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April, 1939

Balanced Incomplete Block and Lattice Square Designs for Testing Yield Differences Among Large Numbers of Soybean Varieties

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SUMMARY

Two quasi-factorial arrangements which are especially well adapted to the testing of differences between large numbers of varieties are described and treated in detail as to their analysis and value.

The arrangements described are balanced incomplete block and lattice square designs. Soybean variety trials are used to illustrate the analysis and the relative precision on soils of varying homogeneity.

Although the efficiency factor of these designs, because of the confounding of variety differences with block effects, is lower than that of randomized complete block designs, yet on soil of normal variability the designs permit the elimination of sufficient variability due to soil differences to more than offset this loss. The merits of the lattice square arrangement are demonstrated on extremely heterogeneous soil where use of the design results in a gain in precision of 150 percent over that of randomized complete blocks. An illustration is also presented in which the design on very uniform soil results in a loss of precision of 31.5 percent.

The use of these designs is recommended in variety trials involving large numbers of varieties when the trials are conducted on variable soil and when differences between the varieties are relatively small.

Balanced Incomplete Block and Lattice Square Designs for Testing Yield Differences Among Large Numbers of Soybean Varieties¹

BY MARTIN G. WEISS AND GERTRUDE M. COX²

Numerous methods of adequately testing varietal differences in field crops have been originated. The control or check plot system has been used by many investigators. The chief faults of this system are the disregard for differential response of varieties to varying fertility levels, the large proportion of experimental plots used as controls, and the difficulty encountered when the control variety differs extremely from some of the varieties tested because of, for instance, virtually complete resistance or susceptibility to a disease. Richey (8) devised a method by which variety yields are adjusted to their regression on a moving average. Analytical procedure differs somewhat with varying numbers of varieties in each moving average group. The semi-Latin square devised by "Student" (10) has been shown by Yates (12) to suffer from a biased error. Although these systems were devised to test large numbers of varieties, their limitations do not permit their universal usage.

The Latin square design requires replicates equal in number to the varieties tested. This design, therefore, is adapted to testing varietal differences only when the number of varieties to be tested is low. The randomized complete block design provides that each variety occurs with equal frequency, usually only once, in every block. The varieties are assigned at random to the plots within a block. Each block should be as uniform in soil characteristics as possible. Thus most of the variance due to soil heterogeneity may fall in the portion ascribable to variation between blocks. However, when testing a large number of varieties on variable soil, because of the large block size, the randomized block design may be

¹Project 186 of the Farm Crops Subsection of the Iowa Agricultural Experiment Station in cooperation with the United States Regional Soybean Industrial Products Laboratory, United States Department of Agriculture, and Project 514 of the Statistical Section of the Iowa Agricultural Experiment Station.

The authors are indebted to Prof. G. W. Snedecor for valuable criticism of the presentation of these experiments.

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expected to be relatively inefficient in eliminating the effects of the major soil irregularities.

In recent years several modifications of the randomized complete block design have been devised in order that a larger portion of the variability due to soil differences may be accounted for in the experiment. This is accomplished by arranging the varieties in small blocks. Instead of every variety occurring with every other variety in the same block, as is the case in the randomized complete block design, every variety occurs with every other variety in the same number of blocks, (but not in every block). This provides a symmetrical arrangement which easily lends itself to statistical analysis. The more adequate elimination of the soil variation, due to the smaller block size, decreases the amount of error variance.

The analysis of variance for statistical reduction of data from randomized complete blocks, Latin squares and similar experiments was designed by Fisher (2). This method facilitates the isolation of variance due to variety mean differences as well as a portion constituting an estimate of error. This estimate of error is used for the F test (9) of significance of differences among the variety means. This analysis is used in these new designs.

The object of this bulletin is to present the statistical features of the designs of two experiments which are particularly well adapted to testing large numbers of varieties on variable soil. These experiments consist of variety trials of soybeans. Since the results in each illustration are for a single year, and are therefore inadequate to warrant publication of the results of the varieties tested, the varieties will be designated by numbers.

BALANCED INCOMPLETE BLOCK DESIGNS

The requirement of the balanced incomplete block design is that every variety occur with every other variety in the same number of blocks. In these balanced designs, since every degree of freedom is confounded equally, all variety comparisons are of equal accuracy. Since the number of replications and block size must be kept within practical limits, it is possible to arrange such designs for only specific numbers of varieties. If the number of varieties to be tested does not coincide with a number for which a good balanced incomplete block design can be written, it is often possible to include a few additional varieties until the proper number of varieties is obtained.

Arrangements which satisfy the conditions for balanced incomplete block designs are provided by taking all possible

combinations of the v (see key on page 300) varieties taken k at a time, when k is less than v . For six varieties taken three at a time there are 20 combinations.* The 20 combinations are 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456. That is, it would take 20 blocks of three plots each to arrange these six varieties in all possible combinations. Observe, however, that half of the combinations, either those in bold face type or the other half in light face type, fulfill the restriction that every variety occur with every other variety in the same number of blocks. Every two varieties occur together twice in either group of blocks. Thus, a balanced incomplete block experiment can be based on either half of the 20 blocks as well as on all 20.

Since the purpose back of the new designs is to keep the block (and therefore k) small, a large number of varieties (v) will result in an inconveniently large number of blocks. Arrangements for fewer blocks are therefore desirable. There are several simple devices for deriving the smallest number of combinations possible yet keeping the balance. Yates (13) gives these combinations for v up to 25 and k from 2 to 12. Fisher and Yates (4) tabulate the arrangements which are known to exist for 10 or less replications.

The structure of balanced incomplete block arrangements is discussed quite thoroughly by Fisher and Yates (4), by Yates (13) and by Goulden (5, 7), numerous examples being presented. The reader who is interested in the procedure of writing out the designs may refer to these papers. We shall give only a brief discussion of the structure of the types of designs which are to be illustrated by experimental results later in this bulletin. Two series of balanced incomplete block arrangements can be derived from completely orthogonalized squares. We shall illustrate the process by means of the following completely orthogonalized 5 x 5 square specified by the digits in italics.

<i>1111</i>	1	<i>2222</i>	2	<i>3333</i>	3	<i>4444</i>	4	<i>5555</i>	5
<i>2345</i>	6	<i>3451</i>	7	<i>4512</i>	8	<i>5123</i>	9	<i>1234</i>	10
<i>3524</i>	11	<i>4135</i>	12	<i>5241</i>	13	<i>1352</i>	14	<i>2413</i>	15
<i>4253</i>	16	<i>5314</i>	17	<i>1425</i>	18	<i>2531</i>	19	<i>3142</i>	20
<i>5432</i>	21	<i>1543</i>	22	<i>2154</i>	23	<i>3215</i>	24	<i>4321</i>	25

$$* b = v^C k = \frac{v!}{k! (v-k)!}$$

$$b = 6^C 3 = \frac{6!}{3! 3!} = \frac{4 \cdot 5 \cdot 6}{2 \cdot 3} = 20$$

The numbers in bold face type designate the varieties which are to be compared in the experiment. These variety numbers are arranged in a set of six orthogonal groups as follows:

Group I (rows)	Group II (columns)	Group III (first number)
1 2 3 4 5	1 6 11 16 21	1 10 14 18 22
6 7 8 9 10	2 7 12 17 22	2 6 15 19 23
11 12 13 14 15	3 8 13 18 23	3 7 11 20 24
16 17 18 19 20	4 9 14 19 24	4 8 12 16 25
21 22 23 24 25	5 10 15 20 25	5 9 13 17 21
Group IV (second number)	Group V (third number)	Group VI (fourth number)
1 9 12 20 23	1 8 15 17 24	1 7 13 19 25
2 10 13 16 24	2 9 11 18 25	2 8 14 20 21
3 6 14 17 25	3 10 12 19 21	3 9 15 16 22
4 7 15 18 21	4 6 13 20 22	4 10 11 17 23
5 8 11 19 22	5 7 14 16 23	5 6 12 18 24

In group I the variety numbers are copied from the rows of the square, each row of the group specifying a block in the field. In like manner, the variety numbers in the blocks of group II are taken from the columns of the square. In group III the varieties in a block are specified by the numbers written first in the cells of the square. Thus, the varieties in the first block are those corresponding to number 1 wherever it occurs first in the cell; as examples, variety 1 is from row 1 column 1 of the completely orthogonalized square, variety 10, from row 2 column 5, variety 14 from row 3 column 4, etc. For group IV, the second numbers in the cells of the square are used to pick out the varieties. Thus, for the third block the number 3 is located in row 1 column 3 (variety 3), in row 2 column 1 (variety 6), etc.

This set of six orthogonal groups constitutes a balanced incomplete block arrangement: in the 30 blocks of 5 plots, each of the 25 varieties occurs six times, once and once only with every other variety. The combination solution in the unreduced form would require a prohibitive number of blocks.*

* $b = v C_k = 25 C_5 = \frac{25!}{5! 20!} = 53,130.$

The arrangement just given is an illustration of the first of two series of balanced incomplete block arrangements which can be made from the completely orthogonalized squares. The second may be written by adding $k + 1$ varieties to the orthogonal groups. For example, the varieties 26, 27, 28, 29, 30 and 31 may be added to the groups given above. An additional block must be added to include these new varieties. The resulting arrangement for 31 varieties in 31 blocks is indicated below. The blocks are made up of six plots each, and each variety appears in a block with every other variety only once.

Block

(1) <u>26 1 2 3 4 5</u>	(6) <u>27 1 6 11 16 21</u>	(11) <u>28 1 10 14 18 22</u>
(2) <u>26 6 7 8 9 10</u>	(7) <u>27 2 7 12 17 22</u>	(12) <u>28 2 6 15 19 23</u>
(3) <u>26 11 12 13 14 15</u>	(8) <u>27 3 8 13 18 23</u>	(13) <u>28 3 7 11 20 24</u>
(4) <u>26 16 17 18 19 20</u>	(9) <u>27 4 9 14 19 24</u>	(14) <u>28 4 8 12 16 25</u>
(5) <u>26 21 22 23 24 25</u>	(10) <u>27 5 10 15 20 25</u>	(15) <u>28 5 9 13 17 21</u>
(16) <u>29 1 9 12 20 23</u>	(21) <u>30 1 8 15 17 24</u>	(26) <u>31 1 7 13 19 25</u>
(17) <u>29 2 10 13 16 24</u>	(22) <u>30 2 9 11 18 25</u>	(27) <u>31 2 8 14 20 21</u>
(18) <u>29 3 6 14 17 25</u>	(23) <u>30 3 10 12 19 21</u>	(28) <u>31 3 9 15 16 22</u>
(19) <u>29 4 7 15 18 21</u>	(24) <u>30 4 6 13 20 22</u>	(29) <u>31 4 10 11 17 23</u>
(20) <u>29 5 8 11 19 22</u>	(25) <u>30 5 7 14 16 23</u>	(30) <u>31 5 6 12 18 24</u>
		(31) <u>26 27 28 29 30 31</u>

The field arrangement of this type of balanced incomplete block design can be explained best with an actual illustration. In fig. 1 is presented the randomized arrangement of the above incomplete block design with 31 varieties replicated six times. Not only are the blocks randomized but the plots in each block are also randomized. In the field plan the first block selected at random was block (5) containing the plots 26, 21, 22, 23, 24, 25. Next came block (3), then (13). For convenience, the blocks in the field arrangement are numbered consecutively.

In field experimentation it has been found that if the land is divided into compact or approximately square blocks, the soil variation between plots within a block will be less than that between blocks. When testing crops which are planted in rows it is usually best to choose a plot which is sufficiently greater in length than in width to make the block as nearly square as possible; thus:

<i>Block 1</i>	Variety 24) Plot
	26	
	21	
	25	
	22	
	23	

The plots of the same block must be kept together in the

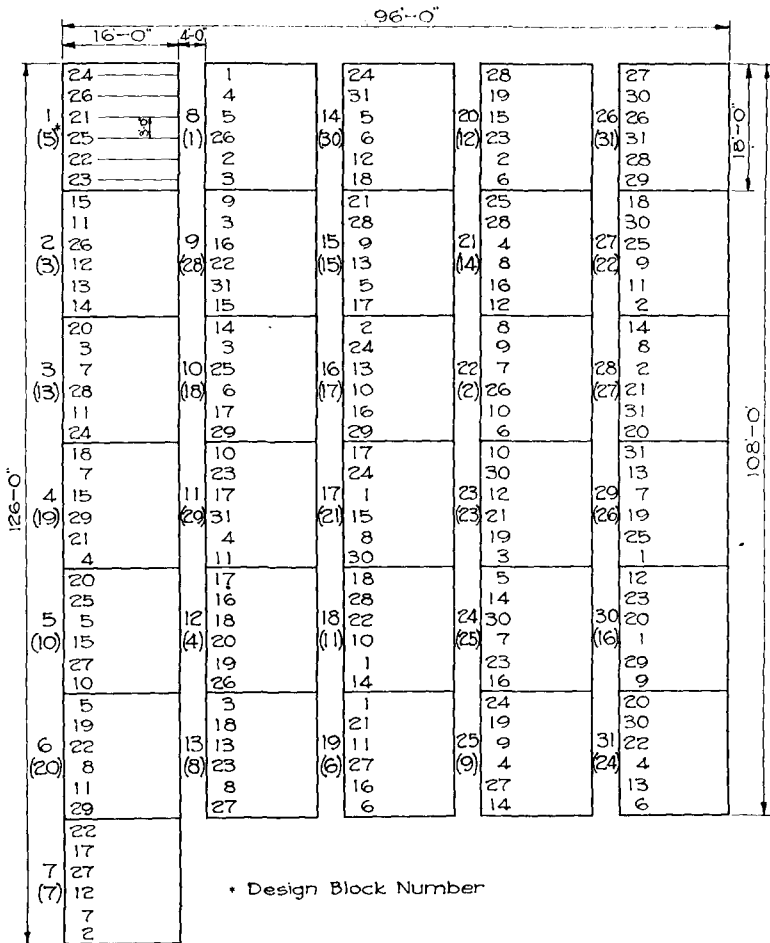


Fig. 1. Field arrangement for a set of balanced incomplete blocks, 31 varieties, 6 replications.

field. The blocks may be planted separately in any order or arrangement desirable, that is, space may be left between two blocks in order to avoid a low place in the field or the blocks may be placed in successive rows. Therefore, this type of design is very convenient for irregularly shaped experimental fields.

Returning to the consideration of fig. 1, the situation of some one variety may be noted. Variety 1, for instance, occurs in blocks number 8, 17, 18, 19, 29 and 30. It occurs with varieties 2, 3, 4, 5, 26 in block 8

8, 15, 17, 24, 30 in block 17

10, 14, 18, 22, 28 in block 18

6, 11, 16, 21, 27 in block 19

7, 13, 19, 25, 31 in block 29

9, 12, 20, 23, 29 in block 30

Variety 1 has, therefore, occurred with every variety once but has never occurred with any variety more than once. This is equally true of any of the other 30 varieties. Before any design of this type is used, it is advisable to make such a check on several or all of the varieties, since errors in construction or in making a copy will cause considerable difficulty when analyzing the results.

To the person who is familiar with the analysis of variance of randomized block and Latin square designs, attention is called to the fact that in balanced incomplete block designs the sum of squares between varieties is not obtained directly from the plot yields, a preliminary step in the analysis being required to obtain this sum. Since variety differences are partially confounded with block effects, it is necessary to compute each variety sum by an appropriate formula involving not only the yields of the plots planted to the variety but also the yields of the blocks in which the variety occurs.

The partial confounding of variety differences with block effects makes it unwise to employ this type of design when comparing varieties which have an extremely large range in yields. Preliminary tests, however, usually eliminate poor varieties, and the balanced incomplete block arrangement provides a means of accurately determining relatively small differences between the remaining select varieties.

In case a few missing plots occur, adjustments can be made without a serious loss of accuracy and with a relatively small amount of computation (1, 11). Caution should be exercised in the selection of this type of field arrangement when the possibility exists of losing a considerable number of plots, because the method of supplying missing data becomes involved when applied to complex experiments.

TABLE 1. PLOT YIELDS OF THE BALANCED INCOMPLETE BLOCKS,
31 SOYBEAN VARIETIES, DESIGNATED BY BOLD FACE TYPE.

Block	(Bushels per acre)						Block totals
	Plots in block						
	1	2	3	4	5	6	
1	24:29.8	26:23.0	21:29.3	25:24.0	22:21.4	23:26.2	153.7
2	15:24.2	11:25.1	26:24.2	12:26.8	13:26.3	14:20.6	147.2
3	20:30.5	3:29.0	7:19.2	28:21.9	11:21.1	24:30.6	152.3
4	18:20.0	7:20.2	15:22.9	29:22.2	21:26.2	4:23.1	134.6
5	20:35.2	25:28.1	5:26.0	15:26.8	27:25.6	10:21.2	162.9
6	5:25.0	19:30.0	22:22.6	8:28.2	11:25.3	29:21.1	152.2
7	22:23.6	17:17.3	27:21.2	12:24.4	7:22.7	2:25.7	134.9
8	1:23.6	4:25.6	5:25.9	26:29.9	4:25.8	3:36.4	167.2
9	9:29.3	3:34.0	16:24.5	22:29.4	31:26.9	15:27.8	171.9
10	14:25.5	3:28.5	25:24.6	6:31.4	17:18.8	29:22.3	151.1
11	10:21.4	23:26.3	17:19.9	31:24.6	4:24.9	11:24.1	141.2
12	17:16.7	16:22.2	18:23.1	20:32.5	19:26.1	26:27.4	148.0
13	3:32.7	18:31.4	13:31.7	23:30.9	8:28.5	27:23.0	178.2
14	24:32.3	31:27.7	5:27.9	6:33.0	12:30.8	18:28.7	180.4
15	21:32.6	28:28.8	9:34.8	13:29.3	5:28.9	17:20.0	174.4
16	2:30.0	24:38.0	13:35.3	10:28.9	16:28.7	29:30.7	191.6
17	17:17.1	24:32.4	1:23.6	15:21.0	8:24.0	30:32.0	150.1
18	18:21.7	26:24.0	22:25.1	10:24.3	1:24.6	14:25.0	144.7
19	1:24.3	21:30.3	11:27.2	27:22.1	16:23.6	6:28.1	155.6
20	28:23.8	19:26.7	15:24.4	23:30.6	2:23.8	6:28.7	158.0
21	25:26.4	28:29.4	4:29.3	8:28.6	16:25.9	12:29.0	168.6
22	8:27.2	9:29.6	7:23.5	26:25.9	10:27.5	6:33.6	167.3
23	10:27.8	30:40.4	12:32.1	21:32.8	19:30.4	3:34.9	198.4
24	5:30.4	14:27.0	30:40.3	7:29.9	23:32.9	16:37.7	198.2
25	24:35.8	19:30.9	9:28.4	4:29.3	27:27.4	14:25.9	177.7
26	27:24.4	30:37.4	26:26.9	31:29.1	28:28.1	29:26.8	172.7
27	18:25.9	30:33.9	25:28.7	9:30.7	11:29.4	2:27.7	176.3
28	14:24.5	8:29.3	2:31.2	21:32.9	31:28.6	20:33.3	179.8
29	31:30.7	13:35.2	7:27.8	19:31.9	25:30.2	1:24.1	179.9
30	12:33.0	23:32.1	20:31.0	1:26.5	29:25.3	9:29.8	177.7
31	20:35.6	30:41.2	22:24.6	4:28.6	13:31.0	6:35.8	196.8
							5143.6

EXAMPLE OF BALANCED INCOMPLETE BLOCK EXPERIMENT

nplete block experiment analyzed in a yield test (bushels of seed per acre) beans grown at Ames during the season as were replicated six times and arranged in six plots per block. The dimensions of the plot were 16 feet by 16 feet, the plot consisting of two rows and 16 blocks 36 feet wide and 16 feet long. The design was similar to the one presented in the discussion of the analysis in section 10, in that there were two rows of soy-

beans. The yields are presented in table 1. Varieties are identified by the numbers, 1, 2 - - - 31, given in bold face type. The values follow the variety identification. The analysis of the data in the discussion of the analysis will be the same as those used by Yates (13):

r = number of replications of each variety, (6)

b = number of blocks, (31)

$N = vr = bk$ = total number of plots, (186)

$\lambda = r(k - 1)/(v - 1)$ = number of times any two varieties occur together in a block, (1)

$E = \frac{1 - 1/k}{1 - 1/v}$ = efficiency factor of design, $\frac{(31)}{(36)}$

G = sum of all N experimental values, (5,143.6)

V = sum of r experimental values for any one variety

B = sum of k experimental values for any block

s^2 = error variance of a single experimental value

The first step in the analysis consists of accumulating the variety sums which are recorded in table 2, column 2. From table 1 collect the yields for each variety. For example, the total yield, V , for variety 1 is:

$$23.6 + 23.6 + 24.6 + 24.3 + 24.1 + 26.5 = 146.7$$

Corresponding to each variety total, there is also a sum of block total (SB) which is recorded in table 2 in column SB. Variety 1 appears in blocks 8, 17, 18, 19, 29 and 30. For variety 1, therefore,

$$SB = 167.2 + 150.1 + 144.7 + 155.6 + 179.9 + 177.7 = 975.2.$$

These block sums are then subtracted from 6 times the variety totals. The resulting values are designated by Q (column 4, table 2). That is,

$$Q = kV - SB$$

As an example, for variety 1,

$$Q = 6(146.7) - 975.2 = -95.0$$

The Q values are divided by 31 giving the values for d in table 2. The general formula is

$$d = \frac{(v - 1)}{N(k - 1)} Q$$

Any d is the deviation of a variety mean from the mean yield of all the varieties in the experiment. The best estimate of variety means is,

$$G/N + d$$

TABLE 2. CALCULATION OF VARIETY MEANS FOR THE EXPERIMENT WITH 31 SOYBEAN VARIETIES, 6 REPLICATIONS.

Variety	Variety totals	Block totals for each V		6V-SB	(1/31)Q	Variety means (bu. per acre)	
	V	SB	Q			d	
1	146.7	975.2	—	95.0	—	3.06	24.6
2	164.2	1007.8	—	22.6	—	.73	26.9**
3	195.5	1019.1	—	153.9	—	4.96	32.6**
4	160.8	986.1	—	21.3	—	.69	27.0**
5	164.1	1035.3	—	50.7	—	1.64	26.0
6	190.6	1009.2	—	134.4	—	4.34	32.0**
7	143.3	967.2	—	107.4	—	3.46	24.2
8	165.8	996.2	—	1.4	—	.05	27.6**
9	182.6	1045.3	—	50.3	—	1.62	29.3**
10	151.1	1006.1	—	99.5	—	3.21	24.4
11	152.2	924.8	—	11.6	—	.37	27.3**
12	176.1	1007.2	—	49.4	—	1.59	29.2**
13	188.8	1068.1	—	64.7	—	2.09	29.7**
14	148.5	998.7	—	107.7	—	3.47	24.2
15	147.1	924.7	—	42.1	—	1.36	26.3*
16	162.6	1033.9	—	58.3	—	1.88	25.8
17	109.8	899.7	—	240.9	—	7.77	19.9**
18	150.8	962.2	—	57.4	—	1.85	25.8
19	176.0	1014.2	—	41.8	—	1.35	29.0**
20	198.1	1017.5	—	171.1	—	5.52	33.2**
21	184.1	996.5	—	108.1	—	3.49	31.1**
22	146.7	954.2	—	74.0	—	2.39	25.3
23	179.0	1007.0	—	67.0	—	2.16	29.8**
24	198.9	1005.8	—	187.6	—	6.05	33.7**
25	162.0	992.5	—	20.5	—	.66	27.0**
26	157.3	956.1	—	12.3	—	.40	27.2**
27	143.7	982.0	—	119.8	—	3.86	23.8
28	156.0	970.7	—	34.7	—	1.12	26.5*
29	148.4	979.9	—	89.5	—	2.89	24.8
30	225.2	1092.5	—	258.7	—	8.35	36.0**
31	167.6	1025.9	—	20.3	—	.65	27.0**
Total	5143.6	30861.6	—	0	—	0	

$$G/N = \frac{5143.6}{186} = 27.65$$

* Yields differing significantly from the yield of the Mukden variety.

** Yields differing highly significantly from the yield of the Mukden variety.

For example, the mean of variety 1 is,

$$G/N + d = 27.65 - 3.06 = 24.6 \text{ bu. per acre}$$

This value is given in the last column of table 2. In the variety means, consideration has been given to the effect of partial confounding of variety differences with block effects. They are the best estimates of the yield performance of these varieties in this experiment.

Several easy verifications of the calculations should be noticed. The sum of the V column is equal to G (5,143.6), while the sum of the SB column is equal to kG. In this illustration $\Sigma SB = 6 \times 5,143.6 = 30,861.6$. The sum of the Q column is zero. Also, the sum of the estimated variety means divided by v will give the general mean.

$$\frac{S(G/N + d)}{v} = G/N$$

It is interesting at this point to notice the exceptional accuracy which was attained in the adjustment of the mean estimates of two of the varieties. As one additional variety was needed in the experiment to fulfill the requirements of the design as to variety number, the Mukden variety was included twice in the trials as varieties 7 and 14. Although plot totals of these varieties were not equal, the correction for blocks adjusted their means to identical values. Such accuracy is not, however, claimed to be a constant characteristic of the design. It should be noted that where several controls are included, these should be kept separate in the analysis and their means combined in the final table.

Derivation of sums of squares:

The correction term (c) is the square of the total divided by the total number of plots:

$$\frac{G^2}{N} = \frac{(5,143.6)^2}{186} = 142,239.90.$$

The total sum of squares is obtained by adding the square of each individual plot yield, table 1, and subtracting therefrom the correction term:

$$(29.8)^2 + (24.2)^2 + \dots + (35.8)^2 - 142,239.90 = 3,932.04$$

The sum of squares between means of blocks is obtained by adding the squares of the block totals, dividing by the number of plots which make up each block total, and subtracting the correction term:

$$\frac{(153.7)^2 + (147.2)^2 + \dots + (196.8)^2}{6} - 142,239.90 = 1,642.60$$

The sum of squares between means of varieties is obtained by squaring each Q value (column 4, table 2), adding these squares and dividing by N:

$$\frac{(-95.0)^2 + (-22.6)^2 + \dots + (-20.3)^2}{186} = 1,841.28$$

The analysis of variance is presented in table 3.

TABLE 3. ANALYSIS OF VARIANCE OF BALANCED INCOMPLETE BLOCK DESIGN.

Source of variation	Degrees of freedom	Sum of squares	Mean square
Total	185	3,932.04	
Blocks	30	1,642.60	
Varieties	30	1,841.28	61.38**
Error	125	448.16	3.585

The test of significance of variety differences:

$$F = 61.38/3.585 = 17.1$$

The standard deviation of the plot yields is,

$$s = \sqrt{3.585} = 1.89$$

This experiment has a mean yield of 27.65 bushels per acre. The ratio of s to the mean (usually called coefficient of variability) is 7 percent. That is, the uncontrolled variation has been reduced to a small percentage of the mean value.

Since there are two controls, the standard error of the difference between a variety and the means of the controls is calculated from the error variance, thus,

$$S_{MD} = \sqrt{\frac{s^2}{E} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

where E is the efficiency factor to be discussed below, n_1 is the number of replications of the controls and n_2 is the number of replications of the other varieties. In this experiment

$$S_{MD} = \sqrt{\frac{(3.585)}{31/36} \left(\frac{1}{12} + \frac{1}{6} \right)} = 1.020.$$

By means of "Student's" t values, the bushel differences required for the 5 percent and 1 percent probability levels can be obtained. Varieties 7 and 14 are the Mukden soybean variety which is considered a standard variety in Iowa and it is desired to compare the yielding ability of the other varieties in this test with the standard, Mukden (24.2 bu. per acre).

Probability	SMD	t(125 D.F.)	Bu. Diff.	Mukden \pm Bu. Diff.	
				High Limits	Low Limits
5%	1.020	1.979	2.0	26.2 *	22.2 *
1%	1.020	2.616	2.7	26.9 **	21.5 **

The significance of the mean varietal differences above or below Mukden is indicated by use of asterisks in column 6, table 2.

EFFICIENCY FACTOR

Assuming soil homogeneity and, consequently, no reduction in the error variance due to the reduction of the block size, the efficiency of the balanced incomplete block design is below that of the randomized complete block arrangement when equal numbers of replications are used. Quoting Yates (13), the loss of information "may be looked on as a measure of the inevitable loss inherent in the arrangement." The efficiency of the incomplete block design as compared to randomized complete blocks is expressed by the fraction $\frac{1 - 1/k}{1 - 1/v}$ when the replication numbers in each arrangement are equal.

In heterogeneous soil, however, the reduction in block size usually more than compensates for the loss of information due to the arrangement.

In an experiment with six randomized blocks of 31 plots, 6 of the 186 degrees of freedom are allotted to blocks and the mean, while the remaining 180 are available for comparing the varieties. In the incomplete blocks, on the other hand, 31 degrees of freedom are allotted to blocks and the mean, leaving 155 for varieties, the efficiency factor being 155/180 or 31/36. This means that if there were no reduction in error variance per plot owing to the reduction of the block size from 31 to 6, a balanced incomplete block design would only give 31/36 of the information that would be given by an arrangement in randomized blocks of 31 plots.

It is difficult to compare the precision of the above incomplete block experiment with a randomized complete block arrangement having the same varieties replicated the same number of times. This is because the replications are not grouped in such a way that a block with 31 plots can be assumed. Such a comparison is readily made in uniformity trial data when blocks of both v and k plots are assumed. Such data are not available from the 1937 trials. Goulden (6) compared the efficiency of balanced incomplete blocks with randomized complete blocks on uniformity trial data, and he concluded that the increase in precision due to the use of balanced incomplete blocks is from 20 to 50 percent.

LATTICE SQUARES (QUASI-LATIN SQUARES)

The quasi-factorial principle was extended by Yates (14) to provide for the elimination of soil differences by the use of two different groupings of the same experimental plots. When variety trials are conducted in quasi-Latin squares, hereafter called lattice squares as suggested by Yates, each replicate of the varieties is arranged in a square from which soil variation as shown by column and row differences may be segregated from varietal differences.

These designs are constructed from completely orthogonalized squares. The six groups on page 296 may be reduced to three by combination in pairs. For each pair a square

Square I	Square II	Square III
<u>1: 2: 3: 4: 5</u>	<u>1:10:14:18:22</u>	<u>1: 8:15:17:24</u>
<u>6: 7: 8: 9:10</u>	<u>23: 2: 6:15:19</u>	<u>25: 2: 9:11:18</u>
<u>11:12:13:14:15</u>	<u>20:24: 3: 7:11</u>	<u>19:21: 3:10:12</u>
<u>16:17:18:19:20</u>	<u>12:16:25: 4: 8</u>	<u>13:20:22: 4: 6</u>
<u>21:22:23:24:25</u>	<u>9:13:17:21: 5</u>	<u>7:14:16:23: 5</u>

may be constructed having its rows formed of the blocks of one group and its columns of the blocks of the other group. For example, square I above is made so that the blocks of group I form the rows and the blocks of group II, the columns. In any one of these lattice squares, the rows and columns may be re-arranged at random. The above three squares were randomized first by rows, then by columns, the resulting field plan being given below.

Square I	Square II	Square III
14:11:15:12:13	10:22:18: 1:14	18: 9: 2:25:11
4: 1: 5: 2: 3	24:11: 7:20: 3	24:15: 8: 1:17
19:16:20:17:18	2:19:15:23: 6	6:22:20:13: 4
24:21:25:22:23	13: 5:21: 9:17	5:16:14: 7:23
9: 6:10: 7: 8	16: 8: 4:12:25	12: 3:21:19:10

In this lattice square arrangement every pair of varieties occurs together once only in either a row or a column of any one of the squares. Also, every variety occurs with every other variety once in one column and one row from each square. Variety 14, for instance, occurs with varieties 11, 15, 12, 13, 4, 19, 24, 9 in the first row and first column of square I, with varieties 10, 22, 18, 1, 3, 6, 17, 25 in the first row and last column of square II and with the remaining varieties 5, 16, 7, 23, 2, 8, 20, 21 in the fourth row and third column of square III.

In the lattice square arrangement since $v = k^2$ the number of varieties must be the square of some integer. However, not all integers can be used either because the completely orthogonalized square is known not to exist, i. e., the 6×6 (3), or because the structure has not yet been worked out. The numbers of varieties and the minimum number of replications for which this type of design has been solved are indicated in table 4. Some of these design illustrations are

TABLE 4. NUMBER OF VARIETIES AND MINIMUM NUMBER OF REPLICATIONS FOR KNOWN LATTICE SQUARES.

No. of varieties	k	Minimum number of replications	λ
9	3	2	1
16	4	5	2
25	5	3	1
49	7	4	1
64	8	9	2
81	9	5	1
121	11	6	1
169	13	7	1

presented by Fisher and Yates (4) and Yates (14). Others will be published shortly.

The advantage noted in balanced incomplete blocks (small

blocks by which much soil variability may be eliminated) is still more pronounced in lattice squares. Since each replicate is arranged in a square from which differences of both columns and rows can be eliminated, if the variability between rows is equal to that between columns more than twice the efficiency is attained for the number of replications used.

Maximum efficiency can be obtained in the lattice square design if the plot and, consequently, the block are nearly square, as a large proportion of variability due to soil heterogeneity is thereby eliminated. On the contrary, if plots are considerably longer than they are wide and if the soil is quite variable, the varieties occurring together in sets in which the sides of the plots are adjacent to each other will be more accurately compared than varieties which occur together in sets in which the ends of the plots are adjacent. A means of partially overcoming this difficulty is possible when the entire lattice square design is replicated. One replication can be turned through 90 degrees thereby causing the varieties which occurred together in rows in one arrangement to occur together in columns in the replicate. Comparisons between varieties should thereby be equal in accuracy.

Each square in the lattice square arrangement must be planted as a unit in the field. The squares themselves may be separated, however, and may even be planted in different fields if soil differences between the fields are not too great. In any one square the rows and columns may be rearranged at random, but it is necessary to keep the specified varieties together in any one row and column. Thus, in square I, varieties 14, 11, 15, 12, 13 must be kept together in some row, and varieties 14, 4, 19, 24, 9 in some column.

Several cautions must be noted regarding the use of the lattice square arrangement. As in the balanced incomplete block designs a special type of statistical analysis is required. Variety differences being confounded with column and row effects, it is necessary to determine the variety means from a suitable formula.

In this arrangement the advisability of testing varieties which have a large range in yield is questionable. The lattice square was designed to determine small differences between a group of selected varieties.

Before this design is selected for use the possibility of losing some plots should also be considered. Although it is possible to adjust for missing plots in the usual way, (1, 11) such adjustment is quite involved.

EXAMPLE OF LATTICE SQUARE EXPERIMENT

The lattice square design which is analyzed in this bulletin consists of a yield test of 49 varieties of soybeans grown

at Ames during 1938. The design consisted of four groups of orthogonal 7 x 7 squares, arranged so the soil variability can be removed by rows and by columns. The plot dimensions were 3 x 16 feet with seven plots in each row and in each column. The plot arrangement and the yield of each are presented in table 5. Varieties are indicated by the number 1, 2, . . . 49 in the bold face type in each cell.

The statistical analysis resembles that of incomplete block designs since variety means are confounded with column and row effects. To secure the variety sums which are recorded as V in table 6, locate the variety yield performance in each square in table 5 and add the four values. For variety 1 the four yield values and sum are:

$$25.9 + 22.1 + 29.0 + 26.0 = 103.0$$

The totals of the column and the row in which each variety occurs (column 3, table 6) are calculated. The column and

TABLE 5. PLOT YIELDS OF 49 SOYBEAN VARIETIES IN LATTICE SQUARE FIELD ARRANGEMENT.
(Bushels per acre.)

Square I.							
45:24.5	47:23.3	49:30.9	48:31.8	43:23.9	46:33.6	44:28.6	196.6
17:18.0	19:22.7	21:29.0	20:36.3	15:23.1	18:25.7	16:24.6	179.4
3:23.8	5:18.0	7:22.5	6:34.5	1:25.9	4:25.1	2:23.4	173.2
24:24.0	26:24.0	28:27.7	27:29.8	22:21.9	25:25.1	23:20.1	172.6
31:21.2	33:22.2	35:32.7	34:35.4	29:27.5	32:19.1	30:22.4	180.5
38:20.7	40:21.8	42:26.6	41:31.8	36:31.5	39:18.0	37:21.3	171.7
10:21.5	12:24.2	14:33.0	13:29.8	8:31.2	11:19.4	9:21.8	180.9
153.7	156.2	202.4	229.4	185.0	166.0	162.2	1254.9
Square II.							
21:23.1	2:22.4	8:28.9	45:31.8	33:31.0	27:19.7	39:23.3	180.2
24:23.0	12:23.6	18:24.5	6:25.9	36:31.3	30:18.0	49:29.8	176.1
1:22.1	38:21.8	44:27.8	32:25.3	20:35.4	14:21.6	26:29.2	183.2
34:29.0	15:22.7	28:30.2	9:29.5	46:31.6	40:26.2	3:28.3	197.5
47:26.5	35:23.6	41:26.9	22:31.0	10:30.1	4:22.5	16:30.7	191.3
37:23.6	25:23.3	31:30.9	19:28.9	7:21.0	43:22.4	13:27.1	177.2
11:23.8	48:21.0	5:23.4	42:38.0	23:24.0	17:33.1	29:27.1	190.4
171.1	158.4	192.6	210.4	204.4	163.5	195.5	1295.9
Square III.							
16:25.7	14:25.6	34:26.0	5:25.9	36:32.5	45:25.4	25:28.4	189.5
11:23.0	2:19.8	22:23.4	49:30.0	31:29.6	40:24.6	20:32.1	182.5
33:23.4	24:25.0	44:29.6	15:32.8	4:38.4	13:22.4	42:26.6	198.2
43:19.2	41:21.5	12:23.9	32:23.6	21:27.7	23:20.9	3:24.8	161.6
38:23.8	29:21.2	7:21.9	27:33.6	9:24.5	18:23.4	47:29.3	177.7
6:15.9	46:26.6	17:21.3	37:28.9	26:26.3	35:26.0	8:29.5	174.5
28:13.9	19:16.0	39:26.2	10:25.9	48:27.8	1:29.0	30:22.2	161.0
144.9	155.7	172.3	200.7	206.8	171.7	192.9	1245.0
Square IV.							
38:18.4	34:19.5	49:24.5	19:32.5	4:37.4	8:30.0	23:28.3	190.6
46:25.3	42:18.9	1:26.0	27:26.6	12:33.6	16:26.3	31:28.1	184.8
22:16.9	18:20.9	33:24.5	3:24.8	37:28.0	43:23.8	14:26.3	165.2
30:19.4	26:17.6	41:23.3	11:31.0	45:33.9	7:21.9	15:22.2	169.3
5:11.6	43:18.2	9:15.1	35:31.2	20:38.6	24:25.6	39:24.4	164.7
21:18.2	10:16.6	25:22.5	44:31.6	29:29.8	40:23.0	6:23.8	165.5
13:12.9	2:16.8	17:21.9	36:31.9	28:33.3	32:21.5	47:23.3	161.6
122.7	128.5	157.8	209.6	234.6	172.1	176.4	1201.7
							4997.5

TABLE 6. CALCULATION OF VARIETY MEANS FOR THE LATTICE SQUARE EXPERIMENT ON 49 SOYBEAN VARIETIES.

Variety	V	SB	$kQ = 7V - SB$	Variety means (bu. per acre)
1	103.0	1387.8	-666.8	27.8
2	82.4	1302.3	-725.5	25.0
3	101.7	1449.2	-737.3	24.4
4	123.4	1524.2	-660.4	28.1
5	78.9	1390.0	-837.7	19.6**
6	100.1	1450.4	-749.7	23.8
7	87.3	1448.6	-837.5	19.6**
8	119.6	1468.8	-631.6	29.4
9	90.9	1458.0	-821.7	20.4**
10	94.1	1386.0	-727.3	24.9
11	97.2	1414.7	-734.3	24.5
12	105.3	1424.9	-687.8	26.8
13	92.2	1437.2	-791.8	21.8
14	106.5	1416.8	-671.3	27.5
15	100.8	1464.9	-759.3	23.3
16	107.3	1419.7	-668.6	27.7
17	94.3	1353.2	-693.1	26.5
18	94.5	1357.2	-695.7	26.4
19	100.1	1440.1	-739.4	24.3
20	142.4	1571.1	-574.3	32.2**
21	98.0	1389.7	-703.7	26.0
22	93.2	1402.0	-749.6	23.8
23	93.3	1429.9	-776.8	22.5
24	97.6	1364.2	-681.0	27.1
25	99.2	1379.9	-684.8	26.9
26	97.1	1386.6	-706.9	25.8
27	109.7	1518.5	-750.6	23.8
28	105.1	1467.2	-731.5	24.7
29	105.6	1484.9	-745.7	24.0
30	82.0	1328.2	-754.2	23.6
31	109.8	1454.5	-685.9	26.3
32	89.5	1436.1	-809.6	21.0*
33	101.1	1387.4	-679.7	27.1
34	109.9	1459.4	-690.1	26.6
35	113.5	1453.1	-658.6	28.1
36	127.2	1504.7	-614.3	30.2*
37	101.8	1457.2	-744.6	24.0
38	84.7	1302.9	-710.0	25.7
39	91.9	1387.8	-744.5	24.0
40	95.6	1380.7	-711.5	25.6
41	103.5	1429.4	-704.9	25.9
42	110.1	1479.3	-708.6	25.8
43	83.7	1322.0	-736.1	24.4
44	117.6	1480.2	-657.0	28.2
45	115.6	1506.0	-696.8	26.3
46	117.1	1402.2	-582.5	31.3**
47	102.4	1423.8	-707.0	25.8
48	104.4	1479.9	-749.1	23.8
49	115.2	1502.2	-695.8	26.4
Total	4997.5	69965.0	-34932.5	
		$\frac{G}{N} = \frac{4997.5}{196} = 25.5$		

* Yields differing significantly from that of Mukden.

** Yields differing highly significantly from that of Mukden.

row totals for variety 1 are,

$$SB = 173.2 + 185.0 + 183.2 + 171.1 + 161.0 + 171.7 + 184.8 + 157.8 = 1,387.8$$

Next, values of kQ are calculated (column 4, table 6). For variety 1,

$$kQ = kV - SB = 7(103.0) - (1,387.8) = -666.8$$

Varietal means are given by the general formula,

$$\frac{2k}{(k-1)} \frac{(G)}{(N)} + \frac{2}{\lambda k(k-1)} kQ$$

For variety 1, the mean is

$$\frac{(2)(7)}{6} \frac{(4,997.5)}{(196)} + \frac{2}{1 \cdot 7 \cdot 6} (-666.8) = 59.50 - 31.75 =$$

27.8 bu. per acre

When the soil is variable, this gives the best estimate of the mean yields of the varieties.

Calculation of sums of squares — total, between lattice squares, between rows and between columns sources — proceeds in the ordinary manner. In calculating sums of squares of rows and columns within squares, caution must be exercised not to include the sum of squares between the lattice squares.

Correction term:

$$c = \frac{(4997.50)^2}{196} = 127,423.50$$

Total sum of squares:

$$(24.5)^2 + (18.0)^2 + \dots + (23.3)^2 - c =$$

$$132,466.63 - 127,423.50 = 5,043.13$$

Sum of squares between squares:

$$\frac{(1254.9)^2 + (1295.9)^2 + (1245.0)^2 + (1201.7)^2}{49} - c =$$

$$\frac{6,248,238.71}{49} - c = 127,515.08 - 127,423.50 = 91.58$$

Sum of squares between columns:

$$\frac{(153.7)^2 + (156.2)^2 + \dots + (176.4)^2}{7} - c =$$

$$\frac{912,999.53}{7} - c = 130,428.50 - 127,423.50 = 3,005.00$$

Sum of squares between columns in squares:

$$3,005.00 - 91.58 = 2913.42$$

Sum of squares between rows:

$$\frac{(196.6)^2 + (179.4)^2 + \dots + (161.6)^2}{7} - c =$$

$$\frac{895,336.97}{7} - c = 127,905.28 - 127,423.50 = 481.78$$

Sum of squares between rows in squares:

$$481.78 - 91.58 = 390.20$$

The sum of squares between varieties consists of the sum of the squared deviations of the $-7Q$ values divided by NE:

$$\frac{1}{NE} \left[S(kQ)^2 - \frac{(SkQ)^2}{n} \right] = \frac{4}{(196)(3)} (25,126,397.75 - 24,975,006.25) = 1,029.87$$

The efficiency factor (E) for a 7 x 7 lattice square design is $(k-1)/(k+1)$ or $3/4$. The analysis of variance is presented in table 7.

TABLE 7. ANALYSIS OF VARIANCE FOR 7 x 7 LATTICE SQUARE.

Source of variation	Degrees of freedom	Sum of squares	Mean square
Total	195	5,043.13	
Columns in squares	24	2,913.42	
Rows in squares	24	390.20	
Squares	3	91.58	30.53**
Varieties	48	1,029.87	21.46**
Error	96	618.06	6.438

The standard error of random differences between two varieties is

$$SMD = \sqrt{\frac{2s^2}{E \cdot r}}$$

$$SMD = \sqrt{\frac{2(6.438)}{3/4(4)}} = 2.072$$

The mean yielding abilities of varieties in this test will again be compared to the mean (25.8 bu. per acre) of the Mukden (26) variety.

Probability	SMD	t(96 D.F.)	Bu. diff.	Mukden \pm Bu. diff.	
				High limit	Low limit
5%	2.072	1.985	4.1	29.9 *	21.7 *
1%	2.072	2.628	5.4	31.2 **	20.4 **

Significance of mean varietal differences is indicated by use of asterisks in column 5, table 6.

PRECISION OF LATTICE SQUARES

What was said concerning the loss of information due to the arrangement of incomplete block designs, when no gain in precision is attained by the reduction of block size, applies equally well to lattice squares. The efficiency of lattice square designs as compared to randomized complete blocks involving equal numbers of replicates is represented by the fraction: $(k - 1)/k + 1$. The efficiency of the above design is, therefore, $\frac{3}{4}$ or .75.

Yates (12, 15) gives a method of securing an estimate of the amount of gain in precision due to confounding. W. G. Cochran gave us suggestions for obtaining an estimate of the gain in precision of lattice squares.

Owing to the representation of all varieties once and only once within a square, an effective comparison can be made between the information obtained from this experiment and that which would have been obtained had it been in randomized complete blocks. The design is analogous to randomized blocks except that varieties are restricted within rows and columns rather than completely random. Since rows and columns are randomized, the comparison should be quite accurate.

In making this comparison it is necessary to find first the sum of squares corresponding to rows and columns unconfounded with varieties. Rows and columns are being considered together for simplicity. This is done in the following table, where the first three rows are taken directly from table 7.

Source of variation	Degrees of freedom	Sum of squares	Mean square
Total	195	5,043.13	
Squares	3	91.58	
Error	96	618.06	6.44
Varieties (confounded)	48	1,863.44	
Row + column (unconfounded)	48	2470.05	51.46

As the squares are analogous to blocks in randomized complete block arrangements, the corresponding sum of squares in the second line represents accountable variation. The calculation of the variety sum of squares is done as is usual in randomized complete block designs. The sums of squares from these sources together with the error sum of squares are subtracted from the total, yielding a sum of squares for rows and columns freed of the confounding with varieties.

The error variance is next deducted from the mean square for row + column.

$$51.46 - 6.44 = 45.02$$

Owing to the adjustments necessary to eliminate varieties from the row and column mean square, this figure, 45.02, is not an estimate of the true variance of rows and columns, but $\frac{3}{4}$ of that amount. Therefore,

$$\frac{45.02}{\frac{3}{4}} = 60.03$$

The error variance is now added back into this mean square, $60.03 + 6.44 = 66.47$,

to get the desired mean square for rows + columns.

We are now ready to compute the error variance which may be expected to have resulted had the experiment been a randomized complete block arrangement. First, the row

Source of variation	Degrees of freedom	Sum of squares	Mean square
Row + column	48	3,190.56	66.47
Variety + error	144	927.36	6.44
Error estimated for randomized complete blocks	192	4,117.92	21.45

+ column mean square, 66.47, is multiplied by the total degrees of freedom for rows and columns to obtain the sum of squares, 3,190.56. Assuming uniformity trial data, the variety mean square should equal the error mean square, except for errors of estimation. Next, therefore, the variety and error degrees of freedom are pooled and multiplied by the error variance to obtain the sum of squares, 927.36. This is now added to the row + column sum of squares to represent a gross sum of squares which would be expected had randomized complete blocks been used. The corresponding mean square for error is obtained by dividing this combined sum of squares, 4,117.92, by 192 (the degrees of freedom for error assuming randomized complete blocks) giving 21.45.

Finally, to make the error variance of the lattice square comparable to the error variance had the randomized complete block design been used, the former must be raised to 100 percent efficiency:

$$\frac{6.44}{.75} = 8.59$$

Assuming the precision obtainable in randomized complete block designs as 100 percent, $\frac{21.45}{8.59} = 250$ percent was obtained with the lattice square, representing a gain of 150 percent.

Thus, the reduction of block size from 49 to 7 plots per block has resulted in a notable increase in precision; that is, in a large decrease in experimental error. Clearly the

efficiency factor (75 percent) was far more than offset by the increased precision of the experiment. This is the result that may be anticipated in fields having extreme soil heterogeneity. The effectiveness of small blocks may be observed in the yield figures of square IV, table 5. The lower left corner of the field bordered on a low area with soil high in carbonates, commonly known as an alkali spot. The high carbonate content resulted in a deficiency of iron available to the plants, causing a curtailment of yield. The column totals in this area are low and, consequently, the variability in yields introduced by the area of high carbonates is largely included in the variance between the columns of the squares. Had square IV been a randomized complete block, a large portion of this variability would have fallen into the error variance as only the variance between squares could have been eliminated from error.

The efficiency factor of quasi-factorial designs, due to partial confounding, may not be compensated by a decrease in the experimental error. An illustration of an experiment in which there was actually a loss of information is found in a rod-row soybean yield test conducted on land near Muscatine, Iowa, on which experimental work had not been previously conducted. The results of the experiment furnish evidence that the soil was exceptionally homogeneous during this season. The analysis of variance is presented in table 8.

TABLE 8. ANALYSIS OF VARIANCE FOR 5 x 5 LATTICE SQUARE.

Source of variation	Degrees of freedom	Sum of squares	Mean square
Total	74	646.05	
Columns	12	171.47	
Rows	12	108.39	
Squares	2	26.86	13.43
Varieties	24	198.76	8.28
Error	24	140.57	5.86

Proceeding as above, an estimate of the unconfounded row + column variation is obtained:

Source of variation	Degrees of freedom	Sum of squares	Mean square
Total	74	646.05	
Squares	2	26.86	
Varieties (confounded)	24	330.37	
Error	24	140.57	5.86
Row + column (unconfounded)	24	148.25	6.18

This row + column variation is raised to full efficiency,

$$\frac{6.18 - 5.86}{2/3} + 5.86 = 6.34$$

The corresponding estimate of error in a randomized complete block experiment is, therefore:

$$\frac{(6.34) (24) + (5.86) (48)}{72} = 6.02$$

The error mean square for the lattice square arrangement is now raised to full efficiency giving, $\frac{5.86}{2/3} = 8.79$. Finally, efficiency of the lattice square design relative to the randomized complete blocks is:

$$\frac{6.02}{8.79} = 68.5 \text{ percent.}$$

Thus a loss of 31.5 percent in information resulted from the use of the lattice square design in this experiment. The rows and columns isolated only random variability. Loss of information in the experiment as compared to randomized complete blocks was therefore almost as great as the loss of efficiency of the design (33.3 percent).

Dr. Yates informs us that he has recently proved that such lattice designs can be analyzed as randomized blocks. His findings are to be reported this year (16). The fact that lattice square designs can be analyzed as randomized blocks increases their attractiveness. The experiment can in no event, therefore, be appreciably less accurate than arrangements in randomized complete blocks.

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