

PROBABILITY
AND
INDUCTION



WILLIAM KNEALE

THIS book is an attempt to clarify the relations between the two notions mentioned in the title. After a preliminary discussion of belief, the author expounds the traditional problem of induction and the reasons for which it is said that inductive conclusions can never be more than probable. He then gives an outline of the theory of chances and an argument to show that this theory cannot be used for the justification of induction. In the last part he tries to explain the special sense in which the results of induction may be described as probable.

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BY

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PREFACE

THIS book is based on lectures I have given to students of philosophy, and it is intended for readers who are interested in the philosophical problems suggested by the title. I do not pretend that it is either a treatise on the mathematical theory of probability or a practical guide to scientific method. For I have not the ability to produce the former, and I do not think it is the business of a philosopher (or perhaps of anyone) to try to provide the latter. So far as mathematics and natural science are concerned, I shall be content if I have avoided howlers.

The philosophical problems discussed are elementary in the sense that they have to do with first principles, and I have tried to make my treatment of them elementary also in the other sense of the word, that is to say, intelligible without much previous reading about the subject. But some of the statements to which I have committed myself are very controversial, and it may be useful to make clear that I do not attach equal importance to them all. In Part I, for example, I have written of knowledge as though it were indefinable, but this is merely because I have seen no satisfactory analysis of knowledge and do not think it necessary for my present purpose to try to find one. In spite of what I have said, I should welcome a new attempt to analyse this notion. At the other extreme, the general account of induction given in Part IV seems to me substantially correct, and I wish to stand by it. The theory of natural necessity in Part II and the range theory of probability in Part III come between these contentions in order of importance. I am acutely conscious of the difficulties of my views and the insufficiency of my arguments, and yet I cannot at present see any other way of describing matters which seems at all plausible. If I am mistaken in what I have said about these topics, I hope that I have at least written clearly enough to be found out quickly.

In accordance with the conventions of the Clarendon Press, logical and mathematical symbols have been printed without quotation marks even where they are themselves the subjects of discourse. I hope no reader will be seriously distressed by this usage, which is almost universal in mathematical texts. In certain contexts it can lead to dangerous confusion, and my own preference is for a rigorous distinction between use and mention at all times ;

but no real ambiguity results here from the omission of inverted commas, and I have therefore not felt justified in pressing for their restoration at the expense of much valuable labour.

My debt to Lord Keynes's *Treatise on Probability* is obvious even where I criticize his doctrines. On the history of induction, or rather of philosophical views about induction, I have learnt much from *Les Théories de l'induction et de l'expérimentation* by M. André Lalande. To my friends Professor Gilbert Ryle, Dr. Friederich Waismann, and Dr. Karl Popper I owe a great deal that I cannot now disentangle from my own thought; but I am afraid they may be surprised and shocked by my conclusions. The fact that I have not made full use of some important recent contributions to the subject, in particular those of Professor R. Carnap and Professor G. H. von Wright, is due to the circumstances in which I have written this book. The first draft was begun in the summer of 1939, laid aside during the war while I was engaged on other work, and finished in the short period of comparative ease before the universities became crowded with men returning from military service. Since the autumn of 1946 I have had no time to do more than remove some of the faults which became evident when the whole was in typescript.

I wish to thank Professor Henry Price very warmly for the great care with which he read my work and advised me about the revision of it. His kindness has saved me (and the reader) from some silly passages and many that were obscure.

Finally, I wish to dedicate this book to my wife, who has helped me with advice and encouragement throughout the making of it.

July 1948

W. K.

CONTENTS

PART I

INTRODUCTORY: KNOWLEDGE AND BELIEF

1. The Scope of the Inquiry	I
2. Subjectivist Theories of Probability: Meanings of 'Belief'	3
3. The Relation of Probability to Evidence	9
4. The Nature of Opinion	13
5. Opinion as a Basis for Action	18
6. Probability and Induction: The Plan of this Book	21

PART II

THE TRADITIONAL PROBLEM OF INDUCTION

7. Aristotle's Definition of Induction	24
8. Summative Induction by Complete Enumeration	25
9. Intuitive Induction: Facts and Principles	30
10. Recursive or Mathematical Induction	37
11. Ampliative Induction and the Search for Causes	43
12. Bacon's Method of Induction by Elimination	48
13. Hume's Doctrine of Antecedent Causes	53
14. The Inductive Methods of J. S. Mill and their Presuppositions	56
15. Limits to the Use of the Notion of Cause in Natural Science	60
16. The Formulation of Natural Laws in Scientific Practice	65
17. Theories about the Logical Character of Natural Laws	70
18. Natural Necessity and Perception	78
19. The Explanation of Natural Laws	89
20. Criticism and Defence of Transcendent Hypotheses	97
21. The Relation between Induction and the Hypothetical Method	103
22. The Consilience of Primary Inductions	106
23. Confirmation and Elimination	110

PART III

THE THEORY OF CHANCES

24. The Notion of Chance	114
25. Probability Rules in Matters of Chance	117
26. Fundamentals of the Calculus of Chances	123
27. The Notion of Relevance and the Inversion Theorem	127
28. Theorems on the Composition of Sets	132
29. Bernoulli's Limit Theorem	136
30. Poincaré's Equalization Theorems	142
31. Equiprobability and Indifference	147

32. The Frequency Theory of Probability	150
33. The Defects of the Frequency Theory	161
34. The Analysis of Equiprobability	167
35. The Notion of Range	173
36. The Variety of the Probability Rules Covered by the Range Definition	190
37. The Determination of Probabilities <i>a Priori</i> and <i>a Posteriori</i>	197
38. Attempts to Justify Induction by Use of the Inversion Theorem	201
39. The Argument from Restriction of Alternatives	207
40. A Fundamental Objection to all Attempts to Justify Induction within the Theory of Chances	211
41. Sampling and the Use of Statistics	214

PART IV

THE PROBABILITY OF INDUCTIVE SCIENCE

42. Restatement of the Problem	223
43. The Policy of Primary Induction	224
44. The Justification of Primary Induction	23.
45. Degrees of Acceptability in Primary Induction	23'
46. The Policy of Secondary Induction	24'
47. The Acceptability of Theories	25
48. Conclusion	25
INDEX	26

INDEX

- absence, table of, 50 ff.
 acceptability, 236 ff.
 accidental universality, 195.
 accumulation, point of, 155 n.
 accumulation of instances, 108, 240.
 addition theorem, 125.
 aesthetic aspect of theories, 252.
 agreement, method of, 57 ff.
 alternatives, 31 f., 36, 171 ff., 212.
 ampliative induction, 44 ff., 195, 214.
 analogy, 208.
 antecedent causes, 53 ff.
 approvability, 20, 169, 190, 225.
a priori conjectures of probability, 197 ff.
 Aristotle, 24 ff., 37, 47 f., 53, 60, 72,
 116 f., 150, 195, 227.
 arithmetical probability, 181.
 arrangements, theory of, 124.
 association, coefficient of, 221; of at-
 tributes, 66 f.; of ideas, 55.
 atomic theory of matter, 97, 249.
 attributes, uniform association of, 66 f.
- Bacon, F., 21, 48 ff., 53 ff., 70, 110 f.,
 113, 211, 223, 250.
 Bayes, T., 124, 129, 201 f.
 'belief', 4 ff.
 Berkeley, G., 21, 85, 87, 94.
 Bernoulli, Jac., 4, 124, 136 ff., 147 ff.,
 163, 166, 201 ff., 244, 255.
 Bernoulli, N., 124.
 Bertrand, J., 184 ff., 198.
 betting, 7.
 Boole, G., 203.
 boundaries of possibility, 78, 80, 227,
 233, 256.
 Broad, C. D., 211.
 Butler, J., 1, 9.
- calculus of chances, 23, 123 ff.
 Cantor, G. F. L. P., 177.
 Carnap, R., vi, 34 n.
 causal theory of perception, 84 f.
 causation, principle of universal, 57 ff.,
 111.
 'cause', 47 ff., 60 ff.
 causes, probability of, 130.
 chance, 114 ff., 146, 162, 180 f., 200.
 chances, 119, 170 f., 181, 212 ff.; *see*
 also calculus.
 clairvoyance, 45.
 Clerk-Maxwell, J., 93, 248 f.
 coherence theory of truth, 108.
 collectives in frequency theory, 154 ff.
 composition of sets, 133 ff.
 'conceive', 79 f.
 concomitant variations, method of, 57 ff.
- Condorcet, M. J. A. N., 125.
 confidence, 6, 13 ff., 194, 226.
 configuration space, 181 f.
 confirmation, 50, 110 ff.
 conjunction theorem, 126.
 consilience of primary inductions, 107 ff.,
 251.
 constancy theory of natural laws, 74 f.,
 151, 254.
 constant conjunction, 54.
 constants in nature, 69 f.
 constitution of sets, 133 n.
 constructibility, mathematical, 161.
 contiguity of cause and effect, 62.
 contingency, 31 ff.
 continuous function, 142 f.
 continuously variable magnitude, 181 ff.
 conventionalism, 257.
 convergence in the theory of collectives,
 155 ff.
 Copeland, A. H., 161.
 correlation coefficient, 221.
 Cournot, A., 170 n.
 Couturat, L., 97 n.
 cumulative evidence, 121 f., 128 f.
 curiosity, 252.
- Dalton, J., 93, 97.
 degrees, table of, 50 ff.
 de Méré, G. A., 123 f.
 Democritus, 52, 92, 94 n.
 De Moivre, A., 119.
 De Morgan, A., 203.
 Descartes, R., 31, 51, 100.
 determinism, 64, 111 f., 115 ff.
 development, uniformity of, 67 f.
 difference, method of, 57 ff.
 disjunction theorem, 125.
 dispersion, 219.
 doubt, 16.
 dyad of characters, 201 ff., 213.
- Eddington, A., 72, 96.
 Einstein, A., 90 n.
 elimination, 50, 56 f., 110 ff., 207.
 Ellis, R. L., 150 f.
 empiricism, 21, 78.
 enumeration, 25 ff., 207.
 epistemology, 3.
 equality, definition of, 177.
 equipossible alternatives, 170, 178,
 181 ff.
 equiprobability, 119, 147 ff., 151, 168 ff.
 equispecific alternatives, 148, 181.
 essential universality, 37, 43.
 Euclid, 177.
 Euler, L., 79, 175.

- evidence, 9 ff., 16, 194.
 existence, mathematical, 161.
 expectation, mathematical, 120.
experimentum crucis, 110.
 explanation, 90 ff., 164, 200, 246 ff., 252.
 extrapolation, 68, 234.
 extravagance of non-inductive hypotheses, 237 ff.
- facts, 32 ff., 73 ff.
 Fermat, P. de, 37, 39, 123.
 field of possibility, 175, 231.
 Findlay, J. N., 12 n.
 finite populations, 152, 214.
 universe, theory of, 72.
 Fisher, R. A., 231 n.
 form, Baconian, 52.
 frequency theory of probability, 150 ff.,
 167, 194, 200, 254, 256.
 fringes of application, 196.
 functional relations, 68 f., 228.
- Galileo, 51.
 gambling apparatus, 146.
 systems, 140, 154.
 generating nature in Bacon's philosophy, 50 ff.
 geometrical probability, 181, 192.
 Goldbach, C., 79 f., 213.
- Hamilton, W., 82 n.
 Heath, T. L., 177 n.
 Hegel, G. W. F., 179.
 Hempel, C. G., 160 n.
 histogram, 219.
 historical hypotheses, 104.
 Hobart, R. E., 75 n.
 Hobbes, T., 54.
 Hume, D., 21, 46, 53 ff., 57, 60, 62, 71,
 74, 78 f., 89, 112 n., 151, 162 f., 223,
 226, 254.
 Huyghens, C., 93, 96 ff., 101, 109.
 hypotheses, 19, 92 ff., 247.
 hypothetical method, 53, 104 ff.
- idealism, 116, 180 n.
 idealization in frequency theory, 157 ff.
 'ideas', 78, 88.
 identity of indiscernibles, 176.
 ignorance, equal distribution of, 148.
 imagination, 79.
 indifference, principle of, 147 ff., 172 f.,
 204.
 'induction', 24, 43 f., 103, 195.
 Infeld, L., 90 n.
 infinitesimal probabilities, 205, 212.
 infinity, 157.
 initial probability, 130, 212.
 insufficient reason, principle of, 147.
 insurance, 122, 141, 165, 167.
 internal relations, 116.
 introspection, 14 ff., 194.
- intuitive induction, 30 ff., 46, 193.
 inversion theorem, 124, 129 ff., 201,
 214 ff.
 Ionian philosophers, 47.
 irrational numbers, 195.
 irrationality of abandoning induction,
 237.
 irregularity of collectives, 153 ff.
 irrelevance, 122, 127 ff.
- Jevons, W. S., 113, 148 n., 203, 205.
 judgement, 18.
 justification of induction, 223 ff., 234 ff.
- Kahle, L. M., 9.
 Kant, I., 47, 55, 78, 253.
 Keynes, J. M., vi, 9, 11 ff., 22, 50, 111,
 113, 133 n., 147 ff., 152, 205 ff., 212,
 214, 233.
 kinetic theory of gases, 141.
 knowledge, 1, 18, 88 f.
 Kronecker, L., 39.
- Lalande, A., vi.
 language, rules of, 33 ff., 80.
 Laplace, P. S. de, 113, 124 f., 149 f.,
 170, 173, 201 ff.
 large numbers, law of, 124, 139.
 laws of nature, 45 f., 65 ff., 70 ff., 211 ff.,
 227.
 Leibniz, G. W., 97 n., 147, 175 f., 201.
 Lewis, C. I., 257 n.
 likelihood, method of maximum, 231 n.
 limit in theory of collectives, 154 ff.
 theorems, 136 ff., 145.
 limitation of independent variety, 210.
 Lindsay, R. B., 157 n.
 loaded die, problem of, 149.
 Locke, J., 21, 54 f., 71, 94 n., 113.
 logic, 256.
 logical constructions, 12, 85.
- Macaulay, T. B., 50.
 Mach, E., 102.
 Margenau, H., 157 n.
 mathematical expectation, 120.
 induction, 37 ff.
 matters of fact, 32 ff., 195.
 maturity of the chances, 140.
 measurement, condition of, 177; of
 probabilities, 15, 22, 119, 214, 243 f.;
 of ranges, 176 ff.
 Meinong, A., 12, 32.
 memory impressions, 17, 194.
 metaphysics, 44, 247, 253.
 Mill, J. S., 45, 55 ff., 70, 85, 111 f., 208,
 211.
 modality, 3 f.; *see also* principles.
 multiplication theorem, 125.
- natural kinds, 66 f., 257.
 necessity, 31 ff., 54, 71, 78 ff., 179,
 211 f., 256, 258.

- negation theorem, 125.
 negative facts, 33.
 instance, 50.
 negligence in primary induction, 237, 243.
 Newton, I., 50, 60, 98 ff., 249.
 Nicod, J., 50, 113, 207, 212.
 numbers, 41 ff., 191 f.
- opinion, 6, 13 ff.
- parameters, 228.
 Pascal, B., 118, 123 ff.
 Pearson, K., 203.
 Peirce, C. S., 44 n., 150, 166.
 percentage, 219.
 perception, 78 ff., 258.
 perceptual object terminology, 83 ff., 248, 257 f.
 perceptual objects, 81 f.
 phenomenalism, 85 ff., 101 ff., 257.
 phenomenology, 44, 256.
 'phenomenon' in scientific usage, 90.
 physicalism, 226.
 place selection, 154 ff.
 Plato, 31, 47, 49.
 plurality of causes, 59.
 Poincaré, H., 142 ff., 184 n., 188 f., 198.
 Poisson, S. D., 139.
 Popper, K. R., vi, 76, 155 n., 163, 230 n., 234 n.
 positivism, 32, 34, 76, 179, 225, 247, 253 f.
 possibility, boundaries of, 78, 80, 227, 233, 256; degree of relative, 170; field of, 175, 231.
 prediction, 234 n., 251.
 prescription theory of natural laws, 76.
 presence, table of, 50 ff.
 Price, H. H., vi, 5 n.
 Price, R., 124.
 primary induction, 104, 106 ff., 226 ff., 250.
 qualities, 94 n.
 set of equipossible alternatives, 178.
 principal set of equipossible alternatives, 179.
 principle, truths of, 32 ff., 168, 193, 195, 212, 254, 258.
 principles of modality, 32 ff., 71, 227.
 probability rules, 22 f., 117 ff., 167, 190 ff., 211 ff., 230, 241.
 'probable', 20, 214, 224 f., 236.
 proportion of instances, 151, 169.
 propositions, 12, 118, 167.
- qualitative laws, 69.
 quantum theory, 111, 157, 254 ff.
- Ramsey, F. P., 76, 167 n.
 'random', 168.
 randomness in collectives, 153 ff.
- range, 174 ff.
 'rational', 7, 13 ff., 17, 19 f., 55, 225 f., 235.
 rationalist metaphysics, 32, 253.
 reciprocal connexion, 70, 209.
 recurrence signs, 42 f.
 recursive definitions, 39 ff.
 induction, 37 ff., 46, 191.
 regularity theory of natural laws, 74;
 see also constancy.
 Reichenbach, H., 163, 236 n.
 relative frequency, 152.
 relevance, 127 ff.
 reliability of results of induction, 106 ff., 236.
 restricted descriptions, 27 ff., 191.
 universal statements, 27 ff., 72 f.
- roulette, 142.
 routines, 64 f.
 rule of succession, 203 f.
- Russell, B., 12.
 Ryle, G., vi.
- sampling, 214 ff.
 scepticism, 55, 75 n., 223.
 Schlick, M., 76 f.
 secondary induction, 104, 106 ff., 246 ff., 250.
 set of equipossible alternatives, 178.
 second-order possibility, 213.
 probability, 22, 195, 214.
 sensation, 82.
 sensum terminology, 83 ff., 257.
 shuffling, 144.
 simplicity, 91, 229, 246 ff.
 Socratic dialectic, 50.
Spielraum theory, 174 n., 194.
 Spinoza, B. de, 179.
 standard deviation, 219.
 statistical induction, 194, 207; *see also* 'induction' and probability rules.
 statistics, 194, 219 ff.
 Stirling, J., 138 n.
 structure, 94.
 subjectivism, 3 ff., 149, 167, 194.
 succession, rule of, 203 f.
 successions in frequency theory, 153, 155 n.
 sufficient reason, principle of, 147.
 summative induction, 25 ff., 45, 191.
 syllogism, 24 ff., 49.
- taking for granted, 5, 16.
 Tannery, P., 177 n.
 telepathy, 200.
 theories, 92, 246 ff.
 Todhunter, I., 118, 123 n., 125.
 tolerance, 197.
 total probability, 203.
 totality proposition, 30.
 transcendent hypotheses, 92 ff.
 object terminology, 95 ff., 248.

- truths, two orders of, 32 ff.; *see also*
principle.
- ultimate alternatives, 171.
- unfulfilled hypothetical propositions,
75, 77, 87 f.
- uniformity of nature, 56, 208.
- universal propositions, varieties of, 71,
190 ff.
- vagueness, 196, 243.
- variables in nature, 182 ff.
- variety of instances, 208, 218.
- Venn, J., 150, 155.
- verifiability, 159.
- von Kries, J., 148, 170 n., 174 n., 194.
- von Mises, R., 152 ff., 170, 194.
- von Wright, G. H., vi, 156 n.
- Waismann, F., vi, 158 n., 174 n.,
197 n.
- Wald, A., 161.
- Weinberg, J. R., 76 n.
- Weyl, H., 230 n.
- Whewell, W., 98, 107, 195.
- Whitehead, A. N., 12, 72 f.
- Wilson, J. C., 5 n.
- Wittgenstein, L., 174 n., 180 n.

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