

The
Pure
Theory
of
Value



RAJAS PARCHURE

TRF

THE TIMES RESEARCH FOUNDATION

This monograph is an investigation into general equilibrium theory on the lines initiated by Piero Sraffa in 1960. A central theme of the monograph is to show how the supply-demand theory of price determination can be made consistent with the theories of capital and economic growth. In so doing, the analysis avoids the fallacies of the neoclassical versions by treating capital as the value of the produced means of production and, correspondingly, profit as rate earned on the value of capital rather than as a 'factor price'.

At the same time the analysis seeks to dissolve the artificial division of economic theory into microeconomics and macroeconomics. The book should therefore be of considerable interest to all economists.

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THE TIMES RESEARCH FOUNDATION

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FOREWORD

The subject of economic theory in the present times seems to be splitting into fragments under the pressure of irreconcilable controversy. There are to be found a plethora of contending 'schools' — neo-classical, Marxian, Keynesian, Monetarist, neo-Ricardian, neo-Keynesian, and what not — each backed with its own models, empirical evidence and policy prescriptions.

While there can be no doubt that some degree of disagreement amongst the working scholars is inevitable and necessary for the progress of ideas in all sciences, the divergence of opinions on economic matters have reached the proportions of a 'crisis' in economic thought. What was once, perhaps, a healthy joke — where there are two economists there are three opinions — is now symptomatic of a crisis. But, what is most agonising about a crisis in economic theory is that it leads, directly or indirectly, to a crisis in practical economic policy worsening in the process the daily lives of the people.

A need has therefore been felt for a synthetic view of the central theoretical issues of economics. Such a synthesis, even if it suffices the purpose of providing a point of reference in the debate of contending opinions on matters of economic policy should, in my opinion, prove immensely useful. I would urge the readers to see this book as a beginning in that direction.

Ram S. Tarneja
Trustee.

PREFACE

This essay is primarily addressed to economic theorists. I hope that it will prove useful to economists engaged in the areas of economic planning and policy-making. For the essay is an investigation into the processes by which prices, outputs, income - distribution, growth and cyclical fluctuations are *simultaneously* determined in a competitive capitalist economy. It's principal aim is synthesize the separate and often incompatible, theories of prices and distribution, and of national income and growth, into a single unified theory. The outcome of this unification should be, provided my argument is itself valid, a parallel unification of the approaches to microeconomics and macroeconomic policies.

My greatest debt is to Piero Sraffa's 1960 classic, *Production of Commodities by Means of Commodities*. Indeed, in my own mind, this monograph is only a report of my attempts to understand Sraffa's system in the light of my understanding of economic reality.

Grateful thanks are due to my teachers

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I am deeply grateful to my parents, my wife and my children who have stretched beyond the reasonable limits of familistic affection to support this work. This book is dedicated to them.

R. K. P.

Pune,
1st May, 1989

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PART. ONE

WORKING CAPITAL

CHAPTER I

ONE COMMODITY

- 1. A non-capitalist all consuming economy.**
- 2. Profits saved and wages consumed.**
- 3. Profits partly consumed.**
- 4. Wages partly saved.**

CHAPTER I

ONE COMMODITY

1. Consider an economy which produces just one commodity. Suppose that the production activity can be described by the following relation

$$A + L \longrightarrow X$$

where A is the input, L is the quantity of homogenous labour used and X is output of the commodity. If we suppose the economy to be non-capitalist and suppose that the workers consume the physical surplus $X - A$ entirely, then the level of consumption per worker is,

$$\frac{X - A}{L} = \frac{F}{L}$$

Such an economy will continue indefinitely at the same scale of activity. Nothing more can be said about this economy.

2. If production takes place under capitalist conditions with workers being paid a wage, the production relation above can be expressed as a

price equation,

$$Ap(1+r) + wL = pX$$

where p is the absolute price of the commodity, w is the absolute wage rate and r is the rate of profit. If we divide through by p , we get

$$A(1+r) + w^*L = X \quad [1]$$

where w^* is the real wage.

With the scale of activity given, equation (1) contains two unknowns, viz. the wage and the rate of profit. It is clearly impossible to solve for both without the aid of additional premises.

A premise that readily comes to mind is to regard the wage to be predetermined, for example, at a physiologically (or socially) determined level of subsistence consumption, *i.e.*

$$w^* = w_s \quad [2]$$

Then, equation (1) solves for the rate of profit.

If it is also assumed that the profit earners do not consume at all, it will follow that the rate of growth of the economy is identical with the rate of profit. To see this, suppose \bar{X} to be the level of

output net of consumption, i.e.

$$\bar{X} = X - wL$$

Then, the rate of growth is equal to,

$$g = \frac{\bar{X} - A}{A} \quad [3]$$

which is precisely the rate of profit solved in equation (1).

3. On the other hand if we allow a portion α of the profits to be consumed, whilst retaining the supposition that the wages are entirely consumed, we get,

$$g = \frac{\bar{X} - A - \alpha rA}{A} = r(1 - \alpha) \quad [4]$$

which is the well-known "Cambridge equation" due to Kaldor (1956), Robinson (1956), Kahn (1959) and Pasinetti (1962).

Equation 4, like equation 3, can also be read in another important way, *viz.*

$$g = \frac{(1 - \alpha)rA/X - A}{A/X - A} = \frac{s}{k} = sR \quad [5]$$

where s is the proportion of saving in national

income, $X - A$, k is the capital to output ratio and R , its inverse, is the maximum rate of profit for the economy, *i.e.* the rate which corresponds to a zero wage. Equation (5) is the famous Harrod-Domar growth equation due to Harrod (1939) and Domar (1946).

4. If we generalise the situation to allow the wage-earners to own a portion of the capital stock and save a portion $(1 - \beta)$ of the aggregate income (*i.e.* wages plus share of profits), then the rate of growth of the economy will be,

$$g = \frac{(1 - \alpha)(1 - \tau)rA + (1 - \beta)[w^*L + \tau rA]}{A} \quad [6]$$

where the numerator is the sum of the annual savings by the capitalists and the workers and the denominator is the initial capital stock.

Now, it is clear that the shares in the ownership of the capital stock of each class depend directly on their respective shares in saving, *i.e.*

$$\frac{\tau}{1 - \tau} = \frac{(1 - \beta)[w^*L + \tau rA]}{(1 - \alpha)(1 - \tau)rA}$$

implying that,

$$\tau = \frac{(1 - \beta)w^*L}{(\beta - \alpha)rA} \quad [7]$$

If we substitute for τ in equation (6) we obtain the Pasinetti (1962) result that,

$$g = r(1 - \alpha) \quad [8]$$

irrespective of the fact of saving by workers.

To solve for the unknowns we may begin with a predetermined wage, w_s , (which, however, loses, in the present case, its 'subsistence' connotation) and solve for r from equation (1), τ from equation (7) and g from equation (8).

We may also begin with a known τ , and solve for r and w^* from equations (2) and (7), and solve for g from equation (8).

Or, alternatively, with Pasinetti (1962), we may consider g to be determined exogenously by the rate of growth of population and determine, r , w^* and τ from equations (8), (1) and (7) respectively. ⁽³⁾

So much for macroeconomics. In the remaining chapters the framework will be complicated by the existence of relative commodity prices, apart from the wage, which will

have to be determined at the same time as the wage and the rate of profit.

NOTES

1. The simple Keynesian model of income determination does not distinguish between wage and profit income. It postulates that a proportion, μ , of the net product, F , is spent on consumption and solves for the quantum of the net product as,

$$F = \frac{\Delta A}{(1 - \mu)}$$

where ΔA , the investment, is the addition to the capital stock A . The corresponding gross product would be,

$$X = \frac{\Delta A}{(1 - a)(1 - \mu)}$$

where $a = A/X$. This analysis holds only if ΔA is regarded as 'autonomous' and not necessarily equal to saving which, of course, is impossible.

2. It will be noticed that in a one-commodity world an increase in α , i.e. the capitalists' propensity to consume lowers the rate of growth but leaves the rate of profit

unaffected. Thus, the capitalists can at their will increase α until the growth rate is zero whilst keeping intact their profits. But they cannot reduce the share of wages to zero as has been argued by Tobin (1959-60) in his criticism of Kaldor's (1956) theory.

3. In all cases, though, the positive solution for the unknowns will exist only when β exceeds α .

CHAPTER II

TWO COMMODITIES

- 5. A “missing” equation.**
- 6. The consuming economy**
- 7. Conditions for the existence of a unique, positive solution.**
- 8. The growing economy : saving from profits.**
- 9. Decomposable economy : a special case.**
- 10. The growing economy : saving from profits and wages.**
- 11. Qualifying remarks.**

CHAPTER II

TWO COMMODITIES

5. Consider now an economic system which produces two commodities. And suppose that the production activities of the industries producing them can be described by the following relations,

$$A_{11} + A_{21} + L_1 \longrightarrow X_1$$

$$A_{12} + A_{22} + L_2 \longrightarrow X_2$$

where the A_{ji} represent the quantity of commodity j required to produce X_i units of commodity i , and L_i represent the quantity of labour required. The price equations of the two commodities are,

$$(A_{11}p_1 + A_{21}p_2)(1+r) + wL_1 = p_1X_1 \quad [1]$$

$$(A_{12}p_1 + A_{22}p_2)(1+r) + wL_2 = p_2X_2$$

where p_1 and p_2 are the absolute prices of commodities 1 and 2.

If we assume the levels of outputs and inputs to be given, the system of equations (1) is an

indeterminate system of simultaneous *non-linear* equations containing two independent equations in four unknowns *viz.* the two prices, the wage and the rate of profit. We can eliminate one of the unknowns by fixing either one of the prices equal to unity (Walras, 1874), or fixing the absolute wage rate equal to unity (Keynes, 1936). There will then remain two equations in three unknowns.

Assuming that the price of commodity 1 has been set equal to unity ⁽¹⁾ and dividing the two equations by p_1 , we get,

$$\begin{aligned} (A_{11} + A_{21}p)(1 + r) + w^*L_1 &= X_1 \\ (A_{12} + A_{22}p)(1 + r) + w^*L_1 &= pX_2 \end{aligned} \quad [2]$$

where $p = p_2 / p_1$ and $w / p_1 = w^*$

The system of equations (2) determines a wage-profit frontier which depicts the possibilities of distributing the net national product between the wage-earners and profit-earners (Sraffa, 1960). But the equations do not determine the point on the wage-profit frontier at which the economy will actually be in equilibrium. Clearly, there is a "missing equation" which, when used along with equation (2), would

enable us to determine the wage rate, the rate of profit and the relative price of commodity 2.

6. The required equation can be filled in by considering the composition of commodities which the respective income recipients *desire* to purchase, *i.e.* the demand equations for the two commodities. Thus, suppose that the profit-earners spend a portion α and the wage-earners spend a portion β of their respective incomes on commodity 1. We then have an independent equation involving the three unknowns in equation (2), *viz.*

$$\alpha rK + \beta w^*L = F_1 \quad (i)$$

where, $K = (A_{11} + A_{12}) + (A_{21} + A_{22})p$ is the capital stock, $L = L_1 + L_2$ is the annual labour and $F_1 = X_1 - A_{11} - A_{12}$ is the physical surplus of commodity 1 over and above its use as means of production.

Since the economy is one which consumes its entire net national product the alternative, linearly dependent equation that can be used to determine the system and obtain the same result is,

$$(1 - \alpha)rK + (1 - \beta)w^*L = pF_2 \quad (ii)$$

where $F_2 = X_2 - A_{21} - A_{22}$. This is because specifying

the one specifies the other automatically since the sum of the two equations leads to the product — income identity,

$$rK + w^*L = F_1 + pF_2$$

Equation (2) along with equation (i) gives the following system of three independent non-linear equations in three unknowns,

$$\begin{aligned} (A_{11} + A_{21}p)(1+r) + w^*L_1 &= X_1 \\ (A_{12} + A_{22}p)(1+r) + w^*L_2 &= pX_2 \\ \alpha rK + \beta w^*L &= F_1 \end{aligned} \quad [3]$$

These can be written in vector-matrix notation as a system of homogeneous equations,

$$\begin{bmatrix} X_1 - A_{11}(1+r) - A_{21}(1+r) - L_1 \\ -A_{12}(1+r) - A_{22}(1+r) - L_2 \\ F_1 - \alpha A_1 r \quad -\alpha A_2 r \quad -\beta L \end{bmatrix} \begin{bmatrix} 1 \\ p \\ w^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $A_1 = A_{11} + A_{12}$ and $A_2 = A_{21} + A_{22}$

A unique non-trivial solution of this system exists if and only if the determinant of the matrix of coefficients is equal to zero. Setting the determinant of the matrix of coefficients equal to zero gives a quadratic polynomial equation in r .

The lowest positive root of the polynomial is the relevant solution. When this is substituted in the price equations, the solutions for the relative price and the wage is obtained.

7. We are naturally interested also in the existence of positive solutions for the price, wage and the rate of profit. One of the necessary conditions for a positive solution to exist is that the industries should produce physical surpluses of both commodities, *i.e.* $F_1 > 0, F_2 > 0$.

Further, it is necessary, in the context of a two-commodity, all consuming economy that the propensities to consume of capitalists, α and of workers β , be unequal. For, if α were to equal β , the demand equations (i) and (ii) would by themselves determine the relative price p which, in general, would not correspond to that consistent with the price equations and lead to a contradiction.

If these conditions are met, the lowest positive rate of profit can be substituted in equations (2) to obtain unique positive solutions for the wage rate and the relative price.

Finally, negative solutions may also be found when the *postulated* levels of output of the two

industries stand in proportions that are very different from the commodity *postulated* to be desired by the consumers. Such situations reflect as it were a lopsidedness in production and consumption plans and are inadmissible in any actual economic system. In all cases, however, the situation can be remedied by changing the levels of activity in the industries in the direction determined by the propensities to consume and obtain a positive solution, *i.e.* by increasing the level of activity of the industry whose product is consumed more intensively and reducing that of the other by simply multiplying each price equation by scalars greater or lesser than unity, respectively.

8. Next consider an economy which saves a part of the income and invests its year after year. To consider the simpler case first, suppose that the wage income is entirely spent for final consumption but that the capitalists save a part of their income, $(1 - \alpha)$. Let α_1 and α_2 be the proportions of profit income spent on commodities 1 and 2, $(\alpha = \alpha_1 + \alpha_2)$, and let β and $(1 - \beta)$ be the proportions of wage incomes spent on the two commodities respectively.

The net physical product of commodity 1 available for final uses (consumption or investment) is F_1 . Of this, a portion βw^*L is consumed by workers and another, $\alpha_1 rK$ is consumed by capitalists. Thus, the quantity available for investment is,

$$F_1 - \beta w^*L - \alpha_1 rK$$

To obtain the demand equation for commodity 1 we shall reason as follows. The total saving available for investment is $(1 - \alpha)rK$. This amount must be allocated between the two industries in proportion to the capital stocks existing in them. Thus, if K_1 and K_2 represent the industry capital stocks, the additions to capital stock in each industry will be,

$$\left(\frac{K_1}{K}\right)(1 - \alpha)rK = (1 - \alpha)rK_1 = r(1 - \alpha)(A_{11} + A_{21}p)$$

$$\left(\frac{K_2}{K}\right)(1 - \alpha)rK = (1 - \alpha)rK_2 = r(1 - \alpha)(A_{12} + A_{22}p)$$

(iii)

These industry-wise allocations will have to be further reapportioned to obtain the investment demands for the individual commodities 1 and 2 which constitute the capital stock of each

industry. These secondary allocations are made in proportion to the *values* in which the commodities are represented in the capital of each industry. Thus the investment demand for commodity 1 by industry 1 is,

$$\left(\frac{A_{11}}{K_1}\right)(1-\alpha)rK_1 = r(1-\alpha)A_{11} = \Delta A_{11}$$

Similarly, the investment demand for commodity 1 by industry 2 is,

$$\left(\frac{A_{12}}{K_2}\right)(1-\alpha)rK_2 = r(1-\alpha)A_{12} = \Delta A_{12}$$

The sum of these investment demands must be equated to the available supply so that,

$$\Delta A_1 = r(1-\alpha)A_1 = F_1 - \alpha_1 rK - \beta w^* L \quad (\text{iv})$$

Equation (iv) along with the two price equations in equations (2) will represent a homogeneous system of three equations in three unknowns. ⁽²⁾ The characteristic polynomial of the coefficients will solve for the rate of profit in equilibrium which, as before, can be substituted in equations (2) to obtain the solutions for the wage and the price.

The relationship of the rate of profit and the

rate of growth of the economy is visible from the capital allocation equation (iv) itself,

$$\frac{\Delta A_1}{A_1} = g = r(1 - \alpha)$$

This relationship is the "Cambridge equation", (Robinson, 1956, Pasinetti, 1962). The classical special case when $\alpha = 0$, gives $g = r$, the equilibrium equality of the rate of growth and the rate of profit.

Further, it will be observed that the *prices* will be such as to ensure that the rate of growth is equal to the ratio of the propensity to save out of national income and the capital to output ratio. (Domar 1946, Harrod 1939)

9. If commodity 1 were a pure capital good while commodity 2 served purely as the means for final consumption, *i.e.* $A_{21} = A_{22} = 0$, we have a special case in which the rate of growth is determined directly by the ratio of the physical surplus of commodity 1, F_1 to its aggregate use in the economy's capital stock, A_1 . The rate of profit is obtained by dividing the rate of growth by the capitalists' propensity to save, $(1 - \alpha)$.

There is, however, a restriction on the

of α if positive solutions for the unknowns are to obtain, viz. that the value of α should be less than $(R - g) / R$, the ratio of the consumed rate of surplus to the rate of total surplus. ⁽³⁾

We are under no necessity to specify this restriction in the general case for, unlike in this special case, the rate of growth itself declines with every increase in the capitalists' propensity to consume and raises the limiting ratio above.

10. Finally, we consider the general case where both workers and capitalists save a part of their incomes. Thus, let β_1 and β_2 be the workers' propensities to consume commodities 1 and 2 respectively and let $(1 - \beta)$ be their propensity to save ($\beta = \beta_1 + \beta_2$). Further, letting τ be the proportion of capital stock owned by workers and $(1 - \tau)$ that owned by the capitalists, the annual incomes of the workers and capitalists are,

$$Y_w = w * L + \tau r K$$

$$Y_c = (1 - \tau) r K$$

The total saving available for new investment every year is,

$$S = (1 - \beta) Y_w + (1 - \alpha) Y_c$$

$$= (1 - \beta) [w * L + \tau r K] + (1 - \alpha) (1 - \tau) r K$$

This amount must be allocated in each industry in proportion to the industry capitals, so that,

$$I_1 = \left(\frac{K_1}{K}\right)S \quad \text{and} \quad I_2 = \left(\frac{K_2}{K}\right)S$$

represent the addition to the industry capitals. Further, we reallocate these to obtain the investment demands for commodity 1 by the respective industries,

$$\begin{aligned} \Delta A_{11} = \left(\frac{A_{11}}{K_1}\right)I_1 &= r [(1-\alpha)(1-\tau) + \tau(1-\beta)]A_{11} \\ &+ \left(\frac{A_{11}}{K}\right)(1-\beta)w^*L \end{aligned} \quad (v)$$

$$\begin{aligned} \Delta A_{12} = \left(\frac{A_{12}}{K_1}\right)I_1 &= r [(1-\alpha)(1-\tau) + \tau(1-\beta)]A_{12} \\ &+ \left(\frac{A_{12}}{K}\right)(1-\beta)w^*L \end{aligned}$$

Summing these gives the investment demand for commodity 1. On the other hand the supply of commodity 1 that is available for the purpose of new investment is,

$$F_1 - \beta_1(w^*L + \tau K) - \alpha_1(1-\tau)rK$$

Equating the investment demand for commodity 1 to its available supply gives,

$$\begin{aligned} r[(1-\alpha)(1-\tau)+\tau(1-\beta)]A_1+\frac{A_1}{K}(1-\beta)w^*L \\ = F_1-\beta_1(w^*L+\tau K)-\alpha_1(1-\tau)rK \end{aligned} \quad (\text{vi})$$

This equation, independent though it is of the two price equations, is not in a form that is amenable for obtaining the solution. This is because of the presence of K , an unknown, in the denominator of the second-term on the left-hand-side of the equation. Its presence makes it impossible to obtain a characteristic polynomial in r alone.

In order to eliminate K from the denominator we argue as follows. Consider the relation between the shares of the ownership in capital stock of the two classes to their respective savings. Clearly, the relative shares in ownership is determined by their relative proportions in saving, *i.e.*

$$\frac{\tau}{1-\tau} = \frac{(1-\beta)[w^*L+\tau K]}{(1-\alpha)(1-\tau)rK}$$

which gives,

$$\frac{w^*L}{K} = \frac{\tau(\beta - \alpha)r}{(1 - \beta)} \quad [4]$$

Substituting this in equation (vi) enables us to rewrite it as follows,

$$F_1 - [(1 - \alpha) + \Phi]A_1r - \Phi A_2pr - \beta_1 w^*L = 0$$

where $\Phi = \beta_1\tau + \alpha_1(1 - \tau)$. This equation along with the price equations (2) will give the solution for the system.

Substituting for w^*L/K from equation (4) into equation (vi) gives us the relationship between the rate of growth and the rate of profit in equilibrium. Evidently, this remains of the same type as in the case in which workers do not save. (Pasinetti, 1962).

11. In all cases, the positive solution will be found, as is seen from equation (4), only when β exceeds α (Kaldor 1956, Pasinetti 1962).

But a positive solution will not be found to exist for all values of τ as the macroeconomic versions seem to suggest. In most cases there will be a minimum value of τ such that the positive solutions correspond only to values higher than it.

Similarly, as in the earlier case of saving

exclusively from profits, the prices will be such as to determine a capital-output ratio that satisfies the Harrod-Domar growth equation. This conclusion, proved as it is irrespective of any flexibility in the techniques of production, makes it unnecessary to consider a flexible 'real' capital to 'real' output ratio defined independently of prices as has, for example, been used by Meade (1963) and Samuelson and Modigliani (1966) with the express purpose of providing a 'neo-classical' solution to this problem.

NOTES

1. Obviously the analysis is not affected by the choice of the *numeraire*. Although considering the wage as *numeraire* has greater expository advantage, enabling as it does the measurement of all prices and values in units of "labour commanded" we shall, in deference to the Walrasian tradition, consider the price of a commodity as *numeraire*.
2. The linearly dependent equation which can be used as the alternative to equation (iv) to yield the same result is,

$$p \Delta A_2 = r(1 - \alpha)pA_2 = pF_2 - \alpha_2 rK - (1 - \beta)w^*L$$

for, the sum of these equations give the product-income identity,

$$rK + w^*L = F_1 + pF_2$$

3. Tobin's (1959-60) criticism of Kaldor's (1956) theory is found to hold in this special case alone—indeed, the capitalists can simply increase their propensity to consume until the rate of profit rise upto R , the maximum and reduce the wage to zero.

CHAPTER III

SEVERAL COMMODITIES

12. Difficulty in the way of generalising to n commodities.
13. The system of equations :
decomposable economy.
14. Procedure for finding the unique
equilibrium solution.
15. Description of the economy in
disequilibrium; the determination
of spot prices.
16. Adjustments with 'sticky' prices.
17. Differences between the equilibrium
system and Sraffa's standard system.
18. Equilibrium solution for a non-decom-
posable economy.
—The Leontief system distinguished
19. Secular dynamics

CHAPTER III

SEVERAL COMMODITIES

12. Any attempt at a straightforward generalisation from the case of two commodities to the case of n commodities is seen to be frustrated due to the problem of overdeterminacy which creeps into the system of equations. For, when we consider n commodities, there are n price equations and $n - 1$ independent demand equations with which to determine, following strictly the reasoning in the two-commodity case, $n + 1$ unknowns after fixing one *numeraire*. There will therefore be $(2n - 1) - (n + 1) = n - 2$ excess equations. And it is only when $n = 2$, as in the special case considered above, that there are no excess equations. In the general case, however, every additional commodity adds *two* equations to the system, *viz.* its price equation and its demand equation, whilst adding but *one* unknown, *viz.* its price relative to the *numeraire*.

This seems at first sight to be a self-contradictory result. We shall, however, see that

in this general case it will be possible to determine not only the relative prices and the distributive variables but also the levels of output of the commodities.

13. Since the argument is long it will be easier to go case by case. At this stage suppose that the system of industries is decomposable, *i.e.* of the n commodities, m are purely intermediate capital goods whilst $n - m$ are purely final consumption goods. These latter are not used directly or indirectly as inputs in the production of any commodity.

We shall also assume throughout that there is at least one 'basic' commodity in the system, *i.e.* a commodity which is used directly or indirectly in the production of *all* commodities.

One point deserves to be mentioned in this context *viz.* that in a system where some commodities are purely capital goods, it is impossible to have an economy that consumes entirely its net national product. Because, if such a system is to exist, there must be physical surpluses in the capital goods industries. ,

And, to finance the purchase of these surpluses there must be saving. Thus, a positive

rate of saving and a positive rate of growth are necessary for the existence of a positive rate of profit.

When the commodities have been split into the two sub-groups of intermediate and final goods, the system of price equations can be represented as follows,

$$(A_{11}p_1 + A_{21}p_2 + \dots + A_{m1}p_m)(1+r) + wL_1 = p_1X_1$$

$$(A_{12}p_1 + A_{22}p_2 + \dots + A_{m2}p_m)(1+r) + wL_2 = p_2X_2$$

..... [1]

$$(A_{1m}p_1 + A_{2m}p_2 + \dots + A_{mm}p_m)(1+r) + wL_m = p_mX_m$$

$$(A_{1,m+1}p_1 + A_{2,m+1}p_2 + \dots + A_{m,m+1}p_m)(1+r) + wL_{m+1} = p_{m+1}X_{m+1}$$

.....

$$(A_{1n}p_1 + A_{2n}p_2 + \dots + A_{nn}p_n)(1+r) + wL_n = p_nX_n$$

There will, in addition, be m investment demand equations for the m capital goods and $(n - m)$ consumption demand equations for the $(n - m)$ consumption goods. The demand equations for the consumption goods are as follows,

$$\alpha_r K + \beta_j W L = p_j X_j \quad j=m+1, \dots, n \quad [2]$$

where,

$$K = \sum_{i=1}^n \sum_{j=1}^m A_{ji} p_j$$

$$\text{and } L = \sum_{j=1}^n L_j$$

The demand equations for the capital goods need to be constructed. Assuming, for the sake of simplicity that workers do not save, the saving available every year from the profit income is $(1 - \alpha)rK$. This saving will be used to purchase the physical surpluses of the m capital goods. Thus the following identity will at all times hold,

$$(1 - \alpha)rK = \sum_{j=1}^m p_j F_j$$

$$\text{where } F_j = X_j - \sum_{i=1}^n A_{ji} \quad j = 1, \dots, m$$

In order to obtain independent equations however, the investment demands for the m individual capital goods must be derived and these must be equated to the physical surpluses of the

individual goods available. Now, the demand for each capital good by each industry depends on the commodity composition of the capital stock of that industry. Thus, the demand by industry i for capital good j is,

$$\Delta A_{ji} = r(1 - \alpha)A_{ji} \quad j = 1, \dots, m, \quad i = 1, \dots, n$$

And the aggregate investment demand for each capital good j by all the n industries will be given as,

$$\Delta A_j = \sum_{i=1}^n A_{ji} = r(1 - \alpha)A_j \quad j = 1, \dots, m$$

The physical surpluses that are available being F_j , there are m equations that equate the demand for capital goods to their supplies. These are,

$$r(1 - \alpha)A_j = F_j \quad j = 1, \dots, m \quad [3]$$

We thus have a total of n demand equations in (2) and (3).

Now, any one of the m equations in (3) will, when used along with the system of price equations yield a solution, so that there are m possible solutions for the relative prices, the wage, and the rate of profit.

14. The multiplicity of such solutions only reflects the fact that the system of equations as set out in (1), (2) and (3), is in a state of disequilibrium. Our objective is to locate a unique solution that is consistent with the requirements of overall economic equilibrium. That such a solution will usually exist may be surmised on the ground that the demand equations (2) and (3) are themselves made up of the same constituents (the A_{ji} and the L_j) as are the price equations themselves. It should therefore be possible to determine a unique solution by changing appropriately the scales of activity in the various industries in accordance with the demands for the various commodities.

A clear reflection of this fact is available if we study the sub-system of the capital goods industries in which the primary balance of demand and supply can be determined and whose results can subsequently be substituted into the sub-system of the consumption goods industries. In the sub-system of the capital goods there are m price equations for the m capital goods industries and m demand equations for these goods. Any one of the demand equations will, when used along with the m price equations, yield a solution for

the rate of profit, the wage rate and the $(m - 1)$ relative prices. (We assume, as in the earlier chapter, commodity 1 to be the numeraire so that $p_1 = 1$ and $w^* = w/p_1$). There will then be m possible solutions,

$$r_j = \frac{1}{1 - \alpha} \left[\frac{F_j}{A_j} \right] \quad j=1, \dots, m$$

with corresponding solutions for the wage and the relative prices.

The cause of the multiplicity of solutions has become visible now; it is the direct result of the fact that at the initially postulated scales of activity of the industries the ratios of the physical surpluses of each capital good available (F_j) to the total use made of the good in the economy (A_j) are unequal to one another. Our immediate task therefore is to determine those scales of activity (or outputs) of the capital goods industries such as to change the various F_j 's and A_j 's to yield a uniform ratio for all the m capital goods.

The formal problem is one of finding scale multipliers $\lambda_1, \dots, \lambda_m$ which when multiplied to the price equations of the capital goods industries will result in the industries' producing physical

surpluses of such magnitudes, and using such quantities of the various capital goods as will equate all the F_j / A_j . To find them we shall use Sraffa's (1960) procedure for constructing the *standard system*. Thus, we set up the following equations,

$$(A_{11} \lambda_1 + A_{12} \lambda_2 + \dots + A_{1m} \lambda_m + \Lambda_1)(1+g) = \lambda_1 X_1$$

$$(A_{21} \lambda_1 + A_{22} \lambda_2 + \dots + A_{2m} \lambda_m + \Lambda_2)(1+g) = \lambda_2 X_2$$

$$\dots \dots \dots [4(a)]$$

$$(A_{m1} \lambda_1 + A_{m2} \lambda_2 + \dots + A_{mm} \lambda_m + \Lambda_m)(1+g) = \lambda_m X_m$$

where, g , is the rate of growth, and,

$$A_j = \sum_{i=m+1}^n A_{ji} \quad j = 1, \dots, m$$

where A_j represents the aggregate use made of the capital good j in the industries producing the final consumption goods. Equations 4(a) represents a system of m homogeneous equations in $m+1$ unknowns *i.e.* the m multipliers and the rate of growth. To them we add one equation,

$$\lambda_1 L_1 + \lambda_2 L_2 + \dots + \lambda_m L_m + \dots + L_n = L$$

which reduces to,⁽¹⁾

$$\lambda_1 L_1 + \lambda_2 L_2 + \dots + \lambda_m L_m = L_B \quad [4(b)]$$

where L_B is the quantity of labour in the system of the intermediate goods industries.

The system of equations (4) will yield, if the system is productive, a unique positive solution for the multipliers and the rate of growth.⁽²⁾ When the price equations of the m capital goods industries are multiplied by $\lambda_1, \dots, \lambda_m$ respectively, the system will be brought into a state in which the demands for and deliveries of capital goods are equal to one another,

$$r(1 - \alpha)A_j^* = F_j^* \quad j = 1, \dots, m$$

where,

$$F_j^* = \lambda_j X_j - \sum_{i=1}^m \lambda_j A_{ji} - \sum_{i=m+1}^n A_{ji}$$

and

$$A_j^* = \sum_{i=1}^m \lambda_j A_{ji} - \sum_{i=m+1}^n A_{ji}$$

The unique rate of profit that emerges from this initial solution is,

$$r = \left[\frac{1}{1 - \alpha} \right] \frac{F_j}{A_j} = \frac{g_j}{1 - \alpha} = \frac{g}{1 - \alpha}$$

Substituting this rate of profit in the price equations for the capital goods industries gives $(m - 1)$ relative prices and the wage rate. These, in turn, can be substituted into the $(n - m)$ price equations of the consumption goods industries to determine the relative prices of the consumption goods. Further, their $n - m$ demand equations can be used to determine the quantities demanded of these commodities,

$$X_j^d = \frac{\alpha_j r K + \beta_j w L}{p_j} \quad j = m + 1, \dots, n \quad [5]$$

These quantities demanded will, in general, be different from those being supplied. To eliminate the disparities between the quantities demanded and the quantities supplied, further iterations will be required. ⁽⁴⁾

It will, of course, not help to plug in directly the aggregate commodity-wise input requirements A_j of the final outputs demanded X_j^d into the equations of the standard system 4(a) and obtain immediately the equilibrium levels of output of the consumption goods industries. Because, the rate of profit and the wage themselves change in the course of the adjustments and with them changes the composition of final demand.

Accordingly, it is necessary to proceed towards the final equilibrium in a series of gradual steps changing in each step the quantities supplied in the direction of the quantities demanded. At each step we calculate the aggregate input requirements of the supply vector being tried out and plug their commodity-wise sums $\Lambda_1', \Lambda_2', \dots, \Lambda_m'$ in equations (4) to determine the corresponding set of multipliers, the rate of profit, the wage and the relative prices of capital goods. These iterations will, of course, be repeated till the excess demands or supplies of the consumption goods are eliminated. What, in effect, is achieved is that the equilibrium levels of output of the consumption goods industries (to find whose value there is no direct method) are arrived at by successive iterations.

It will be noticed at the end of each iteration that the sum of the values of the excess demands for the commodities is equal to zero as required by Walras's Law. Thus, our system contains $2n+1$ equations (n price equations, $n-1$ independent demand equations, 1 "conservation-of-labour equation" (equation 4b above) and Walras's Law) which determine $2n+1$ unknowns ($n-1$ relative prices, n quantities of the commodities and 2

distributive unknowns). A necessary condition for the existence of a unique, positive solution is that the price equations must satisfy the Hawkins-Simon (1949) conditions, *i.e.* $F_i > 0 \quad i = 1, \dots, m$ --

Another necessary condition for the existence of a viable solution is that the equilibrium rate of profit should be less than the maximum rate of profit, R . Since the equilibrium rate of profit depends on the capitalists' propensity to consume, α , this condition sets a limit on the value of α , *viz.* that it should be less than the fraction $(R - g)/R$, the ratio of the rate of consumption to the rate of the maximum possible surplus.

15. It remains now to describe the economic processes in the disequilibrium states of the economy, *i.e.* the process by which the economic system starting from an arbitrary disequilibrium state reattains equilibrium.

Thus, suppose that the economy which is initially in equilibrium is disturbed, *i.e.* the outputs of some commodities rise above their equilibrium levels whilst those of the others fall below. In so far as the capital goods are concerned such a disturbance means that the ratios of their physical surpluses to their aggregate quantities

used become unequal to one another; rising in the case of industries whose outputs have risen and falling in the cases of those whose outputs have fallen.

The immediate consequence of this disturbance is that the spot prices of those commodities the outputs of which have risen, will fall below their equilibrium levels and those of the others, rise above their equilibrium levels. If the disturbance is expected to be temporary, (*i.e.* it is known that the disequilibrium is not the result of a change in the basic parameters of the system; the technical coefficients or the tastes and preferences), the market will expect that the disequilibrium will be corrected say, by the end of the period. Thus, the market will expect that the supply of those commodities whose spot prices have fallen (due to current over-supply) will fall to their competitive levels and that the supply of the other commodities will rise. As a result, the forward prices of the former set of commodities (those whose outputs have risen and spot prices fallen) will rise and these commodities will stand at a *premium* in the forward market. On the other hand, the forward prices of the latter set of commodities will fall so that these commodities

will stand at *discount* in relation to their spot prices.

We define the "own rate of profit" ⁽³⁾ on a commodity j as the sum of the equilibrium rate of profit and the forward premium on the commodity, *i.e.*

$$r_j = r + \frac{p_{Fj} - p_{sj}}{p_{sj}} \quad j = 1, \dots, m \quad [6]$$

where, p_{Fj} is the forward price of commodity j and p_{sj} is its current spot price. When the economy is in disequilibrium these own rates of profit are, in general, different for different commodities. In conditions of equilibrium, however, the forward price of every commodity is equal to its spot price so that all the own rates of profit are equal to one another and equal, in turn, to the equilibrium rate of profit.

It is the values of these own rates that are obtained when the demand equations for capital goods, $r(1 - \alpha)A_j = F_j$ ($j = 1, \dots, m$) are used along with the price equations of the capital goods to solve for the relative prices and distributive variables at the postulated levels of outputs.

If these own rates are substituted in equation (6) it is possible to solve for the disequilibrium spot prices of the m capital goods. To see this, suppose that the market expects equilibrium to be restored by the end of the period. Then, the forward price will coincide with the equilibrium price. And the current spot prices of the capital goods will be determined by equations (6) as,

$$p_{sj} = \frac{P_j}{1+r_j-r} \quad j=1, \dots, m \quad [7]$$

16. The adjustment process that has been described above holds only in the case of those commodities for which forward markets exist and whose spot prices move freely in response to excess supply/ demand situations. In the case of commodities whose prices are "sticky" over a wide range excess supply/ demand situations ('fix prices' as Hicks (1974) has called them), the adjustment will take place exclusively by variations in the output. Thus, it will be found that the multipliers appropriate to those industries whose own rates of profit are high will be relatively lower than for those whose own rates are lower. Those industries whose own rates of profit are high will not realise them because they will be

forced to bear the additional costs of storing their unsold supplies as inventories (if they are storable) and will be forced to reduce their scales of activity in accordance with these multipliers.

Depending upon the various response patterns of prices to the situations of disequilibrium it will be possible to plot the inventory cycles in the course of the adjustments towards equilibrium and the "cobweb" movement of the prices of commodities.

17. A word about the differences between the standard system as presented by Sraffa (1960) and the equilibrium system analysed above is in order. We notice that in the equilibrium system the price equations of *all* intermediate commodities are found to play a role and have positive multipliers appropriate to them. These include the basic intermediate goods as well as the non-basic intermediate goods. Similarly, we observe that the demand and price equations of the exclusively non-basic final consumption goods also play an active role in the determination of all unknowns.

Both the above classes of non-basics play a subordinate role, in so far as the determination of relative prices and rate of profit is concerned, in

Sraffa's system. Indeed, their name, 'non-basics' is derived from their incapacity to be present in Sraffa's standard system. These differences seem to suggest that the distinction between intermediate and final goods supersedes, in some respects, that between the basic and non-basic goods. Whilst the former distinction is relevant to the determination of the equilibrium, the relevance of the latter seems to be restricted to the definition and constitution of the 'standard commodity'.

18. We shall now incorporate the dual-purpose goods in the system *viz.* those goods which serve both as means of production and as means of consumption. If commodity j represents a dual purpose commodity its equation in the standard system will appear in the following form,

$$(A_{j1}\lambda_1 + A_{j2}\lambda_2 \dots + A_{jm}\lambda_m + A_j)(1+g) + C_j = \lambda_j X_j \quad [8]$$

where A_j is the quantity of the commodity used in the production of the consumption goods, and C_j is its quantity that is available for final consumption.

When there are many such commodities the general system of equations will be written as

follows,

$$\begin{bmatrix} X_1 - A_{11}z & -A_{12}z & \dots & -A_{1m}z & -(\Lambda_1 z + C_1) \\ -A_{21}z & X_2 - A_{22}z & \dots & -A_{2m}z & -(\Lambda_2 z + C_2) \\ \vdots & \vdots & & \vdots & \vdots \\ -A_{m1}z & -A_{m2}z & \dots & X_m - A_{mm}z & -(\Lambda_m z + C_m) \\ -L_1 & -L_2 & \dots & -L_m & L_B \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_m \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad [9]$$

where $z=(1+g)$,

This system will be solved by setting the $(m+1) \times (m+1)$ determinant of coefficients equal to zero and by selecting the lowest value of z greater than unity. Further iterations towards the equilibrium solution will follow the procedure outlined earlier.

[The dissimilarities between the system of equations (9) and Leontief's (1941) system must be noted. In the Leontief system the investment requirements of the investment goods industries are not distinguished from those of the final goods industries. Similarly, both the investment and the final demands are clubbed together for all commodities. As a result, an equilibrium rate of growth is usually not present in the system. Instead

a variety of rates of growth will implicitly be targeted when a final demand vector has been targeted for attainment. Moreover it is implicitly assumed that the targeted vector of final demands will be met without requiring any change in the prices of commodities].

19. Observations of the actual processes of economic growth over long periods of time, whether cursory or systematic, invariably reveal that the rates of growth are unequal to one another both over the same and as between different periods of time, that new industries and commodities emerge and supplant earlier ones, and that tastes and habits change continuously. Such observations are not inimical to the tranquil world of uniform growth that has been studied so far. For, the analytical framework can always be adapted to depict how, as new commodities emerge, as new techniques are put into operation and as tastes change, the equilibrium solution is modified period by period. ⁽⁵⁾ Every equilibrium solution is conditional upon the basic data, *viz.* the technical coefficients, the labour coefficients, and the demand coefficients; as these basic data change (exogenously) so will the equilibrium solutions of the prices, outputs, growth and distribution.

NOTES

1. Correctly speaking, this equation must be written as,

$$\left(\sum_{j=1}^m \lambda_j L_j \right) (1 + g_n) = L_B (1 + g_n)$$

where g_n is the 'natural' rate of growth. In equilibrium, the 'natural' rate of growth, *i.e.* the rate of growth of population, must equal the rate of growth of the system. It is convenient to assume, however, that there is an excess supply of labour in the system.

2. The lowest positive root, g , is the one to be selected for this root alone will be consistent with positive values for the multipliers.
3. It is important to note that the iterations to equilibrium in the sphere of consumption goods can also be explained from the side of prices.

Thus,

$$p_j^d = \frac{\alpha_j r K + \beta_j w L}{X_j} \quad j = m+1, \dots, n$$

solves for the "demand price" of the consumption good when the quantity supplied is at the initially postulated level, X_j . In the subsequent iterations, the supply of those goods whose demand prices exceed their costs of production (including profit) will be augmented whilst

those of the others decline until, in final equilibrium, the demand and supply prices of all consumption goods are equal to one another.

4. The term "own rate of profit" derives from similar terms used by Wicksell (1907), Sraffa (1932), Keynes (1936) and Lerner (1952). Sraffa uses the term "natural rate of interest" in defence of Wicksell's original concept. Keynes, referring to Sraffa, introduces the term "own rate of interest" ; a term followed by Lerner who, however, does not refer either to Sraffa or to Wicksell. But the definitions of these authors seem to apply to the process of market arbitrage rather than to production. The term "own rate of profit" as used here denotes the rate of profit that would be possible (warranted) if the economy conformed to the particular commodity's "own rate".
5. See Appendix 4 for a further discussion.

CHAPTER IV
JOINT PRODUCTS

- 20. The system of equations**
- 21. Equilibrium solution for a decomposable system**
- 22. Adjustment towards equilibrium described from the side of prices**
- 23. Consequence of the condition that the number of processes is less than the number of commodities**

CHAPTER IV

JOINT PRODUCTS

20. In this chapter we will consider the case where all "industries" produce a number of joint-products. We will assume that the number of independent processes available to produce the various commodities is equal to the number of commodities.⁽¹⁾ As we shall see presently, this assumption allows us to prove the existence of a competitive equilibrium solution; its formal role being that of raising the number of equations sufficiently so as to ensure that the system of price equations can move with only one degree of freedom.

Following the sequence of the discussion in Chapter III, we will at first suppose that the system is separable into equations for purely intermediate commodities and purely final commodities. Thus, suppose that the first m processes produce capital goods whilst the remaining $(n - m)$ processes produce $(n - m)$ final goods.

The system of the price equations and demand equations for the m capital goods is as follows,

$$\begin{aligned} (A_{11}p_1 + \dots + A_{m1}p_m)(1+r) + wL_1 &= X_{11}p_1 + \dots + X_{m1}p_m \\ (A_{12}p_1 + \dots + A_{m2}p_m)(1+r) + wL_2 &= X_{12}p_1 + \dots + X_{m2}p_m \\ \dots\dots\dots [1] \\ (A_{1m}p_1 + \dots + A_{mm}p_m)(1+r) + wL_m &= X_{1m}p_1 + \dots + X_{mm}p_m \\ r(1-\alpha)A_j &= F_j \quad j=1, \dots, m \end{aligned}$$

where A_{ji} represents the quantity of the j^{th} commodity required by the i^{th} process, X_{ji} represents the quantity of the j^{th} commodity produced by the i^{th} process,

$$A_j = \sum_{i=1}^n A_{ji} \quad j = 1, \dots, m$$

and

$$F_j = \sum_{i=1}^n (X_{ji} - A_{ji}) \quad j = 1, \dots, m$$

As a consequence of our assumption of a decomposable system, the first m processes will not produce final commodities and the $(n - m)$ final commodities will not appear as means of

only such processes may be admitted as will carry positive multipliers appropriate to them. The other "negative" processes may of course exist as technically feasible recipes but will not find a place in a competitive economy. ⁽²⁾

21. System (1), after it has been transformed by the scale multipliers in (2) will give a unique solution for the $m - 1$ relative prices of the capital goods, the wage and the rate of profit. These can be substituted in the price equations of the consumption goods industries.

The quantities demanded of these goods will then be obtained from,

$$X_j^d = \frac{\alpha_i r K + \beta_i w L}{p_i} \quad i=m+1, \dots, n$$

The further iterations will proceed as follows. Since every consumption commodity is, in general, supplied by a number of processes the supply level in the further iterations must first be apportioned between these processes to determine the new levels of capital goods required to produce them *viz.* A'_1, \dots, A'_m . As a matter of efficient computational strategy the apportionment of the change in supply can be done

in a way such that the processes which produce the commodities most intensively bear the onus of the changes.

22. The adjustment process, it may be pointed out, can be explained also from the side of prices. Thus we can evaluate the cost of production (including profit) and evaluate the outputs of every process at the demand prices of the commodities. Some processes will show surpluses and others will show deficits above and below the competitive costs of production. In the successive iterations the processes which exhibit surpluses will expand and those showing deficits will contract till the revenues of each process become equal to the respective competitive costs of production including profits at the equilibrium rate of profit.

The incorporation of dual-purpose goods can be achieved by repeating the procedure outlined in the case of single-product industries.

23. Thus far we have explicitly assumed that the number of independent processes is equal to the number of commodities. This assumption is indispensable for the existence of a unique competitive equilibrium solution. If, as is most likely, the assumption does not hold in reality, the

economic system will not possess a unique equilibrium solution - the rates of profit earned by operating different processes may be different depending upon the demand conditions for their products.

NOTES

1. See Sraffa (1960), Chapter VII for the justification of this assumption.
2. Once again, an important difference between Sraffa's standard system and the equilibrium system considered here must be noted. Sraffa's standard system can admit negative multipliers for, as Sraffa has said, "The *raison d'etre* of the standard system is to give the standard commodity". Since our interest is to use the standard system to find an equilibrium state, negative multipliers cannot be admitted.

PART TWO

DURABLE CAPITAL

CHAPTER V

FIXED CAPITAL

24. Preliminaries
25. 'Reduction' to new machines
26. The system of equations
27. The own rate of profit on the machine
28. Growth cycles and equilibrium
29. Assumption of constant life of machine in all industries dropped
30. Many durable assets
31. Consumer durables incorporated

CHAPTER V

FIXED CAPITAL

24. We shall now consider the behaviour of an economic system which uses durable capital instruments, besides the intermediate capital goods, in the process of production. Durable assets are, by definition, those whose lives exceed "one year". Therefore, their accounting in the price equations that pertain to the production of "a year" requires that they be included amongst the inputs as well as among the outputs. We shall therefore consider every durable asset as going into the production process at the start of every year and emerging from it one year older than before at the end of the year along with the regular output of the process. Every durable asset will therefore appear on both sides of the price equations for all the commodities in whose production it is used and for all the years of its life. And it is only in the last year of its life that it will disappear from the side of the output and resemble, in that respect, an intermediate input.⁽¹⁾

As a rule, we shall assume that all durable assets work with constant efficiency throughout their lives. Although specific conclusions are bound to require amendments to varying degrees if this assumption were replaced in each case by the more realistic ones, the general framework can be adapted suitably to incorporate all patterns of the behaviour of the efficiency of assets over their life-times.

To start with, suppose that there is but one kind of a durable asset, say a machine, designated by commodity 1. Suppose the life of the machine to be s years irrespective of the industry in which it is used. We will also suppose that the scrap value of the machine at the end of its life is nil. Then the price equation for an industry producing commodity i will be,

$$(M_{0i} p_1^0 + M_{1i} p_1^1 + \dots + M_{s-1i} p_1^{s-1})(1+r) + \sum_{j=2}^n (A_{ji} p_j)(1+r)$$

$$+ w L_i = p_i X_i + (M_{0i} p_1^1 + M_{1i} p_1^2 + \dots + M_{s-1i} p_1^s) \quad [1]$$

where p_1^k are the book-prices of a k year-old machines ($k=1, \dots, s$), M_0, M_1, \dots, M_{s-1} are the quantities of the machines of ages 0, ..., $s-1$ used in the production of commodity i during the year, X_i is

the output of commodity i , L_i is the quantity of labour used and A_{ji} are the quantities of the intermediate capital goods required to produce X_i units of output of commodity i .

25. The assumption of constant efficiency makes it possible to rewrite equation (1) in a form in which the various terms representing the machines of various ages are reduced to a single term. This is because all the book-prices p_1^k can then be related to one another in the following manner,⁽²⁾

$$p_1^0 = p_1$$

$$p_1^1 = [(1+r) - \psi]p_1$$

$$p_1^2 = [(1+r)^2 - \psi\{(1+r)+1\}]p_1$$

.....

$$p_1^{s-1} = [(1+r)^{s-1} - \psi\{(1+r)^{s-2} + \dots + (1+r) + 1\}]p_1$$

$$p_1^s = [(1+r)^s - \psi\{(1+r)^{s-1} + \dots + 1\}]p_1 = 0$$

where,

$$\psi = \frac{r(1+r)^s}{(1+r)^s - 1} = \frac{v^s(v-1)}{v^s - 1} = \sum_{k=1}^s \frac{1}{(1+r)^k}$$

Thus, we can, by substitution, obtain the

equations,

$$M_k(1+r)p_1^k - M_{k+1}p_1^{k+1} = M_k p_1 \psi \quad k=0, \dots, s-1$$

which allow us to rewrite equation(1) in the form,

$$A_{1i} p_1 \psi + \left(\sum_{j=2}^n A_{ji} p_j \right) (1+r) + wL_i = p_i X_i \quad [2]$$

where,

$$A_{1i} = M_{0i} + M_{1i} + \dots + M_{s-1,i}$$

is the total number of machines in industry i and,

$$A_1 = \sum_{i=1}^n A_{1i}$$

is the total number of machines in the economy.

26. We now proceed to find the equilibrium solution. As before, we shall assume for convenience that the system is decomposable so that the first m industries produce capital goods and the rest produce consumption goods.

The price equations for the capital goods industries are as follows,

This, we will be required to do in accordance with the number of machines that will be in use in the following year. At the end of the year, a certain number of machines die ; this number is equal to the number of machines of age $s-1$ in the age-distribution shown in equation(1). Let this, as a proportion of the total number of machines in operation during the year, be μ , *i.e.*

$$\mu = \frac{M_{s-1}}{A_1}$$

At the same time, X_1 new machines are added to the stock of machines. Thus, the total number of machines that will be in operation in the following year is,

$$X_1 + A_1 - \mu A_1$$

The quantity of the working capital commodities 2, ..., m will depend on the stock of machines in operation in the following year. These will be given by,

$$A_j \left[\frac{X_1 + A_1 - \mu A_1}{A_1} \right] \quad j=2, \dots, m$$

so that, the incremental demand for each of these commodities will be,

$$A_{1r}p_1\psi + (A_{2r}p_2 + \dots + A_{mr}p_m)(1+r) + wL_n = p_n X_n$$

$$\alpha_j r K + \beta_j w L = p_j X_j \quad j=m+1, \dots, n \quad [7]$$

To the $2n$ equations in (3) to (7) we can add the conservation - of - labour equation to obtain the complete set of $2n+1$ equations which will, in equilibrium, determine the $2n+1$ unknowns, *i.e.* the $n-1$ relative prices, n levels of output and 2 distributive unknowns.

27. Equations (4) and (5) determine the various own rates of profit when the system is in a state of disequilibrium. In so far as the working capital commodities in equation (5) are concerned, their own rates of profit are found, as usual from the equations,

$$r(1 - \alpha)A_j = F_j \quad j = 2, \dots, m$$

where,

$$A_j = \sum_{i=1}^n A_{ji}$$

and

$$F_j = X_j - A_j \quad j = 2, \dots, m$$

Equation (4) which determines the own rate of profit of commodity 1, the durable machine,

deserves a somewhat detailed consideration. We note that the depreciation rate, δ , applicable for the entire economy is an average of the depreciation rates δ_j charged by the industries. And these industry rates, in turn, depend upon the age-distribution of machines operated by them during the year. Since the age-distribution of machines operated itself changes from year to year, the depreciation rate δ changes over time, its value in any one year being obtained from the following equation,

$$\delta = \frac{(\psi - r) \sum_{k=0}^s M_k (1+r)^k}{\sum_{k=0}^s M_k} \quad [8]$$

where,

$$M_k = \sum_{i=1}^n M_{ki}$$

is the number of machines of age k , and

$$\sum_{k=1}^s M_k = A_1$$

is the total number of machines in the economy.

If we substitute the expression for δ from equation (8) into equation (4), we get a polynomial in r whose lowest positive root will represent the own rate of profit of commodity 1.

28. With these preliminaries we can proceed to describe the process by which economic equilibrium is attained. Once again it will be convenient to describe the process in two stages. In the primary stage we shall ensure the balance of demand and supply in the sphere of the working capital goods. To this end, we need to construct a standard system which will yield such scale multipliers $\lambda_1, \dots, \lambda_m$, as will equate, for each working capital commodity, the ratio of the physical surpluses available and the aggregate quantity used of the commodity to the growth rate that is warranted by the net physical surplus of machines available in that year. In other words, we must find $\lambda_1, \dots, \lambda_m$ such that,

$$\frac{F_j}{A_j} = \frac{X_1 - \mu A_1}{A_1} \quad j=2, \dots, m \quad [10]$$

We therefore set up an appropriate system of equations to determine the multipliers consistent with the growth rate in equation (10).

transformed by the multipliers, the sub-system of the working capital goods' industries will be brought into a state of balance. This state of balance, however, will last only for a year. For when the year passes by the age-distribution of machines will change under the joint influence of some machines dying out (μA_1) and new machines being added (X_1). As a result both the mortality rate, μ , and the depreciation rate, δ , will change in the following year. Similarly, the capital formation that takes place in the machine producing industry during the year will increase the productive capacity of the industry to the effect that, in the following year, the output of new machines will be greater.

These influences will also change the growth rate that is warranted by the availability of the machines. In the next iteration, therefore, we must find the multipliers $\lambda'_1, \lambda'_2, \dots, \lambda'_m$ that correspond to the new rate of growth and, in like manner, carry out the iterations for the succeeding years.

It will be found that the entire economy oscillates as the growth rate rises and falls over various durations of time. These growth cycles cease only when the age-distribution of machines

attains such proportions that the mortality rate, μ , and the depreciation rate, δ , are equal to one another and the economy is settled on a balanced growth path. To state the point differently, the cyclical fluctuations in the rate of growth of the economy cease only when the own rate of profit of the machine is equalised with those of all the other commodities in the system.

It must be noted, however, that the procedure of eliminating the demand - supply differences in the sphere of the working capital goods in every year by restoring the system to balanced proportions is only an analytical device which enables us to trace out the cyclical growth path. Its use is a device for making the cyclical growth path visibly free from the aberrations of the imbalances between the demands and deliveries of the working capital commodities.

We now proceed with the second task of determining directly the proportions of the final equilibrium system, *i.e.* the proportions to which the disequilibrium system will converge at the end of its cyclical path. In order to do this we find an initial solution containing multipliers λ_1^* , ..., λ_m^* which are consistent with a uniform balanced rate of growth for the sub-system of all capital goods.

Setting the determinant of the matrix of the coefficients of λ^* s equal to zero solves for the rate of growth. The lowest value of z greater than unity, say z^* is the appropriate solution. Substituting z^* in equations (12) gives the values of the λ^* s.

The values of the multipliers derived apply, in so far as the durable machines are concerned, only to those which fall in the $s-1$ age category. The multipliers appropriate to the other age categories are derived from the following relations,

$$\begin{aligned} \lambda_j^0 &= \lambda_j^*(1+g)^{s-1} \\ \lambda_j^1 &= \lambda_j^*(1+g)^{s-2} \\ &\dots\dots\dots j=1, \dots, n \quad [14] \\ \lambda_j^{s-1} &= \lambda_j^* \end{aligned}$$

Equations (14) describe the age-distribution of machines that are consistent with a uniform rate of annual growth, g^* , for the system.

Finally, there will be the iterations to ensure the equilibrium of demands and supplies of the consumption goods. These iterations follow the sequence of steps described in the earlier chapters.

In the overall equilibrium the Λ 's will be such as to ensure that the system is on a balanced growth path with all markets cleared.

29. One of the simplifying assumptions made for ease of exposition in the preceding discussion was that the life of the machine is equal in all the industries in which it is used. This assumption can be dropped and substituted by the more general situation in which the life of the machine in the respective industries is s_1, \dots, s_n . Accordingly, the factors $\hat{\Omega}$ and Ω will be substituted by $\hat{\Omega}_{s_1}, \Omega_{s_2}$, etc. in equations (12) and (13) respectively. It is clear that the multipliers λ_j^* will be applicable to the machines of age $s_j - 1$. And the multipliers appropriate for the machines in the preceding age groups will be determined in accordance with equations (14).

30. When dealing with the general case of many types of durable assets it will suffice, to begin with, to consider two types of durable machines and to generalise subsequently by repeating the arguments made in the special case.

As regards the reduction of the various terms representing the old machines to a single term, a similar procedure as has been described earlier

will be found to hold. The "reduced" price equations of the capital-goods industries will now be,

$$A_{11}p_1\psi_1 + A_{21}p_2\psi_2 + (A_{31}p_3 + \dots + A_{m1}p_m)(1+r) + wL_1 = p_1X_1$$

$$A_{12}p_1\psi_1 + A_{22}p_2\psi_2 + (A_{32}p_3 + \dots + A_{m2}p_m)(1+r) + wL_2 = p_2X_2$$

..... [15]

$$A_{1m}p_1\psi_1 + A_{2m}p_2\psi_2 + (A_{3m}p_3 + \dots + A_{mm}p_m)(1+r) + wL_m = p_mX_m$$

where ψ_1 and ψ_2 are the annuity factors applicable to commodities 1 and 2, the machines.

There will, in the present case, be two growth rates warranted by the physical surpluses available in the two machines producing industries,

$$g_1 = \frac{X_1 - \mu_1 A_1}{A_1}, \quad g_2 = \frac{X_2 - \mu_2 A_2}{A_2}$$

As a matter of expositional strategy, the lower one of these can be selected in each year for the computation of the quantities of the working

capital goods that will be required in the following year. This rate of growth can be used to find the associated multipliers that ensure a balance between the demands and supplies of the working capital commodities in each year's iterations. The series of these iterations will trace out the oscillatory path towards the final equilibrium.

That the oscillations will cease (in a productive economy for which a standard system exists) can be seen by proceeding in the same steps as in equations (12) and (13). The only difference that will arise in the present case is that the first two equations in each system will contain the factors $\hat{\Omega}_1$, $\hat{\Omega}_2$, and Ω_1 , Ω_2 , respectively.

An identical procedure extends to cover any number of durable assets.

31. To complete the discussion of fixed capital in those respects in which working capital has been treated it remains now to incorporate durable consumption goods.

In so far as the consumer durable is a purely final good no special problem arises; the size of the demand and supply of these goods will be determined in the course of the iterations to final

equilibrium. The stock of consumer durables in equilibrium need not, of course, conform to the balanced age distribution described in equation (14).

If, however, the *services* of the durable commodity are sold, rather than the commodity itself, then this activity will find a place in equations (12), (13) and (14).

It is clear, of course, that the price of the "services" will be related to the price of the durable commodity by the annuity factor, ψ , in addition to the other costs of production.

Finally, if the durable commodity is a dual-purpose good the element $(-A, \Omega)$ in equation (13) must be replaced by the element $(-A, \Omega - C_j)$ before the further iterations can be carried out.

NOTES

- (1) The logic of this treatment of durable assets is explained at length by Sraffa (1960), Chapter 10.
- (2) These book-prices are not notional. They represent the actual market prices if there exist secondhand markets for the commodities to which they refer.

CHAPTER VI

'HUMAN' CAPITAL

- 32. The system of equations.**
- 33. Iterations to equilibrium**
- 34. Generalisations : Opportunity Costs and Hierarchies of Education**
- 35. Consequences of the divergence of the natural and warranted growth rates**

CHAPTER VI

'HUMAN' CAPITAL

32. In this chapter we propose to consider the implications of including 'human' capital in the system of prices and production. The term human capital is used here to describe the various types of labour that are trained to perform various kinds of specialised functions in the process of production.

For purposes of simplicity we shall assume the absence of fixed capital. Of course, this obviously unrealistic assumption can be dropped easily after the salient features of the system have been made clear. We shall also suppose that every type of skilled labour is 'produced' by a *separate* educational industry. This supposition will enable us to avoid the complications attendant upon the introduction of joint-products.

Suppose there are u types of labour used in the economy and denote their quantities by L^1, \dots, L^u . Let L^0 represent the category of untrained/unskilled labour. The u types of labour

will be regarded as the products of u different educational industries and will obviously be included amongst the intermediate commodities in the system. There will therefore be $m+u$ intermediate commodities, the rest, $n-(m+u)$, being the final commodities.

The price equations of the system of $m+u$ intermediate commodities, on the supposition that all types of labour are paid their wages at the end of the year, can be written as follows,

$$(A_{11}p_1 + \dots + A_{m1}p_m)(1+r) + w_o L_1^o + w_1 L_1^1 + \dots + w_u L_1^u = p_1 X_1$$

$$(A_{12}p_1 + \dots + A_{m2}p_m)(1+r) + w_o L_2^o + w_2 L_2^1 + \dots + w_u L_2^u = p_2 X_2$$

$$\dots \dots \dots [1]$$

$$(A_{1m}p_1 + \dots + A_{mm}p_m)(1+r) + w_o L_m^o + w_1 L_m^1 + \dots + w_u L_m^u = p_m X_m$$

$$(A_{1,m+1}p_1 + \dots + A_{m,m+1}p_m)(1+r) + w_o L_{m+1}^o + \dots + w_u L_{m+1}^u = p_{m+1} X_{m+1}$$

$$\dots \dots \dots$$

$$(A_{1,m+u}p_1 + \dots + A_{m,m+u}p_m)(1+r) + w_o L_{m+u}^o + \dots + w_u L_{m+u}^u = p_{m+u} X_{m+u}$$

where A_{ji} ($j=1, \dots, m, i=1, \dots, m$) have the usual meaning, $A_{j,m+1}$ ($j=1, \dots, m, i=1, \dots, u$) denotes the quantity of commodity j required to produce workers of type i , L_k^i ($k=1, \dots, m+u, i=1, \dots, u$) is the quantity of the i^{th} type of labour in the k^{th} intermediate activity, w_0 is the wage rate of basic labour, w_i ($i=1, \dots, u$) represent the wage rate of the various categories of skilled labour, X_{m+i} ($i=1, \dots, u$) represent the number of students produced by the u educational industries and p_{m+1} ($i=1, \dots, u$) denotes the fee per student.

The system of equations (1) contains $m+u$ equations in $2m+3u+1$ unknowns including the $m-1$ commodity prices, the m levels of output of the regular intermediates, the u prices of education (fees), and u levels of output of the educational industries, the u wage rates of trained labour, the wage rate of untrained labour and the rate of profit.

Clearly, the system is indeterminate even in anticipation of the fact that $m+u+1$ equations for the appropriate standard equilibrium system will be available. For, even when these equations are added we have $2m+2u+1$ equations in $2m+3u+1$ unknowns and our system is u equations short of the number required.

We shall therefore add the following u equations whose effect is to replace the u prices of the educational industries by the demand prices of their respective educations,

$$P_{m+i} = \frac{w_i}{\psi_{h-1}} - \frac{w_o}{\psi_h} \quad [2]$$

$$\psi_h = \frac{r(1+r)^h}{(1+r)^h - 1}, \quad \psi_{h-1} = \frac{r(1+r)^{h-1}}{(1+r)^{h-1} - 1}$$

where h is the working life of the student before obtaining the education and $h - 1$ that after obtaining the education.^{(1) (2)}

Finally, there will be $n - (m+u)$ price equations of the regular consumption goods, their $n - (m+u)$ demand equations, m demand equations for the regular capital goods and u demand equations for the educations. The demand equations for the educations need some explanation. If the educations are purchased by households and subsequently sold (or leased) to the producing industries, as they are in reality, their u demand equations will be specified in the same manner as are the demand equations of the regular consumption goods. If, on the other hand,

the educations were purchased directly by the producing industries, the costs of training would rightly find their place amongst the means of production of these industries, and consequently their (investment) demands would have to be represented in a manner akin to those of other means of production.

Irrespective of the specification of the demand function, however, the various educations must be considered as intermediate commodities; the specification of the demand functions depend upon the transactions through which the outputs of the educational industries reach their ultimate industrial users.

33. To bring the system to final equilibrium we construct the equations of a standard system keeping in mind that the various categories of skilled labour will now be included amongst the intermediate commodities in the system,

$$(A_{11}\lambda_1 + A_{12}\lambda_2 + \dots + A_{1m+u}\lambda_{m+u} + \Lambda_1)(1+g) = \lambda_1 X_1$$

$$(A_{21}\lambda_1 + A_{22}\lambda_2 + \dots + A_{2m+u}\lambda_{m+u} + \Lambda_2)(1+g) = \lambda_2 X_2$$

.....

$$(A_{m1}\lambda_1 + A_{m2}\lambda_2 + \dots + A_{m,m+u}\lambda_{m+u} + \Lambda_m)(1+g) = \lambda_m X_m$$

assumptions. Firstly, it is assumed that all students have the opportunity to perform in untrained jobs; an opportunity they must necessarily forego for one year to obtain the education. If such an opportunity does not exist; as in the case of children, or in situations of severe unemployment, the opportunity cost is zero.

Secondly, it is implicitly assumed in equations (2) that all students passing out from every educational industry are of the same age and can look forward to a working life of h years. This too will require modification to accommodate the general situation. The generalisation is achieved by replacing the uniform annuity factor, ψ , pertaining to h years of working life by a weighted average of the factors ψ_{h_1} , ψ_{h_2} , ...etc. where the weights are the proportion of students in the age categories h_1 , h_2 ,...etc.

Thirdly, it is assumed that every kind of education can be obtained in one year. This situation needs generalisation. The generalisation is made, in the present system by introducing a hierarchy of educations each stage of which requires a year.

Thus, if stage 2, for example, is accessible

only after completing stage 1, the demand prices of the two educations will be represented as follows,

$$p_{m+1} = \frac{w_i}{\psi_{h-1}} - \frac{w_o}{\psi_h} \quad [4]$$

$$p_{m+2} = \frac{w_2}{\psi_{h-2}} - \frac{w_1}{\psi_{h-1}}$$

In this manner it should be possible to construct any number of hierarchies of educations all of which start from the category of untrained labour.

35. As has been alluded to earlier, one of the requirements for the economic equilibrium to persist through time is that the rate of growth of the population must, in the absence of technological change, be equal to that of the economy. The multipliers λ_{m+j} must therefore be interpreted, by analogy with the case of fixed capital, as being applicable to the workers of age $h - 1$; those applicable to the earlier age-groups being successively higher by the factor $(1+g)$ where g represents both the rate of growth of the

population as well as that of the economy.

Now, it is clear that neither the age-distribution of the population, nor its growth rate, can satisfy these "knife-edge" requirements. As a result, those regions in which the attainable economic growth rate is greater than that of the population will be forced to adopt some or all of the following devices ; raise the age of retirement, permit immigration of labour, or export capital, the opposite sequence of events happening in the other regions.

NOTES

1. It is assumed that the retirement age is fixed. If this changes with the education the discounting factors must pertain to the appropriate number of working years.
- 2., The expression in equation (2) will become simple if we assume as an approximation that

$$\psi_h \cong \psi_{h-1}$$

so that,

$$P_{m+i} = \frac{w_i - w_0}{\psi_h}$$

i.e. the increment to the wage will, in equilibrium, be equal to the annuity on the fee spent by the student.

CHAPTER VII

CHOICE OF TECHNIQUES

- 36. Technique distinguished from the product.**
- 37. Procedure for the general solution of choice of techniques.**
- 38. Cost-Benefit Analysis : stylised examples.**
- 39. Behaviour of an economy when a new technique is introduced.**
- 40. Cumulative technological progress**

CHAPTER VII

CHOICE OF TECHNIQUES

36. We have so far assumed that in a system of single-products industries each commodity is produced by only one technique. There was, therefore, no occasion to distinguish the technique of production from the description of the product — these were inseparably intermixed with the result that naming the product was alone sufficient. In this chapter we want to consider the general case in which several techniques for the production of one or more of the products are available. Our object is to study which one from amongst the alternative techniques will be chosen in the final equilibrium.

It is therefore necessary to define the technique of producing a given product and to distinguish it from the description of the product itself. In particular, it is important to note that the proportions which the raw material ingredients bear to one another (apart from quality variations of a statistical nature) define the *product*, not the

technique of producing it. Any change in these proportions changes the product. On the other hand, a different *technique* of production will be identified only if it involves the use in different proportions of the "agents of production" *i.e.* of the types of machines and of labour required to process the raw materials into the finished product.

To be sure, a change in the type of machinery will usually impose the requirements of working inputs specific to it, *eg.* electricity in place of human or animal power. Yet, if a comparison between different techniques of production is to be possible without the interfering influences from changes in the nature of the product itself, the technical properties of the product must be assumed to remain the same, irrespective of the technique used to produce it. And this means that the ingredients of the product must be assumed to remain the same when a comparison is to be made between the techniques of production.

The distinction of the technique of production from the specification of the product is thus seen to hinge upon the distinction between the *agents of production* and the *ingredients of the product*; the former do not undergo any change in their physical shape in the course of production,

whilst the latter are, usually irreversibly, transformed into the product.

Thus, in a system with exclusive use of working inputs and one kind of labour (Chapters II and III above), the choice of technique problem is trivial. For, in these conditions different techniques will be identified only by the distinct quantities of labour required by each of them for a given level of output. And the technique that uses the least quantity of labour per unit of output will obviously be chosen as the most profitable. It is only when different types of machines are involved with different types of labour that the problem of choice comes to deserve more detailed consideration. ⁽²⁾

If this line of reasoning is accepted we are led to infer that if the raw material ingredients change, with or without a change in the machines and labour required, the resulting product itself changes. There will then be a change in the Engels' coefficients relevant to the changed products. Such products will have their own prices which will not be directly comparable with the price of *any* commodity and would therefore be considered in relation to those of the others.

37. With these qualifications in view we can proceed to illustrate the procedure for the general solution. Suppose two techniques I and II, specified vectorially by (A_{1j}, L_{1j}) and (A_{2j}, L_{2j}) , are available to produce a given level of output of a commodity j . A_{1j} and A_{2j} represent the number of machines of types I and II respectively and L_{1j} and L_{2j} represent the quantities of homogeneous labour required by the two techniques.

Accordingly, we set up two systems of equations, I and II which, although identical in all other respects, differ only in one, *viz*, that the price equation for commodity j is represented in system I by the equation of technique I and in system II, by that of technique II. Next, we compare the equilibrium solutions of each system and select the system that yields a higher rate of profit.

An identical procedure holds in the general case in which the several techniques of production for the various commodities differ in any one or more of the following respects ; the quantities and types of machines used, their durabilities, the quantities and types of labour required, the quantities of the technique specific working inputs, or the wastages of raw materials characteristic to them.

38. It is possible, of course, to avoid rummaging through all the systems of equations if the purpose is only to know which technique will be selected rather than to know what rate of profit it will earn in the new equilibrium. This is facilitated by Sraffa's criterion for selection, *viz.* the technique which produces at the lowest unit cost, the cost being evaluated at the ruling (equilibrium) levels of the rate of profit, prices and the real wage, is the most profitable technique. In the light of the procedure for the general solution this implies that the adoption of the technique which meets this criterion will raise the rate of profit of the entire economy.

The criterion can be readily applied to a number of stylised examples. Thus consider two alternative techniques for producing commodity j and suppose that $A_{1j} = A_{2j}$ and $L_{1j} = L_{2j}$ but that the durabilities of the machines s_1 and s_2 differ. Under these conditions technique 1 will be selected only if the annuity on machine 1 is lower than machine 2, *i.e.*, if,

$$p_1 \psi_1 < p_2 \psi_2$$

where,

$$\psi_1 = \frac{r(1+r)^{S_1}}{(1+r)^{S_1} - 1}, \psi_2 = \frac{r(1+r)^{S_2}}{(1+r)^{S_2} - 1}$$

If the number of machines and the quantities of labour also differ, technique I will be selected only if

$$A_{1j}p_1\psi_1 + wL_{1j} < A_{2j}p_2\psi_2 + wL_{2j}$$

We can, in like manner, deal with the other possible differences between the techniques. Thus, for example, the wage cost of the different types of labour will appear on the respective sides of the cost equations of the techniques. Similarly, the costs of technique-specific working inputs (energy, maintenance *etc.*) if any, can also be accommodated. Finally, if the wastages of raw materials differ as between the techniques, their net benefit/cost can be included as well.

39. The procedure of the general solution described above conceals the details of the dynamic process observed in the actual economy when a more efficient new technique supplants an existing one. The general solution only tells us which technique will be selected. In fact, however, when a new technique (provided it is more efficient by the criterion explained above) is

introduced it supplants the old technique in steps.

Thus suppose that a new machine substitutes an old one in the production of commodity j . This will have two effects. The first effect is that all expansions of new plant and the replacement of old plant will now embody the new technique. The second is that the size of the industry producing the old machine will shrink because the demand for its product from industry j ceases altogether.

In the years to come the demand for the new machine originating from industry j will rise rapidly in steps determined by the standard multipliers which characterise the equilibrium age-distribution of the existing stock of machines of the old type in industry j . These are,

$$\lambda_j^*, \lambda_j^*(1+g), \dots, \lambda_j^*(1+g)^{s-1}$$

in years 1, 2, ..., $s-1$ respectively. There will thus be a prolonged boom for the industry producing the new machine which will last for producing the new machine which will last for s years, *i.e.* for the duration of the life of old machine. Thereafter, a growth cycle will be observed around the new equilibrium growth path.

The establishment of the new equilibrium growth rate will naturally require a change in the age-distribution of *all* machines in all industries.

If the new machine is as versatile as the old one *i.e.* it substitutes the old machine in all the industries then the industry producing the old machine will cease to operate and its place will be taken by the industry producing the new machine.

40. There is another respect in which much of the above analysis is static. It considers, rather simplistically, the choice of technique problem to be one of selecting a bunch of techniques, one for each commodity, from amongst sets of competing alternatives. An assumption that is implicit in its procedure is that the techniques of production are independent of one another. In reality this is not so. The use of some techniques in one or more lines of production is conditional upon the use of others in the same or in different lines. (For example, the use of the photo-offset process for printing is inconceivable if photography and electronics are not themselves in use). The result is that until the latter are not in a actual use, the former are defunct and must remain outside consideration.

It is precisely in such interdependencies between techniques that is found the key to cumulative technological progress.

In our system these interdependencies must first be identified and placed in a series as sets such that some techniques are brought into consideration as feasible alternatives only after the others, on which they depend, have been brought into *actual* operation.

Once the problem has been considered in this manner it can be seen how the sequence of the adoption of technique results in a cumulative technological boom.

NOTES

1. Strictly speaking the wastages involved in each technique also defines the technique. We have, for simplicity, assumed that the wastages of all techniques are nil.
2. It is important to note that the general procedure for the solution of the final equilibrium holds even if the technique is not distinguished from the product, *i.e.*, it holds even if the usual Leontief-Sraffa definition of technique were adopted.

CHAPTER VIII

NATURAL RESOURCES

- 41. Identification of the 'marginal process'.**
- 42. The system of equations : one type of land.**
- 43. Steps for solving the system.**
- 44. Changes in gradation with occupations.**
- 45. Prices of lands.**
- 46. An alternative method of solving the system.**
- 47. Several types of lands.**
- 48. Exhaustible resources**
- 49. Developments that offset the process of decreasing returns.**

CHAPTER VIII

NATURAL RESOURCES

41. In this chapter we shall incorporate the natural resources, or land, in the system of prices and outputs. For the sake of simplicity we will suppose the absence of joint products, fixed capital and human capital.

It will as a rule be assumed that the "marginal land", *i.e.* the least productive land, can always be identified independently of the rents and/or the other unknowns of the system. But, in view of the fact that the gradation of land changes with its occupations, a land which is deemed marginal in the production of one commodity need not be so when used in the production of another. The identification of the marginal land must therefore be made only with reference to the occupations, *i.e.* the commodities in the system.

This, however, necessitates some criterion for identifying the marginal land. The required criterion is as follows; that land (when equal

quantities of lands are being compared) on which a greater quantity of at least one of the inputs is required, but no less of any other, per unit of the commodity produced than on the other grades of land is the marginal land with reference to that commodity. Being the least productive land it would earn no rent if used in the production of that commodity. As for the other grades of land, their order of productiveness will be determined only after their rents have been determined.

In what follows, therefore, we will assume that corresponding to every commodity in the system there exists a marginal land. The process that produces a commodity on the marginal land will be called the *marginal process* for the commodity. It follows that there will be as many marginal processes as there are commodities in the system.

42. Suppose that there is one type of land of differing grades. For each commodity in whose production land is used there will be as many processes as the relevant grades of land. Thus, if there are v grades of the land there will be nv price equations in the system,

$$\begin{aligned}
 &(A_{11}^1 p_1 + A_{21}^1 p_2 + \dots + A_{m1}^1 p_m)(1+r) \\
 &\qquad\qquad\qquad + \pi_1^1 N_1^1 + wL_1^1 = p_1 X_1^1 \\
 &(A_{11}^2 p_1 + A_{21}^2 p_2 + \dots + A_{m1}^2 p_m)(1+r) \\
 &\qquad\qquad\qquad + \pi_1^2 N_1^2 + wL_1^2 = p_1 X_1^2 \\
 &\dots\dots\dots \\
 &(A_{11}^v p_1 + A_{21}^v p_2 + \dots + A_{m1}^v p_m)(1+r) \\
 &\qquad\qquad\qquad + \pi_1^v N_1^v + wL_1^v = p_1 X_1^v \\
 &(A_{12}^1 p_1 + A_{22}^1 p_2 + \dots + A_{m2}^1 p_m)(1+r) \\
 &\qquad\qquad\qquad + \pi_2^1 N_2^1 + wL_2^1 = p_2 X_2^1 \\
 &(A_{12}^2 p_1 + A_{22}^2 p_2 + \dots + A_{m2}^2 p_m)(1+r) \\
 &\qquad\qquad\qquad + \pi_2^2 N_2^2 + wL_2^2 = p_2 X_2^2 \\
 &\dots\dots\dots [1] \\
 &(A_{12}^v p_1 + A_{22}^v p_2 + \dots + A_{m2}^v p_m)(1+r) \\
 &\qquad\qquad\qquad + \pi_2^v N_2^v + wL_2^v = p_2 X_2^v \\
 &\dots\dots\dots \\
 &(A_{1n}^v p_1 + A_{2n}^v p_2 + \dots + A_{mn}^v p_m)(1+r) \\
 &\qquad\qquad\qquad + \pi_n^v N_n^v + wL_n^v = p_n X_n^v
 \end{aligned}$$

where, in the decomposable system, A_{ji}^k ($j=1, \dots, m$, $i=1, \dots, n$) is the quantity of commodity j required to produce X_i^k units of commodity i on land of grade k ($k=1, \dots, v$), π_i^k is the rent earned by land

of grade k when used in the production of commodity i , N_i^k is the quantity of land and L_i^k the quantity of labour required to produce X_i^k units of commodity i .

Next, there are the m demand equations for the capital goods,

$$r(1 - \alpha)A_j = F_j \quad j=1, \dots, m \quad [2]$$

and $n - m$ consumption demand equations,

$$\alpha_j rK + \beta_j wL + \gamma_j \Pi = p_j X_j \quad j=m+1, \dots, n \quad [3]$$

where γ_j are the proportions of rent income Π spent on the consumption goods. We will assume for simplicity that the wage and rent incomes are entirely spent on consumption goods.

Finally, there are the restrictions that the products of the rents earned on the lands of all grades be equal to zero for each commodity in whose production land is used. These restrictions arise from the fact that the marginal land in each commodity will earn no rent.

$$\pi_j^1 \pi_j^2 \dots \pi_j^v = 0 \quad j=1, \dots, n \quad [4]$$

There are in all $2n + nv$ equations above to which can be added the conservation-of-labour equations. These will solve for $2n + nv + 1$

where A_j^v is the aggregate quantity of commodity j produced by the marginal processes and used in the consumption goods industries. Equations (5) solve for the rate of growth and m scale multipliers. Using these in the price equations of the marginal processes and the demand equations for intermediate goods, we obtain an initial solution for the wage, the rate of profit and the relative prices.

Substituting these latter in the price equations of the intra-marginal processes gives the solutions for nv rents at the postulated levels of activity in these processes. As noted earlier, this reflects a situation of disequilibrium. An essential step towards finding the equilibrium solution is to reallocate the lands, changing thereby the levels of activity of the various processes, in such a way that the rents of each grade of land are rendered uniform.

In order to effect this we shall find it convenient to rewrite the price equations in the following form,

$$(A_{11}^1 p_1 + A_{21}^1 p_2 + \dots + A_{m1}^1 p_m + N_1^1)(1 + \rho_1^1) \\ + wL_1^1 = p_1 X_1^1 + N_1^1$$

.....

$$\begin{aligned}
 & (A_{11}^v p_1 + A_{21}^v p_2 + \dots + A_{m1}^v p_m + N_1^v)(1 + \rho_1^v) \\
 & \qquad \qquad \qquad + wL_1^v = p_1 X_1^v + N_1^v \\
 & (A_{12}^1 p_1 + A_{22}^1 p_2 + \dots + A_{m2}^1 p_m + N_2^1)(1 + \rho_2^1) \\
 & \qquad \qquad \qquad + wL_2^1 = p_2 X_2^1 + N_2^1 \\
 & \dots \dots \dots [6] \\
 & (A_{12}^v p_1 + A_{22}^v p_2 + \dots + A_{m2}^v p_m + N_m^v)(1 + \rho_2^v) \\
 & \qquad \qquad \qquad + wL_n^v = p_2 X_2^v + N_2^v \\
 & \dots \dots \dots \\
 & (A_{1n}^v p_1 + A_{2n}^v p_2 + \dots + A_{mn}^v p_m + N_n^v)(1 + \rho_n^v) \\
 & \qquad \qquad \qquad + wL_n^v = p_n X_n^v + N_n^v
 \end{aligned}$$

In equation (6), the rates of gross return ρ are the weighted averages of the rate of profit and the rents of lands, *i.e.* if K be the value of capital stock in a process and N be the quantity of land, then we define ⁽¹⁾

$$1 + \rho = \frac{K}{K+N} (1+r) + \frac{N}{K+N} (1+\pi) \quad [7]$$

Now ensuring a uniform rate of profit whilst ensuring a uniform rent for each grade of land implies that the rates of gross return ρ are equalised between the processes according to the grades of land used. Thus, we regroup the price

equations in (6) according to the grades of land used. There will therefore be ν groups of equations each containing n price equations,

$$(A_{11}^1 p_1 + A_{21}^1 p_2 + \dots + A_{m1}^1 p_m + N_1^1)(1 + \rho_1^1) \\ + wL_1^1 = p_1 X_1^1 + N_1^1 \\ \dots \dots \dots [8]$$

$$(A_{1n}^1 p_1 + A_{2n}^1 p_2 + \dots + A_{mn}^1 p_m + N_n^1)(1 + \rho_n^1) \\ + wL_n^1 = p_n X_n^1 + N_n^1$$

$$(A_{11}^2 p_1 + A_{21}^2 p_2 + \dots + A_{m1}^2 p_m + N_1^2)(1 + \rho_1^2) \\ + wL_1^2 = p_1 X_1^2 + N_1^2 \\ \dots \dots \dots [9]$$

$$(A_{1n}^2 p_1 + A_{2n}^2 p_2 + \dots + A_{mn}^2 p_m + N_n^2)(1 + \rho_n^2) \\ + wL_n^2 = p_n X_n^2 + N_n^2 \\ \dots \dots \dots$$

$$(A_{11}^\nu p_1 + A_{21}^\nu p_2 + \dots + A_{m1}^\nu p_m + N_1^\nu)(1 + \rho_1^\nu) \\ + wL_1^\nu = p_1 X_1^\nu + N_1^\nu \\ \dots \dots \dots [10]$$

$$(A_{1n}^\nu p_1 + A_{2n}^\nu p_2 + \dots + A_{mn}^\nu p_m + N_n^\nu)(1 + \rho_n^\nu) \\ + wL_n^\nu = p_n X_n^\nu + N_n^\nu$$

It is by transforming these ν groups of equations towards their standard equilibrium

multipliers and the rate of surplus produce. The v^{th} group of equations, the group of marginal processes is, of course, identical with equation (5) with which we began the iterations, whilst the rate e^v in the present notation is identical with the rate of growth, g , in equation (5). Similarly the rate of gross return ρ^v in the price equations of the marginal processes in (10) above is identical with the rate of profit r because the marginal processes earn no rent.

A description of the process by which the final equilibrium solution is found should suffice to complete our discussion.

It is evident that in each iteration the aggregate quantities of the capital goods required by the consumption goods industries will have to be increased or decreased in the direction of excess demands or supplies of the consumption goods. But since, in the present case, every capital good is being supplied by various processes, it will be necessary to apportion the aggregate quantity between these processes. A convenient procedure is to apportion the quantity in proportion to the rates of surplus produce in each process, so that those processes which produce

larger physical surpluses are called upon to bear greater responsibility in meeting the demands of the consumption goods industries.

It is also essential, as mentioned earlier, that at the beginning of every iteration the rates of surplus produce e^k be made uniform in order to uniformise the ρ^k between the commodities that use lands of quality k .

In the final equilibrium, the rate of profit will be determined in the group of price equations of the marginal process. This, as before, will be

$$r = \frac{g}{1-\alpha} = \frac{e^v}{1-\alpha} \quad [13]$$

With the rate of profit, the wage and the prices of capital goods. When these are substituted in the price equations (8), (9), (10) transformed by their respective multipliers as determined in (12), the rates of gross return ρ^k are determined. Finally, using equation (7) we can disentangle the rents π^k from the profits. In this manner the rates of 'gross return', ρ , will be split between the competitive rate of profit r on capital and the rents π^k of the lands.

The pattern of land allocation in the final

equilibrium will clearly be such that every grade of land earns the maximum rent that it possibly can. In other words, there will be no gain by shifting land from one occupation to another.

44. At this stage we shall drop two clearly unrealistic assumptions; firstly, that the relevant grades of land (v in the above discussion) is invariant with the occupations and secondly, that the v^{th} land is marginal in the production of every commodity. We shall now allow the pattern of gradation of land to change and allow the grade which is marginal to also change.

The result is that the number of price equations will now be equal to the product of the number of commodities and the total number of grades distinguished. Apart from this there will be no special problem. All that we require is that the marginal land in each occupation be known and that the marginal land satisfy the criterion stated at the outset.

45. It has been tacitly assumed that the process by which land is transferred from one occupation to another in the course of attaining equilibrium is either through owners themselves changing the proportions of land devoted to various

occupations or through the mechanism of leasing. Because land itself was not supposed to have been transacted there was no occasion to introduce the price of land. In fact, the price of land is determined by the rent and the rate of profit. Thus, in equilibrium, the prices of the various grades of land are determined by

$$P^k = \frac{\pi^k}{r} \quad k=1, \dots, v \quad [14]$$

46. This relationship of the rent of land, the rate of profit and the price of land immediately suggests an alternative, and in some ways a more straightforward approach to solving the system.

The alternative method is to write equations (8), (9), (10) in the following v sets containing n equations each,

$$\begin{aligned} (A_{11}^k p_1 + A_{21}^k p_2 + \dots + A_{m1}^k p_m + P_1^k N_1^k)(1+r) \\ + wL_1^k = p_1^k X_1^k + P_1^k N_1^k \\ (A_{12}^k p_1 + A_{22}^k p_2 + \dots + A_{m2}^k p_m + P_2^k N_2^k)(1+r) \\ + wL_2^k = p_2^k X_2^k + P_2^k N_2^k \\ \dots \dots \dots [15] \\ (A_{1n}^k p_1 + A_{2n}^k p_2 + \dots + A_{mn}^k p_m + P_n^k N_n^k)(1+r) \\ + wL_n^k = p_n^k X_n^k + P_n^k N_n^k \end{aligned}$$

In this reformulation the values of land $P_j^k N_j^k$ have been included amongst the means of production, and like the usual working capital items, marked up by the factor $(1+r)$. Of course, the value of land cancels out on both sides of the equations for each process.

As before, in the marginal processes, the rent π^v being zero, the prices of marginal lands P^v are zero with the effect that the marginal land is entirely eliminated from the means of production. Thus, the marginal processes will determine the rate of profit, the wage and the relative commodity prices just as they did earlier. When these are substituted in equations (15) we get $n(v-1)$ prices for the remaining $(v-1)$ grades of land being used in the n commodities.

In equilibrium, however, the price of land must be uniform irrespective of the commodity in whose production it is used, *i.e.*,

$$P_j^k = P^k \quad j=1, \dots, n \quad [16]$$

And this is assured by reallocating lands in accordance with the scale multipliers determined in equation (12)

The remaining steps are identical with those

outlined in the earlier method except for the circumstance that, under the present method, the rents of the various lands will be determined by equation (14) only after the equilibrium prices of land and the equilibrium rate of profit have been determined.

47. Additional complications arise when we pass on to include the presence of several natural resources each of different grades.

Firstly, the number of price equations for each commodity will now be equal to the number of grades of the maximally graded natural resource used in its production. Secondly, although the assumption of the existence of a marginal process will continue to hold, it will now stand generalised to require that the marginal process be marginal with respect to all the natural resources used in its production. In other words, that process which uses the least fertile grades of all the natural resources will be the marginal process.

The further complications are encountered in the procedure that must be used to allocate the several grades of the several natural resources between the commodities so as to ensure that each earns the maximum possible rent which, in

equilibrium, will be uniform irrespective of the commodity in whose production they may be employed. This, it will be recalled, is effected by ensuring a uniformity of the rates of gross return on the capital-cum-land used in different commodities (equation 11)

To achieve this object it will be necessary to construct appropriate standard systems to allocate the various grades of land between the commodities in such a way as to equalize the rates of gross surplus, e (equation 12). The complication that arises in the present case is that the multipliers which allocate the lands can be derived by using as constraint (the last equation of equation (12)) a variety of grades of natural resources.

In order to get over this difficulty it will be necessary to construct standard systems with all the possible "land-conservation-equations" (last equation in (12)) and select that equation which results in the *lowest* rate of gross surplus. The grade for which the rate of gross surplus is the lowest is the "limitational resource" for the group of processes that produce the various commodities.

48. Uptil this stage we have assumed that the

lands possess, in Ricardo's words, "original and indestructible powers"; or, in other words, that the natural resources are inexhaustible. This assumption is obviously not generally true and needs to be dropped. To accommodate the case of exhaustible natural resources the (annual) reduction in the original productive power must be subtracted from the side of the outputs in equation (1). Thus, if δ_j^k represents the proportion of degradation/irrevocable utilisation, if land of grade k is used in the production of commodity j , then the term $\delta_j^k N_j^k$ will be subtracted from the right-hand-side of the equations (1) (or added to the left-hand-side).

One of the effects of the exhaustion of the natural resources is that the rate of growth of the economy reduces as does the rate of profit, the wage and rents, as compared to the situation in which there is no exhaustion.

49. Periodic exhaustion of the natural resources or the emergence of scarcity in relation to incomes usually has two effects. First, they prompt a search for new sources and second, they prompt the search for such materials as will substitute the expensive resources in some, or all, lines of

production.

The discovery of new natural resources, the discovery of "land-saving techniques" and the discovery of new materials that substitute relatively abundant natural resources for the relatively scarce ones offset over time, the process of decreasing returns, which in the normal course would have led the economy to stagnation.

NOTES

1. It may be wondered whether we have violated the equivalence of the dimensionality of each side of the equations in (6) by including the quantity of land with the other means of production irrespective of the price of land. This is not so. The factors $(1+\rho)$ are pure numbers so that the land N must be read as $N \times 1$, where the number 1 has such dimensions as will convert the "acres" of land into the units of the *numeraire* without affecting the *number* which represents the quantity of land.

PART THREE

MIXED ECONOMY

CHAPTER IX

PUBLIC GOODS

- 50. Consequences of introducing public goods.**
- 51. The system of equation with one public good financed by an income tax.**
- 52. Steps for solving the system.**
- 53. Several public goods.**
- 54. Relation between the rate of profit, rate of taxation and public expenditure.**
- 55. Public expenditure considered 'autonomous'.**
- 56. Tax on wage incomes.**
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- 58. The system with a sales tax charged at**

a uniform rate.

- 59. Different sales tax rates for different commodities.**
- 60. Public sector industry operated on a zero-profit price policy.**
- 61. Appropriate rate of profit for a public sector industry.**

CHAPTER IX

PUBLIC GOODS

50. Public goods which are consumed collectively and are not, as a consequence, bought and sold in markets affect the specification of the system of prices and outputs in two ways.

Firstly, since neither the producing industries nor the consumers purchase public goods from their disposable incomes, there can be no demand equations for them of the types applicable to the regular commodities. It follows at once that the quantities of public goods cannot be determined by market demand and supply. Instead, these must be determined by the government independently of the unknowns of the system.

Secondly, it is evidently impossible to identify the prices of public goods because public goods do not have to be allocated between their users as the private goods are. For, although

public goods are 'economic' from the side production, in as much as they require the use as inputs of economic goods, they are not economic when viewed from the side of consumption. Thus, the term 'cost of production' will apply to these goods but the category of *price* will not.

The result is that neither the quantities nor the prices of the public goods will appear in the system as unknowns.

51. The essentials of the procedure that will be adopted to incorporate public goods can be explained by considering the simple decomposable system of Chapter III. Suppose that there are m (working) capital goods, $n - m - 1$ consumption goods and 1 public good, say defence, represented by the n^{th} good in the system. Further suppose that the government levies an income tax at a uniform rate, t , on the profit incomes in the economy and uses the proceeds to provide defence.

In so doing the government will first be required to determine the adequate supply of defence, *i.e.* the scale at which the activity supplying defence should be operated. In other words, the government must determine the set of

inputs \bar{A}_{1n} , \bar{A}_{2n} , ..., \bar{A}_{n-1n} , \bar{L}_n , viz., rifles, bullets, uniforms, soldiers, *etc.* independently of the prices or the other unknowns, *i.e.*, on the basis of such data as the extent and nature of the territory, the belligerence and the level of armamentation in the enemy countries, the likely pattern of coalitions in the event of war, the extent of crime and terrorism within the country, *etc.* This, it must be noted, presumes that the demand for and the supply of defence are equal at all times — any discrepancy arising between them will be settled by the administrative and political process, independently of the price system.

Our system will contain n price equations; m for the capital goods, $n - m - 1$ for the consumption goods and 1 for defence. The price equation for defence is,

$$\bar{A}_{1n}p_1 + \bar{A}_{2n}p_2 + \dots + \bar{A}_{n-1n}p_{n-1} + w\bar{L}_n = X_n, \quad [1]$$

where X_n is the expenditure on defence. By presumption, the activity of supplying defence will not earn profit. Next, there are m demand equations for the capital goods,

$$(1 - \alpha)(1 - t)rA_j = F_j, \quad j = 1, \dots, m \quad [2]$$

where A_j is the quantity of commodity j used in the

production of all the regular goods (excluding the quantity used in defence) *i.e.*,

$$A_j = \sum_{j=1}^{n-1} A_{ji}$$

and

$$F_j = X_j - A_j - \bar{A}_{jn}$$

Further there are $n - m - 1$ demand equations for the consumption goods,

$$(1-t)\alpha_r K_p + \beta_j w L + p_j \bar{A}_{jn} = p_j X_j \quad j = m + 1, \dots, n - 1 \quad [3]$$

where K_p is the capital stock in the private sector of the economy. Finally, there is a 'collective' demand equation for defence which will be represented by the balanced-budget equation

$$trK_p = X_n \quad [4]$$

When we add to this system the conservation - of - labour equation, we get $2n+1$ equations to solve for $2n+1$ unknowns, *viz.*, $n - 2$ relative prices, $n - 1$ levels of output, the expenditure on defence, the income tax rate, the wage and the rate of profit.

It will be observed that in the peculiar circumstance of the present case two unknowns, *viz.* the price and the quantity of defence, are eliminated and their place is taken by the tax rate and the expenditure on defence.

52. The procedure to solve the system is not without peculiarity either; the main complication arises from the fact that the rate of profit and the rate of tax, both unknowns of the system, are present as a product in equations (2) and (4). Now, the value of the one cannot be known without knowing that of the other. But varying any one not only varies the other but also *all* the unknowns of the system. Hence, the iterations to equilibrium must be more cautious and gradual than would otherwise be the case.

To start with, we formulate the equations to determine an initial solution for the rate of growth and the set of scale multipliers which balance the demands and supplies of the capital goods ensuring, however, that the capital good requirements of the defence industry are also satisfied. These equations are as follows,

$$(A_{11}\lambda_1 + A_{12}\lambda_2 + \dots + A_{1m}\lambda_m + \Lambda_1)(1+g) + \bar{A}_{1n} = \lambda_1 X_1$$

$$(A_{21}\lambda_1 + A_{22}\lambda_2 + \dots + A_{2m}\lambda_m + \Lambda_2)(1+g) + \bar{A}_{2n} = \lambda_2 X_2$$

..... [5]

$$(A_{m1}\lambda_1 + A_{m2}\lambda_2 + \dots + A_{mm}\lambda_m + \Lambda_m)(1+g) + \bar{A}_{mn} = \lambda_m X_m$$

$$\lambda_1 L_1 + \lambda_2 L_2 + \dots + \lambda_m L_m = L_B$$

where Λ_j is the quantity of commodity j used in the production of the regular consumption goods industries at the postulated levels of their outputs.

Equations (5) gives a solution for m multipliers and the rate of growth. It is clear from equation (2) that the rate of profit can be derived from the rate of growth by the equation,

$$r = \frac{g}{(1 - \alpha)(1 - t)} \quad [6]$$

But, since the tax rate, t , is also an unknown, an initial solution for the rate of profit can be found only by plugging in a value for t , which, however, must be such that the resulting rate of profit does not exceed the maximum rate, R .

The initial solution for the rate of profit so arrived at can be substituted in the m price equations of the capital goods to obtain initial solutions for their $m - 1$ relative prices and the wage. When these, in turn, are substituted in the $n - m - 1$ price equations of the final consumption

goods, we get the initial solutions for the prices of the consumption goods. Next, we can substitute the $n - 1$ relative prices in equation (1) to obtain an initial solution for the public expenditure required for defence. Further, substitution of the prices, the rate of profit, the wage and the tax rate into equations (3) will give an initial solution for the quantities demanded of the consumption goods, X_j^d . And, when the same are substituted in equation (4) we get the initial solution for the tax revenue actually collected by the government.

In the next iteration we change the levels of output of the consumption goods, X_j , in the direction of their quantities demanded. At the same time the tax rate must be changed in the direction of the level of public expenditure determined in equation (1). The necessity of this latter step becomes clear when it is observed with reference to equations (1) and (4), that the tax rate in equilibrium is simply the fraction of the tax base (profits, in this case) which is devoted to defence expenditure.

Further iterations will follow the steps that have been explained and continued until a set of values which satisfy all the equations have been

found.

53. When several public goods are considered instead of one, the system of equations will be augmented by an equal number of price equations and an equal number of unknowns, *viz.* the expenditures incurred to supply the public goods.

The equations themselves will require two modifications. Firstly, in equations (2), the A_j will exclude the quantity of commodity j that is required by *all* the public goods industries ; which implies that the scale multipliers determined in equations (5) will be such as to provide for the exogenously determined capital goods requirements of all the public goods industries. Secondly, a similar modification will have to be effected in the consumption demand equations (3) to incorporate the exogenously determined purchases of the consumption goods by the public goods industries.

54. It is worthwhile to consider briefly the relationship between the tax rate, the rate of growth and the rate of profit. An additional facet of this relationship is revealed if we substitute for the tax rate in equation (6), the proportion of public expenditure in the tax base (*i.e.* profits)

$$r = \frac{g}{(1-\alpha)(1-t)} = \frac{g}{(1-\alpha)} + \frac{X_n}{K_p} \quad [7]$$

It is clear from equation (7), and the method of solving the system which has been described earlier, that any increase in the supplies of public goods requiring, of course, an increase in the tax rate, will reduce both the rate of growth and the rate of profit but reduce the rate of growth more than the rate of profit.

Despite the fact that both the before-and-after-tax rates of profit decrease with an increase in the tax rate, the wage does *not* increase, as would have been expected at first glance. On the contrary, the wage also decreases to finance a part of the increased expenditure on public goods even though the wage income is not directly subject to income taxation in the case under discussion.

55. In so far as the supplies of public goods are not required to be increased with increases in the population and the wealth in the economy, the tax rate would, in the normal course of development, decline over time.

But, if, as it usually happens, the government does not reduce the tax rate, the tax proceeds of the

government will increase over time at the rate of growth of the economy. The surpluses in the budget are bound to induce the government to expand the public good industries and result, in particular, in an 'arms-race'.

The last remarks, it will be rightly felt, not only overstep the logical confines of the model but even contradict the stand that has been consistently maintained so far that the government's only role is one of supplying predetermined quantities of public goods. For, it is on this premise that the tax rate derives its status as an *unknown* of the system. But the remarks will be seen to gain validity when we reverse this position and consider instead the tax rate to be predetermined and allow the scales of activity in the public goods industries to be endogenous.

It seems that in practice a combination of the two mechanisms work. The government's revenue increases if the tax rates are not reduced and allows the government to spend more. At the same time the ministries/departments of the government, who have this knowledge, increase their annual demands for grants. In deciding how many of the demands should be satisfied, and to what extent, the government implicitly decides the

tax rate.

An important consequence of an autonomously determined tax rate with endogenised government purchases should be noted. So long as the coefficients of allocation of the tax revenue between the expenditures on the various public goods remain stable there would be no problem for the existence of an equilibrium growth path. But if these coefficients and/or the tax rate were altered frequently, balanced growth is jeopardised.

56. We now turn to consider the effects that would follow if the tax were to be imposed on the wage incomes besides the profits. In the demand equations (3) for the consumption goods there will now be the disposable wage income instead of the entire wage income. Similarly the budget equation (4) will be amended to include the tax revenue on the wages.

We can guess at the outset that a widening of the tax base will result in a lower rate of the tax and higher rates of growth and profit as compared to the original situation.

In like vein, the case of saving from wages can also be included by following the method used

in Chapter II. Once again it will be found that, despite the new fact of the taxation of wages, the relation between the rates of profit, growth and taxation remains just what it is when wages are not taxed. It is obvious though that the rates of growth and profit will now be higher and the rate of tax lower

57. As regards the effect of a tax on the rents of natural resources, it is hardly necessary to repeat the established conclusion that such taxes cannot be 'passed on'.

This, however, should not be considered to mean that taxes on rent have *no* effect on the other values in the system. One effect of the tax on rent is to lower the demand for consumption goods and raise to some extent the rates of growth and profit. The second effect is that taxation of rent makes it possible to finance a given supply of public goods by means of a lower tax rate and results in higher rates of growth and profit.

58. We shall now detail the system when instead of a tax on income, sales taxes (*ad valorem* or excise duties) are used to finance public expenditure.

Thus suppose that a sales tax is levied at a

uniform rate, t , on the sales revenues of the industries producing the regular commodities. The price equations will now be as follows,

$$(A_{1i}p_1 + A_{2i}p_2 + \dots + A_{mi}p_m)(1+t) + wL_i = p_iX_i(1-t) \quad [8]$$

$i = 1, \dots, n-1$

where, the right hand sides are the net sales revenues of the $n-1$ industries.

The price equation of the public good, the demand equations for the capital goods and the demand equations for the consumption goods will remain unchanged. Our final equation, the balanced-budget equation, will be,

$$t(p_1X_1 + p_2X_2 + \dots + p_{n-1}X_{n-1}) = X_n \quad [9]$$

As usual we add to the system the conservation-of-labour equation to get $2n+1$ equations in $2n+1$ unknowns.

59. If the sales tax is imposed at different rates on different commodities, the uniform factor $(1-t)$ in the price equations (8) will be replaced by the factors $(1-t_j)$, where t_j is the rate of tax on the j^{th} commodity.

It is obvious that since all these rates cannot be determined by the system, they must be

determined exogenously. But this means that the number of equations will exceed the number of unknowns by one. To remedy this difficulty two approaches may be considered. The first is to regard one of the tax rates to be an unknown whilst the rest, $(n-2)$, are exogenously given. The second, a more realistic approach, is to consider all $n-1$ tax rates, the scales of activity of the public goods industries and the level of the public expenditure to be exogenously determined.

60. We shall now transgress the limits set by the title to this chapter and consider the government in its role as a producer of a private good.

If the government supplies a good j , following a zero profit pricing policy, it will be necessary to apportion a portion of the tax revenue of each year to enable the public sector industry to add to its capital stock. The tax revenue required by this aspect of the government's role will be determined in such a way that,

$$\Delta K_j = t r K_p \quad [10]$$

where ΔK_j is the addition to the stock of capital K_j in industry j and t is the income tax rate on private profit income which is necessary to finance the

additions to the capital stock in the public sector industry alone.

If balanced growth is to be maintained, the tax rate must be such as to equate the rate of growth of the capital stock in industry j to that of the overall economy. The tax rate which satisfies this condition is,

$$t = \frac{(1 - \alpha)K_j}{K_p + (1 - \alpha)K_j} \quad [11]$$

61. If, however, the public sector industry is to be operated without financial support from the government's annual budget, the price of the public sector industry's product must cover not only the cost of production but also a profit to provide for the expansion of capacity. This, however, need be charged only at the equilibrium rate of growth on the cost of production since public sector industries need not consume their profits at all.

APPENDICES

APPENDIX I

ALTERNATIVE METHOD OF SOLUTION

The method of solving the problem of Chapter II relied on arranging the equations as a homogenous system of equations. We outline below a method which relies on arranging them as a non-homogenous system.

As an illustration, consider a 'classical' economy in which all profits are accumulated and all wages consumed. The equations are,

$$(A_{11}+A_{21}p)(1+r) + 0p + wL_1 = X_1$$

$$(A_{12}+A_{22}p)(1+r) - X_2p + wL_2 = 0$$

$$A_1(1+r) + 0p + \beta wL = X_1$$

They can be arranged in vector-matrix notation,

$$\begin{bmatrix} K_1 & 0 & L_1 \\ K_2 & -X_2 & L_2 \\ A_1 & 0 & \beta L \end{bmatrix} \begin{bmatrix} v \\ p \\ w \end{bmatrix} = \begin{bmatrix} X_1 \\ 0 \\ X_1 \end{bmatrix}$$

where $K_1 = A_{11} + A_{21}p$, $K_2 = A_{12} + A_{22}p$ and $v = 1+r$

By Cramer's rule we can solve for the relative price p ,

$$p = \frac{\begin{vmatrix} K_1 & X_1 & L_1 \\ K_2 & 0 & L_2 \\ A_1 & X_1 & \beta L \end{vmatrix}}{\begin{vmatrix} K_1 & 0 & L_1 \\ K_2 & -1 & L_2 \\ A_1 & 0 & \beta L \end{vmatrix}}$$

This gives a quadratic polynomial in p whose lowest positive root is the equilibrium relative price. It can be substituted in the price equations to obtain the corresponding solution for the rate of profit and the wage.

APPENDIX II

DIFFERING TIMES OF PRODUCTION

1. Throughout the text we have assumed rather simplistically that the period of production of all commodities is "one year" and that the wages are always paid *post-factum*. This appendix shows how the cases more general than this are to be treated. Although we shall use the problem of Chapter II as a working illustration the methods outlined apply to all the chapters in the text.

First, consider the case of the wage being paid *pre-factum*. The price equations will be,

$$(A_{11} + A_{21}p)(1+r) + wL_1(1+r) = X_1$$

$$(A_{12} + A_{22}p)(1+r) + wL_2(1+r) = pX_2$$

If we consider the case of the consuming economy we can write the equations in the following form,

$$\begin{bmatrix} X_1 - A_{11}v & -A_{21}v & -L_1v \\ -A_{12}v & X_2 - A_{22}v & -L_2v \\ F_1 - \alpha A_1(v-1) & -\alpha A_2(v-1) & -\beta + \alpha(v-1)L \end{bmatrix} \begin{bmatrix} 1 \\ p \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and proceed with the solution in the usual manner.

2. It has also been assumed throughout that the period of production of all commodities is one year. We drop this and suppose that commodity 2 requires one-half of a year to produce. This circumstance by itself implies that there must now exist a process which stores commodity 2 for one-half of a year. Thus, the price equations will be,

$$(A_{11} + A_{21}p)(1+r) + wL_1 = X_1$$

$$(A_{12} + A_{22}p)(1+r) + wL_2 = pX_2$$

which, for an all-consuming economy, will give the following system,

$$\begin{bmatrix} X_1 - A_{11}v & -A_{21}v^{3/2} & -L_1 \\ -A_{12}v & X_2 - A_{22}v^{3/2} & -L_2 \\ F_1 - \alpha A_1(v-1) & -\alpha A_2(v^{3/2}-1) & -\beta L \end{bmatrix} \begin{bmatrix} 1 \\ p \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The polynomial which determines the rate of profit will now contain non-integral powers. But it can be transformed by suitable substitutions into a form with integral powers. If commodity 2 was perishable, *i. e.* there was no technique capable of storing it for half a year, then commodity 1 would be stored.

3. A similar procedure holds when the commodity inputs in individual industries have differing 'durabilities', *i.e.* times of application. Thus, suppose, for example, that the time of application of commodity 1 in the production of commodity 2 is one year but that the time of application of commodity 2 is half - a - year. Then that at the end of each year a quantity A_{22} will have to be set aside in storage to be applied six months later. At the end of six months the value of this would be,

$$pA_{22}(1+r)^{1/2}$$

This value would go into production for the latter half of the year and would increase to

$$[pA_{22}(1+r)^{1/2}] (1+r)^{1/2} = pA_{22}(1+r)$$

which is the value which must be recovered from the sales revenue pX_2 of the second industry.

APPENDIX III

REMARKS ON THE DEMAND EQUATIONS

1. The manner in which the demand equations for the consumption goods have been specified may create the impression that it is tacitly assumed that the Engels coefficients do not vary with the level of consumer income - an assumption which is clearly at variance with the facts. No such assumption is required. For, the relationship of the Engels' coefficients and the level of income holds over time. Thus, once the size of the income and its changes over time have been ascertained the relationship can always be incorporated.

2. What has been assumed, however, is that the Engels' functions, whatever may be their shape, invariably pass through the origin. This assumption is restrictive considering that the empirically estimated linear expenditure systems have usually yielded functions with non-negative intercepts.

To incorporate these, the demand equations must be specified as,

$$\alpha_j rK + \beta_j wL = p_j(X_j - X_j^*) \quad j = m+1, \dots, n.$$

where $p_j X_j^*$ are the intercepts of the empirically estimated functions.

3. The major shortcoming, however, lies in the absence of a theory of consumer behaviour which explains the Engels' coefficients themselves. In the circumstance, the Engels' coefficients must be considered as entering the system as data on the same footing as the technical coefficients of production.

4. It may, in this context, seem surprising that no reference should have been made to utility theory to provide the basis for the postulated demand equations. There are two reasons for this. The first reason is that utility theory and general equilibrium are compatible only if it is assumed that all commodities are substitutes of one another. This is clearly an unduly restrictive requirement. To dispense with it, it seemed best to dispense with all references to utility theory.

The second reason is that utility theory, in the form in which it has been developed, is not designed to explain the Engels' coefficients of consumption. And addilog utility functions which

are alleged to provide the basis of linear expenditure systems are merely *post-factum* rationalisations.

5. But the most compelling reason is that a utility-theoretic specification of the demand equations imposes the use of intertemporal utility functions to explain the saving behaviour. The use of intertemporal utility functions is altogether inimical to the spirit of the exercise presented in the text because these functions are based on a spurious theory which discount the future consumptions when they appear in the wealth/budget constraints but choose not to discount them in the utility functions. Indeed, if future consumptions, or the utilities therefrom, are discounted both in the utility function as well as the wealth constraint, as they should be, the entire theory breaks down.

APPENDIX IV

THE SUBSISTENCE WAGE SOLUTION

1. The purpose of this chapter is to outline, and criticise, briefly a solution to our problem based on the assumption of a subsistence wage. This solution, proposed by the classical economists from Ricardo (1817) to Marx (1867), was first used for a mathematically explicit solution by von-Neumann (1945-6) and is followed even to this day in the literature on the von-Neumann and Sraffa systems.

It is assumed that a basket of quantities of consumption goods c_j necessary for a worker's physiological/social survival can be identified independently of the prices so that every worker is paid a value,

$$w_s = \sum_{j=1}^n p_j c_j$$

The price equations can then be rewritten by adding the coefficients of the necessary

consumption of each commodity to the technical coefficients of production, *i.e.*,

$$A_{ji}^* = A_{ji} + c_{ji}L_i$$

and the equations can be arranged in vector - matrix notation as,

$$\begin{bmatrix} A_{11}^* + rA_{11} & A_{21}^* + rA_{21} & \dots & A_{n1}^* + rA_{n1} \\ A_{12}^* + rA_{12} & A_{22}^* + rA_{22} & \dots & A_{n2}^* + rA_{n2} \\ \vdots & \vdots & & \vdots \\ A_{1n}^* + rA_{1n} & A_{2n}^* + rA_{2n} & \dots & A_{nn}^* + rA_{nn} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_n \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} \quad [1]$$

and solved for a unique positive rate of profit and the relative prices.

At the same time the dual of this system, the system of output determination will solve for the outputs that will be required to meet the final

consumption demands.

$$\begin{bmatrix} A_{11}^* + rA_{11} & A_{12}^* + gA_{12} & \dots & A_{1n}^* + gA_{1n} \\ A_{21}^* + rA_{21} & A_{22}^* + gA_{22} & \dots & A_{2n}^* + gA_{2n} \\ \vdots & \vdots & & \vdots \\ A_{n1}^* + rA_{n1} & A_{n2}^* + gA_{n2} & \dots & A_{nn}^* + gA_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ 0 & 0 & \dots & X_n \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} \quad [2]$$

We note however that the system above gives a unique positive solution for the *relative* outputs and the rate of growth. In order to obtain the absolute levels of output it is necessary to add Sraffa's conservation-of-labour equation.

$$\lambda_1 L_1 + \lambda_2 L_2 + \dots + \lambda_n L_n = L \quad [3]$$

2. The most dissatisfying feature of this elegant solution is, of course, the subsistence wage assumption. That its application does not extend beyond a "slave economy" to a modern capitalist economy whose essential feature is the individual freedom to consume (or save) in accordance with

tastes and preferences should be clear at once.

Further, the assumption of a subsistence wage is tantamount to assuming away the institution of markets which co-ordinate investment, consumption and employment decisions.

Moreover, the quantities of commodities that are consumed, in other words the "standard of living", are themselves *unknowns* of economic interest, so that to consider them as *given* at the very outset is like giving out the answer before asking the question.

Finally, when different categories of workers are being paid different wage rates, the assumption of a subsistence wage loses application altogether.

It is, therefore, no wonder that the subsistence wage assumption was criticised so heavily by the early neoclassical writers, notably Menger (1871), Jevons (1871) and Walras (1874) for having no analogue in reality. Indeed it may be said that the rise of utility theory to portray the behaviour of consumers was premised on the "sovereignty" *i.e.* the autonomy of the consumer.

What can be more, Sraffa, writing in 1960,

explicitly allows the wage to be 'variable'*i.e.* consisting of commodities other than the specified quantities of necessities for physiological/social survival of the workers.

3. For these reasons, and on account of the fact that income-consumption demand relations have been the subject of significant empirical enquiry [Klein and Rubin (1947-8), Geary (1949-50), Stone (1954)], resort has been taken to the use of the observable "propensities to consume".

There can, of course, be no doubt that the propensities to consume of individuals themselves depend on the incomes. This relationship, however, is easily accommodated to study the problems of secular dynamics (See Chapter III section 7). As for the *sources* of changes in incomes, we look not towards consumer decisions but rather to developments in the sphere of production. *i.e.* in techniques of production, in the development of skills, and in the discovery of natural resources and other primary products.

4. Besides the subsistence wage solution, which is formally complete, there are to be found in the literature two approaches towards the

problem. The first considers the indeterminate Sraffa system of price equations itself as an adequate portrayal of the modern capitalist economy. We are told that "social" and "political" forces determine the exact between wages and profits and the technological price equations crunch out the price solutions.

The second, also beginning with the indeterminate Sraffa system, suggests that the system can be closed by the Central Bank of the economy determining the basic rate of interest, from which the rate of profit can be derived (by adding transactions costs, risk premia etc.) and the price equations accomplish the rest.

The first view is far-fetched; it conjures an image of two 'classes' in a tug-of-war. The second view leads directly to the ludicrous conclusion that capitalists should find its most reliable ally, for determining its share in the national product in the Central Bank of the economy.

BIBLIOGRAPHY

Domar E.D.(1946) "Capital Expansion, Rate of Growth and Employment." *Econometrica* Vol.14.

Geary R.C. (1949-50) "A Note on a Constant - Utility Index of the Cost of Living" *Review of Economic Studies* Vol. 18.

Harrod R.F.(1939) "An Essay in Dynamic Theory." *Economic Journal* Vol. 49.

Hawkins D. and H.Simon (1949) "Note : Some Conditions of Macroeconomic Stability." *Econometrica* Vol.17.

Hicks J. R. (1974) *Crisis in Keynesian Economics* London : Oxford University Press.

Jevons W.S. (1871) *The Theory of Political Economy*. London.

Kaldor N.(1956) "Alternative Theories of Distribution." *Review of Economic Studies* Vol.23.

Keynes J.M. (1936) *The General Theory of Employment Interest and Money*. London : Macmillan

Klein L.R. & H. Rubin (1947-8) "A Constant - Utility Index of the Cost of Living." *Review of Economic Studies* Vol. 15.

Leontief W.W. (1941) *The Structure of the American Economy*. Cambridge : Harvard University Press.

Lerner A.P. (1952) "The Essential Properties of Interest and Money." *Quarterly Journal of Economics*. May.

Marshall Alfred (1910) *Principles of Economics* London : Macmillan.

Marx Karl (1867) *Capital* Vol. I London : Lawrence and Wishart reprinted. 1970.

Meade J.E. (1963) "The Rate of Profit in a Growing Economy." *Economic Journal*. Vol. 73.

Menger Carl (1871) *Principles of Economics*. trans. & ed. B.F.Hoselitz and J. Dingwall. Glencoe. III. 1950.

von Neumann J. (1945-46) "A Model of General Economic Equilibrium." *Review of Economic Studies*. Vol.13.

Pasinetti L.L. (1962) "Rate of Profit and Income

Distribution in Relation to the Rate of Economic Growth. *Review of Economic Studies*. Vol.29.

Ricardo David (1817) *Principles of Political Economy and Taxation*. reprinted as Vol. I of *David Ricardo Works and Correspondence* ed. P.Sraffa, 1951.

Robinson Joan (1956) *The Accumulation of Capital*. London : Macmillan.

Samuelson P.A. and F. Modigliani (1966) "The Pasinetti Paradox in Neoclassical and More General Models." *Review of Economic Studies*.

Sraffa Piero (1932) "Dr.Hayek on Money and Capital." *Economic Journal*.

Sraffa Piero (1960) *Production of Commodities by Means of Commodities*. London : Cambridge University Press.

Stone R. (1954) "Linear Expenditure Systems and Demand Analysis : An Application to the Pattern of British Demand." *Economic Journal*. Vol. 64.

Tobin J. (1959-60) "Towards a General Kaldorian Theory of Distribution." *Review of Economic Studies*.

Walras Leon (1874) *Elements of Pure Economics*.
trans. & ed. W.Jaffe. London, 1954.

Wicksell Knut (1898) *Interest and Prices*
trans. & ed. R.F.Kahn. New York :
A.M.Kelley, 1936.

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E R R A T A

- P. 16.* In the matrix, the second row second column element should be $X_2 - A_{22} (1 + r)$
- P. 39.* The second to last equation should be;
- $$A_j^* = \sum_{i=1}^m \lambda_j A_{ji} + \sum_{i=m+1}^n A_{ji}$$
- P. 76.* The vector on the left-hand side in (13) contains λ_m^* above the element 1 in place of λ_2^*
- P. 77.* Second para. The values of the multipliers derived apply to the new machines. The multipliers appropriate to the various age categories are derived as,
- $$\begin{aligned} \lambda_j^0 &= \lambda_j^* \\ \lambda_j^1 &= \lambda_j^* / (1+g)^{s-1} \\ \lambda_j^2 &= \lambda_j^* / (1+g)^{s-2} \\ &\dots\dots\dots j = 1, \dots, n, \quad (14) \\ \lambda_j^{s-1} &= \lambda_j^* / (1+g) \end{aligned}$$
- P. 92* Section 35, line 8; Read “applicable to workers entering freshly; those applicable to the later age groups being successively lower by the factor $(1 + g) \dots$ ”
- P. 117.* The first two expressions in the last equation in (12) are, $\lambda_1^k N_1^k$ and $\lambda_2^k N_2^k$
- P 132.* The last term on the left - hand side of equation (3) should be $p_j A_j$
- P 143* Last sentence. Read “capital stock” in place of “cost of production”.

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