INTRODUCTION
TO
ECONOMIC STATISTICS

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ECONOMIC STATISTICS

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PREFACE

This text is a complete revision of a book bearing a similar title which was published in 1925 by the A. W. Shaw Company and subsequently by the McGraw-Hill Book Company, Inc. The revision has included many changes suggested by helpful critics of the former edition and has aimed at bringing the illustrative material up to date and extending it to a somewhat wider range of economic topics. Moreover, certain portions of the present edition—in particular the first three chapters—are almost entirely new. In certain parts of the treatment, materials which appear now appropriate for more advanced texts have been eliminated or reduced in emphasis. The intent throughout has been to preserve the elementary character of the treatment.

The book has been prepared while keeping constantly in mind the needs of those students who are interested in the application of statistical methods to economic problems. This point of view has controlled the selection and arrangement of the topics covered and is responsible for the almost exclusive use of economic statistics for illustrative material. No attempt has been made to discuss numerous recent developments in statistical technique, because they are not regarded as coming within an elementary treatment which aims at expounding fundamentals.

The presentation is designed to enable the student who does not have extensive mathematical training to become familiar with statistical processes and acquire a working knowledge of the limitations in the use of such processes in economic analyses. The reader of the book is assumed to have more than a reminiscence of the ground covered in elementary algebra, and he should also have or shortly acquire some knowledge of logarithms and the slide rule. Further mathematical training is unquestionably desirable for one who hopes to become a statistical specialist; but as nearly all symbolic developments of the book are confined to footnotes, a good understanding of the subject can be secured with no mathematical background beyond the minimum mentioned above.

When the book is used as a text in an organized course, the material of the entire text (except the final chapters in Part II

and in Part III) can be covered in one half year. This presupposes at least three meetings each week, of which one is set aside for exercise work in the "statistical laboratory." To carry out such a program, it will be necessary to cover the chapters of Part I, which is less difficult than the other parts, rather rapidly. If the book is to be used for a year course, the work of the text will need to be supplemented by references to the more elaborate treatments of the advanced topics in other sources or by the assignment of fairly complicated report topics.

We do not believe that anyone, whether a member of a class or a self-taught student, can learn statistics by reading a textbook. Actual performance of the routine details involved in statistical analyses is essential; every student should do a generous number of exercises and early form the habit of completing each task rather than assuming that he understands the unfinished portion.

Every care has been taken to safeguard the quoted data and the computations and charts from error, as well as to ensure textual accuracy. It is not taken for granted, however, that the result is perfect or even as nearly perfect as might reasonably be desired, and the authors will therefore be grateful for information of any errors which are noticed and for all suggestions for the improvement of form or content.

In the work of revision, Mr. Crum has assumed main responsibility for Parts I and II and Mr. Tebbutt for Part III; Mr. Patton has given attention to the entire manuscript. Mr. Frank F. Dodge, of the staff of the Paul Revere Life Insurance Company, Miss Dorothy Wescott, assistant editor of the *Review of Economic Statistics*, and Mrs. Harriet S. Ross, formerly of the staff of that *Review*, have given painstaking and discerning assistance in assembling and criticizing the statistical materials. The authors are grateful also for generous and painstaking aid rendered by Mrs. Esther S. Morton and Miss Eleanor Lyons in preparing the charts, and by Miss Althea MacDonald, Mrs. Anna H. Thorpe, and Mrs. Dorothy Miller in preparing drafts of text and tables.

W. L. C.
A. C. P.
A. R. T.

CAMBRIDGE, MASS.,
WORCESTER, MASS.,
PROVIDENCE, R. I.,
August, 1938.

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APPENDIX A

NUMERICAL DATA FOR CERTAIN OF THE CHARTS OF PARTS I TO III

The data upon which many of the charts in the text are based are given in tabulations in earlier pages. For a large number of the charts, however, such tabular presentations are not essential to the textual development; and, with a few exceptions, they are given herewith to facilitate verification of the several charts and to provide material for further study by graphical or analytical methods.

TABLE A

GROSS INCOME OF SUBGROUPS IN SELECTED GROUPS OF MANUFACTURING CORPORATIONS IN 1934*

Food and kindred products	
Bakery and confectionery products	1,212
Canned products	751
Mill products	1,025
Packing-house products	3,116
Sugar	579
Other food products	1,661
Textiles and their products	
Cotton goods	982
Woolen and worsted goods	377
Silk and rayon goods	478
Other textiles	1,068
Clothing	1,797
Knit goods	547
Metal and its products	
Iron and steel	2,167
Locomotives, etc.	143
Machinery	2,031
Motor vehicles	2,946
Household equipment	385
Office equipment	229
Building material and hardware	1,085
Precious metals, etc.	265
Other metal industries	1,377

* Unit: million dollars. Source: "Statistics of Income, 1934, Part 2," Washington, U. S. Treasury, 1937, pp. 60–63.

TABLE B

UNITED STATES TREASURY BONDS, OF OVER TEN YEARS STATED MINIMUM MATURITY, OUTSTANDING ON DECEMBER 31, 1935*

Rate and dates of maturity	Amount
4¼, 1947–1952	759
3¾, 1946–1956	489
3⅛, 1946–1949	819
3 , 1951–1955	755
3 , 1946–1948	1,036
3⅛, 1949–1952	491
2⅞, 1955–1960	2,611

* Unit: million dollars. Source: "Statistical Abstract of the United States, 1936," Washington, U. S. Department of Commerce, 1936, p. 201.

TABLE C

PERCENTAGE OF NET TO GROSS TON-MILES (EXCLUDING LOCOMOTIVES) FOR UNITED STATES CLASS I RAILROADS IN 1936*

Region	%
New England	37.4
Great Lakes	40.8
Central Eastern	46.3
Pocahontas	53.9
Southern	41.1
Northwestern	40.8
Central Western	36.3
Southwestern	37.2
Entire United States	41.6

* Source: "Statistics of Railways in the United States, 1936," Washington, Interstate Commerce Commission, 1937, p. S 154. (Data pertain to freight train performance.)

TABLE D

UNITED STATES NET IMPORTS OF GOLD IN 1937 FROM COUNTRIES MAKING PRINCIPAL SHIPMENTS*

Belgium	90.9
France[a]	− 13.7
Switzerland	54.5
United Kingdom	891.5
Canada	111.5
Mexico	38.5
Australia	34.7
British India	50.8
Japan	246.5

* Unit: million dollars. Source: *Federal Reserve Bulletin*, March, 1938, p. 206.
[a] Net export to France.

TABLE E

UNITED STATES INTERNAL REVENUE COLLECTIONS FOR FISCAL YEAR ENDING JUNE 30, 1937*

Individual income tax	1,091.7
Corporation income and excess-profits taxes	1,082.0
Unjust enrichment tax	6.1
Dividend tax	•
Alcohol taxes	594.2
Estate and gift taxes	305.5
Tobacco manufacturers tax	552.3
Sales taxes	617.4
Other miscellaneous taxes	138.1
Social security taxes	265.7
Total collections	4,653.2

* Unit: million dollars. Source: "Annual Report of the Commissioner of Internal Revenue, 1937," Washington, 1937, pp. 1–2.
• Less than 1/10 million dollars.

TABLE F

PERCENTAGE DISTRIBUTION OF NATIONAL INCOME OF UNITED STATES IN 1935 ACCORDING TO INDUSTRIAL ORIGIN*

Manufacturing	21.7
Service	15.6
Government	14.5
Trade	12.9
Transportation and other public utilities	9.4
Agriculture	8.8
Finance	8.7
Mining	2.1
Construction	1.7
Miscellaneous	4.5

* Source: Simon Kuznets, "National Income and Capital Formation," 1919–1935, New York, National Bureau of Economic Research, 1937, p. 17.

TABLE G

PERCENTAGE WEIGHTS, BASED ON 1926 VALUES, OF SUBGROUPS IN THE FOODS GROUP OF THE U. S. B. L. S. WHOLESALE PRICE INDEX*

Subgroup	%	Fractions of 360°
Dairy products	18.08	65.1
Cereal products	16.61	59.8
Fruits and vegetables	10.91	39.3
Meats	31.23	112.4
Other foods	23.17	83.4

* Source: "Wholesale Prices, 1931," Washington, U. S. Bureau of Labor Statistics, Bulletin 572, January, 1933, pp. 93–96.

TABLE H

ADJUSTED DEMAND DEPOSITS OF ALL MEMBER BANKS OF THE FEDERAL RESERVE SYSTEM ON DATES OF CALL IN 1934*

March 5	3,648
June 30	3,792
October 17	4,168
December 31	4,292

* Unit: million dollars. Source: "Twenty-Third Annual Report of the Board of Governors of the Federal Reserve System," Washington, 1937, p. 142. (Demand deposits other than interbank and U. S. Government, less cash items reported as in process of collection, and less cash items reported on hand but not in process of collection.)

TABLE I

SILVER PRODUCTION FROM MINES IN CONTINENTAL UNITED STATES, ALASKA, PUERTO RICO, AND PHILIPPINE ISLANDS, 1936*

Alaska	398	New Mexico	1,244
Arizona	8,556	Oregon	103
California	2,037	South Dakota	145
Colorado	6,391	Texas	1,348
Idaho	14,815	Utah	11,204
Michigan	.....	Washington	60
Montana	11,498	Philippine Islands	461
Nevada	5,173	Other	380

* Unit: thousand fine ounces. Source: "Statistical Abstract of the United States, 1937," Washington, U. S. Department of Commerce, 1938, p. 718.

TABLE J

MISCELLANEOUS MONTHLY SERIES FOR 1936 AND 1937*

Month	a.	b.	c.	d.	e.	f.	g.		h.	i.	
							1936	1937		(1)	(2)
January	614.0	811	130	631	72	98	10.18	11.36	1245	5626	15928
February	521.6	721	119	553	76	89	10.17	11.44	1282	5617	14244
March	382.3	820	165	754	90	108	10.18	11.57	1240	7616	19823
April	317.1	786	195	708	89	123	10.22	11.80	1181	7176	20230
May	261.5	834	151	552	95	98	10.40	11.99	1030	8614	22820
June	560.3	670	180	1300	90	69	10.61	12.32	898	8463	22254
July	340.2	618	191	659	65	91	10.65	12.45	736	7706	20409
August	187.3	707	178	556	72	96	10.72	12.57	582	6780	19761
September	223.8	564	127	680	100	45	10.84	12.74	440	7819	24806
October	197.5	768	124	616	103	73	11.04	12.80	394	8957	29990
November	135.3	786	106	542	101	101	11.18	12.77	447	8373	27095
December	163.8	932	94	730	156	142	11.26	12.76	582	16615	38005

* Series (by columns), units, and sources are as follows:

a. Volume of new financing in the United States, 1937. Unit: million dollars. Source: *Commercial and Financial Chronicle*, January 8, 1938, p. 167.

b. Total business failures, 1937. Unit: one failure. Source: *Survey of Current Business*, February, 1938, p. 32.

c. Construction contracts awarded, privately financed, 1937. Unit: million dollars. Source: *Federal Reserve Bulletin*, February, 1938, p. 159.

d. Total expenditures of U. S. Treasury, 1937. Unit: million dollars. Source: *Federal Reserve Bulletin*, February, 1938, p. 149.

e. Department store sales, unadjusted for seasonal variation, 1937. Unit: per cent of 1923–1925 average. Source: *Federal Reserve Bulletin*, February, 1938, p. 152.

f. Sugar meltings, adjusted for seasonal variation, 1937. Unit: per cent of 1923–1925 average. Source: *Federal Reserve Bulletin*, February, 1938, p. 153.

g. Monetary gold stock of United States at end of each month, 1936–1937. Unit: billion dollars. Source: *Federal Reserve Bulletin*, February, 1938, p. 136. Figure for December 31, 1935 is 10.12.

h. Stocks of meats, cold storage, at end of each month, 1937. Unit: million pounds. Source: *Survey of Current Business*, February, 1938, p. 43. Figure for December 31, 1936 is 1149.

i. Sales, two chain-store systems, 1937: (1) W. T. Grant & Co., (2) J. C. Penney Co. Unit: thousand dollars. Source: *Survey of Current Business*, February, 1938, p. 27.

TABLE K

STATUTORY NET INCOME LESS DEFICIT OF ALL CORPORATIONS IN THE UNITED STATES, 1924–1935*

1924	5,363	1930	1,551
1925	7,621	1931	−3,288
1926	7,505	1932	−5,644
1927	6,510	1933	−2,547
1928	8,227	1934	94
1929	8,740	1935	1,696

* Unit: million dollars. Source: "Statistics of Income, 1934, Part 2," Washington, U. S. Treasury, 1937, p. 36. 1935 figure is from preliminary copy of source.

TABLE L

GROSS INCOME OF ALL CORPORATIONS IN THE UNITED STATES, 1919–1933*

1919	99.9	1927	144.4
1920	118.2	1928	152.8
1921	91.2	1929	160.6
1922	100.9	1930	136.1
1923	118.6	1931	107.5
1924	119.2	1932	81.1
1925	134.3	1933	83.6
1926	142.1		

* Unit: billion dollars. Source: "Statistics of Income, 1934, Part 2," Washington, U. S. Treasury, 1937, p. 35.

TABLE M

EXCESS RESERVES OF MEMBER BANKS OF THE FEDERAL RESERVE SYSTEM, 1937*

January 31	2,152	July 31	791
February 27	2,078	August 31	773
March 31	1,398	September 30	1,038
April 30	1,594	October 30	1,055
May 31	918	November 30	1,169
June 30	865	December 31	1,212

* Unit: million dollars. Source: *Federal Reserve Bulletin*, February, 1938, p. 128. December 31, 1936 figure is 1,984.

TABLE N

ISSUES OF CORPORATE LONG-TERM BONDS, FOR NEW CAPITAL, IN THE UNITED STATES, 1919–1937*

1919	499	1929	1,873
1920	1,039	1930	2,460
1921	1,275	1931	951
1922	1,537	1932	271
1923	1,833	1933	24
1924	1,924	1934	112
1925	2,231	1935	323
1926	2,418	1936	816
1927	2,962	1937	743
1928	2,175		

* Unit: million dollars. Source: *Survey of Current Business*, February, 1938, pp. 14–20.

TABLE O

YIELD PER ACRE OF COTTON IN THE UNITED STATES, BY STATES, 1936*

Alabama	236	New Mexico	457
Arizona	438	North Carolina	298
Arkansas	227	Oklahoma	62
California	574	South Carolina	279
Florida	170	Tennessee	250
Georgia	228	Texas	121
Louisiana	260	Virginia	298
Mississippi	305	Other states	313
Missouri	360		

* Unit: one pound. Source: "Statistical Abstract of the United States, 1937," Washington, U. S. Department of Commerce, 1938, pp. 653–655.

TABLE P

NUMBER OF CONCERNS IN BUSINESS IN THE UNITED STATES, BY STATES, 1936*

Maine	1.50	West Virginia	2.04
New Hampshire	0.94	North Carolina	3.08
Vermont	0.65	South Carolina	1.46
Massachusetts	7.85	Georgia	3.03
Rhode Island	1.19	Florida	2.68
Connecticut	2.91		
		Kentucky	3.05
New York	24.54	Tennessee	2.78
New Jersey	8.48	Alabama	2.13
Pennsylvania	16.33	Mississippi	1.71
Ohio	11.13	Arkansas	2.16
Indiana	5.35	Louisiana	2.46
Illinois	13.41	Oklahoma	3.40
Michigan	6.77	Texas	9.20
Wisconsin	5.77		
		Montana	0.93
Minnesota	4.62	Idaho	0.76
Iowa	4.59	Wyoming	0.41
Missouri	6.48	Colorado	1.99
North Dakota	1.05	New Mexico	0.61
South Dakota	1.22	Arizona	0.55
Nebraska	2.71	Utah	0.76
Kansas	3.55	Nevada	0.21
Delaware	0.45	Washington	3.40
Maryland	2.83	Oregon	2.02
District of Columbia	0.86	California	12.09
Virginia	2.93		

* Unit: 10,000 concerns. Source: "Statistical Abstract of the United States, 1937," Washington, U. S. Department of Commerce, 1938, p. 294.

TABLE Q

NUMBER OF NATIONAL BANKS IN OHIO (EXCLUDING "RESERVE CITIES") HAVING TOTAL DEPOSITS WITHIN SPECIFIED RANGES ON DECEMBER 31, 1936*

Class interval[a]	Number of banks	Deposits[b]
0– 500	50	16,647
500– 1,000	71	52,828
1,000– 1,500	33	40,915
1,500– 2,000	28	48,062
2,000– 2,500	12	27,645
2,500– 3,000	10	27,285
3,000– 3,500	8	25,214
3,500– 4,000	4	15,098
4,000– 4,500	1	4,222
4,500– 5,000	2	9,463
5,000– 5,500	3	15,420
5,500– 6,000	3	17,185
6,000– 6,500	1	6,431
6,500– 7,000	3	20,224
7,000– 7,500	0	0
7,500– 8,000	2	15,310
8,000– 8,500	2	16,125
8,500– 9,000	0	0
9,000– 9,500	0	0
9,500–10,000	2	19,647
.....	..	.....
13,500–14,000	1	13,938
.....	..	.....
20,000–20,500	1	20,174
20,500–21,000	1	20,753
.....	..	.....
over 21,000	1	33,392

* Based upon data from "Individual Statements of Condition of National Banks at Close of Business, December 31, 1936" (Table N, issued as supplement to Annual Report of Comptroller of the Currency), Washington, 1937, pp. 121–129. Four banks in Cincinnati, two in Cleveland, three in Columbus, and one in Toledo (the "reserve cities") have been omitted.

[a] Lower limit inclusive. Unit: thousand dollars.

[b] Unit: thousand dollars.

TABLE R

NUMBER OF NATIONAL BANKS IN MICHIGAN (EXCLUDING "RESERVE CITIES") HAVING TOTAL DEPOSITS WITHIN SPECIFIED RANGES ON DECEMBER 31, 1936*

Deposits[a]	Frequency
0– 500	13
500– 1000	22
1000– 1500	9
1500– 2000	4
2000– 2500	6
2500– 3000	4
3000– 3500	3
3500– 4000	2
4000– 4500	0
4500– 5000	0
5000– 5500	3
5500– 6000	2
6000– 7000	3
7000– 8000	2
8000–10000	2
10000–12000	1
12000–14000	2
.....	..
26000–28000	1
Total	79

* Compiled from same source and for same banks as described in Table 61.
[a] Upper limit excluded. Unit: thousand dollars.

TABLE S

NUMBER OF NATIONAL BANKS IN MICHIGAN (EXCLUDING "RESERVE CITIES") HAVING LOANS WITHIN SPECIFIED RANGES ON DECEMBER 31, 1936*

Loans[a]	Frequency	Loans[a]	Frequency
0	12	15	1
1	16	16	1
2	14	17	3
3	4	18	0
4	6	19	1
5	2	20	0
6	3	21	1
7	2	..	..
8	2	26	1
9	2	..	..
10	4	38	1
11	0	..	..
12	0	51	1
13	0	Total	79
14	2		

* Compiled from same source and for same banks as described in Table 61.
[a] Lower limit, inclusive, of class intervals $100,000 wide. Unit: $100,000.

TABLE T

NUMBER OF FEMALE "ROVING-FRAME TENDERS" OVER 16 YEARS OF AGE EMPLOYED IN THE COTTON MILLS OF THE UNITED STATES, RECEIVING SPECIFIED WEEKLY WAGES IN 1890*

Wages[a]	Actual series	Samples				
		(1)	(2)	(3)	(4)	(5)
$2.50	2				1	
3.00	8		1	1	2	
3.50	13	1	1	2	3	3
4.00	9	1	0	0	4	4
4.50	34	2	9	6	11	5
5.00	50	12	18	13	14	14
5.50	52	18	20	12	17	6
6.00	103	27	24	20	19	33
6.50	68	19	16	29	19	16
7.00	31	10	7	10	3	11
7.50	11	5	2	3	1	2
8.00	17	5	2	4	6	6
8.50	1					
9.00	1					
Total	400	100	100	100	100	100
Mean	6.03	6.34	6.00	6.26	5.87	6.19
σ	1.11	.87	.87	.95	1.13	1.02

* Source: "Twelfth Census of the United States, 1900, Special Report on Employees and Wages," Washington, U. S. Census, 1903, p. 31.
[a] Lower limit, inclusive, of class interval. Unit: one dollar.

TABLE U

NUMBER OF NATIONAL BANKS IN MICHIGAN (EXCLUDING "RESERVE CITIES") HAVING UNITED STATES SECURITY HOLDINGS WITHIN SPECIFIED RANGES ON DECEMBER 31, 1936*

Holdings*	Frequency	Holdings*	Frequency
0	20	18	0
1	13	19	0
2	10	20	1
3	5	21	3
4	6	22	1
5	5	23	0
6	1	24	1
7	0	..	..
8	0	27	1
9	2	..	..
10	3	37	1
11	0	..	..
12	1	40	1
13	1	..	..
14	0	60	1
15	0	..	..
16	1	73	1
17	0	Total	79

* Compiled from same source and for same banks as described in Table 61.
* Lower limit, inclusive, of class intervals $100,000 wide. Unit: $100,000.

TABLE V

THE DECILES FOR THE DATA OF TABLES 102 (1) AND 103 (2)*

Decile number	(1)	(2)
1	83	106
2	91	123
3	93	133
4	95	140
5	98	148
6	100	157
7	101	168
8	105	178
9	113	211

* Unit: per cent. Compiled from Tables 102 and 103.

TABLE W

MONTHLY AVERAGE TAX-PAID WITHDRAWALS OF LARGE CIGARS, ANNUALLY, 1919–1937*

Year	Tax-paid withdrawals
1919	589.4
1920	661.4
1921	563.2
1922	574.4
1923	583.2
1924	554.9
1925	541.7
1926	549.1
1927	547.6
1928	537.8
1929	546.0
1930	490.8
1931	443.2
1932	370.2
1933	362.1
1934	383.1
1935	397.0
1936	431.9
1937	443.1

* Unit: million cigars. Source: Annual reports for calendar years of U. S. Treasury, Bureau of Internal Revenue.

TABLE X

PETROLEUM PRODUCED IN THE UNITED STATES, ANNUALLY, 1900–1937*

Year	Production	Year	Production
1900	63.6	1919	378.4
1901	69.4	1920	442.9
1902	88.8	1921	472.2
1903	100.5	1922	557.5
1904	117.1	1923	732.4
1905	134.7	1924	713.9
1906	126.5	1925	763.7
1907	166.1	1926	770.9
1908	178.5	1927	901.1
1909	183.2	1928	901.5
1910	209.6	1929	1007.3
1911	220.4	1930	898.0
1912	222.9	1931	851.1
1913	248.4	1932	785.2
1914	265.8	1933	905.7
1915	281.1	1934	908.1
1916	300.8	1935	996.6
1917	335.3	1936	1099.7
1918	355.9	1937	1277.7

* Unit: million barrels of 42 gallons. Source: "Minerals Yearbook, 1937," Washington, U. S. Department of the Interior, p. 1009, and *Monthly Petroleum Statement No.* 167, U. S. Department of the Interior, February 9, 1938.

TABLE Y

LINK RELATIVES FOR THE RATE OF INTEREST ON CALL LOANS IN THE NEW YORK STOCK EXCHANGE, 1903–1914*

Month	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914
January	84	43	72	52	44	33	62	94	94	60	50	51
February	50	77	97	54	71	38	124	59	72	94	103	75
March	208	97	146	105	146	102	82	104	100	106	127	107
April	70	79	102	195	37	93	105	114	101	124	82	96
May	58	112	74	44	98	97	95	111	100	92	80	97
June	125	73	103	78	135	92	102	76	104	100	82	103
July	82	91	92	91	145	80	110	87	98	105	100	144
August	81	87	89	149	67	87	105	64	98	99	100	236
September	114	170	174	211	131	127	124	129	99	93	129	96
October	116	133	149	55	525	107	160	156	102	203	127	100
November	193	138	145	146	58	122	108	103	117	120	102	90
December	106	112	214	187	119	166	108	105	148	102	123	62

* Unit: 1 per cent. Source: *Review of Economic Statistics*, Prel. Vol. I, 1919, p. 103.

TABLE Z
PRODUCTION OF PORTLAND CEMENT, ANNUALLY, 1910–1926*

Year	Production
1910	76.55
1911	78.53
1912	82.44
1913	92.10
1914	88.23
1915	85.92
1916	91.52
1917	92.81
1918	71.08
1919	80.78
1920	100.02
1921	98.84
1922	114.79
1923	137.46
1924	149.36
1925	161.66
1926	164.53

* Unit: million barrels. Source: "Statistical Abstract of the United States, 1937," Washington, U. S. Department of Commerce, 1938, p. 734.

TABLE AA

DAILY AVERAGE PRODUCTION OF PIG IRON, MONTHLY, 1903–1914*

Month	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914
January	47.509	29.795	57.479	66.739	71.149	33.718	57.986	84.149	56.752	66.384	90.172	60.808
February	49.665	41.668	57.048	68.001	73.039	37.163	60.976	85.616	64.090	72.442	92.369	67.453
March	51.305	46.820	62.460	69.859	71.871	39.619	59.232	84.450	70.039	77.591	89.147	75.739
April	53.614	52.039	64.068	69.107	73.975	38.320	57.963	82.792	68.839	79.181	91.759	75.665
May	55.278	49.580	63.346	67.701	74.049	37.603	60.753	77.103	61.079	81.051	91.039	67.506
June	55.774	43.191	59.773	65.891	74.486	36.404	64.362	75.516	59.586	81.358	87.619	63.926
July	49.877	36.155	56.191	64.948	72.763	39.294	67.853	69.305	57.839	77.771	82.601	63.150
August	50.681	37.830	59.473	62.153	72.594	43.865	72.645	67.963	62.150	81.046	82.121	64.363
September	51.791	45.261	63.317	65.699	72.783	47.300	79.507	68.539	65.903	82.128	83.531	62.753
October	45.989	46.944	66.231	70.865	75.386	50.555	83.856	67.520	67.811	86.772	82.139	57.361
November	34.654	49.554	67.121	72.922	60.939	52.595	84.917	63.659	66.648	87.695	74.453	50.611
December	27.313	52.129	65.991	72.107	39.815	56.158	85.022	57.349	65.912	89.766	63.987	48.896

* Unit: thousand long tons. Source: *Iron Age*, January 3, 1935, p. 276.

TABLE BB

DAILY AVERAGE PRODUCTION OF PIG IRON, MONTHLY, 1919–1924*

Month	1919	1920	1921	1922	1923	1924
January	106.52	97.26	77.94	53.06	104.18	97.38
February	105.01	102.72	69.19	58.21	106.94	106.03
March	99.68	108.90	51.47	65.68	113.67	111.81
April	82.61	91.33	39.77	69.07	118.32	107.78
May	68.00	96.31	39.39	74.41	124.76	84.36
June	70.50	101.45	35.49	78.70	122.55	67.54
July	78.34	98.93	27.89	77.59	118.66	57.58
August	88.50	101.53	30.78	58.59	111.27	60.88
September	82.93	104.31	32.85	67.79	104.18	68.44
October	60.12	106.21	40.22	85.09	101.59	79.91
November	79.74	97.83	47.18	94.99	96.48	83.66
December	84.94	87.22	53.20	99.58	94.22	95.54

* Unit: thousand long tons. Source: *Iron Age*, January 3, 1935, p. 276.

TABLE CC

DAILY AVERAGE PRODUCTION OF BOOTS, SHOES, AND SLIPPERS, MONTHLY*

Month	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938
January	1.092	1.282	1.104	1.109	1.061	1.086	1.140	1.135	1.105	0.846	0.944	0.988	1.085	1.232	1.390	1.651	1.110
February	1.116	1.378	1.167	1.202	1.168	1.241	1.288	1.260	1.117	1.089	1.129	1.199	1.389	1.405	1.469	1.800	1.353
March	1.174	1.463	1.228	1.245	1.198	1.251	1.318	1.315	1.218	1.223	1.227	1.143	1.451	1.457	1.451	1.845	
April	1.193	1.386	1.167	1.228	1.110	1.208	1.158	1.224	1.208	1.246	1.104	1.228	1.497	1.440	1.392	1.679	
May	1.093	1.289	1.074	1.092	1.028	1.114	1.101	1.215	1.043	1.237	0.978	1.374	1.419	1.302	1.316	1.574	
June	1.035	1.203	0.977	0.977	1.043	1.146	1.161	1.250	1.039	1.160	0.981	1.452	1.214	1.210	1.224	1.435	
July	1.008	1.098	0.892	1.011	1.066	1.234	1.223	1.259	1.005	1.168	0.908	1.500	1.234	1.345	1.456	1.483	
August	1.107	1.202	1.083	1.212	1.235	1.402	1.399	1.488	1.210	1.424	1.231	1.481	1.425	1.520	1.731	1.611	
September	1.257	1.282	1.206	1.294	1.377	1.475	1.441	1.583	1.275	1.361	1.473	1.389	1.311	1.543	1.782	1.480	
October	1.266	1.228	1.233	1.267	1.348	1.373	1.336	1.488	1.109	1.036	1.407	1.310	1.258	1.438	1.629	1.238	
November	1.308	1.172	1.178	1.120	1.163	1.129	1.150	1.232	0.862	0.842	1.093	1.030	1.037	1.232	1.380	0.926	
December	1.238	1.007	1.025	1.017	1.038	1.001	0.974	0.977	0.731	0.815	0.856	0.893	1.031	1.259	1.391	0.859	

* Unit: million pairs. Daily average production has been computed from monthly totals by dividing each monthly total by the number of working days in each month. The number of working days in each month has been obtained by making the following allowances for Sundays, Saturdays, and holidays: Sunday, 1 day; Saturday, ½ day; January 1, 1 day; May 30, 1 day; July 4, 1 day; Labor Day, 1 day; Thanksgiving, 1 day; December 25, 1 day. (If holiday falls on Sunday, observance is assumed to come on the following Monday; if holiday falls on Saturday, the holiday allowance plus the Saturday allowance is taken as 1 day.) Source: Monthly reports of the U. S. Department of Commerce, Bureau of the Census. (Rubber-soled footwear is excluded.)

TABLE DD

CYCLES OF STOCK PRICES*

Month	1919	1920	1921	1922	1923	1924
January	0.38	1.44	−0.69	−0.68	−0.12	−0.47
February	0.38	0.78	−0.70	−0.53	0.05	−0.43
March	0.70	1.06	−0.72	−0.38	0.15	−0.58
April	0.90	1.10	−0.68	−0.18	−0.02	−0.81
May	1.40	0.50	−0.67	−0.11	−0.31	−0.88
June	1.76	0.46	−1.15	−0.14	−0.46	−0.82
July	2.01	0.38	−1.20	−0.08	−0.70	−0.58
August	1.50	0.04	−1.32	0.05	−0.65	−0.39
September	1.78	0.11	−1.16	0.10	−0.70	−0.44
October	2.14	−0.04	−1.12	0.10	−0.84	−0.50
November	1.90	−0.44	−0.89	−0.16	−0.72	−0.25
December	1.54	−0.85	−0.70	−0.10	−0.60	−0.02

* Unit: one standard deviation. Source: *Review of Economic Statistics*, May 15, 1932, p. 82.

TABLE EE

HARVARD INDEX OF GENERAL ECONOMIC CONDITIONS*

Month	1925			1926			1927		
	A	B	C	A	B	C	A	B	C
January	0.00	0.47	−1.15	1.05	0.99	−0.20	1.05	0.84	−0.40
February	0.15	0.55	−1.13	1.11	0.88	−0.35	1.04	0.80	−0.59
March	0.17	0.67	−0.84	0.90	0.78	−0.39	1.31	0.79	−0.66
April	−0.21	0.46	−0.90	0.33	0.73	−0.68	1.25	0.95	−0.64
May	−0.12	0.52	−0.95	0.52	0.78	−0.70	1.32	0.88	−0.49
June	0.19	0.66	−0.89	0.55	0.81	−0.72	1.56	0.88	−0.42
July	0.27	0.70	−0.85	0.81	1.04	−0.61	1.39	0.95	−0.46
August	0.41	0.78	−0.69	1.00	0.83	−0.39	1.94	0.90	−0.78
September	0.43	0.81	−0.52	1.11	0.68	−0.26	2.07	0.99	−0.92
October	0.53	0.78	−0.30	1.13	0.70	−0.20	2.42	0.98	−0.90
November	1.01	0.96	−0.24	0.89	0.45	−0.33	2.06	0.81	−0.88
December	0.90	0.80	−0.18	1.07	0.53	−0.33	2.55	0.83	−0.89

* Unit: one standard deviation. Source: files of *Review of Economic Statistics*, by permission. (A, speculation; B, business; C, money.)

TABLE EE (*Continued*)

Month	1928			1929			1930		
	A	B	C	A	B	C	A	B	C
January	2.67	0.99	−0.67	4.41	1.52	1.89	0.74	0.67	+0.06
February	2.51	0.86	−0.58	4.21	1.61	1.89	1.11	0.62	−0.11
March	2.31	0.95	−0.58	4.11	1.73	2.04	1.28	0.67	−0.73
April	2.96	1.28	−0.29	3.70	1.59	2.49	1.83	0.67	−0.90
May	3.37	1.38	0.10	3.84	1.52	2.77	1.59	0.68	−1.11
June	3.38	1.45	0.61	3.17	1.70	2.63	1.40	0.79	−1.47
July	2.82	1.20	0.80	3.88	2.31	2.39	0.13	0.15	−1.81
August	2.89	0.98	1.02	4.25	2.13	2.73	0.25	−0.05	−2.00
September	3.37	1.28	1.23	4.89	2.13	2.70	0.24	−0.22	−2.09
October	3.63	1.21	1.30	4.10	1.96	2.04	−0.51	−0.50	−2.18
November	3.69	1.32	1.30	1.67	1.71	0.69	−1.04	−0.68	−2.37
December	4.39	1.54	1.64	0.68	0.75	0.07	−1.21	−0.72	−2.26

TABLE EE (*Continued*)

Month	1931			1932			1933		
	A	B	C	A	B	C	A	B	C
January	−1.65	−0.95	−2.46	−3.99	−2.67	−1.08	−4.37	−3.98	−4.08
February	−1.35	−1.15	−2.65	−4.02	−3.04	−1.17	−4.31	−4.11	−3.95
March	−0.83	−1.22	−2.64	−3.90	−3.42	−1.58	−4.65	.....	−1.96
April	−1.22	−1.12	−2.75	−4.20	−3.08	−2.04	−4.62	−4.29	−3.14
May	−1.73	−1.14	−2.96	−4.64	−3.38	−2.59	−3.91	−4.08	−3.50
June	−2.35	−1.28	−3.22	−5.05	−3.55	−2.84	−3.33	−3.68	−3.67
July	−1.85	−1.60	−3.20	−5.09	−3.58	−3.06	−2.90	−3.28	−3.61
August	−2.15	−1.85	−3.23	−4.60	−3.80	−3.26	−3.26	−3.72	−3.67
September	−2.18	−1.98	−3.12	−3.86	−3.93	−3.37	−2.89	−3.89	−4.06
October	−3.42	−2.29	−1.87	−3.97	−4.04	−3.71	−3.31	−3.95	−4.07
November	−3.23	−2.68	−1.26	−4.30	−4.21	−3.92	−3.57	−3.94	−3.99
December	−3.54	−2.43	−1.34	−4.42	−3.98	−4.05	−3.33	−3.91	−3.82

TABLE EE (*Continued*)

Month	1934			1935			1936		
	A	B	C	A	B	C	A	B	C
January	−3.27	−3.83	−3.81	−3.21	−3.40	−4.13	−1.93	−2.74	−4.13
February	−2.83	−3.53	−3.88	−3.30	−3.13	−4.14	−1.61	−2.75	−4.14
March	−2.90	−3.56	−4.03	−3.37	−3.02	−4.16	−1.51	−2.65	−4.16
April	−2.89	−3.33	−4.03	−3.51	−2.93	−4.34	−1.51	−2.52	−4.16
May	−2.93	−3.33	−4.02	−3.23	−2.80	−4.51	−1.92	−2.38	−4.04
June	−3.19	−3.21	−4.13	−3.14	−2.75	−4.53	−1.71	−2.11	−4.00
July	−3.13	−3.23	−4.14	−2.97	−2.70	−4.53	−1.61	−2.02	−4.00
August	−3.51	−3.34	−4.15	−2.71	−2.72	−4.55	−1.31	−2.07	−4.02
September	−3.37	−3.38	−4.17	−2.62	−2.80	−4.55	−1.27	−2.22	−4.04
October	−3.40	−3.52	−4.17	−2.55	−2.95	−4.52	−1.23	−2.23	−4.04
November	−3.45	−3.40	−4.16	−2.29	−2.71	−4.16	−0.90	−1.96	−4.03
December	−3.21	−3.32	−4.16	−2.10	−2.60	−4.16	−0.78	−1.42	−4.03

TABLE EE (*Continued*)

Month	1937			1938		
	A	B	C	A	B	C
January	−0.81	−1.92	−4.00	−3.03	−2.53	−3.80
February	−0.64	−1.82	−4.01	−3.03	−2.78	−3.81
March	−0.61	−1.70	−3.99	−2.85	−2.72	−3.83
April	−0.65	−1.67	−3.84	−3.72		
May	−1.11	−1.54	−3.79			
June	−1.17	−1.55	−3.79			
July	−1.44	−1.55	−3.80			
August	−1.03	−1.54	−3.82			
September	−1.31	−1.77	−3.85			
October	−2.04	−1.93	−3.85			
November	−2.48	−2.33	−3.84			
December	−2.85	−2.01	−3.83			

APPENDIX B

LABORATORY PROCEDURE[1]

There are certain general desiderata for all laboratory work in statistics, and chief of these are: uniformity and adequacy of plan, neatness and accuracy in the operation, accessibility and adaptability in the results. The routine worker who fails to govern his operations by a complete plan of procedure, which conforms to some uniform standard for all operations of similar type, runs a serious risk of wasting valuable time and energy and of obtaining results which are of doubtful reliability. The final test of all detailed statistical work is accuracy—not necessarily perfect, but sufficiently close to accuracy for the purpose in hand. Careful planning of the work facilitates accurate operations, and a habit of neatness serves effectively to reduce the number of errors. The only satisfactory general safeguard against error is found in the duplicate performance of the same operation, independently, by two individuals. Comparison of the results, if it shows verification, establishes a very high probability of accuracy. The results of laboratory operations should be in such form that they can readily be found and utilized for interpretation or further analysis. The accomplishing of this end implies a foresight in planning and a thoroughness in operation which are developed only by experience but should be sought from the first.

TRANSCRIPTION

Certain general specifications can be applied to the transcription of series to be used in a statistical study. Of the three main types of series—categorical, time, and frequency—most of the published data upon which computations will be based for the analysis of an economic subject will fall within the class of time series. Undoubtedly the student will at times need to transcribe categorical series, especially if he is interested in conditions of industries or trade at a given time. However, there will be little opportunity actually to transcribe frequency series, as few series

[1] This Appendix was prepared largely, for the original edition, by Ester I. Fismer and Amelia E. Ohse, formerly of the staff of the Harvard University Committee on Economic Research.

of published economic data fall within that class, but rather need to be compiled by selection from the categorical and time series at hand. As the greatest amount of necessary data for a statistical problem falls within the class of time series, methods for transcribing these series will be described in some detail.

Transcriptions—written copies—of the figures to be used as original items in a statistical problem are almost essential. Cases arise when it seems convenient to use material directly from the source at hand without transcribing, but usually this practice is undesirable. It is most important that the original data upon which the computations, tables, and charts of a study are based be preserved; and for this reason the material should always be transcribed so that it may readily be filed with the study, for easy reference. When transcribed, the data can be arranged uniformly and thus are more convenient for use in interpretation and comparison, in drawing up tables and charts, and in computing. Another important reason for transcribing original data is that when later figures appear revising the data originally used, corrections can be entered immediately upon the original copied sheets, and these then become a record of the actual changes made in the series. These changes will then need to be carried over into any corresponding computations, tables, or charts already completed in the study.

Before transcribing data the student must be certain he has secured complete representative series from unquestionable sources, using, whenever possible, a primary source. Secondary sources containing data already tabulated in regular form are usually more easily available, but a careful selection, after research, must be made in order to choose the most reliable secondary sources. A trade journal, or publication specializing in the data desired, is often a practical solution of this problem. For example, sugar data are often taken from *The Sugar Trade Journal*, iron and steel figures from *Iron Age*, and leather prices from *Hide and Leather*.

In order to plan and arrange transcription sheets in the most convenient style, the use to be made of the data should be decided before drawing up the sheets. The form chosen depends on whether the material is to be used for computations, tables, charts, or merely for comparison and observation in deriving conclusions. When planning the sheets, the student should keep in mind any possible future use of the same data either by himself or by someone else, not only in connection with his present study, but for

other and various purposes. Therefore, the transcriber should arrange the sheets so they may easily be understood by others and so they may readily be filed in a general file. For instance, when transcribing time series of monthly production of pig iron and steel for a given number of years, it is best to draw up a separate sheet for pig iron and another for steel, so that later one series may be filed under pig iron and the other under steel.

In the case of monthly time series for a given period, such as January, 1919, to December, 1937, it is a good practice to transcribe each complete series for the specified number of years on a single sheet; but, if similar weekly series are desired, only one year for each series should appear on a sheet, as in the second illustrative table below. If, however, the material is wanted for direct comparison, two or more series may be transcribed on a single sheet. In the case of monthly series, it will then be best to limit the material to one year on a sheet, showing each individual series for the twelve months; in the case of weekly series a quarter of a year on a page is a convenient arrangement, each page showing the various series to be compared during the weeks covering three months in a year. If an index of various items is to be computed, the several series entering the index may, for the sake of convenience, be transcribed on the same sheet for comparison when finally combined in the index.

On the actual process of transcribing data the same general principles apply for all types of series. To prevent discrepancies the original data should be copied by hand, in clearly legible characters. Ink should be used, to insure neatness and to aid in preservation of data. The general tendency, when first transcribing, is to crowd many items on a single page. This frequently causes error, not only in the process of transcribing, but also by producing a confused mass of data upon which to base computations, in the subsequent steps of computing. To avoid crowding, the transcriber should spread his work out, allowing all the space necessary for average-size figures, with ample room for corrections and changes of each item. A legal-size paper, 8½ by 14 inches, most conveniently meets all the necessary requirements of transcription sheets. The following arrangements can well be observed. Space sufficient for title and captions should be allowed at the top of the sheet: for title, at least two inches vertically; for captions, not less than one vertical inch. Lines should be ruled on both sides of each column, allowing about one inch horizontally for a column of items, and more for the stubs if necessary. Not more

than twenty rows of items should be transcribed on a sheet; this spacing allows for only one revision in each space, and more space should be left if possible. At least two vertical inches should be reserved at the bottom of each sheet for footnotes. If the quality of the paper used is poor and the sheet is likely to have considerable use, leave about ½-inch margin at right and left of sheet so that the transcribed figures will not become damaged.

When the data to be transcribed are to be drawn up into tables directly for presentation in a study, or for publication, the above

COTTON CONSUMED

RAW COTTON (EXCLUDING LINTERS) CONSUMED IN THE UNITED STATES*

Month	1919	1920	1921	1922	1923	1924*
January	556.9	591.9	366.5	526.7	610.3	578.5
February	433.3	515.7	395.1	472.3	566.8	507.9
March	433.5	575.8	438.2	519.8	624.3	483.9
April	475.9	566.9	409.2	443.5	576.5	480.0
May	487.9	541.4	440.7	495.3	620.9	413.6
June	474.3	555.2	461.9	509.2	542.0	350.3
July	510.3	525.5	410.1	458.0	462.7	346.7
August	497.3	483.6	467.1	526.4	492.5*	357.5
September	491.1	458.0	484.7	494.0	485.7*	435.2
October	556.0	401.3	494.3	533.7	543.3*	532.6
November	491.3	332.7	527.9	579.2	532.7*	492.2
December	511.7	295.3	510.9	529.3	463.8*	532.0

Unit: thousand bales. Data are for entire month. Source: Original data compiled by U. S. Department of Commerce, Bureau of Census. Transcribed data from "Cotton Production and Distribution" Bulletins, Department of Commerce. January, 1919–July, 1919, data from Bulletin No. 150, p. 49. August, 1919–July, 1923, data from Bulletin No. 153, p. 52. Later data from monthly preliminary report postcards on cotton consumed, sent by Department of Commerce, Bureau of Census.

* Preliminary data to be revised when new issues of *Cotton Bulletin* are available which contain revised figures beginning in August and ending in July of the following year.

suggestions for spacings may be disregarded, and smaller spaces used. Cross-section paper will be found most convenient for this purpose, as it saves the transcriber considerable labor in drawing lines and yet produces sheets which appear clear and concise, and are properly aligned.

These principles apply to categorical series as well as time series; but in the case of monthly time series only twelve or fourteen rows of items will be needed on a sheet, to provide space for each month in the year and for the yearly total and a monthly

*Note that the form is that recommended for transcription sheets but the published table appears on a smaller scale due to the size of the printed sheet: ordinarily legal size paper should be used for transcriptions.

average if desired. In the case of weekly time series, it will be necessary to carry eight columns on a sheet in order to transcribe the data conveniently. One year appears on a single page; and there are two columns for each quarter, one for the stub, containing

AVERAGE PRICE OF COTTON*

RAW COTTON—FAIRCHILD'S AVERAGE PRICE OF NEW YORK SPOT COTTON

1924							
Jan. 5	35.65	Apr. 5	29.50	July 5	30.18	Oct. 4	26.23
12	35.00	12	31.16	12	30.26	11	25.30
19	33.64	19	30.61	19	31.84	18	23.53
26	33.48	26	29.87	26	34.60	25	23.63
Feb. 2	33.81	May 3	29.98	Aug. 2	32.24	Nov. 1	23.99
Feb. 9	34.10	May 10	30.70	Aug. 9	30.88	Nov. 8	23.83
16	32.31	17	31.59	16	29.90	15	24.68
23	30.32	24	32.23	23	27.92	22	24.26
Mar. 1	29.23	31	32.67	30	26.60	29	24.12
Mar. 8	28.48	June 7	30.71	Sept. 6	25.72	Dec. 6	23.30
15	28.79	14	29.46	13	23.93	13	23.43
22	29.00	21	29.69	20	22.53	20	24.01
29	27.27	28	29.90	27	24.46	27	24.22
						Jan. 3	24.58

Unit: cents per pound. (Average of daily prices for week ending Saturday.) Source: Original data compiled by *Fairchild's News Service*. Transcribed data from: *Daily News Records* in issue of Tuesday each week appearing under table "Fairchild Cotton—Average Price and Index Number." Data revised from the annual table in *Daily News Record*, January 20, 1925.

the month and date of week, and the other for the items. Fifteen rows of items are needed for transcribing weekly time series by this method, allowing for the possible fifth week in each month of the quarter.

After the transcription sheets have been planned and prepared, the actual transcribing may be started. In the space allowed at

*Note that the form is that recommended for transcription sheets but the published table appears on a smaller scale due to the size of the printed sheet: ordinarily legal size paper should be used for transcriptions.

the top of the sheet, the title of the series, giving the description of the series in general, should be written in capital letters; and directly under the general title, the full description of the series should appear. Particular series require certain additional elements in the title. For example, in a price series the market, such as Chicago, New York, Philadelphia, and the condition of sale—that is, whether the price is spot or contract, f.o.b mill, or delivered—should be stated in the title.

Units in which the data are transcribed should also be given in a note attached to the title. The determination of the unit to be used is, of course, a step in the planning which precedes actual transcription, and depends upon the use to which the data will be put. The following suggestions may prove of assistance in selecting a unit for transcriptions of various series. In most data it is, of course, not necessary to transcribe figures to as many places as given in the actual source. If the data to be transcribed are for use in computations only, it is usually necessary to carry value and quantity figures to only three or four digits, but if desired for tables of original items and intended for comparison later in the study, it is often advisable to carry some figures to more than four digits. The reason for this is that often the number of digits in the separate items of a series will vary when they are expressed in a single unit: the larger numbers must necessarily be carried out to extra digits so that the small items may not appear too insignificant. In general, the most useful units are those of thousands and millions, which omit three digits or six, according to the size of the items in a series. Units of hundreds, ten thousands, and hundred thousands are confusing, and error is often avoided by selecting units of thousands and millions and carrying the same desired number of digits by the use of a decimal point. For example, if the production of crude petroleum in the United States for a stated period is 56,617,000 barrels, the figure can be handled more conveniently as 56.62 million barrels than by trying to use a unit of ten thousand; and, likewise, 768,903 tons of cottonseed crushed in November, 1924, is better carried as 768.9 thousand tons, than as 7,689 hundred tons, which results in the same number of digits.

The transcriber, no matter how large the actual figures are, should never under any condition use commas. If figures of many digits are to be taken off, a small space allowed between thousands and millions, as 7 835 256, is a far more satisfactory practice.

Moreover, each item should be copied, and no ditto marks should be used.

It is often advisable, when transcribing price data, to transcribe the data as actually given in the source on hand, not omitting any digits, in order to show actual price changes from time to time. For example, iron bars may appear as $0.02875 per pound in one week and $0.02865 in the next. Slight changes in a price series may be important in a study of short-time movements, and for this reason enough digits to be sensitive to small price fluctuations should be retained. When stating the unit used in a series of prices, the value per physical unit should be mentioned on the transcription sheet, as in the case of iron bars, dollars per 100 pounds, or cents per pound.

In time series, the date or interval to which the item applies should also be stated. In monthly series it is necessary to know if transcribed series are items for the entire month, first of month, end of month, or the average for the month. In weekly series it must be stated whether transcribed items are aggregates for the week, or figures as of one day in each week, or an item for the entire week ending on a particular date indicated. For example, pig iron prices in the *Iron Age* are prices as of Tuesday each week, data on car loadings, issued by the American Railway Association, are totals for the entire week ending Friday, and Fairchild's raw cotton price is a weekly average of daily prices. These descriptions of time in monthly and weekly time series are particularly helpful when using data in computing and charting, and should be clearly stated in the title, or in a note to the title, where they will not be overlooked.

Footnotes are an essential part of transcription sheets and should contain all important facts concerning the data. In the footnotes should be given the source of the data, for a record is needed of the actual origin of the transcribed material; and it should be indicated whether the source is primary or secondary. If a secondary source is used, the title, date, volume, and page of the publication should be given. Also, if more than one source is used for the data on a single sheet, the columns of items or exceptional figures should be marked accordingly, so that each item may easily be located when a check on the figures is desired. Any peculiarities in the series, such as a change in description or methods of compilation, or change in number of items included in original reporting of figures, should be noted carefully, since some

adjustment will no doubt be necessary in computations based upon such data.

The actual transcribing of data will be simple and straightforward, involving no complications, if the data are to be taken from a secondary source, where all the desired information appears in one issue of the publication, since the student can see at a glance the material desired. The work will be difficult if the items are to be taken from several issues of the same publication, and even more difficult when they are to be compiled from several sources. When transcribing series from several issues of the same publication, if no overlapping figures are given, it is necessary in each case, before taking a figure, to examine the data carefully in order to be sure the series is homogeneous. For most secondary sources, however, by selecting one item at a time from several issues of the same publication, the transcriber can secure overlapping figures; and, although it is not certain even then that the data are homogeneous, it is generally safe enough to assume homogeneity if the figures check. When overlapping figures are given, the student must use the latest publication first: that is, when a series is to be selected by following through each issue of the publication, if January to December, 1924, data are desired, he should start with the latest publication containing December data and work back to January, 1924. Each issue must be examined when working backward, from the latest to the earliest, even if considerable overlap is given in each issue; for, unless the transcriber systematically examines each issue for changes, a revision of a single item appearing in a later issue may easily be overlooked.

If the data to be transcribed are fairly old—prewar, for instance—it is easy to secure the last revised data at the present time with the assurance they will not again be changed. But if the data desired are current, and the series is one originally compiled some time ago and still being carried in the same form in a publication, the data are likely to appear revised for two or more years at a time, and need to be watched continually for revisions.

The student may be anxious to bring his study up to date before the final data appear in the publication used. Frequently, series may be completed by data appearing in press releases, newspaper clippings, typed or mimeographed preliminary reports. In this form they may readily be used, but the figures taken from such sources should be properly footnoted and marked "preliminary" (*p*). Great care should be taken to revise these figures

as soon as the usual bulletin or publication from which the series is regularly taken becomes available. The student should remember, too, that weekly data may later be revised in monthly issues or annual summaries, and monthly data from monthly publications may later be revised in annual reports issued by the same source. Care should be taken to examine recent data from time to time, and in some way to note on the sheets when the final source has been used.

In transcribing weekly series it is often difficult to specify an item for a week which apparently includes data belonging to either of two months. This problem of allocation of weeks does not arise if the weekly series is one pertaining to only one day in each week, since that item without question falls within the stated month. A figure which is an aggregate or total for a week should be transcribed as belonging to the month in which it has the greater number of days of the week.

When transcriptions are finally completed they should have, if possible, an independent check by someone other than the transcriber. Each item and footnote should be verified carefully before being used for computing or for any other purpose. When error is found or change is necessary, no erasures should be made but the erroneous figure should be crossed out neatly and the correct figure entered directly above the old. Whether revisions are due to error or simply to a revision in the data appearing in a later publication, if computations and charts based upon the data have already been completed, the student should trace through each operation affected, making the necessary corrections. For this reason, replacing figures by erasure is especially dangerous, since it cannot then plainly be seen where such changes occur. Colored ink is useful in making revisions, for the corrections then stand out clearly on the sheet and cannot easily be overlooked.

The student will be able to detect certain discrepancies in his own transcribed data by making a simple audit of the figures when transcribed. After careful examination, some errors will appear quite evident. For example, an item of a series may clearly be out of proportion with the other items (for instance, pig iron, appearing with a monthly production approximating 1,900 long tons, may suddenly appear as 900 long tons). This item needs examination until the transcriber is satisfied that cause exists for such an irregular shift. The explanations for some of the peculiarities in a series are often given in the text accompanying the figures in the source, but if no real reason can be found, the cause

may often quite clearly be due to typographical error. In such cases, the data should be verified elsewhere, either by writing directly to the compilers and questioning the figures, or by referring, whenever possible, to another reliable source.

Many of the foregoing points are illustrated in the two tables shown above (pages 394 and 395).

COMPUTATION

Written instructions should form a part of every computing job: they not only facilitate the planning of the work but are valuable for future reference, because they serve as a record of what has been done. These instructions should give specific directions as to the data to be used, the precise methods to be applied, and the results desired. For example, if a series is to be corrected for trend and seasonal variation and then reduced to cycle form, the instructions should state the type of trend line to be fitted and the period on which the trend should be based; the exact period for which the link relatives are to be computed, and the period on which the standard deviation is to be based. In general, when time series are used, it is necessary that the instructions specify the period of time to which the figures pertain. In fact, any information the computer will need in connection with the computation should be stated clearly. If the instructions have been set down by someone else, it is very important that the computer study and analyze carefully all the directions before he attempts to draw up the computation sheets. If this is done, he will have the entire job clearly in mind and can save himself hours of energy wasted in preparing inappropriate sheets.

The principles involved in the drawing up of computation sheets are very similar to those mentioned in the section pertaining to the drawing up of transcription sheets; but as they are very important, they may bear some repetition. Of these principles, neatness is the most important because it tends to ensure accuracy; and if all work is not done neatly, the result will be poorly arranged sheets, which bring about confusion and cause error. All computations which are to be used a great deal should be done in ink, to facilitate preservation. As in transcribing, all figures should be written legibly. Crowding should be avoided; ample space should be allowed for possible corrections and revisions.

Another detail to which the computer must give attention is the size of computation sheets. These sheets should be of uniform size, preferably 8½ inches by 14 inches, and should allow ample

space for headings, footnotes, and wide columns for computations. Small pieces of paper are unsatisfactory, except for scratch work to be thrown away, because they naturally become crowded, are harder to handle, and are easily lost.

No general rule can be given for the inclusion of specific operations upon specific sheets in computation. Sometimes it is desirable to arrange all of one step on a single sheet, as in computing the link relatives for a given monthly series, or in deriving the averages or aggregates of each of several series. Sometimes it is preferable to arrange several steps, in the analysis of one series or one section of a series, on a single page. For example, if a series is to be corrected for trend and seasonal variation, there should be a sheet for each year. Each sheet would then have a column for original items, one for ordinates of trend, one for ratios of original items to ordinates of trend, one for seasonal indexes, one for percentage deviations, and one for the cycles (or adjusted relatives, if preferred). This method of arranging several steps on one sheet is also desirable when an average or index is computed from the items of several series. In such instances there would be a column for each series, one for the total—whether weighted or simple—of all the series, and one for the average or index. In the analysis of frequency series, it is seldom feasible to place more than one series on a sheet, but often several operations can be applied to a single series.

On the drawing up of computation sheets a final principle to observe is that each sheet used in connection with a computation job should bear a title giving a concise but complete description of the figures appearing on the sheet, a statement of the unit in a conspicuous place, and a statement of the period of time to which the series pertain in case of time series; and the sheet should carry such footnotes as are essential. Essential footnotes include such matters as: change in description; break in series; omission of data; periods on which trend, seasonal indexes, and standard deviation are based; and similar information. In fact, any footnotes which appear on the transcription sheets and affect the original items in any way should appear again on the computation sheets. Although footnotes are important, it is not necessary to enter more than once those which are obviously repeated. It is generally sufficient that one footnote referring to several years be given in detail but once, on the first sheet, or that a note affecting each of several series be stated in full but once. For any succeeding use of the same footnote, the computer may simply refer to it.

After the instructions are clearly in mind and the computation sheets drawn up, actual computing can be started. Accuracy is most essential, and in order to avoid discrepancies, every precaution should be taken. To detect errors, two independent computations should be made. This double system may be carried out by having the two computations done on different-colored sheets, one color being considered the first copy throughout the successive steps, and the other color the second copy; or, the two copies may be done on the same-colored sheets and designated as first and second copies throughout. In cases where there is only one person to do the work, two independent computations may be secured by employing different methods. For example, after one has secured annual averages by dividing the totals of 12 monthly figures by 12, an independent check would consist in multiplying these averages by 12, and then checking the products against the yearly totals.

It is not sufficient that there be two independent computations; these computations should be kept separate throughout successive steps. That is, the person who is working on the first copy should take his data for earlier steps from the first copy and not refer to the second copy, whereas the person working on the second copy should take his figures for earlier steps from the second copy and not refer to the first copy. There are, however, exceptions to this rule, arising ordinarily in cases where a particular step is quite distinct from other operations. For example, in the computation of seasonal indexes, it is the general practice to use what is considered the first copy of link relatives in computing both copies of the medians, and also to use the medians from the first copy in getting the final indexes. The usual reason for making this exception is to prevent cumulative errors.

After the two computations have been completed, they should be carefully compared with each other. The degree of error to be allowed will depend upon the mechanical devices used in computing. For example, if ratios are read to the nearest percentage on a 10-inch slide rule, a discrepancy of 1 per cent should be allowed; whereas, if the ratios are read to tenths on a 20-inch slide rule, a discrepancy of not more than $\frac{2}{10}$ of 1 per cent should be allowed. Any discrepancies falling outside the recognized limits should be noted and corrected. All corrections should be made by drawing a neat line through the incorrect figures and writing the correct figures above—never by erasing, writing over, or altering the figures in any way. In all computation work, it is a good practice to check one step before doing successive steps, in order to avoid

the necessity of carrying possible corrections through later steps already computed.

The decision as to the number of decimal places to carry in intermediate steps cannot be made by a few general rules. The degree of accuracy in the given data sets a maximum upon the accuracy of operations applied to such data. On the other hand, the degree of accuracy desired in the results serves to specify a minimum of accuracy for the steps in computing. Within these limits, the computer must make his decision; and experience will be his chief guide. Examination of the problem will often afford considerable help in deciding how precisely to handle the arithmetic. If the operation to be applied is simple and limited to one or two steps, the answer is usually easy to find. The fundamental algebraic operations have certain calculable effects upon the errors inherent in the data, and allowance can easily be made in these cases.

When several steps are comprised in the operation, it is difficult to assess the manner in which error will be transmitted. An actual test may need to be made, in order to discover how great an allowance for error will be necessary. Frequent use of a particular process of computation may lead to fairly well defined rules for the limitation of error to a specified degree. For example, in computing original items and ordinates of trend in a time series, three significant digits, when the first digit is two or more, are sufficient, and four digits when the first digit is one. For relatives, percentage deviations, and seasonal indexes, it is generally sufficient to carry figures to the nearest per cent only. In cases where the variations are very small, however, one decimal place would be necessary. When deriving the standard deviation of cyclical movements, the percentage deviations should be read to the nearest per cent before squaring, unless they are very small, and then it is essential to take them to tenths before squaring. It is sufficient to carry cycles to one decimal place.

There are a few miscellaneous matters, such as revising data, marking estimated figures, care to use figures from original sheets, and the grouping of overlapping weeks into months, which should be considered. In revising data, the computer should use colored ink, distinct from that used in the original computation, in order to make the revisions stand out clearly. He should also make certain that these revisions have been carried through every necessary step of the computation. If the revision affects the first step but does not alter the succeeding steps, the latter should be marked with a small (*r*) tc show that the revision has been noted.

It is also important to mark all estimated figures. This should be done in cases where the computation is based on data which are not complete for the entire period in question. For example, if a monthly total for car loadings is desired and the data for the entire month are not available at the time of computation, the figure obtained by adding an estimated figure for the missing days to that already available is only a preliminary figure and should, therefore, be marked "estimated." A simple plan is to mark all estimated figures with a small (*e*). Figures should be taken from original sheets. The computer should always refer to original transcriptions and computations rather than to copies.

Another problem which frequently arises in computing work is the method of grouping into months overlapping weekly figures which pertain to an entire week. There are two methods of allocating the overlapping week; one is to assign the entire overlapping week to either the preceding or the following month, and the other is to divide the overlapping week between the two months according to the number of its calendar days falling in each month. The first method is generally used in computing monthly averages of weekly figures, while the second method is more frequently used in securing monthly totals. If the overlapping week is to be allocated according to the first method, the practice generally followed is to assign the week in question to the month in which it has more days. In case the overlapping week has as many days in the preceding month as in the following month, it does not matter in which month it is grouped, provided the same practice is followed throughout the series. When overlapping weeks are allocated by dividing them between two months, holidays should be taken into consideration. For example, if the weekly figure for bank debits pertains to the week ending January 3, this overlapping week should be allocated by distributing three-fifths of the figure to December and two-fifths to January; that is, January 1, being a holiday, must be taken out before allocating.

As an aid in statistical work, the computer will find several mechanical devices will prove helpful. For ordinary multiplying and dividing, when one does not require greater accuracy than three digits, a 10-inch slide rule will be found sufficient. If, however, a degree of accuracy over three digits is desired, it becomes necessary to resort to a 20-inch slide rule for four-digit results, and a cylindrical rule, such as the Thacher, for five-digit results. An adding machine becomes almost essential if there is much adding or subtracting to be done, and a machine which lists is desirable.

Listing is an important function, because it makes possible the preservation of a record of the adding-machine work. Many special time-saving schemes can be worked out for such a machine. For example, in computing ordinates of trend, the computer can set down the first ordinate, clear the machine, push down the repeat key, and then set down the monthly or annual increment and by one touch of the lever secure the successive ordinate. For the greater part of computation work, a slide rule and adding machine are sufficient mechanical equipment, but if more nearly exact results in either multiplication or division are desired, one of the standard calculating machines is helpful.

CHARTING

The bulk of the charting work done in the laboratory will consist in the preparation of study charts rather than of carefully finished charts for publication. Although the same care as respects plan and accuracy is essential in a study chart as in a presentation chart, much of the labor incident to preparing for publication can appropriately be saved in preparing charts of the study type. Whether a chart is to be in pencil or in ink generally depends on whether it is for study or publication, although study charts are sometimes drawn up in ink. In any case, the plotting is done in pencil to begin with, and the procedure is therefore general as far as the completion of the pencil work.

The preparation of a particular chart should be covered by written instructions, and the draftsman should study all the directions carefully before undertaking the task. Decisions must then be made as to the size of the chart, the form of grid rulings to be used, and the scales upon which the measurements are to be made. If the grid is not one of the standard types, it must be ruled up before any plotting can be done. If the chart is to be drawn on transparent paper or cloth, a ruled background may be used as grid. This practice effects a great saving in labor. In general, ruled backgrounds or other grid layouts should be few in number and simple in form. For plotting time series, the choice of time scale is generally limited to years, quarters, months, or weeks. For annual series, vertical rulings are needed for each year, with perhaps each fifth ruling heavier than the rest. For monthly or quarterly series, the annual rulings should be drawn heavy. For weekly series, monthly and annual rulings will not fall exactly on weekly rulings, and should be carefully located. For locating points vertically, a rather closely spaced set of horizontal rulings, either on an arithmetic or on a log scale, is needed, unless each

measurement is to be made with a movable scale. In case a log scale is desired which has a 1:10 ratio in some other length than the standard 5-inch, 10-inch, or 20-inch lengths taken from slide rules, such scale can be made by looking up logarithms, or by adapting a standard scale by the parallel-line construction learned in plane geometry.

For plotting many categorical series and frequency series, particularly of the block diagram type, a background ruled in squares of rather small size is generally effective. If the rulings on such a background are simple fractions of an inch or centimeter, interpolation between the rulings is easy by use of ordinary measuring sticks. Here, as in the case of time series grids, the student will do well to have only a minimum number of different grids; and he will find most plotting can be handled without special and peculiar grid plans.

As soon as the grid is established, the individual points are plotted and other essential measurements laid off. If the chart is designed to show graphic curves, the individual points should be joined by pencil lines. If the chart is of some other type, the appropriate forms should be filled in; for example, the bars of a bar diagram should be filled in with pencil lines. Individual curves (or other forms on the chart) should be labeled with appropriate captions, the scale numbers should be entered, and the title marked upon the chart. The nature and source—the latter presumably a working table—should be noted.

It is then time for the entire chart to be verified, preferably by some one other than the draftsman. The verification consists in checking every part of the chart—plotted points, curves, ruled forms, captions, scale numbers, and title—in the light of the instructions and the original data.

The degree of error to allow depends largely upon the use to which the chart will be put. Perfect graphic presentation is impossible, but a range of allowable error should be determined in advance and adhered to for each chart. Ultimately the degree of accuracy demanded is decided upon the same grounds as the degree of accuracy used in computing work. In charting, the weight of the curve in the final chart limits somewhat the precision with which points are shown, but fundamentally the precision is determined by the inherent accuracy of the data and the purpose of the graphic presentation. As soon as the study chart is verified in pencil form, it is finished and ready for use. Neatness and a passable regularity can be achieved by practice and the exercise of constant care.

APPENDIX C

THE NORMAL CURVE OF ERROR, AND OTHER FREQUENCY CURVES

THE PARAMETER OF THE CURVE

It has been seen (page 209) that the equation of the normal curve of error has the form:

$$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

where N is the total frequency, σ is the standard deviation of the "errors" (deviations from the mean) x, e is the Napierian base

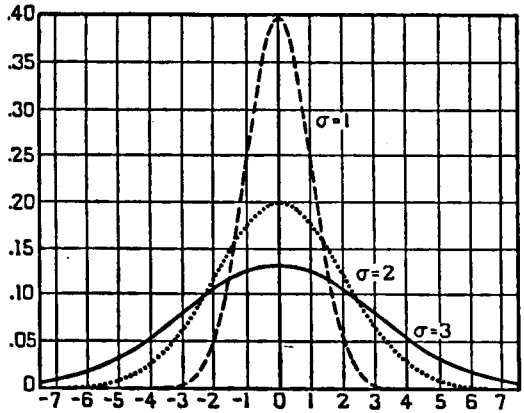

CHART 131.—Normal curves having an identical mean, but different standard deviations.

(Data in Table 147.)

2.71828, and π is the circular constant 3.14159. The equation can be written

$$\frac{y}{N} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

which gives an expression for the *relative* (per cent of the total) frequency y/N. So far as the form of the curve is concerned, the

actual frequency is of no importance, and, therefore, N is not a determinant of the form of the curve.

The forms of two normal curves each showing relative frequencies y/N (or of two curves showing actual frequencies y, if the total frequency N is the same for both curves) differ according as σ is different for the two curves. For this reason σ is called the parameter of the curve. Chart 131 (based on Table 147) shows three normal curves, all having the same N, but different values of σ.

THE QUARTILES OF THE NORMAL CURVE

If the curve is taken in the second form:

$$Y = \frac{y}{N} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

the area under the curve and between the ordinates at the quartiles must equal ½. Therefore, if Q is the quartile, then

$$\int_{-Q}^{Q} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{0}^{Q} e^{-\frac{x^2}{2\sigma^2}} dx = \frac{1}{4}$$

If the variable is changed to z by means of the substitution

$$z = \frac{x}{\sigma\sqrt{2}}$$

and if

$$q = \frac{Q}{\sigma\sqrt{2}}$$

the above equation becomes

$$\frac{2}{\sqrt{\pi}} \int_{0}^{q} e^{-z^2} dz = \frac{1}{2}$$

The value of q, from Peirce's *Table of Integrals*, is found to be 0.47693. And, therefore,

$$Q = \sigma q\sqrt{2} = 0.6744^{+}\sigma$$

The values of the probability integral may be conveniently expressed as

$$\frac{1}{\sqrt{2\pi}} \int_{0}^{u} e^{-\frac{u^2}{2}} du$$

the values of which for various values of the limit u are given in Table 145.

TABLE 145

AREAS AND ORDINATES OF THE PROBABILITY CURVE*

u	$\frac{1}{\sqrt{2\pi}}\int_0^u e^{-\frac{u^2}{2}}du$ a	$\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}$ b	u	$\frac{1}{\sqrt{2\pi}}\int_0^u e^{-\frac{u^2}{2}}du$ a	$\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}$ b
0.00	0.0000	0.3989	2.00	0.4773	0.0540
0.04	0.0160	0.3986	2.04	0.4793	0.0498
0.08	0.0319	0.3977	2.08	0.4812	0.0459
0.12	0.0478	0.3961	2.12	0.4830	0.0422
0.16	0.0636	0.3939	2.16	0.4846	0.0387
0.20	0.0793	0.3910	2.20	0.4861	0.0355
0.24	0.0948	0.3876	2.24	0.4875	0.0325
0.28	0.1103	0.3836	2.28	0.4887	0.0297
0.32	0.1255	0.3790	2.32	0.4898	0.0271
0.36	0.1406	0.3739	2.36	0.4909	0.0246
0.40	0.1554	0.3683	2.40	0.4918	0.0224
0.44	0.1700	0.3621	2.44	0.4927	0.0203
0.48	0.1844	0.3555	2.48	0.4934	0.0184
0.52	0.1985	0.3485	2.52	0.4941	0.0167
0.56	0.2123	0.3411	2.56	0.4948	0.0151
0.60	0.2258	0.3332	2.60	0.4953	0.0136
0.64	0.2389	0.3251	2.64	0.4959	0.0122
0.68	0.2518	0.3166	2.68	0.4963	0.0110
0.72	0.2642	0.3079	2.72	0.4967	0.0099
0.76	0.2764	0.2989	2.76	0.4971	0.0089
0.80	0.2881	0.2897	2.80	0.4974	0.0079
0.84	0.2996	0.2803	2.84	0.4977	0.0071
0.88	0.3106	0.2709	2.88	0.4980	0.0063
0.92	0.3212	0.2613	2.92	0.4983	0.0056
0.96	0.3315	0.2516	2.96	0.4985	0.0050
1.00	0.3413	0.2420	3.00	0.4987	0.0044
1.04	0.3508	0.2323	3.04	0.4988	0.0039
1.08	0.3599	0.2227	3.08	0.4990	0.0035
1.12	0.3686	0.2131	3.12	0.4991	0.0031
1.16	0.3770	0.2036	3.16	0.4992	0.0027
1.20	0.3849	0.1942	3.20	0.4993	0.0024
1.24	0.3925	0.1849	3.24	0.4994	0.0021
1.28	0.3997	0.1759	3.28	0.4995	0.0018
1.32	0.4066	0.1669	3.32	0.4996	0.0016
1.36	0.4131	0.1582	3.36	0.4996	0.0014
1.40	0.4192	0.1497	3.40	0.4997	0.0012
1.44	0.4251	0.1415	3.44	0.4997	0.0011
1.48	0.4306	0.1334	3.48	0.4998	0.0009
1.52	0.4357	0.1257	3.52	0.4998	0.0008
1.56	0.4406	0.1182	3.56	0.4998	0.0007
1.60	0.4452	0.1109	3.60	0.4998	0.0006
1.64	0.4495	0.1040	3.64	0.4999	0.0005
1.68	0.4535	0.0973	3.68	0.4999	0.0005
1.72	0.4573	0.0909	3.72	0.4999	0.0004
1.76	0.4608	0.0848	3.76	0.4999	0.0003
1.80	0.4641	0.0790	3.80	0.4999	0.0003
1.84	0.4671	0.0734	3.84	0.4999	0.0003
1.88	0.4700	0.0681	3.88	0.5000	0.0002
1.92	0.4726	0.0632	3.92	0.5000	0.0002
1.96	0.4750	0.0584	3.96	0.5000	0.0002

* Adapted by permission from Professor James W. Glover's *Tables of Applied Mathematics, Finance, Insurance, Statistics*; published at Ann Arbor, George Wahr, in 1923.

THE AVERAGE DEVIATION FOR THE NORMAL CURVE

The average deviation, *A.D.*, is obtained by summing all the deviations, without regard to sign, and dividing by the total frequency. Hence

$$A.D. = 2\int_0^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} x\,dx$$
$$= \sigma\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-\frac{z^2}{2}} z\,dz, \text{ in which } z = \frac{x}{\sigma}$$
$$= \sigma\sqrt{\frac{2}{\pi}}$$
$$= .7979\sigma$$

Hence, almost exactly, the average deviation of the normal distribution is 0.8 of the standard deviation.

DISTRIBUTIONS WHICH ARE NEARLY NORMAL

The relations between the standard deviation and the quartile deviation, and the average deviation

$$Q = .6745\sigma$$
$$A.D. = .7979\sigma$$

are strictly applicable only to normal distributions. In general, also, distributions which are not normal do not have mean, mode, and median coincident, as is the case with normal distributions.

Ordinarily, the nearest approach to a normal distribution realized in actual statistics appears in the case of data arising in games of pure chance. For example, the data of Table 146 form several series each of which is nearly normal. Even in these cases considerable departures from the error curve appear, but such departures are largely, if not entirely, ascribable to the imperfections of the sampling process. In other words, it may be assumed that, if any one of the series of Table 146 were actually a perfect sample of the underlying frequency series representing the true chance of turning heads, such series would be normal.

A series of data arising in the process of "observation" of a particular magnitude in physical or natural science is also likely to be so nearly normal that the properties of the normal curve may be assumed applicable. It is on this ground that the more common methods of adjustment of the errors of observation have developed about the notion of least squares.

In economic statistics there are numerous instances in which frequency distributions are very nearly normal, but there are great numbers of economic series for which the departure from normal is so great that the properties of the normal curve are not even roughly applicable.

It has been stated in the text (Chap. XIII) that even for a large number of these asymmetrical series the theory of sampling is approximately valid, especially for the simpler characteristics

TABLE 146

DISTRIBUTION OF THE NUMBER OF HEADS RESULTING FROM 100 TOSSES OF 24 COINS, FOR EACH OF 10 SEPARATE SITTINGS

Number of heads	Sitting										Total
	A	B	C	D	E	F	G	H	I	J	
0	...	...	...	...	...	...	...	...	...	...	.....
1	...	...	...	...	...	...	...	...	...	...	.....
2	...	...	...	...	...	...	...	...	...	...	.....
3	...	...	...	...	...	...	...	...	...	...	.....
4	...	...	...	...	...	...	...	...	...	...	.....
5	1	...	...	...	...	2	...	...	...	...	3
6	...	2	1	...	...	2	2	3	...	1	11
7	2	...	1	...	3	3	1	2	4	...	16
8	4	4	3	6	6	5	4	4	3	4	43
9	8	7	10	5	7	7	14	6	5	10	79
10	9	10	16	13	14	8	11	11	15	12	119
11	11	17	17	14	11	14	14	19	14	15	146
12	25	18	16	19	14	14	16	12	18	14	166
13	17	18	12	20	15	16	16	10	20	18	162
14	9	13	11	7	12	11	12	17	9	13	114
15	9	5	5	6	8	6	6	10	6	6	67
16	...	2	2	6	5	4	2	5	4	5	35
17	3	2	5	2	3	4	2	1	2	2	26
18	1	2	1	1	...	2	...	...	...	...	7
19	1	...	...	...	1	2	...	...	...	...	4
20	...	...	...	1	1	...	...	...	...	...	2
21	...	...	...	...	...	...	...	...	...	...	.....
22	...	...	...	...	...	...	...	...	...	...	.....
23	...	...	...	...	...	...	...	...	...	...	.....
24	...	...	...	...	...	...	...	...	...	...	.....
Total	100	100	100	100	100	100	100	100	100	100	1,000

such as the mean. The probable errors of the principal frequency constants, including certain of the characteristics of correlation distributions, have been worked out. Although these probable error formulas have been derived for normal distributions, they can be applied, with appropriate reservations, to series which differ considerably from normal.

The application, to series customarily encountered in economics, of more refined curve-fitting methods than the mere determination of the normal curve which best fits the given data

is not ordinarily justified. Many economic series are so extremely skew that good fits are not readily obtained by any method, and interpretation of the resulting mathematical formulas is in all cases more difficult than for the case of the normal curve.

TABLE 147

ORDINATES OF THE NORMAL CURVE FOR THREE DISTINCT VALUES OF THE STANDARD DEVIATION

Total area under each curve is unity. See Chart 131.

x	$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}}$		
	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$
0.00	0.399	0.199	0.133
0.15	0.394	0.199	0.133
0.30	0.381	0.197	0.132
0.45	0.361	0.194	0.131
0.60	0.333	0.191	0.130
0.75	0.301	0.186	0.129
0.90	0.266	0.180	0.127
1.05	0.230	0.174	0.125
1.20	0.194	0.167	0.123
1.35	0.160	0.159	0.120
1.50	0.130	0.151	0.117
1.65	0.102	0.142	0.114
1.80	0.079	0.133	0.111
1.95	0.060	0.124	0.108
2.10	0.044	0.115	0.104
2.25	0.032	0.106	0.100
2.40	0.022	0.097	0.097
2.55	0.015	0.088	0.093
2.70	0.010	0.080	0.089
2.85	0.007	0.072	0.085
3.00	0.004	0.065	0.081
3.15	0.003	0.058	0.077
3.30	0.002	0.051	0.073
3.45	0.001	0.045	0.069
3.60	0.001	0.039	0.065
3.75	0.000	0.034	0.061
3.90	.....	0.030	0.057
4.05	.....	0.026	0.053
4.20	.....	0.022	0.050
4.35	.....	0.019	0.046

APPENDIX D

LOGARITHMS OF NUMBERS*

Pages 413 and 414 give the common logarithms of numbers between 1 and 10, correct to four places. Moving the decimal point n places to the right (or left) in the number is equivalent to adding n (or $-n$) to the logarithm. Thus, log 0.017453 = 0.2419 − 2(= $\bar{2}$.2419).

	0	1	2	3	4	5	6	7	8	9	10	Tenths of the Tabular Difference 1	2	3	4	5
1.0	0.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	0414					
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	0792					
1.2	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	1139					
1.3	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	1461	To avoid interpolation in the first ten lines, use the special table on pages 415 and 416.				
1.4	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	1761					
1.5	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	2041					
1.6	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	2304					
1.7	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2553					
1.8	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2788					
1.9	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	3010					
2.0	0.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	3222	2	4	6	8	11
2.1	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	3424	2	4	6	8	10
2.2	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	3617	2	4	6	8	10
2.3	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	3802	2	4	5	7	9
2.4	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	3979	2	4	5	7	9
2.5	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	4150	2	3	5	7	9
2.6	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	4314	2	3	5	7	8
2.7	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	4472	2	3	5	6	8
2.8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	4624	2	3	5	6	8
2.9	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	4771	1	3	4	6	7
3.0	0.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	4914	1	3	4	6	7
3.1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	5051	1	3	4	6	7
3.2	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	5185	1	3	4	5	7
3.3	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	5315	1	3	4	5	6
3.4	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	5441	1	3	4	5	6
3.5	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	5563	1	2	4	5	6
3.6	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	5682	1	2	4	5	6
3.7	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	5798	1	2	3	5	6
3.8	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	5911	1	2	3	5	6
3.9	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	6021	1	2	3	4	6
4.0	0.6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	6128	1	2	3	4	5
4.1	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	6232	1	2	3	4	5
4.2	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	6335	1	2	3	4	5
4.3	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	6435	1	2	3	4	5
4.4	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	6532	1	2	3	4	5
4.5	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	6628	1	2	3	4	5
4.6	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	6721	1	2	3	4	5
4.7	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	6812	1	2	3	4	5
4.8	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	6902	1	2	3	4	4
4.9	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	6990	1	2	3	4	4

*Taken, with permission, from E. V. Huntington's *Four Place Tables*, published by Houghton Mifflin Company.

	0	1	2	3	4	5	6	7	8	9	10	Tenths of the Tabular Difference 1	2	3	4	5
5.0	0.6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	7076	1	2	3	3	4
5.1	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	7160	1	2	3	3	4
5.2	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	7243	1	2	2	3	4
5.3	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	7324	1	2	2	3	4
5.4	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	7404	1	2	2	3	4
5.5	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	7482	1	2	2	3	4
5.6	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	7559	1	2	2	3	4
5.7	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	7634	1	2	2	3	4
5.8	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	7709	1	1	2	3	4
5.9	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7782	1	1	2	3	4
6.0	0.7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7853	1	1	2	3	4
6.1	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7924	1	1	2	3	4
6.2	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7993	1	1	2	3	3
6.3	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	8062	1	1	2	3	3
6.4	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	8129	1	1	2	3	3
6.5	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	8195	1	1	2	3	3
6.6	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	8261	1	1	2	3	3
6.7	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	8325	1	1	2	3	3
6.8	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	8388	1	1	2	3	3
6.9	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	8451	1	1	2	3	3
7.0	0.8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	8513	1	1	2	2	3
7.1	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	8573	1	1	2	2	3
7.2	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	8633	1	1	2	2	3
7.3	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	8692	1	1	2	2	3
7.4	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	8751	1	1	2	2	3
7.5	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	8808	1	1	2	2	3
7.6	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	8865	1	1	2	2	3
7.7	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	8921	1	1	2	2	3
7.8	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	8976	1	1	2	2	3
7.9	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	9031	1	1	2	2	3
8.0	0.9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	9085	1	1	2	2	3
8.1	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	9138	1	1	2	2	3
8.2	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	9191	1	1	2	2	3
8.3	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	9243	1	1	2	2	3
8.4	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	9294	1	1	2	2	3
8.5	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	9345	1	1	2	2	3
8.6	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	9395	1	1	2	2	3
8.7	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	9445	0	1	1	2	2
8.8	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	9494	0	1	1	2	2
8.9	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	9542	0	1	1	2	2
9.0	0.9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	9590	0	1	1	2	2
9.1	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	9638	0	1	1	2	2
9.2	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	9685	0	1	1	2	2
9.3	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	9731	0	1	1	2	2
9.4	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	9777	0	1	1	2	2
9.5	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	9823	0	1	1	2	2
9.6	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	9868	0	1	1	2	2
9.7	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	9912	0	1	1	2	2
9.8	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	9956	0	1	1	2	2
9.9	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996		0	1	1	2	2

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1.00	0.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	0043
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1.02	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	0128
1.03	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	0170
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1.06	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	0294
1.07	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	0334
1.08	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	0374
1.09	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	0414
1.10	0.0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	0453
1.11	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	0492
1.12	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	0531
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1.16	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	0682
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1.21	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	0864
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1.23	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	0934
1.24	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	0969
1.25	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	1004
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	0	1	2	3	4	5	6	7	8	9	10
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1.54	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	1903
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1.56	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	1959
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1.59	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	2041
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1.63	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	2148
1.64	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	2175
1.65	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	2201
1.66	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	2227
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1.85	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	2695
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1.99	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	3010

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