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MONEY AND PRICES

An integration of monetary and value theory

by

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Abstract:

The basic aim of this thesis is to provide a concise integration of monetary and value theory. The classical and the neoclassical strands of the literature present this picture in their own constructs and their relative failures provides the motivation for this work. We deviate from the mainstream idea of utility and general equilibrium frameworks as one of the basic assumption on which the general equilibrium framework rests is perfect substitutability between all the commodities in the economy which we feel is highly restrictive an assumption. We draw heavily on the work initiated by Piero Sraffa in 1926 and onwards. We hope to provide a simplified synthesis of the theory of money and value.

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CHAPTER ONE

Introduction and motivation

1. "Money buys goods, goods buy money, but goods do not buy goods"¹. Clower begins his line of attack with this as a refutation to the work proposed by Patinkin in his *Money, interest and prices*, which is a neoclassical synthesis of monetary and micro theory. Patinkin's work was well discarded thus by Clower and Leijonhufvud on the grounds of the method used while Arrow and Hahn attacked it on the grounds of its mathematical existence. This synthesis of a monetary theory of value was attempted by the likes of Hume, Ricardo and of late Keynes. The classical postulate of *dichotomy* between the real and the monetary sectors is well documented and discussed in the literature. The entire synthesis of money and value aims an invalidation of this dichotomy and builds an edifice to get rid of it. The proponents of the neoclassical school of thought like Patinkin continued this dichotomy by incorporating real balances in the utility functions but the dichotomisation is still evident.

2. The motivation of this thesis predominantly dwells in that the question of introducing money in a general equilibrium micro-economic framework is still left open. It is this question that we aim to tackle in a non-Walrasian general equilibrium framework. This work tries to provide a snapshot of a monetary economy at any given point of time. However we do not deny that this implies

¹ R.W.Clower, A reconsideration of micro foundations of monetary theory, *Western Economic Journal*, vol.6, 1967

that the system shall not have dynamic properties, for the reason that those shall be clear in a while. However it is worthy enough to iterate that the apparatus used here is not a standard neoclassical framework. This has no implications that the work has been thought as a critique to the neoclassical economic theory as such. It is simply that the premises of the mainstream economics do not provide an analytical device to model or to present a snapshot of the economy and to take us out of the realm of abstract economies to real economy. We aim to provide the insight on the function of money as a medium of exchange and not merely a *neutral* object of desire required only to determine absolute levels of prices in the economy. Money as such is a commodity that is used in the activities of an economy but is *not used up*. To quote Samuelson, *money is like a catalyst in a chemical reaction, which makes the reaction go faster and better, but which like the oil in the widow's cruse is never used up*. An economy will perennially continue to possess monetary resources and those will always be there and all that can happen is it will match the current needs of the economy's demand for money but still continue to be present in the economy as a perpetual instrument in circulation.

CHAPTER TWO

An excursion into the literature

3. The classical theory of value is a by-product of understanding the debates that the politico-socio setup at the time this doctrine was developed. The substance of Adam Smith's work and the penetration that he requires to emphasise the existence of natural laws in economics must be understood as a way of fighting against mercantilism and every form of interference with the freedom of trade. His polemics against mercantilism led him develop laws and this is to be understood as a by-product of the way in which he advocated a practical policy. Continuing the spirit of Smith, David Ricardo developed his doctrines on the mood to understand the Corn Law debate. The economic interests of the landlord had a view opposite to the views of the urban landlords who believed in the free trade doctrine. Thus Ricardo's pursuit of the theory of rent is a political and social synthesis and hence rent to him did not enter cost of production, which is why it had to be taxed. To put it simply, Ricardo's construct makes a dangerous object for campaigning a rising socialist movement. To add to the contribution of David Ricardo is a feather that he has in his cap: the labour theory of value. Though Smith had analysed it much before him, it was the class conflict between the capitalists and the workers. The analysis of the theory of value of the prominent classical economists thus brings out the fact that economics was not what they were concerned with. It was the political nature of the society than that they were concerned with and this led to their preoccupation towards developing laws that would help explain and understand this prevalent scenario. The ideology gave

vent to develop a value theory though; the classical value doctrines are also faced with critics. Marx had a notion of labour as a commodity and that all other things can be measure in terms of labour to him. On the other hand, Petty singles out food as a measure of value.

4. The cost of production has a lot of debate revolving around it. Petty and the Physiocrats had the concept of cost to be stock of material that is required in the process of production, and this was mainly food for the workers. What is more important is the fact that this stock of food has nothing to do with the pleasant or the unpleasant feelings of the workers. Marx, Ricardo and Smith are to be blamed for corrupting this notion of food and causing a transition to labour. The marginalist notions of cost are totally different and Marshall and the Austrian economists are found to be the propound exponents of these subjectivist notion of cost. For Marshall, the nature of cost is equivalent to the notion of real cost of production. On the other end of the spectrum, for the Austrians, the cost of production is equivalent to what is called the opportunity cost. For Marshall, to add, the notion of cost is a notion of disutility and this when contrasted with the pleasant feeling that he calls utility, can be brought under one single value theory. When it comes to the theory of distribution, the marginalist have a symmetry built in the analysis by requiring every factor optimally employed and hence everyone is paid as per their capacities to add to production. On the contrary, the classicals have an in-built asymmetry in the distribution of property rights between workers and capitalist entrepreneurs.

The premises of the classical model can best be analysed together as a macro model which explains the determination of employment and output, the rate of interest and the division of output between consumption and investment goods and the general level of commodity prices. The paramount concern is, of course, the role of money as a determinant of these variables. The first two sectors of the model can be understood at the outset as they have a vital interdependence which arises because the demand schedule for labour is derived from the production function, which relates the level of real output to the volume of employment. After a monetary sector is added to the model, a unique price level would emerge that allows only one money wage rate to be consistent with equilibrium. This indicates that the only combination such as W_0 , p_0 , or $(W/p)_0$ is a value that will ensure equilibrium in the labour, output and the monetary sectors. To explain the general level of commodity prices, the classical writers formulated what we know to be the crude quantity theory of money. This influence of money on prices was deduced from an analysis of historical experience. Interestingly enough, the quantity theory can alternatively be viewed as a theory of aggregate demand and as a theory of demand for money. When explaining the price level with the quantity theory of money, it is both easier and more direct to use the American or the income velocity approach formulated by Fisher. Thus the analysis consists of the identity more peculiarly called as the *equation of exchange*, which can be stated in terms of income velocity and transactions velocity as well. It expresses the truism that the total value of purchases must equate the total value of sales. This is how Fisher incorporated the notion of money using the price levels of all

the transactions in the economy. However Fisher did not end his analysis merely by formulating an identity. His ultimate contribution was to turn the identity into a theory that explained how and why changes in stock of money produced proportional changes in price level. Perhaps one of the greatest misuses of value theory by classical economists was their assumption that the velocity was a virtual constant- a value that was necessary in preserving equality in an equation. It should be noted that the proposition that the level of prices varies directly and proportionately with the stock of money is essentially one which holds in the long run, full employment equilibrium, if we assume velocity and real output are constant. Fisher's equation of exchange $MV = PT$ can be used to examine the impact of change in money supply. If the level of real output Y is fixed at full employment and velocity V is assumed to be relatively constant, any increase in the money stock should cause a proportionate increase in the prices. Thus the role that money plays in the classical model is that of an important determinant of employment, output and the level of prices. However the orthodox writers' belief in the neutrality of money was evident. The neutrality at first glance appears to follow logically from the classical view on the forces determining the volume of employment and the level and composition of real output. However it was shown that for the strict neutrality to prevail various conditions must be met; prices and wages must be flexible, the participants in the economic activity must be free from money illusion, the distribution of income must be unaltered and the productive factors must operate with market information. It is thus now well understood that the classical writers believed that changes in money brings out

changes in prices. It would be erroneous to believe that that these changes were instantaneous. An early formulation of nonneutrality of money however is seen in Hume. Hume adds that the process of adjustments of prices due to changes in money stocks is not instantaneous and it is this time gap when the money loses its classic neutrality. This neutrality and the time gap for the money to take full effect and translate into no increases in real variables in equilibrium was noted as early as 1752 by Hume, when he observed, *to account, then, for this phenomenon, we must consider, that though the high price of commodities be a necessary consequence of the increase of gold and silver, yet it follows not immediately upon that increase; but some time is required before the money circulates through the whole state, and makes its effect be felt on all ranks of people. At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another; till the whole at last reaches a just proportion with the new quantity of specie which is in the kingdom. In my opinion, it is only in this interval or intermediate situation, between the acquisition of money and rise of prices, that the increasing quantity of gold and silver.*²

5. The classical synthesis took a set back with the analysis of J.M. Keynes. The classical system implicitly laid down that as long as money wages, commodity prices and interest rates were flexible and moved relatively rapidly, the economy was self-adjusting and could maintain a tendency toward continuous full employment in the face of almost any macro disturbance. For the success of the

² Hume D., *Of money. Essays in Honour of Hume.*

Keynes' attack, he denied the classical postulate that commodity prices and wages were independent of each other. The other postulate Keynes denied was that of an upward sloping labour supply schedule. *While workers will resist a reduction of money-wages, it is not their practice to withdraw their labour whenever there is a rise in the price of wage-goods*³. A yet another postulate that Keynes attacked was the classical impotency to recognize involuntary employment, which is a by-product of his fixed wage argument, which debates point out as market imperfections. Having linked wages and commodity prices directly, and denying that labour would withdraw its labour time for every small decrease in the real wage, Keynes posed a problem to the classical economists- the possibility of unemployment equilibrium which could not be disturbed by reductions in money wages. This he called the paradox of poverty in midst of plenty. Crucial to the denial of proportionality proposition was to show that a long-run unemployment could exist in the face of flexible wages and prices. Given such unemployment equilibrium, increases in demand, stimulated perhaps by changes in money stock, might bring forth output changes without changes in the level of prices. The last proposition used to deny proportionality arises from his view that the demand-for-money function is highly unstable, as wealth-owning individuals changed their views periodically as to their normal rate of interest. Each change affected the individual demand schedule for speculative balances and hence shifted the aggregate function as well. Therefore in Keynes' view velocity was not as stable as Fisher and the early quantity theorists supposed. With velocity shifting erratically, one could not predict what effect money-supply changes would have

³ J.M. Keynes, *The general theory of employment, interest and money*, 1936.

on the price level. Keynes was thus able to demonstrate that the long run permanent non-neutrality of money was a ramification of his demonstration of unemployment equilibrium. In such a situation, an expansion in money supply could permanently change real variables like interest rates, employment, output, the rate of capital formation and so forth. In certain situations, the ability to lower interest rates, increase the rate of capital formation, and lower unemployment rates rests clearly on the presence of money illusion. That is the expansion of money stock, which accomplishes nonneutrality, does so in an environment in which inflation takes place while the participants in the economy are oblivious to it. Such behavior was seen to produce temporary nonneutrality in the Fisherian model, but not permanent non-neutrality as the Keynesian model implies. Two interesting situations exist in the Keynesian model, which render money largely neutral. The first is the liquidity trap. In such a state of affair all changes in money stocks are added simply to idle cash balances and have no effect on the real variables; money is thus neutral. Second, in the Keynesian model the only channel through which money can affect the real sector is the interest rate. If expenditures are insensitive to changes in interest rates, money is mostly neutral meaning that it can change the interest rate but will have a minimal impact on all other real variables. In those situations in which money is nonneutral, the magnitude of nonneutrality depends upon the interest and income elasticities of money demand, the interest elasticity of expenditures, the income elasticity of saving schedule (which governs the value of the multiplier), the degree to which the marginal productivity of labour diminishes and the extent to which productive factors

suffer from money illusion. The Keynesian analysis is thus a market imperfection model; labour market imperfections to be precise and a synthesis of wage rigidities. This being the modeling for the real sector, through the links in investment demands and thereby interest rates possessing the clue to the monetary sector. The money in the Keynesian analysis is thus a result of imperfections leading to unemployment equilibrium.

6. A suggested reinterpretation of the Keynesian attempts at a monetary synthesis is the Hicks-Hansen IS-LM analysis. This according to Leijonhufvud is not the economics of Keynes. He believes that the *General Theory* is a continuation of Keynes' *Treatise on Money* and that the essence of his message is to be found by comparing as to how he and the classics analysed the reaction of the economy to a decrease in effective demand. Leijonhufvud is of the opinion that Keynes was trying to break away the implicit classical proposition of a *Walrasian auctioneer* providing costless information to the markets. When aggregate demand declines sellers have no information on whether the initial decline is temporary or of a longer run nature. Since it would be unwise to adjust prices to clear a market or maintain sales if the decline in demand were only temporary, sellers will probably wait to form opinions concerning the permanency of decline before they adjust prices to maintain sales. Thus for both the seller of commodities and seller of labour services, the initial decline in demand will be met by quantity adjustments rather than price adjustments. As Leijonhufvud reads Keynes, this deflationary process with falling output and employment need not occur if only the money rate

of interest would fall sufficiently, for by doing so it would simulate both investment and consumption, restoring full employment. The attractiveness of Leijonhufvud analysis to the quantity theorist is that in order to maintain the economy close to full employment equilibrium, stabilization efforts should rely primarily on a monetary policy to keep the money rate of interest in line with the natural rate, regardless of the initiating cause of cyclical disturbances.

7. As a consequence, we see that the macro synthesis of money was attempted by the likes of the orthodox as well as the Keynesian believers. However, a search for the micro foundations of the macro theory is still debatable. On the other hand Patinkin provided a synthesis of money with the modeling of individual behavior; what we call a synthesis of the monetary and the value theory. Clower reads Patinkin by hypothesizing that market excess demands in terms of individual demand functions for goods and money are obtained as solutions to the problem:

Maximise $U_j(d_1, \dots, d_n, M_j / P)$ subject to $\sum_{i=1}^n p_i (d_{ij} - s_{ij}) + M_j - M'_j = 0$. The

utility functions are supposed to obey their usual curvature and continuity conditions, but as it happens the implications of these conditions are exhausted in certain analytically subtle but empirically trivial restrictions involving the existence of demand functions and the continuity of their partial derivatives. The factual content of the theory depends very largely therefore, on restrictions implicit in the budget equations. *Accordingly, the main question that we have to answer in order to appraise the significance of the contemporary monetary theory is: "Do the budget equations constitute an appropriate definition of choice*

*alternatives in a monetary economy?"*⁴ Archibald and Lipsey⁵ criticize Patinkinian synthesis on the lines of Hicks. Hicks in *value and capital* suggested, a stationary economy '...is in full equilibrium, not merely when demands equal supplies at the currently established prices, but also when the same prices continue to rule at all dates'. To Archibald and Lipsey, Patinkin apparently overlooks this distinction; his analysis never goes beyond the conditions for equilibrium in one week. In the Patinkinsque world the process of adjustment is explained by a set of two budget lines depicting the *monetary* restrictions of two individuals that are aggregated over to explain the society. Trade each week causes the budget lines to move closer together; the price reduction required to equilibrate the market the following week moves both of them to the right; the week's trade again brings them closer together. Ultimately the two budget lines coincide at a point that happens to be the position of full equilibrium for each individual. Only then will the price level cease to vary. This leads us to assimilate the Patinkin thought that a change in money stock leads to a proportionate change in full equilibrium price level. To Archibald and Lipsey, Patinkin does not analyze the model in full equilibrium: he considers a stock of money change which takes place during a process of adjustment as described above. This very crucially rests on the assumption that, when the stock of money is changed, there are no distribution effects. Thus the symmetry of the problem is a prerequisite to attain the result of a *doubling price level*. In case of a non-proportionate

⁴ R.W. Clower, *A reconsideration of the microfoundations of the monetary theory*, *Western Economic Journal*, 1967.

⁵ G.C. Archibald and R.G. Lipsey: *Monetary and Value theory, a critique of Lange and Patinkin*, *Review of Economic Studies*, 1958.

distribution the equilibrium cannot of course be reached merely by a change in price level but will require a redistribution of the real balances as well by a similar process described above. The next axe falls on the use of the Pigovian notion of *real balance* hinted by Patinkin in incorporating them in the utility functions. The role of real balances is to only provide a trajectory to the economy to move from one position to another, and as such the role of real balance is irrelevant, since in full equilibrium, consumption is equal to income, a change in real balances can only change real consumption in the process of adjustment. However a fundamental blow to the Patinkin model is received from Clower, as we had seen earlier when he had raised the question of appropriateness of the budget constraint itself. The fault in the Patinkin budget constraint is that there is no distinction made between money and nonmoney commodities. Clower observed that such a distinction is possible only if we assigned a special role to certain commodities as means of payment. *For any commodity may serve as a unit of account and standard of deferred payment: for every asset is, by its very nature, a potential store of value. If money is to be distinguished by the functions it performs, it is to the medium of exchange function that we must address our attention to*⁶. Clower thus defines a money economy where in certain commodities are denied a role as a potential or actual means of payment. This is when Clower remarks the opening sentence of this work: *money buys goods and goods buy money, but goods do not buy goods*. Owing to this reason Clower denies the neo-Walrasian budget condition as it effectively admits as feasible trades all pairwise combinations of commodities that are traded in the economy. Analytically, this implies that money

⁶ Clower, *ibid.*

be offered or demanded as one of the commodities in every trade. Hence, this entails a clear separation between *goods demanded for purchase (offers to sell money)* and *goods offered for sale (offers to buy money)*. Clower achieves this by dichotomizing the budget constraint into two branches, the first representing a constraint on money expenditure, the second representing a constraint on money

income. Symbolically he has $\sum_{i=1}^n P_i x_{ij} + M_j - M'_j = 0, x_{ij} \equiv d_{ij} - s_{ij}$, where $x_{ij} \geq 0$

as the expenditure constraint and $\sum_{i=1}^n P_i x_{ij} + m_j = 0$, where $x_{ij} < 0$ as the income

constraint, where m_j represents desired intra-period receipts of money income.

The expenditure constraint asserts that the total value of goods demanded cannot in any circumstances exceed the amount of money held by the transactor at the outset of the period. The income constraint asserts that net sale offers involve a demand for just one other commodity, namely money, in exchange. Clower then uses the utility sets described above constrained by his newly *dichotomized* budget sets and derives demand relations accordingly. So much so, the analysis of Clower reduces Patinkin to ashes but it itself does not lead us anyway closer to explaining the monetary economy of a value theory with money.

8. The above discussion was conducted with a view of discussions that were in the order. It is seen that the classical theory started with different perspectives as discovering laws that governed the wealth of nations. The theory of price formations was a secondary object. More so the concise value theory of the orthodox economics is seen only in the labour theory. The attempts of modeling

individual behavior was seen first by Jevons and Bentham, whose utility constructs, are now embedded in the literature. The synthesis of this theory with monetary theory was now primarily required as the question of dichotomy of real and monetary sectors was gaining more grounds than warranted. As a result Patinkin made a commendable attempt among all those who took up the question. However, the construct of Patinkin was demolished first by Archibald et.al and then by Clower. This brings us as a consequence to a point where the question of synthesis of money and value is still open. Our excursion in the literature has brought out this clearly. As a result, here is where lies the motivation to this work to take up this seemingly interesting and widely debated topic and study its implications using a method totally against the current.

CHAPTER THREE

An Alternative Theory Of Value.

9. The analysis of the classical and the neo-classical theories of values can be put in two separate boxes. The classical theory of value is based more on the costs of production and class conflicts, which makes it an objectivist theory, which can be observed or economically measured using numbers. The marginalist notion of theory of value is merely a subjectivist notion, which cannot be observed and can only be indirectly measured. The fundamental logic for this distinction and a semantic shift in the notions of costs of production, or more generally the theory of value is that the two doctrines derive from two different views of nature and goals of economic theory. For the classical economist the goal was to discover the laws that determine the wealth of nations and determine income distribution among various social classes. For the marginalist the purpose is to determine the economic behavior of individual human agents and to determine equilibrium price of individual commodities. The fulfillment of the later goals requires the use of a subjectivist theory. However, we have seen in our literature review that none of the schools have been able to tackle the question of money and value together. In the classical analysis, money has no micro foundations whilst in the Keynesian synthesis it is just there and is permanently non-neutral. The quest for a micro monetary theory also was a failed experiment. As a consequence, we propose to take up the topic and try to resolve this query of attaining a formal synthesis of *money and prices*. As the neoclassical and the classical approaches to this issue

have not been satisfactory, what we aim to do is to propose an alternative theory of value and then in logical sequence, provide for the role of money in such an alternative process.

10. Starting in a logical sequence, that shall in equivalence explain the evolution of money and the analysis of prices should plausibly bring about a fusion between the theory of money and value. This synthesis, as it is now, needless to reiterate is brought about using a non-neoclassical device and more importantly what is called as a *stock-flow model*. The analysis dictates the use of a stock-flow model because of the fact that the variables like commodity money or fiat money for analytical purposes are stock concepts and real variables like commodity sector are flow concepts. Thus we explain the use for a device like the stock-flow model. Bushaw and Clower⁷ were perhaps the first ones to describe that the equilibrium condition for any commodity must include at least two equations; one describing that demand and supply for the flow of that commodity and other for the stock of that commodity. We merge this *double-equilibrium condition* into a single condition as will be explained in a short while. The style of the model borrows heavily from the idea proposed by Piero Sraffa⁸ in his 1960 monograph and its empirical completion by Parchure⁹. The analysis of Sraffa is important for the further aspects of this work and it would be worthwhile to pause and study the

⁷ D.W. Bushaw and R.W. Clower, *introduction to mathematical economics*, Irwin publishers, 1957.

⁸ Sraffa P., *Production of commodities by means of commodities: A prelude to the critique of Economic theory*, Cambridge University Press, 1960.

⁹ Parchure R., *The Pure Theory of Value*, Times Research Foundation, 1989.

Further the production relations described above possess a unique characteristic that follows from the famous theorem of duality in mathematics. However, though the theorem is mathematical it carries out an economic perspective as well. We ask the question as to how much replacements of each industry are required each period in order to achieve the slated rate of profit. The answer that we get for each industry is what we call the output scalars or the multipliers. These by analogy determine the growth rate of their respective industries and competition dictates that these be equal. Lastly the Sraffa postulates that the entire labour force in the economy be preserved as these transitions for adjustments happen and these are to be adjusted as per the output multipliers for each industry. One important point worth noting is that since the capital goods only will be replaced over time, only the basic good industries enter the dual relationship, or what we call the problem of output determination. Thus the Sraffa system simultaneously is a system of determination of price and output; a theory of value in its true spirit. Sraffa's system of output determination can be described as

$$\begin{aligned} &(A_a q_a + A_b q_b + \dots A_k q_k)(1 + g) = A q_a \\ &(B_a q_a + B_b q_b + \dots B_k q_k)(1 + g) = B q_b \\ &..... \\ &(K_a q_a + K_b q_b + \dots K_k q_k)(1 + g) = K q_k \\ &L_a q_a + L_b q_b + \dots L_k q_k = L \end{aligned}$$

In its general form the dual relationship is given in the following manner

$A'q(1 + g) = Bq$. This system is aptly described as Sraffa's system of output determination.

11. Next, we consider the mathematics of the Sraffa system as a whole. Assuming n commodities in the system, there are n price equations and $n+2$ variables. We can eliminate¹⁰ one of the unknowns by fixing either one of the prices equal to unity (Walras, 1874), or by fixing the absolute wage rate equal to unity (Keynes, 1936), and then there will remain n equations in $n+1$ unknowns. Thus there is an equation missing that would help in determining all the *relative* prices in the economy. The required gap between the equations and the variables can be filled in by considering the composition of commodities which the individual agents desire to purchase; the demand equations for the n commodities. Walras' law dictates that only $n-1$ of these will be independent, and that we shall use the empirical demand functions that designed are designed by Stone as

$$\alpha K + \beta wL = B_j p_j \text{ where } K = \sum_{j=1}^n \sum_{i=1}^n A_{ij} p_i$$

is the capital stock, L is the annual labour and the constants alpha and beta are propensities of capitalists and wage earners to consume or spend. Thus now we have $2n-1$ equations in $n+1$ variables, and the system is still overdeterminate. Hence we now use the dual construct of Sraffa that shall help determine the outputs of each industry as well. We introduce n equations for determination of outputs and to do so absolutely we use what Sraffa calls the labour conservation equation, which adds new $n+1$ equations to the system in $n+1$ variables, the n outputs and the growth rate. Closing this system with the relation between the profit rate and the growth rate, we have fully

¹⁰ Note that the construct of the *numeraire* is used only for the Sraffa system and is dropped in the analysis of Money, by definition.

$3n+1$ equations in as many variables and this is what is explained as the complete Sraffa system.

12. A complete overview of this analysis can be analytically examined using a basic *closed Sraffa system*¹¹ comprising of two commodities alone.

$$\begin{aligned} (A_{11} + B_{21}P_2)(1+r) + L_1w &= A \\ (A_{12} + B_{22}P_2)(1+r) + L_2w &= Bp_2 \\ \alpha rK + \beta wL &= A' (= A - A_{11} - A_{12}) \end{aligned}$$

These can be written in vector-matrix notation as a system of homogenous equations,

$$\begin{bmatrix} A - A_{11}(1+r) & -A_{21}(1+r) & -L_1 \\ -A_{12}(1+r) & -A_{22}(1+r) & -L_2 \\ A' - \alpha(A_{11}r + A_{12}r) & -\alpha(A_{21}r + A_{22}r) & -\beta L \end{bmatrix} \begin{bmatrix} 1 \\ p \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A unique non-trivial solution to the above system exists if and only if the determinant of the matrix of coefficients is equal to zero. Setting it thus equal to zero gives us a characteristic polynomial equation in r . The lowest root of the polynomial¹² is the relevant solution. When this is substituted in the price equations, the solutions for the relative prices and wages can be obtained. The “Cambridge Equation” gives the relationship of the growth rate and the profits, and reads as $g = r(1-\alpha)$ ¹³. The algorithm that yields the results to the above *closed Sraffa System* will be made clear

¹¹ As it is called in Parchure.

¹² In Matrix language, it is called the lowest eigenvalue of the characteristic vector or the eigenvector.

¹³ Note that $g = (B-A)/A$ and if we allow a proportion of profits to be consumed equal to alpha, then

$g = \frac{B - A - \alpha r A}{A}$ and we get the equation as above.

in a while when we analyse the augmented Sraffa model to carry on our discussion of money and value theory.

CHAPTER FOUR

A Commodity money

13. After considering the necessary prerequisite of the analysis of value as proposed by Piero Sraffa, we proceed on to explain the evolution of money, by considering first a case with commodity money. A fusion between the theory of money and value should plausibly be brought about by starting in the logical sequence as equivalent to the evolution of money. This synthesis as it is now needless to iterate is brought about using a non-neoclassical device and more importantly a stock-flow model. The analysis dictates the use of a stock-flow model because of the fact that the variables like commodity money or fiat money even for that matter are stock concepts and real variables like commodity sector are flow concepts. Thus we explain the use for a device like the stock-flow model. Bushaw and Clower¹⁴ were perhaps the first ones to describe that the equilibrium condition for any commodity must include at least two equations; one describing that demand and supply for the flow of that commodity and other for the stock of that commodity. We merge this *double-equilibrium condition* into a single condition as will be explained in a short while.

14. We augment the basic model of Piero Sraffa to analyse our concept of equilibrium in a monetary economy. We shall add along with the A matrix of technical flow coefficients, an S matrix of stock technical coefficients. The basic motivation to do this is any industry in the process of production relies on two basic inputs; one that is a stock of raw materials, work-in-process inventories or finished goods stock, the other being its daily

¹⁴ D.W. Bushaw and R.W. Clower, *introduction to mathematical economics*, Irwin publishers, 1957.

and by this virtue we have to determine $2m+2n+3$ variables. And thus we see that this system is not fully solvable and lacks n equations. c.) We aim to close this system of equations, which implies that finding n more equations, by generating demand equations for the n consumption good industries or the non-basic commodities as they are called. The consumption goods industries will be present in an economy only if they find their ways to the consumers and this is our motivation to include them in demand formulations. As a result of this adjustment to close the system, we shall have now $2m+2n+3$ equations in as many variables. d.) a point worth noting about the demand functions to be used is that the demand functions used in the analysis are empirical demand functions generated by Stone¹⁵ and this is done to keep the analysis as empirical as plausible. in general these demand functions will be of the type $\alpha_i wL + \beta_j rS = B_i p_i$, where the coefficients α_i and β_j represent the marginal propensities to consume out of wage incomes and profit earnings respectively.

15. We now move on to explain the working and generating solutions to our model, which includes the commodity money in circulation. This is achieved by simply setting its price equal to one and we have our model in detail as under.

$$\begin{aligned}
&(S_{11}p_1 + S_{12}p_2 + \dots S_{1n})r + A_{11}p_1 + A_{12}p_2 + \dots A_{1n} + L_1w = B_1p_1 \\
&(S_{21}p_1 + S_{22}p_2 + \dots S_{2n})r + A_{21}p_1 + A_{22}p_2 + \dots A_{2n} + L_2w = B_2p_2 \\
&\dots\dots\dots \\
&(S_{n1}p_1 + S_{n2}p_2 + \dots S_{nm})r + A_{n1}p_1 + A_{n2}p_2 + \dots A_{nm} + L_nw = B_n \\
&\alpha_j wL + \beta_j rS = B_j p_j
\end{aligned}$$

¹⁵ Stone R, linear expenditure systems, 1954.

The amount of total capital stock in this economy is given by $\sum_{i=1}^n \sum_{j=1}^n S_{ij} p_j$, and this times the rate of profits shall give us the total profit incomes in the economy. The last equation in the above system is the demand equation for the *ith* non-basic commodity. We now take a numerical example to make the analysis clear and that this numerical is simplified using one basic assumption that the marginal propensity of capitalists is zero, and that this assumption is made for providing mathematical simplicity only.

$$\begin{aligned}(3p_1 + 7p_2 + 15)r + 2p_1 + 3p_2 + 5w &= 20p_1 \\(2p_1 + 7p_2 + 25)r + 4p_1 + 5p_2 + 10w &= 30p_2 \\(5p_1 + 6p_2 + 10)r + 3p_1 + 5p_2 + 5 + 15w &= 40 \\(2p_1 + 4p_2 + 10)r + 5p_1 + 5p_2 + 5 + 10w &= 30p_4\end{aligned}$$

As we can see, the third industry is set to be commodity money and that the fourth is a consumption good industry that will have a demand function of its own. Carrying our assumption that the propensity off the capitalist class is equal to zero, we have a demand function given by $40w = 30p_4$. The dual relation of this problem is given by

$$\begin{aligned}(3q_1 + 2q_2 + 5q_3 + 2)g + 2q_1 + 4q_2 + 3q_3 + 5 &= 20q_1 \\(7q_1 + 7q_2 + 6q_3 + 4)g + 3q_1 + 5q_2 + 5q_3 + 5 &= 30q_2 \\(15q_1 + 25q_2 + 10q_3 + 10)g + 0q_1 + 0q_2 + 5q_3 + 5 &= 40q_3 \\5q_1 + 10q_2 + 15q_3 &= 30\end{aligned}$$

Note the mathematical trick that is applied here in writing the dual relationship. We are interested in finding the solution to the given example and as a result we formulate an algorithm for the analysis.

Step. 1: formulate the dual relationship to the primal problem or importantly set up the output problem from the given price problem. Construct a T matrix of the form $B-A$ where B is now a square matrix. For our problem this T matrix is given by

$$\begin{bmatrix} 18q_1 & 4q_2 & 3q_3 & 5 \\ 3q_1 & 25q_2 & 5q_3 & 5 \\ 0 & 0 & 35q_3 & 5 \\ -5q_1 & -10q_2 & -15q_3 & 30 \end{bmatrix}$$

Step 2: obtain the inverse of this matrix and multiply it to the transposed stock matrix.

The inverse of the above matrix excluding the variables is given by

$$\begin{bmatrix} 0.0557 & -0.0115 & -0.0059 & -0.0064 \\ -0.0078 & 0.0396 & -0.0068 & -0.0042 \\ -0.0009 & -0.0015 & 0.0271 & -0.0041 \\ 0.0063 & 0.0105 & 0.0103 & 0.0288 \end{bmatrix}$$

The multiplication of this with the stock matrix yields a matrix whose characteristic roots will yield us a solution to the growth rate of the above problem. Mathematically speaking we solve the problem as

$$T = B - A'$$

$$S'qg = Tq$$

$$S'T^{-1} = \frac{1}{g}$$

As a result, the solution to the growth rate g is obtained by solving the characteristic equation of the matrix on the left-hand side, which is $S'T^{-1}$, and the lowest non-zero positive root is taken as one over g and obtaining its mathematical inverse gives us the growth rate

Step 3: Substituting the values of this growth rate in the dual, we solve a set of homogenous equations in q 's and multiply them to the primal. In the primal we obtain the

value of r from the Cambridge equation mentioned above¹⁶. Using that we solve for the prices in terms of gold.

Step 4: After obtaining the basic commodity prices we obtain the non-basic commodity prices and then use the demand equation to match the supplies that are $30p_4$ with demands being $15w$. The discrepancy is corrected by moving the supplies towards the demands and the new demands are incorporated in the production relations by adjusting the production equation of the fourth commodity as per the required demands¹⁷. The process shall again start from step one till all supplies match the demands. The above system iteratively yields the following results

Table 1

Iteration number	P_1	P_2	P_4	W	$g=r(\text{since } \beta=0)$	Demand of the forth commodity
1	.69	.83	1.57	2.6993	.5	68.77
2	.46	1.55	1.12	.9998	.49	47.24
3	.19	.29	1.05	1.7733	.5001	43.57
4	.1968	.2892	1.03	1.7736	.5001	43.22

Results of a four good economy with commodity money.

16. One of the aims of this thesis is to produce results that are robust to generalisations and as a result before we proceed, we drop the assumption of zero propensity to save of the working class and yield them a propensity to consume less than one. Further

¹⁶ Refer point 12.

¹⁷ One can use the method of bisection for a faster iterative process.

apparently the above system had at the outset a uniform growth rate¹⁸. Thus our modified system is presented as *system two*.

$$\begin{aligned}(2p_1 + 3p_2 + 5)r + 5p_1 + 3p_2 + 15w &= 30p_1 \\ (3p_1 + 5p_2 + 10)r + 3p_1 + 6p_2 + 10w &= 40p_2 \\ (5p_1 + 10p_2 + 15)r + 2p_1 + 10p_2 + 5 + 15w &= 40 \\ (3p_1 + 5p_2 + 7)r + 3p_1 + 20p_2 + 5 + 16w &= 30p_4\end{aligned}$$

Proceeding the way the above algorithm describes we have the following result sheet.

Table 2

Iteration number	P1	P2	W	P4	r (at $\alpha=0.2$)	g	Demand of the fourth commodity
1	.7395	.5887	.8857	1.0097	.5730	.7162	9.8247
2	.6299	.5744	.4677	.9049	.8782	1.0977	4.5408
3	.5971	.5716	.3841	.8838	.9421	1.7776	3.6349
4	.5913	.5711	.3695	.8799	.9534	1.1917	3.4813
5	.5903	.5710	.3670	.8793	.95535	1.1942	3.4551

A commodity money economy with non-zero propensity to consume of working class.

Lastly, we generalize the system to include three basic commodities while keeping only one consumption good industry. This system is given as *System three*.

$$\begin{aligned}(2p_1 + 3p_2 + 5p_3 + 4)r + 5p_1 + 3p_2 + 12w &= 30p_1 \\ (3p_1 + 5p_2 + 10p_3 + 5)r + 3p_1 + 6p_2 + 10w &= 40p_2 \\ (5p_1 + 10p_2 + 15p_3 + 7)r + 2p_1 + 10p_2 + 5p_3 + 15w &= 40p_3 \\ (3p_1 + 15p_2 + 15p_3 + 8)r + 3p_1 + 20p_2 + 5p_3 + 5 + 16w &= 40 \\ (5p_1 + 12p_2 + 13p_3 + 7)r + 5p_1 + 30p_2 + 7p_3 + 7 + 17w &= 40p_5\end{aligned}$$

The results of system three are presented as under.

¹⁸ Interested readers can verify this by using the relation $r^* = (B_1 - \sum_1^n A_m) / S_m$. This growth rate is called as the *own rate of profit*.

Table 3

Iteration number	P1	P2	P3	W	P5	r	g	Demand for the fifth commodity
1	.6774	.5246	.6945	.8018	1.0045	.2180	.2725	11.1754
2	.5799	.5151	.6931	.3766	.9745	.4837	.6046	4.3899
3	.5562	.5129	.6925	.2848	.9667	.5425	.6782	3.2074
4	.5523	.5125	.6924	.2681	.9661	.5533	.6916	3.0021
5	.5516	.5124	.6924	.2652	.9662	.5552	.6339	2.9654
6	.5515	.5124	.6924	.2648	.9697	.5554	.6942	2.9497
7	.5514	.5124	.6924	.2645	.9662	.5556	.6945	2.9566
8	.5514	.5124	.6924	.2646	.9662	.5556	.6945	2.9566

A general system.

17. To conclude this chapter, we add that the problem of absolute prices can thus be tackled using commodity money. But the commodity money has its inbuilt problems that will be surveyed in the next chapter, where we provide the motivation for introduction of fiat money. But it is the value in exchange function of a standard that we are concentrating upon more dominantly and as a result, we have an intuitive motivation for introducing money in the analysis.

CHAPTER FIVE

The role of fiat money.

18. The time has now come to tackle this all-important discussion on why does money exist in an economy. We prove not. In a later chapter we show that money can assume two forms; either be present as one of the commodities in circulation (as was the case when commodities were used in circulation) or in the form of outside *fiat* money as it is called. Commodity money and its modeling is a neat task and achieves the end objective as desired however a commodity money provide all the predictions of an actual economy. Consider a situation that would prevail if there were no money. Suppose there were N commodities and T traders dealing in them. Suppose, without loss of generality that every trader deals in every commodity. Then there would be a total of $TN(N-1)$ quotes that will have to be made for N commodities. For example, suppose there were 100 traders dealing in 1000 commodities. There would be $100 \times 1000 \times 999 = 99900000$ price quotations in all; about 100 millions! Any individual willing to buy or sell a commodity would have to consider all of them to determine a.) which trader to buy or sell them from and b.) what would be the sequence of commodity transactions? The choice of the trader will depend upon whether he is offering the lowest or the highest quote for the commodity depending upon whether the individual wishes to buy or sell. The choice of the sequence of commodity transactions is important too! There will be several routes for buying or selling the commodity using other commodities as intermediaries. There is always a

possibility of making arbitrage profits by selecting a mispriced sequence of transactions. For example, suppose there are three commodities, wheat, milk and rice and their price quotes are 2 kg. Wheat = 1 kg. Rice; 1 liter milk = 1 kg. Wheat; 1 kg. Rice = 1 liter milk. Suppose an individual has milk to sell and buys wheat. He will not clearly sell milk and buy wheat according to quote 2. It will be profitable for him to sell milk at quote 3, buy rice, sell rice at quote 1 and buy wheat and end up with 1 kg more wheat. Our individual will have to rummage through all such sequences of transactions to find the most profitable sequence for buying/selling. Of course there is an opposite side to this. No single trader will offer quotations, which permit arbitrage profits at his own cost to his customers. This requirement places two restrictions on the price quotations offered by each trader. Ignoring trader margins for the sake of simplicity these restrictions are as follows: 1.) the quote for one commodity for the other must be equal to the reciprocal of the quote for the other commodity in terms of the former, i.e.

$$\frac{P_i}{P_j} = \frac{1}{\frac{P_i}{P_j}} \text{ for all } i, j, i \neq j. \text{ 2.) Every indirect quote must be equal to the direct quote,}$$

i.e. $(\frac{P_i}{P_k})(\frac{P_k}{P_j}) \forall i, j, k, i \neq j \neq k$. The first set of restrictions are $N(N-1)/2$ in

number, the second consists of $(N-1)(N-2)/2$ i.e. a total of $N(N-1)^2$ restrictions.

Every time a price changes the trader will have to make out fresh set of $N(N-1)$

quotations which will have to satisfy $(N-1)^2 N$ equations i.e. to say with our

1000 commodities example then $999 \times 999 = 998001$ computations will need to

be made every time price changes.

19. The designation of one commodity as *numeraire* simplifies all this dramatically. For the N commodities that the trader deals in he need give only $(N-1)$ quotes in terms of the *numeraire* commodity. He need not perform $(N-1)^2 N$ computations at all. Designating a *numeraire* automatically ensures these. All arbitrage opportunities two, three or higher order sequences stand eliminated. [For each trader the number of quotes reduce from $N(N-1)=1000 \times 999=999000$ to 999]. Of course different traders would quote differently so that there will be $T(N-1)$ quotes in the market. However by means of a direct comparison of price quotes of different traders, inter-trader arbitrage will ensure uniform price quotes. The number of effective price quotes will be reduced to $(N-1)$ which is dramatically lesser than $TN(N-1)$. In mathematical terms the degree of complexity has been reduced from a power of three (cube) to a power of one. At the same time everybody's record keeping has become smoother. With all transactions valued in terms of the *numeraire* and with arbitrage possibilities being eliminated, the values e.g. profits do not vary with the choice of the commodity in which the accounts are kept and the intercommodity quotes at which the transactions are made. Implicit in the above is the assumption that all traders accept one commodity as the *numeraire*. If they don't i.e. every trader chooses his own *numeraire*, every trader would give consistent quotes in terms of his own *numeraire* but these may not be consistent as between traders' *numeraires*. That is $T(N-1)$ will have to be processed in the marketplace for every buy/sell decision. ($100 \times 999=99900$ is a large number!) the general acceptance of a *numeraire* solves the computation and the information processing problem, but it

does no more than that. Every individual when he goes to the market will have to carry all sorts of things with him to consummate his trades because the traders he comes across may not be willing to transact some commodities even if they quote for them (if only implicitly). Can those inventory keeping and transactions cost be minimized? Yes, provided the generally accepted *numeraire* commodity can itself be used as the medium of exchange. The properties that a commodity must possess to perform as the role of a *numeraire* are not at all stringent. Almost every commodity can serve as a *numeraire*. But to be a stable medium of exchange a commodity should possess a host of peculiar and self-contradictory properties. Firstly the commodity must itself be useful, yet it should not form too large a proportion of consumption, or have so many uses that it is actually consumed up. It must not be easily producible, yet it should be easily available as the needs of trade augment. It must be durable. The commodity must have a high value in relation to other commodities to keep its own storage and transport costs within limits. Yet it should be desirable without much wastage to facilitate small trades. In short the medium of exchange should have all properties of good medium, viz portability, divisibility, etc and the properties of a good store of value, viz durability, steady demand and supply conditions etc. it is no wonder that gold, silver and other metals served as money for long periods of time in history. The general use of a medium of exchange imparts an additional advantage, viz; the acts of sale of commodities and the acts of purchases of commodities can be separated in time. This separation bestows some economic freedom to individuals.

It reduces the possibilities of their having to make forced sales/purchases. It gives them breathing space to search better prices.

20. In the absence of money, every trader would be required to carry in principle, some stock of N different commodities. (Strictly speaking N_d commodities where N_d denotes durable commodities). There would thus be TN separate hoards. In the presence of a generalized medium, each trader need only carry the money commodity (and not commodity money!). Consequently the number of hoards reduces to T . In terms of our numerical the number reduces from 100000 to 100. Correspondingly there is a dramatic saving of inventory accounting, transport and security costs. Why carry T hoards separately? If the inventory carrying function is centralized on account of economies of scale. Thus arises the need for banks of deposit, which can serve as centralized depositories of hoards of money. The banks of deposits would, quite naturally, function on the basis of giving receipts or notes to depositors of money commodities after ascertaining their quantity and quality.

21. This arrangement, although much better than carrying money inventories separately by individual traders is still inefficient. Because every time a depositor has to make payments he must run to the bank, present his receipt, withdraw the required quantity of the money commodity and get a new receipt for the quantity remaining. If each trader has to make purchases from his $T-1$ counterparts, he has to make $t-1$ trips to the bank. For T trades this amounts to $T(T-1)$ trips even if

each trader synchronises the deposit of his receipts with his withdrawals. This is rather expensive in terms of transport, time and security during transit. It is convenient to allow the depositor to transfer his receipt/note to the persons he makes payments. And that only requires that the bank notes come in several convenient denominations. Thus we see that there are inconveniences with the commodity money in circulation that provides us a rationale for introducing fiat money, and address the issues revolving around its neutrality. A more detailed discussion suggests that commodity monies are produced means of production. Further, they get used up in the process of production. Commodity money as described above has a dual purpose in the economy that it circulates in, namely its value in use and its value in exchange. The value in use of commodity money seen above is its presence in all the production relations as an important ingredient of the process recipe. However a fiat money, as the name suggests, lacks a value in use and has only a value in exchange and this feature of money will be evident when we introduce it in the system.

CHAPTER SIX

Fiat money.

22. Having provided the premises for the failure of commodity money as a generalized medium of exchange, we proceed with the introduction of *money* in the activities of the economy. The activities of an economy involve a process of production, consumption and the distribution of commodities that constitute a nation's national output. These entail a system of exchange of one commodity or at several times all commodities that explains the exchange of above processes as a whole, in return of the accepted medium of exchange. Why this medium of exchange be money is the question that we have already tackled, and that this money be issued by a government to disallow any single individual gain from seignorage. The process of exchange involves a matrix of payments and receipts in lieu of one another. These payments and their receipts are rarely synchronized and as a result the traders who are involved in this economic activity fall short at times of the commodity in exchange or at times fall short of its medium in exchange. This explains why these traders or the producers or any economic agents in the economy find it convenient at all times to carry the stocks of their requirements with them to avoid any unnecessary loss, and that they keep on doing so till the additional loss in trade due to non-carriage of the stocks is more than the cost of carrying the stocks themselves. We therefore at the outset assume that the inconvenience from not hoarding stocks of commodity (here, money to be

precise) is always more than or at least equal to the cost of carrying the stock but never less than it. However, this debate is trivial.

23. Since the argument of integrating money and prices is long it will be easier to go case by case. The initial condition that we begin with is by hypothesizing that the business class requires money stocks for the smoothening of the process of production. These money stocks will be held as a proportion of their turnover value and that these exogenously given proportions shall be designated as k 's. We have a constantly given supply of a *fiat money* stock that cannot be altered by any individual and only the issuing authority can toggle the supply of this money, if it so wishes to. Thus, our new system of equations shall appear as under.

$$\begin{aligned} &(k_1 B_1 p_1 + S_{11} p_1 + S_{12} p_2 + \dots S_{1n} p_n) r + A_{11} p_1 + A_{12} p_2 + \dots A_{1n} p_n + L_1 w = B_1 p_1 \\ &(k_2 B_2 p_2 + S_{21} p_1 + S_{22} p_2 + \dots S_{2n} p_n) r + A_{21} p_1 + A_{22} p_2 + \dots A_{2n} p_n + L_2 w = B_2 p_2 \\ &\dots\dots\dots \\ &(k_n B_n p_n + S_{n1} p_1 + S_{n2} p_2 + \dots S_{nm} p_n) r + A_{n1} p_1 + A_{n2} p_2 + \dots A_{nm} p_n + L_n w = B_n p_n \\ &k_1 B_1 p_1 + k_2 B_2 p_2 + \dots + k_n B_n p_n = M_s \\ &\alpha_i w L = B_i p_i \end{aligned}$$

The only additional equation that we have is the addition of a money market type of an equation that is necessary to determine the absolute level of prices, as we shall see. An important initial condition that we use is the stupid assumption that households do not carry money stocks but this is done for simplicity and that we shall drop this assumption very soon.

24. As has been our procedure, we provide an algorithm¹⁹ for determining the solutions to this newly modeled *monetary economy*. We start from the primal in this case instead of jumping to the dual right away. Using the *basic* system only we attain

¹⁹ Refer the appendix for the detailed programming done using MATLAB.

three matrices namely, A' matrix of basic flows, S' matrix of basic stocks and a B' matrix of basic outputs. Using the eigenvalue construct as above we obtain the maximal rate of profits in the economy, which corresponds to that level of profits when the wage rate is zero. This flows from Sraffa's wage profit frontier $r=R(1-w)$, and when $w=0$, $r=R$ which is called as the maximum rate of profits. Thus from the basic system of equations that contains n basic commodities and hence n equations in n prices, a wage rate and a rate of profits, we aim to solve $n+2$ variables. Any rate of profit less than R can be selected as the starting or the initial condition for the rate of profit, r . With this value, we plug it in the basic system and now are left with $n+1$ unknowns in n equations. We use the money market equation to determine the absolute level of prices and the money wage rate. Using the determined prices of the basic commodities, determine the non-basic prices and match the supplies of the non-basic commodities as was done in the system with commodity money. There will result a set of price equations for the non-basic commodities that are altered as per the demands in each industry. We now proceed with the dual of the problem; this problem will consist of n equations for the basic commodities in as many values for their respective multipliers. As was the case with the general Sraffa system as above, there was a labour conservation equation, since the medium of exchange in that system was a commodity. When we have *outside* fiat money we see no reason to continue with the labour conservation equation and as a result, we introduce a *money reallocation* equation to preserve the money stock in the economy. This equation is required in order to determine a unique growth rate and absolute values of the multipliers. Thus, the dual consists of $n+1$ equations in as many variables. The

system has to move to the next iteration and it has to be iterated as many times as required to clear the consumption goods industries. Since money stocks are present in the economy, the growth-profit relation will also be altered. The new growth profit

relationship shall now be given as $g = \frac{1}{1 - rm^*}$ where m^* is called the *flow money*

ratio. This m^* is defined as $\frac{M}{B_j - \sum_{i=1}^n \sum_{j=1}^n A_{ij} p_j}$ and M is the money supply in the

economy. The relation between the money rate of growth and the rate of profit is instrumental in proceeding in each iteration as it dictates the successive values of r that are required to continue the iterative cycle that may be represented as under

$$r < R \rightarrow P^*, w^* \rightarrow X^d = X^s \rightarrow Q^* \rightarrow g \rightarrow r$$

Once at r the process begins again until demands and supplies (denoted by X above and superscripted respectively) are equalized.

25. The following points are to be observed in this system. a.) As we can see, we use a money reallocation equation in place of the labour conservation equation. This implies that unlike commodity money, fiat money is used only in the process of production but never used up. It thus has a value in use. b.) We do not adhere to the use of a *numeraire* as in doing so we would lose the mathematical determinacy of the system. The economic significance of this argument is that in an economy where fiat money is used as a medium of exchange, the use of *numeraire* loses its motive and fundamentally makes the argument of money as an exchange to lose its importance. c.) Lastly, note the *flow money ratio* or the growth-profit frontier and this implies that

the value of rm^* should be mathematically lesser than one, for the system to possess non-zero and strictly positive rate of profits and the prices a well. Thus, the feasibility of the system rests on the fact that $rm^* < 1$.

26. Having explained the process by which a monetary economy attains its equilibrium, we take up a numerical example to make the results clear.

$$\begin{aligned}(10p_1 + 3p_1 + 2p_2)r + 2p_1 + 5p_2 + 5w &= 20p_1 \\(150p_2 + 2p_1 + 5p_2)r + 5p_1 + 7p_2 + 5w &= 30p_2 \\(60p_3 + 2p_1 + 3p_2)r + 2p_1 + 5p_2 + 10w &= 30p_3 \\20w &= 30p_3 \\10p_1 + 150p_2 + 60p_3 &= 2000 \\ \Rightarrow 10p_1 + 150p_2 + 40w &= 2000\end{aligned}$$

Note the last equation in the above system, which is the money market equation and it, has been converted in terms of the basic equation variables that are here p_1, p_2, w in this example. The money turnover or the velocity of money in industry one is assumed to be half, that in industry two is taken to be five and in industry three is assumed to be two. Note that the households do not hold any money stocks. The system in its dual shall not carry the money stocks in its equations because money being a non-basic commodity, as it is not produced, shall not enter the dual. The dual of the above system is

$$\begin{aligned}(3q_1 + 2q_2 + 2)g + 2q_1 + 5q_2 + 2 &= 20q_1 \\(2q_1 + 5q_2 + 3)g + 5q_1 + 7q_2 + 5 &= 30q_2 \\10q_1 + 150q_2 + 60 &= 2000\end{aligned}$$

Note the last equation of the dual, which is the money allocation equation. This equation reads out that the total money stock in the economy must be preserved.

The result sheet of the above system is presented as *system four*:

$$\begin{aligned}
(10p_1 + 3p_1 + 2p_2 + 5p_3)r + 2p_1 + 5p_2 + 3p_3 + 5w &= 40p_1 \\
(150p_2 + 2p_1 + 5p_2 + 3p_3)r + 5p_1 + 7p_2 + 5p_3 + 5w &= 60p_2 \\
(60p_3 + 2p_1 + 3p_2 + 5p_3)r + 2p_1 + 5p_2 + 3p_3 + 10w &= 50p_3 \\
(3p_1 + 5p_1 + 6p_2 + 15p_4)r + 3p_1 + 2p_2 + 5p_3 + 10w &= 60p_4 \\
(2p_1 + 3p_1 + 5p_2 + 40p_5)r + 2p_1 + 5p_2 + 7p_3 + 10w &= 50p_5 \\
20w &= 60p_4 \\
20w &= 50p_5 \\
\Rightarrow 10p_1 + 150p_2 + 60p_3 + 25w &= 2500
\end{aligned}$$

The results are presented as under.

Table 5A.

Iteration no.	R	P1	P2	P3	X_s^4	X_d^4	X_s^5	X_d^5	m'
Start	.25	4.9242	10.6136	7.6133	60	68	50	44.5	3.4932
1	.2446	4.9529	10.2661	7.672	68	72.05	44.5	46.69	3.3806
2	.2500	4.8207	10.3904	7.4538	72.05	72.53	46.79	46.75	3.4635

Table 5b

Iteration no.	W	P4	P5	G	Q2	Q1	Q3
start	16.0769	4.72 19	7.2245	1.6789	.9164	.8631	1.2905
1	16.7416	4.87 97	7.5146	1.6147	.9992	.9996	1.0018
2	15.7382	4.68 50	7.2667	1.6102	1	.999	1

A five good economy

Note that the demands as per the evaluations through the demand functions are represented by X_d^i and the supplies X_s^i are the bisected supplies obtained using

$$X_s^i = \frac{(\alpha_i w L / p_i) + X_d^i}{2}$$

27. Before conducting the analysis any further, we pause to study the behavior of this system in the disequilibrium states. Thus, suppose that the economy which is initially in equilibrium is disturbed, i.e. the outputs of some commodities rise above their equilibrium levels whilst those of the others fall below. In case of capital goods, such

disequilibria imply that the ratio of the physical surpluses to their aggregate quantities used become unequal to one another. Thus, certain industries will have excess demands and certain industries exhibit excess supplies. Those with excess demands will contract and those with excess supplies will expand, following a reverse movements in their future prices in the next iteration and this will continue happening till again an equilibrium is restored. Parallely, this system implicitly solves for a number of other macroeconomic variables and these are explained as follows. First of all, the system can be used to determine the Harrod-Domar growth rate. Consider the following three good economy, which we use to explain the implicit variables.

$$\begin{aligned}
 (10p_1 + 3p_1 + 2p_2)r + 2p_1 + 5p_2 + 5w &= 20p_1 \\
 (150p_2 + 2p_1 + 5p_2)r + 5p_1 + 7p_2 + 5w &= 30p_2 \\
 (60p_3 + 2p_1 + 3p_2)r + 2p_1 + 5p_2 + 10w &= 30p_3 \\
 20w &= 30p_3 \\
 10p_1 + 150p_2 + 60p_3 + 200w &= 2000 \\
 \Rightarrow 10p_1 + 150p_2 + 40w + 100w &= 2000 \\
 10p_1 + 150p_2 + 140w &= 2000 \\
 M_h &= 100w
 \end{aligned}$$

This economy possesses all the generalisations as mentioned above and the following is the output sheet presented at each iteration.

Table 6

Iteration number	g=r	Q1	Q2	P1	P2	W	P3	Demands
Initial	.1103	.5812	1.4188	4.1552	8.1433	5.264	4.2834	24.5784
1	.1154	.9730	1.011	3.3713	7.0811	3.7174	3.3868	19.9689
2	.1196	.9760	1.0094	3.3457	7.4796	3.1664	3.2728	16.1145
3	.1228	.9789	1.0080	3.3616	7.7986	2.6541	3.1535	12.9371
4	.1259	.9848	1.0056	3.2603	8.0556	2.1846	3.0252	10.3353
5	.1283	.9827	1.0062	3.4014	8.7720	1.8888	3.0873	8.2256
6	.1301	.9885	1.0041	3.2083	8.3745	1.4364	2.8039	6.5274
7	.1319	.9899	1.0035	3.0932	8.4800	1.1478	2.7058	5.1649
8	.1331	.9917	1.0028	3.0497	8.5544	.9140	2.6265	4.079
9	.1340	.9934	1.0023	3.0128	8.6094	.7252	2.5609	3.2168
10	.1350	.9852	1.0050	2.9892	8.6495	.5738	2.5038	2.5374
11	.1354	1.0053	.9982	2.9475	8.6532	.4523	2.4572	1.5965
12	.1359	.9966	1.0011	2.9362	8.7027	.3577	2.4297	1.5702

Three good economy to study implicit variables.

Let us now pen down the picture of this economy at equilibrium.

$$\begin{aligned}
 (4.9878p_1 + 1.4964p_1 + .9976p_2)r + .9976p_1 + 2.4938p_2 + 2.4937w &= 9.9956p_1 \\
 (3.0006p_1 + 225.0063p_2 + 7.5019p_2)r + 7.5019p_1 + 10.5022p_2 + 7.5063w &= 45.075p_2 \\
 (.1331p_1 + .1998p_2 + 3.995p_3)r + .1331p_1 + .3329p_2 + .6657w &= 1.9971p_3 \\
 10.6657w &= 1.5965p_3 \\
 M_h + 4.9878p_1 + 225.0063p_2 + 3.995p_3 &= 2000 \\
 \Rightarrow 4.9878p_1 + 225.0063p_2 + 74.6599w &= 2000
 \end{aligned}$$

The GNP of this system is given by $\sum_{i=1}^n B_i p_i$ and for this system it works out to be

Rs.426.48. Similarly; the NNP of this economy at the current prices given by

$$\sum_{i=1}^n B_i p_i - \sum_{i=1}^n \sum_{j=1}^n A_{ij} p_j \text{ is Rs.281.65. The total capital stock in this system is Rs.2062.10.}$$

Therefore the capital-output ratio is given by 14.23. The savings in the economy are the total income of the capitalists= (.1359*4008.78=) 544.8, and the ratio *savings to NNP* is 1.93.

Therefore the ratio $\frac{\textit{savings to NNP}}{\textit{capital output ratio}}$ gives the Harrod-Domar growth relation, which is exactly equal to the growth rate of the system. Also in this system, we can find out the income velocity (*GNP to money supply*) and transactions velocity (*NNP to money stock*). Lastly note that in the disequilibrium phases of the economy, the Walras' law does not hold.

CHAPTER SEVEN

Nonneutrality of money.

28. Having studied the dynamic properties of the system, we analyse the last leg of this work. This chapter is dedicated to inspecting two special cases where money does not turn out to be neutral. These two cases being a.) Where an economy provides for a public good and b.) When the payment mechanisms are changed for which we provide an intuitive argument.

29. In the system that we have generated to explain the theory of money and prices, we saw that the role of money was merely a facilitator of exchange and thereby was instrumental only in the determination of absolute prices. Money thus was truly neutral. It is here where we provide the first case for nonneutrality of money.

Consider that all the profit incomes in the economy are taxed and out of these tax receipts the authority in the state provides a deficit-financed public good. As a result we augment the system of $2m+2n+3$ equations in as many variables to include one more equation of a public good production and its corresponding demand equation and add additional variables- the tax rate in the economy and the volume of public good provided. Thus, we now have a six good economy and we evaluate the properties of such a system. The system first is presented as

$$\begin{aligned}
(10p_1 + 3p_1 + 2p_2 + 5p_3)r + 2p_1 + 5p_2 + 3p_3 + 5w &= 40p_1 \\
(150p_2 + 2p_1 + 5p_2 + 3p_3)r + 5p_1 + 7p_2 + 5p_3 + 5w &= 60p_2 \\
(60p_3 + 2p_1 + 3p_2 + 5p_3)r + 2p_1 + 5p_2 + 3p_3 + 10w &= 50p_3 \\
(3p_1 + 5p_1 + 6p_2 + 15p_4)r + 3p_1 + 2p_2 + 5p_3 + 10w &= 60p_1 \\
(2p_1 + 3p_1 + 5p_2 + 40p_5)r + 2p_1 + 5p_2 + 7p_3 + 10w &= 50p_1 \\
(3p_1 + 2p_1 + 6p_2 + 20) + 3p_1 + 2p_2 + 9p_3 + 20w &= 200 \\
30w &= 60p_4 \\
30w &= 50p_5 \\
10p_1 + 150p_2 + 60p_3 + 15p_4 + 40p_5 + 20 + 6w &= 2520 \\
\Rightarrow 10p_1 + 150p_2 + 60p_3 + 37.5w &= 2500 \\
t[r(2500 + K)] &= 200
\end{aligned}$$

Notice the changes that have happened in the system with the introduction of a public good. The public good first of all is a non-basic commodity and does not command a rate of profit nor does have a price. It is simply the amount of defence expenditure and the like. The right hand side of this equation determines therefore the scale at which the activity shall be provided. The presence of public goods affects the specification of outputs and prices in the system. The growth profit frontier also is altered by the public good when we consider a uniform tax rate t , and is given by

$$r = \frac{g}{(1 + gm)(1 - t)}$$

There are a lot of variables to be determined in this system and the

result sheet is presented as

Table 7A

It.no	P1	P2	P3	W	P4	P5	G	Q1	Q2	Q3	Xs	Xd
Start	4.5578	9.8239	7.0468	14.8807	4.5255	7.7822	1.0934	.9091	.8774	1.4370	60	98.6457
1	4.7163	7.7994	7.3112	18.9266	5.0173	7.9857	.8919	.9964	.9989	1.0020	79.3228	127.6034
2	4.6372	7.0854	7.1627	19.5902	4.9675	7.7278	.7341	.9963	.9983	1.0028	103.463	146.1845

Table 7B

Iteration no.	Xs	Xd	R	T	m^*
Start	50	61.3030	.25	.2758	6.23
1	55.6515	80.1714	.2085	.3277	5.85
2	67.9115	93.9687	.1936	.3489	5.43

A six good economy and provision of a public good.

One may go on further to determine the state of equilibrium in this system but finding the equilibrium in this case would be trivial as the motive of the analysis is to explain nonneutrality of money. The introduction of public good reduces the volume of currency in circulation, but the prices do not change in proportion to money supply reduction. Compare these results to results in table 5. The system in each case is the same but for the system newly introduced has a public good in it. This does not alter the prices and now money need not determine the level of prices in the system. The public good equation might just serve the purpose of rendering the determinacy to the system. However, without the aid of money, the provision of a level of public good cannot be ascertained. Money thus in this sense is truly *nonneutral* and has functions much beyond ascertaining the level of absolute prices.

30. The second case where money is seen to be nonneutral is analysed intuitively. This happens when the payment matrix in the economy itself undergoes a change, which implies a change in the values of k 's in the economy. The change in the k 's implies a disproportionate change in the value of the *flow money ratio* m^* , the value of the rate of profits from the growth profit frontier also changes disproportionately causing a change in the prices which are not in proportion to the stock of money.

Even without changing the money stock in total and only changing the holding ratios, we see that the system undergoes a change implying nonneutrality.²⁰

31. We conclude this thesis on this lines that; on the whole money is neutral in our system as was found by the classicals and supported by the neoclassicals. However, in exceptional cases as described above, money is nonneutral. Secondly, we find that in a monetary economy the Walras' law does not hold in disequilibrium but by definition holds in equilibrium²¹.

²⁰ This is an intuitive argument though and needs to be tested using the mathematical illustrations as above.

²¹ At any iteration excluding the final one, take the sum of the excess demands in the consumption goods industry and it will be seen that it does not add up to zero.

APPENDIX

MATLAB program for generating solutions of a Fiat money economy

```
a=input('enter the basic stock matrix');
b=a;
c=input('enter the basic flow matrix');
d=c;
e=input('enter the basic output vector');
f=diag(e);
g=inv(f-d);
h=g*b;
i=eig(h);
j=max(max(i));
k=1/j;
disp('the maximal rate of profits in this system is')
disp(k)
l=input('enter an arbitrary value of rate of profits less than this maximum');
m=l;
n=b*m;
o=n+d;
p=o-f;
q=input('enter the labour coefficients');
r=horzcat(p,q);
s=input('enter the money demand side');
t=s;
u=vertcat(r,t');
v=input('enter the convenient output vector');
w=v;
x=u\w;
disp('the basic prices and the wage rate at this specified profit rate is')
disp(x)
y=input('enter the non-basic stock vector as required');
z=y*m;
a1=input('enter the non-basic flow vector with labour coefficients');
a2=a1;
a3=z+a2;
a4=a3*x;
a5=input('enter the non-basic outputs');
a6=a5;
f1=a2;
f2=size(f1);
f1(:,f2(1,2))=[];
d7=vertcat(d,f1);
d9=sum(d7);
d8=(e(1,1)-d9(1,1));
e1=d8*x(1,1);
e2=(e(2,1)-d9(1,2));
e3=e2*x(2,1);
e4=(e(3,1)-d9(1,3));
e5=e4*x(3,1);
e6=e1+e3+e5;
e7=input('enter the money supply');
e8=e7/e6;
disp('the m ratio for this economy is')
```

```

disp(e8)
e9=e8*m;
if e9>1
    disp("")
    error('the system is not feasible, do you want to continue?')
end
a7 = size (a4);
for (a8=1:a7(1,1));
    a9(a8,1)=a4(a8,1)/a6(a8,1);
end
disp('the non-basic prices are')
disp(a9)
b1=y;
b2=size(b1);
b1(:,b2(1,2))=[];
b3=a2;
b4=size(b3);
b3(:,b4(1,2))=[];
g7=input('the changed non basic stock matrix');
i1=sum(g7);
g8=input('the changed non-basic flow matrix');
b6=sum(g8);
b7=vertcat(b,i1);
b8=vertcat(d,b6);
Z1=b8';
z2=(diag(s)*x);
b9=vertcat(Z1,z2');
c1=b9;
d4=q';
c5=vertcat(e,e7);
c6=diag(c5);
c7=inv(c6-c1);
f3=b;
f5=t;
f6=size(f5);
f5(f6(1,1),:)=[];
f9=diag(f5);
f7=f3-f9;
f8=horzcat(f7,i1');
f9=size(f8);
g1=zeros(1,f9(1,2));
g2=vertcat(f8,g1);
g3=g2*c7;
g4=eig(g3);
g5=max(max(g4));
g6=1/g5;
disp('the growth rate is')
disp(g6)
g9=g2*g6;
h1=size(g9);
g9(h1(1,1),:)=[];
j1=Z1+g9
j2=input('enter the last column of j1');
j3=size(j1);
j1(:,j3(1,2))=[];
k1=inv(f-j1);

```

```
k2=(k1*(j2));  
disp('the multipliers in this economy are');  
disp(k2)  
B1=(1+(g6*e8));  
B2=g6/B1;  
disp('the new rate of profits should be')  
disp(B2)
```

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