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A GENERALISED COB-WEB MODEL OF PRICES LEADING  
TO EQUILIBRIUM IN A FINITE NUMBER OF PERIODS

by

V. Mukerji and K. Mukerji\*\*

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\*\* This work was done in 1967-68 and submitted to the Indian  
Econometric Conference at Poona in 1967.

1. In the usual one-variate cob-web model the producer <sup>uses</sup> his knowledge of the supply at time  $t$  (in the period  $t$ ) on his knowledge of the price at which the quantity he produced in the earlier period was demanded in that period ignoring completely his knowledge of points on the demand curve, in the earlier periods. This is not only waste of information but may not also be very realistic.

2. If, on the other hand, the price expected by the farmer at time  $t$  is

$$P_t^* = \frac{1}{t} (P_{t-1} + \dots + P_0) \quad (1)$$

then the model is

$$\begin{aligned} S_t &= S(P_t^*) \\ D_t &= D(P_t) \\ \text{with } P_t^* &= \frac{1}{t} (P_{t-1} + \dots + P_0) \\ \text{and } S_t &= D_t \end{aligned} \quad (2)$$

or simplifying..

$$D(P_t) = S \left( \frac{1}{t} (P_{t-1} + \dots + P_0) \right) \quad (3)$$

If linear demand and supply functions

$$\begin{aligned} D(P_t) &= a - bP_t \quad b > 0 \quad * \\ \text{and } S(P_t) &= \alpha + \beta P_t \quad \beta > 0 \quad * \end{aligned} \quad (3a)$$

then equation (3) becomes

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\* These restrictions are later dropped with certain modifications.

$$a - bP_t = \alpha + \beta \frac{1}{t} (P_{t-1} + \dots + P_0) \quad (4)$$

The equilibrium price level is given by

$$P^* = \frac{a - \alpha}{b + \beta} \quad (5)$$

If  $P_t = P_t - P^*$  denotes the deviation of  $P_t$  from the equilibrium price, then equation (4) reduces to

$$bt P_t = -\beta (P_{t-1} + \dots + P_0) \quad (6)$$

It can be proved by the method of induction or otherwise that the solution of (6) for given  $P_0$  is

$$P_n = \frac{P_0}{n!} b^n \prod_{r=0}^{n-1} (rb - \beta) \quad (7)$$

Hence it is evident that if

$$rb = \beta \text{ for any positive integer } r, \text{ then } P_n = 0$$

for all  $n \geq r$ , or in other words equilibrium is reached at time

For this to happen it is necessary that  $\beta \geq b$ .

If  $\beta = sb$ ,  $s \neq 1$  a positive integer, then  $P_n$  does not become zero for any finite value of  $n$ . If  $k < s < k+1$

then  $P_n$  changes sign till  $n < k$ , and then has the sign of

$$\text{for all } (-1)^{k+1} \text{ for all } n \geq k.$$

Now 
$$P_n = \frac{P_0}{n! b^n} \prod_{i=0}^{n-1} (1-b-i\beta)$$

$$= P_0 \prod_{i=0}^{n-1} \left(1 - \frac{i\beta}{b}\right) \quad (8)$$

For  $1 + \beta/b > 0$  this continued product is monotonically decreasing in magnitude for sufficiently large  $r$  and since  $\sum \frac{1}{r}$  diverges to  $\infty$ , tends to 0 as  $n \rightarrow \infty$  (2) This is true so long as  $\beta + b > 0$  or  $\beta > -b$  provided  $b > 0$ .

If  $\beta + b < 0$  when  $b > 0$ , then  $P_n$  diverges to  $\infty$ .

Hence provided  $\beta + b > 0$ , equilibrium is reached whatever the initial value of the price, or there is global stability of equilibrium. As stated earlier, if in particular  $\beta = rb$  where  $r$  is a positive integer then the equilibrium is reached in  $r$  periods only.

3. In the usual cob-web model with the supply and demand functions given by (3a), the price behaviour around the equilibrium value  $P^*$  is given by

$$P_t = \left(-\frac{\beta}{b}\right)^t P_0 \quad (9)$$

Hence when  $\beta \geq b > 0$  in the cob-web process,  $P_t$  oscillates infinitely as  $t \rightarrow \infty$ . If  $0 \leq \beta < b$  the cob-web process for

$P_t$  oscillates but tends to the equilibrium value. If

$-b < \beta < 0 < b$  then the cob-web process tends to the equilibrium value monotonically.

When the behaviour of the cob-web process is compared with the behaviour of the process considered in section 2, referred to in this paper as the  $umpe$  (unweighted mean price expectation) process we get the following results:

Values of $\alpha$ and $\beta$	Limiting behaviour of	
	cob-web process $P_t$	$umpe$ process $P_t$
$\beta < -b$	diverges to $\infty$ monotonically.	diverges to $\infty$ or to $0$ monotonically.
$-b < \beta < 0$	converges to $0$ monotonically.	converges to $0$ monotonically
$0 < \beta < b$	oscillates but converges to zero.	- do -
$\beta > b$	oscillates infinitely.	- do -
$r$ a positive integer, $\beta = rb$	- do -	reaches equilibrium at time $r$

Hence the  $umpe$  process is a better stabilising process than the cob-web process in that it converges to equilibrium more often than the latter. One would also expect that for values of  $\frac{\beta}{b}$  for which both the processes converge the  $umpe$  process converges faster than the cob-web process. This, however, surprisingly does not happen as the next section shows.

Let  $P_{nc}$  denote the value of  $P_n$  for the cob-web process and  $P_{nu}$  the value of  $P_n$  for the  $umpe$  process. Both the processes converge when  $|\frac{\beta}{b}| < 1$ . We consider the limiting behaviour

of  $|P_{nu}/P_{nc}|$  for  $|b/\beta| < 1$ .

$$\frac{|P_{nu}|}{|P_{nc}|} = \left| \frac{1}{n!} \prod_{r=0}^{n-1} \left(1 - \frac{rb}{\beta}\right) \right| = \frac{1}{n!} \prod_{r=0}^{n-1} \left|1 - \frac{rb}{\beta}\right| = u_n, \text{ say } (10).$$

The sequence  $u_n$  diverges when  $|\frac{\beta}{b}| < 1$  for.  
 $u_{n+1}/u_n \rightarrow |b/\beta| > 1$ , as  $n \rightarrow \infty$

Hence when the cob-web process converges it converges

faster than the umpe process. In other words when the supply reaction coefficient is smaller in magnitude than the demand reaction coefficient Markovian memory (memory of only the previous instant) is a greater stabilizer than the non-Markovian memory (memory of all the previous instants up to the initial instant).

5. If instead of using an unweighted mean price one uses a weighted mean price for the expectation of the price for the supplier and in particular with geometrically decreasing weights  $\lambda, \lambda^2, \lambda^3$  etc. then equation (6) becomes

$$bP_t + \beta \frac{(P_{t-1} + \lambda P_{t-2} + \dots + P_0 \lambda^{t-1})}{1 + \lambda + \dots + \lambda^{t-1}} = 0 \quad 0 < \lambda \leq 1 \quad (11)$$

or  $b(1-\lambda^t) P_t + \beta(1-\lambda)(P_{t-1} + \lambda P_{t-2} + \dots + P_0 \lambda^{t-1}) = 0$

When  $\lambda = 0$  we get the cob-web model and when  $\lambda = 1$  we get the umpe process. It can again be shown by the method of induction or otherwise that the solution of equation is given by

$$P_n = P_0 \prod_{r=0}^{n-1} \left\{ \frac{\lambda(1+\lambda+\dots+\lambda^{r-1}) - \beta/b}{1+\lambda+\dots+\lambda^r} \right\} \quad (12)$$

for  $r=0$  the factor in brackets is  $-\frac{\beta}{b}$   
 (2) can be simplified and expressed in the form:

$$P_n = P_0 \prod_{r=1}^n \left(1 - \frac{x}{1-\lambda r}\right) \quad (13)$$

where  $x = \left(1 + \frac{\beta}{b}\right)(1-\lambda) > 0$  when  $1 + \frac{\beta}{b} > 0$   
 since  $1-\lambda > 0$ .

To investigate the convergence of the continued product in (13),  
 consider

$$\left| \frac{P_{n+1}}{P_n} \right| = \left| \left(1 - \frac{x}{1-\lambda(n+1)}\right) \right| \rightarrow |1-x| \quad 0 \leq \lambda < 1 \quad (14)$$

and  $\rightarrow 1$  for  $\lambda = 1$ , which has

already been considered.

The case of  $\lambda = 0$  has also been considered.

For  $0 \leq \lambda < 1$  (i)  $P_n$  diverges (monotonically) when  $x < 0$   
 that is when  $1 + \frac{\beta}{b} < 0$  or  $\frac{\beta}{b} < -1$ .

Note that for  $\frac{\lambda}{1-\lambda} > \frac{\beta}{b}$ ,  $P_n$  has the same sign for  
 sufficiently large  $n$ .

(ii)  $P_n$  diverges oscillatingly when  $x > 0$  but  $1-x < -1$   
 or  $x > 2$

that is  $\left(1 + \frac{\beta}{b}\right)(1-\lambda) > 2$

$$\text{or } \frac{\beta}{b} > \frac{1+\lambda}{1-\lambda} \quad (15)$$



(iii)  $P_n$  converges to 0 when

$$x > 0 \text{ but } 1-x > -1$$

$$\text{or } \beta/b > -1 \text{ but } < \frac{1+\lambda}{1-\lambda}$$

When  $\frac{\beta}{b} > \frac{\lambda}{1-\lambda}$  it converges oscillatingly and when

$\frac{\beta}{b} < \frac{\lambda}{1-\lambda}$  it converges monotonically.

(iv) When  $1-x = -1$  or  $x = 2$

$$\text{or } \beta/b = \frac{1+\lambda}{1-\lambda}$$

$$\left| \frac{P_{n+1}}{P_n} \right| = \frac{1+\lambda^n}{1-\lambda^n}$$

and  $|P_n| = \prod (1 + 2\lambda^n / (1-\lambda^n))$  and  $P_n$  alternates in sign.

$\sum \lambda^n / (1-\lambda^n)$  converges.

Hence  $P_n$  converges to a non zero limit

but oscillates finitely.

Thus in this case the convergence or divergence when  $\frac{\beta}{b} > -1$  depends on the value of  $\lambda$ . The larger the  $\lambda$  ( $< 1$ )

the larger the range of convergence and the smaller the  $\lambda$

the smaller the range of convergence. The range of

convergence common to all the three processes is the

range given by  $-1 < \frac{\beta}{b} < 1$ .

When  $\frac{\beta}{b} = \lambda(1+\lambda+\dots+\lambda^{r-1})$  the wmp process reaches equilibrium in  $r$  steps.

6. To compare the speed of convergence of the wmp process with that of the cob-web and the <sup>umpe</sup> process in their

common range of convergence we again compare the relevant  $P_n$ 's.

The absolute ratio of  $\rho_n$ 's for the wmp process (in the numerator) and the cob-web process is given by  $|u_n| = \left| \frac{b}{\beta} \right|^n \frac{1}{r} \left( \frac{1-x}{1-\lambda r} \right)^n$

Hence

$$\begin{aligned} \left| \frac{u_{n+1}}{u_n} \right| &= \left| \frac{b}{\beta} \right| \left| \frac{1-x}{1-\lambda} \right| \\ &\rightarrow \left| \frac{b}{\beta} (1-x) \right| \text{ as } n \rightarrow \infty \\ &= \left| (-1 + \lambda + \frac{\lambda b}{\beta}) \right| \\ &= \left| \frac{\lambda b}{\beta} - (1-\lambda) \right| \end{aligned}$$

(17)

If  $\lambda \left( \frac{b}{\beta} + 1 \right) - 1 > 0$  or  $\lambda \left( \frac{b}{\beta} + 1 \right) > 1$   
 $u_n \rightarrow \infty$  or converges to 0 according as  $\lambda \left( \frac{b}{\beta} + 1 \right) \geq 2$

If  $\lambda \left( \frac{b}{\beta} + 1 \right) < 1$  then  $u_n \rightarrow 0$  or  
 converges to  $\infty$  according as

$$\lambda \left( \frac{b}{\beta} + 1 \right) \leq 0$$

When both  $b$  and  $\beta > 0$  and  $\frac{\beta}{b} < 1$  (or  $\frac{b}{\beta} > 1$ )  
 $\lambda \left( \frac{b}{\beta} + 1 \right) > 0$

Hence  $u_n$  converges to 0 or diverges to  $\infty$  according  
 as  $\lambda \left( \frac{b}{\beta} + 1 \right) \leq 2$

or

$$\begin{aligned} \frac{b}{\beta} &\leq \frac{2-\lambda}{\lambda} \\ \text{or } \frac{\beta}{b} &> \frac{\lambda}{2-\lambda} \end{aligned}$$

If  $b > 0$  and  $\beta < 0$  and  $\frac{\beta}{b} > -1$  or  $\frac{b}{\beta} < -1$

$\lambda(b+\beta) > 0$ ; hence the same results as above hold. In

this case however since  $\frac{b}{\beta} < -1$

$$\frac{b}{\beta} \neq \frac{2-\lambda}{\lambda}$$

Hence  $u_n$  converges to zero.

Thus we get <sup>the result that</sup> when  $\frac{b}{\beta} < \frac{2-\lambda}{\lambda}$ , <sup>wmpe</sup> process converges to

equilibrium faster than the cob-web process and when  $\frac{b}{\beta} > \frac{2-\lambda}{\lambda}$

the converse holds.

7. When the ratio  $u_n$  stands for the absolute ratio of the  $p_n$ 's for the wmpe process in the numerator and the umpe process,

then

$$u_n = \frac{1}{1} \left| \frac{1 - \left(1 + \frac{\beta}{b}\right)(1-\lambda)}{1 - \frac{1-\lambda^n}{n}} \right| \quad (18)$$

Hence

$$\frac{u_{n+1}}{u_n} \rightarrow 1 - \alpha \quad \text{as } n \rightarrow \infty$$

$$\text{where } \alpha = \left(1 + \frac{\beta}{b}\right)(1-\lambda).$$

Hence as before if  $\frac{\beta}{b} < \frac{1+\lambda}{1-\lambda}$  the process  $u_n$  converges to zero and if  $\frac{\beta}{b} \geq \frac{1+\lambda}{1-\lambda}$  the process diverges,

$\frac{\beta}{b} < \frac{1+\lambda}{1-\lambda}$  is the common range of convergence for the

umpe and the wmpe processes and hence in this range the process <sup>wmpe</sup> converges faster than the umpe process.

Further since  $\frac{\lambda}{2-\lambda} < \frac{1+\lambda}{1-\lambda}$ , we have from the previous result and the present result, when  $\beta < \frac{\lambda}{2-\lambda}$  the cob-web process converges to equilibrium <sup>faster</sup> than the umpe process and the

umpe process converges faster than the wmpe process. When

$\frac{\lambda}{2-\lambda} < \frac{\beta}{b} < \frac{1+\lambda}{1-\lambda}$  the umpe process converges faster than the cob-web process and the wmpe process converges faster than the umpe process.

If  $\frac{\beta}{b} > \frac{1+\lambda}{1-\lambda}$  the wmpe and cob-web process do not converge to equilibrium but the umpe process converges with the exception that if  $\frac{\beta}{b} = r$  where  $r$  is a positive integer the umpe process converges in finite time  $r$ .

8. In the multi-market case in period analysis the counterparts of the equations for the cob-web process with linear demand and supply functions after simplification are

$$B P_t + C P_{t-1} = 0 \quad (19)$$

where  $P_t = P_t - P^*$   
 and  $D_t = \alpha - B P_t$   
 and  $S_t = \Gamma + C P_t^*$

with  $P_t^* = P_{t-1}$   
 and  $P^* = (B+C)^{-1}(A-I)$ .

$P_{t-1}$ ,  $P_t^*$  and  $P^*$  are the price vector at time  $t-1$ , the expected price-vector at time  $t$ , and the equilibrium price-vector respectively. The solution of (19) is given by

$$P_t = (-B^{-1}C)^t P_0 \quad (20)$$

If the latent roots of  $B^{-1}C$  are less than one in magnitude then  $P_t \rightarrow 0$  as  $t \rightarrow \infty$  or the process converges to equilibrium. If the latent root with the largest magnitude is greater than 1 in magnitude then the process diverges and if it is equal to one in magnitude then in general it oscillates about the equilibrium value. For if  $L$  and  $\Lambda$  are the latent vector and latent root matrices of  $B^{-1}C$

and  $L^{-1}B^{-1}CL = \Lambda$   
 then  $P_t = L^{-1}(-\Lambda)^t L P_0 \quad (21)$

If  $\Lambda \bar{\Lambda} < I$ , then  $P_t \rightarrow 0$  and if  $\Lambda \bar{\Lambda} > I$  the process diverges. If  $\Lambda \bar{\Lambda} \leq I$ , the process oscillates finitely about the equilibrium. If in particular  $B^{-1}C$  is positive definite which follows when  $B$  and  $C$  are both positive-definite,  $\Lambda > 0$  and is real and hence the condition for convergence becomes the matrix  $\Lambda - I$  or  $B^{-1}C - I$  must be negative definite the conditions for which can easily be written down.

For the umpe process, equations (19) are replaced by

$$B P_t + \frac{1}{t} C (P_0 + P_1 + \dots + P_{t-1}) = 0 \quad (22)$$

$$D_t = \alpha - B P_t$$

$$S_t = \Gamma + C P_t^*$$

$$P_t^* = \frac{1}{t} (P_0 + P_1 + \dots + P_{t-1})$$

and

$$P_t = P_t - P_t^*$$

It can again be proved by the method of induction or otherwise that the solution of (22) is given by

$$P_n = A_n P_0 \quad (23)$$

where

$$\begin{aligned} A_n &= \frac{1}{n!} \prod_{r=0}^{n-1} (rI - B^{-1}C) \\ &= \prod_{r=0}^{n-1} \left( I - \frac{I + B^{-1}C}{r+1} \right) \end{aligned} \quad (24)$$

It follows that

$$A_{n+1} = A_n \left( I - \frac{I + B^{-1}C}{n+1} \right)$$

Let

$$B_n = L^{-1} A_n L = \prod_{r=0}^{n-1} \left( I - \frac{I + \Lambda}{r+1} \right)$$

which is a diagonal matrix with

$$b_{s,n} = \prod_{r=0}^{n-1} \left( 1 - \frac{1 + \lambda_s}{r+1} \right) \quad (25)$$

Hence when the real part of it  $1 + \lambda_s > 0$  for all  $s$

$$b_{s,n} \rightarrow 0 \quad \text{for all } s$$

or

$$B_n \rightarrow 0 \quad \text{as } n \rightarrow \infty$$

or

$$A_n \rightarrow 0 \quad \text{as } n \rightarrow \infty$$

and hence  $P_n \rightarrow 0$  as  $n \rightarrow \infty$ .

Hence if  $M = -(I + B^{-1}C)$  is Routhian then

$$P_n \rightarrow 0 \text{ as } n \rightarrow \infty$$

The necessary and sufficient conditions for  $M$  to be Routhian are that a positive definite matrix  $S$  can be found such that  $S B$  is quasi negative definite.

If in particular  $B^{-1}C$  is positive definite then

$$I + \Lambda >> 0 \text{ and hence}$$

$$\text{as } P_n \rightarrow 0 \text{ as } n \rightarrow \infty$$

and hence the process converges provided  $I + \Lambda$  and hence

$I + B^{-1}C$  is positive definite, the conditions for which can be easily written down. A comparison of this result with the earlier result for the cob-web process shows that the range of convergence for the umpe process is very much greater than the range of convergence for the cobweb process. In order to compare the speed of convergence of the two processes when they converge we consider the matrix ratio of the  $B_n$ 's for the two processes.

$$U_n = \left[ (-\Lambda)^{-n} \prod_{r=0}^{n-1} \left( I - \frac{I + \Lambda}{r+1} \right) \right] \quad (26)$$

with the  $P_n$ 's for the umpe process in the numerator.

Each term in the diagonal can be treated separately. When

$\Lambda \ll 1$  the two processes converges and  $U_n$  diverges which implies that the umpe process converges to equilibrium more slowly than the cob-web process.

8. For the multi-market \*swmpe process with linear demand and supply functions we similarly have

$$B p_n + \frac{c}{1+\lambda} + \lambda^{n-1} (p_{n-1} + \lambda p_{n-2} + \lambda^{n-1} p_0) = 0 \quad (27)$$

$\lambda$  is a scalar.

The solution of (27), it can be proved by the method of induction, is given by

$$p_n = \prod_{r=1}^n \left\{ I - \frac{(I+B^{-1}C)(1-\lambda)}{(1-\lambda^r)} \right\} p_0 \quad (28)$$

Let

$$A_n = \prod_{r=1}^n \left\{ I - \frac{(I+B^{-1}C)(1-\lambda)}{(1-\lambda^r)} \right\}$$

Then if  $L^{-1}B^{-1}CL = \Lambda$ ,

where  $L$  and  $\lambda$  are as in the previous section latent vector and latent roots of  $B^{-1}C$

$$B_n \equiv L^{-1}A_nL = \prod_{r=1}^n \left( I - \frac{(I+\Lambda)(1-\lambda)}{(1-\lambda^r)} \right) \quad (29)$$

As before each of the elements in the diagonal can be treated separately. If the largest latent root of  $B^{-1}C$  is  $< \frac{1+\lambda}{1-\lambda}$  then each of the elements in  $B_n$  tends to zero. If the largest of the elements of  $B^{-1}C$  is  $> \frac{1+\lambda}{1-\lambda}$  then one or more of the elements in

$$B_n \rightarrow \infty \quad \text{Hence in the latter case } A_n \rightarrow \infty$$

It is evident that the speed of convergence <sup>d-</sup> the swmpe process can be compared with the speed of convergence of the umpe and the cob-web processes and the results in the one market case can easily be generalised to the multimarket case by comparing the behaviour of the largest elements of  $B_n$  in all the three cases.

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\*swmpe  
stands for scalar weighted mean price expectation process.



Again the process converges to equilibrium in finite time  $r$  only if  $B^{-1}C$  is diagonal and equal to  $I\lambda(1+\lambda+\dots+\lambda^{r-1})$  where  $r$  is a positive integer. For the unimolecular process  $\lambda=1$  and this condition reduces to  $B^{-1}C = rI$ .

If the reaction coefficients instead of being in the form  $\lambda I$ ,  $0 < \lambda < 1$  are given by the matrix  $K$  where  $K$  is positive definite with all its latent roots less than 1 in magnitude equation (27) is replaced by

$$B P_n + C (I + K + \dots + K^{n-1})^{-1} (P_{n-1} + K P_{n-2} + \dots + K^{n-1} P_0) = 0 \quad (30)$$

The solution of (30) it can be verified is given by

$$P_n = \prod_{r=1}^n M_{r-1}^{-1} \{ M_{r-1} K - B^{-1}C \} P_0$$

where

$$M_{r-1} = I + K + \dots + K^{r-1}$$

and

$$M_{-1} = 0$$

Hence

$$\begin{aligned} P_n &= \prod_{r=1}^n M_{r-1}^{-1} (M_{r-1} K - B^{-1}C) P_0 \\ &= \prod_{r=1}^n (I - M_{r-1}^{-1} (I + B^{-1}C)) P_0 \end{aligned}$$

Hence

$$= \prod_{r=1}^n (I - (I - K^r)^{-1} (I - K) (I + B^{-1}C)) P_0 \quad (31)$$

$$P_n = A_n P_0$$

where  $A_n = \prod_{r=1}^n (I - (I - K^r)^{-1} (I - K) (I + B^{-1}C))$

and hence  $A_{n+1} = A_n (I - (I - K^{n+1})^{-1} (I - K) (I + B^{-1}C))$

$$= A_n Q_n \quad \text{say} \quad (32)$$

As  $n \rightarrow \infty$   $Q_n \rightarrow I - (I - K) (I + B^{-1}C) = K - (I - K) B^{-1}C$

If all the latent roots of  $|K - (I-K)B^{-1}C|$  are less than one in magnitude then  $|A_n| \rightarrow 0$  as  $n \rightarrow \infty$  and if one or more of the latent roots are greater than one in magnitude then  $|A_n| \rightarrow \infty$ . Hence all this applies to  $p_n$ . If

all the latent roots of  $|K - (I-K)B^{-1}C|$  are less than or equal to 1 and one or more are = 1 then  $|A_n| \rightarrow$  a finite matrix.

If  $B^{-1}C = M_m K$  then the process converges to equilibrium in time  $t$ .

The speed of convergence of this process can be compared with that of the multi-market cob-web and  $u m p e$  and the scalarweighted  $m p e$  processes.

It should be noticed that when  $K = 0$ , the above process reduces to the multi-market cob-web process.

When  $K = I$  it reduces to the  $u m p e$  process.

When  $K = \lambda I$  it reduces to the  $s w m p e$  process.

The multimarket models reduce to the single market model when the  $m$ , the number of commodities considered is put equal to 1.

9. The generalisations of the above models to the continuous case and the  $\{m\}e$  (limited mean price expectation) processes will be considered in another paper. Limited mean price expectation processes stands for processes of the above type when

$$P_t^* = \frac{\sum_{r=1}^k P_{t-r} w_r}{\sum_{r=1}^k w_r}$$

where  $k$  is fixed.

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