

The Sraffa System for Continuous Industrial Production

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Abstract

This paper articulates a model of industrial production in which technologies are of continuous-input continuous output-type. It is shown that the model nevertheless has all the essential properties of the usual point-input point- output Sraffa system. The dual of the price system is shown to be identical with Leontief's dynamic model for the case of a uniform rate of growth.

I Introductory

The production equations of the Sraffa System (Sraffa 1960) have an 'agrarian' flavour – inputs in all industries are applied at one point of time and outputs of all commodities emerge at the end of the 'season'. This characterization of the production process is somewhat restrictive when the purpose is to describe an economic system in which industries operate continuous - input continuous – output technologies. This paper presents a simple generalization of the Sraffa equations to incorporate this feature. It is shown that the enlarged system has all the properties of the Sraffa system except one, viz. the rates of on-cost markups differ from the rate of profit on invested capital by a factor that depends on the 'capital turnover ratios' in the different industries.

II Equations

It is well-known that production is made continuous because firms carry stocks of raw materials, semi-finished goods and finished goods to close the time gaps between purchases and production, between production at various stages of finish and between the production of the finished good and its sale respectively. When the production process is properly primed the flow of purchases of inputs are exactly matched to the flow of output and its sale so that capital is needed only to finance the holding of stocks. The production equations would then take the form,

$$\begin{aligned}(S_{11}p_1 + S_{21}p_2 + \dots + S_{n1}p_n)r + A_{11}p_1 + \dots + A_{n1}p_n + wL_1 &= B_1p_1 \\(S_{12}p_1 + S_{22}p_2 + \dots + S_{n2}p_n)r + A_{12}p_1 + \dots + A_{n2}p_n + wL_2 &= B_2p_2 \\ \dots & \dots \\(S_{1n}p_1 + S_{2n}p_2 + \dots + S_{nn}p_n)r + A_{1n}p_1 + \dots + A_{nn}p_n + wL_n &= B_n p_n\end{aligned} \quad \dots (1)$$

where S_{ji} is the stock of commodity j held by the i^{th} industry, A_{ji} the flow input requirements, L_i the labour used, B_i the output produced and w and r the wage rate and the rate of profit respectively. Equation (1) contain n equations in $n+2$ unknown;

n prices and the two distributive variables. Either one of the prices or the wage rate can be fixed as *numeraire* so that there will be $n+1$ unknowns.

In free competition the uniform rate of profit that prevails across industries is

$$\begin{aligned} r &= \frac{p_i B_i - \sum_j A_{ji} p_j - w L_i}{\sum_j S_{ji} p_j} \\ &= \frac{p_i B_i - C_i}{K_i} \quad \forall i \end{aligned} \quad \dots (2)$$

where C_i is the total prime cost and K_i is the value of capital employed in industry i . If u_i is the unit prime cost and m_i is the mark-up rate $p_i = u_i(1+m_i)$ and sales $p_i B_i = C_i(1+m_i)$. Substituting in (2) and letting T_i denote the ratio of prime cost to capital gives,

$$m_i = r/T_i \quad \forall i \quad \dots (3)$$

In other words, the mark-up rate varies inversely with the capital turnover ratio so that industries with high turnover ratios (e.g. retail) will need a lower mark-up than industries with low ratios (e.g. aircraft manufacturing) to attain the competitive rate of profit.¹

III Relative Prices

Excepting the above, system (1) has all the properties of the usual Sraffa system. Thus, the behaviour of relative prices due to a change in the rate of profit is seen to depend only on the capital labour ratios in the industries – it is independent of the ratios of stock to flow input requirements or of the flow input to labour requirements. Consider an economy that produces 2 commodities. If commodity 2 is *numeraire* the equations are:

$$\begin{aligned} (S_{11}p + S_{21})r + A_{11}p + A_{21} + w^* L_1 &= B_1 p \\ (S_{12}p + S_{22})r + A_{12}p + A_{22} + w^* L_2 &= B_2 \end{aligned}$$

where $p = p_1/p_2$ and $w^* = w/p_2$. The solution for the relative price is

$$p = \frac{(B_2 - A_{22} - rS_{22})L_1 + (A_{21} + rS_{21})L_2}{(A_{12} + rS_{12})L_1 + (B_1 - A_{11} - rS_{11})L_2}$$

Differentiating with respect to r we find that

$$\frac{dp}{dr} \geq 0 \quad \text{as } p \geq \frac{L_2 S_{21} - L_1 S_{22}}{L_1 S_{12} - L_2 S_{11}}$$

or

$$\frac{S_{11}p + S_{21}}{L_1} \geq \frac{S_{12}p + S_{22}}{L_2} \quad \dots (4)$$

i.e, the price of commodity 1 increases (decreases) with an increase (decreases) in the rate of profit if it is more (less) capital intensively produced.

IV Standard System

Writing equations (1) for unit outputs in matrix notation,

$$SP + AP + wL = P \quad \dots (5)$$

whose solution for the price vector is

$$\begin{aligned} P &= [I - A - rS]^{-1} wL \\ &= [I - (I - A)^{-1} Sr]^{-1} (I - A)^{-1} wL \end{aligned} \quad \dots (6)$$

which is positive if the Hawkins – Simon conditions (Hawkins and Simon 1949) that the principal minors of $I - A$ be positive hold. It is easy to see that the principal minors of $I - (I - A)^{-1} Sr$ will be positive if $(I - A)^{-1}$ is positive. Consider the price system at $w = 0$,

$$P = [(I - A)]^{-1} (S Pr)$$

With $\lambda = 1/r$ the above expression is

$$[\lambda I - (I - A)^{-1} S] P = 0 \quad \dots (7)$$

If $(I - A)^{-1}$ is positive $(I - A)^{-1} S$ must be non-negative. So by the Perron-Frobenius theorem it has a dominant positive eigenvalue λ_d with which is associated a non-negative eigenvector X_d . The maximum rate of profit R is the reciprocal of λ_d . Thus, for values of $r < R$ the matrix $[I - (I - A)^{-1} Sr]$ has positive minors so that the solution of prices must be strictly positive.

Multiplying (5) by the eigenvector X_d we write the system in standard proportions.

$$X_d SP_r + X_d AP + wX_d L = X_d P \quad \dots (8)$$

At $w = 0$,

$$r = R = \frac{X_d (I - A) P}{X_d SP} \quad \dots (9)$$

If $X_d (I - A) P$, the standard national product is set equal to unity and further if $X_d L = 1$ then substituting from (9) into (8) gives

$$r = R(1 - w) \quad \dots (10)$$

which is the linear wage-profit frontier.

limited to the fact that it gives a better description of continuous production, it also gives a better description of the technology of production considering that there are elements amongst the flow coefficients that do not appear in the stocks (e.g. electricity and capital services) and *vice versa*. Besides, it helps to connect the Sraffian theory of prices with empirical work in input-output models, an important development that has recently been initiated by Han and Schefold (2006).

Note

1. The capital turnover ratio is defined in several ways. If it is defined as capital to sales turnover ratio then equation (3) would become

$$m_i = \frac{rT_i}{1 - rT_i}$$

In the Sraffa system the mark-up is charged only on material cost which equals capital invested so that,

$$m_i = \frac{p_i B_i - wL_i}{K_i} - 1 = r$$

If we wish to follow Sraffa's procedure in the present system then,

$$m_i = \frac{r + wL_i / K_i}{T_i}$$

where T_i is defined as the material cost to capital ratio.

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