

Chapter IV: Theory of Interest Rates

10. A bank is a financial intermediary that accepts deposits and channels those deposits into lending activities. Banks are a fundamental component of the financial system, and are also active players in financial markets. The essential role of a bank is to connect those who have capital (such as investors or depositors), with those who seek capital (such as individuals wanting a loan, or businesses wanting to grow). Banking is generally a highly regulated industry, and government restrictions on financial activities by banks have varied over time and location. The current set of global standards are called Basel II. In some countries such as Germany, banks have historically owned major stakes in industrial corporations while in other countries such as the United States banks are prohibited from owning non-financial companies. In Japan, banks are usually the nexus of a cross-share holding entity known as the keiretsu. In France, bancassurance is prevalent, as most banks offer insurance services (and now real estate services) to their clients. The most recent trend has been the advance of universal banks, which attempt to offer their customers the full spectrum of financial services under the one roof. The oldest bank still in existence is Monte dei Paschi di Siena, headquartered in Siena, Italy, which has been operating continuously since 1472. The definition of a bank varies from country to country. Under English common law, a banker is defined as a person who carries on the business of banking, which is specified as
- a. conducting current accounts for his customers
 - b. paying cheques drawn on him, and
 - c. collecting cheques for his customers.

In most English common law jurisdictions there is a Bills of Exchange Act that codifies the law in relation to negotiable instruments, including cheques, and this Act contains a statutory definition of the term banker: banker includes a body of persons, whether incorporated or not, who carry on the business of banking' (Section 2, Interpretation). Although this definition seems circular, it is actually functional, because it ensures that the legal basis for bank transactions such as cheques does not depend on how the bank is organised or regulated. The business

of banking is in many English common law countries not defined by statute but by common law, the definition above. In other English common law jurisdictions there are statutory definitions of the business of banking or banking business. When looking at these definitions it is important to keep in mind that they are defining the business of banking for the purposes of the legislation, and not necessarily in general. In particular, most of the definitions are from legislation that has the purposes of entry regulating and supervising banks rather than regulating the actual business of banking. However, in many cases the statutory definition closely mirrors the common law one. Examples of statutory definitions: "banking business" means the business of receiving money on current or deposit account, paying and collecting cheques drawn by or paid in by customers, the making of advances to customers, and includes such other business as the Authority may prescribe for the purposes of this Act; (Banking Act (Singapore), Section 2, Interpretation).

a. "banking business" means the business of either or both of the following:

i. receiving from the general public money on current, deposit, savings or other similar account repayable on demand or within less than [3 months] ... or with a period of call or notice of less than that period;

ii. paying or collecting cheques drawn by or paid in by customers

A bank can generate revenue in a variety of different ways including interest, transaction fees and financial advice. The main method is via charging interest on the capital it lends out to customers. The bank profits from the differential between the level of interest it pays for deposits and other sources of funds, and the level of interest it charges in its lending activities. This difference is referred to as the spread between the cost of funds and the loan interest rate. Historically, profitability from lending activities has been cyclical and dependent on the needs and strengths of loan customers and the stage of the economic cycle. Fees and financial advice constitute a more stable revenue stream and banks have therefore placed more emphasis on these revenue lines to smooth their financial performance.

In the past 20 years American banks have taken many measures to ensure that they remain profitable while responding to increasingly changing market conditions. First, this includes the Gramm-Leach-Bliley Act, which allows banks again to merge with investment and insurance houses. Merging banking, investment, and insurance functions allows traditional banks to respond to increasing consumer demands for "one-stop shopping" by enabling cross-selling of products (which, the banks hope, will also increase profitability). Second, they have expanded the use of risk-based pricing from business lending to consumer lending, which means charging higher interest rates to those customers that are considered to be a higher credit risk and thus increased chance of default on loans. This helps to offset the losses from bad loans, lowers the price of loans to those who have better credit histories, and offers credit products to high risk customers who would otherwise been denied credit. Third, they have sought to increase the methods of payment processing available to the general public and business clients. These products include debit cards, prepaid cards, smart cards, and credit cards. They make it easier for consumers to conveniently make transactions and smooth their consumption over time (in some countries with underdeveloped financial systems, it is still common to deal strictly in cash, including carrying suitcases filled with cash to purchase a home). However, with convenience of easy credit, there is also increased risk that consumers will mismanage their financial resources and accumulate excessive debt. Banks make money from card products through interest payments and fees charged to consumers and transaction fees to companies that accept the cards. Thus, interest, deposits, loans, credit and debt form the basis of banking activities in the economy

11. In so doing, the banks face major risks of business. In the common parlance, these risks are the interest rate risk, liquidity risk and the credit risk, as well as broader systematic risks. Most of the times, banks tend to describe these risks using probability analysis- we shall also not differ in this regard. Of the major risks faced by banks, one important risk is the risk of solvency- the ability to remain in business and remain liquid. The risks therefore that we are talking about is faced

by the banks because of its customers- the risks of withdrawals. For the sake of simplicity, we would assume away all other risks. In the banking system, the banks face the risk of withdrawals of their deposits such that they may not have any funds left for advancing loans. In this sense, the banks maintain reserves, linked to the withdrawal probabilities that they estimate at the beginning of each period.

<i>Period</i>	0	1	<i>l</i>
0	p_{00}	p_{10}	p_{l0}
1	p_{01}	p_{11}	p_{l1}
<i>k</i>	p_{0k}	p_{1k}	p_{lk}

The withdrawal matrix is decreasing in periods i.e. nearest period deposits have larger probabilities of withdrawal than the higher period deposits. With such a withdrawal system in place, the banks need to determine an optimal reserve base with which they can conduct their operations. Such reserves can be determined using the following matrix

<i>Period</i>	0	1	<i>l</i>
0	p_{00}	p_{10}	p_{l0}
1	$1 - [(1 - p_{00})(p_{01})]$	$1 - [(1 - p_{10})(p_{11})]$	$1 - [(1 - p_{l0})(p_{l1})]$
<i>k</i>	$1 - [(1 - p_{00})(1 - p_{01})(p_{0k})]$	$1 - [(1 - p_{10})(1 - p_{11})(p_{1k})]$	$1 - [(1 - p_{l0})(1 - p_{l1})(p_{lk})]$

This reserve system is generalized for *l* period deposits and *k* period loans. In short, the reserve matrix looks like $R_{lk} = 1 - \prod (1 - p_{l-1,k})p_{lk}$. The theory of interest rates proposed here is an operational theory of loans and deposits wherein the interest rates are determined by the behavior of investors and the behavior of borrowers; behavior here is described in terms of withdrawal probabilities for deposits and technological coefficients for loans.

12. The real question therefore is how do banks determine what interest rates have to be charged on what accounts? Economists have tried to identify theories for this business behavior of the banks; however almost all of the established standard mainstream theories have either failed or have not come close enough to the empirical manner in which the rate of interest is determined. In the present section, we aim to provide a theory characterizing this mechanism of the banking system. Once the reserves are determined per the previous section, the banks need

to match their deposits supply with the demand for loans. For this, we use an assignment based optimization, where in the banks are interested in minimizing the reserves, subject to the efficiency condition that all loans are met. In such a case, we obtain the equations for determining the interest rates in the system

$$\sum_{l=k=0}^b R_{lk} + \sum_{k=1}^b D_{0k} + \sum_{t=1}^b \sum_{l=k=1}^b D_{lk} (1 + i_t)^t = L_t^a i_t$$

13. Let us assume the following loans and deposits schedule to fully understand the interest rate theory

<i>Period</i>	<i>Deposits</i>	<i>Loans</i>
0	97.5	
1	112.5	145
2	100	145
3	125	145

In this case, the banks have a total of 435 deposits and 435 loans to make. In total, the loans and deposits are fully matched. The banks use the period zero deposits first since they are cheapest for the bank- zero period deposits are not paid any interest while if they are lent out, they bring in profits in terms of interest. Thus, banks use zero period deposits, in this case 97.5, to advance one period loans. Since these do not fully satisfy the demand, banks resort to period one deposits (that banks pay interest on) to meet the loan demand for 145. Similarly, the remaining assignments are made to obtain the following (the first subscript denotes the deposit period and the second denotes the loan period advanced- thus the subscript 01 would read as zero period deposit utilized to finance 1 period loan)

$$D_{01} = 97.5; D_{11} = 47.5$$

$$D_{12} = 65; D_{22} = 80$$

$$D_{23} = 20; D_{33} = 125$$

Let us assume a banking probability matrix that assumes that immediate period deposits would have a higher withdrawal probability- on the vertical axis we have time periods 1, 2, and 3 while on the horizontal one we have deposit periods 0, 1,

2 and 3. Using such a probability matrix, we can create the reserve matrix using the formula above- $R_{lk} = 1 - \prod (1 - p_{l-1,k}) p_{lk}$

$$prob = \begin{vmatrix} .05 & .04 & .035 & .03 \\ .107 & .088 & .0736 & .06395 \\ .16951 & .1381 & .1152 & .1014 \end{vmatrix}$$

Using this reserve matrix, we can now estimate the reserves in the banking system for each period

$$\begin{aligned} R_1 &= 6.775 \\ R_2 &= 11.608 \\ R_3 &= 14.979 \end{aligned}$$

Thus, we would now have the equations for the banking system as

$$\begin{aligned} 6.775 + 97.5 + 47.5(1 + i_1) &= 145(1 + i_1) \\ 11.608 + 65(1 + i_1) + 80(1 + i_2)^2 &= 145(1 + i_2)^2 \\ 14.979 + 20(1 + i_2)^2 + 125(1 + i_3)^3 &= 145(1 + i_3)^3 \end{aligned}$$

Solving for these equations, we obtain $i_1=6.9487\%$, $i_2=11.7171\%$ and $i_3=17.6643\%$

14. To conclude this chapter, a few remarks on the proposed theory become imperative. The assignment matrix becomes crucial in determining the level and the slope of the term structure of interest rates. In other words, the way the banks finance their loans determines what rates of interest would prevail in the economy. Let us consider the classic and full assignment solution to any problem to start with. Considering 3 period loans and 3 period deposits, we would ideally have the following assignment solution

<i>Period</i>	0	1	2
1	D_{01}	D_{11}	D_{21}
2	D_{02}	D_{12}	D_{22}
3	D_{03}	D_{13}	D_{23}

For the above case therefore, this matrix would look like

<i>Period</i>	0	1	2	3
0	0	0	0	0
1	97.5	47.5	0	0
2	0	65	80	0
3	0	0	20	125

If the pattern of assignment is carefully noticed, it may be seen that the matrix is lower triangular. The slope of the term structure for this problem is upward as well. In short, it can be generalized that the slope would be upwards in case the matrix is upper triangular.

Consider the other case, where the matrix is upper triangular

<i>Period</i>	0	1	2	3
0	0	0	0	0
1	0	0	110	35
2	0	65	80	0
3	0	0	20	125

The new equations in this case are

$$4.9 + 97.5 + 110(1 + i_2)^2 + 35(1 + i_3)^3 = 145(1 + i_1)$$

$$11.608 + 65(1 + i_1) + 80(1 + i_2)^2 = 145(1 + i_2)^2$$

$$14.979 + 20(1 + i_2)^2 + 125(1 + i_3)^3 = 145(1 + i_3)^3$$

The term structure in this case is upward sloping and necessarily so, therefore it may be concluded that the shape of the matching-matrix would determine the slope of the term structure.