

The Input-Output Theory of Interest Rates

Rajas Parchure

Gokhale Institute of Politics and Economics
Pune 411 004
Maharashtra, India

Abstract

Extant theories of interest rates seek to determine of the level of the interest rate without reference to the structure of interest rates. In contrast, this paper proposes a unified theory of the level and structure of interest rates. The unified theory is based on the simple idea that banks “produce” loans of different maturities from deposits of different maturities by providing the minimum prudential reserves to maintain their solvency while maximizing profitability. The optimal assignment of deposits to loans results in a system of equations (which is formally identical to the Leontief price system) that determines a unique positive solution for the structure of interest rates that banks must quote. Interest rate quotations thus derived normally exhibit their usual upward slopes but can become perverse in economic circumstances that require banks to meet short-term loan demand from their long-term deposits. Generalisations of the basic theory to cover issues like the pricing of default risks, determination of interest rate spreads and the effects of monetary policy have been discussed.

I Introduction

Interest rates theories, whether classical, neo-classical or Keynesian (not Keynes’s theory) have two enigmatic features whose explanation are not found in the theories. The first feature is that interest rate theories seek to determine the level of the regular interest rate without reference to the *structure* of interest rates. Of course all theories mention the structure of interest rates but (a) the relationship of the level to the structure is not explicitly stated and (b) separate hypotheses (e.g. expectations, preferred habitats, etc.) are invoked to explain the structure of interest rates but the relationship of those hypotheses with the theories is not explicitly stated. Nor for that matter are the individual hypotheses of term structure behavior seen to draw their inspiration from particular theories of interest rates. Yet our most elementary intuition suggests that there ought to be some connection between the level and the structure of interest rate – either the interest rate must be some rate belonging to the structure (short, medium or long) or it must be a representative average of some or all the rates in the term structure. However, clarity on this has not been forthcoming.

The second feature, also common to all interest rate theories, is the conviction that the concepts of production and cost of production in their objective and measurable connotation do not apply to interest rates even though interest rates are prices of loans. The underlying belief seems to be that banks only intermediate, they do not produce, that the loanable funds are produced from the disutilities suffered by the

ultimate lenders and that it is the borrowers' offers of interest rates to compensate for those disutilities which will determine the size of loanable funds coming to the market. Banks do incur expenses and expect profit but these are merely costs of intermediation which have no bearing on the determination of the interest rate itself. But, if that were so, banks would function like brokers and exchanges which match buy and sell orders of lenders and borrowers and try to find the best price for them. Yet elementary observation and a huge amount of well documented literature suggests that (a) depositor preferences do not automatically match borrower requirements, thus requiring banks to typically accept payable-on-demand and other low maturity readily-encashable deposits as liabilities but advance illiquid long term loans as assets carrying in the process substantial liquidity / solvency risks on their own account (b) that banks run a whole host of other risks including credit and interest rate risk which they accept by pricing them appropriately or develop hedging techniques to manage them. The acceptance of risks by banks on their own account and the asset-liability management techniques that they operate suggest that banks are not only intermediaries between lenders and borrowers, they are principals themselves, they design products, they manage funds and they underwrite and manage risks. Theories of interest rates have chosen to emphasise only the intermediation role and underplay their production activity.

Accordingly the principal purpose of this paper is to tackle the two issues raised in the foregoing. We shall begin by asking two basic questions, (a) what is the technology that banks use to transform their deposits, considered as their inputs, into loans, their output? and (b) what are the costs of effecting this transformation? It is argued that the cost consists of the liquidity risk that banks are exposed to in the course of advancing loans out of deposits, that is to say, the risk that the timing and size of their asset cash inflows will be insufficient to meet their liability outgoes. The paper then proceeds to tackle other issues such as the determination of the optimal point of the liquidity profitability trade off, the determination of the term structure of interest rates, the pricing of default risks and the effect of monetary policies.

The plan of the paper is as follows. Section II outlines the method for quantifying liquidity risk. Section III formulates the basic assignment problem and the resulting model of interest rate determination. Section IV contains the formal proof of the uniqueness and positivity of the interest rate solution. Section V discusses some properties of the interest rate solution. Section VI deals with the pricing of default risk. Section VII contains some notes on the determination of the asking and bid rates of deposits and loans of depositors, banks and borrowers. Section VIII give numerical illustrations of rising and declining term structures of interest rates. Section IX shows the effects of monetary policy on interest rates. Section X offers concluding observations on the relation of the input-output approach to interest rates with other theories.

II Liquidity Risk

To start at the simplest level, consider a situation in which banks have only demand deposits (by definition non-interest – bearing) but borrowers demand loans of various maturities. Clearly the banks will need to provide reserves to make sure that they have at least sufficient funds to meet withdrawals of depositors. How much reserves should they provide? Suppose the probability of withdrawal of the 0-maturity

(demand) deposits in one period is p_{01} and the bank is to make a 1 period loan. Then, in a competitive scenario the most efficient arrangement would be for the bank to charge interest to cover the withdrawals during the period, i.e.

$$1+i_1 = 1+p_{01} \quad i_1 = p_{01}$$

Anything more than i_1 is acceptable, not any less. The same principle can be carried forward. If the bank wishes to make a 2 – period loan, the interest income must cover withdrawals over two periods. If p_{02} is the probability of withdrawal in the second period then the minimum interest rate that must be charged is obtained from

$$(1+i_2)^2 = p_{01} + p_{02} (1-p_{01})$$

This can be generalised. Let p_{01}, p_{02}, \dots be the probabilities of net withdrawal of 0-period (i.e. instantly withdrawable) demand deposits before periods 1, 2, etc. These probabilities can be used to construct a deposit life table shown in Table 1.

Table 1: Deposit Life Table

Period	Deposit Balance	Probability of withdrawal	Withdrawal	Cumulative Reserve
0	1	p_{01}	p_{01}	p_{01}
1	$1 - p_{01}$	p_{02}	$p_{02}(1 - p_{01})$	$p_{01} + p_{02}(1 - p_{01})$
2	$(1 - p_{01})(1 - p_{02})$	p_{03}	$p_{03}(1 - p_{01})(1 - p_{02})$	\vdots
\vdots	\vdots	\vdots	\vdots	\vdots
t	$\prod_0^t (1 - p_{ot})$	p_{ot}	\vdots	$1 - \prod_{t=0}^t (1 - p_{ot})$

The first column of Table (1) shows the deposits surviving in period t after cumulative withdrawals during the earlier period at the probabilities shown in the second column. In other words each entry in the first column is equal to the earlier entry minus the withdrawal of the earlier period shown in the third column. The fourth column gives the reserve that must be provided to meet cumulative withdrawals up to period t i.e. r_{ot} the reserve that must be provided to make a loan of maturity t from a deposit of maturity 0. In other words, it is the minimum reserve required to maintain solvency. The minimum (asking) rate of interest that banks must charge to make a t-period loan (overnight loans onwards) from 0-period demand deposits must be such as to cover the cumulative withdrawal up to period t so that

$$(1+i_t)^t = 1+r_{ot} \quad \dots 1(a)$$

where i_t is the spot interest rate for t-period loans. Observe that r_{ot} serves the role of a “technical coefficient”; a provision of r_{ot} is used to transform a dollar of deposit of maturity 0 into a dollar of loan of maturity t while enabling the banks to continue operations period after period. Equation (1a) can also be read in the following way; it shows, the banks’ maximum (bid) price for buying a t-period zero-coupon security of \$1 face value issued by the borrowers in

where $R_s = \sum_{ts} r_{ts} D_{ts}$ ($t = 0, \dots, s = 1 \dots T$) and $\sum_0^s D_{ts} = L_s$. The equations in (4)

simply state that interest rates will be charged by competitive banks in such a way that the proceeds from the assets on the right hand side are sufficient to meet the reserves required to meet withdrawals from deposits and repay deposits with interest. Since, in each equation of (4) and all equations taken together, the loans advanced are equal to the sums of deposits used to finance them, it is tempting to infer that banks merely intermediate, that they do not produce a “net product” considered as an excess of outputs over inputs consumed. But such an inference is inadequate. For two interrelated reasons. Firstly, in the usual situation of an upward sloping term structure, it will be found that the average maturity of bank assets will exceed that of bank liabilities,

$$m(A) = \frac{\sum_1 L_t t}{\sum_1 L_t} > \frac{\sum_0 D_t t}{\sum_0 D_t} = m(L)$$

i.e., banks transform low maturity liabilities into longer-term assets. Secondly, the total asset proceeds on the right hand sides will exceed the deposit outgoes on the left hand-sides,

$$\sum R_t = \sum_1^T L_t (1 + i_t) t - \sum_{t=0}^T \sum_{s=1}^T D_{ts} (1 + i_t)^t > 0$$

Accordingly the excess of maturity over that of liabilities may be considered as the physical measures of what banks produce, as distinct from banks’ value-added which is the measure suitable for national income accounts. The reciprocals of the interest factors $1/(1+i_t)^t = P_t$ may be interpreted as the prices of zero-coupon securities quoted in terms of the price of demand deposits (money) $P_0 = 1$ (implies $i_0 = 0$) which in effect serves as the *numeraire* in the financial market.

IV Uniqueness and Positivity of Interest Rates

If the respective equations in (7) are divided by the loan sizes L_t they may be arranged as a system of equations 5(a) whose left hand side has an obvious resemblance with Leontief’s I-A.,

$$\begin{bmatrix} 1 - d_{11} & -d_{21} & \dots & -d_{T1} \\ -d_{12} & 1 - d_{22} & \dots & -d_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ -d_{1T} & -d_{2T} & \dots & 1 - d_{TT} \end{bmatrix} \begin{bmatrix} (1 + i_1) \\ (1 + i_2)^2 \\ \cdot \\ \cdot \\ (1 + i_T)^T \end{bmatrix} = \begin{bmatrix} r_1 + d_{01} \\ r_2 + d_{02} \\ \cdot \\ \cdot \\ r_T + d_{0T} \end{bmatrix} \quad \dots 5(a)$$

be greater than that of its being withdrawn in a week. If so, the rows of the p_{ts} matrix will be decreasing sequences and the columns will be increasing sequences. It follows that the rows of the corresponding r_{ts} matrix of reserve coefficients will be increasing sequences and the columns will be decreasing sequences. In this situation the north-west corner rule will by itself give the optimal assignment. [Mitra and Mohan (1982)] We may now proceed to highlight the general properties of the interest rate solution of equation (4).

1. If the sizes of deposit supplies and loan demands across maturities are fairly matched or, as is usual for banks, the sizes of deposits taper off at increasing maturities but sizes of loan demands tend to balloon, the optimal assignment in equation (4) is lower triangular, i.e. loans of any given maturity t are financed by deposits of maturity less than or equal to t . In this situation, which is the normal case, the structure of interest rates is upward sloping. If, however, in the rare instance of a shortage of short-term loanable deposits (e.g. due to extreme regulatory action) and/or a slackening of long-term loan demand (due to the onset of a recession) the maturity-wise sizes of available deposits and loans exhibit the reverse structures, i.e. if there is a shortage of short term deposits relative to short term loan demand and a shortage of long term loan demand relative to long term deposits the optimal assignment in equation (4) becomes upper-triangular, i.e. loans of maturity t are financed by deposits of equal and greater maturities. In this situation the structure of interest rates exhibits a downward slope (Refer section VIII).

2. When the optimal assignment is lower triangular and the term structure is upward sloping the weighted average maturity of loans exceeds that of the deposits. This is what is meant by banks' production; banks produce longer – maturity loans out of shorter – maturity deposits. The opposite holds when the optimal assignment is upper triangular. Thus the excess of average maturity of bank assets over bank liabilities is quick measure of banks' efficiency in transforming maturities.

3. Term structures obtained are not always regular and monotonic, they exhibit dimples and pimples depending on the deposit supplies D_t and loan demands in the different maturity 'buckets', typically an excess of D_t over L_t will cause a dip in the term structure and an excess of L_t over D_t will cause a crest. (Refer numerical example of section VIII).

4. The term structure solution will contain indeterminate spot rates if situations arise when loan demand of any maturity is met entirely from deposits of the same maturity.

5. The system of equations (4) solves for the structure of interest rates, that is to say, the levels of interest rates in each maturity bucket and the gradient of that structure. Naturally, the level of the interest rate, which is the object of macroeconomic theories, is not found there. However, it is always possible to obtain a summary measure of the level by taking a weighted average of the loan rates in all buckets, viz. $i = \sum i_t L_t / \sum L_t$.

6. An increase in the volume of short-term deposits (money) gives the banks access to low cost funds and will result in a downward shift in the term structure of interest rates as well as in the gradient of the structure. An increase in demand for loans, especially longer term loans, will cause the term structure to shift upwards and raises the gradient of that structure.

7. The system of equations (4) has been formulated on the assumption that the maturities of the deposit supplies available to banks and the demands for loans of the

borrowers are both equal to T so that a square coefficients matrix is obtained. In practice this may not be so; it is quite possible that the maturity of loans demanded exceed the maturity of deposits available. If, in these circumstances, banks are required to quote interest rates for loans of term greater than T , this poses a special problem because banks may have no knowledge of the withdrawal probabilities of deposits during time period greater than T so there is no way to calculate the reserve coefficients for those maturity buckets. Institutions, therefore, have to resort to such financial innovations as “auction rate securities” (ARS) or take-out financing (TOF) by which a new set of financiers will step in to take the place of those who wish to withdraw.

8. The forward rates implied by the spot interest rate solution is readily obtained from the following equations

$$\begin{aligned}
 (1+i_t)^t &= (1+i_1)(1+f_{1t})^{t-1} \\
 &= (1+i_2)^2 (1+f_{2t})^{t-2} \quad \dots(6) \\
 &= (1+i_1)^{t-1} (1+f_{t-1,t})
 \end{aligned}$$

The interpretation that must be given to the forward rates is as follows: Suppose a borrower approaches a bank for a t period loan and asks for alternative quotes e.g.(i) a single interest rate for all t periods, (ii) a rate for the first m periods ($m < t$) and a rate applicable from the m^{th} to t^{th} period, etc. To prevent the borrower from arbitraging at the cost of the bank it must give consistent quotes in accordance with (6), i.e. it must quote the m -period spot rate for the first m periods and the forward rate f_{mt} for the renewed loan after the m^{th} period and going up to the t^{th} period. From this interpretation it follows that forward rates contain no information whatsoever that is not already contained in the schedule of the spot rates. Hence forward rates simply cannot perform the roles that have been ascribed to them in the literature viz. as predictors of future spot rate, as predictors of future inflation rate, as predictors of the future stance of monetary policy, etc. The forward rates are simply arbitrage preventing rates, they do not contain any information that is not already contained in the spot interest rates from which they are derived.

9. Intermediation costs and targeted profits which are recovered by charging a differential between deposit and loan interest rates can also be determined. It is an easy matter to put down a formula for the required interest spread to recover the total intermediation charges E including wages, overheads and the size of expected profits. Then,

$$\sum_{t=1}^T (i_t + x)L_t - \sum_{t=0}^T (i_t - x) D_t = E$$

gives

$$x = \frac{E - \left(\sum_1^T i_t L_t - \sum_0^T i_t D_t \right)}{\sum_1^T L_t + \sum_0^T D_t}$$

so that the spread is $2x$. Thus, i_t serves as a mid-rate and i_t+x and i_t-x are the rates charged to borrowers and paid to depositors respectively.

VI Default Risk Premia

In order to determine the premia to be charged for default risk, banks must first classify their borrowers by their creditworthiness. Let $j=0\dots c$ denote the credit rating going from the best to the worst. Then it may be supposed that the probability of default increases with j . Also the probability of default d_{ij} can be supposed to be an increasing sequence for each t . Table (2) shows the life table of a loan belonging to a particular risk category j .

Table 2: Loan Life Table

Period	Balance	Default Probability	Default	Reserve Co-efficient
0	1	d_{1j}	d_{1j}	d_{1j}
1	$(1 - d_{1j})$	d_{2j}	$d_{2j}(1-d_{1j})$	$d_{1j} + d_{2j}(1-d_{1j})$
2	:	:	:	:
:	:	:	:	:
T	$\Pi(1 - d_{ij})$	d_{nj}	$d_n \Pi(1 - d_{ij})$	$1 - \Pi(1 - d_{ij})$

The logic that holds for withdrawals from deposits is now applied to the loans. If out of a dollar worth of loan only $\Pi(1 - d_{ij})$ is expected to be returned, banks will need to protect themselves by means of a loading on the interest rate charged to cover the default risk in such a way that the expected loan proceeds with default equals that without default. That is to say

$$L_{ij}(1 + i_{ij})^t [\Pi(1 - d_{ij})] = L_{ij}(1 + i_{t0})^t \quad j = 1 \dots c$$

where $j = 0$ implies the loans are default-risk free, e.g. sovereign debt. Then

$$(1 + i_{ij}) = (1 + i_{t0}) \left[\frac{1}{\Pi(1 - d_{ij})} \right]^{1/t} \quad \forall j = 1 \dots c$$

Denoting the second term of the right hand side by $(1 + \delta_t)$ the default risk premia are obtained from

$$i_{ij} - i_{t0} = (1 + i_{t0})(\delta_t) \quad \forall j$$

The structure of interest rates i_{ij} gives the spot yield curve for securities of asset quality j .

VII Ask and Bid Rates

Financial market activity consists of the competitive interactions between depositors and bankers and between bankers and borrowers all of whom will seek to lend and borrow on the best possible terms. In the process the market is required to coordinate four sets of interest rates, depositors' asking rates for the deposits they supply to banks, bankers' bid rates for deposits, bankers' asking rates for loans and borrowers' bid rates for loans. Bankers seek to maximize profits by seeking depositors requiring

the lowest asking rates and seeking borrowers who are willing to pay the highest bid rates for loans. For the sake of completeness some remarks on the determination of these four set of rates are in order.

Depositors' asking rates obviously depend on the probabilities of requiring cash. These probabilities would usually increase as time periods become longer, e.g. the probability of requiring cash in one day is likely to be lower than requiring cash during one month. Accordingly, depositors would seek compensation at successively higher rates for parting with liquidity for longer intervals of time; in other words the term structure of their asking rates will usually slope upwards. Bankers' bid rates for deposit would primarily depend on the probabilities of early (premature) encashment of deposits of various maturities. It is reasonable to expect that these probabilities would be decreasing as the maturity increases, e.g. the probability that a one-period deposit is withdrawn prematurely is likely to be greater than the probability that a five-period deposit is prematurely withdrawn. If so, bankers would be willing to pay the difference between the probabilities of premature encashment at successive maturities as an incentive to induce depositors to place deposits for longer maturities. The resulting term structure bid rates for deposits would be upward sloping. For the extreme case of a zero-probability-of-withdrawal perpetual deposit its bid rate will be equal to the banks' lending rate.

Borrowers' bid rates for loans are a bit more complicated and depend on the type of borrower. In the case of business firms, their bid rate for loans may be considered to be equal to their return on assets. Their bid rates for longer-term loans would depend on their fears/expectations of future increases in the short term rate whose effect would be to erode their return on net worth. To protect the return on net worth they would be willing to pay for insurance. In other words, borrowers would be prepared buy interest rate call options to protect their short positions in debt. And since the call option premium increases with the maturity of the option the term structure of bid rates will have an upward slope. The formula $i_{bt} = r + dc_t$ where d is the proportion of debt in total liabilities and c_t is the interest call option premium quoted in interest rate terms describes the term structure of bid rates. Non-business borrowers like government or households too would seek protection on similar lines. Bankers' asking rates for loans depend, as we have seen earlier, on the reserves they must provide to make longer term loans which depend on the probabilities of cumulative withdrawal from deposits. Since the reserve coefficients are increasing functions of the maturity, the term structure of asking rates too will show an upward slope.

Because all the four sets of rates that form the basis of financial negotiation in the market have upward sloping term structures, the term structure of the market interest rates is usually upward sloping., Perverse structures arise in rare circumstances when the maturity-wise supplies of deposits and the demands for loans are such that long-term deposits with the highest asking and bid rates have to be used to finance short-term loans that carry the lowest asking and bid rates.

VIII Numerical Example

This can be illustrated by a numerical example. Suppose the withdrawal probability matrix and the corresponding reserve coefficients matrix are as shown in Tables 3(a) and 3(b)

Table 3(a): Probabilities of Withdrawal

Loans	Deposits	1	2	3	4	5
1	0.05	0.04	0.035	0.03	0.025	0.02
2	0.06	0.05	0.04	0.035	0.025	0.02
3	0.07	0.055	0.045	0.04	0.025	0.02
4	0.08	0.06	0.05	0.045	0.03	0.02
5	0.085	0.07	0.055	0.045	0.03	0.025

Using the formula for r_{ts} in equation (2) we compute the cost of making a unit loan for maturity s from a deposit of maturity t .

Table 3(b): Reserve Coefficients

Deposits	1	2	3	4	5
Loans					
0	0.050	0.107	0.169	0.236	0.300
1	0.040	0.088	0.138	0.189	0.246
2	0.035	0.073	0.115	0.159	0.205
3	0.030	0.064	0.101	0.141	0.180
4	0.020	0.054	0.087	0.123	0.158

Consider a bank that has the following position of the supplies of deposits and demands for loans

Table 3(c): Supplies and Demands

	Deposits	Loans
0	500	-
1	250	300
2	100	350
3	400	400
4	600	500
5	100	500
Total	1950	1950

The reserves minimizing assignment of deposits to loans is

$$D_{01} = 300 = L_1$$

$$D_{02} + D_{12} = 200 + 150 = 350 = L_2$$

$$D_{13} + D_{23} + D_{33} = 100 + 100 + 200 = 400 = L_3$$

$$D_{34} + D_{44} = 200 + 200 = 400 = L_4$$

$$D_{45} + D_{55} + D_{33} = 400 + 100 = 500 = L_5$$

with a total reserve requirement of 205.

The corresponding interest rate system will be as follows:

$$R_1 + D_{01} = L_1(1+i_1)$$

$$R_2 + D_{02} + D_{12}(1+i_1) = L_2(1+i_2)^2$$

$$R_3 + D_{13}(1+i_1) + D_{23}(1+i_2)^2 + D_{33}(1+i_3)^3 = L_3(1+i_3)^3$$

$$R_4 + D_{34}(1+i_3)^3 + D_{44}(1+i_4)^4 = L_4(1+i_4)^4$$

$$R_5 + D_{45}(1+i_4)^4 + D_{55}(1+i_5)^5 = L_5(1+i_5)^5$$

The reserves are computed from the table for r_{ts} and the assignment D_{ts}

$$R_1 = r_{01} = (0.05)(300) = 15$$

$$R_2 = r_{02} + r_{12} = (0.107)(200) + (0.088)(150) = 21.4 + 13.20 = 34.6$$

$$R_3 = r_{13} + r_{23} + r_{33}, \text{ and so on}$$

We therefore have

$$15 + 300 = 300 (1+i_1)$$

$$34.6 + 200 + 150(1+i_1) = 350 (1+i_2)^2$$

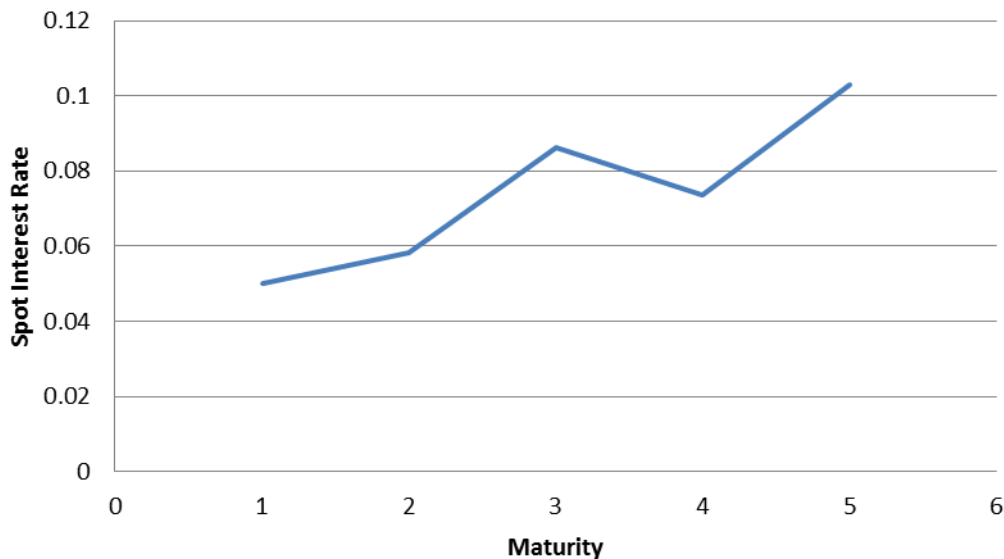
$$45.623 + 100 (1+i_1) + 100 (1+i_2)^2 + 200 (1+i_3)^3 = 400 (1+i_3)^3$$

$$48.55 + 200 (1+i_3)^3 + 200 (1+i_4)^4 = 200 (1+i_4)^4$$

$$61.23 + 400 (1+i_4)^4 + 100 (1+i_5)^5 = 500 (1+i_5)^5$$

These equations solve for the spot interest rate quotes $i_1 = 0.05$, $i_2 = 0.0584$, $i_3 = 0.0864$, $i_4 = 0.0737$, $i_5 = 0.103$

Graph 1: Term Structure of Spot Interest Rate



Observe how the graph shows a dip in the fourth maturity bucket because of the excess supply of deposits over loan demand in that bucket but a sharp crest in the fifth maturity bucket on account of the excess loan demand over the available supply in the bucket. But the same does not happen in the second bucket even though there is an excess demand for loans over deposits in that bucket. That is because of the loan demand of 350, 200 is met by zero cost demand deposits.

Using the same data for withdrawal probabilities and reserve coefficients it may be verified that the configuration of deposit supplies and loan demands shown in Table 4a will lead to a perverse downward sloping term structure of interest rates.

Table 4(a):

	Deposits	Loans
0	150	--
1	150	500
2	200	400
3	400	300
4	500	200
	1400	1400

The configuration shows short-term loan demand is strong in relation to deposit supplies but weakens as the term to maturity increases. This leads to an assignment of deposits to loans that is upper triangular – some portions of long term deposits are assigned to finance short-term loans. This also poses a small difficulty for the solution of the corresponding system of equation 5(a) viz. that, in contrast to the usual lower triangular matching, we will need to solve the interest rate of the longest maturity first and bootstrap backwards to the short-term rate. But the last equation will usually be a contradictory one; it will be of the form

$$R_t + D_{tt} (1+i_t)^t = L_t (1+i_t)^t$$

With $D_{tt} = L_t$, i_t becomes indeterminate. To get round this difficulty we shall suppose that $\sum L_t = \sum D_t + D_{t+1}$ and suppose that i_{t+1} is given exogenously with reference to say a bond issued n periods ago and has $n-(t+1)$ periods' remaining maturity. We will therefore extend Table 4(a) into Table 4(b)

Table 4(b):

	Deposits	Loans
0	150	--
1	150	600
2	200	400
3	400	300
4	500	200
5	100	--
	1500	1500

The assignment by the north-west corner rule will be as follows:

The reserves minimizing assignment of deposits to loans is

$$D_{01} = 150 \quad D_{11} = 150 \quad D_{21} = 200 \quad D_{31} = 100$$

$$D_{32} = 300 \quad D_{42} = 100$$

$$D_{43} = 300$$

$$D_{44} = 100 \quad D_{54} = 100$$

The reserves obtained by applying the reserve coefficients in Table 3(b) give.

$$R_1 = 7.5 + 6 + 7 + 2.5 = 23$$

$$R_2 = 19.2 + 5.4 = 24.6$$

$$R_3 = 26.1$$

$$R_4 = 12.3 + 10 \text{ (say, if } r_{54} = 0.1) = 22.3$$

The interest rate system is

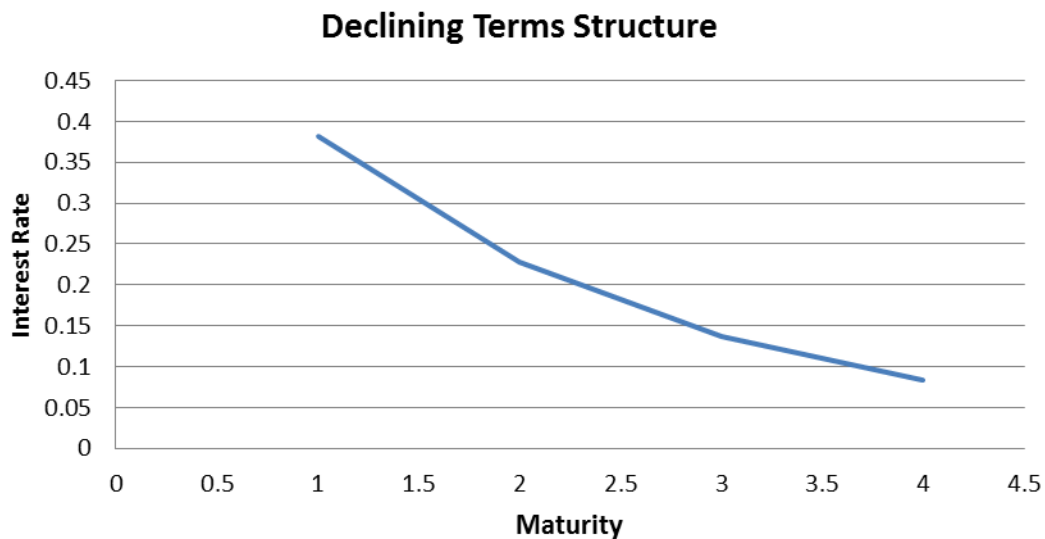
$$23 + 150 + 150 (I+i_1) + 200 (I+i_2)^2 + 100 (I+i_3)^3 = 600 (I+i_1)$$

$$24.6 + 300 (I+i_3) + (100) (I+i_4)^4 = 400 (I+i_2)^2$$

$$26.1 + 300 (I+i_4)^4 = 300 (I+i_3)^3$$

$$22.3 + 100 (I+i_4)^4 + (100) (I+i_5)^5 = 200 (I+i_4)^4$$

Graph 2:



If $i_5 = 0.03$ then the solution obtained for the interest rates is $i_1 = 0.3811$ $i_2 = 0.2281$ $i_3 = 0.1368$ $i_4 = 0.0843$ showing a declining term structure (see Graph No. 2). The configuration of deposit supplies and loan demands that cause term structures to decline would be observed in those rare economic episodes in which the demand for long term loans is weak, which might happen at the end of a long period of buildup of fixed assets by corporates whereafter their loan demand is largely for working capital requirements.

IX Monetary Policy

The effects of monetary policy on the structure of interest rates can be illustrated by considering the operation of the cash reserve ratio (CRR). The effect of the cash reserve ratio is simply to impound a proportion c of the total deposits of the bank so that the loanable funds stand reduced to $(1-c)D$ but reserves to meet withdrawals will continue to be provided on the total deposits. If the central bank does not pay any interest on the CRR funds, banks must recover that amount from the interest rates they charge in the market. The effect, is that the terms D_{ts} in equation (4) are replaced by $(1-c)D_{ts}$ and the terms $cD_{ts}[(1+i_t)^t - 1]$ representing interest foregone is added on to the left hand sides of the t^{th} equation.

Now suppose that banks in the numerical example of the previous section are subjected to cash reserve requirement of 10% of total deposits. Banks would now be

able to service only 90% of the loans in each maturity bucket. The system to determine interest rates would now be as follows:

$$15 + 270 + 30[(1+i_1) - 1] = 270 (1+i_1)$$

$$34.6 + 180 + 135(1+i_1) + 35[1+i_2]^2 - 1] = 315(1+i_2)^2 \text{ etc.}$$

with the solution $i_1 = 0.0625$, $i_2 = 0.07410$, $i_3 = 0.1355$, $i_4 = 0.1807$ and $i_5 = 0.1851$. Imposition of the CRR has not only raised the rates of interest, it has also raised the gradient of the term structure.

X Concluding Remarks

This paper has approached the subject of interest rates from a production perspective. From this perspective the activity of financial intermediation by banks is seen not merely as one of providing a forum to facilitate transactions between ultimate borrowers and ultimate lenders on the same footing as securities brokers working in exchanges. Instead, financial intermediation is accompanied by the activity of producing longer maturity instruments from shorter maturity ones in the course of which the banks are required to perform the task of asset liability matching while accepting substantial risks on their own account. The most significant result of this input-output approach is to yield a model that determines the term structure of interest rates from which the level of the interest rate can be extracted.

Some striking resemblances between the input-output approach and the extant theories may be mentioned. First, there is the direct connection of the probabilities of withdrawal with the liquidity preference hypothesis. Second, there is the explicit influence of the demands and supplies of loanable funds within each maturity bucket and in the aggregate on the interest rates. Third, there is the degree of substitutability of deposits of various maturities to finance loans of various maturities which conditions the slope of the term structure and has some resemblance with the preferred habitats theory of the term structure. Fourth, there is something like the expectations hypothesis at work due to which expectations or even the possibility of future increases in the interest rate creates a demand for insurance against unfavourable increases in interest rates and causes the term structure of bid rates to rise. Fifth, the emphasis that existing theories place on the state of expectations as an active determinant of the observed term structure of interest rates is also present in the theory advanced above. Expectations regarding the future course of interest rates can affect the rates currently quoted in the market in two ways; by changing the maturity composition of the deposit supplies and loan demand and/or by changing the incentive structure that bankers offer to their depositors/borrowers. If the market as a whole expects interest rates to rise in future depositors and borrowers will react in opposite ways; depositors will shift their portfolios towards short-term deposits (maturing long-term deposits will not be renewed and longer-term bonds will be sold and the proceeds placed in short-term deposits) but borrowers would want to lock in loans of longer maturities at the going low rates and/or to issue long term bonds. The effect of these actions will be to raise the gradient of the term structure of interest rates. On the other hand, if the market as a whole expects interest rates to fall depositors will want to lock in the current high rates for longer periods of time and/or shift their portfolios in favour of longer maturity bonds while borrowers would like to unwind their long term loans and shift to short-term loans. The term structure flattens out as a result.

In closing this paper it would be appropriate to reproduce a line from Keynes (1936), Chapter 14 where he states that, “The mistake originates from regarding interest as the reward for waiting as such, instead of as the reward for not hoarding; just as the rates of return on loans on investments involving different degrees of risk, are quite properly regarded as the reward, not of waiting as such, but of running the risk. These is, in truth, no sharp line between these and the so-called “pure” rate of interest, all of them being the reward of running the risk of uncertainty of one kind or another”.

But “rates of return on loans on investments involving different risks” would mean loans carrying different risks of default, is probabilities of non-recovery of interest and principal. That these risks are priced, or loaded in the form of default risk premia in order to obtain commensurate reward. Keynes is telling us that the so-called “pure rate of interest” should be considered to be on the same footing as those risks, viz. as a reward for running a risk. In this paper this risk is identified as the risk of illiquidity as measured by the probability of requiring cash to meet withdrawals from deposits.

References

- Estrella, Arturo (1997), *Why Do Interest Rates Predict Macro Outcomes? A Unified Theory of Inflation, Output, Interest and Policy*, New York, Federal Reserve Bank of New York.
- Fama, Eugene (1984), The Information in the Term Structure, *Journal of Financial Economics*, 13: 509-526.
- Fama, Eugene and Robert Bliss (1987), The Information in Long-Maturity Forward Rates, *American Economic Review*, 77: 680-692.
- Fisher, I. (1930), *Theory of Interest*, New York: The Macmillan Co.
- Hahn, F.H. and F.B.R. Brechling (1965), *The of Interest Rates*, London: Macmillan.
- Keynes, J.M. (1936), *The General Theory of Employment and Money*, London: Macmillan.
- Leontief, W. (1941), *Studies in the Structure of the American Economy*, New Yell: John Willey and Sons.
- Mitra, S.K. and S.R. Mohan (1988), *On the Optimality of North West Corner Solution in Some Application of the Transportation Theory*, Technical Report No. 8801, Indian Statistical Institute, Delhi.
- Rogers, C. (1989), *Money, Interest and Capital: A Study in Monetary Theory*, Cambridge: Cambridge University Press.
- Shiller, R.J., J.Y. Campbell and Schoenhottz (1983), Forward Rates and Future Policy: Interpreting the Term Structure of Interest Rates, *Brookings Papers on Economic Activity*, 1: 73-217.
- Solow, Robert (1952), On the Structure of Linear Models, *Econometrica*, 20(1): 29-46.
- Stiglitz, Joseph E. (1968), *Theory of Liquidity Preferences and the Term Structure of Interest Rates*, New Haven Cowles Foundation for Research in Economics, Yale University.