On Graham's Theory of Multicountry Multicommodity Trade

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Abstract

Mathematical formulations of Frank D. Graham's theory of multicountry multicommodity trade have not provided numerical methods for finding the world trade equilibrium. Graham was in possession of such methods but his writings do not reveal what they were. This paper proposes an algorithm for finding Graham's world trade equilibrium. Modifications to the algorithm that are needed to cover such subjects as intercountry transfers, tariffs and taxes, have been illustrated. Further, it is shown that Graham's theory can be extended to accommodate international trade in intermediate capital goods.

I Introduction

In two contributions to the Quarterly Journal of Economics [Graham (1923, 1932)], and subsequently in his *magnum opus* [Graham (1948)], Frank D. Graham formulated and solved examples of multicountry multicommodity trade and used them not only to criticize classical and early neoclassical economists for wrongly projecting conclusions drawn from the 2x2 apparatus to the general context but also to establish new methods and propositions in trade theory and its applications to the transfer problem, import duties etc. Unfortunately, Graham's work although it was always recognized and celebrated, did not carry through into the subsequent development of international trade theory.[1]

The subsequent lack of interest in the subject of multicountry trade despite its obvious and acknowledged realism may be explained at least in part by the fact that Graham, although he provided a number of extremely painstaking examples of multicountry trade, did not furnish any method or algorithm that others could use to find the equilibrium.[2] He was content to give the final result but omitted to explain the steps by which he arrived at it. Of course, he did explain what the predicament is, "The ratio that will solve the problem can ordinarily be ascertained only through a tedious process of trial and error in which the whole course of trade must be worked out before one can know whether the exchange ratio with which he is experimenting will, in fact, provide a solution … The difficulty is that any shift in the ratio will set in motion kaleidoscopic changes not only in consumption but in production, will immediately take countries completely out of the production of at least one commodity and perhaps put them into others, and will change their consumption in varying proportions according to the varying net changes in the total income of each country and the opportunity cost, in trade, of each of the commodities. The data change unevenly, with every change in the tentative solution". [Graham (1948) p. 95,

footnote 6].

While this description is accurate it is more in the nature of an articulation of the difficulties that will be encountered in solving problems of this kind, it does not give the procedure that must be followed. Elsewhere Graham suggests, "The first approximation lies in the thesis that the largest countries must produce several commodities …" [Graham (1948), p. 72]. However, the steps that must follow are not explained.

Accordingly, the first purpose of this paper to propose an algorithm to find the multicountry trade equilibrium. The workings of the algorithm are illustrated by using Graham's own examples to enable an exact tally with his solutions. Other examples have been given to highlight special points. The second purpose of this paper is to discuss the applications of Graham's theory to issues such as the transfer problem, the consequences of import duties, domestic taxes and expenditures etc. The third purpose is to demonstrate that Graham's theory can be readily generalised to accommodate international trade in intermediate capital goods (See Appendix).

II Autarkic Equilibrium

While discussing the autarkic equilibrium of an economy Graham considered the allocation of "productive resources" in the economy between the various industries but did not describe them in detail. We shall suppose that by productive resources he meant homogenous labour. Further Graham worked strictly within the pure context i.e., without monetary considerations. We shall find it convenient to suppose a given money wage rate in terms of a fiat money. These two aspects are not as great a departure from Graham's theory as they might appear at first glance. Firstly, because Graham nowhere discusses problems that arise in reallocating 'productive resources' between industries due say to the technical specificities of machines or the skill specificities of labour. Indeed, all commentators on Graham's work have also supposed that Graham assumed homogenous labour.[3]

As regards our assumption of a given money wage rate, Graham himself stated, "it is only in the case of independent monetary systems (with debt, fiat or other non-commodity monies not used in any but the jurisdiction of issue) that the introduction of money makes no difference to the normal ratio of exchange. A money which has no use in the arts, and does not circulate in any country but the country of origin is "purer" in the sense, that it serves simply as a *numeraire* and does not disturb the commodity exchange relationships that would evolve under a frictionless form of barter of commodities not including the money material, than any commodity money could possibly be," [Graham (1948, p.152)]. At any rate our assumption of a given money wage rate in each country gives an occasion to test this conjecture of Graham. Except for these two aspects there will be no deviation whatsoever from Graham's framework. Thus we shall suppose with Graham that consumer tastes and preferences for various goods are represented by fixed shares of total income devoted to purchase them[4], and that technology is of fixed coefficients constant returns to scale type.

Autarkic equilibrium in these conditions is easily described. Let L be the total labour, w the money wage rate, α_i the share of total income spent on commodity i and l_i the labour coefficient of production of commodity i. Then, in equilibrium, each industry

must employ $L_i = \alpha_i L$ units of labour and produce $X_{is} = L_i / l_i$ units of commodity *i*. The equilibrium may be written as,

$$\frac{L_i}{l_i} = X_{is} = \frac{\alpha_i L}{l_i} \quad \left(\sum \alpha_i = 1\right) \qquad \ldots (1)$$

No other equilibrium is possible. Because the unit price of each commodity is simply wl_i and the expenditure on the commodity being $\alpha_i wL$, the quantity demanded of commodity *i* is,

$$X_{id} = \frac{\alpha_i wL}{wl_i} = \frac{\alpha_i L}{l_i} \qquad \ldots (2)$$

which will be equal to the quantity supplied given in (1) only if the labour allocated to the production of *i* is $\alpha_i L$. In other words, the demand price of the commodity

$$P_{id} = \frac{\alpha_i wL}{X_{is}}$$

is equal to the supply price $P_{is} = wl_i$ only if the quantity supplied is

$$X_{is} = \frac{\alpha_i wL}{wl_i} = \frac{\alpha_i L}{l_i}$$

The size of the money wage rate affects only the levels of prices, nothing 'real'
As an example consider an economy that produces 4 commodities, has 100 units of labour, pays a wage rate of USD 2 and has the following labour coefficients of production and average (equal to marginal) propensities to consume,

$l_1 = 0.5$	$\alpha_1 = 0.2$
$l_2 = 2$	$\alpha_2 = 0.3$
$l_3 = 1$	$\alpha_3 = 0.1$
$l_4 = 0.8$	$\alpha_4 = 0.4$

Then the equilibrium for the economy is

Industry	Labour	Output	Price (USD)
1	20	40	1.00
2	30	15	4.00
3	10	10	2.00
4	40	50	1.60

III Multicountry Comparative Advantage

The direction of trade of commodities between countries is guided by the principle of comparative advantage. It is mysterious that Graham has left no account of how he determined the pattern of multicountry comparative advantage although he refers to

the principle time and again. Mysterious, because the number of trade possibilities in multicountry multicommodity situations increases very rapidly with both an increase in the number of countries and the number of commodities. For example, even in the simplest case of 2 countries trading in 2 commodities there are 7 possible trade situations one which may be the trade equilibrium, viz. A-1 B-2; A-2 B-1; A-1,2 B-1; A-1,2 B-2; A-1 B-1,2; A-2 B-1,2; A-1,2 B-1,2. Of these, classical theory considered only the first two possibilities of complete specialization and modern neoclassical theory considers only the last possibility of incomplete specialization. With 2 countries trading in 3 commodities the number of trade possibilities (in which every country produces at least one tradable commodity and every commodity is produced by at least one country) rises to 24. And with 4 countries and 5 commodities this number is 693600! Graham must surely have had a method of eliminating most of them before he arrived at a manageable set of feasible alternatives but posterity has no clue about it.

We shall in this paper use the familiar principle of comparative advantage to separate the feasible possibilities from the non-feasible ones. This will be done by means of a restatement of the comparative advantage principle in the manner explained below. In the usual 2-country 2-commodity case, a country A is said to have an advantage in commodity 1 and B in commodity 2 if

$$\frac{P_{1A}}{P_{2A}} < \frac{P_{1B}}{P_{2B}} \qquad \ldots (4)$$

This will be restated in the form

$$E^1_{AB} = \frac{P_{1A}}{P_{1B}} < \frac{P_{2A}}{P_{2B}} = E^2_{AB} \qquad \ldots 5(a)$$

or as

$$E^1_{AB}\ E^2_{BA} < 1 \qquad \ldots 5(b)$$

The idea is that the money prices of the respective goods in the two countries have the dimensions of the currency exchange rate, i.e. E^1_{AB} and E^2_{AB} are the currency exchange rates implied by the money prices of commodities 1 and 2. Thus the usual statement of comparative advantage in real terms (i.e. as comparative costs ratios or the domestic commodity exchange ratios) can be translated into a statement in terms of the currency exchange rates implied by the money prices which we shall call the "natural exchange rates"[5].

Since E^2_{BA} is simply the reciprocal of E^2_{AB}, the statement of comparative advantage in 5(b) is amenable to the following interpretation: If one dollar (say) in country A is used to purchase a quantity of commodity 1 equal to $\left(1/P_{1A}\right)$ which is sold to country B for (say) yen $\left(P_{1B}/P_{1A}\right)$ which proceeds can be used to buy $\left(P_{1B}/P_{1A}\right)\left(1/P_{2B}\right)$ units of commodity 2 in country B and which in turn can be sold for $\left(P_{1B}/P_{1A}\right)\left(P_{2A}/P_{2B}\right)$

dollars in country A then a profit is realized only if $\left(\frac{P_{1B}}{P_{1A}}\right)\left(\frac{P_{2A}}{P_{1A}}\right) > 1$ i.e.

$$E^1_{AB}\ E^2_{BA} < 1$$

We shall then say that country A has a comparative advantage in commodity 1 and country B in commodity 2. In this form the principle of comparative advantage may generalized to any number of countries and commodities. Thus a pattern A-2, B-3, C-1 is feasible on grounds of comparative advantage if

$$E^2_{AB}\ E^3_{BC}\ F^1_{CA} < 1 \qquad \ldots (6)$$

i.e. the purchase of commodity 2 in A, its sale in B, use of the proceeds (in B's currency) to purchase 3 in B, its sale in C, the use of the proceeds (in C's, currency) to purchase 1 in C and its sale in A will result in a profit only if inequality (6) holds.[6] Note that nowhere in the sequence of commodity arbitrage transactions described above is there any exchange of currencies themselves. In other words the pattern of comparative advantage is found from autarkic money prices alone prior to ascertaining the actual exchange rates.

VI Exchange Neutrality Conditions

The actual or market currency exchange rates $E_{ij}\,(i, j = A)(i \neq j)$ must of course adhere to the 'neutrality conditions' brought about by currency arbitrage. Two-currency arbitrage ensures that,

$$E_{ij} = 1/E_{ji}$$

$$\text{i.e. } E_{ij}E_{ij} = 1 \quad \forall i, j \qquad \ldots 7(a)$$

and three-currency arbitrage ensures that direct quotes equal indirect quotes

$$E_{ij}E_{jk} = E_{ik}$$

$$\text{i.e. } E_{ij}E_{jk}E_{ki} = 1 \quad \forall i, j, k \qquad \ldots 7(b)$$

Equations 7(a) set the relations between N(N-1)/2 exchange rates and equations 7(b) set the relations between (N-1) (N-2)/2 exchange rates.

Together they make $(N-1)^2$ exchanges rates "redundant" so that it suffices to know (N-1) currency exchange rates in terms of any one currency E_{Aj} to ascertain all N(N-1) exchange rates. Chacholiades (1971) has proved the remarkable theorem that, "if two-currency and three-currency arbitrage is not profitable then m-currency arbitrage (m > 3) is not profitable either". We shall suppose in what follows that currency exchange rates are always such as to satisfy equation (7).

V Gains from Trade

Given a set of market exchange rates, the quantities that can be purchased of the different commodities in different countries by a unit of any country's currency can be computed. Thus a unit of currency A can purchase $1/P_{iA}$ units of commodity *i* in A, E_{BA}/P_{iB} in B, E_{CA}/P_{iC} in C, etc. where P_{ij} are the money prices of the commodities in the countries *j* and E_{BA}, E_{CA} etc. are the prevailing exchange rates. These may be tabulated as follows:

	A	B	…	Z
1	$1/P_{1A}$	E_{BA}/P_{1B} *	…	E_{ZA}/P_{1Z}
2	$1/P_{2A}$ *	E_{BA}/P_{2B}	…	E_{2A}/P_{2Z}
3	$1/P_{3A}$	E_{BA}/P_{3B}	…	E_{ZA}/P_{3Z}
:	:	:	…	:
N	$1/P_{nA}$	E_{BA}/P_{nB}	…	E_{ZA}/P_{nZ} *

Further suppose that the starred entries give the maximum quantities that a unit of currency A can purchase of the different commodities across countries i.e. it is the maximal entry in each row. The ratio of international commodity exchange, loosely the terms of trade, is E_{BA}/P_{1B} units of $1 = 1/P_{2A}$ units of 2 = … E_{ZA}/P_{nZ} units of commodity n.

The ranking of the elements of each row is identical irrespective of the currency in which they are computed provided the set of exchange rates E_{ij} is consistent, i.e. it satisfies the exchange neutrality conditions in equation (7). Consider row 1 column B element in the table above. Since it is the largest element in its row,

$$\frac{E_{BA}}{P_{1B}} > \frac{1}{P_{1A}}$$

which implies

$$\frac{1}{P_{1B}} > \frac{1}{E_{BA}P_{1A}}$$

so that

$$\frac{1}{P_{1B}} > \frac{E_{AB}}{P_{1B}}$$

provided $E_{AB}E_{BA} = 1$. In the last step the quantities are what a unit of currency B can buy. The same will be seen to hold good for the ranking between any two elements of the row and for any currency. Consider now a comparison with another element, say

$$\frac{E_{BA}}{P_{1B}} > \frac{E_{CA}}{P_{1C}}$$

Then

$$\frac{1}{P_{1B}} > \frac{E_{CA}}{E_{BA}P_{1C}}$$

So that

$$\frac{1}{P_{1B}} > \frac{E_{CB}}{P_{1C}}$$

only if $E_{BA}E_{AB} = 1$ and $E_{CA}E_{AB} = E_{CB}$ i.e. the three currency neutrality condition is satisfied. In short, any set of exchange rates that satisfy the exchange neutrality conditions of equation (7) preserve the ranking of the elements in the gains from trade table.

All the components of the apparatus that we shall require for the algorithm to determine multicountry multicommodity trade equilibrium are now in place. It remains only to state the requirements that a trade assignment should fulfill to qualify as an international trade equilibrium. These are three,

a. Each country must produce positive outputs of the commodities it produces in the post-trade situation and fully employ its labour endowment in the industries that produce those commodities
b. The pattern of gains from trade must exactly match the trade assignment,
c. The world supplies and demands for all commodities must be equal.

VI Graham's 4 Country 3 Commodity Example

Consider Graham's example of 3 commodities being produced in 4 countries whose sizes (measured in terms of the output of commodity 1 in autarkic equilibrium) are in the ratio 1:2:3:4 [Graham (1948), Chapter V]. Graham further supposes that in each country 1/3rd of the income is spent on each commodity. The labour coefficients of production are supposed to be

	A	B	C	D
1	4/10	6/20	9/30	12/40
2	4/19	6/40	9/40	12/112
3	4/42	6/48	9/90	12/160

Then if we suppose that sizes of the labour endowment in the 4 countries are 12, 18, 27 and 36 respectively and the money wage rates are say USD 1, GBP 1, JPY 1 and EUR 1 the autarkic equlibria are,

	A	B	C	D
1	$4w_A = 10P_{1A}$	$6w_B = 20P_{1B}$	$9w_C = 30P_{1C}$	$12w_D = 40P_{1D}$
2	$4w_A = 19P_{2A}$	$6w_B = 40P_{2B}$	$9w_C = 40P_{2C}$	$12w_D = 112P_{2D}$
3	$4w_A = 42P_{3A}$	$6w_B = 48P_{3B}$	$9w_C = 90P_{3C}$	$12w_D = 160P_{3D}$

The first column for each country shows the labour allocated and the second column the output produced (Note that sizes of industry 1 in the four countries are in the ratio

1:2:3:4). The prices of the commodities are simply the money wage rates multiplied by the labour coefficients shown above. The natural exchange rates are as below:

$E^1_{AB}=1.333$	$E^1_{BC}=1.00$	$E^1_{CD}=1.000$	$E^1_{DA}=1.000$
$E^2_{AB}=1.403$	$E^2_{BC}=0.750$	$E^2_{CD}=1.866$	$E^2_{DA}=0.0508$
$E^3_{AB}=0.761$	$E^3_{BC}=1.250$	$E^3_{CD}=1.333$	$E^3_{DA}=0.787$

To identify the trade assignment having the greatest comparative advantage pick up the lowest element in each column and obtain the commodity arbitrage sequence with maximum profit. For the example above that sequence is

$$E^3_{AB}\,E^2_{BC}\,E^1_{CD}\,E^2_{DA}=0.2908$$

indicating a trial trade pattern A-3, B-2, C-1, D-2 on grounds of comparative advantage alone. Since B and D produce commodity 2 in common the implied exchange rate of their currencies is $E_{BD}=E^2_{BD}=P_{2B}/P_{2D}=1.4\,JPY/EUR$ which of course means no gain by mutually trading commodity 2. Since, in the assignment being tried out, each country produces only 1 commodity all of the labour will be employed in its production so that the world's production activities will look as follows:

	A	B	C	D
1	-	-	$27w_C=90P_{1C}$	-
2	-	$18w_B=120P_{2D}$	-	$36w_D=336P_{2D}$
3	$12w_A=126P_{3D}$	-	-	-

This completes the first step. Next we set up the demand-supply equations for the three commodities. They are

$$\frac{L_C}{l_{1C}}=\left(\frac{\alpha_{1A}w_AL_A}{w_Cl_{1C}}\right)E_{CA}+\left(\frac{\alpha_{1B}w_BL_B}{w_Cl_{1C}}\right)E_{CB}+\frac{\alpha_{1C}w_CL_C}{w_Cl_{1C}}+\frac{(\alpha_{1D}w_DL_D)E_{CD}}{w_Cl_{1C}}$$

$$\frac{L_B}{l_{2B}}+\frac{L_D}{l_{2D}}=\left(\frac{\alpha_{2A}w_AL_A}{w_Bl_{2B}}\right)E_{BA}+\frac{\alpha_{2B}w_BL_B}{w_Bl_{2B}}+\left(\frac{\alpha_{2C}w_CL_C}{w_Bl_{2B}}\right)E_{BC}+\left[\frac{\alpha_{2D}w_DL_D}{w_Bl_{2B}}\right]E_{BD}$$

$$\frac{L_{3A}}{l_{3A}}=\frac{\alpha_3w_AL_A}{w_Al_{3A}}+\left(\frac{\alpha_{3B}w_BL_B}{w_Al_{3A}}\right)E_{AB}+\left(\frac{\alpha_{3C}w_CL_C}{w_Al_{3A}}\right)E_{AC}+\left(\frac{\alpha_{3D}w_DL_D}{w_Al_{3A}}\right)E_{AD}$$

The left hand sides of these equations show the total quantities produced of the three commodities and the right hand sides show the total quantities demanded, i.e. the total expenditures on the commodities converted into the currency of the country from which they are imported divided by the price of the commodity in the exporting country. In case of commodity 2 the right hand side is written as if only B exports it but it could indifferently be written as if D exported it or any combination B and D. Using the neutrality conditions (7) some of the exchange rates in (8) may be eliminated and all equations can be expressed in terms of one currency say the currency of country A (e.g. if equation 1 is multiplied by E_{AC}, then

$E_{AC}E_{CA}=1, E_{AC}E_{CB}=E_{AB,}E_{AC}E_{CD}=E_{AD}$, etc.) Substituting the data for w_i, α_{ij} and L_i (8) is reduced to

$$\begin{aligned} 6E_{AB}-18E_{AC}+12E_{AD} &= -4 \\ 45.6E_{AB}-9E_{AC}+0E_{AD} &= 4 \\ 6E_{AB}+9E_{AC}+12E_{AD} &= 8 \end{aligned} \quad \ldots (9)$$

having the solution $E_{AB}=0.1754$, $E_{AC}=0.4444$ and $E_{AD}=0.2456$. ($E_{BD}=E_{BA}E_{AD}=1.4$ as required). The question is whether the countries stand to gain from trade at these exchange rates. To find that we compute the gains from trade table.

	A	B	C	D
1	2.5	19*	7.5	13.57
2	4.75	38*	11.25	38*
3	10.5	45.6	38	54.28*

The starred entries show the countries in which the maximum quantities of the commodities are obtained at the going exchange rates.

They clearly do not support the postulated pattern A-3, B-2, C-1, D-2. Specifically at the going exchange rates B is seen to have an advantage in 1 and D in 3. Keeping the initial assignment intact, it being based on comparative advantage, we make the modifications indicated by the gains from trade table to set up a new trial trade pattern A-3, B-1,2, C-1, D-2,3.

This assignment implies 3 exchange rates

$$\begin{aligned} E_{AD}=E^3_{AD}=P_{3A}/P_{3D}&=1.269 \\ E_{BC}=E^1_{BC}=P_{1B}/P_{1C}&=1.000 \\ E_{BD}=E^2_{BD}=P_{2B}/P_{2D}&=1.400 \end{aligned}$$

From the neutrality conditions we can infer the other rates. Thus $E_{BA}E_{AC}=E_{BC}=1$ implies that $E_{AB}=E_{AC}$ and $E_{BA}E_{AD}=E_{BD}=1.4$ and $E_{AD}=1.269$ implies $E_{BA}=1.1032$. Thus all the relevant exchange rates are 'known'; $E_{AB}=0.9070$, $E_{AC}=0.9070$ and $E_{AD}=1.269$. The commodity demand-supply equations will now contain only the unknown labour allocations in countries B and D both of which produce two commodities each. Thus we write.

$$\frac{L_{1B}x_{1B}}{l_{1B}}+\frac{L_C}{l_{1C}}=\frac{\alpha_{1A}Y_AE_{BA}}{P_{1B}}+\frac{\alpha_{1B}Y_B}{P_{1B}}+\frac{\alpha_{1C}Y_C}{P_{1C}}+\frac{\alpha_{1D}Y_DE_{BD}}{P_{1B}}$$

$$\frac{L_{2B}x_{2B}}{l_{2B}}+\frac{L_{2D}x_{2D}}{l_{2D}}=\frac{\alpha_{2A}Y_A\,\alpha_{2A}Y_AE_{BA}}{P_{2B}}+\frac{\alpha_{2B}Y_B}{P_{2B}}+\frac{\alpha_{2C}Y_CE_{BC}}{P_{2C}}+\frac{\alpha_{2D}Y_D}{P_{2D}}$$

$$\frac{L_A}{l_{3A}} + \frac{L_{3D}x_{3D}}{l_{3D}} = \frac{\alpha_{3A}Y_A}{P_{3A}} + \frac{\alpha_{3B}Y_BE_{AB}}{P_{2A}} + \frac{\alpha_{3C}Y_CE_{AC}}{P_{2A}} + \frac{\alpha_{3D}Y_D}{P_{3D}}$$

where L_{ij} are the autarkic allocations of labour, $Y_i = w_iL_i (i = A...D)$. There are 4 unknowns to be solved, viz., $x_{1B}, x_{2B}, x_{2D}, x_{3D}$. As against these there are only two independent equations since if any two markets clear so should the third. However, there are two full employment equations for countries B and D, viz.

$$L_{1B}x_{1B} + L_{2B}x_{2B} = L_B$$
$$L_{2D}x_{2D} + L_{3D}x_D = L_D$$

Making substitutions from the data we obtain

$$6_{X_{1B}} = 9.2128$$
$$40x_{2B} + 112x_{2D} = 241.40$$
$$160x_{3D} = 218.85$$

and the full employment equations,

$$6x_{1B} + 6x_{2B} = 18$$
$$12x_{2D} + 12x_{3D} = 36$$

These give the solution

$$x_{1B} = 1.535 \quad x_{2B} = 1.464$$
$$x_{2D} = 1.632 \quad x_{3D} = 1.376$$

which shows that positive outputs will be produced of all commodities in all the countries to which they have been assigned thus fulfilling one feasibility condition. However, the gains from trade table shows that at the exchange rates country C exhibits an advantage in commodity 3.

	A	B	C	D
1	2.50	3.675*	3.675*	2.625
2	4.75	7.35*	5.51	7.35*
3	10.50	8.82	11.025*	10.50

Thus the new trial trade pattern indicated is A-3, B-1,2, C-1,3 D-2,3. However, observe that this trade pattern would give rise to contradictory exchange rates. In terms of the currency of A they are

(i) $E_{AB} = 0.9070$ $E_{AC} = 0.9070$ $E_{AD} = 1.269$
(ii) $E_{AB} = 0.9523$ $E_{AC} = 0.9523$ $E_{AD} = 1.269$

The inconsistency has arisen because $E^1_{BC}E^3_{CD} = E_{BD} = (1)(1.3333) = 1.3333$ which is not equal to $E^2_{BD} = 1.4$. The inconsistency should be removed. To remove it we

consider the relative advantages of B, C and D in the production of commodities 1, 2 and 3 at the going exchange rates. They are

	B	C	D	Ratio
1	3.675	3.675	-	1
2	7.35	-	7.35	1
3	-	11.025	10.50	1.05

Observe that C has a distinct relative advantage in producing commodity 3 over D. Therefore C-3 can be retained. But to remove the inconsistency, we do not know whether it is B-1 or C-1 that must go or B-2 or D-2. A further clue is necessary. Consider the two sets of mutually inconsistent exchange rates implied by the pattern A-3; B-1,2; C-1,3; D-2,3.

Next we set up the demand supply and full employment equations,

$$\frac{L_{1B}x_{1B}}{l_{1B}} + \frac{L_{1C}x_{1C}}{l_{1C}} = \sum_{j=A}^{D} \frac{(\alpha_{ij} w_j L_j) E_{Bj}}{w_B l_{1B}}$$

$$\frac{L_{2B}x_{2B}}{l_{2B}} + \frac{L_{2D}x_{2D}}{l_{2D}} = \sum_{j=A}^{D} \frac{(\alpha_{2j} w_j L_j) E_{Bj}}{w_B l_{2B}}$$

$$\frac{L_A}{l_{3A}} + \frac{L_{3C}x_{3C}}{l_{3C}} + \frac{L_{3D}x_{3D}}{l_{3D}} = \sum_{j=A}^{D} \frac{(\alpha_{3j} w_j L_j) E_{Aj}}{w_A l_{2A}}$$

$$L_{1B}x_{1B} + L_{2B}x_{2B} = 18$$
$$L_{1C}x_{1C} + L_{2C}x_{3C} = 27$$
$$L_{2D}x_{2D} + L_{3D}x_{2D} = 36$$

Then substituting the two sets of exchange rates and the other data we obtain two solutions for the market clearing labour allocations,

$$\text{(i)} \quad \begin{array}{ll} x_{1B} = 5.094 & x_{2B} = -2.094 \\ x_{1C} = 0.626 & x_{3C} = 2.373 \\ x_{2D} = 2.903 & x_{3D} = 0.0964 \end{array}$$

$$\text{(ii)} \quad \begin{array}{ll} x_{1B} = 5.209 & x_{2B} = -2.209 \\ x_{1C} = 0.532 & x_{3C} = 2.464 \\ x_{2D} = 2.928 & x_{3D} = 0.071 \end{array}$$

Observe that in both cases the value of $x_{2B} < 0$ showing that at the prices and going exchange rates and in the presence of country D producing 2, the resources of country B have to be stretched to produce commodity 1 beyond its capacity. We therefore allow it to withdraw from producing commodity 2. Accordingly, we strike out B-2 and set the new trial pattern A-3, B-1, C-1,3 D-2,3 with the second set of implied exchange rates above. Solving the equations,

$$\frac{L_B}{l_{1B}} + \frac{L_{1C}x_{1C}}{l_{1C}} = \sum_{j=A}^{D} \frac{(\alpha_{1j} w_j L_j) E_{Bj}}{P_{1B}}$$

$$\frac{L_{2D}x_{2D}}{l_{2D}} = \sum_{j=A}^{D} \frac{(\alpha_{2j} w_j L_j) E_{Dj}}{P_{2D}}$$

$$\frac{L_A}{l_{3A}} + \frac{L_{3C}x_{3C}}{l_{3C}} + \frac{L_{3D}x_{3D}}{l_{3D}} = \sum_{j=A}^{D} \frac{(\alpha_{3j} w_j L_j) E_{Dj}}{P_{3A}}$$

$$L_{1C}x_{1C} + L_{3C}x_{3C} = L_C$$
$$L_{2D}x_{2D} + L_{3D}x_{3D} = L_D$$

gives the solution,

$$x_{1C} = 1.92 \quad x_{3C} = 1.08$$
$$x_{2D} = 2.20 \quad x_{3D} = 0.8$$

The pattern of gains from trade is,

	A	B	C	D
1	2.5	3.5*	3.5*	2.625
2	4.75	7.35*	5.25	7.35*
3	10.5*	8.4	10.5*	10.5*

The trade equilibrium is found since all outputs are positive and at the market clearing exchange rates the pattern of gains from trade is consistent with the trade assignment. The world's production in equilibrium is as follows:

	A	B	C	D
1	-	$18 w_B = 60 P_{1B}$	$17.2 w_C = 57.33 P_{1C}$	-
2	-	-	-	$26.4 w_D = 246.4 P_{2D}$
3	$12 w_A = 126 P_{3A}$	-	$9.8 w_C = 98 P_{3C}$	$12 w_D = 128 P_{3D}$

The international terms of trade can be read from the gains from trade table. It is 3.5 units of commodity 1 = 7.35 units of commodity 2 = 10.5 units of commodity 3, more conveniently expressed as 10: 21:35. The production, consumption and exports/ imports are.

		A	B	C	D	Total
	P	0.0	60	57.33	0	117.33
1	C	14.0	20	30.00	53.33	117.33
	X/M	- 14.0	40	27.33	- 53.33	0.00
	P	0.0	0	0.00	246.40	246.40
2	C	29.4	42	63.00	112.00	246.40
	X/M	-29.4	-42	-63.00	134.40	0.00
	P	126.0	0	98.00	128.00	352.00
3	C	42.0	60	90.00	160.00	352.00
	X/M	84.0	-60	8.00	-32.00	0.00

P-Production C-Consumption X-Exports (+) M-Imports (-)

This is identical to the world trade equilibrium obtained by Graham [Graham (1948) pp 83-84] noting that Graham measures quantities in units of thousands. Also note that the solution given on pages 80-81 is based on the assumption that each country expands its consumption proportionately to its gains from trade, an assumption which he abandons throughout his subsequent discussion].

Graham used this example to show that in the general multicountry context a country may produce a commodity and import it as well (country D, commodity 3) and that a country may have a comparative advantage in a commodity and yet import it (country B, commodity 2), contrary to the assertions of classical theory which based its conclusions on 2 x 2 trade situations.

VII Effects of Changes in Demand

Graham then proceeded to show that in a multicountry multicommodity context in which several countries are likely to be incompletely specialized even fairly wide changes in demand conditions do not cause changes in the international terms of trade which remain anchored to the commodities that are produced in common between the different countries.

Thus Graham showed in the context of the 4 country, 3 commodity example above that even if demand conditions in the countries vary between $\alpha_1 = 0.25 \;\; \alpha_2 = 0.4 \;\;\; \alpha_3 = 0.35$ across $\alpha_1 = 0.4 \;\; \alpha_2 = 0.25 \;\; \alpha_3 = 0.35$ to $\alpha_1 = 0.35 \;\; \alpha_2 = 0.4 \;\;\; \alpha_3 = 0.25$, there is neither a change in the equilibrium trade pattern nor in the international terms of trade. All that happens is a reallocation of labour and changes in the composition of outputs produced in the countries. It is only for large changes ('catastrophic" as Graham called them) that the trade pattern and terms of trade undergo a change. For example if the propensities to consume become $\alpha_1 = 0.5 \;\; \alpha_2 = 0.3 \;\;\; \alpha_3 = 0.2$, the trade pattern A-3, B-1, C-1,3, D-2,3 no longer gives an equilibrium; $x_{3C} = -1.66$ so that industry 3 in country must be closed down in view of the decline in the demand for commodity 3 and the trial trade pattern becomes A-3, B-1, C-1, D-2,3. The implied exchange rates are $E_{AB} = 1.2825 \; E_{AC} = 1.2825$, $E_{AD} = 1.2698$ and the labour allocations in D are $x_{2D} = 2.325$, $x_{3D} = 0.675$ but the pattern of gains from trade indicate that D should be assigned commodity 1.

Accordingly setting the new pattern A-3, B-1, C-1, D-1,2,3 implies the currency exchange rates $E_{AB} = 1.2698 \; E_{AC} = 1.2698$, $E_{AD} = 1.2698$ at which the gains from trade are consistent with the assignment and the labour allocations are positive. The new trade equilibrium is,

	A	B	C	D
1	-	$18\,w_B = 60\,P_{1B}$	$27\,w_C = 90\,P_{1C}$	$0.22\,w_D = 0.75\,P_{1D}$
2	-	-	-	$27.12\,w_D = 253.26\,P_{2D}$
3	$12\,w_A = 126\,P_{3A}$	-	-	$8.64\,w_D = 115.2\,P_{31D}$

The terms of trade change to 10:28:40. The price of commodity 3 has fallen relative to commodity 1 and its output has declined while that of commodity 1 has risen [*See* Graham (1948), p. 87].

VIII Complex Trade

Graham went on to consider a situation of 10 countries trading in 10 commodities [Graham (1948) Chapter VI pp 90-118] to test whether his conclusions carried over to even more complex situations. We shall consider this example to refine the algorithm that we have been formulating. He supposed that in each country $1/10^{th}$ of the income is spent on each commodity.[7] The sizes of the countries measured by the size of industry 1 under autarky are in the ratio 1:2:3:4:5:8:12:20:30:40. The labour coefficients are as tabulated below. The labour endowments are $L_A = 100, L_B = 200, L_C = 300, L_D = 400, L_E = 500, L_F = 800, L_G = 1200, L_H = 2000,$

$L_I = 3000, L_J = 4000$

	A	B	C	D	E	F	G	H	I	J
1	1	1	1	1	1	1	1	1	1	1
2	1/10	1/12	1/14	1/16	1/28	1/36	1/18	1/17	1/32	1/21
3	1/8	1/12	1/3	1/6	1/16	1/5	1/4	1/18	1/7	1/20
4	1/22	1/19	1/15	1/5	1/48	1/17	1/9	1/27	1/13	1/17
5	1/80	1/54	1/21	1/96	1/12	1/45	1/63	1/33	1/43	1/64
6	1/25	1/18	1/50	1/37	1/31	1/23	1/34	1/45	1/12	1/38
7	1/7	1/5	1/11	1/4	1/2	1/12	1/6	1/2	1/16	1/3
8	1/44	1/29	1/31	1/23	1/13	1/38	1/60	1/14	1/80	1/34
9	1/51	1/25	1/30	1/36	1/81	1/37	1/43	1/54	1/64	1/26
10	1/87	1/96	1/32	1/14	1/29	1/31	1/35	1/17	1/52	1/72

If we suppose without loss of generality at $w_j = 1$ in the currency units of each country the labour coefficients are also the money prices of commodities. The natural exchange rates then are,

	E_{AB}	E_{BC}	E_{CD}	E_{DE}	E_{EF}	E_{FG}	E_{GH}	E_{HJ}	E_{IJ}	E_{JA}
1	1	1	1	1	1	1	1	1	1	1
2	1.2	1.16	1.14	1.75	1.28	0.5	0.94	1.88	0.65	0.47
3	1.5	0.25	2	2.66	0.31	0.8	4.5	0.3	2.85	0.90
4	0.86	0.78	0.33	9.6	0.1	1.12	3	0.48	1.13	1.64
5	0.67	0.38	4.57	0.12	3.75	1.40	0.52	1.30	1.48	1.25
6	0.72	2.77	0.74	0.83	0.74	1.47	1.32	0.26	3.16	0.65
7	0.71	2.2	0.36	0.50	6	0.50	0.33	8	0.18	2.33
8	0.65	1.06	0.74	0.56	2.92	1.57	0.23	5.71	0.42	1.29
9	0.49	1.2	1.2	2.25	0.45	1.16	1.25	1.18	0.40	1.96
10	1.10	0.33	0.43	2.07	0.10	1.12	0.48	3.05	1.38	1.20

To find the trade assignment having the greatest comparative advantage pick up the lowest element in each column and consider their product. This gives the assignment A-9, B-3, C-4, D-5, E-10, F-2, G-8, H-6, I-7, J-2. That leaves out commodity 1. Therefore consider the next lowest product that includes the lowest natural exchange

rate corresponding to commodity 1. It is E^1_{JA} so 1 is allocated to J. Thus the first trial trade pattern in which every commodity is assigned to at least one country and every country produces at least one commodity is A-9, B-3, C-4, D-5, E-1, E-10, F-2, G-8, H-6, I-7, J-1,2. Since in this assignment commodity 2 is produced in common by countries F and J, $E^2_{FJ} = 0.5833 = E_{FJ}$ and since $E_{AF}E^2_{FJ} = E_{AJ}$ $0.5833E_{AF} = E_{AJ}$ eliminates one of the 9 unknown exchange rates $E_{AB}...E_{AJ}$. Since all the countries are producing one commodity and J alone is incompletely specialized there will be 2 unknown labour allocations in J, which along with the 8 unknown exchange rates is a total of 10 unknowns. To determine them there are 9 independent demand-supply equations and 1 full employment equation for J. The equations are as follows,

$$\frac{400\,x_{1J}}{l_{1J}} = \frac{10E_{JA} + 20E_{JB} + ... + 400}{P_{1J}}$$

$$\frac{800}{l_{2F}} + \frac{400\,x_{2J}}{l_{2J}} = \frac{10E_{JA} + 20E_{JB} + ... + 400}{P_{2J}}$$

$$\frac{200}{l_{3B}} = \frac{10E_{BA} + 20 + 30E_{BC} + ... + 400E_{BJ}}{P_{3B}}$$

$$\frac{300}{l_{4C}} = \frac{10E_{CA} + 20E_{BC} + ... + 400E_{CJ}}{P_{4C}}$$

$$\frac{400}{l_{5D}} = \frac{10E_{DA} + 20E_{DB} + ... + 400E_{DJ}}{P_{5D}}$$

$$\frac{2000}{l_{6H}} = \frac{10E_{HA} + 20F_{HB} + ... + 400E_{HJ}}{P_{6H}}$$

$$\frac{3000}{l_{7I}} = \frac{10E_{IA} + 20F_{IB} + ... + 400E_{IJ}}{P_{7I}}$$

$$\frac{1200}{l_{8G}} = \frac{10E_{GA} + 20E_{GB} + ... + 400E_{GJ}}{P_{8G}}$$

$$\frac{100}{l_{9A}} = \frac{10 + 20E_{AB} + ... + 400E_{AJ}}{P_{9A}}$$

$$\frac{500}{l_{10E}} = \frac{10E_{EA} + 20E_{EB} + ... + 400E_{EJ}}{P_{10,E}}$$

and

$$400x_{1J} + 400x_{2J} = 4000$$

Multiply the first two equations by E_{AJ}, the third by E_{AB}, the fourth by E_{AC} and so on leaving the ninth as it is and express all equations in terms of currency A. Making substitutions for l_{ij} and P_{ij} and after appropriate cancellations the system will be as follows:

$$(400\,x_{1J})\,E_{AJ} = 10 + 20\,E_{AB} + ... + 400\,E_{AJ}$$

$$(1371.42 + 400\,x_{2J})\,E_{AJ} = 10 + 20E_{AB} + ... + 400E_{AJ}$$

$$200E_{AB} = 10 + 20E_{AB} + ... + 400E_{AJ}$$

.....

$$500E_{AE} = 10 + 20E_{AB} + ... + 400E_{AJ}$$

and

$$400x_{1J} + 400x_{2J} = 4000$$

Observe that the first two equations are non-linear because x_{1J}, x_{2J} and E_{AJ} are unknowns. Nevertheless, the system can be solved by the usual linear methods. Since the right hand sides of the equations are equal, we may simply write,

$$400x_{1J} = 1371.42 + 400x_{2J}$$

which along with the full employment equation solves for $x_{1J} = 3.2857\, x_{2J} = 6.7142$. Substituting these values in the first two equations and using any 8 equations gives the solution of the exchange rates, $E_{AB} = 0.5,\ E_{AC} = 0.333,\ E_{AD} = 0.25,$ $E_{AE} = 0.2,\ E_{AF} = 0.0637,\ E_{AG} = 0.0833,\ \ E_{AH} = 0.5,\ E_{AI} = 0.0333,\ E_{AJ} = 0.0372$
Ascertain the gains from trade at these exchange rates. The new trade pattern indicated is A-9, B-3, C-4, D-5, E-10, F-2, G-8, H-4,6, I-1,2,7,8,9 J-1,2,3,4,6,10
This trade pattern, however, implies an inconsistent set of exchange rates since $E_{IJ} = E^1_{IJ}$ and $E_{IJ} = E^2_{IJ}$ but $E^1_{IJ} \neq E^2_{IJ}$. We will need to reset the trade pattern to eliminate the inconsistency.[8] To reset the trade pattern a clue may be taken from the gains from trade table itself.

	I	J	Ratio
1	30.03	26.88	1.11
2	960.96	564.61	1.70

It shows that I has the relative advantage in producing commodity 2 as compared to J. Accordingly, we eliminate commodity 1 from I's portfolio so that the new trade pattern that presents itself for trial in the first iteration is A-9, B-3, C-4, D-5, E-10, F-2, G-8, H-4,6, I-2,7,8,9, J-1,2,3,5,6,10. All the exchange rates are implied by the trade pattern itself viz. $E^9_{AI}, E^3_{BJ}, E^4_{CH}, E^5_{DJ}, E^6_{CH}, E^{10}_{EJ}, E^2_{FI}, E^8_{GI}, E^6_{HJ}, E^2_{IJ}$ which can be used to ascertain the exchange rates in terms of currency A, $E_{AB} = 0.4941,\ E_{AC} = 0.5417,\ E_{AD} = 1.2352,\ E_{AE} = 0.3317,\ E_{AF} = 1.4117,$ $E_{AG} = 0.9411,\ E_{AI} = 1.2549,\ E_{AJ} = 0.8235$ The right hand sides showing the value of world demand for each commodity in currency A works out to 1228.924. The unknowns are the labour allocations in countries H,I and J; 12 in all. To determine them there are 9 independent world demand supply equalities and 3 full employment equations for H, I and J. The equations are,

$$400x_{1J}E_{AJ} = 1228.924$$
$$(1371.42 + 457.142\,x_{2I} + 400x_{2J})E_{AJ} = 1228.924$$
$$(120 + 400x_{3J})E_{AJ} = 1228.924$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$(201.388 + 400x_{10,J})E_{AJ} = 1228.924$$

and

$$200\,x_{41H} + 200x_{6H} = 2000$$
$$300\,x_{2J} + 300\,x_{7I} + 300\,x_{8I} + 300\,x_{9I} = 3000$$
$$400\,x_{1J} + 400\,x_{2J} + 400\,x_{3J} + 400\,x_{5J} + 400\,x_{6J} + 400\,X_{10J} = 4000$$

The solution is,

$$x_{44} = 5.4681, x_{64} = 4.5318, x_{2I} = 3.4726, x_{7I} = 3.2643, x_{8I} = 0.2643, x_{9I} = 2.9987,$$

$$x_{1J} = 3.7307, x_{2J} = -3.6665, x_{3J} = 3.4307, x_{6J} = 1.0475, x_{10J} = 3.2273.$$

In other words, at the going exchange rates the production and trade pattern is feasible for all countries except country J which need not produce commodity 2 in the presence of F and I producing them. Accordingly, we eliminate 2 from J's portfolio and the rule will be not to assign to it any new commodity irrespective of the gains from trade since in the going situation $(x_{2J} < 0)$ any new assignment to it is beyond its production capacity. Other countries may of course be assigned more commodities depending on the gains from trade. Thus compute the gains from trade and make the new assignments keeping the existing portfolios intact. The new trade pattern is A-9, B-3,5,10, C-4,7, D-5, E-1,2,3,4,6,9,10, F-2, G-8, H-4,6, I-2,7,8,9 and J-1,3,5,6,10. Of course this assignment implies several inconsistent exchange rates, e.g. B-3,5, J-3,5,; B-3,10, J-3,10,; E-1,3, J-1,3,; E-4,6, H-4,6 etc. The inconsistencies must be removed by resetting the trade pattern in accordance with the observed gains from trade. These are shown in the tables below:

Table 1(a): Relative Advantages

	E	J	Ratio
1	3.01	1.21	2.482
3	48.23	24.28	1.978

Table 1(b): Relative Advantages

	E	J	Ratio
1	3.01	1.21	2.48
10	87.42	87.42	1.00

Table 1(c): Relative Advantages

	B	J	Ratio
3	24.28	24.28	1.00
5	109.29	77.77	1.40

Table 1(d): Relative Advantages

	B	J	Ratio
5	109.29	77.71	1.40
10	194.29	87.43	2.22

Table 1(e): Relative Advantages

	E	I	Ratio
2	84.41	25.50	3.31
9	244.2	51	4.78

Table 1(f): Relative Advantages

	E	H	Ratio
4	144.71	27.68	5.22
6	93.45	46.14	2.02

The ratios indicate that 3 must be removed from B's portfolio, 10 from J's portfolio, 4 from H's portfolio, 2 and 6 from E's portfolio to get a pattern A-9, B-5,10, C-4,7, D-5, E-1,4,9, F-2, G-8, H-6, I-2,7,8,9, J-1,3,5,6. But even this trade pattern implies inconsistent exchange rates since

$$E_{CI}^{7}\ E_{JE}^{9} = E_{CE}$$
$$E_{CE}^{4} = E_{CE}$$

give contradictory rates. We need to modify the trade pattern to obtain consistent exchange rates. Consider the relative advantage pattern below:

	C	E	I	Ratio
4	27.69	144.71	-	0.19
7	20.30	-	12.75	1.59
9	-	244.2	51	4.78

It shows that C should retain 7 and E should retain 9. Thus we remove 4 from C's portfolio where its relative advantage is the weakest. The new trial trade pattern is A-9, B-5, 10, C-7, D-5, E-1,4,9, F-2, G-8, H-6, I-2,7,8,9, J-1,3,5,6 and we proceed to perform the second iteration. The implied exchange rates are $E_{AB} = 1.3400, E_{AC} = 0.8627, E_{AD} = 2.3823, E_{AE} = 1.5882, E_{AF} = 1.4117, E_{AG} = 0.94117,$ $E_{AH} = 1.8807, E_{AI} = 1.2549,\ E_{AJ} = 1.5882$, and the value of world demand for each commodity equal to 1851.15 in A's currency. There are 13 unknown labour allocations for countries B, E, I and J and to determine them are the 4 full employment equations for these countries and 9 independent demand supply equations. The solution is $x_{5B} = -59.0788, x_{10,B} = 69.0788, x_{1E} = -26.1838,$
$x_{4E} = 23.3112, x_{9E} = 12.8726, x_{2I} = 1.9171, x_{7I} = 4.2296, x_{8I} = 1.9171, x_{9I} = 1.9361,$

$x_{1J} = 6.1868, x_{3J} = 2.9139, x_{5J} = 3.9063, x_{6J} = -3.0071$Accordingly, we close down industries 5, 1 and 6 in countries B, E and J respectively and make no new allocations to them. The pattern of gains from trade indicate allocation of 5 and 10 to A and 1 and 6 to C so that the trade pattern would stand at A-5,9, 10, B-10, C-1,6,7, E-4,9, F-2, G-8, H-6, I-2,7,8,9 and J-1,3,5. But this gives rise to contradictory exchange rates since $E_{AJ} = E_{AJ}^{5}$ and $E_{AI}^{9} E_{IC}^{7} E_{CJ}^{1} = E_{AJ}$. The pattern must be modified to ensure consistency. The gains from trade pattern is as follows:

	A	C	I	J	Ratio
1	-	1.16	-	0.63	1.84
5	80	-	-	40.29	1.98
7	X	12.75	12.75	-	1
9	51	-	51	-	1

Either C-7 or A-9 can be removed to ensure consistency. If we remove say C-7, the new trial trade pattern for iteration 3 is A-5,9,10, B-10, C-1,6, E-4,9, F-2, G-8, H-6, I-2,7,8,9, J-1,3,5 with implied exchange rates $E_{AB} = 1.1034, E_{AC} = 0.8, E_{AD} = 1.2,$ $E_{AE} = 1.5882, E_{AF} = 1.4117, E_{AG} = 0.94117,\quad E_{AH} = 0.72, E_{AI} = 1.2549, E_{AJ} = 0.8$ and the right-hand-side value of 1249.824.

There are now 14 unknown labour allocations to be solved and 14 independent equations. The solution is $x_{5A} = -33.9876, x_{9,A} = -58.9313, x_{10A} = 102.9189,$ $x_{1C} = 17.9239, x_{6C} = -7.9239, x_{4E} = 15.7388, x_{9E} = -5.7388, x_{2I} = 0.3198, x_{7I} = 3.3198,$ $x_{8I} = 0.3198, x_{9I} = 6.0405, x_{1J} = 2.5614, x_{3J} = 3.9057, x_{5J} = 3.4678$. Thus industries 5 and 9 in A and industry 6 in C will be closed down and no new assignment will be made to A and C. Gains from trade indicate the assignment of commodities 1,3,4 and 9 to H and 2 and 10 to J so that the trade pattern becomes A-10, B-10, C-1, D-5, E-4, F-2, G-8, H-1,3,4,6,9, I-2,7,8,9 J-1,2,3,5,10. This is inconsistent because $E^1_{HJ} \neq E^3_{HJ}$. The relative advantages of H and J in the production of 1 and 3 are as follows:

	H	J	Ratio
1	1.38	1.25	1.10
3	25	25	1

Therefore, 3 may be removed from H's assignment (Not 1 from J's following our rule of keeping the existing assignment intact). There is one more inconsistency, viz. $E^9_{HI} E^2_{IJ} = E_{HJ} \neq E^1_{HJ}$. Once again the relative advantage pattern may be referred.

	H	I	J	Ratio
1	1.38	-	1.25	1.10
2	-	25.5	26.5	0.96
9	75	51	-	1.47

This indicates removing 2 from I's assignment. The new trial trade pattern for iteration 4 is then A-10, B-10, C-1, D-5, E-4, F-2, G-8, H-1,4,6,9, I-7,8,9, J-1,2,3,5,10 with implied exchange rates and the value of world demand for $E_{AB} = 1.1034, E_{AC} = 0.8275, E_{AD} = 1.24125, E_{AE} = 1.4711, E_{AF} = 1.4187, E_{AG} = 0.7355,$ $E_{AH} = 0.8275, E_{AI} = 0.9807, E_{AJ} = 0.8275$. And the value of demand for each commodity of 1172.56 in A's currency. The solution for labour allocation is, $x_{1H} = 2.1298, x_{4H} = 2.6405, x_{6H} = 7.0849, x_{9H} = -1.8554, x_{7I} = 3.9854, x_{8I} = 0.9854,$ $x_{9I} = 5.0291, x_{1J} = 1.7275, x_{2J} = 0.1139, x_{3J} = 3.5424, x_{5J} = 2.0424, x_{10J} = 2.5737.$ Commodity 9 will be removed from H's portfolio. The gains from trade indicate an assignment of 6 to C, 5 to G and 2 to I. But that results in three inconsistencies in the exchange rates. These are $E^1_{GJ} = E^5_{GJ}, E^8_{GI} E^2_{IJ} = E_{GJ}$ and $E^1_{CH} = E^6_{CH}$. The following 3 tables show the relative advantage patterns indicating removal of 5 from G, 2 from J and 1 from C.

	G	J	Ratio
1	1.36	1.20	1.12
5	85.65	77.34	1.10

	G	I	J	Ratio
1	1.36	-	1.20	1.12
2	-	81.57	25.37	3.12
8	81.57	81.57	-	1

	C	H	Ratio
1	1.20	1.20	1
6	60.42	54.38	1.11

The new trial trade pattern for the fifth iteration is A-10, B-10, C-1,6, D-5, E-4, F-2, G-1,8, H-1,4,6, I-2,7,8,9, J-1,3,5,10 with implied exchange rates $E_{AB}=1.1034, E_{AC}=0.9194, E_{AD}=1.24125, E_{AE}=1.4711, E_{AF}=1.24125, E_{AG}=0.8275,$ $E_{AH}=0.8275, E_{AI}=1.1033, E_{AJ}=0.8275$ and value of world demand for each commodity equal to 1208.945.

The solution for 13 unknown labour allocations works out to
$x_{1G}=4.6342, x_{8G}=5.3657, x_{1H}=1.5015, x_{4H}=2.8603, x_{6H}=5.6381, x_{2I}=0.6524,$

$x_{7I}=3.6524, x_{8I}=2.0428, x_{9I}=3.6524, x_{1J}=1.5115, x_{35}=3.6524, x_{5J}=2.1524,$
$x_{10,J}=2.6836$. All the labour allocations are positive. Gains from trade indicate an assignment of commodity 5 to A and commodity 9 to H. That, however, would result in 2 inconsistencies in the exchange rates. $E_{AJ}^{5}=E_{AJ}^{10}$ *and* $E_{HI}^{9}E_{IG}^{8}=E_{HG}^{1}$. These are resolved from the following gains from trade tables.

	A	J	Ratio
5	80	77.34	1.03
10	87	87.00	1.00

	G	H	I	
1	1.20	1.20	-	1
8	72.50	-	72.50	1.000
9	-	65.25	58.00	1.125

Commodity 10 may be removed from A's assignment and commodity 1 from H's assignment giving a new trial trade pattern A-5, B-10, C-6, D-5, E-4, F-2, G-1,8, H-4,6,9, I-2,7,8,9 J-1,3,5,10 for the 6th iteration. The implied exchange rates are
$E_{AB}=1.1034, E_{AC}=1, E_{AD}=1.2, E_{AE}=1.6, E_{AF}=1.2, E_{AG}=0.8, E_{AH}=0.9, E_{AI}=1.0666, E_{AJ}=0.8$

The value of world demand for each commodity is 1201.33. The solution for 13 labour allocations is $x_{1G}=8.4583, x_{8G}=1.5416, x_{4H}=2.2296, x_{6H}=5.0074,$
$x_{9H}=2.7629, x_{2I}=0.7541, x_{7I}=3.7541, x_{8J}=3.2916, x_{9I}=2.2, x_{1J}=1.2166, x_{3J}=3.7541,$
$x_{5J}=1.9416, x_{10,J}=3.0875.$ all of which are positive. Moreover, the pattern from gains from trade exactly supports the trade pattern under trial. The world trade equilibrium has been found. The international ratio of commodity exchange is 1 unit of 1 = 24 units of 2 = 20 units of 3 = 24 units of 4 = 64 units of 5 = 40 units of 6 = 12 units of 7 = 60 units of 8 = 48 units of 9 = 72 units of 10. This tallies exactly with the trade equilibrium found by Graham [Graham (1948) pp. 96-97].

The production and consumption levels of the commodities are shown in Table 2.

Table 2(a): Production

	A	B	C	D	E	F	G	H	I	J
1	-	-	-	-	-	-	101.5	-	-	48.66
2	-	-	-	-	-	2880	-	-	724	-
3	-	-	-	-	-	-	-	-	-	3003.33
4	-	-	-	-	2400	-	-	1204	-	-
5	800	-	-	3840	-	-	-	-	-	4970.66
6	-	-	1500	-	-	-	-	4506.66	-	-
7	-	-	-	-	-	-	1110	-	1802	-
8	-	-	-	-	-	-	-	-	7900	-
9	-	-	-	-	-	-	-	2984	4224	-
10	-	1920	-	-	-	-	-	-	-	8892

Table 2(b): Consumption

	A	B	C	D	E	F	G	H	I	J
1	1.25	2.66	3.75	6	10	12	12	22.5	40	40
2	30	64	90	144	240	288	288	540	960	960
3	25	53.33	75	120	200	240	240	450	800	800
4	30	64	90	144	240	288	288	540	960	960
5	80	170.66	240	384	640	768	768	1440	2560	2560
6	50	106.66	150	240	400	480	480	900	1600	1600
7	15	32	45	72	120	144	144	270	480	480
8	75	160	225	360	600	720	720	1350	2400	2400
9	60	128	180	288	480	576	576	1080	1920	1920
10	90	192	270	432	720	864	864	1620	2880	2880

Graham then proceeded to demonstrate that large changes in demand conditions do not lead to changes in the terms of trade and that the small sized countries stand more to gain from trade than the large countries and that international terms of trade are usually tied down to the domestic cost ratios of commodities prevailing in the larger groups of countries. [Graham (1948), Chapter VII].

On the basis of these examples Graham insisted that international values would be tied down to costs of production of the commodities that are produced in common between the countries and that any disturbance in international equilibrium would be corrected by a reallocation of labour between industries and a change in the volumes and composition of world outputs and trade rather than relative prices of countries' exports and imports. Graham used this theory to explain Taussig's (1927) empirical observation that balance of payments disequilibrium were very rapidly corrected by changes in volumes of exports and imports of countries but were accompanied by very small movements in relative price movements or movements of gold between countries. And it is certainly true that the adjustment mechanics of Graham's theory is much richer in its details than the Keynesian adjustment mechanism formulated by Ohlin (1929), Robinson (1937), Harrod (1939), Metzler (1942), Machlup (1942) and several others to explain Taussig's observations.

Yet it cannot be denied that there have been historical episodes in which substantial terms of trade effects have also been observed in the process of adjustment. In fact if

examples of multicountry trade are constructed in which the number of commodities exceed the number of countries, situations in which countries produce commodities in common become much less rare than in Graham's examples, where, typically the number of commodities is less than or equal to the number of countries and countries themselves differ greatly in economic size. Thus consider a Graham-type example of 4 countries with sizes $L_A = 1400, L_B = 2100, L_C = 2800, L_D = 3500$ producing 7 commodities and spending $1/7^{th}$ of the income on each commodity. Suppose the autarky prices are,

$$
\begin{array}{llll}
P_{1A} = 10 & P_{1B} = 3.3 & P_{1C} = 5 & P_{1D} = 1 \\
P_{2A} = 4 & P_{2B} = 5 & P_{2C} = 2.5 & P_{2D} = 20 \\
P_{3A} = 2.5 & P_{3B} = 3 & P_{3C} = 4 & P_{3D} = 1.5 \\
P_{4A} = 1 & P_{4B} = 10 & P_{4C} = 3.3 & P_{4D} = 6.66 \\
P_{5A} = 3.33 & P_{5B} = 7.5 & P_{5C} = 6.6 & P_{4D} = 1.66 \\
P_{6A} = 2 & P_{6B} = 1.66 & P_{6C} = 10 & P_{6D} = 5 \\
P_{7A} = 5 & P_{7B} = 4 & P_{7C} = 1 & P_{7D} = 4
\end{array}
$$

The international trade equilibrium, found in 3 iterations, is A-4, B-6, C-2,7, D-1,3,5 with exchange rates $E_{AB} = 0.6666, E_{AC} = 1, E_{AD} = 1.2$ and no commodity is produced in common between countries. Thus to sum up with Jones (1976), "Since Graham's work there has apparently emerged an agreement among writers in this area that an electic view is appropriate. In a world of many countries and many commodities, a disturbance to trade may be met primarily by price changes, on the one hand, or production changes, on the other. Limbo price ratios may occur and are not as 'unstable' as Graham would have led us to believe".

IX The Transfer Problem

The purpose of this and the following two sections on protective duties and domestic taxes is not to provide a detailed treatment of their subjects as it is to illustrate how the method of finding the trade equilibrium should be applied in the presence of intercountry transfers and tariffs. Graham devoted a chapter [Graham (1948), Chapter 9, pp157-207] to these subjects. Even though we have thus far used Graham's own examples we shall now be parting company with him because when discussing the transfer problem Graham abandons the multicountry multicommodity trade framework and instead gives a 2 country monetary example in terms of the aggregative monetary values of production, consumption and exports / imports. When discussing protective tariffs, however, he uses the 10 country 10 commodity trade example and offers a very detailed analysis of the effects of tariffs on the trade pattern and terms of trade. However, he does not make any mention of what the tariff levying authorities do with the revenue they earn, that is to say, he implicitly supposes that the manner in which the tariff revenue is spent has no consequences on the results, an assumption that we shall not make.

Therefore, consider an example of 3 countries trading in 4 commodities. Their autarky equilibria are as follows:

	A	B	C
1	$30\,w_A = 120\,P_{1A}$	$100\,w_B = 400\,P_{1B}$	$90\,w_C = 180\,P_{1C}$
2	$20\,w_A = 200\,P_{2A}$	$100\,w_B = 300\,P_{2B}$	$60\,w_C = 160\,P_{2C}$
3	$20\,w_A = 60\,P_{3A}$	$100\,w_B = 600\,P_{3B}$	$60\,w_C = 60\,P_{3C}$
4	$30\,w_A = 240\,P_{4A}$	$100\,w_B = 800\,P_{4B}$	$90\,w_C = 210\,P_{4C}$

Supposing the money wage rates to be 1 in the currencies of the respective countries. Then the money prices are:

	A	B	C
1	$P_{1A} = 0.25$	$P_{1B} = 0.25$	$P_{1C} = 0.5$
2	$P_{2A} = 0.10$	$P_{2B} = 0.333$	$P_{2C} = 0.375$
3	$P_{3A} = 0.333$	$P_{3B} = 0.166$	$P_{3C} = 1$
4	$P_{4A} = 0.125$	$P_{4B} = 0.125$	$P_{4C} = 0.428$

The world trade equilibrium found by the method explained earlier is:

	A	B	C
1	-	$43.75\,w_B = 175\,P_{1B}$	$300\,w_C = 600\,P_{1C}$
2	$100 w_A = 1500 P_{2A}$	-	-
3	-	$162.5\,w_B = 975\,P_{3B}$	-
4	-	$193.75\,w_B = 1550\,P_{4B}$	-

The exchange rates are $E_{AB} = 0.6153$ $E_{BC} = E^1_{BC} = 0.5$, $E_{AC} = 0.3077$ and terms of trade equal to 6.5 units of 1 = 10 units of 2 = 9.75 units of 3 = 13 units of 4.
Now suppose that countries A and C transfer amounts of 20 and 50 respectively in terms of their currencies to country B. The disposable incomes in A and C fall to $Y_{dA} = Y_A - T_{AB,}$ $Y_{dC} = Y_C - T_{CB}$ and that of B rises to $Y_{dB} = Y_B + T_{AB}E_{BA} + T_{CB}E_{BC}$. The post-transfer market clearing equations are,

$$\frac{L_{1B}x_{1B}}{l_{1B}} + \frac{L_C}{l_{1C}} = \frac{\Sigma\alpha_{1i}Y_{di}E_{Bi}}{P_{1B}}$$

$$\frac{L_A}{l_{2A}} = \frac{\Sigma\alpha_{2i}Y_{di}E_{Ai}}{P_{2A}}$$

$$\frac{L_{3B}x_B}{l_{3B}} = \frac{\Sigma\alpha_{3i}Y_{di}E_{Bi}}{P_{3B}}$$

$$\frac{L_{4B}x_{3B}}{l_{4B}} = \frac{\Sigma\alpha_{4i}Y_{di}E_{Bi}}{P_{4B}}$$

to which may be added the full employment equation of country B,

$$L_{1B}x_{1B} + L_{3B}x_{3B} + L_{4B}x_{4B} = L_B$$

The post-transfer trade equilibrium is:

	A	B	C
1	-	$41.93\,w_B = 167.72\,P_{1B}$	$300\,w_C = 600\,P_{1C}$
2	$100\,w_A = 1000\,P_{2A}$	-	-
3	-	$166.19\,w_B = 997.14\,P_{3B}$	-
4	-	$191.93\,w_B = 1535.44\,P_{4B}$	-

The exchange rates are $E_{AB} = 0.6019$ $E_{BC} = E^1_{BC} = 0.5$, $E_{AC} = 0.3009$ and terms of trade are 6.64 units of 1 = 10 units of 2 = 10.38 units of 3 = 13.29 units of 4. The terms of trade have improved in favour of country A.

If we consider the pre and post transfer levels of consumption we get:

	A		B		C	
	Pre	Post	Pre	Post	Pre	Post
1	195	159.49	400	458.22	180	150
2	200	160	500	689.51	180	150
3	195	159.49	600	687.98	180	150
4	390	318.98	800	916.44	360	300

The size of the transfer from A to B was 20 in A's currency and 50 in C's currency, i.e., at the pre-transfer exchange rates these would be 32.5 and 25 respectively in B's currency i.e. a total of 57.5. However, the increase in the disposable income in B is 58.22. The transfer has been over affected by 58.22-57.5=0.72. This is exactly equal to the difference as measured in B's currency of the transfer from A to B measured at the post and pre-transfer exchange rates, i.e. 33.22-32.5 = 0.72. It is a well-known result of international macroeconomics that if the sum of intercountry propensities to import, in this case $\alpha_{1A} + \alpha_{3A} + \alpha_{4A} + \alpha_{2B} = 1.05 > 1$ transfers are over effected. [Metzler (1942), Machlup (1943), Meade (1951), Johnson (1957)]. By the same logic the transfer from C to B should have been over effected since $\alpha_{1C} + \alpha_{3C} + \alpha_{4C} + \alpha_{1B} = 1.05 > 1$ but that does not happen because the exchange rate of their currencies remains fixed at $E^1_{BC} = E_{BC} = 0.5$ on account of commodity 1 being produced in common between B and C. In fact this will be found to be a general conclusion: if the exchange rate between any two countries is fixed at a natural exchange rate in both the pre and post transfer situations, transfers between the two countries will be exactly affected no matter what the sum of propensities to import. [And since Graham was of the opinion that in multicountry multicommodity situations all exchange rates would be tied to natural exchange rates (that is, international terms of trade would be tied to cost ratios of commodities produced in common between countries) transfers would generally not affect the terms of trade, that they would usually be exactly effected [See Graham (1948), p. 198)].

X Tariffs

Next consider the effects of import tariffs. Suppose one of the trading countries, say country A imposes a tariff at the rate t_{iA} on an imported commodity *i.* Clearly, the post-tariff price of the commodity would rise to

$$P_{ij} \Big/ (1-t_{iA})$$

where *j* indexes the countries from which A imports commodity *i.* Then if the citizens of country A spend an amount $\alpha_{iA} w_A L_A$ on the commodity the quantity purchased would reduce to

$$\frac{(\alpha_{iA} w_A L_A) E_{jA}}{P_{ij} \Big/ (1-t_{iA})}$$

so that the total tariff revenue which equals the tariff rate multiplied by the product of the two expressions above would be,

$$t_{iA} \alpha_{iA} w_A L_A$$

We shall suppose in the following example that the government spends the tariff revenue entirely on the non-tradable commodity. But more general patterns of spending can easily be accommodated in the framework.

Consider 3 countries trading in 3 commodities with each country producing a non-tradable commodity 4. Their autarky equlibria are as follows:

	A	B	C
1	$20 w_A = 120 P_{1A}$	$60 w_B = 150 P_{1B}$	$60 w_C = 25 P_{1C}$
2	$50 w_A = 80 P_{2A}$	$600 w_B = 500 P_{2B}$	$60 w_C = 60 P_{2C}$
3	$30 w_A = 60 P_{3A}$	$30 w_B = 100 P_{3B}$	$80 w_C = 800 P_{3C}$
4	$50 w_A = 500 P_{4A}$	$50 w_B = 500 P_{2B}$	$50 w_C = 500 P_{4C}$

The prices of the tradable commodities, supposing the wage rate to be 1 in each country's currency, are

	A	B	C
1	$P_{1A} = 0.166$	$P_{1B} = 0.4$	$P_{1C} = 2.4$
2	$P_{2A} = 0.625$	$P_{2B} = 0.12$	$P_{2C} = 1$
3	$P_{3A} = 0.5$	$P_{3B} = 0.3$	$P_{3C} = 0.1$

The world trade equilibrium is A-1,4 B-2,4 and C-3,4 with equilibrium exchange rates $E_{AB} = 0.8666$, $E_{AC} = 0.4666$ and the terms of trade are 10 units of 1=16 units of 2=35.66 units of 3.

Suppose now that country A imposes a tariff at a 50% rate on commodity 3. The tariff revenue is (0.5) (30) = 15 which we shall suppose is spent on commodity 4. The equations, noting that the price of commodity 3 in country C will now be (0.1/(1-0.5) = 0.2, are

$$\frac{L_{1A}x_{1A}}{l_{1A}} = \frac{\alpha_{1A}Y_A + \alpha_{1B}Y_B E_{AB} + \alpha_{1C}Y_C E_{AC}}{P_{1A}}$$

$$\frac{L_{2B}x_{2B}}{l_{2B}} = \frac{\alpha_{2A}Y_A E_{BA} + \alpha_{2B}Y_B + \alpha_{2C}Y_C E_{BC}}{P_{2B}}$$

$$\frac{L_{3C}x_{3C}}{l_{3C}} = \frac{\alpha_{3A}Y_A E_{CA}}{0.2} + \frac{\alpha_{3B}Y_B E_{CB} + \alpha_{3C}Y_C}{0.1}$$

$$\frac{L_{4A}x_{4A}}{l_{4A}} = \frac{\alpha_{4A}Y_A + 15}{P_{4A}}$$

$$L_{4B}x_{4B} = L_{4B}$$

$$L_{4C}x_{4C} = L_{4C}$$

$$L_{1A}x_{1A} + L_{4A}x_{4A} = L_A$$

$$L_{2B}x_{2B} + L_{4B}x_{4B} = L_B$$

$$L_{3C}x_{3C} + L_{4C}x_{4C} = L_C$$

where the price of 3 in A and the spending of the tariff revenue of 15 on the non-tradable commodity 4 are shown numerically. The solution is $x_{1A} = 4.25, x_{2B} = 2.5, x_{4A} = 1.3, x_{4B} = 1,\ x_{3C} = 2.5, x_{4C} = 1,\ E_{AB} = 0.7666, E_{AC} = 0.3166$. The currency of country A has appreciated and the terms of trade have moved in A's favour. A comparison between pre and post tariff levels of consumption in the three countries are as follows:

	A		B		C		Total	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	120	120	312	276	168	510	600	540
2	480.77	543.47	500	500	269.23	206.52	1250	1250
3	642.85	473.68	557.15	726.31	800	800	2000	2000

Observe that output of commodity 1 in country A has declined due to the tariff and the spending of tariff revenue on commodity 4. The world output of tradables shows a decline and country C is the loser in the redistribution.

XI Domestic Taxes

Similar methods can be devised to deal with other relevant subjects such as export subsidies/taxes, domestic taxes and the like. For instance suppose that the government of country A decides to impose an excise tax on all the commodities it produces, which means that in the example of the previous section, the tax will be imposed on commodities 1 and 4 in the post trade situation. Suppose the tax rate to be 10%. The prices of commodities 1 and 4 will rise to (0.666) / (1-0.10) and (0.1)/(1-0.10), i.e. to 0.1851 and 0.111 respectively. Suppose that 50 per cent of the revenue is spent on each commodity by the government. The equations now are:

$$\frac{L_{1A}x_{1A}}{l_{1A}} = \frac{\Sigma\alpha_{1j}Y_jE_{Aj} + (0.5)\ R}{0.1851}$$

$$\frac{L_{2B}x_{2B}}{l_{2B}} = \frac{\Sigma\alpha_{2j}Y_jE_{Bj}}{p_{2B}}$$

$$\frac{L_{3C}x_{3C}}{l_{3C}} = \frac{\Sigma\alpha_{3j}Y_jE_{Cj}}{p_{3C}}$$

$$\frac{L_{4A}x_{4A}}{l_{4A}} = \frac{\alpha_{4A}Y_A + (0.5)R}{0.1111}$$

$$L_{4B}x_{4B} = L_{4B}$$

$$L_{4C}x_{4C} = L_{4C}$$

$$L_{1A}x_{1A} + L_{4A}x_{4A} = L_A$$

$$L_{2B}x_{2B} + L_{4B}x_{4B} = L_B$$

$$L_{3C}x_{3C} + L_{4C}x_{4C} = L_C$$

Where R is the tax revenue given by

$$R=(0.1851-0.1666)\frac{(L_{1A}x_{1A})}{0.1666} + (0.1111-0.1)\frac{(L_{4A}x_{4A})}{0.1}$$

The solution is $E_{AB} = 0.8666, E_{AC} = 0.4666, x_{1A} = 4.875, x_{4A} = 1.05, x_{2B} = 2.5,$

$x_{4B} = 1, x_{3C} = 2.5, x_{4C} = 1$ and R=16.66. The pattern of gains from trade continues to support the trade pattern at the post-tax price of commodity at the tax rate of 10 per cent so the new trade equilibrium looks as follows:

	A	B	C
1	$97.5w_A = 585P_{1A}$	-	
2	-	$150w_B = 1250P_{2B}$	-
3	-	-	$200w_C = 2000P_{2C}$
4	$52.5w_A = 525P_{4A}$	$50w_B = 500P_{4B}$	$50w_C = 500P_{4C}$

The terms of trade are 10 units of 1 = 17.48 units of 2=39.66 units of 3, which shows a movement in favour of country A. The pre and post –tax levels of consumption in the countries is as follows:

	A		B		C	
	Pre	Post	Pre	Post	Pre	Post
1	120	153	312	280.80	168	151.20
2	480.77	480.77	500	500	269.23	269.23
3	642.85	642.85	557.15	557.15	800	800
4	500	525	500	500	500	500

In case of commodity 1 in country A, 108 units are privately consumed and 45 units by government and for commodity 4,450 units are privately consumed and 75 units by government.

XII Concluding Remarks

The purpose of this paper was to find an algorithm to solve the world trade equilibrium for multicountry multicommodity trade situations of the type formulated by Graham. To that end an algorithm has been proposed. As is only natural, there will usually be several possible algorithms of varying efficiency to solve this problem. This paper has nothing substantial to comment on other possible algorithms and the relative efficiency of the proposed algorithm in relation to those others. As such it may be regarded only as a beginning. However, the methods here outlined have applicability to more general contexts including intercountry transfers and domestic and international taxation. The appendix shows how these methods can be generalized to cover the subject of international trade in intermediate capital goods. Graham's conjecture that the presence of fiat moneys in which wages and prices are denominated does not disturb the real trade equilibrium has also been found to be valid. Not all of our conclusions match those of Graham; in particular it is perfectly possible that international terms of trade will not coincide with domestic ratios of exchange and yet the equilibrium will be stable.

Appendix

International Trade in Capital Goods[9]

1. The purpose of this appendix is to show that Graham's theory can be readily extended to the general case of 'production of commodities by means of commodities and labour' and therefore to international trade in intermediate capital goods. Of course specific conclusions applicable to the special case of 'production by means of labour alone' will not carry over to the general case.

Suppose A to be a *nxn* matrix of technical coefficients and *L* an *nx1* vector of labour coefficients required to produce unit outputs of commodities. As before α_i denotes the share of income spent on the final consumption of the commodities, *w* the money wage rate and L_e the labour endowment. The quantities demanded for final consumption are,

$$F_i = \frac{\alpha_i w L_e}{P_i} \qquad i = 1....n \qquad \ldots (1)$$

The prices of the commodities are obtained from

$$A^T P + wL = P$$

i.e.

$$P = (I - A^T)^{-1} wL \qquad \ldots (2)$$

The gross output vector that must be produced by the economy to satisfy the quantities demanded in (1) is

$$B = (I - A)^{-1} F$$

2. This describes the autarky equilibrium of an economy. Consider an example of 2 countries with labour endowments $L_A = 20$ and $L_B = 25$ each spending half of its net national income on the 2 commodities and whose autarky equilibria are as follows:

A

$$3.052\,P_{1A} + 4.069P_{2A} + 9.156w_A = 20.347P_{1A}$$

$$3.943\,P_{1A} + 4.928P_{2B} + 10.843w_A = 19.715P_{2A}$$

B

$$5.749\,P_{1B} + 4.791P_{2B} + 16.290w_B = 19.165P_{1B}$$

$$5.443\,P_{1B} + 4.354P_{2B} + 8.709w_B = 21.772P_{2B}$$

If the money wage rates are $w_A = \$1$ and $w_B = y1$, the prices and the natural exchange rates are

$P_{1A} = 0.7489$ $\quad P_{1B} = 1.5678$

$P_{2A} = 0.9330$ $\quad P_{2B} = 0.9899$

$E^1_{AB} = 0.4776$ $\quad E^1_{BA} = 2.0933$

$E^2_{AB} = 0.9425$ $\quad E^2_{BA} = 1.0609$

The trade pattern indicated is A-1 B-2. Open the countries to trade. The market clearing equations will be,

$$\frac{L_A}{l_{1A}} = \frac{\alpha_{1A} w_A L_A + \alpha_{1B} w_B L_B E_{AB}}{P_{1A}} + \left[\frac{a_{11A} L_A}{l_{1A}} + \frac{a_{12B} L_B}{l_{2B}} \right]$$

$$\frac{L_B}{l_{2B}} = \frac{\alpha_{1A} w_A L_A E_{BA} + \alpha_{2B} w_B L_B}{P_{2B}} + \left[\frac{a_{21A} L_A}{l_{1A}} + \frac{a_{22B} L_B}{l_{2B}} \right] \quad \ldots (4)$$

where the left hand sides show the outputs produced of the respective commodities when the countries are fully specialized in their production, the first terms on the right hand sides are the quantities demanded for final consumption and the second term are the quantities demanded as inputs to produce the post-trade outputs. The exchange rate E_{BA} can be eliminated by multiplying the second equation by E_{AB}. What remains is one independent equation with which to solve the sole unknown, E_{AB}. The difficulty is that P_{1A} (or P_{2B}) are unknowns unlike in the special case of 'production by means of labour alone'. The post-trade prices of commodities will therefore need to be ascertained. But these cannot be ascertained until the exchange rate is known. So there is circularity here and we must proceed iteratively. Thus substitute initially the autarky price P_{1A} in the first equation so that $E_{AB} = 0.4076$ is the initial solution for the exchange rate.

The equations to solve the post trade prices of commodities are,

$$a_{11A} P_{1A} + a_{21A} P_{2A} E_{AB} + w_A l_{1A} = P_{1A}$$
$$a_{12B} P_{1A} + a_{22B} P_{2B} E_{AB} + w_B l_{2B} E_{AB} = P_{2B} E_{AB}$$

where the prices have for convenience been expressed in the currency of country A. The solution for the prices in own currencies of the countries is,

$$\begin{bmatrix} P_{1A} \\ P_{2B} \end{bmatrix} = \begin{bmatrix} (1 - a_{11A}) & -a_{21A} E_{AB} \\ -a_{12B} & (1 - a_{22B}) E_{AB} \end{bmatrix}^{-1} \begin{bmatrix} w_A l_{1A} \\ w_B l_{2B} E_{AB} \end{bmatrix} \quad \ldots (5)$$

In fact using the solution in (5) the domestic prices of both commodities in each country post-access to trade can be ascertained;

$$a_{11A} P_{1A} + a_{21A} P_{2B} E_{AB} + w_A l_{1A} = P_{1A}$$
$$a_{12A} P_{1A} + a_{22A} P_{2B} E_{AB} + w_A l_{2A} = P_{2A} \quad \ldots 6(a)$$

$$a_{11B} P_{1A} E_{BA} + a_{21B} P_{2B} + w_B l_{1B} = P_{1B}$$
$$a_{12B} P_{1A} E_{BA} + a_{22B} P_{2B} + w_B l_{2B} = P_{2B} \quad \ldots 6(b)$$

The solution of (5) corresponding to the tentative solution $E_{AB} = 0.4076$ is $P_{1A} = 0.6231, P_{2B} = 0.9777$ which can be substituted back into (4) to obtain a new solution for E_{AB} and so on. In 27 iterations the solution obtained (to an accuracy of the 7^{th} place of decimals) is $E_{AB} = 0.2744$, $P_{1A} = 0.6062, P_{2B} = 1.1902$. The domestic

prices of production of the imported goods are $P_{2A} = 0.7529, P_{1B} = 1.8102$. The gains from trade pattern does not support the trade pattern under trial;

	A	B
1	1.649	2.012*
2	1.328	3.060*

It shows that B has the advantage in producing both the commodities

The new trial trade pattern indicated is A-1, B-1,2. There will now be 2 market clearing equations and 1 full employment equation for country B.

$$\frac{L_A}{l_{1A}} + \frac{L_{1B}x_{1B}}{l_{1B}} = \frac{M_1}{P_{1A}} + \left[\frac{a_{11A}L_A}{l_{1A}} + \frac{a_{11B}L_{1B}x_{1B}}{l_{1B}} + \frac{a_{12B}L_{2B}x_{2B}}{l_{2B}}\right]$$

$$\left(\frac{L_{2B}x_{2B}}{l_{2B}}\right)E^1_{AB} = \frac{M_2}{P_{2B}} + \left[\frac{a_{21A}L_A}{l_{1A}} + \frac{a_{21B}L_{1B}x_{1B}}{l_{1B}} + \frac{a_{22B}L_{2B}x_{2B}}{l_{2B}}\right]E^1_{AB} \qquad \ldots(7)$$

$$L_{1B}x_{1B} + L_{2B}x_{2B} = L_B$$

where $M_i = \alpha_{iA}w_A L_A + \alpha_{iB}w_B L_B E_{AB}$

With the exchange rate $E_{AB} = E^1_{AB} = P_{1A}/P_{1B}$ there are two independent equations in (7) with which to solve for 2 labour allocations x_{1B}, x_{2B}. Using the first 2 equations the solution to an accuracy of the 7th place of decimals obtained in 12 iterations is $E_{AB} = 0.3965, x_{1B} = 0.4352, x_{2B} = 3.5937, P_{1A} = 0.6217, P_{2A} = 0.7725, P_{1B} = 1.5678,$ $P_{2B} = 0.9899$

Which, when substituted in the full employment equation of B, shows that the equation is not satisfied;

(16.2908) (0.4352) + (8.709) (3.5937) = 37.91 $\neq 25$

The equations are therefore inconsistent. The issue may viewed from a different angle. If one uses the first and third equations to solve for x_{1B}, x_{2B} and uses the second equation to find the exchange rate, the solution obtained iteratively is $E_{AB} = 0.2744, x_{1B} = 0, x_{2B} = 2.8705$ indicating that industry 1 in country B must be closed down even though the gains from trade at that exchange rate indicate an advantage to B in its production. If may be concluded that the 2 x 2 production and trade example under discussion has no equilibrium solution.

Two remarks are in order. In the simple case of 'production by labour alone', Graham (1948) had conjectured and McKenzie (1954a) proved that international trade equilibrium exists. The example that has just been discussed demonstrates that this conclusion does not carry over to the case of 'production by means of commodities and labour'. Likewise, Graham's conjecture that in multicountry multicommodity trade situations it is most likely that commodities will be produced in common between countries also loses much of its force in the case of 'production by means of

commodities and labour'. In fact it would be most unlikely. To illustrate, if A and B were to both produce commodity 1 in the trade equilibrium, it would require

$$\frac{a_{11}P_{1A} + a_{21A}P_{2B}E_{AB} + w_A l_{1A}}{a_{11B}P_{1A}E_{BA} + a_{21B}P_{2B} + w_B l_{1B}} = E^1_{AB} = E_{AB}$$

i.e.

$$E_{AB} = \frac{(a_{11B} - a_{11A})P_{1A} - w_A l_{1A}}{(a_{21A} - a_{22B})P_{2B} - w_B l_{1B}}$$

A very special set of cost and demand conditions in the two countries alone would make it possible. And when several commodities are involved including non-tradable the condition becomes even more unlikely to occur.

3. If the labour coefficients in country B were to be $l_{1B} = 0.5, l_{2B} = 0.4$ and the labour endowment were to be 18, unique trade equilibrium would be found for the trade pattern A-1 B-2 with the solution $E_{AB} = 0.9160, P_{1A} = 0.6877,$ $P_{2A} = 0.85579, P_{1B} = 0.9088, P_{2B} = 0.7346$ which is obtained in 15 iterations. Observe that the currency exchange rate lies between the 'post-trade natural exchange rates', $E^1_{AB} = 0.7517$ and $E^2_{AB} = 1.164$ as well as the pre-trade ones showing that both countries stand to gain by trade.

A comparison of the production and consumption levels under autarky and trade give an idea of the total gains from trade.

Commodity	Autarky		Trade	
	1	2	1	2
Production A	20.34	19.71	44.44	-
B	19.95	20.05	-	45
Final Use A	13.35	10.71	14.54	14.86
B	8.95	11.05	11.98	12.25
Intermediate Use A	6.99	8.19	6.66	8.88
B	10.99	9.00	11.25	9.00

4. To get a flavour of how the system would look in multicountry multicommodity situations consider an example of three countries A, B, C with labour endowments of 500, 600 and 700 respectively and each country spending 1/4th of its net income on each of the commodities. The technical and labour coefficients are as follows:

$$A_{ijA} \qquad L_{iA}$$

$$\begin{bmatrix} 0.02 & 0.10 & 0.12 & 0.15 \\ 0.03 & 0.08 & 0.14 & 0.12 \\ 0.02 & 0.06 & 0.10 & 0.10 \\ 0.04 & 0.09 & 0.14 & 0.12 \end{bmatrix} \quad \begin{bmatrix} 0.05 \\ 0.10 \\ 0.15 \\ 0.20 \end{bmatrix}$$

$$A_{ijB} \qquad L_{iB}$$

$$\begin{bmatrix} 0.10 & 0.04 & 0.12 & 0.15 \\ 0.08 & 0.03 & 0.14 & 0.12 \\ 0.07 & 0.04 & 0.10 & 0.10 \\ 0.12 & 0.02 & 0.14 & 0.10 \end{bmatrix} \quad \begin{bmatrix} 0.15 \\ 0.05 \\ 0.15 \\ 0.20 \end{bmatrix}$$

$$A_{ijC} \qquad L_{iC}$$

$$\begin{bmatrix} 0.10 & 0.12 & 0.04 & 0.02 \\ 0.12 & 0.16 & 0.03 & 0.03 \\ 0.09 & 0.14 & 0.04 & 0.05 \\ 0.10 & 0.10 & 0.02 & 0.03 \end{bmatrix} \quad \begin{bmatrix} 0.25 \\ 0.20 \\ 0.05 \\ 0.05 \end{bmatrix}$$

If money wage rates are 1 in the respective currencies the autarky prices and natural exchange rates are,

$$P_{1A} = 1.39 \quad P_{1B} = 3.10 \quad P_{1C} = 3.93$$
$$P_{2A} = 1.83 \quad P_{2B} = 1.47 \quad P_{2C} = 3.65$$
$$P_{3A} = 2.13 \quad P_{3B} = 2.31 \quad P_{3C} = 1.76$$
$$P_{4A} = 2.86 \quad P_{4B} = 3.02 \quad P_{4C} = 1.59$$

$$E^1_{AB} = 0.450 \quad E^1_{BC} = 0.787 \quad E^1_{CA} = 2.816$$
$$E^2_{AB} = 1.238 \quad E^2_{BC} = 0.404 \quad E^2_{CA} = 1.997$$
$$E^3_{AB} = 0.925 \quad E^3_{BC} = 1.307 \quad E^3_{CA} = 0.826$$
$$E^4_{AB} = 0.945 \quad E^4_{BC} = 1.901 \quad E^4_{CA} = 0.556$$

The levels of final consumption in autarkic equilibrium are as follows:

	A	B	C
1	89.46	48.37	44.47
2	68.25	101.40	47.83
3	58.46	64.93	99.06
4	43.65	49.54	109.89

The natural exchange rates indicate a trade pattern A-1, B-2, C-3,4 because $E^1_{AB}E^2_{BC}E^4_{CA} = 0.109$ is the lowest product and $E^1_{AB}E^2_{BC}E^3_{CA} = 0.163$ is the next lowest.

When the economies are opened to trade the market clearing equations in the currency of country A and the full employment equation for C corresponding to the trial trade pattern are,

$$\frac{L_A}{l_{1A}} = \frac{M_1}{P_{1A}} + \left[a_{11A}\frac{L_A}{l_{1A}} + a_{12B}\frac{L_B}{l_{2B}} + a_{13C}\frac{L_{3C}x_{3C}}{l_{3C}} + a_{14C}\frac{L_{AC}x_{4c}}{l_{4C}} \right]$$

$$\left(\frac{L_B}{l_{2B}}\right)E_{AB} = \frac{M_2}{P_{2B}} + \left[a_{21A}\frac{L_A}{l_{1A}} + a_{22B}\frac{L_B}{l_{2B}} + a_{23C}\frac{L_{3C}x_{3C}}{l_{3C}} + a_{24C}\frac{L_{4C}x_{4C}}{l_{4C}} \right]E_{AB}$$

$$\left(\frac{L_{3C}x_{3C}}{l_{3C}}\right)E_{AC} = \frac{M_3}{P_{3C}} + \left[a_{31}\frac{L_A}{l_{1A}} + a_{32B}\frac{L_B}{l_{2B}} + a_{33C}\frac{L_{3C}x_{3C}}{l_{3C}} + a_{34C}\frac{L_{4C}x_{4C}}{l_{4C}}\right]E_{AC} \quad \ldots (8)$$

$$\left(\frac{L_{4C}x_{4C}}{l_{4C}}\right)E_{AC} = \frac{M_4}{P_{4C}} + \left[a_{41A}\frac{L_A}{l_{1A}} + a_{42B}\frac{L_B}{l_{2B}} + a_{43C}\frac{L_{3C}x_{3C}}{l_{3C}} + a_{41C}\frac{L_{4C}x_{4C}}{l_{4C}}\right]E_{AC}$$

$$L_{3C}x_{3C} + L_{4C}x_{4C} = L_C$$

where $M_i = \alpha_{iA}w_AL_A + \alpha_{iB}w_BL_BE_{AB} + \alpha_{iC}w_CL_CE_{AC}$. There are 4 independent equations in 4 unknowns, $E_{AB}, E_{AC}, x_{3C}, x_{4C}$ .The equations are non-linear. To obtain a solution by linear methods the third and fourth equations can be used by substituting the autarky prices to eliminate E_{AC} and may be used along with the fifth equation to find a tentative solution for x_{3C} *and* x_{4C}. Substitute, x_{3C}, x_{4C} into the first two equations to obtain a tentative solution for E_{AB} and E_{AC} and then solve for the post-trade prices of goods. The price system is,

$$\begin{bmatrix} (1-a_{11A}) & -a_{21A}E_{AB} & -a_{31A}E_{AC} & -a_{41A} \\ -a_{12B} & (1-a_{22B})E_{AB} & -a_{32B}E_{AC} & -a_{42B} \\ -a_{13C} & -a_{23C}E_{AB} & (1-a_{33C})E_{AC} & -a_{43C} \\ -a_{14C} & -a_{24C}E_{AB} & (1-a_{34C})E_{AC} & -a_{44C} \end{bmatrix}^{-1} \begin{bmatrix} l_{1A} \\ l_{2B}E_{AB} \\ l_{3C}E_{AC} \\ l_{4C}E_{AC} \end{bmatrix} = \begin{bmatrix} P_{1A} \\ P_{2B} \\ P_{3C} \\ P_{4C} \end{bmatrix} \quad \ldots (9)$$

Substitute the new values of prices in (8) and obtain a new solution for x_{3C}, x_{4C}, then E_{AB} and E_{AC}, repeating the process until the results converge. A satisfactory solution (accuracy upto the 5th place of decimals) is obtained in 7 iterations. It is $E_{AB} = 0.7495, E_{AC} = 1.8392, x_{3C} = 4.2617, x_{4C} = 3.6512$. The post trade prices of commodities are:

$$\begin{array}{lll} P_{1A} = 0.6188 & P_{1B} = 2.5222 & P_{1C} = 3.2236 \\ P_{2A} = 0.3305 & P_{2B} = 0.674 & P_{2C} = 2.7824 \\ P_{3A} = 2.0100 & P_{3B} = 2.1804 & P_{3C} = 0.8211 \\ P_{4A} = 2.4879 & P_{4B} = 2.6101 & P_{4C} = 0.8309 \end{array}$$

The pattern of gains from trade exactly supports the trade pattern so trade equilibrium is found. The levels of final consumption in the post-trade situation are as follows:

	A	B	C
1	201.97	181.67	520.07
2	247.74	222.55	637.14
3	82.77	74.44	213.12
4	81.79	73.56	210.61

It shows an overall improvement as do the reduced prices of production in the post trade situation. The country-wise outlays on the commodities in terms of the currency of the exporting countries are as follows:

Country	A		B		C		Total
Commodity	Interme diate	Final	Interme diate	Final	Interme diate	Final	
1	12.377	125.000	29.706	112.439	17.496	321.870	618.890
2	20.329	166.757	24.287	150.000	18.889	429.393	809.566
3	16.423	67.962	39.416	61.132	34.378	175.000	394.313
4	33.238	67.962	19.943	61.132	19.276	175.000	376.553

It is of course possible to extend this analysis to cover the effects of intercountry transfers, import duties, domestic taxes, etc. using methods identical to those employed in the main text. The general methods apply even though specific conclusions will differ depending upon how these policies affect the exchange rates and hence the prices of intermediate goods as well as the extent of the use of imported intermediate goods in the domestic production of the countries.

Notes

1. Graham's (1948) work received a great deal of attention in the form of review articles as well as rigorous refinements and extensions. References to them will be found in the text of this paper as well as the bibliography. But rigorous work on multicountry trade problems seems to have ceased after the early sixties. Thus Ethier (1976) appends this footnote to his masterly survey of the 'Higher Dimensional Issues in Trade Theory'.

   'This essay will not have much to say about the consequences of additional countries because the problems they raise are usually straight forward and sometimes tedious … allowing many goods and many countries introduces problems of its own. The assignment of goods to countries to produce them that will permit the world to obtain an efficient output obviously depends upon the production techniques of all goods in all countries and so cannot be exposed by any sort of chain of bilateral comparisons'.

   The special case of 2 countries trading in several commodities in the context of a Ricardian model has been quite extensively explored. See Dornbusch, Fischer and Samuelson (1977), Wilson (1980).

2. There may be other reasons. One reason may be that McKenzie (1954 a) in a seminal paper proved the existence and uniqueness of equilibrium in Graham's model and may have had the unintended effect of submerging the whole subject of multicountry trade into abstract general equilibrium theory. Another reason for the lack of interest in multicountry trade problems may be that Graham himself endorsed the upcoming neoclassical trade theory as formulated by Ohlin (1933) and Samuelson (1948). He wrote, "… in a well-reasoned article Paul A Samuelson contends that under freedom of trade there would be no tendency for the movement to stop short of the same equalisation of the prices of productive factors of a given trade as would occur in the case of commodities". [Graham (1948), p. 306n].

   This endorsement is surprising because in Graham's own examples (see section 6 of paper) the real wage rates do not get equalized in the post-trade situation. For instance if the real wage rates are computed at the international prices in terms of each commodity in Graham's 4 country 3 commodity example they work out to

	A	B	C	D
1	3.5	3.33	3.33	4.44
2	7.35	7	7	9.33
3	10.5	10	10	13.33

   Only the real wage rates of B and C have been equalized because E_{BC} happens to be equal to

$W_B / W_C = P_{1B} / P_{1C} = 1$, both B and C import commodity 2 from D and $E_{BD} = E_{CD}$ and finally because A, C, D produce commodity 3 in common so that are their exchange rates are locked together by the price of commodity 3, all of which are special circumstances of the particular example.

3. Every commentator on Graham's work without exception has held the same view. See McKenzie (1954a, 1953-54, 1955), Metzler (1950), Whitin (1953), Dorfman, Samuelson and Solow (1958), Jones (1961). Metzler (1950), in his elucidation of Graham's model supposes a given money wage rate in the countries but curiously it is £2 in England and £1 in both France and Hungary. He then goes on to compute the national incomes of all three countries in £ and supposes that fractions of their sum are spent on the commodities.

4. Several notable commentators on Graham's model have supposed that Graham assumed that a fraction of *world income* was spent on each commodity [Metzler (1950), Whitin (1953)]. This is true even of McKenzie (1954a) who otherwise acknowledges that "Each country's labour is confined to its boundaries, and, therefore the labour supplies are distinct resources."[McKenzie (1954a), p. 148]. But that means that the labour endowments of different countries cannot be simply added together without translating them to some common *numeraire*, say money, or one of the commodities to obtain world income. It is only in the special case of uniform propensities to consume in all countries (which Graham has indeed assumed in all his examples) and homogenous labour that

$$\Sigma \alpha_{ij} L_j = \alpha_i \Sigma L_j \text{ only if } \alpha_{ij} = \alpha_j \ \forall_j$$

However, we show below that although Graham's examples have uniform propensities to consume in all countries, his methods apply also to cases where they are not so and of course as McKenzie rightly observed, the labour endowments must be treated as distinct.

There have also been attempts to interpret Graham theory as a linear programming problem in which international trade maximizes the value of the world output (minimize the world cost of production subject to the constraints of country-wise labour endowments). [*See* Whitin (1953), McKenzie (1954a, 1953-54), Chipman (1965), Takayama (1972)]. At the same time, it is noteworthy that no one has used the linear programming formulation to actually solve a numerical multicountry trade problem even though linear programming is essentially an algorithmic technique. And that is because the linear programming problem, while it is suggestive, ignores an important non-linearity in the process of the actual solution, viz. the prices that would prevail in the trade equilibrium cannot be ascertained until the trade pattern is known, the trade pattern cannot be ascertained until the terms of trade are known and the terms of trade cannot be ascertained until the trade pattern is known. Whitin (1953) tried to circumvent this problem by adding among the constraints an equality stating that a fraction of *world* income is spent on each commodity. But then world income which appears as an unknown in the objective function would appear as a (provisional) known in the constraints. As Schumann and Todt (1957) pointed out, with unknown world prices and world outputs, the value of the world output, which is the objective function, becomes a quadratic, not a linear function.

5. The 'natural' exchange rates have been so called because they spring naturally from the autarkic money prices. Apart from that there is nothing natural about them. Competing terms include, "exchange rates implied by commodity prices," 'commodity rates of currency exchange', "threshold", "cross-over", or "watershed" exchange rates.

6. This way of stating the comparative advantage principle is similar to that formulated by Jones (1961) as the 'product of prices' criterion. On this criterion A-1, B-2, C-3 is efficient if, the product $P_{1A}P_{2B}P_{3C}$ is lesser than the product for any other permutation, e.g.

$$P_{1A}P_{2B}P_{3C} < P_{2A}P_{3B}P_{1C}$$

$$P_{1A}P_{2B}P_{3C} < P_{3A}P_{1B}P_{2C}$$

The product of prices criterion holds for the case where the number of countries equals the number of commodities. The criterion based on the product of natural exchange rates applies generally.

7. These specifications of demand suppose that price and income elasticities of demand equal unity for all commodities. It is easily possible to incorporate more general demand equations e.g. linear expenditure systems

$$P_i Q_i = P_i Q_{i0} + \alpha_i (wL - \Sigma P_i Q_{i0})$$

into Graham's model. The autarky equilibrium will be

$$Q_{si} = \frac{L_i}{l_i} = Q_{di} = Q_{i0} + \alpha_i \frac{(wL - \Sigma P_i Q_{i0})}{P_i}$$

So that the equilibrium labour allocation is

$$L_i = l_i \left[Q_{i0} + \alpha_i (wL - \Sigma l_i Q_{i0}) \right] \ \forall \ i$$

where Q_{i0} are the 'subsistence' quantities and $wL - \Sigma P_i Q_{i0}$ is the 'supernumerary income. [*See* Stone (1954) for properties of the linear expenditure system].

8. The presence of inconsistent exchange rates invariably results in an indeterminacy of the system of world demand-supply and full employment equations. For example, consider a trade assignment of the type A-1,2 B-1,2 with $E^1_{AB} \neq E^2_{AB}$. There are 4 unknown labour allocations but with the currency exchange rate set equal to one of E^1_{AB} or E^2_{AB}, there will be only 3 independent market clearing and full employment equations. Likewise, a trade assignment of the type A-1,3 B-2,3 C-1,2 with $E^3_{AB} E^2_{BC} \neq E^1_{AC}$ will have 6 unknowns but only 5 independent equations. And even if, coincidentally $E^1_{AB} = E^2_{AB}$ in the first case or $E^3_{AB} E^2_{BC} = E^1_{AC}$ in the latter, one of the labour allocations must be given an arbitrary value to solve the remaining ones.

9. The subject of trade in capital goods has been vigorously debated from time to time. [Steedman (1979), Sanyal & Jones (1982), Smith (1996)].

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