Chapter VII: Conclusions

20. Section I: Comparisons with Keynesian and Patinkin Syntheses

1. As the build-up and the analysis of monetary theory of value come towards a conclusion, it would be prudent to bring out certain observations made during the course of this study. While it is clear that the Patinkinisque system is dependent upon the relationship between nominal money balances, real money balances, nominal money supply and the equilibrium attained by these forces through equilibrating the labour, money, bond and commodity markets. While Patinkin sought to restore these equilibria in all the markets primarily through the operation of the real balance effect, this leads one into falling in the trap of money in the demand (read utility) functions. "It is obvious from equation.. that if we were to abandon our oversimplified form of labour demand function and instead represent it as dependent also on real balances...." In itself, this captures the essence of Patinkin's thought process. He sought a macro-economic equilibrium thorough the operation of the real balance effect. In the proposed model, no such effect has been used nor is the equilibrium derived using any such effects. As a result, in the Patinkinisque case, the demand functions are utility based and hence, money or wealth appears in the demand functions as utility is said to be dependent upon the money/ wealth/ real balances. As a consequence, it would be only trivial and also faulty to consider a real balance effect in the operation of equilibrium. Therefore, in the theory presented, the demand functions are free from the issues of real balances, nor do commodity, money or labour supplies need to be dependent on such a (trivial) force. It is thus important to note this fact that the equilibrium attained in the presented theory is not through the operation of real balance effect. While we are discussing the nature of demand functions, a point worth a mention is this: Patinkinisque demand functions are utility based whereas the demand functions introduced here have an empirical nature and are not utility based. This is the point where the whole idea of real balance drops off.

- 2. Another point about Patinkinisque demand functions is that Patinkin used the tool of aggregating demand functions and was hence subject to the famous Hahn critique. However, the demand functions employed in the theory presented here are of a social nature and can be thought of to be an aggregated demand function; however, no such explicit assumption is made or needed. The demand functions are merely non-utility based, do not have wealth/ money in the arguments nor do they aggregate implying homogeneity of individuals or the like. However, for the moment let us assume otherwise; that the presented theory would have allowed for money as an argument in the demand functions. In this case, as money balances rose, real wealth would have increased. Now, since demand equates supply, all debtor-creditor relationships remain balanced and hence, if consumers' real balances increase. Consequently, due to the demand supply balancing, the increase in consumers' real balances would also reduce the real wealth of creditors who may have financed the consumers. Eventually, this reduction in real wealth of creditors would reduce investment demand. Therefore, though consumption demand has increased due to increase of consumers' real wealth, the investment demand reduces due to decrease in creditors' (or any other counter party's) real wealth. The net effect on social income is zero. Thus, the mere presence of money in the demand function may also not activate real balances to have a significant effect on the incomes of the society. This not only provides the rationale and the justification for leaving the real balance out of the presented theory but also substantiates that we do not even need it.
- 3. Now that we are in the ambit of comparing and drawing (dis)similarities between the theories of the *standard neo-classical world* and the theory *presented here*, we may not go too far, before we recognize the nature of inter-temporal equilibrium expressed in the presented theory. *"The usual analysis bars this possibility (*of demanding unlimited amounts of commodities each week) *by assuming that there is some imperfection in the capital market which prevents an individual from borrowing all he wants at the going rate of interest. This is undoubtedly a realistic assumption. However, since it is desired to keep the analysis as simple as possible, we shall not employ it here. Instead, we shall accomplish the same result*

by assuming that the individual must formulate his present and future market plans under the additional restriction that on the final Monday of the month, his planned holding s of bonds must be zero.⁶⁰" Multiple comparisons and valid contradictions can be inferred from the paragraph just quoted. In Patinkinisque world, all markets are fully specified in terms of commodities and derivative markets. In fact, all spot and future markets are fully specified. There is an exact equilibrium in all markets at all current and future dates. This is one feature of the Arrow- Debreu world as well. Hence, money has no role in such an economycontrary to this, in fact it should be stated that money is not required in this economy! However, in our theory, we deny the existence of all future markets thereby creating the role for money as a standard of and store of value. We deal only in all spot markets. Hence, role of money is set in through equilibrium requirement in all spot markets of time T₀. All (terminal) money balances are only responsible in pushing the economy to time T_1 , where new spot markets are created. Hence unlike Patinkin, in our world, the terminal balances of all (money) holdings cannot become zero.

4. Patinkin also argues that the presence of money implies that there exist (capital) market imperfections. These imperfections, for these to be corrected, the (economic) agents need to be paid a premium. This premium is what Patinkin regards as *interest*. In the presented theory, we do not explain interest or any other aspect of the economy via imperfection. In fact, imperfections are assumed away by creating near perfect markets. It is worthy a mention that in Patinkinisque world, the sum of excess demands equates the sum of money- the Walras' law operates in Patinkin. This however creates a problem for Clower as he says that if the value of excess demands for factors matches the value of excess supplies for commodities, then money in fact may be rendered redundant. Therefore, Clower proposed a dichotomization with excess demand for commodity being made equal to M while excess demands for factors being made equal to some M¹. The problem here with this is that the velocity of circulation of money would always be equal to 1. This aspect also is absent from our theory and we do not require the

⁶⁰ Patinkin, Don (1965): *Money, Interest & prices: An integration of monetary & value theory* (MIT Press)

velocity of money to be equal to one. Finally, Patinkin had stated "*Thus, shifts in tastes, technology, and the like are in the domain of value theory. Changes in quantity of money and –as we shall see- shifts in liquidity preference are in the domain of monetary theory*⁶¹. The presented theory clearly is an objection to what Patinkin had said in this regard. In fact, it detests all economics that states the dichotomization of economy into real and monetary sectors. The essence of the stated theory is a unification of both- the real and the monetary forces.

- 5. Continuing the chain of comparison and logical reasoning, it must be recognized that the General theory of interest, employment and money was also a Keynesian attempt at unifying the monetary and value theories. Starting with the first of them all, Keynes recognized that the rate of interest is a result of three most important forces defining the reasons (or motives) for money demand- namely the transactions, precautionary and speculative motive of demand for money. These demands in the Keynesian *synthesis* are stated expressly. We do not have the role for speculative motive for money in our theory. We do not provide for presence or absence of explicit stock markets nor do our producers or agents speculate on the volume of inventories with debt capital.
- 6. Standard theory, including the Keynesian one, has always presented mechanism for equating rate of profits and rate of interest. We do not have any such equilibration expressly brought out. However, in our theory, we also have a lot of assets and a lot of interest rates.
- 7. Like the Patinkin case, in the Keynesian system, there is an uncertainty in the bond markets. This uncertainty has a role in explaining the interest rate in Keynesian synthesis. However, this uncertainty is an imperfection to an otherwise perfect economy of Keynes. We, since are in a pursuit of perfectly competitive economy, we do not ascribe the interest rate determination to any imperfection in the economy.
- 8. Unlike the Patinkin story, we do not have to bother about the neutrality of money. Money is not neutral, either in short run or the long run. The presence of money

⁶¹ Patinkin (1965) Ibid, pp 181

has far reaching effects on the economy and money affects output, prices, employment, interest, government and monetary policies equally.

- 9. In the Keynesian world, as Clower pointed out, there is no auctioneer and it was this absence of the auctioneer to which Clower ascribed the imperfections in the Keynesian model. To Clower, the lack of auctioneer caused coordination failures and hence, markets failed to clear giving rise to imperfections. This was the main reason for the Keynesian result of market disequilibrium. However, in our theory, we do not have an auctioneer as well. However, we as well face the situation of disequilibrium in the markets- a result that Keynes obtained which Clower attributed to the absence of an auctioneer.
- 10. The presented theory also makes no room for money wage rigidity and liquidity trap. In fact, while (money) wages are perfectly flexible, as liquidity declines, the rate of interest declines. In our model, unlike the Keynesian case, we do not have the operation of Walras' law as an operative equation for the equilibrium. in our case, the sum of values of outputs less the replacement demand, new demand and consumption demand does not equate zero. In this case, we have introduced deficit financing and Walras' law is brought about and not used as an operative condition as mentioned earlier.
- 11. We do not use the logic of quantity theory of money as well in the synthesis presented here. In quantity theory, the terminal value of money holdings is known always. However, we only have the initial value of money. The terminal value of money cannot be known *apriori*. To put the quantity theory to use therefore, we will have to fix the value of an unknown quantity, which is not possible! Even if it were so, irrelevant solutions are obtained. Further, changes in money have tow parts- one is endogenous and the other is from savings; hence if only savings part is taken, then irrelevant solutions are expected. Also, if quantity theory is used in the price system the solutions to the price system would be available; however the full economic equilibrium cannot be determined on the shoulders of quantity theory.
- 12. Finally, Hahn pointed out that a fully specified non-discontinuous demand function based model would have equilibrium. Even if Hahn is honoured in this

case and we provide a working model of the Hahn specifications, we do not reach equilibrium.

21. Section II: Monetary and Value Theory

- 13. The theory of prices so far developed clearly spells out the following- the theory of prices can be determined without money. Prices technically mean exchange ratios- it does not matter whether these ratios are measured in relation to other commodities as relative prices or as a relation to numeraire good (read money) as absolute prices. Even as we move from the world of micro-price determination, and as we enter the realm of income distribution, the role of money can be silent still. Prices are nothing but exchange ratios. Barter is efficient even without money. The moment one moves into the theories of income and employment determination, money cannot be ignored. The role of money becomes all the more pronounced for employment and income determination. Keynes navigated his entire synthesis through the facets of money, interest and employment. It must be noted that output is aggregate demand. Aggregate demand would involve aggregate consumption demand, aggregate investment demand and aggregate expenditure. The level of aggregate investment would depend upon the level of interest rate in the economy. In essence, for all market principles, the rate of interest would depend on the level of money demand and money supply in the economy. It would be therefore not inept to say that the level of aggregate output depends on the money balances in the economy. Employment depends on the level of aggregate output in the economy. The level of output thus depends on the interest and money and in turn employment depends on the level of output and the wage cost of entrepreneurs.
- 14. It must be also noted that it is therefore not necessary to have an explicit relationship between money and prices- like the quantity theory one. What is necessary and important is that there be a relation between the growth rates of absolute outputs and money. It thus implies that the real and monetary dichotomy, then, of course is a false one. Money affects output and employment. The

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relationship between rate of profits and rate of growth is precisely this one relationship that forms the cornerstone of the integration of monetary and value theories. The equation

$$r = \frac{g\left(\left(e_{w} + e_{k}\left(\sum_{i=1}^{n+m}\sum_{j=1}^{n+m}S_{ij}p_{j} + \sum_{i=1}^{n+m}k_{i}p_{i}B_{i}\right)\right) - \left(k_{e}^{w}(1-\beta)wL\right)}{\left(k_{e}^{w}(1-\beta)e_{w}\left(\sum_{i=1}^{n+m}\sum_{j=1}^{n+m}S_{ij}p_{j} + \sum_{i=1}^{n+m}k_{i}p_{i}B_{i}\right) + k_{e}^{k}(1-\beta)e_{k}\left(\sum_{i=1}^{n+m}\sum_{j=1}^{n+m}S_{ij}p_{j} + \sum_{i=1}^{n+m}k_{i}p_{i}B_{i}\right)\right)}$$

is the most fundamental equation of this synthesis. Whilst the first parenthesis in the numerator term describes the monetary aspect, the second one is the savings block out of the workers incomes. This explicitly models the relationship between the rate of growth of absolute outputs, money and the rate of profits in the economy. Nowhere in the theory are wages assumed to be rigid. The assumption of a perfectly mobile labour (that fits in with the theory with perfect markets) does not fit in with the assumption of rigid wages. Labour is always not a growable stock as well. The economy has to employ the available stock of labour if it were to maintain its growth momentum. Therefore, effective demand has been abandoned in favour of full demand. It had to be abandoned. Given the level of employment, all people should work, "earn" money and hence "determine" output. A one line conclusion that this exploration leads to is this: Output grows, money does not constrain labour (it cannot) and prices do not constrain distribution; in effect, they all determine level of new money, new outputs, new interest, new employment, new prices and new income distribution. Individuals create wealth by being employed and hence contribute to savings, hence to investment and hence to growth. All this because they are in constant pursuit of at least maintaining their wealths. They are not the Walrasian wealth maximisers. In fact, individual wealth in a monetary economy is a by-product of national wealth/ income. In a monetary economy, money alone is able to make entrepreneurs produce and workers work. It is an enabler to the entire economic activity. It is like a catalyst in a chemical reaction.

- 15. In a monetary economy, a valid question is does the interest rate get a liquidity trap? The answer could be "it may". But as we have pointed out in the course of analysis, in a monetary economy, liquidity trap may not have harmful prescriptions for the economic activity. The government and the central monetary authority would ensure that in this situation, enough support would be forthcoming such that it would have minimal implications for inflation. Finally, money is or can never be a veil in a monetary economy. Real balances cannot be an explanation for disequilibrium in a monetary economy.
- 16. A monetary economy will always face a disequilibrium if let loose. A regulator is required to manage the entire economic activity. Money calls for a truly integrated economic system with individual roles for producers, workers, monetary & fiscal authorities.
- 17. While the current work completely rests on using a hypothetical numerical example to illustrate and provide results for a monetary economy, the following points must be noted
 - a. Providing solutions to the equations using the construct of a numerical example is a means to provide more accurate quantitative discussion on the topics raised in this work. Without such construct, the discussions would only had been qualitative.
 - b. The fact that the disequilibrium gaps of NNP at factor costs and NNP at market prices, saving and investment and finally the labour market gap, all are equal (numerically) is not a matter of coincidence neither construct. With the help of numerical example, it becomes all the more clear that these gaps exists and the numerical helps to quantify the amount of deficit financing to be done by the state.
 - c. The purpose of using numerical examples gets satisfied through the logic of mathematical induction. While this work has resorted to using a single numerical example across chapters, it is done only to provide continuity and relevant points of comparison to the readers. However, multiple simulations (over 1000 different model equations) have been tested to provide similar results.

- d. In so doing the simulations, all that is required is the following conditions should hold true:
 - i. Hawkins-Simon conditions for stocks, flows and output values
 - ii. Kaldor Passineti theorem that MPS of capitalists is more than that of workers
- e. If these conditions are honoured, any model produced on these lines will provide same results and hence, though the construct is numerical, it is also easily worthy of generalization

Limitations of the proposed theory:

Finally, as we conclude, it would only be imperative to present certain limitations of the presented model/ theory.

- a. The ever predominant real balance effect plays no role in this system described so far. The real balance effect is seen to operate in the industrial equations where (real) money balances are held by entrepreneurs in the process of production; however, the consuming class does not have a money balance variable in the consumption functions. The reason for this dichotomization is obvious in the fact that the presented theory is fairly and to a large extent empirical in nature. This empiricism leads one to search for an empirical relationship for the consumption functions that involves real money balances. Such an empirical relationship is absent from the present economic literature.
- b. Almost all the markets are explicitly states, except the labour market. By explicit statement, we mean the famous Marshallian cross here, where labour demand and labour supply interact to determine the price of labour. Such a formulation is absent from the theory. However, we have presented the labour market in a fairly subtle manner. The famous

$$N_d = f \left[\frac{w}{p} \right], N_s = f \left[\frac{w}{p} \right] \& N_d = N_s$$
 is a macroeconomic formulation and

applies where there is one good; however in a general equilibrium system N_d , i.e. labour demand would come from various sources. Every entrepreneur is employing/ retrenching labour. Therefore, the famous cross of partial economics is also absent. If one carefully looks at the growth-profit relation, L_s i.e. supply of labour is present and in the dual relations, individual L_d ie labour demand is present. Thus, L_s has an impact on the rate of profits, r, and L_d has an impact on the rate of growth, g.

- c. The role of uncertainty and expectations is absent in the theory. However, whether one needs such a role is a question of epistemology in the current context. Even without providing for the assumption of uncertainty and expectations, it is shown that the desired results are obtained; those of presenting a disequilibrium in a monetary economy and the means of addressing the same. Even if we assume that such a role for uncertainty is provided for, a qualitative account of the scenario can be provided- in the face of uncertainty, people tend to hoard money balances. The end result of this would be that the Keynesian gap would increase and the result similar to increasing money demand would be obtained. As a consequence, no new result would be achieved by assuming for the role of uncertainty and expectations, except for the fact that if one assumes the same, the qualitative aspect would only be enhanced quantitatively.
- d. Real balance and wealth effects are not considered in the theory; however, the theory is robust enough for someone who intends to consider the same. the theory also leaves out the principles of international values and is out of scope for the current work.
- e. Finally, Graham had proposed a model of commodity reserve currency during the later stages of the American depression. However, owing to high transaction cost and supply conditions, such a model was not adopted. The exposition of such a currency in the theory is not been considered

Annexure to Chapter 1: Survey of Classical, Neoclassical & Monetary Theories

Section I: Theories other than Keynesian General Theory

Karl Marx's attempts at providing a theory of value had repercussions for a theory of money and (hence) a theory of business cycles that developed through it. The most important point to emerge from Marx's theory of money is the idea that money is a form of value. The difficulty with this idea is that we are more familiar with money itself than with value in other forms. But value does appear in forms other than money. For example, the balance sheet of a capitalist firm estimates the value of goods in process and of fixed capital which has not yet been depreciated, as well as the value of inventories of finished commodities awaiting sale. Each of these aggregations of commodities has a value, usually expressed as the equivalent of a certain amount of money, but it is clear that neither goods in process nor fixed capital is money. Marx views the value of commodities in this sense as analytically prior to money; money can be explained according to Marx only on the basis of an understanding of the value of commodities. Marx follows Smith in regarding value as the property of exchangeability of commodities. In a society where exchange is common, products come to have a dual character as use values and as values. They have two powers: first, to satisfy particular human needs and wants; and second, to exchange for other products. This second power can be thought of quantitatively, as an amount of exchangeability or command over other commodities. The classical economists viewed value as a real, though socially determined, entity, with its own laws of conservation and motion. Value in this sense bears the same relation to commodities as mass bears to physical objects. Marx regards value, the general power of exchangeability that resides in commodities, as an expression of the labor expended in the production of the commodities. Marx was clever in describing that it would not matter if one attributed this general exchangeability to any one commodity, say gold, and start treating it as money. The only caution that he had advised, which is noteworthy is that while attributing the *moneyness* to any one commodity, it should be borne in mind that the commodity under consideration itself has two values: one its value in use and secondly its value in circulation. Whilst a commodity is used in circulation, it should never be used up in the process. This is the puzzle Marx sets himself to resolve in his discussions of the money form in the first pages of Capital, and in his Contribution to the Critique of Political Economy. How can gold

simultaneously be a concrete commodity and the form of money? If we use the word "labor" for the more accurate phrase, "abstract, socially necessary, simple labor," this theory suggests that the value in aggregate collections of commodities is proportional to the quantity of labor expended in their production. This proportion is very important to the theory of money, because it implies that each unit of money value can be regarded as expressing a certain amount of labor time. I call this ratio the "value of money," the amount of social labor time expressed on average by a unit of money. (This idea should not be confused with the concept of the "value of the money commodity", which is the labor time embodied in a unit of a particular commodity that may be functioning as money.) The value of money is not the inverse of the wage rate in a capitalist system of production; it is the ratio of the total labor time expended to the total value added in the commodities produced. The average wage rate is the ratio of aggregate wages, which are only a part of the value added, to the total labor time. The integrity of the idea of value, however, requires us to think of exchange as a process which conserves value. This means that although one trader may gain and another loses in exchange; no value is either created or destroyed. The sum of the values they begin with is the same as the sum they end up with; what one gains the other loses. This law of the conservation of value is of the utmost importance in grasping Marx's use of the theory of value in analyzing capitalist production. When we apply the idea of value separate from price to transactions involving money, the concept of the value of money, the ratio of total labor time to total value added, plays a central role. Only with this convention for defining the value of money will we be able consistently to maintain the ideas that money is a form of value; that value is conserved in exchange; and that the expenditure of labor creates value. It is unfortunate that the general equivalent theory suggests that the value of money is always determined by the conditions of production of the money commodity. In the development of Marxist theory the problem of the determination of the value of money separate from the value of the money commodity has not attracted much attention. Most Marxist theorists assume that the problem of the value of money has been settled by the general equivalent theory and the idea of the standard of price. They see no substantial difference between the value of money and

the value of the money commodity. The moral of this thought is simple: the seeds of a quantity theory consistent with a Walras' law could be found in Marxian analysis of money. The law of conservation of value, in its modern parlance, assumes the form of a Patinkinsque or a Clowerian dual decision hypothesis that we shall touch upon sooner. What is lacking in Marx's theory of money is one crucial aspect: the role of credit or of money of the future periods. The value of money is not determined only due to exchange of commodities or circulation of money, but also due to an important property of money being a store of value. Marxian monetary system takes cognizance of money being a medium of exchange but not of its store of value function. The second issue with Marxian monetary theory is already pointed out above. Where Marx highlights that the two values: use and exchange value of commodities need to be distinguished, he himself is unable to provide a logical reasoning to overcome this predicament. Marx's discussion of this issue in the second chapter of the Contribution suggests that the value of money depends ultimately on the conditions of exchange between gold and other commodities at the point of production of gold. Thus arbitrage, minting, and melting of gold coin for export seem to be the mechanisms Marx has in mind for maintaining the relation between the value of the money commodity and the value of money. It is important to recognize that this arbitrage is costly, and works only up to a pointing any commodity-producing society; there is always some margin within which the value of money can vary in relation to the value of the money commodity. Thus there is always some further question as to the exact determination of the value of money.

Commodities have inherent in them a natural value to remain in existence for a definitive period of time. This is often regarded as their value in store. Though, as described above, Marx ignored this, Wicksell was apt in recognizing this in his monumental work⁶². Own rates of interest of a particular commodity was ingenious and Wicksell could only have come up with that. Knut Wicksell's (1898, 1906) theory of the "cumulative process" of inflation remains the first decisive swing at the idea of money as a "veil" as well as Say's Law. The Quantity Theory still held in his system, but the dynamics of adjustment of prices to money supply, the "reason" for the

⁶² Wicksell, K (1898), Interest and Prices (Great Britain: R & R Clark Limited, Edinburgh)

Quantity Theory to hold, is fundamentally based on money having very real short-run effects. Recall that Fisher's Quantity Theory spoke of exogenous increases in supplies of money leading to "bidding wars" for commodities, as agents try to get rid of excess money holdings, thereby raising their prices. However, as Wicksell noted, there was nothing inherent in the neoclassical theories of value and output which implied any of this could make sense. In fact, he clearly recognized that Say's Law, which prevents aggregate demand for goods and factors from exceeding real aggregate supply under all circumstances, implied that the Quantity Theory mechanism was contradictory. A general rise in prices is therefore only conceivable on the supposition that the general demand has for some reason become, or is expected to become, greater than supply. This may seem paradoxical, because we have accustomed ourselves, with J.B. Say, to regard goods themselves as reciprocally constituting and limiting the demand for each other. And indeed ultimately they do so; here, however, we are concerned with precisely what occurs, in the first place, with the middle link. Any theory of money worthy of the name must be able to show how and why the monetary or pecuniary demand for goods exceeds or falls short of the supply of goods in given conditions⁶³. We can see this differently. Say's Law says that real aggregate demand (Yd) is derived from real aggregate supply (Ys), thus Yd = Ys at all times. Yet, in a Walras' Law constraint, we must remember that (Yd - Ys) + (Md - Ms)/p = 0. where Md and Ms is money demand and supply respectively. Thus, by Say's Law, left side falls to zero, and thus Md = Ms at all times: there can never be excess or insufficient money supply necessary to make the Quantity Theory work. We can look at this in terms of investment and savings. Now, by definition, Yd = C + I + G where C is consumption, I is investment and G is government spending and Ys = C + S + T where S is savings and T is taxation, then assuming a balanced government budget, (G=T), to claim that Say's Law states that Yd = Ys at all times is the same as saying that I = S, i.e. investment is equal to saying at all times. Our Walras's Law constraint becomes (I - S) + (Md - Ms)/p = 0. which is identical to our previous constraint. However, again, by Say's Law, I = S so that necessarily Md = Ms, i.e. money demand is always equal to money supply. This way we can see the

⁶³ Wicksell, K (1906): Lectures on Political Economy, Vol.2, pp.159-60

force of Wicksell's criticism of Say's Law and its inoperability in a theory of money. Say's Law is in essence "dichotomy" as it separates the real and monetary sides completely - i.e. disequilibria in money markets cannot spill over into disequilibria in goods markets. But then, Fisher's whole story of the Quantity Theory arising from a "bidding war" for goods as a result of an excess supply of money is precisely why Fisher contradicted himself: as Wicksell claims, you cannot simultaneously assume Say's Law and the Quantity Theory. This fundamental insight of Wicksell's was resurrected in the Patinkin Controversy of the 1950s and 1960s. Wicksell's process has its roots in that of Henry Thornton (1802). Recall that the start of the Quantity Theory's mechanism is a helicopter drop of cash: an exogenous increase in the supply of money. Wicksell's theory claims, indeed, that increases in the supply of money leads to rises in price levels, but the original increase is endogenous, created by the relative conditions of the financial and real sectors. With the existence of credit money, Wicksell argued, two interest rates prevail: the "natural" rate and the "money" rate. The natural rate is the return on capital - or the real profit rate. It can be roughly considered to be equivalent to the marginal product of new capital, therefore let us simply call it r. The money rate, which we shall refer to as i, in turn, is the loan rate, an entirely financial construction. Credit, then, is perceived quite appropriately as "money". Banks provide credit, after all, by creating deposits upon which borrowers can draw. Since deposits constitute part of real money balances, therefore the bank can, in essence, "create" money. This idea was put simply in later years by Dennis Robertson. By a wave, apparently, of the bank's magic wand the farmer and his men [the borrowers] have been enabled to live for six months at the expense of the rest of the community: the bank has give them a claim on the community's real income of food and clothing and tools and cinema shows. And for rendering this service to the farmer the bank charges him something called 'interest'. Our first impulse surely is to *cry out on the whole proceeding as a piece of fraudulent legerdemain*⁶⁴. Indeed it might be considered a "sleight-of-hand". But, as Robertson and Wicksell go on to note, without this type of "fraud" one remains constrained by Say's Law - and this is inconsistent with the implied "bidding war" mechanism of the Quantity Theory. It is

⁶⁴ Robertson, D. H. (1922): Money, Volume 2 of Cambridge Economic Handbooks, pp.71

finance, Wicksell argued, which liberates investment from a given supply of saving to become the wild card that can take aggregate demand above (or below) aggregate supply - a maneuver which anticipates and influences Keynes. Wicksell's "cumulative process" works as follows. Put simply, the finance demand for money is set by the difference between the money and natural rates of interest. Let us propose that the natural rate is greater than the money rate (i.e. r > i). In short, the marginal product of capital is greater than its cost. Consequently, it will be to the advantage of every entrepreneur to borrow funds from the bank and invest it in capital. That means I > S, i.e. finance investment will rise above savings as the bank, by its "magic wand", can create the deposits upon which borrowers can draw. In short, the money supply increases as a result. Now one may accept that investment is independent of savings at least initially. Banks, after all, give credit out first and then verify if the funds are available. Thus, like Keynes and unlike modern Neoclassical economics, Wicksell does not think investment is constrained by savings. But eventually, surely, the savings have to come eventually to equality - the goods market must eventually clear. Keynes had his multiplier to do this. What did Wicksell have? Wicksell actually had no self-correcting mechanism other than a reserve constraint. The logic works as follows: when r > i, then I > S. This extra investment demand then bears down on the capital goods industry. Assuming full employment, the extra demand for capital goods by loan-backed entrepreneurs cannot be met by the makers of capital goods. On the contrary, the extra volume of demand will have to be siphoned off by raising the price of capital goods. But just as they rise in the capital goods industry, so too must they rise elsewhere - including consumer goods and, as a result, wage demands by workers. A spiral ensues, a "cumulative process" whereas prices will rise and rise without limit as long as loan-backed entrepreneurs keep borrowing from the banks and coming to market. And they will continue doing so as long as the natural rate of interest (the marginal product of capital) remains above the money rate of interest (the loan rate). Thus, the demand for loans will continue accumulating, and the banking system's deposit creation forthcoming, indefinitely - with savings never really catching up. Money supply will expand endogenously without limit and prices will rise also without end. Nonetheless, adhering to Wicksell's main thesis, the

disequilibrium engendered by real changes leads endogenously to an increase in the demand for money - and, simultaneously, its supply as banks try to accommodate it perfectly. Given full employment, (a constant Y) and payments structure (constant V), then in terms of the equation of exchange, MV = PY, a rise in M leads only to a rise in P. Thus, the story of the Quantity Theory, the long-run relationship between money and inflation, is kept in Wicksell. Finally, the endogenous creation of money, and how it leads to changes in the real market (i.e. increase real aggregate demand) is fundamentally a breakdown of the Neoclassical tradition of a dichotomy between monetary and real sectors. Money is not a "veil" - agents do react to it and this is not due to some irrational "money illusion". However, we should remind ourselves that, for Wicksell, in the long run, the Quantity Theory still holds: money is still neutral in the long run, although to do so, we have broken the cherished Neoclassical principles of dichotomy, money supply exogeneity and Say's Law.

Simon Newcomb's and Irving Fisher's Quantity Theory, as we noted, relies entirely on the idea of a stable transactions demand for money. This requires that money is desired only for its medium of exchange function and this is institutionally imposed. An alteration on this point was brought in by several Cambridge economists in the earlier part of this century. In particular, A.C.Pigou (1917), Alfred Marshall (1923), D.H. Robertson (1922), John Maynard Keynes (1923), R.G. Hawtrey and Frederick Lavington (1921, 1922). These were the joint creators of what has since become known as the "Cambridge cash-balance" approach. The proposition they advance is that money is desired as a store of value. The Cambridge story, then, is fundamentally different from the Fisher story. In Fisher, money is desired by agents in some fixed amount solely because it happens to be the medium of exchange. As Fisher noted, money yields no gains to the holder. However, in the Cambridge story, this is not the case. Money does increase utility in a way: namely, by enabling the divorce of sale and purchase as well as a hedge against uncertainty. The first reason resembles that outlined by Adam Smith, W.S. Jevons (1875) and Carl Menger (1892) - where money is necessary to overcome transaction costs and coincidence of wants problems. As they note, in simultaneous, multilateral exchange with no transaction costs, the need for money by traders is not apparent. The advantage of money, in that it overcomes

the need to obtain coincidence of wants; it implies that an agent can sell his good at one time for "money" and then extend his leisurely search for the best price, then trading his "money" for the goods he finally wishes to purchase. The Cambridge lesson is that the sale and purchase of commodities are not simultaneous and thus there is a need for a "temporary abode" of purchasing power, i.e. some temporary store of wealth. In particular, A.C. Pigou (1917) also allowed for money demand to involve a precautionary motive - with money holdings acting as a hedge against uncertain situations. As it is in its store-of-wealth and precautionary modes that money yields utility to the consumer, then it is demanded for itself in a way. How much of it is demanded depends partly on income and partly on other items, notably wealth and interest rates. The first part is obviously implied in transactions terms: the higher the volume of income, the greater the volume of purchases and sales, hence the greater the need for money as a temporary abode to overcome transactions costs. Thus, Cambridge theorists regarded real money demand as a function of real income, i.e. M/P = kY where k is the famous "Cambridge constant". However, this is really misleading for the "constant" k is not constant at all. Rather, it relies on other components, such as interest (the opportunity cost of money) and wealth. We can compare this to Fisher's system by simply recognizing that real income (Y) and transactions (T) are, in equilibrium, identical. Of course there are transactions in wealth (e.g. the sale of existing assets such as a house) which do not count as part of income or output proper since they are only transferrals of ownership. The way around this is, as Pigou (1927) notes, is to recognize that, properly valued, the sale value of a home is really the discounted value of rents (which are income). Thus, the transactions in wealth represent transactions in discounted streams of income. Thus, we can claim that at least in some long-run, perfect world, T = Y. Therefore we can rewrite Fisher's equation as M/P = (1/V)Y, such that k = 1/V. Thus, in sum, one equation can be implied from the other. However, the theories are quite different. Firstly, money is here conceived in store-of-value, uncertain, utility-yielding terms. In Fisher, it was just the institutional medium-of-exchange that enabled transactions. Secondly, they advanced the possibility that k (and thus V) is not necessarily instutitionally fixed but rather changing. However, the dichotomy between the real

and monetary sectors cannot really be said to have been broken down by this given the ambiguity as to what is contained in k - and their creators' reluctance to make much of this (see Patinkin, 1974). More than anything else, they considered the issue of uncertainty and confidence entering k and thus leading to real fluctuations - an idea which had already been contained in Marshall (1890: 591-2). However, this explanation lacked deterministic power for they placed forth no theory of expectation formation in such circumstances - and therefore, as a theory of fluctuations, it can be regarded (however stretched) as a short-run phenomena. But this is not very interesting. Indeed, had not Fisher's (1911) credit cycle and his "dance of the dollar" demonstrated the breakdown of the Quantity Theory in the face of short-run adjustment costs? Nonetheless, the main points of the Cambridge approach were two: (1) neutrality remains but dichotomy is doubtful; (2) money yields services and is demanded by choice

Utility theory is an important value theoretic concept that had started evolving due to the likes of Marshall, Jevons, Walras during these times. Walrasian pure exchange refers to a price-mediated exchange process of endowments of goods, i.e. no production as initially outlined by Jevons. To put it simple: People are endowed with goods and have preferences over bundles of goods and so may desire to exchange the goods they are endowed with for other goods. People don't trade with each other (they don't even "look" at each other), but rather, they trade exclusively with an abstract entity called a "market". (i.e. if we want people to actually look at and try to trade directly with each other, we must turn to non-Walrasians exchange processes). People take the prices announced by the market as "given" and make their net demands and offers to the market in full confidence that these will be met at the stated prices (i.e. people do not make "strategic" offers or demands in an effort to get the market to change its prices). In order to demand goods from the market, they must offer it goods which have the same monetary value (i.e. they must "sell" something to the market in order to get purchasing power to "buy" something from it). If the market cannot balance the offers of goods with the demands for goods it receives from consumers, then there is a disequilibrium. Under the process of tatonnement, there will be no trade and another list of prices will be announced by "the market" where

the price of those goods that were in excess demand will be raised, and of those goods that were in excess supply will be lowered, and the process begins anew. (under nontatonnement processes, as many trades as possible will be conducted before it becomes necessary to announce a new set of prices). If the market can balance the offers and demands so that they net out to zero, only then will the market fulfill the demands and pay for the offers. This is a Walrasian equilibrium. But again the question remains: where is money in the system? A general equilibrium monetary theory in the true modern or non-classical sense has this biggest limitation. That does not, in any manner, ask the question: How to make money appear without making standard theory disappear?⁶⁵ – or there are also statements of the fashion: the most serious challenge the existence of money poses to the theorist is this- even the best developed models of the economy cannot find room for it⁶⁶. John Maynard Keynes attempted a formal theory of money first with his Treatise on Money and then with his General Theory. What Keynes tried to achieve was truly a "Monetary Theory of Production⁶⁷". Keynes identified a monetary economy as a one in which expectations of the future influence decisions taken today, or, one in which money is a subtle device for linking the present and the future, or one in which production begins with money on the expectation of ending with more money later. *Finally, a monetary* economy is also a one in which Say's law need not hold because of the existence of a non-produced sink-hole of purchasing power⁶⁸. Keynes was himself aware of the dichotomy the contemporary monetary theory presented. 4. The discussion around forces which determine physical output and the determination of price-level has traditionally been regarded as separate departments in economics. In the later department, money has played the dominant role. The total volume of transactions effected with a given stock of money, M, in a given unit of time is T. The average velocity or rapidity of money circulation is V and P is the general index of prices which enter into the transactions T. Therefore, MV=PT. This famous and yet so

⁶⁵ Ostroy, J.M.(1973), "The informational efficiency of monetary exchange", *American Economic Review*, Vol 63 (No.4): pp. 597-610

⁶⁶ Hahn, F.H. (1982), "The Neo-Ricardians", *Cambridge Journal of Economics*, Vol. 6, pp. 353-374.

⁶⁷ See section II below for a survey of General Theory

⁶⁸ Wray, L.R. (1999), "Theories of Value and Monetary Theories of Production", WP 261 (Jerome Levy Economics Institute)

simple identity (or equation?) is known as the Yale equation of the Quantity Theory of Money (QTM), in honour of Irving Fisher. The theory is also presented in the form of proportion, k, of the value of what Pigou called the 'total resources enjoyed by the community' which the public desire to hold in form of money. These total resources Pigou denoted by R, so that, M=kPR. This form of the QT equation is known as the Cambridge equation, in honour of Marshall and Pigou. Though the Yale and the Cambridge versions can be converted through algebraic manipulations in either form, the underlying concepts are rather different. Fisher and the Yale school thought of money as a means of effecting transactions; Pigou thought of money as a form of holding wealth necessary for effecting the ordinary transactions of life without trouble. Kahn has rightly pointed out that in either form, since the QTM recognizes that alternate liquid assets can well be used as substitutes to money this is my reason for maintaining that the only sense in which the so-called quantity theory can be given a casual is not really a quantity theory at all but an exercise in portfolio theory. No wonder, Kahn records that with respect to the various versions of QTM, the Yale equation is a truism, and the Cambridge equation a delusion. John Locke and Hume believed in this relation to an extent as well, primarily due to the reason that the monetary theory of their times was concerned with commodity money systems. Locke explains wrote Keynes that money has two values: its value in use given by the rate of interest and its value in exchange...but he was confused. Professor Arthur Leigh also maintains that Locke's theory of money's value in exchange contains all the elements of Fisher's equation. To interpret his theory of demand for money, the Cambridge equation is also useful. David Hume, sixty years later was not really a quantity theorist at all. In his treatment, there is a causal factor. It is an increasing a stock of money, which so long as the increase continues, raises the level of demand. In its modern form, the QTM was attributed by Marshall to J.S. Mill. Schumpeter emphasizes Mill's claim to be the first strict quantity theorist in the strict sense. The following passage, quoted by Kahn, from Mill's Principles of Political Economy expands the doctrine as he saw it. the value of money[...] varies inversely as its quantity; every increase of quantity lowering the value, and every diminution raising it, in a ratio exactly equivalent. [....] If we assume the quantity of goods on sale, and

the number of times those goods are resold, to be fixed quantities, the value of money will depend upon its quantity, together with the average number of times that each piece changes hands in the process.... And the quantity of money in circulation is equal to the money value of all the goods sold divided by the number which expresses the rapidity of circulation. Marshall was prompt and quick in establishing that the Mill's system had a weak link in the rapidity of money circulation. Marshall's version of the QTM is best examined through the eyes of Keynes; and to this account, Keynes' early writings are best evidences. We would return to this topic in a short while. Returning to the Cambridge versus the Yale debate, it was mentioned that the two equations can be algebraically translated into each other. However, even Keynes failed to take cognizance of the fact that mere ease of algebraic translation does not mean that the two equations are the same. In fact Keynes pointed out that it comes out to the same thing in the end and it is easy to pass from the above formula to Professor Fisher's. Pigou suggested the real advantage, because it brings us at once into relation with volition-an ultimate cause of demand. Dennis Robertson as well could not keep himself away from the most amazing debates of his times. In order to secure the symmetry between his exposition of the Yale and the Cambridge concepts, Robertson suggested a proportion of annual real turnover as an alternative to the proportion of real national income.

Section II: A Survey of General Theory

Economic thought, especially after the 1920s took a significant turn with the likes of Keynes, Marshall, Robertson, Fisher, Pigou and to an extent Sraffa becoming the fore-runners of the economic theory. Major changes were seen in the thought process of these great minds during the course of years to come after 1920s. To this fact, even Keynes was no exception. His thought and works underwent significant changes through the years 1924-1939. There were several changes in the line of thought; more so, there were attitudinal changes especially post 1930s, as Keynes himself points it out. The General Theory of Employment, Interest & Money (GT) is undoubtedly the greatest efforts in the history of economic thought towards providing a general theory

involving the three variables- employment, interest and Money. Evidences for this shift in attitude can be found right from the preface of the GT. In the preface, Keynes warns us that the GT is an attempt at dealing with the difficult questions of the theory and only in second place, test the applications of this theory to practice. Keynes goes ahead and makes another point clear in the preface itself: his distinction between the classicals and the neo-classicals. Having said this, it should therefore be noted that the GT is an attempt not to find that if the orthodox economics is at fault, the error is to be found not in the superstructure, which has been erected with great care for logical consistency, but in lack of clearness and generality of premisses⁶⁹. This contention of Keynes should become clear in a while from now. Keynes himself was aware that those, who are strongly wedded to what I shall call "the classical theory", will fluctuate, I expect, between a belief that I am quite wrong and a belief that I am saying nothing new⁷⁰. All this while, Keynes was perhaps hinting at an important point: that orthodox economics was good, but the exposition was just not good enough. However, this may not be true as well: Keynes himself attacked the postulates of classical economics in his first chapter in the GT. It is important to note however the historical context in which the GT was written. Wide-spread depression and chronic recession was the order of the day. Keynes was busy drafting the report of the Macmillan committee and also making his taxations dictums public. Also, he had just finished writing an epic putting in six years of his life: A Treatise on Money. Given this set-up, the GT surely achieved its purpose of providing prescriptions for the distressed economy. And to help him out in this distressed conditions, Keynes relied on the expertise of R. F. Kahn, Dennis Robertson, A.C. Pigou and Mrs. Joan Robinson to an extent. All of these eminent scholars have left considerable impact on the attitude of Keynes and significantly led to the publication of the GT. Since the times of the Treatise, Keynes was aimed at developing a pure theory of money. The organization of the treatise is no better an evidence of this statement. Keynes begins the treatise with a proper definition of money where in the various forms of money are explained. He then moves on to concentrate on the fact that there are primarily

⁶⁹ Keynes, J.M. (1934), *General Theory of Interest, Employment & Money*, pp v (Cambridge University Press) pp v

⁷⁰ Ibid

four types of money: commodity money, managed money, fiat money and bank money. Of these, Keynes (1930) suggests that the first three are money-proper and the fourth one is not money-proper but an acknowledgement of debts. When acknowledgements of debts are used in this way, we may call them Bank Money- not forgetting however that they are not Money Proper. Bank Money is simply an acknowledgement of private debt expressed in the money-of-account which is used by passing from one hand to another, alternatively with Money-Proper, to settle a *transaction*⁷¹. Here, Keynes recognizes that the amounts or transactions in the form of loans or debts are also to be regarded as money. The second chapter of Treatise talks about bank money, creation of bank money and other forms of bank money. This is an important chapter as it lays down the fundamental role of banking in an economy. To Keynes, creation of deposits by the bank is in a way accepting to honour a claim some time at a future date. The Treatise also lays down the fundamental properties of a banking system which can move ahead in an economy. The third chapter of the Treatise is an important chapter that details the analysis of bank money. This chapter carries some important bearings for this work as well. Keynes identifies three major reasons of the public to hold money and these three reasons give rise to three specific types of bank accounts. These accounts are income deposits, business deposits and savings deposits. A savings deposit also corresponds to what used to be called in theories of money, which were stated with primary reference to commodity money, the use of money as a store of value. This question of the value of money bothered Keynes significantly during the Treatise. The quantity theory of money was at the centre point in this botheration. As a result, taking actual data from business deposits and income deposits, Keynes aimed at ascertaining the "velocities of circulation⁷²" of income and business deposits. In the two chapters that follow, Keynes describes another fundamental (missing) link in the orthodox theory. It is concerned with measuring the value of money. Some kind of an index number normally would have been instrumental in providing this answer. The question however to Keynes was not the construction of such an index, but which index is accurate! Edgeworth supposedly

 ⁷¹ Keynes, J.M. (1930), A Treatise on Money: The pure theory of money, pp 6 (Macmillan)
⁷² Ibid, p 49

is to blame for different types of price-levels. Edgeworth distinguished six different standards of leading types- the Capital Standard, the Consumption Standard, the Currency Standard, the Income Standard, the Indefinite Standard and the Production Standard. Forty years later, Edgeworth classified index numbers in three leading types- Index numbers representing welfare, un-weighted index numbers and the labour standard. This plurality of index numbers, or the value of money, or the purchasing power of money was one of the issues of orthodox economics that Keynes aimed at resolving. Keynes had devoted an entire chapter to the value of money in his Treatise. To Keynes, the fundamental problem of Monetary Theory is not merely to establish identities or statical equations relating (e.g.) the turnover of monetary instruments to the turnover of things traded for money. The real task of such a Theory is the treat the problem dynamically, analyzing the different elements involved, in such a manner as to exhibit the causal process by which the price level is determined, and the method of transition from one position of equilibrium to another⁷³. As a matter of fact, to Keynes during the writings of Treatise, the natural doctrine of the orthodox theory was more appealing and hence, his monetary theory of the Treatise was more towards the investigation of the equilibrium price level or more so, in determining the (equilibrium) value of money. On the train to this journey, Keynes made significant discoveries and broke away from the shackles of the received doctrine: more so, in 18 months from publishing the Treatise, the GT had begun taking shape.

The utility of the wage when a given volume of labour is employed is equal to the marginal disutility of that amount of employment. The GT sets out the context through the denials of the classical postulates. The one we aim to start off with is the second classical postulate that Keynes denies. The principal links in Keynes' argument are the following: a.) the classical theory recognizes only voluntary employment. It is necessary to concentrate on three key factors- how very widely Keynes defined voluntary employment; his concern was entirely with the residual category of involuntary employment. With regard to the definition of involuntary employment, Keynes followed up his definition of second postulate by noting that

⁷³ Keynes, J.M. (1930) Ibid

disutility must here be understood to cover every kind of reason which might lead a man to withhold their labour rather than accept a wage which had to them a utility below a certain minimum. The list of "every kind of reason" supplied by Keynes is very long. Most importantly here, the second postulate:is also compatible with "voluntary" unemployment due to the refusal of a unit of labour of slow response to change or of mere human obstinacy, to accept a reward corresponding to the value of the product attributable to its marginal productivity. Leijonhufvud describes this definition of unemployment as "income-expenditure" unemployment. It is not, I think, unfair to do so-Keynes' followers have had persistent difficulties in assigning a clear meaning to his definition of involuntary employment..... This to Keynes was a classical idea. Leijonhufvud further quotes a relevant passage from the GT: *Thus writers in the* classical tradition...have been driven inevitably to the conclusion...that apparent unemployment....must be due at bottom to a refusal by the unemployed factors to accept a reward which corresponds to their marginal productivity. A classical economist may sympathize with labour in refusing to accept a cut in its moneywage....; but scientific integrity forces him to declare that this refusal is, nevertheless, at the bottom of the trouble. The essence of GT is therefore the denial of the second postulate of classical economics by Keynes. Keynes had two separate objections to the second Classical postulate and the denial of the possibility of involuntary employment that it implied. The first of this relates to the actual behavior of labour. It concerns the resistance to money wage cuts. All that Keynes needed to assert is that the worker who is threatened with a lay-off will not offer to take any cut necessary to retain his job. Nor, having been laid off, will he immediately resign himself to shining shoes or selling apples. With the train of thought towards understanding the GT rightly set out, it is important to summarize the classical postulates that Keynes rightfully denied: a. that the real wage is equal to the marginal disutility of the existing employment b. that there is no such thing as involuntary unemployment in the strict sense and c. that supply creates its own demand. Having analyzed the second one and understanding the fact that Keynesian definition of involuntary employment is a residual definition

of employment attempted by Keynes by lumping together frictional, seasonal and voluntary motives of job-seeking. The residual portion out of the totally ablebodied employable portion is the Keynesian involuntary unemployment (IU). Keynes in his GT has suggested that IU is chronic and present everywhere in every economy. Men are involuntarily employed if, in the event of a small rise in the price of wage-goods relatively to the money-wage, both the aggregate supply of labour willing to work for the current money-wage and the aggregate demand for it at that wage would be greater than the existing volume of employment. Keynes' theoretically fundamental objection to the classical theory of labour market is that it misrepresents the nature of wage bargain in presuming that it does not matter whether the analysis of wage determination is done is real terms or money terms; and that this point is pivotal to the current discussion around the scope, content and context of involuntary unemployment as well. Money wages do not affect the labour markets and instead it is the real wages that do so. Keynes recognized this early on and in his version of Pigou's theory of unemployment that real wages matter. The fact that workpeople in fact stipulate, not for a real rate of wages, but for money-rate is not ignored; but in effect, it is assumed that the actual money-rate of wages divided by the price of wage-goods can be taken to measure the real rate demanded. The attack on Pigou's theory of unemployment continues in the GT when Keynes is able to demonstrate that in the edifice of equations developed by Pigou, it is essential to assume that the labour is always in a position to determine its own real wage. This implies, as Keynes rightly points out, that the adjustments take place in the right spirit so as to preserve full employment. Without this assumption Professor Pigou's analysis breaks down and provides no means of determining what the volume of employment will be....His title the "Theory of Unemployment" is, therefore something of a misnomer. The attack on Say's law follows directly upon the definition of involuntary unemployment. There are two prongs to the attack. Both arguments dispute the same Classical notion: that excess supplies must have their counterpart somewhere in effective excess demands of the same total value. The indictment of Say's law is a topic of central theme for the Keynesian analysis.

This is also revalidated by Rogers through: *the distinction between a Say's law or a co-operative commodity money economy and a capitalist bank money economy proves to be fundamental to understanding monetary analysis. Needless to say, the distinction is lost in real analysis. The first odd feature of a Say's law economy is that individuals produce for themselves; hence they may demand their own outputs if they cannot sell it, and consumers and producers are identical. Therefore in terms of Say's law, productions buy productions. But, as both Marx and Keynes argued, this interpretation of production is not compatible with capitalist production. However, the important problem of Say's law when applied to a capitalist economy is that it implies that there is no limit to the profitable expansion of output.* This also therefore implies that output will expand therefore to a point of full employment. Therefore, denial of Say's law also implies denial of full employment and hence one flows from the other.

So long as economists are concerned with what is called the theory of value, they have been accustomed to teach that prices are governed by the conditions of supply and demand; and, in particular, changes in marginal cost and the elasticity of short-period supply have played a prominent part. But when they pass in volume II, or more often in a separate treatise, to the theory of money and prices, we hear no more of these homely but intelligible concepts and move into a world where prices are governed by the quantity of money, by its income-velocity, by the velocity of circulation relatively to the volume of transactions, by hoarding, by forced saving, by inflation and deflation et hoc genus omne; and little or no attempt is made to relate these vaguer phrases to our former notions of the elasticities of supply and demand. If we reflect on what we are being taught and try to rationalize it, in the simpler discussions it seems that the elasticity of supply must have become zero and demand proportional to the quantity of money; whilst in the more sophisticated we are lost in a haze where nothing is clear and everything is possible. We have all of us become used to finding ourselves sometimes on the one side of the moon and sometimes on the other, without knowing what route or journey connects them, related, apparently, after the fashion of our waking and our dreaming lives. One of the objects of the foregoing

chapters has been to escape from this double life and to bring the theory of prices as a whole back to close contact with the theory of value. The division of economics between the theory of value and distribution on the one hand and the theory of money on the other hand is, I think, a false division. The right dichotomy is, I suggest, between the theory of the individual industry or firm and of the rewards and the distribution between different uses of a given quantity of resources on the one hand, and the theory of output and employment as a whole on the other hand. So long as we limit ourselves to the study of the individual industry or firm on the assumption that the aggregate quantity of employed resources is constant, and, provisionally, that the conditions of other industries or firms are unchanged, it is true that we are not concerned with the significant characteristics of money. But as soon as we pass to the problem of what determines output and employment as a whole, we require the complete theory of a monetary economy. Quoting Keynes directly in this manner in this section would help us setting the motivation of this work right from the very beginning. A truly integrated theory of money and prices would therefore need and call for a role of money, not merely as a facilitator of exchanges between the individual agents but also help in determining the level of aggregate volume of outputs and the level of prices simultaneously. Whilst determining the level of prices, we are also determining the individual prices and hence, there seems to be an abrupt confusion to Keynes as well in regarding the classical dichotomy as genuine. The dichotomy however, as we will see further, had played an important role in the theory of value and it is to this role that we believe we can lay our finger on in distracting us from the point- an integration of monetary and value propositions. For when an economy operates, it never determines an absolute level first, then a monetary level is set only to determine the absolute level of prices- this is ridiculous! Money has a far better role in the economy other than merely being a medium of trade. Keynes was late in seeing this; notwithstanding to say that finally he saw this. The first intimation that Keynes provided while declaring that he was publishing a new book came in the preface of Japanese edition of the Treatise, dated 5th April 1932. *I propose [....] to publish a short book on a purely*

theoretical character, extending and correcting the theoretical views as set forth in books III and IV below. ['The Fundamental Equations' and 'The Dynamics of Price Level']. Such a book taking the *treatise* as its basis would be a waste of an attempt; more so the General Theory of 1935 was a much different book. Till 1932, Keynes had informed his mother: I have written nearly a third of my new book on monetary theory. Keynes' belief structure however was changing rapidly. Until 1929, Keynes delivered a set of university lectures titled 'Pure Theory of Money'. By the October term of 1932, he had changed it to Monetary theory of production'- an indication of the marked change in emphasis. With due help from Dennis Robertson, the true book that Keynes was also in search of saw the light of the day by 1936. During these formative years, Keynes had finally given up the classical idea of dichotomy, several classical concepts of wages and supply schedules and had truly integrated the monetary and real forces through marvelous designs like the multiplier, the principle of effective demand and the marginal efficiency of capital. However so, something was incomplete and we would provide a flavour of what it was. However, from a classical to a Keynesian, Keynes had to journey a lot- the final Keynes had given up the dichotomy and the general theory had capital, goods, bonds, labour all in the same model and everything determining everything else.

The discussion around forces which determine physical output and the determination of price-level has traditionally been regarded as separate departments in economics. In the later department, money has played the dominant role. The total volume of transactions effected with a given stock of money, M, in a given unit of time is T. The average velocity or rapidity of money circulation is V and P is the general index of prices which enter into the transactions T. Therefore, MV=PT. This famous and yet so simple identity (or equation?) is known as the Yale equation of the Quantity Theory of Money (QTM), in honour of Irving Fisher. The theory is also presented in the form of proportion, *k*, of the value of what Pigou called the 'total resources enjoyed by the community' which the public desire to hold in form of money. These total resources Pigou denoted by R, so that, M=*k*PR. This form of the QT equation is

known as the Cambridge equation, in honour of Marshall and Pigou. Though the Yale and the Cambridge versions can be converted through algebraic manipulations in either form, the underlying concepts are rather different. Fisher and the Yale school thought of money as a means of effecting transactions; Pigou thought of money as a form of holding wealth necessary for effecting the ordinary transactions of life without trouble. Kahn has rightly pointed out that in either form, since the QTM recognizes that alternate liquid assets can well be used as substitutes to money this is my reason for maintaining that the only sense in which the so-called quantity theory can be given a casual is not really a quantity theory at all but an exercise in portfolio theory. No wonder, Kahn records that with respect to the various versions of QTM, the Yale equation is a truism, and the Cambridge equation a delusion. John Locke and Hume believed in this relation to an extent as well, primarily due to the reason that the monetary theory of their times was concerned with commodity money systems. Locke explains wrote Keynes that money has two values: its value in use given by the rate of interest and its value in exchange...but he was confused. Professor Arthur Leigh also maintains that Locke's theory of money's value in exchange contains all the elements of Fisher's equation. To interpret his theory of demand for money, the *Cambridge equation is also useful.* David Hume, sixty years later was not really a quantity theorist at all. In his treatment, there is a causal factor. It is an increasing a stock of money, which so long as the increase continues, raises the level of demand. In its modern form, the QTM was attributed by Marshall to J.S. Mill. Schumpeter emphasizes Mill's claim to be the first strict quantity theorist in the strict sense. The following passage, quoted by Kahn, from Mill's Principles of Political Economy expands the doctrine as he saw it. *the value of money*[...] varies inversely as its quantity; every increase of quantity lowering the value, and every diminution raising it, in a ratio exactly equivalent. [....] If we assume the quantity of goods on sale, and the number of times those goods are resold, to be fixed quantities, the value of money will depend upon its quantity, together with the average number of times that each piece changes hands in the process.... And the quantity of money in circulation is equal to the money value of all the goods

sold divided by the number which expresses the rapidity of circulation. Marshall was prompt and quick in establishing that the Mill's system had a weak link in the rapidity of money circulation. Marshall's version of the QTM is best examined through the eyes of Keynes; and to this account, Keynes' early writings are best evidences. We would return to this topic in a short while. Returning to the Cambridge versus the Yale debate, it was mentioned that the two equations can be algebraically translated into each other. However, even Keynes failed to take cognizance of the fact that mere ease of algebraic translation does not mean that the two equations are the same. In fact Keynes pointed out that *it comes out to the* same thing in the end and it is easy to pass from the above formula to Professor Fisher's. Pigou suggested the real advantage, because it brings us at once into relation with volition-an ultimate cause of demand. Dennis Robertson as well could not keep himself away from the most amazing debates of his times. In order to secure the symmetry between his exposition of the Yale and the Cambridge concepts, Robertson suggested a proportion of annual real turnover as an alternative to the proportion of real national income.

I have called this book the General Theory of Employment, Interest and Money; and the third feature to which I may call attention is the treatment of money and prices. The following analysis registers my final escape from the confusions of Quantity Theory, which once entangled me. I regard the price level as a whole as being determined in precisely the same way as individual prices; that is to say, under the influence of supply and demand. Technical conditions, the level of wages, the extent of unused capacity of plant and labour, and the state of the markets and competition determine the supply conditions of individual products and of products as a whole. The decisions of entrepreneurs, which provide the incomes of individual producers and the decision of those individuals as to the disposition of such incomes determine the demand conditions. And prices-both individual prices and the price-level-emerge as the resultant of these two factors. Money, and the quantity of money, are not direct influences at this stage of the proceedings [.....] The quantity of money determines the supply of liquid resources, and hence the rate of interest, and in conjunction with other factors (particularly that of confidence) the inducement to invest, which in turn fixes the equilibrium level of incomes, output and employment and (at each stage in conjunction with other factors) the price-level as a whole through the influences of supply and demand thus established⁷⁴.. Towards the end of the General Theory, as Kahn rightly notes, Keynes provided a symbolic expression of four elasticities of response, which he wrote 'can be regarded as the generalized statement of the Quantity Theory of Money'. Keynes added: 'I do not myself attach much value to manipulations of this kind [...] I doubt if they carry us any further than ordinary discourse can.' From the days of treatise, Keynes' major predicament, as the documented literature points out to has been his long fight for the release from the shackles of the Quantity Theory. In the early drafts of the Treatise of Money, the Quantity Theory of Money continued for a time to dominate Keynes' thinking. Keynes' long struggle over a period of six years to produce a version of the Treatise worthy of publication was directed partly to an escape from the stranglehold of QTM in its crude form, Kahn notes. In the end Keynes was able to write that *The forms of quantity theory* [....] *are but ill adapted for this purpose* of exhibiting the casual process by which the price level is determined, and the method of transition from one position to another. [....] they do not, any of them, have the advantage of separating out those factors through which [....] the casual process actually operates during a period of change. Five pages later, Keynes wrote that the conclusions he drew from his Fundamental Equations are, of course, obvious and may serve to remind us that all these equations are purely formal; they are mere identities; truisms which tell us nothing in themselves. In this respect they resemble all other versions of the quantity theory of money. Their only point is to analyze and arrange our material in what will turn out to be a useful way for tracing cause and effect, when we have vitalized them by the introduction of extraneous facts from the actual world. Kahn quotes the following on the above passage: Keynes did not explain how the introduction of facts could convert a truism into a causal relationship. This is the first occasion on which Keynes admitted that the OTM is a truism. Nevertheless, Keynes seems to have

⁷⁴ Keynes, J.M. (1939), General Theory, Preface to the French edition, dated February 1939

been so much under the spell of the QTM that he could write about his fundamental equations as though they were versions of the QTM; although, up to this point in his book, the QTM does not figure in them in any sense. This documentation, we believe is sufficient to summarize that from 1924 to at least 1931 or so, Keynes had changed attitudinally. He had dropped the fascination of QTM (that cannot be missed by the reader of Treatise in chapter 14 of the book!) and a more *General Theory* had started taking shape in his minds. Keynes, in his treatise as well, had hinted of what was going to be the General theory of Interest: this was the liquidity preference theory- Keynes' attempt of reconciling the Treatise with the QTM. The liquidity preference theory explains how the quantity of money exercises a causative influence by helping to determine the rate of interest- or more generally as we would put it now, the state of credit and the price-level of securities, both fixed-interest and equities. Dating slightly back to the Keynesian Tract on Monetary Reform, Keynes noted that the QTM is fundamental. Four pages after his statement, Keynes denied the validity of the QTM, in the form in which it is normally presented, except in the long run in which we are all dead. The distinguishing feature of Quantity Theories is simply the idea that the most convenient method of analyzing income movements is to define a collection of assets, called money, and to organize the determinants of money income in terms of their effects on the supply of and the demand for money. One cannot require that the quantity theory should postulate either pure price-level adjustment or continuous constancy of velocity over time- if these criteria were imposed on short run analysis, we might well find that history is *devoid of pure quantity theorists*⁷⁵. Keynes could not accept the assumption that aggregate real output can be unambiguously defined. The price-theoretic matter therefore in Keynesian constructs revolves around the inducement to invest and the marginal efficiency of capital. As will be seen shortly, Keynes more advocated quantity adjustments than price movements. The denial of the QTM or Keynes' struggle to get out of the theory therefore, does not sound surprising.

⁷⁵ Leijonhufvud A (1968), On Keynesian economics and the economics of Keynes, (Oxford University Press) pp. 27

Talking about the *General Theory*, Kahn notes that *there is no separate compartment labeled 'monetary theory'. The quantity theory of Money had finally been abandoned.*⁷⁶ Keynes, therefore had finally moved on from a quantity theory approach to a theory of flows of money. In his chapter on the General Theory of the rate of Interest, Keynes begins with saying that whilst there are forces causing the rate of investment to rise or fall so as to keep the marginal efficiency of capital equal to the rate of interest, yet the marginal efficiency of capital is, in itself; a different thing from the ruling rate of interest. The schedule of the marginal efficiency of capital may be said to govern the terms on which loanable funds are demanded for the purpose of new investment; whilst the rate of interest governs the terms on which funds are being currently supplied. To complete our theory, therefore, we need to know what determines the rate of interest. In his quest for ascertaining the factors determining the rate of interest, Keynes puts his first finger on the major causes of holding money. It is here where Keynes actually discovered that liquidity and more so, parting with the liquidity could be regarded as one of the important causes of the rise of interest rates. To Keynes, thus the rate of interest at any time, being the reward for parting with liquidity, is a measure of the unwillingness of those who possess money to part with their liquid control over it. The rate of interest is not the 'price' which brings into equilibrium the demand for resources to invest with the readiness to abstain from present consumption. It is the 'price' which equilibrates the desire to hold wealth in the form of cash with the available quantity of cash;—which implies that if the rate of interest were lower, i.e. if the reward for parting with cash were diminished, the aggregate amount of cash which the public would wish to hold would exceed the available supply, and that if the rate of interest were raised, there would be a surplus of cash which no one would be willing to hold. If this explanation is correct, the quantity of money is the other factor, which, in conjunction with liquidity-preference, determines the actual rate of interest in given circumstances. Liquidity-preference is a potentiality or functional tendency, which fixes the quantity of money which the public will hold when the rate of

⁷⁶ Kahn R.F. (1984), *The making of Keynes' General Theory*, (Cambridge University Press), pp. 123

interest is given; so that if r is the rate of interest, M the quantity of money and L the function of liquidity-preference, we have M = L(r). This is where, and how, the quantity of money enters into the economic scheme. Therefore, in the Keynesian construct of the liquidity preference, the analysis boils down to understanding why such a thing as liquidity preference exists as a leader to the question of what determines interest rate. Keynes here suggests us returning to the ancient distinction between the use of money for the transaction of current business and its use as a store of wealth. In the later macroeconomic literature, however, the term liquidity preference has become synonymous with 'demand for money'. However, in this regard, we therefore thought it is important and look back at what Keynes was suggesting. Similarly, Keynesian definition of money is much broader in the sense that it included money as well as non-money assets. In the reasoning for the liquidity preference, Keynes highlights the notions of the opportunity cost of funds, or the cost of moving from cash to other forms of nonmoney assets. Therefore, including the interest theory in this discussion around money and value provides completeness to the argument. However, of late, the money demand function is usually conceived as a stable relationship between the demand for cash balances and the observed rate of interest. Econometric analysis suggests this is true as well. However true it may be from the lines of best fitting lines and technical statistics, Keynes definitely predicts that this relationship will be unstable in the longer run: the demand for money at a given level of income will not have definite quantitative relation to a given rate of interest of 'r;-what matters is not the absolute level of 'r' but the degree of its divergence from what is considered a fairly safe level of 'r'77. Over time, as Leijonhufvud suggests, opinions of this fairly safe level will be revised in the light of experience. In Keynes' theory, such revisions imply shifts in money demand function commonly used in Keynesian macro-models. Yet again, the Keynesian exposition of the theory of interest rate points out that the general theory is an explanation provided without resorting to any rigidities. Wage-price flexibility, interest rate flexibility and quantity adjustments are Keynesian explanations towards the general theory

⁷⁷ Leijonhufvud A (1968), *Ibid*
of interest, employment and money. The principle of effective demand is a logical extension of the flexible adjustments provided by Keynes. Lack of quantity adjustments and hence, under-full employment leading to *wasteful* government spends to bridge the gap provides the theory for unemployment. Whilst wage-price flexibility leads to the denial of the classical postulates of upward sloping labour supply curves, interest rate flexibility tackles and establishes the link for the money market. Commodity and labour markets adjust in commodities through the multiplier.

An increase (or decrease) in the rate of investment will have to carry with it an increase (or decrease) in the rate of consumption; because the behaviour of the public is, in general, of such a character that they are only willing to widen (or narrow) the gap between their income and their consumption if their income is being increased (or diminished). That is to say, changes in the rate of consumption are, in general, in the same direction (though smaller in amount) as changes in the rate of income. The relation between the increments of consumption which has to accompany a given increment of saving is given by the marginal propensity to consume. The ratio, thus determined, between an increment of investment and the corresponding increment of aggregate income, both measured in wage-units, is given by the investment multiplier. Keynes claimed that additional expenditure on public works can be financed by creation of additional money, instead of borrowings from the public, though if the programme is heavy, some pumping may be resorted to from the banking system. An important point is that the increase in employment is a result of necessarily an increase in the amount of money, contrary to what Robertson believed. Keynesian theory of multiplier is based on an important principle of marginal propensity to consume. The marginal propensity relates to money income and psychological reasons to consume. This implies that as income increases, the gap between income and consumption increases faster. The level of money income is determined simultaneously along with the system and hence, the dynamics of the Keynesian world can be ascertained using the variable, MPC. Changes in money income or the marginal propensities to consume lead to changes in income levels

in a defined manner. Estimating the MPC for the world, for USA, for Orange County, for males, for females etc has been always the spirit of many econometrically driven economists. Empirical estimations of propensity however can provide only a leader to the entire process of economic tatonnement. The 'Widow's Cruse' and the 'Danaid Jar' fallacy are also peculiar extensions of the propensity to consume principle. *There is one peculiarity of profits (or losses)* which we may note in passing, because it is one of the reasons why it is necessary to segregate them from income proper, as a category apart. If entrepreneurs choose to spend a portion of their profits on consumption[...] the effect is to increase the profit on the sale of liquid consumption goods by an amount exactly equal to the amount of profits which have thus been expended[....] Thus, however much of their profits entrepreneurs spend on consumption, the increment of wealth belonging to entrepreneurs remains the same as before. Thus profits, as a source of capital increment for entrepreneurs, are a widow's cruse which remains undepleted however much of them may be devoted to riotous living. Where on the other hand, entrepreneurs are making losses, and seek to recoup these losses by curtailing their normal expenditure on consumption, i.e. by saving more, the cruse becomes a Danaid Jar which can never be filled up. The consumption thus lies at the bottom of entire analysis of the multiplier and therefore the determination of income. The general theory at this point truly achieves its definitive character of being a theory formalizing money and value. So much for the story of John Maynard Keynes; so what went wrong? Or did we even ever ponder on that question. As students of economics, we were told that the GT was one of the greatest epics of the modern world. But even epics fail to achieve certain things. First, it would be prudent to point out the major limitations of the GT instead of heavily criticizing it. The book contains almost no reference to international trade and the problem of acceptable balance of trade or payments with a high level of activity. And yet, problems of macroeconomics, national and international, engaged Keynes for the most of his life. Keynes however provided a flavour of this in his chapter on 'Notes of Mercantilism' wherein he has touched a variety of historical topics. Another point worthy of mention here is the

formalization of the GT. Great minds like even Hicks tried to provide concrete boundaries, mathematical expositions, algebraic and geometric, but faced immense difficulties. Keynes' letter to Hicks' article on 'Mr. Keynes and the Classics' held a mild criticism though it had a friendly tone: at one time I tried the equations, as you have done, with I (Income) in all of them. The objection to this is that it over-emphasizes current income. In the case of inducement to invest, expected income for the period of investment is the relevant variable. Keynes' criticism clearly pointed towards the IS-LM model that Hicks had developed and claimed that it was a true exposition of the GT. The result has been that the elementary teaching of Keynesian economics has been a victim of IS-LM and related diagrams and algebra. It is tragic that Keynes made no public protest when they began to appear⁷⁸. Also, as John Robinson put it, modern teaching has been confused by Hicks' attempt to reduce the GT to a version of static equilibrium with the formula IS/LM. Hicks has now repented and changed his name from J.R. to John, but it will take a long time for the effects of his teaching to wear off. Of late, in 1973, Hicks has pointed out however that, the General Theory [....] provides a model on which the academic economists can comfortable perform their accustomed tricks. Haven't they just? With ISLM I myself fell into the trap. All said, the GT still awaits a more formalization of the conjectures pointed out by Keynes. Till date, the general theory stands as a badly written book. In his extreme hurry to bring out his propositions to the public, Keynes completely forgot and lost sight of the fact that what was going to come out was a strong integration of monetary and value theory. But many economists of his times believe that Keynes had a very little understanding of microeconomic tools. Though he made significant contributions to these through his index number theories and theory of forward markets, it can only be understood as his effort to develop building blocks for his macroeconomic structure. Mrs. Robinson echoes to this fact by pointing out an old canard: Gerald Shove used to say that Maynard had never spent the twenty minutes necessary to understand the theory of value. The assumption that Keynes lacked an adequate working knowledge of the value

⁷⁸ R.F. Kahn (1984), Ibid

theory grants the interpreter of the GT to read into in practically whatever he wants. To complete the confusion, L.R. Klein was found quoting: *as in the Treatise, Keynes did not really understand what he had written.* Keynesian literature has developed beyond the life and times of Keynes by people who claimed to understand Keynes and by even those who actually understood it. That literature is vast and ranges from Hicks to date. There is not enough space neither the need to document it here.

Annexure to Chapter 2: Sraffa's Economics

The analysis of the classical and the neo-classical theories of values can be put in two separate boxes. The classical theory of value is based more on the costs of production and class conflicts, which makes it an objectivist theory, which can be observed or economically measured using numbers. The marginalist notion of theory of value is merely a subjectivist notion, which cannot be observed and can only be indirectly measured. The fundamental logic for this distinction and a semantic shift in the notions of costs of production, or more generally the theory of value is that the two doctrines derive from two different views of nature and goals of economic theory. For the classical economist the goal was to discover the laws that determine the wealth of nations and determine income distribution among various social classes. For the marginalist the purpose is to determine the economic behavior of individual human agents and to determine equilibrium price of individual commodities. The fulfillment of the later goals requires the use of a subjectivist theory. The existing notions of cost calls for a relook or prefabrication in terms of the structure. In his two articles published around 1925-1926, Piero Sraffa was able to demonstrate the most important notions of his times: the relation between costs and quantity produces and the natural of extension of it- the laws of returns under competitive conditions. Coming to a point in his 1925 article, Sraffa highlights Clapham's 'empty economic boxes' and this is where he launches his attack on the mainstream economic thought. What these circumstances might be, from the point of view of variation of costs in relation to the variations in quantity produced, has not been established, so that the curiosity of anyone wanting to see the empty economic boxes of constant, increasing and decreasing returns filled with concrete industries, remains more than ever unsatisfied. Here, Sraffa is clearly hinting at the law of returns to scale but points out an important feature of the law- there are very few industries which in fact can be classified and may be well aligned to the law. Sraffa crisply in this article points out that this inability of true classification can be attributed easily to lack of data on costs, quantity or lack of genuine scholars to do so. However, it is not the case. It is simply the fact that fundamentals of the topic on which the law is based are itself shaky. In particular, it remains to be seen whether the 'fundamentum divizionis' is formed by objective circumstances inherent in the various industries, or instead is dependent on the point of view of the person acting as observer[...] or to put it other way, the increasing and decreasing costs

are nothing other than different aspects of one and the same thing that can occur at the same time, for the same industry so that the industry can be classified arbitrarily in one or the other category according to the definition of the *'industry' that is considered preferable for each particular problem, and* according to whether long or short periods are considered. Sraffa aims at discussing these particular problems at length and the valid argument that he lays his thesis on is the fact that any industry at any point in time may be classified as an increasing cost industry or a decreasing cost one, depending upon at what point in time one views the industry. Sraffa points out that the classical believed in independence of costs and quantity produced; it is the neoclassical thought that put the issue of interdependence of costs and quantity produced in the front-line of economic thought. The idea of interdependence of costs and quantity produced is in fact a result of the change in the basis of the theory of value, from cost of production to utility. The fact remains that only after the studies of marginal utility had called attention to the relation between price and quantity (consumed), did there emerge by analogy the symmetrical concept of a connection between cost and quantity produced. Sraffa always hinted that that marginal notion of cost, profits, revenues and the like had weaker underlying foundations. Weaker still was the microeconomic device of *ceteris paribus* according to Sraffa. Weaker because of the fact that if one decides to analyze the price of coal, it would be very difficult to conduct the analysis without considering the impact on the demand for railways (say). The point that Sraffa made was under competitive conditions, it was always impossible to conduct the ceteris paribus. To Sraffa therefore, all commodities and all prices were related to one another. His belief was further strengthened by the works of Quesnay and to an extent von-Neumann. Sraffa, after his attack on Marshall's ideas of the microeconomic thought went into complete economic seclusion as the Librarian at the university. During those days, Sraffa made the most of them by discovering the true David Ricardo through his correspondences. So strong was the Sraffian discovery that dedicated series on the works and correspondences of David Ricardo was entrusted with him by the university. As meticulous as he can be, the works stands today as one

of the best edited collection in the history of any subject. During this, he came closer to various classical notions and like the maximum rate of profits of Marx and also the concept of a standard measure of value as a medium between two extremes primarily borrowed from Ricardo. It was during these times that he had started believing in the cyclical nature of production and consumption. Sraffa very well points out that Tableau Economique is a correct manifestation of a system of production and consumption as a circular process and it stands in striking contrast to the view presented by modern theory, of a one-way avenue that leads from the 'Factors of production' to 'Consumption goods'. Production of Commodities by Means of Commodities therefore is not a wild-card entry into the quintessential Sraffian literature. In the Preface to Production of Commodities by Means of *Commodities Sraffa begins by stating that the investigation [in the book] is* concerned exclusively with such properties of an economic system as do not depend on changes in the scale of production or in the proportions of 'factors'. This standpoint, which is that of the old classical economists from Adam Smith to *Ricardo, has been submerged and forgotten since the advent of the 'marginal'* method. The reason is obvious. The marginal approach requires attention to be focused on change, for without change either in the scale of an industry or in the 'proportions of the factors of production' there can be neither marginal product nor marginal cost. In a system in which, day after day, production continued unchanged in those respects, the marginal product of a factor (or alternatively the marginal cost of a product) would not merely be hard to find—it just would not be there to be found. Thus, at the very outset Sraffa is pointing out that his investigations in the book are not of the usual nature discovering the causes of apparent phenomena, as a causal explanation can only be called for when there is a change. This could also point to the Humean notion of time, and thus an absence of time in his theory. As Hume argued, for the quality of the co-existence of parts belongs to extension, and is what distinguishes it from duration. Now as time is composed of parts, that are not coexistent; an unchangeable object, since it produces none but co-existent impressions, produces none that can give us the idea of time; and consequently that idea must be derived from a succession of

changeable objects, and time in its first appearance can never be severed from such a succession. The second point to note here is that Sraffa attributes "this stand point" to classical economists from Adam Smith to Ricardo. This, however, should not be interpreted as complete endorsement of classical theory, as we shall see later. All that is acknowledged here is the absence of laws of returns and returns to scale as tools of analysis in classical theory. Chapter one of the book is entitled, 'Production for Subsistence'. This chapter deals with a simple subsistence economy with specialization. Thus, the production process requires distributions of commodities given by the requirements of the technology (for subsistence economy consumption is part of technical requirement) where as, commodities are concentrated in the hands of separate industries after the production process is over. In this case Sraffa finds that there is a set of exchange ratios or prices of commodities that spring directly from the methods of production which can restore the original distribution of the commodities and make it possible for the system to repeat itself at the same scale. Chapter Two complicates the world by considering the case of a system that produces more than its minimum requirements (A system that produces less than its minimum requirements is not considered by Sraffa since such a system cannot have historical viability). Once a 'surplus' is admitted in the system, it becomes, in Sraffa's words, "self-contradictory". The required distribution of the commodities after production is no longer entirely determined by the methods of production. The problem of distribution of the 'surplus' must be solved. He argues that the surplus cannot be distributed prior to the determination of prices because "the surplus (or profit) must be distributed in proportion to the means of production (or capital) advanced in each industry; and such a proportion between two aggregates of heterogeneous goods (in other words, the rate of profits) cannot be determined before we know the prices of the goods" (p. 6). The upshot of the argument is that both the prices and the rate of profits must be determined simultaneously by the same mechanism. Accordingly, he adds a uniform rate of profits to his system of equations as an unknown, which gives him a system of n independent equations with n unknowns (n-1 prices and one rate of

profits) that has an economically meaningful solution. One effect of the emergence of surplus is that commodities can be divided into two separate categories. There can now be some commodities that appear in the system only as outputs but do not enter the system as inputs. Such commodities can be characterized as non-basics whereas the commodities that enter the system both as inputs and outputs can be characterized as basics. Any change in the conditions of production of the basics would have an impact on the prices of all the commodities through its influence as input in the system. Whereas, any such change in the production of non-basics can affect only its own price. Sraffa further complicates the system by arguing that workers' remuneration may contain a part of 'surplus', thus adding another unknown to the system as wages. It is necessary to comment on some of the above propositions at this stage. Within the same Chapter we find that the measure of the 'surplus' has changed. In the beginning only profits were calculated as surplus whereas workers' remuneration was considered to be necessities. By the end of the Chapter, both profits and wages are counted as 'surplus'. So the question arises, what is this surplus and how is it determined? As a matter of fact the notion of surplus is not self-evident. It exists only in relation to the notion of 'necessity'. And the notion of necessity has definite meaning only from the subject's point of view. For a capitalist as a subject, the wages must constitute a necessity and only the profit over which s/he has total control can be taken as surplus. On the other hand, from a technical standpoint all the output over and above whatever has been used up in the production process must be characterized as surplus. From an entirely objective scientific point of view, however, there cannot be any surplus since there cannot be any effect without a sufficient cause or there cannot be any product without an equivalent cost. Thus, it appears that Sraffa takes a technical standpoint towards his subject matter rather than either a class or a pure scientific standpoint. Secondly, Sraffa identifies his surplus producing system with a capitalist system by identifying the form of surplus appropriation with profits. But not only that. Without any further ado he claims that the rate of profits "must be uniform for all industries". Soon after that he goes on to say that "Such classical terms as

'necessary price', 'natural price' or 'price of production' would meet the case, but value and price have been preferred as being shorter and in the present context (which contains no reference to market prices) no more ambiguous". This has led to an almost unanimous opinion among Sraffa scholars that Sraffa's imposition of a uniform rate of profits on the system is an implicit acceptance of the notion of a centre of gravitation of classical economics. As is well known, the 'natural prices' of Smith and Ricardo and the 'prices of production' of Marx are the centres of gravitation around which the market prices fluctuate. The gravitational point or the 'centre of gravitation' comes about because of competition and mobility of capital, given that capital seeks the highest profit rate⁷⁹.

The cyclical nature of commodities was an observation made by earlier writers including the likes of Leontief and von Neumann. However, it was Sraffa who developed the model for portraying a system where production is carried out by the means. The aim of Sraffa was as clear as a crystal as was two-fold- one to provide a concise theory of value and two to provide a basic infrastructure for launching a full fledged critique of the economic theory. Sraffa intended to develop a device through which price movements and the issue of relative prices could be solved forever. This was the underlying basis for a theory of value. The necessity of having to express the price of one commodity in terms of the other which is arbitrarily chosen as a standard complicates the study of price movements which accompany a change in distribution. Here, Sraffa is searching for a measurable and an invariable standard for the understanding of the peculiarities of a system of which such a commodity is a part of. In so doing, Sraffa explored various angles of the production relationships in the first place- a production where the economy is a subsistence economy, followed by a surplus bearing economy. In the later sections, Sraffa also extended the discussion to fixed, circulating capital and to the cases of joint and by-products. All of them highlighting one important fact-Sraffa's intriguing quest for an invariable

⁷⁹ Sinha, A. (2001): *Reading Sraffa: The Philosophical Underpinnings of Production of Commodities by Means of Commodities*, Gokhale Institute of Politics and Economics WP231

standard of measure. It is this measure that Sraffa says to be the foundations and *if the foundation holds, a critique may be attempted later by the writer or by someone younger and better equipped to do the task.* The production-price equations of Sraffa would be the most logical starting point for the nature of work we intend to conduct. These sets of equations can be represented as:

$$(A_{a}p_{a} + B_{a}p_{b} + \dots K_{a}p_{k})(1+r) + L_{a}w = Ap_{a}$$
$$(A_{b}p_{a} + B_{b}p_{b} + \dots K_{b}p_{k})(1+r) + L_{b}w = Bp_{b}$$
$$\dots$$
$$(A_{k}p_{a} + B_{k}p_{b} + \dots K_{k}p_{k})(1+r) + L_{k}w = Kp_{b}$$

In the above model, there are *K* commodities that are represented using as many production equations. On the left hand side of each equation, we have the aggregate value of inputs that determine the value of outputs produced. As the model is seen, a commodity enters the production of other commodities valued at its own prices that are represented as p_i with the subscripts representing the number of the commodity. An important definition is in the order. *Commodities that enter the production of every other commodity are called basic commodities and that which do not enter the production of other commodities are called consumption commodities*. It is worthwhile to note this definition, as only the determination of basic commodity prices is important as these are by definition the capital goods prices. The producer of each commodity enjoys a competitive rate of profits *r* on the volume of capital invested. The labour terms L_i dictate a uniform wage rate the economy. In simple matrix notations therefore the above Sraffa model can be characterized by

$$Ap(1+r) + wL = Bp$$

This system involves usage of circulating capital only. Sraffa devotes a separate section in itself to analyze the characteristics of a system with fixed capital. The economy is seen to produce more than what is required for subsistence and there is a surplus to be distributed. Sraffa says that in this regard the system becomes self-contradictory. Sraffa has introduced wages on the same footing as the fuel for engines and feed for cattle. Sraffa assumes that the whole of this wage is variable

and he does not intend to tamper with the traditional definition of wages. Sraffa suggests that such an assumption would have its own drawback and *that is it* involves relegating the necessaries of consumption to the limbo of non-basic products. This is due to their no longer appearing among the means of production on the left-hand side of the equations: so that improvement in methods of production of necessaries of life will no longer directly affect the rate of profits and the prices of other products. Necessaries however are essentially basic and if they are prevented from exerting their influence on prices and profits under that label, they must do so in devious ways. Sraffa, for labour assumes that the whole labour in the economy may be taken to be unity and that L_a , L_b etc would be annual quantities of labour defined as a fraction of total annual labour which is one. More so, it is assumed that $L_a + L_b + ... L_k = 1.$ A uniform rate of profits is presented in the system, may be with a view to exhibit competition amongst industries. Prices and rate of profits are determined simultaneously in this system, for without knowing the one, the other cannot be known. The next item on the bill of enquiry for Sraffa is even more intricate. After the prices and other variables of interest can be discovered, the important question is how to determine the standard which is invariable to any economic fluctuations. Sraffa points out that for the standard to be truly invariant; there need be a ratio of the net product to the means of production of the system. This ratio we shall call the standard ratio. Thus, in the standard system, the ratio of net product to the means of production would remain the same whatever variations occur in the division of net products between wages and profits and whatever the consequent price changes. In so doing, we would have truly acknowledged Sraffa's quest. We ask the question as to how much replacements of each industry are required each period in order to achieve the slated rate of profit. The answer that we get for each industry is what we call the output scalars or the multipliers. The problem for the standard system can be stated in general terms as: the problem of constructing a standard commodity amounts to finding a set of k suitable multipliers to be applied respectively to production of commodities *a*,*b*,...*k*. The multipliers must be such that the resulting quantities of various commodities will bear the same

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proportions to one another on the right hand sides of the equations (as products) as they do on the aggregate of left hand sides (as means of production). These by analogy determine the maximum rate of profits-that rate which corresponds to zero profits- of their respective industries and competition dictates that these be equal. Lastly the Sraffa postulates that the entire labour force in the economy be preserved as these transitions for adjustments happen and these are to be adjusted as per the output multipliers for each industry. One important point worth noting is that since the capital goods only will be replaced over time, only the capital goods good industries enter the dual relationship, or what we call the problem of output determination. Thus the Sraffa system simultaneously is a system of determination of price and output; a theory of value in its true spirit. Sraffa's system of output determination can be described as

$$(A_{a}q_{a} + A_{b}q_{b} + \dots A_{k}q_{k})(1+R) = Aq_{a}$$

$$(B_{a}q_{a} + B_{b}q_{b} + \dots B_{k}q_{k})(1+R) = Bq_{b}$$

.....

$$(K_{a}q_{a} + K_{b}q_{b} + \dots K_{k}q_{k})(1+R) = Kq_{b}$$

$$L_{a}q_{a} + L_{b}q_{b} + \dots L_{k}q_{k} = L$$

In its general form the dual relationship is given in the following manner A'q(1+R) = Bq. This system is aptly described as Sraffa's system of output determination. From this, we obtain the necessary multipliers and apply it to the equations of the production system and transform it into a Standard system as follows:

$$q_{a}[(A_{a}p_{a} + B_{a}p_{b} + \dots K_{a}p_{k})(1+r) + L_{a}w] = q_{a}Ap_{a}$$

$$q_{b}[(A_{b}p_{a} + B_{b}p_{b} + \dots K_{b}p_{k})(1+r) + L_{b}w] = q_{b}Bp_{b}$$

$$\dots$$

$$q_{k}[(A_{k}p_{a} + B_{k}p_{b} + \dots K_{k}p_{k})(1+r) + L_{k}w] = q_{k}Kp_{b}$$

From these sets of equations, we can conveniently derive the standard national income. For the remaining course of Sraffa's book, he aims to use the standard national income as a unit of wages and prices in the original system of production. He has truly achieved his desired objective! The foundations surely seem rock solid.

Next, we consider the mathematics of the Sraffa system as a whole. Assuming n commodities in the system, there are *n* price equations and n+2 variables. We can eliminate⁸⁰ one of the unknowns by fixing either one of the prices equal to unity (Walras, 1874), or by fixing the absolute wage rate equal to unity (Keynes, 1936), and then there will remain *n* equations in n+1 unknowns. Thus there is an equation missing that would help in determining all the *relative* prices in the economy. The Sraffa system in its current form is incomplete and is open! In Sraffa's own terms, the system "moves with one degree of freedom". We are one equation away from actually and mathematically solving it: not only that, we are just one equation away from determining a general production-price equilibrium in this Sraffian edifice. A degree of freedom in the system implies that the system is indeterminate unless one variable is given from outside the system. As Hahn⁸¹ has correctly pointed out, taking either wages or the rate of profits or a price given from outside can formally solve Sraffa's system. One could think of a price of a basic good being fixed by the government. Sraffa, however, considers only wages or the rate of profits as given from outside. Most likely this is because taking a price determined by the government could only give an arbitrary solution to the system. On the other hand, wages or profits have distinct status from the rest of prices given that they are income categories. Sraffa's position appears to be that the same complex socio-historical processes that have given the technical configuration and the surplus of the system also determine the income categories. *Giving one income category is tantamount to determining the other income* category simultaneously, given the surplus. In the classical tradition real wages were generally taken as given by the socio-historical forces at any given time. The classical economists (particularly Ricardo and Marx) took the standpoint of the capitalist in analysing the capitalist mode of production and identified surplus with non-wage incomes only. Sraffa's technical standpoint, on the other hand, leaves it open. As Sraffa later in the book argues, "The rate of profits, as a ratio,

⁸⁰ Note that the construct of the *numeraire* is used only for the Sraffa system and is dropped in the analysis of Money, by definition.

⁸¹ Hahn, F.H. (1982), "The Neo-Ricardians", Cambridge Journal of Economics, Vol. 6, pp. 353-374.

has a significance which is independent of any prices, and can well be 'given' before the prices are fixed. It is accordingly susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest". This suggests that Sraffa's position could be that the rate of profits is conventionally determined in relation to the going rate of interest, which of course is uniformly given by the monetary authorities. As Sraffa wrote, "It is possible to conceive of it [the rate of profits] as being 'given' from outside the system of production, such as conforming to the pattern of money rates of interest determined independently by the banking system or the Stock Exchange" (PSP D3/12/78, quoted in Ranchetti, 1998). This may explain the introduction of a uniform rate of profits in his system. Unfortunately Sraffa did not elaborate on this crucial point. This notwithstanding, Sraffa's contention that the uniform rate of profits is "susceptible of being determined from outside the system of production" is vet another evidence against the 'centre of gravitation' interpretation. For, if the uniform rate of profits is the result of a gravitational mechanism then it cannot be conceived of being independent of the system of production, as it must depend upon the level of outputs in conformity with the effectual demand. A uniform rate of profits given from outside the system of production could, however, be applied to a system not necessarily in equilibrium. In this case disequilibrium would imply an unplanned fall or rise in the inventories of various sectors⁸².

The required missing degree of freedom between the equations and the variables can be also filled in by considering the composition of commodities which the individual agents desire to purchase; the demand equations for the *n* commodities. Walras' law dictates that only n-1 of these will be independent, and that we shall use the empirical demand functions that designed are designed by Stone as

$$\alpha r K + \beta w L = B_j p_j$$
 where $K = \sum_{j=1}^n \sum_{i=1}^n A_{ij} p_i$ is the capital stock, L is the annual

labour and the constants alpha and beta are propensities of capitalists and wage earners to consume or spend. Thus now we have 2n-1 equations in n+1 variables,

⁸² Sinha A (2001), Ibid

and the system is still over-determinate. Hence we now use the dual construct of Sraffa that shall help determine the outputs of each industry as well. We introduce *n* equations for determination of outputs and to do so absolutely we use what Sraffa calls the labour conservation equation, which adds new n+1 equations to the system in n+1 variables, the *n* outputs and the growth rate. Closing this system with the relation between the profit rate and the growth rate, we have fully 3n+1 equations in as many variables and this is what is explained as the complete Sraffa system. Demand and even the slightest hint of demand for a commodity is seen missing from Sraffa's analysis. A more specific reason amongst the many offered by a lot of economists is the fact that Sraffa's quest was towards developing a theory of value, truly capable of providing a standard of value as seen above. In this context, Sraffa also hence did not bother about closing the one degree of freedom that his system lay open. His intention was clear- to provide a device for the critique of the mainstream. More so, what we are doing is taking this device to its ultimate aim- develop a theory of value that now requires filling the gap and leaving no degrees of freedom. In so doing, we propose a logical method- introducing consumption commodity demand functions. We, as described above, would aim to use Stone's linear expenditure systems. A complete overview of this analysis can be analytically examined using a capital goods *closed Sraffa system* comprising of two commodities alone.

$$(A_{11} + B_{21}p_2)(1+r) + L_1w = A$$

$$(A_{12} + B_{22}p_2)(1+r) + L_2w = Bp_2$$

$$\alpha r K + \beta w L = A'(= A - A_{11} - A_{12})$$

These can be written in vector-matrix notation as a system of homogenous equations,

$$\begin{bmatrix} A - A_{11}(1+r) & -A_{21}(1+r) & -L_{1} \\ -A_{12}(1+r) & -A_{22}(1+r) & -L_{2} \\ A' - \alpha(A_{11}r + A_{12}r) & -\alpha(A_{21}r + A_{22}r) & -\beta L \end{bmatrix} \begin{bmatrix} 0 \\ p \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A unique non-trivial solution to the above system exists if and only if the determinant of the matrix of coefficients is equal to zero. Setting it thus equal to zero gives us a characteristic polynomial equation in *r*. The lowest root of the

polynomial⁸³ is the relevant solution. When this is substituted in the price equations, the solutions for the relative prices and wages can be obtained. The "Cambridge Equation" gives the relationship of the growth rate and the profits, and reads as $g = r(1-\alpha)^{84}$. The algorithm that yields the results to the above *closed Sraffa System* will be made clear in a while when we analyze the augmented Sraffa model to carry on our discussion of money and value theory.

Annexure to Chapter 3- A Model of Commodity Money

Money has dominated the economic systems for ages. Right from stone to gold to paper and plastic money, civilizations have been formed and destroyed because of money. Such a pivotal aspect to the economic systems yet not a single model is able to provide for the role that money is expected to perform in the real monetary economies. Before we can move any further, consider a situation that would prevail if there were no money. Suppose there were N commodities and T traders dealing in them. Suppose, without loss of generality that every trader deals in every commodity. Then there would be a total of TN (N-1) quotes that will have to be made for N commodities. For example, suppose there were 100 traders

$$g = \frac{B - A - \alpha r A}{A}$$
 and we get the equation as above.

⁸³ In Matrix language, it is called the lowest eigenvalue of the characteristic vector or the eigenvector and hence we can easily resort to Perron - Frobenius theorem

⁸⁴ Note that g=(B-A)/A and if we allow a proportion of profits to be consumed equal to alpha, then

dealing in 1000 commodities. There would be 1000X999X100=99900000 price quotations in all; about 100 millions! Any individual willing to buy or sell a commodity would have to consider all of them to determine a.) Which trader to buy or sell them from and b.) What would be the sequence of commodity transactions? The choice of the trader will depend upon whether he is offering the lowest or the highest quote for the commodity depending upon whether the individual wishes to buy or sell. The choice of the sequence of commodity transactions is important too! There will be several routes for buying or selling the commodity using other commodities as intermediaries. There is always a possibility of making arbitrage profits by selecting a mispriced sequence of transactions. For example, suppose there are three commodities, wheat, milk and rice and their price quotes are 2 kg. Wheat =1 kg. Rice; 1 liter milk =1 kg. Wheat; 1 kg. Rice =1 liter milk. Suppose an individual has milk to sell and buys wheat. He will not clearly sell milk and buy wheat according to quote 2. It will be profitable for him to sell milk at quote 3, buy rice, sell rice at quote 1 and buy wheat and end up with 1 kg more wheat. Our individual will have to rummage through all such sequences of transactions to find the most profitable sequence for buying/selling. Of course there is an opposite side to this. No single trader will offer quotations, which permit arbitrage profits at his own cost to his customers. This requirement places two restrictions on the price quotations offered by each trader. Ignoring trader margins for the sake of simplicity these restrictions are as follows: 1.) the quote for one commodity for the other must be equal to the reciprocal of the quote for the other commodity in terms of the former, i.e.

$$\frac{P_i}{P_j} = \frac{1}{\frac{P_i}{P_j}}$$
 for all i,j, i j. 2.) Every indirect quote must be equal to the direct

quote, i.e. $\binom{P_i}{P_k} \binom{P_k}{P_j} \forall i, j, k, i \neq j \neq k$. The first set of restrictions are N (N-

1)/2 in number, the second consists of (N-1)(N-2)/2 i.e. a total of $N(N-1)^2$ restrictions. Every time a price changes the trader will have to make out fresh set of N (N-1) quotations which will have to satisfy $(N-1)^2 N$ equations i.e. to say

with our 1000 commodities example then 999X999=9980001 computations will need to be made every time price changes. The designation of one commodity as numeraire simplifies all this dramatically. For the N commodities that the trader deals in he need give only (N-1) quotes in terms of the numeraire commodity. He need not perform $(N-1)^2$ N computations at all. Designating a numeraire automatically ensures these. All arbitrage opportunities two, three or higher order sequences stand eliminated. [For each trader the number of quotes reduce from N(N-1)=1000 999=999000 to 999]. Of course different traders would quote differently so that there will be T(N-1) quotes in the market. However by means of a direct comparison of price quotes of different traders, inter-trader arbitrage will ensure uniform price quotes. The number of effective price quotes will be reduced to (N-1) which is dramatically lesser than TN (N-1). In mathematical terms the degree of complexity has been reduced from a power of three (cube) to a power of one. At the same time everybody's record keeping has become smoother. With all transactions valued in terms of the numeraire and with arbitrage possibilities being eliminated, the values e.g. profits do not vary with the choice of the commodity in which the accounts are kept and the inter-commodity quotes at which the transactions are made. Implicit in the above is the assumption that all traders accept one commodity as the numeraire. However, an important point of distinction between a commodity money economy and a barter economy is in order. A barter, as described above permits all inter-commodity transactions and allows all pair-wise exchanges of goods and services. However, in the current set-up, in case of commodity money, all pair-wise exchanges of goods between themselves other than the money commodity (and not commodity money!) must be excluded. In precise terms, the following should hold. We may now proceed to build a model with commodity money, ascertain the process to equilibrium in such an economy and demonstrate criticalities of such an economy. Every individual when he goes to the market will have to carry all sorts of things with him to consummate his trades because the traders he comes across may not be willing to transact some commodities even if they quote for them (if only implicitly). Can those inventory keeping and transactions cost be minimized? Yes,

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provided the generally accepted numeraire commodity can itself be used as the medium of exchange. The properties that a commodity must possess to perform as the role of a numeraire are not at all stringent. Almost every commodity can serve as a numeraire. But to be a stable medium of exchange a commodity should possess a host of peculiar and self-contradictory properties. Firstly the commodity must itself be useful, yet it should not form too large a proportion of consumption, or have so many uses that it is actually consumed up. It must not be easily producible, yet it should be easily available as the needs of trade augment. It must be durable. The commodity must have a high value in relation to other commodities to keep its own storage and transport costs within limits. Yet it should be desirable without much wastage to facilitate small trades. In short the medium of exchange should have all properties of good medium, viz portability, divisibility, etc and the properties of a good store of value, viz durability, steady demand and supply conditions etc. it is no wonder that gold, silver and other metals served as money for long periods of time in history. The general use of a medium of exchange imparts an additional advantage, viz; the acts of sale of commodities and the acts of purchases of commodities can be separated in time. This separation bestows some economic freedom to individuals. It reduces the possibilities of their having to make forced sales/purchases. It gives them breathing space to search better prices. In the absence of money, every trader would be required to carry in principle, some stock of N different commodities. (Strictly speaking commodities where denotes durable commodities). There would thus be TN separate hoards. In the presence of a generalized medium, each trader need only carry the money commodity (and not commodity money!). Consequently the number of hoards reduces to T. In terms of our numerical the number reduces from 100000 to 100. All this simplification due to the adoption of generalized medium or a money commodity. The introduced money commodity surely reduces the complications of barter and hence, provides a logical point in the theory of value. We need to analyze the theory of value in light of the commodity money that shall be introduced in this chapter.

We start with explaining a model of commodity money. We stick to the definition of commodity money as to be a unit of account and medium of exchange. The store of value function will be taken up later in order to keep away the complexities in the current system. By commodity money most economists mean one or other of the precious metals such as gold or silver, although shells or other scarce items have served as commodity money in some societies. Two characteristics of commodity money are that it is a commodity subject to the "laws" of production and that it is in relatively inelastic supply. For example, in a particular economy using gold as commodity money, at any point in time the quantity of gold is effectively fixed. The growths in the supply of commodity money is small in comparison to the existing stock and subject either to developments on the balance of payments or gold mining output if the economy happens to possess some gold mines. Commodity money introduced in this system is thus a produced means of commodity exchange. A basic Sraffa model extended by incorporating stock-flow variables and explained in the previous chapter is used here. Standard assumptions of the Sraffa system are thus retained with respect to uniformity of rate of profits and other symmetry conditions. However, the standard Sraffa system has an agrarian flavour in the sense that the production of commodities is carried by means of other commodities. It is rare that in an industrial economy and where (commodity) money is present, the role of capital has to be divided into fixed, circulating and current capital. The circulating capital is what is meant by the daily expenses of production, material, wages and other administration expenses. The current capital is the current bank accounts, idle cash and reserve materials i.e. these constitute the stocks of various production materials (and money) and these along with the circulating capital comprises the stock-flow constituents of the production system that we would concentrate upon. In this entire analysis, we do not include factory sheds, plant and machinery etc that make up the fixed capital. All forms of the discussed capitals would be held in terms of the respective stocks, flows and the cash balances and liquid cash expenses would be met through commodity money. This commodity money and its properties would be revisited throughout the course of

this chapter. As we started off, we noted that the commodity money of this chapter is a standard and a medium of exchange. However, the necessary properties that the system must bestow on this commodity to be truly money are: firstly that it should be commonly accepted as a means of exchange and any exchange without it should be made impossible. Secondly, it should be necessarily used in every activity of the economy, from production to consumption and investment. It is important albeit it is used in every activity, it should never be used up. Thirdly, it should have a value in exchange: the exchange value of money is defined in terms of its purchasing power. Lastly, the medium of exchange should be able to make trades possible and markets exist: it in itself should be a good hedge for inflation. We would aim to demonstrate that if the commodity to which the 'money-ness' is ascribed adheres to these properties. We assume a production economy with n industries producing n outputs. Capital and labour are perfectly mobile in this system and hence across industries, can be assumed to command a uniform rate of profits and a uniform wage-rate. The individual commodities command an exchange value in the market- this value in exchange of the individual commodities is measured through the prices of these commodities. Therefore, it is important to note that for the exchange of commodities, we assume a market to exist and it is in this market that the prices of commodities, the wage rate are valued, expressed and quoted in terms of the commodity money. In this economy as well, it is true that: goods buy money, money buys goods but goods would not buy goods! In the production system thus described that assumes a commodity money, it is important to introduce a money commodity. This money commodity is a produced means of exchange and is governed by similar laws of production as other commodities. A system of equations or a model representing an economy with commodity money is therefore a logical starting point to this train of thought. We would resort to the stock-flow model of the type introduced in an earlier chapter. Assuming there are *n* capital goods commodities and *m* consumption commodities, we can set out to determine the production price equations for this system.

$$(S_{11}p_1 + S_{21}p_2 + \dots + Sk_1B_t)r + (A_{11}p_1 + A_{21}p_2 + \dots + Ak_1B_t) + wL_1 = p_1B_1 (S_{12}p_1 + S_{22}p_2 + \dots + Sk_2B_t)r + (A_{12}p_1 + A_{22}p_2 + \dots + Ak_2B_t) + wL_2 = p_2B_2 \dots (S_{1n+m}p_1 + S_{2n+m}p_2 + \dots + Sk_{n+m}B_t)r + (A_{1n+m}p_1 + A_{2n+m}p_2 + \dots + Ak_{n+m}B_t) + wL_{n+m} = p_{n+m}B_{n+m}$$

These equations are the crux of the value theory that we aim to propose. They form the basis for determining the economic equilibrium and also most critical and relevant economic values such as prices, wage rate, profits, national income, employment and the like. Since the equations are similar to the stock-flow model introduced before, we do not feel the need to reiterate the explanation. However, a new entry in the price equations is the variable k. For the time being, it would suffice to know that it is an important variable in the commodity money system. Along with the equations for determination of outputs make up a system for determination of other values such as growth rate, outputs and industry sizes. The output system for this model that is developed from the production-price relations is given by

$$\left(S_{11}x_1 + S_{12}x_2 + \dots S_{1n}x_n + \sum_{j=1}^m S_{1j} \right) g + \left(A_{11}x_1 + A_{12}x_2 + \dots A_{1n}x_n + \sum_{j=1}^m A_{1j} \right) = B_1 x_1$$

$$\left(S_{21}x_1 + S_{22}x_2 + \dots S_{2n}x_n + \sum_{j=1}^m S_{2j} \right) g + \left(A_{21}x_1 + A_{22}x_2 + \dots A_{2n}x_n + \sum_{j=1}^m A_{2j} \right) = B_2 x_2$$

$$\left(S_{n1}x_1 + S_{n2}x_2 + \dots S_{nn}x_n + \sum_{j=1}^m S_{nj} \right) g + \left(A_{n1}x_1 + A_{n2}x_2 + \dots A_{nn}x_n + \sum_{j=1}^m A_{nj} \right) = B_n x_n$$

$$L_1 x_1 + L_2 x_2 + \dots L_n x_n = L$$

As Sraffa had noted, commodities that enter the production of other commodities are called as capital goods; we call them capital commodities. Commodities that do not enter the production of every other commodity shall be deemed as consumption gods; they are not used in the production of every other commodity. Also, a money commodity has to be introduced in this set-up. We call this commodity with a subscript *t* and labeled as B_t . It would soon be seen that this commodity does not have any price; it is a money commodity! All other transactions and exchanges would be quoted, expressed and conducted in terms of

the commodity money- a commodity which has no price of its own (mathematically, this price is 1!). The equation for the commodity money industry has to be introduced. It is given by

$$(S_{1t}p_1 + S_{2t}p_2 + \dots + Sk_tB_t)r + (A_{1t}p_1 + A_{2t}p_2 + \dots + Ak_tB_t) + wL_t = B_t$$

Therefore, this system involves *m* capital goods and *n* consumption goods. The production of these commodities is carried out using current capital, circulating capital, wages and money. All the industries in the economy are Sraffian, enjoying a uniform rate of profits on their current capital- the stocks of commodities and money. The last term in the first parenthesis stands for the stock of money held by all the entrepreneurs in terms of the money commodity. Specifically, $Sk_{n+m}B_t$ represent the commodity money holdings of all the industrialists. The term k_{n+m} is referred to as the money-turnover ratio and is defined as the value of money holdings or desired money stocks divided by the value of the industrial output. There may be several reasons to keep money as a part of the commodity stocks. From mere balance sheet perspectives, these may be understood as cash balances kept for meeting unforeseen business circumstances. Entrepreneurs are assumed to hold cash balances as a proportion to their current turnover levels. These can be also referred to as the money demands of the industrialists. These along with the stocks of other commodities make up the capital of the industrialists that earns profits. The term, $Ak_{n+m}B_t$, is used to represent the (flow) money commodity coefficients. It should however be noted that these flow coefficients in terms of the money commodity are necessarily nonmonetary uses of the money commodity. Any flows of the money commodity are meant to be its pure value in use and not used for any payments or other purposes. Lastly, these production activities are assumed to generate employment to Llabourers that earns an industry average wage rate. We suppose labour to be uniform in quality or, what amounts to the same thing, we assume any differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labour receives the same wage⁸⁵. This wage rate is the actually

⁸⁵ Sraffa, P, Production of commodities by means of commodities, 1960

received wage by the labourers for participating in production- also it is assumed that only those labourers that aim to work get the respective jobs⁸⁶. Here, the concept of wages does not require measuring the workers' utility and disutility. These wages are same ex-post and ex-ante as well. We shall also hereafter assume that the wage is paid post factum as a share of annual product, thus abandoning the classical economists' idea of a wage 'advanced' from capital^{8/}. Thus, the total operating expenses, the wage bills, the material requirements in terms of flows and only the profits are assumed to make up the total expenditures of producing commodities. Notice that the stock matrix in the above set-up is post-multiplied with r only and not the Sraffian (1+r). This implies that producers aim to cover their margins on the total capital alone and that the capital thus introduced is a commodity that is hardly replenished or is hardly used-up in the production process. The terms on the right-hand side of the equation represent the outputs of individual industries valued at the going market prices. In order to determine demand for consumption goods, Stone's linear expenditure system is used. The advantage of using this system in the current set-up is that it provides measurable demand equations in terms of the variables involved in the model. Stone describes that the demand for any commodity is driven by the incomes, and if we know the incomes of all the individuals and the marginal propensity to consume, we would immediately come to know the total consumption expenditure. We follow the similar logic and start by making the assumption that only workers consume a defined proportion of from their incomes. This proportion appears in the demand equations through the parameter α . Using this parameterization, we obtain the demand equations given by $\alpha_i wL = p_m B_m$ where it is assumed that consumption takes place only out of wage incomes and entire profits are saved. The outputs are determined using Sraffa's system for multipliers or what Sraffa described as the q-system. Sraffa had used the maximum rate of

⁸⁶ This is different than the Keynesian version of involuntary employment. To Keynes, if the wages were not satisfactory, labourers would withdraw from production. However, there may be cases when labour would want to withdraw from production not only for wages. Personal reasons, choice and ego could just be a few reasons for not working.

⁸⁷ Ibid

profits, R, in his q-system; this we replace by the growth rate of the industry. The idea is this- production should be carried out in such a manner that it meets the above mentioned line items in its bill and also provides for its own existence. It should aim to maintain a consistency in size which can be through growths in its capital stocks. Growth in capital stocks implies an increase in the demand for capital (and money as well). This increase in demand for other commodities makes the economy grow as a whole. The story is similar when the demands for capital goods fall as well. Therefore, the model of the economy presented herein comprises of multiple technologies- the production technology, the consumption technology, investment technology and the implicit savings technology. These involve m+n production equations, m demand equations for consumption goods, *n* output equations for capital goods and 1 growth-profit closing equation⁸⁸. Therefore, the model thus specified has 2m+2n+1 equations and unknowns. An important question worth a mention at the outset is this: is money commodity or the industry producing money commodity a capital goods industry or a consumption one? The answer to this question is in the affirmative. Money enters in the production of every other commodity and hence is a necessary to the production activity. As a result, it assumes the form of a basic industry and hence is also a part of the q-system or the system of outputs⁸⁹. Commodity money is the simplest form of money and an exposition of the same would be provided here in. Fixed values, consumer preferences, national income, growth rate and distribution of income along with the determination of absolute prices and price levels measured in terms of the adopted commodity money standard would provide the first step to the theory. The second step would be to test the comparative statics of the system and hence deduce properties with commodity money. The famous MV=PT or a similar relation has been explored with this system. However, the adoption of a quantity theory kind of an equation provides no solutions to this

⁸⁸ More on this later.

⁸⁹ Clearly, it should be remembered here that the system of equations can be broken down into two fundamental sub-systems. One sub-system is the production-price system which is the primal of the economy. The dual of this economy is the output system or Sraffa's output system. Necessarily enough, the price system and the output system together comprise of the theory of value. Truly so, it would be difficult to analyze price and output together in a system where micro and macro economics divide no longer holds is indeed complicated. On top of that, we have introduced additional complication of commodity money.

system and hence we have to conclude that resorting to QTM may not be possible in this system. We would also claim that the excess demands at each time do not necessarily equate the value of money or in other terms, the Walrasian law does not hold as well.

We now begin providing more flesh to the system. We would follow similar methodologies in subsequent chapters where we would introduce an economic model depicting the specific case and then follow it up with an illustrative economy and actual numbers. The more important point being since most of the theory we build can be tested in this manner; we would use the conclusions from these models for deducing the properties of the actual economies. A numerical example of an imaginary economy will be used to study the properties of the greater and the actual economy with a commodity standard.

$$\begin{aligned} (3p_1 + 2p_2 + 10)r + (2p_1 + 5p_2 + 3) + 5w &= 40p_1 \\ (2p_1 + 5p_2 + 30)r + (5p_1 + 7p_2 + 5) + 5w &= 60p_2 \\ (2p_1 + 3p_2 + 50)r + (2p_1 + 5p_2 + 3) + 10w &= 50 \\ (3p_1 + 5p_2 + 90)r + (3p_1 + 2p_2 + 5) + 10w &= 60p_4 \\ (2p_1 + 3p_2 + 12.5)r + (2p_1 + 5p_2 + 7) + 10w &= 50p_5 \\ (2r + 2)p_1 + (3r + 5)p_2 + ??? \\ 0.45(40w + 20r) &= 60p_4 \\ 0.45(40w + 20r) &= 50p_5 \\ (3q_1 + 2q_2 + 2q_3 + 5)g + (2q_1 + 5q_2 + 2q_3 + 5) &= 40q_1 \\ (2q_1 + 5q_2 + 3q_3 + 8)g + (5q_1 + 7q_2 + 5q_3 + 7) &= 60q_2 \\ (5q_1 + 3q_2 + 5q_3 + 11)g + (3q_1 + 5q_2 + 3q_3 + 12) &= 50q_3 \\ 5q_1 + 5q_2 + 10q_3 &= 20 \\ 321r &= 660g - 2w \end{aligned}$$

In this economy, invariably, the rate of profit and the rate of growth will be equal to one another; only due to the simplifying assumption that the capitalists save everything and workers consume everything. If we also allow capitalists to consume with a propensity of α , the growth-profit relationship would then be modified to $r = g(1 - \alpha)$. We would soon look at this case as well. In the current context, it would be useful to study the properties of an economy where capitalist savings are absent. The production price equations are presented in the first five

equations of the system. The production activities of the economy are carried out using stocks and flows as described. Along with commodity stocks and commodity flow coefficients, we also have stock and flow coefficients in terms of the money commodity. These represent the individual industries' money balances. The closing equation of this system is the pivotal equation. Talking about the sixth equation in the above system, this equation takes into account the (commodity) money flow in the system. Where a case of circulating money coefficients is observed, the quantity theory of money becomes invalid an equation to be used. In such a scenario where there are stocks of money and commodities and flows of money and commodities, it may not be prudent to circumscribe the economy within the quantity theory tradition. Needless to add, if we superimpose this condition on the economy, the economic equilibrium is out of the window. The seventh and the eighth equation are the demand equations for the consumption goods industries. The demands for capital goods industries are accurately depicted in the production-price relations. The demands for money are also presented in the production price equations through the money-turnover ratios. Equations 9-12 form the output system or the system of determination of multipliers. The maximum rate of profits in Sraffa's output system has been translated into the growth rate here. A point of mention is the uniformity of rates of profits and rates of growth across all industry sectors. However, at the outset it is not so. At the outset, the own rates of growth and also the own rates of profits are unequal. Own rate of growth in the system is defined

as $\left(B_i - \sum_{i=1}^n \sum_{j=1}^n A_{ij}\right) / \sum_{i=1}^n \sum_{j=1}^n S_{ij}$. By this definition, for industry one, the own rate of

growth works out to be $\{[40-(2+5+2+3+2)]/[3+2+2+3+2]\}=2.166$. Similarly, for industry two, the own rate of growth is 2. For the money commodity industry, the rate of growth is 0.14. The own rates of growth apply only to capital goods or capital goods industries. These rates are called as growth rates because they provide signals whether in future periods, a particular industry is bound to expand or contract. Whilst industries with growth rates above 1 are expected to contract in size, industries with rates lesser than 1 are expected to expand. The demand pattern and the capital technology coefficients harmonize the growth rates across the system. As a matter of fact, this also is the first step towards obtaining the economic equilibrium of the system. At each step, we would first ten to equalize the own rates of growth across the system. This would help us determine the uniform rate of growth of the system as a whole. This rate of growth will be used as rate of profits since rate of growth is equal to the rate of profits. Along with the determination of rate of growth, the multipliers of the system are also determined simultaneously. These multipliers will also be applied to the production price system. At the initial stages, in the first step itself, the rate of growth of the system works out to be 0.306 and the capital goods industry multipliers are 0.352, 0.408 and 1.619 for industry 1, 2 and 3 respectively. This is the case since as described; industry 1 and 2 had growth rates more than 1. Hence, these industries will shrink in size and the converse is true for the money commodity industry. These quantity adjustments will happen in the course of the algorithm. Industries which use relatively lesser capital or means of production will have a deficit and those with relatively higher usage of means of production have a surplus. This theorem is also a direct result of the standard Sraffa system. Having determined the multipliers for the capital goods industries, we would apply these to the respective industry sectors and move an inch closer to the equilibrium. This process of applying the multipliers to the respective industries was also advocated by Sraffa as a move towards developing the standard system. We would be doing this as a move towards developing a system where the rates of growth are equalized. This happens to be Sraffa's standard ratio as well. Pre-multiplying all the capital goods equations with the three multipliers described as above, we obtain a new set of the production-price system wherein the own rates of growth are equalized. What we have achieved in the process is elimination of any economic reason for flight of capital. As a result, this step also helps determine the equilibrium in the capital goods industries by matching the respective demands with the available supplies. All this while, the commodity money or the money commodity had been out of the discussion. It should be noted that the commodity money industry has its own rate of growth, is an influential member in the price determination process

and participates in every economic activity. Having determined the growth rate and the associated multipliers for this economic system, the rate of profits becomes a known variable from the growth profit relation. The growth profit relation is hence a crucial closing equation of the system⁹⁰. Thus, using the rate of profits as a known variable, we may proceed to determine the prices and the wage rate in the system. It should be noted that there are 5 prices and 1 wage rate to be determined. Of these, the price of the commodity money industry is known and assumed to be 1. Nevertheless, this price is also an unknown to the system as the price of commodity money determines its value in circulation in terms of the purchasing power. It also provides the necessary benchmark for measuring and converting relative prices to absolute prices. The commodity money also helps resolve this issue of price determination and discovering other values in the process. The initial set of prices in this system is 0.612, 0.60, 1, 1.06 and 0.85 respectively for each of the five industries. Along with prices, the wage rate is also determined which is equal to 2.65. It is important to check whether at these prices, the markets clear or not. To validate market clearing, we would resort to using the commodity demand equations. The total initial income is (40w+20r)=112.12. Using the MPC coefficients of 0.45, the commodity demands turn out to be 50.454. Accordingly, it should be noted that there is excess supply in industry 4 where the supply is 60 and marginal excess demand in industry 5 where the supply is 50. There is an excess supply created altogether. As an important step towards the economic equilibrium, we move the supplies towards the demands. Replacing the right hand sides of the consumption goods industries by numbers 55.227 for industry 4 and 50.227 for industry 5, we have used bisected demands and created new consumption goods industry sub-systems. Since the RHS of the equations are altered, the LHS will be changed proportionately in terms of the new supplies. Consequently, the own rates of growth will again be thrown out of equality and the process has to start again

⁹⁰ Digressional to the current topic but worth a mention here is the fact that the system has four implicit closing equations- one closing the production-price equations, a labour conservation equation closing the output system of equations, a growth profit relation closing the loop between price and output relations and finally, an overall closing equation for the system-namely the demand equations. Every equation in its own right merits a special mention.

from step one where the harmonization of own rates of growth had to be achieved. We start off there again, determine prices, excess demands and keep circulating in this closed loop till the markets clear and we have the sum of value of excess demands equal to zero. Market clearing situations dictate that the equilibrium is attained. The way the algorithm is designed, it is important only that the commodity markets clear since for each *iteration*, the remaining markets are made to definitionally clear. The following tables summarize the iterative history of the solutions.

Iteration	P1	P2	P3	P4	P5	W	r=g
1	0.612	0.6	1	1.06	0.85	2.65	0.306
10	0.604	0.602	1	1.074	0.834	2.54	0.327
25	0.604	0.602	1	1.07	0.83	2.53	0.329
38	0.604	0.602	1	1.076	0.833	2.531	0.329

Table: Results of commodity money system-A

Table: Results of commodity money system-B

Iteration	B1	B2	B3	B4	B5	NNP
1	14.08	24.49	80.99	55.02	56.2	171.2
10	14.13	25.48	80.54	47.46	60.99	166.95
25	14.09	25.45	80.58	47.06	60.79	166.41
38	14.09	25.45	80.58	47.05	60.78	166.406

Table A above summarizes the results of the production-price system where the prices are determined. The outputs determined through the output system are presented in table B. The price of commodity 3, P3, is seen to be one as expected since commodity 3 is a money commodity. Say, if this commodity were *bushels of wheat* for that matter now, the price of commodity one would be .612 *bushels of wheat* and on similar lines, the labour would be paid 2.65 *bushels of wheat*. It should be noted that after 38 iterations, all markets are said to be cleared and hence the general economic equilibrium of the system is determined. One point worthy of mention here is though this is the simplest case of commodity money, arranging equations or depicting the economy in this manner leads to determining

the monetary equilibrium in the realms of value theory. We face no hurdles in so doing; expect for the fact that we leave out Quantity theory and the Walras' law. Important properties of the system can be ascertained by determining important values in this system. The gross national product is given by 205.71 (say bushels of wheat). The capital stock given by the value of the stock coefficients is equal to 197.78. Accordingly, the capital-output ratio is equal to 0.961. The savings in this economy are equal to 65.134 and the ratio savings/GNP is equal to 0.3166. The ratio of savings to GNP divided by capital output ratio is equal to the 0.329, exactly equal to the system's growth rate and also happens to be the Harrod-Domar rate of growth. Being a Harrod-Domar rate of growth, it obeys all the principles of Harrod-Domar. The value of excess demands, right from iteration 1 which was equal to 10.32 did not have any other equivalent and hence, the Walras' law did not hold; at least it would not be wrong to state that the validation of Walras' law does not happen in this system. The velocity of circulation of money also is of greater economic significance. The transactions velocity is given by the ratio NNP/Money Supply and the income velocity is given by GNP/Money

Supply. The GNP and the NNP are respectively given by $\sum_{i=1}^{n} B_i p_i$

and $\sum_{i=1}^{n} B_i p_i - \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} p_j$. Using these, the transactions velocity happens to be

2.06 and the income velocity is equal to 2.55. The algorithm and the depiction of the economy in this fashion lead us to be able to determine all the relevant values and also the economic equilibrium. The real wage rate in terms of price of commodity 4 is 2.35 and in terms of commodity 5 is 3.038. These would be used to measure the price level in the economy. The commodity money supply at the outset in this system was 50. We can now think of altering this money supply and tracing out the effects. From accepted theory, a doubling of money supply is expected to double all prices such that the level of relative prices remains unaltered. This famous idea is called as the *Neutrality of Money* principle. However, if money were neutral and its only role in the economy was to enable determination of absolute prices, any commodity such as *bushels of wheat* of this

chapter can do the job. However, we would propose an even shocking result: where money plays a dominant role in the economic activity, it is always nonneutral, be it any form of money. This non-neutrality implies doubling the supply of commodity money in this case, would change absolute prices for sure and not necessarily in the same proportion. The following tables summarize the results for an increase in money supply from 50 to 75:

Table: Increase of Commodity Money Supply: Table A

P1	P2	P3	P4	P5	W	r=g
0.757	0.761	1	1.386	1.025	3.171	0.4438

Table: Increase of Commodity Money Supply: Table B

B1	B2	B3	B4	B5	NNP
15.41	27.54	118.33	45.76	61.86	233.04

It should be noted here at once that the supply of commodity 3, which is the money commodity has been increased from 50 to 75. However, no definite movements happen in absolute prices. In terms of the real price levels, the real wage in terms of commodity 4 falls to 2.29 (base case 2.35) and in terms of commodity 5 increases to 3.09 (base case 3.03). It cannot be certainly said that this would happen always. However, it should be noted that commodity 4 is a cash-intensive industry and industry 5 is a relatively lesser cash intensive industry. However, having said this, it should also be noted that the increments in money supply would not get fully distributed to all the industries; in fact, all the industries are operating under conditions of fixed technological coefficients that would deny the possibility of increased money supply percolating via money demand equations to the respective industries, unless the money demand coefficients themselves change. However, this is also not the case. Comparing

with the base case, however, it can be concluded that an overall inflationary condition is observed in this system. The rates of profits and growth both increase in this scenario compared to the base case system when commodity money supply was 50. The prices increase as well in absolute terms and the absolute level of output increases from 227 (tons, say) to 268 (tons, say). This causes the level of NNP to increase from 166 to 233 (say, *bushels of wheat*). Overall, it can be safely concluded that an increase in commodity money supply causes inflationary conditions in the economy. An increase in commodity money supply causes the initial own rate of growth to increase to 0.27 from 0.14 described above. This increased own rate of growth pushes the growth rate of the system upwards and hence, the rate of profits in the system increases. This rate of profits also increases due to expansionary movements across the economy. An increased rate of profits further pushes the prices upwards and hence causes an inflationary condition. Another aspect worth an enquiry is the case when all commodity money demands are decreased. The controlling variable for commodity money demand happens to the money-turnover ratios. In the base case, these money turnover ratios were equal to 0.25, 0.5, 1, 1.5 and 0.25 respectively for each of the industries. We now aim to reduce these commodity money requirements or commodity money demand to new values of 0.2, 0.5, 0.5, 1 and 0.05. An immediate result of decreasing the commodity money demand would be an increase in the own rate of growth and hence, an increase in the system's overall rate of growth. This would also therefore cause inflationary conditions in the economy. The new set of prices is given below.

P1	P2	P3	P4	P5	W	r=g
0.676	0.618	1	1.205	0.817	2.612	0.582

Table: Decrease commodity money demand: Results table A

Table: Decrease commodity money demand: Results table B

B1	B2	B3	B4	B5	NNP
17.02	30.13	76.8	43.34	63.88	169.86

The inflationary condition is similar to the one caused by increasing commodity money supply. It can therefore be concluded that money cannot be neutral and impacts every sector of the economy. Commodity money also is not merely a veil enabling the solution to absolute prices but is a commodity which has a value in use and also a value in circulation. Also, it can be noted that a reduction in the demand for money commodity leads to contraction in its size. As a result, compared to the base case, the quantity of the commodity money in circulation reduces from 80.58 to 76.8 as above. Also, the income velocity increases due to its reduction in demand for commodity money. It should also be noted therefore, that changes in commodity money demand also has an impact on the real variables along with the monetary variables. Monetary commodity is said to have a value in exchange and also a value in use itself. Its use value is given by the coefficients A_{it} where the subscript *i* refers to the industry where the money is used and the subscript t refers to the index for commodity money, as before. A good case may be presented and is worth an exploration- the case where the nonmonetary uses of commodity money are eliminated. These uses specifically are for non-monetary purposes and hence, they may be easier to eliminate. What would happen in this case is that the flow money coefficients are easily removed since then industries would depend only on the non-money commodities for meeting their flow requirements. By eliminating the non-monetary uses of commodity money, its demands in terms of flow requirements are reduced, following which its amount in circulation would increase. An increased money circulation leads to a increase in its velocity of circulation as well. Reduction or elimination of non-monetary uses of commodity money causes a fall in the prices of capital goods goods industry. A common reason for this is elimination of nonmonetary uses of commodity money translates into elimination of any expenditure on this account as well. This causes the overall cost of production to decline leading to a possible fall in the prices of commodities. The consumption goods industries behave in a manner depicted by the demand equations and nonmonetary uses of commodity money have little impact on those. The following tables summarize the results of a system where non-monetary uses of commodity

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money are eliminated. The production price system takes the following form in this case:

$$(3p_1 + 2p_2 + 10)r + (2p_1 + 5p_2) + 5w = 40p_1 (2p_1 + 5p_2 + 30)r + (5p_1 + 7p_2) + 5w = 60p_2 (2p_1 + 3p_2 + 50)r + (2p_1 + 5p_2) + 10w = 50 (3p_1 + 5p_2 + 90)r + (3p_1 + 2p_2) + 10w = 60p_4 (2p_1 + 3p_2 + 12.5)r + (2p_1 + 5p_2) + 10w = 50p_5$$

The own rate of growth of commodity money increases to 0.26 due to elimination of non-monetary uses of the standard.

Table: Elimination of non-monetary uses of commodity money-Table A

P1	P2	P3	P4	P5	W	r=g
0.516	0.549	1	1.131	0.657	2.214	0.4572

Table: Elimination of non-monetary uses of commodity money-Table B

B1	B2	B3	B4	B5	NNP
15.42	28.02	78.67	39.13	67.38	173.37

Elimination of non-monetary uses is tantamount to reduction of commodity money demand. As a result, it has consequences similar to those under reduction of commodity money demand. The overall volume of commodity money in circulation increases due to its relative reduction in size. Hence, its velocity would increase. The income velocity increases to 2.20 from 1.96 under the bases case scenario. The real wages in terms of commodity 4 reduce and in terms of commodity 5 increases. Overall, it can be said the inflationary conditions may be experienced in the economy. As a conclusion to this chapter, we may say that a monetary and value theory can be integrated in a manner depicted in the system proposed here. Such an integrated theory can be tested for various other conditions like changes in technology of production and consumption, changes in outputs etc. The theory developed so far is robust to generalizations.
Annexure to Chapter 3: A Model of Currency Money with deficit financing

We would now present the picture of such an economy and then characterize and analyze the properties of this system, post the deficit financing. $(S_{11}p_1 + S_{21}p_2 + \dots + k_1p_1B_1)r + (A_{11}p_1 + A_{21}p_2 + \dots + A_1G) + wL_1 = p_1B_1$ $(S_{12}p_1 + S_{22}p_2 + \dots + k_2p_2B_2)r + (A_{12}p_1 + A_{22}p_2 + \dots + A_2G) + wL_2 = p_2B_2$

 $(S_{1n+m}p_1 + S_{2n+m}p_2 + \dots + k_{n+m}p_{m+n}B_{n+m})r + (A_{1n+m}p_1 + A_{2n+m}p_2 + \dots + A_{n+m}G) + wL_{n+m} = p_{n+m}B_{n+m}$

The additional flow terms are the terms introduced to depict deficit financing and provide a complete model of currency money economy. The prices post the provision of deficit financing tends to increase; the deficit financing tends to create a demand effect. It creates additional demand and by creating additional demand tends to employ the unemployed labour in the economy. We would now

present the revised initial system below:

 $\begin{aligned} (3p_1 + 0.4p_1 + 2p_2 + 5p_3)r + (2p_1 + 5p_2 + 3p_3) + 1.5p_1 + 5w &= 40p_1 \\ (2p_1 + 5p_2 + 1.5p_2 + 3p_3)r + (5p_1 + 7p_2 + 5p_3) + 1.5p_2 + 5w &= 60p_2 \\ (2p_1 + 3p_2 + 6p_3 + 5p_3)r + (2p_1 + 5p_2 + 3p_3) + 1.5p_3 + 10w &= 50p_3 \\ (3p_1 + 5p_2 + 6p_3 + 0.6p_4)r + (3p_1 + 2p_2 + 5p_3) + 1.5p_4 + 10w &= 60p_4 \\ (2p_1 + 3p_2 + 5p_3 + 2.5p_5)r + (2p_1 + 5p_2 + 7p_3) + 1.5p_5 + 10w &= 50p_5 \\ (12.4p_1 + 19.5p_2 + 30p_3 + 0.6p_4 + 2.5p_5)(1 + g) &= 1493 \\ 0.45(40w + 20r) &= 60p_4 \\ 0.45(40w + 20r) &= 50p_5 \\ (3q_1 + 2q_2 + 2q_3 + 5)g + (2q_1 + 5q_2 + 2q_3 + 5) &= 40q_1 \\ (2q_1 + 5q_2 + 3q_3 + 8)g + (5q_1 + 7q_2 + 5q_3 + 7) &= 60q_2 \\ (5q_1 + 3q_2 + 5q_3 + 11)g + (3q_1 + 5q_2 + 3q_3 + 12) &= 50q_3 \\ 5q_1 + 5q_2 + 10q_3 &= 20 \\ 321r &= 660g - 2w \end{aligned}$

The terms introduced following the parenthesis of flow variables are the new policy variables in the revised model- the level of deficit financing. While iterating for equilibrium, the final levels of necessary deficit financing are determined by the system in such a manner that the employment gap and the distortionary gap between NNP market prices and factor costs is eliminated. This revised system re-attains equilibrium with a different set of prices:

Table: Results of complete Currency Economy Model-Table A

P1	P2	P3	P4	P5	r	g	W
9.56	7.40	12.32	9.76	12.12	1.35	1.27	28.38
				1.0.0.1			0

In this economy, the real wage rate is 2.91 and 2.34 respectively in terms of commodity 4 and 5. Thus, this provision of deficit financed expenditures creates inflationary conditions in terms of the purchasing power of the wage rate. However, in terms of relative prices, there is an overall decrease of the prices. The increase in real wages tends to attract more labour to close the disequilibrium gap. A new final equilibrium can thus be attained through the process of deficit financing. More so, there is also a change in the rate of profits and rate of growth. An increase in deficit financing is necessary when the labour demand is less than the labour supply. In this case, deficit finance is necessary to bridge this gap. The

profits and the rate of growth are 1.35 and 1.26. As purchases by the government in terms of the own commodities are introduced in the model, the own rate of growth of all the industries reduces. This causes a fall in the overall growth rate and hence, a fall in the rate of profits. This fall in the rate of profits causes the prices in absolute terms as well to reduce. There can be a case where there is excess government expenditure as well. This case would also merit some discussion. During such cases where there is excessive deficit financing activity in the economy, the labour demand would exceed the labour supply. In this connection, we would see a reverse gap; the NNP market prices would exceed the NNP factor costs. Let us consider a case where excess government expenditure through the way of deficit financing is seen in the economy. In such a case, there would be over-employment in the economy. There would be, say, 48 resources employed but only 40 resources are being paid-a fraud of a second nature. In such a case, the prices would be much lower than the economy with equilibrium level of deficit financing. The real wages in this economy are 3.55 and 2.83 and to reduce the volume of employment to restore it to the level of 40, there has to be a reduction in real wages. This can be forthcoming through reduction in the volume of deficit financing.

Table: Special case- Excess government expenditure

P1	P2	P3	P4	P5
6.89	5.36	9.10	7.35	9.21

In general, the increase in deficit financing leads to increase the real wage rate and hence increase the level of employment in the economy. This increased volume of employment leads to closing the gap between the valuations of NNP at market prices and factor costs. Reverting back, it should be noted that the government has this tool at its disposal to correct any employment gap and eliminate any abnormalities in the pricing process. It should be noted that initially as we begin with the economy, the own rates of growth were 1.97, 1.77 and 0.85 respectively for industry 1, 2 and 3. As the prices and wages are determined, the income levels in the economy would be determined. It is observed that in the first iteration, there is excess supply in both the consumption goods industries. As a result, in the next iteration the prices of consumption goods industries fall and their demand changes accordingly. However, it is seen that there is now an excess demand in industry 4 and an excess supply in industry 5. Quantity adjustments keep on happening till appoint where all excess demands and supplies are cleared through changes in wealth, income and other price variables. The level of deficit financing has to be accurately determined by governments in such a manner that the gap is exactly eliminated. This also turns out be an important feature of monetary economy- it in itself provides for a role of government. In proper equilibrium, when all distortions are cleared from the system, the NNP of the economy is 2028. This value, when compared to the value 1875 when no deficit financing is used looks higher. The reason for this is that deficit financing creates additional demand and income so that more labour is actually employed in the system. In a system where no deficit financing was provided for, the income velocity of money was 0.79. Due to provisioning of deficit financing, two monetary effects are seen: one, the level of currency in the system increases from 1470 to 1497; secondly, due to this increase, the velocity of circulation of money reduces to 0.73. This also explains why prices fall. The level of deficit financing at equilibrium is 190.93. The growth rate of the system is also the Harrod-Domar growth rate. In all the cases, there would exist always an inverse relationship between rate of profits and wages: as wages rise, profits necessarily fall and the converse is true. As we can conclude this section, it would be prudent to provide the iterative solutions of the above economy with deficit financing, providing for the accurate role of the state.

Iteration	P1	P2	P3	P4	P5	W	r
1	17.13	13.25	21.98	17.34	21.51	48.19	1.308
5	9.52	7.37	12.27	9.72	12.08	28.29	1.351
20	9.55	7.40	12.32	9.76	12.12	28.37	1.352
27	9.55	7.40	12.32	9.76	12.12	28.37	1.352

Table: Iterative results of a currency money economy: Price System

Iteration	B1	B2	B3	B4	B5	g
1	28.96	47.02	62.30	58.73	48.81	1.289
5	29.01	47.06	62.25	60.42	49.94	1.267
20	29	47.04	62.27	60.29	49.83	1.268
27	29	47.04	62.27	60.29	49.83	1.268

Table: Iterative results of a currency money economy: Quantity System

Table: Iterative results of a currency money economy: Other parameters

Iteration	Savings	NNP	Currency	Capital/output	Per Capita Income
1	919.85	3572	1580	0.1234	90
5	836.69	2023	1496	0.1857	51
20	837.36	2028	1497	0.1854	50
27	837.38	2028	1497	0.1854	50

A few derived variables can be ascertained. The real wages in terms of commodity 4 and 5 respectively at equilibrium are 2.91 and 2.34. The currency velocity defined as the ratio of NNP to total currency is 1.35.

The economic properties of this system could be analyzed in greater details through various simulations that we plan to introduce. The effect of parameters of the system on the equilibrium properties of the system have to be analyzed in the context of currency money. Let us begin by changing the parameters of the system one by one and trace the direction of impact on the economic variables. It is important to note that the parameters in this system are currency demand functions (or the money turnover ratios), propensities to consume, propensities to save asset-wise (propensity for currency and equity) and the wealth of capitalists and workers. Let us begin by changing the easiest of all- the wealth of capitalists and workers. We would increase the wealth of capitalists and workers. Also, we would at each time compare the results of our economic simulations after adjusting for the appropriate deficit financing level so that we at each time are comparing economic systems across the equilibrium positions. Let us begin by changing the wealth of capitalists and workers to 1000 and 400 from previous levels of 800 and 200. This implies disproportionate change in the wealth parameters- while capitalist wealth is increased by 25%, the workers wealth is increased by 50%. The prices would change and increase due to changes in the equity holdings which are a proportion of the workers' and capitalists' individual wealth. Also, the absolute wage rate increases. The rate of profits and rate of growth increases as well.

Table: Impact of increase in wealth- Table A

P1	P2	P3	P4	P5	r	g	W
10.42	8.07	13.44	10.65	13.24	1.35	1.26	31.03

The real wage rate decreases marginally in this scenario to 2.91 and 2.34 in terms of commodity 4 and commodity 5. These comparisons are performed against the case of complete model of currency money with deficit financing. The value of real wages was similar compared to the base case. The NNP of this economy is 2212, higher than the NNP of the base-case economy of 2028. An increase in the overall incomes and a reduction in absolute prices is an immediate wealth effect in this economy. The value of the commodity-wise outputs also changes in this system. The new outputs are as below

Table: Impact of increase in wealth-Table B

B1	B2	B3	B4	B5
29.00	47.05	62.27	60.28	49.81

It can be seen that due to an autonomous increase of wealth, profits decline and wages increase. Also, the prices increase relatively and the outputs decline. This would happen because with an increased wealth, consumption may not increase in the same proportion and in fact, given the savings propensities, the individual commodity demands increase. It should be noted here that the volume of deficit financing in this economy is relatively higher at 208.11 compared to the level in the base case economy of 190. In general, changes in wealth coefficients do not cause major changes in real variables of the economy. The absolute outputs, growth rate and real wages remain do not change to a greater extent. However, reallocations of wealth and hence capital cause profits and level of absolute prices

or change. It should be noted here therefore that, changes in wealth and any attempts to alter social wealth of all the economic agents may lead to only increments in prices without any real impact.

We would now restore the wealth coefficients to the previous level and make changes the marginal propensity to consume of the workers. The current economic system assumes a workers' MPC of .90. We would decrease this to 0.60. An immediate result of the decrease in the MPC would be that the demands and the sizes of the consumption goods industries would reduce. This would cause the rate of profits and the rate of growth to reduce and real wages to increase. Also, a reduction in the MPC increases the savings in the economy and the rate of capital formation. Overall, a reduction in the NNP would be seen due to a fall in the MPC of the workers.

Table: Impact of changes in MPC-Table A

P1	P2	P3	P4	P5	r	g	w
9.38	7.30	12.41	10.04	12.59	0.95	1.27	36.01

In net effect, the outputs of the individual capital goods industries do not change much, compared to the outputs of the consumption goods industries. Table: Impact of changes in MPC-Table B

B1	B2	B3	B4	B5
29.00	47.05	62.27	59.35	50.53

In value terms, the outputs of the consumption goods industries increases to 1232.04 from 1193.92, measured in terms of the currency. An important impact of the reduction in MPC can be presented here. As the MPC in the economy reduces, thereby causing a decrease of the total demand as indicated by a fall form 1166 to 1041, the level of employment in the economy also falls considerably. Therefore, with this decline in the MPC by the workers, the government has to step in the system again and tweak its policy variable, the volume of deficit financing. There is an increase in deficit financing from 190 to 450. If this increase is not resorted to, there would be under-employment in the economy and an overall shortfall of (aggregate) demand. The government therefore would increase its deficit

financing in the event of an economy wide reduction in consumption demand. The above results should be compared to a system where the level of deficit financing is kept at previous levels and the MPC decreases. In this case, the real wage rate is lower, the level of absolute prices is lower and also, the profit and growth rate is lower.

Table: Results of decrease in MPC without adequate deficit financing

P1	P2	P3	P4	P5	r	g	W
9.38	7.30	12.41	10.04	12.59	0.95	1.27	36.01

The government aims to stimulate demand and hence, at current levels, the sizes of individual industries are also smaller. Accordingly, in this system, the level of NNP is also around 1794 and the full effect of a decline in the MPC is not seen in its totality- one due to the uniqueness of the labour and secondly, due the nature of the algorithm which clears only the commodity markets.

A classic case simulation would be seen if the money turnover ratios are changed in this economy. These specifically are the currency demand functions. In the asis conditions, the currency demand function are as under:

Table: As-is money-turnover technology

K1	K2	K3	K4	K5
0.01	0.025	0.12	0.01	0.05

These, when multiplied with the respective outputs give the exact currency requirements of the form $k_i p_i B_i$. We would now change these K_i s. We would aim to present two cases- one where the K_i s increase and the other case where in the K_i s decrease. Let us begin with the following assumption for the K_i s: Table: Simulation money-turnover technology

K1	K2	K3	K4	K5
0.02	0.05	0.15	0.02	0.1

The immediate impact of this would be the volume of currency circulating in the economy reduces from 1293 to 1255, providing lesser liquidity to the people to make their purchases of commodities. This reduces the income velocity of money circulation to 0.75 from 0.78 under the base case scenario. As a result, the NNP in the economy reduces considerably to 1587. An autonomous demand gap is

therefore created which needs to be eliminated by stimulating further demand. Hence, the amount of deficit financing increases. The volume of deficit financing in this economy is slightly lower at 183 from the volume of deficit financing in the base case economy of 190.

Lastly, we would now change the asset-wise propensities to save of the workers, keeping the aggregate savings propensity equal to the base case economy. The following table should make the assumption for this simulation clear.

	Kwc (Currency propensity to save)	Kpe (Equity Propensity to save)
Base Case	0.5	0.5
New	0.4	0.6

As indicated, we have reduced the propensity to hoard currency and increased the propensity to add to the equity. The immediate effect should be that there would be more capital available and hence the outputs would be higher. This would now create a situation of excess demand and the government would now aim to reduce its expenditure in order to eliminate the situation of over-employment. The volume of deficit financing reduces to 153 from the base case scenario of 190. The level of NNP reduces as well. The effect on prices is certain; and a result of this reallocation of savings by the workers in the various assets in favour of the equity capital causes prices to reduce. There is more capital stock leading to a reduction in the rate of profits; the profit rate reduces from 1.33 to 1.31.

P1	P2	P3	P4	P5	r	g	W
8.24	6.39	10.66	8.46	10.52	1.31	1.27	25.14

The wages increase marginally and the outputs of the individual industries do not change drastically. The growth rate as a result of minuscule changes in outputs remains unchanged.

In summary, the following can be presented as marquee observations in a currency money system.

	Simulations							
Impact		Decrease	Increase	Reallocate				
variable	Increase of (absolute) wealth	MPC of	Money	asset-wise				
	of agents	workers	Demand	MPS				
Prices	Increase	Decrease	Increase	Unchanged				
Profit rate	Increase	Decrease	Increase	Unchanged				

Wage rate	Increase	Decrease	Decrease	Unchanged
NNP	Increase	Increase	Decrease	Increase
Deficit				
Financing	Increase	Increase	Increase	Decrease
Absolute				
outputs	Increase	Increase	Increase	Increase
Real wages	Decrease	Increase	Increase	Increase
Growth Rate	Increase	Unchanged	unchanged	Unchanged

The above table illustrates the impact of changes in certain key parameters in the economy on the economic variables. It can be concluded that changes in any parameters leading to a demand reduction in the economy would entail an increment in the deficit financing and the impact on the remaining variables of interest can be traced out accordingly. Also, a point worth noting is this: in a currency money economy, all the markets *may not clear* autonomously without any external corrections. An externally introduced agent, system or even a catalyst like money is enough to disturb the processes of various markets, if the market for any of the economic variables fails to exist. Another point worth note is that we have left out the labour and the money market out of analysis not by choice but by reasoning and economic consistency of the system. In a system where labour enters production on the same footing as capital and money, there may not be a separate device to identify its price; in fact it will always be determined as a part of the production process. In effect, it should be concluded that in a currency money system, deficit financing plays a dominant role in closing the system. However, deficit financing and its use is necessary only in a system where an external form of a monetary commodity is introduced. What may need elaboration at the end of this chapter therefore is the device of pulling the economy out of disequilibrium phases. It should be noted that we had started off with an economy where the level of deficit financing was zero and hence had experienced a distortionary gap of -133. We would now increase the coefficients attached to the deficit financing, $A_i G$ to 10; this leads to sign reversal in the gap and the gap increases to 59. This implies that the value of deficit financing has to be between 0 and 10. We now take the value to be 5. At 5, the gap reduces to -31indicating that the level of deficit financing should lie between 5 and 10; we try

7.5. At 7.5, the gap increases to 14 and hence, the level of deficit financing has to be between 5 and 7.5; we try a value 6. At a value 6, the gap is -12 indicating the



value of deficit financing has to be between 6 and 7.5; the accurate value happens to be 6.7045. This is crude and a rudimentary method of finding equilibrium of the system. It should be noted that whilst we are aiming to reduce the gap to zero, in the process we are also aiming to eliminate the gap between NNP valuations at factor costs and market prices. This implies a correction of outputs, prices and employment brought simultaneously with the help of deficit financing i.e. currency money. Thus, this process of eliminating the gap is the core process of a monetary economy.

Annexure⁹¹ to Chapter 5: The failure of Quantity Theory Equation as a closing equation

In this brief note, the conceptual properties of the system introduced in chapter 3 remain intact. However, we would like to demonstrate the failure of using a quantity theoretic equation in determining the solutions to an economy with money in the system. It should be remembered that quantity theory formed the backbone of the entire monetary synthesis and was thought of as a means to determine the "absolute" prices- though what it determined was an absolute "level" of prices and not the individual commodity prices. Nevertheless, the use of such an equation of the quantity theory nature in any of the neoclassical frameworks is missing and where used, it is only found to provide contradictory results. This emphasizes the only point- the quantity theory equation cannot be used as a closing equation for the price system- in the sense that such an equation will not provide "meaningful" solutions to the system⁹². The quantity theory of money says that the value of transactions in an economy is restricted by the volume of money circulating in the economy. We would use a similar equation in this note to demonstrate the failure of the quantity theory. In the economy presented in the chapter 3, money was held as capital stock with a relationship with the turnover. The money-turnover ratio Ψ_i of chapter 5 is the actual money holding in the economy. To introduce the quantity theory equation in the system, we need to introduce an exogenously given money supply- say we fix this at 2,500. In this case, the closing equation would take the following form

⁹¹ Though it reads "Annexure to chapter 5", the conclusions derived in this note also apply to economic system with only currency money- those of chapter 3. The only difference is mentioned in the end of this note

⁹² Notice the use of the adjective "meaningful". We will clearly see the context in which this is being said in a while. However, it is important to state here that though we replace the closing equation in the price system of chapter 3 with the equation of quantity theory of the type to be introduced in this note, it may lead to "closing" the system *mathematically* still- however, the economic logic of such a system will be lost!

 $\sum_{i=1}^{m+n} \psi_i p_i B_i (1+g) = 2500$. In this regard, Ψ_i can also be regarded as the inverse of

velocity of circulation of money, to an extent. Thus, the equations would change in the following

manner

$$\begin{array}{l} (0.25p_1+3p_1+2p_2+5p_3)r + (0.66p_1+0.40p_2+1.02p_3)(1+i_1) + (0.66p_1+0.40p_2+1.02p_3)i_2 + \\ (0.66p_1+0.40p_2+1.02p_3)i_3 + (2p_1+5p_2+3p_3) + 5_w = 40p_1 \\ (2p_1+0.5p_2+5p_2+3p_3)r + (0.40p_1+1.12p_2+0.61p_3)(1+i_1) + (0.40p_1+1.12p_2+0.61p_3)i_2 + \\ (0.40p_1+1.12p_2+0.61p_3)i_3 + (5p_1+7p_2+5p_3) + 5_w = 60p_2 \\ (2p_1+3p_2+1p_3+5p_3)r + (0.40p_1+0.61p_2+1.22p_3)(1+i_1) + (0.40p_1+0.61p_2+1.22p_3)i_2 + \\ (0.40p_1+0.61p_2+1.22p_3)i_3 + (2p_1+5p_2+3p_3) + 10_w = 50p_3 \\ (3p_1+5p_2+6p_3+1.5p_4)r + (0.61p_1+1.02p_2+1.22p_4)(1+i_1) + (0.61p_1+1.02p_2+1.22p_4)i_2 + \\ (0.61p_1+1.02p_2+1.22p_4)i_3 + (3p_1+2p_2+5p_3) + 10_w = 60p_4 \\ (2p_1+3p_2+5p_3+0.25p_3)r + (0.40p_1+0.61p_2+1.02p_5)(1+i_1) + (0.40p_1+0.61p_2+1.02p_5)i_2 + \\ (0.40p_1+0.61p_2+1.02p_5)i_3 + (2p_1+5p_2+7p_3) + 10_w = 50p_5 \\ (0.25p_1(1+g)+0.5p_2(1+g)+1p_3(1+g)+1.5p_4(1+g)+0.25p_5(1+g)) = 2500 \\ 104.275+47.5(1+i_1) = 145(1+i_1) \\ 11.608+65(1+i_1)+80(1+i_2)^2 = 145(1+i_2)^2 \\ 14.979+20(1+i_2)^2 + 125(1+i_2)^3 = 145(1+i_2)^3 \\ 853 = B_4p_4 \\ 848 = B_5p_5 \\ (3.25q_1+2q_2+2q_3+5.73)g + (2q_1+5q_2+2q_3+5.73) = 40q_1 \\ (2q_1+5.5q_2+3q_3+9.18)g + (5q_1+7q_2+5q_3+7.96) = 60q_2 \\ (5q_1+3q_2+6q_3+12.59)g + (3q_1+5q_2+3q_3+13.68) = 50q_3 \\ 5q_1+5q_2+10q_3 = 20 \end{array}$$

The only change above that we have made is in equation 6, which is the closing equation for the price system. The entire algorithm of arriving at solutions of the system remains entirely similar to the one detailed in chapter V. However, the results of this system are much different than the one in chapter V. In the first place, this system does not solve itself fully since we obtain one or a few economic variables as negative values. Since a change is made in the closing equations for price system, invariably these negative values occur either for prices, wages or rate of profits or any combination of these three variables. As the prices become negative, the value of debt also becomes negative and interest rates become bizarre. Lastly, if such an equation be used in the system with currency

money only, it should be noted that we do not encounter negativity of economic variables; however the disequilibrium gap still exists- only to reiterate that solutions with quantity theory equation are also not possible for a monetary economy!

Annexure to Chapter 6: Inter-temporal Equilibrium

Previous sections detailed out the process of equilibrium within the purview of a monetary theory of value. In effect, we studied the properties of a system where money in its various forms was involved. It was observed that in this type of an economy, deficit financing provided the only measure to clear any sort of disequilibrium noted. This effect behaves exactly akin to the impacts Keynes had predicted in his general theory with respect to wasteful spending. Two important points worth to be noted before we can proceed any further: in this capitalist monetary economy, any form of debt, external or fiat money will cause a disequilibrium gap to exist and deficit financing would have to be introduced as the only alternative if equilibrium has to be restored. Secondly, in this economy, the standard quantity theory does not turn out to be an operative equation-in fact if introduced it defies its own purpose- the much debated "determination of absolute prices". Also, the sum of excess demands even at equilibrium does not equate the excess demand for money- a direct violation of the Walras' law. More specifically, these two requirements of a general equilibrium theory of money are seen to be the root causes of general disequilibrium with money. In essence, equilibrium can be restored by parting with these necessities. Having done so, it is important to note that all this while, we were assuming a one period analysis. The true Walrasian auctioneer was present on every Monday morning crying out quotes and matching demands and supplies to seek the value-more so the money value of goods and services. What needs to be explored is the behaviour of the system outside the realm of this one-period analysis. We would therefore present an analysis of inter-temporal equilibrium beginning with an economy with only deposit money and currency of the previous chapters.

$$\begin{array}{l} (0.25p_1+3p_1+2p_2+5p_3)r + (0.66p_1+0.40p_2+1.02p_3)(1+i_1) + (0.66p_1+0.40p_2+1.02p_3)i_2 + \\ (0.66p_1+0.40p_2+1.02p_3)i_3 + (2p_1+5p_2+3p_3) + 5_w = 40p_1 \\ (2p_1+0.5p_2+5p_2+3p_3)r + (0.40p_1+1.12p_2+0.61p_3)(1+i_1) + (0.40p_1+1.12p_2+0.61p_3)i_2 + \\ (0.40p_1+1.12p_2+0.61p_3)i_3 + (5p_1+7p_2+5p_3) + 5_w = 60p_2 \\ (2p_1+3p_2+1p_3+5p_3)r + (0.40p_1+0.61p_2+1.22p_3)(1+i_1) + (0.40p_1+0.61p_2+1.22p_3)i_2 + \\ (0.40p_1+0.61p_2+1.22p_3)i_3 + (2p_1+5p_2+3p_3) + 10_w = 50p_3 \\ (3p_1+5p_2+6p_3+1.5p_4)r + (0.61p_1+1.02p_2+1.22p_4)(1+i_1) + (0.61p_1+1.02p_2+1.22p_4)i_2 + \\ (0.61p_1+1.02p_2+1.22p_4)i_3 + (3p_1+2p_2+5p_3) + 10_w = 60p_4 \\ (2p_1+3p_2+5p_3+0.25p_5)r + (0.40p_1+0.61p_2+1.02p_5)(1+i_1) + (0.40p_1+0.61p_2+1.02p_5)i_2 + \\ (0.40p_1+0.61p_2+1.02p_5)i_3 + (2p_1+5p_2+7p_3) + 10_w = 50p_5 \\ (15.31p_1+23.12p_2+31.25p_3+1.875p_4+0.312p_5) = 725 \\ 104.275 + 47.5(1+i_1) = 145(1+i_1) \\ 11.608 + 65(1+i_1) + 80(1+i_2)^2 = 145(1+i_2)^2 \\ 14.979 + 20(1+i_2)^2 + 125(1+i_2)^3 = 145(1+i_2)^3 \\ 853 = B_4p_4 \\ 848 = B_5p_5 \\ (3.25q_1+2q_2+2q_3+5.73)g + (2q_1+5q_2+2q_3+5.73) = 40q_1 \\ (2q_1+5.5q_2+3q_3+9.18)g + (5q_1+7q_2+5q_3+7.96) = 60q_2 \\ (5q_1+3q_2+6q_3+12.59)g + (3q_1+5q_2+3q_3+13.68) = 50q_3 \\ 5q_1+5q_2+10q_3 = 20 \end{array}$$

We had already presented the results of this system under conditions of market clearing i.e. with deficit financing. The only difference is in this case a using a different numerical example as compared to the one used in before. We would start at with the solution for this economy at period 1.

i1	i2	i3	Loans	Deposits
5.71%	9.61%	14.08%	20.93	20.93

r	g	P1	P2	P3	P4	P5	w
0.46	0.29	0.266	0.204	0.298	0.245	0.318	0.695

B1	B2	B3	B4	B5
54.18	65.90	67.89	80.39	62.13

The period one solution is presented above. It is important to pause for a moment and understand that the process of period 1 equilibrium also has had serious impacts on the economy in the sense of market clearing. Whilst all market aim to attain equilibrium, the shape of the economy is considerably changed. Demand equations change the production equations significantly, that causes prices, profits, wages and hence incomes to change. As these significant variables change, deposits and loans also change drastically. Hence, it would be worthwhile to look at the shape of this changed economy before we can explain the path to period 2.

$$\begin{array}{l} (0.25p_1+3p_1+2p_2+5p_3)r + (\ 0.66p_1+0.40p_2+1.02p_3)(1+i_1) + (\ 0.66p_1+0.40p_2+1.02p_3)i_2 + \\ (0.66p_1+0.40p_2+1.02p_3)i_3 + (\ 2p_1+5p_2+3p_3) + 5_w = 40p_1 \\ (2p_1+0.5p_2+5p_2+3p_3)r + (\ 0.40p_1+1.12p_2+0.61p_3)(1+i_1) + (\ 0.40p_1+1.12p_2+0.61p_3)i_2 + \\ (0.40p_1+1.12p_2+0.61p_3)i_3 + (\ 5p_1+7p_2+5p_3) + 5_w = 60p_2 \\ (2p_1+3p_2+1p_3+5p_3)r + (\ 0.40p_1+0.61p_2+1.22p_3)(1+i_1) + (\ 0.40p_1+0.61p_2+1.22p_3)i_2 + \\ (0.40p_1+0.61p_2+1.22p_3)i_3 + (\ 2p_1+5p_2+3p_3) + 10_w = 50p_3 \\ (3p_1+5p_2+6p_3+1.5p_4)r + (\ 0.61p_1+1.02p_2+1.22p_4)(1+i_1) + (\ 0.61p_1+1.02p_2+1.22p_4)i_2 + \\ (0.61p_1+1.02p_2+1.22p_4)i_3 + (\ 3p_1+2p_2+5p_3) + 10_w = 60p_4 \\ (2p_1+3p_2+5p_3+0.25p_5)r + (\ 0.40p_1+0.61p_2+1.02p_5)(1+i_1) + (\ 0.40p_1+0.61p_2+1.02p_5)i_2 + \\ (0.40p_1+0.61p_2+1.02p_5)i_3 + (\ 2p_1+5p_2+7p_3) + 10_w = 50p_5 \\ (15.31p_1+23.12p_2+31.25p_3+1.875p_4+0.312p_5) = 725 \\ 6.2661+1.05(1+i_1) = 6.9782(1+i_1) \\ 0.5705+3.9495(1+i_1) + 3.0287(1+i_2)^2 = 6.9782(1+i_2)^2 \\ 0.734+1.9712(1+i_2)^2 + 5(1+i_2)^3 = 6.9782(1+i_2)^3 \\ 2.67 = B_4p_4 \\ 2.67 = B_5p_5 \\ (3.25q_1+2q_2+2q_3+5.73)g + (2q_1+5q_2+2q_3+5.73) = 40q_1 \\ (2q_1+5.5q_2+3q_3+9.18)g + (5q_1+7q_2+5q_3+7.96) = 60q_2 \\ (5q_1+3q_2+6q_3+12.59)g + (3q_1+5q_2+3q_3+13.68) = 50q_3 \\ 5q_1+5q_2+10q_3 = 20 \\ \end{array}$$

With this as the final state of the economy at the end of period 1, all which is left now is explanation of the process of equilibrium. From the final equilibrium condition, the economy before it goes into the markets for the next period grows in size in the first place. It grows per the growth rates attributed to capital and consumption goods industries. Thus, new stock, flow matrices are created using the growth rate obtained. As an example, while moving from one time period to the next, the following augmentation has to be performed on the production-price relations.

$$\begin{cases} \varepsilon_{11}(\psi_{1}B_{1}p_{1} + S_{11}p_{1} + S_{12}p_{2} + \dots + S_{1n}p_{n})r + (\delta_{11})(\psi_{1}B_{1}p_{1} + S_{11}p_{1} + S_{12}p_{2} + \dots + S_{1n}p_{n})(1 + i_{1}) \\ + (\delta_{12})(\psi_{1}B_{1}p_{1} + S_{11}p_{1} + S_{12}p_{2} + \dots + S_{1n}p_{n})(i_{2}) + \dots + (\delta_{1})(\psi_{1}B_{1}p_{1} + S_{11}p_{1} + S_{12}p_{2} + \dots + S_{1n}p_{n})(i_{t}) \\ + (\delta_{1t})(\psi_{1}B_{1}p_{1} + S_{11}p_{1} + S_{12}p_{2} + \dots + S_{1n}p_{n})(i_{t}) + A_{11}p_{1} + A_{12}p_{2} + \dots + A_{1n}p_{n} + L_{1}w \\ = B_{i}p_{i}(1 + g^{*}) \\ \forall i = 1 \dots + n \end{cases}$$

Here, g^* is the equilibrium growth rate of the previous period. As a consequence of this alternation, the attained equilibrium of the previous period gets distorted. A new equilibrium has to be restored. In the process, the demand for loans and the money holdings also get altered. There is a new set of loan demands that now has to be matched to deposits. It is important to point out here that the discussion on parameters introduced earlier had exhibited the savings process in this economy as well. With given savings proportions spread over asset holding preferences, new deposits are determined using these ratios and the new equilibrium incomes of capitalists and workers. The savings are a part of incomes and are distributed across deposits and equities. Thus, new levels of deposits are determined. These new deposits are matched to the new loan demands from the producers. A new set of interest rate equations are determined using the probability matrix. With these new interest rates, the production-price equation determines the new set of prices, rate of profits and wages. New level of currency is also determined in the process. Demands are recalibrated and outputs are determined; the process continues till equilibrium has been restored in period 2. We have solved out the process of this inter-temporal equilibrium until eight periods (for want of space; nonetheless the process can be continued to eternity) of economic analysis and we present the results below. In effect, it can be added that the model presented above has similar properties across and within time.

Period	P1	P2	P3	P4	P5	R	g	W
1	0.2664	0.2045	0.2982	0.2449	0.3182	0.4585	0.2911	0.6945
2	0.2653	0.2037	0.2988	0.2455	0.3192	0.4398	0.2919	0.7184
3	0.2644	0.203	0.2991	0.2459	0.3198	0.4265	0.2924	0.7356
4	0.2637	0.2025	0.2993	0.2461	0.32	0.4174	0.2928	0.7474
5	0.2633	0.2022	0.2994	0.2462	0.3205	0.4114	0.2933	0.7549
6	0.2629	0.2019	0.2994	0.2463	0.3206	0.4069	0.2934	0.7609
7	0.2628	0.2017	0.2994	0.2463	0.3207	0.4042	0.2936	0.7642
8	0.2625	0.2017	0.2994	0.2463	0.3208	0.4022	0.2938	0.7667

These are the solutions of the real economy where it can be seen that this system has properties of long-term convergence. All the key variables tend to converge to their long term values over a course of time. It should be noted that the real wage rate in terms of commodity 4 and 5 increase from 2.83 and 2.18 respectively to 3.11 and 2.39 respectively over the course of the eighth iteration. There is therefore seen that there is a relatively booming economy. In terms of the interest rate equations, the following is seen:

Period	i1	i2	i3	Deposits/Loans
1	5.71%	9.61%	14.08%	20.940
2	5.82%	9.81%	14.47%	26.690
3	5.92%	9.98%	14.73%	34.190
4	6%	10.12%	14.91%	43.949
5	6.07%	10.23%	15.08%	56.616
6	6.13%	10.32%	15.21%	73.030
7	6.17%	10.41%	15.37%	94.370
8	6.21%	10.46%	15.40%	122.010

It should also be pointed out that the amount of "gap", which we had pointed out while explaining the nature of monetary disequilibrium, increases over a period of time. The level of deficit financing accordingly required for clearing the "gap" increases over the period of time. The value of deficit financing changes from 1.577 in the first period to 9.057 indicating that as the economy grows in size, the level of deficit financing increases. Thus, our model of a monetary economy has general equilibrium property of inter-temporal equilibrium and long tern stability. As we draw towards the closure of this synthesis, it becomes imperative to study the inter-temporal properties through simulated changes in parameters of the model. We may begin with changes in tastes and preferences represented by changes in consumption propensities for the consumption goods industries. At the

current levels, the current observations, the workers are assumed to consume 0.80 of their incomes and the capitalists consume 0.10 of their incomes. We would begin by reducing these propensities. A reduction in demand would reduce prices and thereby affect the real and the monetary variables. An overall reduction in the size of the economy at the end of the eighth iteration can be expected. Changes in propensity to consume, here an overall reduction in MPC, reduces the demand in each period and at the same time increases savings. A reduction in demand causes prices to fall whereas an increase in savings either in form of equities or deposits increases the supply of loans. At new loan supplies, the loan demands not being unchanged, eases the pressure on interest rates. As a consequence, interest rates decline. Rate of profits also decreases due to reduction in economic activity and more wages have to be paid to labour since a substitution happens between cheap capital and labour. We now present the results of the system for the first few iterations post the changes in the MPC

Period	P1	P2	P3	P4	P5	R	g	W
1	0.2641	0.2029	0.3020	0.2488	0.3224	0.409	0.2914	0.7675
2	0.2629	0.2020	0.3023	0.2492	0.3230	0.3931	0.2917	0.7888
3	0.2621	0.2014	0.3025	0.2494	0.3235	0.3828	0.2921	0.8011
4	0.2617	0.2011	0.3025	0.2495	0.3237	0.3763	0.2927	0.8093
5	0.2614	0.2009	0.3026	0.2496	0.324	0.3715	0.293	0.8155
6	0.2612	0.2007	0.3027	0.2497	0.3241	0.3683	0.2933	0.8198

The monetary economy necessarily is impacted with the changes in the MPC. Changes in MPC impact the savings behavior of the economy and hence affect interest rates through changes in deposits and thereby related investments. Since the consumption expenditure is replaced with savings, the NNP of the economy does not change drastically. The real wages increase from 3.09 to 3.28 over six iterations in terms of fourth commodity and from 2.37 to 2.53 in terms of commodity five. Due to overall price reduction that is seen, the real purchasing power of money is seen to increase. This causes the level of deficit financing to reduce. It in fact falls to 4.177 as compared to 7.864 in sixth iteration of the economy with not MPC changes. Interest rate movements are also not absurd. The following table summarizes the monetary side of the economy.

Period	i1	i2	i3	Deposits/Loans
1	5.71%	9.60%	14.05%	20.940
2	5.82%	9.81%	14.45%	26.797
3	5.93%	9.98%	14.73%	34.421
4	6.01%	10.12%	14.94%	44.3125
5	6.08%	10.25%	15.07%	57.18
6	6.13%	10.34%	15.17%	73.88

The important observation while concluding this is as follows: a change in MPC in any period affects the long term equilibrium of the economy by reducing prices and interest rates, thereby reducing profit rates and increasing real wages. These effects are sustained and they do not affect the long term equilibrium- the system moves to another level of inter-temporal equilibrium.

The next step in our simulations we take is changes in the technological matrix. By technological matrix, we would mean to change only the production-price equations by changing the stock-flow coefficients, labour and output coefficients. There may be alternate ways to produce similar products; this fact we have seen in a previous chapter and the choice of technology shall depend upon the cost of producing that particular product. However, here, we would explore the long term properties of a system where in changes in technology takes place and as a result, a new level of equilibrium is attained. The following represents the results of the system when there is a technology improvement- lesser input coefficients are required to produce the same level of physical output. A general feel before we present the results is that an improvement in technology would increase the level of prices overall and increase the rate of profits. The following table presents the results for the production-price system with improvements in technology:

Period	P1	P2	P3	P4	P5	R	g	W
1	0.3639	0.2605	0.3846	0.3096	0.4251	0.7779	0.4498	0.8220
2	0.3605	0.2589	0.3874	0.3124	0.4279	0.721	0.4514	0.8935
3	0.3585	0.2580	0.3888	0.3139	0.4296	0.6883	0.4529	0.9348
4	0.3574	0.2575	0.3897	0.3148	0.4306	0.669	0.4539	0.9594
5	0.3565	0.2571	0.3900	0.3152	0.4309	0.6569	0.4545	0.9745
6	0.3558	0.2568	0.3901	0.3153	0.4311	0.6496	0.4549	0.9837

Changes in technology also require debt component in the capital stock. However, with prices increasing, the value of national income in real terms increases and hence more savings are forthcoming at same propensities. With increased savings and increased deposits, the interest rates relatively decline though there is an increase in the deposits. In fact, the banking equilibrium or the monetary equilibrium occurs at higher level of deposits and loans but relatively lower interest rates. The following table presents the interest rate solutions in this economy:

Period	i1	i2	i3	Deposits/Loans
1	5.59%	9.36%	13.67%	21.21
2	5.75%	9.69%	14.25%	29.75
3	5.89%	9.91%	14.55%	42.27
4	6.02%	10.12%	14.85%	60.59
5	6.10%	10.26%	15.03%	87.35
6	6.17%	10.37%	15.18%	126.12

One point is important to mention here: though there is a change in technology and a technical improvement is introduced in the economy, the real wages remain unchanged at 3.11 and 2.29. Changes in technology therefore do not affect the purchasing power of money in short term as well as in long term. Before we conclude this chapter, we would present the results of changing the monetary variables-namely by changing the money-turnover ratios thereby affecting money demand. We would present the results by increasing the money turnover ratios. In essence, this increases the money holdings, especially the current account deposits are increased. An increase in money supply thus, in form of current deposits, causes marginal increases in prices-a standard theorem of money supply increase. But it should be noted that there may not be an increase in prices in proportion to the changes in money supply. However, for sure even in short and long run equilibrium of the system, it remains true that an increase in money supply increases absolute prices, relative prices remain reasonably unchanged. The following table presents the *real* solutions of the system when money turnover ratios i.e. demand for current deposits by the capitalists have been increased.

Period	P1	P2	P3	P4	P5	R	g	W
1	0.2663	0.2047	0.2984	0.2449	0.3180	0.4610	0.2898	0.6924
2	0.2656	0.2041	0.2995	0.2460	0.3196	0.4414	0.2908	0.7186
3	0.2651	0.2036	0.3001	0.2467	0.3207	0.4274	0.2915	0.7374
4	0.2647	0.2033	0.3006	0.2472	0.3214	0.4183	0.2921	0.7499
5	0.2645	0.2032	0.3009	0.2475	0.3220	0.4119	0.2926	0.7586
6	0.2644	0.2030	0.3011	0.2477	0.3223	0.4075	0.2930	0.7645

The interest rates are also impacted with changes in money supply in this manner. As money supply increases, the interest rate reduces in the economy as expected. The following presents the solutions to the interest rate equations:

Period	i1	i2	i3	Deposits/Loans
1	5.44%	9.15%	13.42%	21.44
2	5.59%	9.42%	13.89%	27.22
3	5.73%	9.65%	14.25%	34.74
4	5.85%	9.85%	14.51%	44.49
5	5.95%	10.02%	14.77%	57.18
6	6.03%	10.15%	14.95%	73.67

As money supply increase, the level of deficit financing also reduces to 3.42 as compared to 10.77 in the base case economy that we set out with at the beginning of this chapter.

In conclusion, it may be said that the long term monetary equilibrium exists and it can be verified from the various simulations above and the values of prime variables as presented. As the economy moves through time, the variables tend to converge to some definite long term values. Also, the short term and the long term properties of the system are consistent with important results from standard theory. These simulations and an understanding of the economic behaviour under various conditions presented so far would be important in developing the train of thought for the process of achieving macro-economic stabilization through interplay of monetary and real variables.