830	·00780	·00735
3-0	18-101	·02038	·01875	·01732	·01608	·01497	·01399	·01311	·01231	·01159	·01094
3-1	14-802	·02910	·02685	·02488	·02315	·02160	·02022	·01898	·01786	·01684	·01590
3-2	12-211	·04053	·03753	·03488	·03254	·03044	·02856	·02686	·02531	·02390	·02261
3-3	10-159	·05505	·05117	·04772	·04465	·04188	·03939	·03712	·03506	·03317	·03143
3-4	8-521	·07297	·06809	·06374	·05982	·05628	·05307	·05014	·04745	·04498	·04271
3-5	7-205	·09441	·08848	·08313	·07829	·07389	·06987	·06618	·06278	·05965	·05674
3-6	6-1394	·11034	·11232	·10595	·10014	·09481	·08991	·08540	·08122	·07734	·07373
3-7	5-2705	·14753	·13945	·13205	·12525	·11898	·11318	·10780	·10279	·09812	·09376
3-8	4-5571	·17854	·16947	·16111	·15336	·14616	·13946	·13321	·12736	·12187	·11672
3-9	3-9676	·21179	·20185	·19260	·18398	·17591	·16836	·16127	·15460	·14831	·14237
4-0	3-4770	·24656	·23589	·22589	·21649	·20766	·19933	·19146	·18402	·17698	·17029

TABLE II (cont.)

Percentage natural mortality, O

Y	Q/Z	11	12	13	14	15	16	17	18	19	20
4-1	3-0665	·28204	·27081	·26020	·25017	·24068	·23168	·22314	·21501	·20728	·19991
4-2	2-7206	·31742	·30578	·29473	·28421	·27420	·26465	·25554	·24684	·23852	·23055
4-3	2-4276	·35186	·33998	·32864	·31779	·30740	·29744	·28789	·27873	·26992	·26145
4-4	2-1780	·38457	·37261	·36112	·35008	·33945	·32922	·31937	·30986	·30069	·29184
4-5	1-9640	·41482	·40292	·39142	·38033	·36960	·35922	·34919	·33947	·33006	·32094
4-6	1-7797	·44198	·43025	·41887	·40784	·39713	·38674	·37664	·36683	·35729	·34802
4-7	1-6202	·46551	·45405	·44289	·43202	·42144	·41113	·40109	·39129	·38174	·37242
4-8	1-4814	·48496	·47385	·46299	·45239	·44202	·43190	·42199	·41231	·40284	·39357
4-9	1-3599	·50001	·48931	·47883	·46855	·45849	·44862	·43894	·42945	·42015	·41102
5-0	1-2533	·51044	·50020	·49014	·48026	·47054	·46100	·45162	·44240	·43333	·42441
5-1	1-1593	·51614	·50639	·49680	·48735	·47804	·46887	·45984	·45094	·44217	·43354
5-2	1-0759	·51709	·50787	·49876	·48978	·48091	·47216	·46353	·45500	·44658	·43827
5-3	1-0018	·51340	·50471	·49612	·48762	·47923	·47092	·46271	·45459	·44657	·43863
5-4	0-9357	·50524	·49709	·48903	·48104	·47313	·46529	·45754	·44985	·44224	·43470
5-5	0-8764	·49289	·48529	·47775	·47028	·46286	·45551	·44822	·44099	·43382	·42671
5-6	0-8230	·47669	·46963	·46262	·45567	·44876	·44190	·43509	·42832	·42161	·41494
5-7	0-7749	·45706	·45054	·44406	·43761	·43120	·42484	·41851	·41222	·40597	·39975
5-8	0-7313	·43447	·42847	·42250	·41656	·41066	·40478	·39893	·39311	·38733	·38157
5-9	0-6917	·40942	·40393	·39846	·39302	·38761	·38221	·37684	·37149	·36617	·36087
6-0	0-6557	·38245	·37745	·37248	·36752	·36258	·35766	·35275	·34787	·34300	·33815
6-1	0-6227	·35410	·34958	·34508	·34059	·33611	·33165	·32720	·32276	·31834	·31393
6-2	0-5926	·32493	·32087	·31681	·31277	·30874	·30472	·30071	·29671	·29272	·28874
6-3	0-5649	·29546	·29183	·28821	·28460	·28099	·27739	·27380	·27022	·26664	·26308
6-4	0-5394	·26620	·26298	·25977	·25656	·25335	·25016	·24696	·24378	·24060	·23742
6-5	0-5158	·23760	·23476	·23193	·22910	·22628	·22346	·22064	·21783	·21502	·21222

6-6	0-4940	-21007	-20759	-20511	-20264	-20016	-19770	-19523	-19277	-19031	-18785
6-7	0-4739	-18394	-18179	-17964	-17749	-17535	-17320	-17106	-16892	-16679	-16465
6-8	0-4551	-15950	-15765	-15580	-15395	-15210	-15026	-14841	-14657	-14473	-14289
6-9	0-4376	-13694	-13536	-13378	-13220	-13063	-12905	-12748	-12591	-12433	-12276
7-0	0-4214	-11640	-11506	-11373	-11239	-11106	-10973	-10840	-10707	-10574	-10441
7-1	0-4062	-09794	-09682	-09570	-09458	-09347	-09235	-09123	-09012	-08900	-08789
7-2	0-3919	-08157	-08064	-07971	-07878	-07786	-07693	-07600	-07508	-07415	-07323
7-3	0-3786	-06724	-06647	-06571	-06495	-06419	-06342	-06266	-06190	-06114	-06038
7-4	0-3661	-05485	-05423	-05361	-05299	-05237	-05175	-05113	-05051	-04989	-04927
7-5	0-3543	-04428	-04378	-04328	-04278	-04228	-04178	-04128	-04078	-04028	-03978
7-6	0-3432	-03537	-03498	-03458	-03418	-03378	-03338	-03298	-03258	-03218	-03178
7-7	0-3327	-02796	-02765	-02733	-02702	-02670	-02639	-02607	-02576	-02544	-02513
7-8	0-3228	-02187	-02163	-02138	-02114	-02089	-02064	-02040	-02015	-01990	-01966
7-9	0-3134	-01693	-01674	-01655	-01636	-01617	-01598	-01579	-01560	-01541	-01521
8-0	0-3046	-01297	-01282	-01267	-01253	-01238	-01224	-01209	-01194	-01180	-01165
8-1	0-2962	-00982	-00971	-00960	-00949	-00938	-00927	-00916	-00905	-00894	-00883
8-2	0-2882	-00737	-00728	-00720	-00712	-00704	-00695	-00687	-00679	-00670	-00662
8-3	0-2806	-00547	-00540	-00534	-00528	-00522	-00516	-00510	-00504	-00497	-00491
8-4	0-2734	-00401	-00397	-00392	-00388	-00383	-00379	-00374	-00370	-00365	-00361
8-5	0-2666	-00291	-00288	-00285	-00282	-00278	-00275	-00272	-00269	-00265	-00262
8-6	0-2600	-00209	-00207	-00205	-00202	-00200	-00198	-00195	-00193	-00191	-00188
8-7	0-2538	-00149	-00147	-00145	-00144	-00142	-00140	-00139	-00137	-00135	-00134
8-8	0-2478	-00105	-00104	-00103	-00101	-00100	-00099	-00098	-00097	-00096	-00094
8-9	0-2421	-00073	-00072	-00071	-00071	-00070	-00069	-00068	-00067	-00067	-00066
9-0	0-2367	-00050	-00050	-00049	-00049	-00048	-00047	-00047	-00046	-00046	-00045

TABLE II (cont.)

Percentage natural mortality, *C*

<i>Y</i>	<i>Q/Z</i>	21	22	23	24	25	26	27	28	29	30
1-1	5034	—	—	—	—	—	—	—	—	—	—
1-2	3425	—	—	—	—	—	—	—	—	—	—
1-3	2354	—	—	—	—	—	—	—	—	—	—
1-4	1634	—	—	—	—	—	—	—	—	—	—
1-5	1146	—	—	—	—	—	—	—	—	—	—
1-6	811-2	·00001	·00001	·00001	—	—	—	—	—	—	—
1-7	580-2	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001
1-8	419-1	·00002	·00002	·00002	·00002	·00002	·00002	·00002	·00001	·00001	·00001
1-9	305-8	·00004	·00004	·00004	·00003	·00003	·00003	·00003	·00003	·00003	·00002
2-0	225-3	·00007	·00007	·00007	·00006	·00006	·00006	·00005	·00005	·00005	·00005
2-1	167-69	·00013	·00013	·00012	·00011	·00011	·00010	·00010	·00009	·00009	·00008
2-2	126-02	·00023	·00022	·00021	·00020	·00019	·00018	·00017	·00016	·00015	·00015
2-3	95-63	·00040	·00038	·00036	·00034	·00032	·00031	·00029	·00028	·00026	·00025
2-4	73-28	·00069	·00065	·00061	·00058	·00055	·00052	·00049	·00047	·00045	·00043
2-5	56-70	·00114	·00107	·00101	·00096	·00091	·00086	·00082	·00078	·00075	·00071
2-6	44-288	·00185	·00174	·00165	·00156	·00148	·00141	·00134	·00127	·00121	·00116
2-7	34-923	·00293	·00277	·00262	·00248	·00236	·00224	·00213	·00203	·00194	·00185
2-8	27-797	·00456	·00431	·00408	·00387	·00368	·00349	·00333	·00317	·00302	·00288
2-9	22-330	·00694	·00657	·00622	·00590	·00561	·00533	·00508	·00484	·00462	·00441
3-0	18-101	·01034	·00979	·00928	·00881	·00838	·00797	·00760	·00725	·00692	·00661
3-1	14-802	·01505	·01426	·01354	·01287	·01224	·01166	·01112	·01061	·01014	·00969
3-2	12-211	·02143	·02033	·01932	·01838	·01751	·01669	·01593	·01522	·01455	·01392
3-3	10-159	·02983	·02834	·02697	·02569	·02450	·02338	·02234	·02136	·02044	·01957
3-4	8-521	·04060	·03864	·03682	·03512	·03354	·03205	·03065	·02934	·02810	·02693
3-5	7-205	·05404	·05153	·04918	·04698	·04492	·04299	·04117	·03945	·03782	·03629

3-6	6-1394	-07037	-06722	-06427	-06150	-05889	-05644	-05412	-05193	-04985	-04788
3-7	5-2705	-08966	-08582	-08221	-07881	-07569	-07255	-06967	-06695	-06435	-06189
3-8	4-5571	-11187	-10730	-10298	-09890	-09503	-09136	-08787	-08455	-08130	-07838
3-9	3-9676	-13676	-13145	-12641	-12163	-11708	-11275	-10862	-10468	-10091	-09732
4-0	3-4770	-16394	-15791	-15216	-14668	-14145	-13645	-13167	-12710	-12271	-11851
4-1	3-0665	-19288	-18616	-17974	-17360	-16771	-16207	-15665	-15145	-14645	-14164
4-2	2-7206	-22291	-21559	-20855	-20180	-19531	-18906	-18304	-17725	-17166	-16626
4-3	2-4276	-25331	-24546	-23790	-23061	-22358	-21679	-21023	-20389	-19776	-19182
4-4	2-1780	-28329	-27503	-26704	-25930	-25181	-24456	-23753	-23072	-22411	-21769
4-5	1-9640	-31210	-30352	-29520	-28712	-27927	-27165	-26424	-25703	-25002	-24319
4-6	1-7797	-33900	-33022	-32167	-31335	-30524	-29734	-28963	-28212	-27479	-26764
4-7	1-6202	-36332	-35444	-34578	-33731	-32904	-32095	-31305	-30533	-29777	-29038
4-8	1-4814	-38450	-37562	-36693	-35841	-35007	-34190	-33390	-32605	-31836	-31082
4-9	1-3599	-40206	-39327	-38464	-37617	-36785	-35968	-35166	-34378	-33604	-32843
5-0	1-2533	-41564	-40702	-39853	-39019	-38197	-37389	-36593	-35810	-35039	-34279
5-1	1-1593	-42502	-41663	-40836	-40020	-39216	-38423	-37641	-36870	-36109	-35359
5-2	1-0759	-43007	-42196	-41396	-40606	-39825	-39054	-38292	-37540	-36796	-36062
5-3	1-0018	-43077	-42300	-41532	-40772	-40020	-39276	-38540	-37812	-37091	-36378
5-4	0-9357	-42724	-41984	-41252	-40526	-39807	-39094	-38388	-37689	-36996	-36309
5-5	0-8764	-41966	-41266	-40572	-39884	-39201	-38524	-37852	-37185	-36524	-35868
5-6	0-8230	-40832	-40174	-39521	-38873	-38229	-37590	-36954	-36324	-35697	-35075
5-7	0-7749	-39357	-38743	-38133	-37526	-36923	-36323	-35727	-35134	-34545	-33959
5-8	0-7313	-37584	-37014	-36447	-35883	-35322	-34763	-34207	-33655	-33104	-32557
5-9	0-6917	-35559	-35033	-34510	-33989	-33470	-32954	-32439	-31927	-31417	-30909
6-0	0-6557	-33332	-32850	-32370	-31892	-31416	-30941	-30469	-29997	-29528	-29060
6-1	0-6227	-30954	-30516	-30079	-29643	-29209	-28776	-28344	-27914	-27485	-27057
6-2	0-5926	-28477	-28081	-27666	-27292	-26899	-26507	-26116	-25726	-25337	-24949
6-3	0-5649	-25952	-25596	-25242	-24888	-24535	-24182	-23831	-23480	-23130	-22780
6-4	0-5394	-23425	-23109	-22793	-22477	-22163	-21848	-21535	-21221	-20909	-20597
6-5	0-5158	-20942	-20662	-20383	-20104	-19825	-19547	-19270	-18992	-18715	-18439

TABLE II (cont.)

Y	Q/Z	Percentage natural mortality, C									
		21	22	23	24	25	26	27	28	29	30
6-6	0-4940	-18540	-18294	-18049	-17805	-17561	-17317	-17073	-16829	-16586	-16343
6-7	0-4739	-16252	-16039	-15826	-15613	-15401	-15188	-14976	-14764	-14552	-14341
6-8	0-4551	-14105	-13921	-13738	-13554	-13371	-13188	-13005	-12822	-12639	-12457
6-9	0-4376	-12119	-11962	-11805	-11649	-11492	-11336	-11179	-11023	-10866	-10710
7-0	0-4214	-10308	-10175	-10042	-99909	-99777	-99644	-99512	-99379	-99247	-99114
7-1	0-4062	-08677	-08566	-08455	-08343	-08232	-08121	-08010	-07899	-07787	-07676
7-2	0-3919	-07230	-07137	-07045	-06953	-06860	-06768	-06675	-06583	-06491	-06398
7-3	0-3786	-05962	-05886	-05809	-05733	-05657	-05581	-05505	-05429	-05353	-05277
7-4	0-3661	-04865	-04803	-04741	-04679	-04617	-04555	-04493	-04431	-04369	-04307
7-5	0-3543	-03928	-03878	-03828	-03778	-03728	-03678	-03628	-03578	-03528	-03479
7-6	0-3432	-03139	-03099	-03059	-03019	-02979	-02939	-02899	-02860	-02820	-02780
7-7	0-3327	-02481	-02450	-02418	-02387	-02355	-02324	-02292	-02261	-02229	-02198
7-8	0-3228	-01941	-01917	-01892	-01867	-01843	-01818	-01793	-01769	-01744	-01720
7-9	0-3134	-01502	-01483	-01464	-01445	-01426	-01407	-01388	-01369	-01350	-01331
8-0	0-3046	-01151	-01136	-01122	-01107	-01092	-01078	-01063	-01049	-01034	-01019
8-1	0-2962	-00872	-00861	-00850	-00839	-00828	-00817	-00806	-00795	-00784	-00773
8-2	0-2882	-00654	-00646	-00637	-00629	-00621	-00612	-00604	-00596	-00588	-00579
8-3	0-2806	-00485	-00479	-00473	-00467	-00461	-00454	-00448	-00442	-00436	-00430
8-4	0-2734	-00356	-00352	-00347	-00343	-00338	-00334	-00329	-00325	-00320	-00316
8-5	0-2666	-00259	-00255	-00252	-00249	-00246	-00242	-00239	-00236	-00232	-00229
8-6	0-2600	-00186	-00184	-00181	-00179	-00176	-00174	-00172	-00169	-00167	-00165
8-7	0-2538	-00132	-00130	-00129	-00127	-00125	-00124	-00122	-00120	-00119	-00117
8-8	0-2478	-00093	-00092	-00091	-00090	-00088	-00087	-00086	-00085	-00084	-00083
8-9	0-2421	-00065	-00064	-00063	-00062	-00062	-00061	-00060	-00059	-00058	-00057
9-0	0-2367	-00045	-00044	-00044	-00043	-00042	-00042	-00041	-00041	-00040	-00040

Y	Q/Z	31	32	33	34	35	36	37	38	39	40
1-1	5034	—	—	—	—	—	—	—	—	—	—
1-2	3425	—	—	—	—	—	—	—	—	—	—
1-3	2354	—	—	—	—	—	—	—	—	—	—
1-4	1634	—	—	—	—	—	—	—	—	—	—
1-5	1146	—	—	—	—	—	—	—	—	—	—
1-6	811-2	—	—	—	—	—	—	—	—	—	—
1-7	580-2	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001
1-8	419-1	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001	·00001
1-9	305-8	·00002	·00002	·00002	·00002	·00002	·00002	·00002	·00002	·00002	·00002
2-0	225-3	·00004	·00004	·00004	·00004	·00004	·00003	·00003	·00003	·00003	·00003
2-1	167-69	·00008	·00008	·00007	·00007	·00007	·00006	·00006	·00006	·00006	·00005
2-2	126-02	·00014	·00013	·00013	·00012	·00012	·00011	·00011	·00010	·00010	·00009
2-3	95-63	·00024	·00023	·00022	·00021	·00020	·00019	·00018	·00018	·00017	·00016
2-4	73-28	·00041	·00039	·00037	·00036	·00034	·00033	·00031	·00030	·00029	·00028
2-5	56-70	·00068	·00065	·00062	·00059	·00057	·00054	·00052	·00050	·00048	·00046
2-6	44-288	·00111	·00106	·00101	·00097	·00092	·00089	·00085	·00081	·00078	·00075
2-7	34-923	·00176	·00169	·00161	·00154	·00148	·00142	·00136	·00130	·00125	·00120
2-8	27-797	·00276	·00263	·00252	·00241	·00231	·00221	·00212	·00204	·00195	·00188
2-9	22-330	·00422	·00403	·00386	·00370	·00354	·00339	·00325	·00312	·00300	·00288
3-0	18-101	·00632	·00605	·00579	·00555	·00532	·00510	·00489	·00469	·00451	·00433
3-1	14-802	·00927	·00888	·00850	·00815	·00782	·00750	·00720	·00691	·00664	·00637
3-2	12-211	·01333	·01276	·01223	·01173	·01126	·01080	·01037	·00996	·00957	·00920
3-3	10-159	·01875	·01797	·01724	·01654	·01588	·01525	·01465	·01408	·01354	·01302
3-4	8-521	·02582	·02478	·02378	·02284	·02194	·02109	·02027	·01949	·01875	·01804
3-5	7-205	·03483	·03345	·03214	·03089	·02970	·02856	·02748	·02645	·02546	·02451

TABLE II (cont.)

Percentage natural mortality, *C*

Y	Q/Z	31	32	33	34	35	36	37	38	39	40
3-6	6-1394	·04601	·04423	·04254	·04093	·03938	·03791	·03651	·03516	·03387	·03263
3-7	5-2705	·05954	·05731	·05517	·05313	·05118	·04932	·04753	·04581	·04417	·04259
3-8	4-5571	·07551	·07276	·07013	·06761	·06520	·06289	·06067	·05853	·05648	·05451
3-9	3-9676	·09387	·09057	·08741	·08437	·08145	·07865	·07595	·07335	·07085	·06844
4-0	3-4770	·11447	·11059	·10687	·10328	·09983	·09650	·09329	·09020	·08721	·08432
4-1	3-0665	·13701	·13255	·12825	·12410	·12010	·11623	·11249	·10888	·10538	·10200
4-2	2-7206	·16106	·15603	·15116	·14646	·14191	·13751	·13324	·12910	·12509	·12120
4-3	2-4276	·18608	·18051	·17512	·16989	·16481	·15989	·15511	·15046	·14595	·14156
4-4	2-1780	·21146	·20541	·19953	·19382	·18826	·18285	·17758	·17246	·16747	·16261
4-5	1-9640	·23655	·23008	·22377	·21763	·21164	·20580	·20010	·19454	·18911	·18382
4-6	1-7797	·26066	·25384	·24719	·24069	·23433	·22812	·22205	·21611	·21031	·20462
4-7	1-6202	·28315	·27607	·26914	·26236	·25572	·24921	·24283	·23659	·23046	·22446
4-8	1-4814	·30342	·29617	·28905	·28206	·27521	·26848	·26188	·25539	·24902	·24276
4-9	1-3599	·32096	·31361	·30638	·29928	·29229	·28542	·27866	·27201	·26547	·25904
5-0	1-2533	·33532	·32795	·32070	·31356	·30652	·29958	·29275	·28602	·27938	·27284
5-1	1-1593	·34619	·33889	·33168	·32457	·31756	·31063	·30380	·29705	·29039	·28381
5-2	1-0759	·35336	·34618	·33909	·33209	·32516	·31832	·31155	·30486	·29824	·29170
5-3	1-0018	·35672	·34974	·34282	·33598	·32921	·32250	·31587	·30930	·30279	·29635
5-4	0-9357	·35629	·34954	·34286	·33624	·32967	·32317	·31672	·31032	·30399	·29770
5-5	0-8764	·35217	·34571	·33930	·33294	·32663	·32037	·31416	·30799	·30187	·29580
5-6	0-8230	·34457	·33843	·33233	·32628	·32026	·31428	·30835	·30245	·29659	·29077
5-7	0-7749	·33377	·32798	·32222	·31650	·31081	·30515	·29953	·29393	·28837	·28284
5-8	0-7313	·32012	·31470	·30931	·30394	·29860	·29329	·28800	·28273	·27750	·27229
5-9	0-6917	·30403	·29899	·29398	·28898	·28401	·27906	·27413	·26921	·26432	·25945
6-0	0-6557	·28594	·28129	·27666	·27205	·26746	·26288	·25831	·25377	·24923	·24472

6-1	0-6227	-26631	-26206	-25782	-25359	-24938	-24518	-24099	-23681	-23265	-22850
6-2	0-5926	-24561	-24175	-23790	-23406	-23022	-22640	-22259	-21878	-21499	-21120
6-3	0-5649	-22431	-22083	-21736	-21390	-21044	-20699	-20354	-20010	-19667	-19325
6-4	0-5394	-20285	-19974	-19663	-19353	-19044	-18735	-18426	-18119	-17811	-17504
6-5	0-5158	-18163	-17887	-17612	-17337	-17062	-16788	-16514	-16240	-15967	-15695
6-6	0-4940	-16101	-15858	-15616	-15374	-15133	-14891	-14650	-14410	-14169	-13929
6-7	0-4739	-14129	-13918	-13707	-13496	-13286	-13075	-12865	-12655	-12445	-12236
6-8	0-4551	-12274	-12092	-11910	-11728	-11546	-11364	-11182	-11001	-10819	-10638
6-9	0-4376	-10554	-10398	-10242	-10086	-9931	-9775	-9619	-9464	-9309	-9153
7-0	0-4214	-08982	-08850	-08718	-08586	-08454	-08322	-08190	-08058	-07926	-07794
7-1	0-4062	-07565	-07454	-07343	-07232	-07122	-07011	-06900	-06789	-06679	-06568
7-2	0-3919	-06306	-06214	-06121	-06029	-05937	-05845	-05753	-05661	-05569	-05477
7-3	0-3786	-05201	-05125	-05049	-04974	-04898	-04822	-04746	-04670	-04594	-04519
7-4	0-3661	-04245	-04183	-04122	-04060	-03998	-03936	-03874	-03812	-03751	-03689
7-5	0-3543	-03429	-03379	-03329	-03279	-03229	-03179	-03129	-03079	-03030	-02980
7-6	0-3432	-02740	-02700	-02660	-02620	-02581	-02541	-02501	-02461	-02421	-02382
7-7	0-3327	-02166	-02135	-02103	-02072	-02041	-02009	-01978	-01946	-01915	-01883
7-8	0-3228	-01695	-01670	-01646	-01621	-01597	-01572	-01547	-01523	-01498	-01474
7-9	0-3134	-01312	-01293	-01274	-01255	-01236	-01217	-01198	-01179	-01160	-01141
8-0	0-3046	-01005	-00990	-00976	-00961	-00947	-00932	-00917	-00903	-00888	-00874
8-1	0-2962	-00762	-00750	-00739	-00728	-00717	-00706	-00695	-00684	-00673	-00662
8-2	0-2882	-00571	-00563	-00554	-00546	-00538	-00530	-00521	-00513	-00505	-00497
8-3	0-2806	-00424	-00418	-00411	-00405	-00399	-00393	-00387	-00381	-00375	-00368
8-4	0-2734	-00311	-00307	-00302	-00298	-00293	-00289	-00284	-00279	-00275	-00270
8-5	0-2666	-00226	-00223	-00219	-00216	-00213	-00210	-00206	-00203	-00200	-00196
8-6	0-2600	-00162	-00160	-00158	-00155	-00153	-00151	-00148	-00146	-00144	-00141
8-7	0-2538	-00115	-00114	-00112	-00110	-00109	-00107	-00105	-00104	-00102	-00100
8-8	0-2478	-00081	-00080	-00079	-00078	-00077	-00075	-00074	-00073	-00072	-00071
8-9	0-2421	-00057	-00056	-00055	-00054	-00053	-00053	-00052	-00051	-00050	-00049
0-0	0-2367	-00039	-00038	-00038	-00037	-00037	-00036	-00036	-00035	-00034	-00034

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TABLE II (cont.)

Percentage natural mortality, *C*

Y	Q/Z	41	42	43	44	45	46	47	48	49	50
1.8	419.1	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001
1.9	305.8	-00002	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001
2.0	225.3	-00003	-00003	-00003	-00002	-00002	-00002	-00002	-00002	-00002	-00002
2.1	167.69	-00005	-00005	-00005	-00005	-00004	-00004	-00004	-00004	-00004	-00004
2.2	126.02	-00009	-00009	-00008	-00008	-00008	-00007	-00007	-00007	-00007	-00006
2.3	95.63	-00016	-00015	-00014	-00014	-00013	-00013	-00012	-00012	-00011	-00011
2.4	73.28	-00026	-00025	-00024	-00023	-00023	-00022	-00021	-00020	-00019	-00018
2.5	56.70	-00044	-00042	-00041	-00039	-00038	-00036	-00035	-00033	-00032	-00031
2.6	44.288	-00072	-00069	-00066	-00064	-00061	-00059	-00056	-00054	-00052	-00050
2.7	34.923	-00115	-00110	-00106	-00102	-00098	-00094	-00090	-00087	-00083	-00080
2.8	27.797	-00180	-00173	-00166	-00160	-00153	-00147	-00142	-00136	-00131	-00126
2.9	22.330	-00276	-00265	-00255	-00245	-00236	-00226	-00218	-00209	-00201	-00194
3.0	18.101	-00416	-00399	-00384	-00369	-00355	-00341	-00328	-00315	-00303	-00292
3.1	14.802	-00613	-00589	-00566	-00544	-00523	-00503	-00484	-00466	-00448	-00431
3.2	12.211	-00885	-00851	-00818	-00787	-00757	-00728	-00701	-00674	-00649	-00624
3.3	10.159	-01252	-01204	-01159	-01115	-01073	-01033	-00994	-00957	-00921	-00886
3.4	8.521	-01736	-01671	-01609	-01549	-01491	-01436	-01382	-01331	-01282	-01234
3.5	7.205	-02360	-02273	-02189	-02109	-02031	-01957	-01885	-01816	-01749	-01685
3.6	6.1394	-03144	-03030	-02920	-02815	-02713	-02615	-02521	-02429	-02342	-02257
3.7	5.2705	-04107	-03961	-03820	-03684	-03554	-03427	-03306	-03188	-03074	-02964
3.8	4.5571	-05261	-05078	-04901	-04730	-04566	-04407	-04253	-04105	-03961	-03821
3.9	3.9676	-06611	-06386	-06169	-05959	-05757	-05560	-05370	-05186	-05008	-04835
4.0	3.4770	-08153	-07883	-07622	-07369	-07124	-06887	-06657	-06433	-06217	-06006

4-1	3-0665	-09872	-09554	-09246	-08948	-08658	-08376	-08103	-07838	-07579	-07328
4-2	2-7206	-11743	-11376	-11020	-10674	-10337	-10010	-09692	-09382	-09080	-08786
4-3	2-4276	-13729	-13314	-12910	-12516	-12133	-11759	-11395	-11040	-10694	-10357
4-4	2-1780	-15787	-15325	-14874	-14434	-14005	-13587	-13178	-12778	-12388	-12007
4-5	1-9640	-17864	-17359	-16865	-16382	-15910	-15448	-14997	-14555	-14122	-13699
4-6	1-7797	-19906	-19362	-18829	-18307	-17796	-17295	-16804	-16323	-15852	-15390
4-7	1-6202	-21857	-21280	-20713	-20158	-19612	-19077	-18552	-18036	-17530	-17032
4-8	1-4814	-23662	-23058	-22464	-21881	-21307	-20744	-20189	-19644	-19108	-18581
4-9	1-3599	-25270	-24647	-24033	-23428	-22833	-22247	-21670	-21102	-20542	-19990
5-0	1-2533	-26639	-26003	-25376	-24757	-24148	-23546	-22953	-22368	-21790	-21221
5-1	1-1593	-27732	-27091	-26458	-25832	-25215	-24605	-24002	-23407	-22819	-22237
5-2	1-0759	-28524	-27884	-27252	-26626	-26008	-25396	-24790	-24192	-23599	-23013
5-3	1-0018	-28998	-28366	-27741	-27122	-26509	-25901	-25300	-24704	-24114	-23530
5-4	0-9357	-29148	-28530	-27918	-27311	-26709	-26113	-25521	-24935	-24353	-23776
5-5	0-8764	-28977	-28379	-27785	-27196	-26611	-26031	-25454	-24882	-24314	-23751
5-6	0-8230	-28499	-27925	-27354	-26787	-26224	-25664	-25108	-24555	-24006	-23461
5-7	0-7749	-27734	-27187	-26644	-26103	-25565	-25030	-24499	-23970	-23444	-22921
5-8	0-7313	-26710	-26194	-25680	-25169	-24660	-24154	-23650	-23149	-22650	-22153
5-9	0-6917	-25460	-24977	-24496	-24017	-23539	-23064	-22591	-22120	-21650	-21183
6-0	0-6557	-24022	-23573	-23127	-22681	-22238	-21795	-21355	-20916	-20478	-20042
6-1	0-6227	-22436	-22023	-21611	-21201	-20792	-20384	-19977	-19572	-19167	-18764
6-2	0-5926	-20742	-20366	-19990	-19615	-19241	-18868	-18496	-18125	-17755	-17385
6-3	0-5649	-18984	-18643	-18302	-17963	-17624	-17286	-16949	-16612	-16276	-15940
6-4	0-5394	-17198	-16892	-16587	-16282	-15978	-15674	-15371	-15068	-14766	-14464
6-5	0-5158	-15422	-15150	-14879	-14608	-14337	-14066	-13796	-13527	-13257	-12989
6-6	0-4940	-13689	-13450	-13210	-12971	-12732	-12494	-12255	-12017	-11780	-11542
6-7	0-4739	-12026	-11817	-11608	-11399	-11191	-10982	-10774	-10566	-10358	-10150
6-8	0-4551	-10457	-10276	-10095	-09914	-09734	-09553	-09373	-09193	-09013	-08833
6-9	0-4376	-08998	-08843	-08688	-08533	-08378	-08224	-08069	-07914	-07760	-07606
7-0	0-4214	-07663	-07531	-07400	-07268	-07137	-07005	-06874	-06743	-06611	-06480

TABLE II (cont.)

Percentage natural mortality, C

Y	Q/Z	41	42	43	44	45	46	47	48	49	50
7-1	0-4062	-06457	-06347	-06236	-06126	-06015	-05905	-05794	-05684	-05574	-05463
7-2	0-3919	-05384	-05292	-05201	-05109	-05017	-04925	-04833	-04741	-04649	-04557
7-3	0-3786	-04443	-04367	-04291	-04215	-04140	-04064	-03988	-03913	-03837	-03761
7-4	0-3661	-03267	-03565	-03504	-03442	-03380	-03318	-03257	-03195	-03133	-03072
7-5	0-3543	-02930	-02880	-02830	-02780	-02731	-02681	-02631	-02581	-02531	-02482
7-6	0-3432	-02342	-02302	-02262	-02222	-02183	-02143	-02103	-02063	-02023	-01984
7-7	0-3327	-01852	-01820	-01789	-01757	-01726	-01695	-01663	-01632	-01600	-01569
7-8	0-3228	-01449	-01424	-01400	-01375	-01351	-01326	-01301	-01277	-01252	-01228
7-9	0-3134	-01122	-01103	-01084	-01065	-01046	-01026	-01007	-00988	-00969	-00950
8-0	0-3046	-00859	-00845	-00830	-00815	-00801	-00786	-00772	-00757	-00743	-00728
8-1	0-2962	-00651	-00640	-00629	-00618	-00607	-00596	-00585	-00574	-00563	-00552
8-2	0-2882	-00488	-00480	-00472	-00463	-00455	-00447	-00439	-00430	-00422	-00414
8-3	0-2806	-00362	-00356	-00350	-00344	-00338	-00332	-00325	-00319	-00313	-00307
8-4	0-2734	-00266	-00261	-00257	-00252	-00248	-00243	-00239	-00234	-00230	-00225
8-5	0-2666	-00193	-00190	-00187	-00183	-00180	-00177	-00174	-00170	-00167	-00164
8-6	0-2600	-00139	-00136	-00134	-00132	-00129	-00127	-00125	-00122	-00120	-00118
8-7	0-2538	-00099	-00097	-00095	-00094	-00092	-00090	-00089	-00087	-00085	-00084
8-8	0-2478	-00070	-00068	-00067	-00066	-00065	-00064	-00062	-00061	-00060	-00059
8-9	0-2421	-00048	-00048	-00047	-00046	-00045	-00044	-00044	-00043	-00042	-00041
9-0	0-2367	-00033	-00033	-00032	-00032	-00031	-00031	-00030	-00029	-00029	-00028

Y	Q/Z	51	52	53	54	55	56	57	58	59	60
1-8	419-1	-00001	-00001	-00001	—	—	—	—	—	—	—
1-9	305-8	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001
2-0	225-3	-00002	-00002	-00002	-00002	-00002	-00002	-00001	-00001	-00001	-00001
2-1	167-69	-00003	-00003	-00003	-00003	-00003	-00003	-00003	-00003	-00002	-00002
2-2	126-02	-00006	-00006	-00006	-00005	-00005	-00005	-00005	-00005	-00004	-00004
2-3	95-63	-00010	-00010	-00010	-00009	-00009	-00009	-00008	-00008	-00008	-00007
2-4	73-28	-00018	-00017	-00016	-00016	-00015	-00015	-00014	-00013	-00013	-00012
2-5	56-70	-00030	-00028	-00027	-00026	-00025	-00024	-00023	-00022	-00021	-00021
2-6	44-288	-00048	-00046	-00045	-00043	-00041	-00039	-00038	-00036	-00035	-00034
2-7	34-923	-00077	-00074	-00071	-00068	-00066	-00063	-00061	-00058	-00056	-00054
2-8	27-797	-00121	-00116	-00112	-00107	-00103	-00099	-00095	-00091	-00088	-00084
2-9	22-330	-00186	-00179	-00172	-00165	-00159	-00153	-00147	-00141	-00135	-00130
3-0	18-101	-00280	-00270	-00259	-00249	-00240	-00230	-00221	-00213	-00204	-00196
3-1	14-802	-00414	-00399	-00383	-00369	-00354	-00341	-00327	-00314	-00302	-00290
3-2	12-211	-00600	-00578	-00556	-00534	-00514	-00494	-00475	-00456	-00438	-00421
3-3	10-159	-00853	-00821	-00790	-00760	-00731	-00703	-00676	-00649	-00624	-00599
3-4	8-521	-01188	-01144	-01101	-01059	-01019	-00981	-00943	-00907	-00871	-00837
3-5	7-205	-01623	-01563	-01505	-01449	-01395	-01342	-01291	-01242	-01194	-01147
3-6	6-1394	-02174	-02095	-02018	-01944	-01872	-01802	-01734	-01668	-01605	-01543
3-7	5-2705	-02858	-02755	-02655	-02559	-02465	-02374	-02286	-02200	-02117	-02036
3-8	4-5571	-03686	-03556	-03429	-03306	-03186	-03070	-02958	-02848	-02742	-02638
3-9	3-9676	-04667	-04504	-04346	-04193	-04044	-03899	-03758	-03620	-03487	-03357
4-0	3-4770	-05802	-05603	-05410	-05222	-05040	-04862	-04689	-04520	-04356	-04196
4-1	3-0665	-07084	-06847	-06615	-06390	-06170	-05956	-05748	-05545	-05346	-05153
4-2	2-7206	-08500	-08221	-07949	-07684	-07425	-07172	-06926	-06685	-06450	-06220
4-3	2-4276	-10027	-09705	-09391	-09084	-08785	-08492	-08206	-07926	-07652	-07384
4-4	2-1780	-11634	-11270	-10914	-10565	-10224	-09890	-09563	-09244	-08930	-08623
4-5	1-9640	-13285	-12879	-12481	-12092	-11710	-11336	-10970	-10610	-10258	-09912

TABLE II (cont.)

Percentage natural mortality, *C*

<i>Y</i>	<i>Q/Z</i>	51	52	53	54	55	56	57	58	59	60
4.6	1.7797	·14936	·14491	·14055	·13627	·13207	·12794	·12390	·11992	·11602	·11218
4.7	1.6202	·16544	·16061	·15592	·15129	·14673	·14225	·13785	·13352	·12926	·12507
4.8	1.4814	·18062	·17551	·17049	·16554	·16067	·15588	·15116	·14652	·14194	·13744
4.9	1.3599	·19447	·18911	·18383	·17863	·17350	·16844	·16346	·15854	·15369	·14891
5.0	1.2533	·20658	·20104	·19556	·19016	·18482	·17956	·17436	·16923	·16416	·15915
5.1	1.1593	·21663	·21096	·20535	·19981	·19433	·18891	·18356	·17827	·17304	·16787
5.2	1.0759	·22434	·21860	·21292	·20730	·20175	·19624	·19080	·18541	·18007	·17479
5.3	1.0018	·22951	·22377	·21809	·21246	·20688	·20135	·19588	·19045	·18508	·17975
5.4	0.9357	·23204	·22636	·22074	·21516	·20962	·20413	·19868	·19328	·18792	·18261
5.5	0.8764	·23101	·22636	·22084	·21536	·20993	·20453	·19917	·19385	·18856	·18332
5.6	0.8230	·22919	·22380	·21845	·21313	·20785	·20259	·19737	·19218	·18703	·18191
5.7	0.7749	·22401	·21884	·21369	·20858	·20349	·19843	·19340	·18839	·18341	·17846
5.8	0.7313	·21658	·21166	·20677	·20189	·19704	·19221	·18741	·18262	·17786	·17312
5.9	0.6917	·20717	·20253	·19791	·19331	·18873	·18417	·17962	·17509	·17059	·16609
6.0	0.6557	·19607	·19174	·18742	·18312	·17884	·17456	·17030	·16606	·16183	·15762
6.1	0.6227	·18362	·17961	·17562	·17163	·16766	·16370	·15974	·15580	·15188	·14796
6.2	0.5926	·17017	·16649	·16282	·15917	·15552	·15188	·14825	·14462	·14101	·13740
6.3	0.5649	·15606	·15272	·14938	·14606	·14274	·13943	·13612	·13282	·12953	·12624
6.4	0.5394	·14163	·13862	·13562	·13262	·12963	·12665	·12366	·12069	·11771	·11475
6.5	0.5158	·12720	·12452	·12184	·11916	·11640	·11383	·11116	·10850	·10585	·10319
6.6	0.4940	·11305	·11068	·10831	·10595	·10359	·10123	·9887	·9652	·9417	·9182
6.7	0.4739	·9942	·9735	·9528	·9321	·9114	·8908	·8701	·8495	·8289	·8083
6.8	0.4551	·8653	·8473	·8294	·8114	·7935	·7756	·7577	·7398	·7219	·7040
6.9	0.4376	·7451	·7297	·7143	·6989	·6835	·6681	·6528	·6374	·6220	·6067
7.0	0.4214	·6340	·6218	·6087	·5956	·5826	·5695	·5564	·5434	·5303	·5172

7-1	0-4062	-05353	-05243	-05133	-05023	-04913	-04803	-04693	-04583	-04473	-04363
7-2	0-3919	-04466	-04374	-04282	-04190	-04099	-04007	-03916	-03824	-03732	-03641
7-3	0-3786	-03686	-03610	-03535	-03459	-03383	-03308	-03232	-03157	-03081	-03006
7-4	0-3661	-03010	-02948	-02887	-02825	-02763	-02702	-02640	-02578	-02517	-02455
7-5	0-3543	-02432	-02382	-02332	-02283	-02233	-02183	-02133	-02084	-02034	-01984
7-6	0-3432	-01944	-01904	-01864	-01825	-01785	-01745	-01705	-01666	-01626	-01586
7-7	0-3327	-01537	-01506	-01475	-01443	-01412	-01380	-01349	-01317	-01286	-01255
7-8	0-3228	-01203	-01179	-01154	-01129	-01105	-01080	-01056	-01031	-01006	-00982
7-9	0-3134	-00931	-00912	-00893	-00874	-00855	-00836	-00817	-00798	-00779	-00760
8-0	0-3046	-00713	-00699	-00684	-00670	-00655	-00641	-00626	-00611	-00597	-00582
8-1	0-2962	-00541	-00530	-00519	-00508	-00497	-00485	-00474	-00463	-00452	-00441
8-2	0-2882	-00405	-00397	-00389	-00381	-00372	-00364	-00356	-00348	-00339	-00331
8-3	0-2806	-00301	-00295	-00289	-00282	-00276	-00270	-00264	-00258	-00252	-00246
8-4	0-2734	-00221	-00216	-00212	-00207	-00203	-00198	-00194	-00189	-00185	-00180
8-5	0-2666	-00160	-00157	-00154	-00151	-00147	-00144	-00141	-00138	-00134	-00131
8-6	0-2600	-00115	-00113	-00111	-00108	-00106	-00104	-00101	-00099	-00096	-00094
8-7	0-2538	-00082	-00080	-00079	-00077	-00075	-00074	-00072	-00070	-00069	-00067
8-8	0-2478	-00058	-00057	-00055	-00054	-00053	-00052	-00051	-00050	-00048	-00047
8-9	0-2421	-00040	-00039	-00039	-00038	-00037	-00036	-00035	-00034	-00034	-00033
9-0	0-2367	-00028	-00027	-00027	-00026	-00025	-00025	-00024	-00024	-00023	-00023

TABLE II (cont.)

Percentage natural mortality, *C*

<i>Y</i>	<i>Q/Z</i>	61	62	63	64	65	66	67	68	69	70
1-0	305.8	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	—	—
2-0	225.3	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001
2-1	167.69	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002	.00002
2-2	126.02	.00004	.00004	.00004	.00004	.00003	.00003	.00003	.00003	.00003	.00003
2-3	95.63	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005	.00005
2-4	73.28	.00012	.00011	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008
2-5	56.70	.00020	.00019	.00018	.00017	.00017	.00016	.00015	.00015	.00014	.00013
2-6	44.288	.00032	.00031	.00030	.00028	.00027	.00026	.00025	.00024	.00023	.00022
2-7	34.923	.00052	.00049	.00047	.00045	.00043	.00042	.00040	.00038	.00036	.00035
2-8	27.797	.00081	.00078	.00074	.00071	.00068	.00065	.00062	.00060	.00057	.00054
2-9	22.330	.00125	.00119	.00114	.00110	.00105	.00101	.00096	.00092	.00088	.00084
3-0	18.101	.00188	.00180	.00173	.00166	.00159	.00152	.00145	.00139	.00133	.00127
3-1	14.802	.00278	.00267	.00256	.00245	.00235	.00225	.00215	.00206	.00197	.00188
3-2	12.211	.00404	.00388	.00372	.00356	.00342	.00327	.00313	.00299	.00286	.00273
3-3	10.159	.00575	.00552	.00530	.00508	.00487	.00466	.00446	.00427	.00408	.00389
3-4	8.521	.00804	.00772	.00741	.00710	.00681	.00652	.00624	.00597	.00571	.00545
3-5	7.205	.01102	.01058	.01016	.00975	.00934	.00895	.00857	.00820	.00784	.00749
3-6	6.1394	.01483	.01424	.01367	.01312	.01258	.01206	.01155	.01106	.01057	.01010
3-7	5.2705	.01958	.01881	.01807	.01735	.01664	.01595	.01529	.01463	.01400	.01338
3-8	4.5571	.02538	.02440	.02344	.02251	.02161	.02072	.01986	.01902	.01820	.01740
3-9	3.9676	.03230	.03107	.02987	.02870	.02755	.02644	.02535	.02429	.02325	.02224
4-0	3.4770	.04039	.03887	.03739	.03594	.03452	.03314	.03179	.03047	.02919	.02793

4-1	3-0665	·04964	·04779	·04599	·04423	·04251	·04083	·03919	·03758	·03601	·03447
4-2	2-7206	·05996	·05776	·05562	·05352	·05146	·04946	·04749	·04557	·04368	·04184
4-3	2-4276	·07122	·06865	·06614	·06368	·06128	·05892	·05661	·05434	·05212	·04995
4-4	2-1780	·08323	·08028	·07739	·07456	·07178	·06906	·06639	·06377	·06120	·05867
4-5	1-9640	·09572	·09240	·08913	·08592	·08277	·07968	·07664	·07366	·07073	·06785
4-6	1-7797	·10841	·10471	·10107	·09750	·09398	·09053	·08713	·08379	·08050	·07727
4-7	1-6202	·12095	·11690	·11291	·10899	·10513	·10132	·09758	·09389	·09026	·08669
4-8	1-4814	·13300	·12863	·12432	·12007	·11589	·11177	·10770	·10370	·09975	·09585
4-9	1-3599	·14420	·13954	·13496	·13043	·12596	·12155	·11720	·11291	·10867	·10449
5-0	1-2533	·15421	·14933	·14451	·13975	·13504	·13039	·12580	·12126	·11678	·11234
5-1	1-1593	·16275	·15769	·15269	·14775	·14285	·13802	·13323	·12850	·12381	·11918
5-2	1-0759	·16957	·16439	·15927	·15419	·14917	·14420	·13927	·13440	·12957	·12478
5-3	1-0018	·17447	·16923	·16405	·15891	·15381	·14876	·14375	·13879	·13387	·12899
5-4	0-9357	·17733	·17210	·16691	·16176	·15665	·15158	·14655	·14156	·13661	·13169
5-5	0-8764	·17811	·17293	·16780	·16270	·15763	·15260	·14760	·14264	·13771	·13281
5-6	0-8230	·17681	·17175	·16672	·16172	·15675	·15181	·14690	·14202	·13717	·13235
5-7	0-7749	·17353	·16863	·16376	·15891	·15409	·14929	·14452	·13977	·13505	·13035
5-8	0-7313	·16840	·16371	·15903	·15438	·14975	·14514	·14055	·13598	·13143	·12690
5-9	0-6917	·16162	·15716	·15273	·14831	·14390	·13952	·13515	·13080	·12646	·12214
6-0	0-6557	·15342	·14923	·14506	·14090	·13676	·13263	·12851	·12441	·12032	·11624
6-1	0-6227	·14405	·14016	·13628	·13240	·12854	·12469	·12085	·11702	·11321	·10940
6-2	0-5926	·13381	·13022	·12664	·12307	·11951	·11595	·11241	·10887	·10534	·10182
6-3	0-5649	·12296	·11969	·11642	·11316	·10991	·10666	·10342	·10018	·09696	·09374
6-4	0-5394	·11179	·10883	·10588	·10293	·09999	·09705	·09412	·09119	·08827	·08535
6-5	0-5158	·10055	·09790	·09526	·09262	·08999	·08736	·08473	·08211	·07948	·07687
6-6	0-4940	·08947	·08713	·08479	·08245	·08012	·07779	·07546	·07313	·07080	·06848
6-7	0-4739	·07878	·07672	·07467	·07262	·07057	·06852	·06648	·06443	·06239	·06035
6-8	0-4551	·06862	·06683	·06505	·06327	·06149	·05971	·05793	·05616	·05438	·05261
6-9	0-4376	·05913	·05760	·05607	·05454	·05301	·05148	·04995	·04842	·04690	·04537
7-0	0-4214	·05042	·04912	·04781	·04651	·04521	·04390	·04260	·04130	·04000	·03870

TABLE II (cont.)

Percentage natural mortality, *C*

<i>Y</i>	<i>Q/Z</i>	61	62	63	64	65	66	67	68	69	70
7-1	0-4062	·04253	·04143	·04033	·03924	·03814	·03704	·03595	·03485	·03376	·03266
7-2	0-3919	·03849	·03458	·03366	·03275	·03183	·03092	·03001	·02909	·02818	·02727
7-3	0-3786	·02930	·02855	·02780	·02704	·02629	·02553	·02478	·02403	·02327	·02252
7-4	0-3661	·02304	·02332	·02271	·02209	·02147	·02086	·02024	·01963	·01901	·01840
7-5	0-3543	·01934	·01885	·01835	·01785	·01736	·01686	·01636	·01586	·01537	·01487
7-6	0-3432	·01547	·01507	·01467	·01427	·01388	·01348	·01308	·01269	·01229	·01189
7-7	0-3327	·01223	·01192	·01160	·01129	·01098	·01066	·01035	·01003	·00972	·00941
7-8	0-3228	·00957	·00933	·00908	·00884	·00859	·00834	·00810	·00785	·00761	·00736
7-9	0-3134	·00741	·00722	·00703	·00684	·00665	·00646	·00627	·00608	·00589	·00570
8-0	0-3046	·00568	·00553	·00539	·00524	·00509	·00495	·00480	·00466	·00451	·00437
8-1	0-2962	·00430	·00419	·00408	·00397	·00386	·00375	·00364	·00353	·00342	·00331
8-2	0-2882	·00323	·00314	·00306	·00298	·00290	·00281	·00273	·00265	·00256	·00248
8-3	0-2806	·00239	·00233	·00227	·00221	·00215	·00209	·00203	·00196	·00190	·00184
8-4	0-2734	·00176	·00171	·00167	·00162	·00158	·00153	·00149	·00144	·00140	·00135
8-5	0-2666	·00128	·00124	·00121	·00118	·00115	·00111	·00108	·00105	·00102	·00098
8-6	0-2600	·00092	·00089	·00087	·00085	·00082	·00080	·00078	·00075	·00073	·00071
8-7	0-2538	·00065	·00064	·00062	·00060	·00059	·00057	·00055	·00054	·00052	·00050
8-8	0-2478	·00046	·00045	·00044	·00042	·00041	·00040	·00039	·00038	·00037	·00035
8-9	0-2421	·00032	·00031	·00030	·00030	·00029	·00028	·00027	·00026	·00025	·00025
9-0	0-2367	·00022	·00021	·00021	·00020	·00020	·00019	·00019	·00018	·00018	·00017

	Y	Q/Z	71	72	73	74	75	76	77	78	79	80
P P A	2-0	225-3	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-
	2-1	167-69	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001
	2-2	126-02	-00003	-00002	-00002	-00002	-00002	-00002	-00002	-00002	-00002	-00002
	2-3	95-63	-00004	-00004	-00004	-00004	-00004	-00003	-00003	-00003	-00003	-00003
	2-4	73-28	-00008	-00007	-00007	-00007	-00006	-00006	-00006	-00005	-00005	-00005
	2-5	56-70	-00013	-00012	-00011	-00011	-00010	-00010	-00009	-00009	-00008	-00008
	2-6	44-288	-00021	-00020	-00019	-00018	-00017	-00016	-00015	-00014	-00013	-00013
	2-7	34-923	-00033	-00031	-00030	-00028	-00027	-00026	-00024	-00023	-00022	-00020
	2-8	27-797	-00052	-00049	-00047	-00045	-00042	-00040	-00038	-00036	-00034	-00032
	2-9	22-330	-00080	-00076	-00072	-00069	-00065	-00062	-00059	-00055	-00052	-00049
3-0	18-101	-00121	-00115	-00109	-00104	-00099	-00094	-00088	-00084	-00079	-00074	
3-1	14-802	-00179	-00170	-00162	-00154	-00146	-00139	-00131	-00124	-00117	-00110	
3-2	12-211	-00260	-00248	-00236	-00224	-00213	-00202	-00191	-00181	-00170	-00160	
3-3	10-159	-00371	-00354	-00337	-00320	-00304	-00288	-00273	-00258	-00243	-00229	
3-4	8-521	-00520	-00496	-00472	-00449	-00426	-00404	-00383	-00362	-00341	-00321	
3-5	7-205	-00715	-00681	-00649	-00617	-00586	-00556	-00526	-00498	-00469	-00442	
3-6	6-1394	-00964	-00920	-00876	-00833	-00792	-00751	-00711	-00673	-00635	-00598	
3-7	5-2705	-01278	-01219	-01161	-01105	-01050	-00996	-00944	-00893	-00843	-00794	
3-8	4-5571	-01662	-01586	-01512	-01439	-01368	-01298	-01231	-01164	-01099	-01035	
3-9	3-9676	-02125	-02028	-01934	-01841	-01751	-01663	-01576	-01492	-01409	-01328	
4-0	3-4770	-02669	-02549	-02431	-02316	-02203	-02093	-01985	-01879	-01775	-01673	
4-1	3-0665	-03296	-03149	-03005	-02864	-02725	-02590	-02457	-02327	-02199	-02074	
4-2	2-7206	-04003	-03826	-03652	-03482	-03315	-03152	-02991	-02834	-02680	-02528	
4-3	2-4276	-04781	-04572	-04367	-04165	-03968	-03774	-03583	-03396	-03213	-03032	
4-4	2-1780	-05620	-05377	-05138	-04903	-04673	-04446	-04224	-04006	-03791	-03580	
4-5	1-9640	-06502	-06224	-05951	-05682	-05418	-05158	-04903	-04651	-04404	-04161	

TABLE II (cont.)

Y	Q/Z	Percentage natural mortality, <i>C</i>									
		71	72	73	74	75	76	77	78	79	80
4-6	1-7797	·07409	·07096	·06788	·06485	·06187	·05893	·05604	·05319	·05039	·04763
4-7	1-8202	·08317	·07970	·07629	·07292	·06960	·06633	·06311	·05994	·05681	·05372
4-8	1-4814	·09201	·08822	·08449	·08081	·07717	·07359	·07005	·06656	·06311	·05971
4-9	1-3599	·10036	·09628	·09226	·08828	·08436	·08048	·07665	·07287	·06913	·06544
5-0	1-2583	·10796	·10364	·09936	·09513	·09095	·08681	·08272	·07868	·07469	·07074
5-1	1-1593	·11459	·11006	·10557	·10113	·09673	·09238	·08808	·08382	·07960	·07543
5-2	1-0759	·12005	·11535	·11071	·10610	·10154	·09702	·09255	·08811	·08372	·07937
5-3	1-0018	·12416	·11936	·11461	·10990	·10522	·10059	·09599	·09144	·08692	·08244
5-4	0-9357	·12681	·12197	·11717	·11240	·10767	·10298	·09832	·09369	·08910	·08454
5-5	0-8764	·12795	·12312	·11833	·11356	·10883	·10413	·09946	·09482	·09021	·08563
5-6	0-8230	·12756	·12279	·11806	·11335	·10867	·10402	·09939	·09479	·09022	·08567
5-7	0-7749	·12568	·12103	·11640	·11180	·10723	·10267	·09814	·09364	·08915	·08469
5-8	0-7313	·12240	·11791	·11344	·10900	·10457	·10016	·09578	·09141	·08706	·08273
5-9	0-6917	·11784	·11356	·10929	·10504	·10080	·09659	·09238	·08820	·08403	·07987
6-0	0-6557	·11218	·10813	·10410	·10008	·09607	·09208	·08809	·08412	·08017	·07623
6-1	0-6227	·10560	·10182	·09804	·09428	·09053	·08678	·08305	·07933	·07562	·07192
6-2	0-5926	·09831	·09481	·09132	·08783	·08435	·08088	·07742	·07397	·07052	·06708
6-3	0-5649	·09052	·08731	·08411	·08092	·07773	·07454	·07137	·06820	·06503	·06187
6-4	0-5394	·08244	·07953	·07662	·07372	·07083	·06794	·06506	·06218	·05930	·05643
6-5	0-5158	·07426	·07165	·06904	·06644	·06384	·06124	·05865	·05606	·05348	·05090
6-6	0-4940	·06616	·06384	·06153	·05922	·05691	·05460	·05230	·05000	·04770	·04540
6-7	0-4739	·05831	·05628	·05424	·05221	·05018	·04815	·04612	·04410	·04207	·04005
6-8	0-4551	·05084	·04907	·04730	·04553	·04376	·04200	·04023	·03847	·03671	·03495
6-9	0-4376	·04384	·04232	·04080	·03927	·03775	·03623	·03471	·03319	·03168	·03016
7-0	0-4214	·03741	·03611	·03481	·03351	·03222	·03092	·02963	·02833	·02704	·02574

7-1	0-4062	·03157	·03047	·02938	·02829	·02719	·02610	·02501	·02392	·02283	·02174
7-2	0-3919	·02636	·02544	·02453	·02362	·02271	·02180	·02089	·01997	·01906	·01815
7-3	0-3786	·02177	·02101	·02026	·01951	·01876	·01800	·01725	·01650	·01575	·01500
7-4	0-3661	·01778	·01717	·01655	·01594	·01533	·01471	·01410	·01348	·01287	·01226
7-5	0-3543	·01437	·01388	·01338	·01289	·01239	·01189	·01140	·01090	·01040	·00991
7-6	0-3432	·01149	·01110	·01070	·01030	·00991	·00951	·00911	·00872	·00832	·00792
7-7	0-3327	·00909	·00878	·00846	·00815	·00784	·00752	·00721	·00690	·00658	·00627
7-8	0-3228	·00712	·00687	·00663	·00638	·00613	·00589	·00564	·00540	·00515	·00491
7-9	0-3134	·00551	·00532	·00513	·00494	·00475	·00456	·00437	·00418	·00399	·00380
8-0	0-3046	·00422	·00408	·00393	·00378	·00364	·00349	·00335	·00320	·00306	·00291
8-1	0-2962	·00320	·00309	·00298	·00287	·00276	·00265	·00254	·00243	·00232	·00221
8-2	0-2882	·00240	·00232	·00223	·00215	·00207	·00199	·00190	·00182	·00174	·00165
8-3	0-2806	·00178	·00172	·00166	·00160	·00153	·00147	·00141	·00135	·00129	·00123
8-4	0-2734	·00131	·00126	·00122	·00117	·00113	·00108	·00104	·00099	·00095	·00090
8-5	0-2666	·00095	·00092	·00088	·00085	·00082	·00079	·00075	·00072	·00069	·00065
8-6	0-2600	·00068	·00066	·00064	·00061	·00059	·00056	·00054	·00052	·00049	·00047
8-7	0-2538	·00048	·00047	·00045	·00043	·00042	·00040	·00038	·00037	·00035	·00033
8-8	0-2478	·00034	·00033	·00032	·00031	·00029	·00028	·00027	·00026	·00025	·00024
8-9	0-2421	·00024	·00023	·00022	·00021	·00021	·00020	·00019	·00018	·00017	·00016
9-0	0-2367	·00016	·00016	·00015	·00015	·00014	·00014	·00013	·00012	·00012	·00011

TABLE II (cont.)

Percentage natural mortality, *C*

Y	Q/Z	81	82	83	84	85	86	87	88	89	90
2-1	167-69	-00001	-00001	-00001	-00001	-00001	-00001	-00001	—	—	—
2-2	126-02	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001	-00001
2-3	95-63	-00003	-00002	-00002	-00002	-00002	-00002	-00002	-00001	-00001	-00001
2-4	73-28	-00004	-00004	-00004	-00004	-00003	-00003	-00003	-00003	-00002	-00002
2-5	56-70	-00007	-00007	-00006	-00006	-00005	-00005	-00005	-00004	-00004	-00003
2-6	44-288	-00012	-00011	-00010	-00010	-00009	-00008	-00008	-00007	-00006	-00006
2-7	34-923	-00019	-00018	-00017	-00015	-00014	-00013	-00012	-00011	-00010	-00009
2-8	27-797	-00030	-00028	-00026	-00024	-00022	-00021	-00019	-00017	-00016	-00014
2-9	22-330	-00046	-00043	-00040	-00037	-00035	-00032	-00029	-00027	-00024	-00022
3-0	18-101	-00070	-00065	-00061	-00057	-00052	-00048	-00044	-00041	-00037	-00033
3-1	14-802	-00103	-00097	-00090	-00084	-00078	-00072	-00066	-00060	-00055	-00049
3-2	12-211	-00150	-00141	-00131	-00122	-00113	-00105	-00096	-00088	-00080	-00072
3-3	10-159	-00215	-00201	-00188	-00175	-00162	-00150	-00137	-00125	-00114	-00102
3-4	8-521	-00301	-00282	-00264	-00245	-00228	-00210	-00193	-00176	-00160	-00144
3-5	7-205	-00415	-00389	-00363	-00338	-00314	-00289	-00266	-00243	-00220	-00198
3-6	6-1394	-00561	-00526	-00491	-00457	-00424	-00392	-00360	-00329	-00298	-00269
3-7	5-2705	-00746	-00699	-00653	-00608	-00564	-00521	-00479	-00438	-00397	-00357
3-8	4-5371	-00973	-00912	-00853	-00794	-00737	-00681	-00626	-00572	-00519	-00467
3-9	3-9676	-01248	-01170	-01094	-01020	-00946	-00875	-00804	-00735	-00667	-00601
4-0	3-4770	-01574	-01476	-01380	-01287	-01195	-01104	-01016	-00929	-00844	-00760
4-1	3-0665	-01951	-01831	-01713	-01597	-01483	-01371	-01262	-01154	-01049	-00945
4-2	2-7206	-02379	-02233	-02090	-01950	-01811	-01676	-01542	-01411	-01282	-01156
4-3	2-4276	-02855	-02681	-02510	-02342	-02177	-02015	-01855	-01698	-01544	-01392
4-4	2-1780	-03372	-03168	-02967	-02770	-02575	-02384	-02196	-02011	-01829	-01650
4-5	1-9640	-03921	-03685	-03453	-03225	-03000	-02779	-02561	-02346	-02134	-01926

4-6	1-7797	-04491	-04223	-03959	-03699	-03442	-03190	-02941	-02695	-02453	-02214
4-7	1-6202	-05068	-04767	-04472	-04180	-03892	-03608	-03328	-03051	-02778	-02509
4-8	1-4814	-05636	-05305	-04978	-04655	-04337	-04022	-03711	-03404	-03101	-02802
4-9	1-3599	-06180	-05820	-05464	-05112	-04764	-04421	-04081	-03745	-03414	-03085
5-0	1-2533	-06683	-06296	-05914	-05536	-05162	-04792	-04426	-04064	-03705	-03351
5-1	1-1593	-07129	-06720	-06315	-05914	-05517	-05124	-04735	-04349	-03967	-03589
5-2	1-0759	-07505	-07078	-06654	-06235	-05819	-05407	-04998	-04593	-04192	-03794
5-3	1-0018	-07799	-07358	-06921	-06488	-06057	-05631	-05208	-04788	-04371	-03958
5-4	0-9357	-08002	-07553	-07107	-06665	-06226	-05790	-05357	-04927	-04500	-04076
5-5	0-8764	-08108	-07656	-07208	-06762	-06318	-05878	-05441	-05006	-04574	-04145
5-6	0-8230	-08116	-07666	-07219	-06775	-06334	-05895	-05458	-05024	-04592	-04163
5-7	0-7749	-08025	-07584	-07144	-06707	-06272	-05839	-05409	-04980	-04554	-04130
5-8	0-7313	-07842	-07413	-06986	-06560	-06137	-05715	-05295	-04877	-04461	-04047
5-9	0-6917	-07573	-07161	-06751	-06341	-05934	-05528	-05123	-04720	-04319	-03919
6-0	0-6557	-07230	-06838	-06448	-06058	-05671	-05284	-04899	-04514	-04132	-03750
6-1	0-6227	-06823	-06454	-06087	-05721	-05356	-04992	-04629	-04267	-03906	-03546
6-2	0-5926	-06365	-06023	-05682	-05342	-05002	-04663	-04325	-03987	-03651	-03315
6-3	0-5649	-05872	-05558	-05244	-04930	-04618	-04306	-03994	-03683	-03373	-03063
6-4	0-5394	-05357	-05071	-04785	-04500	-04215	-03931	-03647	-03364	-03081	-02799
6-5	0-5158	-04832	-04575	-04318	-04061	-03805	-03548	-03293	-03037	-02782	-02528
6-6	0-4940	-04311	-04082	-03853	-03624	-03396	-03168	-02940	-02712	-02485	-02258
6-7	0-4739	-03803	-03601	-03400	-03198	-02997	-02796	-02595	-02395	-02194	-01994
6-8	0-4551	-03319	-03143	-02967	-02792	-02616	-02441	-02266	-02091	-01916	-01741
6-9	0-4376	-02864	-02713	-02561	-02410	-02259	-02107	-01956	-01805	-01654	-01504
7-0	0-4214	-02445	-02316	-02187	-02058	-01929	-01800	-01671	-01542	-01413	-01284
7-1	0-4062	-02065	-01956	-01847	-01738	-01629	-01520	-01411	-01302	-01194	-01085
7-2	0-3919	-01724	-01633	-01542	-01451	-01361	-01270	-01179	-01088	-00997	-00906
7-3	0-3786	-01425	-01349	-01274	-01199	-01124	-01049	-00974	-00899	-00824	-00749
7-4	0-3661	-01164	-01103	-01041	-00980	-00919	-00857	-00796	-00735	-00674	-00612
7-5	0-3543	-00941	-00892	-00842	-00792	-00743	-00693	-00644	-00594	-00545	-00495

TABLE II (cont.)

Percentage natural mortality, *C*

<i>Y</i>	<i>Q/Z</i>	81	82	83	84	85	86	87	88	89	90
7·6	0·3432	·00753	·00713	·00673	·00634	·00594	·00555	·00515	·00475	·00436	·00396
7·7	0·3327	·00596	·00564	·00533	·00501	·00470	·00439	·00407	·00376	·00345	·00313
7·8	0·3228	·00466	·00442	·00417	·00393	·00368	·00343	·00319	·00294	·00270	·00245
7·9	0·3134	·00361	·00342	·00323	·00304	·00285	·00266	·00247	·00228	·00209	·00190
8·0	0·3046	·00277	·00262	·00247	·00233	·00218	·00204	·00189	·00174	·00160	·00146
8·1	0·2962	·00210	·00199	·00188	·00176	·00165	·00154	·00143	·00132	·00121	·00110
8·2	0·2882	·00157	·00149	·00141	·00132	·00124	·00116	·00108	·00099	·00091	·00083
8·3	0·2806	·00117	·00110	·00104	·00098	·00092	·00086	·00080	·00074	·00068	·00061
8·4	0·2734	·00086	·00081	·00077	·00072	·00068	·00063	·00059	·00054	·00050	·00045
8·5	0·2666	·00062	·00059	·00056	·00052	·00049	·00046	·00043	·00039	·00036	·00033
8·6	0·2600	·00045	·00042	·00040	·00038	·00035	·00033	·00031	·00028	·00026	·00024
8·7	0·2538	·00032	·00030	·00028	·00027	·00025	·00023	·00022	·00020	·00018	·00017
8·8	0·2478	·00022	·00021	·00020	·00019	·00018	·00017	·00015	·00014	·00013	·00012
8·9	0·2421	·00016	·00015	·00014	·00013	·00012	·00011	·00011	·00010	·00009	·00008
9·0	0·2367	·00011	·00010	·00010	·00009	·00008	·00008	·00007	·00007	·00006	·00006

TABLE III. Maximum and Minimum Working Probits and Range

Minimum working probits		Range $1/Z$	Maximum working probits	
Expected probit Y	$y_0 = Y - P/Z$		$y_{100} = Y + Q/Z$	Expected probit Y
1-1	0-8579	5034	9-1421	8-9
1-2	0-9522	3425	9-0478	8-8
1-3	1-0462	2354	8-9538	8-7
1-4	1-1400	1634	8-8600	8-6
1-5	1-2334	1146	8-7666	8-5
1-6	1-3266	811-5	8-6734	8-4
1-7	1-4194	580-5	8-5806	8-3
1-8	1-5118	419-4	8-4882	8-2
1-9	1-6038	308-1	8-3962	8-1
2-0	1-6954	225-6	8-3046	8-0
2-1	1-7866	168-00	8-2134	7-9
2-2	1-8772	126-34	8-1228	7-8
2-3	1-9673	95-96	8-0327	7-7
2-4	2-0568	73-62	7-9432	7-6
2-5	2-1457	57-05	7-8543	7-5
2-6	2-2339	44-654	7-7661	7-4
2-7	2-3214	35-302	7-6786	7-3
2-8	2-4081	28-189	7-5919	7-2
2-9	2-4938	22-736	7-5062	7-1
3-0	2-5786	18-5216	7-4214	7-0
3-1	2-6624	15-2402	7-3376	6-9
3-2	2-7449	12-6662	7-2551	6-8
3-3	2-8261	10-6327	7-1739	6-7
3-4	2-9060	9-0154	7-0940	6-6
3-5	2-9842	7-7210	7-0158	6-5
3-6	3-0606	6-6788	6-9394	6-4
3-7	3-1351	5-8354	6-8649	6-3
3-8	3-2074	5-1497	6-7928	6-2
3-9	3-2773	4-5903	6-7227	6-1
4-0	3-3443	4-1327	6-6557	6-0
4-1	3-4083	3-7582	6-5917	5-9
4-2	3-4687	3-4519	6-5313	5-8
4-3	3-5251	3-2025	6-4749	5-7
4-4	3-5770	3-0010	6-4230	5-6
4-5	3-6236	2-8404	6-3764	5-5
4-6	3-6643	2-7154	6-3357	5-4
4-7	3-6982	2-6220	6-3018	5-3
4-8	3-7241	2-5573	6-2759	5-2
4-9	3-7407	2-5192	6-2593	5-1
5-0	3-7467	2-5066	6-2533	5-0
5-1	3-7401	2-5192	6-2599	4-9
5-2	3-7186	2-5573	6-2814	4-8
5-3	3-6798	2-6220	6-3202	4-7
5-4	3-6203	2-7154	6-3797	4-6
5-5	3-5360	2-8404	6-4640	4-5
5-6	3-4220	3-0010	6-5780	4-4
5-7	3-2724	3-2025	6-7276	4-3
5-8	3-0794	3-4519	6-9206	4-2
5-9	2-8335	3-7582	7-1665	4-1
6-0	2-5230	4-1327	7-4770	4-0
6-1	2-1324	4-5903	7-8676	3-9
6-2	1-6429	5-1497	8-3571	3-8
6-3	1-0295	5-8354	8-9705	3-7
6-4	0-2806	6-6788	9-7394	3-6
6-5	-0-7052	7-7210	10-7052	3-5

The working probit, y , may be obtained as $y = (Y - P/Z) + p/Z$ or $y = (Y + Q/Z) - q/Z$, whichever is the more convenient, where $p = 1 - q$ is the observed proportion killed.

TABLE IV (cont.)

(Y = 3-0-3-9; 0-50% kill)

% kill	Expected probit, Y									
	3-0	3-1	3-2	3-3	3-4	3-5	3-6	3-7	3-8	3-9
0	2-579	2-682	2-745	2-826	2-906	2-984	3-061	3-135	3-207	3-277
1	2-764	2-815	2-872	2-932	2-996	3-061	3-127	3-193	3-259	3-323
2	-949	-967	-998	3-039	3-086	-139	-194	-252	-310	-369
3	3-134	3-120	3-125	-145	-176	-216	-261	-310	-362	-415
4	-319	-272	-252	-251	-267	-293	-328	-369	-413	-461
5	-505	-424	-378	-358	-357	-370	-395	-427	-465	-507
6	3-690	3-577	3-505	3-464	3-447	3-447	3-461	3-485	3-516	3-553
7	-875	-729	-632	-570	-537	-525	-528	-544	-568	-599
8	4-060	-882	-758	-677	-627	-602	-595	-602	-619	-645
9	-246	4-034	-885	-783	-717	-679	-662	-660	-671	-690
10	-431	-186	4-012	-889	-808	-756	-728	-719	-722	-736
11	4-616	4-339	4-138	3-996	3-898	3-834	3-795	3-777	3-774	3-782
12	-801	-491	-265	4-102	-988	-911	-862	-835	-825	-828
13	-986	-644	-391	-208	4-078	-986	-929	-894	-877	-874
14	5-172	-796	-518	-315	-168	4-065	-996	-952	-928	-920
15	-357	-948	-645	-421	-258	-142	4-062	4-010	-980	-966
16	5-542	5-101	4-771	4-527	4-348	4-220	4-129	4-069	4-031	4-012
17	-727	-253	-898	-634	-439	-297	-196	-127	-083	-058
18	-913	-406	5-025	-740	-529	-374	-263	-185	-134	-104
19	6-098	-558	-151	-846	-619	-451	-330	-244	-186	-149
20	-283	-710	-278	-953	-709	-528	-396	-302	-237	-195
21	6-468	5-863	5-405	5-059	4-799	4-606	4-463	4-361	4-289	4-241
22	-653	6-015	-531	-165	-889	-683	-530	-419	-340	-287
23	-839	-168	-658	-272	-979	-760	-597	-477	-392	-333
24	7-024	-320	-785	-378	5-070	-837	-664	-536	-443	-379
25	-209	-472	-911	-484	160	-914	-730	-594	-495	-425
26	7-394	6-625	6-038	5-591	5-250	4-992	4-797	4-652	4-546	4-471
27	-580	-777	-165	-697	-340	5-069	-864	-711	-598	-517
28	-765	-930	-291	-803	-430	-146	-931	-769	-649	-563
29	-950	7-082	-418	-910	-520	-223	-997	-827	-701	-608
30	8-135	-234	-545	6-016	-610	-300	5-064	-886	-752	-654
31	8-320	7-387	6-671	6-122	5-701	5-378	5-131	4-944	4-804	4-700
32	-506	-539	-798	-229	-791	-455	-198	5-002	-855	-746
33	-691	-692	-925	-335	-881	-532	-265	-061	-907	-792
34	-876	-844	7-051	-441	-971	-609	-331	-119	-958	-838
35	9-061	-996	-178	-548	6-061	-687	-398	-177	5-010	-884
36	9-247	8-149	7-305	6-654	6-151	5-764	5-465	5-236	5-061	4-930
37	-432	-301	-431	-760	-242	-841	-532	-294	-113	-976
38	-617	-454	-558	-867	-332	-918	-599	-353	-164	5-022
39	-802	-606	-685	-973	-422	-995	-665	-411	-216	-068
40	-987	-758	-811	7-079	-512	6-073	-732	-469	-267	-113
41	—	8-911	7-938	7-186	6-802	6-150	5-799	5-528	5-319	5-159
42	—	9-063	8-065	-292	-692	-227	-866	-586	-370	-205
43	—	-216	-191	-398	-782	-304	-932	-644	-422	-251
44	—	-368	-318	-505	-873	-381	-999	-703	-473	-297
45	—	-520	-445	-611	-963	-459	6-066	-761	-525	-343
46	—	9-673	8-571	7-717	7-053	6-536	6-133	5-819	5-576	5-389
47	—	-825	-698	-824	-143	-613	-200	-878	-628	-435
48	—	-978	-825	-930	-233	-690	-266	-936	-679	-481
49	—	—	-951	8-036	-323	-787	-333	-994	-731	-527
50	—	—	9-078	-143	-414	-845	-400	6-053	-782	-572

TABLE IV (cont.)
(Y = 4.0-4.9; 0-50% kill)

% kill	Expected probit, Y									
	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9
0	3.344	3.408	3.469	3.525	3.577	3.624	3.664	3.698	3.724	3.741
1	3.386	3.446	3.503	3.557	3.607	3.652	3.691	3.724	3.750	3.766
2	.427	.487	.538	.589	.637	.680	.719	.751	.775	.791
3	.468	.521	.572	.621	.667	.709	.746	.777	.801	.816
4	.510	.559	.607	.653	.697	.737	.773	.803	.826	.841
5	.551	.596	.641	.685	.727	.766	.800	.829	.852	.867
6	3.592	3.634	3.676	3.717	3.757	3.794	3.827	3.856	3.878	3.892
7	.634	.671	.710	.749	.787	.822	.854	.882	.903	.917
8	.675	.709	.745	.781	.817	.851	.882	.908	.929	.942
9	.716	.747	.779	.813	.847	.879	.909	.934	.954	.967
10	.758	.784	.814	.845	.877	.908	.936	.960	.980	.993
11	3.799	3.822	3.848	3.877	3.907	3.936	3.963	3.987	4.005	4.018
12	.840	.859	.883	.909	.937	.964	.990	4.013	.031	.043
13	.882	.897	.917	.941	.967	.993	4.017	.039	.057	.068
14	.923	.934	.952	.973	.997	4.021	.044	.065	.082	.093
15	.964	.972	.986	4.005	4.027	.050	.072	.092	.108	.119
16	4.006	4.010	4.021	4.038	4.057	4.078	4.099	4.118	4.133	4.144
17	.047	.047	.056	.070	.087	.106	.126	.144	.159	.169
18	.088	.085	.090	.102	.117	.135	.153	.170	.184	.194
19	.130	.122	.125	.134	.147	.163	.180	.196	.210	.219
20	.171	.160	.159	.166	.177	.192	.207	.223	.236	.245
21	4.212	4.198	4.194	4.198	4.207	4.220	4.235	4.249	4.261	4.270
22	.253	.235	.228	.230	.237	.248	.262	.275	.287	.295
23	.295	.273	.263	.262	.267	.277	.289	.301	.312	.320
24	.336	.310	.297	.294	.297	.305	.316	.327	.338	.345
25	.377	.348	.332	.326	.327	.334	.343	.354	.363	.370
26	4.419	4.385	4.366	4.358	4.357	4.362	4.370	4.380	4.389	4.396
27	.460	.423	.401	.390	.387	.391	.397	.406	.415	.421
28	.501	.461	.435	.422	.417	.419	.425	.432	.440	.448
29	.543	.498	.470	.454	.447	.447	.452	.459	.466	.471
30	.584	.536	.504	.486	.477	.476	.479	.485	.491	.496
31	4.625	4.573	4.539	4.518	4.507	4.504	4.506	4.511	4.517	4.522
32	.667	.611	.573	.550	.537	.533	.533	.537	.542	.547
33	.708	.649	.608	.582	.567	.561	.560	.563	.568	.572
34	.749	.686	.642	.614	.597	.589	.588	.590	.594	.597
35	.791	.724	.677	.646	.627	.618	.615	.616	.619	.622
36	4.832	4.761	4.711	4.678	4.657	4.646	4.642	4.642	4.645	4.648
37	.873	.799	.746	.710	.687	.675	.669	.668	.670	.673
38	.915	.836	.780	.742	.717	.703	.696	.695	.696	.698
39	.956	.874	.815	.774	.747	.731	.723	.721	.721	.723
40	.997	.912	.849	.806	.777	.760	.750	.747	.747	.748
41	5.039	4.949	4.884	4.838	4.807	4.788	4.778	4.773	4.773	4.774
42	.080	.987	.918	.870	.837	.817	.805	.799	.798	.799
43	.121	5.024	.953	.902	.867	.845	.832	.826	.824	.824
44	.163	.082	.988	.934	.897	.873	.859	.852	.849	.849
45	.204	.899	5.022	.966	.927	.902	.886	.878	.875	.874
46	5.245	5.137	5.057	4.998	4.957	4.930	4.913	4.904	4.900	4.900
47	.287	.175	.091	5.030	.987	.959	.941	.931	.926	.925
48	.328	.212	.126	.062	5.017	.987	.968	.957	.952	.950
49	.369	.250	.160	.094	.047	5.015	.995	.983	.977	.975
50	.411	.287	.195	.126	.078	.044	5.022	5.009	5.003	5.000

TABLE IV (cont.)

(Y = 5-0-5-9; 0-50% kill)

% kill	Expected probit, Y									
	5-0	5-1	5-2	5-3	5-4	5-5	5-6	5-7	5-8	5-9
0	3-747	3-740	3-719	3-680	3-620	3-536	3-422	3-272	3-079	2-834
1	3-772	3-765	3-744	3-706	3-647	3-564	3-452	3-304	3-114	2-871
2	.797	.790	.770	.732	.675	.593	.482	.336	.148	.909
3	.822	.816	.795	.758	.702	.621	.512	.368	.183	.946
4	.847	.841	.821	.785	.729	.650	.542	.400	.217	.984
5	.872	.866	.846	.811	.756	.678	.572	.433	.252	3-021
6	3-897	3-891	3-872	3-837	3-783	3-706	3-602	3-465	3-287	3-059
7	.922	.916	.898	.863	.810	.735	.632	.497	.321	.097
8	.947	.942	.923	.890	.838	.763	.662	.529	.356	.134
9	.972	.967	.949	.916	.865	.792	.692	.561	.390	.172
10	.997	.992	.974	.942	.892	.820	.722	.593	.425	.209
11	4-022	4-017	4-000	3-988	3-919	3-848	3-752	3-625	3-459	3-247
12	.047	.042	.025	.994	.948	.877	.782	.657	.494	.284
13	.073	.068	.051	4-021	.973	.905	.812	.689	.528	.322
14	.098	.093	.077	.047	4-000	.934	.842	.721	.563	.360
15	.123	.118	.102	.073	.028	.962	.872	.753	.597	.397
16	4-148	4-143	4-128	4-099	4-055	3-990	3-902	3-785	3-632	3-435
17	.173	.168	.153	.126	.082	4-019	.932	.817	.666	.472
18	.198	.194	.179	.152	.109	.047	.962	.849	.701	.510
19	.223	.219	.204	.178	.136	.076	.992	.881	.735	.548
20	.248	.244	.230	.204	.163	.104	4-022	.913	.770	.585
21	4-273	4-269	4-256	4-230	4-191	4-132	4-052	3-945	3-804	3-623
22	.298	.294	.281	.257	.218	.161	.082	.977	.839	.660
23	.323	.320	.307	.283	.245	.189	.112	4-009	.873	.698
24	.348	.345	.332	.309	.272	.218	.142	.041	.908	.735
25	.373	.370	.358	.335	.299	.246	.172	.073	.942	.773
26	4-398	4-395	4-383	4-362	4-326	4-275	4-202	4-105	3-977	3-811
27	.423	.420	.409	.388	.353	.303	.232	.137	4-011	.848
28	.449	.445	.435	.414	.381	.331	.262	.169	.046	.886
29	.474	.471	.460	.440	.408	.360	.292	.201	.080	.923
30	.499	.496	.486	.466	.435	.388	.322	.233	.115	.961
31	4-524	4-521	4-511	4-493	4-462	4-417	4-352	4-265	4-149	3-999
32	.549	.546	.537	.519	.489	.445	.382	.297	.184	4-036
33	.574	.571	.563	.545	.516	.473	.412	.329	.219	.074
34	.599	.597	.588	.571	.544	.502	.442	.361	.253	.111
35	.624	.622	.614	.598	.571	.530	.472	.393	.288	.149
36	4-649	4-647	4-639	4-624	4-598	4-559	4-502	4-425	4-322	4-186
37	.674	.672	.665	.650	.625	.597	.532	.457	.357	.224
38	.699	.697	.690	.676	.652	.615	.562	.489	.391	.262
39	.724	.723	.716	.702	.679	.644	.592	.521	.426	.299
40	.749	.748	.742	.729	.706	.672	.622	.553	.460	.337
41	4-774	4-773	4-767	4-755	4-734	4-701	4-652	4-585	4-495	4-374
42	.799	.798	.793	.781	.761	.729	.682	.617	.529	.412
43	.825	.823	.818	.807	.788	.757	.712	.649	.564	.450
44	.850	.849	.844	.833	.815	.786	.742	.682	.598	.487
45	.875	.874	.869	.860	.842	.814	.772	.714	.633	.525
46	4-900	4-899	4-895	4-886	4-869	4-843	4-802	4-746	4-667	4-562
47	.925	.924	.921	.912	.897	.871	.832	.778	.702	.600
48	.950	.949	.946	.938	.924	.899	.862	.810	.738	.637
49	.975	.975	.972	.965	.951	.928	.892	.842	.771	.675
50	5-000	5-000	.997	.991	.978	.956	.922	.874	.805	.713

TABLE IV (cont.)

(Y = 6-0-6-9; 0-50% kill)

kill	Expected probit, Y									
	6-0	6-1	6-2	6-3	6-4	6-5	6-6	6-7	6-8	6-9
0	2-523	2-132	1-643	1-030	0-261	—	—	—	—	—
1	2-564	2-178	1-694	1-088	0-327	—	—	—	—	—
2	-606	-224	-746	-146	-394	—	—	—	—	—
3	-647	-270	-797	-205	-461	—	—	—	—	—
4	-688	-316	-849	-263	-528	—	—	—	—	—
5	-730	-362	-900	-321	-595	—	—	—	—	—
6	2-771	2-408	1-952	1-380	0-661	—	—	—	—	—
7	-812	-454	-2-003	-438	-728	—	—	—	—	—
8	-854	-500	-455	-496	-795	—	—	—	—	—
9	-895	-546	-106	-555	-862	—	—	—	—	—
10	-936	-591	-158	-613	-928	0-067	—	—	—	—
11	2-978	2-637	2-209	1-671	0-995	0-144	—	—	—	—
12	3-019	-683	-261	-730	1-062	-221	—	—	—	—
13	-060	-729	-312	-788	-129	-299	—	—	—	—
14	-102	-775	-364	-846	-196	-376	—	—	—	—
15	-143	-821	-415	-905	-262	-453	—	—	—	—
16	3-184	2-867	2-467	1-963	1-329	0-530	—	—	—	—
17	-226	-913	-518	-2-022	-396	-607	—	—	—	—
18	-267	-959	-570	-060	-463	-685	—	—	—	—
19	-308	3-005	-621	-138	-530	-762	—	—	—	—
20	-350	-050	-673	-197	-596	-839	—	—	—	—
21	3-391	3-096	2-724	2-255	1-663	0-916	—	—	—	—
22	-432	-142	-776	-313	-730	-993	0-062	—	—	—
23	-474	-188	-827	-372	-797	1-071	-152	—	—	—
24	-515	-234	-879	-430	-664	-148	-243	—	—	—
25	-556	-280	-930	-488	-930	-225	-333	—	—	—
26	3-598	3-326	2-982	2-547	1-997	1-302	0-423	—	—	—
27	-639	-372	3-033	-605	2-064	-379	-513	—	—	—
28	-680	-418	-065	-663	-131	-457	-603	—	—	—
29	-721	-464	-136	-722	-197	-534	-693	—	—	—
30	-763	-509	-188	-780	-264	-611	-784	—	—	—
31	3-804	3-555	3-239	2-838	2-331	1-688	0-874	—	—	—
32	-845	-601	-291	-897	-398	-766	-964	—	—	—
33	-887	-647	-342	-955	-465	-843	1-054	0-050	—	—
34	-928	-693	-394	3-014	-531	-920	-144	-156	—	—
35	-969	-739	-445	-072	-598	-997	-234	-262	—	—
36	4-011	3-785	3-497	3-130	2-665	2-074	1-324	0-369	—	—
37	-052	-831	-548	-189	-732	-152	-415	-475	—	—
38	-093	-877	-600	-247	-799	-229	-505	-581	—	—
39	-135	-923	-651	-305	-865	-306	-595	-688	—	—
40	-176	-969	-703	-364	-932	-383	-685	-794	—	—
41	4-217	4-014	3-754	3-422	2-999	2-460	1-775	0-900	—	—
42	-259	-060	-806	-480	3-066	-538	-865	1-007	—	—
43	-300	-106	-857	-539	-132	-615	-955	-113	0-035	—
44	-341	-152	-909	-597	-199	-692	2-046	-219	-162	—
45	-383	-198	-960	-655	-266	-769	-136	-326	-289	—
46	4-424	4-244	4-012	3-714	3-333	2-846	2-226	1-432	0-415	—
47	-465	-290	-063	-772	-400	-924	-316	-538	-542	—
48	-507	-336	-115	-830	-466	3-001	-406	-645	-669	—
49	-548	-382	-166	-889	-533	-078	-496	-751	-795	—
50	-589	-428	-218	-947	-600	-155	-586	-857	-922	—

TABLE IV (cont.)
($Y = 3.0-3.9; 51-100\%$ kill)

% kill	Expected probit, Y									
	3-0	3-1	3-2	3-3	3-4	3-5	3-6	3-7	3-8	3-9
51	—	—	9-205	8-249	7-504	6-922	6-467	6-111	5-834	5-618
52	—	—	-331	-355	-594	-999	-534	-170	-685	-664
53	—	—	-458	-462	-684	7-076	-600	-228	-937	-710
54	—	—	-585	-568	-774	-154	-667	-286	-988	-756
55	—	—	-711	-674	-864	-231	-734	-345	6-040	-802
56	—	—	9-838	8-781	7-954	7-308	6-801	6-403	6-091	5-848
57	—	—	-965	-887	8-045	-385	-868	-461	-143	-894
58	—	—	—	-993	-135	-462	-934	-520	-194	-940
59	—	—	—	9-100	-225	-540	7-001	-578	-246	-986
60	—	—	—	-206	-315	-617	-068	-636	-297	6-031
61	—	—	—	9-312	8-405	7-694	7-135	6-695	6-349	6-077
62	—	—	—	-419	-495	-771	-201	-753	-400	-123
63	—	—	—	-525	-585	-848	-268	-811	-452	-169
64	—	—	—	-631	-676	-926	-335	-870	-603	-215
65	—	—	—	-738	-766	8-003	-402	-928	-555	-261
66	—	—	—	9-844	8-856	8-080	7-469	6-986	6-606	6-307
67	—	—	—	-950	-946	-157	-535	7-045	-658	-353
68	—	—	—	—	9-036	-234	-602	-103	-709	-399
69	—	—	—	—	-126	-312	-669	-162	-761	-445
70	—	—	—	—	-216	-369	-736	-220	-612	-491
71	—	—	—	—	9-307	8-466	7-803	7-278	6-864	6-536
72	—	—	—	—	-397	-543	-869	-337	-915	-582
73	—	—	—	—	-487	-621	-936	-395	-967	-628
74	—	—	—	—	-577	-698	8-003	-453	7-018	-674
75	—	—	—	—	-667	-775	-070	-512	-070	-720
76	—	—	—	—	9-757	8-852	8-136	7-570	7-121	6-766
77	—	—	—	—	-848	-929	-203	-628	-173	-812
78	—	—	—	—	-938	9-007	-270	-687	-224	-856
79	—	—	—	—	—	-084	-337	-745	-276	-904
80	—	—	—	—	—	-161	-404	-803	-327	-950
81	—	—	—	—	—	9-238	8-470	7-862	7-379	6-995
82	—	—	—	—	—	-315	-537	-920	-430	7-041
83	—	—	—	—	—	-393	-604	-978	-482	-087
84	—	—	—	—	—	-470	-671	8-037	-533	-133
85	—	—	—	—	—	-547	-738	-095	-585	-179
86	—	—	—	—	—	9-624	8-804	8-154	7-636	7-225
87	—	—	—	—	—	-701	-871	-212	-688	-271
88	—	—	—	—	—	-779	-938	-270	-739	-317
89	—	—	—	—	—	-856	9-005	-329	-791	-363
90	—	—	—	—	—	-933	-072	-367	-842	-409
91	—	—	—	—	—	—	9-138	8-445	7-894	7-454
92	—	—	—	—	—	—	-205	-504	-945	-500
93	—	—	—	—	—	—	-272	-562	-997	-546
94	—	—	—	—	—	—	-339	-620	8-048	-592
95	—	—	—	—	—	—	-405	-679	-100	-636
96	—	—	—	—	—	—	9-472	8-737	8-151	7-684
97	—	—	—	—	—	—	-539	-795	-203	-730
98	—	—	—	—	—	—	-606	-854	-254	-776
99	—	—	—	—	—	—	-673	-912	-306	-822
100	—	—	—	—	—	—	-739	-970	-357	-868

TABLE IV (cont.)
($F=4.0-4.9$; 51-100% kill)

% kill	Expected probit, F									
	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9
51	5.452	5.325	5.229	5.158	5.108	5.072	5.049	5.035	5.028	5.025
52	.493	.363	.264	.190	.138	.101	.076	.062	.054	.051
53	.535	.400	.298	.222	.168	.129	.103	.088	.079	.076
54	.576	.438	.333	.254	.198	.157	.131	.114	.105	.101
55	.617	.475	.367	.286	.228	.186	.158	.140	.131	.126
56	5.659	5.513	5.402	5.318	5.258	5.214	5.185	5.167	5.156	5.151
57	.700	.550	.436	.351	.288	.243	.212	.193	.182	.177
58	.741	.588	.471	.383	.318	.271	.239	.219	.207	.202
59	.783	.626	.505	.415	.348	.299	.266	.245	.233	.227
60	.824	.663	.540	.447	.378	.328	.294	.271	.258	.252
61	5.865	5.701	5.574	5.479	5.408	5.356	5.321	5.298	5.284	5.277
62	.907	.738	.609	.511	.438	.385	.348	.324	.310	.303
63	.948	.776	.643	.543	.468	.413	.375	.350	.335	.328
64	.989	.814	.678	.575	.498	.441	.402	.376	.361	.353
65	6.031	.851	.712	.607	.528	.470	.429	.402	.386	.378
66	6.072	5.889	5.747	5.639	5.558	5.498	5.456	5.429	5.412	5.403
67	.113	.926	.781	.671	.588	.527	.484	.455	.437	.429
68	.155	.964	.816	.703	.618	.555	.511	.481	.463	.454
69	.196	6.001	.851	.735	.648	.583	.538	.507	.489	.479
70	.237	.039	.885	.767	.678	.612	.565	.534	.514	.504
71	6.279	6.077	5.920	5.799	5.708	5.640	5.592	5.560	5.540	5.529
72	.320	.114	.954	.831	.738	.669	.619	.586	.565	.555
73	.361	.152	.989	.863	.768	.697	.647	.612	.591	.580
74	.402	.189	6.023	.895	.798	.725	.674	.638	.617	.605
75	.444	.227	.058	.927	.828	.754	.701	.665	.642	.630
76	6.485	6.265	6.092	5.959	5.858	5.782	5.728	5.691	5.668	5.655
77	.526	.302	.127	.991	.888	.811	.755	.717	.693	.680
78	.568	.340	.161	6.023	.918	.839	.782	.743	.719	.706
79	.609	.377	.196	.055	.948	.868	.809	.770	.744	.731
80	.650	.415	.230	.087	.978	.896	.837	.796	.770	.756
81	6.692	6.452	6.265	6.119	6.006	5.924	5.864	5.822	5.796	5.781
82	.733	.490	.299	.151	.038	.953	.891	.848	.821	.806
83	.774	.528	.334	.183	.068	.981	.918	.874	.847	.832
84	.816	.565	.368	.215	.098	6.010	.945	.901	.872	.857
85	.857	.603	.403	.247	.128	.038	.972	.927	.898	.882
86	6.898	6.640	6.437	6.279	6.158	6.066	6.000	5.953	5.923	5.907
87	.940	.678	.472	.311	.188	.095	.027	.979	.949	.932
88	.981	.716	.506	.343	.218	.123	.054	6.006	.975	.958
89	7.022	.753	.541	.375	.248	.152	.081	.032	6.000	.983
90	.064	.791	.575	.407	.278	.180	.108	.058	.026	6.008
91	7.105	6.828	6.610	6.439	6.308	6.208	6.135	6.084	6.051	6.033
92	.146	.866	.644	.471	.338	.237	.162	.110	.077	.058
93	.188	.903	.679	.503	.368	.265	.190	.137	.102	.084
94	.229	.941	.713	.535	.398	.294	.217	.163	.128	.109
95	.270	.979	.748	.567	.428	.322	.244	.189	.154	.134
96	7.312	7.016	6.783	6.600	6.458	6.350	6.271	6.215	6.179	6.159
97	.353	.054	.817	.632	.488	.379	.298	.242	.205	.184
98	.394	.091	.852	.664	.518	.407	.325	.268	.230	.210
99	.436	.129	.886	.696	.548	.436	.353	.294	.256	.235
100	.477	.166	.921	.728	.578	.464	.380	.320	.281	.260

TABLE IV (cont.)
($Y = 5.0-5.9; 51-100\% \text{ kill}$)

%	Expected probit, Y									
	kill	5-0	5-1	5-2	5-3	5-4	5-5	5-6	5-7	5-8
51	5-025	5-025	5-023	5-017	5-005	4-985	4-953	4-906	4-840	4-750
52	-060	-060	-048	-043	-032	5-013	-983	-938	-874	-788
53	-075	-075	-074	-069	-059	-041	5-013	-970	-909	-825
54	-100	-100	-100	-096	-087	-070	-043	5-002	-943	-863
55	-125	-126	-125	-122	-114	-098	-073	-034	-978	-901
56	5-150	5-151	5-151	5-148	5-141	5-127	5-103	5-066	5-012	4-938
57	-175	-176	-176	-174	-168	-155	-133	-098	-047	-978
58	-201	-201	-202	-201	-195	-183	-163	-130	-062	5-013
59	-226	-226	-227	-227	-222	-212	-193	-162	-116	-051
60	-251	-252	-253	-253	-250	-240	-223	-194	-151	-068
61	5-276	5-277	5-279	5-279	5-277	5-269	5-253	5-228	5-185	5-126
62	-301	-302	-304	-305	-304	-297	-283	-258	-220	-164
63	-326	-327	-330	-332	-331	-325	-313	-290	-254	-201
64	-351	-352	-355	-358	-358	-354	-343	-322	-289	-239
65	-376	-378	-381	-384	-385	-382	-373	-354	-323	-276
66	5-401	5-403	5-406	5-410	5-412	5-411	5-403	5-386	5-358	5-314
67	-426	-428	-432	-437	-440	-439	-433	-418	-392	-351
68	-451	-453	-458	-463	-467	-467	-463	-450	-427	-389
69	-476	-478	-483	-489	-494	-496	-493	-482	-461	-427
70	-501	-504	-509	-515	-521	-524	-523	-514	-496	-464
71	5-526	5-529	5-534	5-541	5-548	5-553	5-553	5-546	5-530	5-502
72	-551	-554	-560	-568	-575	-581	-583	-578	-565	-539
73	-577	-579	-585	-594	-603	-609	-613	-610	-599	-577
74	-602	-604	-611	-620	-630	-638	-643	-642	-634	-615
75	-627	-630	-637	-646	-657	-666	-673	-674	-668	-652
76	5-652	5-655	5-662	5-673	5-684	5-695	5-703	5-706	5-703	5-690
77	-677	-680	-688	-699	-711	-723	-733	-738	-737	-727
78	-702	-705	-713	-725	-738	-752	-763	-770	-772	-765
79	-727	-730	-739	-751	-765	-780	-793	-802	-806	-802
80	-752	-755	-764	-777	-793	-808	-823	-834	-841	-840
81	5-777	5-781	5-790	5-804	5-820	5-837	5-853	5-866	5-875	5-878
82	-802	-806	-816	-830	-847	-865	-883	-898	-910	-915
83	-827	-831	-841	-856	-874	-894	-913	-930	-944	-953
84	-852	-856	-867	-882	-901	-922	-943	-962	-979	-990
85	-877	-881	-892	-908	-928	-950	-973	-995	6-014	6-028
86	5-902	5-907	5-918	5-935	5-956	5-979	6-003	6-027	6-048	6-066
87	-927	-932	-943	-961	-983	6-007	-033	-059	-083	-103
88	-953	-957	-969	-987	6-010	-036	-063	-091	-117	-141
89	-978	-982	-995	6-013	-037	-064	-093	-123	-152	-178
90	6-003	6-007	6-020	-040	-064	-092	-123	-155	-186	-216
91	6-028	6-033	6-046	6-066	6-091	6-121	6-153	6-187	6-221	6-253
92	-063	-068	-071	-092	-118	-149	-183	-219	-255	-291
93	-078	-083	-097	-118	-146	-178	-213	-251	-290	-329
94	-103	-106	-122	-144	-173	-206	-243	-283	-324	-366
95	-128	-133	-148	-171	-200	-234	-273	-315	-359	-404
96	6-153	6-159	6-174	6-197	6-227	6-263	6-303	6-347	6-393	6-441
97	-178	-184	-199	-223	-254	-291	-333	-379	-428	-479
98	-203	-209	-225	-249	-281	-320	-363	-411	-462	-517
99	-228	-234	-250	-278	-309	-348	-393	-443	-497	-554
100	-253	-259	-276	-302	-336	-376	-423	-475	-531	-592

TABLE IV (cont.)
(Y = 6.0-6.9; 51-100% kill)

%	Expected probit, Y									
	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9
51	4.631	4.473	4.269	4.006	3.667	3.233	2.677	1.964	1.049	—
52	6.72	5.19	3.21	0.64	7.34	3.10	7.67	2.070	1.175	0.022
53	7.13	5.65	3.72	1.22	8.00	3.87	8.57	1.176	3.02	1.175
54	7.55	6.11	4.24	1.81	8.67	4.64	9.47	2.83	4.29	3.27
55	7.96	6.57	4.75	2.39	9.34	5.41	3.037	3.89	5.55	4.80
56	4.837	4.703	4.527	4.297	4.001	3.619	3.127	2.495	1.682	0.632
57	8.79	7.49	5.78	3.56	0.68	8.96	2.18	6.02	8.09	7.84
58	9.20	7.95	6.30	4.14	1.34	7.73	3.08	7.06	9.35	9.37
59	9.61	8.41	6.81	4.72	2.01	8.50	3.98	8.14	2.062	1.089
60	5.003	8.87	7.33	5.31	2.68	9.27	4.88	9.21	1.69	2.42
61	5.044	4.932	4.784	4.589	4.335	4.005	3.578	3.027	2.315	1.394
62	0.65	9.78	8.36	6.47	4.01	0.82	6.68	1.33	4.42	5.46
63	1.27	5.024	8.87	7.06	4.68	1.59	7.58	2.40	5.69	6.99
64	1.68	0.70	9.39	7.64	5.35	2.36	8.49	3.46	6.95	8.51
65	2.09	1.16	9.90	8.23	6.02	3.13	9.39	4.52	8.22	2.004
66	5.251	5.162	5.042	4.881	4.669	4.391	4.029	3.559	2.949	2.156
67	2.92	2.08	0.93	9.39	7.35	4.68	1.19	6.65	3.075	3.08
68	3.33	2.54	1.45	9.98	8.02	5.45	2.09	7.71	2.02	4.61
69	3.75	3.00	1.96	5.056	8.69	6.22	2.99	8.78	3.29	6.13
70	4.16	3.46	2.48	1.14	9.36	7.00	3.90	9.84	4.55	7.66
71	5.457	5.392	5.299	5.173	5.003	4.777	4.480	4.090	3.582	2.918
72	4.99	4.37	3.51	2.31	0.69	8.54	5.70	1.97	7.09	3.070
73	5.40	4.83	4.02	2.89	1.36	9.31	6.60	3.03	8.35	2.23
74	5.81	5.29	4.54	3.48	2.03	5.006	7.50	4.09	9.62	3.75
75	6.23	5.75	5.05	4.06	2.70	0.66	8.40	5.16	4.089	5.28
76	5.664	5.621	5.557	5.464	5.336	5.163	4.930	4.622	4.215	3.680
77	7.05	6.67	6.08	5.23	4.03	5.021	2.40	7.28	3.42	8.32
78	7.47	7.13	6.60	5.81	4.70	3.17	1.11	8.35	4.69	9.85
79	7.88	7.59	7.11	6.39	5.37	3.94	2.01	9.41	5.95	4.137
80	8.29	8.05	7.63	6.98	6.04	4.72	2.91	5.047	7.22	2.90
81	5.870	5.851	5.814	5.756	5.670	5.549	5.381	5.154	4.849	4.442
82	9.12	8.96	8.66	8.15	7.37	6.26	4.71	2.60	9.75	5.94
83	9.53	9.42	9.17	8.73	8.04	7.03	5.61	3.66	5.102	7.47
84	9.94	9.88	9.69	9.31	8.71	7.80	6.52	4.73	2.29	8.99
85	6.036	6.034	6.020	5.990	5.938	5.858	5.742	5.79	3.55	5.052
86	6.077	6.080	6.072	6.048	6.004	5.935	5.832	5.685	5.482	5.204
87	1.18	1.26	1.23	1.06	0.71	6.012	9.22	7.92	6.09	3.56
88	1.60	1.72	1.75	1.65	1.38	0.89	6.012	8.98	7.35	5.09
89	2.01	2.18	2.26	2.23	2.05	1.66	1.02	6.004	8.62	6.61
90	2.42	2.64	2.78	2.81	2.72	2.44	1.92	1.11	9.88	8.14
91	6.284	6.310	6.329	6.340	6.338	6.321	6.283	6.217	6.115	5.966
92	3.25	3.55	3.81	3.98	4.05	3.98	3.73	3.23	2.42	6.118
93	3.66	4.01	4.32	4.56	4.72	4.75	4.63	4.30	3.68	2.71
94	4.08	4.47	4.84	5.15	5.39	5.53	5.53	5.36	4.95	4.23
95	4.49	4.93	5.35	5.73	6.05	6.30	6.43	6.42	6.22	5.76
96	6.490	6.539	6.587	6.631	6.672	6.707	6.733	6.749	6.748	6.728
97	5.32	5.85	6.38	6.90	7.39	7.84	8.24	8.55	8.75	8.80
98	5.73	6.31	6.90	7.48	8.06	8.61	9.14	9.61	7.002	7.033
99	6.14	6.77	7.41	8.07	8.73	9.39	7.004	7.068	1.28	1.85
100	6.56	7.23	7.93	8.65	9.39	7.016	0.94	1.74	2.55	3.38

TABLE IV (cont.)
(Y = 7.0-7.9; 51-100% kill)

% kill	Expected probit, Y									
	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9
51	—	—	—	—	—	—	—	—	—	—
52	—	—	—	—	—	—	—	—	—	—
53	—	—	—	—	—	—	—	—	—	—
54	—	—	—	—	—	—	—	—	—	—
55	—	—	—	—	—	—	—	—	—	—
56	—	—	—	—	—	—	—	—	—	—
57	—	—	—	—	—	—	—	—	—	—
58	—	—	—	—	—	—	—	—	—	—
59	—	—	—	—	—	—	—	—	—	—
60	0.013	—	—	—	—	—	—	—	—	—
61	0.198	—	—	—	—	—	—	—	—	—
62	.383	—	—	—	—	—	—	—	—	—
63	.568	—	—	—	—	—	—	—	—	—
64	.753	—	—	—	—	—	—	—	—	—
65	.939	—	—	—	—	—	—	—	—	—
66	1.124	—	—	—	—	—	—	—	—	—
67	.309	0.003	—	—	—	—	—	—	—	—
68	.494	.231	—	—	—	—	—	—	—	—
69	.680	.458	—	—	—	—	—	—	—	—
70	.865	.685	—	—	—	—	—	—	—	—
71	2.050	0.913	—	—	—	—	—	—	—	—
72	.235	1.140	—	—	—	—	—	—	—	—
73	.420	.367	—	—	—	—	—	—	—	—
74	.606	.595	0.263	—	—	—	—	—	—	—
75	.791	.822	.545	—	—	—	—	—	—	—
76	2.976	2.050	0.827	—	—	—	—	—	—	—
77	3.161	.277	1.108	—	—	—	—	—	—	—
78	.347	.504	.390	—	—	—	—	—	—	—
79	.532	.732	.672	0.265	—	—	—	—	—	—
80	.717	.959	.954	.618	—	—	—	—	—	—
81	3.902	3.186	2.236	0.971	—	—	—	—	—	—
82	4.087	.414	.518	1.324	—	—	—	—	—	—
83	.273	.641	.800	.677	0.175	—	—	—	—	—
84	.458	.868	3.082	2.030	.621	—	—	—	—	—
85	.643	4.096	.364	.383	1.068	—	—	—	—	—
86	4.828	4.323	3.645	2.736	1.514	—	—	—	—	—
87	5.014	.551	.927	3.089	.961	0.438	—	—	—	—
88	.199	.778	4.209	.442	2.408	1.008	—	—	—	—
89	.384	5.005	.491	.795	.854	.579	—	—	—	—
90	.569	.233	.773	4.148	3.301	2.149	0.581	—	—	—
91	5.754	5.460	5.055	4.501	3.747	2.720	1.317	—	—	—
92	.940	.687	.337	.854	4.194	3.290	2.054	0.356	—	—
93	6.125	.915	.619	5.207	.640	.861	.790	1.316	—	—
94	.310	6.142	.901	.560	5.087	4.431	3.526	2.275	0.542	—
95	.495	.369	6.182	.914	.533	5.002	4.262	3.235	1.806	—
96	6.681	6.597	6.464	6.287	5.980	5.572	4.998	4.194	3.069	1.493
97	.866	.824	.746	.620	6.426	6.143	5.735	5.154	4.333	3.173
98	7.051	7.051	7.028	.973	.873	.713	6.471	6.114	5.596	4.853
99	.236	.279	.310	7.326	7.319	7.284	7.207	7.073	6.859	6.533
100	.421	.506	.592	.679	.766	.854	.943	8.033	8.123	8.213

TABLE V. The Probability, P , the Ordinate, Z , and Z^2

Y	P	Z	Z^2
5.0	0.50000	0.39894	0.15915
5.1	0.53983	0.39695	0.15757
5.2	0.57926	0.39104	0.15291
5.3	0.61791	0.38139	0.14546
5.4	0.65542	0.36827	0.13562
5.5	0.69146	0.35207	0.12395
5.6	0.72575	0.33322	0.11104
5.7	0.75804	0.31225	0.09750
5.8	0.78814	0.28969	0.08392
5.9	0.81594	0.26609	0.07080
6.0	0.84134	0.24197	0.05855
6.1	0.86433	0.21785	0.04746
6.2	0.88493	0.19419	0.03771
6.3	0.903200	0.17137	0.02937
6.4	0.919243	0.14973	0.02242
6.5	0.933193	0.12952	0.01677
6.6	0.945201	0.11092	0.01230
6.7	0.955435	0.094049	0.00885
6.8	0.964070	0.078950	0.00623
6.9	0.971283	0.065616	0.00431
7.0	0.977250	0.053991	0.00292
7.1	0.982136	0.043984	0.00193
7.2	0.986097	0.035475	0.00126
7.3	0.989276	0.028327	0.00080
7.4	0.9918025	0.022395	0.00050
7.5	0.9937903	0.017528	0.00031
7.6	0.9953388	0.013583	0.00018
7.7	0.9965330	0.010421	0.00011
7.8	0.9974449	0.0079155	0.00006
7.9	0.9981342	0.0059525	0.00004
8.0	0.9986501	0.0044318	0.00002
8.1	0.99903240	0.0032668	0.00001
8.2	0.99931286	0.0023841	0.00001
8.3	0.99951658	0.0017226	0.00000
8.4	0.99966307	0.0012322	0.00000
8.5	0.99976737	0.00087268	0.00000
8.6	0.99984089	0.00061190	0.00000
8.7	0.99989220	0.00042478	0.00000
8.8	0.999927652	0.00029195	0.00000
8.9	0.999951904	0.00019866	0.00000
9.0	0.999968329	0.00013383	0.00000

Values of P and Z are tabulated above, to 5 significant digits, in order to aid the formation of functions that may be required in any new developments of probit methods; Z^2 is the weighting coefficient introduced in § 47.

When Y is less than 5.0, read the entries for $(10 - Y)$ as Q , Z , Z^2 .

TABLE VI. Distribution of χ^2

Degrees of freedom	Probability								
	.90	.70	.50	.30	.10	.05	.02	.01	.001
1	.016	.15	.45	1.1	2.7	3.8	5.4	6.6	10.8
2	.21	.71	1.4	2.4	4.6	6.0	7.8	9.2	13.8
3	.58	1.4	2.4	3.7	6.3	7.8	9.8	11.3	16.3
4	1.1	2.2	3.4	4.9	7.8	9.5	11.7	13.3	18.5
5	1.6	3.0	4.4	6.1	9.2	11.1	13.4	15.1	20.5
6	2.2	3.8	5.3	7.2	10.6	12.6	15.0	16.8	22.5
7	2.8	4.7	6.3	8.4	12.0	14.1	16.6	18.5	24.3
8	3.5	5.5	7.3	9.5	13.4	15.5	18.2	20.1	26.1
9	4.2	6.4	8.3	10.7	14.7	16.9	19.7	21.7	27.9
10	4.9	7.3	9.3	11.8	16.0	18.3	21.2	23.2	29.6
12	6.3	9.0	11.3	14.0	18.5	21.0	24.1	26.2	32.9
14	7.8	10.8	13.3	16.2	21.1	23.7	26.9	29.1	36.1
16	9.3	12.6	15.3	18.4	23.5	26.3	29.6	32.0	39.3
18	10.9	14.4	17.3	20.6	26.0	28.9	32.3	34.8	42.3
20	12.4	16.3	19.3	22.8	28.4	31.4	35.0	37.6	45.3
22	14.0	18.1	21.3	24.9	30.8	33.9	37.7	40.3	48.3
24	15.7	19.9	23.3	27.1	33.2	36.4	40.3	43.0	51.2
26	17.3	21.8	25.3	29.2	35.6	38.9	42.9	45.6	54.1
28	18.9	23.6	27.3	31.4	37.9	41.3	45.4	48.3	56.9
30	20.6	25.5	29.3	33.5	40.3	43.8	48.0	50.9	59.7

When χ^2 is based on more than 30 degrees of freedom, the quantity $\sqrt{(2\chi^2)} - \sqrt{(2f-1)}$ (where f is the number of degrees of freedom) has approximately the following distribution:

> 30 | -1.28 -0.52 0.00 0.52 1.28 1.64 2.05 2.33 3.09

I am indebted to Professor R. A. Fisher and Dr F. Yates, and also to Messrs Oliver and Boyd, Ltd., of Edinburgh, for permission to print Table VI as an abridgement of Table IV of their book *Statistical Tables for Biological, Agricultural and Medical Research*.

TABLE VII. Distribution of t

Degrees of freedom	Probability								
	.90	.70	.50	.30	.10	.05	.02	.01	.001
1	.16	.51	1.00	1.96	6.31	12.7	31.8	63.7	637.
2	.14	.44	.82	1.39	2.92	4.30	6.96	9.92	31.6
3	.14	.42	.76	1.25	2.35	3.18	4.54	5.84	12.9
4	.13	.41	.74	1.19	2.13	2.78	3.75	4.60	8.61
5	.13	.41	.73	1.16	2.02	2.57	3.36	4.03	6.86
6	.13	.40	.72	1.13	1.94	2.45	3.14	3.71	5.96
7	.13	.40	.71	1.12	1.90	2.36	3.00	3.50	5.40
8	.13	.40	.71	1.11	1.86	2.31	2.90	3.36	5.04
9	.13	.40	.70	1.10	1.83	2.26	2.82	3.25	4.78
10	.13	.40	.70	1.09	1.81	2.23	2.76	3.17	4.59
12	.13	.40	.70	1.08	1.78	2.18	2.68	3.06	4.32
14	.13	.39	.69	1.08	1.76	2.14	2.62	2.98	4.14
16	.13	.39	.69	1.07	1.75	2.12	2.58	2.92	4.02
18	.13	.39	.69	1.07	1.73	2.10	2.55	2.88	3.92
20	.13	.39	.69	1.06	1.72	2.09	2.53	2.84	3.85
22	.13	.39	.69	1.06	1.72	2.07	2.51	2.82	3.79
24	.13	.39	.68	1.06	1.71	2.06	2.49	2.80	3.74
26	.13	.39	.68	1.06	1.71	2.06	2.48	2.78	3.71
28	.13	.39	.68	1.06	1.70	2.05	2.47	2.76	3.67
30	.13	.39	.68	1.06	1.70	2.04	2.46	2.75	3.65
40	.13	.39	.68	1.05	1.68	2.02	2.42	2.70	3.55
60	.13	.39	.68	1.05	1.67	2.00	2.39	2.66	3.46
120	.13	.39	.68	1.04	1.66	1.98	2.36	2.62	3.37
∞	.126	.385	.674	1.036	1.645	1.960	2.326	2.576	3.291

I am indebted to Professor R. A. Fisher and Dr F. Yates, and also to Messrs Oliver and Boyd, Ltd., of Edinburgh, for permission to print Table VII as an abridgement of Table III of their book *Statistical Tables for Biological, Agricultural and Medical Research*.

TABLE VIII. The Weighting Coefficient in Wadley's Problem

Expected probit Y	Weighting coefficient <i>w</i>	Expected probit Y	Weighting coefficient <i>w</i>
1-1	0-00000 004	5-1	0-34242
1-2	0-00000 009	5-2	0-36344
1-3	0-00000 02	5-3	0-38069
1-4	0-00000 04	5-4	0-39359
1-5	0-00000 08	5-5	0-40173
1-6	0-00000 2	5-6	0-40488
1-7	0-00000 3	5-7	0-40296
1-8	0-00000 6	5-8	0-39612
1-9	0-00001	5-9	0-38466
2-0	0-00002	6-0	0-36904
2-1	0-00004	6-1	0-34983
2-2	0-00006	6-2	0-32770
2-3	0-00011	6-3	0-30338
2-4	0-00019	6-4	0-27760
2-5	0-00031	6-5	0-25109
2-6	0-00051	6-6	0-22452
2-7	0-00081	6-7	0-19848
2-8	0-00128	6-8	0-17348
2-9	0-00197	6-9	0-14993
3-0	0-00298	7-0	0-12813
3-1	0-00443	7-1	0-10829
3-2	0-00647	7-2	0-09051
3-3	0-00926	7-3	0-07482
3-4	0-01302	7-4	0-06118
3-5	0-01798	7-5	0-04948
3-6	0-02439	7-6	0-03958
3-7	0-03251	7-7	0-03132
3-8	0-04261	7-8	0-02452
3-9	0-05491	7-9	0-01899
4-0	0-06959	8-0	0-01455
4-1	0-08677	8-1	0-01103
4-2	0-10648	8-2	0-00827
4-3	0-12863	8-3	0-00614
4-4	0-15300	8-4	0-00451
4-5	0-17926	8-5	0-00327
4-6	0-20692	8-6	0-00235
4-7	0-23540	8-7	0-00167
4-8	0-26398	8-8	0-00118
4-9	0-29189	8-9	0-00082
5-0	0-31831	9-0	0-00057

TABLE IX. Minimum Working Probit, Range, and Weighting Coefficient for Inverse Sampling

Expected probit Y	Minimum working probit $Y - Q/Z$	Range Q^2/Z	Weighting coefficient Z^2/PQ^2
5-0	3-7467	0-62666	1-2732
5-1	3-9407	0-53346	1-3784
5-2	4-1241	0-45269	1-4912
5-3	4-2982	0-38279	1-6124
5-4	4-4643	0-32241	1-7428
5-5	4-6236	0-27039	1-8831
5-6	4-7770	0-22572	2-0342
5-7	4-9251	0-18750	2-1970
5-8	5-0687	0-15493	2-3724
5-9	5-2083	0-12732	2-5613
6-0	5-3443	0-10403	2-7647
6-1	5-4773	0-084485	2-9833
6-2	5-6074	0-068187	3-2181
6-3	5-7351	0-054679	3-4700
6-4	5-8606	0-043557	3-7395
6-5	5-9842	0-034460	4-0275
6-6	6-1060	0-027073	4-3346
6-7	6-2261	0-021117	4-6613
6-8	6-3449	0-016352	5-0081
6-9	6-4624	0-012568	5-3753
7-0	6-5786	0-0095862	5-7633
7-1	6-6938	0-0072558	6-1721
7-2	6-8081	0-0054491	6-6019
7-3	6-9214	0-0040600	7-0528
7-4	7-0339	0-0030007	7-5247
7-5	7-1457	0-0021999	8-0177
7-6	7-2568	0-0015996	8-5315
7-7	7-3673	0-0011534	9-0661
7-8	7-4772	0-00082480	9-6214
7-9	7-5866	0-00058484	10-1971
8-0	7-6954	0-00041117	10-7933
8-1	7-8038	0-00028660	11-4097
8-2	7-9118	0-00019805	12-0463
8-3	8-0194	0-00013567	12-7030
8-4	8-1266	0-000092128	13-3797
8-5	8-2334	0-000062011	14-0762
8-6	8-3400	0-000041372	14-7926
8-7	8-4462	0-000027357	15-5289
8-8	8-5522	0-000017929	16-2859
8-9	8-6579	0-000011645	17-0607
9-0	8-7633	0-0000074951	17-8563

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