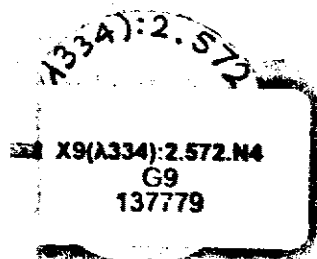


STUDIER FRA AARHUS UNIVERSITETS
ØKONOMISKE INSTITUT, Nr. 4

**A DYNAMIC STUDY OF
PIG PRODUCTION IN DENMARK**

BY

TRYGVE HAAVELMO



EJNAR MUNKSGAARD
1939

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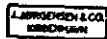
EJNAR MUNKSGAARD

1939

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FOREWORD:

The present study is worked out at the Institute of Economics, University of Aarhus, Denmark, during my stay in Aarhus as a temporary teacher of statistics. My most sincere thanks are due to the director of the Institute, Professor, Dr. polit. Jørgen Pedersen, for all his kind help and for the encouragement and interest he has shown to my work. Without all his practical information and suggestive ideas, I should, as a foreigner, have been quite helpless in this field of specific Danish problems.

I am also greatly indebted to cand. polit. K. Philip, Aarhus, who very kindly permitted me to draw heavily on his valuable collection of data and facts on Danish economic policy. Besides, I wish to thank him for numerous stimulating discussions.

To the secretary of »De samvirkende danske Andels-Slagteriers Fælleskontor«, Mr. C. Trautmann, and the director of the »Aarhus Flæskehøst«, Mr. M. Storgaard, I wish to express my gratitude for their kindness in answering practical questions. They should not, however, be made responsible for possible errors in the present publication, the author assumes sole responsibility.

Mrs. V. Friis Olsen and stud. økon. B. Andersson have carried through a great part of the numerical work; the former also typed the manuscript and corrected the language on several points. I want to extend my thanks to both of them.

Aarhus, April 1939.

Trygve Haavelmo.

I. INTRODUCTION

1. The factual background of the present study¹⁾.

The Danish pig-production is essentially an export industry, being almost entirely dependent on the English bacon market. During the last ten years the quantity exported has been around two or three times the total quantity (secondary qualities included) absorbed by the home market. Nearly all killings take place in slaughter-houses, which mostly are cooperative organizations owned by the farmers. Home killings are not allowed except for private consumption.

Until 1933 the pig production was a free trade, apart from ordinary restrictions as to a certain standard of quality, weight limits, common price quotations etc. But as a result of the adoption of the quota system in Great Britain it was found necessary to regulate the pig production by an act. The first act came in 1933 (April 24th). It has later been amended by several supplementary acts (especially by the act of December 22nd, 1937), but the changes are not fundamental, at least not to the problems attacked in the present study. The main lines of the regulation policy run in brief as follows:

¹⁾ This is only a very short review of some important facts, which it is absolutely necessary to keep in mind in order to follow the idea of the present study. The reader is referred to the new book of K. Philip: »En Fremstilling og Analyse af den danske Kiselovgivning 1931—38« (København 1939), ch. V. 4., giving a detailed description of the Danish pig production problems.

The farmers are still, formally, in a perfectly free position as to the number of pigs they wish to keep and to put on the market. The restrictions take place by means of a particular type of pricing system by which a fixed quantity may be delivered at a high price, and the surplus at a very low price. (These prices are quoted weekly by the »Andelsslagteriernes Landsnoteringsudvalg« in cooperation with special government institutions).

The standard price is the weekly quotation for first-class bacon hogs within the ideal weight limits (usually somewhere between 60 and 70 kg slaughtered weight). A certain range above or below the ideal weight class is allowed, but then the price is reduced at a fixed scale, from which it is obvious that the ideal weight is the most profitable. The standard price covers very high profits to the producers. *To obtain this price* each pig delivered at the slaughter houses — apart from fulfilling the above quality conditions — must be followed by a so-called »pig-card« (a licence), and this is just the point of the whole regulation policy. The pig-cards are issued to *order, not to name*, and valid only within fixed time intervals, mostly 1—2 months. There are also cards valid for six months, but these usually amount to only about 10—15 % of all cards.

The cards are distributed (free) to farmers by local subordinate institutions of the »Landbrugsministeriets Svinereguleringsudvalg«, a permanent committee especially established to manage the regulations. The main policy as to the total number of cards issued has been, firstly, to cover the English import-quota, and secondly, to leave such a quantity of pork for the home market which may be sold at a price near to the export price. (Besides, the possibility of export to Germany or other secondary markets plays some role). The total number of cards issued per year since 1933 has been around 4 millions or about 20 cards per farmer. The distribution among the individual producers is fixed through a rather complicated set of criteria, such as the size and quality of the farm, the previous scale of pig production, milk production, etc. The main result of the individual distribution policy has been a considerable shifting of the production in favour of small farms (especially by the new act of 1937). Due to the high price of slaughter-hogs *with* cards, the allotting of cards means a real gift of profit.

For ordinary slaughter-hogs delivered *without* cards the price is fixed so far below the standard price that it obviously does not cover production costs, and reductions for over- and underweights are made at the same scale as for »card-pigs«. Therefore deliveries of pigs without cards take place only in two cases, namely, either when a pig does not fulfill the required quality conditions and weight limits of »card-pigs«, or for lack of cards.

The cards being issued to order not to name, the result is of course a regular *card trade* between farmers. We may indeed speak of a well-defined *pig-card-market*. To a large extent cards are bought and sold by the slaughter houses themselves. A farmer, lacking cards, may then bring his pigs to the slaughter houses and buy the cards needed there. Prices of pig-cards are found in the newspapers. These prices are of course extremely sensible as to the difference between the total number of cards available and the number of pigs ready for slaughtering. This will be studied in detail later. If the price of pig-cards is very high, covering nearly the whole difference between the prices of deliveries with and without cards, there is an increasing tendency to home killings for private consumption. The farmers may also have pigs slaughtered free at the slaughter houses for private consumption. In this case of course no cards are needed.

Sows and boars are excepted from the above price restrictions, they may be slaughtered and sold in the home market at »free competition« prices. The idea is that sows and boars are production means, not products. This may however have some influence on profit calculations at production starting, as is shown in a later part of this study.

2. *The problems and the general lines of attack.*

The pig production being of vital importance to Danish farmers, the present regulation policy is a matter of daily discussions. One can hardly open a newspaper without finding criticisms, replies and new proposals on this subject. The main difficulties of a clear discussion are the same here as in all other discussions of economic policy: there are too many variables to be kept apart from each other in a purely verbal treatment. The discussions are therefore filled up with conclusions as to what is »right« or »wrong«, based on insufficient knowledge and unrealistic assumptions as to the network of relationships ruling the whole system.

A mapping of the different interrelations must be made if a rational discussion shall be possible, and this is the problem of the present study. We do not intend to raise a general discussion of the practical regulation policy, our problem is mainly to procure *tools* for such a discussion. Some conclusions are however drawn incidentally.

This problem is essentially a matter of econometrics. What we need are quantitative measures of the different effects and dependencies. The theoretical solutions of the problems in this field are mostly rather simple, as long as we do not ask for statistical verification. We shall see that many of the interrelations are different from what might be expected,

which just shows how difficult it is to make realistic assumptions without a glance at the concrete observations.

Our analysis will be a short-time study, which is justified by the fact that the most characteristic and interesting movements within the pig production apparatus are rapid changes. As statistical testing period we have chosen monthly data for the three years 1935—37, in order not to use the data from the first time of the regulation, which certainly enclose a sort of adaptation, and to save the year 1938 for testing the forecasting-effectivity of the estimated relations.

Statistical regression analysis will play an important role in this study. We here follow the method of »bunch analysis«¹⁾, and we do not calculate any standard errors or the like. We shall however add the correlation matrices and the standard deviations of the variates, from which all ordinary multiple correlation statistics may be calculated. (The correlation coefficients we denote by r_{ij} , i and j being the enumeration of the variates taken in the sequence in which they stand in the corresponding regression equation. Correspondingly the standard deviations are termed σ_i).

II. THE FUNDAMENTAL RELATIONS AND THEIR STATISTICAL DETERMINATION.

3. *The variables introduced and their statistical measurement.*

To fix the ideas, it is practical at once to give a list of symbols for the variables occurring in our study. They will be discussed in detail later. Capital letters are used for stock terms, small letters for flow terms. All variables are functions of time, which is indicated by the term (t) behind the symbols. (In certain unambiguous cases the functional sign (t) may for convenience be left out). The variables are:

- (3.1) $Y_0(t)$ = the stock of first time breeding sows.
- (3.2) $y_0(t)$ = the inflow to the stock $Y_0(t)$ per month.
- (3.3) $Y_1(t)$ = the stock of other breeding sows (having at least bred once).
- (3.4) $y_1(t)$ = the inflow to the stock $Y_1(t)$ per month.
- (3.5) $Y(t)$ = $Y_0(t) + Y_1(t)$ = total stock of breeding sows.
- (3.6) $y(t)$ = $y_0(t) + y_1(t)$ = total inflow of breeding sows per month.

¹⁾ See Ragnar Frisch: Statistical confluence analysis by means of complete regression systems. Publ. No. 5 from the Institute of Economics, Oslo 1934.

- (3.7) $X_i(t)$ = stock of feeding pigs in the weight group No. i ($i = 0$: sucking pigs, $i = 1$, weaned pigs below 35 kg, etc.).
- (3.8) $x_i(t)$ = the inflow to the stock $X_i(t)$ per month.
- (3.9) $x(t)$ = the outflow of finished pigs (at the average slaughter weight) per month.
- (3.10) $x^*(t)$ = the flow of pigs delivered with cards per month (usually almost equal to the number of cards per month).
- (3.11) $s(t)$ = the price of small weaned living pigs sold in the market.
- (3.12) $p_1(t)$ = the standard price (per animal) of pigs delivered *with* cards.
- (3.13) $p_2(t)$ = the standard price (per animal) of pigs delivered *without* cards.
- (3.14) $v(t)$ = the price of pig-cards in trade between farmers.

In the present case we are in an exceptionally good position as to statistical information, especially through the six-weekly pig countings arranged to give a base for the regulation policy. (These countings regularly cover a sample of about $\frac{1}{6}$ of all farms. Once a year (in July) *all* pigs are counted, as a base for the sampling estimation, and experiences show that the error of estimation lies within very narrow limits. By each counting we get the following specification (with our symbols added):

| | Number in thousands |
|--------------------------------------|---------------------|
| Boars | |
| Sows: First time breeding sows | $Y_0(t)$ |
| Other breeding sows | $Y_1(t)$ |
| Sows with sucking pigs | |
| Barren sows | |
| Sows to be slaughtered | |
| Sucking pigs | $X_0(t)$ |
| Weaned pigs below 35 kg | $X_1(t)$ |
| Pigs 35—60 kg | $X_2(t)$ |
| Pigs above 60 kg | $X_3(t)$ |

By drawing the series graphically it is easy to read off the values of the stocks at every point of time. This procedure has been used in our calculations. The possible errors thereby introduced may safely be neglected, as the interval between the countings is only 6 weeks. These series give only stock terms, but the monthly flows may be computed by methods described below.

Information about the number of cards issued and used and slaughter-

ings with and without cards at slaughter houses, as well as the price quotations and weight limits, are given by statistics of the regulation authorities.

In table 1 we have compiled the series involved in the present study, using the system of notations listed above. (Some small supplementary series will be added in the text). The unit of time t is always 1 month. In the cases of lagged series, the values of the variate are placed at the point of time indicated by t . So, for instance, in column (7) the value placed at $t = \text{Jan. 15th, 1935}$ is the value of y at $(t-6) = \text{July 15th, 1934}$. This is convenient for computing regressions.

Table 1:

Column:

- (1) The stock of first time teeming sows. Graphical interpolation on the series $Y_0(t)$ given by the six-weekly countings.
 - (2) The stock of other teeming sows. Interpolation as in (1).
 - (3) The stock of weaned pigs below 35 kg at $(t+4.4)$. Interpolation as in (1).
 - (4) The stock of pigs above 60 kg at $(t-0.7)$. Interpolation as in (1).
- Source of these four series: »Statistiske Efterretninger«, (bulletin of the Statistical Department) 1935—38, the tables: »Svinebestanden«.
- (5) The inflow of first time teeming sows (calculated from (1) by method to be explained later).
 - (6) The inflow of other teeming sows (calculated from (2) by method to be explained later).
 - (7) The inflow of all teeming sows 6 months back (calculated from (1) and (2) by method to be explained later).
 - (8) The inflow of weaned pigs below 35 kg (calculated from $X_1(t)$ by method to be explained later).
 - (9) The inflow of weaned pigs 4.65 months back (calculated from $X_1(t)$ by the same proceeding as for (8)).
 - (10) The regular outflow of finished pigs (calculated from the series (9) by the formula $x(t) = 0.95 \cdot x_1(t-4.65)$ and afterwards reduced for exceptional slaughtering of small pigs from May 1936 to January 1937. This is explained later in the text.).
 - (11) Number of pigs per month delivered with cards at the slaughter houses. Calculated from the series of weekly deliveries. In the interval considered this series is practically equal to the number of pig-cards issued. Source: »Regnskaber fra Andelsslagteriernes Fælleskontor 1935—38«. Section: »Statistiske Oplysninger«.

- (12) The difference between the regular outflow of finished pigs and number of pigs delivered with cards per month (column (10) minus column (11)).
- (13) The price of small living pigs. Monthly averages of the quotations on the »Køge« market. Source: »Landbrugsraadets Meddelelser« 1935—38.
- (14) Price per animal of pigs delivered *with* cards. Monthly averages. Calculated by multiplying the standard quotation per kg by the upper limit of the ideal slaughter-weight class minus 1 kg (as an estimate of the typical weight within the class). The ideal weight class has varied as follows:

| | Ideal weight limits | Estimated typical weight |
|----------------------------|------------------------|-----------------------------|
| 1935—1937..... | 58—64 kg | 63 kg |
| Jan. 1938—Febr. 1938 | 58—64 » | 63 » |
| Febr. 1938—March 1938..... | 60—66 » | 65 » |
| March 1938—July 1938 | 62—68 » | 67 » |
| July 1938—Aug. 1938 | 60—66 » | 65 » |
| Rest of 1938 | 58—64 » | 63 » |

- (15) The price per animal of first class pigs delivered *without* cards. Calculated in the same way as (14).
Sources of data for (14) and (15): »Landbrugsraadets Meddelelser« 1935—38.
- (16) The pig-card prices. Monthly averages. This is not an official series, it is collected from card-sellers' advertisements in several newspapers by cand. polit. K. Philip¹⁾. The series is used here with his kind permission.
- (17) The relative pig-card price. Calculated as indicated in the table.

4. The characteristic lag-relations in the production process.

The breeding of hogs is a typical example of a timerequiring production process. Accordingly, the length of the production period will play a central role in our analysis. Taking slaughter hogs to be the final product, the production period is defined as the time elapsing between the starting of production (the mating of sows) and the attainment of the slaughter-weight required. Now the period of gestation of sows is practically a constant. If therefore the weight of pigs were a function of age only, the production period would be a function of the slaughter-weight, and therefore a constant when the slaughter-weight is fixed. But several other things may break this simple connection,

¹⁾ See his new book, l.c.p. 102—104.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | |
|---------------------|----------|----------|--|--|----------|----------|----------------------------------|----------|------|-----|
| | $Y_0(t)$ | $Y_1(t)$ | $X_1\left(t+\frac{\theta_1}{2}+\theta_0+\frac{\theta_1}{2}\right)$ $= X_1(t+4.4)$ | $X_2\left(t-\frac{\theta_2}{2}\right)$ $= X_2(t-0.7)$ | $u_0(t)$ | $u_1(t)$ | $\frac{v(t-\alpha)}{v(t-\beta)}$ | $z_1(t)$ | | |
| Unit of Measurement | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| Day of the Month | 1st | 1st | 1st | 15th | 15th | 15th | 15th | 15th | 15th | |
| 1935 | J | 65 | 185 | 777 | 532 | 24 | 43 | 58 | 400 | 409 |
| | F | 79 | 176 | 787 | 463 | 23 | 41 | 58 | 395 | 393 |
| | M | 89 | 166 | 771 | 501 | 22 | 44 | 60 | 396 | 398 |
| | A | 89 | 155 | 750 | 483 | 22 | 48 | 65 | 416 | 403 |
| | M | 85 | 157 | 764 | 471 | 22 | 50 | 68 | 420 | 405 |
| | J | 83 | 175 | 808 | 500 | 20 | 50 | 68 | 412 | 397 |
| | J | 84 | 186 | 876 | 472 | 21 | 48 | 67 | 401 | 395 |
| | A | 78 | 188 | 885 | 481 | 24 | 48 | 64 | 409 | 402 |
| | S | 75 | 182 | 860 | 545 | 26 | 48 | 66 | 432 | 424 |
| | O | 84 | 178 | 817 | 537 | 26 | 49 | 70 | 468 | 418 |
| 1936 | N | 94 | 180 | 824 | 548 | 25 | 48 | 72 | 473 | 409 |
| | D | 98 | 181 | 839 | 540 | 26 | 46 | 70 | 460 | 397 |
| | J | 97 | 180 | 846 | 454 | 30 | 46 | 69 | 437 | 416 |
| | F | 95 | 176 | 826 | 500 | 33 | 47 | 71 | 441 | 445 |
| | M | 107 | 173 | 840 | 535 | 34 | 49 | 74 | 449 | 472 |
| | A | 120 | 174 | 906 | 559 | 33 | 50 | 75 | 452 | 474 |
| | M | 126 | 182 | 970 | 561 | 31 | 49 | 73 | 442 | 450 |
| | J | 128 | 188 | 1024 | 560 | 24 | 48 | 72 | 449 | 438 |
| | J | 120 | 188 | 1014 | 554 | 18 | 48 | 76 | 484 | 442 |
| | A | 100 | 181 | 942 | 552 | 15 | 49 | 80 | 519 | 452 |
| 1937 | S | 77 | 178 | 850 | 604 | 14 | 49 | 83 | 548 | 449 |
| | O | 61 | 180 | 773 | 576 | 14 | 50 | 84 | 542 | 444 |
| | N | 54 | 183 | 747 | 554 | 14 | 50 | 80 | 504 | 459 |
| | D | 52 | 187 | 741 | 548 | 15 | 46 | 72 | 455 | 501 |
| | J | 53 | 188 | 755 | 528 | 17 | 44 | 66 | 413 | 530 |
| | F | 55 | 179 | 770 | 520 | 19 | 43 | 64 | 399 | 545 |
| | M | 60 | 167 | 765 | 522 | 20 | 43 | 64 | 396 | 537 |
| | A | 68 | 161 | 792 | 534 | 19 | 44 | 64 | 404 | 487 |
| | M | 74 | 159 | 825 | 524 | 18 | 42 | 64 | 412 | 440 |
| | J | 72 | 162 | 847 | 514 | 15 | 41 | 61 | 409 | 408 |
| 1938 | J | 69 | 161 | 841 | 502 | 12 | 37 | 61 | 424 | 395 |
| | A | 61 | 154 | 772 | 490 | 12 | 35 | 62 | 441 | 399 |
| | S | 48 | 146 | 680 | 572 | 13 | 37 | 63 | 453 | 406 |
| | O | 43 | 132 | 588 | 532 | 15 | 38 | 63 | 450 | 414 |
| | N | 45 | 133 | 573 | 540 | 20 | 38 | 60 | 413 | 411 |
| | D | 53 | 140 | 620 | 580 | 26 | 38 | 56 | 364 | 430 |
| | J | 65 | 143 | 680 | 500 | 29 | 35 | 50 | 314 | 447 |
| | F | 87 | 143 | 695 | 508 | 28 | 33 | 46 | 306 | 452 |
| | M | 104 | 136 | 715 | 528 | 24 | 36 | 49 | 332 | 444 |
| | A | 107 | 125 | 723 | 532 | 18 | 38 | 53 | 364 | 398 |
| M | 97 | 129 | 718 | 472 | 15 | 41 | 58 | 372 | 347 | |
| J | 71 | 139 | 702 | 442 | 12 | 42 | 64 | 362 | 312 | |
| J | 60 | 150 | 680 | 428 | 12 | 42 | 63 | 387 | 303 | |
| A | 48 | 158 | 655 | 454 | 15 | 41 | 61 | 384 | 345 | |
| S | 40 | 160 | 630 | 540 | 17 | 40 | 59 | 375 | 366 | |
| O | 50 | 157 | 618 | 522 | 20 | 38 | 56 | 364 | 374 | |
| N | 62 | 152 | — | 534 | 24 | 38 | 56 | 350 | 394 | |
| D | 70 | 145 | — | 555 | 29 | 39 | 54 | 337 | 358 | |

the Variables Introduced.

| (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | |
|--------|----------|-------------------------------------|--------|----------|----------|--------|--------------------------------|---------------------|
| $x(t)$ | $x^*(t)$ | $\frac{x(t) - x^*(t)}{(10) - (11)}$ | $s(t)$ | $P_1(t)$ | $P_2(t)$ | $v(t)$ | $\frac{v(t)}{P_1(t) - P_2(t)}$ | |
| 1000 | 1000 | 1000 | Kr. | Kr. | Kr. | Kr. | — | Unit of Measurement |
| 15th | 15th | 15th | 15th | 15th | 15th | 15th | 15th | Day of the Month |
| 386 | 434 | -48 | 33.4 | 101.3 | 50.3 | 36.3 | .71 | J 1935 |
| 373 | 264 | 109 | 23.9 | 98.1 | 45.5 | 43.3 | .82 | F |
| 378 | 362 | 16 | 21.7 | 97.6 | 44.0 | 42.8 | .80 | M |
| 383 | 290 | 93 | 24.5 | 100.1 | 44.0 | 41.1 | .73 | A |
| 385 | 375 | 10 | 28.6 | 107.0 | 44.0 | 30.3 | .48 | M |
| 377 | 366 | 11 | 31.3 | 110.8 | 44.0 | 32.9 | .49 | J |
| 375 | 332 | 43 | 30.7 | 107.0 | 44.0 | 37.9 | .60 | J |
| 382 | 358 | 24 | 31.0 | 100.1 | 44.0 | 37.2 | .66 | A |
| 403 | 353 | 50 | 29.8 | 98.2 | 44.0 | 36.5 | .67 | S |
| 397 | 332 | 65 | 26.7 | 104.5 | 44.0 | 37.1 | .61 | O |
| 389 | 307 | 82 | 23.0 | 100.0 | 49.0 | 36.3 | .71 | N |
| 377 | 277 | 100 | 22.0 | 97.2 | 60.0 | 35.1 | .94 | D |
| 395 | 301 | 94 | 22.8 | 101.3 | 50.3 | 38.5 | .75 | J 1936 |
| 423 | 306 | 117 | 26.4 | 109.5 | 50.3 | 41.6 | .70 | F |
| 448 | 399 | 49 | 29.3 | 110.8 | 50.3 | 38.5 | .64 | M |
| 450 | 341 | 109 | 27.2 | 110.2 | 50.3 | 44.7 | .75 | A |
| 428 | 364 | 64 | 22.0 | 107.0 | 50.3 | 47.4 | .84 | M |
| 416 | 315 | 101 | 20.0 | 105.7 | 44.0 | 52.8 | .86 | J |
| 404 | 315 | 89 | 15.7 | 104.5 | 39.3 | 58.8 | .90 | J |
| 408 | 297 | 111 | 12.0 | 110.9 | 37.8 | 67.7 | .93 | A |
| 398 | 377 | 19 | 10.7 | 110.9 | 37.8 | 73.3 | 1.00 | S |
| 377 | 222 | 155 | 9.2 | 108.4 | 37.8 | 67.6 | .96 | O |
| 369 | 310 | 59 | 8.5 | 102.1 | 37.8 | 63.1 | .98 | N |
| 388 | 270 | 118 | 10.2 | 103.3 | 37.8 | 57.6 | .86 | D |
| 426 | 296 | 130 | 10.8 | 102.1 | 37.8 | 60.4 | .94 | J 1937 |
| 420 | 300 | 120 | 10.9 | 99.5 | 37.8 | 56.8 | .92 | F |
| 419 | 379 | 40 | 14.3 | 104.0 | 37.8 | 52.4 | .79 | M |
| 433 | 318 | 115 | 14.0 | 109.6 | 37.8 | 57.8 | .81 | A |
| 418 | 348 | 70 | 13.2 | 109.0 | 37.8 | 63.2 | .89 | M |
| 388 | 305 | 83 | 13.0 | 102.1 | 37.8 | 62.6 | .97 | J |
| 375 | 348 | 27 | 12.0 | 109.6 | 37.8 | 67.9 | .95 | J |
| 379 | 353 | 26 | 12.9 | 116.6 | 37.8 | 72.9 | .93 | A |
| 386 | 366 | 20 | 13.1 | 118.4 | 37.8 | 80.5 | 1.00 | S |
| 393 | 248 | 145 | 13.2 | 111.2 | 37.8 | 66.4 | .90 | O |
| 390 | 353 | 37 | 21.3 | 97.0 | 56.7 | 37.5 | .93 | N |
| 409 | 344 | 65 | 26.8 | 107.1 | 63.0 | 40.7 | .92 | D |
| 425 | 309 | 116 | 32.5 | 108.4 | 63.0 | 43.5 | .96 | J 1938 |
| 429 | 313 | 116 | 38.1 | 113.8 | 65.7 | 37.0 | .77 | F |
| 422 | 300 | 122 | 44.7 | 120.6 | 67.0 | 27.4 | .51 | M |
| 378 | 369 | 9 | 42.6 | 126.7 | 67.0 | 22.8 | .38 | A |
| 330 | 305 | 25 | 39.3 | 123.3 | 67.0 | 26.4 | .47 | M |
| 296 | 326 | -30 | 38.0 | 116.6 | 67.0 | 22.9 | .46 | J |
| 288 | 275 | 13 | 32.9 | 117.8 | 64.0 | 42.4 | .79 | J |
| 323 | 322 | 6 | 29.5 | 111.5 | 63.0 | 42.3 | .87 | A |
| 348 | 348 | 0 | 31.7 | 109.7 | 47.3 | 52.4 | .84 | S |
| 355 | 292 | 63 | 31.0 | 108.4 | 47.3 | 46.0 | .75 | O |
| 365 | 275 | 90 | 30.0 | 102.1 | 47.3 | 40.0 | .73 | N |
| 369 | 287 | 82 | 29.0 | 107.1 | 63.0 | 32.0 | .73 | D |

especially the feeding technique, accidental events, change of breeds, etc. In the present analysis we will for simplicity have to consider the period as constant. This is justified by the following facts.

Firstly, the ideal weight of slaughter hogs is officially fixed and varies within narrow limits, usually between 60 and 70 kg. This fixing depends on long experiences as to bacon quality, the rate of increase in weight in relation to feeding costs, etc., and is not likely to be significantly changed. For pigs below or above the ideal weight region the price is cut down so much that it is an obvious loss to *aim* at such deviations. Secondly, accidental variations will practically disappear in the great masses we are considering. And thirdly, whatever may happen, the positive correlation between the age and weight is bound to be strong, so that a constant period of production at least may be considered as a highly significant average when the ideal slaughter weight is given.

To estimate the production period and get an idea of the whole mechanism, it is necessary to consider the successive steps in the production process. This subdivision plays an important role in the present study. For this purpose it is convenient to use a schematic representation as obtained by following the Danish pig census specification. See (4.1).

From this scheme it is seen that the *flows* (the streams of entrance and departure) through different groups will be linked together by a chain of lag-relations, and our problem is to determine their analytic expressions. As however, the statistics do not give the flow quantities but only the stocks in the different groups at the date of the counting, we shall first have to study some relations between these two concepts. When all the sub-periods in the scheme (4.1) are constants, the relations are very simple. Indeed, taking the flow terms to be continuous functions of time, and indicating by $P(\tau)$ a continuous function showing the probability of still being present in the group τ time units after entrance, we have:

$$(4.2) \quad Y_0(t) = \int_0^{\theta_0} P_{y_0}(\tau) \cdot y_0(t - \tau) d\tau$$

(= the stock of first time breeding sows)

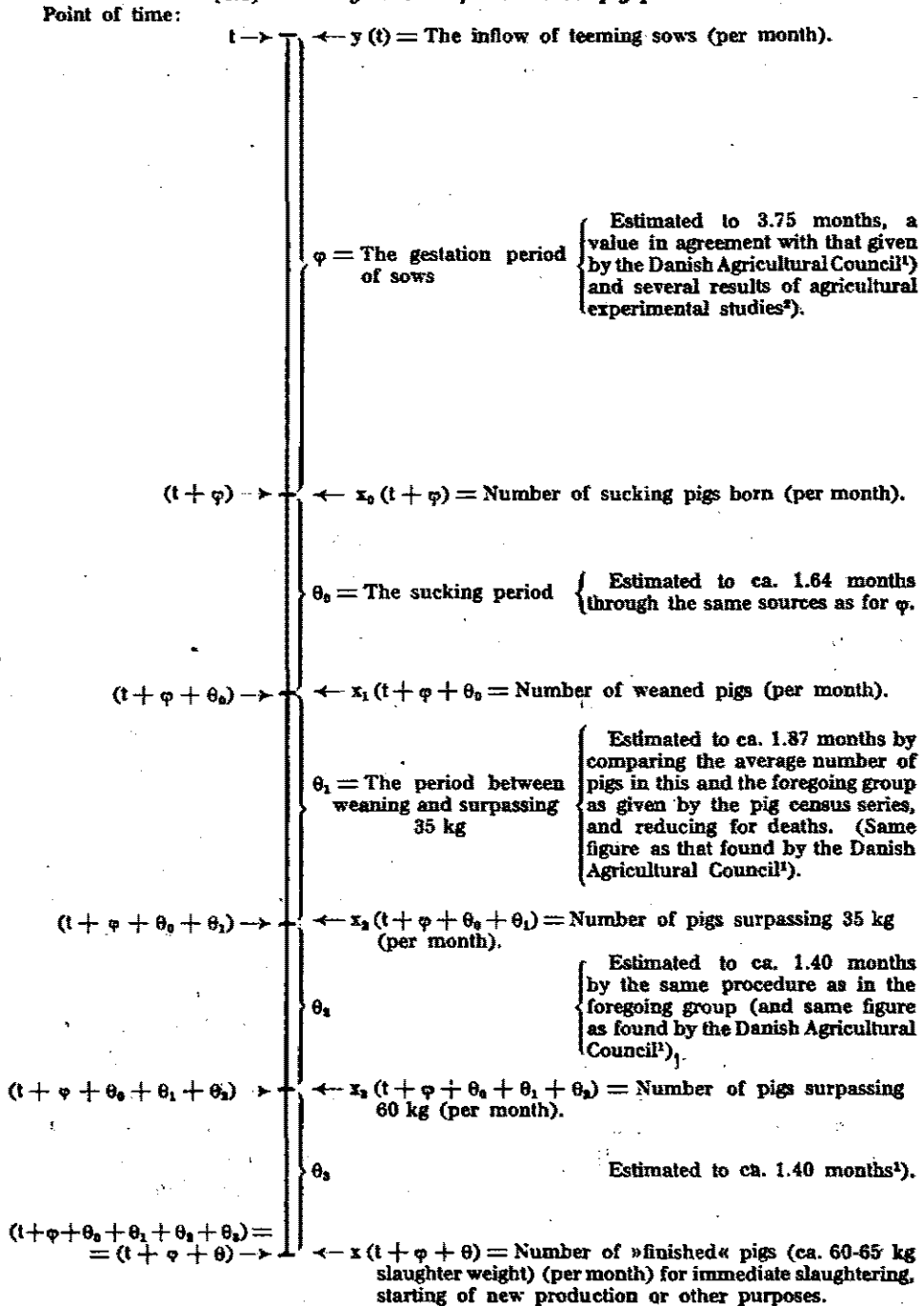
$$(4.3) \quad Y_1(t) = \int_0^{\theta_1} P_{y_1}(\tau) \cdot y_1(t - \tau) d\tau$$

(= the stock of other breeding sows).

$$(4.4) \quad X_1(t) = \int_0^{\theta_1} P_{x_1}(\tau) \cdot x_1(t - \tau) d\tau$$

(= the stock of weaned pigs below 35 kg).

(4.1) *The lag scheme of the Danish pig production.*



¹⁾ »Landbrugsraadets Meddelelser« 1935, p. 1330.

²⁾ See for instance: »Landbrugets Ordbog«, København 1938, p. 524.

Consequently the stock terms are nothing but continuous moving averages taken on the flow terms, the result of which is a sort of smoothing effect. The formulae above may be approximated by the following expressions:

$$(4.5) \quad Y_0(t) = \text{const.} \cdot \varphi \cdot y_0\left(t - \frac{\varphi}{2}\right), \text{ or } y_0(t) = \text{const.} \cdot \frac{Y_0\left(t + \frac{\varphi}{2}\right)}{\varphi}$$

$$(4.6) \quad Y_1(t) = \text{const.} \cdot \varphi \cdot y_1\left(t - \frac{\varphi}{2}\right), \text{ or } y_1(t) = \text{const.} \cdot \frac{Y_1\left(t + \frac{\varphi}{2}\right)}{\varphi}$$

$$(4.7) \quad X_1(t) = \text{const.} \cdot \theta_1 \cdot x_1\left(t - \frac{\theta_1}{2}\right), \text{ or } x_1(t) = \text{const.} \cdot \frac{X_1\left(t + \frac{\theta_1}{2}\right)}{\theta_1}$$

that is, the stock terms will be proportional to the corresponding flow terms half the period before the point of observation. These simplified formulae have been used in our numerical work below. The series of $y_0(t)$, $y_1(t)$ and $x_1(t)$ given in the compiled table 1, columns (5), (6) and (8) are calculated from these formulae by putting the term »const» equal to 1 in each case. This means a reduction in advance for deaths and accidental departure from the group. Several more complicated formulae were tried but with little significance. The use of more complicated formulae would certainly mean to stretch the exactness of the statistical data more than it can bear.

Turning now to our task of studying the characteristic lag-relations, the first problem is to find the fecundity of the sows. This depends to a considerable extent on the age of the sows, which is practically proportional to their number of earlier periods of gestation. Experience shows that the size of the litters is, within practical limits, an increasing function of the age. However, the most marked difference comes in between the *first* litter and the later ones, and since the countings only give this bipartition, it is of no use to operate with more detailed distinctions. As to the measure of the fecundity, it would seem natural to chose the number of sucking pigs. Since, however, this group has a very high and rapidly falling death rate, and the real interest centers around the part which is capable of living, it is more practical to take the next group, i. e. weaned pigs below 35 kg. Putting the relation in a linear form, we should then have:

$$(4.8) \quad x_1(t + \varphi + \theta_0) = ay_0(t) + by_1(t)$$

where a and b are constants to be determined by statistics. By the formulae (4.5) to (4.7) this equation may be expressed in terms of the stock variables. Indeed, we have

$$(4.9) \quad X_1 \left(t + \frac{\varphi + \theta_1}{2} + \theta_0 \right) = AY_0(t) + BY_1(t)$$

If the flow terms x_1 , y_0 and y_1 are considered as the part of the inflow to each group which actually will be present in the group on the average after the elapse of half the staying period, we may take the coefficients a and b to be

$$(4.10) \quad a = \frac{\varphi}{\theta_1} \cdot A, \quad b = \frac{\varphi}{\theta_1} \cdot B.$$

The series involved in (4.9) are given in the compiled table 1, columns (1), (2) and (3). A regression analysis on the equation (4.9), the bunch map of which is drawn in fig. 1, shows a highly significant fit. (The

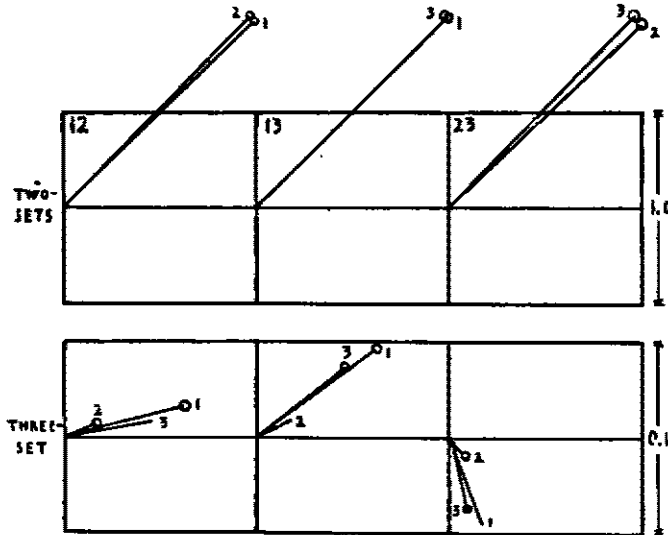


Fig. 1. Bunch map of equation (4.9). (1: $x_1(t+4.4)$, 2: $Y_0(t)$, 3: $Y_1(t)$).

matrix of *origin*-correlations was: $r_{12} = 0.981$, $r_{13} = 0.996$, $r_{23} = 0.968$ and the origin square moments (divided by 36): $M_{11} = 661427.$, $M_{22} = 6738.$, $M_{33} = 29529.$). Taking elementary regression coefficients¹⁾ and introducing our values of φ and the θ 's, (4.9) takes the form

$$(4.11) \quad X_1(t+4.4) = 2.61 Y_0(t) + 3.50 Y_1(t)$$

Using (4.10) we obtain, as an estimate of (4.8), the equation

¹⁾ See Frisch: l. c. p. 65.

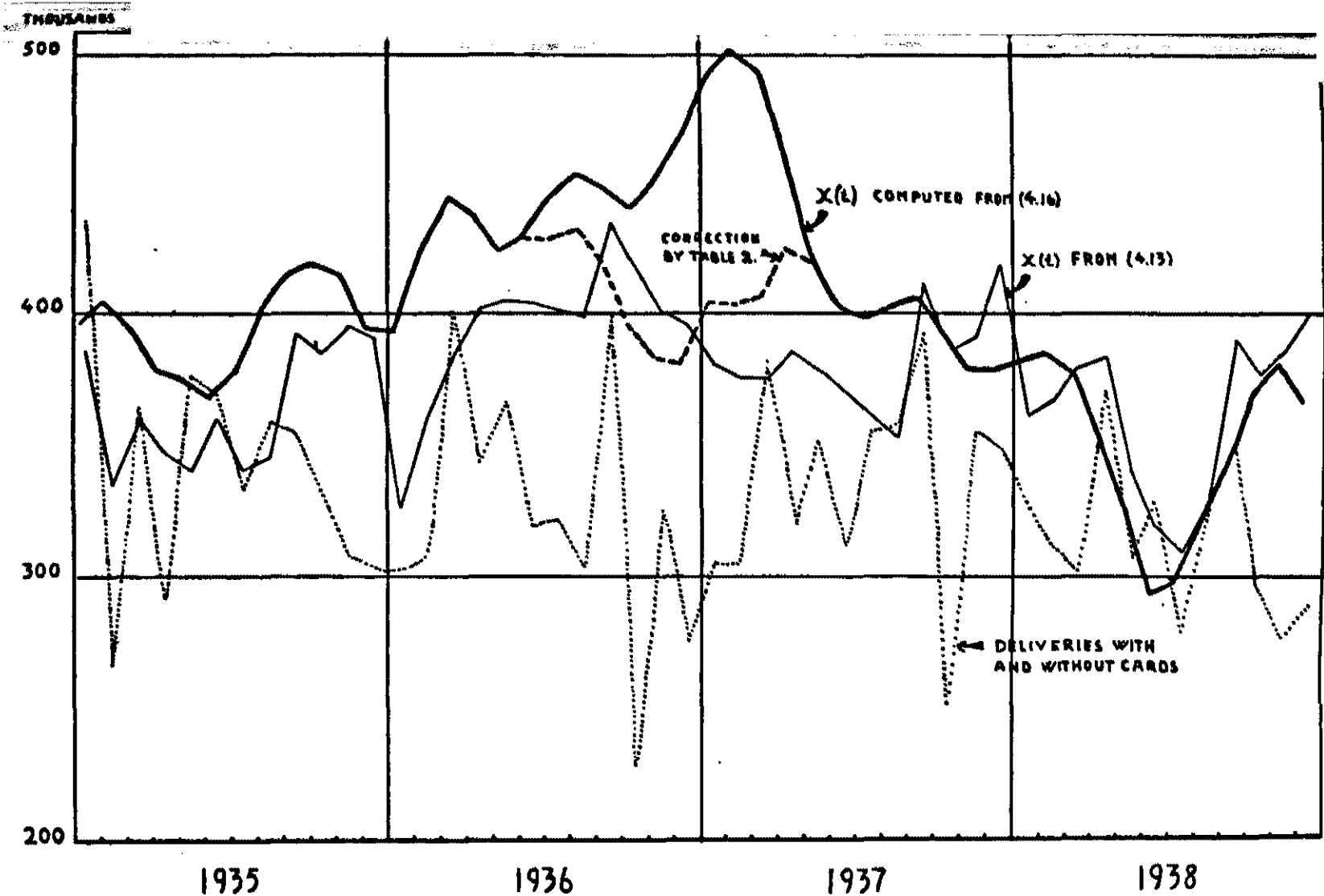


Fig. 3. Monthly outflow of finished slaughterhogs (equations (4.13) and (4.16)) and deliveries to slaughterhouses.

extraordinary slaughtering of pigs below 50 kg living weight¹⁾, in order to reduce the stock of pigs. The series of $x(t)$ given in table 1, column (10), is that calculated from formula (4.14) *reduced* for the slaughtering of small pigs.

Statistics of the extraordinary slaughtering of small pigs below 50 kg living weight are given by the cooperating slaughter houses²⁾. Most of these pigs have been near to 50 kg and would thus have been finished slaughter pigs about 2 months later. Deaths during these 2 months may be neglected. Under these assumptions the corresponding reductions in the number of finished pigs will be those given in table 2.

Table 2.

| t | Reduction of $x(t)$ because of slaughtering of small pigs below 50 kg living weight (in thousands) |
|----------------|--|
| 1936..... July | 16 |
| Aug. | 21 |
| Sept. | 31 |
| Oct. | 45 |
| Nov. | 67 |
| Dec. | 88 |
| 1937..... Jan. | 78 |
| Febr. | 98 |
| March | 91 |
| April | 30 |

These figures subtracted from the series calculated by (4.14) give the series in table 1, column (10). The broken line in fig. 3 shows the effect of this reduction.

Writing the right side of (4.16) in the corresponding stock terms we obtain the elementary forecast formula

$$(4.17) \quad x(t+8.18) = 1.33 Y_0(t) + 1.78 Y_1(t)$$

This formula which by our method gives the same curve as that calculated from (4.16) (fig. 3, thick line), permits a direct forecast of the outflow of finished pigs (per month) at the point of time $(t+8.18)$ months when t is the date of a pig census.

The forecast formulae above give the total number of pigs which will *regularly* be finished at the end of a *normal* feeding period. They do not of course give the actual slaughterings at the slaughter houses.

¹⁾ In the period mentioned such pigs were accepted without cards to ordinary market prices.

²⁾ »Regnskaber for Andelsslagteriernes Fælleskontor«, 1936—37, statistical part.

This last series will depend almost entirely on the number of cards, at least when there are more pigs than cards. The slaughterings for sale without cards are usually small, the surplus being mostly absorbed by producers' home consumption, and adding of new teeming sows. There is also the possibility of stretching or shortening the feeding period. Therefore, a forecast of the number of slaughterings in the slaughter houses is not very interesting¹⁾. Indeed, if the future number of cards is given, the main determinant of the future slaughterings at slaughter houses (pigs delivered with cards) is already known, at least when the changes in the number of cards are so small as they have actually been during the period of the regulation policy. The main interest really centers around the number of pigs *available*, because this is a means of judging the effect of the regulation policy. The total monthly deliveries of ordinary slaughter pigs with and without cards is given by the dotted curve in fig. 3. This is the series (11) in table 1 plus pigs delivered without cards as given by the statistics of the slaughter houses²⁾. These last ones amount to only about 1 or 2% of the deliveries.

5. *The equations of the »production starting«.*

The starting of the production process is the mating of the sows, and the number of teeming sows added per month is a measure of the rise or fall in the planned production activity. Our next task will be to study the factors affecting the intensity of the production starting. Such a study is highly important because the production starting will rule the whole carry-on-activity as is seen from the characteristic lag-relations studied in the foregoing section.

It is not difficult to find factors affecting the production starting, the difficulty lies in picking out the important ones. *Apriori* at least the following factors would claim to be taken account of:

1. The price of small living pigs. This is an indicator of the profit situation as to the first step in the production process and may be supposed to be that price element, which has the closest contact with the production of small pigs.

2. The indicators of movements in the future market situation of finished pigs, for instance the present stock of teeming sows, certain information about the future export- and price-policy, etc.

3. The stock of old sows.

4. The number of finished pigs available for production starting, which again will depend on

¹⁾ Such a forecast is published after each pig counting by the »Landbrugsraadets«, see for example »Landbrugsraadets Meddelelser« 1935, p. 1330.

²⁾ See »Regnskaber for Andelsslagteriernes Fælleskontor«, 1935—38, statistical part.

5. the number of pig-cards and thereby the profit of selling slaughter pigs instead of taking them for breeding.

6. The price of sow-pork. If this is high and expected to hold, and the instantaneous profit situation as to ordinary slaughter pigs is unfavourable, it may for *that* reason pay to add new sows. Indeed, there have not been restrictions on the sow-pork trade in the same way as for ordinary slaughter hogs.

7. The production costs.

8. All these factors must be assumed to have different effects on the two categories of sows.

In spite of these complications we have found it safe and reasonable to represent the production starting by the following two very simple relations

$$\left. \begin{aligned} (5.1) \quad y_0(t) &= c_1(x(t) - \hat{x}(t)) + c_2s(t) + c_0 \\ (5.2) \quad y_1(t) &= k_1y(t - \alpha) + k_2s(t) + k_0 \end{aligned} \right\} \text{The equations of the production starting.}$$

where the c 's, the k 's and α are certain constants to be described and determined below. But this is not, as might be thought, an arbitrary choice. It is, indeed, the final result of a comprehensive statistical testing process where almost every reasonable possibility of including other factors in a linear way has been tried. In all cases, except those represented by (5.1) and (5.2) above, the factors included had little or no significance at all, apart from the price of small pigs (s) which came out as essentially important in every case. The two equations adopted may be given a very reasonable interpretation.

Firstly, the price of small pigs very naturally comes in as an essential barometer of profit possibilities for both kinds of production starting. It may be taken as a catch-all for the whole set of expected profit elements.

Secondly, as might be expected, the other influences are of different kinds in the two relations. In the first equation comes in the difference between the number of finished pigs and the number of pigs delivered with cards. This (or more correctly about half of it) indicates the number of pigs available for breeding when assuming that the utilization of pig-cards will be stretched to the highest possible limit. On the other hand a large $(x(t) - \hat{x}(t))$ indicates bad market possibilities which may have a braking effect on the production starting. $(x(t) - \hat{x}(t))$ may be taken as a catch-all for these and similar effects. The first member to the right in (5.2) is simply the part of the total production starting (y) coming back after the weaning of the small pigs. The lag α is estimated to 6 months by the following considerations: The gestation period is 3.75 months, and the sucking period about 1.64 months, giving a total

of about 5.4 months. Taking account of the time elapsing between the weaning of the pigs and the next pregnancy, the estimate of $\alpha = 6$ months seems reasonable. The percentage coming back is of course not actually a constant. Firstly it surely depends on the age distribution of the sows, and secondly it also might be expected to vary considerably

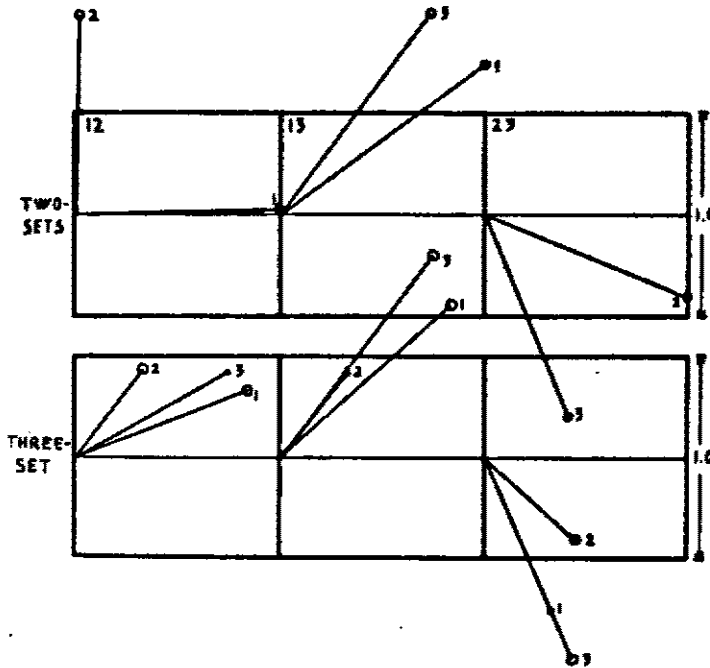


Fig. 4a. Bunch map of equation (5.1). (1: $y_0(t)$, 2: $(x(t) - \bar{x}(t))$, 3: $s(t)$).

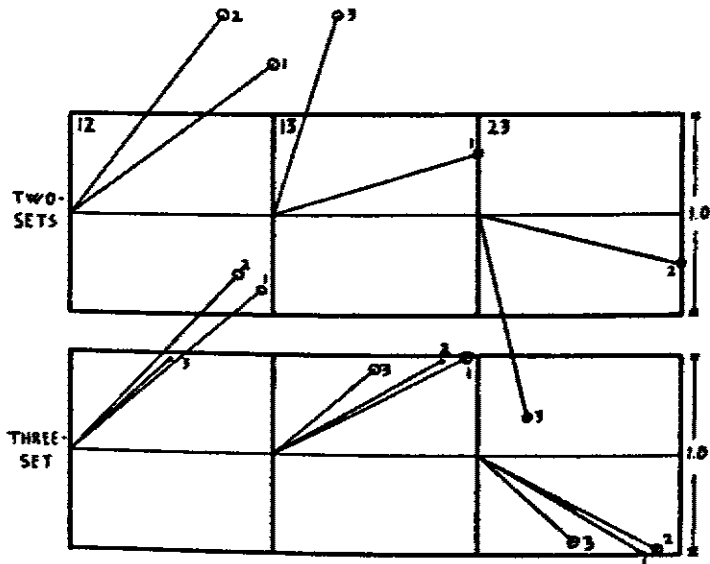


Fig. 4b. Bunch map of equation (5.2). (1: $y_1(t)$, 2: $y(t-6)$, 3: $s(t)$).

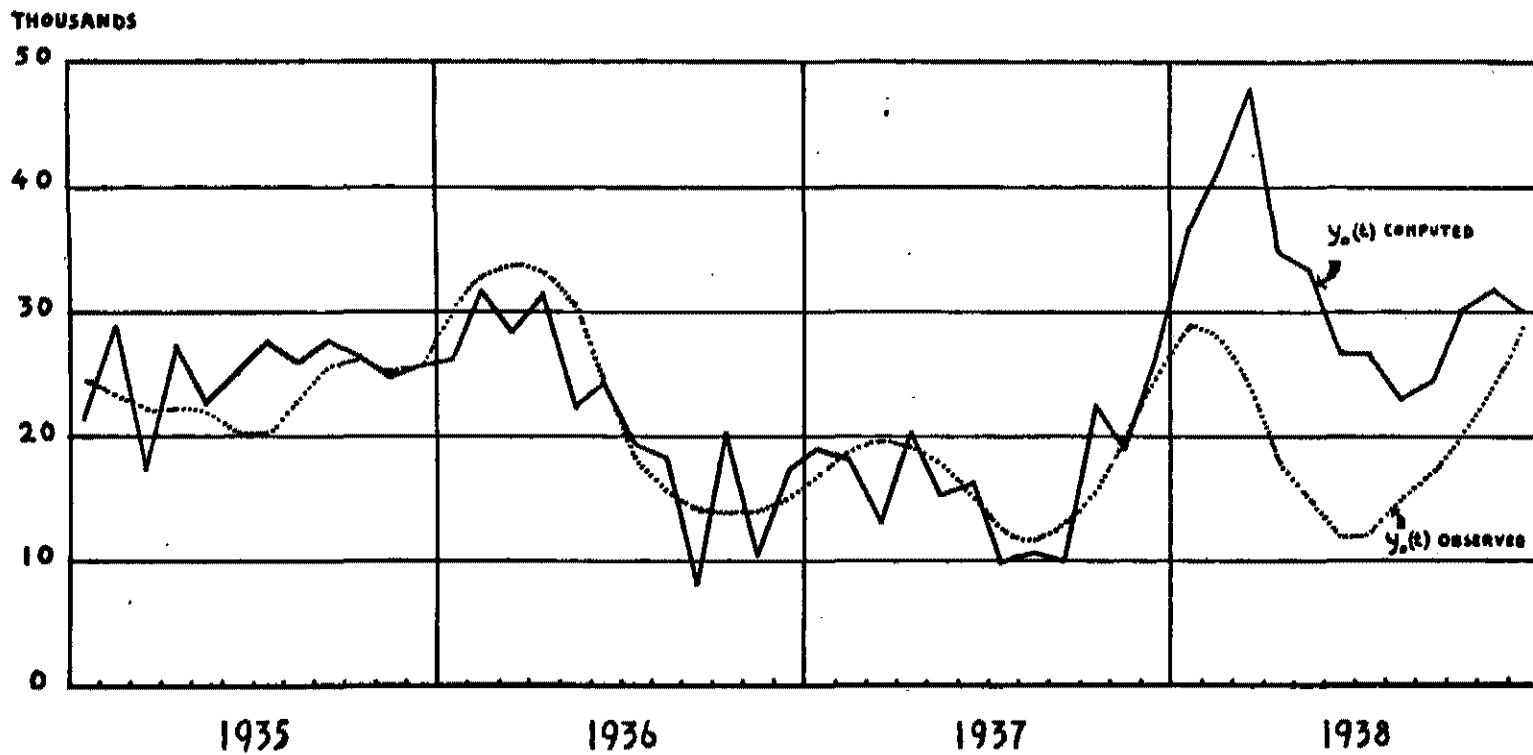


Fig. 5a. The inflow of new breeding sows per month (equation (5.8)).

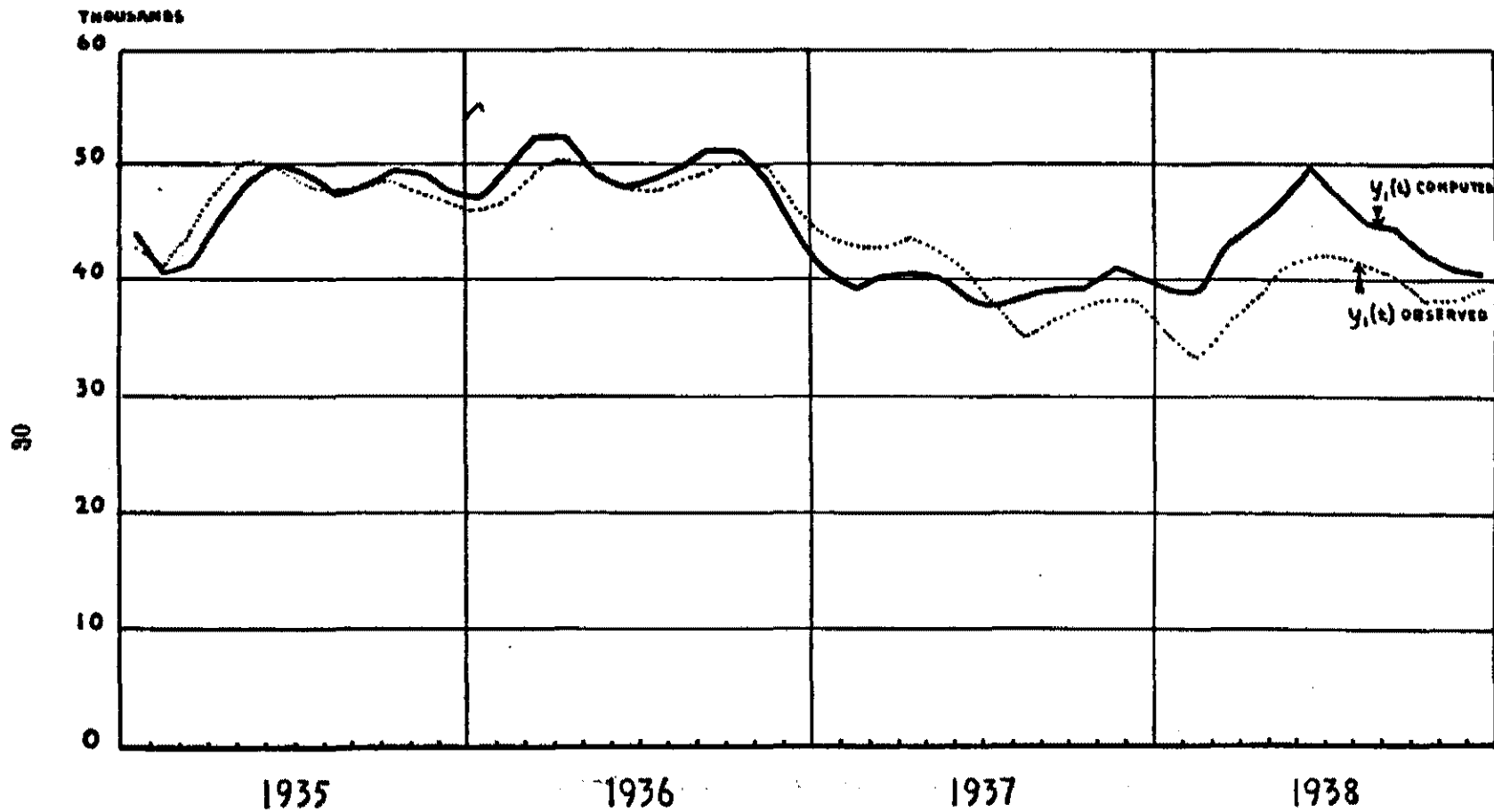


Fig. 5b. The inflow of old breeding sows per month (equation (5.4)).

with the favourable or unfavourable profit situations. But our statistical trials gave no significant results as to this specification.

The necessity of including a constant term in each equation is obvious.

The statistical series involved in the equations (5.1) and (5.2) are given in table 1, columns (5), (6), (7) and (13). By regression analysis, the bunch maps of which are shown in fig. 4a and fig. 4b, we get the following regression equations (taking diagonal regression coefficients)¹⁾

$$(5.3) \quad y_0(t) = 0.10 (x(t) - \bar{x}(t)) + 0.87 s(t) - 3.2$$

$(r_{11}=0.023, r_{12}=0.744, r_{22}=-0.410; \sigma_1=6.09, \sigma_2=44.7, \sigma_3=7.64)$

$$(5.4) \quad y_1(t) = 0.62 \cdot y(t-6) + 0.39 s(t) - 4.6$$

$(r_{11}=0.752, r_{12}=0.312, r_{22}=-0.239; \sigma_1=4.44, \sigma_2=7.04, \sigma_3=7.64)$

An inspection of the bunch maps shows that the statistical significance of these results is clear.

A comparison between the observed and calculated values of $y_0(t)$ and $y_1(t)$ is shown in fig. 5a and 5b.

As might be expected, the inflow of new sows is much more sensitive to variations in the prices of small pigs than is the inflow of old sows. Indeed, one »Krone« added to the price s (the other terms being constant) would raise y_0 by 0.87 thousands, but y_1 only by 0.39 thousands. The equation (5.3) shows that, s being fixed, about 10% of a change in the surplus $(x - \bar{x})$ are made sows. The first term to the right in (5.4) is the element of »inertia« in the production starting. It is seen that — s being constant — about 60% of a previous increase in the inflow of sows will be »reinvested«.

6. The Market Price of Small Pigs.

From the view point of the individual producer the small (weaned) pigs are an intermediate product, and it has its own market. At each point of time the total number available of pigs at a certain age is *given* (neglecting import possibilities), and so the whole market business of small pigs is only a matter of changing ownership for a greater or smaller part of a fixed quantity. And there is really no characteristic difference between the supply side and the demand side. In the case of an *un*-regulated pig production the supply situation for small pigs will be as follows: The producers of small pigs may either feed them to finished pigs or sell them in the small-pig-market. If the price of small pigs is so high that it covers the difference between the price of finished pigs and the total costs in the rest of the feeding period, the question of

¹⁾ Frisch: l.c.p. 108—109.

selling or keeping small pigs is a matter of indifference. The reasoning on the demand side runs in the same way. If the cost per unit (i. e. per finished pig) were constant and the same for all producers, the number of small pigs sold and bought would be fixed only through speculation activity or pure chance. If the producers have different, but *given technical cost functions*, the supply and the demand of small pigs will be functions of their market price, the expected price of finished pigs and the prices of feeding stuff. For short periods the assumption of a *given distribution of fixed technical cost functions* is sufficiently realistic. The result of these two functions, the demand and the supply relation, will then be a *confluent market relation* of the form

$$(6.1) \quad \text{price of small pigs} = f(\text{price of finished pigs, cost}).$$

Let us now see how this connection is to be formulated for the Danish market. The total flow of »small pigs« may here be taken to be the flow term $x_1(t)$. Since the price of finished slaughter pigs delivered with cards is a monopoly price, it is obvious that all producers of small pigs will at least retain a number corresponding to the allotted number of cards, as long as the market price of small pigs does not cover the whole monopoly profit. And if, as a whole, the number of small pigs is less than the number of cards available at the end of the feeding period, the price of small pigs will actually cover nearly the whole monopoly profit, since in such cases the pig cards will change hands at almost zero prices, and the total turn over of small pigs will depend on the differences in the individual technical cost functions.

On the other hand, if there is an abundant number of pigs as compared with the number of cards available, the risk of paying a high card price at the end of the production period will press the price of small pigs downwards, and the whole loss will fall on the producers having a superfluous number of small pigs. The difference between free market and the Danish market will be a change in the meaning of the term »price of finished pigs« in (6.1). Indeed, we will have to replace it by the expression $(p_1(t) - v(t))$, where p_1 is the standard price of slaughter pigs delivered with cards (price per animal) and $v(t)$ is the price of pig-cards (see section 3). Actually the price $(p_1 - v)$ should of course be considered as a forecast price. However, all experiences show that the reasoning of the producers runs mostly in the instantaneous prices at the marketing of small pigs. Taking the relationship to be approximately linear, we may then write:

$$(6.2) \quad s(t) = h_1(p_1(t) - v(t)) + h_2 \cdot (\text{feeding costs}) + h_3$$

where $s(t)$ is the market price of small pigs, and the h 's are constants to be estimated by statistics.

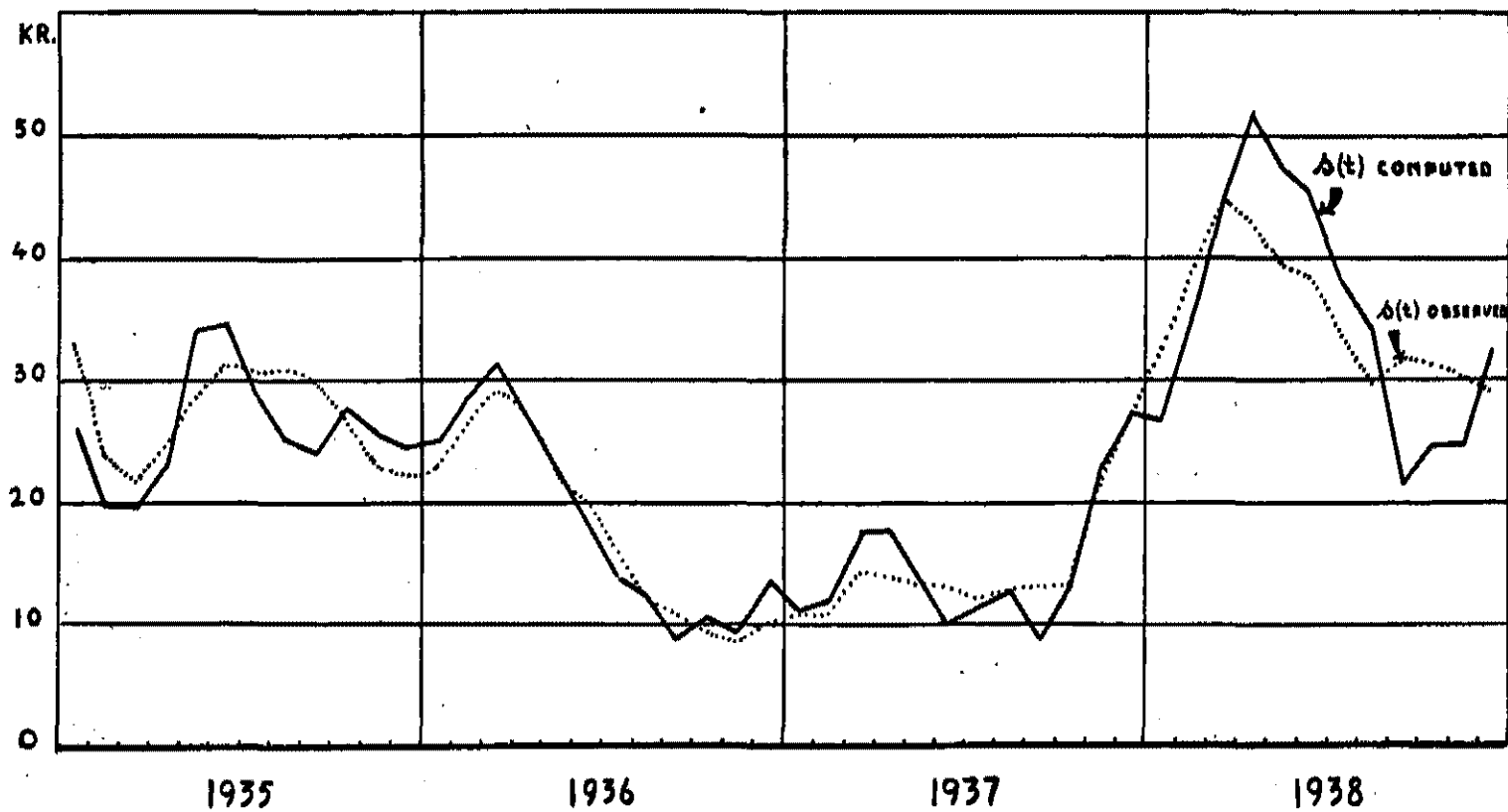


Fig. 6. The price of small weaned pigs below 85 kg (equation 6.4).

It appeared, however, to be actually impossible to get any statistical information about the cost term. Attempts were made to include prices of important feeding stuffs as a variable, but it came out entirely insignificant. The statistical explanation of this is that the feeding stuff prices have been varying slowly, and this variation has been almost exactly linearly connected with the slowly varying term p_1 , and in the expression (p_1-v) the variations in v are far the most important. Therefore, the inclusion of the feeding stuff prices could add no new statistical information. If the feeding stuff prices should happen to move significantly in other directions, it would probably be possible to obtain a statistical measure of its effect. The result of such variations would probably only be a correction of s corresponding to the difference in cost-level as compared with that covered by the statistics in our chosen testing period.

Accordingly, we will have to adopt the more simple equation

$$(6.3) \quad s(t) = h_1 (p_1(t) - v(t)) + h_0$$

The statistical series involved are given in table 1, columns (13), (14), (16). The regression analysis of (6.3) gives an almost surprising close connection. The correlation coefficient between the two variates s and (p_1-v) came out to $r_{12} = 0.93$ ($\sigma_1 = 7.64$, $\sigma_2 = 11.74$). Taking the diagonal regression coefficient, we obtain the following estimate of the small pig prices:

$$(6.4) \quad s(t) = 0.65 (p_1(t) - v(t)) - 15.80$$

This means that each »Krone« added to the price $(p_1(t)-v(t))$ would raise the prices of small pigs by Kr. 0.65. The meaning of the constant term is that if the price $(p_1(t)-v(t))$ is not higher than Kr. 24.30 ($0.65 \cdot 24.30 = 15.80$) the price of small pigs would be zero. But it is of course doubtful to draw such extreme conclusions since situations of this kind are not covered by statistics.

The correspondance between the observed and the computed values of $s(t)$ is shown in fig. 6.

7. The Demand for Pig-cards.

The foregoing discussions show that the price of pig-cards will influence the motion of the whole system. Indeed, the price of pig-cards is the chief determinant of price of small pigs, which again is the main factor determining the production starting (compare equations (5.1), (5.2) and (6.3)). Therefore, the determination of the pig-card price is a highly important problem. But it is also a very complicated problem, as will be seen from the discussion below.

We first notice that the price of pig-cards, whatever else may happen to it, must lie between two well-defined limits. Indeed if p_1 is the price (per animal) of pigs delivered *with* cards, p_2 the corresponding price of pigs delivered *without* cards and v the pig-card price, we must have

$$(7.1) \quad 0 \leq v \leq (p_1 - p_2)$$

Of course nobody buys cards when he will lose less by selling his pigs without cards. On the other hand he is willing to pay any price within the limits (7.1) instead of selling without cards. Now pig-cards are »perishable goods« as they are only valid within fixed time intervals. Therefore, all farmers having more cards than pigs which they wish to sell will either sell the superfluous cards or buy half-finished pigs; and correspondingly, farmers having less cards than pigs which they wish to sell will either buy cards or sell half finished pigs (or finished pigs without cards). In a situation where there are as a whole more pigs to be sold than the number of cards available, the owners of superfluous cards will take the whole profit $(p_1 - p_2)$ from the card buyers or sellers of half-finished pigs. Then there is competition between card buyers, pushing the card price near to maximum. On the other hand, if the number of cards is abundant, there is competition between card sellers (but not between card buyers), pushing the card price down nearly to zero. Then the whole profit $(p_1 - p_2)$ goes to card buyers or sellers of half-finished pigs.

Let x' be the total number of pigs to be sold in a given period and x^* the total number of cards available. The theoretical connection between the difference $(x' - x^*)$ and v would then be as illustrated by the thick line in fig. 7.

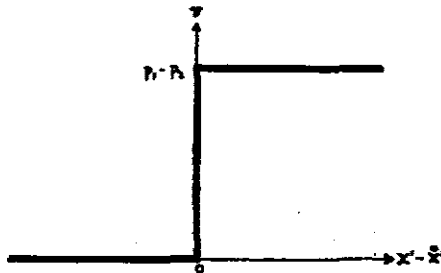


Fig. 7.

This is the theoretical market demand function of pig-cards, and it is also the market supply function. The *number* of cards sold and bought is given by the total number of pigs to be sold and the total number and distribution of cards.

But this simple connection is far from being that actually observed, as will be seen below. Indeed, we may already apriori add a great number of modifications.

Firstly, the farmers in lack of cards have the possibility of using the superfluous pigs for home consumption. The higher the card price (with a fixed p_1) the lower will be the price of home consumption (i. e. the money the farmer loses by not selling). On farms with a great household the possibility of home consumption represents a rather elastic element.

Secondly, there is some possibility of exporting great living pigs to Germany, and those deliveries do not require cards. However, this quantity has been rather constant (about 150 thousand a year) and can not help very much in a precarious situation.

Thirdly, some of the superfluous pigs may be used for breeding as already discussed in section 5.

And, lastly, there is the possibility of a stretching or shortening of the feeding period. This may indeed have an important effect upon the card price, even by a slight stretching or shortening, which may be illustrated by the following theoretical considerations:

Suppose that the number of finished pigs (per month) which are to be sold is constant from a certain point of time t_0 and equal to a constant number of pig-cards (per month). Then it would be possible to buy pig-cards for almost nothing. But suppose now that there had been a lack of pig-cards in the month just before t_0 . In that month the price of cards would then be almost at maximum. Then it is likely — and it is actually what happens — that some of the superfluous pigs would be held back for the next month, their weight being kept below the ideal weight by means of a »subsistence minimum« rate of feeding. In our case the result will of course be a certain number of superfluous pigs also in the month t_0 to t_0+1 . Accordingly, a certain number of new pigs which actually should have been slaughtered in this month will be held back for the next month (t_0+1) to (t_0+2) and so on. This theoretical case is illustrated in fig. 8.

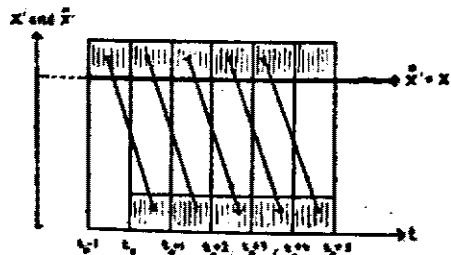


Fig. 8.

This illustrates what may be termed the *queue-phenomenon* in its pure and most simple form. Suppose, next, that the flow of cards is constant while the regular flow of finished pigs is rising or falling from the point of time t . Then we have the situations illustrated in fig. 9 and fig. 10 respectively.

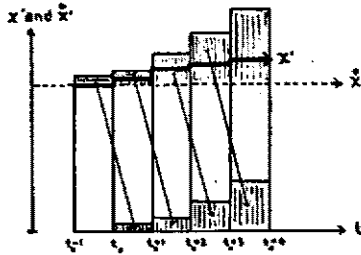


Fig. 9.

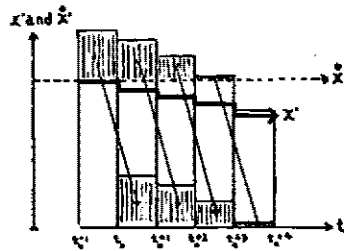


Fig. 10.

In the first case the queue is increasing as long as the regular flow of finished pigs increases, i. e. the average age of pigs ready for sale in each month is rising and will continue to rise until either the number of cards is increased to an amount equal to or greater than the regular flow of finished pigs, or a sufficiently great number is slaughtered without cards. As long as the number of pigs to be sold surpasses the number of cards, the card price will be near to maximum. In the second case (fig. 10) the card price would keep near maximum until the month (t_0+3) to (t_0+4) and then fall, more or less rapidly, down to almost zero. As long as this situation holds, there will, in opposition to the case above, be a tendency to intensify the feeding to utilize the favourable profit situation, so that the average age of pigs slaughtered would decrease (within certain possible technical limits). It must be observed that even very small changes in the feeding period is sufficient to give the effects discussed above, so that these schemes are not in contradiction to the assumption of an average constant feeding period.

In addition to these disturbances come a lot of other irregular influences such as: 1) a prolongation of the period of validity of a card emission, 2) public information as to the future market possibilities and probable increase or decrease in the future number of cards, 3) publication of statistics from pig countings, 4) possible alterations in the regulation laws, and many other things.

In our preliminary work a large number of hypotheses were tested in trying to isolate the effects of these several factors. Regression analyses were tried in all directions and with several different types of mathematical specification. But no simple functional connections could

be found of statistical significance. The only positive result was that very great surpluses of pigs mean almost maximal card prices, while very great surpluses of cards mean almost zero card prices. Indeed, the card price seems to be a true *shock variate*, being very sensible to all events having any connection with the economic field we are studying.

This being so we have found it plausible and more realistic to choose the following type of solution:

In our table 1, column (12), are given the differences $(x(t) - \hat{x}(t))$ between the number of finished pigs and the number of pigs delivered with cards (which in the period studied was practically equal to the number of cards, as in these years practically every card was used). The corresponding values of the ratio

$$(7.2) \quad \frac{v(t)}{P_1(t) - P_2(t)} \quad (\text{The relative card price})$$

are given in table 1, column (17).

As all these observations belong to a period where all cards were used, it is found necessary to add some supplementary data from a period with a superfluous number of cards. The only period giving significant information about such cases is the 5 months July-Nov. of 1934. The supplementary data for this period are given in table 3 (the sources are the same as for the rest of the series).

Table 3.

| t | $(x(t) - \hat{x}(t))$ (000) | $\frac{v(t)}{P_1(t) - P_2(t)}$ |
|--------------|--------------------------------|--------------------------------|
| | | |
| 1934... July | -21 | 0.11 |
| Aug. | -56 | 0.11 |
| Sept. | 1 | 0.19 |
| Oct. | -35 | 0.15 |
| Nov. | 3 | 0.31 |
| Total... | -108 | - |

Plotting the series $(x - \hat{x})$ and $\frac{v}{P_1 - P_2}$ against each other, we obtain the spot diagram given in fig. 11 (the small crosses are the data for 1938).

It is seen that the difference $(x(t) - \hat{x}(t))$ is sometimes negative. This would of course be impossible if $x(t)$ were exactly the number of finished pigs available per month at t . But according to our definition above, $x(t)$ is the outflow of finished pigs corresponding to a normal feeding

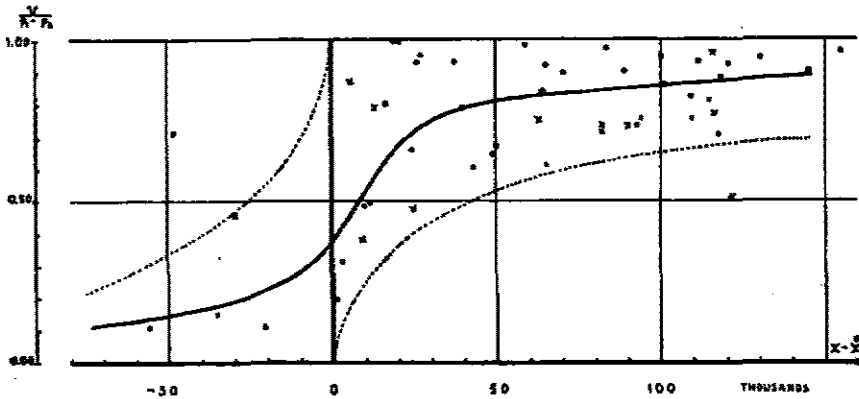


Fig. 11. The demand for pig cards.

period. The difference between $x(t)$ and the flow of finished pigs actually observed is caused partly by random variations and partly by a stretching or shortening of the feeding period as discussed above. This introduces an element of elasticity which makes it quite possible that the relative card price may be above zero also to the left of the origin in fig. 11. The S-shaped curve (drawn in free-hand) in fig. 11 indicates a sort of *average* demand function,

$$(7.3) \quad \frac{v(t)}{p_1(t) - p_2(t)} = f(x(t) - \bar{x}(t))$$

The free-hand fitting is, I think, the best proceeding in this case. Trials were indeed made of fitting different types of mathematical functions, but, at least to my eye, they gave less plausible results than the curve drawn in fig. 11. In any case, a curve-fitting to this material will be rather arbitrary. I think the best solution is to consider the relationship as random variation within a certain random region, such as indicated by the two dotted curves in fig. 11. (The two points not included are results of quite exceptional shiftings from one month to the next (December-January)). The vertical breadth of the region may be taken to indicate the normal *shock intensity* of the relative card price, being greatest near to origin and diminishing constantly to the left and to the right.

It must be observed that the shape of the connection will depend on the *time unit chosen*. The connection indicated in fig. 11 is only valid for *monthly data*. If longer periods are chosen, the connection would probably come nearer to the form indicated in fig. 7. Of course the card price can not be significantly above zero when there is a constant deficit of finished pigs; and the card price would, for a longer period, even have to be zero until some positive value of $(x - \bar{x})$, as new sows are needed for the regular maintenance of the production. Due to the

imperfection of the pig card market, the relative card price will usually not reach exactly the upper or lower bounds except for very great absolute values of the difference $(x - \bar{x})$.

Let us now see how the number of cards available affects the card price. From fig. 11 it is seen that the card price may be above zero also at some distance to the left of $(x - \bar{x}) = 0$. There is therefore the possibility of all cards being used also when the difference $(x - \bar{x})$ is temporarily negative. This must be assumed to be the case as long as the market price of cards is significantly above zero. If the flow of cards available per month is denoted by \bar{x}' (statistics of this series are not obtainable), we will have

$$(7.4) \quad \bar{x} \sim \bar{x}' \text{ when } (x - \bar{x}) > \delta$$

where δ is some negative figure.

If the difference $(x - \bar{x}')$ (difference between regular flow of finished pigs and the flow of cards) is constantly negative for a longer period, the amount of deliveries \bar{x} will depend on x . But sufficient statistics are not available to estimate the shape of this relation. Some information is given through the 1934-data. During the months included in table 3 there were about 130 thousand unused cards. Now the average value of the difference $(x - \bar{x})$ for the whole period 1935—38 was about 65 thousands per month or about 325 thousands for 5 months. The corresponding actual difference for the period July-Nov. 1934 was -108 (see table 3), or about $325 + 108 = 433$ thousands below the average. Adding to this figure the number of unused cards, we get a «normal» deficit equal to 563. Of this $433/563$ or about 77 % is covered by shortening the feeding period and only 23 % by unused cards. From this follows that the amount of deliveries with cards will be closely connected with the number of cards available as long as the deficit of finished pigs is moderate. Thus the number of slaughterings will regularly be known when the number of cards available is given (compare the remarks on the forecast problems in section 4).

II. THE SIMULTANEOUS SYSTEM OF EQUATIONS AND SOME OF ITS CONFLUENCE EFFECTS

8. *The system of fundamental relations and its degrees of freedom.*

Collecting together the different conditions laid upon the variates studied, we obtain the following simultaneous system of equations:

$$\begin{aligned}
(8.1) \quad x_1(t + \varphi + \theta_0) &= ay_0(t) + by_1(t) && \{\text{Eq. (4.8)}\} \\
(8.2) \quad x(t) &= mx_1(t - \theta_1 - \theta_2 - \theta_3) && \{\text{Eq. (4.14)}\} \\
(8.3) \quad y_0(t) &= c_1(x(t) - \dot{x}(t)) + c_2s(t) + c_0 && \{\text{Eq. (5.1)}\} \\
(8.4) \quad y_1(t) &= k_1y(t - \alpha) + k_2s(t) + k_3 && \{\text{Eq. (5.2)}\} \\
(8.5) \quad s(t) &= h_1(p_1(t) - v(t)) + h_0 && \{\text{Eq. (6.3)}\} \\
(8.6) \quad \frac{v(t)}{p_1(t) - p_2(t)} &= f(x(t) - \dot{x}(t)) && \{\text{Eq. (7.3)}\}
\end{aligned}$$

To this system we should add the identity

$$(8.7) \quad y(t) = y_0(t) + y_1(t)$$

Inspecting this system we see that there are in all 10 unknown variates, viz.

$$y_0, y_1, y; x_1, x, \dot{x}; s; p_1, p_2, v.$$

There are 7 equations, accordingly the system has still 3 degrees of freedom. It is obvious that we cannot make the system determinate without grasping outside the special field we are studying. Indeed, the three variates: p_1 (the maximal price of »card-pigs«), p_2 (the price of pigs without cards) and \dot{x} (the number of pigs delivered with cards) are essentially affected by things outside our system, such as the international export conditions, and special aims of the whole regulation policy. These three variates may be taken as data in the present analysis, and the main problem is to study the effects which their variations or fixing have upon the *internal* variates of the system.

9. Some approximate information about the natural tendencies of the system.

Even if the three variates p_1 , p_2 and \dot{x} are given as constants or simple time functions, it is not possible to give explicite solutions of the system, because of the complicated pig-card demand function. But some approximate information may be obtained through numerical step-by-step calculation. This is however a rather tedious job.

As an example we shall consider the series $x_1(t)$ (small weaned pigs below 35 kg). From the four equations (8.1), (8.3), (8.4) and (8.7) we get, by an elimination process:

$$(9.1) \quad x_1(t + \varphi + \theta_0) = k_1 x_1(t + \varphi + \theta_0 - \alpha) + a c_1 (x(t) - \hat{x}(t)) \\ + (c_1 b k_1 - a c_1 k_1) (x(t - \alpha) - \hat{x}(t - \alpha)) + (b k_2 + a c_0) s(t) \\ + (b c_2 k_1 - a c_2 k_1) s(t - \alpha) + (a c_0 + b k_0 + c_0 b k_1 - a c_0 k_1)$$

Inserting here the equation (8.2) we get

$$(9.2) \quad x_1(t + \varphi + \theta_0) = k_1 x_1(t + \varphi + \theta_0 - \alpha) + a c_1 m x_1(t - \theta_1 - \theta_2 - \theta_3) \\ - a c_1 \hat{x}(t) + (c_1 b k_1 - a c_1 k_1) m x_1(t - \theta_1 - \theta_2 - \theta_3 - \alpha) \\ - (c_1 b k_1 - a c_1 k_1) \hat{x}(t - \alpha) + (b k_2 + a c_0) s(t) \\ + (b c_2 k_1 - a c_2 k_1) s(t - \alpha) + (a c_0 + b k_0 + c_0 b k_1 - a c_0 k_1)$$

Further, from (8.5) and (8.6) we get

$$(9.3) \quad s(t) = h_1 [p_1(t) - (p_1(t) - p_2(t)) f(x(t) - \hat{x}(t))] + h_0$$

By combining (8.2), (9.3) and (9.2) we see that the time expansion of x_1 is fixed when p_1 , p_2 and \hat{x} are given. For the purpose of step-by-step calculations it is most convenient to keep the two equations (9.2) and (9.3) separated. Inserting our numerical values of the coefficients in (9.2) we get

$$(9.5) \quad x_1(t + 5.4) = 0.62 x_1(t - 0.6) + 0.50 x_1(t - 4.65) - 0.52 \hat{x}(t) \\ + 0.10 x_1(t - 10.65) - 0.11 \hat{x}(t - 6) + 7.27 s(t) \\ + 0.96 s(t - 6) - 52.58.$$

To make a step-by-step calculation we need some initial conditions. Indeed, it is seen from the equation (9.5) (paying regard to (9.3) and (8.2)) that we must know — apart from \hat{x} , p_1 and p_2 — the series x_1

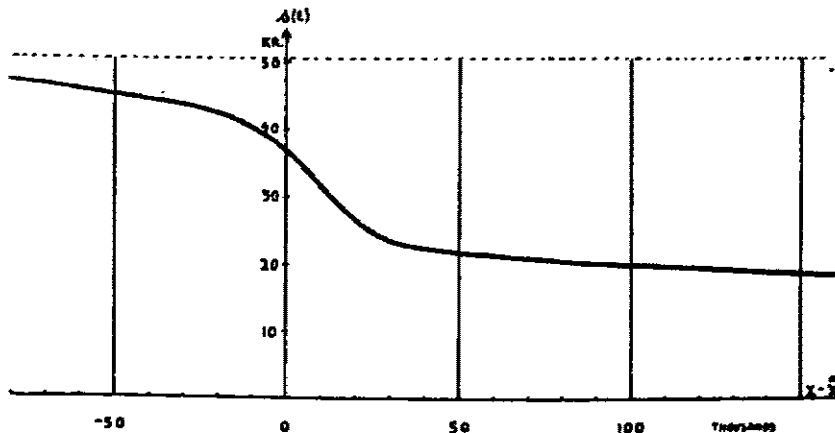


Fig. 12. The confluent relation (9.6) between $s(t)$ and the surplus $(x(t) - \hat{x}(t))$.

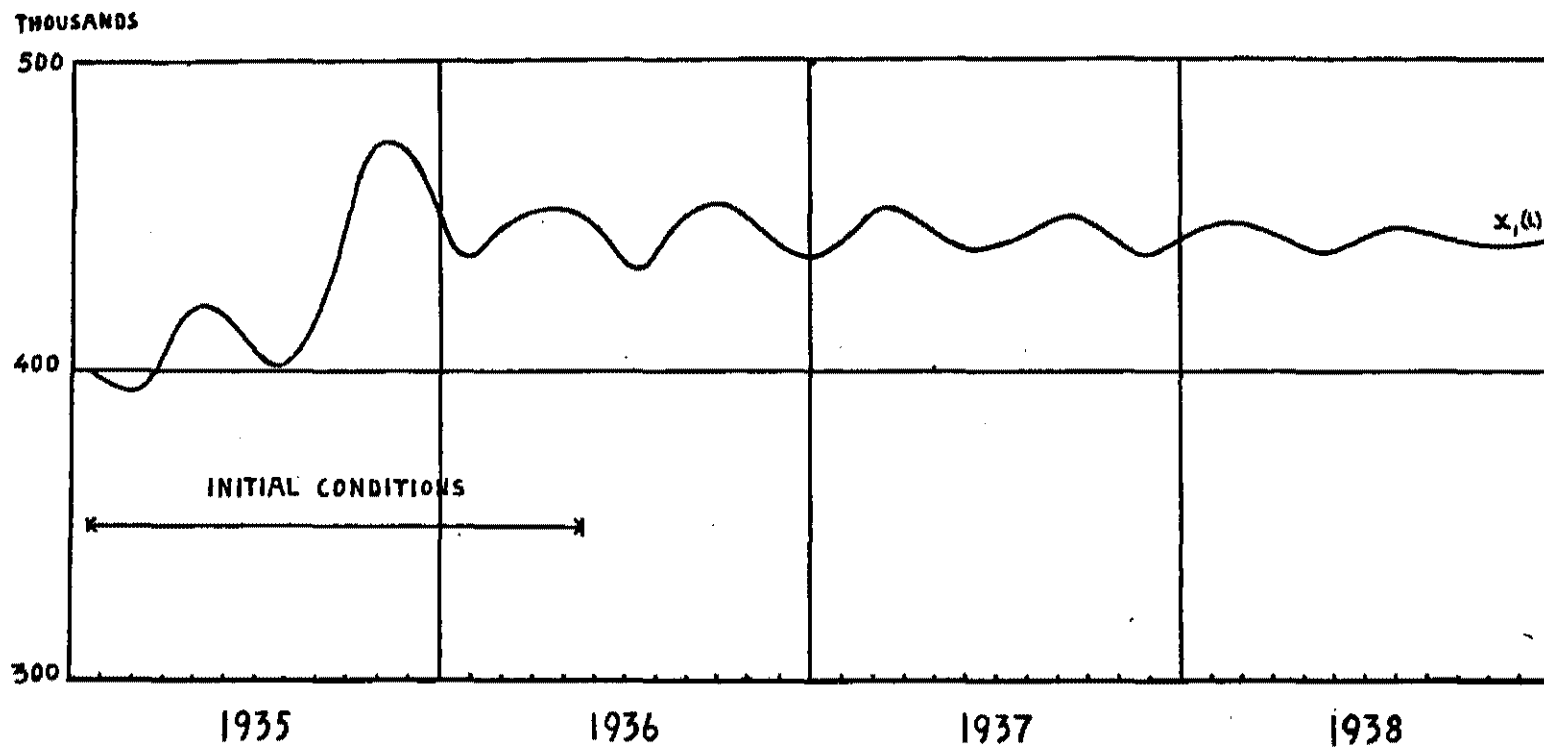


Fig. 13. Step-by-step calculation of x_1 by equation (9.7).

for about 16 months. We have chosen the period Jan. 1935—April 1936. (The series x_1 is found in table 1, column (8)). p_1 , p_2 and \bar{x} were chosen as constants equal to the average of 1935 which gives $\bar{p}_1 = 101.8$, $\bar{p}_2 = 46.40$ and $\bar{x} = 337.5$. This seems to be an interesting assumption in this case, and it is not very far from reality. Inserting \bar{p}_1 and \bar{p}_2 and the values of h_1 and h_2 in (9.3) we get

$$(9.6) \quad s(t) = -36.00 f(x(t) - \bar{x}(t)) + 50.37.$$

Reading off graphically the function $f(x - \bar{x})$ from fig. 11 in section 7, we may draw the corresponding curve of s by the formula (9.6). This curve is shown in fig. 12. Inserting $\bar{x} = 337.5$ into (9.5) we get

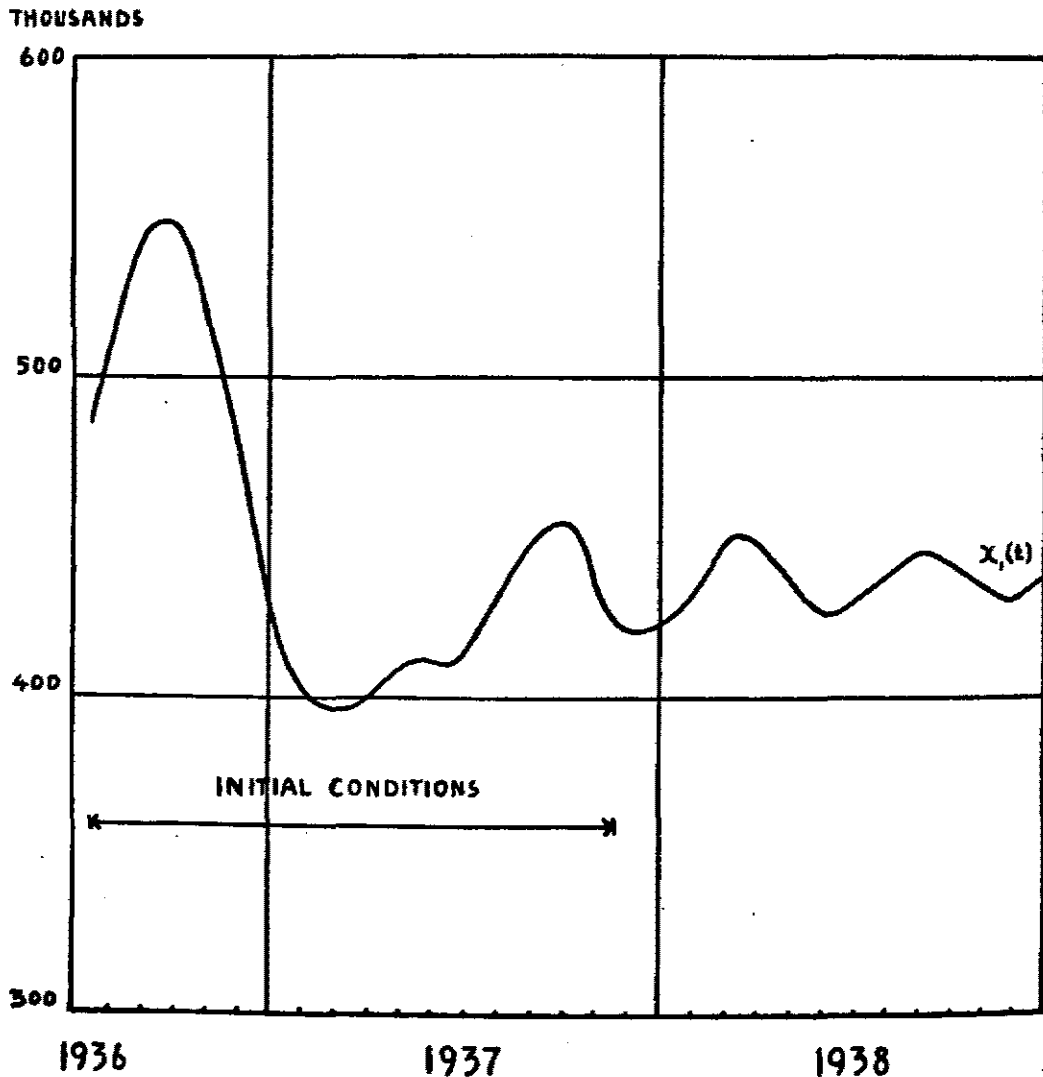


Fig. 14. Step-by-step calculation of x_1 by equation (9.7) with the constant term changed to 360.

$$(9.7) \quad x_1(t + 5.4) = 0.62 x_1(t - 0.6) + 0.50 x_1(t - 4.65) \\ + 0.10 x_1(t - 10.65) + 7.27 s(t) + 0.96 s(t - 6) - 265.$$

By this equation a step-by-step calculation was made, reading off graphically the values of $s(t)$ from fig. 12 and using (8.2). The result is shown in fig. 13.

It is seen to involve a characteristic cycle of about 5.5 months and slightly damped. This cycle may indeed be seen in nearly all the series we have discussed above (compare the different graphs given). But the observed series also show other cycles, for instance one of about 10—11 months (see for example fig. 2). This may be explained by regular or irregular influences from external elements or the autonomic elements p_1 , p_2 and \bar{x} . Left to itself — by constant p_1 , p_2 and \bar{x} as chosen — the system would expand as in fig. 13. One might perhaps think that the 5.5 months cycle in fig. 13 only is a repetition of the initial conditions chosen, because we here see the same cycle. However, this is not the case, as may be seen by choosing another set of initial conditions. In fig. 14 we have calculated part of the x_1 -series by choosing the period July 1936-October 1937 as initial conditions. \bar{p}_1 and \bar{p}_2 were put equal to the same values as above (in order to use the same graph of s) and \bar{x} was put equal to 330 which is the average of \bar{x} for 1937. Here the initial conditions show no similarity to those in fig. 13 but the natural tendency of the system is so strong that it almost immediately presses the movements into its own mould.

I do not think it is possible to give an elementary *explanation* of the 5.5 month cycle. It is a confluence effect of the whole simultaneous system of fundamental equations. Accepting this system we have, implicitly, accepted a 5.5 month cycle.

10. The effect of changes in pork prices.

It has sometimes been argued that changes in the price of pigs delivered with cards does not affect the production activity, since the price is a monopoly price, and the production, therefore, is entirely dependent on the number of cards issued. This is however a hasty conclusion, as will be seen from our system of equations. From this system it follows that the production starting is essentially dependent on the price of small pigs which in turn is closely connected with the difference $(p_1 - v)$. (Eq. (8.3), (8.4) and (8.5)). From equation (8.6) we have

$$(10.1) \quad v = (p_1 - p_2) f(x - \bar{x})$$

If the price p_2 is constant, the change in v will be equal to a change in p_1 when — and only when — the relative card price is equal to unity.

In this case the change in p_1 does not affect the price of small pigs (s) (see eq. (8.5)) and accordingly not the production starting. In all other cases with constant p_2 , the absolute value of a change in the price p_1 will be greater than the absolute value of the corresponding change in v . Then the value of s in (8.5) will change and thereby affect the production starting and the whole resulting carry-on activity.

In the same way a change in p_2 may affect the production activity. Indeed, if p_2 is raised nearer to p_1 , the card price will fall, and therefore the price of small pigs will rise. The difference between the two prices p_1 and p_2 is therefore an important tool in the whole regulation policy.

11. The sensitivity of the system to erratic shocks.

One of the main problems of artificial regulation policy is to obtain a »shock proof« system. This means that the forces acting towards the type of equilibrium in aim must be strong. Some systems may theoretically fulfil the conditions of giving the equilibrium desired, but they may nevertheless be quite unsuitable for practical regulation purposes because of lack of stability.

The present system is evidently not very »shock proof«. Indeed, we have just seen how the highly shocklike variable v (the card price) rules the whole system. The chain of characteristic lag-relations in the system is a typical example of the most perfect shock-collector. The shocks in v affect essentially the price of small pigs (eq. (8.5)) which in turn carries them into the production activity where they are preserved for a long period, and lead to new shocks when the finished pigs are to be sold.

DANSK RESUME

Hensigten med nærværende Arbejde har været at faa kvantitative Maal for *Sammenhængene* mellem de forskellige Faktorer, som kommer i Betragtning ved Forsøg paa en objektiv Vurdering af den nuværende danske Svineregulering. I det endelige System af Relationer er ialt 10 forskellige Størrelser trukket ind, nemlig første Gang drægtige og andre drægtige Søer, Summen af disse, Antal Torvegrise, Antal slagtefærdige Svin, Antal Svin leveret med Kort, Pris paa Torvegrise, Pris paa Svin med og uden Kort samt Pris paa Svinekort. Ved statistisk Regressionsanalyse paa Maanedtsdata for de tre Aar 1935—37 har vi fastlagt syv forskellige fundamentale Relationer mellem disse Størrelser. Dette vil sige, at dersom Flæskepriserne og Antallet af Svinekort er givet, saa kan vi beregne, hvor store alle de øvrige Størrelser vil blive (bortset fra en vis statistisk Usikkerhed). Paa denne Maade er det muligt at anslaa paa Forhaand Virkningen af Ændringer i de Faktorer, som Svinereguleringsudvalget har i sin Haand.

De vigtigste Resultater, vi har fundet, er følgende:

1) *Forecast af Antal slagtefærdige Svin.* Ved statistisk Regressionsanalyse paa Data fra de 6-ugentlige Svinetællinger har vi bestemt følgende Forecast-Formel (Formel (4.17)):

$$\begin{aligned} & 1.33 \times \text{Antal 1. G. drægtige Søer paa Tællingsdagen} \\ & + 1.78 \times \text{Antal andre drægtige Søer paa Tællingsdagen} \\ & = \text{Antal slagtefærdige Svin pr. Maaned omkring et Tidspunkt ca.} \\ & \quad 8.2 \text{ Maaneder senere.} \end{aligned}$$

(Se Fig. 3 foran). Konstanterne 1.33 og 1.78 giver god Overensstemmelse for 1938, som ikke er taget med i Regressionsanalysen. Det kan dog senere blive nødvendigt at korrigere disse Tal ved at medtage nyere Data i Regressionsanalysen, men dette er statistisk-teknisk set meget enkelt.

2) *De Faktorer, som indvirker paa Tilgangen af drægtige Søer.* Disse Faktorer viser sig at være forskellige for 1. Gang drægtige og andre drægtige. Vi har fundet, at Totaltilgangen af 1. Gang drægtige Søer vil

stige med ca. 870, naar Prisen paa Torvegrise stiger med en Krone, og dernæst, at Totaltilgangen vil stige med ca. 100, naar Overskudet af slagtefærdige Svin (d.v.s. Forskellen mellem Antallet af færdige Svin og Antallet af Leveringer med Kort) stiger med 1000 (se Ligning (5.3)). For andre drægtige Søer har vi fundet, at Totaltilgangen vokser med ca. 390, naar Torvegrisprisen stiger med 1 Kr., og dernæst, at Tilgangen i Dag afhænger af Tilgangen af drægtige Søer 6 Maaneder tidligere. Hvis Tilgangen af drægtige Søer (1. Gang drægtige + andre drægtige) øges med 1000 Søer, og Torvegrisprisen er uforandret, saa vil gennemsnitligt 62% af denne Forøgelse komme igen som »andre drægtige Søer« ca. 6 Maaneder senere (Ligning (5.4)).

3) *Prisen paa Torvegrise.* Der er en nøje Sammenhæng mellem denne Pris og *Forskellen* mellem Værdien af et Svin med Kort og Prisen paa Svinekort. Hvis denne Forskel stiger med 1 Kr., saa vil Torvegrisprisen stige med ca. 65 Øre (se Ligning (6.4)).

4) Ved at kombinere de forskellige Relationer og tildele Flæskeprieserne og Kortantallet visse Gennemsnitsværdier kan vi beregne det karakteristiske Tidsforløb af de øvrige Størrelser. Eksempler er givet for Tilgangen af fravænnede Svin under 35 kg (Torvegrise). Vi har her fundet en typisk Bølgebevægelse paa ca. 5 1/2 Maaned, som ogsaa kan ses tydeligt i de fleste af de betragtede Rækker. Denne Bølgebevægelse synes at være et karakteristisk Træk ved Svineproduktionen i Danmark under det nuværende Reguleringsystem.

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