

DEPARTMENT OF STATISTICS  
(COMPUTING SECTION)  
UNIVERSITY OF LONDON, UNIVERSITY COLLEGE

## TRACTS FOR COMPUTERS

EDITED BY KARL PEARSON, F.R.S.

No. I

Tables of the Digamma and Trigamma Functions

By ELEANOR PAIRMAN, M.A.

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# TRACTS FOR COMPUTERS

## No. I

### PREFATORY NOTE

During the course of the past five years the Department of Applied Statistics in the University of London (University College) has carried out a great deal of computing work of one kind or another bearing on special war problems of a physical character. Its members have been struck by the absence of any simple text-book for the use of computers and still more by the absence of obviously necessary auxiliary tables. The present series of *Tracts for Computers* will endeavour to fill this gap as far as it lies in our power. It will not concern itself with the higher mathematical theory, but solely with the practical difficulties of the computer, or rather such difficulties as we have met with in our own experience. The first tract will be followed not only by others containing recently computed tables or by the republication of old tables at present very inaccessible, but by tracts dealing with interpolation, quadrature, mechanical integration, calculating machines, tabling machines, and bibliographies of memoirs and of tables having special value to the practical computer. In regard to the present tract, giving the values of the digamma and trigamma functions, we should ourselves have been saved many weeks of work had it been in existence four years ago. Further, we believe it will be of help not only in many physical problems, other than those we have had to deal with ourselves, but to the schoolmaster, who grasps the urgent importance of teaching practical mathematics to the modern schoolboy. The table of logarithms is not the only table that a schoolboy should learn to handle. In most modern computing laboratories a table of logarithms is very rarely used—and when used it is generally one to 10 or 14 figures\* where multiplications are necessary which exceed the range of the ordinary multiplying machine. Nowadays the schoolboy ought to be practised in computing, and this practice should run parallel with his algebraic work. He should be exercised in the use of tables which are not becoming obsolete

\* And where in seeking the antilog. the schoolboy's knowledge of the process is idle!

like the smaller tables of logarithms. He comes at a fairly early stage to the summation of series and he is liable to regard certain series as unsummable because he has not approached them numerically, just as he unfortunately regards certain integrals as unintegrable, because he is not introduced at a quite early stage to graphical, mechanical and numerical methods of quadrature. The present tract covers a very wide class of numerically summable series, and we can conceive no better practice than the schoolmaster could provide for his pupils by teaching them to sum all such series by tabular aid. If the pupil be asked at the same time to compare the result obtained by summing directly 15 to 20 terms of the series set (using tables of logarithms if he likes!), he will have learnt during the process a good deal of the practical value of logarithms, of tests for convergency, of partial fractions, of interpolation and of the value of tabular aids to the computer. He will further have realised that "proportional parts" are neither the sole nor necessarily adequate method of entering a table;—a belief not indeed infrequently found to dominate the post-graduate as well as the schoolboy mind and probably arising from the same limitation of experience—the very words "mathematical tables" being treated as synonymous with the smaller tables of common and trigonometrical logarithms.

K. P.

*August 1919.*

TABLES OF THE DIGAMMA AND TRIGAMMA FUNCTIONS

$$\frac{d}{dz} \log \Gamma(1+z) \text{ AND } \frac{d^2}{dz^2} \log \Gamma(1+z),$$

TO FACILITATE THE SUMMATION OF SERIES OF THE FORM

$$S = \sum_{i=1}^{\infty} \frac{a_0 + a_1 i + a_2 i^2 + \dots + a_{n-1} i^{n-1}}{(p_1 i + q_1)(p_2 i + q_2) \dots (p_n i + q_n)},$$

BY ELEANOR PAIRMAN, M.A.

In work recently undertaken by the Biometric Laboratory on the torsion-flexure of aeroplane propeller blades it was found necessary to sum a large number of series of the form

$$S = \sum_{i=1}^{\infty} \frac{a_0 + a_1 i + a_2 i^2 + \dots + a_{n-1} i^{n-1}}{(p_1 i + q_1)(p_2 i + q_2) \dots (p_n i + q_n)},$$

where the  $a$ 's,  $p$ 's and  $q$ 's are numerical quantities, and any number of pairs of factors in the denominator may be equal. By multiplying numerator and denominator by a suitable numerical factor this can always be thrown into the form

$$S = \sum_{i=1}^{\infty} \frac{b_0 + b_1 i + b_2 i^2 + \dots + b_{m+n-1} i^{m+n-1}}{(i+r_1)(i+r_2) \dots (i+r_m)(i+s_1)^2 (i+s_2)^2 \dots (i+s_n)^2}.$$

As it stands this series is only very slowly convergent, as many as from 60 to 80 terms being sometimes required to calculate it to eight figure accuracy.

The expression can always be broken up into partial fractions and we have

$$S = \sum_{i=1}^{\infty} \left[ \frac{A_1}{i+r_1} + \frac{A_2}{i+r_2} + \dots + \frac{A_m}{i+r_m} + \frac{B_1}{(i+s_1)^2} + \frac{B_2}{(i+s_2)^2} + \dots + \frac{B_n}{(i+s_n)^2} \right. \\ \left. + \frac{C_1}{(i+s_1)^2} + \frac{C_2}{(i+s_2)^2} + \dots + \frac{C_n}{(i+s_n)^2} \right],$$

where the  $A$ 's,  $B$ 's and  $C$ 's are readily determined.

We cannot, however, re-arrange the terms in  $S$  and sum each of the series  $\sum \frac{A_1}{i+r_1}$ , ...,  $\sum \frac{B_1}{i+s_1}$ , ...,  $\sum \frac{C_1}{(i+s_1)^2}$  ... separately, since a series of the form  $\sum \frac{1}{i+\alpha}$  is divergent.

Calling the  $i$ th term in the series  $v(i)$  we have

$$\begin{aligned} v(i) &= \frac{A_1}{i+r_1} + \frac{A_2}{i+r_2} + \dots + \frac{A_m}{i+r_m} + \frac{B_1}{i+s_1} + \frac{B_2}{i+s_2} + \dots + \frac{B_n}{i+s_n} \\ &\quad + \frac{C_1}{(i+s_1)^2} + \frac{C_2}{(i+s_2)^2} + \dots + \frac{C_n}{(i+s_n)^2} \\ &= A_1 \left( \frac{1}{i+r_1} - \frac{1}{i} \right) + A_2 \left( \frac{1}{i+r_2} - \frac{1}{i} \right) + \dots + A_m \left( \frac{1}{i+r_m} - \frac{1}{i} \right) \\ &\quad + B_1 \left( \frac{1}{i+s_1} - \frac{1}{i} \right) + B_2 \left( \frac{1}{i+s_2} - \frac{1}{i} \right) + \dots + B_n \left( \frac{1}{i+s_n} - \frac{1}{i} \right) \\ &\quad + \frac{C_1}{(i+s_1)^2} + \frac{C_2}{(i+s_2)^2} + \dots + \frac{C_n}{(i+s_n)^2} \\ &\quad + \frac{1}{i} (A_1 + A_2 + \dots + A_m + B_1 + B_2 + \dots + B_n). \end{aligned}$$

Also from the general theory of Partial Fractions we have\*

$$A_1 + A_2 + \dots + A_m + B_1 + B_2 + \dots + B_n = 0,$$

which gives 
$$v(i) = -\frac{A_1 r_1}{i(i+r_1)} - \frac{A_2 r_2}{i(i+r_2)} - \dots - \frac{A_m r_m}{i(i+r_m)} - \frac{B_1 s_1}{i(i+s_1)} - \frac{B_2 s_2}{i(i+s_2)} - \dots - \frac{B_n s_n}{i(i+s_n)} + \frac{C_1}{(i+s_1)^2} + \frac{C_2}{(i+s_2)^2} + \dots + \frac{C_n}{(i+s_n)^2}.$$

\* If

$$\frac{b_0 + b_1 i + b_2 i^2 + \dots + b_{m+2n-2} i^{m+2n-2}}{(i+r_1)(i+r_2)\dots(i+r_m)(i+s_1)^2(i+s_2)^2\dots(i+s_n)^2}$$

$$\equiv \frac{A_1}{i+r_1} + \dots + \frac{A_m}{i+r_m} + \frac{B_1}{i+s_1} + \dots + \frac{B_n}{i+s_n} + \frac{C_1}{(i+s_1)^2} + \dots + \frac{C_n}{(i+s_n)^2},$$

then

$$b_0 + b_1 i + b_2 i^2 + \dots + b_{m+2n-2} i^{m+2n-2}$$

$$\begin{aligned} &\equiv A_1 (i+r_1) \dots (i+r_m) (i+s_1)^2 \dots (i+s_n)^2 + \dots + A_m (i+r_1) \dots (i+r_{m-1}) (i+s_1)^2 \dots (i+s_n)^2 \\ &\quad + B_1 (i+r_1) \dots (i+r_m) (i+s_1) (i+s_2)^2 \dots (i+s_n)^2 + \dots \\ &\quad \quad \quad + B_n (i+r_1) \dots (i+r_m) (i+s_1)^2 \dots (i+s_{n-1})^2 (i+s_n)^2 \\ &\quad + C_1 (i+r_1) \dots (i+r_m) (i+s_1)^2 \dots (i+s_n)^2 + \dots + C_n (i+r_1) \dots (i+r_m) (i+s_1)^2 \dots (i+s_{n-1})^2. \end{aligned}$$

Equating the coefficients of  $x^{m+2n-1}$  on each side of this identity we obtain the result

$$A_1 + A_2 + \dots + A_m + B_1 + B_2 + \dots + B_n = 0.$$

But  $\sum_{i=1}^{\infty} \frac{1}{i(i+a)}$  and  $\sum_{i=1}^{\infty} \frac{1}{i(i+b)}$  are both absolutely convergent series. Hence we can now re-arrange the terms of  $S$  and have

$$\begin{aligned} S = & -A_1 \sum_{i=1}^{\infty} \frac{r_1}{i(i+r_1)} - A_2 \sum_{i=1}^{\infty} \frac{r_2}{i(i+r_2)} - \dots - A_m \sum_{i=1}^{\infty} \frac{r_m}{i(i+r_m)} \\ & - B_1 \sum_{i=1}^{\infty} \frac{s_1}{i(i+s_1)} - B_2 \sum_{i=1}^{\infty} \frac{s_2}{i(i+s_2)} - \dots - B_n \sum_{i=1}^{\infty} \frac{s_n}{i(i+s_n)} \\ & - C_1 \sum_{i=1}^{\infty} \frac{1}{(i+s_1)^2} - C_2 \sum_{i=1}^{\infty} \frac{1}{(i+s_2)^2} - \dots - C_n \sum_{i=1}^{\infty} \frac{1}{(i+s_n)^2}. \end{aligned}$$

Now it is known\* that

$$\begin{aligned} \frac{d}{dt} \log \Gamma(1+t) &= -\gamma + \sum_{n=1}^{\infty} \frac{t}{n(n+t)}, \\ \frac{d^2}{dt^2} \log \Gamma(1+t) &= \sum_{n=1}^{\infty} \frac{1}{(n+t)^2}, \end{aligned}$$

where  $\gamma = \text{Euler's Constant} = 0.5772157\dots$

Therefore

$$\begin{aligned} S = & -A_1 \left\{ \frac{d}{dr_1} \log \Gamma(1+r_1) + \gamma \right\} - A_2 \left\{ \frac{d}{dr_2} \log \Gamma(1+r_2) + \gamma \right\} - \dots \\ & \qquad \qquad \qquad - A_m \left\{ \frac{d}{dr_m} \log \Gamma(1+r_m) + \gamma \right\} \\ & - B_1 \left\{ \frac{d}{ds_1} \log \Gamma(1+s_1) + \gamma \right\} - B_2 \left\{ \frac{d}{ds_2} \log \Gamma(1+s_2) + \gamma \right\} - \dots \\ & \qquad \qquad \qquad - B_n \left\{ \frac{d}{ds_n} \log \Gamma(1+s_n) + \gamma \right\} \\ & + C_1 \frac{d^2}{ds_1^2} \log \Gamma(1+s_1) + C_2 \frac{d^2}{ds_2^2} \log \Gamma(1+s_2) + \dots + C_n \frac{d^2}{ds_n^2} \log \Gamma(1+s_n); \end{aligned}$$

or putting  $\frac{d}{dr_i} \log \Gamma(1+r_i) = F(r_i)$ ,

$$\frac{d^2}{dr_i^2} \log \Gamma(1+r_i) = F(r_i) \dagger,$$

and remembering that  $A_1 + \dots + A_m + B_1 + \dots + B_n = 0$ , we have finally

$$\begin{aligned} S = & -A_1 F(r_1) - A_2 F(r_2) - \dots - A_m F(r_m) \\ & - B_1 F(s_1) - B_2 F(s_2) - \dots - B_n F(s_n) \\ & + C_1 F(s_1) + C_2 F(s_2) + \dots + C_n F(s_n), \end{aligned}$$

\* Cf. Whittaker and Watson, *Modern Analysis* (2nd Edition), § 12.16.

† It is convenient to speak of these functions as the "digamma and trigamma functions."

where  $F(x)$  and  $\mathbb{F}(x)$  are the functions here tabled for values of  $x$  from  $x=0\cdot00$  to  $x=20\cdot00$ , the increment in  $x$  being  $\cdot02$ .

The tables were calculated by means of the asymptotic expansions for the logarithmic derivatives of the Gamma-function

$$F(z) = \frac{d}{dz} \log \Gamma(1+z) = \log(1+z) - \frac{1}{2} \frac{1}{z+1} + \sum_{r=1}^{\infty} \frac{(-)^r B_r}{2r(1+z)^{2r}},$$

$$\mathbb{F}(z) = \frac{d^2}{dz^2} \log \Gamma(1+z) = \frac{1}{1+z} + \frac{1}{2} \frac{1}{(z+1)^2} + \sum_{r=1}^{\infty} \frac{(-)^{r+1} B_r}{(1+z)^{2r+1}},$$

in conjunction with the formulae

$$F(z-1) = -\frac{1}{z} + F(z),$$

$$\mathbb{F}(z-1) = \frac{1}{z^2} + \mathbb{F}(z).$$

If values of the functions outside the range of the tables are desired they may be obtained from the first of these pairs of equations, or, alternatively, from the formulae

$$F(nz) = \log_e n + \frac{1}{n} \left\{ F\left(z - \frac{n-1}{n}\right) + F\left(z - \frac{n-2}{n}\right) + \dots + F\left(z - \frac{1}{n}\right) + F(z) \right\}$$

and

$$\mathbb{F}(nz) = \frac{1}{n^2} \left\{ \mathbb{F}\left(z - \frac{n-1}{n}\right) + \mathbb{F}\left(z - \frac{n-2}{n}\right) + \dots + \mathbb{F}\left(z - \frac{1}{n}\right) + \mathbb{F}(z) \right\},$$

which are deduced from Gauss' multiplication-theorem for the Gamma-function.

This latter pair are the more useful for values of  $x$  lying between 20 and 40.

$$\begin{aligned} \text{E.g.} \quad F(37\cdot68) &= \log_e 2 + \frac{1}{2} \{F(18\cdot34) + F(18\cdot84)\} \\ &= \quad \cdot69314718 \\ &\quad + 1\cdot46804980 \\ &\quad + 1\cdot48114342 \\ &= \quad 3\cdot64244040 \end{aligned}$$

$$\begin{aligned} F(37\cdot68) &= \frac{1}{2} \{F(18\cdot34) + F(18\cdot84)\} \\ &= \frac{1}{2} \left\{ \begin{array}{l} \cdot05306611 \\ + \cdot05169480 \end{array} \right\} \\ &= \frac{1}{2} [1\cdot0476091] \\ &= \quad \cdot02619023. \end{aligned}$$



Besides the values of the functions there are tabled their second and fourth differences, and the formula of interpolation to be used is Everitt's Central Difference Formula:

$$f(a + \theta\omega) = \theta f_1 + \phi f_0 - \frac{\theta\phi}{6} \{(\theta + 1)\delta^2 f_1 + (\phi + 1)\delta^2 f_0\} + \frac{\theta\phi(\theta + 1)(\phi + 1)}{120} \{(\theta + 2)\delta^4 f_1 + (\phi + 2)\delta^4 f_0\}$$

where  $\omega =$  increment in  $x = .02$ ,  
 $f_1 = f(a + .02)$ ,  
 $f_0 = f(a)$ ,  
 $\theta + \phi = 1$ .

E.g.  $F(8.615)$ ,  
 $x = 8.60$ .  $\theta = \frac{1}{2}$ .  $\phi = \frac{1}{2}$ .

$F(8.615)$ $= \frac{1}{4}(2.20877652) + \frac{\frac{1}{2} \cdot \frac{1}{2}}{6} \left\{ \frac{1}{4}(479) \right.$ $+ \frac{\frac{1}{2}}{4}(2.21096971) \left. + \frac{1}{4}(482) \right\}$ $= 2.21042186.$	$F(8.615)$ $= \frac{1}{4}(.10977999) - \frac{\frac{1}{2} \cdot \frac{1}{2}}{6} \left\{ \frac{1}{4}(106) \right.$ $+ \frac{\frac{1}{2}}{4}(.10953972) \left. + \frac{1}{4}(105) \right\}$ $= .10959969.$
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It was not discovered until the present tables were almost completed that a certain portion of the work had been already performed. Gauss (*Werke*, Bd III, pp. 161, 162, Göttingen, 1863 etc.) gives tables of  $\frac{d}{dx} \log \Gamma(1 + x)$  correct to 18 decimal places for values of  $x$  between 0 and 1, the increment in  $x$  being .01. Prof. G. N. Watson (*Report of the British Association*, 1916, pp. 125, 126) gives tables of the same function correct to 13 decimal places for all integers and halves of odd integers from 0 to 100. These two valuable tables have been made use of as a check on the present 8-figure tables.

So far as has been discovered there are no other tables of  $\frac{d^2}{dx^2} \log \Gamma(1 + x)$ .

The following illustrations are some which have recently occurred in practice. They were previously calculated by the longer method, and the results obtained from the two methods agree exactly.

$$(i) \quad S = \sum_{i=1}^{\infty} \frac{1}{(4i+3)(4i+2)4i(4i+4)}$$

$$= \sum_{i=1}^{\infty} \left[ \frac{1}{3} \frac{1}{4i+3} - \frac{1}{4} \frac{1}{4i+2} + \frac{1}{24} \frac{1}{4i} - \frac{1}{8} \frac{1}{4i+4} \right]$$

$$= \sum_{i=1}^{\infty} \left[ \frac{1}{12} \frac{1}{i+.75} - \frac{1}{16} \frac{1}{i+.50} + \frac{1}{96} \frac{1}{i} - \frac{1}{32} \frac{1}{i+1.00} \right]$$

$$= -\frac{1}{12} F(.75) + \frac{1}{16} F(.50) - \frac{1}{96} F(.00) + \frac{1}{32} F(1.00).$$

$$F(.75) = \frac{1}{2} \left\{ \begin{array}{l} .23980321 \\ + .25508551 \end{array} \right\} + \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{2}}{6} \left\{ \begin{array}{l} 22802 \\ + 22161 \end{array} \right\} - \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{5}{2}}{120} \left\{ \begin{array}{l} 27 \\ + 26 \end{array} \right\}$$

$$= .24744436 + 2810 - 1 = .24747245.$$

Hence  $S = .00228062 - .02062270$

$$\begin{array}{r} 601266 \\ 1321201 \\ \hline = .02150529 - .02062270 = .00088259. \end{array}$$

$$(ii) S = \sum_{i=1}^{\infty} \frac{1}{(4i+2)(4i+1)(4i+3)^2}$$

$$= \sum_{i=1}^{\infty} \left[ -\frac{1}{4i+2} + \frac{1}{4} \frac{1}{4i+1} + \frac{3}{4} \frac{1}{4i+3} + \frac{1}{2} \frac{1}{(4i+3)^2} \right]$$

$$= \frac{1}{4} F(.50) - \frac{1}{16} F(.25) - \frac{3}{16} F(.75) + \frac{1}{8} F(.75).$$

$$F(.25) = -\frac{1}{2} \left\{ \begin{array}{l} .23949368 \\ + .21554617 \end{array} \right\} + \frac{1}{16} \left\{ \begin{array}{l} 54251 \\ + 52024 \end{array} \right\} - \frac{3}{16} \left\{ \begin{array}{l} 140 \\ + 129 \end{array} \right\}$$

$$= -.22751993 + 6642 - 3 = -.22745354.$$

$$F(.75) = \frac{1}{2} \left\{ \begin{array}{l} .23980321 \\ + .25508551 \end{array} \right\} + \frac{1}{16} \left\{ \begin{array}{l} 22802 \\ + 22161 \end{array} \right\} - \frac{3}{16} \left\{ \begin{array}{l} 27 \\ + 26 \end{array} \right\}$$

$$= .24744436 + 2810 - 1 = .24747245.$$

$$F(.75) = \frac{1}{2} \left\{ \begin{array}{l} .76976109 \\ + .75852269 \end{array} \right\} - \frac{1}{16} \left\{ \begin{array}{l} 32695 \\ 31363 \end{array} \right\} + \frac{3}{16} \left\{ \begin{array}{l} 75 \\ + 70 \end{array} \right\}$$

$$= .76414189 - 4004 + 2 = .76410187.$$

Hence  $S = .00912249 - .04640108$

$$\begin{array}{r} .01421585 \\ .02387818 \\ \hline = .04721652 - .04640108 = .00081544. \end{array}$$

$$(iii) S = \sum_{i=1}^{\infty} \frac{30}{(60i+29)(60i+30)(60i+31)^2}$$

$$= \sum_{i=1}^{\infty} \left[ \frac{30}{4} \frac{1}{60i+29} - \frac{30}{60i+30} + \frac{90}{4} \frac{1}{60i+31} + \frac{30}{2} \frac{1}{(60i+31)^2} \right]$$

$$= -\frac{1}{8} F\left(\frac{29}{60}\right) + \frac{1}{2} F\left(\frac{30}{60}\right) - \frac{3}{8} F\left(\frac{31}{60}\right) + \frac{1}{4} F\left(\frac{31}{60}\right).$$

$$F\left(\frac{31}{60}\right) = F\left(48 + \frac{1}{60}\right)$$

$$= \frac{1}{60} (.01762627) + \frac{\frac{1}{60} \cdot \frac{1}{60}}{6} \left\{ \begin{array}{l} \frac{1}{60} (34312) \\ + \frac{1}{60} (33157) \end{array} \right\} - \frac{\frac{1}{60} \cdot \frac{1}{60} \cdot \frac{1}{60} \cdot \frac{1}{60}}{120} \left\{ \begin{array}{l} \frac{1}{60} (60) \\ + \frac{1}{60} (55) \end{array} \right\}$$

$$= .02077022 + 2352 - 1 = .02079373.$$

$$\begin{aligned} F\left(\frac{3}{6}\right) &= F\left(\cdot 50 + \frac{1}{6}(\cdot 02)\right) \\ &= \frac{1}{6}(\cdot 03648997) + \frac{\frac{1}{6} \cdot \frac{1}{6}}{6} \left\{ \frac{1}{6}(33157) \right\} - \frac{\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}}{120} \left\{ \frac{1}{6}(55) \right\} \\ &\quad + \frac{1}{6}(\cdot 05502211) \left\{ + \frac{1}{6}(32056) \right\} \\ &= \cdot 05193342 + 2256 - 1 = \cdot 05195597. \end{aligned}$$

$$\begin{aligned} F\left(\frac{5}{6}\right) &= F\left(\cdot 50 + \frac{1}{6}(\cdot 02)\right) \\ &= \frac{1}{6}(\cdot 93480220) - \frac{\frac{1}{6} \cdot \frac{1}{6}}{6} \left\{ \frac{1}{6}(56379) \right\} - \frac{\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}}{120} \left\{ \frac{1}{6}(178) \right\} \\ &\quad + \frac{1}{6}(\cdot 91850353) \left\{ + \frac{1}{6}(53685) \right\} \\ &= \cdot 92121998 - 3801 + 2 = \cdot 92118199. \end{aligned}$$

$$\begin{aligned} \text{Hence } S &= \cdot 01824499 - \cdot 00259922 \\ &\quad \frac{383826}{1948349} \\ &= \cdot 02208325 - \cdot 02208271 = \cdot 00000054. \end{aligned}$$

$$\begin{aligned} \text{(iv) } S &= \sum_{i=1}^{\infty} \frac{1}{(2i+1)(6i-7)(6i+13)^2} \\ &= \sum_{i=1}^{\infty} \left[ -\frac{1}{10^2} \frac{1}{2i+1} + \frac{3}{4} \frac{1}{10^2} \frac{1}{6i-7} + \frac{9}{4} \frac{1}{10^2} \frac{1}{6i+13} + \frac{3}{2} \frac{1}{10^2} \frac{1}{(6i+13)^2} \right] \\ &= \frac{1}{10^2} \left[ \frac{1}{2} F\left(\frac{1}{50}\right) - \frac{1}{8} F\left(-\frac{7}{6}\right) - \frac{3}{8} F\left(\frac{13}{6}\right) + \frac{5}{12} F\left(\frac{13}{6}\right) \right], \end{aligned}$$

$$\begin{aligned} F\left(-\frac{7}{6}\right) &= \frac{7}{6} + F\left(-\frac{7}{6}\right) = \frac{7}{6} + \frac{7}{6} + F\left(\frac{7}{6}\right) \\ &= 6\frac{7}{6} + F\left(82 + \frac{1}{6}(\cdot 02)\right) \\ &= 6\cdot 85714286 + \frac{1}{6}(\cdot 29962710) + \frac{\frac{1}{6} \cdot \frac{1}{6}}{6} \left\{ \frac{1}{6}(20390) \right\} - \frac{\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}}{120} \left\{ \frac{1}{6}(21) \right\} \\ &\quad + \frac{1}{6}(\cdot 31405886) \left\{ + \frac{1}{6}(19845) \right\} \\ &= 6\cdot 85714286 + \cdot 30924827 + 2232 = 7\cdot 16641345. \end{aligned}$$

$$\begin{aligned} F\left(\frac{13}{6}\right) &= F\left(2\cdot 16 + \frac{1}{6}(\cdot 02)\right) \\ &= \frac{1}{6}(\cdot 98407878) + \frac{\frac{1}{6} \cdot \frac{1}{6}}{6} \left\{ \frac{1}{6}(5468) \right\} \\ &\quad + \frac{1}{6}(\cdot 99148578) \left\{ + \frac{1}{6}(5389) \right\} \\ &= \cdot 98654778 + 604 = \cdot 98655382. \end{aligned}$$

$$\begin{aligned} F\left(\frac{13}{6}\right) &= F\left(2\cdot 16 + \frac{1}{6}(\cdot 02)\right) \\ &= \frac{1}{6}(\cdot 37171057) - \frac{\frac{1}{6} \cdot \frac{1}{6}}{6} \left\{ \frac{1}{6}(3982) \right\} \\ &\quad + \frac{1}{6}(\cdot 36899640) \left\{ + \frac{1}{6}(3897) \right\} \\ &= \cdot 37080585 - 438 = \cdot 37080147. \end{aligned}$$

$$\begin{aligned} \text{Hence } S &= \frac{1}{10^2} \left[ \cdot 01824499 - \cdot 89580168 \right] \\ &\quad \frac{\cdot 15450061 \quad \cdot 36995768}{\cdot 17274560 - 1\cdot 26575936} \\ &= \frac{1}{10^2} [-17274560 - 1\cdot 26575936] = -\cdot 00109301. \end{aligned}$$

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TABLES OF  $F(x)$  AND  $F'(x)$

$x=2.00$  to  $4.00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
2.00	.92278434	6165	.39493407	4758	3.00	1.25611767	3201	.28382296	1795
2.02	.93065235	6071	.39187540	4650	3.02	1.26177818	3166	.28223108	1765
2.04	.93845966	5979	.38886323	4546	3.04	1.26740703	3131	.28065686	1736
2.06	.94620718	5889	.38589653	4445	3.06	1.27300457	3097	.27910000	1708
2.08	.95389582	5801	.38297428	4347	3.08	1.27857114	3063	.27756022	1680
2.10	.96152644	5715	.38009550	4252	3.10	1.28410709	3029	.27603723	1653
2.12	.96909992	5631	.37725923	4159	3.12	1.28961274	2997	.27453077	1626
2.14	.97661709	5549	.37446456	4069	3.14	1.29508843	2964	.27304056	1600
2.16	.98407878	5468	.37171057	3982	3.16	1.30053447	2933	.27156636	1574
2.18	.99148578	5389	.36899640	3897	3.18	1.30595119	2901	.27010790	1550
2.20	.99883889	5312	.36632119	3814	3.20	1.31133889	2871	.26866494	1525
2.22	1.00613888	5237	.36368413	3734	3.22	1.31669789	2840	.26723723	1501
2.24	1.01338651	5163	.36108440	3656	3.24	1.32202848	2811	.26582453	1478
2.26	1.02058250	5090	.35852123	3580	3.26	1.32733097	2781	.26442661	1455
2.28	1.02772759	5020	.35599385	3506	3.28	1.33260564	2752	.26304323	1432
2.30	1.03482249	4950	.35350154	3434	3.30	1.33785279	2724	.26167418	1410
2.32	1.04186788	4882	.35104357	3364	3.32	1.34307270	2696	.26031923	1389
2.34	1.04886446	4816	.34861924	3296	3.34	1.34826565	2668	.25897817	1368
2.36	1.05581287	4750	.34622788	3230	3.36	1.35343192	2641	.25765078	1347
2.38	1.06271379	4686	.34386881	3166	3.38	1.35857177	2615	.25633686	1327
2.40	1.06956784	4624	.34154140	3103	3.40	1.36368548	2588	.25503621	1307
2.42	1.07637565	4562	.33924501	3042	3.42	1.36877331	2562	.25374862	1287
2.44	1.08313784	4502	.33697905	2982	3.44	1.37383551	2537	.25247391	1268
2.46	1.08985501	4443	.33474290	2924	3.46	1.37887235	2512	.25121187	1249
2.48	1.09652775	4385	.33253599	2867	3.48	1.38388407	2487	.24996233	1231
2.50	1.10315664	4328	.33035776	2812	3.50	1.38887003	2462	.24872510	1213
2.52	1.10974225	4273	.32820765	2759	3.52	1.39383316	2438	.24750000	1195
2.54	1.11628513	4218	.32608512	2706	3.54	1.39877101	2414	.24628685	1178
2.56	1.12278583	4164	.32398966	2655	3.56	1.40368471	2391	.24508548	1161
2.58	1.12924489	4112	.32192075	2605	3.58	1.40857450	2368	.24389572	1144
2.60	1.13566284	4060	.31987790	2557	3.60	1.41344062	2345	.24271741	1128
2.62	1.14204018	4009	.31786061	2510	3.62	1.41828327	2323	.24155037	1112
2.64	1.14837743	3960	.31586842	2463	3.64	1.42310270	2301	.24039445	1096
2.66	1.15467508	3911	.31390087	2418	3.66	1.42789913	2279	.23924949	1081
2.68	1.16093363	3863	.31195749	2374	3.68	1.43267276	2258	.23811534	1065
2.70	1.16715354	3816	.31003786	2331	3.70	1.43742381	2236	.23699184	1051
2.72	1.17333529	3770	.30814154	2289	3.72	1.44215250	2216	.23587885	1036
2.74	1.17947935	3724	.30626811	2248	3.74	1.44685903	2195	.23477622	1022
2.76	1.18558617	3680	.30441717	2208	3.76	1.45154362	2175	.23368380	1008
2.78	1.19165619	3636	.30258831	2169	3.78	1.45620645	2155	.23260146	994
2.80	1.19768985	3593	.30078114	2131	3.80	1.46084774	2135	.23152906	980
2.82	1.20368758	3551	.29899529	2094	3.82	1.46546768	2115	.23046647	967
2.84	1.20964980	3509	.29723038	2058	3.84	1.47006646	2096	.22941354	954
2.86	1.21557693	3468	.29548604	2022	3.86	1.47464428	2077	.22837014	941
2.88	1.22146937	3428	.29376193	1987	3.88	1.47920133	2059	.22733616	928
2.90	1.22732754	3389	.29205768	1953	3.90	1.48373779	2040	.22631146	916
2.92	1.23315181	3350	.29037298	1920	3.92	1.48825385	2022	.22529592	904
2.94	1.23894258	3312	.28870747	1888	3.94	1.49274969	2004	.22428942	892
2.96	1.24470024	3275	.28706084	1856	3.96	1.49722549	1986	.22329184	880
2.98	1.25042514	3238	.28543277	1825	3.98	1.50168142	1969	.22230305	869
3.00	1.25611767	3201	.28382296	1795	4.00	1.50611767	1952	.22132296	857

Interpolation Formula:

$$f(s + \phi) = \phi f_0 + \phi f_1 - \frac{\phi^2}{6} \{(\phi + 1) \delta^2 f_0 + (\phi + 1) \delta^2 f_1\} + \frac{\phi^3}{120} \{(\phi + 1) \delta^3 f_0 + (\phi + 1) \delta^3 f_1\} \text{ where } \theta + \phi = 1.$$

TABLES OF  $F(x)$  AND  $F'(x)$

$x=4.00$  to  $6.00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
4.00	1.50611767	1952	.22132296	857	5.00	1.70611767	1312	.18132296	473
4.02	1.51053440	1935	.22035143	846	5.02	1.70973759	1302	.18066952	468
4.04	1.51493178	1918	.21938836	835	5.04	1.71334448	1293	.18002076	463
4.06	1.51930999	1901	.21843364	824	5.06	1.71693845	1284	.17937663	458
4.08	1.52366918	1885	.21748717	814	5.08	1.72051958	1275	.17873709	453
4.10	1.52800953	1869	.21654883	803	5.10	1.72408796	1266	.17810208	448
4.12	1.53233119	1853	.21561852	793	5.12	1.72764369	1257	.17747155	444
4.14	1.53663432	1837	.21469614	783	5.14	1.73118685	1248	.17684546	439
4.16	1.54091909	1821	.21378160	773	5.16	1.73471754	1239	.17622376	435
4.18	1.54518564	1806	.21287478	763	5.18	1.73823583	1230	.17560640	430
4.20	1.54943413	1791	.21197560	754	5.20	1.74174182	1222	.17499335	425
4.22	1.55366471	1776	.21108395	744	5.22	1.74523559	1213	.17438455	421
4.24	1.55787754	1761	.21019975	735	5.24	1.74871723	1205	.17377996	417
4.26	1.56207275	1746	.20932290	726	5.26	1.75218682	1197	.17317954	412
4.28	1.56625050	1732	.20845331	717	5.28	1.75564444	1188	.17258324	408
4.30	1.57041093	1718	.20759089	708	5.30	1.75909018	1180	.17199103	404
4.32	1.57455418	1704	.20673555	700	5.32	1.76252411	1172	.17140286	400
4.34	1.57868040	1690	.20588721	691	5.34	1.76594632	1164	.17081869	396
4.36	1.58278972	1676	.20504578	683	5.36	1.76935688	1156	.17023847	392
4.38	1.58688228	1662	.20421117	674	5.38	1.77275588	1149	.16966218	388
4.40	1.59095821	1649	.20338331	666	5.40	1.77614340	1141	.16908976	384
4.42	1.59501765	1636	.20256212	658	5.42	1.77951950	1133	.16852119	380
4.44	1.59906074	1623	.20174751	650	5.44	1.78288427	1126	.16795642	376
4.46	1.60308760	1610	.20093940	643	5.46	1.78623778	1118	.16739541	373
4.48	1.60709836	1597	.20013772	635	5.48	1.78958011	1111	.16683813	369
4.50	1.61109315	1584	.19934239	628	5.50	1.79291133	1104	.16628454	365
4.52	1.61507210	1572	.19855333	620	5.52	1.79623152	1096	.16573460	362
4.54	1.61903532	1560	.19777048	613	5.54	1.79954074	1089	.16518828	358
4.56	1.62298296	1547	.19699376	606	5.56	1.80283907	1082	.16464554	355
4.58	1.62691511	1535	.19622310	599	5.58	1.80612658	1075	.16410635	351
4.60	1.63083192	1523	.19545843	592	5.60	1.80940335	1068	.16357067	348
4.62	1.63473349	1512	.19469967	585	5.62	1.81266943	1061	.16303847	344
4.64	1.63861995	1500	.19394677	578	5.64	1.81592491	1054	.16250971	341
4.66	1.64249140	1488	.19319964	572	5.66	1.81916985	1047	.16198437	338
4.68	1.64634797	1477	.19245824	565	5.68	1.82240431	1041	.16146240	335
4.70	1.65018977	1466	.19172249	559	5.70	1.82562836	1034	.16094378	331
4.72	1.65401691	1455	.19099232	552	5.72	1.82884208	1027	.16042848	328
4.74	1.65782950	1444	.19026768	546	5.74	1.83204552	1021	.15991646	325
4.76	1.66162765	1433	.18954850	540	5.76	1.83523876	1014	.15940768	322
4.78	1.66541147	1422	.18883472	534	5.78	1.83842185	1008	.15890213	319
4.80	1.66918107	1412	.18812629	528	5.80	1.84159487	1002	.15839977	316
4.82	1.67293656	1401	.18742313	522	5.82	1.84475787	995	.15790057	313
4.84	1.67667803	1391	.18672520	516	5.84	1.84791091	989	.15740450	310
4.86	1.68040560	1380	.18603243	511	5.86	1.85105407	983	.15691153	307
4.88	1.68411937	1370	.18534476	505	5.88	1.85418739	977	.15642163	304
4.90	1.68781943	1360	.18466215	500	5.90	1.85731095	971	.15593477	302
4.92	1.69150588	1350	.18398454	494	5.92	1.86042480	965	.15545093	299
4.94	1.69517884	1340	.18331186	489	5.94	1.86352901	959	.15497008	296
4.96	1.69883839	1331	.18264408	483	5.96	1.86662363	953	.15449219	293
4.98	1.70248464	1321	.18198112	478	5.98	1.86970872	947	.15401743	291
5.00	1.70611767	1312	.18132296	473	6.00	1.87278434	941	.15354518	288

Interpolation Formula:

$$f(a + \theta\delta) = f_0 + \theta f_1 - \frac{\theta^2}{2} \{ (\theta + 1) \delta^2 f_0 + (\theta + 1) \delta^2 f_1 \} + \frac{\theta^3 (\theta + 1) (\theta + 2)}{6} \{ (\theta + 2) \delta^3 f_0 + (\theta + 2) \delta^3 f_1 \} \text{ where } \theta + \phi = 1.$$

TABLES OF  $F(x)$  AND  $F'(x)$

$x=6.00$  to  $8.00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
6.00	1.87278434	941	.15354518	288	7.00	2.01564148	708	.13313701	188
6.02	1.87585054	935	.15307600	285	7.02	2.01830068	704	.13278396	186
6.04	1.87890739	930	.15260968	283	7.04	2.02095285	700	.13243277	185
6.06	1.88195495	924	.15214619	280	7.06	2.02359801	697	.13208343	184
6.08	1.88499326	919	.15168550	278	7.08	2.02623620	693	.13173593	182
6.10	1.88802239	913	.15122758	275	7.10	2.02886746	690	.13139025	181
6.12	1.89104238	908	.15077242	273	7.12	2.03149182	686	.13104637	179
6.14	1.89405330	902	.15031998	270	7.14	2.03410932	682	.13070429	178
6.16	1.89705520	897	.14987024	268	7.16	2.03672000	679	.13036398	177
6.18	1.90004813	891	.14942318	265	7.18	2.03932390	675	.13002545	175
6.20	1.90303214	886	.14897878	263	7.20	2.04192103	672	.12968866	174
6.22	1.90600730	881	.14853701	261	7.22	2.04451145	668	.12935366	172
6.24	1.90897364	876	.14809784	258	7.24	2.04709519	665	.12902028	171
6.26	1.91193123	871	.14766126	256	7.26	2.04967228	662	.12868867	170
6.28	1.91488011	865	.14722725	254	7.28	2.05224275	658	.12835875	168
6.30	1.91782034	860	.14679577	252	7.30	2.05480664	655	.12803052	167
6.32	1.92075196	855	.14636681	250	7.32	2.05736398	651	.12770396	166
6.34	1.92367502	851	.14594034	247	7.34	2.05991481	648	.12737906	165
6.36	1.92658959	846	.14551635	245	7.36	2.06245915	645	.12705581	163
6.38	1.92949569	841	.14509481	243	7.38	2.06499705	642	.12673419	162
6.40	1.93239340	836	.14467570	241	7.40	2.06752853	638	.12641419	161
6.42	1.93528274	831	.14425900	239	7.42	2.07005363	635	.12609581	160
6.44	1.93816377	826	.14384469	237	7.44	2.07257237	632	.12577902	159
6.46	1.94103654	822	.14343275	235	7.46	2.07508480	629	.12546382	157
6.48	1.94390110	817	.14302316	233	7.48	2.07759094	626	.12515019	156
6.50	1.94675748	812	.14261590	231	7.50	2.08009082	623	.12483812	155
6.52	1.94960575	808	.14221094	229	7.52	2.08258447	619	.12452760	154
6.54	1.95244594	803	.14180828	227	7.54	2.08507193	616	.12421862	153
6.56	1.95527810	799	.14140788	225	7.56	2.08755323	613	.12391117	152
6.58	1.95810227	794	.14100974	223	7.58	2.09002839	610	.12360524	151
6.60	1.96091850	790	.14061383	221	7.60	2.09249745	607	.12330081	149
6.62	1.96372684	785	.14022013	219	7.62	2.09496043	604	.12299788	148
6.64	1.96652732	781	.13982863	218	7.64	2.09741737	601	.12269642	147
6.66	1.96932000	776	.13943930	216	7.66	2.09986830	598	.12239644	146
6.68	1.97210491	772	.13905214	214	7.68	2.10231324	596	.12209793	145
6.70	1.97488209	768	.13866711	212	7.70	2.10475222	593	.12180086	144
6.72	1.97765160	764	.13828420	211	7.72	2.10718528	590	.12150523	143
6.74	1.98041348	760	.13790341	209	7.74	2.10961244	587	.12121103	142
6.76	1.98316775	755	.13752470	207	7.76	2.11203373	584	.12091826	141
6.78	1.98591448	751	.13714806	205	7.78	2.11444918	581	.12062689	140
6.80	1.98865369	747	.13677347	204	7.80	2.11685882	579	.12033692	139
6.82	1.99138543	743	.13640093	202	7.82	2.11926267	576	.12004834	138
6.84	1.99410974	739	.13603040	200	7.84	2.12166076	573	.11976114	137
6.86	1.99682666	735	.13566188	199	7.86	2.12405312	570	.11947530	136
6.88	1.99953623	731	.13529334	197	7.88	2.12643978	568	.11919083	135
6.90	2.00223849	727	.13493078	196	7.90	2.12882077	565	.11890771	134
6.92	2.00493347	723	.13456818	194	7.92	2.13119610	562	.11862593	133
6.94	2.00762123	719	.13420752	193	7.94	2.13356581	560	.11834548	132
6.96	2.01030179	716	.13384878	191	7.96	2.13592993	557	.11806635	131
6.98	2.01297519	712	.13349195	190	7.98	2.13828847	554	.11778853	130
7.00	2.01564148	708	.13313701	188	8.00	2.14064148	552	.11751201	129

Interpolation Formula:

$$f(s + \phi) = \phi f_0 + \phi f_1 - \frac{\phi^2}{6} \{(\phi + 1) \delta^2 f_0 + (\phi + 1) \delta^2 f_1\} + \frac{\phi^2(\phi + 1)(\phi + 1)}{120} \{(\phi + 2) \delta^2 f_0 + (\phi + 2) \delta^2 f_1\} \text{ where } \theta + \phi = 1.$$

TABLES OF  $F(x)$  AND  $F(x)$

$x=8.00$  to  $10.00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
8.00	2.14064148	552	.11751201	120	9.00	2.25175259	442	.10516634	93
8.02	2.14298896	549	.11723679	128	9.02	2.25385371	440	.10494580	92
8.04	2.14533096	547	.11696286	128	9.04	2.25595043	438	.10472619	92
8.06	2.14766749	544	.11669020	127	9.06	2.25804276	436	.10450750	91
8.08	2.14999857	542	.11641880	126	9.08	2.26013073	435	.10428971	90
8.10	2.15232425	539	.11614867	125	9.10	2.26221436	433	.10407283	90
8.12	2.15464453	537	.11587978	124	9.12	2.26429365	431	.10385685	89
8.14	2.15695945	534	.11561214	123	9.14	2.26636864	429	.10364176	89
8.16	2.15926902	532	.11534572	122	9.16	2.26843933	427	.10342757	88
8.18	2.16157328	529	.11508053	121	9.18	2.27050575	426	.10321425	88
8.20	2.16387225	527	.11481656	121	9.20	2.27256790	424	.10300181	87
8.22	2.16616595	524	.11455379	120	9.22	2.27462582	422	.10279025	87
8.24	2.16845441	522	.11429222	119	9.24	2.27667952	421	.10257955	86
8.26	2.17073765	519	.11403184	118	9.26	2.27872901	419	.10236971	86
8.28	2.17301569	517	.11377265	117	9.28	2.28077431	417	.10216073	85
8.30	2.17528856	515	.11351463	117	9.30	2.28281545	415	.10195260	85
8.32	2.17755629	512	.11325777	116	9.32	2.28485242	414	.10174531	84
8.34	2.17981888	510	.11300208	115	9.34	2.28688526	412	.10153887	84
8.36	2.18207638	508	.11274753	114	9.36	2.28891398	410	.10133326	83
8.38	2.18432879	506	.11249413	114	9.38	2.29093860	409	.10112848	82
8.40	2.18657615	503	.11224186	113	9.40	2.29295913	407	.10092452	82
8.42	2.18881847	501	.11199072	112	9.42	2.29497559	405	.10072139	81
8.44	2.19105579	499	.11174070	111	9.44	2.29698799	404	.10051907	81
8.46	2.19328811	497	.11149179	111	9.46	2.29899635	402	.10031755	81
8.48	2.19551547	494	.11124399	110	9.48	2.30100070	401	.10011686	80
8.50	2.19773788	492	.11099729	109	9.50	2.30300103	399	.09991696	80
8.52	2.19995536	490	.11075168	108	9.52	2.30499738	397	.09971785	80
8.54	2.20216795	488	.11050715	108	9.54	2.30698975	396	.09951954	79
8.56	2.20437566	486	.11026369	107	9.56	2.30897817	394	.09932201	78
8.58	2.20657851	484	.11002131	106	9.58	2.31096264	393	.09912526	78
8.60	2.20877652	482	.10977999	106	9.60	2.31294318	391	.09892929	77
8.62	2.21096971	479	.10953972	105	9.62	2.31491982	390	.09873410	77
8.64	2.21315811	477	.10930050	104	9.64	2.31689255	388	.09853967	76
8.66	2.21534174	475	.10906233	103	9.66	2.31886141	387	.09834601	76
8.68	2.21752061	473	.10882519	103	9.68	2.32082640	385	.09815310	75
8.70	2.21969475	471	.10858907	102	9.70	2.32278754	384	.09796095	75
8.72	2.22186418	469	.10835398	101	9.72	2.32474484	382	.09776955	75
8.74	2.22402892	467	.10811990	101	9.74	2.32669832	381	.09757800	74
8.76	2.22618899	465	.10788683	100	9.76	2.32864800	379	.09738698	74
8.78	2.22834440	463	.10765477	100	9.78	2.33059389	378	.09719681	73
8.80	2.23049518	461	.10742369	99	9.80	2.33253600	376	.09700713	73
8.82	2.23264135	459	.10719361	98	9.82	2.33447435	375	.09682365	72
8.84	2.23478293	457	.10696451	98	9.84	2.33640895	373	.09663666	72
8.86	2.23691994	455	.10673639	97	9.86	2.33833982	372	.09644509	72
8.88	2.23905240	453	.10650923	96	9.88	2.34026697	370	.09626484	71
8.90	2.24118032	451	.10628304	96	9.90	2.34219042	369	.09608000	71
8.92	2.24330372	450	.10605781	95	9.92	2.34411018	368	.09589587	70
8.94	2.24542264	448	.10583353	95	9.94	2.34602626	366	.09571244	70
8.96	2.24753707	446	.10561019	94	9.96	2.34793868	365	.09552971	70
8.98	2.24964705	444	.10538780	93	9.98	2.34984745	363	.09534768	69
9.00	2.25175259	442	.10516634	93	10.00	2.35175259	362	.09516634	69

Interpolation Formula:

$$f(a + .02\theta) = f_0 + \theta f_1 - \frac{\theta^2}{6} \{(\phi + 1)\delta^2 f_0 + (\theta + 1)\delta^2 f_1\} + \frac{\theta^3}{120} \{(\phi + 1)(\phi + 1)\delta^3 f_0 + (\theta + 2)\delta^3 f_1\} \text{ where } \theta + \phi = 1.$$



TABLES OF  $F(x)$  AND  $F'(x)$

$x=10.00$  to  $12.00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
10.00	2.35173259	362	.09516634	69	11.00	2.44266168	302	.08690187	52
10.02	2.35365411	361	.09498568	68	11.02	2.44439821	301	.08675119	52
10.04	2.35555202	359	.09480571	68	11.04	2.44613173	300	.08660103	52
10.06	2.35744634	358	.09462642	68	11.06	2.44786225	299	.08645139	52
10.08	2.35933708	357	.09444781	67	11.08	2.44958979	298	.08630226	51
10.10	2.36122426	355	.09426987	67	11.10	2.45131435	297	.08615365	51
10.12	2.36310788	354	.09409260	67	11.12	2.45303594	296	.08600554	51
10.14	2.36498797	353	.09391599	66	11.14	2.45475457	295	.08585795	51
10.16	2.36686453	351	.09374005	66	11.16	2.45647026	294	.08571086	50
10.18	2.36873757	350	.09356476	65	11.18	2.45818301	293	.08556427	50
10.20	2.37060712	349	.09339013	65	11.20	2.45989283	292	.08541819	50
10.22	2.37247318	347	.09321614	65	11.22	2.46159974	291	.08527260	50
10.24	2.37433577	346	.09304280	64	11.24	2.46330374	290	.08512750	49
10.26	2.37619490	345	.09287011	64	11.26	2.46500485	289	.08498200	49
10.28	2.37805058	343	.09269806	64	11.28	2.46670306	288	.08483879	49
10.30	2.37990283	342	.09252664	63	11.30	2.46839840	287	.08469517	49
10.32	2.38175165	341	.09235585	63	11.32	2.47009087	286	.08455203	48
10.34	2.38359706	340	.09218569	63	11.34	2.47178048	285	.08440938	48
10.36	2.38543908	338	.09201616	62	11.36	2.47346725	284	.08426720	48
10.38	2.38727771	337	.09184725	62	11.38	2.47515118	283	.08412551	48
10.40	2.38911207	336	.09167896	61	11.40	2.47683227	282	.08398428	47
10.42	2.39094488	335	.09151128	61	11.42	2.47851055	281	.08384354	47
10.44	2.39277343	334	.09134422	61	11.44	2.48018602	280	.08370326	47
10.46	2.39459865	332	.09117776	60	11.46	2.48185868	279	.08356345	47
10.48	2.39642054	331	.09101191	60	11.48	2.48352856	278	.08342410	46
10.50	2.39823913	330	.09084666	60	11.50	2.48519565	277	.08328522	46
10.52	2.40005442	329	.09068201	60	11.52	2.48685997	276	.08314681	46
10.54	2.40186641	328	.09051796	59	11.54	2.48852153	275	.08300885	46
10.56	2.40367514	326	.09035449	59	11.56	2.49018033	274	.08287134	45
10.58	2.40548060	325	.09019162	59	11.58	2.49183638	273	.08273430	45
10.60	2.40728281	324	.09002933	58	11.60	2.49348970	272	.08259770	45
10.62	2.40908177	323	.08986762	58	11.62	2.49514029	271	.08246155	45
10.64	2.41087751	322	.08970650	58	11.64	2.49678817	270	.08232586	45
10.66	2.41267004	321	.08954595	57	11.66	2.49843333	270	.08219060	44
10.68	2.41445936	319	.08938597	57	11.68	2.50007579	269	.08205580	44
10.70	2.41624548	318	.08922656	57	11.70	2.50171557	268	.08192143	44
10.72	2.41802842	317	.08906772	56	11.72	2.50335265	267	.08178750	44
10.74	2.41980819	316	.08890945	56	11.74	2.50498707	267	.08165401	43
10.76	2.42158480	315	.08875173	56	11.76	2.50661882	266	.08152095	43
10.78	2.42335827	314	.08859458	56	11.78	2.50824791	265	.08138833	43
10.80	2.42512859	313	.08843798	55	11.80	2.50987435	264	.08125613	43
10.82	2.42689579	312	.08828193	55	11.82	2.51149816	263	.08112437	43
10.84	2.42865987	310	.08812643	55	11.84	2.51311933	262	.08099303	42
10.86	2.43042085	309	.08797148	54	11.86	2.51473788	261	.08086212	42
10.88	2.43217874	308	.08781707	54	11.88	2.51635382	261	.08073162	42
10.90	2.43393354	307	.08766320	54	11.90	2.51796715	260	.08060155	42
10.92	2.43568527	306	.08750987	54	11.92	2.51957788	259	.08047190	42
10.94	2.43743394	305	.08735708	53	11.94	2.52118603	258	.08034266	41
10.96	2.43917953	304	.08720481	53	11.96	2.52279159	257	.08021384	41
10.98	2.44092213	303	.08705308	53	11.98	2.52439459	256	.08008543	41
11.00	2.44266168	302	.08690187	52	12.00	2.52599501	256	.07995743	41

Interpolation Formula:

$$f(a + \phi h) = \phi f_0 + \phi_1 \delta f_0 + \frac{\phi \phi_1}{6} \{ (\phi + 1) \delta^2 f_0 + (\phi + 1) \delta^2 f_1 \} + \frac{\phi \phi_1 (\phi + 1) (\phi + 1)}{120} \{ (\phi + 2) \delta^3 f_0 + (\phi + 2) \delta^3 f_1 \} \text{ where } \theta + \phi = 1.$$

TABLES OF  $F(x)$  AND  $F(x)$

$x=12^{\circ}00$  to  $14^{\circ}00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
12.00	2.52599501	256	.07995743	41	13.00	2.60291809	219	.07404027	32
12.02	2.52759289	255	.07982984	41	13.02	2.60439780	219	.07393084	32
12.04	2.52918821	254	.07970265	40	13.04	2.60587533	218	.07382174	32
12.06	2.53078099	253	.07957587	40	13.06	2.60735067	217	.07371295	32
12.08	2.53237125	252	.07944949	40	13.08	2.60882385	217	.07360449	32
12.10	2.53395898	252	.07932351	40	13.10	2.61029485	216	.07349635	32
12.12	2.53554419	251	.07919793	40	13.12	2.61176370	215	.07338852	32
12.14	2.53712690	250	.07907275	39	13.14	2.61323040	215	.07328101	31
12.16	2.53870710	249	.07894796	39	13.16	2.61469494	214	.07317381	31
12.18	2.54028482	248	.07882357	39	13.18	2.61615735	214	.07306693	31
12.20	2.54186005	248	.07869956	39	13.20	2.61761762	213	.07296035	31
12.22	2.54343280	247	.07857595	39	13.22	2.61907577	212	.07285409	31
12.24	2.54500309	246	.07845272	39	13.24	2.62053179	212	.07274814	31
12.26	2.54657001	245	.07832988	38	13.26	2.62198570	211	.07264249	31
12.28	2.54813629	245	.07820742	38	13.28	2.62343749	210	.07253715	30
12.30	2.54969921	244	.07808535	38	13.30	2.62488718	210	.07243212	30
12.32	2.55125970	243	.07796365	38	13.32	2.62633478	209	.07232739	30
12.34	2.55281776	242	.07784234	38	13.34	2.62778028	209	.07222296	30
12.36	2.55437340	242	.07772140	38	13.36	2.62922370	208	.07211883	30
12.38	2.55592662	241	.07760083	37	13.38	2.63066504	207	.07201500	30
12.40	2.55747743	240	.07748064	37	13.40	2.63210430	207	.07191147	30
12.42	2.55902585	239	.07736082	37	13.42	2.63354150	206	.07180824	30
12.44	2.56057187	238	.07724137	37	13.44	2.63497663	206	.07170530	29
12.46	2.56211551	238	.07712229	37	13.46	2.63640971	205	.07160266	29
12.48	2.56365676	237	.07700358	37	13.48	2.63784074	204	.07150031	29
12.50	2.56519565	236	.07688522	36	13.50	2.63926973	204	.07139826	29
12.52	2.56673218	236	.07676724	36	13.52	2.64069667	203	.07129649	29
12.54	2.56826634	235	.07664961	36	13.54	2.64212159	203	.07119501	29
12.56	2.56979816	234	.07653234	36	13.56	2.64354447	202	.07109383	29
12.58	2.57132764	233	.07641544	36	13.58	2.64496534	202	.07099293	29
12.60	2.57285478	233	.07629888	36	13.60	2.64638419	201	.07089231	28
12.62	2.57437960	232	.07618269	35	13.62	2.64780104	200	.07079198	28
12.64	2.57590209	231	.07606684	35	13.64	2.64921587	200	.07069193	28
12.66	2.57742227	231	.07595135	35	13.66	2.65062872	199	.07059217	28
12.68	2.57894015	230	.07583621	35	13.68	2.65203956	199	.07049269	28
12.70	2.58045572	229	.07572142	35	13.70	2.65344842	198	.07039348	28
12.72	2.58196901	229	.07560697	35	13.72	2.65485530	198	.07029456	28
12.74	2.58348000	228	.07549287	34	13.74	2.65626021	197	.07019591	28
12.76	2.58498872	227	.07537911	34	13.76	2.65766314	196	.07009754	28
12.78	2.58649517	226	.07526569	34	13.78	2.65906411	196	.06999944	27
12.80	2.58799935	226	.07515262	34	13.80	2.66046312	195	.06990162	27
12.82	2.58950128	225	.07503988	34	13.82	2.66186018	195	.06980407	27
12.84	2.59100095	224	.07492748	34	13.84	2.66325529	194	.06970679	27
12.86	2.59249838	224	.07481542	33	13.86	2.66464845	194	.06960979	27
12.88	2.59399357	223	.07470369	33	13.88	2.66603968	193	.06951305	27
12.90	2.59548653	222	.07459230	33	13.90	2.66742898	193	.06941658	27
12.92	2.59697726	222	.07448124	33	13.92	2.66881635	192	.06932038	27
12.94	2.59846578	221	.07437050	33	13.94	2.67020179	192	.06922445	27
12.96	2.59995209	220	.07426010	33	13.96	2.67158533	191	.06912878	26
12.98	2.60143619	220	.07415002	33	13.98	2.67296695	191	.06903337	26
13.00	2.60291809	219	.07404027	32	14.00	2.67434666	190	.06893823	26

Interpolation Formula:

$$f(a + .02\theta) = \phi f_0 + \theta f_1 - \frac{\theta\phi}{6} \{(\phi + 1)\delta^2 f_0 + (\theta + 1)\delta^2 f_1\} + \frac{\theta\phi(\theta + 1)(\phi + 1)}{120} \{(\phi + 2)\delta^3 f_0 + (\theta + 2)\delta^3 f_1\} \text{ where } \theta + \phi = 1.$$

TABLES OF  $F(x)$  AND  $F'(x)$

$x=14.00$  to  $16.00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
14.00	2.67434666	190	.06893823	26	15.00	2.74101333	166	.06449378	21
14.02	2.67572448	190	.06884335	26	15.02	2.74230237	166	.06441073	21
14.04	2.67710040	189	.06874873	26	15.04	2.74358976	165	.06432789	21
14.06	2.67847443	188	.06865436	26	15.06	2.74487549	165	.06424526	21
14.08	2.67984057	188	.06856026	26	15.08	2.74615957	165	.06416285	21
14.10	2.68121684	187	.06846642	26	15.10	2.74744201	164	.06408065	21
14.12	2.68258523	187	.06837283	26	15.12	2.74872280	164	.06399865	21
14.14	2.68395175	187	.06827950	25	15.14	2.75000195	163	.06391687	21
14.16	2.68531641	186	.06818642	25	15.16	2.75127947	163	.06383529	21
14.18	2.68667921	185	.06809359	25	15.18	2.75255537	163	.06375392	21
14.20	2.68804016	185	.06800102	25	15.20	2.75382963	162	.06367276	21
14.22	2.68939926	184	.06790870	25	15.22	2.75510228	162	.06359181	21
14.24	2.69075651	184	.06781663	25	15.24	2.75637331	161	.06351106	21
14.26	2.69211192	183	.06772480	25	15.26	2.75764272	161	.06343052	20
14.28	2.69346550	183	.06763323	25	15.28	2.75891053	160	.06335018	20
14.30	2.69481725	182	.06754190	25	15.30	2.76017673	160	.06327004	20
14.32	2.69616718	182	.06745082	25	15.32	2.76144133	160	.06319011	20
14.34	2.69751529	181	.06735999	24	15.34	2.76270434	159	.06311037	20
14.36	2.69886158	181	.06726940	24	15.36	2.76396575	159	.06303084	20
14.38	2.70020607	180	.06717905	24	15.38	2.76522557	158	.06295151	20
14.40	2.70154874	180	.06708894	24	15.40	2.76648381	158	.06287238	20
14.42	2.70288962	180	.06699908	24	15.42	2.76774047	158	.06279345	20
14.44	2.70422871	179	.06690945	24	15.44	2.76899555	157	.06271471	20
14.46	2.70556600	179	.06682007	24	15.46	2.77024906	157	.06263617	20
14.48	2.70690151	178	.06673092	24	15.48	2.77150100	156	.06255783	20
14.50	2.70823524	178	.06664201	24	15.50	2.77275137	156	.06247968	20
14.52	2.70956720	177	.06655334	24	15.52	2.77400019	156	.06240173	19
14.54	2.71089738	177	.06646490	23	15.54	2.77524744	155	.06232397	19
14.56	2.71222579	176	.06637670	23	15.56	2.77649315	155	.06224641	19
14.58	2.71355245	176	.06628873	23	15.58	2.77773730	155	.06216904	19
14.60	2.71487734	175	.06620100	23	15.60	2.77897991	154	.06209186	19
14.62	2.71620049	175	.06611350	23	15.62	2.78022098	154	.06201487	19
14.64	2.71752189	174	.06602622	23	15.64	2.78146050	153	.06193807	19
14.66	2.71884154	174	.06593918	23	15.66	2.78269850	153	.06186147	19
14.68	2.72015945	173	.06585237	23	15.68	2.78393496	153	.06178505	19
14.70	2.72147564	173	.06576578	23	15.70	2.78516990	152	.06170882	19
14.72	2.72279009	173	.06567942	23	15.72	2.78640332	152	.06163278	19
14.74	2.72410281	172	.06559329	23	15.74	2.78763522	151	.06155692	19
14.76	2.72541382	172	.06550738	22	15.76	2.78886560	151	.06148126	19
14.78	2.72672311	171	.06542170	22	15.78	2.79009447	151	.06140577	19
14.80	2.72803069	171	.06533624	22	15.80	2.79132183	150	.06133048	18
14.82	2.72933656	170	.06525101	22	15.82	2.79254769	150	.06125536	18
14.84	2.73064073	170	.06516600	22	15.84	2.79377204	150	.06118043	18
14.86	2.73194320	169	.06508121	22	15.86	2.79499491	149	.06110569	18
14.88	2.73324398	169	.06499663	22	15.88	2.79621627	149	.06103112	18
14.90	2.73454307	168	.06491228	22	15.90	2.79743615	149	.06095674	18
14.92	2.73584047	168	.06482815	22	15.92	2.79865454	148	.06088254	18
14.94	2.73713620	168	.06474423	22	15.94	2.79987145	148	.06080852	18
14.96	2.73843024	167	.06466053	22	15.96	2.80108689	147	.06073468	18
14.98	2.73972262	167	.06457705	22	15.98	2.80230084	147	.06066102	18
15.00	2.74101333	166	.06449378	21	16.00	2.80351333	147	.06058753	18

Interpolation Formula:

$$f(a + .02\theta) = f_0 + \theta f_1 - \frac{\theta^2}{6} \{(\theta + 1)^2 f_2 + (\theta + 1) \delta^2 f_1\} + \frac{\theta^3(\theta + 1)(\theta + 2)}{120} \{(\theta + 2) \delta^2 f_2 + (\theta + 1) \delta^3 f_1\} \text{ where } \theta + \phi = 1.$$

TABLES OF  $F(x)$  AND  $F(x)$

$x=16.00$  to  $18.00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
16.00	2.80351333	147	.06058753	18	17.00	2.86233686	131	.05712733	15
16.02	2.80472435	146	.06051423	18	17.02	2.86347875	130	.05706215	15
16.04	2.80593390	146	.06044110	18	17.04	2.86461934	130	.05699712	15
16.06	2.80714199	146	.06036815	18	17.06	2.86575864	130	.05693224	15
16.08	2.80834863	146	.06029537	17	17.08	2.86689664	130	.05686750	15
16.10	2.80955381	145	.06022277	17	17.10	2.86803334	129	.05680291	15
16.12	2.81075754	145	.06015034	17	17.12	2.86916875	129	.05673847	15
16.14	2.81195982	144	.06007809	17	17.14	2.87030288	129	.05667418	15
16.16	2.81316066	144	.06000601	17	17.16	2.87143572	128	.05661003	15
16.18	2.81436006	144	.05993410	17	17.18	2.87256728	128	.05654602	14
16.20	2.81555803	143	.05986237	17	17.20	2.87369756	128	.05648216	14
16.22	2.81675456	143	.05979081	17	17.22	2.87482657	127	.05641845	14
16.24	2.81794966	143	.05971941	17	17.24	2.87595430	127	.05635488	14
16.26	2.81914334	142	.05964819	17	17.26	2.87708076	127	.05629145	14
16.28	2.82033559	142	.05957714	17	17.28	2.87820596	126	.05622816	14
16.30	2.82152642	142	.05950626	17	17.30	2.87932989	126	.05616502	14
16.32	2.82271584	141	.05943554	17	17.32	2.88045256	126	.05610201	14
16.34	2.82390385	141	.05936499	17	17.34	2.88157397	126	.05603915	14
16.36	2.82509044	141	.05929461	17	17.36	2.88269413	125	.05597643	14
16.38	2.82627563	140	.05922440	17	17.38	2.88381303	125	.05591385	14
16.40	2.82745942	140	.05915435	17	17.40	2.88493068	125	.05585141	14
16.42	2.82864181	140	.05908447	17	17.42	2.88604709	124	.05578911	14
16.44	2.82982280	139	.05901476	16	17.44	2.88716225	124	.05572694	14
16.46	2.83100240	139	.05894520	16	17.46	2.88827617	124	.05566492	14
16.48	2.83218061	139	.05887581	16	17.48	2.88938885	124	.05560303	14
16.50	2.83335743	138	.05880659	16	17.50	2.89050029	123	.05554128	14
16.52	2.83453287	138	.05873752	16	17.52	2.89161050	123	.05547967	14
16.54	2.83570693	138	.05866862	16	17.54	2.89271948	123	.05541819	14
16.56	2.83687962	137	.05859988	16	17.56	2.89382723	123	.05535685	14
16.58	2.83805093	137	.05853130	16	17.58	2.89493375	122	.05529565	14
16.60	2.83922087	137	.05846289	16	17.60	2.89603905	122	.05523458	13
16.62	2.84038945	136	.05839463	16	17.62	2.89714314	122	.05517365	13
16.64	2.84155666	136	.05832653	16	17.64	2.89824600	121	.05511285	13
16.66	2.84272251	136	.05825859	16	17.66	2.89934765	121	.05505218	13
16.68	2.84388700	135	.05819080	16	17.68	2.90044809	121	.05499165	13
16.70	2.84505014	135	.05812318	16	17.70	2.90154732	121	.05493125	13
16.72	2.84621193	135	.05805571	16	17.72	2.90264534	120	.05487098	13
16.74	2.84737237	134	.05798840	16	17.74	2.90374216	120	.05481084	13
16.76	2.84853147	134	.05792124	16	17.76	2.90483777	120	.05475084	13
16.78	2.84968922	134	.05785424	15	17.78	2.90593219	120	.05469097	13
16.80	2.85084564	134	.05778739	15	17.80	2.90702541	119	.05463123	13
16.82	2.85200072	133	.05772070	15	17.82	2.90811744	119	.05457161	13
16.84	2.85315447	133	.05765416	15	17.84	2.90920828	119	.05451213	13
16.86	2.85430689	133	.05758778	15	17.86	2.91029793	119	.05445278	13
16.88	2.85545798	132	.05752154	15	17.88	2.91138639	118	.05439356	13
16.90	2.85660775	132	.05745546	15	17.90	2.91247367	118	.05433446	13
16.92	2.85775620	132	.05738954	15	17.92	2.91355977	118	.05427550	13
16.94	2.85890333	131	.05732376	15	17.94	2.91464469	117	.05421666	13
16.96	2.86004915	131	.05725813	15	17.96	2.91572844	117	.05415795	13
16.98	2.86119366	131	.05719266	15	17.98	2.91681101	117	.05409936	13
17.00	2.86233686	131	.05712733	15	18.00	2.91789241	117	.05404091	13

Interpolation Formula:

$$f(a + \theta\delta) = f_0 + \theta f_1 - \frac{\theta\phi}{6} \{(\phi + 1)\delta^2 f_0 + (\theta + 1)\delta^2 f_1\} + \frac{\theta\phi(\theta + 1)(\phi + 1)}{120} \{(\phi + 2)\delta^3 f_0 + (\theta + 2)\delta^3 f_1\} \text{ where } \theta + \phi = 1.$$

TABLES OF  $F(x)$  AND  $F(x)$

$x=18.00$  to  $20.00$

$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +	$x$	$F(x)$	$\delta^2$ -	$F(x)$	$\delta^2$ +
18.00	2.91789241	117	.05404091	13	19.00	2.97052399	105	.05127082	11
18.02	2.91897265	117	.05398257	13	19.02	2.97154888	105	.05121831	11
18.04	2.92005172	116	.05392437	13	19.04	2.97257273	105	.05116591	11
18.06	2.92112962	116	.05386629	13	19.06	2.97359552	104	.05111362	11
18.08	2.92220637	116	.05380833	12	19.08	2.97461727	104	.05106143	11
18.10	2.92328196	115	.05375050	12	19.10	2.97563798	104	.05100935	11
18.12	2.92435639	115	.05369280	12	19.12	2.97665765	104	.05095738	11
18.14	2.92542967	115	.05363521	12	19.14	2.97767627	104	.05090551	11
18.16	2.92650180	115	.05357776	12	19.16	2.97869387	103	.05085374	11
18.18	2.92757278	115	.05352042	12	19.18	2.97971042	103	.05080208	10
18.20	2.92864262	114	.05346320	12	19.20	2.98072595	103	.05075053	10
18.22	2.92971131	114	.05340611	12	19.22	2.98174045	103	.05069908	10
18.24	2.93077886	114	.05334914	12	19.24	2.98275391	103	.05064774	10
18.26	2.93184528	114	.05329229	12	19.26	2.98376636	102	.05059650	10
18.28	2.93291056	113	.05323557	12	19.28	2.98477778	102	.05054536	10
18.30	2.93397470	113	.05317896	12	19.30	2.98578817	102	.05049432	10
18.32	2.93503771	113	.05312247	12	19.32	2.98679755	102	.05044339	10
18.34	2.93609960	113	.05306611	12	19.34	2.98780591	102	.05039256	10
18.36	2.93716036	112	.05300986	12	19.36	2.98881325	101	.05034184	10
18.38	2.93822000	112	.05295373	12	19.38	2.98981958	101	.05029121	10
18.40	2.93927851	112	.05289772	12	19.40	2.99082490	101	.05024069	10
18.42	2.94033591	112	.05284183	12	19.42	2.99182921	101	.05019027	10
18.44	2.94139218	111	.05278606	12	19.44	2.99283251	101	.05013995	10
18.46	2.94244735	111	.05273040	12	19.46	2.99383481	100	.05008973	10
18.48	2.94350140	111	.05267486	12	19.48	2.99483610	100	.05003961	10
18.50	2.94455434	111	.05261944	12	19.50	2.99583639	100	.04998959	10
18.52	2.94560618	111	.05256414	12	19.52	2.99683569	100	.04993967	10
18.54	2.94665691	110	.05250895	12	19.54	2.99783398	100	.04988985	10
18.56	2.94770654	110	.05245387	12	19.56	2.99883128	99	.04984013	10
18.58	2.94875507	110	.05239892	12	19.58	2.99982759	99	.04979051	10
18.60	2.94980250	110	.05234407	11	19.60	3.00082290	99	.04974099	10
18.62	2.95084883	109	.05228934	11	19.62	3.00181723	99	.04969157	10
18.64	2.95189407	109	.05223473	11	19.64	3.00281057	98	.04964224	10
18.66	2.95293822	109	.05218023	11	19.66	3.00380292	98	.04959301	10
18.68	2.95398128	109	.05212584	11	19.68	3.00479429	98	.04954388	10
18.70	2.95502325	108	.05207157	11	19.70	3.00578467	98	.04949485	10
18.72	2.95606414	108	.05201741	11	19.72	3.00677408	98	.04944591	10
18.74	2.95710395	108	.05196336	11	19.74	3.00776251	98	.04939707	10
18.76	2.95814268	108	.05190943	11	19.76	3.00874997	97	.04934833	10
18.78	2.95918033	108	.05185560	11	19.78	3.00973645	97	.04929968	10
18.80	2.96021690	107	.05180189	11	19.80	3.01072195	97	.04925113	10
18.82	2.96125240	107	.05174829	11	19.82	3.01170649	97	.04920267	10
18.84	2.96228684	107	.05169480	11	19.84	3.01269006	97	.04915431	9
18.86	2.96332020	107	.05164142	11	19.86	3.01367266	96	.04910605	9
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19.00	2.97052399	105	.05127082	11	20.00	3.02052399	95	.04877082	9

Interpolation Formula:

$$f(a + .02\theta) = \phi f_0 + \theta f_1 - \frac{\theta\phi}{6} \{(\phi + 1)\delta^2 f_0 + (\theta + 1)\delta^2 f_1\} + \frac{\theta\phi(\theta + 1)(\phi + 1)}{120} \{(\phi + 2)\delta^3 f_0 + (\theta + 2)\delta^3 f_1\} \text{ where } \theta + \phi = 1.$$

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# THE COMPLETE $\Gamma$ -FUNCTION

BY EGON S. PEARSON, B.A.

## I. *Methods of Calculation.*

The publication of the Tables of the Incomplete Gamma Function\*, or rather of the ratio

$$I_x(p) = \int_0^x e^{-x} x^{p-1} dx / \int_0^\infty e^{-x} x^{p-1} dx = \int_0^x e^{-x} x^{p-1} dx / \Gamma(p),$$

renders desirable the issue of a Table of the Complete Gamma Function in readily accessible form. In many statistical problems the value of  $\Gamma(p)$  is required for high values of  $p$ , amounting perhaps to several hundreds, and in such cases it is practically essential to deal with  $\text{Log } \Gamma(p)$ ; it is this function which is tabled below. The most important Tables of  $\text{Log } \Gamma(p)$  that have been published are:

(i) That of Gauss, first issued in 1813† and to be found in his *Werke*, Band III. s. 161–162, Göttingen 1866; this Table gives to twenty places of decimals the value of  $\text{Log } \Gamma(p)$  in the range 1.00 to 2.00 for an argument interval of 0.01;

(ii) That of Legendre originally published in Tom. II. (pp. 490–499) of his *Traité des Fonctions Elliptiques, avec des Tables pour en faciliter le calcul numérique*, Paris 1825; this Table is calculated to twelve places of decimals for the range 1.000 to 2.000 with an argument interval of 0.001. A facsimile reproduction has been issued as No. IV of the present series of Tracts for Computers.

The value of  $\text{Log } \Gamma(p)$  for values of  $p$  lying outside the range of these two tables may be calculated by various methods:

(a) By successive use of the reduction relation

$$\Gamma(p) = (p-1) \Gamma(p-1),$$

} a laborious process if  $p$  is much above 2.

\* Tables of the Incomplete Gamma Function; issued by His Majesty's Stationery Office, 1922.

† *Disquisitiones Generales Circa Seriem Infinitam*

$$1 + \frac{\alpha \cdot \beta}{1 \cdot \gamma} x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1 \cdot 2 \cdot \gamma(\gamma+1)} x^2 + \dots$$

*Commentationes Societatis regiae Scientiarum Göttingensis recentiores*, Vol. II. Göttingae MD.CCCXIII.

(b) By Stirling's asymptotic expansion

$$\begin{aligned} \text{Log}_e \Gamma(p) &= (p - \frac{1}{2}) \log_e p - p + \frac{1}{2} \log_e 2\pi \\ &+ \frac{1}{12p} - \frac{1}{360p^3} + \frac{1}{1260p^5} - \frac{1}{1680p^7} + \frac{1}{1188p^9} - \dots \end{aligned}$$

(c) By Professor Forsyth's formula

$$\Gamma(p+1) = \sqrt{2\pi} \left\{ \frac{\sqrt{p^2 + p + \frac{1}{4}}}{e} \right\}^{p+\frac{1}{2}},$$

which may be used for calculation of  $\text{Log } \Gamma(p+1)$  to 5 or 6 places of decimals.

(d) By Professor Pearson's formula\*

$$\text{Log} \frac{\Gamma(p+1)}{p^p e^{-p}} = 0.399,0899 + \frac{1}{2} \log p + .080,929 \sin \frac{25^\circ.623}{p},$$

which is particularly convenient in many statistical problems where it is the ratio of  $\Gamma(p+1)$  to  $p^p e^{-p}$  that occurs.

(e) By interpolation in a Table of the Logarithms of factorials†.

These methods are none of them simple, and may be extremely laborious if a high degree of accuracy is required. It is hoped, therefore, that the computer's problem will be simplified by the issue of the present Table containing the values of  $\text{Log } \Gamma(p)$  to ten places of decimals from  $p=2$  to  $p=1200$ , together with central differences for purposes of interpolation‡.

\* *Biometrika*, Vol. vi. pp. 118-119.

† As for example Degen's Table of the Logarithms of factorials up to 1200, calculated to eighteen decimal places. *Tabularum ad faciliorem et breviorum probabililitatis computationem utilium Enneas*. Havniæ mccccxxv. The Logarithms of factorials up to 1000 to seven decimal places are also given in *Tables for Statisticians and Biometricians*. Cambridge University Press.

‡ To make full use of the present Table, ten figure logarithms are of course required. These are unfortunately not readily accessible. Vega's *Thesaurus Logarithmorum completus*, Leipzig, 1794, is now a rare book; it also contains certain last figure errors. Andoyer, in the introduction to his *Nouvelles Tables Trigonométriques Fondamentales*, states that photozincographic reproductions of Vega's Tables were issued by the Instituto Geografico Militare at Florence in 1889, 1892 and 1910; the original errors are reproduced in these facsimile reprints. Duffield's ten figure Table calculated independently of Vega's, and issued for the U.S. Coast and Geodetic Survey, Washington, 1897, has been long out of print.

Briggs' Table (to fourteen figures) may be used, and there are several other tables going to more than ten figures, but their range is small and lengthy interpolation may be required in using them.

II. *Construction and Scope of the present Table.*

The Table has been constructed as follows :

(i) The values of  $\text{Log } \Gamma(p)$  for the range  $p = 2$  to  $p = 70$  were obtained by method (a) above, by the successive addition of the logarithms of ordinary numbers to the logarithms of  $\Gamma(p)$  (range  $p = 1$  to  $p = 2$ ). The work was first carried out starting from Legendre's twelve figure Table\*, but as it was found that in certain cases this led to a unit error in the tenth place of a 4th or 6th difference, the results were checked by working to fifteen places of decimals from Gauss' Table, and finally cutting off to ten places. This portion of the Table is divided into two parts: from  $p = 2.0$  to  $5.0$ , the argument interval is  $0.1$ , while from  $p = 5.0$  to  $70.0$  it is  $0.2$ .

(ii) For the range  $p = 70$  to  $p = 1200$ ,  $\text{Log } \Gamma(p)$  is given for every integral value of  $p$ ; the values of the function have been taken from Degen's Table of the Logarithms of factorials†, where of course  $\text{Log } (p!) = \text{Log } \Gamma(p+1)$ . The differences were worked out using twelve decimal places, and the last two figures then cut off, but here again when doubtful cases arose (i.e. when the 11th figure was nearly 5) it was necessary to examine the 13th or 14th figures.

The Table may of course be used to find values of the Logarithms of Factorials. To obtain from it values of  $\text{Log } \Gamma(p)$ , interpolation will in general be necessary, and the differences tabled are the second, fourth and sixth differences— $\delta^2$ ,  $\delta^4$  and  $\delta^6$ —which should be used in Everett's Central Difference Interpolation Formula‡:

$$z_\theta = \phi z_0 + \theta z_1 - \frac{1}{8} \phi \theta \{ (1 + \phi) \delta^2 z_0 + (1 + \theta) \delta^2 z_1 \} \\ + \frac{1}{144} \phi \theta (1 + \phi) (1 + \theta) \{ (2 + \phi) \delta^4 z_0 + (2 + \theta) \delta^4 z_1 \} \\ - \frac{1}{1080} \phi \theta (1 + \phi) (1 + \theta) (2 + \phi) (2 + \theta) \{ (3 + \phi) \delta^6 z_0 + (3 + \theta) \delta^6 z_1 \} + \dots,$$

where  $z_0 = f(x_0)$ ,  $z_1 = f(x_0 + h)$ ,  $z_\theta = f(x_0 + \theta h)$ ,

$h$  is the argument interval and  $\theta$  and  $\phi$  are two positive quantities such that  $\theta + \phi = 1$ .

If this formula be written

$$z_\theta = \phi z_0 + \theta z_1 - [e_2(\phi) \delta^2 z_0 + e_2(\theta) \delta^2 z_1] + [e_4(\phi) \delta^4 z_0 + e_4(\theta) \delta^4 z_1] \\ - [e_6(\phi) \delta^6 z_0 + e_6(\theta) \delta^6 z_1] + \dots,$$

\* A number of last (i.e. 12th) figure errors have been found in Legendre's Table. In testing his values against those of Gauss (who goes to twenty figures), an error of a unit was found in twenty-three cases out of a hundred; the errors were both positive and negative.

† See footnote † to p. iv above.

‡ *Journal of the Institute of Actuaries*, Vol. xxxv. p. 452.

the functions  $\epsilon_2$ ,  $\epsilon_4$  and  $\epsilon_6$  have been tabled for values of  $\theta$  (and therefore of  $\phi$ ) lying between zero and unity by Mr A. J. Thompson in Tract No. V of this Series\*. Using his Table the whole process of an interpolation can be carried out by a practised computer in one continuous operation on the machine. The functions are tabled to ten figures; if the value of  $\text{Log } \Gamma(p)$  be required to the full ten places of decimals, it is advisable to take out  $\epsilon_2$  to eight figures, while for seven place accuracy in  $\text{Log } \Gamma(p)$ , only five figures of  $\epsilon_2$  need be used. For  $\epsilon_4$  less figures are actually needed, but if the operation is to be a continuous one on the machine, little is gained and mistakes may be introduced by taking out fewer figures than for  $\epsilon_2$ .

It may be stated as a rough guide, that when working with Everett's Formula it is unnecessary to use a 2nd difference under 4, a 4th difference under 20, or a 6th difference under 100 (where these differences correspond to the last figures of the tabled entries). Thus in working to ten decimal places, the 6th differences of  $\text{Log } \Gamma(p)$  are only required within the range  $p = 2$  to  $p = 10$ , while 4th differences become negligible at about  $p = 800$ . If working to seven decimal places only, 6th differences will not be required at all, and 4th differences only between  $p = 2$  and  $p = 10$ , and again from  $p = 70$  to  $p = 80$ . These rules will serve for practical purposes, although an error of a unit in the last place may occur in unfavourable circumstances; in fact it is obvious that in any process involving the addition of quantities cut off at a certain figure, this possibility cannot be eliminated altogether although the chance of the occurrence of a unit last figure error may be reduced by retaining differences below the limits given above.

### III. Illustrative Examples.

#### Example 1.

To find  $\text{Log } \Gamma(6.8234)$  to ten places of decimals. At this point of the Table the argument interval is 0.2; we find

$$\begin{array}{ll} z_0 = \log \Gamma(6.8) = 2.696\ 012\ 0302, & z_1 = \log \Gamma(7.0) = 2.857\ 332\ 4964, \\ \delta^2 z_0 = & 2\ 752\ 1435, & \delta^2 z_1 = & 2\ 667\ 7699, \\ \delta^4 z_0 = & 5\ 5003, & \delta^4 z_1 = & 5\ 0111, \\ \delta^6 z_0 = & 657, & \delta^6 z_1 = & 562. \end{array}$$

$$\theta = \frac{.234}{2} = .117, \quad \phi = .883.$$

$$\begin{array}{ll} \epsilon_2(\phi) = .03242\ 244, & \epsilon_2(\theta) = .01923\ 306, \\ \epsilon_4(\phi) = 522\ 052, & \epsilon_4(\theta) = 383\ 345, \\ \epsilon_6(\phi) = 102\ 177, & \epsilon_6(\theta) = 82\ 020. \end{array}$$

\* *Tracts for Computers*, No. V. Cambridge University Press.



Hence we find, taking the terms separately,

$$\begin{aligned} \phi z_0 + \theta z_1 &= + 2.714\ 886\ 5247(5) \\ -[\epsilon_2(\phi) \delta^2 z_0 + \epsilon_2(\theta) \delta^2 z_1] &= - 140\ 5405(9) \\ +[\epsilon_4(\phi) \delta^4 z_0 + \epsilon_4(\theta) \delta^4 z_1] &= + 479(2) \\ -[\epsilon_6(\phi) \delta^6 z_0 + \epsilon_6(\theta) \delta^6 z_1] &= - 1(1) \\ \hline \text{Log } \Gamma(6.8234) &= 2.714\ 746\ 0320 \end{aligned}$$

By the addition of the logarithms of 1.8234, 2.8234, 3.8234, 4.8234 and 5.8234 to the value of  $\log \Gamma(1.8234)$  interpolated from Legendre's Table, the same result was obtained\*. Were Stirling's asymptotic expansion used, it would be necessary to include the term  $\frac{1}{1188p^2}$  to be certain of tenth figure accuracy.

*Example 2.*

To find  $\text{Log } \Gamma(691.43)$  to ten places of decimals.

$$\begin{aligned} z_0 = \log \Gamma(691) &= 1660.961\ 247\ 0260, & z_1 = \log \Gamma(692) &= 1663.800\ 725\ 0734, \\ \delta^2 z_0 &= 628\ 9566, & \delta^2 z_1 &= 628\ 0471, \\ \delta^4 z_0 &= 26, & \delta^4 z_1 &= 26. \\ \phi &= .57, & \theta &= .43. \\ \phi z_0 + \theta z_1 &= 1662.182\ 222\ 5863(8) \\ -[\epsilon_2(\phi) \delta^2 z_0 + \epsilon_2(\theta) \delta^2 z_1] &= - 77\ 0255(0) \\ +[\epsilon_4(\phi) \delta^4 z_0 + \epsilon_4(\theta) \delta^4 z_1] &= (6) \\ \hline \text{Log } \Gamma(691.43) &= 1662.182\ 145\ 5609 \end{aligned}$$

*Example 3.*

If  $\theta$  contains more than three decimal places it becomes necessary to interpolate into Thompson's Table to obtain the values of the  $\epsilon$  functions. This of course lengthens the process of computation, but if  $\text{Log } \Gamma(p)$  is required to seven decimal places only,  $\epsilon_6$  need only be taken to five places and linear interpolation for its value is sufficient throughout the Table. The process is illustrated in the following example.

To find  $\text{Log } \Gamma(72.38365)$  to seven places of decimals.

$$\begin{aligned} z_0 = \text{Log } \Gamma(72) &= 101.929\ 6634, & z_1 = \log \Gamma(73) &= 103.786\ 9959, \\ \delta^2 z_0 &= 6\ 0741, & \delta^2 z_1 &= 5\ 9904, \\ \delta^4 z_0 &= 24, & \delta^4 z_1 &= 23. \end{aligned}$$

\* The process is however lengthy, since for last figure accuracy it is necessary to work with logarithms to eleven places, and even using Briggs' Table interpolation may be necessary.

$$\begin{array}{rcl} \phi = \cdot 61635, & \theta = \cdot 38365. & \\ \epsilon_2(616) = \cdot 063709 & \epsilon_2(\cdot 383) = \cdot 054470 & \\ \epsilon_2(617) = \cdot 063686 & \epsilon_2(\cdot 384) = \cdot 054563 & \\ \Delta = - & 23 & \Delta = + & 93 \end{array}$$

Hence  $\epsilon_2(\phi) = \cdot 06370,$   $\epsilon_2(\theta) = \cdot 05453,$   
 $\epsilon_4(\phi) = \cdot 01153$  (at sight),  $\epsilon_4(\theta) = \cdot 01050$  (at sight).

$$\begin{aligned} \phi z_0 + \theta z_1 &= 102\cdot 642\ 2290(1) \\ -[\epsilon_2(\phi) \delta^2 z_0 + \epsilon_2(\theta) \delta^2 z_1] &= - & 7135(8) \\ [\epsilon_4(\phi) \delta^4 z_0 + \epsilon_4(\theta) \delta^4 z_1] &= & (5) \\ \text{Log } \Gamma(72\cdot 38365) &= \underline{102\cdot 641\ 5155} \end{aligned}$$

#### IV. *The Derivatives of Log $\Gamma(p)$ .*

It is also possible to use the present Table to calculate the values of the successive Derivatives of the Logarithm of the Gamma Function, or of

$$F(p) = \frac{d}{dp} \log_e \Gamma(1+p) = -\gamma + \sum_{n=1}^{\infty} \frac{p}{n(n+p)},$$

where

$$\gamma = 0\cdot 5772157\dots, \text{ or Euler's Constant.}$$

$$F(p) = \frac{d^2}{dp^2} \log_e \Gamma(1+p) = \sum_{n=1}^{\infty} \frac{1}{(n+p)^2},$$

$$\frac{d^3}{dp^3} \log_e \Gamma(1+p) = -2 \sum_{n=1}^{\infty} \frac{1}{(n+p)^3}, \text{ etc.}$$

Values of the first function,  $\frac{d}{dp} \log_e \Gamma(1+p)$ , have been given by Gauss\* to eighteen decimal places for the range  $p = 0$  to  $p = 1$  with argument interval  $\cdot 01$ , and also by Prof. G. N. Watson† to thirteen decimal places for all integers and halves of odd integers from 0 to 100. *Tracts for Computers*, No. 1‡, contains Tables to eight decimal places of both  $F(p)$  and  $F(p)$ , for the range  $p = 0$  to  $p = 20$ , and with argument interval  $\cdot 02$ .

The value of all these functions can however be obtained by interpolation from the present Table, with the help of the following central difference interpolation formulae for the derivatives of  $z_\theta = f(x_\theta + \theta h)$ .

\* *Werke*, Bd III. S. 161, 162. Göttingen, 1866.

† *Report of British Association*, 1916, pp. 125 and 126.

‡ Cambridge University Press. The symbols  $F$  and  $F$  and the names Digamma and Trigamma Functions were there first suggested.  $\frac{d^3}{dp^3} \log_e \Gamma(1+p)$  might be called the Tetragamma Function.

$$(i) \quad h \frac{dx_0}{dx} = \phi \frac{x_1 - x_{-1}}{2} + \theta \frac{x_2 - x_0}{2} \\ - \left[ \left\{ \frac{1}{2} \phi + \epsilon_2(\phi) \right\} \frac{\delta^2 x_1 - \delta^2 x_{-1}}{2} + \left\{ \frac{1}{2} \theta + \epsilon_2(\theta) \right\} \frac{\delta^2 x_2 - \delta^2 x_0}{2} \right] \\ + \left[ \left\{ \frac{1}{30} \phi + \frac{1}{6} \epsilon_2(\phi) + \epsilon_4(\phi) \right\} \frac{\delta^4 x_1 - \delta^4 x_{-1}}{2} + \left\{ \frac{1}{30} \theta + \frac{1}{6} \epsilon_2(\theta) + \epsilon_4(\theta) \right\} \frac{\delta^4 x_2 - \delta^4 x_0}{2} \right] \\ - \dots\dots,$$

$$(ii) \quad h^2 \frac{d^2 x}{dx^2} = \phi \delta^2 x_0 + \theta \delta^2 x_1 - \left[ \left\{ \frac{1}{12} \phi + \epsilon_2(\phi) \right\} \delta^4 x_0 + \left\{ \frac{1}{12} \theta + \epsilon_2(\theta) \right\} \delta^4 x_1 \right] \\ + \left[ \left\{ \frac{1}{30} \phi + \frac{1}{12} \epsilon_2(\phi) + \epsilon_4(\phi) \right\} \delta^6 x_0 + \left\{ \frac{1}{30} \theta + \frac{1}{12} \epsilon_2(\theta) + \epsilon_4(\theta) \right\} \delta^6 x_1 \right] \\ - \dots\dots,$$

$$(iii) \quad h^3 \frac{d^3 x}{dx^3} = \phi \frac{\delta^3 x_1 - \delta^3 x_{-1}}{2} + \theta \frac{\delta^3 x_2 - \delta^3 x_0}{2} \\ - \left[ \left\{ \frac{1}{4} \phi + \epsilon_2(\phi) \right\} \frac{\delta^5 x_1 - \delta^5 x_{-1}}{2} + \left\{ \frac{1}{4} \theta + \epsilon_2(\theta) \right\} \frac{\delta^5 x_2 - \delta^5 x_0}{2} \right] \\ + \left[ \left\{ \frac{1}{120} \phi + \frac{1}{4} \epsilon_2(\phi) + \epsilon_4(\phi) \right\} \frac{\delta^7 x_1 - \delta^7 x_{-1}}{2} + \left\{ \frac{1}{120} \theta + \frac{1}{4} \epsilon_2(\theta) + \epsilon_4(\theta) \right\} \frac{\delta^7 x_2 - \delta^7 x_0}{2} \right] \\ - \dots\dots,$$

where  $h$  is the argument interval,  $\theta$ ,  $\phi$ ,  $\epsilon_2$ , and  $\epsilon_4$  are the Everett coefficients referred to above, and

$$x_{-1} = f(x_0 - h); \quad x_0 = f(x_0); \quad x_1 = f(x_0 + h); \quad x_2 = f(x_0 + 2h).$$

The following rules provide a rough guide to the order of differences which can be neglected:

- In (i) it is unnecessary to use the term containing the 4th differences if 5th differences are less than 6;
- in (ii) it is unnecessary to use 6th differences less than 10;
- in (iii) the term containing 6th differences is negligible if 7th differences are less than 4;

where the differences correspond to the last figures of the tabled entries.

It will be noted that if  $\theta = 0$  and  $\phi = 1$ , or we require the value of a derivative of the function for a value of  $x$  at which the function is tabled, the equations (i), (ii) and (iii) reduce to much simpler forms, for in this case

$$\epsilon_2(\phi) = \epsilon_2(\theta) = 0, \quad \epsilon_4(\phi) = \epsilon_4(\theta) = 0.$$

*Example.*

To find  $F(37.68) = \frac{d^2}{dp^2} \log_{10} \Gamma(38.68) \times \log_e 10.$

Here  $z_0 = \log \Gamma(38.6), \quad z_1 = \log \Gamma(38.8),$   
 $\delta^2 z_0 = .000\ 455\ 9281, \quad \delta^2 z_1 = .000\ 453\ 5476,$   
 $\delta^4 z_0 = \quad \quad 251, \quad \delta^4 z_1 = \quad \quad 247.$

The sixth difference will be less than 10 and is therefore negligible.

$h = .2,$	$\phi = .6,$	$\theta = .4.$
$\epsilon_2(\phi) = .06400\ 00$		$\epsilon_2(\theta) = .05600\ 00$
$\frac{1}{12}\phi = .05000\ 00$		$\frac{1}{12}\theta = .03333\ 33$
$.11400\ 00$		$.08933\ 33$

$$\begin{aligned} & \phi \delta^2 z_0 + \theta \delta^2 z_1 = .000\ 454\ 9759(0) \\ - [ \{ \frac{1}{12}\phi + \epsilon_2(\phi) \} \delta^4 z_0 + \{ \frac{1}{12}\theta + \epsilon_2(\theta) \} \delta^4 z_1 ] = & \quad \quad \quad 50(7) \\ & \quad \quad \quad \underline{\underline{.000\ 454\ 9708(3)}} \end{aligned}$$

Multiplying .000 454 97083 by  $\log_e 10 (= 2.302\ 585\ 0930)$  and dividing by  $h^2 (= .04)$  we have finally

$F(37.68) = .0261\ 9023$  to eight decimal places ;

this result agrees with that given by Miss Pairman as an example on p. 6 of the introduction to her Table\*.

\* See footnote to p. viii above.

TABLE OF THE LOGARITHMS OF THE COMPLETE  
 $\Gamma$ -FUNCTION, ARGUMENTS 2 TO 1200

Complete  $\Gamma$ -Functions.

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$	$\delta^6$	$p$
2.0	0.000 000 0000	2 802 7032	21 6033	9266	2.0
2.1	0.019 733 3591	2 637 0316	18 1288	6976	2.1
2.2	0.042 103 7499	2 489 4889	15 3520	5227	2.2
2.3	0.066 963 6296	2 357 2980	13 1078	4121	2.3
2.4	0.094 180 8071	2 238 2150	11 2757	3225	2.4
2.5	0.123 636 1997	2 130 4077	9 7662	2552	2.5
2.6	0.155 222 0001	2 032 3666	8 5118	2039	2.6
2.7	0.188 840 1671	1 942 8373	7 4614	1644	2.7
2.8	0.224 401 1713	1 860 7894	6 5764	1337	2.8
2.9	0.261 822 9450	1 785 2769	5 8231	1096	2.9
3.0	0.301 029 9957	1 715 6076	5 1804	905	3.0
3.1	0.344 952 6539	1 661 1186	4 6282	752	3.1
3.2	0.384 626 4307	1 619 2680	4 1513	629	3.2
3.3	0.428 691 4656	1 585 5486	3 7372	529	3.3
3.4	0.474 392 0488	1 458 5763	3 3760	448	3.4
3.5	0.521 576 2084	1 484 9801	3 0697	381	3.5
3.6	0.570 196 3481	1 389 4435	2 7814	326	3.6
3.7	0.620 203 9312	1 346 6883	2 5357	280	3.7
3.8	0.671 589 2027	1 306 4688	2 3179	241	3.8
3.9	0.724 220 9429	1 268 5672	2 1243	209	3.9
4.0	0.778 151 2304	1 232 7899	1 9515	182	4.0
4.1	0.833 314 3477	1 198 9640	1 7969	158	4.1
4.2	0.889 676 4090	1 166 9360	1 6581	139	4.2
4.3	0.947 206 4054	1 136 6641	1 5332	122	4.3
4.4	1.006 870 9668	1 107 7264	1 4205	107	4.4
4.5	1.066 644 2628	1 080 3092	1 3185	96	4.5
4.6	1.126 497 8488	1 054 2104	1 2260	84	4.6
4.7	1.188 406 6563	1 029 2376	1 1419	75	4.7
4.8	1.251 342 7993	1 006 6067	1 0658	67	4.8
4.9	1.315 285 5600	982 9411	9954	60	4.9
5.0	1.380 211 2417	961 2709	9316	53	5.0
5.0	1.380 211 2417	3 846 0153	14 9466	2457	5.0
5.2	1.512 926 6994	3 683 4852	13 1401	2791	5.2
5.4	1.649 323 6423	3 524 0963	11 6127	2273	5.4
5.6	1.789 258 6806	3 396 3180	10 3126	1867	5.6
5.8	1.932 584 0367	3 288 8632	9 1991	1544	5.8
6.0	2.079 181 2460	3 150 6876	8 2401	1286	6.0
6.2	2.228 929 0430	3 040 6621	7 4096	1078	6.2
6.4	2.381 717 4022	2 937 9463	6 6869	909	6.4
6.6	2.537 443 7075	2 842 0173	6 0651	770	6.6
6.8	2.696 012 0302	2 752 1435	5 5003	657	6.8
7.0	2.857 332 4964	2 667 7699	5 0111	562	7.0
7.2	3.021 320 7325	2 588 4075	4 5783	484	7.2
7.4	3.187 897 3761	2 513 6233	4 1938	418	7.4
7.6	3.356 987 6431	2 443 0329	3 8511	363	7.6
7.8	3.528 520 9429	2 376 2936	3 5447	316	7.8
8.0	3.702 430 5364	2 313 0990	3 2700	276	8.0
8.2	3.878 663 3290	2 253 1744	3 0228	243	8.2
8.4	4.057 129 0959	2 196 2726	2 8000	214	8.4
8.6	4.237 801 2354	2 142 1708	2 5985	189	8.6
8.8	4.420 616 5456	2 090 6675	2 4169	167	8.8
9.0	4.606 520 5234	2 041 6801	2 2500	146	9.0
9.2	4.792 467 0813	1 994 7427	2 0989	132	9.2
9.4	4.981 408 3819	1 950 0041	1 9610	118	9.4
9.6	5.172 299 6866	1 907 2266	1 8349	106	9.6
9.8	5.365 098 2178	1 866 2839	1 7195	96	9.8
10.0	5.569 763 0329	1 827 0607	1 6136	86	10.0

## Tables of the Logarithms

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
10.0	5.559 763 0329	1 827 0607	1 6156
10.2	5.756 254 9087	1 789 4510	1 5160
10.4	5.954 536 2355	1 753 3573	1 4262
10.6	6.154 570 9196	1 718 6897	1 3434
10.8	6.356 324 2935	1 685 3666	1 2668
11.0	6.559 763 0329	1 653 3082	1 1960
11.2	6.764 656 0805	1 622 4468	1 1303
11.4	6.971 569 6748	1 592 7157	1 0694
11.6	7.179 876 7849	1 564 0540	1 0127
11.8	7.389 748 0490	1 536 4050	9600
12.0	7.601 156 7180	1 509 7160	9109
12.2	7.814 078 1081	1 483 9379	8661
12.4	8.028 474 4262	1 459 0249	8223
12.6	8.244 334 7741	1 434 9342	7822
12.8	8.461 630 0603	1 411 6257	7448
13.0	8.680 336 9641	1 389 0619	7096
13.2	8.900 432 9338	1 367 2078	6767
13.4	9.121 896 1113	1 346 0304	6458
13.6	9.344 706 3192	1 325 4988	6167
13.8	9.568 840 0259	1 305 5838	5893
14.0	9.794 280 3154	1 286 2582	5636
14.2	10.021 006 8650	1 267 4961	5393
14.4	10.249 000 9097	1 249 2732	5164
14.6	10.478 244 2276	1 231 5668	4947
14.8	10.708 719 1123	1 214 3550	4743
15.0	10.940 408 3521	1 197 6176	4550
15.2	11.173 295 2094	1 181 3361	4367
15.4	11.407 363 4018	1 165 4892	4193
15.6	11.642 597 0634	1 150 0627	4029
15.8	11.878 980 8277	1 135 0391	3873
16.0	12.116 499 6111	1 120 4028	3725
16.2	12.356 138 7973	1 106 1391	3585
16.4	12.594 884 1228	1 092 2328	3452
16.6	12.836 721 6818	1 078 6738	3325
16.8	13.077 637 9147	1 065 4462	3204
17.0	13.320 619 5938	1 052 5390	3089
17.2	13.564 663 8119	1 039 9407	2979
17.4	13.809 727 9707	1 027 6403	2876
17.6	14.056 829 7698	1 015 6276	2776
17.8	14.302 947 1954	1 003 8922	2680
18.0	14.551 068 5152	992 4249	2590
18.2	14.800 182 2588	981 2185	2503
18.4	15.050 277 2190	970 2585	2420
18.6	15.301 342 4376	959 5424	2341
18.8	15.553 367 1987	949 0605	2265
19.0	15.806 341 0203	938 8050	2192
19.2	16.060 233 6468	928 7687	2123
19.4	16.315 095 0420	918 9447	2066
19.6	16.570 855 3818	909 3263	1992
19.8	16.827 526 0480	899 9071	1931
20.0	17.085 094 6212	890 6810	1872

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
20.0	17.085 094 6212	890 6810	1872
20.2	17.343 554 8755	881 6422	1816
20.4	17.602 896 7719	872 7849	1762
20.6	17.863 111 4532	864 1038	1710
20.8	18.124 190 2382	855 5936	1660
21.0	18.386 124 6189	847 2494	1612
21.2	18.648 906 2449	839 0664	1565
21.4	18.912 528 9393	831 0398	1521
21.6	19.176 978 6736	823 1654	1478
21.8	19.442 253 5732	815 4388	1437
22.0	19.708 343 9116	807 8558	1397
22.2	19.975 242 1058	800 4126	1359
22.4	20.242 940 7127	793 1052	1322
22.6	20.511 432 4247	785 9301	1286
22.8	20.780 710 0668	778 8835	1252
23.0	21.050 766 5924	771 9622	1219
23.2	21.321 595 0803	765 1628	1187
23.4	21.593 188 7310	758 4821	1156
23.6	21.865 540 8639	751 9171	1127
23.8	22.138 644 9138	745 4647	1098
24.0	22.412 494 4285	739 1221	1070
24.2	22.687 083 0652	732 8866	1043
24.4	22.962 404 5824	726 7552	1017
24.6	23.238 452 8668	720 7256	992
24.8	23.515 221 8709	714 7953	968
25.0	23.792 705 6702	708 9617	944
25.2	24.070 898 4312	703 2226	922
25.4	24.349 794 4148	697 5756	900
25.6	24.629 387 9739	692 0186	878
25.8	24.909 673 5517	686 5494	858
26.0	25.190 645 6788	681 1660	838
26.2	25.472 298 9720	675 8663	818
26.4	25.754 628 1314	670 6485	799
26.6	26.037 627 9392	665 5105	781
26.8	26.321 293 2576	660 4508	763
27.0	26.606 619 0268	655 4673	746
27.2	26.890 600 2632	650 5585	730
27.4	27.176 232 0682	645 7227	714
27.6	27.462 509 5769	640 9582	698
27.8	27.749 428 0517	636 2635	683
28.0	28.036 982 7910	631 6371	666
28.2	28.325 169 1673	627 0774	654
28.4	28.613 982 6211	622 5831	640
28.6	28.903 418 6679	618 1528	626
28.8	29.193 472 8476	613 7851	613
29.0	29.484 140 8223	609 4786	600
29.2	29.776 418 2756	605 2322	588
29.4	30.067 300 9611	601 0445	575
29.6	30.359 784 6911	596 9143	564
29.8	30.652 868 3354	592 8406	552
30.0	30.946 538 8202	588 8220	541

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
30.0	30.946 538 8202	588 8220	541
30.2	31.240 801 1271	684 8576	530
30.4	31.535 648 2915	580 9462	520
30.6	31.831 076 4021	577 0867	509
30.8	32.127 081 5994	573 2782	499
31.0	32.423 660 0749	569 5196	490
31.2	32.720 808 0700	565 8100	480
31.4	33.018 521 8761	562 1484	471
31.6	33.316 797 8286	558 5339	462
31.8	33.615 632 3159	554 9655	453
32.0	33.915 021 7688	551 4425	444
32.2	34.214 962 6641	547 9639	436
32.4	34.515 451 5232	544 5289	428
32.6	34.816 494 9112	541 1367	420
32.8	35.118 059 4359	537 7868	412
33.0	35.420 171 7471	534 4775	405
33.2	35.722 818 5357	531 2090	397
33.4	36.025 995 5334	527 9802	390
33.6	36.329 702 5113	524 7905	383
33.8	36.633 933 2796	521 6390	376
34.0	36.938 685 6870	518 5252	370
34.2	37.243 956 6195	515 4483	363
34.4	37.549 743 0002	512 4077	357
34.6	37.856 041 7987	509 4028	350
34.8	38.162 849 9799	506 4329	344
35.0	38.470 164 6040	503 4974	338
35.2	38.777 982 7255	500 5958	333
35.4	39.086 301 4428	497 7274	327
35.6	39.395 117 8876	494 8917	321
35.8	39.704 429 2238	492 0881	316
36.0	40.014 232 6484	489 3161	311
36.2	40.324 525 3890	486 5752	305
36.4	40.635 304 7048	483 8648	300
36.6	40.946 567 8854	481 1844	295
36.8	41.258 312 2505	478 5338	290
37.0	41.570 535 1491	475 9118	286
37.2	41.883 233 9595	473 3188	281
37.4	42.196 406 0885	470 7534	277
37.6	42.510 048 9708	468 2160	272
37.8	42.824 100 0692	466 7057	268
38.0	43.138 736 8782	465 2222	263
38.2	43.453 776 8994	460 7651	259
38.4	43.769 277 6907	458 3338	255
38.6	44.085 236 3158	455 9281	251
38.8	44.401 651 8690	453 5476	247
39.0	44.718 520 4698	451 1917	243
39.2	45.035 840 2623	448 8602	240
39.4	45.353 608 9150	446 5527	236
39.6	45.671 824 1204	444 2688	232
39.8	45.990 483 5946	442 0081	229
40.0	46.309 585 0768	439 7703	225

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
40.0	46.309 585 0768	439 7703	225
40.2	46.629 126 3293	437 5550	222
40.4	46.949 105 1369	435 3620	219
40.6	47.269 519 3084	433 1908	215
40.8	47.590 366 6657	431 0412	212
41.0	47.911 545 0682	428 9128	209
41.2	48.233 352 3824	426 8053	206
41.4	48.555 486 5020	424 7184	203
41.6	48.878 045 3399	422 6518	200
41.8	49.201 026 8298	420 6053	197
42.0	49.524 428 9249	418 5785	194
42.2	49.848 249 5984	416 5711	192
42.4	50.172 486 8431	414 5828	189
42.6	50.497 138 6706	412 6135	186
42.8	50.822 203 1118	410 6628	184
43.0	51.147 678 2153	408 7304	181
43.2	51.473 562 0494	406 8161	178
43.4	51.799 852 6997	404 9197	176
43.6	52.126 548 2697	403 0409	173
43.8	52.453 646 8806	401 1794	171
44.0	52.781 146 6709	399 3351	169
44.2	53.109 045 7962	397 5076	166
44.4	53.437 342 4292	395 6968	164
44.6	53.766 034 7589	393 9024	162
44.8	54.095 120 9911	392 1242	160
45.0	54.424 599 3473	390 3620	158
45.2	54.754 468 0668	388 6155	156
45.4	55.084 725 3993	386 8846	153
45.6	55.415 369 6177	385 1691	151
45.8	55.746 399 0051	383 4687	149
46.0	56.077 811 8611	381 7832	148
46.2	56.409 606 5004	380 1125	146
46.4	56.741 781 2522	378 4564	144
46.6	57.074 334 4603	376 8146	142
46.8	57.407 264 4831	375 1870	140
47.0	57.740 569 6928	373 5734	138
47.2	58.074 248 4759	371 9736	136
47.4	58.408 299 2327	370 3875	135
47.6	58.742 720 3770	368 8148	133
47.8	59.077 510 3361	367 2555	131
48.0	59.412 667 5507	365 7092	130
48.2	59.748 190 4746	364 1760	128
48.4	60.084 077 5744	362 6556	126
48.6	60.420 327 3297	361 1477	125
48.8	60.756 938 2327	359 6523	123
49.0	61.093 908 7881	358 1693	122
49.2	61.431 237 8128	356 6985	120
49.4	61.768 922 9360	355 2397	119
49.6	62.106 963 5990	353 7928	117
49.8	62.445 358 0547	352 3576	116
50.0	62.784 104 8681	350 9341	115



## Tables of the Logarithms

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
50.0	62.784 104 8681	350 9341	115
50.2	63.123 202 6166	349 5219	113
50.4	63.462 649 8850	348 1211	112
50.6	63.802 446 2765	346 7315	110
50.8	64.142 587 3975	345 3589	109
51.0	64.483 074 8725	343 9863	108
51.2	64.823 906 3327	342 6284	107
51.4	65.165 080 4214	341 2822	105
51.6	65.506 895 7923	339 9466	104
51.8	65.848 451 1008	338 6213	103
52.0	66.190 648 0486	337 3064	102
52.2	66.533 176 2937	336 0016	101
52.4	66.876 042 5404	334 7068	99
52.6	67.219 245 4939	333 4220	98
52.8	67.562 780 8695	332 1471	97
53.0	67.906 648 3922	330 8818	96
53.2	68.250 846 7967	329 6262	95
53.4	68.596 374 8274	328 3800	94
53.6	68.940 231 2381	327 1433	93
53.8	69.285 414 7921	325 9168	92
54.0	69.630 924 2618	324 6975	91
54.2	69.976 768 4290	323 4882	90
54.4	70.322 916 0844	322 2880	89
54.6	70.669 396 0278	321 0966	88
54.8	71.016 197 0677	319 9140	87
55.0	71.363 318 0216	318 7400	86
55.2	71.710 757 7155	317 5747	85
55.4	72.058 514 9841	316 4178	84
55.6	72.406 588 6705	315 2693	83
55.8	72.754 977 6262	314 1292	82
56.0	73.103 680 7111	312 9972	81
56.2	73.452 696 7933	311 8734	80
56.4	73.802 024 7489	310 7577	80
56.6	74.151 663 4621	309 6498	79
56.8	74.501 611 8762	308 5499	78
57.0	74.851 868 7381	307 4577	77
57.2	75.202 433 1088	306 3733	76
57.4	75.553 303 8528	305 2964	75
57.6	75.904 479 8933	304 2272	75
57.8	76.255 960 1609	303 1653	74
58.0	76.607 743 5938	302 1109	73
58.2	76.959 829 1376	301 0638	72
58.4	77.312 215 7452	300 0239	72
58.6	77.664 902 3767	298 9911	71
58.8	78.017 887 9993	297 9655	70
59.0	78.371 171 5874	296 9468	69
59.2	78.724 752 1223	295 9351	69
59.4	79.078 628 5923	294 9303	68
59.6	79.432 799 9927	293 9323	67
59.8	79.787 265 3254	292 9410	67
60.0	80.142 023 5990	291 9564	66

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
60.0	80.142 023 5990	291 9564	66
60.2	80.497 073 8290	290 9783	65
60.4	80.852 415 0373	290 0068	66
60.6	81.208 046 2624	289 0418	64
60.8	81.563 966 5094	288 0831	63
61.0	81.920 174 8494	287 1308	63
61.2	82.276 670 3203	286 1848	62
61.4	82.633 451 9759	285 2450	62
61.6	82.990 518 8766	284 3113	61
61.8	83.347 870 0886	283 3838	60
62.0	83.705 504 6844	282 4622	60
62.2	84.063 421 7424	281 5467	59
62.4	84.421 620 3471	280 6370	59
62.6	84.780 099 5888	279 7332	58
62.8	85.138 858 5637	278 8353	57
63.0	85.497 896 3739	277 9430	57
63.2	85.857 212 1271	277 0565	56
63.4	86.216 804 9368	276 1758	56
63.6	86.576 673 9220	275 3002	55
63.8	86.936 818 2075	274 4304	55
64.0	87.297 236 9234	273 5661	54
64.2	87.657 929 2054	272 7072	54
64.4	88.018 694 1946	271 8537	53
64.6	88.380 131 0376	271 0066	53
64.8	88.741 638 8862	270 1628	52
65.0	89.103 416 8973	269 3249	52
65.2	89.465 464 2334	268 4924	51
65.4	89.827 780 0620	267 6651	51
65.6	90.190 363 5556	266 8428	50
65.8	90.553 213 8920	266 0255	50
66.0	90.916 330 2540	265 2133	49
66.2	91.279 711 8292	264 4069	49
66.4	91.643 357 8103	263 6035	49
66.6	92.007 267 3950	262 8080	48
66.8	92.371 439 7857	262 0132	48
67.0	92.735 674 1895	261 2232	47
67.2	93.100 569 8186	260 4420	47
67.4	93.466 625 8897	259 6634	46
67.6	93.833 741 6242	258 8895	46
67.8	94.196 216 2481	258 1201	46
68.0	94.561 948 9922	257 3554	45
68.2	94.927 939 0917	256 5961	45
68.4	95.294 185 7862	255 8393	44
68.6	95.660 688 3201	255 0880	44
68.8	96.027 445 9420	254 3411	44
69.0	96.394 457 9049	253 5985	43
69.2	96.761 723 4663	252 8602	43
69.4	97.129 241 8880	252 1263	42
69.6	97.497 012 4358	251 3966	42
69.8	97.865 034 3802	250 6710	42
70.0	98.233 306 9957	249 9497	41

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
70	98.233 306 9957	6 248 9493	2 5882
71	100.078 405 0357	6 160 3087	2 4796
72	101.929 663 3844	6 074 1477	2 3770
73	103.786 995 8808	5 990 3637	2 2799
74	105.650 318 7410	5 908 8596	2 1881
75	107.519 550 4607	5 829 5437	2 1012
76	109.394 611 7241	5 752 3289	2 0188
77	111.275 425 3184	5 677 1329	1 9406
78	113.161 916 0415	5 603 8776	1 8665
79	115.054 010 6442	5 532 4886	1 7960
80	116.951 637 7355	5 462 8957	1 7291
81	118.854 727 7225	5 395 0319	1 6654
82	120.763 212 7414	5 328 8335	1 6049
83	122.677 026 6939	5 264 2400	1 5472
84	124.596 104 6861	5 201 1937	1 4923
85	126.520 383 9722	5 139 6397	1 4399
86	128.449 802 8979	5 079 5255	1 3900
87	130.384 301 3492	5 020 8014	1 3423
88	132.323 820 6018	4 963 4195	1 2968
89	134.268 303 2739	4 907 3345	1 2533
90	136.217 693 2806	4 852 5028	1 2118
91	138.171 935 7900	4 798 8829	1 1721
92	140.130 977 1823	4 746 4350	1 1340
93	142.094 766 0097	4 695 1212	1 0977
94	144.063 247 9582	4 644 9050	1 0628
95	146.036 376 8118	4 595 7517	1 0294
96	148.014 099 4171	4 547 6278	9974
97	149.996 370 6502	4 500 5012	9667
98	151.983 142 3844	4 454 3414	9373
99	153.974 368 4601	4 409 1189	9090
100	155.970 008 6547	4 364 8054	8819
101	157.970 003 6547	4 321 3738	8558
102	159.974 325 0285	4 278 7980	8308
103	161.982 925 2003	4 237 0529	8067
104	163.995 762 4250	4 196 1146	7835
105	166.012 796 7643	4 156 9598	7612
106	168.033 986 0633	4 116 5662	7398
107	170.059 290 9286	4 077 9124	7192
108	172.088 674 7063	4 039 9778	6993
109	174.122 098 4618	4 002 7425	6801
110	176.159 524 9597	3 966 1872	6616
111	178.200 917 6449	3 930 2936	6438
112	180.246 240 6237	3 895 0439	6267
113	182.295 488 6463	3 860 4208	6101
114	184.348 537 0898	3 826 4079	5941
115	186.406 441 9411	3 792 9890	5787
116	188.466 139 7915	3 760 1489	5638
117	190.530 597 7707	3 727 8725	5494
118	192.598 783 6326	3 696 1456	5355
119	194.670 665 6398	3 664 9541	5220
120	196.746 212 6012	3 634 2847	5090

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
120	196.746 212 6012	3 634 2847	5090
121	198.825 393 8472	3 604 1243	4966
122	200.908 179 2175	3 574 4604	4843
123	202.994 539 0482	3 546 2808	4726
124	205.084 444 1597	3 518 5737	4612
125	207.177 865 8448	3 488 3278	4501
126	209.274 775 8578	3 460 6321	4395
127	211.375 146 4029	3 433 1758	4291
128	213.478 950 1239	3 406 2487	4191
129	215.586 160 0935	3 379 7407	4094
130	217.696 749 8038	3 353 6420	4000
131	219.810 693 1561	3 327 9433	3909
132	221.927 964 4518	3 302 6356	3820
133	224.048 538 3850	3 277 7098	3734
134	226.172 390 0240	3 253 1574	3651
135	228.299 494 8223	3 228 9701	3570
136	230.429 828 5908	3 205 1399	3492
137	232.563 367 4992	3 181 6688	3415
138	234.700 088 0664	3 158 5192	3341
139	236.839 967 1528	3 135 7139	3270
140	238.982 981 9530	3 113 2334	3200
141	241.129 109 9887	3 091 0770	3132
142	243.278 329 1014	3 069 2317	3066
143	245.430 617 4457	3 047 6931	3002
144	247.586 983 4832	3 026 4546	2940
145	249.744 315 9753	3 005 5101	2879
146	251.905 683 9775	2 984 8535	2820
147	254.070 036 8333	2 964 4790	2763
148	256.237 354 1681	2 944 3806	2707
149	258.407 616 8835	2 924 5530	2653
150	260.580 802 1519	2 904 9906	2600
151	262.756 893 4109	2 885 6882	2548
152	264.935 870 3582	2 866 6407	2498
153	267.117 713 9462	2 847 8429	2449
154	269.302 406 3770	2 829 2900	2402
155	271.489 926 0978	2 810 9773	2355
156	273.680 257 7960	2 792 9002	2310
157	275.873 382 3945	2 775 0541	2266
158	278.069 282 0468	2 757 4345	2223
159	280.267 939 1337	2 740 0374	2181
160	282.469 336 2580	2 722 8583	2141
161	284.673 456 2407	2 705 8934	2101
162	286.880 282 1167	2 689 1385	2062
163	289.089 797 1313	2 672 5890	2024
164	291.301 984 7357	2 656 2436	1987
165	293.516 628 5637	2 640 0962	1951
166	295.734 312 5279	2 624 1438	1916
167	297.954 420 6160	2 608 3831	1882
168	300.177 137 0871	2 592 8106	1848
169	302.402 448 3688	2 577 4229	1816
170	304.630 333 0735	2 562 2168	1784

## Tables of the Logarithms

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$	$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
170	304.630 333 0735	2 562 2168	1784	220	419.016 246 8613	1 978 5660	821
171	306.860 781 9948	2 547 1890	1753	221	421.358 669 5421	1 969 5929	810
172	309.093 778 1052	2 532 3365	1722	222	423.703 061 8158	1 960 7008	799
173	311.329 306 5521	2 517 6562	1692	223	426.049 414 7903	1 951 8886	789
174	313.567 352 6563	2 503 1452	1663	224	428.397 719 6533	1 943 1553	778
175	315.807 901 9035	2 488 8004	1635	225	430.747 967 6717	1 934 4998	768
176	318.050 939 9522	2 474 6191	1607	226	433.100 150 1898	1 925 9210	758
177	320.296 452 6200	2 460 5985	1580	227	435.454 258 6289	1 917 4180	748
178	322.544 425 8864	2 446 7359	1553	228	437.810 284 4861	1 908 9898	738
179	324.794 846 8887	2 433 0287	1527	229	440.168 219 3331	1 900 6353	728
180	327.047 698 9197	2 419 4741	1502	230	442.528 054 8154	1 892 3537	719
181	329.302 971 4248	2 406 0698	1477	231	444.889 782 6515	1 884 1439	709
182	331.560 649 9997	2 392 8131	1453	232	447.253 394 6314	1 876 0050	700
183	333.820 721 3876	2 379 7017	1429	233	449.618 882 6162	1 867 9361	691
184	336.083 172 4774	2 366 7333	1406	234	451.986 238 5373	1 859 9364	682
185	338.347 990 3004	2 353 9054	1383	235	454.355 454 3947	1 852 0049	674
186	340.615 162 0288	2 341 2158	1361	236	456.726 522 2570	1 844 1407	665
187	342.884 674 9730	2 328 6623	1339	237	459.099 434 2599	1 836 3430	657
188	345.156 616 5795	2 316 2427	1318	238	461.474 182 6059	1 828 6110	648
189	347.430 674 4288	2 303 9549	1297	239	463.850 759 5630	1 820 9439	640
190	349.707 136 2330	2 291 7968	1276	240	466.229 157 4639	1 813 3408	632
191	351.985 889 8339	2 279 7663	1256	241	468.609 368 7056	1 805 8009	624
192	354.256 923 2012	2 267 8615	1237	242	470.991 385 7482	1 798 3234	617
193	356.550 224 4299	2 256 0803	1218	243	473.375 201 1142	1 790 9076	609
194	358.835 781 7389	2 244 4209	1199	244	475.760 807 3878	1 783 5527	602
195	361.123 583 4688	2 232 8814	1181	245	478.148 197 2141	1 776 2580	594
196	363.413 618 0802	2 221 4600	1162	246	480.537 363 2985	1 769 0227	587
197	365.705 874 1515	2 210 1548	1145	247	482.928 298 4056	1 761 8462	580
198	368.000 340 3777	2 198 9641	1128	248	485.320 995 3589	1 754 7276	573
199	370.297 005 5680	2 187 8861	1111	249	487.715 447 0397	1 747 6663	566
200	372.596 858 6444	2 176 9193	1094	250	490.111 646 3868	1 740 6616	559
201	374.896 888 6400	2 166 0618	1078	251	492.509 586 3955	1 733 7128	553
202	377.200 084 6976	2 155 3120	1062	252	494.909 260 1169	1 726 8193	546
203	379.505 436 0569	2 144 6685	1046	253	497.310 660 6577	1 719 9804	540
204	381.812 932 1048	2 134 1295	1031	254	499.713 781 1789	1 713 1954	533
205	384.122 562 2722	2 123 6936	1016	255	502.118 614 8955	1 706 4638	527
206	386.434 316 1333	2 113 3593	1001	256	504.525 155 0760	1 699 7849	521
207	388.748 183 3537	2 103 1251	986	257	506.933 395 0413	1 693 1580	515
208	391.064 153 6991	2 092 9895	972	258	509.343 328 1646	1 686 5826	509
209	393.382 217 0341	2 082 9511	958	259	511.754 947 8706	1 680 0581	503
210	395.702 363 3202	2 073 0086	945	260	514.168 247 6346	1 673 5839	497
211	398.024 582 6149	2 063 1606	931	261	516.583 220 9826	1 667 1594	491
212	400.348 865 0702	2 053 4058	918	262	518.999 861 4900	1 660 7840	486
213	402.675 200 9312	2 043 7425	905	263	521.418 182 7813	1 654 4572	480
214	405.003 580 5348	2 034 1699	893	264	523.838 118 5298	1 648 1784	475
215	407.333 994 3080	2 024 6866	880	265	526.259 722 4566	1 641 9471	469
216	409.666 432 7679	2 015 2912	868	266	528.682 968 3306	1 635 7627	464
217	412.000 886 5190	2 005 9827	856	267	531.107 849 9672	1 629 6247	459
218	414.337 346 2529	1 996 7598	844	268	533.534 361 2286	1 623 5327	454
219	416.675 802 7465	1 987 6212	833	269	535.962 496 0226	1 617 4860	449
220	419.016 246 8613	1 978 5660	821	270	538.392 248 3026	1 611 4842	444

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
270	538.392 248 3026	1 611 4842	444
271	540.823 612 0568	1 606 5267	439
272	543.256 681 3576	1 609 6132	434
273	545.691 150 2617	1 593 7430	429
274	548.127 312 9087	1 587 9168	425
275	550.566 068 4715	1 582 1310	420
276	553.004 396 1654	1 576 3882	415
277	555.445 308 2474	1 570 6870	411
278	557.887 785 0165	1 565 0269	406
279	560.331 829 8124	1 559 4074	402
280	562.777 434 0167	1 553 8281	398
281	565.224 692 0470	1 548 2886	394
282	567.673 298 3669	1 542 7884	389
283	570.123 547 4752	1 537 3272	385
284	572.576 333 9108	1 531 9045	381
285	575.028 662 2508	1 526 5200	377
286	577.483 497 1108	1 521 1731	373
287	579.939 863 1439	1 515 8636	369
288	582.397 746 0407	1 510 5910	366
289	584.857 137 6284	1 505 3550	362
290	587.318 036 3712	1 500 1551	358
291	589.780 433 3691	1 494 9911	354
292	592.244 326 3561	1 489 8625	351
293	594.709 709 2095	1 484 7689	347
294	597.176 676 8299	1 479 7101	344
295	599.644 924 1603	1 474 6866	340
296	602.114 746 1763	1 469 6961	337
297	604.586 667 8873	1 464 7383	333
298	607.068 794 3367	1 459 8148	330
299	609.553 010 6007	1 454 9242	327
300	612.048 681 7891	1 450 0664	323
301	614.546 803 0438	1 445 2409	320
302	617.046 369 6394	1 440 4474	317
303	619.544 376 4923	1 435 6855	314
304	622.042 819 1108	1 430 9561	311
305	624.540 692 8944	1 426 2567	308
306	627.038 992 6338	1 421 5871	305
307	629.537 713 9603	1 416 9490	302
308	632.036 862 3357	1 412 3410	299
309	634.534 403 0522	1 407 7629	296
310	637.032 161 6317	1 403 2144	293
311	639.530 336 723 2255	1 398 6952	290
312	642.028 493 6145	1 394 2060	287
313	644.522 638 2086	1 389 7435	285
314	647.018 182 6461	1 385 3105	282
315	649.516 112 1942	1 380 9067	279
316	652.013 422 7480	1 376 5288	277
317	654.513 109 8306	1 372 1796	274
318	657.014 169 0928	1 367 8578	271
319	659.516 696 2128	1 363 5631	269
320	662.018 386 8968	1 359 2953	266

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
320	661.820 386 8968	1 359 2953	266
321	664.325 536 8742	1 355 0541	264
322	666.832 041 9066	1 350 8393	261
323	669.339 897 7783	1 346 6606	259
324	671.849 100 3006	1 342 4879	257
325	674.359 646 3108	1 338 3508	254
326	676.871 528 6718	1 334 2391	252
327	679.384 746 2718	1 330 1526	250
328	681.899 294 0245	1 326 0911	247
329	684.415 167 6682	1 322 0542	245
330	686.932 363 7682	1 318 0419	243
331	689.450 877 7060	1 314 0539	241
332	691.970 706 6998	1 310 0899	238
333	694.491 843 7835	1 306 1498	236
334	697.014 288 0170	1 302 2333	234
335	699.538 034 4838	1 298 3402	232
336	702.063 079 2909	1 294 4704	230
337	704.589 418 5683	1 290 6236	228
338	707.117 048 4691	1 286 7994	226
339	709.646 965 1694	1 282 9979	224
340	712.176 164 8676	1 279 2189	222
341	714.707 643 7847	1 275 4620	220
342	717.240 398 1636	1 271 7271	218
343	719.774 424 2697	1 268 0140	216
344	722.309 718 3897	1 264 3225	214
345	724.846 276 8323	1 260 6525	212
346	727.384 096 9274	1 257 0037	211
347	729.923 172 0262	1 253 3760	209
348	732.463 501 6010	1 249 7692	207
349	735.006 080 7449	1 246 1830	206
350	737.547 906 1719	1 242 6174	203
351	740.091 974 2162	1 239 0721	202
352	742.637 281 3327	1 235 5470	200
353	745.183 823 9962	1 232 0419	198
354	747.731 598 7016	1 228 5566	197
355	750.280 601 9635	1 225 0910	196
356	752.830 830 3166	1 221 6449	193
357	755.382 260 3146	1 218 2181	192
358	757.934 948 5307	1 214 8105	190
359	760.488 831 6574	1 211 4219	189
360	763.043 926 0060	1 208 0522	187
361	765.600 228 5067	1 204 7011	185
362	768.157 735 7086	1 201 3686	184
363	770.716 444 2792	1 198 0545	182
364	773.276 350 9042	1 194 7586	181
365	775.837 452 2878	1 191 4808	179
366	778.399 745 1523	1 188 2209	178
367	780.963 226 2377	1 184 9789	176
368	783.527 892 3019	1 181 7544	175
369	786.093 740 1206	1 178 5475	174
370	788.660 766 4868	1 175 3579	172

Tables of the Logarithms

p	Log Γ(p)	δ <sup>2</sup>	δ <sup>4</sup>
370	788-660 766 4868	1 175 3579	172
371	791-228 968 2108	1 172 1855	171
372	793-798 342 1205	1 169 0303	169
373	796-368 885 0603	1 166 8919	168
374	798-940 593 8922	1 162 7704	167
375	801-513 465 4944	1 159 6655	165
376	804-087 496 7621	1 156 5772	164
377	806-662 684 6070	1 153 5053	163
378	809-239 025 9572	1 150 4496	161
379	811-816 517 7571	1 147 4101	160
380	814-395 156 9670	1 144 3866	159
381	816-974 940 5636	1 141 3791	158
382	819-555 865 5393	1 138 3872	156
383	822-137 928 9022	1 136 4111	155
384	824-721 127 6762	1 132 4504	154
385	827-305 458 9006	1 129 5051	153
386	829-890 919 6301	1 126 5752	152
387	832-477 506 9347	1 123 6603	150
388	835-065 217-8998	1 120 7606	149
389	837-654 049 6254	1 117 8757	148
390	840-243 999 2267	1 115 0057	147
391	842-835 063 8337	1 112 1504	146
392	845-427 240 5911	1 109 3096	145
393	848-020 526 6581	1 106 4834	144
394	850-614 919 2085	1 103 6715	143
395	853-210 415 4303	1 100 8738	141
396	855-807 012 5259	1 098 0903	140
397	858-404 707 7119	1 095 3208	139
398	861-003 498 2186	1 092 5653	138
399	863-603 381 2907	1 089 8236	137
400	866-204 354 1864	1 087 0956	136
401	868-806 414 1777	1 084 3813	135
402	871-409 558 6503	1 081 6805	134
403	874-013 784 6034	1 078 9931	133
404	876-619 089 6496	1 076 3190	132
405	879-225 471 0147	1 073 6581	131
406	881-832 926 0379	1 071 0104	130
407	884-441 452 0715	1 068 3756	129
408	887-051 046 4807	1 065 7539	128
409	889-661 706 6438	1 063 1449	127
410	892-273 429 9518	1 060 5487	126
411	894-886 213 8085	1 057 9652	125
412	897-500 056 6304	1 056 3942	125
413	900-114 952 8464	1 052 8356	124
414	902-730 902 8981	1 050 2896	123
415	905-347 903 2392	1 047 7558	122
416	907-965 951 3359	1 045 2339	121
417	910-585 044 6655	1 042 7243	120
418	913-208 180 7215	1 040 2268	119
419	915-826 357 0033	1 037 7412	119
420	918-448 571 0263	1 035 2674	118

p	Log Γ(p)	δ <sup>2</sup>	δ <sup>4</sup>
420	918-448 571 0263	1 035 2674	118
421	921-071 820 3167	1 032 8054	117
422	923-696 102 4125	1 030 3551	116
423	926-321 414 8635	1 027 9164	115
424	928-947 755 2308	1 025 4892	114
425	931-575 121 0874	1 023 0735	114
426	934-203 510 0175	1 020 6691	113
427	936-832 919 6166	1 018 2759	112
428	939-463 347 4916	1 015 8940	111
429	942-094 791 2606	1 013 5232	110
430	944-727 248 5528	1 011 1634	110
431	947-360 717 0084	1 008 8146	109
432	949-995 194 2785	1 006 4767	108
433	952-630 678 0254	1 004 1495	107
434	955-267 165 9217	1 001 8332	107
435	957-904 655 6512	999 5274	106
436	960-543 144 9082	997 2323	105
437	963-182 631 3974	994 9477	104
438	965-823 112 8344	992 6735	104
439	968-464 585 9449	990 4097	103
440	971-107 061 4652	988 1562	102
441	973-750 504 1416	985 9130	102
442	976-394 942 7311	983 6799	101
443	979-040 365 0005	981 4569	100
444	981-686 768 7267	979 2439	100
445	984-334 151 6968	977 0409	99
446	986-982 511 7078	974 8477	98
447	989-631 846 5665	972 6644	98
448	992-282 154 0896	970 4909	97
449	994-933 432 1036	968 3270	96
450	997-585 678 4446	966 1728	96
451	1000-238 890 9584	964 0281	95
452	1002-893 067 5003	961 8929	94
453	1005-548 205 9351	959 7672	94
454	1008-204 304 1371	957 6508	93
455	1010-861 359 9900	955 5438	93
456	1013-519 371 3866	953 4460	92
457	1016-178 336 2293	951 3574	91
458	1018-838 252 4293	949 2779	91
459	1021-499 117 9074	947 2075	90
460	1024-160 930 5929	945 1461	90
461	1026-823 688 4246	943 0937	89
462	1029-487 389 3500	941 0502	88
463	1032-152 031 3255	939 0155	88
464	1034-817 612 3165	936 9895	87
465	1037-484 130 2971	934 9723	87
466	1040-151 583 2600	932 9638	86
467	1042-819 969 1567	930 9639	86
468	1045-489 286 0472	928 9725	85
469	1048-159 531 9003	926 9895	84
470	1050-830 704 7430	925 0152	84

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
470	1050.830 704 7430	925 0152	84
471	1053.502 802 8010	923 0492	83
472	1056.175 823 5081	921 0915	83
473	1058.849 765 5067	919 1421	82
474	1061.524 828 6475	917 2009	82
475	1064.200 404 9891	915 2680	81
476	1066.877 098 5988	913 3431	81
477	1069.554 706 5515	911 4263	80
478	1072.233 223 9308	909 5176	80
479	1074.912 661 8271	907 6168	79
480	1077.592 987 3405	905 7240	79
481	1080.274 228 5779	903 8390	78
482	1082.956 373 6843	901 9619	78
483	1085.639 420 6925	900 0925	77
484	1088.323 267 8233	898 2309	77
485	1091.008 213 1849	896 3770	76
486	1093.693 954 9235	894 5307	76
487	1096.380 591 1928	892 6920	75
488	1099.068 120 1540	890 8608	75
489	1101.756 539 9760	889 0371	75
490	1104.445 848 8351	887 2209	74
491	1107.136 044 9182	886 4121	74
492	1109.827 126 4073	883 6106	73
493	1112.519 091 5101	881 8165	73
494	1115.211 938 4293	880 0296	72
495	1117.906 665 3783	878 2500	72
496	1120.600 270 5772	876 4776	71
497	1123.295 752 2537	874 7122	71
498	1125.992 108 6424	872 9540	70
499	1128.689 337 9852	871 2029	71
500	1131.387 438 5308	869 4587	70
501	1134.086 408 5351	867 7216	69
502	1136.786 246 2610	865 9913	69
503	1139.486 949 9781	864 2679	68
504	1142.188 517 9632	862 5514	68
505	1144.890 948 4996	860 8417	68
506	1147.594 239 8778	859 1387	67
507	1150.298 390 3946	857 4425	67
508	1153.003 398 3539	855 7530	66
509	1155.709 262 0662	854 0701	66
510	1158.415 979 8486	852 3938	66
511	1161.123 560 0247	850 7240	65
512	1163.831 970 9248	849 0608	65
513	1166.541 240 8958	847 4041	66
514	1169.251 358 2509	845 7539	64
515	1171.962 321 3699	844 1100	64
516	1174.674 128 5989	842 4726	63
517	1177.386 778 3005	840 8416	63
518	1180.100 268 8436	839 2167	63
519	1182.814 598 6034	837 5981	62
520	1185.529 765 9612	835 9858	62

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
520	1185.529 765 9612	835 9858	62
521	1188.245 769 3049	834 3797	62
522	1190.962 607 0282	832 7797	61
523	1193.680 277 5312	831 1859	61
524	1196.398 779 2200	829 5981	61
525	1199.118 110 5070	828 0164	60
526	1201.838 269 8104	826 4407	60
527	1204.559 255 5546	824 8711	60
528	1207.281 066 1698	823 3073	59
529	1210.003 700 0923	821 7495	59
530	1212.727 155 7644	820 1976	59
531	1215.451 431 6340	818 6515	58
532	1218.176 526 1550	817 1112	58
533	1220.902 437 7873	815 5767	58
534	1223.629 164 9964	814 0480	57
535	1226.356 706 2534	812 5250	57
536	1229.085 060 0354	811 0077	57
537	1231.814 224 8251	809 4960	56
538	1234.544 199 1108	807 9900	56
539	1237.274 981 3865	806 4895	56
540	1240.006 570 1517	804 9946	55
541	1242.738 963 9115	803 5053	55
542	1245.472 161 1766	802 0214	55
543	1248.206 160 4631	800 5431	54
544	1250.940 960 2927	799 0701	54
545	1253.676 559 1924	797 6026	54
546	1256.412 955 6947	796 1404	54
547	1259.150 148 3374	794 6836	53
548	1261.888 135 6637	793 2322	53
549	1264.626 916 2222	791 7860	53
550	1267.366 488 5667	790 3450	52
551	1270.106 851 2562	788 9094	52
552	1272.848 002 8550	787 4789	52
553	1275.589 941 9327	786 0536	52
554	1278.332 667 0640	784 6334	51
555	1281.076 176 8288	783 2184	51
556	1283.820 469 8119	781 8085	51
557	1286.565 544 6035	780 4036	50
558	1289.311 399 7987	779 0038	50
559	1292.058 033 9976	777 6089	50
560	1294.806 446 8055	776 2191	50
561	1297.553 633 8325	774 8342	49
562	1300.302 598 6937	773 4543	49
563	1303.052 333 0093	772 0793	49
564	1305.802 841 4042	770 7091	49
565	1308.554 120 5081	769 3438	48
566	1311.306 168 9560	767 9834	48
567	1314.058 985 3871	766 6277	48
568	1316.812 568 4460	765 2768	48
569	1319.566 916 7818	763 9307	47
570	1322.322 029 0481	762 5893	47

Tables of the Logarithms

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
570	1322.322 029 0481	762 5893	47
571	1325.077 903 9038	761 2526	47
572	1327.834 540 0121	759 9205	47
573	1330.591 936 0409	758 5932	46
574	1333.350 090 8628	757 2704	46
575	1336.109 002 5552	755 9523	46
576	1338.868 670 3999	754 6387	46
577	1341.629 092 8833	753 3297	45
578	1344.390 268 6965	752 0253	45
579	1347.152 195 5349	750 7253	45
580	1349.914 875 0986	749 4298	45
581	1352.678 303 0922	748 1388	44
582	1355.442 479 2246	746 8523	44
583	1358.207 402 2092	745 5701	44
584	1360.973 070 7640	744 2924	44
585	1363.739 483 6111	743 0190	43
586	1366.506 639 4772	741 7499	43
587	1369.274 537 0932	740 4852	43
588	1372.043 175 1945	739 2248	43
589	1374.812 552 8206	737 9687	43
590	1377.582 667 8153	736 7169	42
591	1380.353 619 8270	735 4692	42
592	1383.125 107 3079	734 2258	42
593	1385.897 429 0146	732 9866	42
594	1388.670 483 7079	731 7516	42
595	1391.444 270 1529	730 5207	41
596	1394.218 787 1185	729 2940	41
597	1396.994 033 3784	728 0714	41
598	1399.770 007 7095	726 8529	41
599	1402.546 708 8935	725 6384	41
600	1405.324 135 7159	724 4280	40
601	1408.102 286 9663	723 2216	40
602	1410.881 161 4383	722 0193	40
603	1413.660 757 9295	720 8209	40
604	1416.441 075 2417	719 6265	40
605	1419.222 112 1803	718 4360	39
606	1422.003 867 5550	717 2495	39
607	1424.786 340 1791	716 0669	39
608	1427.569 528 8702	714 8882	39
609	1430.353 432 4495	713 7134	39
610	1433.138 049 7421	712 5424	38
611	1435.923 379 5771	711 3752	38
612	1438.709 420 7874	710 2119	38
613	1441.496 172 2095	709 0524	38
614	1444.283 632 6840	707 8966	38
615	1447.071 801 0552	706 7446	37
616	1449.860 876 1709	705 5964	37
617	1452.650 255 8831	704 4519	37
618	1455.440 542 0471	703 3111	37
619	1458.231 530 5222	702 1739	37
620	1461.023 221 1712	701 0405	37

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
620	1461.023 221 1712	701 0405	37
621	1463.815 612 8607	699 9107	36
622	1466.608 704 4609	698 7845	36
623	1469.402 494 8456	697 6620	36
624	1472.195 982 8923	696 5430	36
625	1474.992 167 4619	695 4277	36
626	1477.788 047 4993	694 3159	35
627	1480.584 621 8325	693 2076	35
628	1483.381 889 3733	692 1029	35
629	1486.179 849 0171	691 0017	35
630	1488.978 499 6625	689 9040	35
631	1491.777 840 2120	688 8098	35
632	1494.577 869 5712	687 7190	34
633	1497.378 586 6495	686 6317	34
634	1500.179 990 3595	685 5479	34
635	1502.982 079 6174	684 4674	34
636	1505.784 853 3427	683 3904	34
637	1508.588 310 4583	682 3167	34
638	1511.392 449 8907	681 2464	34
639	1514.197 270 5694	680 1794	33
640	1517.002 771 4278	679 1158	33
641	1519.808 951 4015	678 0556	33
642	1522.615 809 4311	676 9986	33
643	1525.423 344 4591	676 9449	33
644	1528.231 555 4320	674 8944	33
645	1531.040 441 2994	673 8473	32
646	1533.850 001 0140	672 8034	32
647	1536.660 233 5320	671 7627	32
648	1539.471 137 8127	670 7252	32
649	1542.282 712 8186	669 6909	32
650	1545.094 957 5154	668 6598	32
651	1547.907 870 8720	667 6319	32
652	1550.721 451 8606	666 6072	31
653	1553.535 699 4563	665 5856	31
654	1556.350 612 6376	664 5670	31
655	1559.166 190 3859	663 5517	31
656	1561.982 431 6859	662 5394	31
657	1564.799 335 5253	661 5302	31
658	1567.616 900 8948	660 5241	31
659	1570.435 128 7885	659 5210	30
660	1573.254 012 2031	658 5209	30
661	1576.073 556 1386	657 5239	30
662	1578.893 757 5981	656 5300	30
663	1581.714 615 6975	655 5390	30
664	1584.536 129 1159	654 5510	30
665	1587.358 297 1953	653 5669	30
666	1590.181 118 8406	652 5859	29
667	1593.004 593 0698	651 6047	29
668	1595.828 718 9037	650 6286	29
669	1598.653 495 3662	649 6553	29
670	1601.478 921 4839	648 6849	29

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
670	1601.478 921 4839	648 6849	29
671	1604.304 996 2866	647 7175	29
672	1607.131 718 8068	646 7529	29
673	1609.959 088 0798	645 7912	29
674	1612.787 103 1441	644 8323	28
675	1615.615 763 0406	643 8763	28
676	1618.445 066 8134	642 9231	28
677	1621.275 013 5094	641 9727	28
678	1624.105 602 1781	641 0252	28
679	1626.936 831 8719	640 0804	28
680	1629.768 701 6462	639 1384	28
681	1632.601 210 5589	638 1992	28
682	1635.434 357 6708	637 2627	27
683	1638.268 142 0455	636 3290	27
684	1641.102 562 7492	635 3980	27
685	1643.937 618 8509	634 4698	27
686	1646.773 309 4224	633 5442	27
687	1649.609 633 6381	632 6214	27
688	1652.446 590 2752	631 7012	27
689	1655.284 178 7134	630 7837	27
690	1658.122 397 9353	629 8688	26
691	1660.961 247 0260	628 9566	26
692	1663.800 725 0734	628 0471	26
693	1666.640 831 1679	627 1402	26
694	1669.481 564 4025	626 2358	26
695	1672.322 923 8729	625 3341	26
696	1675.164 908 6775	624 4350	26
697	1678.007 517 9171	623 5385	26
698	1680.850 750 6952	622 6445	26
699	1683.694 606 1179	621 7531	25
700	1686.539 083 2936	620 8643	25
701	1689.384 181 3336	619 9780	25
702	1692.229 899 3516	619 0942	25
703	1695.076 236 4637	618 2129	25
704	1697.923 191 7887	617 3341	25
705	1700.770 764 4479	616 4578	25
706	1703.618 953 5649	615 5841	25
707	1706.467 758 2659	614 7127	25
708	1709.317 177 6797	613 8439	25
709	1712.167 210 9374	612 9775	24
710	1715.017 867 1726	612 1136	24
711	1717.869 115 5213	611 2520	24
712	1720.720 985 1220	610 3929	24
713	1723.573 466 1187	609 5362	24
714	1726.426 554 5455	608 6819	24
715	1729.280 252 8673	607 8300	24
716	1732.134 558 8991	606 9805	24
717	1734.989 471 9214	606 1334	24
718	1737.844 991 0771	606 2886	24
719	1740.701 116 5213	604 4461	23
720	1743.557 844 4117	603 6060	23

$p$	$\text{Log } \Gamma(p)'$	$\delta^2$	$\delta^4$
720	1743.557 844 4117	603 6060	23
721	1746.415 176 9081	602 7683	23
722	1749.273 112 1728	601 9329	23
723	1752.131 649 3704	601 0997	23
724	1754.990 787 6677	600 2689	23
725	1757.850 526 2339	599 4404	23
726	1760.710 864 2406	598 6141	23
727	1763.571 800 8612	597 7902	23
728	1766.433 335 2720	596 9685	23
729	1769.295 466 6614	596 1490	22
730	1772.158 194 1797	595 3318	22
731	1775.021 517 0398	594 5168	22
732	1777.885 434 4167	593 7041	22
733	1780.749 945 4978	592 8936	22
734	1783.615 049 4724	592 0853	22
735	1786.480 746 5324	591 2792	22
736	1789.347 032 8714	590 4753	22
737	1792.213 910 6858	589 6735	22
738	1795.081 378 1736	588 8740	22
739	1797.949 434 5355	588 0766	22
740	1800.818 078 9739	587 2813	21
741	1803.687 310 6936	586 4882	21
742	1806.557 128 9016	585 6973	21
743	1809.427 532 8069	584 9085	21
744	1812.298 621 6206	584 1218	21
745	1815.170 094 5562	583 3372	21
746	1818.042 250 8289	582 5547	21
747	1820.914 989 6664	581 7743	21
748	1823.788 310 2582	580 9960	21
749	1826.662 211 8561	580 2198	21
750	1829.536 693 6738	579 4457	21
751	1832.411 754 9372	578 6736	21
752	1835.287 394 8742	577 9036	20
753	1838.163 612 7147	577 1356	20
754	1841.040 407 6909	576 3697	20
755	1843.917 779 0368	575 6059	20
756	1846.795 725 9884	574 8439	20
757	1849.674 247 7839	574 0840	20
758	1852.553 343 6634	573 3261	20
759	1855.433 012 8691	572 5703	20
760	1858.313 254 6450	571 8164	20
761	1861.194 068 2373	571 0646	20
762	1864.075 452 8940	570 3146	20
763	1866.957 407 8654	569 5666	20
764	1869.839 932 4033	568 8206	20
765	1872.723 025 7619	568 0766	19
766	1875.606 687 1971	567 3345	19
767	1878.490 915 9667	566 5943	19
768	1881.375 711 3305	565 8561	19
769	1884.261 072 5507	565 1198	19
770	1887.146 998 8906	564 3854	19



Tables of the Logarithms

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
770	1887.146 998 8905	564 3854	19
771	1890.033 489 6156	563 6529	19
772	1892.920 543 9937	562 9223	19
773	1895.808 161 2940	562 1936	19
774	1898.696 340 7879	561 4668	19
775	1901.585 081 7486	560 7418	19
776	1904.474 383 4511	560 0188	19
777	1907.364 245 1724	559 2975	19
778	1910.254 668 1912	558 5782	18
779	1913.145 645 7882	557 8607	18
780	1916.037 183 2459	557 1450	18
781	1918.929 277 8485	556 4312	18
782	1921.821 928 8824	555 7192	18
783	1924.715 135 6355	555 0090	18
784	1927.608 897 3975	554 3006	18
785	1930.503 213 4602	553 5941	18
786	1933.398 083 1170	552 8893	18
787	1936.293 505 6630	552 1863	18
788	1939.189 480 3954	551 4851	18
789	1942.086 006 6129	550 7857	18
790	1944.983 083 6161	550 0881	18
791	1947.880 710 7074	549 3922	18
792	1950.778 887 1909	548 6981	18
793	1953.677 612 3724	548 0057	17
794	1956.576 885 5598	547 3151	17
795	1959.476 706 0622	546 6262	17
796	1962.377 073 1908	545 9391	17
797	1965.277 986 2586	545 2537	17
798	1968.179 444 5800	544 5700	17
799	1971.081 447 4713	543 8880	17
800	1973.983 994 2506	543 2077	17
801	1976.887 084 2376	542 5291	17
802	1979.790 716 7537	541 8522	17
803	1982.694 891 1220	541 1770	17
804	1985.599 606 6673	540 5035	17
805	1988.504 862 7160	539 8316	17
806	1991.410 659 5964	539 1614	17
807	1994.316 993 6582	538 4929	17
808	1997.223 867 1729	537 8261	16
809	2000.131 278 5337	537 1608	16
810	2003.039 227 0553	536 4973	16
811	2005.947 712 0742	535 8353	16
812	2008.856 732 9284	535 1750	16
813	2011.766 288 9576	534 5164	16
814	2014.676 379 6032	533 8593	16
815	2017.587 003 9081	533 2039	16
816	2020.498 161 5169	532 5500	16
817	2023.409 851 6756	531 8978	16
818	2026.322 073 7322	531 2471	16
819	2029.234 827 0368	530 5981	16
820	2032.148 110 9376	529 9506	16

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
820	2032.148 110 9376	529 9506	16
821	2035.061 924 7900	529 3047	16
822	2037.976 267 9471	528 6604	16
823	2040.891 139 7646	528 0177	16
824	2043.806 639 5998	527 3765	16
825	2046.722 466 8115	526 7369	15
826	2049.638 920 7601	526 0988	15
827	2052.555 900 8074	525 4622	15
828	2055.473 406 3170	524 8272	15
829	2058.391 436 6537	524 1938	15
830	2061.309 991 1843	523 5618	15
831	2064.229 069 2767	522 9314	15
832	2067.148 670 3005	522 3025	15
833	2070.068 793 6267	521 6751	15
834	2072.989 438 6282	521 0492	15
835	2075.910 604 6788	520 4248	15
836	2078.832 291 1543	519 8020	15
837	2081.754 497 4317	519 1806	15
838	2084.677 222 8897	518 5606	15
839	2087.600 466 9083	517 9422	15
840	2090.524 228 8692	517 3252	15
841	2093.448 508 1552	516 7097	15
842	2096.373 304 1510	516 0957	15
843	2099.298 616 2425	515 4831	15
844	2102.224 443 8171	514 8720	14
845	2105.150 786 2638	514 2623	14
846	2108.077 642 9727	513 6541	14
847	2111.005 013 3358	513 0473	14
848	2113.932 896 7461	512 4419	14
849	2116.861 292 5984	511 8380	14
850	2119.790 200 2686	511 2355	14
851	2122.719 619 2143	510 6344	14
852	2125.649 548 7744	510 0347	14
853	2128.579 989 3692	509 4364	14
854	2131.510 937 4003	508 8395	14
855	2134.442 395 2710	508 2440	14
856	2137.374 361 3857	507 6499	14
857	2140.306 835 1504	507 0572	14
858	2143.239 815 9723	506 4659	14
859	2146.173 303 2602	505 8760	14
860	2149.107 296 4240	505 2874	14
861	2152.041 794 8753	504 7002	14
862	2154.976 798 0267	504 1144	14
863	2157.912 305 2925	503 5299	14
864	2160.848 316 0883	502 9468	13
865	2163.784 829 8307	502 3650	13
866	2166.721 845 9382	501 7846	13
867	2169.659 363 8302	501 2056	13
868	2172.597 382 9277	500 6277	13
869	2175.535 902 6529	500 0513	13
870	2178.474 922 4293	499 4762	13

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
870	2178.474 922 4293	499 4762	13
871	2181.414 441 8819	498 9024	13
872	2184.354 459 8370	498 3299	13
873	2187.294 976 3219	497 7589	13
874	2190.235 990 5656	497 1889	13
875	2193.177 501 9982	496 6204	13
876	2196.119 510 0512	496 0531	13
877	2199.062 014 1574	495 4872	13
878	2202.005 018 7508	494 9225	13
879	2204.948 508 2667	494 3592	13
880	2207.892 497 1418	493 7971	13
881	2210.836 979 8139	493 2363	13
882	2213.781 956 7223	492 6767	13
883	2216.727 424 3075	492 1184	13
884	2219.673 385 0110	491 5614	13
885	2222.619 837 2760	491 0057	13
886	2225.566 780 5467	490 4512	13
887	2228.514 214 2686	489 8979	12
888	2231.462 137 8885	489 3459	12
889	2234.410 550 8542	488 7952	12
890	2237.359 452 6152	488 2457	12
891	2240.308 842 6219	487 6974	12
892	2243.258 720 3259	487 1503	12
893	2246.209 086 1803	486 6045	12
894	2249.169 936 6392	486 0599	12
895	2252.111 274 1580	485 5165	12
896	2255.063 097 1933	484 9743	12
897	2258.015 406 2029	484 4334	12
898	2260.968 197 8460	483 8936	12
899	2263.921 473 9826	483 3551	12
900	2266.876 233 6744	482 8177	12
901	2269.829 476 1838	482 2815	12
902	2272.784 200 9748	481 7466	12
903	2275.739 407 5123	481 2128	12
904	2278.695 095 2526	480 6802	12
905	2281.651 263 6931	480 1487	12
906	2284.607 912 2723	479 6185	12
907	2287.565 040 4700	479 0894	12
908	2290.522 647 7571	478 5616	12
909	2293.480 733 8056	478 0347	12
910	2296.439 297 4888	477 5091	12
911	2299.398 338 8811	476 9847	12
912	2302.357 857 2581	476 4614	11
913	2305.317 852 0964	475 9392	11
914	2308.278 322 8740	475 4182	11
915	2311.239 269 0697	474 8983	11
916	2314.200 690 1638	474 3796	11
917	2317.162 585 6374	473 8620	11
918	2320.124 954 9731	473 3455	11
919	2323.087 797 6543	472 8302	11
920	2326.051 113 1657	472 3160	11

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
920	2326.051 113 1657	472 3160	11
921	2329.014 900 9930	471 8029	11
922	2331.979 160 6232	471 2909	11
923	2334.943 891 5443	470 7800	11
924	2337.909 093 2453	470 2702	11
925	2340.874 765 2165	469 7615	11
926	2343.840 906 9493	469 2539	11
927	2346.807 517 9360	468 7475	11
928	2349.774 597 6701	468 2421	11
929	2352.742 145 6463	467 7378	11
930	2355.710 161 3603	467 2346	11
931	2358.678 644 3089	466 7324	11
932	2361.647 593 9898	466 2314	11
933	2364.617 009 9022	465 7314	11
934	2367.586 891 5459	465 2325	11
935	2370.557 238 4222	464 7346	11
936	2373.528 050 0331	464 2379	11
937	2376.499 326 8818	463 7421	11
938	2379.471 066 4727	463 2475	11
939	2382.443 268 3111	462 7539	11
940	2385.416 933 9033	462 2613	10
941	2388.389 061 7569	461 7698	10
942	2391.362 661 3803	461 2794	10
943	2394.336 702 2831	460 7899	10
944	2397.311 213 9759	460 3016	10
945	2400.286 185 9702	459 8142	10
946	2403.261 617 7787	459 3279	10
947	2406.237 508 9151	458 8426	10
948	2409.213 868 8941	458 3583	10
949	2412.190 667 2314	457 8751	10
950	2415.167 933 4439	457 3929	10
951	2418.145 657 0491	456 9116	10
952	2421.123 837 5661	456 4314	10
953	2424.102 474 5145	455 9523	10
954	2427.081 567 4151	455 4741	10
955	2430.061 115 7898	454 9969	10
956	2433.041 119 1614	454 5207	10
957	2436.021 577 0637	454 0455	10
958	2439.002 488 9914	453 5713	10
959	2441.983 854 5005	453 0981	10
960	2444.965 673 1077	452 6259	10
961	2447.947 944 3407	452 1546	10
962	2450.930 667 7284	451 6844	10
963	2453.913 842 8004	451 2151	10
964	2456.897 469 0876	450 7468	10
965	2459.881 546 1215	450 2794	10
966	2462.866 073 4348	449 8131	10
967	2465.851 050 5612	449 3477	10
968	2468.836 477 0553	448 8832	10
969	2471.822 352 3926	448 4197	10
970	2474.808 676 1697	447 9572	10

Tables of the Logarithms

p	Log Γ(p)	δ <sup>2</sup>	δ <sup>4</sup>
970	2474.808 676 1697	447 9572	10
971	2477.795 447 9039	447 4956	10
972	2480.782 667 1338	447 0350	9
973	2483.770 333 3988	446 5753	9
974	2486.758 446 2390	446 1166	9
975	2489.747 005 1959	445 6588	9
976	2492.736 009 8116	445 2020	9
977	2495.725 459 6293	444 7461	9
978	2498.715 354 1930	444 2911	9
979	2501.705 693 0478	443 8370	9
980	2504.696 476 7396	443 3839	9
981	2507.687 701 8153	442 9317	9
982	2510.679 370 8227	442 4804	9
983	2513.671 482 3105	442 0300	9
984	2516.664 035 8283	441 5805	9
985	2519.657 030 9267	441 1321	9
986	2522.650 467 1572	440 6844	9
987	2525.644 344 0722	440 2377	9
988	2528.638 661 2248	439 7919	9
989	2531.633 418 1694	439 3470	9
990	2534.628 614 4610	438 9030	9
991	2537.624 249 6566	438 4599	9
992	2540.620 323 3101	438 0177	9
993	2543.616 834 9822	437 5763	9
994	2546.613 784 2307	437 1359	9
995	2549.611 170 6151	436 6963	9
996	2552.608 993 6959	436 2577	9
997	2555.607 253 0343	435 8199	9
998	2558.605 948 1926	435 3830	9
999	2561.605 078 7339	434 9469	9
1000	2564.604 644 2221	434 5118	9
1001	2567.604 644 2221	434 0775	9
1002	2570.605 078 2996	433 6441	9
1003	2573.605 946 0211	433 2115	9
1004	2576.607 246 9642	432 7798	9
1005	2579.608 980 6670	432 3489	9
1006	2582.611 146 7287	431 9190	9
1007	2586.613 744 7094	431 4898	9
1008	2588.616 774 1800	431 0616	8
1009	2591.620 234 7121	430 6341	8
1010	2594.624 125 8783	430 2075	8
1011	2597.628 447 2521	429 7818	8
1012	2600.633 198 4077	429 3569	8
1013	2603.638 378 9202	428 9329	8
1014	2606.643 988 3686	428 5096	8
1015	2609.650 026 3206	428 0873	8
1016	2612.656 492 3628	427 6657	8
1017	2615.663 386 0708	427 2450	8
1018	2618.670 707 0237	426 8251	8
1019	2621.678 484 8017	426 4060	8
1020	2624.686 628 9857	425 9878	8

p	Log Γ(p)	δ <sup>2</sup>	δ <sup>4</sup>
1020	2624.686 628 9857	425 9878	8
1021	2627.695 229 1575	425 5703	8
1022	2630.704 254 8996	425 1537	8
1023	2633.713 705 7954	424 7379	8
1024	2636.723 581 4291	424 3229	8
1025	2639.733 881 3867	423 9088	8
1026	2642.744 605 2511	423 4954	8
1027	2645.755 752 6119	423 0828	8
1028	2648.767 323 0555	422 6711	8
1029	2651.779 316 1701	422 2601	8
1030	2654.791 731 5449	421 8499	8
1031	2657.804 568 7696	421 4406	8
1032	2660.817 827 4349	421 0320	8
1033	2663.831 507 1322	420 6242	8
1034	2666.845 607 4537	420 2172	8
1035	2669.860 127 9925	419 8110	8
1036	2672.875 068 3422	419 4056	8
1037	2675.890 428 0977	419 0010	8
1038	2678.906 206 8540	418 5971	8
1039	2681.922 404 2076	418 1940	8
1040	2684.939 019 7551	417 7917	8
1041	2687.956 053 0944	417 3902	8
1042	2690.973 503 8239	416 9895	8
1043	2693.991 371 5429	416 5895	8
1044	2697.009 655 8513	416 1902	8
1045	2700.028 356 3500	415 7918	8
1046	2703.047 472 6404	415 3941	8
1047	2706.067 004 3250	414 9971	8
1048	2709.086 951 0066	414 6010	8
1049	2712.107 312 2893	414 2055	8
1050	2715.128 087 7775	413 8109	8
1051	2718.149 277 0765	413 4170	7
1052	2721.170 879 7926	413 0238	7
1053	2724.192 895 5324	412 6314	7
1054	2727.215 323 9036	412 2397	7
1055	2730.238 164 5145	411 8488	7
1056	2733.261 416 9741	411 4586	7
1057	2736.285 080 8923	411 0691	7
1058	2739.309 155 8796	410 6804	7
1059	2742.333 641 5473	410 2924	7
1060	2745.358 537 5074	409 9062	7
1061	2748.383 843 3727	409 5186	7
1062	2751.409 558 7566	409 1328	7
1063	2754.435 683 2733	408 7478	7
1064	2757.462 216 5378	408 3634	7
1065	2760.489 158 1658	407 9798	7
1066	2763.516 507 7736	407 5969	7
1067	2766.544 264 9783	407 2147	7
1068	2769.572 429 3977	406 8333	7
1069	2772.601 000 6604	406 4525	7
1070	2775.629 978 3556	406 0725	7

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
1070	2778-629 978 3556	406 0725	7
1071	2778-659 362 1333	405 6931	7
1072	2781-689 151 6041	405 3145	7
1073	2784-719 346 3895	404 9366	7
1074	2787-749 946 1114	404 5594	7
1075	2790-780 950 3928	404 1829	7
1076	2793-812 358 8570	403 8071	7
1077	2796-844 171 1284	403 4320	7
1078	2799-876 386 8317	403 0576	7
1079	2802-909 005 5925	402 6838	7
1080	2805-942 027 0372	402 3108	7
1081	2808-975 450 7927	401 9385	7
1082	2812-009 276 4866	401 5668	7
1083	2815-043 508 7474	401 1959	7
1084	2818-078 132 2040	400 8256	7
1085	2821-113 151 4862	400 4560	7
1086	2824-148 591 2244	400 0871	7
1087	2827-184 421 0497	399 7188	7
1088	2830-220 650 5938	399 3513	7
1089	2833-257 279 4691	398 9844	7
1090	2836-294 307 3689	398 6182	7
1091	2839-331 733 8668	398 2526	7
1092	2842-369 558 6174	397 8878	7
1093	2845-407 781 2558	397 5236	7
1094	2848-446 401 4177	397 1600	7
1095	2851-485 418 7397	396 7972	7
1096	2854-524 832 8589	396 4350	7
1097	2857-564 643 4131	396 0734	7
1098	2860-604 850 0406	395 7125	7
1099	2863-645 452 3807	395 3523	7
1100	2866-686 450 0732	394 9927	7
1101	2869-727 842 7683	394 6338	7
1102	2872-769 630 0773	394 2756	6
1103	2875-811 811 6718	393 9179	6
1104	2878-854 387 1843	393 5610	6
1105	2881-897 356 2576	393 2046	6
1106	2884-940 718 5357	392 8489	6
1107	2887-984 473 6626	392 4939	6
1108	2891-028 621 2835	392 1395	6
1109	2894-073 151 0439	391 7858	6
1110	2897-118 092 5901	391 4326	6
1111	2900-163 415 5688	391 0802	6
1112	2903-209 129 6278	390 7283	6
1113	2906-255 234 4150	390 3771	6
1114	2909-301 729 5794	390 0265	6
1115	2912-348 614 7702	389 6765	6
1116	2915-395 889 6376	389 3272	6
1117	2918-443 553 8322	388 9785	6
1118	2921-491 607 0053	388 6304	6
1119	2924-540 048 8089	388 2830	6
1120	2927-588 878 8954	387 9361	6

$p$	$\text{Log } \Gamma(p)$	$\delta^2$	$\delta^4$
1120	2927-588 878 8954	387 9361	6
1121	2930-638 096 9181	387 5899	6
1122	2933-687 702 5306	387 2443	6
1123	2936-737 695 3876	386 8993	6
1124	2939-788 075 1438	386 5550	6
1125	2942-838 841 4551	386 2112	6
1126	2945-889 993 9775	385 8681	6
1127	2948-941 532 3680	385 5255	6
1128	2951-993 456 2841	385 1836	6
1129	2955-045 765 3837	384 8423	6
1130	2958-098 459 3256	384 5016	6
1131	2961-151 537 7691	384 1614	6
1132	2964-205 000 3741	383 8219	6
1133	2967-258 846 8009	383 4830	6
1134	2970-313 076 7108	383 1447	6
1135	2973-367 689 7663	382 8070	6
1136	2976-422 685 6269	382 4698	6
1137	2979-478 063 9592	382 1333	6
1138	2982-533 824 4229	381 7974	6
1139	2985-589 966 6850	381 4620	6
1140	2988-646 490 4091	381 1273	6
1141	2991-703 395 2604	380 7931	6
1142	2994-760 580 9048	380 4596	6
1143	2997-818 347 0087	380 1266	6
1144	3000-876 393 2371	379 7941	6
1145	3003-934 819 2636	379 4622	6
1146	3006-993 624 7502	379 1310	6
1147	3010-052 809 3679	378 8003	6
1148	3013-112 372 7858	378 4702	6
1149	3016-172 314 6738	378 1406	6
1150	3019-232 634 7025	377 8117	6
1151	3022-293 382 6429	377 4833	6
1152	3025-354 407 8665	377 1556	6
1153	3028-415 860 3456	376 8282	6
1154	3031-477 689 6529	376 5015	6
1155	3034-539 895 4617	376 1754	6
1156	3037-602 477 4459	375 8499	6
1157	3040-665 435 2800	375 5249	6
1158	3043-728 768 6390	375 2004	6
1159	3046-792 477 1984	374 8766	6
1160	3049-856 560 6343	374 5533	6
1161	3052-921 018 6236	374 2305	6
1162	3055-986 850 8433	373 9083	6
1163	3059-051 056 9714	373 5867	6
1164	3062-116 636 6861	373 2656	6
1165	3065-182 589 6664	372 9450	6
1166	3068-248 915 5918	372 6251	6
1167	3071-315 614 1422	372 3056	6
1168	3074-382 684 9982	371 9867	5
1169	3077-450 127 8410	371 6684	5
1170	3080-517 942 3522	371 3506	5

Complete  $\Gamma$ -Functions.

$p$	Log $\Gamma(p)$	$\delta^2$	$\delta^4$	$p$	Log $\Gamma(p)$	$\delta^2$	$\delta^4$
1170	3080.517 942 3522	371 3506	5	1185	3126.579 545 4049	366 6480	5
1171	3083.586 128 2139	371 0333	5	1186	3129.653 263 7653	366 3387	5
1172	3086.654 685 1090	370 7166	5	1187	3132.727 348 4443	366 0299	5
1173	3089.723 612 7207	370 4004	5	1188	3135.801 799 1632	365 7217	5
1174	3092.792 910 7328	370 0848	5	1189	3138.876 615 6089	365 4140	5
1175	3095.862 578 8297	369 7697	5	1190	3141.951 797 4585	365 1068	5
1176	3098.932 616 6963	369 4551	5	1191	3145.027 344 4199	364 8001	5
1177	3102.003 024 0180	369 1411	5	1192	3148.103 256 1814	364 4939	5
1178	3105.073 800 4809	368 8276	5	1193	3151.179 532 4368	364 1883	5
1179	3108.144 945 7713	368 5146	5	1194	3154.256 172 8805	363 8831	5
1180	3111.216 459 5764	368 2022	5	1195	3157.333 177 2072	363 5785	5
1181	3114.288 341 5837	367 8903	5	1196	3160.410 545 1125	363 2744	5
1182	3117.360 591 4813	367 5789	5	1197	3163.488 276 2922	362 9708	5
1183	3120.433 208 9579	367 2681	5	1198	3166.566 370 4426	362 6676	5
1184	3123.506 193 7025	366 9578	5	1199	3169.644 827 2606	362 3650	5
1185	3126.579 545 4049	366 6480	5	1200	3172.723 646 4437	362 0629	5

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(COMPUTING SECTION)  
UNIVERSITY OF LONDON, UNIVERSITY COLLEGE

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Log  $\Gamma(x)$  from  $x = 1$  to 50.9 by intervals of .01

By JOHN BROWNLEE, M.D., D.Sc.

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## PREFATORY NOTE

**T**HE following table, due to Dr John Brownlee's initiative, rounds off the work of this series of *Tracts* on the  $\Gamma$ -function. It will enable the computer to obtain somewhat more easily (but to 7 instead of 10 figures) the value of the  $\Gamma$ -function for values of the argument lying between 1 and 50·9.

K. P.



The following Table of  $\log \Gamma(x)$  gives the values to seven figures for values of  $x$  from unity to fifty-one. The compact form of this Table is rendered possible by the introduction of formulae for interpolating by means of central differences. Though the first differences are of six figures, from the beginning of the table to the end, the fourth difference is zero. The method of construction has been as follows. The values of  $\log \Gamma(x)$  were taken to ten places of decimals from the facsimile of *Legendre's* tables published lately by Professor Pearson in the Tracts for Computers Series. The subsequent values were obtained by adding on a Burroughs' Adding Machine the logarithms of the appropriate numbers as given by Vega, in the *Thesaurus Logarithmorum Completus*, and the consecutive additions were carried on to the limit of the table. The final figures were then checked by the ordinary *Euler-Maclaurin* formula for  $\log \Gamma(x)$ . It was found in every case, in spite of the fact that fifty odd consecutive additions had been made, that at least eight figures of the results were correct, while in a number of cases the ninth was also accurate.

The whole of the work was carried out under the supervision of Mr W. T. Russell, chief statistical clerk to the National Institute for Medical Research, who himself checked all the differences. The whole credit for the table is thus his.

JOHN BROWNLEE.

NATIONAL INSTITUTE FOR MEDICAL RESEARCH.

Feb. 1st, 1923.

# TABLE

OF

LOG  $\Gamma(x)$  FROM  $x = 1$  TO 50·9 BY INTERVALS OF ·01

	0		1		2		3		4	
1.0	0.0000000	714	9.9975287	704	9.9951279	694	9.9927964	684	9.9905334	675
1.1	9.9783467	622	9.9765313	615	9.9747834	607	9.9730962	599	9.9714689	592
1.2	9.9629225	550	9.9616946	544	9.9605212	538	9.9594015	532	9.9583350	526
1.3	9.9530203	493	9.9523100	487	9.9516485	482	9.9510353	477	9.9504608	473
1.4	9.9480528	445	9.9478084	441	9.9476081	437	9.9474515	433	9.9473382	429
1.5	9.9475449	406	9.9477237	402	9.9479426	399	9.9482015	394	9.9484998	392
1.6	9.9511020	373	9.9516680	370	9.9522710	367	9.9529107	364	9.9535867	361
1.7	9.9583912	344	9.9593141	342	9.9602712	339	9.9612622	337	9.9622869	334
1.8	9.9691287	320	9.9703823	318	9.9716678	316	9.9729848	313	9.9743331	311
1.9	9.9830693	299	9.9846311	297	9.9862226	295	9.9878436	293	9.9894938	291
2.0	.0000000	280	.0018501	278	.0037280	277	.0056336	274	.0075667	273
2.1	.0197334	264	.0218543	263	.0240014	260	.0261746	259	.0283737	257
2.2	.0421038	249	.0444800	247	.0468810	246	.0493066	245	.0517567	243
2.3	.0669636	236	.0695813	234	.0722225	233	.0748869	232	.0775746	231
2.4	.0941808	224	.0970275	223	.0998964	222	.1027875	220	.1057007	219
2.5	.1236362	213	.1267006	212	.1297862	211	.1328929	210	.1360206	209
2.6	.1552220	203	.1584939	202	.1617860	201	.1650983	200	.1684306	200
2.7	.1888402	194	.1923102	193	.1957997	193	.1993083	192	.2028362	191
2.8	.2244012	186	.2280609	185	.2317392	184	.2354359	184	.2391510	183
2.9	.2618229	178	.2656645	178	.2695238	177	.2734009	176	.2772956	176
3.0	.3010300	172	.3050462	171	.3090794	170	.3131297	170	.3171969	169
3.1	.3419527	165	.3461368	164	.3503373	164	.3545542	163	.3587875	163
3.2	.3845264	159	.3888723	159	.3932340	158	.3976115	157	.4020047	157
3.3	.4286915	154	.4331933	153	.4377105	152	.4422428	152	.4467904	151
3.4	.4743920	148	.4790445	148	.4837118	147	.4883938	147	.4930905	146
3.5	.5215762	143	.5263743	143	.5311867	143	.5360134	142	.5408543	142
3.6	.5701953	139	.5751344	138	.5800873	138	.5850540	138	.5900345	137
3.7	.6202039	135	.6252795	134	.6303686	134	.6354710	133	.6405867	133
3.8	.6715592	131	.6767672	130	.6819883	130	.6872223	129	.6924693	129
3.9	.7242209	127	.7295575	126	.7349067	126	.7402685	126	.7456429	125
4.0	.7781513	123	.7836127	123	.7890864	123	.7945723	122	.8000705	122
4.1	.8333143	120	.8388971	120	.8444919	119	.8500986	119	.8557171	119
4.2	.8896764	117	.8953773	116	.9010898	116	.9068140	116	.9125497	115
4.3	.9472054	114	.9530213	113	.9588485	113	.9646871	113	.9705369	112
4.4	1.0058710	111	1.0117989	110	1.0177379	110	1.0236879	110	1.0296489	110
4.5	1.0656443	108	1.0716814	108	1.0777294	107	1.0837881	107	1.0898575	107
4.6	1.1264978	105	1.1326416	105	1.1387959	105	1.1449607	105	1.1511359	104
4.7	1.1884057	103	1.1946535	103	1.2009115	102	1.2071798	102	1.2134583	102
4.8	1.2513428	101	1.2576922	100	1.2640516	100	1.2704211	100	1.2768005	100
4.9	1.3152856	98	1.3217343	98	1.3281928	98	1.3346611	98	1.3411391	97
5.0	1.3802112	96	1.3867570	96	1.3933124	96	1.3998774	95	1.4064519	95
5.1	1.4460982	94	1.4527390	94	1.4593891	94	1.4660486	93	1.4727175	93
5.2	1.5129257	92	1.5196594	92	1.5264023	92	1.5331544	91	1.5399156	91
5.3	1.5806739	90	1.5874986	90	1.5943323	90	1.6011750	90	1.6080266	89
5.4	1.6493236	88	1.6562375	88	1.6631602	88	1.6700916	88	1.6770319	88
5.5	1.7188568	87	1.7258580	86	1.7328678	86	1.7398863	86	1.7469134	86
5.6	1.7892557	85	1.7963425	85	1.8034379	85	1.8105416	84	1.8176539	84
5.7	1.8605035	83	1.8676744	83	1.8748535	83	1.8820410	83	1.8892367	83
5.8	1.9325840	82	1.9398373	82	1.9470987	81	1.9543682	81	1.9616459	81
5.9	2.0054816	80	2.0128157	80	2.0201579	80	2.0275080	80	2.0348661	80
6.0	2.0791812	79	2.0865948	79	2.0940161	78	2.1014453	78	2.1088824	78

6		7		8		9		10		
9.9883379	666	9.9862089	656	9.9841455	648	9.9821469	639	9.9802123	631	1.0
9.9909007	584	9.9683910	577	9.9669390	570	9.9655440	564	9.9642054	557	1.1
9.9573211	520	9.9563592	514	9.9554487	509	9.9545891	503	9.9537798	498	1.2
9.9499515	468	9.9494800	463	9.9490549	459	9.9486756	454	9.9483417	450	1.3
9.9472677	425	9.9472397	421	9.9472539	417	9.9473097	413	9.9474068	410	1.4
9.9488374	389	9.9492139	386	9.9496289	382	9.9500822	379	9.9505733	376	1.5
9.9542989	358	9.9550468	355	9.9558303	353	9.9566491	350	9.9575028	347	1.6
9.9633451	332	9.9644364	329	9.9655606	327	9.9667176	325	9.9679070	322	1.7
9.9757126	309	9.9771230	307	9.9785640	305	9.9800356	303	9.9815374	301	1.8
9.9911732	289	9.9928815	287	9.9946185	285	9.9963840	284	9.9981779	282	1.9
.0095272	272	.0115147	270	.0135293	268	.0155707	267	.0176388	265	2.0
.0305085	256	.0328490	255	.0351248	253	.0374260	252	.0397524	250	2.1
.0542311	242	.0567297	242	.0592524	239	.0617990	238	.0643695	237	2.2
.0802853	230	.0830190	228	.0857755	227	.0885547	226	.0913565	225	2.3
.1086357	218	.1115926	217	.1145712	216	.1175714	215	.1205931	214	2.4
.1391691	208	.1423385	207	.1455286	206	.1487392	205	.1519704	204	2.5
.1717828	199	.1751549	198	.1785468	197	.1819584	196	.1853895	195	2.6
.2063831	190	.2099490	189	.2135339	188	.2171376	188	.2207600	187	2.7
.2428843	182	.2466359	181	.2504056	181	.2541934	180	.2579992	179	2.8
.2812078	175	.2851375	174	.2890847	174	.2930492	173	.2970310	172	2.9
.3212810	168	.3253820	168	.3294906	167	.3336340	166	.3378051	166	3.0
.3630370	162	.3673027	161	.3715846	161	.3758825	160	.3801965	160	3.1
.4064136	156	.4108382	156	.4152783	155	.4197339	155	.4242050	154	3.2
.4513531	151	.4559310	150	.4605238	150	.4651316	149	.4697544	149	3.3
.4978018	146	.5025277	145	.5072681	145	.5120231	144	.5167924	144	3.4
.5457093	141	.5505785	141	.5554617	140	.5603589	140	.5652702	139	3.5
.5950287	137	.6000366	136	.6050581	136	.6100931	135	.6151418	135	3.6
.6457158	133	.6508581	132	.6560137	132	.6611824	131	.6663642	131	3.7
.6977292	129	.7030019	128	.7082875	128	.7135859	128	.7188971	127	3.8
.7510298	125	.7564292	125	.7618411	124	.7672655	124	.7727022	124	3.9
.8055809	122	.8111034	121	.8166380	121	.8221848	121	.8277435	120	4.0
.8613476	118	.8669898	118	.8726438	118	.8783096	117	.8839872	117	4.1
.9182070	115	.9240558	115	.9298260	115	.9356077	114	.9414009	114	4.2
.9763980	112	.9822702	112	.9881537	112	.9940483	111	.9999541	111	4.3
1.0356209	109	1.0416038	109	1.0475976	109	1.0536023	109	1.0596179	108	4.4
1.0959377	107	1.1020285	106	1.1081299	106	1.1142420	106	1.1203646	106	4.5
1.1573216	104	1.1635176	104	1.1697241	104	1.1759410	103	1.1821682	103	4.6
1.2197471	102	1.2260460	101	1.2323550	101	1.2386742	101	1.2450035	101	4.7
1.2831899	99	1.2895893	99	1.2959985	99	1.3024177	99	1.3088467	99	4.8
1.3476269	97	1.3541244	97	1.3606316	97	1.3671485	97	1.3736751	96	4.9
1.4130359	95	1.4196294	95	1.4262324	95	1.4328449	94	1.4394668	94	5.0
1.4793956	93	1.4860831	93	1.4927799	93	1.4994859	92	1.5062012	92	5.1
1.5466859	91	1.5534654	91	1.5602539	91	1.5670515	91	1.5738582	90	5.2
1.6148872	89	1.6217507	89	1.6286351	89	1.6355224	89	1.6424186	89	5.3
1.6839809	87	1.6909387	87	1.6979051	87	1.7048803	87	1.7118642	87	5.4
1.7539491	86	1.7609933	86	1.7680461	85	1.7751075	85	1.7821773	85	5.5
1.8247745	84	1.8319036	84	1.8390410	84	1.8461868	84	1.8533410	83	5.6
1.8964407	82	1.9036529	82	1.9108734	82	1.9181021	82	1.9253390	82	5.7
1.9689317	81	1.9762255	81	1.9835275	81	1.9908375	81	1.9981555	80	5.8
2.0422321	79	2.0496061	79	2.0569880	79	2.0643779	79	2.0717756	79	5.9
2.1163273	78	2.1237799	78	2.1312404	78	2.1387086	78	2.1461846	78	6.0

	0		1		2		3		4	
6.0	2.0791812	79	2.0865948	79	2.0940161	78	2.1014453	78	2.1088824	78
6.1	2.1536684	77	2.1611599	77	2.1686591	77	2.1761660	77	2.1836806	77
6.2	2.2289290	76	2.2364971	76	2.2440728	76	2.2516560	76	2.2592469	75
6.3	2.3049497	75	2.3125931	75	2.3202439	74	2.3279022	74	2.3355679	74
6.4	2.3817174	73	2.3894348	73	2.3971595	73	2.4048915	73	2.4126308	73
6.5	2.4592195	72	2.4670096	72	2.4748069	72	2.4826114	72	2.4904232	72
6.6	2.5374437	71	2.5453054	71	2.5531742	71	2.5610500	71	2.5689330	71
6.7	2.6163784	70	2.6243105	70	2.6322495	70	2.6401956	70	2.6481486	69
6.8	2.6960120	69	2.7040134	69	2.7120217	69	2.7200368	69	2.7280587	68
6.9	2.7763336	68	2.7844032	68	2.7924796	68	2.8005627	68	2.8086525	67
7.0	2.8573325	67	2.8654692	67	2.8736126	66	2.8817627	66	2.8899193	66
7.1	2.9389982	66	2.9472011	66	2.9554105	65	2.9636265	65	2.9718490	65
7.2	3.0213207	65	3.0295887	65	3.0378632	65	3.0461441	64	3.0544314	64
7.3	3.1042903	64	3.1126225	64	3.1209610	64	3.1293059	63	3.1376571	63
7.4	3.1878974	63	3.1962928	63	3.2046945	63	3.2131024	63	3.2215167	62
7.5	3.2721328	62	3.2805906	62	3.2890545	62	3.2975246	62	3.3060009	62
7.6	3.3569876	61	3.3655069	61	3.3740322	61	3.3825636	61	3.3911011	61
7.7	3.4424532	60	3.4510330	60	3.4596188	60	3.4682106	60	3.4768085	60
7.8	3.5285209	59	3.5371605	59	3.5458060	59	3.5544575	59	3.5631148	59
7.9	3.6151827	59	3.6238813	59	3.6325857	58	3.6412959	58	3.6500120	58
8.0	3.7024305	58	3.7111872	58	3.7199497	58	3.7287180	58	3.7374920	58
8.1	3.7902566	57	3.7990707	57	3.8078905	57	3.8167160	57	3.8255472	57
8.2	3.8786532	56	3.8875240	56	3.8964004	56	3.9052824	56	3.9141700	56
8.3	3.9676131	56	3.9765398	56	3.9854721	55	3.9944099	55	4.0033532	55
8.4	4.0571291	55	4.0661110	55	4.0750984	55	4.0840913	55	4.0930896	55
8.5	4.1471941	54	4.1562305	54	4.1652724	54	4.1743196	54	4.1833723	54
8.6	4.2378012	54	4.2468915	53	4.2559871	53	4.2650881	53	4.2741944	53
8.7	4.3289439	53	4.3380874	53	4.3472361	53	4.3563901	53	4.3655494	53
8.8	4.4206155	52	4.4298116	52	4.4390128	52	4.4482192	52	4.4574309	52
8.9	4.5128098	52	4.5220578	52	4.5313108	52	4.5405691	51	4.5498325	51
9.0	4.6055205	51	4.6148198	51	4.6241241	51	4.6334335	51	4.6427480	51
9.1	4.6987416	50	4.7080915	50	4.7174465	50	4.7268065	50	4.7361716	50
9.2	4.7924671	50	4.8018672	50	4.8112722	50	4.8206822	50	4.8300972	50
9.3	4.8866912	49	4.8961409	49	4.9055954	49	4.9150549	49	4.9245192	49
9.4	4.9814084	49	4.9909070	49	5.0004105	49	5.0099188	49	5.0194320	49
9.5	5.0766130	48	5.0861601	48	5.0957119	48	5.1052686	48	5.1148301	48
9.6	5.1722997	48	5.1818947	48	5.1914944	48	5.2010989	48	5.2107082	47
9.7	5.2684632	47	5.2781055	47	5.2877526	47	5.2974044	47	5.3070609	47
9.8	5.3650982	47	5.3747875	47	5.3844814	47	5.3941799	47	5.4038832	46
9.9	5.4621998	46	5.4719355	46	5.4816757	46	5.4914205	46	5.5011700	46
10.0	5.5597630	46	5.5695446	46	5.5793306	46	5.5891213	46	5.5989165	45
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10.2	5.7562549	45	5.7661268	45	5.7760031	45	5.7858839	45	5.7957692	45
10.3	5.8551742	44	5.8650905	44	5.8750113	44	5.8849365	44	5.8948661	44
10.4	5.9545362	44	5.9644966	44	5.9744614	44	5.9844305	44	5.9944040	44
10.5	6.0543366	43	6.0643406	43	6.0743489	43	6.0843615	43	6.0943785	43
10.6	6.1545709	43	6.1646181	43	6.1746695	43	6.1847252	43	6.1947852	43
10.7	6.2552349	43	6.2653247	43	6.2754189	42	6.2855172	42	6.2956198	42
10.8	6.3563243	42	6.3664505	42	6.3765929	42	6.3867335	42	6.3968783	42
10.9	6.4578350	42	6.4680091	42	6.4781874	42	6.4883698	42	6.4985564	42
11.0	6.5597630	41	6.5699786	41	6.5801984	41	6.5904222	41	6.6006502	41



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12.3	7.9210898	37	7.9318118	37	7.9425374	37	7.9532667	37	7.9639997	37
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12.5	8.1362237	36	8.1470186	36	8.1578171	36	8.1686192	36	8.1794249	36
12.6	8.2443348	36	8.2551657	36	8.2660001	36	8.2768382	36	8.2876798	36
12.7	8.3528045	36	8.3636711	36	8.3745413	36	8.3854150	35	8.3962922	35
12.8	8.4616301	35	8.4725321	35	8.4834376	35	8.4943467	35	8.5052592	35
12.9	8.5708085	35	8.5817456	35	8.5926863	35	8.6036304	35	8.6145780	35
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13.1	8.7902127	34	8.8012193	34	8.8122293	34	8.8232427	34	8.8342596	34
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13.5	9.2331338	33	9.2442759	33	9.2554214	33	9.2665703	33	9.2777225	33
13.6	9.3447053	33	9.3558807	33	9.3670595	33	9.3782415	33	9.3894269	33
13.7	9.4566082	33	9.4678167	33	9.4790284	33	9.4902434	33	9.5014616	33
13.8	9.5688400	33	9.5800812	33	9.5913256	33	9.6025733	33	9.6138243	33
13.9	9.6813982	32	9.6926719	32	9.7039488	32	9.7152289	32	9.7265123	32
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14.3	10.1348466	31	10.1462479	31	10.1576524	31	10.1690600	31	10.1804707	31
14.4	10.2490009	31	10.2604336	31	10.2718693	31	10.2833082	31	10.2947502	31
14.5	10.3634675	31	10.3749313	31	10.3863981	31	10.3978681	31	10.4093411	31
14.6	10.4782442	31	10.4897389	31	10.5012366	31	10.5127374	31	10.5242412	31
14.7	10.5933288	31	10.6048541	31	10.6163825	31	10.6279139	31	10.6394484	30
14.8	10.7087191	30	10.7202749	30	10.7318337	30	10.7433955	30	10.7549604	30
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15.3	11.2901826	29	11.3018875	29	11.3135954	29	11.3253062	29	11.3370199	29
15.4	11.4073634	29	11.4190975	29	11.4308346	29	11.4425745	29	11.4543174	29
15.5	11.5248355	29	11.5365987	29	11.5483647	29	11.5601337	29	11.5719055	29
15.6	11.6425971	29	11.6543801	29	11.6661840	29	11.6779817	29	11.6897823	29
15.7	11.7606461	29	11.7724668	29	11.7842903	29	11.7961166	29	11.8079458	29
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15.9	11.9975993	28	12.0094766	28	12.0213568	28	12.0332399	28	12.0451257	28
16.0	12.1164996	28	12.1284051	28	12.1403134	28	12.1522244	28	12.1641383	28

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6.6108823	41	6.6211185	41	6.6313588	41	6.6416032	41	6.6518517	41	11.0
6.7134288	41	6.7237059	41	6.7339871	41	6.7442724	41	6.7545617	41	11.1
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8.4071730	35	8.4180573	35	8.4289452	35	8.4398366	35	8.4507316	35	12.7
8.5161753	35	8.5270949	35	8.5380181	35	8.5489447	35	8.5598748	35	12.8
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8.7352316	35	8.7462209	35	8.7572137	35	8.7682099	35	8.7792096	34	13.0
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16.7	12.9565458	27	12.9686430	27	12.9807428	27	12.9928454	27	13.0049506	27
16.8	13.0776379	27	13.0897618	27	13.1018884	27	13.1140176	27	13.1261494	27
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17.1	13.4425059	26	13.4547090	26	13.4669146	26	13.4791229	26	13.4913338	26
17.2	13.5646538	26	13.5768829	26	13.5891146	26	13.6013489	26	13.6135858	26
17.3	13.6870601	26	13.6993167	26	13.7115743	26	13.7238345	26	13.7360973	26
17.4	13.8097280	26	13.8220088	26	13.8342921	26	13.8465780	26	13.8588665	26
17.5	13.9326512	26	13.9449576	26	13.9572665	26	13.9695780	25	13.9818920	25
17.6	14.0558298	25	14.0681616	25	14.0804960	25	14.0928329	25	14.1051724	25
17.7	14.1792023	25	14.1916194	25	14.2039791	25	14.2163413	25	14.2287060	25
17.8	14.3029472	25	14.3153295	25	14.3277143	25	14.3401017	25	14.3524915	25
17.9	14.4268831	25	14.4392904	25	14.4517003	25	14.4641126	25	14.4765274	25
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18.1	14.6755020	25	14.6879590	25	14.7004184	25	14.7128803	25	14.7253446	25
18.2	14.8001823	25	14.8126638	25	14.8251478	25	14.8376342	24	14.8501231	24
18.3	14.9251078	24	14.9376138	24	14.9501222	24	14.9626331	24	14.9751464	24
18.4	15.0502772	24	15.0628075	24	15.0753403	24	15.0878754	24	15.1004130	24
18.5	15.1756892	24	15.1882437	24	15.2008006	24	15.2133599	24	15.2259216	24
18.6	15.3013424	24	15.3139210	24	15.3265019	24	15.3390852	24	15.3516710	24
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18.9	15.6797361	24	15.6923860	24	15.7050383	24	15.7176929	24	15.7303498	24
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19.5	16.4428610	23	16.4556501	23	16.4684416	23	16.4812353	23	16.4940314	23
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19.7	16.6990771	23	16.7119118	23	16.7247487	23	16.7375878	23	16.7504292	23
19.8	16.8275250	23	16.8403822	22	16.8532417	22	16.8661033	22	16.8789673	22
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20.3	17.4731162	22	17.4860844	22	17.4990548	22	17.5120274	22	17.5250022	22
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26.7	26.1793777	17	26.1935618	17	26.2077476	17	26.2219350	17	26.2361241	17
26.8	26.3212933	17	26.3354939	17	26.3496962	17	26.3639001	17	26.3781057	16
26.9	26.4633739	16	26.4775910	16	26.4918098	16	26.5060302	16	26.5202522	16
27.0	26.6056190	16	26.6198526	16	26.6340877	16	26.6483245	16	26.6625630	16
27.1	26.7480280	16	26.7622779	16	26.7765294	16	26.7907826	16	26.8050374	16
27.2	26.8906003	16	26.9048664	16	26.9191342	16	26.9334037	16	26.9476747	16
27.3	27.0333351	16	27.0476176	16	27.0619016	16	27.0761872	16	27.0904745	16
27.4	27.1762321	16	27.1905306	16	27.2048308	16	27.2191326	16	27.2334361	16
27.5	27.3192904	16	27.3336051	16	27.3479214	16	27.3622393	16	27.3765588	16
27.6	27.4625096	16	27.4768403	16	27.4911727	16	27.5055066	16	27.5198421	16
27.7	27.6058890	16	27.6202357	16	27.6345840	16	27.6489340	16	27.6632855	16
27.8	27.7494281	16	27.7637907	16	27.7781550	16	27.7925208	16	27.8068882	16
27.9	27.8931262	16	27.9075047	16	27.9218848	16	27.9362665	16	27.9506498	16
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28.1	28.1809973	16	28.1954074	16	28.2098191	16	28.2242324	16	28.2386472	16
28.2	28.3251692	16	28.3395950	16	28.3540224	16	28.3684513	16	28.3828818	16
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28.4	28.6139826	16	28.6284397	16	28.6428983	16	28.6573584	16	28.6718202	16
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30.7	31.9790071	14	31.9938081	14	32.0086105	14	32.0234144	14	32.0382197	14
30.8	32.1270816	14	32.1418969	14	32.1567137	14	32.1715319	14	32.1863516	14
30.9	32.2752994	14	32.2901291	14	32.3049601	14	32.3197926	14	32.3346266	14
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31.3	32.8695945	I4	32.8844809	I4	32.8993687	I4	32.9142579	I4	32.9291485	I4
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31.9	33.7652579	I4	33.7802281	I4	33.7951996	I4	33.8101726	I4	33.8251469	I4
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32.7	34.9672047	I3	34.9822841	I3	34.9973649	I3	35.0124470	I3	35.0275305	I3
32.8	35.1180594	I3	35.1331523	I3	35.1482465	I3	35.1633421	I3	35.1784390	I3
32.9	35.2690486	I3	35.2841549	I3	35.2992625	I3	35.3143715	I3	35.3294818	I3
33.0	35.4201717	I3	35.4352914	I3	35.4504124	I3	35.4655348	I3	35.4806585	I3
33.1	35.5714285	I3	35.5865615	I3	35.6016959	I3	35.6168316	I3	35.6319686	I3
33.2	35.7228185	I3	35.7379648	I3	35.7531125	I3	35.7682615	I3	35.7834117	I3
33.3	35.8743413	I3	35.8895009	I3	35.9046618	I3	35.9198240	I3	35.9349876	I3
33.4	36.0259965	I3	36.0411693	I3	36.0563434	I3	36.0715189	I3	36.0866956	I3
33.5	36.1777837	I3	36.1929697	I3	36.2081570	I3	36.2233456	I3	36.2385355	I3
33.6	36.3297025	I3	36.3449016	I3	36.3601020	I3	36.3753037	I3	36.3905068	I3
33.7	36.4817525	I3	36.4969647	I3	36.5121782	I3	36.5273930	I3	36.5426091	I3
33.8	36.6339333	I3	36.6491585	I3	36.6643851	I3	36.6796130	I3	36.6948421	I3
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35.3	38.9320797	I2	38.9474963	I2	38.9629141	I2	38.9783331	I2	38.9937534	I2
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35.5	39.2406476	I2	39.2560891	I2	39.2715318	I2	39.2869757	I2	39.3024209	I2
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35.9	39.8592696	I2	39.8747604	I2	39.8902524	I2	39.9057456	I2	39.9212401	I2
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33.0930383	I4	33.1079458	I4	33.1228547	I4	33.1377650	I4	33.1526767	I4	31.4
33.2421763	I4	33.2570978	I4	33.2720207	I4	33.2869450	I4	33.3018707	I4	31.5
33.3914542	I4	33.4063897	I4	33.4213265	I4	33.4362648	I4	33.4512044	I4	31.6
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33.8401226	I4	33.8550997	I4	33.8700781	I4	33.8850580	I4	33.9000392	I4	31.9
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34.1399259	I4	34.1549305	I4	34.1699365	I4	34.1849439	I4	34.1999526	I4	32.1
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35.0426153	I3	35.0577014	I3	35.0727889	I3	35.0878777	I3	35.1029679	I3	32.7
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35.4957835	I3	35.5109098	I3	35.5260375	I3	35.5411665	I3	35.5562969	I3	33.0
35.6471069	I3	35.6622466	I3	35.6773876	I3	35.6925299	I3	35.7076736	I3	33.1
35.7985634	I3	35.8137163	I3	35.8288706	I3	35.8440262	I3	35.8591831	I3	33.2
35.9501524	I3	35.9653186	I3	35.9804861	I3	35.9956549	I3	36.0108251	I3	33.3
36.1018737	I3	36.1170530	I3	36.1322337	I3	36.1474157	I3	36.1625991	I3	33.4
36.2537267	I3	36.2689192	I3	36.2841131	I3	36.2993082	I3	36.3145047	I3	33.5
36.4057111	I3	36.4209168	I3	36.4361237	I3	36.4513320	I3	36.4665416	I3	33.6
36.5578266	I3	36.5730453	I3	36.5882653	I3	36.6034867	I3	36.6187093	I3	33.7
36.7100726	I3	36.7253044	I3	36.7405374	I3	36.7557718	I3	36.7710075	I3	33.8
36.8624489	I3	36.8776936	I3	36.8929397	I3	36.9081871	I3	36.9234357	I3	33.9
37.0149549	I3	37.0302127	I3	37.0454717	I3	37.0607320	I3	37.0759936	I3	34.0
37.1675904	I3	37.1828611	I3	37.1981330	I3	37.2134063	I3	37.2286808	I3	34.1
37.3203550	I3	37.3356386	I3	37.3509234	I3	37.3662095	I3	37.3814969	I3	34.2
37.4732482	I3	37.4885446	I3	37.5038423	I3	37.5191413	I3	37.5344415	I3	34.3
37.6262698	I3	37.6415790	I3	37.6568894	I3	37.6722012	I3	37.6875142	I3	34.4
37.7794192	I3	37.7947412	I3	37.8100644	I3	37.8253889	I3	37.8407147	I3	34.5
37.9326962	I3	37.9480309	I3	37.9633669	I3	37.9787041	I3	37.9940426	I3	34.6
38.0861003	I3	38.1014477	I3	38.1167964	I3	38.1321463	I3	38.1474975	I3	34.7
38.2396313	I3	38.2549913	I3	38.2703526	I3	38.2857152	I3	38.3010791	I3	34.8
38.3932886	I3	38.4086613	I3	38.4240352	I3	38.4394104	I3	38.4547869	I3	34.9
38.5470720	I3	38.5624573	I3	38.5778438	I3	38.5932316	I3	38.6086206	I3	35.0
38.7009812	I3	38.7163790	I3	38.7317780	I3	38.7471783	I3	38.7625799	I3	35.1
38.8550156	I3	38.8704259	I2	38.8858375	I2	38.9012503	I2	38.9166644	I2	35.2
39.0091750	I2	39.0245978	I2	39.0400218	I2	39.0554471	I2	39.0708737	I2	35.3
39.1634590	I2	39.1788942	I2	39.1943307	I2	39.2097685	I2	39.2252074	I2	35.4
39.3178673	I2	39.3333149	I2	39.3487638	I2	39.3642139	I2	39.3796653	I2	35.5
39.4723994	I2	39.4878595	I2	39.5033207	I2	39.5187832	I2	39.5342469	I2	35.6
39.6270551	I2	39.6425275	I2	39.6580010	I2	39.6734759	I2	39.6889519	I2	35.7
39.7818341	I2	39.7973167	I2	39.8128046	I2	39.8282917	I2	39.8437800	I2	35.8
39.9367358	I2	39.9522327	I2	39.9677309	I2	39.9832302	I2	39.9987308	I2	35.9
40.0917601	I2	40.1072692	I2	40.1227796	I2	40.1382912	I2	40.1538040	I2	36.0



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36.0	40.0142326	I2	40.0297357	I2	40.0452399	I2	40.0607454	I2	40.0762521	I2
36.1	40.1693180	I2	40.1848333	I2	40.2003498	I2	40.2158674	I2	40.2313864	I2
36.2	40.3245254	I2	40.3400528	I2	40.3555815	I2	40.3711113	I2	40.3866424	I2
36.3	40.4798544	I2	40.4953940	I2	40.5109348	I2	40.5264768	I2	40.5420200	I2
36.4	40.6353047	I2	40.6508564	I2	40.6664093	I2	40.6819634	I2	40.6975187	I2
36.5	40.7908760	I2	40.8064397	I2	40.8220047	I2	40.8375709	I2	40.8531383	I2
36.6	40.9465679	I2	40.9621437	I2	40.9777207	I2	40.9932989	I2	41.0088783	I2
36.7	41.1023801	I2	41.1179679	I2	41.1335569	I2	41.1491472	I2	41.1647386	I2
36.8	41.2583123	I2	41.2739121	I2	41.2895131	I2	41.3051152	I2	41.3207186	I2
36.9	41.4143640	I2	41.4299758	I2	41.4455887	I2	41.4612029	I2	41.4768182	I2
37.0	41.5705351	I2	41.5861588	I2	41.6017837	I2	41.6174097	I2	41.6330369	I2
37.1	41.7268252	I2	41.7424608	I2	41.7580975	I2	41.7737354	I2	41.7893745	I2
37.2	41.8832340	I2	41.8988813	I2	41.9145299	I2	41.9301797	I2	41.9458306	I2
37.3	42.0397610	I2	42.0554202	I2	42.0710806	I2	42.0867422	I2	42.1024049	I2
37.4	42.1964061	I2	42.2120771	I2	42.2277492	I2	42.2434226	I2	42.2590971	I2
37.5	42.3531688	I2	42.3688516	I2	42.3845355	I2	42.4002206	I2	42.4159068	I2
37.6	42.5100490	I2	42.5257434	I2	42.5414391	I2	42.5571358	I2	42.5728338	I2
37.7	42.6670462	I2	42.6827523	I2	42.6984596	I2	42.7141681	I2	42.7298777	I2
37.8	42.8241601	I2	42.8398779	I2	42.8555968	I2	42.8713170	I2	42.8870383	I2
37.9	42.9813904	I2	42.9971198	I2	43.0128504	I2	43.0285822	I2	43.0443151	I2
38.0	43.1387369	I2	43.1544779	I2	43.1702201	I2	43.1859634	I2	43.2017079	I2
38.1	43.2961991	I2	43.3119517	I2	43.3277055	I2	43.3434604	I2	43.3592164	I2
38.2	43.4537769	I2	43.4695410	I2	43.4853063	I2	43.5010727	I2	43.5168403	I2
38.3	43.6114699	II	43.6272455	II	43.6430222	II	43.6588002	II	43.6745792	II
38.4	43.7692777	II	43.7850648	II	43.8008530	II	43.8166421	II	43.8324329	II
38.5	43.9272001	II	43.9429986	II	43.9587983	II	43.9745991	II	43.9904011	II
38.6	44.0852368	II	44.1010468	II	44.1168578	II	44.1326701	II	44.1484834	II
38.7	44.2433875	II	44.2592088	II	44.2750313	II	44.2908549	II	44.3066796	II
38.8	44.4016519	II	44.4174845	II	44.4333184	II	44.4491533	II	44.4649894	II
38.9	44.5600296	II	44.5758736	II	44.5917188	II	44.6075650	II	44.6234124	II
39.0	44.7185205	II	44.7343758	II	44.7502322	II	44.7660897	II	44.7819484	II
39.1	44.8771241	II	44.8929907	II	44.9088584	II	44.9247272	II	44.9405971	II
39.2	45.0358403	II	45.0517181	II	45.0675970	II	45.0834770	II	45.0993582	II
39.3	45.1946686	II	45.2105576	II	45.2264477	II	45.2423390	II	45.2582313	II
39.4	45.3536089	II	45.3695091	II	45.3854104	II	45.4013126	II	45.4172163	II
39.5	45.5126608	II	45.5285722	II	45.5444846	II	45.5603981	II	45.5763128	II
39.6	45.6718241	II	45.6877466	II	45.7036701	II	45.7195948	II	45.7355205	II
39.7	45.8310985	II	45.8470320	II	45.8629666	II	45.8789024	II	45.8948392	II
39.8	45.9904836	II	46.0064282	II	46.0223739	II	46.0383207	II	46.0542686	II
39.9	46.1499792	II	46.1659349	II	46.1818916	II	46.1978494	II	46.2138084	II
40.0	46.3095851	II	46.3255517	II	46.3415194	II	46.3574883	II	46.3734582	II
40.1	46.4693009	II	46.4852785	II	46.5012572	II	46.5172370	II	46.5332179	II
40.2	46.6291263	II	46.6451149	II	46.6611046	II	46.6770953	II	46.6930872	II
40.3	46.7890612	II	46.8050607	II	46.8210612	II	46.8370629	II	46.8530657	II
40.4	46.9491051	II	46.9651155	II	46.9811270	II	46.9971396	II	47.0131532	II
40.5	47.1092579	II	47.1252792	II	47.1413015	II	47.1573250	II	47.1733495	II
40.6	47.2695193	II	47.2855514	II	47.3015846	II	47.3176188	II	47.3336542	II
40.7	47.4298890	II	47.4459319	II	47.4619759	II	47.4780210	II	47.4940671	II
40.8	47.5903667	II	47.6064204	II	47.6224751	II	47.6385310	II	47.6545879	II
40.9	47.7509521	II	47.7670166	II	47.7830821	II	47.7991487	II	47.8152164	II
41.0	47.9116451	II	47.9277203	II	47.9437965	II	47.9598739	II	47.9759523	II

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40.0917601	I2	40.1072692	I2	40.1227796	I2	40.1382912	I2	40.1538040	I2	36.0
40.2469065	I2	40.2624278	I2	40.2779504	I2	40.2934742	I2	40.3089992	I2	36.1
40.4021747	I2	40.4177082	I2	40.4332429	I2	40.4487789	I2	40.4643160	I2	36.2
40.5575644	I2	40.5731100	I2	40.5886569	I2	40.6042050	I2	40.6197542	I2	36.3
40.7130752	I2	40.7286330	I2	40.7441919	I2	40.7597521	I2	40.7753134	I2	36.4
40.8687069	I2	40.8842667	I2	40.8998477	I2	40.9154199	I2	40.9309933	I2	36.5
41.0244590	I2	41.0400408	I2	41.0556238	I2	41.0712080	I2	41.0867935	I2	36.6
41.1803312	I2	41.1959250	I2	41.2115200	I2	41.2271162	I2	41.2427136	I2	36.7
41.3363232	I2	41.3519290	I2	41.3675360	I2	41.3831441	I2	41.3987535	I2	36.8
41.4924347	I2	41.5080524	I2	41.5236713	I2	41.5392914	I2	41.5549127	I2	36.9
41.6486653	I2	41.6642949	I2	41.6799257	I2	41.6955577	I2	41.7111909	I2	37.0
41.8050148	I2	41.8206563	I2	41.8362989	I2	41.8519427	I2	41.8675878	I2	37.1
41.9614827	I2	41.9771360	I2	41.9927905	I2	42.0084462	I2	42.0241030	I2	37.2
42.1180688	I2	42.1337339	I2	42.1494002	I2	42.1650677	I2	42.1807363	I2	37.3
42.2747728	I2	42.2904496	I2	42.3061277	I2	42.3218069	I2	42.3374873	I2	37.4
42.4315943	I2	42.4472829	I2	42.4629726	I2	42.4786636	I2	42.4943557	I2	37.5
42.5885329	I2	42.6042333	I2	42.6199347	I2	42.6356374	I2	42.6513412	I2	37.6
42.7455885	I2	42.7613005	I2	42.7770137	I2	42.7927280	I2	42.8084434	I2	37.7
42.9027607	I2	42.9184843	I2	42.9342091	I2	42.9499350	I2	42.9656621	I2	37.8
43.0600491	I2	43.0757844	I2	43.0915208	I2	43.1072583	I2	43.1229970	I2	37.9
43.2174535	I2	43.2332004	I2	43.2489483	I2	43.2646974	I2	43.2804477	I2	38.0
43.3749736	I2	43.3907320	I2	43.4064915	I2	43.4222521	I2	43.4380139	I2	38.1
43.5326090	I2	43.5483789	I2	43.5641499	I2	43.5799221	I2	43.5956954	I2	38.2
43.6903594	I2	43.7061408	I2	43.7219233	I2	43.7377069	I2	43.7534917	I2	38.3
43.8482246	I2	43.8640174	I2	43.8798114	I2	43.8956065	I2	43.9114027	I2	38.4
44.0062042	I2	44.0220084	I2	44.0378138	I2	44.0536203	I2	44.0694280	I2	38.5
44.1642979	I2	44.1801136	I2	44.1959303	I2	44.2117483	I2	44.2275673	I2	38.6
44.3225055	I2	44.3383325	I2	44.3541606	I2	44.3699899	I2	44.3858203	I2	38.7
44.4808266	I2	44.4966649	I2	44.5125044	I2	44.5283450	I2	44.5441868	I2	38.8
44.6392609	I2	44.6551106	I2	44.6709614	I2	44.6868133	I2	44.7026663	I2	38.9
44.7978082	I2	44.8136691	I2	44.8295312	I2	44.8453944	I2	44.8612587	I2	39.0
44.9564681	I2	44.9723340	I2	44.9882136	I2	45.0040880	I2	45.0199636	I2	39.1
45.1152404	I2	45.1311238	I2	45.1470084	I2	45.1628940	I2	45.1787808	I2	39.2
45.2741248	I2	45.2900194	I2	45.3059151	I2	45.3218119	I2	45.3377099	I2	39.3
45.4331209	I2	45.4490267	I2	45.4649336	I2	45.4808415	I2	45.4967506	I2	39.4
45.5922286	I2	45.6081455	I2	45.6240635	I2	45.6399826	I2	45.6559028	I2	39.5
45.7514474	I2	45.7673754	I2	45.7833045	I2	45.7992347	I2	45.8151660	I2	39.6
45.9107772	I2	45.9267163	I2	45.9426564	I2	45.9585977	I2	45.9745401	I2	39.7
46.0702176	I2	46.0861677	I2	46.1021190	I2	46.1180713	I2	46.1340247	I2	39.8
46.2297684	I2	46.2457295	I2	46.2616918	I2	46.2776551	I2	46.2936195	I2	39.9
46.3894292	I2	46.4054014	I2	46.4213746	I2	46.4373489	I2	46.4533244	I2	40.0
46.5491999	I2	46.5651830	I2	46.5811672	I2	46.5971525	I2	46.6131389	I2	40.1
46.7090801	I2	46.7250741	I2	46.7410693	I2	46.7570655	I2	46.7730628	I2	40.2
46.8690695	I2	46.8850745	I2	46.9010805	I2	46.9170876	I2	46.9330958	I2	40.3
47.0291679	I2	47.0451838	I2	47.0612007	I2	47.0772187	I2	47.0932378	I2	40.4
47.1893751	I2	47.2054017	I2	47.2214295	I2	47.2374584	I2	47.2534883	I2	40.5
47.3496906	I2	47.3657281	I2	47.3817667	I2	47.3978064	I2	47.4138471	I2	40.6
47.5101143	I2	47.5261626	I2	47.5422120	I2	47.5582625	I2	47.5743140	I2	40.7
47.6706459	I2	47.6867050	I2	47.7027652	I2	47.7188264	I2	47.7348887	I2	40.8
47.8312852	I2	47.8473550	I2	47.8634259	I2	47.8794979	I2	47.8955709	I2	40.9
47.9920318	I2	48.0081123	I2	48.0241939	I2	48.0402766	I2	48.0563604	I2	41.0

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41.0	47.9116451	II	47.9277203	II	47.9437965	II	47.9598739	II	47.9759523	II
41.1	48.0724452	II	48.0885311	II	48.1046181	II	48.1207062	II	48.1367953	II
41.2	48.2333524	II	48.2494490	II	48.2655466	II	48.2816453	II	48.2977451	II
41.3	48.3943662	II	48.4104735	II	48.4265818	II	48.4426911	II	48.4588016	II
41.4	48.5554865	II	48.5716044	II	48.5877233	II	48.6038433	II	48.6199644	II
41.5	48.7167130	II	48.7328414	II	48.7489710	II	48.7651016	II	48.7812332	II
41.6	48.8780453	II	48.8941844	II	48.9103245	II	48.9264657	II	48.9426079	II
41.7	49.0394834	II	49.0556330	II	49.0717836	II	49.0879354	II	49.1040881	II
41.8	49.2010268	II	49.2171870	II	49.2333481	II	49.2495104	II	49.2656737	II
41.9	49.3626754	IO	49.3788461	IO	49.3950177	IO	49.4111905	IO	49.4273643	IO
42.0	49.5244289	IO	49.5406100	IO	49.5567922	IO	49.5729754	IO	49.5891596	IO
42.1	49.6862871	IO	49.7024786	IO	49.7186712	IO	49.7348649	IO	49.7510596	IO
42.2	49.8482496	IO	49.8644516	IO	49.8806546	IO	49.8968587	IO	49.9130638	IO
42.3	50.0103163	IO	50.0265287	IO	50.0427421	IO	50.0589565	IO	50.0751720	IO
42.4	50.1724868	IO	50.1887096	IO	50.2049334	IO	50.2211582	IO	50.2373841	IO
42.5	50.3347611	IO	50.3509942	IO	50.3672283	IO	50.3834635	IO	50.3996997	IO
42.6	50.4971387	IO	50.5133821	IO	50.5296266	IO	50.5458721	IO	50.5621186	IO
42.7	50.6596194	IO	50.6758732	IO	50.6921279	IO	50.7083837	IO	50.7246406	IO
42.8	50.8222031	IO	50.8384671	IO	50.8547322	IO	50.8709982	IO	50.8872653	IO
42.9	50.9848895	IO	51.0011637	IO	51.0174390	IO	51.0337153	IO	51.0499927	IO
43.0	51.1476782	IO	51.1639627	IO	51.1802482	IO	51.1965348	IO	51.2128223	IO
43.1	51.3105692	IO	51.3268639	IO	51.3431596	IO	51.3594563	IO	51.3757541	IO
43.2	51.4735620	IO	51.4898669	IO	51.5061728	IO	51.5224798	IO	51.5387877	IO
43.3	51.6366566	IO	51.6529717	IO	51.6692877	IO	51.6856048	IO	51.7019229	IO
43.4	51.7998527	IO	51.8161779	IO	51.8325041	IO	51.8488313	IO	51.8651595	IO
43.5	51.9631500	IO	51.9794853	IO	51.9958216	IO	52.0121589	IO	52.0284972	IO
43.6	52.1265483	IO	52.1428936	IO	52.1592400	IO	52.1755874	IO	52.1919358	IO
43.7	52.2900473	IO	52.3064027	IO	52.3227592	IO	52.3391166	IO	52.3554751	IO
43.8	52.4536469	IO	52.4700124	IO	52.4863788	IO	52.5027463	IO	52.5191148	IO
43.9	52.6173467	IO	52.6337222	IO	52.6500987	IO	52.6664762	IO	52.6828547	IO
44.0	52.7811467	IO	52.7975322	IO	52.8139186	IO	52.8303061	IO	52.8466946	IO
44.1	52.9450464	IO	52.9614419	IO	52.9778383	IO	52.9942358	IO	53.0106342	IO
44.2	53.1090458	IO	53.1254512	IO	53.1418576	IO	53.1582650	IO	53.1746734	IO
44.3	53.2731445	IO	53.2895599	IO	53.3059762	IO	53.3223935	IO	53.3388118	IO
44.4	53.4373424	IO	53.4537677	IO	53.4701939	IO	53.4866211	IO	53.5030493	IO
44.5	53.6016392	IO	53.6180744	IO	53.6345105	IO	53.6509475	IO	53.6673856	IO
44.6	53.7660348	IO	53.7824797	IO	53.7989257	IO	53.8153726	IO	53.8318206	IO
44.7	53.9305287	IO	53.9469836	IO	53.9634393	IO	53.9798961	IO	53.9963539	IO
44.8	54.0951210	IO	54.1115856	IO	54.1280512	IO	54.1445178	IO	54.1609853	IO
44.9	54.2598113	IO	54.2762857	IO	54.2927611	IO	54.3092374	IO	54.3257148	IO
45.0	54.4245993	IO	54.4410835	IO	54.4575687	IO	54.4740548	IO	54.4905419	IO
45.1	54.5894859	IO	54.6059789	IO	54.6224738	IO	54.6389697	IO	54.6554666	IO
45.2	54.7544681	IO	54.7709717	IO	54.7874763	IO	54.8039819	IO	54.8204885	IO
45.3	54.9195483	IO	54.9360616	IO	54.9525759	IO	54.9690912	IO	54.9856075	IO
45.4	55.0847254	IO	55.1012484	IO	55.1177724	IO	55.1342974	IO	55.1508233	IO
45.5	55.2499993	IO	55.2665320	IO	55.2830656	IO	55.2996002	IO	55.3161358	IO
45.6	55.4153696	IO	55.4319120	IO	55.4484553	IO	55.4649995	IO	55.4815447	IO
45.7	55.5808363	IO	55.5973882	IO	55.6139411	IO	55.6304950	IO	55.6470498	IO
45.8	55.7463990	IO	55.7629606	IO	55.7795231	IO	55.7960865	IO	55.8126510	IO
45.9	55.9120576	IO	55.9286287	IO	55.9452008	IO	55.9617738	IO	55.9783478	IO
46.0	56.0778119	IO	56.0943925	IO	56.1109742	IO	56.1275568	IO	56.1441403	IO

5		6		7		8		9		
47.9920318	II	48.0081123	II	48.0241939	II	48.0402766	II	48.0563604	II	41.0
48.1528855	II	48.1689767	II	48.1850690	II	48.2011624	II	48.2172569	II	41.1
48.3138460	II	48.3299479	II	48.3460509	II	48.3621549	II	48.3782600	II	41.2
48.4749131	II	48.4910256	II	48.5071393	II	48.5232539	II	48.5393697	II	41.3
48.6360865	II	48.6522097	II	48.6683339	II	48.6844592	II	48.7005855	II	41.4
48.7973659	II	48.8134997	II	48.8296345	II	48.8457704	II	48.8619073	II	41.5
48.9587512	II	48.9748955	II	48.9910409	II	49.0071873	II	49.0233348	II	41.6
49.1202419	II	49.1363968	II	49.1525527	II	49.1687097	II	49.1848677	II	41.7
49.2818380	II	49.2980034	II	49.3141698	II	49.3303373	II	49.3465058	IO	41.8
49.4435391	IO	49.4597150	IO	49.4758919	IO	49.4920698	IO	49.5082489	IO	41.9
49.6053449	IO	49.6215313	IO	49.6377187	IO	49.6539071	IO	49.6700965	IO	42.0
49.7672553	IO	49.7834521	IO	49.7996499	IO	49.8158488	IO	49.8320487	IO	42.1
49.9292699	IO	49.9454771	IO	49.9616854	IO	49.9778946	IO	49.9941049	IO	42.2
50.0913886	IO	50.1076062	IO	50.1238248	IO	50.1400444	IO	50.1562651	IO	42.3
50.2536110	IO	50.2698390	IO	50.2860679	IO	50.3022979	IO	50.3185290	IO	42.4
50.4159370	IO	50.4321752	IO	50.4484145	IO	50.4646549	IO	50.4808963	IO	42.5
50.5783662	IO	50.5946148	IO	50.6108644	IO	50.6271150	IO	50.6433667	IO	42.6
50.7408984	IO	50.7571573	IO	50.7734172	IO	50.7896782	IO	50.8059401	IO	42.7
50.9035335	IO	50.9198026	IO	50.9360728	IO	50.9523440	IO	50.9686162	IO	42.8
51.0662710	IO	51.0825504	IO	51.0988308	IO	51.1151123	IO	51.1313947	IO	42.9
51.2291109	IO	51.2454005	IO	51.2616912	IO	51.2779828	IO	51.2942755	IO	43.0
51.3920529	IO	51.4083527	IO	51.4246535	IO	51.4409553	IO	51.4572582	IO	43.1
51.5550966	IO	51.5714066	IO	51.5877176	IO	51.6040296	IO	51.6203426	IO	43.2
51.7182420	IO	51.7345621	IO	51.7508832	IO	51.7672054	IO	51.7835285	IO	43.3
51.8814887	IO	51.8978189	IO	51.9141502	IO	51.9304824	IO	51.9468157	IO	43.4
52.0448365	IO	52.0611769	IO	52.0775182	IO	52.0938605	IO	52.1102039	IO	43.5
52.2082852	IO	52.2246356	IO	52.2409870	IO	52.2573395	IO	52.2736929	IO	43.6
52.3718345	IO	52.3881950	IO	52.4045565	IO	52.4209189	IO	52.4372824	IO	43.7
52.5354843	IO	52.5518548	IO	52.5682263	IO	52.5845988	IO	52.6009723	IO	43.8
52.6992342	IO	52.7156147	IO	52.7319962	IO	52.7483787	IO	52.7647622	IO	43.9
52.8630841	IO	52.8794746	IO	52.8958660	IO	52.9122585	IO	52.9286520	IO	44.0
53.0270337	IO	53.0434341	IO	53.0598355	IO	53.0762380	IO	53.0926414	IO	44.1
53.1910828	IO	53.2074931	IO	53.2239045	IO	53.2403169	IO	53.2567302	IO	44.2
53.3552311	IO	53.3716514	IO	53.3880727	IO	53.4044949	IO	53.4209182	IO	44.3
53.5194785	IO	53.5359087	IO	53.5523398	IO	53.5687720	IO	53.5852051	IO	44.4
53.6838247	IO	53.7002647	IO	53.7167058	IO	53.7331478	IO	53.7495908	IO	44.5
53.8482695	IO	53.8647194	IO	53.8811702	IO	53.8976221	IO	53.9140749	IO	44.6
54.0128126	IO	54.0292723	IO	54.0457330	IO	54.0621947	IO	54.0786574	IO	44.7
54.1774539	IO	54.1939234	IO	54.2103939	IO	54.2268654	IO	54.2433378	IO	44.8
54.3421931	IO	54.3586724	IO	54.3751527	IO	54.3916339	IO	54.4081161	IO	44.9
54.5070300	IO	54.5235191	IO	54.5400091	IO	54.5565001	IO	54.5729921	IO	45.0
54.6710644	IO	54.6884632	IO	54.7049629	IO	54.7214637	IO	54.7379654	IO	45.1
54.8369980	IO	54.8535045	IO	54.8700140	IO	54.8865245	IO	54.9030359	IO	45.2
55.0021247	IO	55.0186429	IO	55.0351621	IO	55.0516822	IO	55.0682033	IO	45.3
55.1673503	IO	55.1838781	IO	55.2004070	IO	55.2169368	IO	55.2334675	IO	45.4
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55.4980909	IO	55.5146381	IO	55.5311862	IO	55.5477352	IO	55.5642853	IO	45.6
55.6636056	IO	55.6801624	IO	55.6967201	IO	55.7132788	IO	55.7298384	IO	45.7
55.8292163	IO	55.8457827	IO	55.8623500	IO	55.8789182	IO	55.8954871	IO	45.8
55.9949228	IO	56.0114987	IO	56.0280756	IO	56.0446534	IO	56.0612321	IO	45.9
56.1607248	IO	56.1773102	IO	56.1938966	IO	56.2104840	IO	56.2270723	IO	46.0

	0		1		2		3		4	
46.0	56.0778119	10	56.0943925	10	56.1109742	10	56.1275568	10	56.1441403	10
46.1	56.2436616	10	56.2602518	10	56.2768429	10	56.2934351	10	56.3100281	10
46.2	56.4096065	10	56.4262062	9	56.4428069	9	56.4594085	9	56.4760111	9
46.3	56.5756465	9	56.5922557	9	56.6088658	9	56.6254770	9	56.6420890	9
46.4	56.7417813	9	56.7583999	9	56.7750196	9	56.7916401	9	56.8082617	9
46.5	56.9080107	9	56.9246388	9	56.9412679	9	56.9578979	9	56.9745289	9
46.6	57.0743345	9	57.0909720	9	57.1076105	9	57.1242500	9	57.1408904	9
46.7	57.2407525	9	57.2573994	9	57.2740474	9	57.2906962	9	57.3073460	9
46.8	57.4072645	9	57.4239208	9	57.4405781	9	57.4572364	9	57.4738956	9
46.9	57.5738703	9	57.5905360	9	57.6072027	9	57.6238703	9	57.6405388	9
47.0	57.7405697	9	57.7572448	9	57.7739208	9	57.7905977	9	57.8072756	9
47.1	57.9073625	9	57.9240469	9	57.9407322	9	57.9574185	9	57.9741057	9
47.2	58.0742485	9	58.0909422	9	58.1076368	9	58.1243324	9	58.1410289	9
47.3	58.2412275	9	58.2579305	9	58.2746344	9	58.2913393	9	58.3080450	9
47.4	58.4082992	9	58.4250115	9	58.4417247	9	58.4584388	9	58.4751539	9
47.5	58.5754636	9	58.5921851	9	58.6089076	9	58.6256309	9	58.6423552	9
47.6	58.7427204	9	58.7594511	9	58.7761828	9	58.7929154	9	58.8096489	9
47.7	58.9100694	9	58.9268093	9	58.9435502	9	58.9602920	9	58.9770347	9
47.8	59.0775103	9	59.0942595	9	59.1110096	9	59.1277605	9	59.1445124	9
47.9	59.2450431	9	59.2618015	9	59.2785607	9	59.2953208	9	59.3120819	9
48.0	59.4126676	9	59.4294350	9	59.4462034	9	59.4629727	9	59.4797429	9
48.1	59.5803834	9	59.5971600	9	59.6139375	9	59.6307159	9	59.6474953	9
48.2	59.7481905	9	59.7649762	9	59.7817628	9	59.7985504	9	59.8153388	9
48.3	59.9160886	9	59.9328834	9	59.9496791	9	59.9664758	9	59.9832733	9
48.4	60.0840776	9	60.1008815	9	60.1176863	9	60.1344920	9	60.1512986	9
48.5	60.2521572	9	60.2689702	9	60.2857840	9	60.3025988	9	60.3194144	9
48.6	60.4203273	9	60.4371493	9	60.4539722	9	60.4707960	9	60.4876207	9
48.7	60.5885877	9	60.6054187	9	60.6222506	9	60.6390834	9	60.6559171	9
48.8	60.7569382	9	60.7737782	9	60.7906191	9	60.8074609	9	60.8243036	9
48.9	60.9253786	9	60.9422276	9	60.9590775	9	60.9759283	9	60.9927799	9
49.0	61.0939088	9	61.1107667	9	61.1276256	9	61.1444853	9	61.1613459	9
49.1	61.2625285	9	61.2793954	9	61.2962631	9	61.3131318	9	61.3300014	9
49.2	61.4312375	9	61.4481133	9	61.4649900	9	61.4818676	9	61.4987461	9
49.3	61.6000357	9	61.6169204	9	61.6338061	9	61.6506926	9	61.6675799	9
49.4	61.7689229	9	61.7858165	9	61.8027110	9	61.8196064	9	61.8365027	9
49.5	61.9378990	9	61.9548014	9	61.9717048	9	61.9886090	9	62.0055142	9
49.6	62.1069036	9	62.1238749	9	62.1408781	9	62.1577702	9	62.1746142	9
49.7	62.2761167	9	62.2930369	9	62.3099579	9	62.3268798	9	62.3438027	9
49.8	62.4453581	9	62.4622870	9	62.4792169	9	62.4961476	9	62.5130793	9
49.9	62.6146875	9	62.6316253	9	62.6485640	9	62.6655035	9	62.6824439	9
50.0	62.7841049	9	62.8010514	9	62.8179989	9	62.8349472	9	62.8518964	9
50.1	62.9536100	9	62.9705653	9	62.9875215	9	63.0044786	9	63.0214365	9
50.2	63.1232026	9	63.1401667	9	63.1571316	9	63.1740975	9	63.1910642	9
50.3	63.2928826	9	63.3098555	9	63.3268291	9	63.3438037	9	63.3607791	9
50.4	63.4626499	9	63.4796314	9	63.4966138	9	63.5135970	9	63.5305812	9
50.5	63.6325042	9	63.6494944	9	63.6664854	9	63.6834774	9	63.7004702	9
50.6	63.8024453	9	63.8194442	9	63.8364439	9	63.8534445	9	63.8704460	9
50.7	63.9724731	9	63.9894806	9	64.0064890	9	64.0234983	9	64.0405084	9
50.8	64.1425874	9	64.1596036	9	64.1766206	9	64.1936385	9	64.2106573	9
50.9	64.3127881	9	64.3298129	9	64.3468385	9	64.3638651	9	64.3808925	9

$x=46.0$  to  $50.9$  LOG  $\Gamma(x)$  FROM  $x=1.0$  TO  $x=50.9$  BY  $.01$

28

5		6		7		8		9	
56.1607248	10	56.1773102	10	56.1938966	10	56.2104840	10	56.2270723	10
56.3266221	10	56.3432171	10	56.3598130	10	56.3764099	10	56.3930077	10
56.4926146	9	56.5092191	9	56.5258245	9	56.5424309	9	56.5590382	9
56.6587020	9	56.6753160	9	56.6919309	9	56.7085467	9	56.7251635	9
56.8248841	9	56.8415076	9	56.8581319	9	56.8747572	9	56.8913835	9
56.9911608	9	57.0077936	9	57.0244274	9	57.0410622	9	57.0576978	9
57.1575317	9	57.1741740	9	57.1908172	9	57.2074613	9	57.2241064	9
57.3239967	9	57.3406484	9	57.3573010	9	57.3739546	9	57.3906091	9
57.4905557	9	57.5072167	9	57.5238787	9	57.5405416	9	57.5572055	9
57.6572083	9	57.6738787	9	57.6905501	9	57.7072223	9	57.7238955	9
57.8239544	9	57.8406342	9	57.8573149	9	57.8739965	9	57.8906790	9
57.9907938	9	58.0074729	9	58.0241729	9	58.0408638	9	58.0575557	9
58.1577264	9	58.1744247	9	58.1911240	9	58.2078242	9	58.2245254	9
58.3247518	9	58.3414594	9	58.3581680	9	58.3748775	9	58.3915879	9
58.4918099	9	58.5085866	9	58.5253046	9	58.5420233	9	58.5587430	9
58.6590805	9	58.6758066	9	58.6925337	9	58.7092616	9	58.7259905	9
58.8263834	9	58.8431187	9	58.8598550	9	58.8765922	9	58.8933303	9
58.9937784	9	59.0105229	9	59.0272684	9	59.0440148	9	59.0607021	9
59.1612653	9	59.1780190	9	59.1947737	9	59.2115292	9	59.2282857	9
59.3288439	9	59.3456068	9	59.3623706	9	59.3791353	9	59.3959010	9
59.4965141	9	59.5132861	9	59.5300591	9	59.5468329	9	59.5636077	9
59.6642755	9	59.6810567	9	59.6978388	9	59.7146218	9	59.7314057	9
59.8321282	9	59.8489184	9	59.8657096	9	59.8825017	9	59.8992947	9
60.0000717	9	60.0168711	9	60.0336714	9	60.0504725	9	60.0672746	9
60.1681061	9	60.1849145	9	60.2017238	9	60.2185340	9	60.2353452	9
60.3362310	9	60.3530484	9	60.3698668	9	60.3866861	9	60.4035063	9
60.5044463	9	60.5212727	9	60.5381001	9	60.5549284	9	60.5717576	9
60.6727517	9	60.6895872	9	60.7064236	9	60.7232609	9	60.7400991	9
60.8411472	9	60.8579917	9	60.8748371	9	60.8916834	9	60.9085306	9
61.0096325	9	61.0264860	9	61.0433403	9	61.0601956	9	61.0770517	9
61.1782074	9	61.1950699	9	61.2119332	9	61.2287974	9	61.2456625	9
61.3468718	9	61.3637432	9	61.3806154	9	61.3974886	9	61.4143626	9
61.5156255	9	61.5325058	9	61.5493869	9	61.5662690	9	61.5831519	9
61.6844682	9	61.7013574	9	61.7182474	9	61.7351384	9	61.7520302	9
61.8533999	9	61.8702979	9	61.8871968	9	61.9040967	9	61.9209974	9
62.0224202	9	62.0393271	9	62.0562349	9	62.0731436	9	62.0900532	9
62.1915291	9	62.2084449	9	62.2253615	9	62.2422790	9	62.2591974	9
62.3607264	9	62.3776509	9	62.3945764	9	62.4115027	9	62.4284300	9
62.5300118	9	62.5469452	9	62.5638794	9	62.5808146	9	62.5977506	9
62.6993852	9	62.7163274	9	62.7332704	9	62.7502144	9	62.7671592	9
62.8688465	9	62.8857974	9	62.9027492	9	62.9197019	9	62.9366555	9
63.0383954	9	63.0553551	9	63.0723156	9	63.0892771	9	63.1062394	9
63.2080317	9	63.2250002	9	63.2419695	9	63.2589397	9	63.2759107	9
63.3777554	9	63.3947325	9	63.4117106	9	63.4286895	9	63.4456692	9
63.5475062	9	63.5645520	9	63.5815387	9	63.5985263	9	63.6155148	9
63.7174639	9	63.7344584	9	63.7514538	9	63.7684501	9	63.7854473	9
63.8874484	9	63.9044516	9	63.9214556	9	63.9384606	9	63.9554664	9
64.0575194	9	64.0745313	9	64.0915440	9	64.1085576	9	64.1255721	9
64.2276769	9	64.2446974	9	64.2617188	9	64.2787410	9	64.2957641	9
64.3979207	9	64.4149498	9	64.4319798	9	64.4490106	9	64.4660423	9

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