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BUREAU OF BUSINESS RESEARCH

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BULLETIN NO. 16

(Distributed, September, 1927)

THE NATURE OF CYCLICAL FLUCTUATIONS IN ELECTRIC POWER PRODUCTION DATA

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

1927

It is the purpose of the Bureau of Business Research of the University of Illinois—

(1) to study and as far as possible to explain economic and industrial conditions within the State;

(2) to direct attention to experience-tested practices of good business management; and,

(3) to investigate methods for securing the best executive control of business.

BUREAU OF BUSINESS RESEARCH
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3. Composite curve of selected companies in the Seventh Federal Reserve District.

4. Several individual companies.

These various curves will be compared and discussed with respect to their relation to the so-called general business cycle as presented by the Harvard B-Curve.

The general plan of this study is to compare the "cycle" of the total electric power production for the United States with the "general business cycle," to separate the total of power production by geographical divisions, and lastly, compare the "cycles" of several individual companies with the "general cycle." Attention will be focused upon the changing characteristics of the "cycle" curves of power production, as the analysis proceeds from the curve of total power production to the curves of individual companies. In general, the purpose of this bulletin will be to discover whether the movements of the cycles of small territorial subdivisions and of individual companies in the electric light and power industry are comparable to the so-called general business cycle.

The Harvard B-Curve is used as a "curve of reference" in this study. It is plotted along with each physical production curve in order to show how these curves differ *from each other* by referring all of them to a common "curve of reference." The curve representing the total electric power production for the United States, shown in Chart I, if it had been plotted along with each of the other production curves, would have served the purpose equally as well as the B-Curve. The Harvard B-Curve, however, is one of the well known indicators of the so-called "general business cycle," and it was felt that its use in this way might add to the reader's interest, although the difference between the various production curves on the one hand and the Harvard B-Curve on the other is not the focus of this bulletin.

In Appendix A a brief outline of the procedure in refining these sequences is presented. Appendix B contains a brief discussion on the applicability of correlation to time-ordered sequence, together with a working description of a coefficient for determining similarity. With Appendix C are presented the original data and the "cycle" fluctuations of all of the sequences used in this study.

In the preparation of the bulletin, acknowledgments are due to O. Gressens, Ph.D., and to J. R. Ramser. The latter undertook the immediate supervision of the tedious analyses which were a necessary part of the work connected with this bulletin.

I. TOTAL MONTHLY PRODUCTION OF ELECTRIC POWER IN THE UNITED STATES

Before entering into a discussion of the "cycles" of electric power production, it is advisable to comment on the derivation and significance of the curves presented in the following pages of this bulletin. All of these curves present the so-called cycle contained in the original data. These "cycles" have been isolated according to the general procedure of eliminating from the original data the secular trend and the seasonal variation.[1] These isolated cycle curves do not present exact quantitative measurements. They represent residual magnitudes after the long-time movements, impounded by the secular trend, and the seasonal have been approximately eliminated. These isolated "cycles" are, therefore, mechanically separated residuals which reflect, as nearly as it is now possible to determine, the short-time movements of the data.[2] They are comparable to the Harvard B-Curve representing the so-called general business cycle.

It might be objected that in these comparisons the Harvard B-Curve contains price data, and that it cannot, therefore, be judiciously compared with sequences of purely physical data. In the case of the public utility industry, however, this objection loses most of its weight, for changes in business conditions are more readily traceable in the physical production data of these companies than in the Gross Revenues. Public utility companies, as is well known, are subject to price regulation. This, together with the inherent tendency towards local monopoly, eliminates that resiliency towards price changes which can so often be observed in the case of industrials which are more or less dominated by competition, and which are not subject to the interference of regulatory bodies. The Gross Revenue data of public utility companies contain, therefore, an arbitrary element owing to the comparative inflexibility of rate structures. Consequently, it is through changes in production that the effects of those short-time movements, which have come to be called the business cycle, are most noticeable in these companies rather than in adjustments of Gross Revenue to changing prices.

Such effects as are introduced by rate changes do not bear a direct relationship to changing prices. They are abrupt, not gradual, and are based on "fair returns" to the companies in question rather than on price changes *per se*. Unless rate structures are changed, therefore, to

[1] See also Bulletin No. 8, Bureau of Business Research, University of Illinois.

[2] The procedure of isolating these "cycles" is discussed in Appendix A.

meet changed price conditions, the effects of variations in prices will only be reflected in these data as the value of consumption goods are changed, relatively to each other. Particularly, in the case of those companies which carry a heavy industrial load, the reactions to the short-time movements of the "business cycle" are to be sought in the data of physical production rather than in the data of Gross Revenues.

In those cases, however, in which it is desired to obtain a picture of the "business cycle" apart from direct price changes, the curve of power production is more desirable than a composite curve made up partly of price data and partly of other components containing reflections of price changes. The fluctuations of data, such as the "cycles" of electric power production in the United States, do not directly reflect changes in prices and are, in this respect, less "general" than such data as presented by the Harvard B-Curve. On the other hand, "general business conditions" is a cloudy concept, as Prof. Fisher has pointed out,[3] and the more complex the composition of a curve, the more difficult does it become to ascribe its fluctuations to particular causes.

In general, however, as was indicated above, there is a striking similarity between the "cycle" curve of total electric power production in the United States, and the Harvard B-Curve. The Harvard B-Curve, of course, is a compound of volume of business, represented by bank debits, and of price fluctuations. The gradual decline of this curve in the latter years is probably ascribable to the price factor. This illustrates one of the greatest difficulties in analyzing the fluctuations of a compound curve. It is impossible, without going back of the composite picture, to ascribe a movement to the influence of a particular factor. This, in fact, is one of the most telling criticisms against the usefulness of any summary description of business conditions.

Another effect of these movements makes itself felt in Production Expense and in Operating Expense. But owing to the difficulty of getting sufficient quantities of these data and of the lack of homogeneity resulting from arbitrary accounting classifications, they are not studied here; the comparisons of the short-time movements of public utility companies with the "general business cycle" is limited to physical production data.

In Chart I[4] is presented the Harvard B-Curve together with the "cycle" of the total electric power production in the United States. The latter data were available only since 1919 and the analysis covers, therefore, a period of seven years. Some difficulty would probably have been

[3]Our Unstable Dollar and the So-Called Business Cycle, Jour. Am. Statistical Ass'n., June, 1925.

[4]See pp. 24-36.

experienced in analyzing a longer period, because of the unusual condition introduced by the War.

Upon the examination of Chart I, it can be seen that there is a great deal of similarity between these "cycle" curves. This is particularly true prior to the end of 1924. The unusual peak of prosperity in 1920 is reflected in the power data somewhat earlier than in the data representing the general business cycle. The severe depression of 1921 is faithfully reflected although the general cycle movement again precedes the similar movement in the power data. The recovery in 1923 and the depression in 1924 are also clearly shown by both of these curves. There is a sudden rise in the "general cycle" data at the close of 1924 while the rise in the "cycles" of the power data is considerably more gradual. Since that time, the "general cycle" data have persistently, though slowly, declined, while the power data have steadily risen. Nevertheless, there is on the whole a great deal of similarity between these two curves. This similarity lends some weight to the contention that total electric power production is a fairly good indicator of general business conditions.

II. GEOGRAPHICAL DIVISIONS OF TOTAL MONTHLY POWER PRODUCTION

The similarity between the curve of total power production in the Eastern States (Chart II) and the general business curve, while not so striking as the likeness evidenced in Chart I, is still very apparent. This may be accounted for, in part, by the fact that the total power production of the East comprises more than a third of the total for the United States. As long as the "cycle" is isolated from a considerable accumulation of data, it can be expected to resemble the "general business cycle" quite closely. That is, the "cycle" curve of such an accumulation would show relatively smooth "cycle" movements, which are comparatively free from the rapid fluctuations that are typical of the "cycle" movements isolated from individual company data. The inevitable cancellations resulting from massing a large number of sequences has a tendency to make the fluctuations less violent. The less general the data, the greater the variations of the isolated cycle may be expected to be from the general business curve.

The depression in 1921 and 1922 is more marked in the curve representing power production in the East than in the general business curve. The rise in 1924 portrayed by the business curve is not nearly so great in the power production curve, and the latter has a tendency to continue at about the same level, while the general curve turns downward.

In Chart III are presented the curves of electric power production in the Middle West and the "general business curve." The similarity in this case is less than that found by the comparisons made in Charts I and II. It may again be noted that the power production represented by this curve is less than power production represented by the curve for the Eastern States.

Beginning with 1922, the "cycle" movements of the curve representing the electric power production of the Middle West are obscured by irregular fluctuations. When Charts IV and V are considered, this tendency becomes more evident; particularly the curve representing the power production of the Southern States. It has very little in common with the movements of the "general business curve." This curve represents the smallest amount of power production of the four geographical divisions. It is important to note that the curves of these geographical divisions become increasingly dissimilar from the "general business curve" directly as the amount of the power production, which they represent, decreases. An explanation can be found for this in the fact that the larger the number of sequences which are summed to give one general picture, the more are the characteristics of the individual sequences immersed in the totals. This fact tends to give to the curve representing the larger totals the appearance of a greater degree of smoothness. This is, thus far, excellently illustrated by the curves of power production which have been considered.

It will be interesting and important to see whether or not this tendency evidenced by the curves for the geographical divisions is supported by still smaller divisions of the electric power figures. For this purpose, a combined group of selected electric power companies in the Seventh Federal Reserve District are compared with the "general business curve" in the next section.

III. COMPARISON OF K.W.H. PRODUCTION OF LARGE AND SMALL COMPANIES

In the previous sections of this bulletin, the comparisons were made between the so-called general business cycle and the total production of electric power for the entire country and sub-divisions thereof. It will be important to note whether the similarity to this so-called cycle persists where the comparison has been narrowed down to a few companies. For this purpose, two groups of companies have been considered: (1) a group of large companies, and (2) a group of small companies.

In Chart VI is presented a comparison of the "general business" curve with the corresponding curve of a group of three large companies. Two of the companies represented furnish electric power to a large number of communities through an extensive interconnected system of transmission lines. One of the companies (shown separately in Chart VIII) operates in and around a large city. The electric power production curve in Chart VI shows a rather close resemblance to the Harvard Curve. The range of movement of the entire curve is relatively small and the monthly fluctuations are slight.[1] In the first few years, however, the production curve has the tendency to precede the B-Curve, while from about 1923 the production curve lags behind the B-Curve.

Chart VII sets in contrast the "general business cycle" and the production curve of a group of small companies operating for the most part in small towns or rural communities. The production curve has a relatively small range of fluctuation and the monthly movements are slight, but the general movement of the curve does not express as much similarity to the "general business cycle" curve as that found in Chart VI.

Even though the production figures used in each of these illustrations have about the same magnitude, there is a difference in the curves because of the characteristics of the consumers served in each case. The production curve in Chart VI represents a considerable amount of power used for industrial purposes. It is only logical to expect production of this character in a large city to follow the movement of the B-Curve more closely than the production curve in Chart VII which is composed of companies producing power for rural communities and

[1]That is, the standard deviation of this sequence is smaller than that of the other sequences. The "cycle" fluctuations of the original data are, therefore, confined within narrower limits.

small towns, where the industrial load is not an important factor.

This characteristic of the "cycles" can be generalized as follows: The greater the number of accumulated sequences that enter in as components of a compound sequence, the more will the curve of this compound sequence be similar to a curve of the "general business cycle." If, however, of two such compound and similar sequences, whose individual observations are of about the same magnitude, one of them is composed of data from a great number of small businesses operating in small localities, and the other is composed of data from a few large businesses operating in large markets, the curve of the latter sequence will tend to be more similar to a "general business cycle," such as that represented by the Harvard B-Curve, than will the curve of the former.

Concretely, this means that if two monthly sales sequences are taken, and the annual total sales of these sequences are computed; and if the annual total sales of these sequences average to the same figure over a period of years, one of them being made up of sales of a great number of *small businesses* while the other is composed of the sales of a few *large businesses,* operating in *large markets,* the "cycle" curve of the latter sequence will look more like a "general business cycle curve" than the former.

This point is illustrated by Charts VI and VII. The former is a compound of a few large companies while the latter is composed of a larger number of small companies. The total electric power production represented by these curves is about the same. The "cycle" curve presented in Chart VI, however, is more like the Harvard B-Curve than the "cycle" curve of Chart VII. The explanation lies, it would seem, in the fact that each individual company entering into the electric power production totals of the data of Chart VI is affected by a greater number of the same causes that determine the behavior of the Harvard B-Curve than are the companies which give the data for the curve presented in Chart VII.

IV. THE POWER PRODUCTION CYCLE OF INDIVIDUAL COMPANIES

In Chart VIII is presented the power production "cycle" curve for one of the large companies included in the production curve of Chart VI. This company furnishes electric power for a large city and much of the total production is put to industrial uses. This curve, in general movement, is somewhat closer to the B-Curve than when the data is combined with the other companies in Chart VI.[1] This is probably due, as was pointed out before, to the fact that in a company which carries a large industrial load, the production "cycle" curve will look more like the Harvard B-Curve than the company that has no industrial load. The former type of company is affected indirectly, by numerous factors influencing the course of the industries with which it is connected, while the latter type of company is dominated by influences more peculiarly local in character. The range of movement of the curve of Chart VIII, however, is wider than the production curve shown in Chart VI, and the consecutive monthly fluctuations are greater. The movement of the curve becomes smoother as more companies are massed in the analysis.

The company, whose production curve is represented in Chart IX also operates in certain cities and has, therefore, some industrial load, but it has not as large a load from this source as the company in Chart VIII. This fact would seem to explain some of the difference between the production curves in Charts IX and X. The company represented in Chart X carries very little industrial load. Both the curves of Charts IX and X show a large range of movement with a wide fluctuation in the monthly items. Both vary more from the Harvard B-Curve than the curve of the company presented with Chart VIII.

With Charts XI, XII, are presented two of the production "cycle" curves which are representative of the companies included in the group production curve presented by Chart VII. The production curve shown in Chart XI represents the largest number of kilowatt hours generated by any of these three companies. When this curve is compared with the curve of the "general business cycle," it is impossible to recognize any similarity. The fluctuations of the production curve of this company are much more violent and much more irregular than the compound curve that represents the "general business cycle." The same irregular and extreme fluctuation can be noticed in Chart XII.

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[1]This conclusion is substantiated by the coefficients presented in Appendix B, Table I, items 6 and 7.

It appears, therefore, that not only do the so-called cycles of these individual companies show great dissimilarity when compared with the so-called general business cycle curve but this dissimilarity seems enhanced when the size of the company is decreased. From the comparisons of Charts VIII and IX, the conclusion seems warranted that the heavier the industrial load which a company carries the more nearly does its so-called production cycle curve agree with the similar curve for general business. When the industrial East and Middle West were compared with the "general business cycle," this same fact was noted in contrast to the curves for the less industrial West and South.

V. FURTHER COMPARISONS OF INDIVIDUAL COMPANY DATA WITH THE SO-CALLED BUSINESS CYCLE

In Tables XIV and XV, Appendix C, are presented the "cycle" fluctuations of Gross Revenue and Production Expense of the Company whose physical production in kilowatt hours is represented in Chart XII. These curves show a wide range of monthly movement and their general appearance is very unlike that of the so-called general business cycle. Tables XVI and XVII show the cycle fluctuations for the Production Expense and Gross Revenue for the company whose physical production "cycle" is given in Table XIII. Again these curves show very little relation to the general "cycle" curve.

The "cycle" fluctuations representing Production Expense, shown in Tables XV and XVI, are not as reliable as the similar fluctuations based on physical production. Expense data of this kind are very likely to lack homogeneity because of changing accounting classifications and other arbitrary factors. Where the data lacks homogeneity, its analysis is, of course, unreliable. In the case of the expense data, particularly, the lack of this characteristic is evident. The data are composed of wages, fuel expense, salaries, indirect expense connected with production, etc., etc. These items vary in their reactions to changing conditions, some of them, like wages, offering a considerable resistance to changes, while others, like fuel expense, reacting much more quickly to the short-time movements called the business cycle. A sequence of data such as Production Expense, therefore, is much less homogeneous than are sequences of physical production, or even Gross Revenues. The factors which make the latter kind of data generally less valuable for comparative purposes have been discussed in a previous section of this bulletin. There is little doubt but that the regulation of rates reflected in the Gross Revenue data, and the heterogeneity of the Production Expense data contribute to the dissimilarity of these data from the "general business cycle." The curves presented in this section are, therefore, included merely as additional evidence for the general conclusion that the "cycle" movements which affect individual companies are not necessarily comparable to the "general business cycle."

VI. SUMMARY

1. There is, no doubt, a tendency for the "cycle" curves of large groups of business data to indicate similar movements. Industrial organization has reached such a degree of complexity and interdependence that it is impossible to isolate, for purpose of analysis, the various component phenomena. This ramified interrelation accounts for the fact that many massed sequences of industrial data look very much alike. Before this similarity is evident, however, it may become necessary to accumulate a considerable amount of data in any one industry; a small amount of data from one or a few enterprises does not suffice. Bank clearings of a large number of banks, total car loading of the United States, total electric power production, all have "cycle" curves which are very similar. Other sequences of data could be mentioned which would be similar also—provided a sufficient mass was accumulated and included in the analysis.

2. When large masses of data are broken down, and their various components analyzed, the curves of these parts look less like the curve of the whole and have distinct characteristics of their own. This was demonstrated by separating the total electric power production for the United States into four parts and comparing each of these divisions with the Harvard B-Curve as a curve of reference. A further analysis of individual companies shows that the smaller companies have "cycle" curves which have very little in common with the general "cycle" curve from the point of view of their general behavior. Because of the fact that any general curve representing the so-called business cycle is necessarily a composite indicating the general movement of a mass of data, it is not very usable for many individual companies. The "cycle" of any particular company will depend upon the specific conditions which affect the operation of that company. A general composite curve cannot reflect specific conditions pertaining to a particular locality or industry and can therefore only be suggestive, or "background" to the individual company in its budgeting or forecasting problems. The larger the company, however, and the wider its range of operation, the more closely will its production data follow a general business curve. Unless, therefore, a particular company has a sufficiently large and diversified market so that all or a large part of the influences which determine the behavior of the "general business cycle" are also reflected in the experience of the company, it cannot profitably compare its own short-time movements with those of the "general business cycle."

3. A "cycle" curve of general business will indicate the "cycle" movements of one part of the country closer than those of other parts. It was pointed out above that those large utility companies which carry a considerable industrial load will have a "cycle" curve looking more like the general "cycle" curve than those furnishing power for rural or domestic use. The East and Middle West which contain large industrial centers show "cycle" curves that resemble the "general cycle" curve much more closely than do the similar curves for the West and South.

4. In general, it may be said that the monthly movements of "cycles" of individual utility companies have a wider range and are more rapid in their oscillations than the "general cycle" curve. The extreme fluctuations often present in the "cycle" curves of small companies are subdued if not entirely immersed in large accumulations of such data. In other words, the peculiarities of circumscribed localities are lost in the net results of a great many such peculiarities. They are swallowed up, so to speak, in a general picture in which they become lost details.

APPENDIX A

I. GENERAL PROCEDURE OF ANALYSIS

1. The first step in the analysis of the data presented in this bulletin is the plotting of the monthly items. With electric production data, the semi-logarithmic graph paper was found to be more suitable for trend determinations because in most cases it yielded more simple trends.

2. Trends were fitted to the data by the method of least squares. In most cases the trend, with the data plotted on semi-log paper, was a straight line, the equation of which is a simple exponential function. As a general rule, the straight line will give the best results when the data used cover only a short period.

3. The monthly trend items were calculated and the ratio of the actual items to the corresponding trend ordinates were computed. These ratios were used in finding an index of the seasonal variation.

TABLE 1

COMPUTATION OF SEASONAL INDICES

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
117.3	108.7	109.5	102.4	97.7	95.4	95.9	96.8	94.0	99.9	105.1	112.6
116.2	107.8	108.1	100.6	97.9	93.9	94.9	95.3	93.8	98.6	104.1	117.7
115.4	105.7	107.4	96.3	94.4	93.5	92.9	89.2	92.6	96.5	103.3	110.6
112.1	105.0	103.6	96.5	93.7	91.9	89.9	89.8	91.9	94.4	102.9	110.3
461.0*	427.2	428.6	395.8	383.7	374.7	373.6	371.1	372.3	389.4	415.4	445.2
114.3**	106.0	106.3	98.2	95.2	92.9	92.7	92.0	92.3	96.6	103.0	110.4

Average of totals in line 5 $= \frac{4838.0}{12} = 403.17$

461.0÷403.17 = 114.3 Seasonal index for January.

*The figures in line 5 are the totals of the monthly items.

**Seasonal Indices. For additional notes on the method of computation see Bulletin No. 8 of the Bureau of Business Research of the University of Illinois, pp. 35-37.

4. In Table 1 is presented the method used in computing the index of seasonal variation. The monthly "actual-trend" ratios are charted on coordinate paper as shown in Chart XIII. The advantage of charting these items in this manner is that such a procedure clearly shows whether or not the data analyzed has a distinct seasonal swing. If the monthly ratios group themselves rather closely, as shown in Chart XIII, and do not fall in a straight line, there is a seasonal in the data which may be safely used in analysis. A wide scatter of the items on the chart indicates a lack of seasonal.

TABLE 2

COLUMNAR HEADINGS USED IN CYCLE COMPUTATION

	a	b	c	d	e	f	g	h*
	Actual	Trend	a ÷ b	Seasonal Index	c − d	e ÷ d	f²	f ÷ σ
Jan.	15,706	14,155	111.0	114.3	−3.3	−2.9	8.41	− .8
Feb.	14,235	14,290	99.6	106.0	−6.4	−6.0	36.00	−1.6
Mar	14,270	14,425	98.9	106.3	−7.4	−7.0	49.00	−1.8

*The figures found in column "h" cannot be computed from the data given in the table. See Bulletin No. 8 of the Bureau of Business Research of the University of Illinois, pp. 25-34 for additional notes on the methods of computation.

The "actual-trend" ratios are carried to one decimal place, as 115.2, and shown in Chart XIII. Here, it will be noted, the digits of a given ratio are to be read from the scale on the left margin while the decimal portion of the ratio is to be found in the body of the chart. The decimal figure therefore indicates by its size the fraction, and by its placement the graphic location, of the ratio in question.

The next step in the computation was to take the arithmetic average of the four central items of each monthly column. These items are easily selected from the chart by inspection. The arithmetic mean of the twelve averages is found, and the ratio of each average to this mean is computed. The resulting ratios are used as the seasonal indices for the respective months. Actually, this process was shortened as in Table 1. The four central items for each month were added and the average of these sums found (403.17). Each sum was in turn divided by this average to give the seasonal indices. Seasonal indices computed in this manner do not allow for trend adjustment or a variation in the seasonal from year to year. These items of change, however, are negligible when considered for the short sequences analyzed in this bulletin.[1]

5. Table 2 shows the columnar headings on the computation sheets used in the procedure of isolating the cycle movement. A further refinement is sometimes made by the use of the moving median with the cycle figures. The object in using the moving median is to eliminate items which are thought to be irregular or accidental. With this method, however, there is a danger of throwing out items which are essential to the final analysis. The crudeness of this sort of refinement makes its value very doubtful. While it was tested in the preparation of this bulletin, the results did not justify its use.

[1]For the method of eliminating seasonal variation from long sequences of data, see Bulletin No. 8, Bureau of Business Research, University of Illinois.

APPENDIX B

In the text of this bulletin, a number of curves were discussed from the point of view of their similarity to the so-called general business cycle curve. In this connection, the only evidence introduced to establish the several conclusions was a direct graphic comparison of the several "cycle" curves with the "general cycle" curve. No coefficients of correlation were introduced to demonstrate statistically that the conclusions drawn from direct comparisons are correct. There is a valid reason for this omission, based on the assumptions involved in the theory of correlation.

It is generally accepted, following particularly upon the work of Hooker, Persons, and Yule, that the oscillations of time-ordered sequences of economic data which remain after the secular trends and the seasonal variations have been eliminated can be compared to discover, in general, the degree of similarity of such data. Professor Persons, among others, has been careful to state that the coefficient of correlation in these cases serves only to determine the maximum lag at which there is the greatest similarity between the "cycle" movements of such sequences. He attaches no further significance to the numerical magnitude of the coefficient, or the probable error of the coefficient. Persons' coefficient of correlation, in fact, is an empirical measure, independent of the assumptions underlying the theory of correlation.

The assumptions underlying the derivation of the product moment coefficient of correlation introduced by Pearson are:

1. The causes which determine the magnitude of the observations of the correlated data are *independent.*

2. The variations in intensity of these causes are small as compared to their absolute intensity.

3. The variations distribute themselves according to the normal law of error.

The exchange of relationships which obtain between economic goods invalidate the first of these assumptions for economic data. The causes which determine the magnitudes of any sequence of economic data have, through the phenomena of exchanges, been transmitted to become a part of the nexus of causes which affect innumerable other sequences of data. This complex of interdependence created by exchange relationships makes it impossible to postulate independent causes. The great bulk of the nexus of causes which determine the magnitudes of the ob-

TABLE 3

COMPARISON OF PERSONS' COEFFICIENT OF CORRELATION WITH THE COEFFICIENTS BASED ON AVERAGE DIFFERENCES OF "CYCLES" OF TIME-ORDERED SEQUENCES

Sequences correlated and compared with the Harvard B-Curve	Coefficient based on Average Differences	Persons' Coefficient of Correlation
1. Total Monthly Production of K.W.H. in the United States	.35	.73
2. Total Monthly Production of K.W.H. in the Eastern States	.38	.72
3. Total Monthly Production of K.W.H. in Middle Western States	.40	.63
4. Total Monthly Production of K.W.H. in Western States	.43	.61
5. Total Monthly Production of K.W.H. in Southern States	.54	.36
6. Total Monthly Production of K.W.H. of companies in Seventh Federal Reserve District	.43	.53
7. Total Monthly Production of K.W.H. of Company A	.42	.55
8. Total Monthly Production of K.W.H. of Company B	.56	.16
9. Total Monthly Production of K.W.H. of Company C	.62	.04

servations of time-ordered sequences are made up of *interdependent*, interrelated causes.

The variations in intensity of these causes, as compared with their absolute intensity, are not necessarily small, and in fact, furnish some of the most difficult of all economic problems. Technical improvements in production, sudden changes in demand caused by the introduction of new consumption goods or the increased efficiency in production of old goods, changes in population characteristics owing to immigration, discovery of new resources or sudden cessation of old resources, and all of the multitudes of changes, swift and unforseen, which root in the dynamic characteristic of economic development cause significant variations in the intensities of the causes that determine the *magnitudes* of economic data. The second assumption involved in the derivation of the coefficient of correlation is, therefore, not tenable in the case of time-ordered sequences.

The third assumption, that the variations of the causes affecting the data under consideration distribute themselves according to the normal law of error, is not necessarily true. Such an assumption is, in fact, arbitrary.

The fundamental assumptions involved in the theory of correlation are, therefore, not tenable in the case of economic sequences. Correlation between these sequences, as Prof. Persons has pointed out, does not allow of the *interpretation usually made* in connection with data to which these assumptions can be applied. As was suggested above, Persons' coefficient of correlation is strictly an empirical coefficient.

The important difference between his coefficient and Pearson's product-moment coefficient is that the former is not based on any *a priori* assumptions and is, therefore, above criticism in this respect. This accounts, however, in part, for the impossibility of attaching any exact significance to particular numerical magnitudes of the coefficient. This defect becomes more evident when it is taken into consideration that the product of two unusually large fluctuations in the same direction results in a large product term, thereby unduly increasing the magnitude of the coefficient. A few large fluctuations in the same direction at the same time might be sufficient to indicate rather high correlation, when, as a matter of fact, there is very little or negligible correlation between all the remaining observations. Persons' coefficient of correlation, therefore, does not yield a dependable measure of the degree of similarity between time-ordered sequences.

A coefficient not subject to this criticism has been used as supplementary evidence to the direct comparisons of the several "cycle" curves. It is calculated as follows:

1. The "cycle" movements of the sequences to be compared are isolated by the usual methods, and put into units of their standard deviations.

2. The differences between corresponding "cycle" items of the sequences to be compared are then obtained.

3. The average deviation of these differences is calculated.

4. This average deviation is divided by the sum of the average deviations of the sequences which are being compared. This last step makes the coefficients of several such comparisons comparable to each other.[1]

If the "cycle" movements of two time-ordered sequences are exactly alike, this coefficient is zero, for the differences of the "cycle" curves, in terms of standard or average deviations, are all zero. The coefficient increases in magnitude as the sequences that are being compared become more dissimilar. The upper limit of this coefficient is unity, and indicates maximum dissimilarity.

The advantages of using this coefficient as supplementary evidence of the comparative similarity of the "cycles" of time sequences instead of Persons' coefficient are:

1. Its derivation is completely divorced from the derivation of the coefficient of correlation, and, therefore, from those assumptions involved in the theory of correlation which are untenable in the case of time-ordered economic sequences.

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[1]The calculation of this coefficient is illustrated in Table 4.

TABLE 4

ILLUSTRATION OF THE CALCULATION OF THE COEFFICIENT OF CORRELATION (PERSONS') AND THE COEFFICIENT BASED ON AVERAGE DIFFERENCES BETWEEN "CYCLE" CURVES OF TWO TIME SEQUENCES

Year and Month	Harvard B-Curve*	Total Monthly K.W.H. Production*	Product 2 and 3	Differences 2 and 3	Year and Month	Harvard B-Curve*	Total Monthly K.W.H. Production*	Product 2 and 3	Differences 2 and 3
1919					*1923*				
Jan.	−.9	.8	−.72	−1.7	Jan.	.5	.5	.25	.0
Feb.	−1.1	.4	−.44	−1.5	Feb.	.7	.6	.42	.1
Mar.	−1.5	.2	−.30	−1.7	Mar.	.7	1.0	.70	−.3
Apr.	−1.2	.3	−.36	−1.5	Apr.	.7	.8	.56	−.1
May	−.4	1.3	−.52	−1.7	May	1.0	1.2	1.20	−.2
June	−.2	.9	−.18	−1.1	June	.6	.9	.54	−.3
July	.4	.6	.24	−.2	July	.2	.8	.16	−.6
Aug.	.5	1.3	.65	−.8	Aug.	.4	.7	.28	−.3
Sept.	.2	.5	.10	−.3	Sept.	−.1	.4	−.04	−.5
Oct.	.3	.4	.12	−.1	Oct.	−.1	.5	−.05	−.6
Nov.	.3	.2	.06	.1	Nov.	−.3	.3	−.09	−.6
Dec.	.8	.5	.40	.3	Dec.	−.2	−.1	.02	−.1
1920					*1924*				
Jan.	1.1	1.8	1.98	−.7	Jan.	−.1	.4	−.04	−.5
Feb.	1.0	1.7	1.70	−.7	Feb.	.2	1.0	.20	−.8
Mar.	1.2	1.8	2.16	−.6	Mar.	.1	.2	.02	−.1
Apr.	1.4	1.9	2.66	−.5	Apr.	−.1	.2	−.02	−.3
May	1.8	1.5	2.70	.3	May	−.3	.0	.0	−.3
June	1.9	1.6	3.04	.3	June	−.4	−.6	.24	.2
July	2.0	1.8	3.60	.2	July	−.4	−.7	.28	.3
Aug.	1.7	1.6	2.72	.1	Aug.	.1	−.8	−.08	.9
Sept.	1.5	1.5	2.25	.0	Sept.	−.1	−.2	.02	.1
Oct.	.8	.5	.40	.3	Oct.	.2	−.4	−.08	.6
Nov.	.4	.5	.20	−.1	Nov.	.6	−.5	−.30	1.1
Dec.	.0	−.1	−.00	.1	Dec.	.8	.3	.24	.5
1921					*1925*				
Jan.	−.6	−1.4	.84	.8	Jan.	1.4	.1	.14	1.3
Feb.	−.9	−1.8	1.62	.9	Feb.	1.3	−.2	−.26	1.5
Mar.	−1.3	−1.7	2.21	.4	Mar.	1.5	−.1	−.15	1.6
Apr.	−1.3	−1.7	2.21	.4	Apr.	1.0	.1	.10	.9
May	−1.1	−2.0	2.20	.9	May	1.0	.2	.20	.8
June	−1.1	−1.9	2.09	.8	June	1.2	.0	.0	1.2
July	−1.4	−1.9	2.66	.5	July	1.4	.4	.56	1.0
Aug.	−1.2	−1.7	2.04	.5	Aug.	1.5	−.1	−.15	1.6
Sept.	−1.1	−1.6	1.76	.5	Sept.	1.4	.2	.28	1.2
Oct.	−1.1	−2.1	2.31	1.0	Oct.	1.5	.5	.75	1.0
Nov.	−1.4	−1.5	2.10	.1	Nov.	1.4	.2	.28	1.2
Dec.	−1.4	−1.5	2.10	.1	Dec.	1.3	.5	.65	.8
1922					*1926*				
Jan.	−1.7	−1.9	3.23	.2	Jan.	1.7	.1	.17	1.6
Feb.	−1.3	−1.8	2.34	.5	Feb.	1.3	.2	.26	1.1
Mar.	−1.3	−1.3	1.69	.0	Mar.	1.1	.8	.88	.3
Apr.	−1.2	−1.5	1.80	.3	Apr.	1.1	.5	.55	.6
May	−.6	−.8	.48	.2	May	1.0	.2	.20	.8
June	−.4	−.5	.20	.1	June	1.0	.6	.60	.4
July	−.4	−.5	.20	.1	July	1.1	.6	.66	.5
Aug.	−.1	−.2	.02	.1	Aug.	.8	.6	.48	.2
Sept.	−.1	−.0	.00	−.1	Sept.	.7	1.1	.77	−.4
Oct.	−.1	−.3	.03	.2	Oct.	1.0	.6	.60	.4
Nov.	−.3	.3	−.09	−.6	Nov.	.6	.6	.36	.0
Dec.	.1	.3	.03	−.2	Dec.	....	....	....	....
					Totals	80.3**	75.5**	68.89	54.2**

*"Cycles" in units of standard deviations.

**Arithmetic Sum.

$$\text{Average Deviation of Harvard B-Curve} = \frac{80.3}{95} = .85$$

$$\text{Average Deviation of T.M.K.W.H.P.} = \frac{75.5}{95} = .79$$

$$\text{Coefficient of Correlation} = \frac{68.89}{95} = .73$$

$$\text{Coefficient based on differences} = \frac{54.2}{95(.85 + .79)} = .35$$

2. Unusual fluctuations of paired observations in the same direction do not unduly influence the magnitude of the coefficient, and thus indicate similarity between two sequences, which, in fact, is due to only a few sporadic fluctuations.

Table 3 compares Persons' coefficient of correlation for all of the comparisons which were made in the text of this bulletin with the coefficients calculated according to the above method. It can be seen from this table that the coefficient of correlation increases in magnitude when the coefficient based on the average differences of the cycle movements of two sequences decreases in magnitude. Both of these coefficients, however, supplement the argument and conclusions presented in the text of this bulletin.

The greatest similarity to the curve of the "general business cycle" is found to obtain in the "cycle" curve of Total Monthly K.W.H. Production for the United States. In this case, the coefficient of correlation has the highest value of all of the correlations (.73), while the coefficient based on the average differences between two sequences has the lowest value of all of the similar coefficients calculated (.35). The smallest of the individual companies which were compared with the "general business cycle" curve, and which was found to be least like this general curve, yields a correlation coefficient of only .04, the lowest of all of these coefficients. The coefficient based on the average differences between the "cycle" curve of this company and the "general business cycle" curve, however, has the highest numerical value of all of the similar coefficients calculated in this connection (.62).

The general argument presented in the text of this bulletin is, therefore, substantiated by this statistical evidence. The more restricted the universe from which the data are drawn, the smaller the company and the more circumscribed its markets, the less likely are these data to resemble the "general business cycle." Not much of value, therefore, can be expected from a comparison of the data of an individual company with general data unless the company making the comparison is sufficiently large to experience in its own business at least the greater part of those influences which determine the behavior of a "general curve."

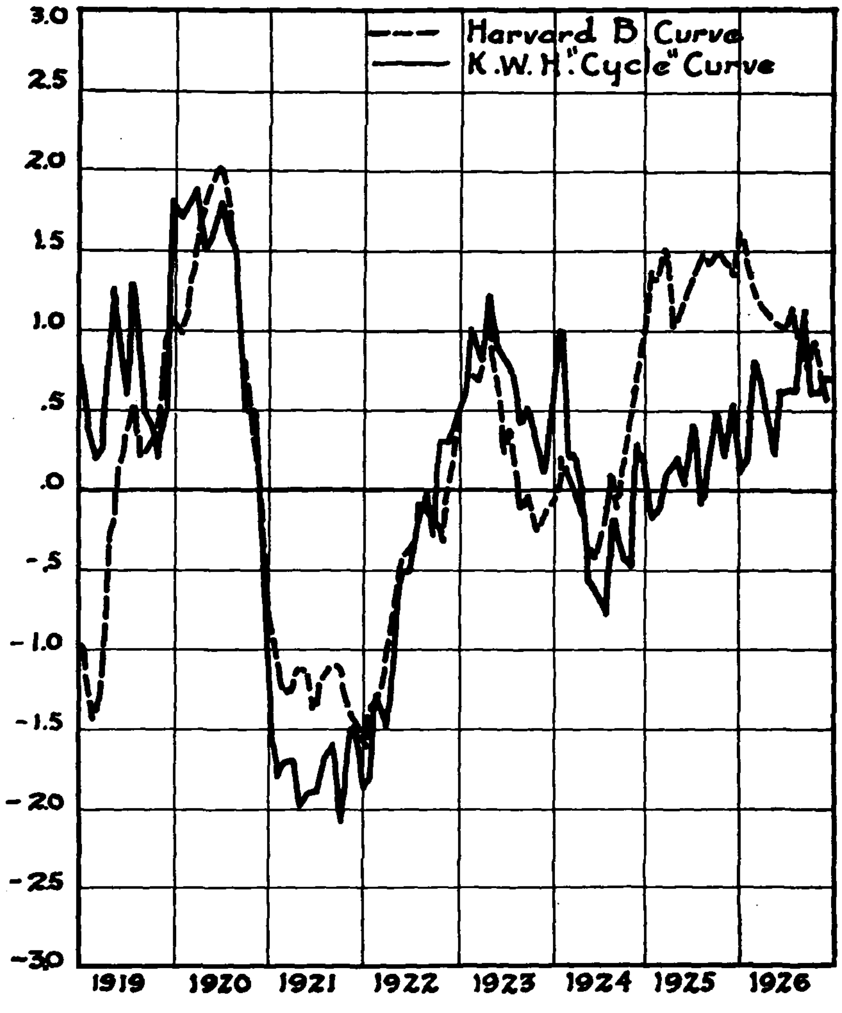

CHART I

COMPARISON OF THE TOTAL MONTHLY PRODUCTION OF ELECTRIC POWER IN THE UNITED STATES WITH THE GENERAL BUSINESS CYCLE

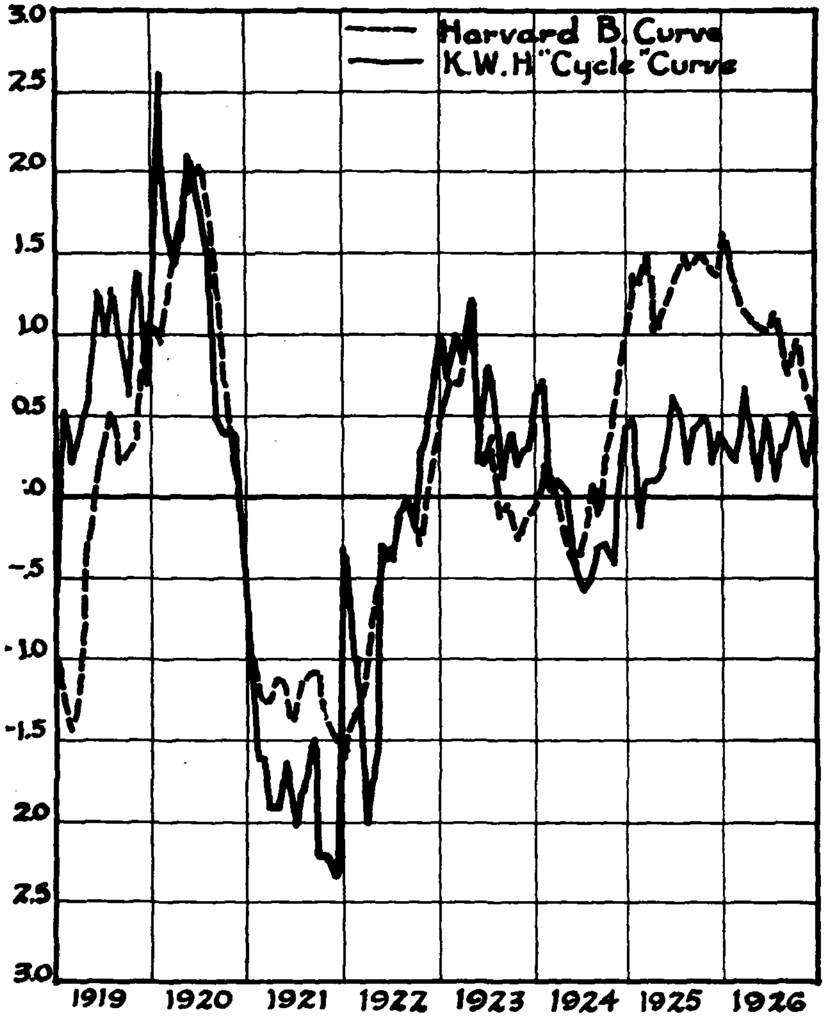

CHART II

COMPARISON OF THE TOTAL MONTHLY PRODUCTION OF ELECTRIC POWER IN THE EAST WITH THE GENERAL BUSINESS CYCLE

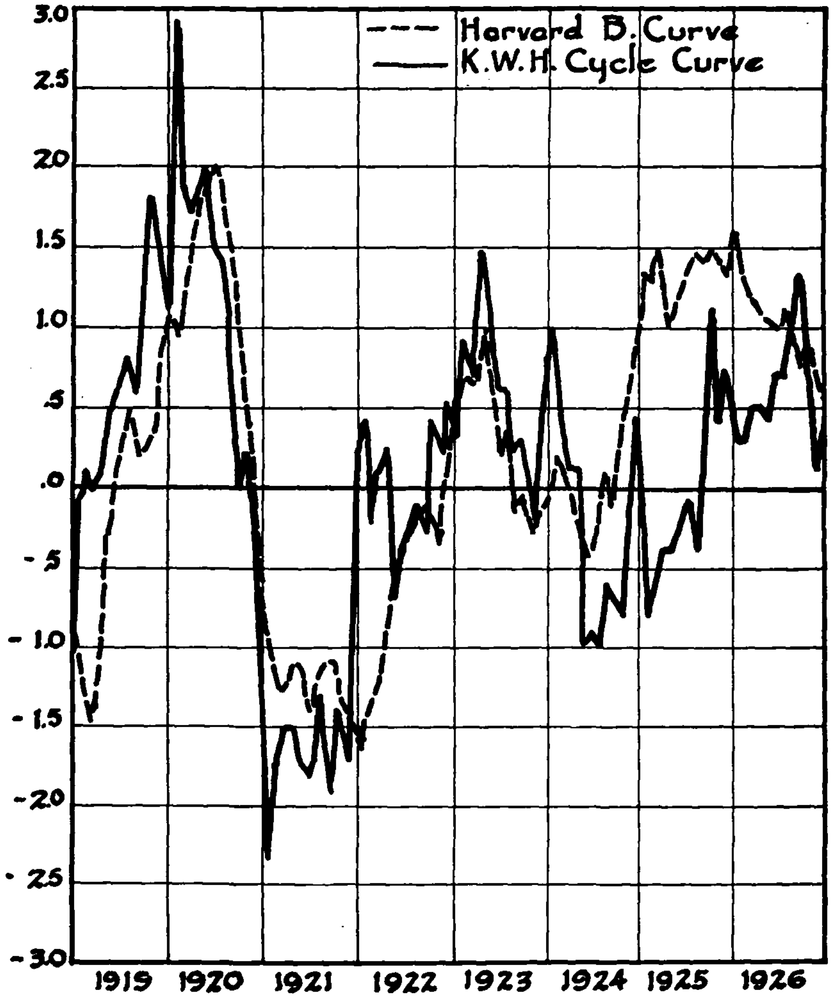

CHART III

COMPARISON OF THE TOTAL MONTHLY PRODUCTION OF ELECTRICAL POWER IN THE MIDDLE WEST WITH THE GENERAL BUSINESS CYCLE

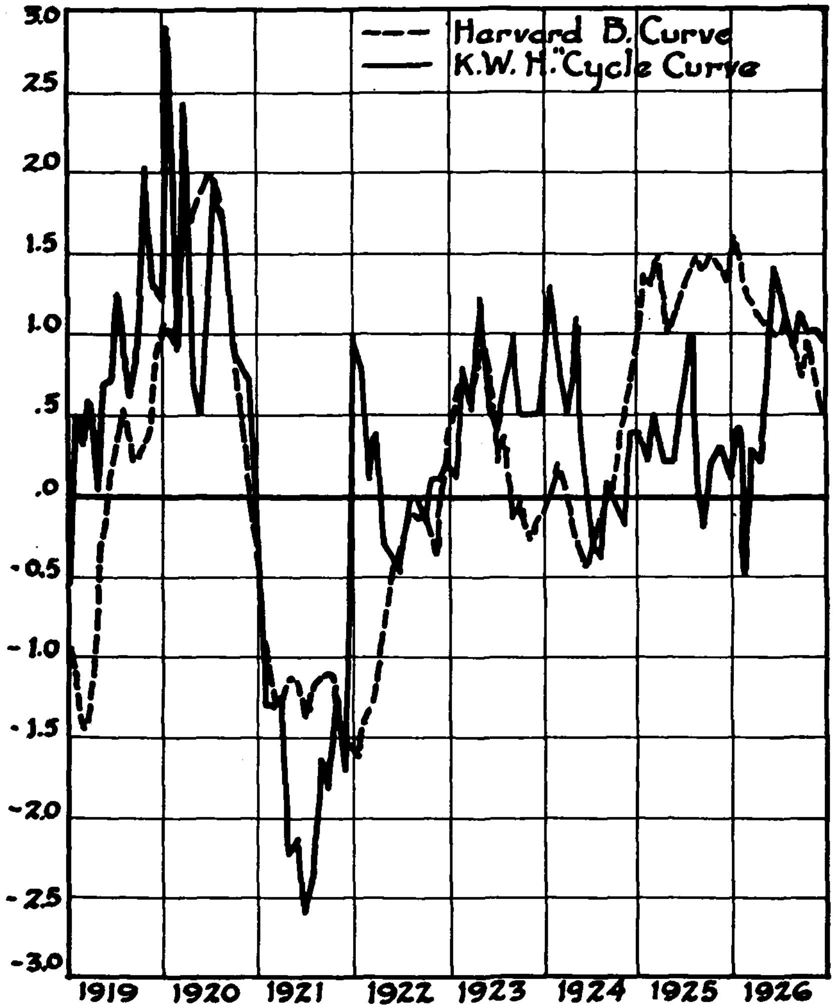

CHART IV

COMPARISON OF THE TOTAL MONTHLY PRODUCTION OF ELECTRICAL POWER IN THE WEST WITH THE GENERAL BUSINESS CYCLE

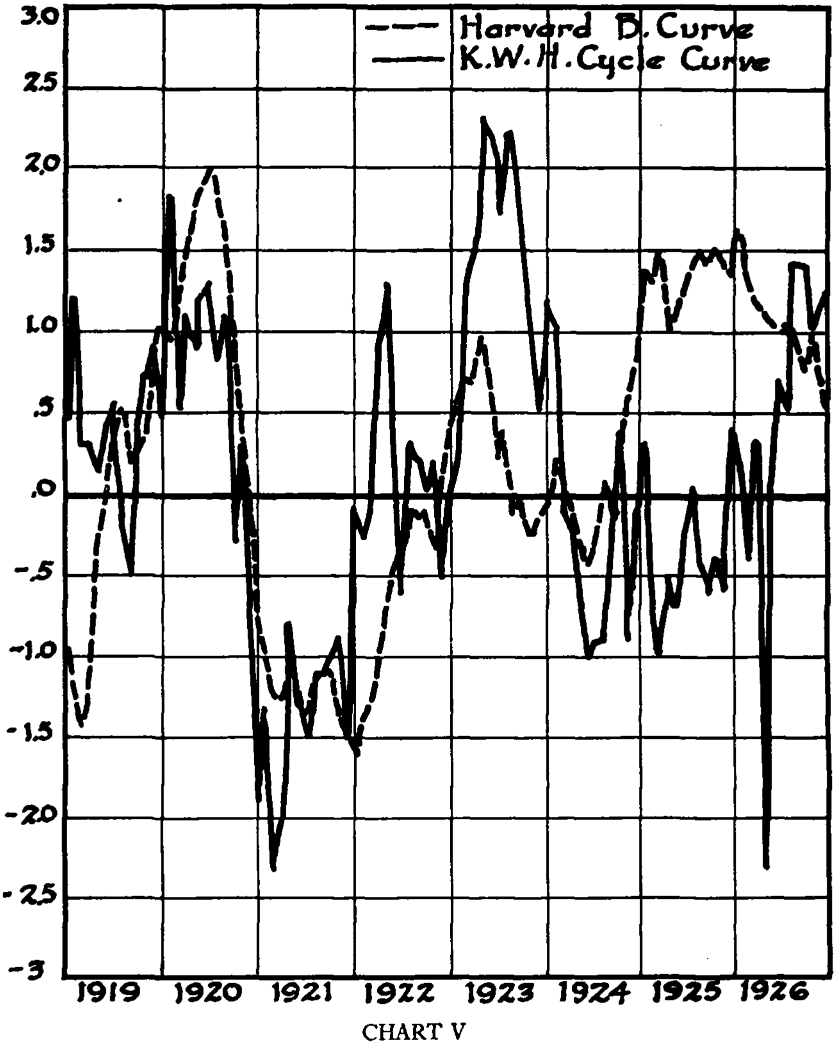

CHART V

COMPARISON OF THE TOTAL MONTHLY PRODUCTION OF ELECTRICAL POWER IN THE SOUTH WITH THE GENERAL BUSINESS CYCLE

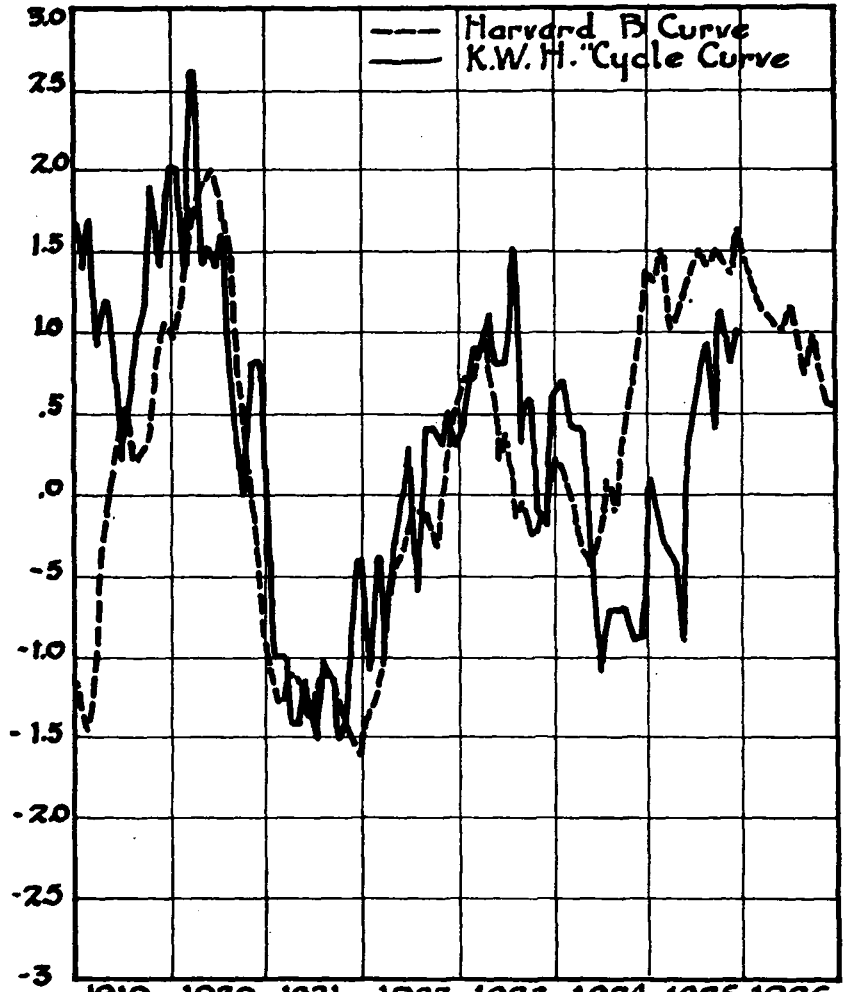

CHART VI

COMPARISON OF THE GENERAL BUSINESS CYCLE WITH THE TOTAL K.W.H. PRODUCTION OF A GROUP OF LARGE PUBLIC UTILITY COMPANIES

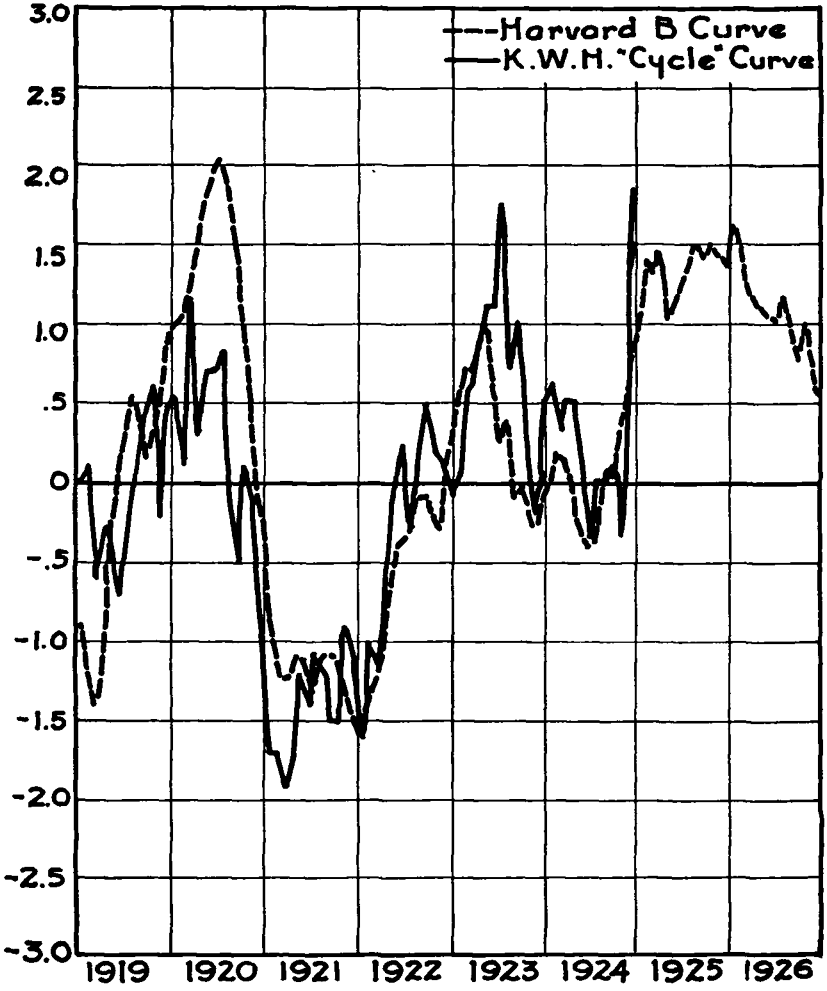

CHART VII

COMPARISON OF THE GENERAL BUSINESS CYCLE WITH THE TOTAL K.W.H. PRODUCTION OF A GROUP OF SMALL PUBLIC UTILITY COMPANIES

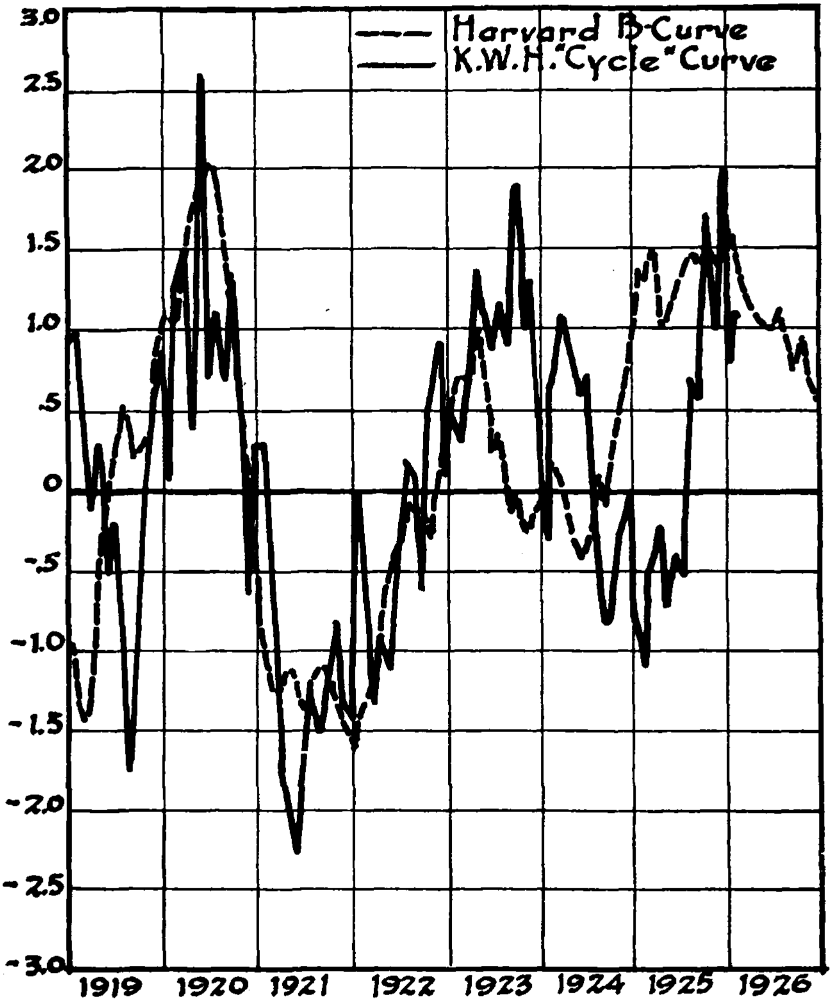

CHART VIII

COMPARISON OF THE GENERAL BUSINESS CYCLE WITH THE TOTAL K.W.H. PRODUCTION OF A LARGE INDIVIDUAL COMPANY

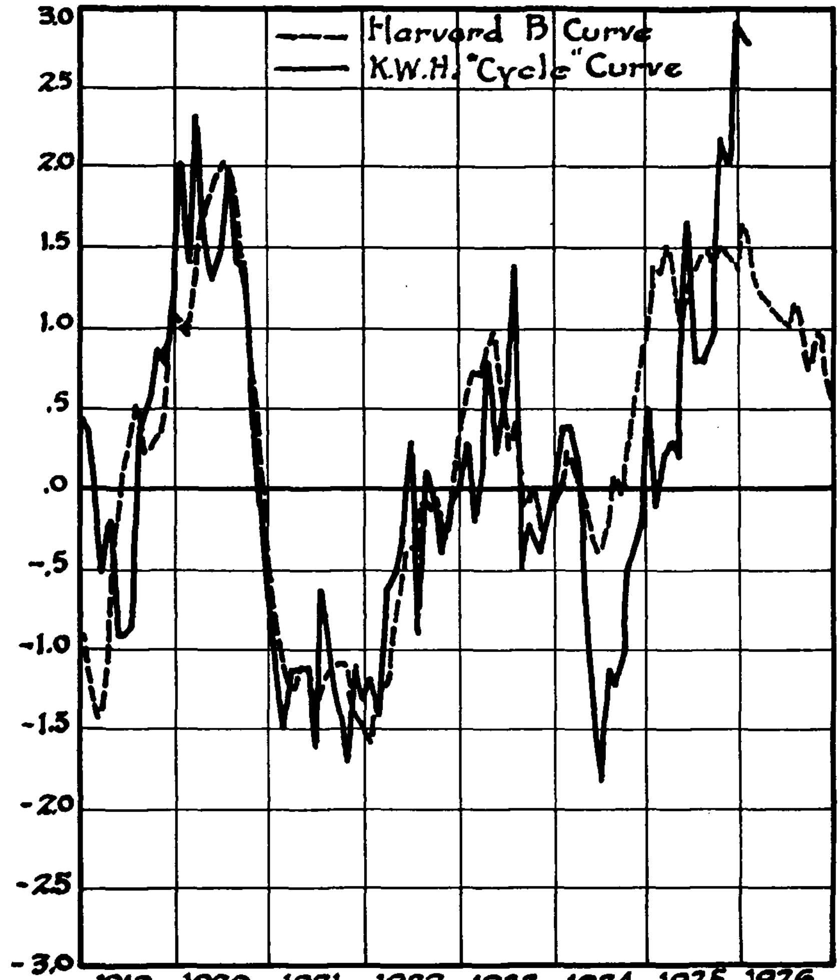

CHART IX

COMPARISON OF THE GENERAL BUSINESS CYCLE WITH THE TOTAL K.W.H. PRODUCTION OF A LARGE INDIVIDUAL COMPANY

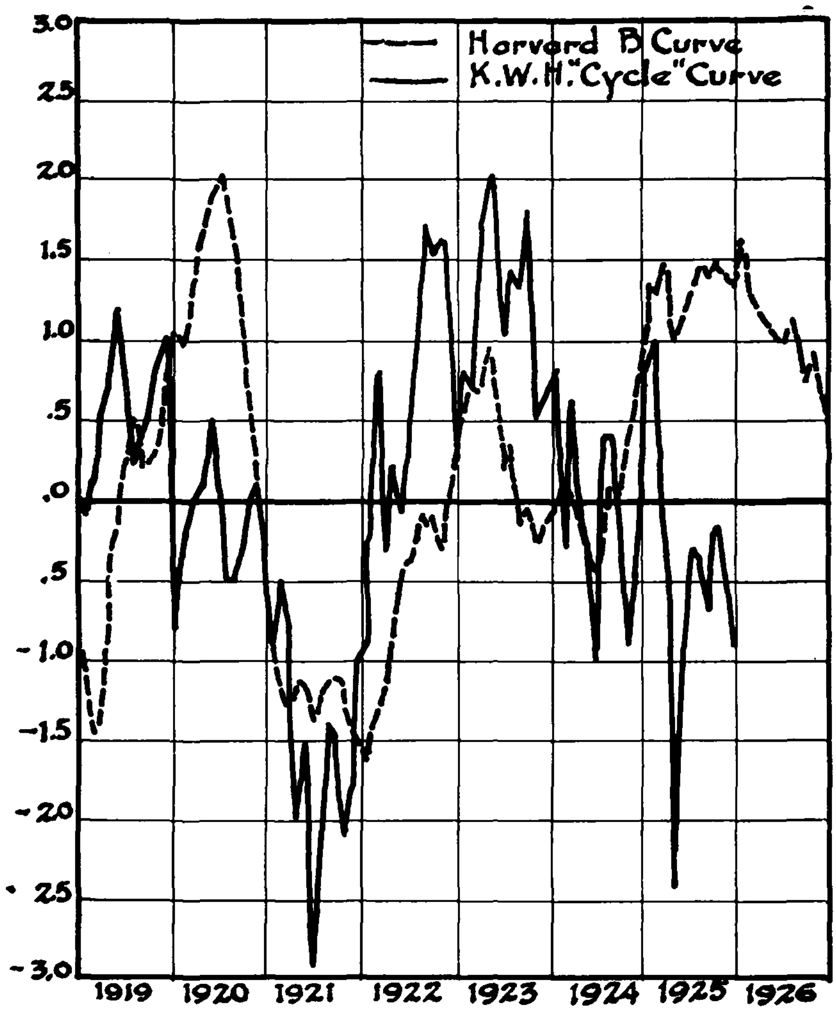

CHART X

COMPARISON OF THE GENERAL BUSINESS CYCLE WITH THE TOTAL K.W.H. PRODUCTION OF A LARGE INDIVIDUAL COMPANY

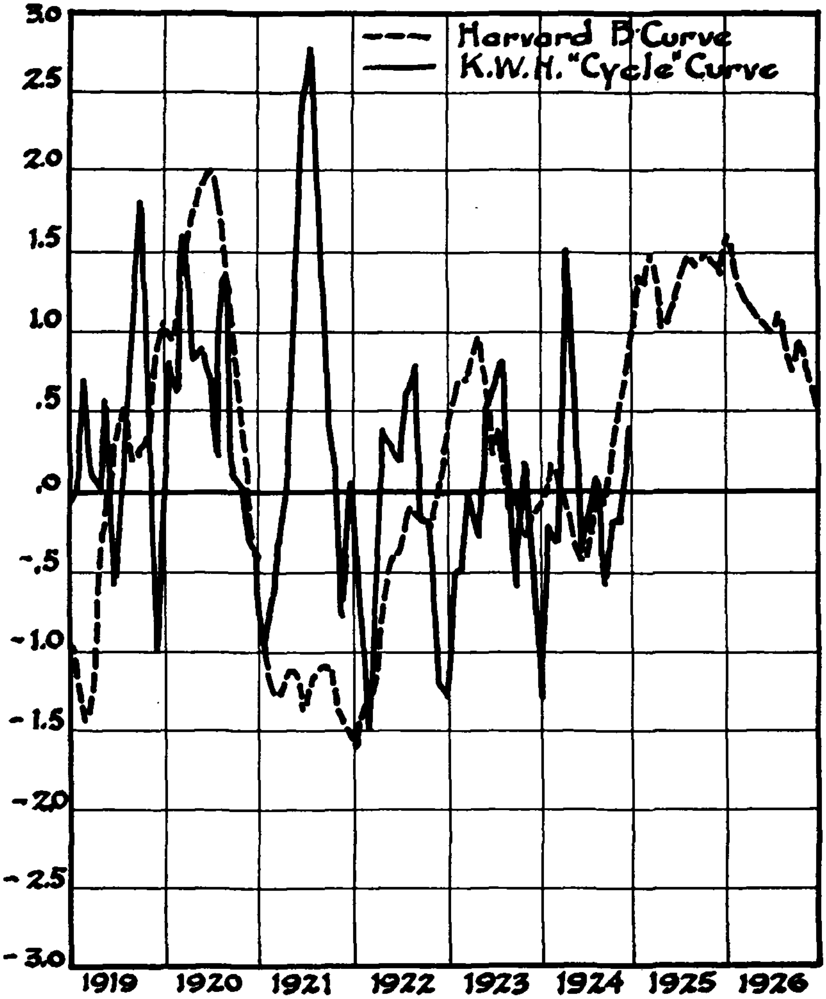

CHART XI

COMPARISON OF THE GENERAL BUSINESS CYCLE WITH THE TOTAL K.W.H. PRODUCTION OF A SMALL INDIVIDUAL COMPANY

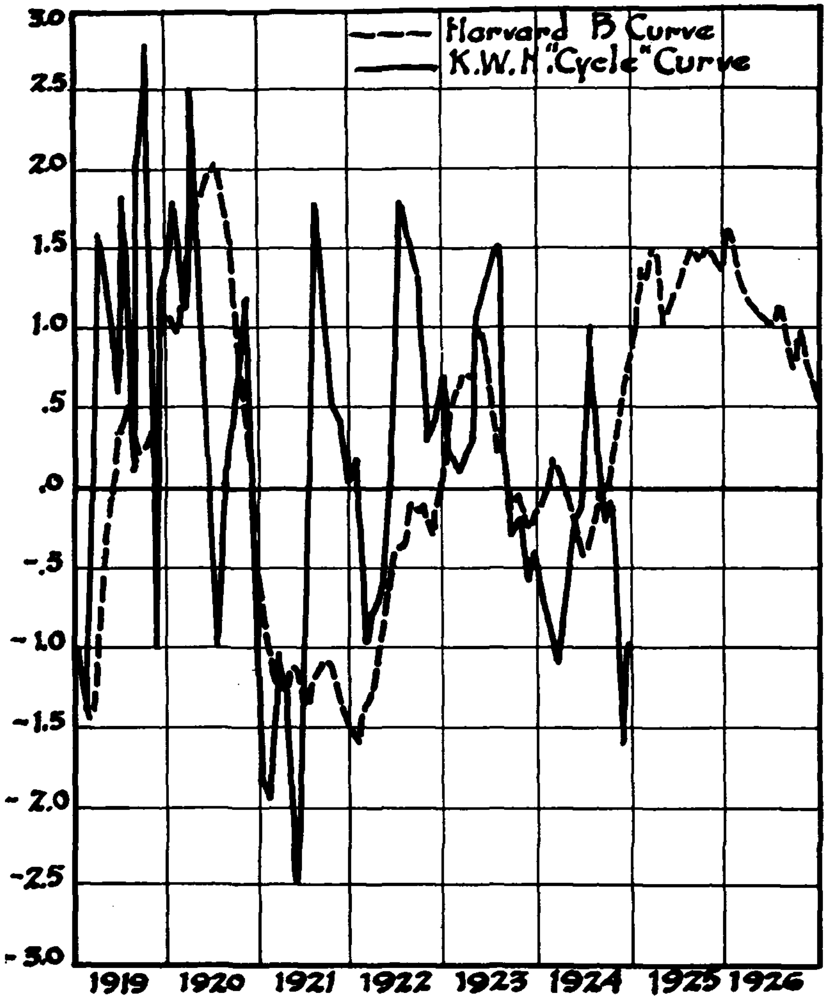

CHART XII

COMPARISON OF THE GENERAL BUSINESS CYCLE WITH THE TOTAL K.W.H. PRODUCTION OF A SMALL INDIVIDUAL COMPANY

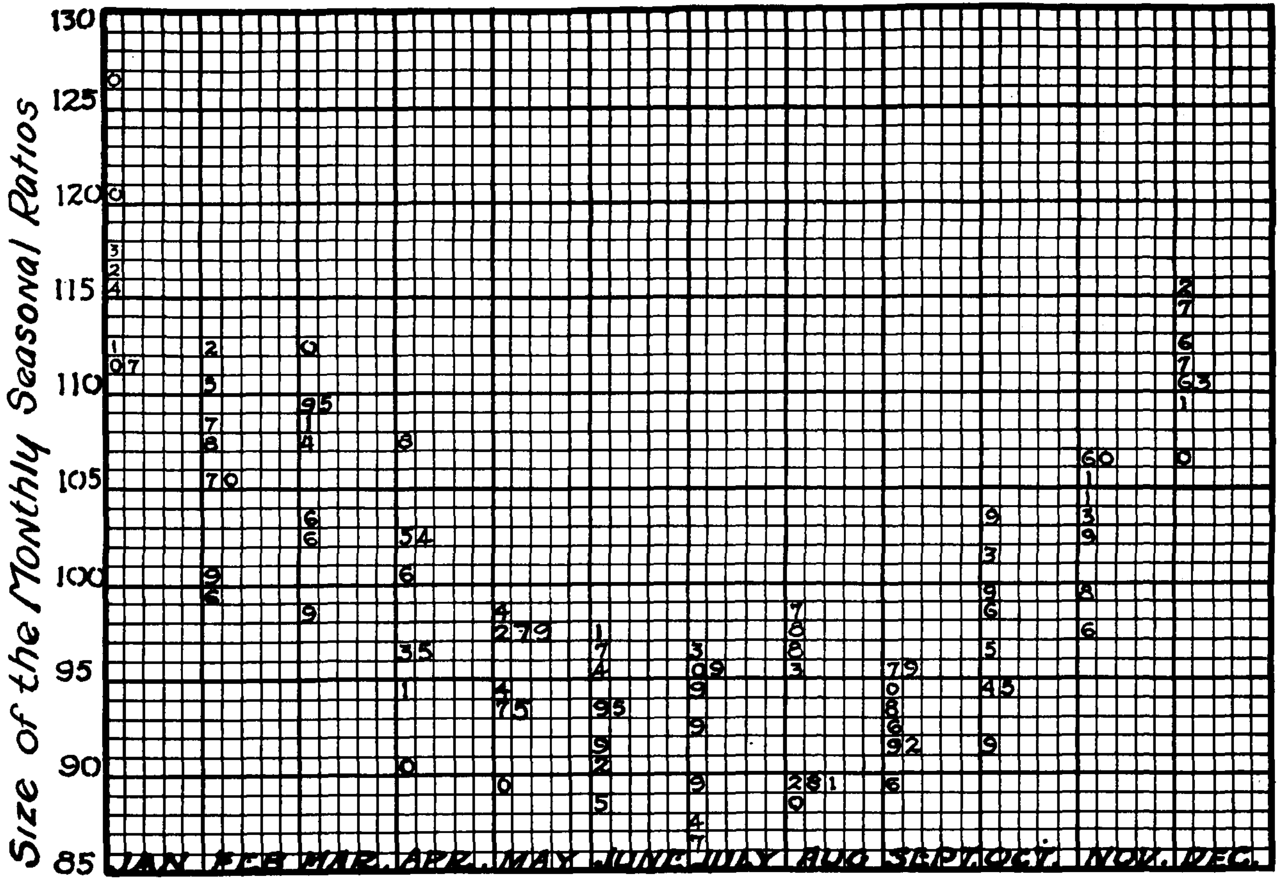

CHART XIII
THE GRAPHIC INDICATION OF SEASONAL VARIATION

APPENDIX D

TABLE I

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART I

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	3340	.8	3558	−1.4	4754	.5	5572	.1
Feb.	2976	.4	3166	−1.8	4324	.6	4982	− .2
Mar.	3138	.2	3394	−1.7	4728	1.0	5362	− .1
Apr.	3015	.3	3239	−1.7	4473	.8	5159	.1
May	3246	1.3	3264	−2.0	4653	1.2	5185	.2
June	3140	.9	3244	−1.9	4523	.9	5204	.0
July	3129	.6	3270	−1.9	4536	.8	5338	.4
Aug.	3343	1.3	3411	−1.7	4670	.7	5367	− .1
Sept.	3166	.5	3375	−1.6	4535	.4	5386	.2
Oct.	3419	.4	3574	−2.1	4950	.5	5932	.5
Nov.	3327	.2	3640	−1.5	4834	.3	5772	.2
Dec.	3535	.5	3820	−1.5	4947	− .1	6108	.5
	1920		*1922*		*1924*		*1926*	
Jan.	3856	1.8	3805	−1.9	5189	.4	6117	.1
Feb.	3480	1.7	3468	−1.8	4834	1.0	5593	.2
Mar.	3745	1.8	3821	−1.3	4981	.2	6149	.8
Apr.	3577	1.9	3596	−1.5	4751	.2	5774	.5
May	3583	1.5	3824	− .8	4797	.0	5806	.2
June	3569	1.6	3836	− .5	4579	− .6	5879	.6
July	3626	1.8	3871	− .5	4599	− .7	5914	.6
Aug.	2717	1.6	4075	− .2	4734	− .8	6120	.6
Sept.	3631	1.5	4049	.0	4806	− .2	6176	1.1
Oct.	3752	.5	4330	− .3	5187	− .4	6559	.6
Nov.	3705	.5	4414	.3	5065	− .5	6445	.6
Dec.	3761	− .1	4612	.3	5545	.3	6767	.7

*000,000 omitted.

TABLE II

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART II

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	1145	− .6	1339	−1.0	1812	1.0	2102	.5
Feb.	1135	.5	1195	−1.6	1651	.7	1856	− .3
Mar.	1186	.2	1259	−1.6	1777	1.0	2013	.1
Apr.	1133	.4	1168	−1.9	1656	.8	1908	.1
May	1136	.6	1157	−1.9	1684	1.2	1902	.2
June	1138	1.3	1139	−1.6	1540	.2	1877	.6
July	1139	1.0	1134	−2.0	1620	.8	1910	.5
Aug.	1205	1.3	1187	−1.8	1651	.5	1939	.2
Sept.	1182	.9	1220	−1.5	1621	.1	1972	.4
Oct.	1281	.6	1286	−2.2	1818	.4	2189	.5
Nov.	1326	1.4	1283	−2.2	1792	.2	2145	.2
Dec.	1363	.7	1343	−2.3	1845	−.3	2296	.4
	1920		*1922*		*1924*		*1926*	
Jan.	1400	1.3	1527	− .3	1933	.6	2275	.3
Feb.	1400	2.6	1356	−1.0	1811	.7	2099	.2
Mar.	1400	1.6	1433	−1.0	1834	.0	2287	.7
Apr.	1312	1.4	1273	−2.0	1742	.1	2115	.3
May	1312	1.6	1305	−1.5	1718	.0	2071	.1
June	1312	2.1	1359	− .3	1615	− .4	2048	.5
July	1309	1.8	1373	− .4	1630	− .6	2041	.1
Aug.	1333	1.5	1450	− .1	1692	− .5	2141	.3
Sept.	1262	.5	1466	.0	1731	− .3	2169	.5
Oct.	1383	.4	1602	− .2	1907	− .3	2360	.3
Nov.	1382	.4	1654	.3	1888	− .4	2358	.2
Dec.	1405	− .3	1767	.6	2099	.4	2528	.5

*000,000 omitted.

TABLE III

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART III

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	896	−1.5	1061	−1.8	1436	.5	1677	− .1
Feb.	885	− .1	951	−2.3	1309	.3	1495	− .8
Mar.	940	.1	1028	−1.7	1415	.9	1586	− .6
Apr.	893	.0	990	−1.5	1338	.7	1529	− .4
May	893	.1	987	−1.5	1372	1.4	1514	− .4
June	900	.5	966	−1.7	1333	.9	1520	− .2
July	918	.6	972	−1.8	1328	.6	1547	− .1
Aug.	965	.8	1021	−1.7	1390	.6	1586	− .4
Sept.	958	.6	1048	−1.3	1365	.2	1634	.1
Oct.	1072	1.1	1108	−1.9	1498	.3	1879	1.1
Nov.	1101	1.8	1127	−1.4	1462	.0	1798	.4
Dec.	1125	1.5	1150	−1.7	1494	− .2	1884	.7
	1920		*1922*		*1924*		*1926*	
Jan.	1122	1.1	1295	.2	1598	.8	1878	.3
Feb.	1122	2.9	1200	.4	1490	1.0	1731	.3
Mar.	1122	1.9	1224	− .2	1513	.4	1831	.5
Apr.	1063	1.7	1185	.1	1424	.1	1752	.5
May	1063	1.8	1184	.2	1416	.1	1727	.4
June	1063	2.0	1148	− .7	1327	−1.0	1746	.7
July	1056	1.5	1152	− .4	1355	− .9	1767	.7
Aug.	1094	1.4	1218	− .2	1404	−1.0	1868	1.0
Sept.	1076	1.0	1222	− .1	1435	− .6	1897	1.3
Oct.	1118	.0	1323	− .3	1559	− .7	2018	.7
Nov.	1117	.2	1356	.4	1536	− .8	1944	.1
Dec.	1140	− .1	1391	.2	1596	.4	2036	.4

*000,000 omitted.

TABLE IV

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART IV

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	523	− .7	635	− .3	771	.2	923	.4
Feb.	512	.5	554	−1.3	708	.1	843	.2
Mar.	559	.3	609	−1.3	806	.8	943	.5
Apr.	557	.6	603	−1.2	782	.5	911	.2
May	587	.0	615	−2.2	873	1.2	981	.2
June	615	.7	629	−2.1	855	.5	1008	.4
July	636	.7	632	−2.6	882	.4	1074	1.0
Aug.	655	1.2	643	−2.3	895	.6	1033	.1
Sept.	595	.6	628	−1.6	856	1.0	954	− .2
Oct.	612	.9	629	−1.8	845	.5	983	.2
Nov.	620	1.9	629	−1.2	815	.5	952	.3
Dec.	625	1.3	637	−1.7	845	.5	980	.1
	1920		*1922*		*1924*		*1926*	
Jan.	627	1.2	736	1.0	864	.8	1006	.4
Feb.	627	2.9	674	.8	817	1.3	884	− .5
Mar.	627	.9	716	.1	873	.7	1013	.3
Apr.	660	2.4	712	.4	851	.5	994	.2
May	660	.7	744	− .3	943	1.1	1100	.7
June	660	.5	751	− .4	919	.3	1150	1.4
July	709	1.2	773	− .5	924	− .3	1179	1.1
Aug.	738	1.9	797	.0	922	− .4	1169	.9
Sept.	679	1.6	745	.0	888	.1	1107	1.1
Oct.	667	.9	748	− .2	894	.0	1114	1.0
Nov.	639	.8	734	.1	853	− .2	1078	1.0
Dec.	660	.7	761	.1	915	.4	1111	.9

*000,000 omitted.

TABLE V

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART V

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan	445	.0	506	−1.9	690	.0	870	.3
Feb	444	1.2	467	−1.3	656	.2	788	−.6
Mar	453	.3	499	−2.3	729	1.3	819	−1.0
Apr	428	.3	479	−2.0	697	1.5	793	−.5
May	428	.1	510	−.8	725	2.3	793	−.7
June	428	.4	492	−1.3	713	2.2	799	−.3
July	436	.6	492	−1.5	706	1.7	815	.0
Aug	430	−.2	513	−1.1	739	2.2	818	−.4
Sept	431	−.5	520	−1.1	728	1.6	825	−.6
Oct	485	.4	565	−1.0	785	1.5	898	−.4
Nov	491	.7	569	−.9	765	.9	892	−.6
Dec	501	.9	561	−1.5	762	.5	947	.4
	1920		*1922*		*1924*		*1926*	
Jan	506	.4	615	−.1	808	1.2	962	.2
Feb	506	1.8	575	−.3	759	1.0	884	−.4
Mar	508	.5	616	−.1	766	−.1	970	.3
Apr	495	1.1	608	.8	722	−.2	811	−2.3
May	495	.9	625	1.3	716	−.5	911	.0
June	495	1.2	576	−.3	693	−1.0	931	.7
July	501	1.3	573	−.6	703	−.9	931	.5
Aug	502	.8	610	.3	718	−.9	990	1.4
Sept	514	1.1	616	.2	749	−.3	1003	1.4
Oct	524	−.3	659	.0	833	.4	1067	1.0
Nov	538	.3	666	.2	788	−.9	1071	1.1
Dec	531	−.3	655	−.5	827	−.1	1093	1.3

*000,000 omitted.

TABLE VI

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART VI

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan	17352	2.1	20264	.1	26277	.3	33207	.1
Feb	15871	1.4	18191	−1.0	24925	.5	31095	−.1
Mar	16359	1.7	18450	−1.0	25907	.9	31283	−.3
Apr	15271	.9	17439	−1.4	24966	.9	30018	−.4
May	15309	1.2	17266	−1.4	24867	1.1	28961	−.9
June	14873	.8	17404	−1.1	24462	.8	30580	.3
July	14416	.2	16982	−1.5	24320	.8	30792	.6
Aug	14663	.5	17445	−1.0	25134	1.5	31433	.9
Sept	15466	.9	17996	−1.1	24668	.3	31601	.4
Oct	16440	1.2	18536	−1.5	26233	.6	34437	1.1
Nov	17865	1.9	19571	−1.4	26695	−.1	35639	.8
Dec	18492	1.4	21851	−.4	28158	−.2	38125	1.0
	1920		*1922*		*1924*		*1926*	
Jan	19537	2.0	22372	−.4	30098	.6		
Feb	18456	2.0	20460	−1.1	28450	.7		
Mar	18245	1.4	21467	−.4	28518	.4		
Apr	18605	2.6	20293	−.9	27542	.4		
May	17439	1.4	20670	−.3	27067	.2		
June	17435	1.5	20712	−.1	25716	−.7		
July	17250	1.4	20907	.3	25110	−1.1		
Aug	17485	1.6	20382	−.6	25674	−.7		
Sept	17276	.6	21816	.4	26560	−.7		
Oct	17615	.0	23004	.4	27878	−.7		
Nov	19249	.8	24038	.3	29075	−.9		
Dec	20355	.8	25703	.5	30877	−.9		

*0000 omitted.

TABLE VII

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART VII

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	17991	.2	21071	−.9	27381	−.1		
Feb.	16719	.0	18945	−1.7	25974	.0		
Mar.	17007	.1	19194	−1.7	26957	.5		
Apr.	15722	−.6	18192	−1.9	25964	.6		
May	15631	−.3	17999	−1.7	25890	.9		
June	15227	−.5	18186	−1.2	25573	1.1		
July	14919	−.7	17782	−1.4	25406	1.1		
Aug.	15368	−.3	18230	−1.1	26289	1.7		
Sept.	16195	.1	18798	−1.2	25812	.7		
Oct.	17204	.4	19326	−1.5	27503	1.0		
Nov.	18448	.6	20447	−1.5	28016	.3		
Dec.	19042	−.2	22747	−.9	29471	−.2		
	1920		*1922*		*1924*		*1926*	
Jan.	20247	.4	23273	−1.1	31513	.5		
Feb.	19153	.5	21247	−1.6	29786	.6		
Mar.	18937	.1	22281	−1.0	29780	.3		
Apr.	19271	1.2	21132	−1.2	28768	.5		
May	18074	.3	21483	−.5	28249	.5		
June	18041	.7	21548	−.1	26959	.0		
July	17881	.7	21733	.2	26248	−.4		
Aug.	18149	.8	21258	−.3	26887	.0		
Sept.	17975	.0	22749	.3	27804	.0		
Oct.	18343	−.5	23975	.5	29224	.1		
Nov.	20011	.1	25063	.2	30470	−.3		
Dec.	21162	−.2	26748	.1	42345	5.2		

*0000 omitted.

TABLE VIII

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART VIII

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	13394	.4	15706	−.8	20064	.3	23936	−.5
Feb.	12305	−.1	14235	−1.6	19080	.7	22624	−.2
Mar.	12617	.3	14270	−1.8	19841	1.4	22528	−.7
Apr.	11414	−.5	13107	−2.2	18300	1.1	21192	−.4
May	11291	−.2	13082	−1.7	17762	.9	20710	−.5
June	10896	−.8	13128	−1.2	17690	1.2	21341	.7
July	10565	−1.7	13073	−1.5	17615	.9	21420	.6
Aug.	10974	−.8	13292	−1.1	18293	1.9	22261	1.7
Sept.	11487	.1	13648	−.8	17912	1.0	22042	1.0
Oct.	12347	.6	14131	−1.3	19119	1.3	24076	2.0
Nov.	13456	.9	15135	−1.4	19602	.0	24786	.8
Dec.	14085	.1	17256	.0	20964	−.3	27158	1.1
	1920		*1922*		*1924*		*1926*	
Jan.	15329	1.3	17520	−.6	22601	.7		
Feb.	14466	1.5	15980	−1.3	21476	1.1		
Mar.	14067	.4	16401	−.9	21493	.8		
Apr.	14164	2.6	15191	−1.1	19940	.6		
May	12950	.7	15258	−.4	19580	.7		
June	12943	1.1	15358	.2	18539	−.3		
July	12835	.7	15400	.1	18308	−.8		
Aug.	13186	1.4	15025	−.6	18315	−.8		
Sept.	12889	.4	15871	.5	18912	−.3		
Oct.	13102	−.6	17011	.9	20190	.0		
Nov.	14566	.3	17740	.1	21058	−.8		
Dec.	15772	.3	19503	.5	22561	−1.1		

*0000 omitted.

TABLE IX

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART IX

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	1758	.5	2121	-.5	2792	.0	3715	.5
Feb.	1616	.4	1866	-1.1	2629	.3	3285	-.1
Mar.	1642	.1	1869	-1.5	2634	-.2	3472	.2
Apr.	1557	-.5	1910	-1.1	2671	.2	3441	.3
May	1588	-.2	1914	-1.1	2787	.8	3419	.2
June	1577	-.9	1995	-1.1	2789	.2	3915	1.7
July	1601	-.9	1958	-1.6	2923	.7	3790	.8
Aug.	1639	-.8	2127	-.6	3092	1.4	3831	.8
Sept.	1879	.4	2161	-1.2	2908	-.5	4114	1.0
Oct.	1943	.6	2184	-1.4	3039	-.2	4513	2.2
Nov.	2069	.9	2206	-1.7	3107	-.4	4647	2.0
Dec.	2007	.8	2256	-1.1	3078	-.2	4780	2.9
	1920		*1922*		*1924*		*1926*	
Jan.	2082	1.2	2258	-1.3	3173	.0		
Feb.	2010	2.0	2090	-1.2	2997	.4		
Mar.	2013	1.4	2141	-1.4	3088	.4		
Apr.	2100	2.3	2229	-.6	3029	.2		
May	2018	1.6	2259	-.5	2922	-.3		
June	2058	1.3	2387	-.3	2849	-1.3		
July	2121	1.5	2518	.3	2789	-1.8		
Aug.	2224	2.0	2350	-.9	2980	-1.1		
Sept.	2265	1.4	2672	.1	3131	-1.2		
Oct.	2320	1.4	2713	-.1	3238	-1.0		
Nov.	2307	.7	2757	-.4	3524	-.4		
Dec.	2143	-.1	2742	-.1	3502	-.2		

*0000 omitted.

TABLE X

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART X

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	2200	.1	2437	-.3	3421	.3	5556	.8
Feb.	1950	-.1	2090	-.9	3216	.8	5186	1.0
Mar.	2100	.1	2311	-.5	3432	.7	5283	.2
Apr.	2300	.5	2422	-.8	3995	1.7	5385	-.5
May	2430	.7	2270	-2.0	4318	2.0	4832	-2.4
June	2400	1.2	2281	-1.5	3983	1.5	5324	-1.0
July	2250	.7	1951	-2.9	3782	1.0	5582	-.3
Aug.	2050	.2	2026	-2.0	3749	1.4	5341	-.4
Sept.	2100	.3	2187	-1.4	3848	1.3	5445	-.7
Oct.	2150	.5	2221	-1.5	4075	1.8	5848	-.2
Nov.	2340	.8	2230	-2.1	3986	.5	6206	-.4
Dec.	2400	1.0	2339	-1.8	4116	.6	6187	-.9
	1920		*1922*		*1924*		*1926*	
Jan.	2126	-.8	2594	-1.0	4324	.7		
Feb.	1980	-.5	2390	-.9	3977	.8		
Mar.	2165	-.2	2925	.8	3937	-.3		
Apr.	2341	.0	2873	-.3	4573	.6		
May	2471	.1	3153	.2	4565	-.1		
June	2434	.5	2967	-.1	4328	-.3		
July	2294	.1	2989	.3	4016	-1.0		
Aug.	2075	-.5	3007	.9	4379	.4		
Sept.	2122	-.5	3273	1.7	4517	.4		
Oct.	2193	-.3	3280	1.5	4450	-.2		
Nov.	2376	.0	3541	1.6	4493	-.9		
Dec.	2440	.1	3458	.9	4814	-.5		

*0000 omitted.

TABLE XI

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART XI

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	625	− .1	811	− .4	966	−1.3		
Feb.	578	.0	685	−1.0	986	− .5		
Mar.	608	.7	691	− .7	957	− .5		
Apr.	589	.1	752	− .3	1039	.0		
May	567	.0	790	.3	985	− .3		
June	691	.6	1066	1.9	1229	.5		
July	623	− .6	1149	2.4	1280	.6		
Aug.	659	− .1	1180	2.8	1301	.8		
Sept.	721	.6	1118	2.0	1252	.2		
Oct.	808	1.8	941	.5	1117	− .6		
Nov.	699	− .1	932	− .1	1304	.2		
Dec.	666	−1.0	918	− .8	1272	− .6		
	1920		*1922*		*1924*		*1926*	
Jan.	746	.1	983	.0	1119	−1.3		
Feb.	729	.8	843	− .5	1178	− .2		
Mar.	693	.6	716	−1.5	1130	− .3		
Apr.	796	1.6	843	− .5	1427	1.5		
May	715	.8	917	.4	1243	.3		
June	832	.9	1035	.3	1284	− .4		
July	836	.7	1054	.2	1384	− .1		
Aug.	785	.2	1098	.6	1408	.1		
Sept.	903	1.3	1142	.8	1321	− .6		
Oct.	781	.1	1003	− .2	1361	− .2		
Nov.	821	.0	1071	− .2	1444	− .2		
Dec.	849	− .3	1009	−1.2	1654	.4		

*000 omitted.

TABLE XII

ACTUAL DATA AND CALCULATED CYCLE USED IN THE PRODUCTION CURVE SHOWN IN CHART XII

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	215	− .9	254	−1.8	465	.2		
Feb.	193	−1.4	237	−2.0	453	.1		
Mar.	254	.2	289	−1.0	470	.2		
Apr.	271	1.6	248	−1.3	428	.3		
May	275	1.4	208	−2.5	443	.3		
June	244	.6	216	−2.1	482	1.1		
July	281	1.8	312	.1	496	1.3		
Aug.	235	.1	392	1.8	514	1.5		
Sept.	331	2.1	426	1.6	490	.2		
Oct.	389	2.8	408	.5	491	− .3		
Nov.	233	−1.0	400	.4	492	− .2		
Dec.	371	1.2	424	− .1	535	− .6		
	1920		*1922*		*1924*		*1926*	
Jan.	348	1.5	399	.2	500	− .4		
Feb.	352	1.8	330	−1.0	468	− .7		
Mar.	332	1.1	350	− .8	451	−1.1		
Apr.	350	2.5	323	− .6	437	− .6		
May	317	1.3	368	.0	487	− .2		
June	265	1.1	393	.7	478	− .1		
July	229	−1.0	448	1.8	556	1.0		
Aug.	269	.0	444	1.5	518	.3		
Sept.	312	.3	481	1.3	538	− .2		
Oct.	355	.7	459	.3	593	− .1		
Nov.	370	1.2	456	.4	453	−1.6		
Dec.	317	−1.0	548	.7	590	−1.0		

*000 omitted.

TABLE XIII

ACTUAL DATA AND CALCULATED CYCLE OF MONTHLY PRODUCTION DATA OF A SMALL COMPANY

Month	Actual*	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan	1433	− .5	2210	.4	2508	.1		
Feb	1868	.9	2128	.1	1910	−1.3		
Mar	1917	1.5	2144	.6	1698	−1.4		
Apr	1002	− .8	1836	1.2	1568	− .7		
May	840	−1.3	1468	.3	1643	.0		
June	1150	.1	1682	1.0	1614	− .2		
July	926	− .9	1608	.8	1721	.3		
Aug	1092	− .9	1841	.7	1936	.1		
Sept	1459	.1	1911	.3	2004	− .2		
Oct	1409	− .4	1776	− .4	2174	− .1		
Nov	1784	− .1	2083	− .5	2498	− .2		
Dec	1546	−1.1	2235	− .4	2816	.2		
	1920		*1922*		*1924*		*1926*	
Jan	1819	− .1	2202	− .2	2513	− .1		
Feb	1972	.4	2355	.2	2270	− .7		
Mar	1520	− .7	1789	− .9	1986	− .9		
Apr	1623	1.1	1528	− .5	1836	− .1		
May	1586	1.5	1290	− .9	1845	.5		
June	1549	1.2	1573	.0	1655	− .3		
July	1690	1.9	1388	− .5	1580	− .4		
Aug	1764	1.1	1859	.2	1786	− .5		
Sept	2261	2.1	2163	.5	2128	− .1		
Oct	2488	2.4	2137	.2	2505	.6		
Nov	2250	.5	2259	− .5	3436	1.6		
Dec	2510	.9	2666	.2	3404	1.2		

*0 omitted.

TABLE XIV

ACTUAL DATA AND CALCULATED CYCLE OF THE MONTHLY GROSS REVENUES OF A SMALL COMPANY

Month	Actual	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan	9092	.1	11483	− .4	15203	− .1		
Feb	7956	− .9	10171	−1.2	15293	.8		
Mar	8901	.4	10613	− .8	15462	.8		
Apr	8108	.4	9315	−1.3	13807	.5		
May	8013	.4	8343	−2.4	13596	.5		
June	7613	.3	8294	−2.0	13503	1.0		
July	8244	1.7	9444	− .2	13198	.8		
Aug	7577	− .1	11034	1.6	14032	1.2		
Sept	9710	2.1	11753	1.0	14471	.2		
Oct	10283	.8	12829	.3	16533	.1		
Nov	9835	− .2	13545	.7	16569	− .2		
Dec	13321	1.9	15215	.0	18968	− .6		
	1920		*1922*		*1924*		*1926*	
Jan	11335	1.3	13771	.3	17871	.3		
Feb	10749	1.5	11837	− .9	16189	− .2		
Mar	10808	1.3	12498	− .4	15504	−1.0		
Apr	10171	1.8	11328	− .4	15205	.0		
May	9403	.8	11798	.3	14646	− .3		
June	8542	.1	11933	1.1	14317	.0		
July	8816	.7	11845	1.2	14419	.2		
Aug	8852	.3	12250	1.2	14990	.3		
Sept	9909	.4	13248	.8	16330	.1		
Oct	11363	.4	14493	.1	17129	−1.2		
Nov	12410	1.3	15159	.4	16492	−1.9		
Dec	12606	− .8	18589	1.0	20532	−1.3		

TABLE XV

ACTUAL DATA AND CALCULATED CYCLE OF THE MONTHLY PRODUCTION EXPENSES OF A SMALL COMPANY

Month	Actual	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	3404	−.7	3814	−2.6	8457	.8		
Feb.	3119	−1.3	4915	−.6	8125	.6		
Mar.	3933	.6	5039	−.5	8441	.8		
Apr.	3920	.7	4536	−1.3	8058	.5		
May	3864	.4	4182	−2.0	8207	.5		
June	3723	.3	4235	−1.7	8511	1.2		
July	4214	1.2	5587	.3	8618	.9		
Aug.	3942	.3	6266	1.2	8800	.8		
Sept.	4791	2.0	6812	1.7	8777	.5		
Oct.	5391	2.7	6813	1.2	8951	.2		
Nov.	3686	−.9	6422	.7	8488	.1		
Dec.	4240	.1	6077	.0	8923	.1		
	1920		*1922*		*1924*		*1926*	
Jan.	5365	1.9	6307	−.2	9159	−.1		
Feb.	4893	1.1	6190	−.2	8623	−.6		
Mar.	5003	1.2	6410	.0	8705	−.6		
Apr.	5021	1.3	6186	−.3	8019	−1.2		
May	4667	.4	6462	.0	8445	−.9		
June	4318	.0	6956	1.0	8357	−.7		
July	3609	−1.8	8066	2.2	8839	−.6		
Aug.	4465	−.3	7773	1.5	9590	−.1		
Sept.	3726	.0	8191	1.7	9195	−.7		
Oct.	5146	.2	8326	1.3	9633	−.8		
Nov.	4994	.1	8490	1.7	8904	−1.3		
Dec.	4122	−1.7	8812	1.6	9407	−1.2		

TABLE XVI

ACTUAL DATA AND CALCULATED CYCLE OF MONTHLY PRODUCTION EXPENSE OF A SMALL COMPANY

Month	Actual	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	654	−.4	873	.4	698	−.4		
Feb.	640	−.1	734	.1	584	−.7		
Mar.	705	.0	823	.2	526	−1.2		
Apr.	496	−.4	721	.8	513	−.5		
May	1183	4.2	642	.2	564	−.2		
June	853	1.6	675	.2	552	−.5		
July	791	.5	643	−.5	525	−1.0		
Aug.	782	.6	668	−.3	607	−.5		
Sept.	821	.5	658	−.6	598	−.7		
Oct.	989	1.3	669	−.6	647	−.5		
Nov.	1005	1.0	744	−.4	591	−1.0		
Dec.	955	.7	715	−.6	564	−1.1		
	1920		*1922*		*1924*		*1926*	
Jan.	920	.8	686	−.6	541	−1.1		
Feb.	687	−.1	684	−.2	455	−1.3		
Mar.	731	−.1	653	−.6	511	−1.1		
Apr.	682	.7	551	−.3	440	−.8		
May	747	.9	499	−.7	463	−.7		
June	698	.4	572	−.4	460	−.9		
July	816	.4	595	−.8	830	.9		
Aug.	863	.8	649	−.4	940	1.8		
Sept.	1046	1.5	885	.7	892	1.2		
Oct.	896	.6	790	.1	1245	3.1		
Nov.	1020	.9	750	−.3	1357	3.3		
Dec.	964	.6	769	−.3	1182	2.4		

TABLE XVII

ACTUAL DATA AND CALCULATED CYCLE OF THE MONTHLY GROSS REVENUES OF A SMALL COMPANY

Month	Actual	Cycle	Actual	Cycle	Actual	Cycle	Actual	Cycle
	1919		*1921*		*1923*		*1925*	
Jan.	1650	− .3	2386	1.1	2701	.3		
Feb.	1658	.0	2287	.9	2283	− .8		
Mar.	1665	1.3	2164	1.8	2034	− .5		
Apr.	1331	.8	1960	1.7	1893	− .4		
May	1137	− .6	1772	1.4	1901	.1		
June	1367	.4	1923	1.6	1942	− .1		
July	1273	− .3	1890	1.3	2007	.0		
Aug.	1318	− .8	1901	.4	2219	.0		
Sept.	1357	− .8	2112	1.2	2219	− .2		
Oct.	1558	− .3	2055	.3	2332	− .4		
Nov.	1786	− .1	2424	.7	2878	.3		
Dec.	1641	−1.3	2542	.4	2992	.0		
	1920		*1922*		*1924*		*1926*	
Jan.	1781	− .5	2440	.4	2858	− .1		
Feb.	1912	.3	2372	.3	2560	− .7		
Mar.	1581	− .1	1976	.0	2248	− .6		
Apr.	1627	.9	1803	.0	2122	− .3		
May	1547	1.1	1654	− .1	2071	.0		
June	1460	.1	1740	− .2	1974	− .8		
July	1559	.5	1632	− .8	1941	−1.1		
Aug.	1679	.3	1982	− .1	2081	−1.3		
Sept.	1915	1.2	2210	.6	2333	− .6		
Oct.	2073	1.3	2347	.5	2654	− .2		
Nov.	2132	.4	2450	− .1	3213	.4		
Dec.	2372	.7	2821	.4	3342	.1		

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