

The Linear Matrix-Valued Cost Functions as a Source of Leontief and Ghosh Models

Vladimir Motorin

Variations in exogenous elements of input–output model lead to changes of price and quantity proportions in the resulting supply and use table that are formally described by two nonlinear multiplicative patterns. These patterns can be linearized and adjusted for evaluating the input–output model at constant prices and level of production.

The pattern for assessing at constant prices provides an exact identifiability of the model when the Leontief technical coefficients and the product-mix matrix are invariable. In contrast, the model based on the other pattern is exactly identifiable when the Ghosh allocation coefficients and the market shares matrix stay invariant. The regular (rectangular case) and supplementary (square case) solutions for both types of input–output models are obtained. Supplementary solutions are used to formulate generalized versions of Leontief demand-driven model and Ghosh supply-driven model.

For symmetric input-output table, the properties of diagonal production matrix allow transforming the generalized models into the “classical” Leontief and Ghosh input–output models. The equivalence of Leontief price model and Ghosh supply-driven model as well as the equivalence of Leontief demand-driven model and Ghosh quantity model is proven. The obtained formulas demonstrate a remarkable set of duality properties.

I A General Formulation of Linear Input-Output Model

The general linear input–output model of an economy with N products (commodities) and M industries (sectors) for the certain time period leans on a pair of rectangular matrices, namely, supply (production) matrix \mathbf{X} and use for intermediates (intermediate consumption) matrix \mathbf{Z} of the same dimension $N \times M$ both. In mathematical notation, the model includes the vector equation for material balance of products' intermediate and final uses, i.e.,

$$\mathbf{X}\mathbf{e}_M = \mathbf{Z}\mathbf{e}_M + \mathbf{y}, \quad \dots(1)$$

and the following vector equation for financial balance of industries' intermediate and primary (combined into value added) inputs:

Vladimir Motorin, National Research University Higher School of Economics, Moscow, Russian Federation, Email: motoriny@gmail.com