

On the Relevance of the Median Voter to Resource Allocation amongst Jurisdictions^{*}

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Abstract

This paper examines allocation of local public good over three jurisdictions with individuals with heterogeneous tastes, in a model with democratic institutions and majority rule. The nature of electoral uncertainty, the expectations of individuals from government captured by their reservation utilities, and heterogeneity in tastes within a jurisdiction are observed to affect resource allocation.

JEL classifications: H41; H72

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I Introduction

The problem of optimal public good supply given heterogeneous individuals has aroused considerable interest in the literature. When individuals differ in taste, the Lindahl-Wicksell, Groves-Loeb and Groves Ledyard mechanisms¹ posit a solution to the problem of optimal public good supply and the implied tax liabilities of individuals. However, as pointed out by Hurwicz (1979), any mechanism to solicit true preference for a public good will fail to simultaneously satisfy the following three criteria: Pareto-Optimality, incentive compatibility and a balanced budget. Given this failure of demand revelation methods, an alternative is the political, democratic mechanism encapsulated as the Median Voter Theorem (MVT). This theorem states that given single peaked preferences and majority voting, the preferences of the median voter will triumph. Practical applications of the MVT have been severely limited, given that MVT does not hold for multidimensional issue space, multi-peaked voter preference functions, vote abstentions and multiparty elections (see Rowley, 1994 and Aransson, 1996). This paper presents conditions under which political competition, given individuals with heterogeneous tastes will lead to the jurisdiction with the median voter getting the largest share of public resources. Preference for the

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median voter emerge from the incentive to win from a majority of jurisdictions rather than from a pairwise comparison of votes between alternatives as in the original MVT. The problem for the democratic government is then to decide on the tax rate and the optimal amounts of local public good to be supplied to such jurisdictions in the single dimension case with one individual in each jurisdiction. Allocation rules are extended for certain multi-dimensional situations, and for the case of heterogeneous individuals within any jurisdiction that may arise in such a context. We resort to simulation exercises to illustrate our results when analytical solutions become intractable.

In the context of redistributive politics, Dixit and Londregan (1996) model situations when voters compromise their political affinities in response to offers by competing parties. They conclude that groups that are likely to have advantage in redistributive politics are: (a) those that are indifferent to party ideology relative to private consumption benefits and (b) low income groups whose marginal utility of income is higher, making them more willing to compromise their political preferences for additional private consumption. Warskett, Winer and Hettich (1998) model income heterogeneity, to decide on the problem of a tax structure and on the level of public good provision, by a democratic government in order to economize the costs of tax administration. Similar but not identical voters may be clubbed in the same tax bracket, depending on the gain in expected votes from discriminating between taxpayers who have different economic and political characteristics to the loss in votes that result from diverting resources from public service provision to tax administration. Alesina, Baqir and Easterly (1999) try to link heterogeneity of preferences across ethnic groups in a city, to the amount and on the type of public good the city supplies. They show that for any positive amount and type of public good supplied, the median voter's preference wins. The larger the heterogeneity in preferences, captured by the median distance from the type most preferred by the median voter, the less is the amount of public good that is supplied. We incorporate the notion of the type of public good supplied along the same lines as Alesina, Baqir and Easterly (1999) and study its implications for resource allocation given heterogeneity within and across jurisdictions.

We assume individuals with identical endowments having an additively, separable utility function defined over a private and local public good². Individuals differ in tastes and the type of public good they would ideally like, which is perfectly correlated to taste. A unit of any type of public good can be supplied at the same cost. A central government decides on a lumpsum tax rate, the amount and the type of local public good to be supplied to jurisdictions. Under homogeneous tastes, both Utilitarian, as well as, a Rawlsian social planner would maximize the welfare of a representative individual. However, the same would not be the case when individuals have heterogeneous tastes. We see that discrimination is inevitable irrespective of whether the planner is Rawlsian or Utilitarian. With a single resident in each jurisdiction, each one can be given its most preferred type of local public good, under any scheme of allocation, the problem then shifts to the amount of local public good to be supplied to each jurisdiction. A Utilitarian social planner would maximize the sum of welfare of all individuals, if each individual is given the same weight. Hence, it would favor the individual with the highest weight on the local public good. In contrast, a Rawlsian planner's allocation by the maximin strategy would give the

highest allocation to the individual with the least weight and would favor precisely those individuals against whom a Utilitarian planner discriminates.

One of the questions of interest is who would a democratic planner choose to favor in resource allocation given heterogeneity of individuals, when this planner has to win by majority rule. With a single individual in each jurisdiction, each with the same reservation utility and with perfectly correlated electoral uncertainty across individuals, the democratic planner would choose to give the highest allocation of local public good to the jurisdiction with the median voter. However, this is not always true if electoral uncertainty is i.i.d. Once we allow individuals with higher weights on local public good (who can be satisfied more easily) have higher reservation utilities, one cannot say a priori which jurisdiction will be favored. With common electoral uncertainty across individuals, the jurisdiction with the median voter may not longer get the highest allocation. With i.i.d uncertainty, the allocation seems similar to a Utilitarian social planner's allocation.

We finally address the question of there being many individuals with different tastes within a jurisdiction. In this situation the type of local public good to be supplied becomes an important decision variable, and all individuals may not receive their most preferred type of public good. Variance in tastes within a jurisdiction would matter and the median voter may no longer remain as the representative individual for the democratic government. With i.i.d uncertainty, a democratic planner's allocation would match that of a Utilitarian social planner, and would favor those jurisdictions³ with individuals with high weight on local public good. Those jurisdictions with individuals having a high variance in their weights on local public good would correspondingly get a lower allocation. With heterogeneity within a jurisdiction and common electoral uncertainty, it is optimal to deny the jurisdiction which receives the highest allocation from a Utilitarian social planner from any allocation of the local public good.

This paper is organized as follows. The next section describes the basic model. Section III discusses the alternative allocations of local public good with a Rawlsian and a Utilitarian social planner, as well as, a democratic planner with one individual in each jurisdiction. Section IV deals with the problem of resource allocation with heterogeneous individuals within a jurisdiction. Section V concludes.

II The Model

Consider an economy with three⁴ jurisdictions with a single individual in each jurisdiction. We are interested in the relevance of the median voter to the allocation of local public good in this context. This model builds in the notion of population heterogeneity affecting the type and the amount of public good to be supplied as discussed by Alesina, Baqir and Easterly (1999), with some variation. The voting model incorporates the notion of reservation utility as in Seabright (1996) and Gupta (2001). All individuals have identical ability but differ in their tastes for the local public good. The taste parameter not only captures the valuation of the local public good by the individual, but also indicates the type of local public good most desired by the individual. Therefore, the individual's valuation of the local public good and the type of local public good most desired are perfectly correlated. Individuals are assumed to be immobile across jurisdictions. The central government has to satisfy a

majority of jurisdictions (in this case two) in order to get re-elected. Since we are interested in examining the distortions that come about with institutional structures, we make the simplifying assumption that there exist no labor supply distortions.

The utility function of a representative individual in jurisdiction i , $i \in j, k, l$ is given by:

$$W_m = ax^\alpha + \frac{b_{im}}{1 + d(c_i - b_{im})^2} g_i^\beta$$

where x is the private good consumed and g_i is the amount of local public good supplied to a jurisdiction i . $\alpha, \beta, a, b_{im}, c_i, d$ are parameters, $0 < \alpha, \beta < 1$, and $a, b_{im}, c_i, d \geq 0$ and a is the weight on private good. The weight on local public good depends on b_{im}, c_i and d . Since the weight on private good (a) is the same across all individuals, b_{im} captures the relative preference or the relative weight of the m^{th} individual residing in jurisdiction i over the local public good, it also denotes the type of local public good most preferred by the individual. Therefore, there is the implicit assumption that individuals with large preference over public good also prefer a different type of public good⁵. c_i denotes the type of local public good supplied to the jurisdiction⁶, d is the extent to which individuals are sensitive to the type of local public good supplied to the jurisdiction, and the type most preferred by the individual. This functional form helps us focus on the problem of local public good provision given heterogeneity within jurisdictions. Given heterogeneity, targeting of public good by type becomes an issue, wide differences in tastes also has implications on the amount actually spent on public service provision. If $d > 0$, then $1 + d(c_i - b_{im})^2 > 1$, the utility from local public good is less than what it would be if $d = 0$, for any given level of local public good. If $d = 0$, an individual is not sensitive to the type of local public good supplied and the utility function is of a standard form

$$W_m = ax^\alpha + b_{im}g_i^\beta \quad \dots(1)$$

Each individual is endowed with an amount of resource y obtained from labor income. The central government is constrained to use the same lump-sum tax rate t for all jurisdictions and therefore its budget constraint is given by:

$$\sum_i g_i = 3t \quad \dots(2)$$

Therefore, the amount of private good consumed by any individual is

$$x = y - t \quad \dots(3)$$

The uncertainty regarding an incumbent government's re-election is captured by an electoral uncertainty ε , which is a random variable following a uniform distribution over the range $[-q, q]$ and a mean of zero. Let e_m denote the event that the welfare of a representative individual, net of electoral uncertainty be greater than a reservation utility V_m , which can be interpreted as the welfare expected from a rival political

party. That is if the welfare of the individual is equal to or above V_m the individual is satisfied with the government and votes in its favor. A representative individual in jurisdiction m would be satisfied with the government if

$$W_m + \varepsilon \geq V_m$$

and the probability of the individual voting for the incumbent government is given by

$$p(m) = \frac{q - (V_m - W_m)}{2q} \quad \dots(4)$$

It can be seen from the above expression that if the government just manages to provide the reservation utility, it wins with a probability of 0.5, if it provides more it wins with a probability more than 0.5, and the converse holds true. It should be noted that the electoral uncertainty ε is common across all individuals and hence will be referred to as *common uncertainty*. Electoral uncertainty is then perfectly correlated across individuals in the jurisdictions. In contrast, if the electoral uncertainty between individuals may be independently, identically distributed (i.i.d), it will be denoted by ε_i . The two types of electoral uncertainty have starkly different implications in respect to resource allocation, which will be discussed at length in the paper.

III Alternative Allocations to the Individual

If there exists only one individual in each jurisdiction, consumption of a local public good is non-excludable so it is almost like a private good. It is then possible to match exactly each individual's most preferred choice on the type of local public good by any allocation (private or government), one can set $c_i^* = b_{im}$ to get the maximum utility from the local public good. A role for government is not necessary and an individual can choose to divide his/her endowment on private and local public good in a manner that yields the maximum utility. We may like to compare this allocation with government allocations made by a Utilitarian social planner, a Rawlsian social planner, as well as, a democratic planner who has to win by majority rule.

Private Allocation

An individual would allocate his endowment between private and public good by solving the problem:

Maximize

$$W_i = ax^\alpha + b_i g_i^\beta$$

subject to

$$x_i + g_i = y$$

Let (x^p, g_i^p) be the optimal allocation giving a welfare of W_i^p to a jurisdiction i . It should be noted that this allocation involves no discrimination and each individual maximizes utility subject to his/her budget constraint.

Utilitarian Social Planner's Allocation

A Utilitarian social planner's allocation problem will be given by a solution to the problem:

Maximize

$$\sum_i W_i = \sum_i (ax^\alpha + b_i g_i^\beta) \quad \dots(5)$$

subject to

$$\sum_i g_i = 3t$$

Let (x^u, g_i^u) be the optimal allocation in this scenario and the welfare of an individual being W_i^u . This allocation would involve discrimination against the individual with the least weight on local public good (i.e. least b_i) and would favor the individual with the highest weight i.e. (highest b_i).

Rawlsian Social Planner's Allocation

A Rawlsian social planner's allocation problem will be given by:

Maximize

$$\min(W_i)$$

subject to

$$\sum_i g_i = 3t$$

The optimal allocation in this case is given by (x^R, g_i^R) with the level of welfare for any individual being W_i^R . This allocation would involve discrimination against the individual with the highest weight on local public good (i.e., highest b_i) and would favor the individual with the least weight i.e., (lowest b_i).

Democratic Planner's Allocation

The central government will distribute resources for local public good to the jurisdictions in order to maximize the probability of getting re-elected from any two jurisdictions. This would depend not only on the weights on private good and on local public good, but also on the level of reservation utility of individuals and the type of electoral uncertainty, (whether common or i.i.d). Given common uncertainty across all individuals, if the reservation utility is set at the level of welfare obtained by a Rawlsian social planner, then all individuals will have their reservation utility set at the same level. Given that reservation utility and the weights on private good are the same for all individuals, the probability of winning will be the highest if allocation is concentrated on the two jurisdictions with the highest weights on local public good (for a proof, see Appendix 1) until the probability of winning becomes equal for both these jurisdictions. It would imply discrimination against the jurisdictions with the individual with least weight. The jurisdiction with the individual with the median

weight gets favored the most. The tax setting problem is similar to that in the identical individual case (i.e., maximizing the probability of winning from the jurisdiction with the individual with the median weight given the resource constraint). More specifically if k and l are two jurisdictions with the highest weight on local public good, the optimal tax rate would be found by solving:

$$\max_t [Min(pk, pl)] \text{ s.t } gk + gl = 3t \quad \dots(6)$$

where p_k and p_l are the probabilities of getting elected from jurisdictions k and l respectively.

It would be interesting to analyze a situation where reservation utility is not the same across individuals. With the same reservation utility for all individuals, it is easier to satisfy the individuals with the higher weights for the same level of local public good allocation, and hence they are more likely to vote for the government. If individuals with higher weights are also those with lower reservation utilities, then again it would be easier to satisfy those individuals with higher weight. The only case where the same need not be true is when people with higher weights have higher reservation utilities. One possible instance of such a situation would arise if the reservation utilities of individuals are at the level of welfare obtained from a Utilitarian social planner⁷. Who would be favored under such circumstances, becomes an empirical question and simulations have to be done to gain some insight.

Given common uncertainty and given reservation utility of individuals set at the level one obtains with a Utilitarian social planner, one has to solve an optimization problem similar to equation (1), for each combination of favored jurisdictions: (j, k) , (j, l) and (k, l) . Let the highest probability of winning for each of these combinations be denoted by p^{*jk} , p^{*jl} and p^{*kl} respectively. Then the optimum tax rate and the combination of jurisdictions to be favored are given by:

$$\arg \max_t [p^{*jk}, p^{*jl}, p^{*kl}]$$

In the simulation results presented in Table 1, b_i represents the weight on local public good for the individuals, W_i^U is the welfare provided to these individuals by a Utilitarian social planner and g_i is the amount of local public good provided by the Utilitarian social planner, e_i and p_i are the amounts of local public good provided by a democratic planner and the corresponding probability of winning from each jurisdiction respectively if it favors a combination of two jurisdictions (j, k) , (j, l) , (k, l) . From the results in Table 1, it is seen that it is best to favor j and k : the jurisdictions with the individuals with the least and the median weights. For the values of b_l varying from 100-70, given $b_j = 1$, and $b_k = 50$, the highest allocation goes to the jurisdiction with the least weight on local public good. This situation is similar to that discussed in Baron and Ferejohn (1989), where the representative in parliament with the lowest probability of being the agenda setter can do better than other members because he/she is a less costly member of any majority⁸. When the highest weight is very close to the median (for $b_j = 1$, $b_k = 50$, $b_l = 52$), favoring the extremes j and l may turn out to be as good as favoring the least and the median. If such extreme allocations are also optimal and but still chosen, we could infer that the median is the one left out of resource allocation.

Table 1: Allocation by a Democratic Planner

i	b_i	W_i^U	g_i	<i>favored</i> $j \& k$		<i>favored</i> $j \& l$		<i>favored</i> $k \& l$	
				E_i	p_i	e_i	p_i	e_i	p_i
j	1	0.310	0.024	175.849	0.504	32.433	0.502	0.001	0.500
k	50	387.391	59.981	64.11	0.504	0.001	0.404	60.012	0.500
l	100	1549.100	239.923	0.001	0.115	241.643	0.502	239.955	0.500
j	1	0.336	0.028	170.259	0.504	50.761	0.502	0.001	0.500
k	50	420.668	70.728	75.127	0.504	0.001	0.396	70.765	0.500
l	90	1362.587	229.159	0.001	0.161	231.505	0.502	229.196	0.500
j	1	0.367	0.034	157.188	0.504	60.451	0.503	0.001	0.500
k	50	489.073	84.232	88.828	0.504	0.001	0.386	84.276	0.500
l	80	1174.941	215.633	0.001	0.208	218.429	0.503	215.678	0.500
j	1	0.403	0.041	151.88	0.504	72.693	0.503	0.001	0.500
k	50	503.432	101.297	106.236	0.504	0.001	0.375	101.349	0.500
l	70	986.534	198.541	0.001	0.255	201.907	0.503	198.594	0.500
j	1	0.443	0.049	127.833	0.504	88.168	0.503	0.001	0.500
k	50	554.456	122.87	127.834	0.504	0.001	0.363	122.935	0.500
l	60	798.319	176.933	0.001	0.302	181.022	0.503	176.998	0.500
j	1	0.480	0.058	111.931	0.503	103.325	0.503	0.001	0.500
k	50	600.259	144.009	149.015	0.503	0.001	0.351	144.086	0.500
l	52	649.221	155.76	0.001	0.339	160.561	0.503	155.837	0.500

$$W_i = ax^\alpha + b_i g_i^\beta; a = 1; \alpha = \beta = 0.5$$

Thus, the results may be summarized as:

Proposition 1 With common electoral uncertainty and same reservation utility, a demo-cratic planner's optimum allocation would imply the maximum allocation of local publicgood to the jurisdiction with the individual with the median weight on local public good, followed by the highest and no allocation to the least. Therefore, the median voter is favored the most. With the reservation utilities set at the level obtained from a Utilitarian social planner, it is always at least as good to favor jurisdictions with individuals with least and median weight on local public good.

With i.i.d uncertainty denying a jurisdiction completely from any allocation of local public good does not emerge as a general solution as is the case with common uncertainty. The objective function may then be written as:

$$\Pi = p_j \cdot p_k (1 - p_l) + p_j \cdot p_l (1 - p_k) + p_k \cdot p_l (1 - p_j) + p_j \cdot p_k \cdot p_l \quad \dots(7)$$

Given the resource allocation constraint $\sum_i g_i = 3t$, partial differentiation with respect to t and g_i would give us the optimum allocation. Table 2 compares the Utilitarian social planner's allocation (U) in the second column, to a democratic planner's allocation given that reservation utility is set at the level obtained from a Rawlsian (labeled as Rawlsian expectations) and from a Utilitarian social planner (labeled as Utilitarian expectations). The social planner's allocation is invariant to changes in the level of electoral uncertainty, but the democratic planner's is not. We compute the local public good allocation, the tax rate and the probability of winning for noise levels q over a range, from 50 to 2000. If reservation utility is set at the level expected from a Rawlsian social planner (with all individuals having the same reservation utility) and the extent of noise is low (i.e. $q = 50$) the government can win with certainty from two jurisdictions. It is easiest to hedge from the two jurisdictions with highest weights, some more resources are needed to hedge from the jurisdiction with

the individual with the median weight. So the jurisdiction with the median voter gets favored the most. As the extent of uncertainty increases to $q = 100$, the probability of winning declines to 0.842, but the jurisdiction with the median voter continues to be the most favored. As the extent of uncertainty increases to 1000 and beyond, the allocation seems to come closer to that of a social planner. The tax rate also seems to converge to a Utilitarian social planner's tax rate. If we compare this allocation with the situation when reservation utility is set at the level of a Utilitarian social planner, then the probability of winning remains fairly constant at 0.5, and does not vary with the extent of uncertainty, unlike in the case earlier. The tax rate, as well as, the allocation does not vary much with the change in the extent of uncertainty except when very low ($q = 50$), and is similar and close to the social planner's allocation in all cases. Thus, at high levels of noise, both Utilitarian and Rawlsian levels of reservation utility gives us similar allocation.

Table 2: Allocation by a Democratic Planner Given Jurisdiction Specific Uncertainty

	q	2000	1000	500	250	100	50
U				Rawlsian Expectations			
p	0.500	0.526	0.551	0.602	0.694	0.842	1.000
t	28.571	28.472	28.195	27.311	22.363	23.584	8.764
g_j	2.857	2.859	2.864	2.851	2.968	0.558	0.001
g_k	11.429	11.532	11.834	12.79	19.893	56.408	22.606
g_l	71.429	71.026	69.886	66.113	44.137	13.788	3.617
U				Utilitarian Expectations			
p	0.500	0.5	0.5	0.5	0.5	0.5	0.512
t	28.571	28.573	28.573	28.571	28.57	28.571	22.849
g_j	2.857	2.858	2.857	2.857	2.857	2.857	3.917
g_k	11.429	11.429	11.43	11.429	11.429	11.429	13.943
g_l	71.429	71.432	71.432	71.429	71.425	71.429	50.688

$$W_i = ax^\alpha + b_i g_i^\beta; a = 1; \alpha = \beta = 0.5$$

Thus, we have:

Proposition 2 Given i.i.d uncertainty, and reservation utility set at the level obtained from a Rawlsian social planner for individuals, the jurisdiction with the median voter is most favored as long as the extent of uncertainty is low. With increase in the extent of uncertainty, allocation tends more towards a Utilitarian social planner's. When reservation utility is set at the level obtained from a Utilitarian social planner, allocation is not very sensitive to the extent of uncertainty and tends towards a social planner's.

IV Resource Allocation with Heterogeneous Individuals within a Jurisdiction

We now consider the case with heterogeneous individuals within each jurisdiction. In this situation the choice of the type of public good c_i to be supplied by the central government becomes an important decision variable, since it would no longer match exactly everyone's most preferred type (denoted by b_i) for the local public good. The preference for local public good, denoted by b_i , and the loss in utility from the type of local public good that is supplied [captured by $d(c_i - b_{im})^2$] will jointly determine the actual weight on the local public good. The determination of the median voter will not only depend on b_i but also on the type of local public good (c_i) supplied⁹. The population composition within the jurisdiction determines c_i .

It would be interesting to examine how the introduction of the type of local public good affects both social planner, as well as, democratic allocations. We therefore, make a comparison of these two situations with $d = 0$ where type does not matter and with $d = 1$ where it does. With $d = 0$, with heterogeneous individuals within a jurisdiction, one can identify the jurisdiction with the median voter if only all individuals with low, middle and high weights live in separate jurisdictions¹⁰. To investigate this problem further, we construct three population profiles denoted in Tables 3 and 4, where the median voter always resides in jurisdiction k . Each jurisdiction has three individuals: people with low weight on local public good (low b_i) are in jurisdiction j , those with high weights (high b_i) in l , and the middle ones in k . Table 3 indicates that individuals are not sensitive to the type of local public good supplied given $d = 0$. Hence in all three population profiles (from I to III), with a Utilitarian social planner's allocation, individuals in l enjoy higher welfare than those in k , who, in turn enjoy higher welfare than those in j . In the example in Table 4 with $d = 1$ a Utilitarian planner's allocation with population profile I would imply that the highest allocation of local public good is given to l , ($g_l = 616.078$) where individuals have the highest b_i 's and the least variance in the weights. With population profile II, jurisdiction k gets the highest allocation, the relatively lower b_i 's are more than offset by the relatively lower variance in b_i 's. For the same reason, with population profile III, allocation of local public good is highest in j , the jurisdiction with individuals with the least b_i 's. Optimal choice of c_i^* will always be between the least and the highest b_{im} of individuals in that jurisdiction. Individuals who enjoy the highest welfare with population profile I are those in l , with population profile II, are those in k and with population profile III, are those in j .

Table 3: Utilitarian Planner's Allocation with Heterogeneous Individuals within a Jurisdiction

	Profile I			Profile II			Profile III		
b_{mj}	18	18.9	19	5	10	19	18.8	18.9	19
b_{mk}	19	19.25	19.8	19.7	19.8	19.9	19.1	23.1	16.1
b_{ml}	19.9	20	20.1	19.9	25	30	30.2	34.2	39.2
W	4448.629			4488.86			5926.46		
x	16.372			16.08			9.225		
t	183.628			183.92			190.775		
g_j	511.583			185.879			296.565		
g_k	551.692			567.343			430.324		
g_l	589.38			902.062			990.088		
W_{j1}	447.59			108.268			354.128		
W_{j2}	467.956			176.437			355.851		
W_{j3}	470.208			299.141			357.573		
W_{k4}	486.736			509.533			426.588		
W_{k5}	492.608			511.715			509.565		
W_{k6}	505.52			514.097			571.797		
W_{l7}	523.777			637.783			980.635		
W_{l8}	526.005			790.958			1106.498		
W_{l9}	528.432			941.13			1263.626		

$$W_i = ax^\alpha + b_i g_l^\beta$$

$$a = 10, d = 0, \alpha = \beta = 0.5, y = 200, q = 5000$$

Table 4: Utilitarian Planner's Allocation with Heterogeneous Individuals within a Jurisdiction

	Profile I			Profile II			Profile III		
b_{mj}	18	18.9	19	5	10	19	18.8	18.9	19
b_{mk}	19	19.25	19.8	19.7	19.8	19.9	19.1	23.1	16.1
b_{ml}	19.9	20	20.1	19.9	25	30	30.2	34.2	39.2
W	4322.45			3208.53			3440.069		
x	17.341			31.473			27.379		
t	182.659			168.527			172.621		
g_j	550.173			115.385			868.61		
g_k	477.677			1095.856			227.308		
g_l	616.078			305.506			457.675		
c_j	18.9			18.999			18.9		
c_k	19.318			19.8			26.071		
c_l	20			29.993			39.192		
W_{j1}	478.214			56.373			600.877		
W_{j2}	484.947			57.411			609.349		
W_{j3}	482.921			260.193			606.791		
W_{k4}	418.738			701.743			58.131		
W_{k5}	960.424			711.534			87.773		
W_{k6}	392.841			708.386			445.488		
W_{l7}	530.656			59.482			60.217		
W_{l8}	538.061			72.951			80.547		
W_{l9}	535.637			580.438			890.846		

$$W_i = ax^\alpha + \frac{b_{im}}{1 + d(c_i - b_{im})^2} g_i^\beta$$

$$a = 10, d = 1, \alpha = \beta = 0.5, y = 200, q = 5000$$

If on the other hand, allocation was done by a Rawlsian social planner, given heterogeneity within the jurisdiction and a uniform tax rate, individuals within a jurisdiction will not experience the same level of welfare (this will vary with the b_i s) for any given level of local public good supplied. However, in equilibrium the worst-off individual in each jurisdiction enjoys the same level of welfare across all jurisdictions. Given that all individuals need not enjoy the same welfare in equilibrium even with a Rawlsian allocation, we analyze only the case where people set their reservation utilities at the level of welfare obtained from a Utilitarian social planner.

Now given a democratic planner, the type of uncertainty (whether common or i.i.d), as well as, the level of reservation utilities of individuals would matter in resource allocation. Given common uncertainty across individuals, and the level of reservation utility of individuals set at the level obtained from a Utilitarian social planner, it is still true that the optimum is to concentrate resources on local public good to two of the three jurisdictions¹¹. For a given tax rate, and for any division of resources on local public good across two jurisdictions, we compute the probability with which each individual will vote for the government. From any jurisdiction, the probability of getting elected is the median of the probabilities, with which the individuals vote for the incumbent. Optimization requires that this probability be maximum and equal across any two jurisdictions. This would determine the probability of winning the election. One has to work this out for all three combinations of two jurisdictions and

examine the combination, which yields the maximum probability of winning (i.e., the one that is finally chosen for resource allocation). The optimal tax rate is the one which solves the following optimization problem:

$$\arg \max_t [p_{*jk}, p_{*jb}, p_{*kl}]$$

where

$$p_{i_1 i_2} = \text{Max}[\min(p_{i_1}, p_{i_2})]$$

subject to

$$g_{i_1} + g_{i_2} = 3t$$

$$i_1, i_2 \in j, k, l, i_1 \neq i_2$$

$$p_{i_1} = \text{median}(p_{i_1 m})$$

$$p_{i_2} = \text{median}(p_{i_2 m})$$

$p_{i_1 m}$ probability of m^{th} individual in jurisdiction i_1 voting for the government.

We examine the allocation by a democratic planner with population profiles I, II and III in Tables 5 and 6 with $d = 0$ and $d = 1$ respectively. With $d = 0$, when individuals are indifferent to the type of local public good supplied, we notice that it is always jurisdiction l (the jurisdiction with the individuals with highest b_i 's) that is denied any local public good for all population profiles. This is similar to the situation with a single individual in each jurisdiction where the jurisdiction with the individual with the highest weight on local public good was discriminated because his/her reservation utility was very high. When $d = 1$, an individual's utility from a local public good may be reduced by a variation between the type of good preferred by the individual, and the type actually supplied. The optimal choice of c_i^* will now be done keeping the welfare of the b_i 's of only two of the three individuals in mind in a jurisdiction, given that it has to win by majority rule. From Table 6, we see that jurisdictions j and k receive local public good allocations in Profile I, j and l receive the same in Profile II, and k and l in Profile III. Thus, given common uncertainty, any combination of jurisdictions may be favored with the introduction of heterogeneity in tastes with respect to the type of local public good supplied. It should however, be interesting to note that the jurisdiction that is discriminated against is always the one that receives the highest allocation from a Utilitarian social planner. This planner chooses to favor jurisdictions, which have relatively higher proportion of individuals with a high relative weight on local public good to private good and low variance in the relative weight on local public good within the jurisdiction. In the examples constructed from Profiles I to III, the median voter is always in jurisdiction k , however this jurisdiction may not receive the highest allocation of local public good, and may be discriminated against in the sense of receiving no allocation of local public good. This situation is different from the one discussed in Besley and Coate (1997), where agents run for political office, get elected and in turn frame policies. The interests of the democratic planner do not in any way coincide with that of its citizens. There is no concept of parliament with representatives from each jurisdiction, but it is the democratic planner which decides on allocation after taking into account the characteristics of the median voter in each jurisdiction.

Table 5: Democratic Planner's Allocation with Heterogeneous Individuals within a Jurisdiction

Population		Profile I			Profile II			Profile III		
	b_{mj}	18	18.9	19	5	10	19	18.8	18.9	19
	b_{mk}	19	19.25	19.8	19.7	19.8	19.9	19.1	23.1	16.1
Govt. type	b_{ml}	19.9	20	20.1	19.9	25	30	30.2	34.2	39.2
UP	W	4448.629			4488.86			5926.46		
UP	t	183.628			183.92			190.775		
UP	g_j	511.583			185.879			296.565		
UP	g_k	551.692			567.343			430.324		
UP	g_l	589.38			902.062			990.088		
DC	p	0.511			0.513			0.521		
DC	t	188.153			178.254			190.273		
DC	g_j	824.601			691.16			813.247		
DC	g_k	868.772			913.125			899.206		
DC	g_l	0.001			0.001			0.001		
DI	p	0.5			0.5			0.5		
DI	t	183.622			183.927			190.776		

DI	g_j	511.565	185.867	296.561
DI	g_k	551.644	567.599	430.221
DI	g_l	589.39	901.875	990.199

$$W_i = ax^\alpha + b_i g_i^\beta$$

$$a = 10, d = 0, \alpha = \beta = 0.5, y = 200, q = 5000$$

UP: Utilitarian Planner's allocation

DC: Democratic Planner's allocation with common uncertainty

DI: Democratic Planner's allocation with i.i.d uncertainty

Given i.i.d uncertainty and Utilitarian reservation utilities, although the objective function would remain the same as before, the probability of getting re-elected from any jurisdiction i would be given by:

$$p_i = p_{i1} \cdot p_{i2} \cdot (1 - p_{i3}) + p_{i1} \cdot p_{i3} \cdot (1 - p_{i2}) + p_{i2} \cdot p_{i3} \cdot (1 - p_{i1}) + p_{i1} \cdot p_{i2} \cdot p_{i3}$$

The probability of winning for the incumbent government will still be given by equation (2). Simulation results with population profiles I to III, for allocation by a democratic planner given i.i.d uncertainty are reported in Tables 5 and 6. We find that although the tax rate seems to converge to the Utilitarian social planner's in most cases, the actual allocation of local public good may not be exactly identical with that of the social planner's¹². With $d = 0$, we find that jurisdiction l consistently gets the highest allocation of local public good, followed by k , and then j . This is because the relative weights on local public good (b_i 's) is all that matters for resource allocation. However with $d = 1$, the highest allocation with population profile I goes to jurisdiction l , then j and k . With population profile II, the highest goes to k then j and l . Finally with population profile III, the highest goes to j , then l and k . So in this situation a democratic planner, like a Utilitarian social planner gives the highest allocation to jurisdictions populated by individuals with relatively higher weight on local public good and relatively low variance in their weights. This denotes a convergence in tastes for the type of local public good to be supplied. It is interesting to note that the jurisdiction favored the most by a social planner and a democratic planner (given jurisdiction specific uncertainty) is precisely the one discriminated heavily against by a democratic planner in the case of common uncertainty.

Table 6: Democratic Planner's Allocation with Heterogeneous Individuals within a Jurisdiction

Population		Profile I			Profile II			Profile III		
	b_{mj}	18	18.9	19	5	10	19	18.8	18.9	19
	b_{mk}	19	19.25	19.8	19.7	19.8	19.9	19.1	23.1	16.1
Govt. type	b_{ml}	19.9	20	20.1	19.9	25	30	30.2	34.2	39.2
UP	W	4322.45			3208.53			3440.069		
UP	t	182.659			168.527			172.621		
UP	g_j	550.173			115.385			868.61		
UP	g_k	477.677			1095.856			227.308		
UP	g_l	616.078			305.506			457.675		
UP	c_j	18.9			18.999			18.9		
UP	c_k	19.318			19.8			26.071		
UP	c_l	20			29.993			39.192		
DC	p	0.51			0.508			0.507		
DC	t	165.531			196.667			33.928		

DC	g_j	793.214	10.654	0.001
DC	g_k	696.566	0.001	25.383
DC	g_l	0.001	19.341	279.965
DC	c_j	18.8	5.055	18.8
DC	c_k	19.145	19.7	24.791
DC	c_l	20.1	24.907	39.095
DI	p	0.5	0.5	0.5
DI	t	182.671	168.527	172.621
DI	g_j	549.854	115.385	868.609
DI	g_k	477.392	1095.861	227.31
DI	g_l	616.796	305.5	457.675
DI	c_j	18.9	18.999	18.9
DI	c_k	19.318	19.993	26.071
DI	c_l	20	29.993	39.192

$$W_i = ax^\alpha + \frac{b_{im}}{1 + d(c_i - b_{im})^2} g_i^\beta$$

$a = 10, d = 1, \alpha = \beta = 0.5, y = 200, q = 5000$

UP: Utilitarian Planner's allocation

DC: Democratic Planner's allocation with common uncertainty

DI: Democratic Planner's allocation with i.i.d uncertainty

V Conclusion

This paper presents conditions under which political competition, given individuals with heterogeneous tastes will lead to the jurisdiction with the median voter getting the largest share of public resources. As in the original MVT, the favored position of the median voter holds only under very stylized assumptions. With one individual in each jurisdiction, each can be given his/her most preferred type of local public good. In such a situation, with reservation utility being equal across individuals and common electoral uncertainty, the jurisdiction with the median voter receives the highest allocation of local public good. Those with weights above the median get some allocation of local public good, while those below get none at all. However, when individuals set their reservation utilities at the level they would obtain from a Utilitarian social planner, simulation results reveal that it is always at least as good to favor jurisdictions with the individuals with the least and median weight, but the highest allocation may at times go to the individual with the least weight. The optimal tax rate in such circumstances can be found only after calculating the maximum probability of re-election for each combination of favored jurisdictions. With i.i.d electoral uncertainty and all individuals with the same reservation utility, the jurisdiction with the median voter gets the highest allocation as long as electoral uncertainty is low. If high, the allocation tends towards a Utilitarian social planner's. With Utilitarian reservation utilities, the allocation does not seem to be too sensitive to the magnitude of electoral uncertainty and is similar to the Utilitarian social planner's allocation.

In the case of more than one individual in a jurisdiction and with heterogeneous individuals, the choice of the type of public good to be supplied becomes an important decision variable. With i.i.d uncertainty, a democratic planner's allocation would

match that of a Utilitarian social planner's giving high allocation of local public good to jurisdictions with individuals with high weight on local public good, and to those jurisdictions with individuals having a low variance in their weights on local public good. With common electoral uncertainty, it is still optimal to deny the jurisdiction which receives the highest allocation from a Utilitarian social planner from any allocation of local public good. Thus with i.i.d uncertainty, a democratic planner favors jurisdictions with individuals with homogeneity in tastes on the type of local public good, the very same factor may work against it, in the case of common uncertainty.

A major limitation of this analysis is that the reservation utilities of individuals, are taken as exogenous, we do not suggest a mechanism as to how these reservation utilities are arrived at by the individual voter, nor a mechanism by which the government can know the expectations of individuals. Future work must attempt to address this problem. In our situation, the government acts out of its own selfish perspective which is divorced from that of its citizens as in Niskanen (1971) as against Besley and Coate (1997) where citizens run for political office and implement their own policy choice. Benefit spillovers occur for individual types not targeted, but living in favored jurisdictions. Inefficiencies, that arise here are similar to that discussed in Weingast et. al. given that one has individual specific benefits and collective costs. Nevertheless majority rule need not necessarily imply pork barrel politics as seen in the case with i.i.d uncertainty where a democratic planner adopts an allocation similar to a Utilitarian social planner as a risk hedging strategy.

Appendix 1: Local Public Good Allocation with Common Uncertainty

We assume each jurisdiction has a single individual and the reservation utility is the same for all individuals at \underline{V} . The central government has to decide on the allocation of local public good to jurisdictions. It will maximize the probability of winning in any two of the three jurisdictions, the objective function given by

$$p(e_j \cap e_k \cap -e_l) + p(e_j \cap -e_k \cap e_l) + p(-e_j \cap e_k \cap e_l) + p(e_j \cap e_k \cap e_l)$$

where $-e_i$ is the event of not satisfying jurisdiction i . In a completely centralized scenario, the government will maximize the above objective function subject to the budget constraint, to get the optimal resource allocation.

Given common electoral shock, the event e_j , e_k , or e_l will occur, when

$$\varepsilon \geq \delta_i$$

where $\delta_i = \underline{V} - ax^\alpha - b_i g_i^\beta$ and $i \in j, k$ or l .

Now for a given tax rate t , the amount of expenditure on local public goods is, $\sum_{j,k,l} g_i = Q$. Let the resources be divided in such a manner that the resulting allocation leads to $\delta_j \geq \delta_k \geq \delta_l$. So when the event e_l occurs, events e_j , e_k , also occur. Therefore,

$$p(-e_j \cap e_k \cap e_l) = p(e_j \cap -e_k \cap e_l) = 0$$

and the objective function reduces to

$$p(e_j \cap e_k \cap -e_l) + p(e_j \cap e_k \cap e_l) = p(e_j \cap e_k)$$

By a similar reasoning as before e_j implies e_k , therefore $p(e_j \cap e_k) = p(e_k)$. Therefore, the value of the objective function is highest when p_k is the highest.

Given that $\delta_j \geq \delta_k \geq \delta_l$ implies that $p_j \leq p_k \leq p_l$, the maximal value of p_k is highest when $p_l = p_k$, and no resources are allocated for jurisdiction j , i.e. $g_j = 0$

Since

$$p(i) = \frac{q - (V_i - W_i)}{2q}$$

given $p_l = p_k$ would imply $W_l = W_k$. Since $W_i = ax^\alpha + b_i g_l^\beta$, it would imply that $b_l g_l^\beta = b_k g_k^\beta$. For p_k to be a maximum $b_k, b_l > b_j$. Therefore, if $b_k < b_l$ it would imply $g_k > g_l$. Therefore, the individual in jurisdiction k emerges out as the median voter and receives the maximum allocation of local public good.

Endnotes

- 1 A comprehensive discussion of all these models appears in Chapter 5 of the book 'Modern Public Economics' by Jha (1998)
- 2 These are simplifying assumptions which would help us highlight the result better.
- 3 They will be favored in terms of local public good allocation.
- 4 The model can be easily extended to n jurisdictions where n is odd.
- 5 One can interpret this as individuals who prefer a larger government also prefer public healthcare and education, while those who prefer a smaller government, want would like only public provision of basic infrastructure. The range of services demanded is being captured by the type.
- 6 In Alesina, Baqir and Easterly (1999) utility from the public good is given by the functional form $U_i = g^\alpha (1 - l_i)$ where g is the public good and l_i is the preference distance between individual i 's most preferred type of public good and the actual public good.
- 7 This again might be seen as a constitutional requirement, where no government can explicitly promise a reservation utility lower than what an individual would get from a Utilitarian social planner. Such provisions may exist in the system to prevent discrimination against one with least weight, given, we tax by ability (which is same for all), but give according to tastes. Channels of discrimination nevertheless exist through discretionary grants and delays by the central government.
- 8 Baron and Ferejohn (1989) has a discussion on government formation with three parties in a parliamentary democracy. In case of a tie between the two largest

parties, government formation will normally be between one large and the smallest party rather than between the two largest parties since in the former, the smallest party would demand a fewer number of ministerial berths.

9 This measure of the median would therefore be sensitive to changes in extreme values which is not the case with the normal median.

10 If jurisdictions were not so fully differentiated, the median of the median voter in each of the three jurisdictions may be identified as the jurisdiction with the median voter. This would be true only if we have common uncertainty and the same reservation utility for all individuals. However, as explained later in the text, the latter will not prevail.

11 Proof along similar lines as in Appendix 1.

12 Although close it is not identical after the first y decimal place.

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