Irving Fisher

Mathematical Investigations in the Theory of Value and Price (1892) Appreciation and Interest (1896)

REPRINTS OF ECONOMIC CLASSICS

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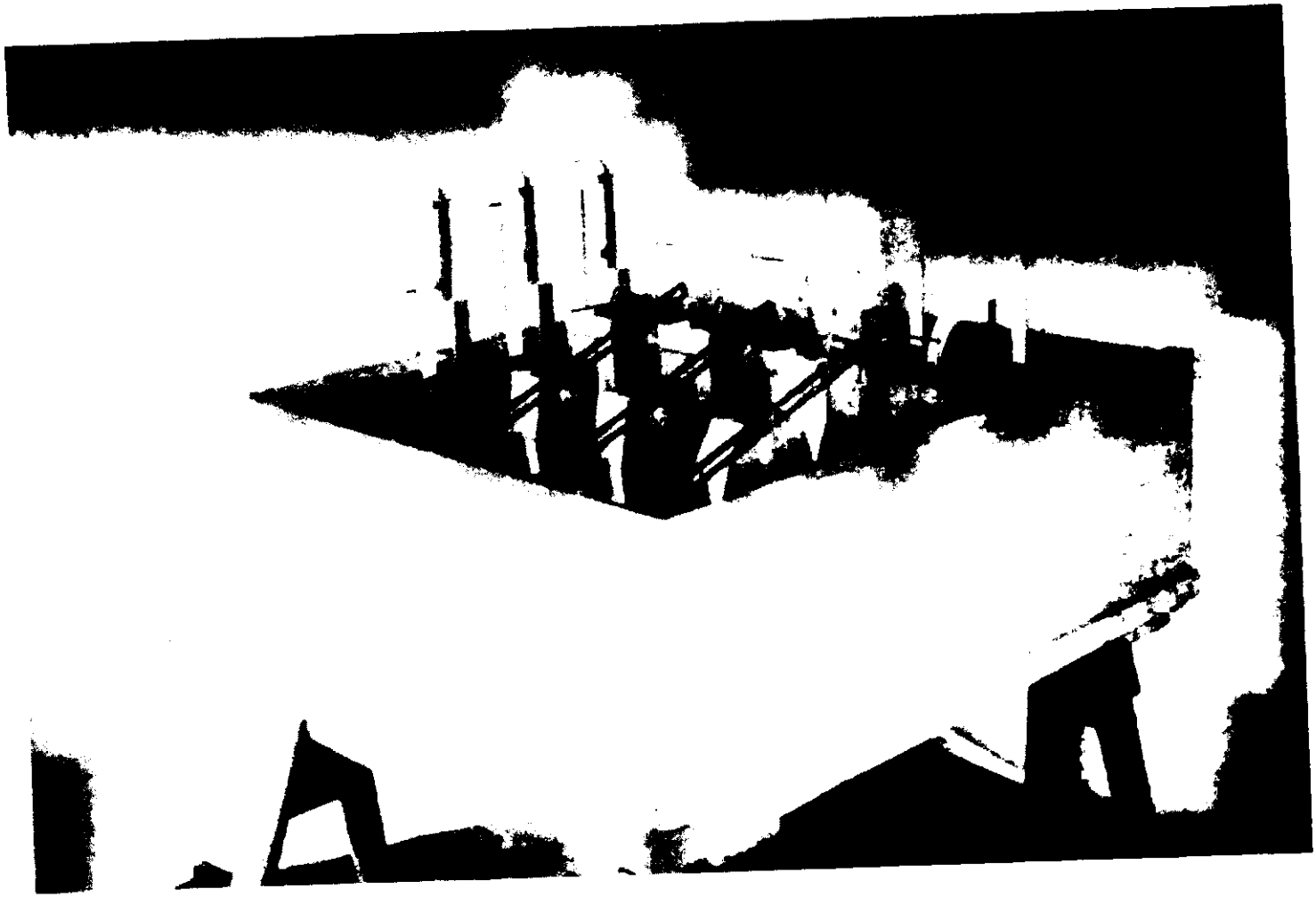

First Model of Mechanism Pictured on Page 38, Constructed in 1893.

SECOND MODEL OF MECHANISM PICTURED ON PAGE 38, CONSTRUCTED IN 1925.

MATHEMATICAL INVESTIGATIONS

IN THE

THEORY OF VALUE AND PRICES

BY

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for the Degree of Ph.D. in Yale University
1891.

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PREFACE

TO THIS REPRINT.

The following booklet is a photo-engraved reprint of my doctor's thesis, long since out of print, and first published in the *Transactions* of the Connecticut Academy of Arts and Sciences, in 1892.

Twenty-five years later, in the midst of the World War, much to my surprise and pleasure, a French translation was made by Jacques Moret.*

Occasionally mathematical economists have suggested that the original American edition should be reprinted, one of them maintaining that "the book has become recognized as an economic classic and should be more available." So here it is, a third of a century after its first publication.

I cannot honestly recommend that it be read entire by anybody. But in my own classes I have, for many years, found use for pages 11-54 and pages 64-79. The book was written when I was more interested in mathematics than in economics. The vector analysis notation on page 81 was a tribute to J. Willard Gibbs, whose student I then was and who took a lively interest in the work.

The "Bibliography on Mathematico-Economic Writings" at the end was superseded years ago by my "Bibliography of Mathematical Economics" on pages 173-209 of Nathaniel T. Bacon's English translation of Cournot ("Researches into the Mathematical Principles of the Theory of Wealth") in the *Economic Classics Series,* edited by Professor W. J. Ashley and published by Macmillan in 1897.

The hydrostatic mechanism pictured on page 38 has been actually constructed—twice in fact. In 1893, the year after the thesis was published, my colleague, Professor Henry W. Farnam, generously defrayed the expense of constructing a model. This was used in my classes for many years. Eventually it wore out and was recently (1925) replaced by a second model somewhat improved and simplified. Photographs of both these models are reproduced herewith.

As I look back on this youthful production I take some satisfaction in finding that my treatment of so-called "marginal utility" and criticism of the then current notions of a "calculus of pleasure and pain" are in tune with modern psychology and that this fact has been favorably commented on by Professor Wesley C. Mitchell.

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* *Recherches Mathématiques sur la Théorie de La Valeur et Des Prix.* M. Giard & É. Brière, Paris, 1917.

The suggestion in this booklet that so-called "marginal utility" may be measurable statistically is now being followed up Within a few months, I expect to publish the first attempt of this sort and to discuss its possible practical use in deciding on the proper rate of progression for an income tax.

IRVING FISHER.

Yale University, August, 1925.

[From Transactions of the Connecticut Academy, Vol. IX. July, 1892]

[The following memoir is in substance the writer's thesis for the degree of Ph.D. at Yale University, 1891.]

I.—MATHEMATICAL INVESTIGATIONS IN THE THEORY OF VALUE AND PRICES.

By Dr. IRVING FISHER.

[Read April 27, 1892.]

PREFACE.

John Stuart Mill* asserted that he had left nothing in the laws of value for any future economist to clear up. Until 1871 this statement doubtless had much the force of dogma. Even Jevons made preliminary obeisance before proceeding to break the ground afresh with the mathematical instrument. Jevons with characteristic candor expressly disclaimed finality ;† but few of his followers have realized with his clearness and honesty the need of further analysis along the lines which he laid down.

The truth is, most persons, not excepting professed economists, are satisfied with very hazy notions. How few scholars of the literary and historical type retain from their study of mechanics an adequate notion of force ! Muscular experience supplies a concrete and practical conception but gives no inkling of the complicated dependence on space, time, and mass. Only patient mathematical analysis can do that. This natural aversion to elaborate and intricate analysis exists in Economics and especially in the theory of value. The very foundations of the subject require new analysis and definition. The dependence of value on utility, disutility, and commodity, the equality of utilities, the ratio of utilities, the utility of a commodity as a function of the quantity of that commodity solely, or of that commodity and others conjointly, are subjects, the neglect of which is sure to leave value half understood, and the mastery of which claims, therefore, the first and most patient effort of the economic scientist.

These form the subject matter of the following memoir which is *a study by mathematical methods of the determination of value and prices.*

Much germane to the subject has been omitted because already elaborated by others. Cases of discontinuity belong to almost every step, to modify or extend the continuous case. But the application of this correction has been thoroughly worked out by Auspitz und Lieben. Multiple equilibrium and monopoly value are omitted for a similar reason.

The two books which have influenced me most are Jevons : " *Theory of Political Economy,*" and Auspitz und Lieben: " *Untersuch-*

* Pol. Econ., Bk. III, Ch. I, § 1.

† Pol. Econ., Pref. 3rd ed.

ungen ueber die Theorie des Preises." To the former I owe the idea of marginal utility and of mathematical treatment in general, to the latter the clear conception of the "symmetry" of supply and demand and the use of *rate of commodity* in place of absolute commodity, and to both many minor obligations.

The equations in Chapter IV, § 10, were found by me two years ago, when I had read no mathematical economist except Jevons. They were an appropriate extension of Jevons' determination of exchange of *two* commodities between *two* trading bodies to the exchange of *any number* of commodities between *any number* of traders and *were obtained as the interpretation of the mechanism* which I have described in Chapter IV. That is, the determinateness of the mechanism was expressed by writing as many equations as unknowns. These equations are essentially those of Walras in his *Éléments d'économie politique pure.* The only fundamental differences are that I use marginal utility throughout and treat it as a function of the quantities of commodity, whereas Professor Walras makes the quantity of each commodity a function of the prices. That similar results should be obtained independently and by separate paths is certainly an argument to be weighed by those skeptical of the mathematical method. It seemed best not to omit these analytical portions of Part I, both because they contribute to an understanding of the other portions of the work and because they were in a proper sense my own.

Three days after Part II was finished I received and saw for the first time Prof. Edgeworth's *Mathematical Psychics.* I was much interested to find a resemblance between his surface on page 21 and the total utility surfaces* described by me. The resemblance, however, does not extend far. It consists in the recognition that in an exchange, utility is a function of *both* commodities (not of one only as assumed by Jevons), the use of the surface referred to as an interpretation thereof and the single phrase (Math. Psych., p. 28) "and similarly for larger numbers in hyperspace" which connects with Part II, Ch. II, § 5.

There is one point, however, in which, as it seems to me, the writer of this very suggestive book has gone far astray. Mathe-

* His result, which translated into my notation is

$$\left(\frac{dU}{dA_1}\right)\left(\frac{dU}{dB_2}\right) - \left(\frac{dU}{dB_1}\right)\left(\frac{dU}{dA_2}\right) = 0,$$

becomes by transposition and division identical with part of the continuous proportion, Part I, Ch. IV, § 3.

matical economists have been taunted with the riddle: What is a unit of pleasure or utility? Edgeworth, following the Physiological Psychologist Fechner, answers: "Just perceivable increments of pleasure are equatable" (p. 99). I have always felt that utility must be capable of a definition which shall connect it with its positive or objective commodity relations. A physicist would certainly err who defined the unit of force as the minimum sensible of muscular sensation. Prof. Edgeworth admits his perplexity: "It must be confessed that we are here leaving the *terra firma* of physical analogy" (p. 99). Yet he thinks it is "a principle on which we are agreed to act but for which it might be hard to give a reason;" and again: [such equality] "it is contended, not without hesitation is appropriate to our subject."

This foisting of Psychology on Economics seems to me *in*appropriate and vicious. Others besides Prof. Edgeworth have done it. Gossen* and Jevons appeared to regard the "calculus of Pleasure and Pain"† as part of the profundity of their theory. They doubtless saw no escape from its use. The result has been that "mathematics" has been blamed for "restoring the metaphysical entities previously discarded."‡

These writers with Cournot,§ Menger,‖ and Marshall¶ appear to me to have contributed the most to the subject in hand. With the exceptions noted I have endeavored not to repeat them but to add a little to them, partly in the theory of the subject and partly in the mode of representing that theory. Readers to whom the subject is new will find the present memoir exceedingly condensed. In the attempt to be brief, the possible uses of the diagrams and mechanisms have been merely sketched, and elaborate explanations and illustrations have been omitted. I have assumed that my readers are already familiar with (say) Jevons, Walras, Menger or Wieser where illustrations and explanations regarding "final utility" abound. Much of Part II and Appendix I may not be thoroughly intelligible to those not familiar with higher geometrical analysis. These parts are made as brief as possible.

My especial thanks are due to Prof. Gibbs and Prof. Newton for valuable criticism.

IRVING FISHER.

Yale University, May, 1892.

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* Menschlich Verhehr., *Braunschweig*, 1854. † Jevons, p. 23, also pp. 8–9.

‡ Dr. Ingram. § Théorie des Richesses, *Paris*, 1838.

‖ Volkswirthschaftslehre, *Wien*, 1871. ¶ Prin. of. Econ., *Macmillan*, 1890.

SYLLABUS.

PART I. UTILITY OF EACH COMMODITY ASSUMED TO BE DEPENDENT ONLY ON THE QUANTITY OF THAT COMMODITY.

CHAPTER I. *Utility as a Quantity.*

CHAPTER II. *Mechanism.*

PART II. UTILITY OF ONE COMMODITY A FUNCTION OF THE QUANTITIES OF ALL COMMODITIES.

APPENDIX I. Miscellaneous remarks on Part I.

APPENDIX II. Limitations of the preceding analyses.

APPENDIX III. THE UTILITY AND HISTORY OF MATHEMATICAL METHOD IN ECONOMICS.

APPENDIX IV. BIBLIOGRAPHY OF MATHEMATICO-ECONOMIC WRITINGS.