

Central Board of Irrigation, India.
Publication No. 12. (Reprint)

DESIGN OF WEIRS
ON
PERMEABLE FOUNDATIONS

BY

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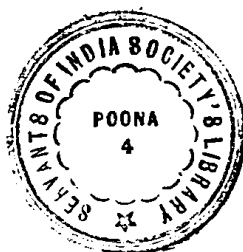
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FOREWORD

CENTRAL BOARD OF IRRIGATION,
SIMLA.

The Central Board of Irrigation published in December 1934 its eighth publication under the title "Observation and record of Pressures below Works on Permeable Foundations", which was edited by Rai Bahadur A.N. Khosla, I.S.E., B.A., M.Am.S.C.E., A.M.I.E. (India). The object of that publication was, primarily, to set forth the factors involved in the design of hydraulic works on permeable foundations, the up-to-date knowledge on the subject, and the information further required for the solution of the problem. That Publication also described the methods by which further information could be acquired, and recommended the introduction of observation pressure pipes on all new works of that type. The Publication was given a wide distribution and recipients were asked to provide the Board with any information likely to prove useful in further study of the problem. Very little information has been received however, chiefly owing to the fact that few works of this nature have recently been constructed and provided with the necessary observation pipes. Fortunately, however, exhaustive investigations have been carried out by the Authors of this new Publication, with the result that they are now able to present a complete solution to the problem, which is supported by data from prototypes in the field, models in the laboratory, and by mathematical theory. The contents of this Publication are the result of an investigation in the Punjab Irrigation Department over a period of nine years, and the Board is indebted to the Punjab Members of the Board, and to the Authors, for the opportunity of publishing this valuable treatise on one of the first problems which received the attention of the Board at the time of its constitution in November 1930.

This Publication has been compiled by Rai Bahadur A. N. Khosla, who has also been responsible for the field research, for the engineering interpretation of the laboratory investigations and their application to design. The laboratory investigations at the Punjab Irrigation Research Institute have been carried out by Dr. V. I. Vaidhianatban, Dr. H. L. Uppal and Mr. J. K. Malhotra under the direction of Dr. E. McKenzie Taylor. The theory of subsoil flow and the mathematics of weir design is the work of Dr. N. K. Bose.

M. T. GIBLING,
Secretary,
Central Board of Irrigation.

Dated 1st September, 1936.

IN RODUCTION

The Design of Weirs on Permeable Foundations was one of the first subjects taken up for enquiry by the Central Board of Irrigation in their first¹ meeting in November, 1930.

The flow of water through the subsoil flow below Weirs, with its attendant hydraulic gradients and uplift pressures, has been recognised as the determining factor in design, ever since 1895, when Col. Clibborn carried out his classic experiments with Khanki sand at Roorkee. Until recently, the so-called Bligh Creep Theory—which was no more than an imperfect adaptation of Col. Clibborn's work—held the field. It has been realised, eventually that the subject is more complex than the simple formula of Bligh indicated.

After some considerable discussion at the Central Board meeting of November 1933² the following resolution was adopted unanimously:—

- (a) that further intensive study of subsoil flow beneath actual works was assential,
- (b) that in view of the active part already taken by Mr. Khosla, and his published papers on the subject, the compilation of a comprehensive note on the subject should be entrusted to him,
- (c) that Mr. Khosla should be supplied with all available observations on the subject from India,
- (d) that in order to secure uniformity of observations he should prepare a note on the nature of observations required and the form in which they should be tabulated; this note to be communicated to all interested, through the office of the Board.

In compliance with item (d) of this resolution, Rai Bahadur A. N. Khosla, I.S.E., presented a note on the Observations and Record of Pressures below Works on Permeable Foundations, which was printed in December, 1934, as Publication No. 8 of the Central Board of Irrigation.

All available copies of this Publication were soon distributed and there was demand for more. It was, at first, proposed to issue a reprint, but in view of the considerable advance that had been made, in the meantime, on the study of the subject, both in the field and in the laboratory, the Executive Committee of the Central Board resolved (20th July, 1935) to have the Publication No. 8 revised or to have a new paper written on the subject. Sir Bernard Darley in his letter No. 9453, dated the 29th July, 1935, suggested that a tentative set of rules for guidance in design be framed by Mr. Khosla and discussed at the November meeting of the Board with a view to their incorporation in the new publication. During the interval before November, the field and laboratory researches reached a stage when the final solution of the problem came within sight so that it was possible to lay down definite, and not merely tentative, rules for guidance in design. This fact along with a general review of the state of knowledge to date, was presented by Mr. Khosla³ at the meeting of the Central Board held at Delhi in November, 1935. As a result of discussion of this, and in view of item (b) of the Resolution No. 7 of November, 1933, requiring the writing up of a comprehensive note on the subject, it was decided that the new publication should embody all available information on the subject in addition to the essential matter already contained in the Publication No. 8. The President stressed the necessity for the revised Publication being complete in itself and not in any way supplementary to the first addition⁴.

The present publication incorporates the essence of Field Research in the Punjab since 1927 and of Laboratory Research carried out at the Punjab Irrigation Research Institute since 1932—the latter, under the direction of Dr. E. McKenzie Taylor. An attempt has been made to trace from its inception the history of development of the science of subsoil hydraulics as related to the design of Weirs on Permeable Foundations. Notice has been taken of all contemporary literature within the Authors' knowledge, and any help derived from such literature has been duly acknowledged in the text.

(1) Proceedings of the Central Board of Irrigation, first meeting, November, 1930, page 8.

(2) Minutes of the Proceedings of the 4th Annual Meeting of the Central Board of Irrigation held in Lahore, 13-15 November, 1933, page 12.

(3) Appendix II, Central Board of Irrigation. Publication No. 11, Annual Report 1934-35.

(4) Publication No. 11, page 50.

The publication has been divided into a number of self-contained Chapters. Of these Chapter VI is more or less a reproduction of the relevant matter in Publication No. 8. The remaining Chapters contain mostly new matter. The entire mathematics has been collected in one Chapter, No. VII. Chapter VIII, on exit gradients, deals with some fundamental principles in design.

The Plates, Diagrams, Equations and Tables in each Chapter bear the number of that Chapter in addition to their own distinctive numbers. Thus Plate VII. 6 indicates Plate No. 6 of Chapter VII. Similarly Equation No. 10. 2 indicates Equation No. 2 of Chapter X.

Every effort has been made to make the publication complete in itself, so that designs can be worked out in complete detail without much reference to outside literature. The incorporation in Chapter XI, of the essential diagrams relating to Energy of Flow and Standing Wave, originally prepared by Messrs. Montagu and Crump and modified to suit special problems by Mr. Kanwar Sain, will considerably facilitate the work of the designer.

While it can now be claimed that the design of Weirs on sand foundations in relation to flow of water through the foundation soil can be determined with great exactness in a rational and scientific manner from the simple rules given in Chapter X, its determination in relation to flow over the surface still depends mainly on empiricism. This latter aspect of the subject is dealt with in Chapter XI and calls for further intensive study in the field and more particularly in the Laboratory.

Acknowledgments

Dr. Vaidhianathan, Head of the Physics Section, Punjab Irrigation Research Institute, developed the technique of the Electrical Method used for determination of uplift pressures. The Authors are indebted to him and to his Assistant, Mr. Gurdas Ram, for the valuable investigations that they have carried out in connection with this paper.

The Authors are also indebted to Mr. Harbans Lal of the Hydraulics section for the investigation he carried out on the Hydraulic Models, the results of which form an essential part of this paper.

The Authors wish to thank Bhai Naranjan Singh, Subordinate, for the careful observations and record of field data at Panjuad and Khanki.

The Authors are indebted to Mr. A. Murphy, O. B. E.; Mr. J. B. G. Smith C. I. E. and Sir Bernard Darley, Kt., C.I.E., whose keen interest in the subject has been of very material assistance, and to Messrs. Kanwar Sain, Montagu and Haigh, for their valuable suggestions in Chapters X and XI.

Design of Weirs on Permeable Foundations

CHAPTER I

History of Development of the Science of Sub-soil Hydraulics in Relation to the Design of Weirs on Permeable Foundations

The law of flow of water through permeable soils was enunciated for the first time in 1856 by H. Darcy¹ who, as a result of experiments, found that the velocity of flow varied directly as the head and inversely as the length of the path of flow. This law is expressed by the equation,

$$V = K \frac{H}{L}$$

where

V = velocity

H = Head

L = Length of path of flow

K = A constant called the "transmission constant".

Darcy's Law is of the same form as the law for movement of water in capillaries, enunciated by Poiseuille in 1841-42.

The validity of this law in relation to weir design was tested by Col. Clibborn in 1896 in connection with the proposals for repairs to the damage, in 1895, to the Khanki Weir² on the Chenab River. This Weir which feeds the Lower Chenab Canal, was completed in February 1892. In January 1895, 100 ft. of the weir crest in Bay No 1 subsided about 2 ft. This was the first major weir to be constructed on the alluvial bed of a Punjab river and this damage, occurring so soon after the construction, gave food for thought to the engineers responsible for its construction and maintenance. Sir John (then Colonel) Ottley, Inspector-General of Irrigation, visited Khanki on 14th February 1895 and had a discussion with Mr. (later Sir) Thomas Higham, Chief Engineer. In order to investigate the causes of this damage and the means of ensuring future safety, the Government of India ordered³, in their letter No. 851 dated the 19th March 1895, that a series of experiments be carried out with Khanki sand at the Civil Engineering College, Roorkee, with a view to investigate the laws of flow of water through sand in relation to weir design.

These historic experiments were carried out by Lt.- Col. Clibborn, I.S.C., Principal of the Thomason Civil Engineering College, during 1895-96 and 97, with a tube 120 ft. long and 2 ft. internal diameter filled with Khanki sand. The relationship obtained from these experiments between velocity, head and length of path of flow, was in keeping with that of Darcy, except that at very high heads slight departures were noticed.

An account of these experiments and of the different points raised by certain selected officers, *viz.*, Sir John Benton, Messrs. Beresford; Kennedy, Gordon, etc., is printed up in the Technical Publication No. 97³ (1902). The discussions contained in it refer to the relative effectiveness of vertical cut-offs and horizontal length of floor, to a consideration of the standing wave and the mechanical action of falling water in determining the length of floor and to the necessity of vertical cut-offs at the upstream and, more especially, at the downstream end, to prevent washing out of sand causing dangerous subsidence and eventual failure. But this investigation, though affording *the first rational basis for design*, embracing as it did the conception of failure by undermining and by uplift due to flow of water through the sub-soil of the weir, did not materially advance knowledge for purposes of practical design. It gave only qualitative indications which, however, were generally in the right direction.

(1) Recherches Hydrauliques, Paris 1856, by H. Darcy.

(2) History of Khanki Headworks, Lower Chenab Canal, Pages 6-10 and 11.

(3) Technical Paper No. 97, Government of India, 1902.

The following extract from a Note¹ by Mr. J. S. Beresford, C.I.E., Chief Engineer, Punjab Irrigation, dated 30th January 1901, on Colonel Clibborn's experiments, will be of interest:—

"The experiments carried out by Colonel Clibborn, Principal of the Thomason College at Roorkee, in 1896, although not leading to the discovery of any new principle regarding the distribution of pressure in a tube filled with sand and operated on in the manner and under the conditions described in Colonel Clibborn's Note on the subject, have attracted attention to certain important features in the design of weirs which had escaped the notice of many distinguished Engineers."

"That the statement just made above is true is fully borne out by the recent history of the Narora Weir across the Ganges river in the North Western Provinces. I read the account of Colonel Clibborn's experiments for the first time in December 1896, and in writing to Colonel Clibborn on the subject in the following month, January 1897, I suggested that experiments should be made by drilling a few holes at selected points in the floor of the Narora Weir, and thus test the actual percolation pressure in the case of a large existing weir founded on sand with a head of 12 or 13 feet of water above the surface of a horizontal floor composed of concrete and brickwork 5 feet thick. For one reason or another the experiments were not made until March 1898, when, on visiting Narora as Inspector-General of Irrigation, I ordered them to be made. On the 27th March two pipes were ready and the pressures given in sketch Plate XIII,* were observed."

"This first experiment showed clearly that the upward pressure at the point operated on had reached an intensity which rendered the stability of the weir very precarious, and orders were given to test the pressure at a few other points and then to consider the question of strengthening the work, or rather, reducing the percolation pressure, by the addition of the puddle apron on the upstream side of the work as put in above the Khanki Weir. However, unfortunately, on the 29th of March, two days after the first experiment was made, a length of about 200 feet of the weir and some 600 or 700 feet from the site of the latter floor was blown up. It was fortunate the failure of the floor did not take place at or very near to the site of the experiment, otherwise the uninitiated would never have believed that the failure was not due to the holes drilled in the masonry."

"Had the experiment suggested in January 1897 been made at the time, it is probable that measures would at once have been taken which would have prevented any such failure of the work as occurred more than a year later. However, the failure of the Khanki Weir gave great prominence to the subject of percolation water pressure, discussed by Colonel Clibborn in his Note and the valuable practical suggestions contained in that Note."

The Hydraulic Gradient Theory for Weir Design, apparently originated between Sir John Ottley and Thomas Higham and was developed as a result of experiments by Col. Clibborn (1895-97).

A considerable amount of research on the flow of water through soils, with special regard to the Transmission Constant (K) was carried out in 1898 by Slichter and King² of the U. S. Geological Survey. The main object of this investigation, however, was to determine the amount of water percolating through various types of soils with a view to ascertain the extent and behaviour of ground waters of the United States. It did not deal with the distribution of pressures across structures built on previous soils.

With the publication of the results of Col. Clibborn's experiments in 1902 the Hydraulic Gradient Theory came to be generally accepted in India. The following passage from Buckley's Book³ will bear testimony to this:—

"It has been maintained that, in those cases where the chief danger to a weir is from under scour and not from parallel currents, *the true measure of security of a weir in a permeable bed is the distance through the soil which a current of water would have to travel before it could rise up below the Weir, and that it is of little consequence whether masonry is laid horizontally on the Weir bed or sunk vertically below it, so long as currents passing through the soil below the structure are exposed to the friction of the same length of passage* †. This view appears to be sound, but it is essential to attach to the application of this principle, the condition that the weir must be protected from horizontal scour on the face of the toe, that it must have sufficient weight to resist the horizontal pressure of the head it supports, and also sufficient weight to oppose the upward pressure in the base of the foundation which that head may produce and, further, that the surface of it which is exposed to erosion should be of material sufficiently hard, and sufficiently heavy to resist that erosion."

In 1907. Bligh, in his book on "Practical Design of Irrigation Works," believed, rather, that the stability of a weir depended on its weigh. But in the 1910 edition of his book⁴ he admitted the fallacy of his original belief and became converted to the "Hydraulic Gradient Theory" of Ottley, Higham and Clibborn. Bligh's enunciation of this theory—which later came to be known as the "Bligh Creep Theory"—is almost fully contained in the extract from Buckley's book given above. But, in his attempt to clarify ideas, Bligh went a step forward.

(1) Technical Paper No. 97, pages 26 and 27.

(2) U. S. Geological Survey, Annual Report for 1898. "Movement of Ground waters" by F. H. King and G.S. Slichter.

(3) "Irrigation Works in India" by R. B. Buckley (1905), page 175.

(4) "Practical Design of Irrigation Works" by W. G. Bligh. Second Edition, Revised and Enlarged.

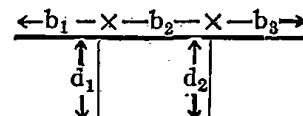
* Not reproduced in this publication

† Italics are ours.

He stated that the length of the path of flow had the same effectiveness, length for length, in reducing uplift pressures whether it was along the horizontal or the vertical. Herein lies the danger of applying Bligh's otherwise simple formula. The only experimental data available at the time were those of Col. Clibborn and perhaps the only field observations, those at the Narora Weir. From these meagre data Bligh evolved a simple formula—which, however, fitted neither the Clibborn result with sheet piles nor those at the Narora Weir—and recommended it for general use to the profession. Following Buckley, he assumed the percolation water to "creep" along the contact of the base profile of the weir with the sub-soil, losing head *en route*, proportional to the length of its travel.

The length of travel in a weir profile as given in the sketch below would be:—

$$L = b_1 + d_1 + d_1 + b_2 + d_2 + d_2 + b_3$$

$$= b_1 + 2d_1 + b_2 + 2d_2 + b_3$$


If H = total head over the Weir, the loss of head per unit length of creep would be:—

$$C = \frac{H}{b_1 + 2d_1 + b_2 + 2d_2 + b_3}$$

This loss of head per unit length, or what is the same thing as the average hydraulic gradient (C), he called the percolation co-efficient. He assigned a safe value to C for each of the different classes of soils. Thus, if in a given weir design, the value of C were less than the safe value assigned to it for the given class of sub-soil, the design would be considered safe.

Because of its simplicity, this theory found general acceptance. Some works designed on this theory failed while others stood, depending on the extent to which they ignored or took note of the importance of vertical cut-offs at the upstream and downstream ends. A fairly comprehensive summary of failures with an analysis of creep ratios has been given by Lane¹.

Colman² carried out, for the first time in 1915, tests with models of weirs resting on sand, to determine the distribution of pressure under the base and the relative effect of sheet piling at the heel and at the toe of the weir. These experiments demonstrated the greater relative effectiveness, length for length, of vertical contacts as against horizontal ones, but as the technique of experiments was crude no real advance was made.

Pavlovsky³ (1920) approached the problem of the flow of water through sub-soils of hydraulic structures from the analogy of flow of electricity through a conductor. According to Ohm's law:—

$$\text{Current } C = \frac{\text{Potential difference}}{\text{Resistance}} = \frac{E}{A \cdot L}$$

where

E = Potential difference.

A = Area of section of conductor.

and L = Length of conductor.

This is identical with Darcy's equation for flow of water through sand, *viz.*,

$$V = K \frac{H}{L}$$

The work was published in Russian. Pavlovsky achieved success in solving a number of problems, but as the laboratory results could not be shown to agree with field results, this method did not inspire confidence among the engineers and remained more or less of academic interest.

In 1926-27 trouble at the syphons under the Upper Chenab Canal (Punjab) became acute. Cracks appeared at the upstream and downstream ends due to undermining of the sub-soil. Repairs were carried out on the accepted Bligh theory but the trouble persisted. A set of pressure pipes with well points were inserted in the floors of two of these syphons and the observations disclosed that the pressures indicated by these pipes had absolutely no relationship with those calculated from the Bligh Theory. These researches were carried out by

(1) Paper No. 1919 American Soc. of Civil Engineers, "Security from under Seepage", by E. W. Lane, M. Am. Soc. C. E. (1935).

(2) "The Action of Water under Dams", by J. B. T. Colman, Assoc. M. Am. Soc. C. E. Paper 1356, 1916.

(3) "The Theory of Ground Waters flow under "Hydrotechnical Structures" Leningrad U. S. S. R." by N. N. Pavlovsky (in Russian) 1922.

Khosla and are embodied in Punjab Engineering Congress Papers Nos. 138¹ and 142² of 1930. The simple creep theory of Bligh was repudiated and some provisional and important conclusions³ were arrived at, notable among which were:—

- (a) The outer faces of the end sheet piles were much more effective than the inner ones and the horizontal length of floor.
- (b) The intermediate piles if smaller in length than the outer ones were ineffective except for local redistribution of pressures.
- (c) Undermining of floors started from the tail end. If the hydraulic gradient at exit was more than the critical gradient for the particular sub-soil, the soil particles would move with the flow of water thus causing progressive degradation of the subsoil, resulting in cavities and ultimate failure.
- (d) It was absolutely essential to have a reasonably deep vertical cut-off at the downstream end to prevent undermining.
- (e) There was urgent necessity for research work with regard to pressures under existing and new structures and in the laboratory. The former could be done by inserting suitably located pressure pipes in these structures and by maintaining a continuous and comprehensive record of the observations of pressures from those pipes.

Another useful paper⁴ appeared at the same time by Bose dealing with the mathematics of certain aspects of flow of seepage water.

The investigation of uplift pressures under dams was being actively pursued about the same time in the United States of America. Parsons⁵ and Julian Hinds⁶ presented two valuable papers before the American Society of Civil Engineers in 1928.

In 1929, a notable contribution came from Karl Terzaghi⁷ (U. S. A. and Vienna). He stated and proved by laboratory experiments that failures occurred by undermining if the hydraulic gradient at exit was in excess of what he called the "flotation gradient". This was the same as the critical gradient enunciated by Khosla, but it was more explicit in so far that it implied a state of flotation of the soil mass at the toe of the work if the exit gradient there exceeded the limit of 1:1 at which the upward force due to the flow of water was almost exactly counter-balanced by the weight of the soil. This conception of failure was possibly enunciated by Terzaghi in an earlier publication⁸ in German.

Dr. Rehbock⁹ (1931) produced a photographic demonstration of flow of water through porous soil in stream lines. He used a scale model of weir on sand and injected aniline dyes at certain points at the upstream end where the seepage water entered the sub-soil under the weir. These dyes left a coloured trace along the various lines of flow which were consequently capable of being photographed.

Forchheimer¹⁰ &¹¹ had developed a geometrical method of plotting stream lines and pressure lines as far back as 1911. This was a method of trial and error, said to be sufficiently accurate for practical purposes.

In 1929, it was decided to extend the Panjnad Weir¹² (Punjab). This afforded the opportunity of putting in a comprehensive set of pressure pipes and of conducting full scale experiments as suggested in Punjab Engineering Congress Papers 138 and 142 of 1930. This was the first full size experiment in the world, and the results obtained from it (1932) paved the way

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- (1) "Hydraulic Gradients in sub-soil Water Flow in Relation to Stability of Structures Resting on Saturated Soils" by Ajudhia Nath Khosla, I. S. E., Paper No. 138, Punjab Engineering Congress (1930).
 - (2) "Stability of Weirs and canal Works. An Application of the New Theory of Hydraulic Gradient", by Ajudhia Nath Khosla, I. S. E., Paper No. 142, Punjab Engineering Congress (1930).
 - (3) Central Board of Irrigation Publication No. 11 (pages 68-69).
 - (4) Punjab Engineering Congress Paper No. 140 by Dr. N. K. Bose, "Exponential Law of sub-soil Flow".
 - (5) "Hydraulic uplift in previous soils," by H. De B. Parsons, M. Am. Soc. C. E. Paper No. 1713 (1929).
 - (6) "Upward Pressures under Dams. Experiments by the United States Bureau of Reclamation" by Julian Hinds, M. Am. Soc. C. E. Paper No. 1717 (1929).
 - (7) "Effect of Minor Geological details on the safety of Dams" by Charles Terzaghi. Technical Publication of the Institute of Mining and Metallurgical Engineers, No. 215, February 1929.
 - (8) "Erd bau mechanic" by Von Karl Terzaghi, M. Am. Soc. C. E. Vienna, 1925.
 - (9) Punjab Engineering Congress Paper No. 162 of 1933. Figures 2 and 3. "Pressure pipe observations at Panjnad Weir" by A. N. Khosla, M. Am. Soc. C. E.
 - (10) "Hydraulik" by P. Forchheimer, 2nd Edition, Berlin, 1924.
 - (11) "Percolation under aprons of Irrigation Works" by S. Leliavsky, Cairo—Egypt, 1929.
 - (12) "Pressure pipe Observations at Panjnad Weir" by Ajudhia Nath Khosla, I. S. E., M. Am. Soc. C. E. Punjab Engineering Congress Paper No. 162.

to the final solution of the problem. The main conclusions of 1928-29 derived from researches at the Upper Chenab Siphons were confirmed and the following facts were established :—

- (a) The flow of water through the sub-soil is in stream lines and therefore susceptible of mathematical treatment.
- (b) The ratio (ϕ) of uplift pressure (P), at any point along the base of a particular weir founded on permeable soil to the total head (H) is constant and independent of :—
 - (i) Head (H),
 - (ii) Class of sub-soil so long as it is homogeneous,
 - (iii) Upstream and downstream water levels,
 - (iv) Temperature, provided it is uniform throughout the sub-soil, but it varies with :—
 - (i) Silt deposits or scour upstream or downstream of the impervious floor.
 - (ii) Temperature, which varies from point to point in the sub-soil and in different seasons of the year.

From the above conclusions it is obvious that the problem of seepage flow under weirs should be well adapted for model work, subject to the limitations imposed by the varying silt, scour and temperature.

About the time when full size experiments were under way at the Panjnad Weir, Professor Warren Weaver, ¹ Head of the Department of mathematics at the University of Wisconsin and at the time working with the Rockefeller Foundation, developed his mathematical treatment of the flow of water through the permeable sub-soil under dams. Weaver's work, as will be seen later, provided the inspiration for the complete solution of the problem, contained in Chapter VII.

With the indications from observations at Panjnad that the problem was well adapted for model work, "it was decided to determine whether a small scale model was capable of giving consistent results".² A scale model of the Rasul Weir (Punjab) was set up at the Punjab Irrigation Research Institute, Lahore, by Harbans Lal Uppal and Gunn, and experiments were carried out to determine uplift pressures. The results are published in Research Publication, Vol. II,² No. 2, January 1933.

The results though not conclusive were sufficiently encouraging. The investigation was then extended to a study of stream lines under simple types of weir floors. A description of the details of developing the stream lines will be found in Chapter IV. An account of these studies is contained in Research Publications, Vol. II, Nos. 3³ and 4⁴ (1934). These researches were carried out by Dr. E. McKenzie Taylor, Director of the Institute, and Harbans Lal Uppal. The pressures under these simple forms were investigated⁵ at the same time.

At the February 1933 Session of the Punjab Engineering Congress, Mr. (now Sir) R. P. Hadow, Chief Engineer, Punjab Irrigation, during the discussion of Paper No. 162, by Khosla, questioned as to whether deductions made from the Panjnad experiments would hold good on other works with different arrangement of floor and sheet piles and if they were enough and suitable to form the basis of future designs. He suggested that the field results be tested on a scale model in the Lahore Research Laboratory.

It was not till late in 1934 that a model of the Panjnad Weir was set up. This investigation was carried out by Harbans Lal Uppal and the results were presented at the Punjab Engineering Congress in February 1935 (Paper No. 185).⁶

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- (1) "Uplift pressure on Dams" by Warren Weaver, Journal of Mathematics and Physics, Massachusetts Institute of Technology Press, Vol. XI, No. 2, June 1932.
 - (2) "Study on Sub-soil Hydraulics. Investigation of Observational methods for models" by Harbans Lal Uppal, M.Sc., and J. P. Gunn, M. A., B. Sc. Research Publication, Vol. II, No. 2, January, 1933.
 - (3) "A study of the Flow of Water under Works on sand Foundation by Means of Models," by E. McKenzie Taylor, M.B.E., Ph.D., D.Sc., F.I.C. and Harbans Lal Uppal, M. Sc. Research Publication, Vol. II, No. 3 (1934).
 - (4) "A study of the Flow of Water under Works on Sand Foundations by means of Models, Part II, by E. McKenzie Taylor, M.B.E., Ph.D., D.Sc., F.I.C., Director, and Harbans Lal Uppal, M.Sc., Research Publication, Vol. II. No. 4 (1934).
 - (5) "An Investigation of the Pressures on Works on Sand Foundations" by E. McKenzie Taylor, M.B.E. Ph.D., D.Sc., F.I.C., Director, and Harbans Lal Uppal, M.Sc. Research Publication, Vol. II, 5 (1934).
 - (6) "Pressure under model of the Panjnad Weir and under the Prototype" by Harbans Lal Uppal, Paper No. 185, Punjab Engineering Congress, 1935.

Field observations continued to be made at the Panjnad Weir and fresh observations were started at the Khanki Weir where a series of pressure pipes had been installed during its reconstruction¹ (1934-35).

The years 1932 to 1935 were marked by special activity in respect of the study of sub-soil hydraulics in relation to weir design, both in India, the United States of America, Europe and elsewhere.

In September 1932, Lane² analysed over 200 dams, all over the world, and evolved his *Weighted Creep Theory*, which in effect might be called the Bligh Creep Theory corrected for vertical cut-offs and sloping faces. Lane proposed a weight of three for vertical creep and one for horizontal creep. While this theory was an improvement on the original Bligh theory, it was empirical and lacked the background for a rational basis for design. His memorandum on the subject appeared later, as Paper No. 1919³, in the 1935 Transactions of the American Society of Civil Engineers. This paper is valuable for the wealth of information about the numerous weirs it deals with.

A number of useful papers on the subject were presented at the International Commission on Dams⁴ in July 1933.

In May 1934, Hebert⁵ carried out experiments for various weirs on the electric model. He tried the Panjnad Model but failed to secure results in agreement with those given in Paper No. 162⁶ (Punjab Engineering Congress, 1933). He ascribed this lack of agreement to non-homogeneity of the sub-soil at the Panjnad Weir and decided that the application of electric models to problems of practical designs must be made with care as the results appeared to be of a qualitative rather than of a quantitative nature.

In the same year Haigh⁷ and Harza⁸ independently produced two very useful papers on almost similar lines. Fundamentals were once again introduced and a definite attempt was made to break away from empiricism and to lay a rational basis for design. They took note of the exit gradients as a controlling factor in stability and discussed the distribution of pressures which could be considered as safe,

Harza got agreement between theoretical values of uplift pressures and those obtained from the electric models for some of the simple cases dealt with by Weaver⁹ in his mathematical solutions. But as will be seen from the criticism on his Paper, the engineers were frankly sceptical and remained unconvinced as no agreement was shown to exist between the model and field results.

In his experiments on the Panjnad model, carried out at the Lahore Research Institute, Harbans Lal succeeded in getting almost exact reproduction of the field results by suitable superimposition of silt and temperature. The profession would not accept theory, nor the results of models so long as both could not be conclusively shown to conform exactly to those obtained in the field. The results obtained on the Panjnad model in conjunction with the researches of Dr. Vaidhianathan on the electric models (mentioned later) have shown conclusively, *for the first time in the history of research on this subject that* :—

- (a) The distribution of pressures under works on sand foundations can be exactly reproduced on Hydraulic or Electric models.

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- (1) "Reconstruction of the Khanki Weir" by R. B. A. N. Khosla, M. Am. Soc. C. E., Paper No. 195 of Punjab Engineering Congress (1930).
- (2) Technical Memorandum No. 303, of the United States Bureau of Reclamation. "Security of Masonry Dams on earth Foundations from under Seepage or Piping" by E. W. Lane, M. Am. Soc. C. E.
- (3) Paper No. 1919 American Soc. of Civil Engineers, "Security from under Seepage", by E. W. Lane, M. Am. Soc. C. E. (1935).
- (4) Commission Internationale Des Grands Barrages. Stockholm, 1933, 5 Volumes.
- (5) U. S. A. Bureau of Reclamation Technical Memorandum No. 384, "Hydrostatic Uplift Pressures under Dams on Pervious earth Foundations" by D. J. Hebert, Junior Engineer.
- (6) "Pressure Pipe Observations at Panjnad Weir" by Ajudhia Nath Khosla, I. S. E., M. Am. Soc. C. E., Punjab Engineering Congress, Paper No. 162.
- (7) "Design of Weirs on sand Foundations" by F. F. Haigh, M. I. C. E. (London) Punjab Engineering Congress Paper No. 182, 1935.
- (8) "Uplift and Seepage under Dams on sand" by L. F. Harza, M. Am. Soc. C. E., Paper No. 1920. American Society of Civil Engineers, 1935.
- (9) "Uplift Pressure on Dams" by Warren Weaver, Journal of Mathematics and Physics, Massachusetts Institute of Technology Press, Vol. XI, No. 2, June 1932.

(b) All seasonal and other variations from the normal conditions can be reproduced on Hydraulic Models by superimposition of silt, temperature, or both, and by simulating the stratification.

(c) The problem is susceptible of mathematical treatment.

This conclusive proof about the reliability of model and mathematical results in application to field conditions marks a great advance and a definite landmark in the development of this branch of engineering and one that has led to a final solution of the problem of design of weirs on permeable foundations.

A review of the advance in knowledge of sub-soil hydraulics would be incomplete without mention of the valuable work done by Dr. V. I. Vaidhianathan, Head of the Physics Section at the Punjab Irrigation Research Institute. The high class technique¹ with the electric model developed by him in experimental work is described in Chapter IV. The experiments carried out by him on a series of standard forms, specially planned for the purpose by Khosla, are dealt with at length in Chapter IX. The results obtained from them have helped to bridge the gap between theory and practice and have enabled a more complete presentation of the subject, than would have been possible otherwise.

(1) "The Electric Method of investigating the Uplift Pressures under Dams and Weirs" by Dr. V. I. Vaidhianathan and Gurdas Ram. Research Publication, Vol. V. No. 4, Irrigation Research Institute, Lahore

CHAPTER II

Theory of Seepage Flow

Darcy's Law and its Extension

In the vast plains of India the sub-soil consists of alluvium whose particle size varies from that of very fine colloidal clay to very coarse sand. The movement of water through a soil obeys the laws governing the movement of a fluid in capillaries. Movement of water in capillaries was first investigated by Poiseuille¹ in 1841-42. He found that the time of efflux of a given volume of water varied directly as the length of the tube, inversely as the difference of pressure at the two ends and inversely as the fourth power of the diameter. This has also been worked out from theoretical considerations by G. G. Stokes.² If $2a$ is the diameter of the tube, l its length, p the difference of pressure at its ends, and f the discharge in cc., per second,

$$f = \frac{\pi a^4}{8\mu} \cdot \frac{p}{l} \quad (2.1)$$

where μ is the co-efficient of viscosity of water. As the viscosity of water changes with temperature the amount of flow will vary with the temperature. The viscosity of water has been given by Helmholtz as follows:—

$$\mu = \frac{0.01778}{1 + .0337 \theta + .000221 \theta^2} \quad (2.2)$$

so that as the temperature θ rises, μ falls and from equation (2.1) we get a higher discharge. Thus in summer the movement of water in capillaries is faster than in winter.

Theory and experiments tend to shew that the equation (2.1) will hold only for very small pressure differences. Poiseuille's experiments give indications that as the pressure is increased, the flow increases more rapidly than is given by equation (2.1). These experiments were repeated by Meyer³ in 1873 with air and he also found that at high heads there was a departure from equation (2.1). Later experiments have shewn that the equation holds good for Reynold's Number $R < 600$,

$$\text{where } R = \frac{a v}{\nu}$$

$2a$ = diameter of capillaries,
 v = velocity of flow through the capillaries, and
 ν = Kinematic viscosity, μ/ρ

As the dimensions of the pore spaces in the alluvial soil resemble very closely those of capillaries, the movement of water in that medium would naturally be expected to follow some law similar to equation (2.1). In 1856 Darcy⁴ carried out some experiments with soil and found that the velocity of water flowing through a bed of sand was directly proportional to the thickness of the bed (l), measured in the direction of flow, which may be represented by the equation:—

$$v = K \cdot \frac{h}{l} \quad (2.3)$$

where the proportionality factor K represents the permeability of the sand medium and is known as the co-efficient of permeability or the Transmission Constant. This relation (2.3) is very similar to relation (2.1) of Poiseuille and has similar limitations at high heads. In 1884 Welitschkowsky⁵ published the results of a series of experiments on the movement of water in a sand medium and stated:—

“It is to be observed, in the first place, that the column of sand 25 cm. long, gives a flow under each of the ten different pressures increasing faster than the pressure by amounts ranging from 3.9 per cent. under a pressure of 45 cm. of water to 11.3 per cent. when the pressure becomes 125 cm. When the column has its

- (1) *Poiseuille* . . . “Recherches sur le mouvement des liquides dans les tubes de tres petits diametres.” Acad. Sciences Savants estrangers, 1842, Vol. IX. p. 433.
 (2) *G. G. Stokes* . . . “On the Theories of the internal friction of fluid in motion and of the equilibrium and motion of Elastic Solids.” Trans. of the Camb. Phil. Society, 8, 287, 1845,
 (3) *O. E. Meyer* . . . “Uber die innere Riebung der Gase.” Pogg. Annalen, 1866, Vol. CXXVII.
 (4) *H. Darcy* . . . Recherches Hydrauliques, Paris, 1856.
 (5) *Welitschkowsky* . . . Archiv fur Hygiene, Vol. II, 1884.

length doubled or made 50 cm. high, there is under the lowest pressure, 70 and 80 cm., a nearly complete agreement with the law; but as the pressure is increased the flow becomes faster than is required until, under a pressure of 150 cm., it exceeds the computed amount by 3 per cent. Then when this column was given a length of 75 cm. there became practically an almost complete agreement of observation with theory, the percentage departure in no case reaching as much as 0.5 of 1 per cent."

In 1928, G. Von. Heidiken conducted some experiments in the Royal Technical University, Stockholm, on the movement of water in a medium consisting of lead shots of uniform size. Lindquist analysing these data comes to the conclusion, "that Darcy's Law $v=Kj$ is not valid as soon as the Reynold's Number computed on the mean velocity in the filter bed, the size of the grains and the Kinematic Viscosity (=Viscosity/Density) is greater than 4.* For instance, with water at 8°C it means that the product of the mean velocity in the filter bed (cm./sec.) and the size of the grain (mm.) must be smaller than 0.50 when Darcy's law is applied. The summing up of many years of experiments is that Darcy's law only applies up to a limit of velocity which at 15°C apparently varies from 0.003 to 0.004 meter per second."

The experimental evidence about the range of validity of Darcy's Law tends to set an upper limit to the velocity at 0.01 to 0.015 ft./sec., and to the maximum grain size of the medium between 1.5 to 2.0 mm. For the alluvial soil that is generally met with in the beds of Indian rivers these limits are seldom exceeded excepting perhaps at the tail end of the works where the pressure gradient may be locally high or where shingle beds are met with.

C. S. Schlichter¹ in his theoretical treatment of the problems of sub-soil flow started with the assumption that the movement of water in the pore space of the soil follows Poiseuille's Law of Capillary Flow. In his analysis he states "It is true, of course, that few water washed sands actually approach the condition of uniformity of size and sphericity of forms that it will be necessary to hypothecate for the ideal soil which is to be made the subject of investigation. Nevertheless in taking account of the important elements of structure in a medium which is made up of a mass of spheres, we are probably considering in their due proportion the different factors that control the water movements in a well sorted sand, and notwithstanding the great variety in the sizes and arrangements of contiguous grains in a natural sand, there probably exists a tendency in every such soil towards a certain average size and mean arrangement of grains which the theories of probabilities would justify us in setting up as an ideal soil to replace a given soil in the investigation of its hydraulic properties".

Having made this assumption about the uniformity and sphericity of the sand particles Schlichter proceeds to find the area of the section of the pore space of such a hypothetical medium. He obtains:—

$$A = \frac{\sin \theta - \pi/4}{2} d^2 \quad \dots \dots \dots (2.4)$$

where d is the diameter of the sphere and θ the face angle of a rhombohedron that is formed by lines joining the centres of eight contiguous grains that may be separated from the mass of the soil. If the grains of the soil are arranged in the most compact manner possible, each grain will touch surrounding grains at twelve points and the element of volume will be a rhombohedron having face angles equal to 60° and 120°. If the grains are not arranged in the most compact manner the rhombohedron will have its face angles greater than 60° and each sphere will touch other spheres in but six points, but will nearly touch in six other points. The most open arrangement of the soil grains which is possible with the grains in contact is when the rhombohedron is a cube.

The pores through such an ideal soil are capillary tubes of approximately triangular section of area given by equation (2.4). The length of those capillary tubes is evidently greater than the length of the column of soil. Thus for a length of soil column h is given by:—

$$\frac{h(1 + \cos \theta)}{\sin \theta \sqrt{1 + 2 \cos \theta}} \quad \dots \dots \dots (2.5)$$

The pore enlarges slightly in area as it follows the surfaces of the spherical grains and then diminishes again to its former value given by equation (2.4).

Having obtained the length and area of cross-section of these pores Schlichter proceeded to find the flux through such pores with the help of a theorem of Greenhill who had shewn that the expression for the velocity of a viscous fluid at any point in the cross-section can

*This number is considerably smaller than the one for movement in capillaries.
(1) C. S. Schlichter . . . "Movement of Groundwater," 1899,

be found if the motion of a frictionless liquid relating to the boundary is known for a rotating prismatic cylindrical vessel of the same form of section. The flux per second

$$f = \frac{\rho A^2}{20\sqrt{3} \mu l}$$

where A is the area of cross-section and l the length of the triangular cylinder. The mean velocity is $v = 0.02887 \rho A / \mu l$. For a circular tube the mean velocity is

$$v = 0.03979 \rho A / \mu l \quad (2.6)$$

Substituting in this the values of A and l as given by equations (2.4) and (2.5) Schlichter obtains:—

$$v = \frac{\rho d^2}{16\pi \mu h} \left(\frac{\sin \theta \sqrt{1+2 \cos \theta} (\sin \theta - \pi/4)}{1 + \cos \theta} \right) \quad (2.7)$$

If s be the section of the cylinder containing the soil and substituting the value of μ the viscosity $\mu_{10} = 0.01315$ for 10°C , he obtains for the flux across the section

$$q = 1.1846 \frac{\rho d^2 s}{h K} \text{ cubic feet per minute}$$

where K is a function of θ which determines the porosity of the soil. So that

$$v = \frac{q}{s} = \left(1.1846 \frac{d^2}{K} \right) \cdot \frac{\rho}{h} = K' \frac{\rho}{h}$$

which is of the same form as Darcy's Law given by equation (2.3).

Having thus derived a relationship similar to Darcy's Law, Schlichter proceeded to generalise it by writing equation (2.3) in the form:—

$$v = K \frac{\delta p}{\delta s} \quad (2.8)$$

where v is the velocity-vector, p the pressure in the fluid driving the water particles in the length s . The pores of the soil are arranged in such a haphazard manner that we can assume that at any point the fluid can move in any direction.

Writing the above vector equation (2.8) in the cartesian form he obtained the following relation:—

$$\begin{aligned} u &= K \frac{\delta p}{\delta x} \\ v &= K \frac{\delta p}{\delta y} \\ w &= K \frac{\delta p}{\delta z} \end{aligned} \quad (2.9)$$

Schlichter then proceeds to make the assumption that "The flow of a fluid in a soil, is not different from the flow of a hypothetical fluid which we may suppose to replace both the fluid in the soil and the soil itself". With this assumption the equation of continuity

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0 \quad (2.10)$$

will hold good. This equation is simply the mathematical statement that a given mass of the liquid does not change its volume during the given motion. Substituting (2.9) in (2.10), he obtains:—

$$\frac{\delta^2 p}{\delta x^2} + \frac{\delta^2 p}{\delta y^2} + \frac{\delta^2 p}{\delta z^2} = 0 \quad (2.11)$$

Equations similar to (2.9) and (2.11) were obtained by Bose¹ in 1929, who assuming, that the motion in the sub-soil was extremely slow, replaced "the conditions of flow in a porous medium by those of flow of a fictitious fluid of suitable density and viscosity so as to reproduce the whole motion hydrodynamically. This motion is supposed to be laminar and the fluid obeys all the laws of hydrodynamics and the whole motion is executed under gravity only".

This equation (2.11) is the well-known Laplace's equation which occurs in nearly all branches of applied mathematics. The pressure function p which satisfies this differential equation is called the "Potential Function". This function will be dealt with further in the next chapter.

(1) N. K. Bose Memoir, Vol. II, No. 1, Punjab Irrigation Research Institute, Lahore (India) 1929.

It is well-known that the same equation of Laplace governs the steady flow of heat, and electricity through a conductor and of a perfect fluid. It is thus clear that the problem of flow of water through the sub-soil is analogous to that of the flow of a viscous fluid or that of electricity through a conductor. This analogy has been used in Chapter III for the solution of the various problems connected with seepage flow.

The potential function ϕ , determined from this partial differential equation, will be subject to the boundary conditions present in each particular problem. This function is finite, continuous, and single valued at all points of the hypothetical medium and the general methods of solution usual in potential theory will apply. This will be elaborated in the next chapter.

It must be remembered that as the Potential Law (2.11) has been derived from Darcy's Law (2.3) it will be subject to all the limitations of the latter law. These are:—

1. The velocities in the medium should not exceed 0.015 ft./sec., and the maximum grain size should not be more than 2 mm., in diameter.
2. The soil medium in which the flow takes place must be completely *saturated* with water. In this medium gravity is the only force acting on the fluid, and the transmission of pressure follows the equation (2.11)

Unsaturated Flow

When the medium is not fully saturated the laws of movement of water in such an *unsaturated* zone are altogether different from those given by (2.3) or (2.11). Movements in such cases are not governed by gravity but mostly by the force of capillarity, and they take place in direction leading away from the pore spaces which are wholly or partly filled with water and toward those which are entirely empty or are less filled than those from which they move. Pressure exerted on such a medium is transmitted almost fully through this medium and not gradually lost in transit as in the case of saturated flow, so that, if there be an unsaturated zone above the spring level of a country-side and there be some extensive rainfall on the area it will close all the pore spaces on the natural surface, and almost the full pressure equivalent to the depth of rainfall will be transmitted to the water-table, which will be manifested immediately by a rise in wells and observation pipes. If however, there be a canal running through an unsaturated zone and there be no saturated connection between the bed of the canal and the spring level, then whether a canal is in closure or not will not affect the water-table to any great extent, as only a fraction of the pressure transmitted through the bed of the canal to the unsaturated zone will be carried to the water-table; most of it will be relieved through the pore spaces of the natural surface surrounding the canal.

CHAPTER III

Potential Law and its Applications to the Problems of Weir Design

In the previous chapter it has been shewn that a problem in the steady motion of ground water is mathematically analogous to a problem in the steady motion of a perfect fluid (irrotational movement), and to those of the steady flow of heat and electricity. In all these problems the unknown function ϕ is determined by a solution of the differential equation (2.11) of Chapter II, *viz.*,

$$\frac{\delta^2\phi}{\delta x^2} + \frac{\delta^2\phi}{\delta y^2} + \frac{\delta^2\phi}{\delta z^2} = 0 \quad (2.11)$$

that will satisfy the boundary conditions present in each particular problem. In a determinate problem, the value of the function (ϕ) must be known over a closed boundary or along a given axis in space, from which it will be possible to derive the value of the function ϕ at any point x, y, z , of the region. After ϕ has been determined by an appropriate solution of equation (2.11) for a given problem, the relation

$$\phi = \text{constant}$$

will give, for different values of the constant, a series of surfaces upon each one of which the function ϕ will have the constant assigned value. These are the equipotential surfaces and in the case of sub-soil flow, the equi-pressure surfaces. Orthogonal to these series of surfaces there will be a series of lines in space along which the particles of liquid actually move. These lines are the lines of flow or stream lines.

Before proceeding to find the solutions of the equation (2.11) for the problems of seepage flow under weirs it will be interesting to mention the various other fields of mathematical physics where this differential equation of Laplace's finds application.

Laplace's Equation—

$$\frac{\delta^2\phi}{\delta x^2} + \frac{\delta^2\phi}{\delta y^2} + \frac{\delta^2\phi}{\delta z^2} = 0$$

was originally introduced by Laplace¹ in a memoir on Saturn's rings.

Some of the fields of mathematical physics, to which this equation applies, are mentioned below :—

1. The Gravitational Field-potential in regions not occupied by attracting matter ;
2. The Electrostatic Field-potential in uniform dielectric.
3. The Magnetic Field-potential in free aether in the theory of magnetostatics.
4. The Temperature Distribution in the theory of thermal equilibrium in solids.
5. The velocity distribution at points of a homogenous liquid moving irrotationally in hydrodynamical problems ;
6. The pressure and flow lines for extremely slow movement in a viscous medium like glycerine, treacle, etc. ;
7. The Electric Fields (potential and flow lines) in the theory of the steady flow of electric currents in conductors.

To these might now be added :—

8. The pressure distribution at points in a saturated porous medium such as a soil through which the liquid is moving.

Of these, case (5) forms the basis of the hydraulics of surface flow where turbulence is absent so that Bernoulli's equation applies. The applications of cases 6, 7 and 8 will be discussed in Chapter IV.

Though the physical theories involved in the above cases are different from one another the mathematical investigations are much the same for all; thus in the problem of sub-soil flow the determination of the equi-pressure surfaces is identical with that of the equipotential surfaces in the problems of the steady flow of electric currents through conductors—the boundaries being equi-pressure lines and stream lines in one case, and equipotential lines and lines of flow in the other case. This analogy has been utilised in the electrical method developed in the next chapter for the determination of the pressure distribution in the experiment.

(1) Laplace Mem. de L'Acad. des Sciences 1787.

Application of the Potential Law to Problems of Weir Design

Equation (2.11) deals with three dimensional flow. In weirs the flow will be mainly two dimensional as the width of a river is considerable so that flow at any cross-section of the weir is not appreciably influenced by any cross flow from the sides. At the flanks of a weir and at syphons, or other narrow structures, the influence from the sides will be considerable, and the flow will approximate to three dimensional.

For two dimensional flow, equation (2.11) reduces to $\frac{\delta^2 p}{\delta x^2} + \frac{\delta^2 p}{\delta y^2} = 0$. . . (3.1)

for which the boundaries, in the case of weirs, are given by :—

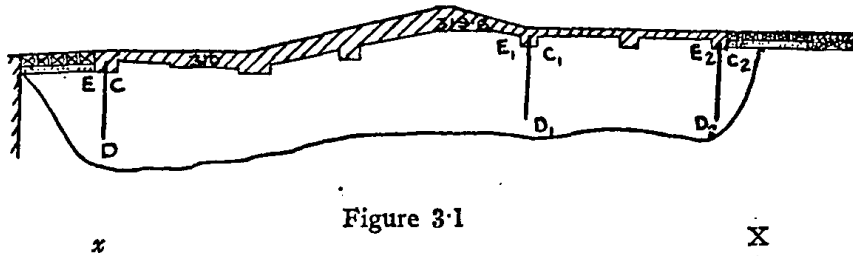


Figure 3.1

(a) The foundation profile B E D C E₁ D₁ C₁ E₂ D₂ C₂ A forms the inner boundary and represents the stream line immediately along the profile so that the seepage discharge between this stream line and the foundation profile is equal to zero. Between this stream line and the one next below if we assume a seepage discharge of *q* cusecs to flow, then the space between the two stream lines will form a stream tube carrying a discharge *q*. If *u* defines the stream line function, the stream line along the foundation profile will be given by *u*=0 and the one next below by *u*=*q*.

(b) For a total discharge of *Q* cusecs passing under the structure, the extreme boundary will be defined by the stream line

$$u = Q$$

The other two boundaries will be defined by :—

(c) at the upstream end, the sand surface Ax' along which pressure *v*=*H* representing the total head (*H*).

(d) at the downstream end, the sand surface Bx along which pressure *v*=0.

The first solution of a problem of two dimensional motion, in which the motion is bounded partly by fixed plain walls, was given by Helmholtz. A solution of equation (3.1) for any boundary conditions has been made possible by the method of conformal transformations devised by Schwarz and Christoffel¹ in 1869. This method consists of transforming an area in the *Z*-plane bounded by any rectilinear polygon to the lower half of *ζ*-plane bounded by a straight line, so that if we know the solution of the Laplace's Equation (3.1) for the *ζ*-plane with the real axis (*ξ*-axis) as the boundary, this transformation *Z*=*f* (*ζ*) will give the solution of the Laplace's equation for the rectilinear polygon in the *Z*-plane as a boundary.

So that if *Z*=*x*+*iy* and *ζ*=*ξ*+*iη*, then any polygon bounded by straight lines in the *Z*-plane can be transformed into the axis of *ξ* in the *ζ*-plane by the following transformation :—

$$\frac{dz}{d\zeta} = A (\zeta - \zeta_1)^{\lambda_1} (\zeta - \zeta_2)^{\lambda_2} (\zeta - \zeta_3)^{\lambda_3} \dots \dots \dots (3.2)$$

where $\lambda_1\pi, \lambda_2\pi, \lambda_3\pi \dots$ are the external angles of the polygon in the *Z*-plane and $\zeta_1, \zeta_2, \zeta_3 \dots$ are the points in the *ζ*-axis that corresponds to the angular points of the polygon of the *Z*-plane.

This method has been used in Chapter VII to solve the general cases of subsoil flow occurring in weir design.

(1) Schwarz and Christoffel. *Über einige Abbildungsaufgaben*. Crelle 70, p. 105, 1869.

CHAPTER IV

Experimental Verification of the Potential Law by means of Models

In the previous chapters it has been stated that the movement of water through permeable subsoils is analogous to the movement of water through capillaries, to the extremely slow movement of a viscous fluid, and to the flow of electricity through a conductor, and that all such movements are governed by the Potential Law. But these statements, the engineering profession will not accept for practical application to designs, unless their validity can be experimentally established. This chapter provides the necessary experimental verification. In Chapter VI it will be shown that what applies to models, applies equally well to actual full-sized field structures. Once it is established that the theoretical deductions from the Potential Law apply in full to the flow of water through permeable homogeneous subsoils under actual weirs, the problem of designing of these weirs in relation to seepage flow will have been solved.

The various methods employed to establish the validity of the above analogies and of the Potential Law are described below:—

I.—MATHEMATICAL

The mathematical solutions have been worked out in complete detail, for the various determinate forms of weir profiles, in Chapter VII. The results so obtained will, as shown later, be found to agree almost exactly with those obtained by the other methods, the difference, if any, being due to experimental error.

II.—TRIAL, AND ERROR OR GRAPHICAL METHOD

This method is approximate but in many cases gives results which are sufficiently accurate for practical purposes. This approximate solution has been derived from the Potential Law, mentioned in Chapter III, by Prof. Forchheimer¹. A brief description of this method is reproduced² below:—

“Suppose that the lines AB and CD shown on the Figure 4.1, are two stream lines and let AC, ac, a₁c₁, a₂c₂..... represent a series of lines of constant head. The latter are supposed to be drawn in such a manner that the head lost in between each pair of consecutive

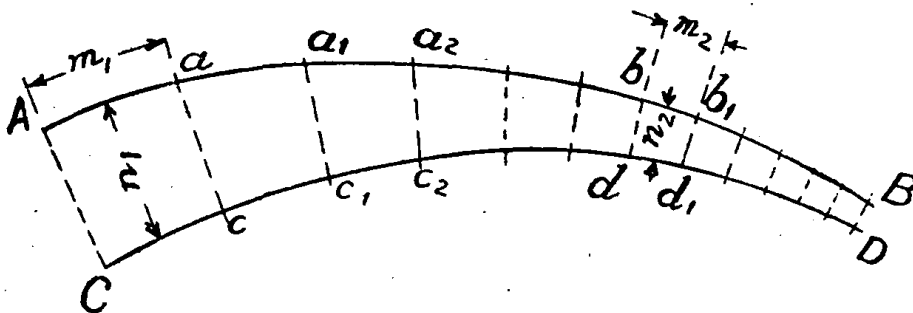


Figure 4.1

lines is always the same. Thus, for instance, if h represents the head for the line AC and Δh , represents the head lost in between the lines AC and ac, then the head for the line ac will be $h - \Delta h$, the head for the line a₁c₁ will be $h - 2\Delta h$, the head for the line a₂c₂ will be $h - 3\Delta h$, and so on.

Further let it be supposed that in drawing the lines AB and CD the distance m has been chosen approximately equal to the distance n , so that the figure A a c C represents approximately a “curvilinear square”.

Forchheimer shows that in this case for any other two lines of constant head, for instance for the lines b d and b₁ d₁, the distance m_2 will be nearly equal to n_2 , that is to say, we obtain then a diagram consisting of a series of “curvilinear squares”.

An approximate proof of this can be given in an elementary manner.

Let q represent the discharge in between the lines AB and CD. As explained earlier, this discharge remains constant over all the length of the two stream lines, therefore, the

(1) “Hydraulik” by P. Forchheimer.

(2) “Percolation under aprons of irrigation works” by S. Leliavesky. Page 33.

velocity will be inversely proportional to the distance between these lines. Thus the average velocity over the reach m_1 is about:—

$$u_1 = \frac{q}{n_1 \varepsilon}$$

whereas the average velocity in the reach m_2 is*

$$u_2 = \frac{q}{n_2 \varepsilon}$$

The average slope over the length m_1 is about $\frac{\Delta h}{m_1}$ and as the slope is proportional to the velocity

$$\frac{\Delta h}{m_1} = K u_1 = K \frac{q}{n_1 \varepsilon}$$

But as $m_1 = n_1$

$$\Delta h = \frac{K q}{\varepsilon}$$

Now, the average slope over the reach m_2 is about $\frac{\Delta h}{m_2}$ so that

$$\frac{\Delta h}{m_2} = K u_2 = K \frac{q}{n_2 \varepsilon}$$

$$\text{and } m_2 = \Delta h \frac{\varepsilon}{K q} n_2;$$

or, as $\Delta h = \frac{K q}{\varepsilon}$

$$m_2 = n_2$$

that is to say, the figure $bb_1 dd_1$ will represent approximately a curvilinear square, which it was required to prove.

Forchheimer explains his method on a diagram which is reproduced on Figure 4.2. The diagram shows a section of a weir built over a pervious foundation. In the particular case illustrated on the diagram it was known that an impervious stratum existed at a certain depth below the bed of the channel.

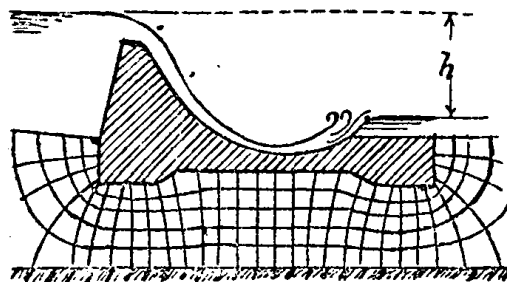


Figure 4.2.

The graphical process followed by Forchheimer in drawing out the stream lines can be described as a "method of trial and error". The first line of flow is drawn out free hand following approximately the outline of the section of the apron. All the sharp angles are, however, rounded out and the general shape of the flow line is more smooth than that of the apron.

The space in between the flow line and the apron is then divided in sections in such a way that the width of each section is approximately equal to its height, that is to say, each of the sections thus obtained will be a curvilinear square.

Under each of these squares a new square is drawn out. The outer sides of these new squares will form the second line of flow.

In the same way all the other flow lines can be drawn one after the other.

The last flow line must coincide with the line representing the limit of the previous material. This serves to check the assumed alignment of the first flow line. In the majority of cases the first trial will not be found satisfactory and it will, therefore, be required to re-arrange the diagram.

The method can be directly applied when the depth of the permeable layer is limited by a bed of impervious material, as is shown, for instance, on the Figure 4.2.

In the case of unlimited depth it will be necessary to take into account that the flow lines at greater depth from the apron tend gradually to change into semicircles.

Therefore, to check the diagram in this case it may be assumed that at a sufficient depth below the apron the last flow line must be nearly a semicircle.

* In the direction perpendicular to the plane of the drawing the width of the layer of porous material is assumed to be unity. The letters represents the porosity.

III.—VISCIOUS FLUID METHOD

A neat stream line apparatus has been devised for the purpose by Harbans Lal Uppal and Mushtaq Ahmed (Punjab Research Institute) which consists of a 4" x 3" tank with parallel sides of glass plate spaced 1 mm. apart, which is connected by small holes to two small chambers, one containing water and the other a solution of potassium permanganate. A small ebonite model of the weir is inserted between the glass plates and the flow started. The model which is a good fit between the plates is greased on the sides to ensure water tightness. The water and the permanganate solution flow in definite and distinct stream lines, as shown in Photo 4.1.

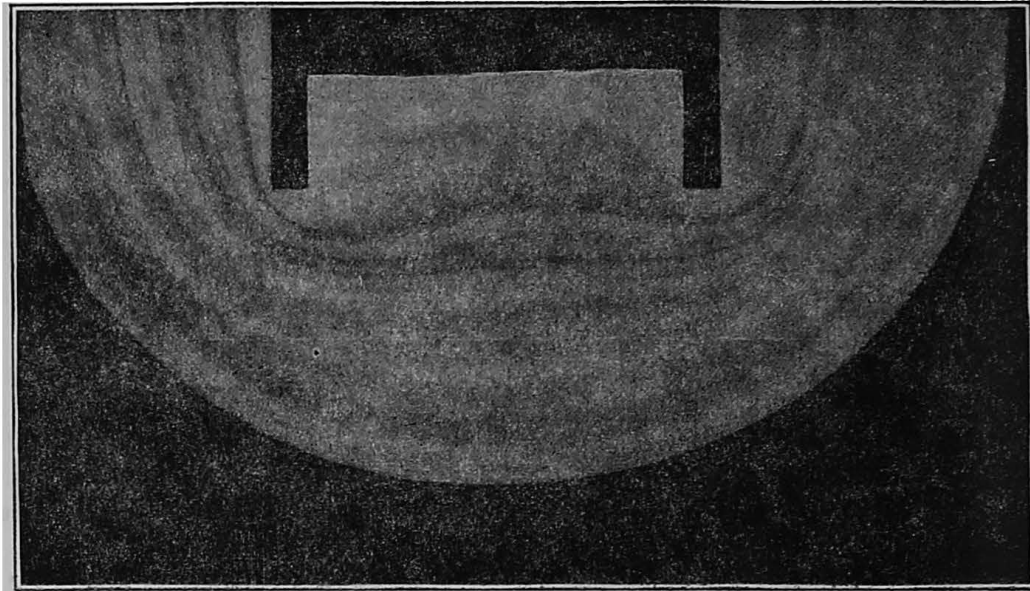


PHOTO 4.1

Stream lines can be obtained by this method in a few minutes and the pressure lines can be computed from these. Direct determination of pressures by this method has not so far been tried and it is doubtful whether it will be possible, with any pretence to accuracy.

This apparatus is similar to the one used by Hele Shaw¹ in 1897 and described² in connection with the determination of stream lines and pressure lines in spill-way siphons at Hanover.

The flow of water through this apparatus is similar to that of viscous liquids flowing at a rate below the critical velocity.

Stream lines have similarly been obtained by passing glycerine under the weir model between glass plates spaced 1 mm. apart. Red stained glycerine is introduced at points, and flows along with the plain glycerine in well defined stream lines which can be photographed. Photo 4.2, reproduced below, is taken from the work done by Ionides of the Irrigation Directorate, Baghdad (Mesopotamia), in 1932.

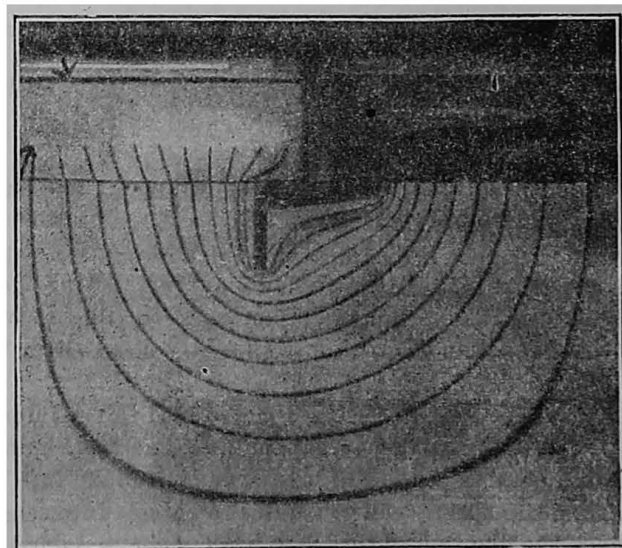


PHOTO 4.2.

(1) Hele Shaw :—"Engineering" July 16, 1897 and April 8, 1898.

(2) "Hydraulic Laboratory Practice" by John R. Freeman (1929), Page 608.

The equi-pressure lines can be plotted from these stream lines by the graphical method of Forchheimer.

The application of this method is obviously very limited. It is, however, useful in giving a quick picture of the stream lines under any work.

IV.—THE HYDRAULIC SCALE MODEL

The scale model was first tried by Colman¹, in 1915. As remarked in Chapter I, Colman's experiments failed to produce any results due to faulty technique.

The technique of hydraulic scale models has been perfected in the Punjab Irrigation Research Institute by Harbans Lal Uppal who has succeeded in obtaining on the models an almost exact reproduction of the field results of the Panjnad Weir, including the seasonal variations. This is described² below :—

EXPERIMENTAL TANK

The experiments were carried out in a series of tanks which were 3 ft. 10 in. in length, 2 ft. 6 in. in depth and 2 inches wide. Figure 4·3 shows the arrangement. Photograph 4·3 shows the bigger tank, 8 ft. × 3 ft. × 5 in. wide inside, for the Panjnad Model to a scale of 1 in 50.

THE EXPERIMENTAL TANK

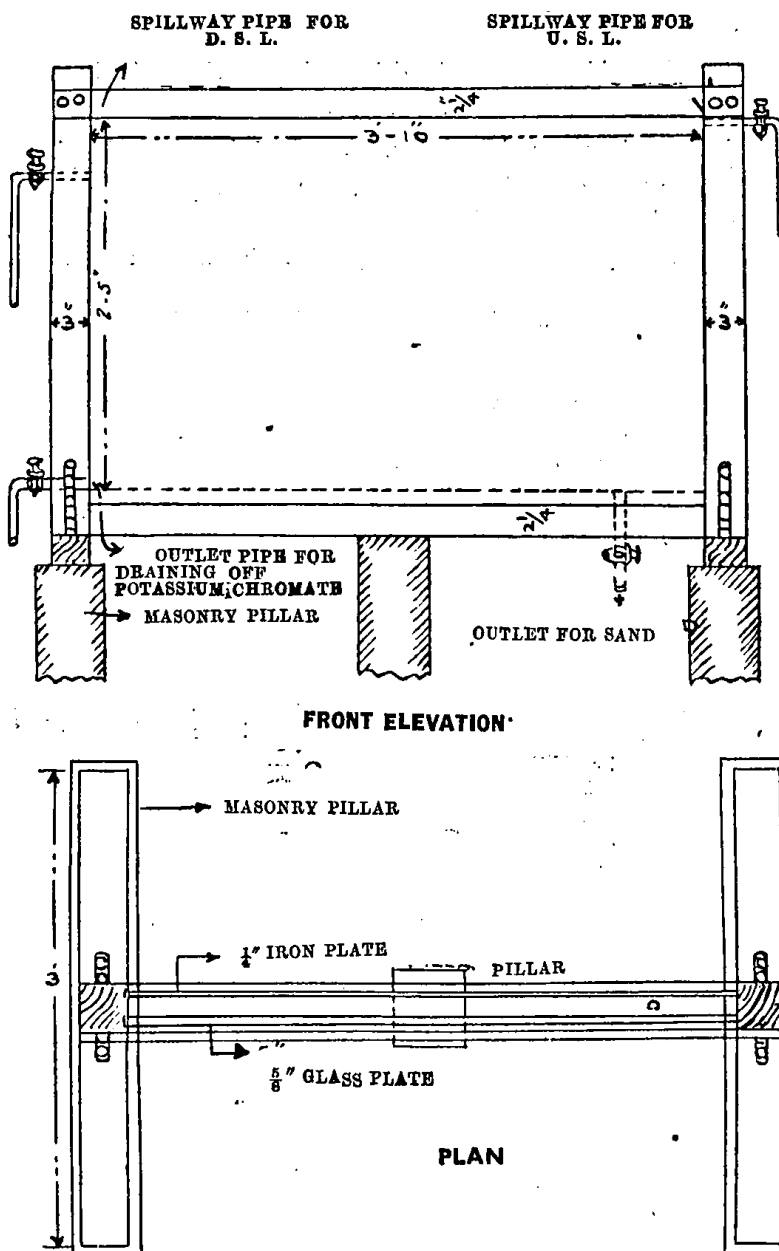
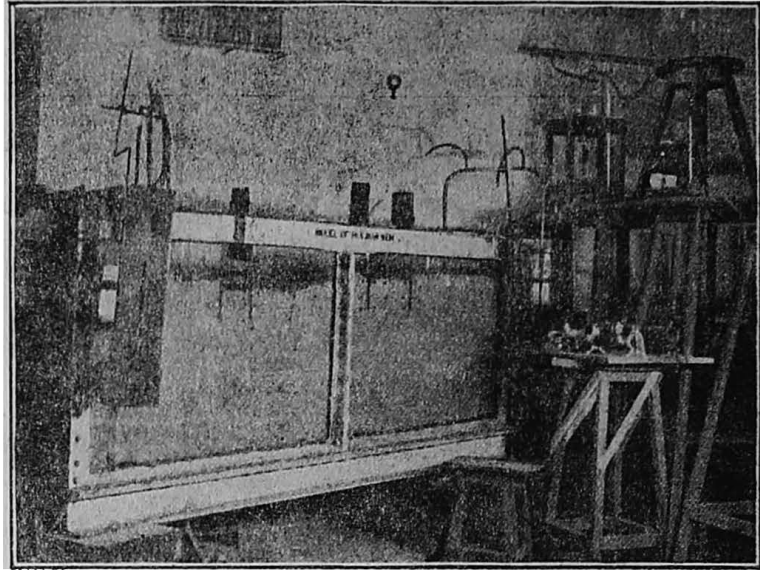


Figure 4·3

- (1) "The action of water under Dams" by J. B. T. Colman, A.M.Am.Soc.C.E., Paper 1356, (1916) American Society of Civil Engineers.
- (2) "A study of the flow of water under works on sand foundations by means of models" by E. McKenzie Taylor, Director, and Harbans Lal Uppal, Assistant Research Officer. Research Publication, Vol. II, No. 3. (1934).



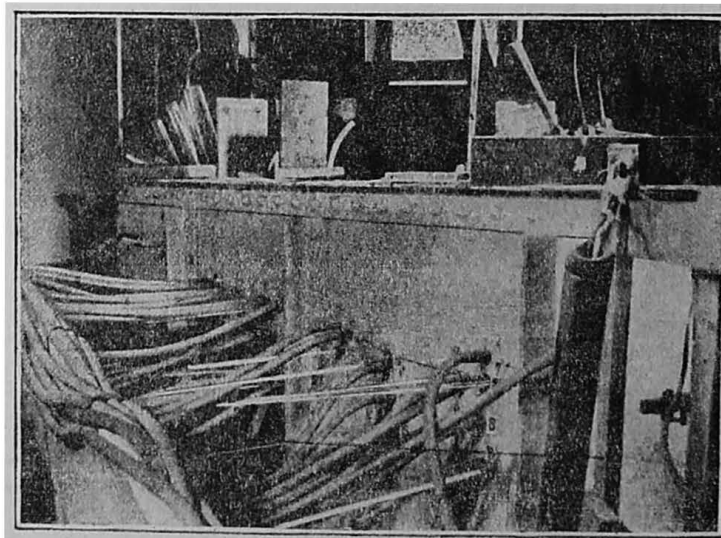
Showing glass side of the tank with Panjnad Weir Model in position

PHOTO 4.3

The frame of the tank was made of teak. The tank itself consisted, on one side, of a glass plate $\frac{5}{8}$ inch in thickness, and, on the other side, of a steel plate $\frac{1}{2}$ inch thick and similar in size to the glass plate, the ends of the tank being formed by the framework. The frame was grooved near both edges for the insertion of the glass and the steel plates. The tank was mounted on planks of wood placed at right angles to the bottom of the tank to give it stability.

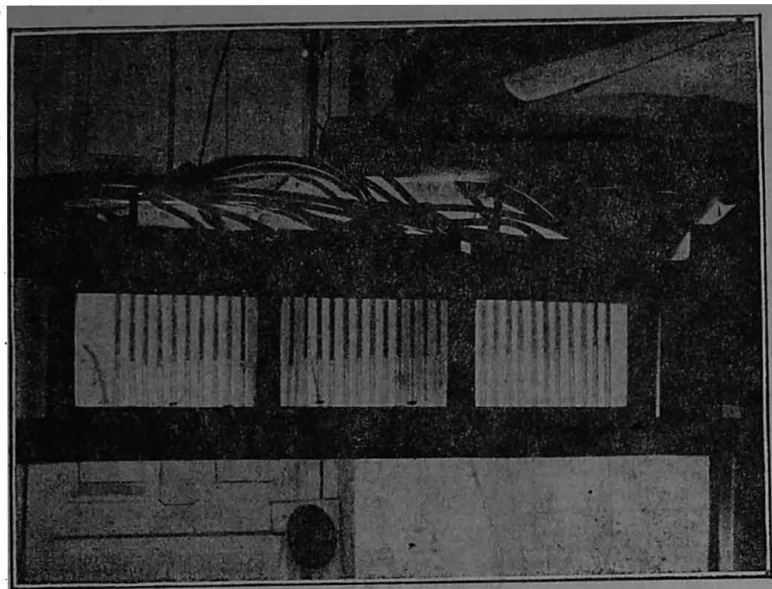
One difficulty experienced was in connection with the making of watertight joints between the glass and steel plates and the frame. After a number of trials the following procedure was adopted to secure watertight joints. One side of the frame was mounted on the base. Two right-angled steel plates were screwed to the base and the end frame to hold the frame together. The grooves in these two members were then filled with a hot mixture of paraffin wax, bitumen and rosin, and the edges of the plates inserted in the grooves. The grooves in the other end of the frame were then filled with the wax-bitumen-rosin mixture and the frame placed in position on the base and slowly forced against the ends of the glass and steel plates by an iron clamp. As a result, the excess of the cementing mixture was squeezed out of the grooves. The glass and the steel plates were also forced by the press against the grooves in the base. By this method watertight joints were secured. As a precautionary measure a further quantity of the cementing mixture was applied to joints inside the tank after assembly.

In the bottom of the tank, at the end which was to be the upstream of the model, a hole one inch in diameter was bored to provide an outlet for the removal of the sand at the conclusion of the experiment. Towards the bottom of the end frame piece which was to be the downstream of the model a glass tube fitted with a strainer was provided for draining off the liquid contents of the tank. To the steel back of this tank, the necessary apparatus was attached for pressure observations. Photographs 4.4 and 4.5 show the general arrangement.



Showing back of tank with rubber tubing connections to glass pipes.

PHOTO 4.4



Showing observation glass pipes.

PHOTO 4.5

Holes were drilled in the steel plate at the back of the tank at points under the model corresponding to the places at which pressure observations were to be made. Nipples made from brass tubing $\frac{1}{4}$ " in diameter were screwed into the holes in the steel plate so that they projected into the tank for a distance of two inches. The nipples were provided with strainers of copper gauze of 100 mesh. The nipples were connected outside the tank to a series of glass tubes by means of rubber pressure tubing. All joints were made watertight.

The pressures were to be observed in the glass tubes. These were made from specially selected barometer tubes of approximately 7 mm., diameter and calibrated for a series of heads before the model was constructed in the tank. The calibration was done with reference to two standard pipes each of two inches uniform diameter. The second pipe was used as a check on the first. The standard pipes and the observation pipes were all filled with water and any air bubbles sticking to these or the rubber connections were removed by sucking through the observation pipes. Observations were taken along the vertical at an interval of 1 cm. The difference between the standard pipe and the observation pipe gave the correction for the pipe at that point. The calibration curve for each pipe was plotted and reference was made to these corrections when the actual pressures were computed. All observations were made with a cathetometer and are accurate to $\pm .005$ cms.

Two cement pillars $5' \times 2' \times 2'$ and ten feet apart were erected, one on either back corner of the tank. Three angle iron pieces $11" \times 2" \times \frac{3}{8}"$ were fixed on to the pillars, two at the top and the third 18 inches below. The two top pieces were at the same level, one on the inner and the other at the outer edge of the pillar. This arrangement was made for holding observation pipes for their lighting arrangement. The observation pipes were divided into five sets each of twenty-five pipes fitted into a brass frame. Each brass frame holding a set of twenty-five pieces could be raised or lowered independently on to the angle iron pieces. The advantage of this arrangement was that no difficulty was experienced in recording pressure of down-stream pipes at low levels.

Two straight edges were fixed on to the top of the tank, one on either side. A "spitzentester" scale, mounted on a carrier, could move along the tank on the straight edges. By means of this arrangement the position of any pressure observation point or the R. L.'s of the model could be read accurately.

The meniscus of the water in the observation pipes was illuminated by means of diffused light from behind. A series of tubular bulbs fixed on to a frame, such that they could be raised or lowered, were placed behind each set of observation pipes. An opaque glass plate was placed between the pipes and the bulbs to obtain diffused light.

The distribution curves for the sand which was used in these experiments is given in Figure 4.4

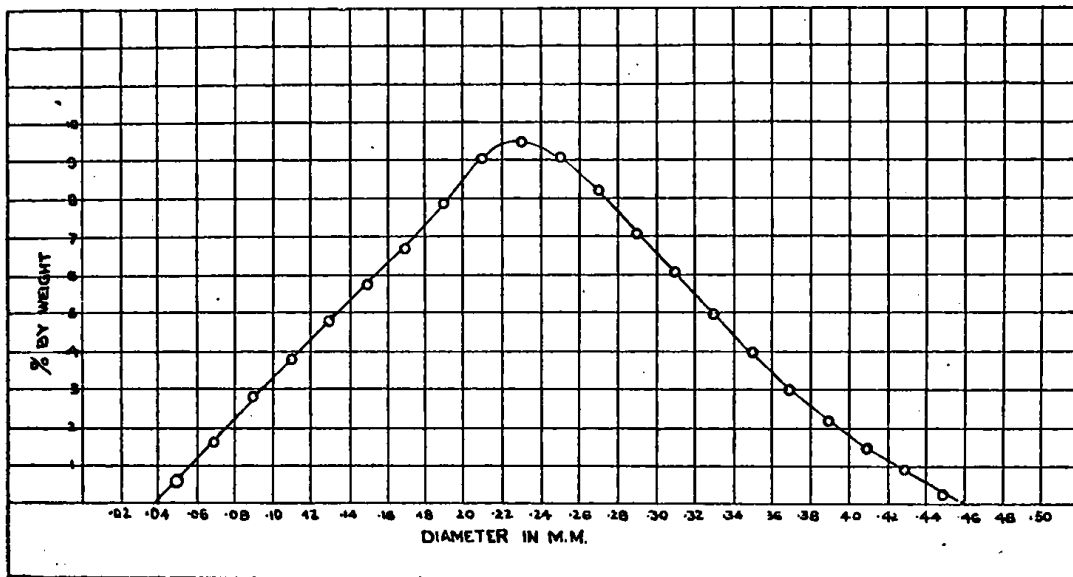


Figure 4.4

The sand was placed in the tank and the model was then constructed.

CONSTRUCTION OF THE MODEL

The method of construction of the model is described below: —

First of all a line representing the profile of the model was drawn on the glass plate such that the model was centrally located along the length of the tank. The position of the sheet piles, and all other important points were marked. The sand was then introduced into the tank using water instead of potassium chromate solution, which was employed in the earlier experiments. When the sand level in the tank was slightly lower than the lowest point of the model, the sheet piles were constructed at their proper position. The method of construction of sheet piles has been improved considerably. These are made of $\frac{1}{8}$ " teak board with a brass plate $\frac{1}{8}$ inch on each side. These plates project half an inch beyond the wood and form grooves in which rubber pressure tubes are inserted. The brass plates are $\frac{1}{2}$ " narrower than the tank and the overall dimension of the pile is raised to $\frac{1}{8}$ " more than the width of the tank by the rubber tubes. When the pile is inserted, the rubber tubes form a watertight seal with the front and back of the tank. The piles are tested for watertightness by filling in sand to about three inches above the bottom of the pile and applying a head of water. The model is constructed by filling the tank with sand to about two inches above the final sand level and is allowed to settle for 4 hours. After this time the top of the sand is formed according to the required shape and cement poured in before the sand has had time to dry. With this improved technique the time taken for the construction of the model has been much reduced and the accuracy of the observations has been considerably increased. It may be mentioned that the actual time taken for the construction of the model in the improved technique was less than 4 hours. The rapid hardening cement took three hours to set. So in a period of 7 to 8 hours any complicated model could be made ready for experimentation.

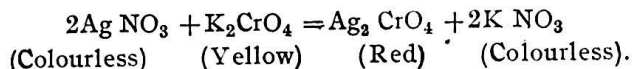
A constant head of water was maintained upstream and a constant level downstream for a period of two hours before starting the pressure observations. Another precaution of great importance was taken. *The water for feeding the upstream was passed through a series of glass wool filters before it was led into a reservoir.* By this means, all silt and suspended matter in the water was completely removed. The observation points were labelled carefully on the glass plate and the corresponding numbers were marked on the observation pipes.

PRESSURE OBSERVATIONS AND STREAM LINES

A number of pressure observations, at different heads of water, were taken for each model and entered in a register for recording observations.

After completing the pressure observations the stream lines were traced. The basis for tracing the course of the stream lines was the reaction between potassium chromate and

silver nitrate. When solutions of these two substances are brought into contact, a red precipitate of silver chromate is obtained according to the following equation:—

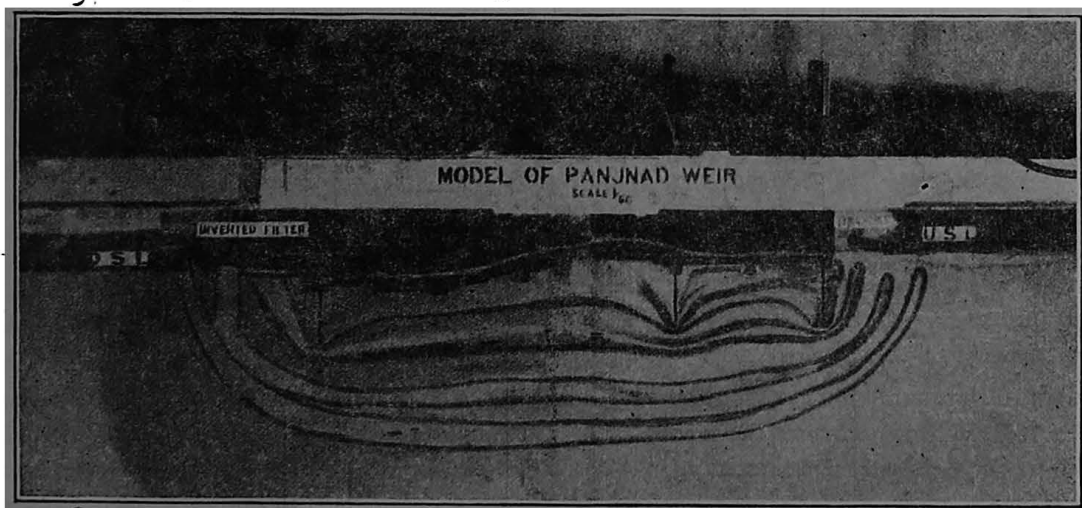


The solutions of potassium chromate and silver nitrate used in these experiments were of 1 per cent. strength.

The solution of potassium chromate was contained in an aspirator and placed in such a position that the potassium chromate could be introduced into the top of the tank and maintained at a constant level at the upstream end of the model. The level of the potassium chromate solution in the aspirator was also kept constant. Before starting any experiment, the solution of potassium chromate was run through the sand to ensure that the model was watertight, the sand saturated with the solution, and the flow uniform.

The solution of silver nitrate was contained in a series of vertical glass tubes provided with stop-cocks. The lower ends of these tubes were drawn out into fine jets. The top ends of the tubes were fused at regular intervals on to a horizontal tube which was connected to the reservoir containing silver nitrate solution. The level of silver nitrate in the reservoir was maintained constant by means of a spill-way pipe. The capillary points were introduced into the sand on the upstream side half an inch below the surface and close up to the glass side of the tank.

The head of silver nitrate in the tube was slightly greater than the head of the main bulk of potassium chromate solution to ensure that a stream of silver nitrate was delivered. This additional head was dissipated soon after it left the point of the capillary. The experiment was commenced by opening the stop-cock of the tube at a known time. The silver nitrate reacted with the solution of the potassium chromate and gave precipitate along the lines of flow. The photographs of the development of the stream lines were taken at intervals, suitable illumination being provided. It was found necessary to take a series of photographs at intervals as points of interest occurred during the experiment which were likely to be obscured by later developments. Photo 4·6 shows the stream lines, so obtained, on the Panjnad weir model. Other photographs of stream lines are given in Chapter IX.



Showing stream lines under the Panjnad weir model.

PHOTO 4·6

A description of the experiments on the hydraulic model to determine the effect of silt and temperature will be found in Chapter VI.

A comparison of the pressures as obtained on this and the electric models with those derived mathematically will be given later in this chapter.

V.—ELECTRIC ANALOGY METHOD

This method was first tried by Professor N. N. Pavlovsky¹ in Russia in 1920. The technique as developed in the Punjab Irrigation Research Institute

(1) "Motion of water under Dams" by Prof. N. N. Pavlovsky, Paper No. 30, International Commission of Dams. (1933.)

by Dr. V. I. Vaidhianathan, ^{1 & 2} and as described in the Proceedings of the Indian Academy of Sciences, is reproduced below:—

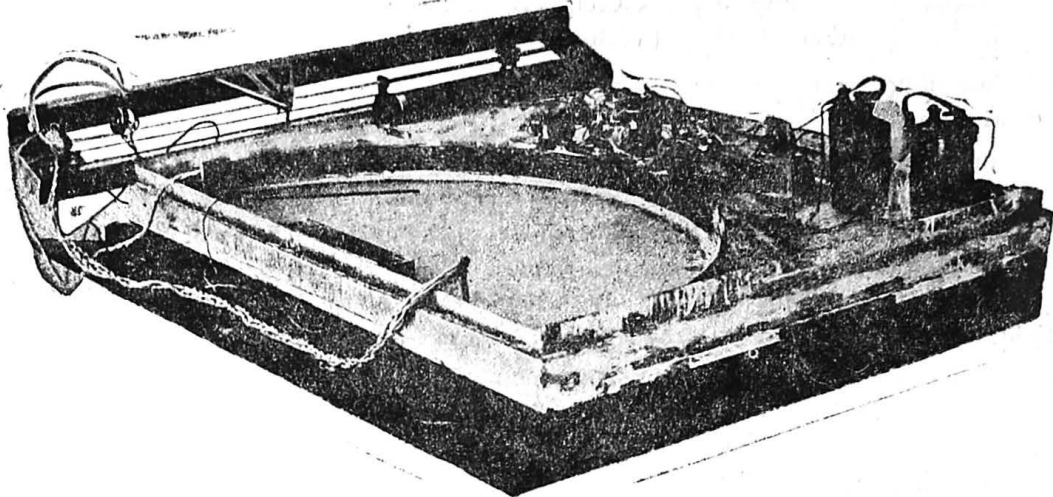


PHOTO 4.7

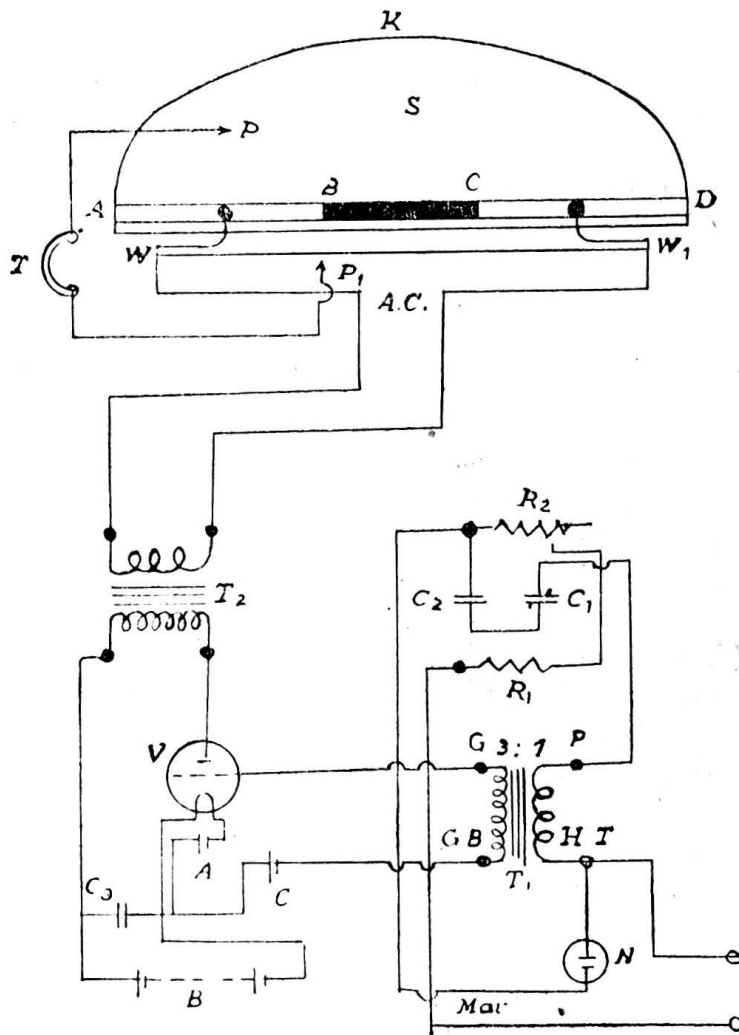


Figure 4.5

C₃—Fixed condenser
 V—Triode Valve
 T₂—Transformer
 A—Filament Battery
 B—Plate Battery
 C—Grid Battery

N—Neon Bulb
 R₁—High Resistance
 R₂—Rheostat
 C₁—Variable Condenser
 C₂—Fixed Condenser
 T₁—Transformer.

(1) "Potential Distribution in infinite Conductors and Uplift Pressure on Dams" by Gurdas Ram, M. Sc. Dr. V. I. Vaidhianathan, D. Sc., F. Inst. P. and Dr. E. McKenzie Taylor, Ph. D., D. Sc., F. I. C., Proceedings of the Indian Academy of Sciences, Vol. II, No. 1, July 1935.

(2) "On the Electrical Method of investigating the Uplift Pressures under Dams and Weirs" by Dr. V. I. Vaidhianathan, M. A., D. Sc., F. Inst. P. Physicist and Gurdas Ram, M. Sc., Research Assistant, Research Publication, Vol. V. No. 4, April, 1935.

The tank is 4 ft. long and 3 inches deep. (Figure 4.5 and photo 4.7.) AB and CD are two thick conducting plates of copper each 1.5 feet long representing the surfaces of the porous strata on the upstream and downstream of the dam. BC is an ebonite plate 1 foot long and flush with the conductors representing the non-porous masonry of the dam.

In the first set of experiments the tank was rectangular, being about 4'×3'×3". It was thought that the potential distribution round BC would not be affected by the outer boundary, but it was found that this introduced an error of about 5 per cent. The outer boundary AKD of the tank was then made elliptical in shape with B and C at foci. The elliptical shape of the boundary conforms to the theoretical condition, that the boundary is at an infinite distance away. The tank S is filled with a dilute solution of ammonium chloride. P is a probe made by fusing a platinum wire into a glass tube, the end of the wire alone being exposed. P₁ is a jockey moving on a ten-metre potentiometer wire WW₁. T is a headphone connected as shown in the figure.

The potentiometer wire serves as a potential divider. The ends of the potentiometer are connected in parallel with the AC source and with the conducting plates as is shown in the diagram (Figure 4.5). The wire was accurately calibrated.

The AC source (Figure 4.5) is obtained from a Neon oscillator with an amplifier. This arrangement which is shown completely in Figure 4.5, is a very convenient source of oscillating current, giving a sharp null point in the phone. A high accuracy in determining the null point is required in this experiment, because it can be seen from the arrangement that the total fall of potential between the ends of BC is equal to the fall at the ends of the ten-metres potentiometer and the problem is to determine very accurately the potential of the various points along BC. The present arrangement for determining the potential was found so good that the null points in the phone could be determined with absolute certainty. The success of the experiment was mainly due to this part of the technique. The method may be employed with advantage for measurement of resistances where AC is to be used.

In the preliminary experiments where buzzers were used, no definite results were obtained.

The details of the arrangement can be understood from the diagrammatic sketch in Figure 4.5.

METHOD OF OBSERVATION

In carrying out the experiments, the moving contact P₁ in the potentiometer (Figure 4.5) is placed at 1/20th of the potential and by probing with the help of P, the series of null points corresponding to this potential are marked in the tank. P₁ is then moved to 2/20th of the potential and the observations are then repeated. A complete set of equipotential lines for decrements of 5 percent of the total potential drop, are thus traced on the bottom of the tank. To facilitate the direct marking of the points, the bottom of the tank was made of ground glass. Tracings of the points are then made and reduced to a convenient size for comparison.

COMPARISON OF RESULTS—MODELS AND THEORY

Pressure observations were done in the hydraulic and electric models, on the standard forms given in Chapter IX. The agreement in all cases was very satisfactory between the results obtained from models and those derived mathematically.

Figure 4·6, shows the pressure contours under a plain horizontal floor as derived from theory and the hydraulic and electric models.

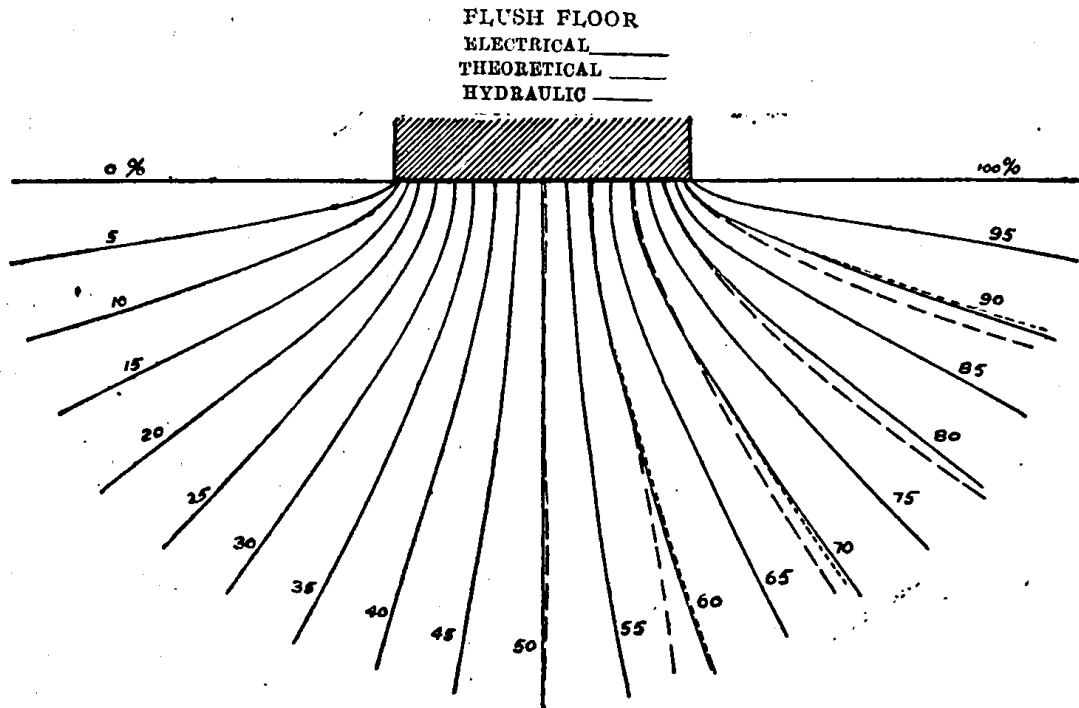


Figure 4·6

Figure 4·7 shows the uplift—pressure lines under a plain floor and under a plain floor with a pile line at one end for various ratios of base length to pile-depth. The agreement in all cases is as good as could be desired.

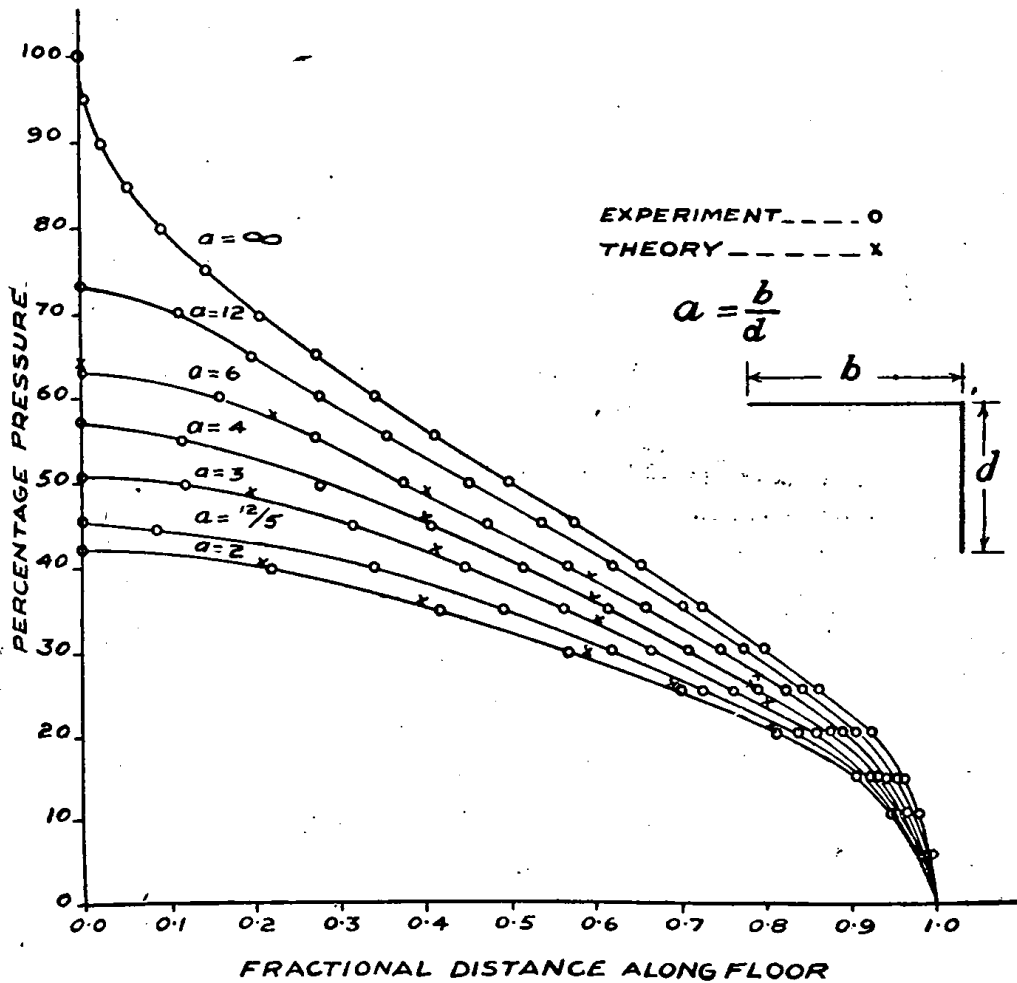


Figure 4·7

Table IV-1 shows the comparison of pressures along a pile line with no aprons but with equal or unequal fill on the upstream and downstream sides. In this case the depths at which 5 per cent., 10 per cent., 20 per cent., etc., pressure occur have been determined

mathematically (Chapter VII, Case 2) and on the electric model with a 5 inch depth of pile. The agreement could be no better.

Experiment were also carried out with an upstream apron and pile line at the downstream end, with different depths of fill downstream of the pile line. The results are given in Plate IV-1, and can be easily checked against theoretical values from tables in Chapter VII.

Similar experiments were carried out with a stepped floor having a pile line at the step. This is the general case of Chapter VII. The experimental values as obtained from the electric model and those derived from Chapter VII are given in Table IV-2. See also table for General Case in Chapter IX.

TABLE IV-1

Φ	d ₁ =5 d ₂ =0		d ₁ =5 d ₂ =1		d ₁ =5 d ₂ =2		d ₁ =5 d ₂ =3		d ₁ =5 d ₂ =4		d ₁ =5 d ₂ =5	
	Y _T	Y _E	Y _T	Y _E	Y _T	Y _E	Y _T	Y _E	Y _T	Y _E	Y _T	Y _E
5	5.00	5.00	4.24	4.12	3.39	3.30	2.54	2.44	1.68	1.68	0.80	0.78
10	5.00	5.00	4.44	4.40	3.72	3.69	3.04	2.99	2.31	2.25	1.55	1.50
15	4.95	5.00	4.60	4.47	4.05	4.08	3.50	3.48	2.90	2.90	2.25	2.22
20	4.95	5.00	4.80	4.76	4.38	4.36	3.96	3.88	3.48	3.45	2.95	2.91
25	4.90	4.95	4.92	4.88	4.62	4.60	4.34	4.29	3.98	3.98	3.55	3.51
30	4.80	4.79	5.00	4.88	4.80	4.76	4.64	4.60	4.38	4.36	4.05	3.99
35	4.65	4.65	5.00	4.98	4.92	4.88	4.86	4.81	4.70	4.69	4.45	4.50
40	4.50	4.48	4.96	4.96	4.98	5.00	5.00	5.00	4.92	4.90	4.75	4.80
45	4.35	4.28	4.88	4.88	4.94	5.00	5.06	5.00	5.00	5.00	4.95	5.00
50	4.10	4.02	4.68	4.76	4.80	4.86	5.00	5.00	5.00	5.00	5.00	5.00
55	3.85	3.72	4.48	4.48	4.65	4.66	4.86	4.81	4.95	4.61	4.95	5.0
60	3.50	3.44	4.16	4.20	4.38	4.41	4.60	4.60	4.72	4.74	4.75	4.80
65	3.15	3.08	3.80	3.83	4.02	4.04	4.26	4.28	4.40	4.40	4.45	4.50
70	2.80	2.70	3.40	3.40	3.60	3.65	3.84	3.83	3.98	4.00	4.05	3.99
75	2.40	2.31	2.92	2.92	3.12	3.20	3.34	3.32	3.48	3.50	3.55	3.51
80	1.95	1.90	2.40	2.36	2.58	2.64	2.76	2.76	2.88	2.92	2.95	2.91
85	1.45	1.46	1.80	1.84	1.96	2.04	2.10	2.14	2.20	2.25	2.25	2.22
90	1.00	1.00	1.24	1.24	1.32	1.40	1.44	1.55	1.51	1.52	1.55	1.50
95	0.50	0.52	0.64	0.68	0.69	0.74	0.74	0.89	0.78	0.79	0.80	0.78
00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
d ₁	5	5	5	5	5	5	5	5	5	5	5	
d ₂	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	
Φ _D	0	29.3	34.3	38.2	41.2	42.7	45.5	46.3	48.1	49.1	50.0	

$\Phi = \frac{P}{H} \times 100 = \text{Percentage Pressure.}$

Y = Depth along pile at which a certain percentage pressure occurs.

Y_T = Depth derived from theory.

Y_E = Depth derived from electric model, experimentally.

Φ_D = Percentage pressure at D.

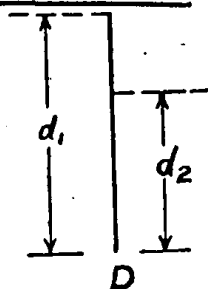
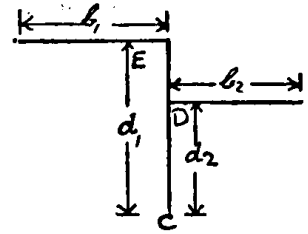


Figure 4.8

TABLE IV-2

$$\delta = \frac{d_2}{d_1 - d_2}; \delta_1 = \frac{b_1}{d_1 - d_2}; \delta_2 = \frac{b_2}{d_1 - d_2}$$



δ	δ ₁	δ ₂	Φ _E		Φ _D		Φ _C	
			Exp.	Theor.	Exp.	Theor.	Exp.	Theor.
4.0	4	0	77.0	76.3	43.4	43.8	0.0	0.0
4.0	4	1	77.1	76.5	44.0	44.3	6.8	6.9
4.0	4	2	77.4	76.9	45.6	45.6	12.4	13.7
4.0	4	3	78.2	77.1	47.1	47.3	18.7	19.4
4.0	4	4	78.6	78.1	48.9	49.2	24.2	24.6
4.0	4	5	79.3	78.8	50.5	51.2	27.9	28.9
4.0	4	6	79.9	79.5	53.3	53.1	31.9	32.7
4.0	4	7	80.7	80.0	55.4	54.8	37.5	36.0
4.0	4	8	81.7	80.8	56.5	56.4	39.4	38.9
0.0	7	7	56.7	56.8	53.7	53.9	53.7	53.9
1.0	7	7	60.2	59.8	51.4	51.7	45.6	46.6
2.0	7	7	63.2	63.2	50.6	50.9	40.4	41.6
3.0	7	7	66.9	66.6	50.3	50.4	36.2	37.2
4.0	7	7	69.7	69.6	49.8	50.0	32.3	33.4
5.0	7	7	72.6	72.4	49.5	49.8	29.2	30.0

Similarly, cases of depressed floor have been tried and the following table gives a comparison of the results obtained mathematically and from the electric model : -

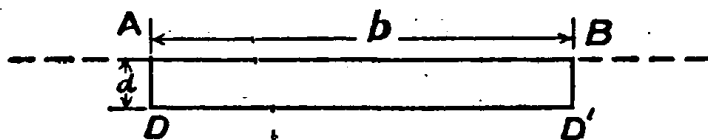


Figure 4.9

TABLE IV-3

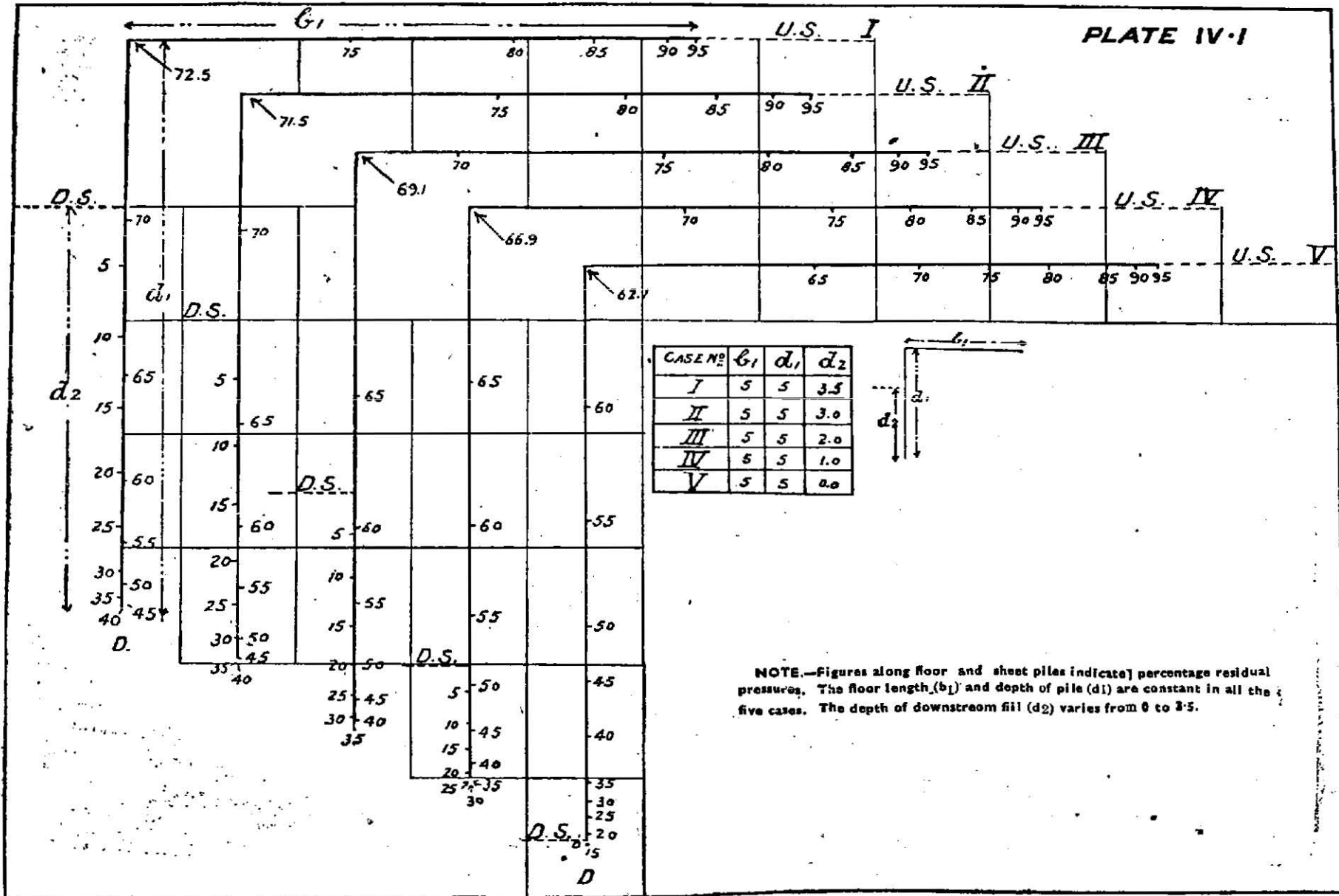
$\frac{d}{b}$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{12}{12}$	$\frac{5}{2.5}$
	0.042	0.083	0.167	0.25	0.333	0.42	0.50	1.00	2.00
φ _{D'} Electric	10.0	13.2	17.4	20.0	22.7	24.3	25.3	30.0	34.6
φ _{D'} Theoretical	9.8	13.1	17.4	20.1	22.2	23.7	25.0	30.0	34.6

See also Table VII-5 of Chapter VII and Appendix to Chapter VII, page 88.

Most of the other standard forms have been similarly tried and the results agree in all cases. The comparative values given above are enough to show that:—

- the results obtained mathematically,
 - the results obtained from the hydraulic model, and
 - the results obtained from the electric model.
- agree between themselves.

It may, therefore, be taken as proved that the flow of water through permeable soils below weirs is analogous to the flow of electricity through a conductor, and to the flow of a viscous



fluid and that the potential law applies in all cases. So that, in cases where the equations are determinate, a complete solution is possible mathematically.

That the results from models agree with those on their prototype field structures, will be proved in Chapter VI.

MODEL CHARACTERISTICS IN SUB-SOIL WORK

I.—Hydraulic Model

(a) *Effect of porosity, grain size, packing etc., of sub-soil.*—Experiments were carried out with sands of different particle diameter, different porosity and different packing under the same model but homogeneity was maintained in each set of experiments. The distribution of pressures and the stream lines remained unaltered.

Hence the configuration of equi-pressure and stream lines—generally known as the flow net—is independent of the class and structure of sub-soil so long as the latter is homogeneous.

(b) *Effect of Scale.*—Experiments were carried out for the case of a depressed floor and floor with sheet pile at one end by changing the floor length but by keeping constant the ratio of floor length to depression in the case of the depressed floor and to depth of pile in the other case. The flow nets were identical in all cases.

Hence the flow net is independent of the scale of the model.

(c) *Effect of Temperature.*—Experiments were carried out at different temperatures with the same model—the temperature remaining uniform throughout the sub-soil. The flow nets were identical in all cases.

Hence the flow net is independent of temperature so long as the latter is uniform throughout the medium.

(d) *Effect of Head.*—Experiments were carried out by varying the head through a fairly wide range. The stream lines remained unchanged. The potential function $\Phi = \frac{P}{H}$, where

P = indicated residual head at any point, and

H = total head,

remained unchanged. In other words if H was doubled, P at any point was doubled; if H was halved, P was halved.

Hence the flow net is independent of head.

(e) *Effect of Upstream and Downstream Levels.*—Experiments were carried out by varying the upstream and/or downstream levels. The pressure lines remained unaltered and so also the stream lines.

Hence the flow net is independent of upstream and downstream levels.

II.—Electric Model.

The above five sets of experiments were repeated on the electric model with similar results.

(a) The variations in class and structure of sub-soil were reproduced by changing the concentration of the salt solution forming the electrolyte.

(b) The scale of the model was changed so that the ratio of floor length to depth of depression or sheet pile remained unaltered.

(c) Experiments were carried out at various temperatures which remained uniform throughout the medium during any one experiment.

(d) Different potentials were employed.

These experiments prove that the flow net of equi-pressure and stream lines, for any particular weir profile, is independent of:—

(a) Class and structure of subsoil so long as it is homogeneous.

(b) Scale or size of structure.

(c) Temperature so long as it is uniform throughout the medium.

(d) Applied head.

(e) Upstream and downstream water levels.

Conclusions (c), (d) and (e) were arrived at by Khosla¹ in 1932 from a study of the pressure observations at the Panjnad Weir.

(1) "Pressure Pipe Observations at Panjnad Weir" by A. N. Khosla, I. S. E., M. A. Soc. C. E., Punjab Engineering Congress, Paper No. 162 (1933).

Since the models show that the flow net is independent of the class and structure of the subsoil and of scale, it can at once be inferred that the flow net obtained on a model should conform exactly to that under the prototype field structures provided that the subsoil of the latter is homogeneous. This conclusion is perfectly valid without further proof but actual comparisons of the model and field results have been given in Chapter VI, so as to place the validity of this conclusion beyond question.

STRATIFICATION

It has been proved above that the flow net remains unchanged in a homogeneous sub-soil irrespective of the class of soil, the head, the temperature, *etc.* This flow net under a given weir profile will change if the subsoil becomes non-homogeneous or stratified. Some work has been done on this aspect in the Punjab Research Institute in connection with uplift pressures below the clay substratum under Bay 4 of the Khanki Weir¹ and the investigation of effect of silt upstream of the Panjnad Weir² floor, mentioned in Chapter VI. Schaffernak³ has done a good deal of work on stratification and his conclusions are reproduced below:—

“Investigation of the laws of the flow of water through dams or through the subsoil, can be carried out by means of experiments with appropriate models.

Experiments with models require the ratio between the co-efficients of permeability of the individual layers used in the model to be in strict conformity with that of the corresponding strata in nature.

For some special cases, however, the solution of the problem can also be found by theory.

There is no need to establish special rules for carrying out the laboratory work, because the experimental investigation concerning the flow of water through non-homogeneous materials can be carried out in the same manner as the model-tests concerning the flow of water through homogeneous materials.”

Haigh⁴ has also dealt with some aspects of stratification.

RELATIVE MERITS OF THE HYDRAULIC AND ELECTRIC MODELS

From the above it will be seen that either model is suitable for the determination of the essential factors governing weir designs. The difference in the two is one of degree. This will be considered under:—

- (a) Speed.
 - (b) Accuracy.
 - (c) Scope.
- (a) *Speed.*—The electric model is much the quicker of the two. The equipment is brief and simple to set up. The readings can be taken very quickly.
- (b) *Accuracy.*—While both methods can be equally accurate, there are a greater number of sources of error in the hydraulic model. The scale and shape of the model and the relative size of pressure points are among the major sources. In the electric model the probe is a fine point which can locate the position of any pressure point with great precision. In all the comparisons so far made, the error from the theoretical has rarely exceeded 1 per cent. and in an extreme case it has been slightly above 2 per cent.
- The electric method is, therefore, more accurate.
- (c) *Scope.*—So far as our present knowledge goes, the hydraulic model scores in this respect. It can reproduce effect of stratification and temperature variation. If set up in a suitable flume, it can reproduce tail erosion and can effectively show the behaviour of the standing wave and the damage likely to occur from various conditions of surface flow. It permits of an investigation of the limiting gradients at exit and the true significance of such gradients. It is capable of reproducing stream lines.

On the electric model, stratification has been studied by Pavlovsky⁵ and Hazra⁵. The effect of varying temperature can be studied by putting in the same electrolyte with different temperatures in compartments separated by copper strips. The temperature variation can

(1) Research Publication, Vol. II, No 10. By A. N. Khosla, *etc.* (1935).

(2) Paper 185, Punjab Engineering Congress (1935). By Harbans Lal Uppal.

(3) Paper No. 31 by Dr. Friedrich Schaffernak of Vienna. International Commission on Dams, (1933).

(4) “Design of Wier on Sand Foundations.” By F. F. Haigh, Paper 182, Punjab Engineering congress.

(5) “Uplift and Seepage under Dams on Sand,” By L. F. Hazra, M.A.M.Soc., C.E., Paper No. 1020 (1935).

either be in steps or graded. With a rise in temperature of the inflowing water the pressures increase. With a rise in temperature of the electrolyte, the latter falls in conductivity and pressures drop. Apparently the results are contrary in the two cases but, really speaking, they are in agreement. A rise in temperature of inflowing water is the same thing as a drop in temperature of the soil medium with the temperature of the inflowing water remaining constant. Thus a drop in temperature of the subsoil medium results in a rise in pressures. Similarly a drop in the temperature of an electrolyte results in a rise in pressures.

Scour at either the upstream or the downstream end of a work can be equally well reproduced on the electric models. The super-imposition of silt of a grade different from that forming the subsoil, is equivalent to stratification which can be reproduced on this class of model by adding at the upstream end a suitable length of electrolyte of different conductivity.

It is in the investigation of problems of surface flow that the hydraulic model is distinctly superior to the electric model. The latter method appears, however, to possess vast potentialities and a sustained effort to explore these will be amply repaid by the extraordinarily rapid and accurate results that it will yield.

CHAPTER V

The Location and Erection of pressure Pipes and Standard Methods of Observation and Record

This chapter is essentially a reproduction of the first part of Publication No. 8¹ of the Central Board of Irrigation, of which the second part is contained in Chapter VI.

Pressure pipes were first of all put in the Narora Weir in March 1898. The observations on these showed that the uplift pressure under the floor as indicated by the level of water in the pipes was in excess of the weight of the latter, suggesting possibility of failure. Two days later, part of the weir actually failed. Since then pressure pipes have been installed on a number of weirs and dams in India and abroad, with two objects in view :—

firstly, to watch the stability of works, and

secondly, to investigate if the actual pressures at various points on the structure were in conformity with those assumed for purposes of design.

Observations of uplift pressure have been made at these works from time to time and a record of these exists in published papers^{2 to 5} and official records. Lane⁶ gives a comprehensive list of such documents.

In the Punjab, pressure pipes were inserted in the Khanki Weir⁷ and the Rasul Weir about 1917 and in the Sulemanki⁴ Weir in 1925. Between 1926 and 1928, series of pressure pipes were put in the various siphons⁸ under the Upper Chenab Canal. The results obtained from these showed a wide departure from those given by the Bligh Creep Theory. In 1929, it was decided to extend the Panjnad Weir. This afforded an opportunity to put in a comprehensive set of pressure pipes on scientific lines to enable a thorough investigation of the uplift pressures under an actual field structure to be made. This was the first experiment of its kind in the world and one that has been instrumental in furnishing the final solution to the problem of Weir Design on Sand Foundations. The results of this investigation are contained in Paper No. 162⁵ (1933) by Khosla. Early in 1932, a fresh set of pressure pipes were installed at the Khanki Weir. The observations on these showed little drop in pressure (due to flow through sand) below the weir crest in some of the bays, indicating unstable conditions of the weir floor (see Plate V. 1). Serious damage occurred during the following monsoon that same year; the floor was undermined at several places and huge cavities were discovered (see Photo 1 of Chapter VIII). The history of damage, repairs and reconstruction of the Khanki Weir are described by Khosla in Paper No. 195⁸ (1936).

A comprehensive set of pressure pipes on the lines of Panjnad were erected at the Khanki Weir during reconstruction. These are discussed in the next chapter.

Like the Sulemanki Weir, the Khanki Weir has a substratum of clay, in some of the bays. Bay 4 is a typical case. The location of pressure pipes, the interpretation of results

(1) "The Location and Erection of Pressure Pipes and Standard Methods of Observations and Record." By R. B. A. N. Khosla, I. S. E., M. Am. Soc. C. E.

(2) "Upward Pressures under Dams. Experiments by the United States Bureau of Reclamation." By Julian Hinds, M. Am. Soc. C. E., Paper No. 1717 (1920). American Society of Civil Engineers.

(3) "Hydraulic Gradients in sub-soil water flow in relation to Stability of Structures Resting on Saturated Soils." By A. N. Khosla, I. S. E., A. M. I. E. (India), Paper No. 138, Punjab Engineering Congress (1930).

(4) "Stability of Weirs and Canal Works. An application of the new theory of Hydraulic Gradients." By A. N. Khosla, I. S. E., Paper No. 142, Punjab Engineering Congress (1930).

(5) "Pressure Pipe Observations at Panjnad Weir." By A. N. Khosla, I. S. E., M. Am. Soc. C. E., Punjab Engineering Congress, Paper No. 162.

(6) Paper No. 1919, American Soc. of Civil Engineers, "Security from under seepage". By E. W. Lane, M. Am. Soc. C. E. (1935).

(7) "Design of Irrigation Works." Third edition. By W. G. Bligh. Page 197. Publishers, Constable & Co., London.

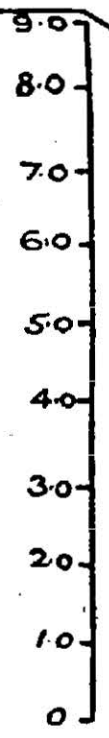
(8) "Reconstruction of the Khanki Weir." By R. B. A. N. Khosla, I. S. E., Paper No. 195 of Punjab Engineering Congress (1936).

U/S W-LEVEL

PLATE V.I

KHANKI WEIR PEIR NO. 5 BAY NO. 6

D/9.9.33



P=7.0

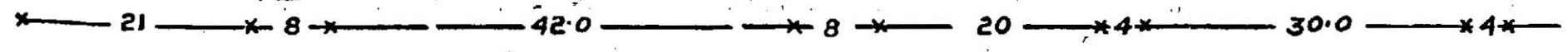
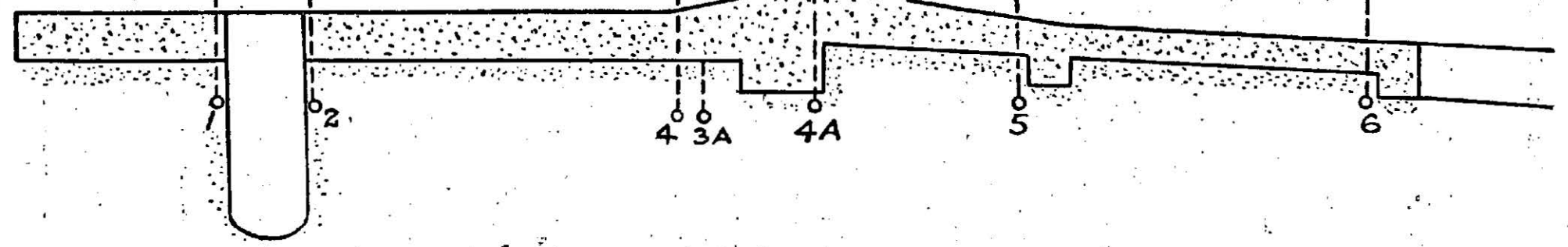
P=3.65

P=0.95

P=0.6

P=0.4

D/S W-LEVEL



obtained from them and the fundamental principles involved in design in such cases will be dealt with in the next chapter and in Chapters X and XI.

From the cases of Narora and Khanki Weirs, it is obvious that the pressure pipes acted as "tell-tales". They made it possible to predict undesirable developments under the weir floor in the neighbourhood of the pipes. Their usefulness for scientific research has been established by the Panjnad results. The pressure pipes recently fixed at the Merala Weir have helped to indicate the extent to which reconstruction is necessary.

The erection of pressure pipes may, therefore, be considered as an essential part of any newly constructed hydraulic structure and a useful addition to an old one. A systematic record of their observations—apart from its scientific value—will be as necessary for the maintenance of such structures as a record of the usual soundings and probings.

GEOLOGICAL FORMATION OF SUBSOIL

For a correct interpretation of the results of observation, it is necessary to know the geological formation of the subsoil. If the subsoil is homogeneous, the results, under normal conditions, will be identical with those obtained from models (see Chapter VI). Any considerable difference between the two sets of results must be due either to some local fault or to non-homogeneity in the subsoil. The exact location of pressure pipes will be determined by the nature and extent of such non-homogeneity. If a weir is under-laid with a continuous bed of impervious clay (Khanki Weir Bay 4 and Sulemanki¹ Weir, see Plate V. 3) the pressure in the permeable stratum below the clay bed will have absolutely no relation with the pressure above that bed. It is the latter pressure which will act against the weir floor. But a puncture in the impervious bed may materially alter the uplift pressures by affording, to the underlying high pressure water, access to the subsoil immediately under the weir floor, thus subjecting the latter to pressures much in excess of the designed. Local clay bands or lenticles will not interfere with the general distribution of pressures, but may cause local abnormalities which must be considered in design. The existence of very porous beds of shingle and their position relative to the weir floor may call for substantial modifications in the normal design. In certain positions they might add to safety and, in others, detract from it.

The formation of the subsoil can be found by making bore holes at suitable points at the site of work and by keeping a careful record of the class of soil met with at various depths in each bore hole.

In homogeneous soils and in soils with small lenticles of impervious or very porous material, borings can be made anywhere with safety. Where there is a continuous substratum of clay below the permeable soil underlying the weir floor (Khanki Weir Bay 4) great care must be exercised in locating the position and depth of bores. It will be dangerous to puncture the clay as that will provide access for the high pressure water under it to the underside of the weir floor. But punctures must be made to investigate soil formation. Such punctures, if made, should be carefully sealed. Where the sealing is likely to be doubtful, the bore pipe should be left in position and plugged. Similarly, where extremely pervious substrata underlie the subsoil below the weir floor, the position of punctures must be carefully considered. They might lead to dangerous springs if done at the tail end of the work. Such formations, with proper manipulation, can just as well afford good drainage and thus cause reduction in uplift pressures, with consequent economy in floor thickness.

But it must be remembered that while the borings will furnish a general idea of the nature of the subsoil, they cannot possibly indicate all the minor geological details. In most cases an intelligent guess will be possible. Due consideration must, therefore, be given in designs for this element of uncertainty by allowing a suitable factor of safety.

(1) "Stability of Weirs and Canal Works." by A. N. Khosla, Paper No. 142, Punjab Engineering Congress, 1930, Plate V.

LOCATION OF PRESSURE POINTS

A comprehensive arrangement of pressure points under weirs on homogeneous (or nearly so) subsoils and those on homogeneous subsoils underlaid with a continuous impervious substratum (of clay, *etc.*) is given in Plates V. 2 and V. 3 (Plate V. 2 is the same as Plate 1 of Publication 8). The Panjnad Weir and Bay 4 of the Khanki Weir have been selected as representative types.

The pressure points may be divided into three groups :—

- (1) along and immediately beneath the horizontal floor,
- (2) at different points along the deep vertical cut-offs, and
- (3) at different depths in the subsoil.

The suitable points under the horizontal floor are the upstream and downstream ends, immediately upstream and downstream of each vertical cut-off and other intermediate points at regular intervals. These will give uplift pressures at these determined points below the floor, a knowledge of which is essential to watch the safety of the work.

Along the deep vertical cut-offs, the pressure points should be placed or driven close against the upstream and downstream faces at suitable depths. In a homogeneous subsoil, in addition to the points upstream and downstream of the cut-off just under the floor, it will be enough to have one point half-way along the depth on either side and one just at the bottom. An additional point some 8 to 10 feet below the bottom will be desirable. In a subsoil where definite stratification exists, more especially where clay beds are encountered, the location of these points will be determined by the nature of the stratification. The pressure points so located enable a correct determination to be made of the effect on uplift pressures of the depth and spacing of cut-offs and of the stratification of the subsoil. It will also be possible to find out the consequent distortion of the stream-lines.

In the subsoil, the pressure points should be located at suitable depths and intervals under the pervious as well as impervious floors with due regard to stratification. The pressures at these points will give the normal distribution of pressure and stream-lines away from the effect of the distortion mentioned above.

In new works under construction any scheme of pressure observation points can be carried out in full, but in existing structures the extent to which it can be done will depend upon local considerations.

The location of pressure pipes in any work, existing or new, should be planned with great care and thought. The stratigraphy of the subsoil, especially the presence of clay beds and other geological features, must be duly considered together with the profile of the foundation of the structure. A knowledge of the geological sections of the subsoil is essential for working out a suitable design for the locality and for a proper appreciation of the subsequent pressure observations.

The arrangement of pressure points shown in Plates V. 2 and 3, is required for a detailed study of the flow of water through the subsoil under a particular weir. Such a large number of pressure points need only be placed at either flank and at a few lines in the body of the weir depending on its size. Certain key points under the horizontal floor, *viz.*, upstream and downstream of each vertical cut-off and just under the crest, may be fitted with pressure points at more frequent intervals. These will enable the safety of the work to be watched along the entire length. Thus, on the Panjnad Weir, Pipes Nos. 31, 24, 21, 13, 10 (see Plate V. 2) may with advantage be put in every alternate pier and Pipes Nos. 21, 10 in each pier.

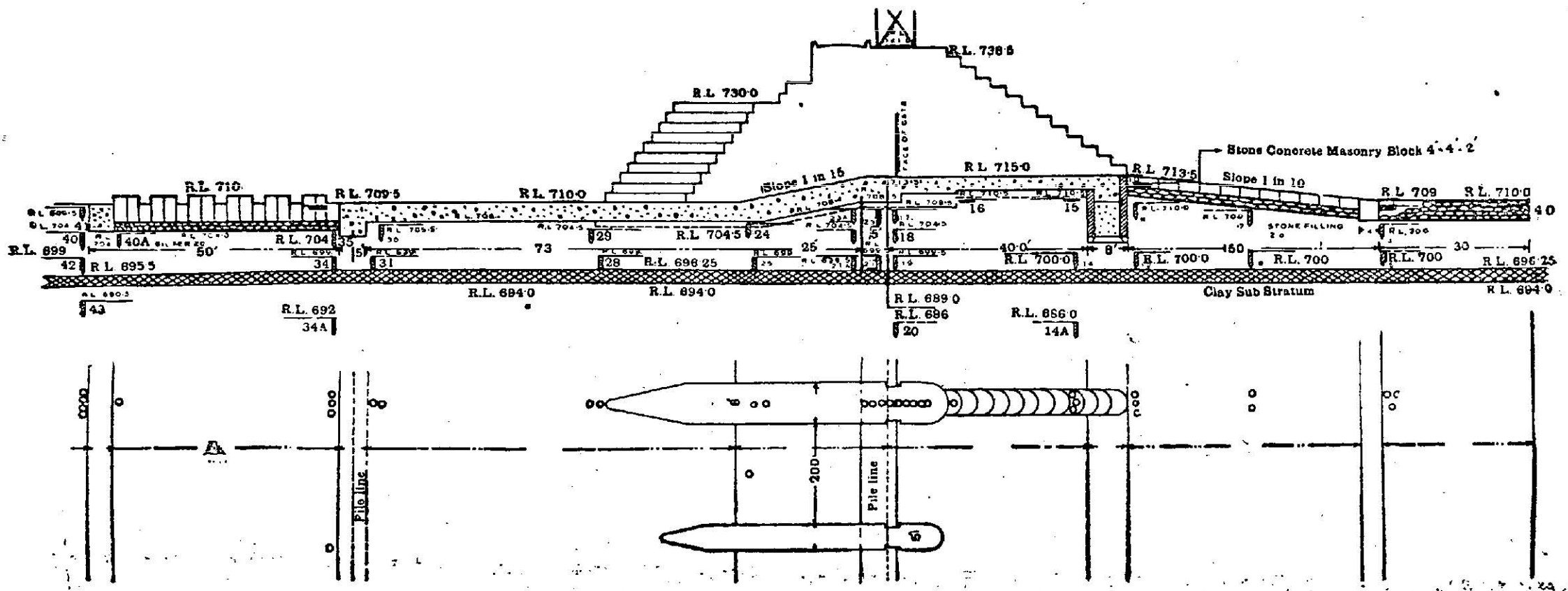
ERECTION OF PRESSURE PIPES

To allow of the pressures at the selected points below the work being read, $1\frac{1}{2}$ " diameter filters are placed at these points which are connected by $1\frac{1}{2}$ " G. I. pipes to suitable locations

KHANKI WEIR BAY IV

CROSS SECTION OF RIGHT AND LEFT DIVIDE PIER

SHOWING LOCATION OF PRESSURE POINTS



on the superstructure. These filters (Plate V. 2) are available in the market in standard lengths of about 3 ft. They are fitted with a driving point at one end and a threaded $1\frac{1}{2}$ " blind pipe at the other, with a length of about $2\frac{1}{2}$ feet of 50 mesh wire gauze strainer in between. These filter points should be laid horizontal where excavation permits. Otherwise they should be driven down to the proper level by means of a monkey. When the depth is too great or when the soil is hard and liable to cause damage to the filter in driving, the filter and the connecting blind pipe can be inserted in a bore hole made by a 4" diameter casing pipe. The casing pipe is ordinarily withdrawn, but when a clay bed exists below the sandy subsoil, the extraction of the casing pipe may lead to troublesome springs.

The pressure conditions below a clay bed are unaffected by the pumping above, so that if that bed be punctured a spring may result, leading to cavities and rendering the pressure observation point of doubtful value. In such cases the casing pipe may be withdrawn just clear of the $2\frac{1}{2}$ feet length of filter, and the space between the casing and the $1\frac{1}{2}$ " pipe filled with cement grout. Such conditions were met with in the reconstruction of the Khanki Weir Bay 4.

PRECAUTIONS TO BE OBSERVED IN ERECTION OF PRESSURE PIPES

(i) A damaged filter should not be used. If one is damaged in driving, it should be replaced.

(ii) The jointing of the filter and the blind pipes should be done with great care and made absolutely watertight as any leak may give entirely unreliable observations. Wherever such joints are accessible they should be sealed with a coating of rich cement concrete.

(iii) When more than one pipe are driven at the same place to different depths, they should be spaced not closer than one foot. This will avoid any direct connection between any two filters due to driving or extraction where such may become necessary.

(iv) The horizontal piping between the filter and the observation point on the superstructure should be slightly inclined downwards in the direction of the filter to avoid possible air lock.

(v) During erection, the ends of all pipes should be kept closed by standard caps to avoid foreign matter finding its way into the pipe and thus making them useless for purposes of observation. This is a most important precaution and cannot be too strongly stressed. Careless workmen or mischievous boys may ruin the pipes by choking them with grit, sand or other foreign material. All vertical pipes must be kept dead vertical and all kinks must be carefully avoided. Failure in this respect may make it impossible to lower the bell-sounder or the thermometer to the right place for observations.

(vi) Each pressure observation point should be given a distinctive number and that number should be marked on the filter and on each length of connecting pipe. The greatest care must be exercised in this respect, for once confusion arises there will be no means to determine to which filter point a given stand pipe refers. These distinctive numbers should be stamped on the caps at the end of the stand pipes and on the masonry or concrete platform where they are collected.

(vii) Pipes from the filter points are led into piers or flank-walls to enable readings to be taken throughout the year. Points directly under the piers or flank-walls are connected to the observation platform by a single vertical lengths of piping. But those removed from piers and flank-walls have to be connected by horizontal length of piping. These latter should be placed well below the lowest pressure level that is likely to occur at the respective points. Otherwise, no observations will be possible during certain conditions of supply when the observation stand-pipe will be dry.

(viii) Each pipe should be tested to see that the strainer is not choked. This can be best done by connecting the top of each pipe with an ordinary hand pump and by working the pump for one-quarter to half an hour till water comes out with freedom. A choked filter will not respond to variations of pressure and will give erratic results.

After the initial test, should any pipe or pipes give erratic results they should be pumped as above. After that they will be found to give consistent results.

(ix) It is futile to lay a pressure point in impervious clay. This will show no response to any variation of pressure. If it is desired to find out whether the clay is actually impervious or not, a pressure point will be of use.

OBSERVATIONS AND INSTRUMENTS USED

The following observations for pressure data should be made simultaneously :—

1. Upstream water level.
2. Downstream water level.
3. Shade temperature ; maximum and minimum.
4. The temperature of the river water, at a depth from the surface below which it remains approximately constant.
5. Temperature of sub-soil water in a few selected observation pipes.
6. Water level in all pipes.
7. The depth of silt on the upstream and downstream floors and if possible the nature and grade of silt.

Items 1 and 2 are read from gauges suitably fixed. Item 3 is read by means of the ordinary maximum and minimum thermometer.

It has been found that the surface temperature of the river water is influenced by the atmospheric temperature. Below the surface, this effect is less marked and at a depth of 4 to 5 feet the temperature appears to remain approximately constant. This is the temperature which should be observed and it follows that a common thermometer is useless as the reading will be affected by the varying temperature in the upper 4 to 5 feet during withdrawal. This observation should, therefore, be made with a maximum-minimum thermometer.

The water temperature in selected pipes can be observed only with special thermometers, capable of passing freely through a $1\frac{1}{2}$ " pipe. A suitable type is the Six's Pattern max.-min. thermometer manufactured by Messrs. Negretti and Zambra (London) which can be obtained through the Indian Stores Department. Long distance recording thermometers are even more satisfactory though more expensive. These may be either electric or mercury-in-steel type. This latter type, when let down the pipe to any depth, will indicate the temperature at that depth on a dial. The temperature can thus be read directly at the observation platform. In the electric type the resistance thermometer is connected to the dial by means of an insulated and water-proof cable. In the mercury-in-steel type, the mercury bulb is connected to the dial by means of a steel capillary. Both types are capable of automatic record. Where a max.-min. thermometer is used it must be raised very steadily as any jerk would disturb the indicator. This is quite a common source of error.

The water level in the pipes can best be read by means of a *bell-sounder* (Plate V, 2) lowered into the pipe by a steel tape. Electric devices have been tried but the bell-sounder has been found to be the most accurate, reliable and simple. The bell-sounder consists of a solid brass rod about $3\frac{1}{2}$ " long and $\frac{7}{8}$ " diameter ending in an inverted cup of $1\frac{1}{4}$ " diameter. A swivel is screwed on to the upper end to which the steel tape is attached. The length of the sounder below the zero of the tape should be measured carefully and added to each reading of the tape to get the true depth from the top of the stand-pipe to the water level in the pipe. Directly the cup of the sounder hits the water surface in the pipe a definite sound is produced which cannot be mistaken or drowned even in the roar of water passing over the weir crest. For exact reading, the cup is moved up and down and the precise position, where the sound occurs, is read. These readings can be accurate to within $\frac{1}{8}$ of an inch, but $\frac{1}{8}$ " may be taken as the normal degree of accuracy. Each observation takes from 1 to 2 minutes.

The reduced level of the top of each stand pipe being known, the reduced level of the water in the stand pipe is obtained by subtracting the measured depth from the known reduced level.

The depth of silt is obtained by soundings.

POINT OF STRAINER TO WHICH A PIPE READING RELATES

For a strainer laid horizontal, the centre of the strainer will be the position of the pressure point to which the reading in the connecting stand pipe relates. If there is drop of pressure along the 3 ft. length of the strainer, the pressure point will be at the farthest position along the direction of flow. Similarly in a vertical strainer, the pipe reading will refer to the top or bottom end of the strainer whichever is furthest downstream along the direction of flow and has the lowest pressure. Thus at the upstream end, the reading will refer to the bottom of the strainer and, at the downstream, to the top.

FREQUENCY OF OBSERVATIONS

At the Panjnad Weir, these observations are taken twice a week; at the Khanki Weir once a week. The frequency of observations will depend on local requirements. For investigation of a particular problem, the observations will have to be rather frequent but for watching the safety of a weir, once a week for the key points and once a fortnight for other pipes should be enough.

TIME LAG

When there are large fluctuations in upstream and downstream water-levels, for instance during rising or falling floods or when the river supply is being ponded up to feed supplies into the canals, the results are likely to be influenced by time lag. A rise in the upstream level will give relatively lower readings and *vice versa*. If the water level rises upstream, the total head increases but the water levels in the pipes have not had time to adjust themselves to the raised pond level and therefore, the reading will be relatively low. Similarly when the pond level falls, the total head is decreased but the levels in the pipes remain high and, therefore, give a relatively higher reading. When observations are taken at regular intervals, due allowance can be made for such time lag and the true pressures obtained.

What applies to variations in upstream levels, applies also in the case of variations in downstream levels. A sudden rise in the downstream level will give relatively lower pipe readings and *vice versa*. In heading up there is a rise upstream and a drop in downstream river level. The effect of lag in such a case may be non-existent or may result in the pipe reading being higher or lower than the normal, depending on the relative rate of change of upstream and downstream levels.

In a rising flood, when both upstream and downstream levels are rising, the pressure pipes will read relatively low. In a falling flood, when both the upstream and downstream levels are falling, they will read high.

The response to any variation in head due to change in the upstream or downstream levels is almost instantaneous throughout the work but as the rate of movement of subsoil water is very, very slow, the water in the pressure pipes takes some little time to deplete or recuperate according as the pressure falls or rises. Some tests on time lag will be found in Chapter X. But more systematic and comprehensive tests are required to enable an accurate study of time lag being made. If the gauges upstream and downstream remain steady for 24 hours the results of observations will, it is believed, be reliable. The exact period after which the effect of time lag disappears remains yet to be investigated.

According to Slichter¹ "in a uniform sand of diameter of grain of one-half mm. the ground water will reach within one percent of its final maximum velocity due to a sudden application of pressure or head in approximately 30 seconds".

(1) Water Supply, Paper No. 140., U. S. Geological Survey. "Field measurements of the rate of movement of Ground Waters." By Charles S. Slichter (page 71), (1905).

METHOD OF RECORD AND STANDARD FORMS USED

The record consists of the following data:—

1. Date of observation.
2. Upstream water level.
3. Downstream water level.
4. H = Total Head, *i. e.*, difference in levels; upstream and downstream.
5. Maximum and minimum shade temperatures.
6. Temperature of river water.
7. Temperature of water in selected pipes.
8. Depths of silt on upstream and downstream floors.
9. Water level in all pipes.
10. P = Pressure in each pipe, *i. e.*, difference between water in stand-pipe and downstream river level.
11. $\Phi = \frac{P}{H} \times 100$, *i. e.*, pressure in each pipe expressed as a percentage of the total head H .

The form illustrated in from V. 1 is specially prepared for use in this connection.

NAME OF WORK

Date		Water Level		Head—H	Shade Temperature	River Water Temperature 4 feet below surface	Depth of silt on pervious floor	
		U/S	D/S				U/S	D/S
1	Line No.							
2	Pipe No.							
3	Reduced Level of Measuring point							
4	Temperature of (Deg. F.)	Water in Pipe						
5	Depth to (feet)							
6	Reduced Level of (Col. 3—Col. 5)							
7	P=Pressure (Col. 6—D/S Water Level)							
8	$\Phi = \frac{P}{H} \times 100$							
4	Temperature of (Deg. F.)	Water in Pipe						
5	Depth to (feet)							
6	Reduced Level of (Col. 3.—Col. 5)							
7	P=Pressure (Col. 6—D/S Water Level)							
8	$\Phi = \frac{P}{H} \times 100$							
4	Temperature of (Deg. F.)	Water in Pipe						
5	Depth to (feet)							
6	Reduced Level of (Col. 3—Col. 5)							

DIVISION

CANAL

	7	P=Pressure (Col. 6—D/S Water Level)	
	8	$\Phi = \frac{P}{H} \times 100$	
	4	Temperature of (Deg. F)	Water in Pipe
	5	Depth to (feet).	
	6	Reduced Level of (Col. 3—Col. 5.)	
	7	P=Pressure (Col. 6—D/S Water Level)	
	8	$\Phi = \frac{P}{H} \times 100$	
	4	Temperature of (Deg. F.)	Water in Pipe
	5	Depth to (feet).	
	6	Reduced Level of Col. 3—Col. 5	
	7	P=Pressure (Col. 6—D/S Water Level).	
	8	$\Phi = \frac{P}{H} \times 100$	
	4	Temperature of (Deg. F.)	Water in Pipe
	5	Depth to (feet)	
	6	Reduced Level of (Col. 3—Col. 5)	
	7	P=Pressure (Col. 6—D/S Water Level).	
	8	$\Phi = \frac{P}{H} \times 100$	

For purposes of analysing the observations, pipes are grouped by "lines", *i.e.* pipes of a single section from upstream to downstream of the weir.

One page of the register should be devoted to one line. Sufficient pages should be reserved consecutively for each line to admit of all observations for that line being recorded consecutively for a standard period of, say, one year.

The registers should be maintained in duplicate, one for permanent record in the office of origin and the other for periodical despatch to the central organization where these data will be scrutinized.

These registers should be printed on standard forms and made available to the various Provinces for uniformity of record.

PLOTTING OF GRAPHS

For purposes of ready reference, the mass of data, collected and recorded as explained in previous paragraphs, should be plotted as graphs. If properly plotted these will give, without much mental effort, a quick visual idea of the behaviour of different pipes at any time of the year and will bring to light any special features or abnormalities. In the past, graphs have been generally plotted either with absolute levels or with indicated pressures (P), with the result that the interpretation of data and their comparison became extremely difficult on account of the variations in upstream and downstream water levels and in the total head (H).

It has been shown in the previous chapter, and will be dealt with in the next chapter,

that $\Phi = \frac{P}{H}$ remains constant for any weir provided the temperature of the inflowing water and

the nature and depth of silt deposit or scour on the upstream or downstream pervious floors does not alter. The function Φ should, therefore, form the basis of plotting, as any variation in its value from the normal will connote damage unless this variation can be explained by temperature and silt.

Plate VI. 6 shows a suitable method of plotting on a seasonal basis. The dates of observation are plotted on the horizontal base and the variable factors on the vertical axis. These latter are:—

1. $\Phi = \frac{P}{H}$ per cent.
2. River temperature.
3. Sub-soil temperature.
4. Head= H
5. Downstream water level.
6. Depth of silt or scour at the pervious floors, upstream and downstream.

This graph will bring out the salient factors that are likely to influence the values of Φ .

These graphs should be kept posted up to date for all the key points, *viz.*, points just under the weir floor upstream and downstream of each vertical cut-off and under or just downstream of the crest, so that any unfavourable developments in the sub-soil can be discovered as soon as they occur.

CHAPTER VI

Models and the Prototypes

Comparison of Results

In Chapter IV it has been shown how the results obtained from the hydraulic and electric models agree between themselves and with those derived from mathematics. As this agreement holds irrespective of the scale of the model, the nature of subsoil or of the electrolyte and of the head, it is rational to conclude that it will hold between models and their prototypes in the field. But the engineer naturally, and rightly, hesitates to accept such conclusions wholesale unless their validity can be established from actual observation in the field. With that in view, experiments were conducted by Harbans Lal Uppal¹ at the Punjab Research Institute with a model of the Panjnad Weir to see if the results obtained at the actual weir as presented by Khosla² (1933), and as observed subsequently, could be reproduced on models. Paper No 185¹ shows the degree of success attending these experiments. The results of the hydraulic model were found to agree almost exactly with those of the electric model and all the field observations at Panjnad were reproduced on the hydraulic model within very close limits, by superimposing more or less silt at the upstream end and by changing the temperature of the inflowing water. The relevant results are shown in Table VI. 1, and have been plotted in a somewhat unusual but effective manner in Plate VI. 1. The vertical axis denotes the value of ϕ , i.e., $P/H \times 100$. The horizontal axis is made any arbitrary length—in this case twice the vertical. The two extremities A B are joined across by a straight line. The normal values of ϕ for each point are taken from the results of the hydraulic or electric model with no silt or temperature effect. These are projected on to the diagonal line A B. The verticals through these intersections are then taken to represent the "proportional" relative positions of these points. Thus the normal value for pipe 35 is 87.12 per cent. The projection of 87.12 per cent. from the vertical scale cuts the diagonal AB at point X. The vertical X (35) will represent the standard line for pipe 35 along which all other values for that pipe will be plotted. It will be noticed that observations at all points relating to one particular set of conditions of silt and temperature fall on one straight line passing through A. This shows that:—

- (a) The effect of silt is to cut off a part of pressure H permanently at or near the intake reducing it to H' so that if P' be the pressure at a point where it would normally be P, the ratio ϕ'/ϕ , where $\phi' = P'/H'$ and $\phi = P/H$, will remain constant for all points under the floor for that particular condition of silt deposit. See Curves 2, 4, 6, 8, 15, Plate VI. 1.
- (b) The effect of temperature is somewhat similar to the above. Its differential character will be dealt with later. See Curves 8 and 11—Plate VI. 1 and Plate VI. 7.

This diagram incidently shows the relative effectiveness of the horizontal floor and the vertical cut-offs in the case of Panjnad Weir.

It will be noticed from Table VI. 1 that pressures in the hydraulic model for pipes 8 and 7 are distinctly higher than those on the electric model. This is due to the existence of the inverted filter in the hydraulic model and its absence in the electric model.

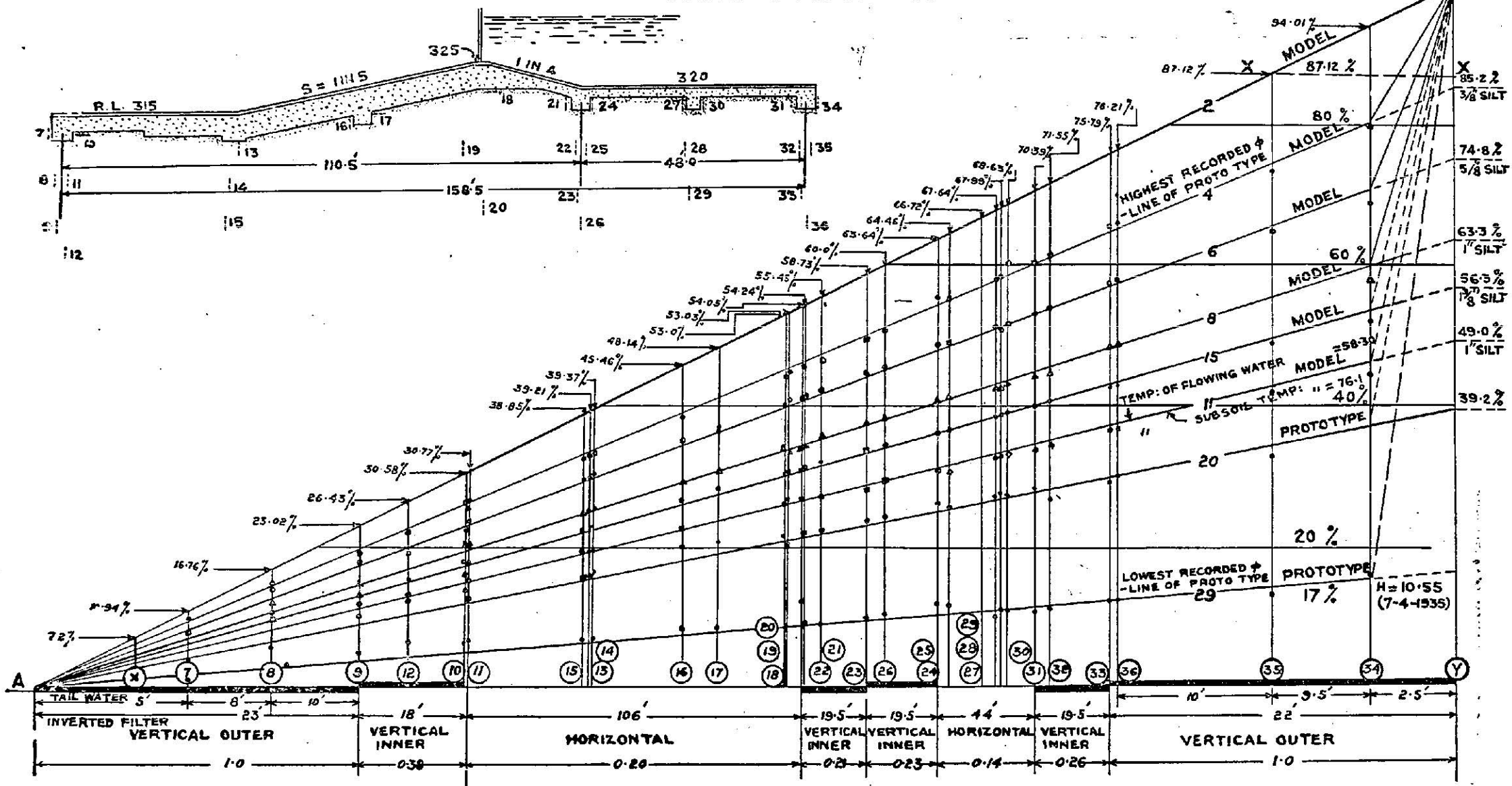
Plate VI. 2 and Table VI. 2 show the comparative results obtained on the hydraulic and electric models of the Khanki Weir (Bay VIII) and on the actual work at Khanki. The pressure in the tail pipes in the hydraulic model are higher in this case, as in that of Panjnad, than those in the electric model due to the presence of the inverted filter in the former model and its absence in the latter. The inverted filter at Khanki is comparatively less pervious than the one at Panjnad due to the smaller clearance between the deep blocks overlying the kankar and shingle filter bed.

(1) "Pressures under a model of the Panjnad Weir and under the prototype" by Harbans Lal Uppal. Paper No. 185, Punjab Engineering Congress, 1935.

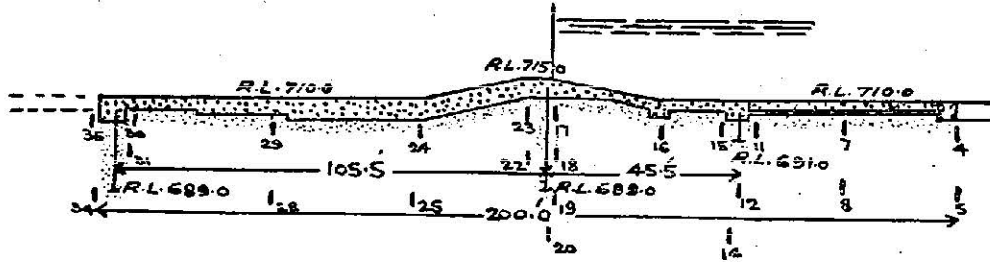
(2) "Pressure pipe observations at Panjnad Weir" by A. N. Khosla, I.S.E., M.Am. Soc., C.E., Punjab Engineering Congress, paper No. 182.

PANJNAD WEIR PRESSURES UNDER FLOOR

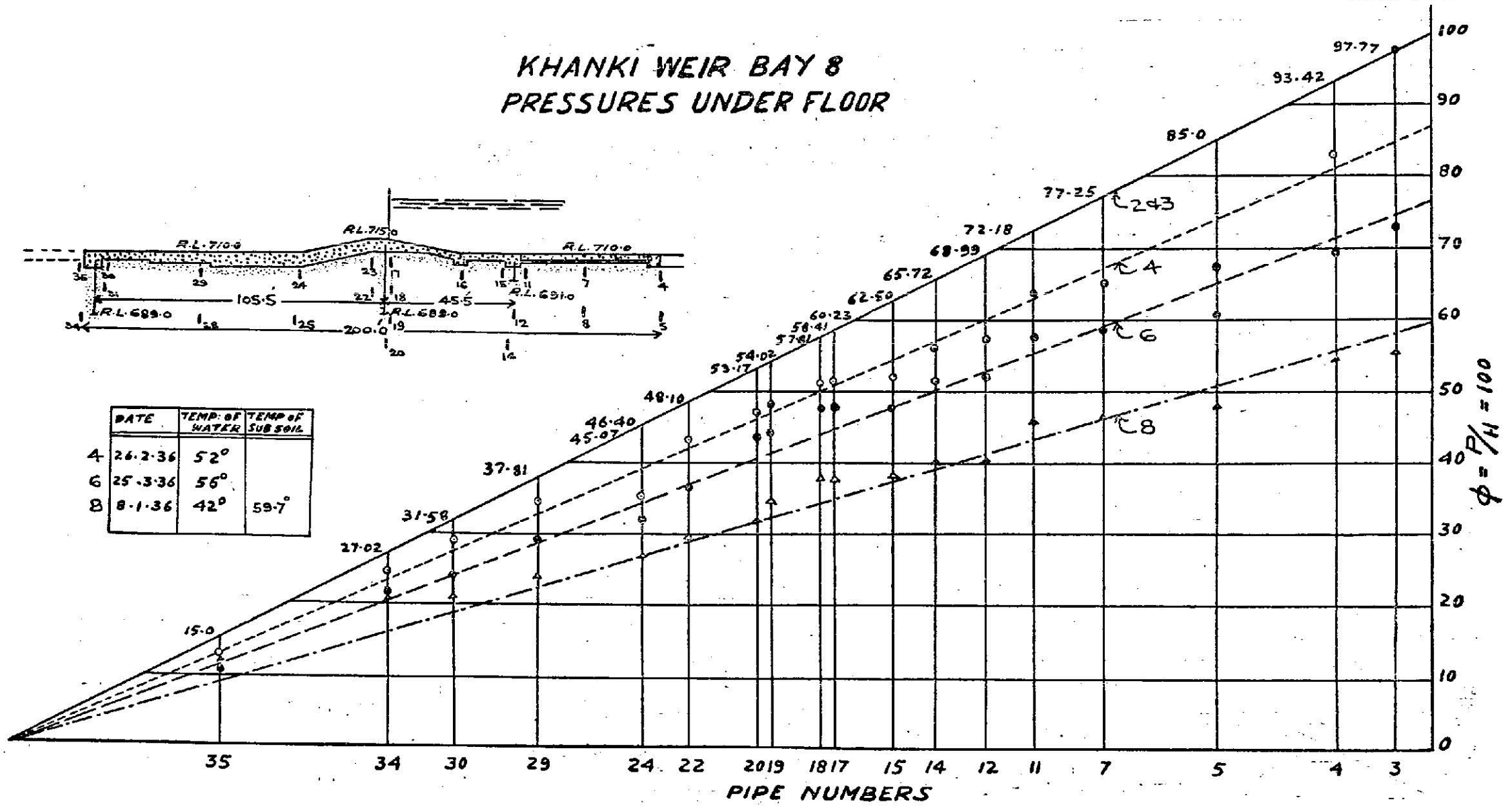
PLATE VI-1



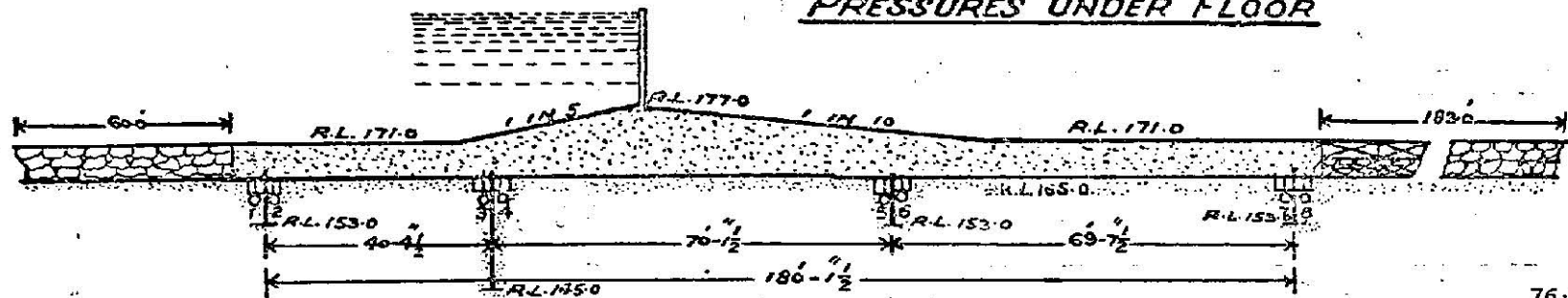
KHANKI WEIR BAY 8 PRESSURES UNDER FLOOR



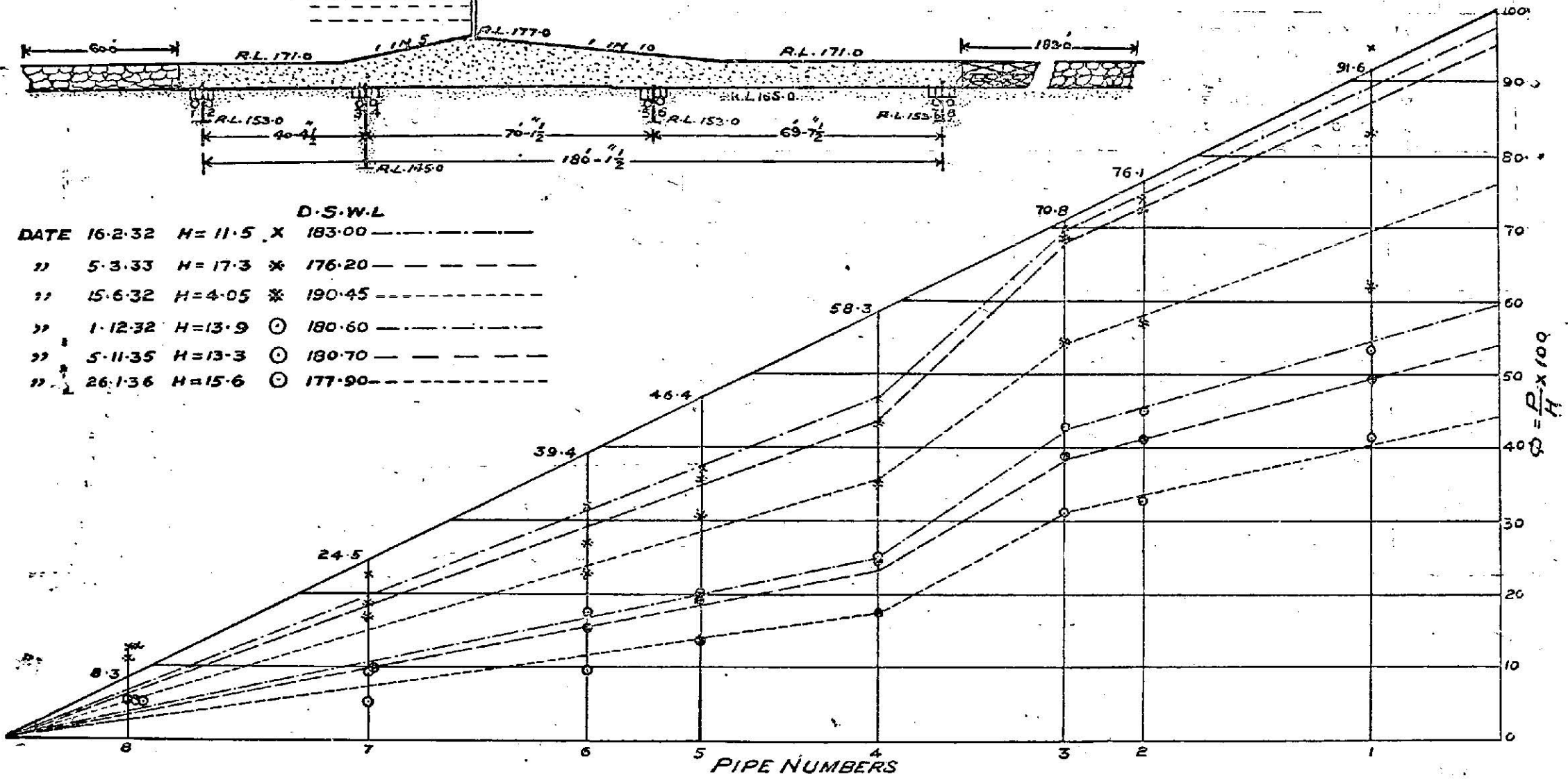
DATE	TEMP. OF WATER	TEMP. OF SUB SOIL
4	26.2.36	52°
6	25.3.36	56°
8	8.1.36	42°
		59.7°



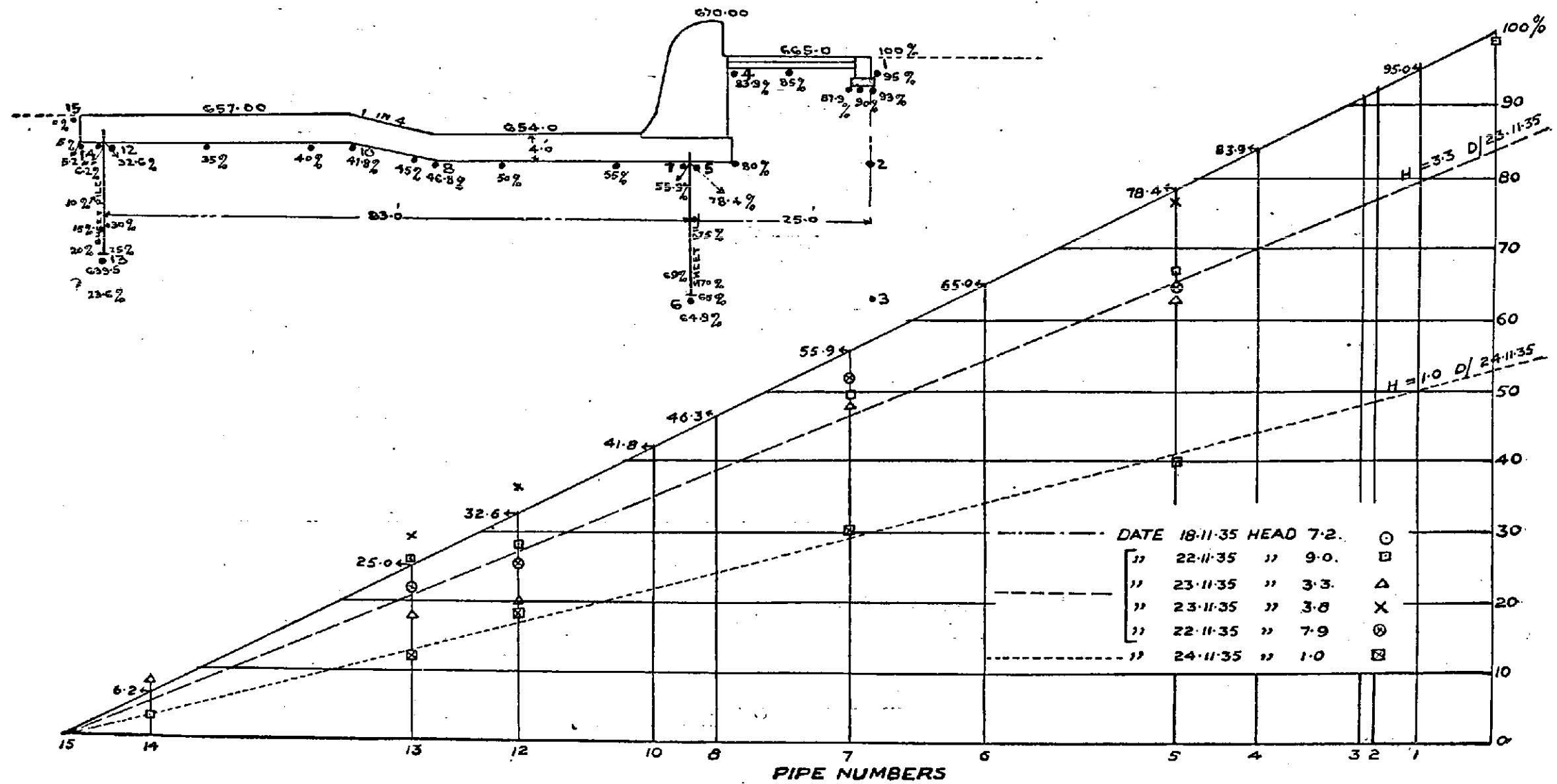
PRESSURES UNDER FLOOR

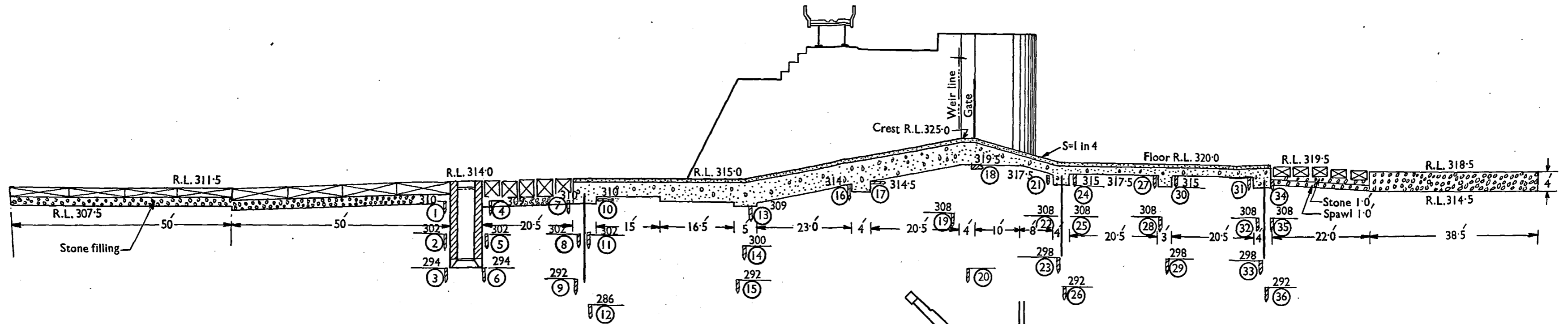


DATE	H	D.S.W.L	X
16-2-32	11.5	X	183.00
5-3-33	17.3	*	176.20
15-6-32	4.05	*	190.45
1-12-32	13.9	○	180.60
5-11-35	13.3	○	180.70
26-1-36	15.6	○	177.90

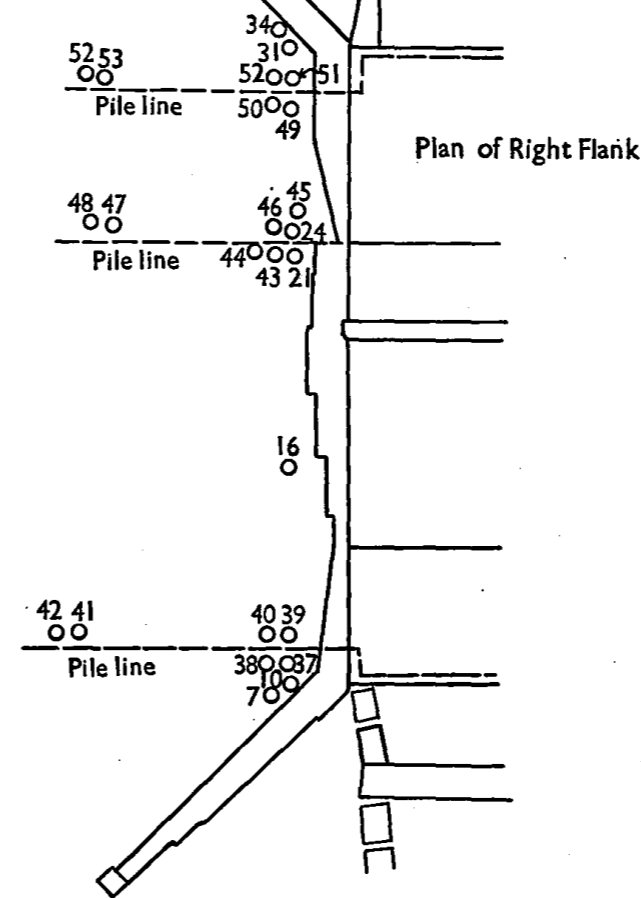
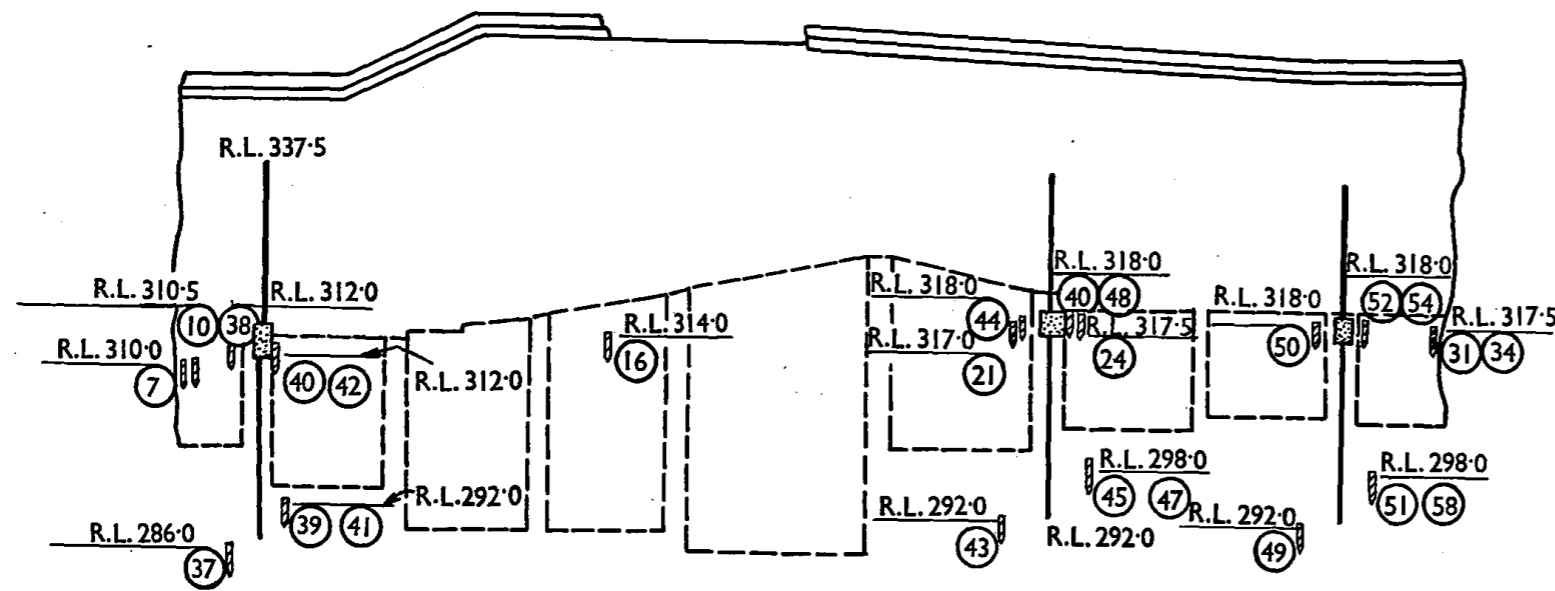


DEG DIVERSION HEAD PRESSURES UNDER FLOOR



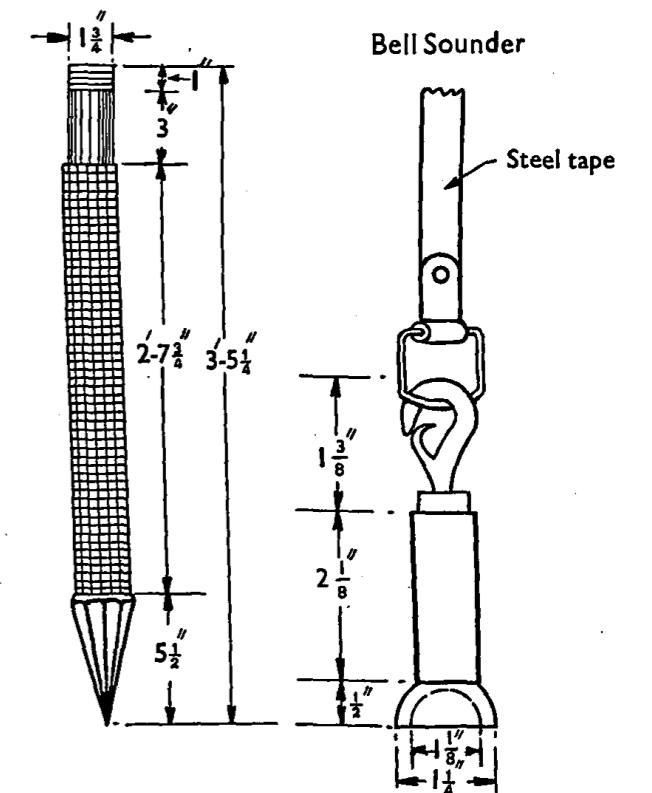


Sectional Elevation of Right Flank



Plan of Right Flank

Drive Point Well Strainer



Showing location of pressure points

TABLE VI. 2.

Date	26th February, 1936.	25th March, 1936.	8th January, 1936.
Head H	11.2	12.3	8.1
Silt U/S	1.0	1.2	2.0
Silt D/S	<i>Nil.</i>	<i>Nil.</i>	<i>Nil.</i>
Temp. } River	52°	56°	42°
	Subsoil		

Pipe No.	Electric Analogy	Scale Model	PROTOTYPE					
			Actual	Calculated*	Actual	Calculated*	Actual	Calculated*
1	2	3	4	5	6	7	8	9
3	..	97.77	98.2	84.8	72.7	74.6	53.6	58.2
4	93.42	90.55	82.3	80.8	69.3	71.4	52.7	55.8
5	86.70	84.28	67.5	74.0	60.5	65.0	48.0	50.8
7	77.25	74.70	65.0	67.2	58.9	53.0	46.5	45.9
8	76.12	73.90	66.5	..	59.8	..	64.4	..
11	72.18	71.17	63.8	62.9	57.8	55.3	46.0	43.2
12	68.99	69.40	57.0	60.0	52.4	52.8	40.2	41.0
14	65.72	67.37	56.2	57.1	51.7	50.1	40.2	39.1
15	62.50	..	52.0	54.2	47.8	47.9	38.2	37.4
16	60.23	59.48	52.3	..	49.0	..	38.2	..
17	58.41	..	51.7	50.8	48.1	44.6	38.2	34.8
18	57.81	..	51.5	50.1	47.6	43.9	37.9	34.1
19	54.02	..	48.0	46.8	44.2	41.1	34.7	32.0
20	53.17	50.11	47.2	46.0	43.7	40.2	31.6	31.7
22	48.10	..	43.2	41.9	36.2	36.6	29.3	28.8
23	47.15	46.40	43.3	..	36.2	..	29.3	..
24	43.52	45.07	35.0	39.1	32.0	34.1	26.3	26.6
25	43.26	..	42.5	..	35.4	..	28.4	..
28	34.60	38.00	58.0	..	43.2	..	58.5	..
29	34.12	37.81	34.6	32.6	28.3	28.7	23.8	22.2
30	25.96	31.58	28.8	27.4	24.1	24.0	20.5	18.8
34	20.12	27.02	24.6	23.2	21.1	20.1	20.8	16.0
35	5.02	7.04	13.2	13.2	10.7	11.4	12.1	9.0

Plate VI. 3 gives comparative results for the Lloyd Barrage and its electric model. In this case the pressure drop between pipes 3 and 4 at the actual Barrage is much more than that indicated by the model, otherwise, the law of proportional variation holds in every case. This comparatively bigger drop is presumably due to the deepest pile line bedding into a layer of lesser porosity and the other pile lines remaining above it. This point is being further investigated.

Plate VI. 4 gives comparative results for the Deg Escape Head and its model.

The above comparisons relate to works resting on more or less homogeneous subsoils. A comparison of the results of model of Bay IV (Khanki Weir), which has a substratum of clay.

* Calculated by the Method of Independent Variables,

with the field results is given in Research Publication Vol. II, No. 10¹ (See Plate VI. 5). The field observations relate to 29th September 1934 with H=10.06, when the subsoil temperature of the river water was 70.25°F. and not much different from that of the subsoil which ranged from 72° to 74° F.

The following table shows the satisfactory agreement between the model and field (prototype) results.

Khanki Weir

Model				Prototype			
Head 13.64 I 12.96 II				10.06			
Pipe	Per cent			Pipe	Per cent		
	I	II	Mean		Right	Left	Mean
20	5.89	7.76	6.83	1	8.1	1.7	4.9
27	7.14	9.15	8.15	18	8.6	9.4	9.0
33	79.52	79.85	79.69	21	82.9	78.4	80.7
32	78.83	79.59	79.21				

Other cases of stratification have not been tried but there is no reason to believe that an equally satisfactory agreement will not be forthcoming between the model and prototype results.

THEORY AND MODELS OFFER SAFE GUIDE FOR DESIGNS

From the above comparisons it may be concluded that *the models can reproduce exact field conditions* and that *the results obtained from models can be accepted as a safe guide for designs*. Since the models agree with theory, therefore, *the design of works can be safely based on theory*.

In Table VI. I, it will be noticed from Col. 27, that in very high heads the values of ϕ for the points under the floor do not exceed 82 per cent. of the normal values. The results in the field cover the period from April 1932 to March 1936. Thus during the 4 years the weir has been in operation, the uplift pressures have not exceeded 82 per cent of the designed. This is *the factor of safety which nature provides*. For the flood period, silt deposits if any are of a coarse nature so that the blanketing effect is small. The values of ϕ are high but those of head (H) are low. The absolute uplift pressures (in feet above the tail-water level) are, in consequence low. In the months when heads are high, the supply in the river is ponded up and the fine clay and silts in suspension settle on to the upstream bed. This results in effective blanketing causing a substantial reduction in the otherwise high uplift pressures.

SEASONAL VARIATIONS IN VALUE OF ϕ .

Plate VI. 6 illustrates the seasonal variations in values of ϕ at the Panjnad Weir. For a correct interpretation of field results, it is most important that the fact of seasonal variations be duly appreciated and their genesis known.

These seasonal variations exist to a greater or less degree on all the structures studied so far. They are due to the following two causes:—

1. Silt or scour at the upstream or downstream of pervious floors.
2. Difference in temperature of the river (seepage) water and the subsoil.

Silt upstream will reduce uplift pressures, that downstream will increase them.

Scour upstream will increase uplift pressure, and scour downstream will reduce them. The latter may, however, lead to dangerous conditions otherwise.

(1) "An investigation of the uplift pressures on a model of Bay IV, Khanki Weir and the prototype". by A. N. Khosla, E. McKenzie Taylor, and H. L. Uppal.

If the temperature of the inflowing water is higher than that of the sub-soil, the uplift pressures will rise and *vice versa*.

EFFECT OF SILT AND SCOUR

The effect of silt or scour is easy to explain. A deposit of silt is equivalent to an increase in the length of the sheet pile at the upstream or downstream end where the deposit occurs. The equivalent length of increase will depend on the nature of silt. A 6" clay deposit at the upstream end may cut off practically all pressure under the floor whereas a 6 ft. deposit of sand may make a comparatively small difference. Compare Cois. 24 and 29, Table VI. I. The occurrence of scour will introduce a contrary effect.

Some engineers are of the opinion that the colloids of the clayey silts, depositing at the upstream end of weirs, penetrate into the pores of the sub-soil, thus causing progressive choking of the latter and consequently a progressive reduction in uplift pressures under the work. This view is entirely erroneous. Silt deposits, of whatever nature, merely form a surface blanket which influences uplift pressures so long as it lasts. The subsoil composition remains completely unaffected. A conclusive proof of this is furnished by Plate VI. 6. The uplift pressures fluctuate within a very wide range attaining the minimum values followed by the maximum each year.

Experiments were carried out by the Development and Research Division (Sind) in 1934 to see the effect of injecting clay under pressure in a sandy subsoil with a view to making it impervious. It was found that in spite of the high pressure, the clay had absolutely no spread laterally or in the depth. If injection under pressure fails to produce any penetration into the subsoil the normal deposits of silts can have no possibility of penetration into the subsoil. These deposits can have therefore only a temporary and seasonal effect.

The effect of silt deposits has been demonstrated on the model of the Panjnad Weir referred to above. See Plate VI. 1, Curves 2, 4, 6, 8 and 15.

EFFECT OF TEMPERATURE

The effect of temperature on the flow of water appears to have been investigated first by Allen Hazen ¹ & ² 1892. He found the following formula for the velocity of flow through sand or other soils:—

$$V = 3.28 c d^2 \frac{h}{l} \left(\frac{t + 10}{60} \right).$$

where

V = the velocity of flow, in feet per 24 hours, of a column of water of the same area as that of the sand.

d = effective size of sand,

c = a coefficient which is usually taken as 1,000.

$\frac{h}{l}$ = hydraulic gradient = $\frac{\text{head producing the flow}}{\text{length of the column of sand}}$

t = temperature of the water in degrees Fahrenheit

The formula is only considered applicable to sand of which the "Effective Size" is between 0.10 and 3.0 millimeters. For gravels with "Effective Size" above 3.0 millimeters* the velocity with a given head does not, Hazen says, "increase as rapidly as the square of the "Effective Size", and with coarse gravels the velocity varies as the square root of the head instead of directly with the head as in sands. The influence of temperature is also less marked with the coarse gravels."

The increase in discharge with increase in temperature and *vice versa*, is due to viscosity (or fluid resistance) which varies inversely with temperature. If μ = viscosity and t = temperature in degrees Fahrenheit,

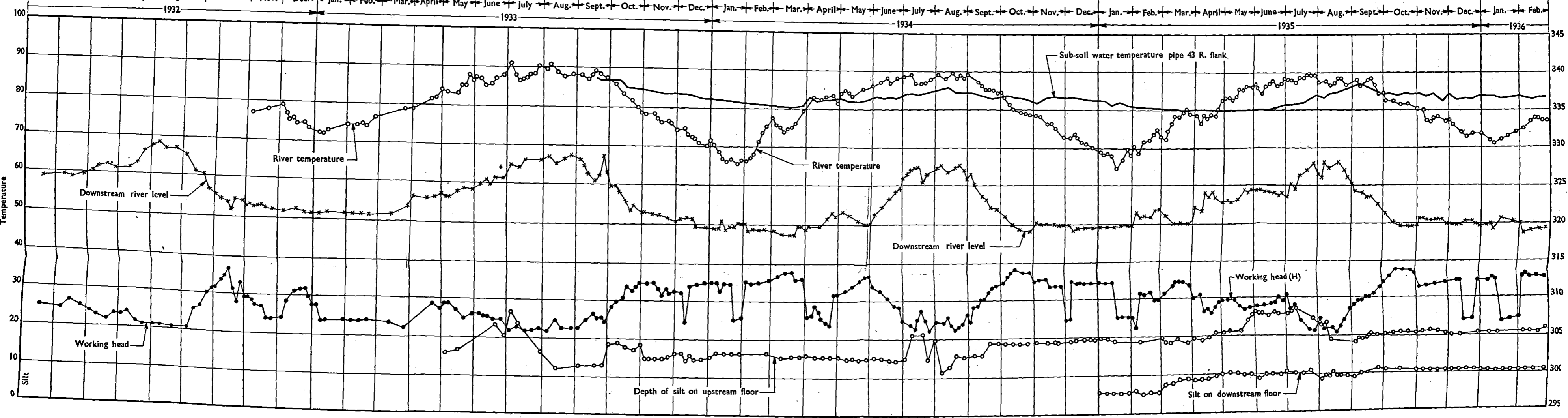
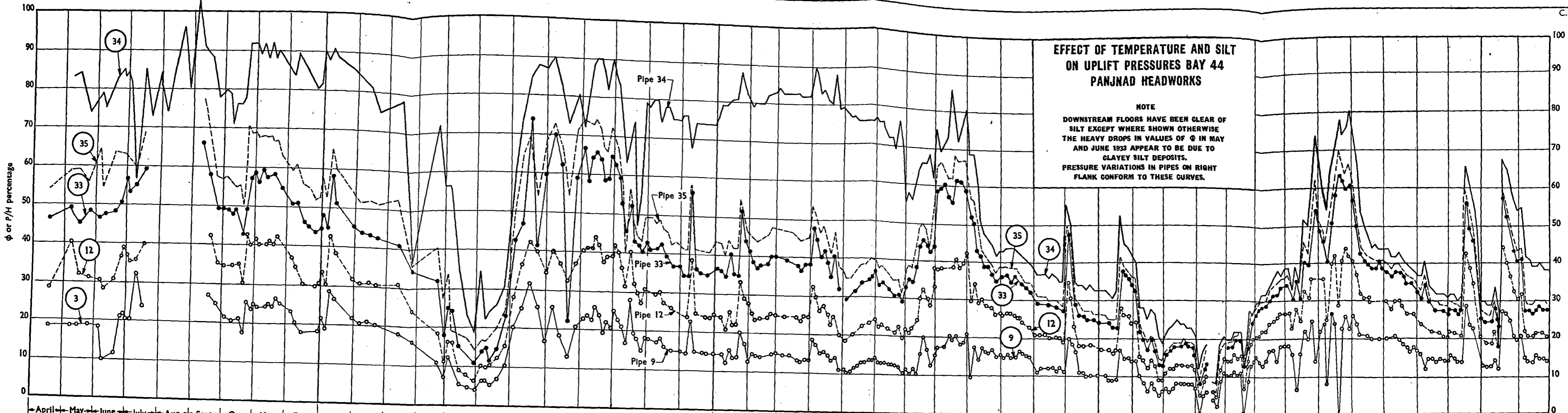
$$\mu = \frac{.00003716}{.4712 + .01435 t + .000682 t^2}$$

For comparing the effect of viscosity, the determining factor is μ/ρ where ρ is the density of the fluid. This ratio μ/ρ is denoted by symbol ν and is known as "Kinematic Viscosity."

(1) "Some Physical Properties of Sand and Gravels," by Allen Hazen, Report of Massachusetts State Board of Health 1892, P. 541. (Reference copied from 2 below.)

(2) Irrigation Pocket Book, by R. B. Buckley, Third Edition 1920, Page 493.

* Lindquist finds the upper limit 2 mm. (See Chapter II.)



The following table and Figure 6.0 give the relative values of temperature, viscosity and discharge :-

Temperature °F.	Relative* Discharge q	Kinematic Viscosity $\nu \times 10^5$	$\nu \times q \times 10^5$
32°	0.70	1.93	1.351
41°	0.85	1.64	1.394
50°	1.00	1.41	1.410
59°	1.15	1.23	1.415
68°	1.30	1.08	1.404
77°	1.45	0.96	1.392
86°	1.60	0.865	1.384

* Discharge at 50°F is taken as unity

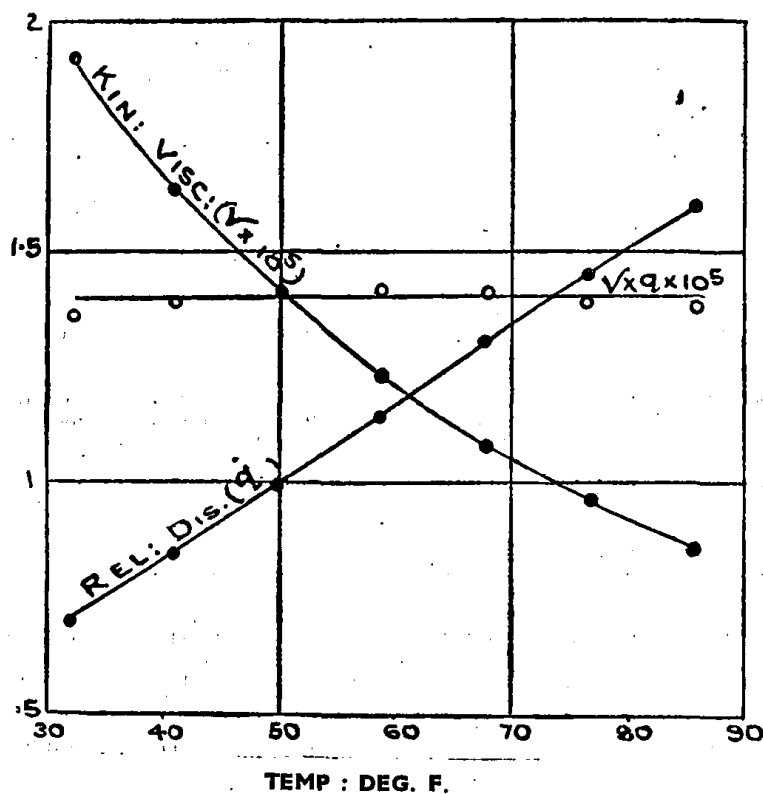


Figure 6.0

As a point of interest it will be noticed from the above table that

$$q \cdot \nu = \text{constant (very nearly)} = C$$

$$\text{But } q = a \nu \text{ and } \nu = K \cdot dp/ds.$$

where a = effective area of section of flow. This is a measure of the relative permeability of the soil medium.

ν = velocity.

p = pressure.

s = length along stream line and

K = a constant.

$$\text{Hence } a K \frac{dp}{ds} \nu = C$$

$$\text{or } \frac{dp}{ds} = \frac{C}{K} \cdot \frac{1}{a} \cdot \frac{1}{\nu}$$

Thus the pressure gradient varies with $\frac{1}{a}$ and $\frac{1}{\nu}$ or inversely with the kinematic viscosity ν , and inversely with the effective area of flow or relative permeability.

Thus when water is passing through a less permeable stratum, the gradient steepens, so that the loss of head per unit length is greater. This explains the large drop of head through the silt deposits at the Panjnad Weir before the water gets into the subsoil below the floor. As the stratum becomes more permeable the gradient flattens and the loss of head per unit length decreases.

Similarly when the temperature is non-uniform, the pressure gradient flattens in the region of higher temperature of the inflowing water and steepens in that of lower temperature.

Thus the effect of variation in permeability on pressures is the same whether that variation is due to natural stratification or to a non-uniform temperature.

A uniform rise or fall in the temperature throughout the medium will have the same effect as uniform increase or decrease in the effective size of the soil particle, *viz.* the discharge will increase or decrease, but the pressure gradients will remain unaltered. *Thus the pressure distribution under a work will be identical whether the sub-soil is coarse sand or silt so long as it is homogeneous, and whether the temperature is 40° or 100° so long as that temperature is uniform, through out the foundation of that work. This characteristic of flow through sub-soil lends it to experiments on models.*

The relative permeability of various silts, sands and fine gravels at a uniform temperature of 60°F. is given in the following table (VI. 3)¹ :—

TABLE VI. 3.—Transmission constants from which the velocity of water in sands and various effective sizes of grain can be obtained.
Table computed for temperature of 64°F.

Diameter of soil grain in mm.	Porosity						Kind of soil
	30 per cent.	32 per cent.	34 per cent.	36 per cent.	38 per cent.	40 per cent.	
.01	0.000033	0.000040	0.000050	0.000060	0.000072	0.000085	Silt
.02	.000131	.000162	.000198	.000230	.000286	.000330	
.03	.000296	.000364	.0004460	.000538	.000645	.000763	Silt
.04	.000527	.000648	.0007940	.000958	.001145	.001355	
.05	.000822	.001012	.001240	.001495	.001790	.002120	Very fine sand.
.06	.001182	.001458	.001784	.002150	.002580	.003050	
.07	.001610	.001983	.002430	.002930	.003510	.004155	
.08	.002105	.002590	.003175	.003825	.004585	.005425	
.09	.002660	.003280	.004018	.004845	.005800	.006860	
.10	.003282	.004050	.004960	.005980	.007170	.008480	
.12	.004725	.005830	.007130	.008620	.01032	.01229	Fine sand.
.14	.006430	.007940	.009720	.01172	.01404	.01662	
.16	.007390	.009120	.0115	.01345	.01611	.01910	
.16	.008410	.01036	.01268	.01531	.01835	.02170	Fine sand.
.18	.1064	.01311	.01605	.01940	.02320	.02745	
.20		.01620	.01983	.02390	.02865	.03390	
.25	.02050	.02530	.03100	.03740	.04480	.05300	
.30	.02960	.03640	.04460	.05380	.06430	.07630	Medium sand.
.35	.04025	.04960	.06075	.07330	.08790	.10390	
.40	.05270	.06480	.07940	.09575	.1145	.1365	
.45	.06650	.08200	.1005	.1211	.1450	.1718	
.50	.08220	.1012	.1240	.1495	.1780	.2120	
.55	.09940	.1225	.1500	.1810	.2165	.2565	Coarse sand.
.60	.1182	.1458	.1784	.2150	.2580	.3050	
.65	.1390	.1710	.2095	.2530	.3030	.3580	
.70	.1610	.1983	.2430	.2930	.3510	.4165	
.75	.1850	.2278	.2785	.3365	.4030	.4770	
.80	.2105	.2590	.3175	.3825	.4585	.5425	
.85	.2375	.2925	.3580	.4325	.5175	.6125	Fine gravel.
.90	.2660	.3280	.4018	.4845	.5800	.6860	
.95	.2965	.3650	.4470	.5400	.6460	.7650	
1.00	.3282	.4050	.4960	.5980	.7170	.8480	
2.00	1.315	1.620	1.933	2.390	2.865	3.390	
3.00	2.960	3.640	4.460	5.380	6.450	7.630	
4.00	5.270	6.480	7.940	9.575	11.45	13.55	
5.00	8.220	10.12	13.40	14.95	17.90	21.20	

(1) "The Rate of movements of underground Waters" by Charles S. Slichter (1905), U. S. Geological Survey. Water supply and Irrigation paper No. 140.

For the pressure distribution under a work to deviate from the normal, it is necessary that the homogeneity of the subsoil must alter. The natural homogeneous formation of the sub-soil will not alter. But a condition of non-homogeneity can be induced by the differential temperatures of the inflowing water as it moves through the sub-soil which is at a very different temperature. The kinematic viscosity and therefore the relative permeability will in such a case vary from point to point, and the subsoil—although originally homogeneous—will, under the conditions imposed by the differential temperature, behave as a non-homogeneous medium. This condition of non-homogeneity imposed artificially on an otherwise homogeneous medium may be called *Induced Stratification*. The effect on pressure due to induced stratification will be similar to that due to natural stratification.

If the subsoil is originally non-homogeneous, the setting up of temperature gradients by difference in temperature of the inflowing water and the sub-soil, will alter the original pressure distribution by *inducing* changes in the existing non-homogeneity by modifying, in varying degrees, the original permeability of its various strata.

In nature, the temperature of the inflowing water and that of the sub-soil vary from day to day so that the pressure distribution under any work must also vary from day to day and from season to season.

Plate VI. 6 shows the seasonal variations at the Panjnad Weir, in river and sub-soil temperatures, silt deposits, working head, pressures, etc. The pressures at any particular pipe go up and down through a very considerable range. These variations are—as explained before—due to two causes :—

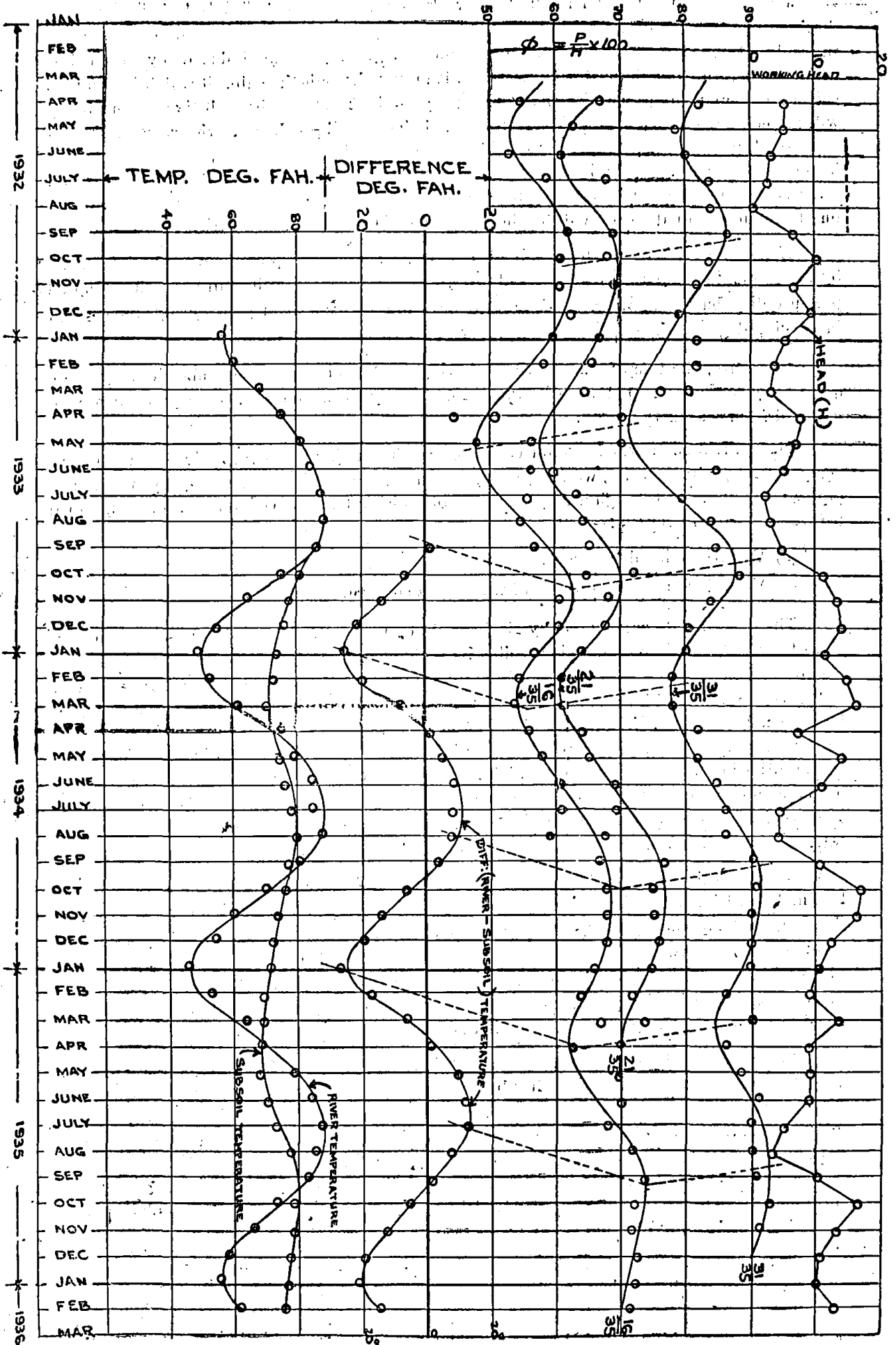
- (a) Variations in silt or scour.
- (b) Variations in temperatures of water and sub-soil.

The experiments carried out by Harbans Lal (Plate VI. 1 and Table VI. 1) clearly show that a change in pressures is possible by a change in the depth and quality of silt and also by a change in the temperature of the water relative to that of the sub-soil medium. See Curves 8 and 11, Plate VI. 1. The exact reproduction of the temperature effect would be extremely difficult, if not impossible in a model. There are temperature gradients in the subsoil even apart from the temperature gradients set up by the difference in the temperature of the inflowing water and the subsoil. While the effect of silt on pressures will be uniformly proportional throughout the foundation sub-soil, the effect of change in temperature will be differential. With careful and laborious experiments it may be possible to obtain a qualitative demonstration of this differential effect.

But does this differential effect really exist on field structures? The observations at Panjnad and Khanki furnish a conclusive reply to this. Only the Panjnad results where the range of temperature is large have been discussed here. Taken as a percentage of the total head H , the silt and temperature effect both come in. But as will be seen from Plate VI. 1, the effect of silt disappears if the pressures in the various pipes are taken as a percentage of that in pipe 34. Curves were plotted for $P/P_{34} \times 100$ and showed distinct and consistent seasonal characteristics very similar to those in Plate VI. 7.

But as Pipe 34 is too close to the surface, it is likely to be influenced at times by silt. To get conclusive evidence Pipe 35 has been selected as the reference point and curves of $P/P_{35} \times 100$ have been plotted on Plate VI. 7. The seasonal trend of variation in pressures due to the differential effect of temperature is unmistakable. The plotting has been done with monthly means and covers the entire range of observations since their beginning in 1932. The monthly means of head (H) of the river and sub-soil temperatures and of the difference in these latter have been plotted on a monthly basis. For distinction, the points for H have been joined straight across and not by smooth curves.

Dotted parallel lines show the time lag from pipe to pipe and from the temperature-difference curve. Since all the peaks and furrows are very nearly connected by parallel lag lines, it is permissible to infer that the lag does not vary materially from year to year and that the differential seasonal variation is due to the temperature effect only. The lag appears to be about a month up to Pipe 31 and $1\frac{1}{2}$ to 2 months to Pipes 21 and 16,



It will be noticed that the range of variation is fairly big. The temperature variation ranges between -25° and -24° and $+10^{\circ}$ to $+14^{\circ}$ or 35° to 38° . The corresponding variation in P/P_{35} is 12 to 17 per cent. Expressed as a percentage of the ratio of normal pressure at any point to P_{35} —no temperature effect—the range comes to about 20 per cent. Thus a range of 35° to 38° F. in temperature has caused a range of 20 per cent. in pressures or approximately *half per cent. for every degree (Fah) differential variation in temperature.* This range applies to pipes 31 and 21 and nearly so to pipe 16. The temperature effect is, therefore, quite considerable.

It may be argued that just as the pressure curves follow the trend of the temperature-difference curve with a certain time lag, they follow, though in a lesser degree, the river temperature curve and, therefore, it would appear that the pressures are influenced by the absolute temperature of the river water. But the river temperature cannot influence pressures if it is the same as the sub-soil temperature. Experiments show that if the temperatures of the soil medium and the inflowing water are uniform and equal, the pressures will remain unchanged, no matter what the actual temperatures are. It is not the absolute temperature but the temperature gradients that influence the pressures and pressure gradients. There is obviously no temperature gradient with a uniform temperature.

TEMPERATURE EFFECT ON MODELS

Two sets of model tests have been carried out to investigate the effect of temperature on pressures. The first was by Dr. Vaidhianathan on a plain pipe filled with sand. The investigation was then extended to the case of the Panjnad model by Harbans Lal. Both of these experiments are described below.

VAIDHIANATHAN'S EXPERIMENTS

The following experiment was conducted by Vaidhianathan and Chanan Singh in the laboratory of the Irrigation Research Institute, to illustrate the effect of the temperature gradient in the sub-soil on the subsoil pressures.

In the following diagrammatic sketch, A B is a brass tube about 50 cms. long, 3 cms. in diameter packed with sand. P_1 , P_2 and P_3 are glass tubes fitted to A B and intended to

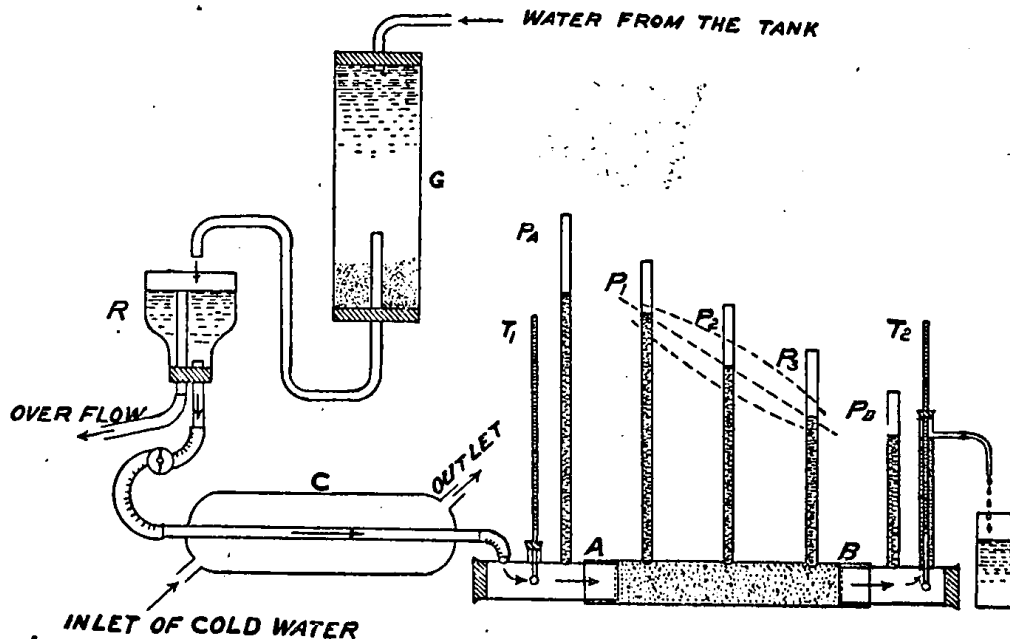


Figure 6.1

(1) "Observation and Record of Pressures below Works on Permeable Foundations," by A. N. Khosla, I.S.E., Central Board of Irrigation Publication No. 8 (1934), p. 13,

measure the pressures in the sand, when water is allowed to flow through A B P_A measures the upstream pressure, P_B the downstream pressure. A and B are wire gauzes keeping the sand in position after packing. T_1 and T_2 are two thermometers indicating the upstream and downstream temperatures. C is a double-walled jacket. By passing water, cooled by a freezing mixture, through the outside jacket the water that enters A B can be cooled considerably. The small reservoir R is fitted with an overflow and an arrangement to lower or raise it which helps to keep the required upstream pressure steady. G is a long glass-wool filter to remove traces of fine material in suspension from the water. It is very important that the water should be carefully filtered. Before entering the filter tube, the water is kept in a reservoir for 3 days to remove air bubbles from the same.

The experiment is conducted as follows :—

A steady head of water is maintained in P and water at atmospheric temperature is passed through A B. P_1 , P_2 and P_3 are kept under observation for about 4 hours. With the precautions mentioned above, the pressures in P_1 , P_2 and P_3 remain steady and when the menisci indicating the pressures are joined, they give a straight line making an angle with the axis of A B. The thermometers T_1 and T_2 are observed. As an illustration, the actual readings of T_1 and T_2 were 23°C in one of the experiments.

Water, cooled by means of the freezing mixture, is now passed through the jacket. This cools the water that enters A B. In the experiment cited above, the temperature fell to 14° C as indicated by T_1 . The temperature at T_2 remained at 23°C for a long time, this depending on the sand and the velocity with which water passes through the tube A B. A temperature difference of 9°C, between the two ends of A B was thus introduced. The line joining the menisci P_1 , P_2 and P_3 then became convex to the axis of A B as a consequence of this temperature gradient. The pressure P_A was not affected since the density change for 9°C is only 0.00156 and the total head being about 51 cms. the total change in head due to this cause could only be 0.079 cms. and was negligible. While P_A remained within this limit, P_1 changed in the particular case mentioned here by 12 mms., P_2 by 17 mms., and P_3 by 5 mms., P_B was not affected, giving the same downstream pressure as before. When the water in the whole tube attained the same temperature, this convexity of the pressure distribution disappeared. Now when water at atmospheric temperature, which was relatively warmer than the cold water in the tube was introduced, the line joining the menisci in P_1 , P_2 and P_3 became slightly concave to the axis of A B. These phenomena are indicated in the figure, which is not to scale. Qualitatively these phenomena were repeated many times, the gradients of temperatures being different in the different cases.

These results lend support to the hypothesis of induced stratification explained above and show how temperature variations of the river and the sub-soil water affect the pressures under weir.

HARBANS LAL'S EXPERIMENTS WITH THE PANJNAD MODEL.¹

A number of thermometers were introduced into the sand in the same position as the pressure observation pipes. Two thermometers were placed in the water at the upstream and downstream ends. One thermometer was placed outside close to the tank to record the temperature of the air. Before actually starting the experiment the temperatures at all the thermometers were recorded. The temperature of the inflowing water was then raised by 5°C and the experiment was allowed to continue for a number of hours. The temperature and pressure readings were taken simultaneously after regular intervals. The temperature of the flowing water was then raised by 10°C and the process repeated. This experiment was continued for 30 hours, and readings were taken after definite interval of time. The effect of the fall in temperature of the flowing water on the pressures was then studied. The temperature and pressure readings were taken in the same manner as described before. The variation in the temperature of the upstream water was about 20°C.

The hydraulic gradients at this model for various temperatures of the inflowing water and the temperature gradients are plotted on Plate VI. 8. The pressures on the upstream portion of the work are high when the temperature of the inflowing water is high. When the temperature of the inflowing water is low, the pressures are also low.

(1) Paper No. 185 of Punjab Engineering Congress. "Pressures under a Model of Panjnad Weir and under the prototype," by H. L. Uppal, M. Sc., Assistant Research Officer.

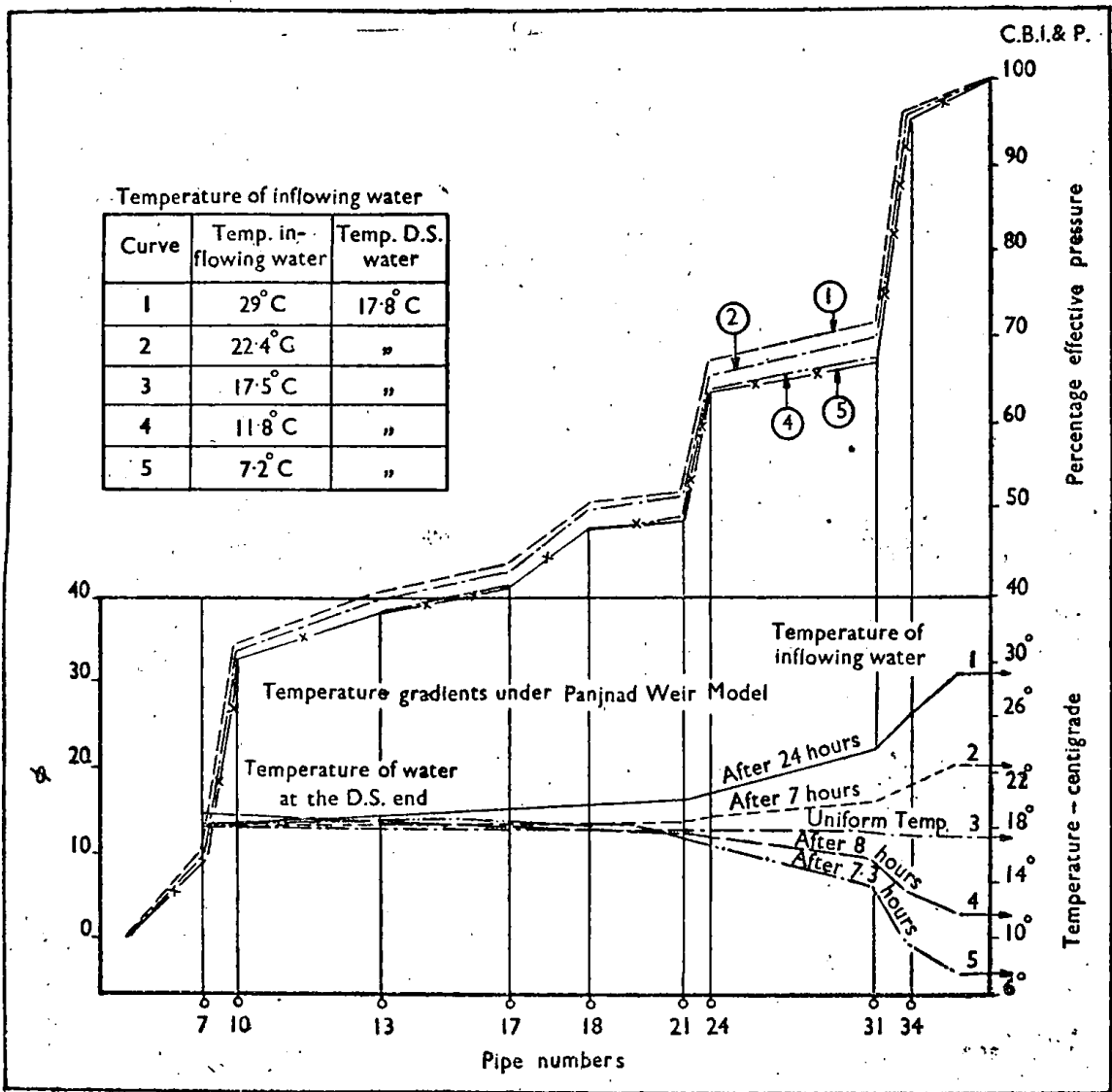


PLATE VI. 8:—Showing Hydraulic Gradients under Panjnad Weir Model at different temperatures of the inflowing water.

The temperature variations on the actual weir at Panjnad will be somewhat as given in Figure 6. 2.

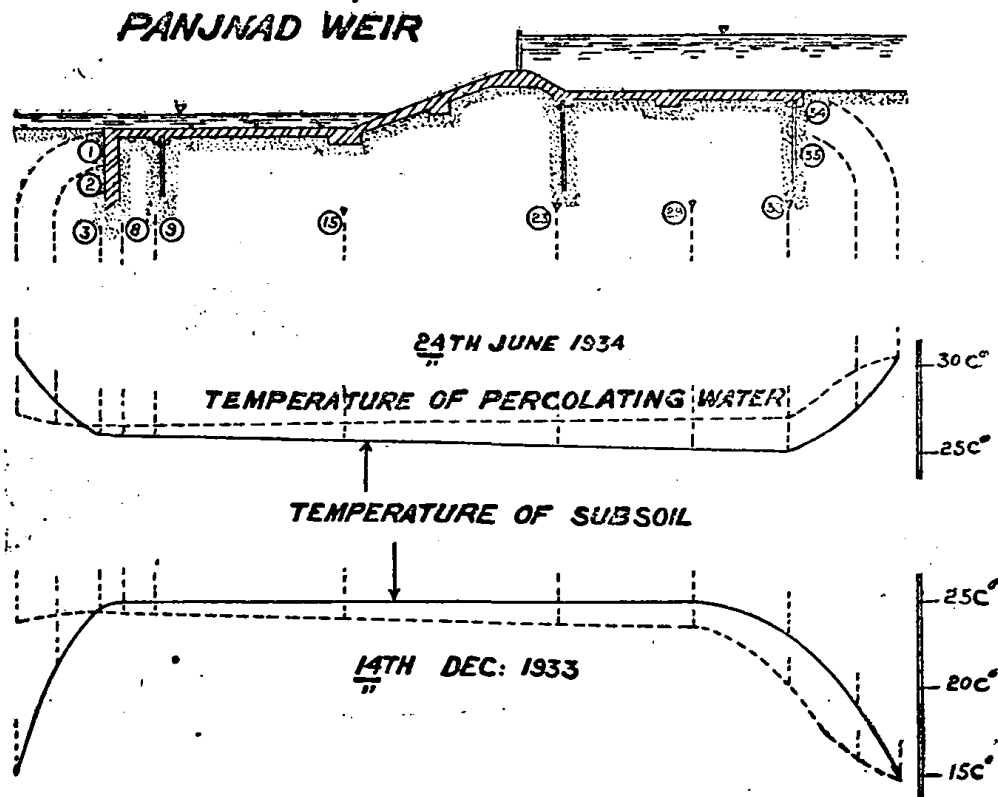


Figure 6. 2

PLATE VI. 7 shows the pressure variations.

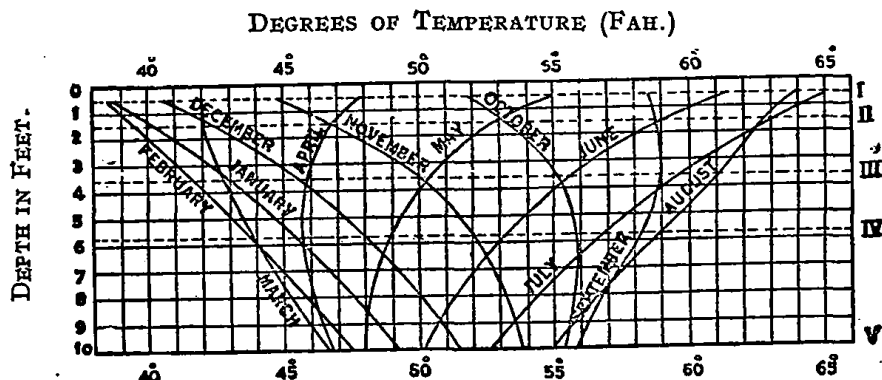
These two sets of experiments, with the sand filled tube and the Panjnad Weir model, demonstrate that :-

1. With the inflowing water temperature higher than that of the soil, the uplift pressures rise.
2. With that temperature less than that of the subsoil, the uplift pressures fall.
3. When the temperature of the inflowing water is the same as that of the subsoil, the pressures remain steady irrespective of the actual temperature.

SEASONAL VARIATIONS IN SUBSOIL TEMPERATURES

The following diagram¹ shows the seasonal variations in subsoil at various depths. There is a time lag between the hot wave transmitted from the surface and the one occurring in the subsoil. The wave travels slowly and its amplitude decreases rapidly with the depth.

MEAN MONTHLY TEMPERATURE GRADIENTS IN SOIL TEMPERATURE IN GRASS COVERED SOIL.



The Positions of the Thermometers are indicated by the Roman Numerals at the right of the Diagram.

Figure 6. 3

(1) "Physical Properties of Soil," by B. A. Keen (1931), pages 305.

Figure 6.4 shows the normal variations in temperature of river water and that observed on Pipe 34A of Bay 8 of Khanki Weir. It also shows the temperature variation in the same pipe at different depths.

KHANKI WEIR BAY 8 LEFT FLANK

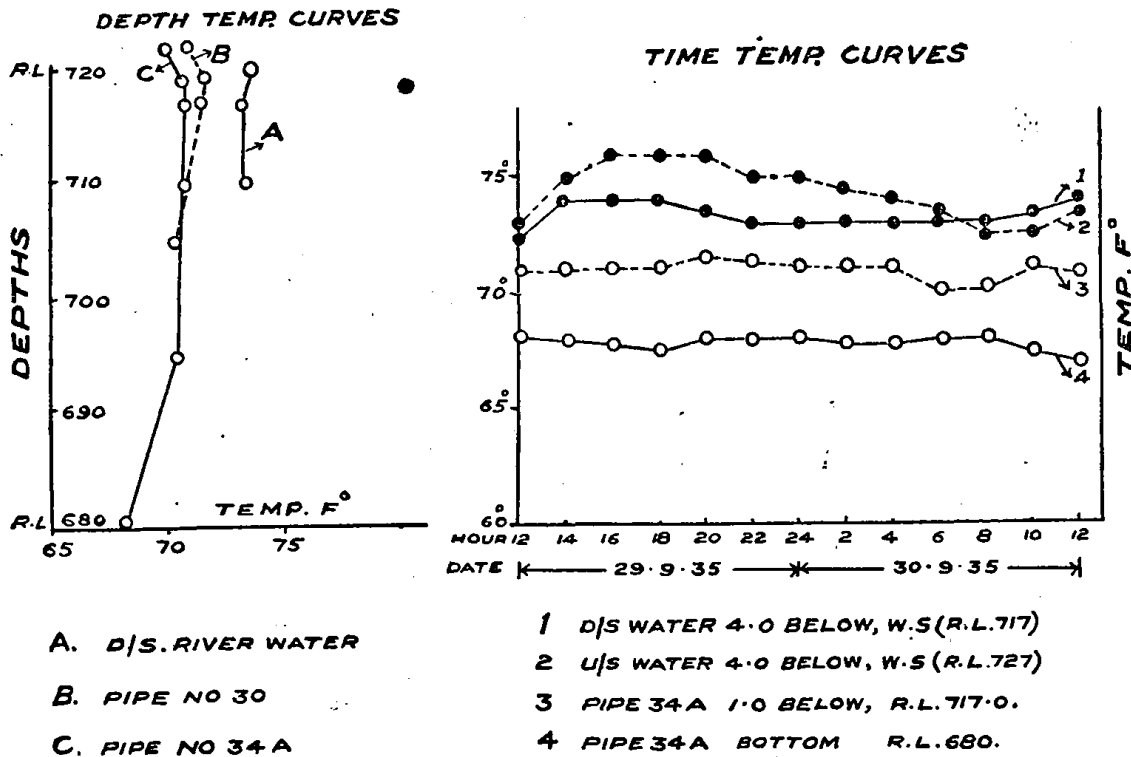


Figure 6. 4

The variation of pressure with temperature was noticed by H. De B. Parsons¹ (1929) in connection with the Sherman Island Dam (Figure 6. 5).

SEASONAL VARIATION OF UPLIFT HEAD UNDER MAIN DAM, AND WATER TEMPERATURE, SHERMAN ISLAND DAM.

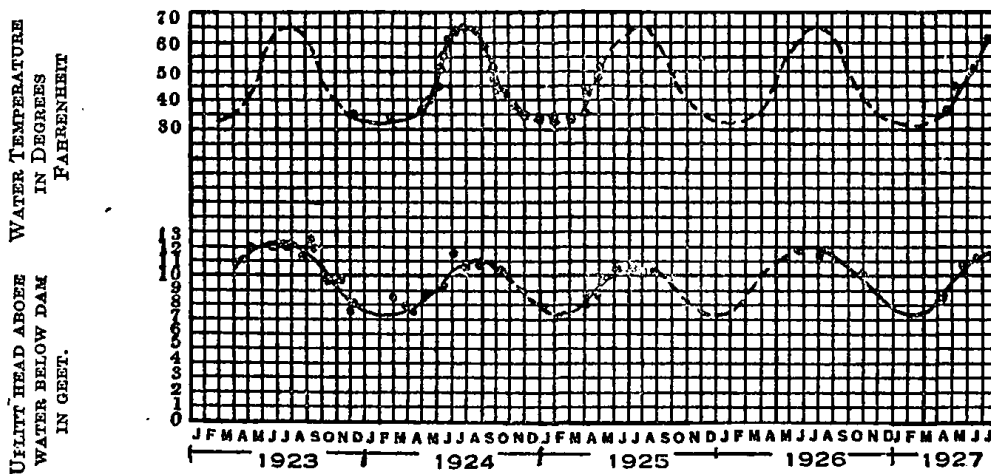


Figure 6. 5

SEASONAL VARIATIONS OF SILT AND TEMPERATURE AS INFLUENCING STABILITY OF WORKS.

In the Punjab, clayey silts deposit when the heads are high in the winter months and sandy silts deposit in the flood months when the heads are small. This combination affords a natural factor of safety. Under high heads, the clayey silts cut off a considerable fraction of the total head before it operates under the weir.

(1) Uplift Pressures under Dams. Experiments by the United States Bureau of Reclamation, by Julian Hinds, M.Am. Soc. C. E. Paper No. 1717 (1920). American Society of Civil Engineers.

The most unfavourable conditions for design occur when:—

- (a) the upstream floor is free from silt,
- (b) the temperature of the inflowing water is above that of the sub-soil by the maximum amount, and
- (c) the Head (H) is a maximum.

In the Punjab, the collective effect of (a) and (c) and of (b) with allowance for time lag occurs in the months of September and October. It is in these months, following immediately the period of flood damage, that failures are most likely to occur.

INTERPRETATION OF SEASONAL VARIATIONS

With a proper appreciation of what has been said in the foregoing pages, and with knowledge of local peculiarities, it should be possible to interpret the results of pressure observations and to predict any abnormalities. A lack of such appreciation might lead to a false sense of security or unnecessary panic.

A useful method of analysing data is by plotting them as in Plates VI. 1, 2, 3 and 4. In Figure 6.6 let the line A B represent normal pressures. If the pressures fall on line BC, it will indicate no silt upstream, but silt downstream, and possibly a higher temperature of inflowing water than that of the subsoil.

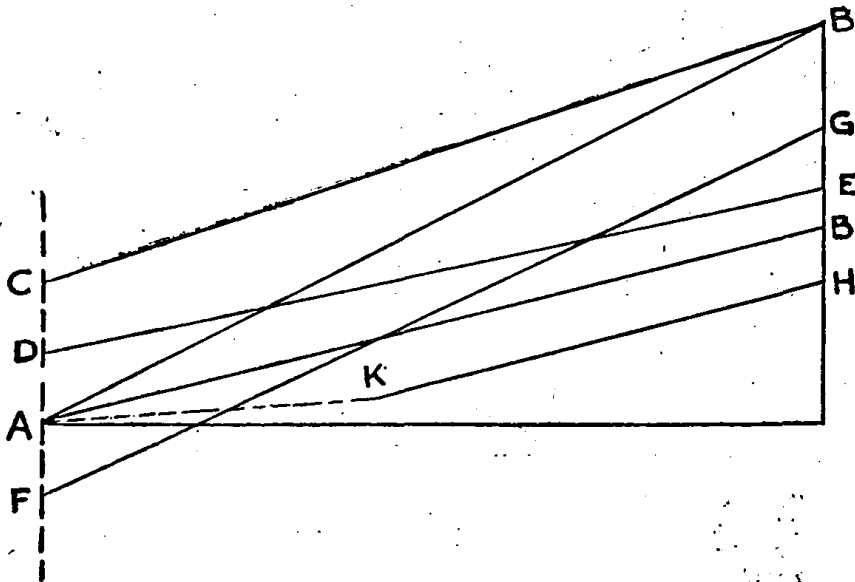


PLATE 6. 6

Line D E indicates silt downstream, and silt upstream, with possibility of temperature of the inflowing water being lower than that of the subsoil.

A B₁ indicates no silt downstream but silt upstream, or lower temperature of inflowing water. F G indicates scour downstream with silt upstream or lower temperature of inflowing water. Line A K H indicates cavities, as the pressure drop between A and K is disproportionately smaller than in K H.

As a rule the resistance from point to point will rise or fall more or less proportionately throughout the subsoil under a weir. If suddenly the resistance becomes disproportionately small, in any particular portion, the possibility of cavities under that portion will be indicated. The pipes in that case should be checked to see that they are functioning. If necessary they may be pumped out. If the pipes show no sign of chocking, the natural conclusion would be that an abnormality of some sort has occurred.

A knowledge of local features will be necessary in each case. Where clay bands occur, the interpretation of data will have to be done with due regard to the geological formation of the soil and the base profile of the weir.

CHAPTER VII

THE MATHEMATICS OF WEIR DESIGN

Determination of Uplift Pressures and Exit Gradients

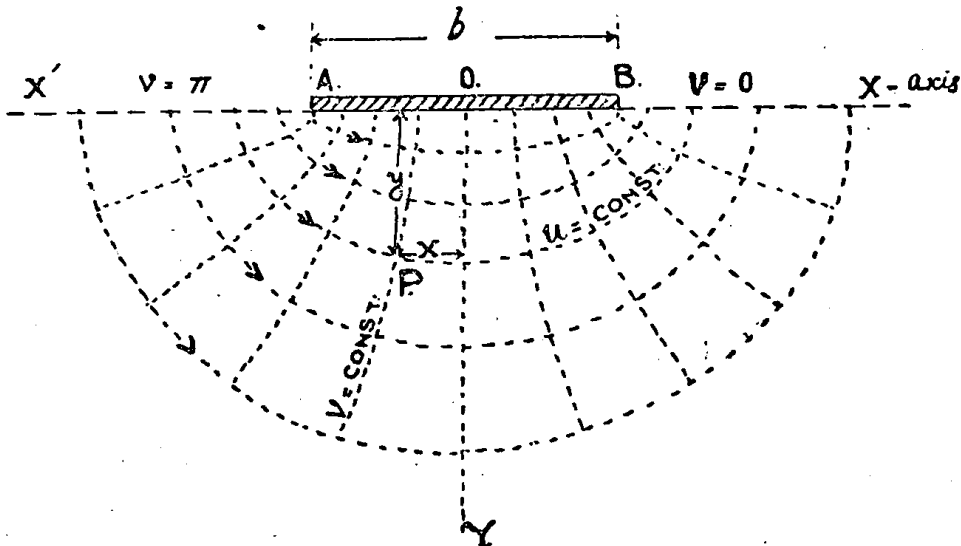


Figure 7.1

It is generally known that the stream lines or lines of flow under a floor $A B$ as given in Fig. 7.1 are confocal ellipses with the centre at O , the middle of the floor $A B (=b)$ the major axis and with foci at A and B . One of the stream lines will be $A B$, which is the limiting form of this family of ellipses.

These ellipses are given by the equation

$$\frac{x^2}{\left(\frac{b}{2} \cosh u\right)^2} + \frac{y^2}{\left(\frac{b}{2} \sinh u\right)^2} = 1 \quad \dots \dots \dots (7.1)$$

where u is stream line function.

From this equation any particular stream line can be determined by giving a suitable value to u , so that equation 7.1 will take the form:—

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$$

The stream line along the floor $A B$ is given by putting

$$u = 0,$$

from which

$$\cosh u = 1$$

$$\sinh u = 0$$

and equation 7.1 reduces to $Y = 0$, that is the line $A B$.

The lines of equal pressure are at right angles to the lines of flow and are in the above case given by

$$\frac{x^2}{\left(\frac{b}{2} \cos v\right)^2} - \frac{y^2}{\left(\frac{b}{2} \sin v\right)^2} = 1 \quad \dots \dots \dots (7.2)$$

where v is the pressure function.

From this equation any particular equi-pressure line can be obtained by giving suitable values to v so that the equation 7.2 will take the form:—

$$\frac{x^2}{A_1^2} - \frac{y^2}{B_1^2} = 1.$$

At the upstream end A , $v = \pi$ the total head or pressure.

Substituting this value in 7.2 we get $Y = 0$, or that the full pressure line is $A X$ which is horizontal and in plane of the floor at the upstream end.

The entire head or pressure $v = \pi$ is gradually lost along the direction of flow through the sand medium till it is finally reduced to zero at its exit along $B X$. The rate of loss is given by the spacing of the eqi-pressure lines shown in the Figure 7.1.

If P is any point in the medium, and the pressure and stream line functions at this point are given by u and v , then the relationships (7.1) and (7.2) can be expressed as below:—

$$Z = \frac{b}{2} \cosh w \quad \dots \dots \dots (7.3)$$

where $Z = x+iy$ and $w = u+iv$.

The position of a point P in a plane (say Z -plane) is usually denoted by the two cartesian co-ordinates x and y . It is convenient to denote this number pair (x, y) by the compound symbol $x+iy$; and this number pair is conveniently called (after Gauss) a complex number. In the fundamental operations of arithmetic, the complex number $x+i$. 0 may be replaced by the real number x ; defining i to mean $0+i$. 1, we have $i^2 = (0, 1) \times (0, 1) = (-1, 0)$ and so i^2 may be replaced by -1

$$\therefore i^2 = -1.$$

Substituting these values in 7.3 we get

$$\begin{aligned} x+iy &= \frac{b}{2} \cosh (u+iv) \\ &= \frac{b}{2} \cosh u \cos v + i \sinh u \sin v \quad \dots \dots \dots (7.4) \end{aligned}$$

Equating the reals with the reals and the imaginaries with the imaginaries we get:—

$$x = \frac{b}{2} \cosh u \cos v \quad \dots \dots \dots (7.5)$$

$$\text{and} \quad y = \frac{b}{2} \sinh u \sin v \quad \dots \dots \dots (7.6)$$

Or

$$\cos v = \frac{x}{\frac{b}{2} \cosh u}$$

$$\sin v = \frac{y}{\frac{b}{2} \sinh u}$$

Squaring and adding we get:—

$$\cos^2 v + \sin^2 v = 1 = \frac{x^2}{\left(\frac{b}{2} \cosh u\right)^2} + \frac{y^2}{\left(\frac{b}{2} \sinh u\right)^2}$$

which is the same as 7.1.

Similarly we can get equation 7.2 as:—

$$\cosh^2 u - \sinh^2 u = 1 = \frac{x^2}{\left(\frac{b}{2} \cos v\right)^2} - \frac{y^2}{\left(\frac{b}{2} \sin v\right)^2}.$$

In the case of a simple floor the pressure at any point along it (where $y = 0$) is given by equation (7.5) by substituting:—

$$u = 0$$

Or

$$x = \frac{b}{2} \cos v$$

as

$$\cosh 0 = 1$$

Or

$$v = \cos^{-1} \frac{2x}{b}$$

Or as

$$v = \pi \phi, \quad \phi = \frac{1}{\pi} \cos^{-1} \left(\frac{2x}{b} \right) \quad \dots \dots \dots (7.7)$$

General Form.—If under a floor we introduce a vertical obstruction like a line of sheet piles or wells, the configuration of stream and pressure lines of Figure (7.1) will be distorted. But it is possible to bring back this distortion to the normal configuration of Figure 7.1 by means of a transformation—known as Schwarz Christoffel transformation. (See Chapter III.)

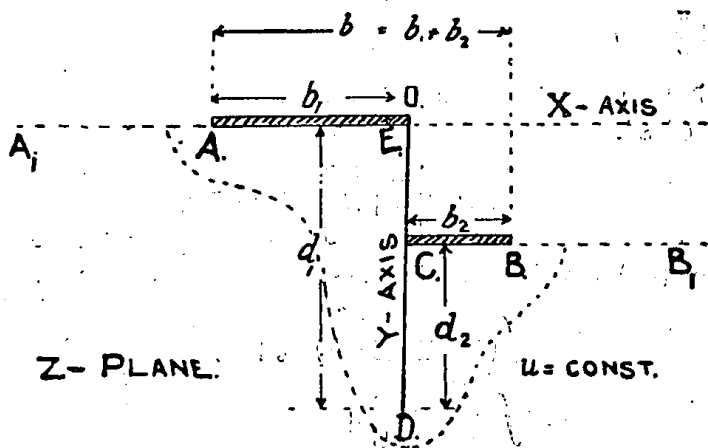


Figure 7.2.

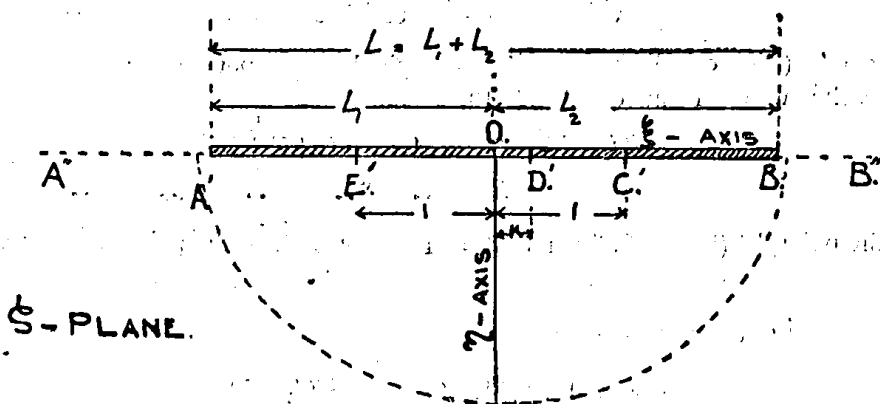


Figure 7.3

Figure 7.2 is a typical general case. The problem before us is, therefore, that of representing the infinite plane below the profile of Figure 7.2 uniformly, continuously and conformally on the lower half of the plane represented by Figure 7.3.

Let Figure 7.2 be called the Z-plane, so that any point in this plane is given by

$$Z = x + iy$$

and Figure 7.3 be called the ζ -plane so that any point in that plane is given by

$$\zeta = \xi + i\eta$$

In such transformation, the system of pressure and stream lines will continue to cut each other at right angles. In simple language it amounts to a pulling out and adjusting the profile $A_1 A E D C B B_1$ of Figure 7.2 in such a way that it conforms to $A' A' E' D' C' B' B''$ in Figure 7.3 so that the distorted stream line $A E D C B$ is correctly represented in its different sections by the stream line $A' E' D' C' B'$. Similarly any other stream line U that cuts the X-axis in Figure 7.2 at right angles will be represented by a stream line U' which will cut the ξ -axis in Figure 7.3 also at right angles.

Thus the distorted contour of u will have been conformally transformed into an ellipse.

This transformation is given by the following equation:—

$$Z = A \int \frac{d\zeta}{(\zeta - \xi_1)\lambda^1 (\zeta - \xi_2)\lambda^2 (\zeta - \xi_3)\lambda^3} \dots \dots \dots (7.8)$$

where $\lambda_1\pi$, $\lambda_2\pi$, and $\lambda_3\pi$ are the changes in the angles at vertices C , D , and E as one traverses along the sides of the polygon $B_1 B C D E A A_1$ in the positive sense. ξ_1 , ξ_2 , ξ_3 are the coordinates of the points C' , D' , E' in the ξ -plane (Figure 7.3) on which the points C , D , E of the Z-plane (Figure 7.2) map.

Now at the first turning C , the angle of turn is $+\frac{\pi}{2}$ so that $\lambda_1\pi = +\frac{\pi}{2}$ or $\lambda_1 = +\frac{1}{2}$

Similarly $\lambda_2\pi$ at D is equal to $-\pi$ or

$$\begin{aligned} \lambda_2 &= -\pi \\ \therefore \lambda_2 &= -1 \end{aligned}$$

and $\lambda_3\pi$ at E is $+\frac{\pi}{2}$ or $\lambda_3 = +\frac{1}{2}$.

The origin in Figure 7.2 is at E but that in Figure 7.3 has been chosen at O which is midway between C' E' .

Now
$$\left. \begin{aligned} \zeta_1 &= OC' = +1 \\ \zeta_2 &= OD' = +K \\ \zeta_3 &= OE' = -1 \end{aligned} \right\} \text{Figure 7.3.}$$

The equation (7.8) now simplifies into :—

$$\begin{aligned} Z &= A \int \frac{d\zeta}{(\zeta-1)^{\frac{1}{2}} (\zeta-K)^{-1} (\zeta+1)^{\frac{1}{2}}} + B \quad \dots \dots \dots (7.9) \\ &= A \int \frac{(\zeta-K)d\zeta}{\sqrt{\zeta(\zeta^2-1)}} + B \end{aligned}$$

Or

$$Z = A\sqrt{\zeta^2-1} - AK \log \{ \zeta + \sqrt{\zeta^2-1} \} + B \quad \dots \dots (7.10)$$

The point C' (Figure 7.3) where $\zeta=1$, corresponds to the point C (Figure 7.2) where $Z = v i(d_1 - d_2)$, so that we get from equation (7.10)

$$\begin{aligned} i(d_1 - d_2) &= A \times 0 - AK \log (1+0) + B \\ &= B \quad \dots \dots \dots (7.11) \end{aligned}$$

Similarly the point E' (Figure 7.3) where $\zeta = -1$ corresponds to the point E (Figure 7.2) where $Z=0$.

From equation (7.10) we get :—

$$\begin{aligned} 0 &= A \times 0 - AK \log (-1+0) + i(d_1 - d_2) \\ &= -AK \log (-1) + i(d_1 - d_2) \\ &= -AK\pi i + i(d_1 - d_2) \end{aligned}$$

Hence
$$AK = \frac{d_1 - d_2}{\pi} \quad \dots \dots \dots (7.12)$$

Similarly the point D' (Figure 7.3) where $\zeta = K < 1$ corresponds to the point D (Figure 7.2) where $Z = id_1$.

From equation (7.10) we get:—

$$\begin{aligned} id_1 &= A\sqrt{K^2-1} - AK \log \{ K + \sqrt{K^2-1} \} + i(d_1 - d_2) \\ &= Ai\sqrt{1-K^2} - AK \log \{ K + i\sqrt{1-K^2} \} + i(d_1 - d_2) \end{aligned}$$

as $K < 1$.

Let

$$\begin{aligned} K &= \cos \theta, \text{ the equation becomes} \\ id_1 &= Ai \sin \theta - AK \log (\cos \theta + i \sin \theta) + i(d_1 - d_2) \\ &= Ai \sin \theta - AK \log e^{i\theta} + i(d_1 - d_2) \\ &= Ai \sin \theta - AK \cdot i\theta + i(d_1 - d_2) \end{aligned}$$

Or $d_2 = A \sin \theta - AK\theta$.

Substituting values of A and AK from (7.12)

and because

$K = \cos \theta$, we get

$$\begin{aligned} d_2 &= \frac{d_1 - d_2}{\pi} \cdot \frac{\sin \theta}{\cos \theta} - \frac{d_1 - d_2}{\pi} \theta \\ \frac{d_2}{d_1 - d_2} \cdot \pi &= \tan \theta - \theta \quad \dots \dots \dots (7.13) \end{aligned}$$

Let $\frac{d_2}{d_1-d_2} = \delta$ then $\tan \theta - \theta = \pi \delta \dots \dots \dots (7.14)$

Solving this equation (7.14) we get θ in terms of δ , or $K = \cos \theta$ in terms of $\delta = d_2/d_1 - d_2$. This relationship is plotted on Plate VII-1, and tabulated in Table VII-1.

With the constants A , AK and B worked out above, equation (7.10) becomes:—

$$Z = \frac{d_1-d_2}{\pi K} \sqrt{\zeta^2-1} - \frac{d_1-d_2}{\pi} \log \{ \zeta + \sqrt{\zeta^2-1} \} + i(d_1-d_2) \dots \dots (7.15)$$

where $K = \cos \theta$ is obtained from (7.14) and Plate VII-1 and Table VII-1.

The next step is to determine the constants L_1 and L_2 , Figure 7.3. Let us consider points A and B in Figure 7.2 and the corresponding points A' and B' in Figure 7.3.

Point A in Figure 7.2, $Z = -b_1$, corresponds to point A' in Figure 7.3, $\zeta = -L_1$. Substituting these in equation (7.15) we get

$$\begin{aligned} -b_1 &= \frac{d_1-d_2}{\pi K} \sqrt{L_1^2-1} - \frac{d_1-d_2}{\pi} \log \{ -L_1 + \sqrt{L_1^2-1} \} + i(d_1-d_2) \\ &= \frac{d_1-d_2}{\pi K} \sqrt{L_1^2-1} - \frac{d_1-d_2}{\pi} \log (-1) \\ &\quad - \frac{d_1-d_2}{\pi} \log \{ L_1 - \sqrt{L_1^2-1} \} + i(d_1-d_2); \end{aligned}$$

hence $-b_1 = \frac{d_1-d_2}{\pi K} \sqrt{L_1^2-1} - \frac{d_1-d_2}{\pi} \log \{ L_1 - \sqrt{L_1^2-1} \} \dots \dots (7.16)$

Similarly point B , in Figure 2, $Z = b_2 + i(d_1-d_2)$ corresponds to point B' in Figure 3, $\zeta = L_2$.

Substituting these in equation (7.15), we get

$$b_2 + i(d_1-d_2) = \frac{d_1-d_2}{\pi K} \sqrt{L_2^2-1} - \frac{d_1-d_2}{\pi} \log \{ L_2 + \sqrt{L_2^2-1} \} + i(d_1-d_2)$$

or

$$b_2 = \frac{d_1-d_2}{\pi K} \sqrt{L_2^2-1} - \frac{d_1-d_2}{\pi} \log \{ L_2 + \sqrt{L_2^2-1} \} \dots \dots (7.17)$$

In equations 7.16 and 7.17 substituting

$$\begin{aligned} L_1 &= \cosh \gamma_1 \text{ and} \\ L_2 &= \cosh \gamma_2 \text{ we get} \end{aligned}$$

$$-b_1 = \frac{d_1-d_2}{\pi K} \sinh \gamma_1 + \frac{d_1-d_2}{\pi} \gamma_1$$

Or $\sinh \gamma_1 + K\gamma_1 = -\pi K \cdot \frac{b_1}{d_1-d_2} = -\pi K \cdot \delta_1 \dots \dots (7.18)$

Similarly $\sinh \gamma_2 - K\gamma_2 = \pi K \delta_2 \dots \dots (7.19)$

where $\delta_1 = \frac{b_1}{d_1-d_2}$ and $\delta_2 = \frac{b_2}{d_1-d_2}$.

In these equations, all quantities except γ_1 and γ_2 are known and therefore γ_1 can be found from equation 7.18 and γ_2 from 7.19. Knowing γ_1 and γ_2 , $\cosh \gamma_1 = L_1$ and $\cosh \gamma_2 = L_2$ are known. These values of L_1 and L_2 are given in curves (Plate VII/2 and Table VII/2).

Thus the equation 7.15 along with 7.14, 7.16 and 7.17 give a complete solution for the transformation of the general case of Figure 7.2 into Figure 7.3.

The stream lines and pressure lines for Figure 7.3 can be worked out from equation 7.3. The origin in that case is in the middle of length $A'B'$ whereas in Figure 7.2 it is at a distance of $(\frac{L_1-L_2}{2})$ from the middle point. The equation 7.3 will in this case be

$$\zeta + \frac{L_1-L_2}{2} = \frac{L_1+L_2}{2} \cosh w \dots \dots (7.20)$$

By eliminating ζ between (7.15) and (7.20) and separating the variables in $Z = x + iy$ and in $w = u + iv$ and equating the reals to reals and imaginaries to imaginaries, the stream lines and pressure lines of the general case in Figure 7.2 can be obtained.

Now the equation (7.20) can be written in the form

$$\zeta = \lambda \cosh w - \lambda_1 \dots \dots \dots \dots \dots (7.21)$$

where
$$\lambda = \frac{L_1 + L_2}{2}, \lambda_1 = \frac{L_1 - L_2}{2} \dots \dots \dots (7.22) \text{ \& (7.23)}$$

Substituting this in 7.15, we get

$$Z = \frac{d_1 - d_2}{\pi K} \sqrt{(\lambda \cosh w - \lambda_1)^2 - 1} - \frac{d_1 - d_2}{\pi} \log \{(\lambda \cosh w - \lambda_1) + \sqrt{(\lambda \cosh w - \lambda_1)^2 - 1}\} + i(d_1 - d_2) \dots (7.24)$$

For purposes of designs we are mainly concerned with the pressure distribution along the foundation profile of the work, and the pressure gradients mainly at the exit end. For the stream line which hugs the base of the work, we have $u = 0$ so that

$$w = u + iv, \\ = iv.$$

Substituting this in equation (7.24) we get

$$x + iy = \frac{d_1 - d_2}{\pi K} \sqrt{(\lambda \cos v - \lambda_1)^2 - 1} - \frac{d_1 - d_2}{\pi} \log \{(\lambda \cos v - \lambda_1) + \sqrt{(\lambda \cos v - \lambda_1)^2 - 1}\} + i(d_1 - d_2) \dots (7.25)$$

- For points along *AE* (Figure 7.2), $y = 0$ and x varies from 0 to $-b_1$ in this equation, and for those along *CB*, $y = (d_1 - d_2)$ and x varies from 0 to b_2 .
 - For points along *ED*, $x = 0$ and y varies from 0 to d_1 .
 - For points along *DC*, $x = 0$, y varies from $d_1 - d_2$ to d_1 .
- For the floor *AE* equation (7.25) reduces to

$$x = \frac{d_1 - d_2}{\pi K} \sqrt{(\lambda \cos v - \lambda_1)^2 - 1} - \frac{d_1 - d_2}{\pi} \log \{(\lambda \cos v - \lambda_1) + \sqrt{(\lambda \cos v - \lambda_1)^2 - 1}\} + i(d_1 - d_2).$$

From Figure 7.3 it is apparent that $\zeta = \lambda \cos v - \lambda_1$ for the portion *A'E'* which corresponds to the floor *AE* is greater than unity and negative.

Hence substituting

$$\lambda \cos v - \lambda_1 = -\cosh \beta'_1 \dots \dots \dots (7.26)$$

we get

$$x = \frac{d_1 - d_2}{\pi K} \sinh \beta'_1 - \frac{d_1 - d_2}{\pi} \log \{-\cosh \beta'_1 + \sinh \beta'_1\} + i(d_1 - d_2) \\ = \frac{d_1 - d_2}{\pi K} \sinh \beta'_1 + \frac{d_1 - d_2}{\pi} \beta'_1 - \frac{d_1 - d_2}{\pi} \log (-1) + i(d_1 - d_2) \\ = \frac{d_1 - d_2}{\pi K} \sinh \beta'_1 + \frac{d_1 - d_2}{\pi} \beta'_1 \\ \therefore \frac{x}{d_1 - d_2} \cdot \pi K = \beta'_1 \cdot \pi K = \sinh \beta'_1 + K\beta'_1 \dots \dots \dots (7.27)$$

which has the same form as (7.18).

This equation (7.27) gives corresponding to any point x under the floor *AE* the value of β'_1 which substituted in (7.26) will give pressure for that point. Hence along the upstream floor,

$$v = \pi \cdot \frac{P}{H} = \cos^{-1} \left(\frac{\lambda_1 - \cosh \beta'_1}{\lambda} \right)$$

or

$$P = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 - \cosh \beta'_1}{\lambda} \right) \dots \dots \dots (7.28)$$

Similarly along the downstream floor *CB*, since $\zeta = \lambda \cos v - \lambda_1$ is greater than unity and positive we can substitute

$$\lambda \cos v - \lambda_1 = \cosh \beta'_2 \dots \dots \dots (7.29)$$

and get

$$\sinh \beta'_2 - K\beta'_2 = \pi K \cdot \frac{x}{d_1 - d_2} \dots \dots \dots (7.30)$$

which has the same form as (7.19).

Hence along the downstream floor pressure is given by

$$P = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 + \cosh \beta'_2}{\lambda} \right) \dots \dots \dots (7.31)$$

Hence for the pressure along the floor

$$P = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 \pm \cosh \beta'_{1,2}}{\lambda} \right) \dots \dots \dots (7.32)$$

where the negative sign refers to the upstream floor and the positive sign to the downstream portion. Values of $\cosh \beta_{1,2}$ have been worked out in Table VII/2 where $L = \cosh \beta$ has been tabulated for different values of $x/(d_1-d_2)$.

For the sheet pile equation (7.25) reduces to

$$iy = \frac{d_1-d_2}{\pi K} \sqrt{(\lambda \cos v - \lambda_1)^2 - 1} - \frac{d_1-d_2}{\pi} \log \{ (\lambda \cos v - \lambda_1) + \sqrt{(\lambda \cos v - \lambda_1)^2 - 1} \} + i(d_1-d)_2.$$

From Figure 7.3 it is apparent that $\zeta = \lambda \cos v - \lambda_1$ for the portion between EC is always less than unity varying from -1 to +1. Hence substituting

$$\lambda \cos v - \lambda_1 = \cos \beta \dots \dots \dots (7.33)$$

we get

$$iy = \frac{d_1-d_2}{\pi K} i \sin \beta - \frac{d_1-d_2}{\pi} i \beta + i(d_1-d_2)$$

or

$$\left(\frac{y}{d_1-d_2} - 1 \right) \pi K = \sin \beta - K\beta \dots \dots \dots (7.34)$$

This equation (7.34) gives corresponding to any point y on the pile line the value of β which substituted in equation (7.33) will give pressure for the point. Hence along the pile line.

$$v = \pi \cdot \frac{P}{H} = \cos^{-1} \left(\frac{\lambda_1 + \cos \beta}{\lambda} \right)$$

$$\therefore P = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 + \cos \beta}{\lambda} \right) \dots \dots \dots (7.35)$$

For design, it is generally enough to know the pressure at E, D and C (Figure 7.2). It is now proposed to determine them:—

For the point E, Figure (7.3):—

$$\zeta = \lambda \cos v - \lambda_1 = -1$$

$$\therefore v_E = \cos^{-1} \left(\frac{\lambda_1 - 1}{\lambda} \right)$$

or

$$P_E = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 - 1}{\lambda} \right) \dots \dots \dots (7.36)$$

For the point C, Figure (7.3):—

$$\zeta = \lambda \cos v - \lambda_1 = +1$$

$$\therefore P_C = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 + 1}{\lambda} \right) \dots \dots \dots (7.37)$$

For the point D, Figure (7.3):—

$$\zeta = \lambda \cos v - \lambda_1 = K$$

$$\therefore P_D = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 + K}{\lambda} \right) \dots \dots \dots (7.38)$$

The fundamental general equations for pressure distribution under the foundation profiles may be summarised as below:—

I. $x+iy = \frac{d_1-d_2}{\pi K} \sqrt{(\lambda \cos v - \lambda_1)^2 - 1} - \frac{d_1-d_2}{\pi} \log \{ (\lambda \cos v - \lambda_1) + \sqrt{(\lambda \cos v - \lambda_1)^2 - 1} \} + i(d_1-d_2) \dots \dots \dots (7.25)$

II. Pressure at E:—

$$P_E = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 - 1}{\lambda} \right) \dots \dots \dots (7.36)$$

III. Pressure at C:—

$$P_C = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 + 1}{\lambda} \right) \dots \dots \dots (7.37)$$

IV. Pressure at D:—

$$P_D = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 + K}{\lambda} \right) \dots \dots \dots (7.38)$$

where $K = \cos \theta$ is given by

$$\tan \theta - \theta = \pi \delta \quad \dots \quad \dots \quad \dots \quad (7.14)$$

(Plate VII/1,
Table VII/1)

and

$$\lambda = \frac{L_1 + L_2}{2} \quad \dots \quad \dots \quad \dots \quad (7.22)$$

$$\lambda_1 = \frac{L_1 - L_2}{2} \quad \dots \quad \dots \quad \dots \quad (7.23)$$

where the values of L_1 and L_2 are given by equations (7.18) and (7.19) and Plate VII/2 and Table VII/2 for various values of δ , δ_1 and δ_2 .

The following relations that will be obtained later on under Case 1 may as well be summarised here, as they are also of general application.

V. G_E or exit gradient at C, where downstream apron CB is absent, is given by

$$G_E \text{ at } C = \frac{dh}{dy(\text{EXIT})} = \frac{H}{(d_1 - d_2)\sqrt{\lambda}} \cdot \frac{K}{1-K} \quad \dots \quad \dots \quad (7.44)$$

$$\text{VI. } Li \left(\frac{K}{d_1 - d_2} \right)_{d_1 = d_2 = d} = \frac{1}{\pi d} \quad \dots \quad \dots \quad \dots \quad (7.45)$$

We shall now deal with particular cases of this general form. As we are mainly concerned with the values of the pressures at the key point, E, D and C and the values of the exit gradients we shall obtain only the values of these quantities.

CASE 1.

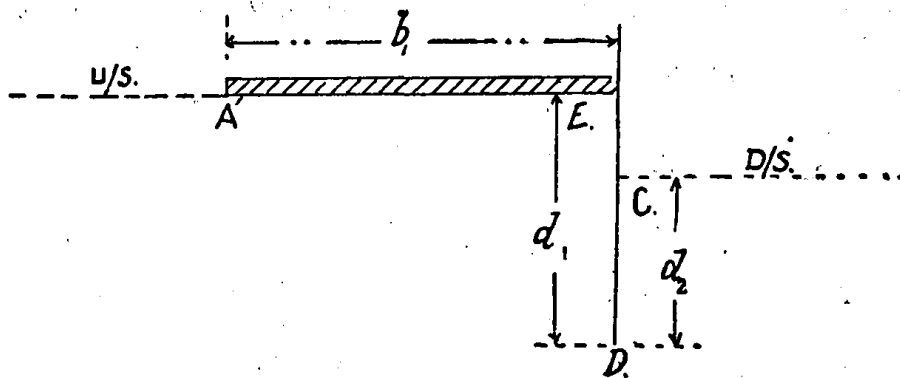


Figure 7.4

Floor with pile line at downstream end with step.

In this case $CB = b_2 = 0$ and equation (7.19) gives

$$\sinh \gamma_2 - \gamma_2 = 0, \text{ i.e. } \gamma_2 = 0$$

so that

$$L_2 = \cosh 0 = 1.$$

Equation 7.25 will give the distribution of pressure along the base and the pile line, the value of L_1 will be obtained from Plate VII/2 and Table VII/2 for $\delta_2 = 0$ and $\delta_1 = b_1 / (d_1 - d_2)$.

From equations (7.22) and (7.23) we obtain

$$\lambda = (L_1 + 1) / 2$$

$$\lambda_1 = (L_1 - 1) / 2 = \frac{L_1 + 1}{2} - 1 = \lambda - 1$$

$$\therefore \lambda_1 - 1 = \frac{L_1 - 3}{2}$$

$$\lambda_1 + 1 = (L_1 + 1) / 2.$$

Hence

$$\left[Lt \left(\frac{\sin v}{\sin \beta} \right) \right]^2 = \frac{1}{\lambda}$$

Or

$$Lt_{\substack{v=0 \\ \beta=0}} \frac{\sin v}{\sin \beta} = \frac{1}{\sqrt{\lambda}} \dots \dots \dots (7.43)$$

Substituting in (7.42), we get for $v=0, \beta=0$

$$1 = \frac{d_1 - d_2}{\pi K} \cdot \frac{\lambda}{\sqrt{\lambda}} \cdot \left(\frac{dv}{dy} \right)_{v=0} - \frac{d_1 - d_2}{\pi} \cdot \frac{\lambda}{\sqrt{\lambda}} \cdot \left(\frac{dv}{dy} \right)_{v=0}$$

$$\left(\frac{dv}{dy} \right)_{\text{at } C} = \frac{\pi K}{(d_1 - d_2) \sqrt{\lambda}} \cdot \frac{1}{1 - K}$$

$$\therefore \left(\frac{dP}{dy} \right)_{\text{EXIT}} = \frac{H}{\pi} \left(\frac{dv}{dy} \right)_{\text{at } C} = \frac{HK}{(d_1 - d_2) \sqrt{\lambda}} \cdot \frac{1}{1 - K}$$

Or

$$\left(\frac{dP}{dy} \right)_{\text{EXIT}} = \frac{H}{d_2} \cdot \frac{d_2}{d_1 - d_2} \cdot \frac{K}{1 - K} \cdot \frac{1}{\sqrt{\lambda}} \dots \dots \dots (7.44)$$

Values of exit gradients are given in Plate VII/3C and Table VII/3 for various values of δ and δ_1 . For cases where

$$d_1 = d_2$$

$$K = 0$$

we have

as OD' in Figure 7.3 vanishes, see also Plate VII/I. For such cases

$$\frac{K}{d_1 - d_2} = \frac{0}{0}$$

This relationship occurs in all the above equations so it is proposed to work out its limiting value where $d_1 = d_2 = d$.

From equation 7.14

$$\tan \theta - \theta = \pi \delta = \frac{\pi d_2}{d_1 - d_2}$$

also $\cos \theta = K$ and

multiplying the left side by $\cos \theta$ and the right side by K we get

$$\sin \theta - \theta \cos \theta = \pi d_2 \frac{K}{d_1 - d_2}$$

for $K=0, \theta = \frac{\pi}{2}$.

Hence the above equation becomes

$$1 - 0 = \pi d_2 \left(\frac{K}{d_1 - d_2} \right) Lt_{d_1=d_2}$$

Hence

$$Lt \left(\frac{K}{d_1 - d_2} \right)_{d_1=d_2} = \frac{1}{\pi d_2} = \frac{1}{\pi d} \dots \dots \dots (7.45)$$

CASE 2.

Single Pile Line with Fall—No Apron Upstream or Downstream.

In Figure 7.2 $b_1 = b_2 = 0$ and in Figure 7.3 $A'E' = C'B' = 0$ so that $L_1 = L_2 = 1$.

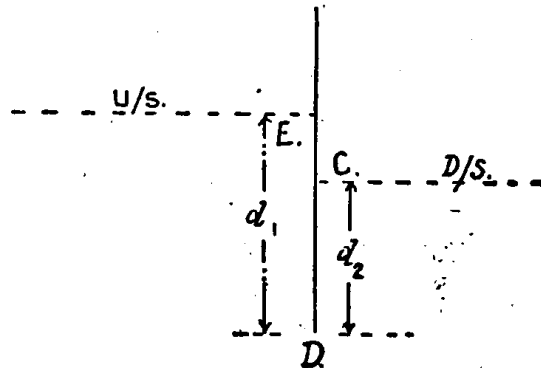


Figure 7.5

From equations 7.22 and 7.23

$$\lambda = \frac{L_1 + L_2}{2} = 1$$

$$\lambda_1 = \frac{L_1 - L_2}{2} = 0.$$

The equation of flow (7.25) for giving pressure distribution along faces of pile line will become for $x=0$

$$iy = \frac{d_1 - d_2}{\pi K} \sqrt{\cos^2 v - 1} - \frac{d_1 - d_2}{\pi} \log (\cos v + i \sin v) + i(d_1 - d_2)$$

or
$$iy = \frac{d_1 - d_2}{\pi K} i \sin v - \frac{d_1 - d_2}{\pi} iv + i(d_1 - d_2)$$

or
$$y = \frac{d_1 - d_2}{\pi K} \sin v - \frac{d_1 - d_2}{\pi} v + (d_1 - d_2) \quad \dots \quad \dots \quad \dots \quad (7.46)$$

As previously stated

$$v = \pi \phi = \pi \cdot \frac{P}{H}.$$

When d_1 and d_2 are known, K is known from Plate VII/I and v can be determined for any value of y .

A simple solution is to assume values of v as 0.05, 0.1, 0.15 to 0.9, 0.95 and 1.0 and work out corresponding values of y .

This has been done in Table IV/2, which also contains values of y obtained from experiments for the particular cases.

Exit Gradient at C.

In this case $\lambda=1$.

Hence equation (7.44) becomes

$$G_{\text{EXIT at C}} = \left(\frac{dp}{dy} \right)_{\text{EXIT}} + \frac{H}{1-K} \cdot \frac{K}{d_1 - d_2} = \frac{H}{d_2} \cdot \frac{d_2}{d_1 - d_2} \cdot \frac{K}{1-K} \quad \dots (7.47)$$

Values of G_E are given in Table VII/I and Plate VII/4.

CASE 3.

Single Sheet Pile—Equal Fill—No Aprons.

Here

$$d_1 = d_2 = d$$

and from equation (7.45) we have

$$\text{Lt. } \left(\frac{K}{d_1 - d_2} \right)_{d_1 = d_2 = d} = \frac{1}{\pi d}.$$

Equation (7.46) will become $y = d \sin v$

or
$$P = \frac{H}{\pi} \sin^{-1} \left(\frac{y}{d} \right) \quad \dots \quad \dots \quad \dots \quad (7.48)$$

This will give the pressure distribution along the faces of the pile line.

Exit Gradient at C.

Equation (7.47) becomes

$$G_E \text{ at C} = \left(\frac{dp}{dy} \right)_{\text{EXIT}} = \frac{H}{1-K} \cdot \frac{K}{d_1 - d_2} = \frac{H}{\pi d} \text{ as } K=0 \quad \dots (7.49)$$

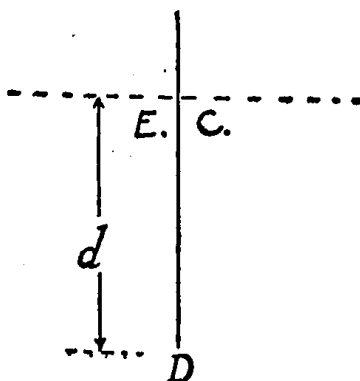


Figure 7.6

This equation can also be derived from simple differentiation of equation (7.1i).

CASE 4.

Floor with Pile Line not at end.

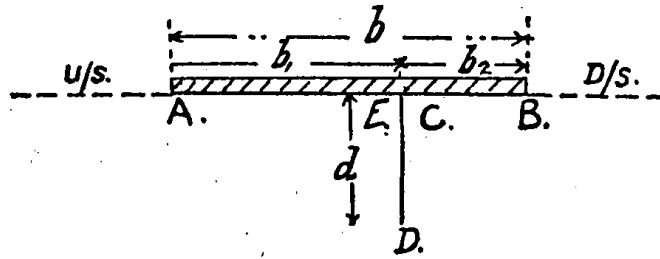


Figure 7.7

For this case so that $K=0$ and

$$d_1 = d_2 = d$$

$$\lim_{\substack{d_1 \rightarrow 0 \\ d_2 \rightarrow 0}} \frac{K}{d_1 - d_2} = \frac{1}{\pi d}$$

This is Weaver's general case. Weaver's solutions have really furnished the inspiration for this Chapter.

Equation (7.24) in this case becomes

$$Z = \frac{d_1 - d_2}{\pi K} \sqrt{(\lambda \cosh w - \lambda_1)^2 - 1}$$

as with $d_1 = d_2$, the other two terms of the equation (7.24) disappear and

$$\lim_{\substack{d_1 \rightarrow d_2 \\ d_2 \rightarrow d_1}} \frac{K}{d_1 - d_2} = \frac{1}{\pi d}$$

$$\therefore Z = d \sqrt{(\lambda \cosh w - \lambda_1)^2 - 1} \quad \dots \quad (7.50)$$

when $w = u + iv$.

The pressure determination along the base and the pile line will be given by equation (7.25) which now becomes :-

$$x + iy = d \sqrt{(\lambda \cos v - \lambda_1)^2 - 1} \quad \dots \quad (7.51)$$

From equations 7.16 and 7.17

$$-b_1 = \frac{d_1 - d_2}{\pi K} \sqrt{L_1^2 - 1} = d \sqrt{L_1^2 - 1}$$

$$\text{and } b_2 = d \sqrt{L_2^2 - 1}$$

or

$$L_1 = \sqrt{\left(\frac{b_1}{d}\right)^2 + 1}$$

$$L_2 = \sqrt{\left(\frac{b_2}{d}\right)^2 + 1}$$

$$\frac{b_1}{d} = \alpha_1 \quad \dots \quad (7.52)$$

$$\frac{b_2}{d} = \alpha_2 \quad \dots \quad (7.53)$$

Hence

$$L_1 = \sqrt{1 + \alpha_1^2} \quad \dots \quad (7.54)$$

$$L_2 = \sqrt{1 + \alpha_2^2} \quad \dots \quad (7.55)$$

From equations (7.22) and (7.23) we get

$$\lambda = \frac{L_1 + L_2}{2} = \frac{\sqrt{1 + \alpha_1^2} + \sqrt{1 + \alpha_2^2}}{2} \quad \dots \quad (7.56)$$

$$\lambda_1 = \frac{L_1 - L_2}{2} = \frac{\sqrt{1 + \alpha_1^2} - \sqrt{1 + \alpha_2^2}}{2} \quad \dots \quad (7.57)$$

Equation (7.50) can be written as

$$(\lambda \cosh w - \lambda_1)^2 = 1 + \left(\frac{z}{d}\right)^2 = 1 + \left(\frac{x + iy}{d}\right)^2 = 1 + \frac{x^2 - y^2}{d^2} + \frac{2ixy}{d} \quad \dots \quad (7.58)$$

This is the general equation which will give stream lines and pressure lines.

Along the base $u = 0, y=0$ and equation (7.50) becomes

$$\lambda \cos v - \lambda_1 = \pm \int \frac{d^2 + x^2}{d^2}$$

Or
$$\cos v = \frac{\lambda_1 d \pm \sqrt{d^2 + x^2}}{\lambda d}$$

Or
$$v = \cos^{-1} \frac{\lambda_1 d \pm \sqrt{d^2 + x^2}}{\lambda d}$$

As
$$v = \pi \phi$$

$$= \pi \cdot \frac{P}{H}$$

Or
$$P = \frac{H}{\pi} v = \frac{H}{\pi} \cos^{-1} \frac{\lambda_1 d \pm \sqrt{d^2 + x^2}}{\lambda d} \dots \dots \dots (7.59)$$

Along the sheet pile $u = 0$ and $x = 0$ we get,

$$P = \frac{H}{\pi} \cos^{-1} \frac{\lambda_1 d \pm \sqrt{d^2 - y^2}}{\lambda d} \dots \dots \dots (7.60)$$

In equations (7.59) and (7.60) the values of pressure on the upstream of the pile will be given by the negative sign of the radical and those on the downstream side by the positive sign.

From equations 7.36 and 7.37 and also by substituting the values of the co-ordinates at E and C, $x = y = 0$ in equations 7.59 and 7.60 we get

$$P_E = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 - 1}{\lambda} \right) \dots \dots \dots (7.36)$$

and

$$P_C = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1 + 1}{\lambda} \right) \dots \dots \dots (7.37)$$

where λ and λ_1 in this particular case are given by equations 7.56 and 7.57.

For point D at bottom of pile $y = d$, so that from equation (7.60), and also from equation 7.38 as $K = 0$ in this case, we get

$$P_D = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda_1}{\lambda} \right) \dots \dots \dots (7.38)$$

These values of P_E, P_C and P_D are given in Plate VII/6.

CASE 5.

Floor with pile line at end.

Here

$$d_1 = d_2 = d, b_2 = 0 \text{ and } b_1 = b.$$

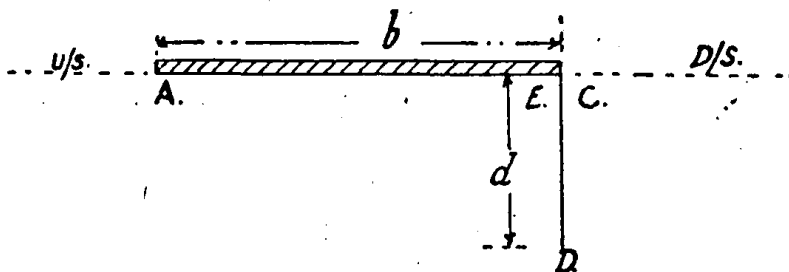


Figure 7.8

$\therefore K = 0$

and

$$Lt \left(\frac{K}{d_1 - d_2} \right)_{d_1 = d_2 = d} = \frac{1}{\pi d}$$

Equations 7.39 and 7.40 will become

$$P_E = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda-2}{\lambda} \right) \dots \dots \dots (7.61)$$

$$P_D = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda-1}{\lambda} \right) \dots \dots \dots (7.62)$$

$$P_C = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda}{\lambda} \right) = 0,$$

where $\lambda = \frac{1 + \sqrt{1 + \alpha_1^2}}{2}$ as derived from 7.56 because $\alpha_2 = \frac{b_2}{d} = 0$. Values of P_E and P_D are given in Plate VII/6 and Table VII/5.

Exit Gradient at C.

With $K = 0$ and

$$Lt \left(\frac{K}{d_1 - d_2} \right)_{d_1 = d_2 = d} = \frac{1}{\pi d}$$

equation (7.44) becomes

$$G_{EXIT} = \frac{H}{\pi d} \cdot \frac{1}{\sqrt{\lambda}} \dots \dots \dots (7.63)$$

Values of G_E are given in Plate VII/5 and Table VII/5.

When the floor is absent,

$$\lambda = \frac{\sqrt{1 + \alpha_1^2} + \sqrt{1 + \alpha_2^2}}{2} = 1 \text{ as } \alpha_1 = \alpha_2 = 0$$

and equation (7.63) becomes

$$G_{EXIT} = \frac{H}{\pi d} \dots \dots \dots (7.64)$$

CASE 6.

Simple floor—No pile line.

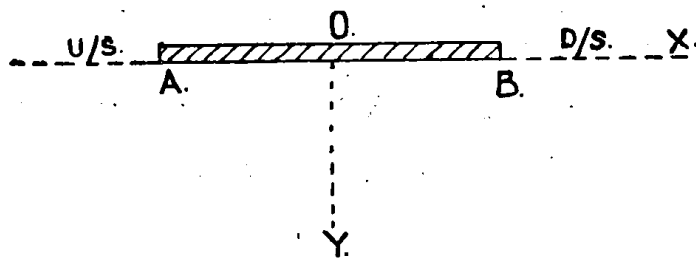


Figure 7.9

The stream and pressure lines for this case are given by equations (7.5) and (7.6). The pressures along the base are given by

$$P = \frac{H}{\pi} \cos^{-1} \left(\frac{2x}{b} \right) \dots \dots \dots (7.7)$$

Exit Gradient.

This can be simply derived from equation (7.49) by putting $d = 0$

$$G_{EXIT} = \frac{H}{\pi \times 0} \times \frac{1}{\sqrt{\lambda}} = \infty \dots \dots \dots (7.64)$$

In a simple floor, the exit gradient at the immediate toe is infinity, which as will be explained in the next chapter connotes a condition of definite instability. The particles at the toe must move.

But this value of infinity does not persist any distance from B. It rapidly falls off. The infinite value holds only for the infinitesimal length of floor δx . This fact must be kept in mind as a lack of proper appreciation of this will lead to undue exaggeration of the significance of infinite gradients.

The following solution will help in determining the exit gradient at any point at a distance x from the toe (B).

Equation (7.6) is :-

$$2y = b \sinh u \sin v.$$

Differentiating with respect to y along the stream line $u = \text{const.}$

$$b \sinh u \cos v \cdot \frac{dv}{dy} = 2.$$

Equation (7.5) gives :-

$$b \cosh u \cos v = 2x.$$

Substituting value of $\cos v$ in the first equation we get :-

$$b \sinh u \frac{2x}{b \cosh u} \cdot \frac{dv}{dy} = 2.$$

Hence

$$\frac{dv}{dy} = \frac{2b \cosh u}{2x} \times \frac{1}{b \sinh u} = \frac{\cosh u}{\sinh u} \cdot \frac{1}{x}.$$

Now

$$v = \pi\phi = \pi P/H.$$

Hence

$$\frac{dv}{dy} = \frac{\pi}{H} \cdot \frac{dP}{dy} = \frac{\cosh u}{\sinh u} \cdot \frac{1}{x}.$$

Or

$$\frac{d\phi}{dy} = \frac{H}{\pi x} \frac{\cosh u}{\sinh u} \dots \dots \dots (7.65)$$

Now from equation (7.5)

$$x = \frac{b}{2} \cosh u \cos v.$$

Or

$$\frac{\cosh u}{x} = \frac{2}{b \cos v}.$$

Substituting this in the above equation :-

$$\frac{d\phi}{dy} = \frac{2H}{\pi b \cos v} \times \frac{1}{\sinh u} \dots \dots \dots (7.66)$$

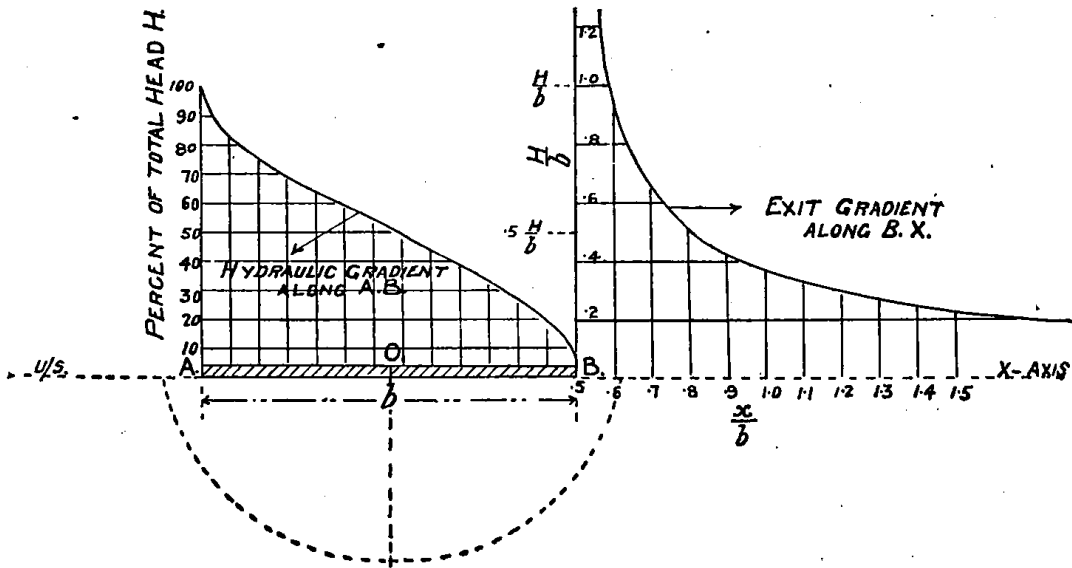


Figure 7.10

Now for points along X-axis beyond B, $v=0$.

Hence (7.66) becomes for the exit where $v=0$.

$$\left(\frac{d\phi}{dy}\right)_{\text{EXIT}} = \frac{2H}{\pi b} \times \frac{1}{\sinh u} \dots \dots \dots (7.67)$$

For

$$v=0, x = \frac{b}{2} \cosh u \cos v$$

$$= \frac{b}{2} \cosh u.$$

$$\therefore \cosh u = \frac{2x}{b}.$$

Hence $\sinh u = \sqrt{\left(\frac{2x}{b}\right)^2 - 1}$.

Equation 7.67 then becomes :—

$$\left(\frac{dp}{dy}\right)_{EXIT} = \frac{2H}{\pi b} \times \frac{1}{\sqrt{\left(\frac{2x}{b}\right)^2 - 1}} \dots \dots \dots (7.68)$$

At B,

$$x = \frac{b}{2}, \frac{dp}{dy} = \infty \dots \dots \dots (7.69)$$

The following table gives the values of $\left(\frac{dp}{dy}\right)_{EXIT}$ for other points along BX(Figure 7.10):—

$\frac{x}{b}$.5	.6	.7	.8	.9	1.0
$\frac{dp}{dy}$	$\frac{H}{b} \times \infty$	$\frac{H}{b} \times .961$	$\frac{H}{b} \times .649$	$\frac{H}{b} \times .509$	$\frac{H}{b} \times .424$	$\frac{H}{b} \times .368$
$\frac{x}{b}$	1.1	1.2	1.3	1.4	1.5	1.6
$\frac{dp}{dy}$	$\frac{H}{b} \times .325$	$\frac{H}{b} \times .292$	$\frac{H}{b} \times .266$	$\frac{H}{b} \times .243$	$\frac{H}{b} \times .225$	$\frac{H}{b} \times .210$

These results are plotted in Figure (7.10).

From this it will be noticed that although the exit gradient is infinite immediately at point B, it rapidly falls off at the stream lines emerging further away.

In this connection it may be remembered that in cases where there is a sheet pile at the toe, the exit gradients which are never infinite, decrease as the point of exit recedes from the toe, but not as rapidly as in the above case.

CASE 7.

Depressed Floor

It has been proved above that if the soil at the exit surface is flush with the bottom of the floor, the pressure gradient at the immediate toe will be infinite. [As shown in the next chapter, this is an unstable condition and is never permitted in design. The floor in practice is depressed in the sub-soil, so that the latter at surface is flush with the top of the floor.

It has not been possible,* so far, to obtain the mathematical solution for this case but the following formula has been derived from laboratory experiments :—

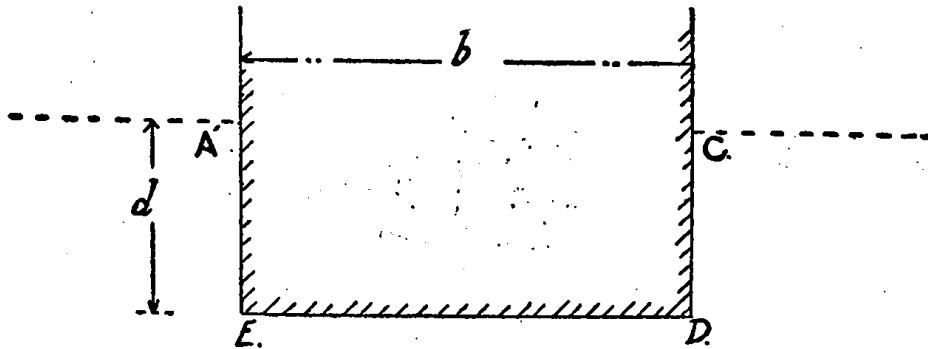


Figure 7.11

$$\phi_{D'} = \phi_D - \frac{2}{3}(\phi_E - \phi_D) + \frac{3}{a^2} \dots \dots \dots (7.70)$$

where ϕ_D and ϕ_E are given by equations 7.61 and 7.62 or can be read off from curves of Plate VII/6.

The values of $\phi_{D'}$ are given in Plate VII/6 and Table VII/5.

Values of the exit gradients can also be obtained from

$$G_{EXIT} = \frac{0.84}{d} \cdot P_{D'} \dots \dots \dots (7.71)$$

* A solution of this case has since been worked out by J. K. Malhotra, M.A., Statistical Officer, Punjab Irrigation Research Institute, and is given as appendix to this Chapter. The same officer has worked out a general solution for hydraulic gradients from which all particular cases can be simply derived. This solution is given after Case 8.

CASE 8.

Floor with multiple lines of piles.

(a) Floor and two piles at end.

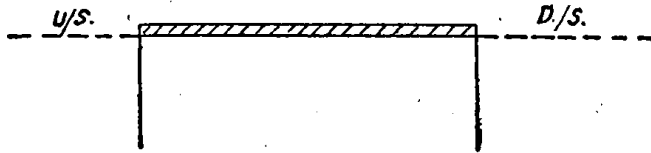


Figure 7.12

(b) Floor and three piles.

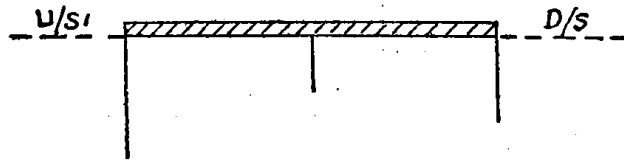


Figure 7.13

(c) Floor and two piles not at end.

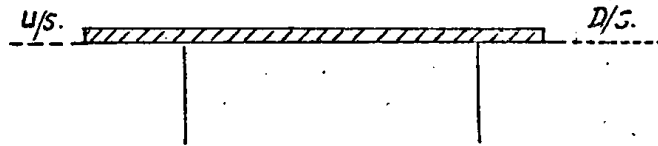


Figure 7.14

The theoretical solutions for these have not been worked out but as will be shown in Chapter X the solutions of the above forms can be obtained empirically with a reasonable degree of accuracy from the general solutions given in the above pages.

A General Solution for Exit Gradient by J.K. Malhotra, M.A., Offg. Mathematical Officer, Punjab Irrigation Research Institute.

By the general theory of exit gradient we have

$$G = \left[\frac{\partial P}{\partial s} \right]_{\text{EXIT}}$$

$$= \frac{H}{\pi} \cdot \left[\frac{\partial v}{\partial s} \right]_{\text{EXIT}}$$

Now

$$\partial s = \sqrt{\partial x^2 + \partial y^2}$$

$$= \partial y \sqrt{1 + \left(\frac{\partial x}{\partial y} \right)^2}$$

So if $\frac{dx}{dy} = \cot \theta$, where θ is the angle at which the stream line in the Z-plane meets the x-axis, we get $ds = dy \cdot \text{cosec } \theta$.

Also

$$\partial z = \partial x + i \partial y = \partial y \left(\frac{\partial x}{\partial y} + i \right)$$

$$= \partial y (\cot \theta + i)$$

$$\therefore \frac{\partial z}{\partial s} = \cos \theta + i \sin \theta.$$

Now

$$G_E = \frac{H}{\pi} \left[\frac{\partial v}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial z} \cdot \frac{\partial z}{\partial s} \right]_{\text{EXIT}} \dots \dots \dots (7.72)$$

Where for a conformal transformation $\frac{dz}{ds}$ must equal i , as the exit angle for the stream lines cannot be anything but $\frac{\pi}{2}$.

Also

$$\frac{\partial \zeta}{\partial z} = \frac{1}{A} : (\zeta - \zeta_1)^{\lambda_1} (\zeta - \zeta_2)^{\lambda_2} \dots \dots \dots,$$

and

$$\zeta - \frac{\zeta_{e_1} + \zeta_{e_2}}{2} = \frac{\zeta_{e_1} - \zeta_{e_2}}{2} \cosh w,$$

where ζ_{e_1} , ζ_{e_2} are the values of ζ for the extremities of the structure:

But for any particular stream line $u = \text{const.}$, i. e. $dw = idv$, which leads to

$$idv = d \left[\cosh^{-1} \frac{\zeta - \frac{\zeta_{e_1} + \zeta_{e_2}}{2}}{\frac{\zeta_{e_1} - \zeta_{e_2}}{2}} \right] \\ = \frac{d\zeta}{\sqrt{(\zeta - \zeta_{e_1})(\zeta - \zeta_{e_2})}}.$$

Hence

$$G_E = \frac{H}{\pi A} \cdot \left[\frac{(\zeta - \zeta_1)^{\lambda_1} (\zeta - \zeta_2)^{\lambda_2} \dots}{\sqrt{(\zeta - \zeta_{e_1})(\zeta - \zeta_{e_2})}} \right] \zeta = \zeta_{\text{EXIT}} \dots \dots \dots (7.73)$$

which is the general expression for exit gradients. Just at the downstream end of the structure, $\zeta_1 = \zeta_{\text{EXIT}}$ and it is at once seen from (7.73), that unless one of the ζ 's, say ζ_1 equals ζ_{e_1} , and unless at the same time λ equals $\frac{1}{2}$ (this will always be the case as the stream line is taken to cut the x -axis at right angles after turning through a right angle from its direction along the floor), the gradient at the toe will be infinite. For points beyond the heel, G_E is given on substituting the exit value of ζ in (7.73).

The values of G_E are evaluated below for a few particular cases :—

Case I. Sheet Pile at Step with Aprons.

Here $\zeta_{e_2} = L_2$, $\zeta_{e_1} = -L_1$, $\zeta_1 = 1$, $\zeta_2 = k$, $\zeta_3 = -1$, $\lambda_1 = \frac{1}{2}$, $\lambda_2 = -1$, $\lambda_3 = \frac{1}{2}$.

$$\therefore G_E = \frac{H}{\pi A} \cdot \frac{(\zeta - 1)^{\frac{1}{2}} (\zeta - k)^{-1} (\zeta + 1)^{\frac{1}{2}}}{\sqrt{(\zeta - L_2)(\zeta + L_1)}} \\ = \frac{Hk}{d_1 - d_2} \cdot \frac{(\zeta^2 - 1)^{\frac{1}{2}}}{\sqrt{(\zeta - L_2)(\zeta + L_1)}} \cdot \frac{1}{\zeta - k} \quad (\zeta \geq L_2) \dots \dots \dots (7.74)$$

Immediately at the toe $\zeta = L_2$ and $G_{ET} = \infty$.

Case II. Sheet Pile at Downstream End and Step.

Here $L_1 = 1$, and therefore

$$G_E = \frac{Hk}{d_1 - d_2} \cdot \left(\frac{\zeta + 1}{\zeta + L_1} \right)^{\frac{1}{2}} \cdot \frac{1}{\zeta - k} \quad (\zeta \geq 1) \quad \dots \dots \dots (7.75)$$

Immediately at the toe $\zeta = L_2 = 1$ and

$$G_{ET} = \frac{Hk}{d_1 - d_2} \cdot \left(\frac{2}{1 + L_1} \right)^{\frac{1}{2}} \cdot \frac{1}{1 - k} \\ = \frac{H}{d_1 - d_2} \cdot \frac{k}{1 - k} \cdot \frac{1}{\sqrt{\lambda}}, \text{ if } \lambda = \frac{1 + L_1}{2} \quad \dots \dots \dots (7.44)$$

Case III. Sheet Pile in Unequal Filling. No Aprons.

Here

$$L_1 = 1, L_2 = 1$$

and

$$G_E = \frac{Hk}{d_1 - d_2} \cdot \frac{1}{\zeta - k} \quad (\zeta \geq 1) \quad \dots \dots \dots (7.76)$$

Just at the toe

$$\zeta = L_2 = 1 \\ \therefore G_{ET} = \frac{H}{d_1 - d_2} \cdot \frac{k}{1 - k} \quad \dots \dots \dots (7.47)$$

Case IV. Floor with Pile Line not at End.

$$d_1 = d_2 = d.$$

Also as

$$\tan \theta - \theta = \pi \cdot \frac{d_2}{d_1 - d_2},$$

and

$$\cos \theta = k.$$

So

$$\sin \theta - \theta \cos \theta = \pi d_2 \cdot \frac{k}{d_1 - d_2}$$

$$\lim_{\substack{k \rightarrow 0 \\ d_1 \rightarrow d_2}} \frac{k}{d_1 - d_2} = \frac{1}{\pi d_2} \lim_{\theta = \frac{\pi}{2}} (\sin \theta - \theta \cos \theta) = \frac{1}{\pi d_2} = \frac{1}{\pi d}.$$

Then (7.74) gives

$$G_E = \frac{H}{\pi d} \cdot \frac{(\zeta^2 - 1)^{\frac{1}{2}}}{\sqrt{(\zeta - L_2)(\zeta + L_1)}} \cdot \frac{1}{\zeta - k} \quad (\zeta \geq L_2) \quad \dots \quad (7.77)$$

At the heel

$$\zeta = L_2$$

$$G_{EH} \text{ is infinite} \quad \dots \quad (7.78)$$

Case V. Single Sheet Pile—Equal Fill—No Aprons.

This is a particular case of Case III, when $k=0$ and $d_1=d_2$.

It gives

$$G_E = \frac{H}{\pi d} \cdot \frac{1}{\zeta} \quad (\zeta \geq 1) \quad \dots \quad (7.79)$$

At the heel

$$G_{EH} = \frac{H}{\pi d}, \text{ as } \zeta = L_2 = 1 \quad \dots \quad (7.49)$$

Case VI. Floor with Pile Line at Downstream End.

Here also $d_1=d_2=d$, and hence $k=0$. This is a particular case of Case II, when the step is absent.

$$G_E = \frac{H}{\pi d} \cdot \left(\frac{\zeta + 1}{\zeta + L_1} \right)^{\frac{1}{2}} \cdot \frac{1}{\zeta} \quad \dots \quad (7.80)$$

$$G_{EH} = \frac{H}{\pi d} \cdot \left(\frac{2}{1 + L_1} \right)^{\frac{1}{2}} = \frac{H}{\pi d} \cdot \frac{1}{\sqrt{\lambda}} \quad \dots \quad (7.63)$$

Since $\zeta = L_2 = 1$.

Case VII. Simple Floor—No Pile Line.

Here λ_1, λ_2 are all zero (7.73), and so

$$\frac{\partial \zeta}{\partial z} = \frac{1}{A}, \text{ i.e. } \zeta = \frac{z}{A} + B.$$

Taking

$$\zeta = 0, \frac{b}{2}, -\frac{b}{2} \text{ for } z = 0, \frac{b}{2}, -\frac{b}{2}.$$

we get

$$B = 0 \text{ and } \zeta = z = x. \text{ Also } \zeta_{e_1} = \frac{b}{2}, \zeta_{e_2} = -\frac{b}{2}$$

∴ from (7.73)

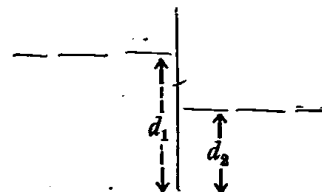
$$G_E = \frac{H}{\pi} \cdot \frac{1}{\sqrt{\left(x - \frac{b}{2}\right)\left(x + \frac{b}{2}\right)}} = \frac{H}{\pi} \cdot \frac{2}{\sqrt{4x^2 - b^2}} \quad \dots \quad (7.68)$$

At heel

$$x = \frac{b}{2} \text{ and } G_{EH} = \infty \quad \dots \quad (7.69)$$

TABLE VII/1

$\tan \theta - \theta = n\delta \quad \dots (7.14)$
 where $\delta = \frac{d_2}{d_1 - d_2}$ and $K = \cos \theta$
 (see Plate VII/I).



$$G_E = \frac{H}{d_2} \cdot \frac{K}{1-K} \cdot \frac{d_2}{d_1 - d_2} \quad (7.47)$$

δ	K	G_E in terms of H/d_2
0	1.000	
0.1	0.645	0.182 H/d_2
0.2	0.516	0.213 H/d_2
0.3	0.437	0.233 H/d_2
0.4	0.380	0.245 H/d_2
0.5	0.335	0.252 H/d_2
0.6	0.302	0.260 H/d_2
0.7	0.275	0.265 H/d_2
0.8	0.254	0.270 H/d_2
0.9	0.233	0.274 H/d_2
1.0	0.217	0.278 H/d_2
2.0	0.129	0.295 H/d_2
3.0	0.091	0.301 H/d_2
4.0	0.071	0.305 H/d_2
5.0	0.058	0.307 H/d_2
6.0	0.049	0.310 H/d_2
7.0	0.042	0.310 H/d_2
8.0	0.038	0.311 H/d_2
9.0	0.033	0.311 H/d_2
10.0	0.030	0.312 H/d_2

TABLE VII/2

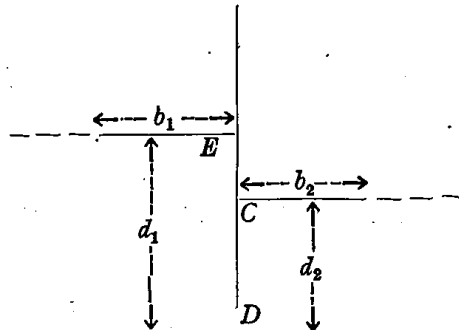
$$\sinh \gamma_1 + K\gamma_1 = -\pi K \frac{b_1}{d_1 - d_2} = -\pi K \delta_1 \quad \dots \quad (7.18)$$

$$\sinh \gamma_2 - K\gamma_2 = \pi K \frac{b_2}{d_1 - d_2} = \pi K \delta_2 \quad \dots \quad (7.19)$$

$$L_1 = \cosh \gamma_1$$

$$L_2 = \cosh \gamma_2$$

$$\left. \begin{aligned} \lambda &= \frac{L_1 + L_2}{2} \\ \lambda_1 &= \frac{L_1 - L_2}{2} \end{aligned} \right\} \delta = \frac{d_2}{d_1 - d_2}$$



δ_1 or δ_2	$\delta=0$		$\delta=1$		$\delta=2$		$\delta=3$		$\delta=4$	
	L_1	L_2	L_1	L_2	L_1	L_2	L_1	L_2	L_1	L_2
1	2.055	5.645	1.149	1.313	1.062	1.101	1.034	1.048	1.022	1.028
2	4.271	9.252	1.524	1.922	1.233	1.354	1.130	1.180	1.084	1.109
3	6.884	12.660	2.023	2.597	1.480	1.683	1.276	1.369	1.180	1.229
4	9.656	16.084	2.585	3.290	1.773	2.048	1.458	1.594	1.304	1.380
5	12.534	19.444	3.179	3.987	2.097	2.431	1.666	1.840	1.449	1.550
6	15.455	22.586	3.795	4.690	2.439	2.823	1.892	2.098	1.610	1.735
7	18.500	25.977	4.423	5.385	2.794	3.222	2.128	2.365	1.782	1.929
8	21.486	29.287	5.062	6.084	3.158	3.624	2.376	2.638	1.964	2.130
9	24.466	32.366	5.707	6.782	3.531	4.026	2.628	2.914	2.151	2.335
10	27.308	35.768	6.354	7.481	3.907	4.432	2.887	3.191	2.346	2.544
11	30.482	38.746	7.008	8.179	4.283	4.834	3.149	3.474	2.542	2.755
12	33.351	41.972	7.669	8.873	4.667	5.239	3.414	3.755	2.743	2.970
13	36.490	45.466	8.327	9.570	5.052	5.645	3.683	4.041	2.948	3.170
14	39.528	48.762	8.989	10.270	5.438	6.054	3.952	4.325	3.152	3.405
15	42.819	51.777	9.646	10.902	5.826	6.462	4.226	4.608	3.359	3.620
16	45.923	54.978	10.373	11.690	6.218	6.870	4.497	4.896	3.568	3.839
17	48.762	58.377	11.011	12.287	6.610	7.276	4.773	5.183	3.780	4.061
18	51.777	61.370	11.690	13.044	7.001	7.684	5.047	5.465	3.991	4.279
19	54.978	64.516	12.410	13.711	7.392	8.091	5.322	5.752	4.201	4.502
20	57.797	67.823	13.044	14.412	7.783	8.494	5.601	6.042	4.415	4.722

TABLE VII/2

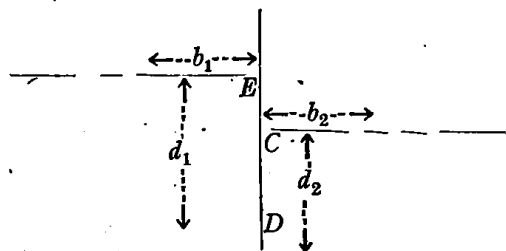
$$\sinh \gamma_1 = K\gamma_1 = -\pi K\delta_1 \quad \dots \quad \dots \quad \dots \quad (7-18)$$

$$\sinh \gamma_2 = K\gamma_2 = \pi K\delta_2 \quad \dots \quad \dots \quad \dots \quad (7-19)$$

$$L_1 = \cosh \gamma_1$$

$$L_2 = \cosh \gamma_2$$

$$\left. \begin{aligned} \lambda &= \frac{L_1 + L_2}{2} \\ \lambda &= \frac{L_1 - L_2}{2} \end{aligned} \right\} \delta = \frac{d_2}{d_1 - d_2}$$



δ_1 or δ_2	$\delta=5$		$\delta=6$		$\delta=7$	
	L_1	L_2	L_1	L_2	L_1	L_2
1	1.015	1.018	1.011	1.013	1.009	1.010
2	1.058	1.072	1.042	1.052	1.031	1.039
3	1.126	1.155	1.094	1.113	1.073	1.085
4	1.215	1.261	1.161	1.192	1.123	1.144
5	1.322	1.385	1.240	1.287	1.186	1.219
6	1.442	1.522	1.337	1.393	1.263	1.303
7	1.573	1.670	1.443	1.509	1.346	1.393
8	1.712	1.826	1.555	1.629	1.433	1.497
9	1.858	1.983	1.669	1.752	1.531	1.604
10	2.009	2.147	1.796	1.888	1.629	1.709
11	2.164	2.314	1.921	2.023	1.737	1.826
12	2.323	2.484	2.040	2.170	1.841	1.937
13	2.484	2.655	2.170	2.310	1.954	2.058
14	2.647	2.828	2.310	2.440	2.076	2.170
15	2.815	3.001	2.462	2.601	2.189	2.290
16	2.981	3.179	2.601	2.750	2.310	2.417
17	3.149	3.353	2.725	2.882	2.417	2.531
18	3.321	3.531	2.882	3.021	2.531	2.675
19	3.490	3.708	3.021	3.167	2.650	2.802
20	3.662	3.888	3.167	3.321	2.776	2.909

TABLE VII/2

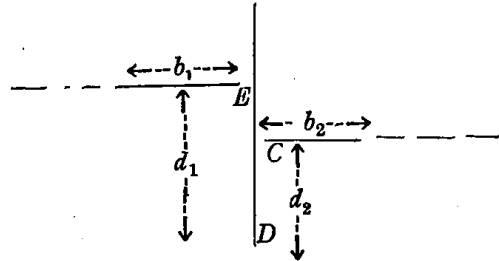
$$\sinh \gamma_1 + K \gamma_1 = -\pi K \delta_1 \quad \dots \quad (7.18)$$

$$\sinh \gamma_2 - K \gamma_2 = \pi K \delta_2 \quad \dots \quad (7.19)$$

$$L_1 = \cosh \gamma_1$$

$$L_2 = \cosh \gamma_2$$

$$\left. \begin{aligned} \lambda &= \frac{L_1 + L_2}{2} \\ \lambda_1 &= \frac{L_1 - L_2}{2} \end{aligned} \right\} \delta = \frac{d_2}{d_1 - d_2}$$



δ_1 or δ_2	$\delta=8$		$\delta=9$		$\delta=10$	
	L_1	L_2	L_1	L_2	L_1	L_2
1	1.006	1.007	1.005	1.006	1.004	1.005
2	1.027	1.029	1.020	1.024	1.016	1.018
3	1.055	1.066	1.045	1.052	1.037	1.042
4	1.098	1.113	1.081	1.090	1.066	1.073
5	1.149	1.173	1.123	1.138	1.103	1.113
6	1.212	1.240	1.173	1.192	1.144	1.161
7	1.279	1.311	1.226	1.255	1.192	1.212
8	1.356	1.393	1.287	1.320	1.240	1.271
9	1.433	1.486	1.356	1.393	1.303	1.329
10	1.520	1.579	1.433	1.475	1.365	1.393
11	1.604	1.669	1.509	1.555	1.423	1.465
12	1.696	1.766	1.579	1.642	1.497	1.543
13	1.796	1.873	1.669	1.723	1.567	1.616
14	1.888	1.971	1.752	1.811	1.642	1.696
15	1.988	2.076	1.826	1.905	1.709	1.766
16	2.095	2.189	1.921	1.988	1.781	1.841
17	2.189	2.290	2.005	2.076	1.873	1.937
18	2.290	2.396	2.095	2.170	1.937	2.005
19	2.396	2.507	2.189	2.269	2.023	2.095
20	2.507	2.626	2.290	2.374	2.113	2.189

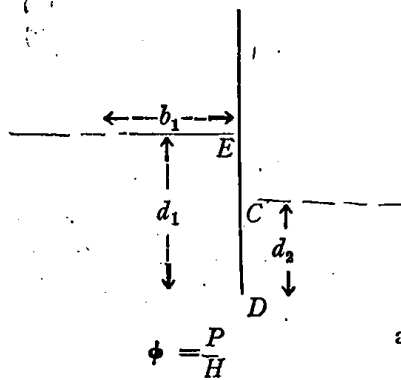


TABLE VII/3

$$P = \frac{H}{\pi} \cos^{-1} \left(\frac{L_1 - \delta}{L_1 + 1} \right) \dots \dots \dots (7.39)$$

$$P_D = \frac{H}{\pi} \cos^{-1} \left(\frac{2K + L_1 - 1}{L_1 + 1} \right) \dots \dots \dots (7.40)$$

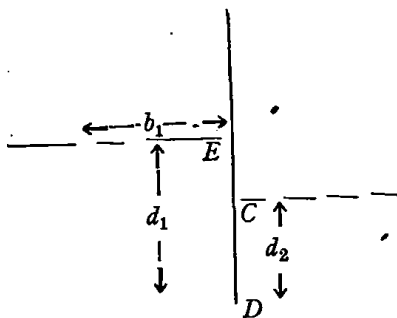
$$G = \frac{H}{d_2} \cdot \frac{d_2}{d_1 - d_2} \cdot \frac{1}{\sqrt{\lambda}} \cdot \frac{K}{1 - K} \dots \dots \dots (7.44)$$

$$\lambda = \frac{L_1 + 1}{2}$$

where K is given in Plate VII/1 and Table VII/1, and L_1 in Table VII/2 and Plate VII/2.

δ_1	$\delta=0$	$\delta=1$			$\delta=2$			$\delta=3$		
	ϕ_E	ϕ_E	ϕ_D	G_E	ϕ_E	ϕ_D	G_E	ϕ_E	ϕ_D	G_E
1	0.600	0.830	0.413	0.268 $\frac{H}{d_2}$	0.889	0.451	0.290 $\frac{H}{d_2}$	0.918	0.466	0.299 $\frac{H}{d_2}$
2	0.423	0.699	0.376	0.247 ,,	0.791	0.429	0.278 ,,	0.841	0.453	0.292 ,,
3	0.336	0.605	0.340	0.226 ,,	0.710	0.404	0.264 ,,	0.774	0.436	0.282 ,,
4	0.285	0.537	0.309	0.208 ,,	0.646	0.379	0.250 ,,	0.716	0.416	0.272 ,,
5	0.251	0.486	0.285	0.192 ,,	0.592	0.356	0.236 ,,	0.668	0.397	0.261 ,,
6	0.227	0.448	0.265	0.180 ,,	0.552	0.336	0.224 ,,	0.625	0.379	0.250 ,,
7	0.208	0.415	0.248	0.169 ,,	0.517	0.318	0.213 ,,	0.590	0.362	0.241 ,,
8	0.193	0.390	0.234	0.160 ,,	0.488	0.303	0.204 ,,	0.558	0.347	0.232 ,,
9	0.181	0.368	0.222	0.152 ,,	0.463	0.289	0.195 ,,	0.533	0.334	0.223 ,,
10	0.171	0.349	0.212	0.145 ,,	0.441	0.277	0.188 ,,	0.508	0.321	0.216 ,,
11	0.162	0.333	0.202	0.139 ,,	0.422	0.266	0.181 ,,	0.489	0.310	0.209 ,,
12	0.155	0.319	0.194	0.133 ,,	0.405	0.257	0.175 ,,	0.470	0.300	0.203 ,,
13	0.148	0.307	0.187	0.129 ,,	0.390	0.248	0.169 ,,	0.453	0.291	0.197 ,,
14	0.143	0.295	0.181	0.124 ,,	0.376	0.240	0.164 ,,	0.438	0.282	0.191 ,,
15	0.137	0.285	0.175	0.120 ,,	0.364	0.233	0.159 ,,	0.425	0.274	0.186 ,,
16	0.132	0.275	0.169	0.116 ,,	0.353	0.226	0.155 ,,	0.412	0.267	0.182 ,,
17	0.129	0.268	0.164	0.113 ,,	0.343	0.219	0.151 ,,	0.401	0.260	0.177 ,,
18	0.125	0.260	0.160	0.110 ,,	0.333	0.214	0.147 ,,	0.390	0.253	0.173 ,,
19	0.121	0.252	0.150	0.107 ,,	0.325	0.209	0.143 ,,	0.380	0.248	0.169 ,,
20	0.118	0.246	0.152	0.077 ,,	0.317	0.204	0.140 ,,	0.371	0.242	0.166 ,,

TABLE VII/3



$$P_E = \frac{H}{\pi} \cos^{-1} \left(\frac{L_1 - 3}{L_1 + 1} \right) \dots \dots \dots (7.30)$$

$$P_D = \frac{H}{\pi} \cos^{-1} \left(\frac{2K + L_1 - 1}{L_1 + 1} \right) \dots \dots \dots (7.40)$$

$$G_E = \frac{H}{d_2} \cdot \frac{d_2}{d_1 - d_2} \cdot \frac{1}{\sqrt{\lambda}} \cdot \frac{K}{1 - K} \dots \dots \dots (7.44)$$

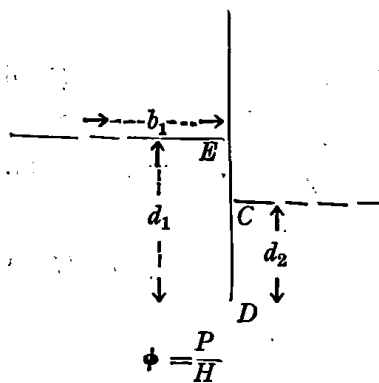
$$\phi = \frac{P}{H} \quad \text{and} \quad \lambda = \frac{L_1 + 1}{2}$$

where K is given in Plate VII/I and Table VII/I, and L_1 in Table VII/2 and Plate VII/2.

δ_1	$\delta=4$			$\delta=5$			$\delta=6$			$\delta=7$		
	ϕ_E	ϕ_D	G_E^*	ϕ_E	ϕ_D	G_E^*	ϕ_E	ϕ_D	G_E^*	ϕ_E	ϕ_D	G_E^*
1	0.933	0.474	0.304	0.945	0.479	0.306	0.953	0.483	0.309	0.957	0.485	0.310
2	0.871	0.466	0.299	0.893	0.474	0.303	0.908	0.478	0.307	0.921	0.482	0.308
3	0.814	0.453	0.292	0.843	0.464	0.298	0.864	0.471	0.303	0.880	0.476	0.305
4	0.763	0.438	0.284	0.798	0.452	0.292	0.824	0.462	0.298	0.846	0.469	0.301
5	0.718	0.422	0.276	0.757	0.439	0.285	0.788	0.452	0.293	0.812	0.461	0.297
6	0.679	0.407	0.267	0.720	0.427	0.278	0.752	0.441	0.286	0.778	0.451	0.292
7	0.644	0.392	0.259	0.687	0.414	0.271	0.720	0.429	0.280	0.749	0.441	0.287
8	0.614	0.378	0.251	0.658	0.401	0.264	0.691	0.418	0.274	0.723	0.432	0.282
9	0.587	0.366	0.243	0.631	0.389	0.257	0.666	0.407	0.268	0.697	0.422	0.276
10	0.563	0.353	0.236	0.607	0.378	0.250	0.642	0.397	0.262	0.674	0.413	0.271
11	0.541	0.342	0.229	0.585	0.367	0.244	0.621	0.387	0.256	0.653	0.403	0.266
12	0.522	0.332	0.223	0.565	0.357	0.238	0.602	0.378	0.251	0.634	0.394	0.261
13	0.504	0.322	0.217	0.547	0.348	0.233	0.584	0.369	0.246	0.615	0.386	0.256
14	0.488	0.314	0.212	0.531	0.339	0.227	0.567	0.360	0.241	0.597	0.377	0.251
15	0.474	0.306	0.207	0.515	0.331	0.222	0.549	0.351	0.235	0.582	0.369	0.246
16	0.460	0.298	0.202	0.502	0.323	0.218	0.536	0.343	0.231	0.567	0.362	0.242
17	0.448	0.291	0.197	0.489	0.316	0.213	0.523	0.337	0.227	0.554	0.355	0.238
18	0.436	0.284	0.193	0.476	0.309	0.209	0.509	0.329	0.222	0.542	0.349	0.234
19	0.426	0.278	0.189	0.465	0.303	0.205	0.498	0.323	0.218	0.531	0.342	0.230
20	0.416	0.272	0.186	0.455	0.297	0.201	0.487	0.317	0.215	0.519	0.336	0.226

*The values of G_E should be multiplied by $\frac{H}{d_2}$ to get the actual gradient at exit:

TABLE VII/3



$$P = \frac{H}{\pi} \cos^{-1} \left(\frac{L_1 - 3}{L_1 + 1} \right) \dots \dots \dots (7.30)$$

$$P_D = \frac{H}{\pi} \cos^{-1} \left(\frac{2K + L_1 - 1}{L_1 + 1} \right) \dots \dots \dots (7.40)$$

$$G_E = \frac{H}{d_2} \cdot \frac{d_2}{d_1 - d_2} \cdot \frac{1}{\sqrt{\lambda}} \cdot \frac{K}{1 - K} \dots \dots \dots (7.44)$$

$$\phi = \frac{P}{H} \quad \text{and} \quad \lambda = \frac{L_1 + 1}{2}$$

* where K is given in Plate VII/1 and Table VII/1, and L_1 , in Table VII/2 and Plate VII/2.

δ_1	$\delta=8$			$\delta=9$			$\delta=10$		
	ϕ_E	ϕ_D	G_E^*	ϕ_E	ϕ_D	G_E^*	ϕ_E	ϕ_D	G_E^*
1	0.965	0.487	0.312	0.968	0.489	0.311	0.972	0.511	0.490
2	0.926	0.484	0.310	0.937	0.487	0.310	0.943	0.488	0.310
3	0.896	0.480	0.308	0.905	0.483	0.308	0.914	0.485	0.308
4	0.861	0.474	0.305	0.873	0.478	0.305	0.886	0.481	0.306
5	0.831	0.467	0.301	0.845	0.472	0.302	0.858	0.475	0.303
6	0.799	0.458	0.297	0.818	0.465	0.299	0.833	0.470	0.300
7	0.772	0.450	0.292	0.793	0.458	0.295	0.809	0.463	0.297
8	0.746	0.441	0.288	0.769	0.451	0.291	0.788	0.457	0.294
9	0.723	0.433	0.283	0.746	0.443	0.287	0.764	0.449	0.290
10	0.700	0.424	0.278	0.723	0.434	0.282	0.743	0.443	0.286
11	0.680	0.416	0.273	0.703	0.426	0.278	0.726	0.436	0.282
12	0.661	0.408	0.269	0.686	0.419	0.274	0.706	0.428	0.278
13	0.642	0.399	0.264	0.666	0.411	0.270	0.688	0.422	0.275
14	0.626	0.392	0.260	0.650	0.404	0.266	0.672	0.414	0.271
15	0.610	0.384	0.255	0.636	0.398	0.262	0.658	0.408	0.267
16	0.594	0.377	0.251	0.621	0.390	0.258	0.644	0.402	0.264
17	0.582	0.370	0.247	0.607	0.384	0.254	0.628	0.395	0.259
18	0.569	0.364	0.243	0.594	0.378	0.250	0.618	0.390	0.257
19	0.557	0.357	0.239	0.582	0.371	0.247	0.605	0.383	0.253
20	0.545	0.351	0.236	0.569	0.365	0.243	0.592	0.377	0.249

* The values of G should be multiplied by $\frac{H}{d_2}$ to get the actual exit gradients.

TABLE VII/4. (1)

Floor with Pile not at End.

$$\phi_E = \frac{P_E}{H} \times 100$$

$$= 100 \times \frac{1}{\pi} \cos^{-1} \left(\frac{\lambda_1 - 1}{\lambda} \right) \dots \dots \dots (7.36)$$

$$\alpha_1 = \frac{b_1}{d}, \alpha_2 = \frac{b_2}{d}, \alpha = \frac{b}{d}$$

$\frac{b_1}{b}$	$\frac{b_1}{b}$ or base rates.											
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
α	Values of ϕ_E											
0.1	2.16	2.82	2.55	2.22	1.91	1.59	1.28	0.96	0.63	0.35	0	
0.2	6.22	5.67	5.06	4.45	3.81	3.17	2.52	1.87	1.26	0.63	0	
0.25	7.86	7.10	6.31	5.53	4.75	3.98	3.17	2.38	1.58	0.78	0	
1/3	9.95	9.29	8.27	7.26	6.24	5.39	4.15	3.11	2.08	0.99	0	
0.5	14.93	13.74	12.30	10.83	9.37	7.82	6.22	4.66	3.11	1.56	0	
0.75	21.90	20.00	18.00	15.80	13.70	11.50	9.00	6.50	4.50	2.00	0	
1.0	27.18	25.46	22.81	20.03	17.38	14.93	11.87	8.92	5.87	2.92	0	
1.5	35.80	33.70	30.90	27.40	24.10	20.50	16.50	12.50	8.30	4.20	0	
2.0	42.48	39.93	36.79	33.29	29.13	24.91	20.32	15.44	10.24	5.03	0	
2.5	47.40	44.80	41.50	37.60	33.20	28.50	23.50	17.90	12.20	6.10	0	
3.0	51.22	48.67	45.21	41.03	36.42	31.32	25.89	19.84	13.46	6.85	0	
4.0	56.98	54.37	50.48	45.86	40.74	35.25	29.35	22.92	15.83	8.08	0	
5.0	61.07	58.17	54.08	49.01	43.62	37.90	31.76	25.09	17.63	9.27	0	
6.0	64.22	61.34	56.64	51.32	45.79	39.77	33.48	26.76	19.06	10.09	0	
7.0	66.83	63.65	58.89	52.92	47.11	41.15	34.48	27.98	19.62	10.90	0	
8.0	68.84	65.43	60.00	54.13	48.22	42.28	35.93	28.96	21.11	11.60	0	
9.0	70.48	67.00	61.13	55.41	49.15	43.04	36.67	29.76	21.88	12.00	0	
10.0	72.01	68.12	62.09	55.96	50.13	43.72	37.09	30.33	22.57	12.74	0	
11.0	73.24	69.11	62.89	56.60	50.47	44.28	37.87	30.96	23.00	13.22	0	
12.0	74.37	69.96	63.44	57.16	50.96	44.76	38.33	31.43	23.58	13.68	0	
14.0	76.23	71.38	64.54	58.01	51.74	45.48	39.00	32.15	24.32	14.37	0	
16.0	77.72	72.27	65.28	58.64	52.68	46.24	39.60	32.74	24.92	14.93	0	
18.0	78.93	73.20	65.86	59.17	53.00	46.50	39.84	33.41	25.37	15.48	0	
20.0	80.00	73.84	66.37	59.43	53.14	46.83	40.41	33.71	25.87	15.88	0	
25.0	82.10	75.01	67.04	60.30	53.81	47.46	41.02	34.06	26.45	16.68	0	
30.0	84.21	75.78	67.77	60.75	54.23	47.88	41.44	34.63	26.95	17.32	0	

TABLE VII/4. (2).

Floor with Pile not at End.

$$\phi_D = \frac{P_D}{H} \times 100 = \frac{100}{\pi} \cos^{-1} \left(\frac{\lambda^1}{\lambda} \right) \dots \dots \dots (7.38)$$

b_1/b	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
α	Values of ϕ_D										
0.1	50.08	50.07	50.04	50.03	50.02	50.00	49.98	49.97	49.96	49.93	49.92
0.2	50.32	50.25	50.19	50.13	50.07	50.00	49.92	49.87	49.81	49.75	49.68
0.25	50.48	50.40	50.30	50.20	50.10	50.00	49.90	49.80	49.70	49.60	49.52
1/3	50.83	50.65	50.50	50.34	50.17	50.00	49.83	49.66	49.50	49.35	49.17
0.5	51.72	51.44	51.13	50.72	50.38	50.00	49.62	49.28	48.87	48.56	48.28
0.75	53.70	52.94	52.34	51.62	50.80	50.00	49.20	48.38	47.66	47.06	46.30
1.0	55.46	54.50	53.61	52.43	51.22	50.00	48.78	47.57	46.39	45.50	44.54
1.5	59.20	58.00	56.37	54.44	52.29	50.00	47.71	45.56	43.63	42.00	40.80
2.0	62.50	60.99	58.77	56.11	53.17	50.00	46.83	43.89	41.23	39.01	37.50
2.5	65.18	63.50	60.70	57.50	53.82	50.00	46.18	42.50	39.30	36.50	34.82
3.0	67.29	65.46	62.44	58.61	54.37	50.00	45.63	41.39	37.56	34.54	32.71
4.0	71.06	68.61	64.75	60.02	55.07	50.00	44.93	39.98	35.25	31.39	28.94
5.0	73.56	70.87	66.26	60.94	55.50	50.00	44.50	39.06	33.74	29.13	26.44
6.0	75.47	72.53	67.07	61.55	55.83	50.00	44.17	38.45	32.93	27.47	24.53
7.0	77.08	73.81	68.38	61.93	55.89	50.00	44.11	38.07	31.62	26.19	22.92
8.0	78.43	74.68	68.50	62.17	56.02	50.00	43.98	37.83	31.50	25.32	21.57
9.0	79.57	75.44	68.86	62.33	56.07	50.00	43.93	37.67	31.14	24.56	20.43
10.0	80.53	76.01	69.11	62.39	56.15	50.00	43.85	37.61	30.89	23.99	19.47
11.0	81.33	76.52	69.34	62.57	56.21	50.00	43.79	37.43	30.66	23.48	18.67
12.0	82.16	76.92	69.47	62.65	56.22	50.00	43.78	37.35	30.53	23.08	17.84
14.0	83.39	77.25	69.77	62.79	56.28	50.00	43.72	37.21	30.23	22.75	16.61
16.0	84.44	77.93	69.93	62.85	56.28	50.00	43.72	37.15	30.07	22.07	15.56
18.0	85.14	78.22	70.04	62.89	56.29	50.00	43.71	37.11	29.96	21.78	14.86
20.0	86.01	78.40	70.12	62.93	56.34	50.00	43.66	37.07	29.88	21.60	13.99
25.0	87.43	78.78	70.26	63.02	56.37	50.00	43.63	36.98	29.74	21.22	12.57
30.0	88.74	79.02	70.32	63.03	56.40	50.00	43.60	36.97	29.68	20.98	11.26

TABLE VII/5

$$P_E = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda - 2}{\lambda} \right) \quad \dots \quad \dots \quad \dots \quad (7.61)$$

$$P_D = \frac{H}{\pi} \cos^{-1} \left(\frac{\lambda - 1}{\lambda} \right) \quad \dots \quad \dots \quad \dots \quad (7.62)$$

$$G_{EXIT} = \frac{H}{\pi d} \frac{1}{\sqrt{\lambda}} \quad \dots \quad \dots \quad \dots \quad (7.63)$$

$$\phi_{D'} = \phi_D - \frac{2}{3}(\phi_E - \phi_D) + \frac{3}{\alpha^2} \quad \dots \quad \dots \quad (7.70)^*$$

$$\phi = \frac{P}{H} \times 100.$$

$\frac{1}{\alpha}$	ϕ_D	ϕ_E	$\phi_{D'}$ Eqn. 7.70	$\phi_{D'}$ Theoretical	α	λ	$\frac{1}{\pi\sqrt{\lambda}}$
0.02	8.9	12.7	6.40	6.92	50	25.545	0.063
0.04	12.6	17.9	9.10	9.56	45	23.006	0.066
0.06	15.3	21.8	11.01	11.34	40	20.506	0.070
0.08	17.6	25.1	12.62	12.90	35	18.007	0.075
0.10	19.4	28.0	13.73	14.12	30	15.508	0.080
0.12	21.2	30.6	14.94	15.16	25	13.010	0.088
0.14	22.7	32.9	15.96	16.12	20	10.512	0.098
0.16	24.1	35.0	16.88	16.96	18	9.514	0.103
0.18	25.3	37.0	17.60	17.85	16	8.516	0.109
0.20	26.6	38.8	18.68	18.61	14	7.518	0.116
0.22	27.6	40.6	19.05	19.27	12	6.520	0.125
0.24	28.7	42.2	19.87	19.80	11	6.020	0.130
0.28	30.5	45.2	20.94	20.91	10	5.52	0.136
0.32	32.1	47.9	21.91	21.84	9	5.03	0.142
0.36	33.6	50.4	22.79	22.63	8	4.53	0.150
0.40	34.8	52.7	23.38	23.41	67	4.04	0.158
0.44	36.0	54.7	24.08	24.07	5	3.54	0.169
0.48	37.1	56.7	24.69	24.71	4	3.05	0.182
0.52	37.9	58.5	25.01	25.30	3	2.561	0.199
0.56	38.8	60.1	25.54	25.81	2	2.08	0.225
0.60	39.6	61.7	26.08	26.30	1½	1.62	0.252
0.64	40.3	63.2	26.23	26.72	1	1.401	0.269
0.68	40.9	64.5	26.39	27.18	¾	1.207	0.289
0.72	41.6	65.8	26.96	27.61	½	1.125	0.300
0.76	42.1	66.9	27.33	28.00	¼	1.057	0.309
0.80	42.6	68.1	27.52	28.35	0	1.015	0.316
0.90	43.7	70.7	28.13	29.21		1.000	0.318
1.00	44.6	72.8	28.80	30.00			
1.10	45.2	74.7	29.13	30.65			
1.20	45.8	76.4	29.72	31.21			
1.30	46.3	77.9	30.27	31.75			
1.40	46.7	79.3	30.88	32.23			
1.50	47.1	80.4	31.65	32.70			
1.60	47.3	81.4	32.28	33.10			
1.70	47.7	82.5	33.17	33.52			
1.80	47.9	83.3	34.02	33.90			
1.90	48.1	84.1	34.93	34.21			
2.00	48.2	84.9	35.80	34.60			

*The approximate formula 7.70 will not hold much beyond $\frac{1}{\alpha} = 2$

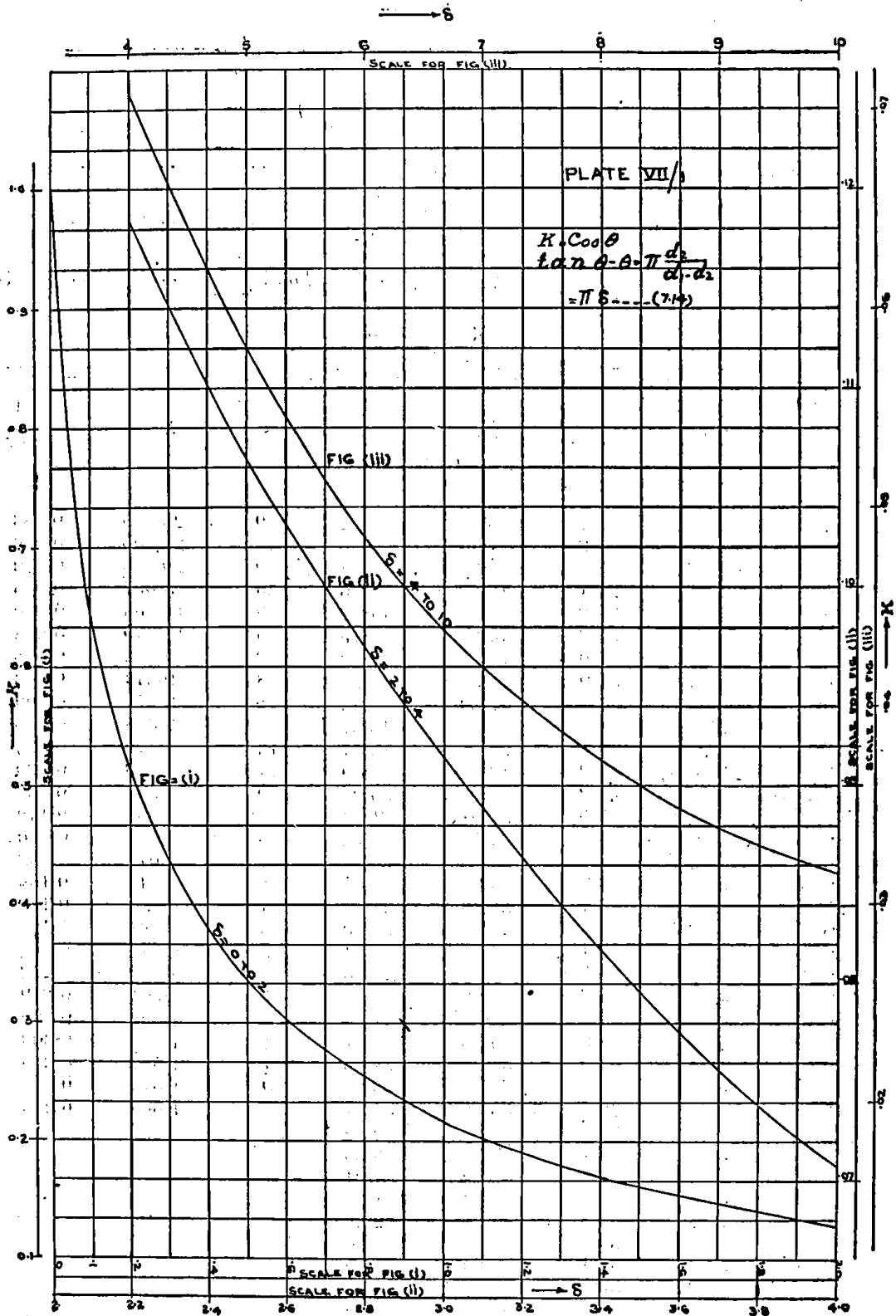


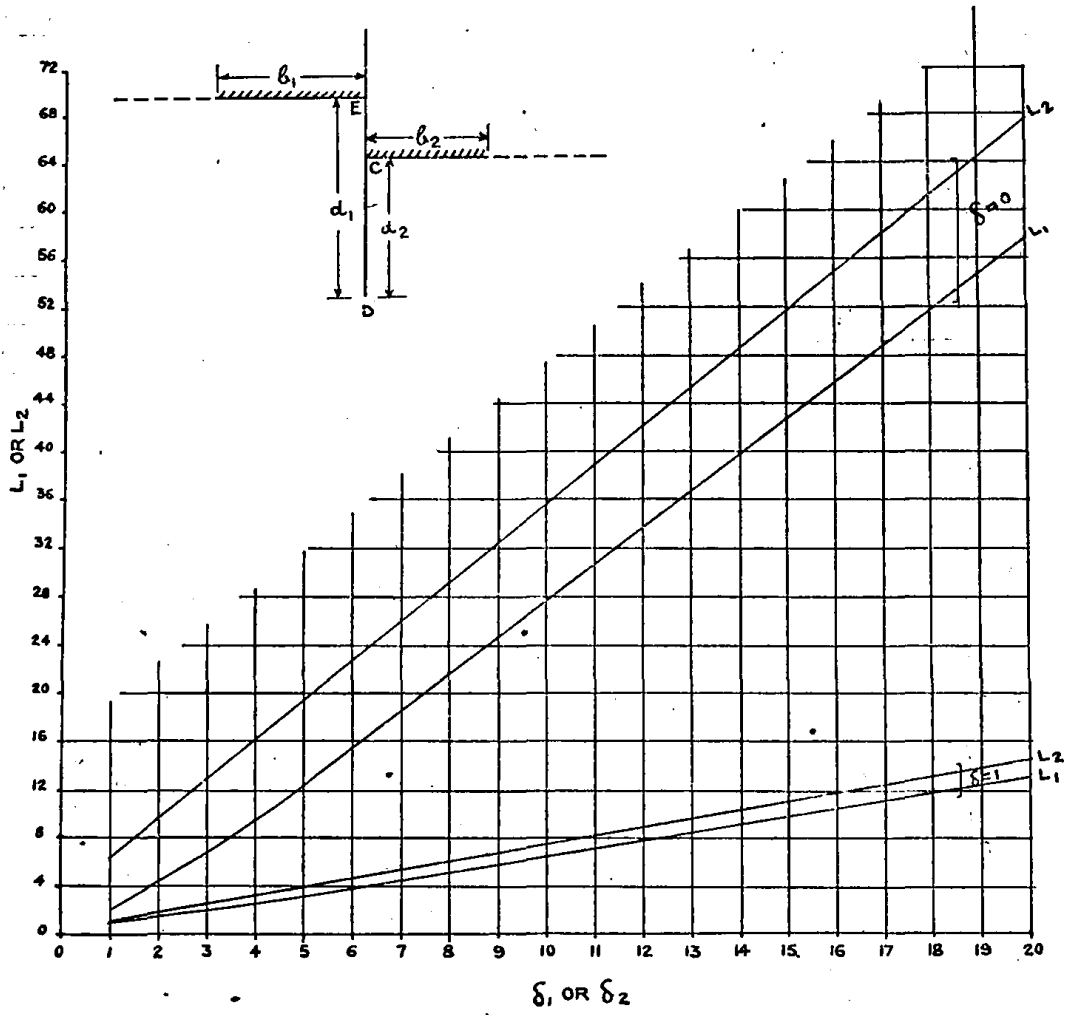
PLATE VII/2A

$$\text{SINH } \gamma_1 + K \gamma_1 = -\pi K \frac{b_1}{d_1 - d_2} = -\pi K \delta_1 \quad (7.18)$$

$$\text{SINH } \gamma_2 - K \gamma_2 = \pi K \frac{b_2}{d_1 - d_2} = \pi K \delta_2 \quad (7.19)$$

$$L_1 = \text{COSH } \gamma_1$$

$$L_2 = \text{COSH } \gamma_2$$



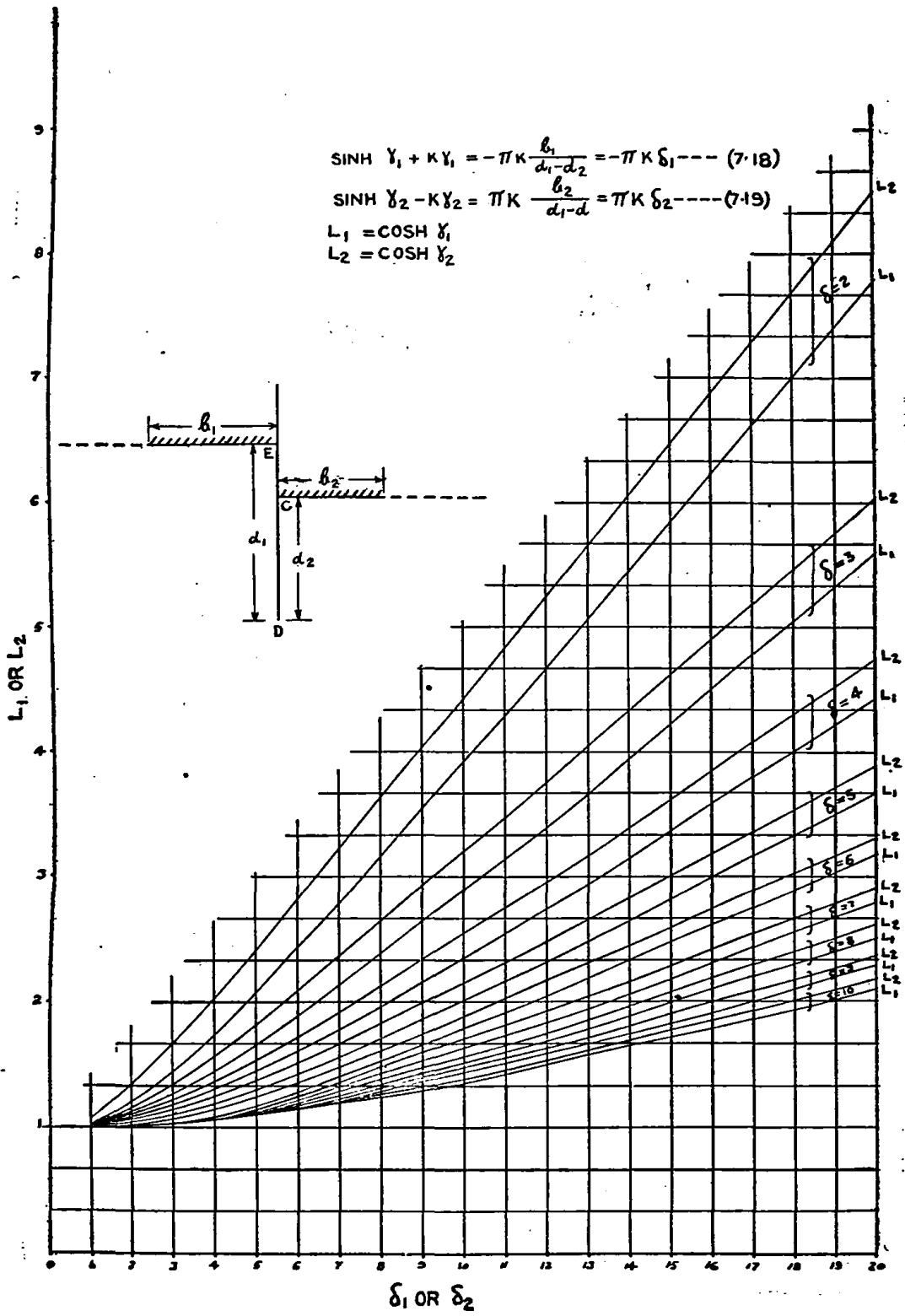
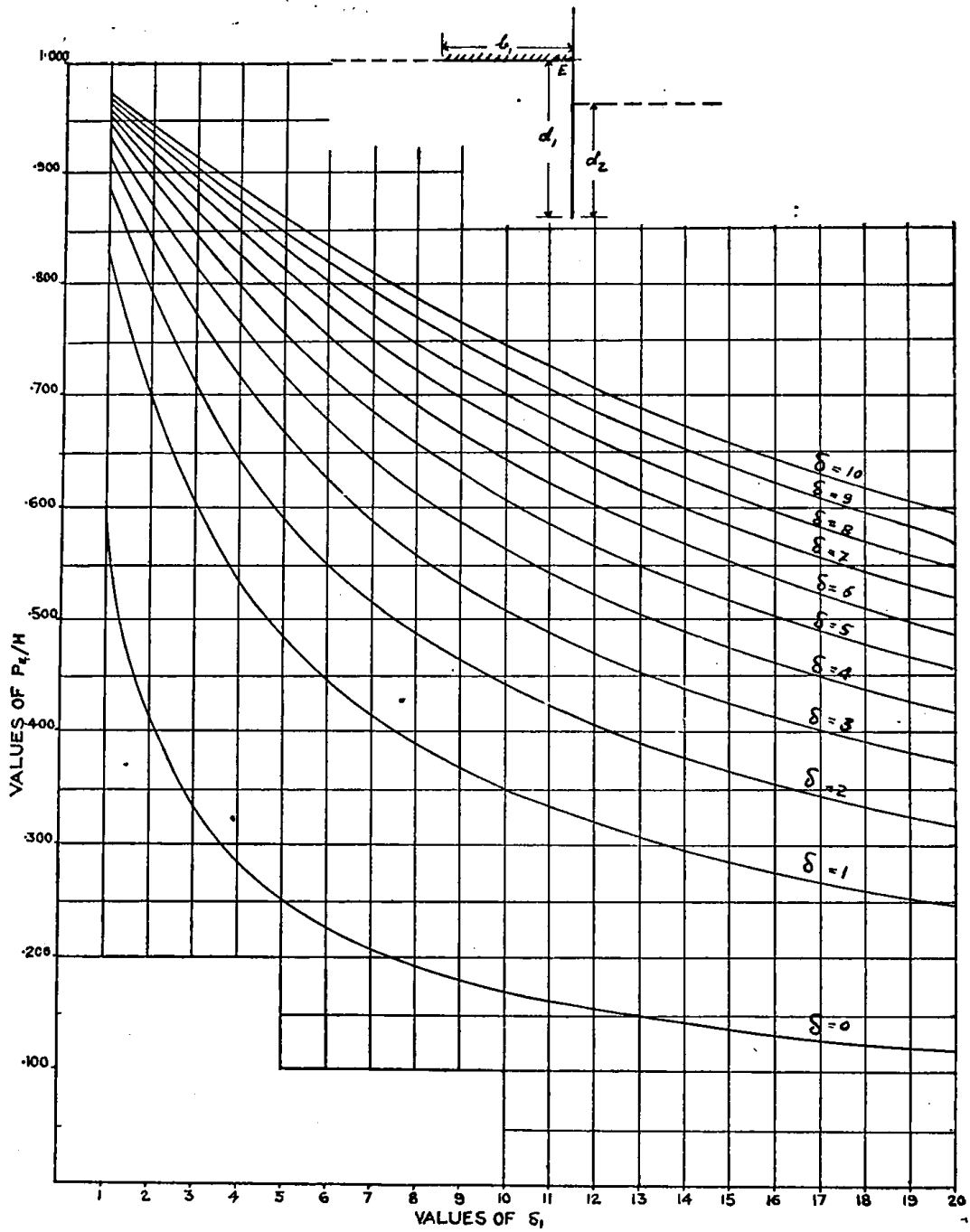


PLATE VII/3A

$$P_c = \frac{H}{\pi} \cos^{-1} \left(\frac{L_1 - 3}{L_1 + 1} \right) \text{---7.39}$$



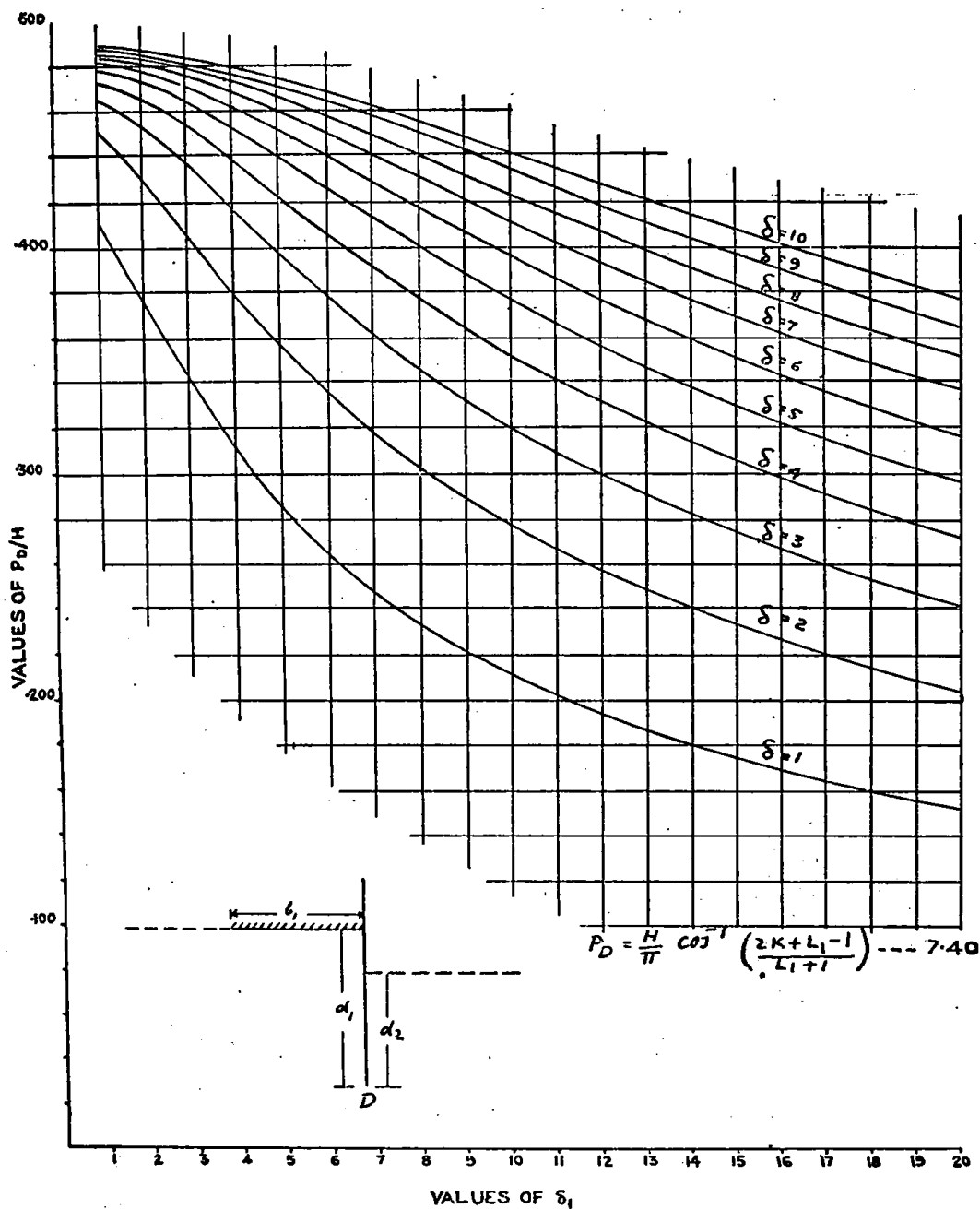
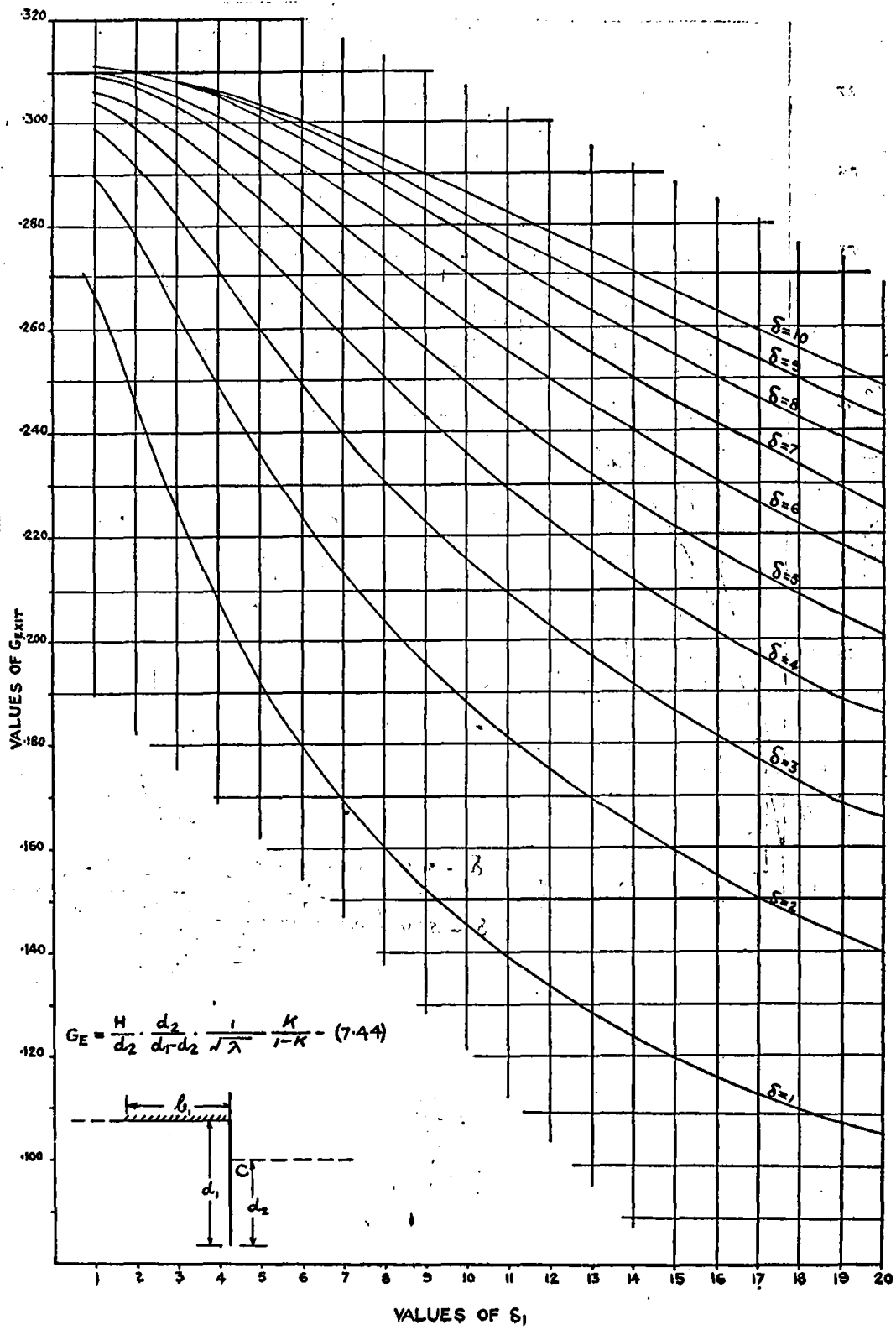


PLATE VII/3C



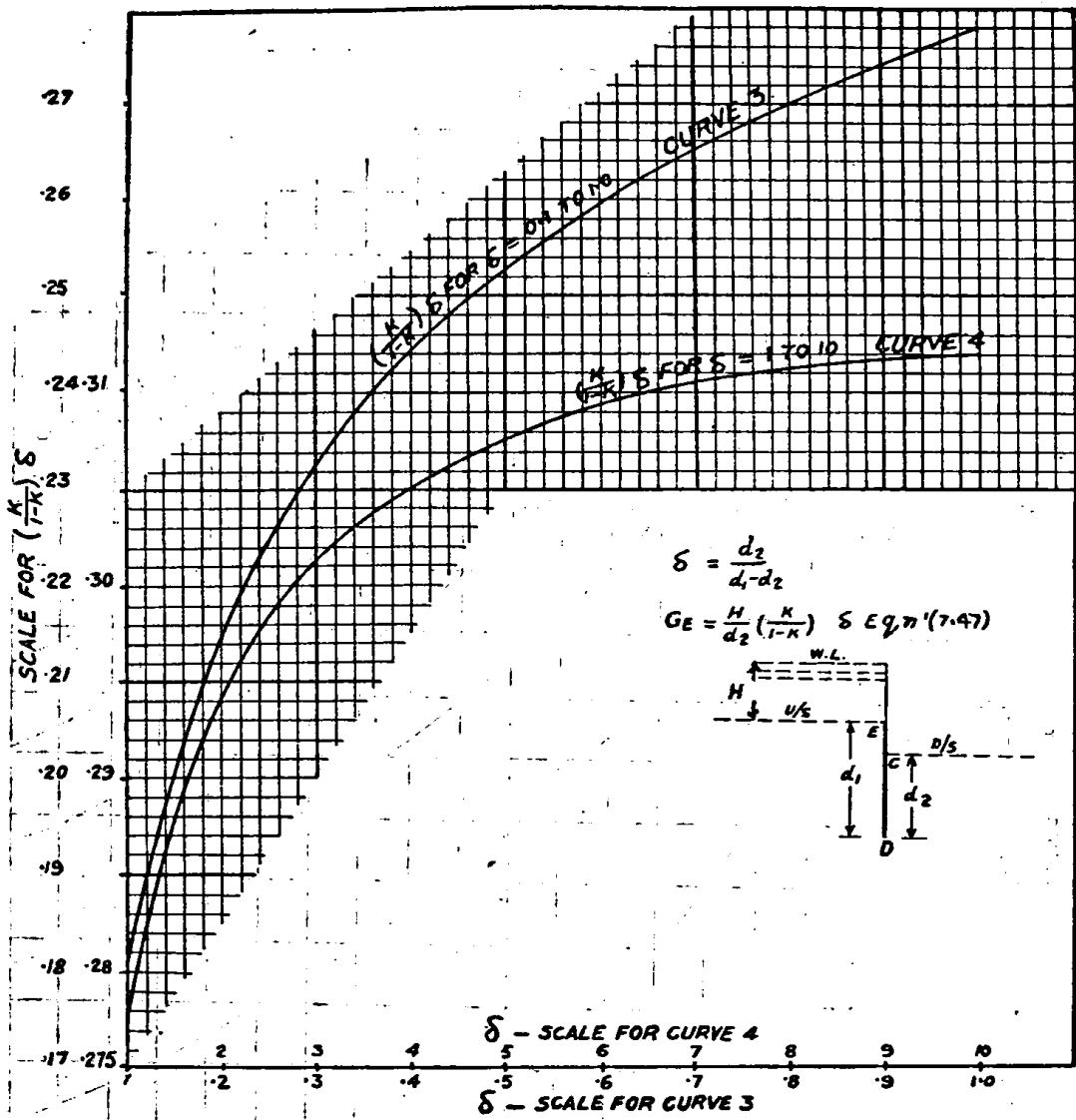
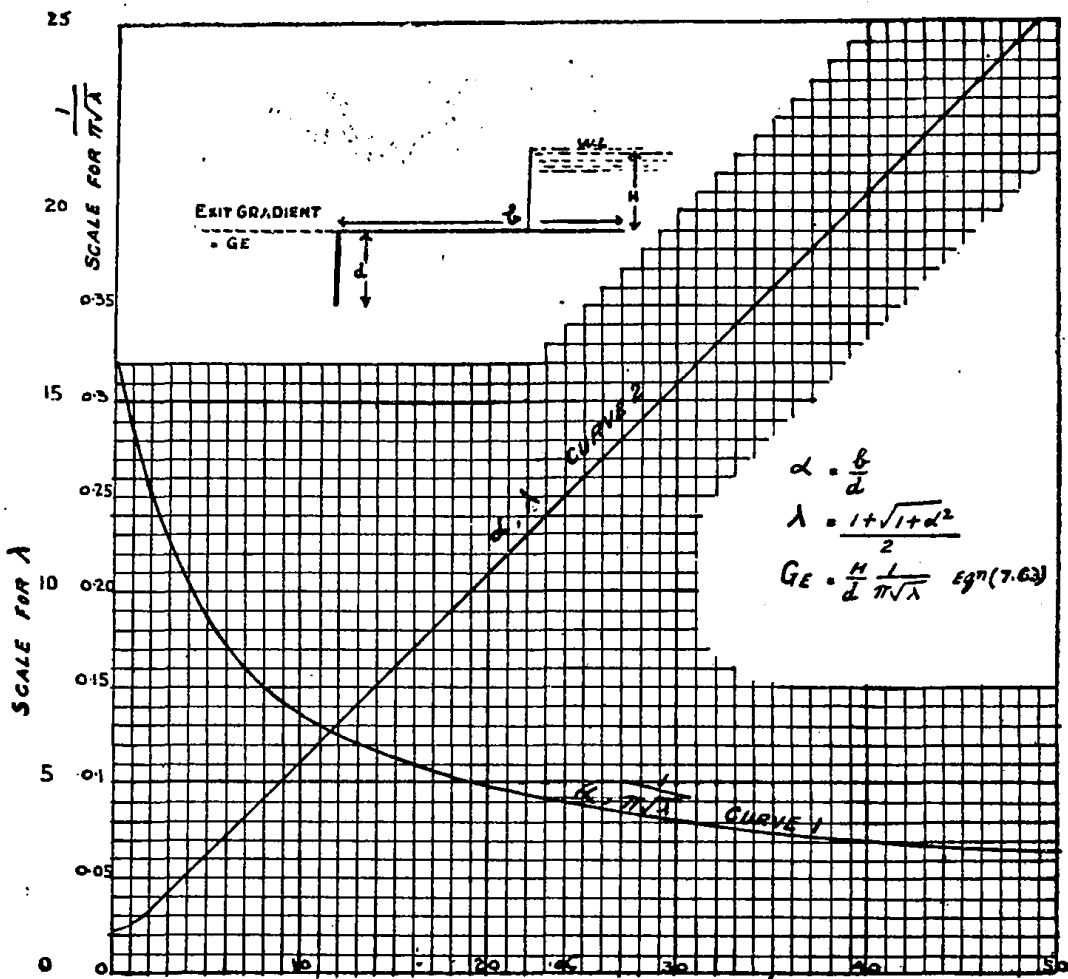


PLATE VII/5

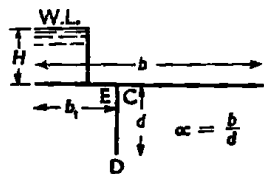


Sheet pile not at end

$$\phi_E = \frac{1}{\pi} \cos^{-1} \left(\frac{\lambda - 1}{\lambda} \right) \dots \text{Eqn. 7-36}$$

$$\phi_C = \frac{1}{\pi} \cos^{-1} \left(\frac{\lambda + 1}{\lambda} \right) \dots \text{Eqn. 7-37}$$

$$\phi_D = \frac{1}{\pi} \cos^{-1} \left(\frac{\lambda}{\lambda} \right) \dots \text{Eqn. 7-38}$$

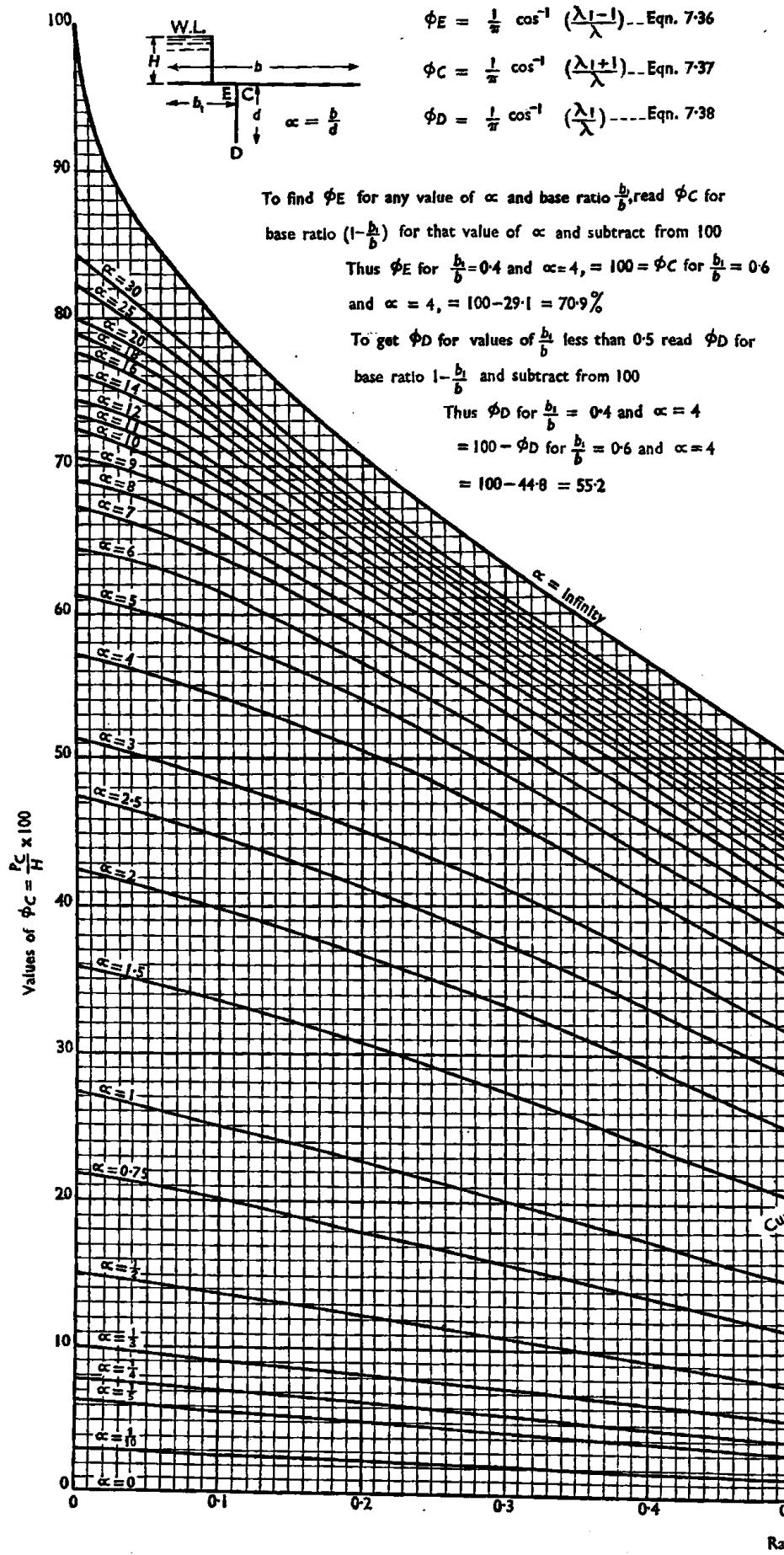


To find ϕ_E for any value of α and base ratio $\frac{b_1}{b}$, read ϕ_C for base ratio $(1 - \frac{b_1}{b})$ for that value of α and subtract from 100

Thus ϕ_E for $\frac{b_1}{b} = 0.4$ and $\alpha = 4$, = 100 - ϕ_C for $\frac{b_1}{b} = 0.6$ and $\alpha = 4$, = 100 - 29.1 = 70.9%

To get ϕ_D for values of $\frac{b_1}{b}$ less than 0.5 read ϕ_D for base ratio $1 - \frac{b_1}{b}$ and subtract from 100

Thus ϕ_D for $\frac{b_1}{b} = 0.4$ and $\alpha = 4$
 = 100 - ϕ_D for $\frac{b_1}{b} = 0.6$ and $\alpha = 4$
 = 100 - 44.8 = 55.2



Values of $\frac{1}{\alpha} = \frac{d}{b}$

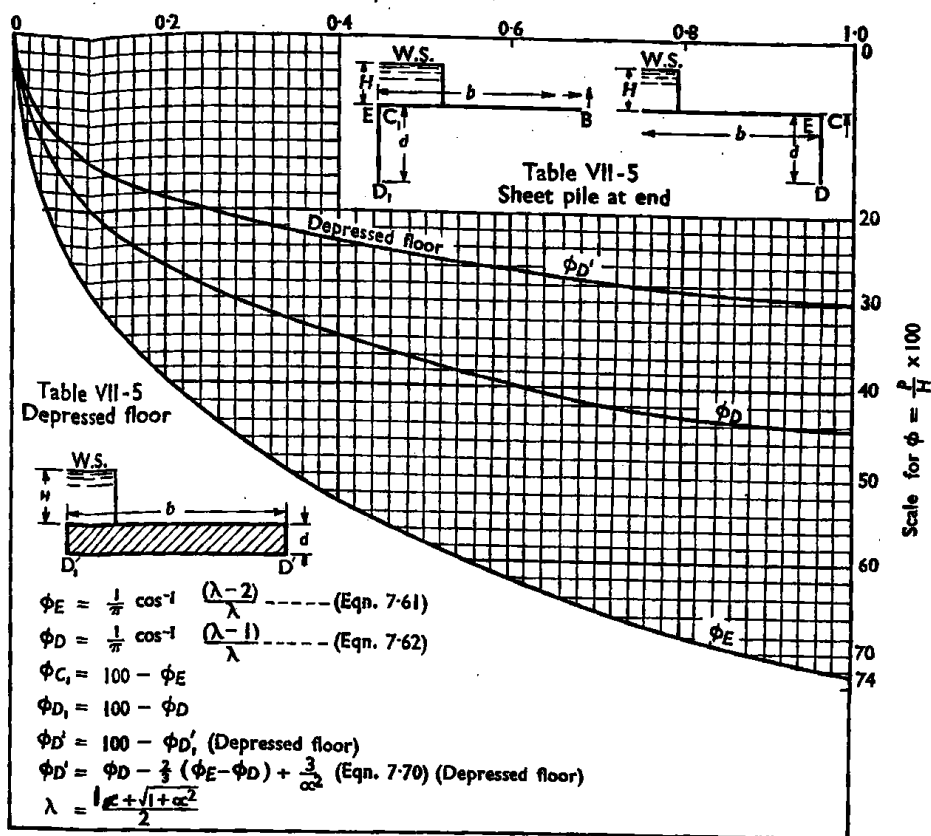


Table VII-5 Depressed floor

$$\phi_E = \frac{1}{\pi} \cos^{-1} \left(\frac{\lambda - 2}{\lambda} \right) \dots \text{(Eqn. 7-61)}$$

$$\phi_D = \frac{1}{\pi} \cos^{-1} \left(\frac{\lambda - 1}{\lambda} \right) \dots \text{(Eqn. 7-62)}$$

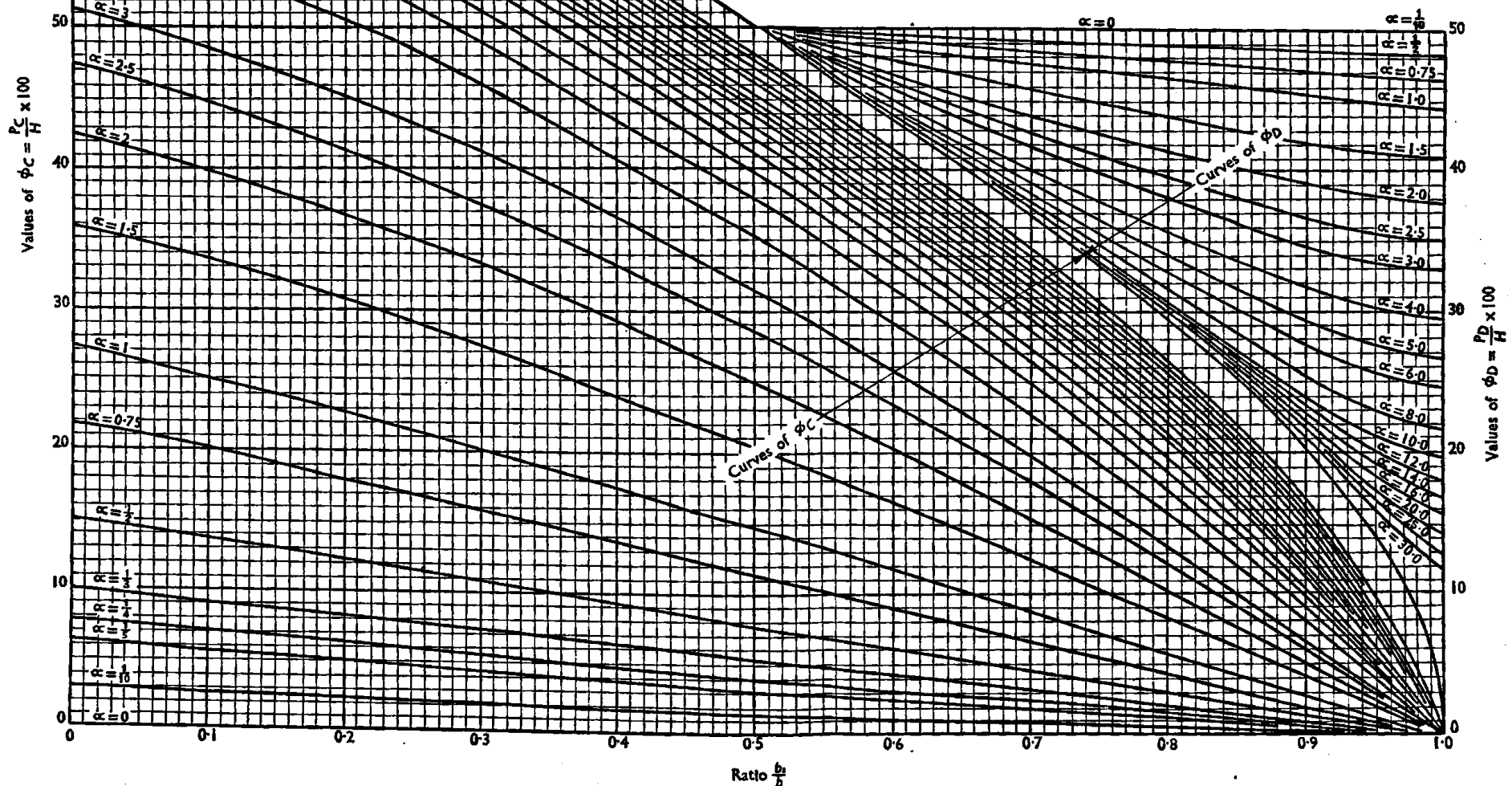
$$\phi_{C_1} = 100 - \phi_E$$

$$\phi_{D_1} = 100 - \phi_D$$

$$\phi_{D'} = 100 - \phi_{D_1} \text{ (Depressed floor)}$$

$$\phi_{D'} = \phi_D - \frac{2}{3} (\phi_E - \phi_D) + \frac{3}{\alpha^2} \text{ (Eqn. 7-70) (Depressed floor)}$$

$$\lambda = \frac{1}{2} (\alpha^2 + \sqrt{1 + \alpha^2})$$



APPENDIX TO CHAPTER VII

*Mathematical Investigations of the sub-soil flow under two standard forms of structures, by Jai Krishan Malhotra, M.A., Officiating Mathematical Officer, Punjab Irrigation Research Institute.

The structures proposed for investigation consist of a simple depressed floor without aprons and a pair of equal sheet piles placed at the heel and toe of a flush floor. They will be obtained as particular cases of a more composite structure which consists of two equal sheet piles placed anywhere under a flush floor, with the floor in between the piles depressed uniformly.

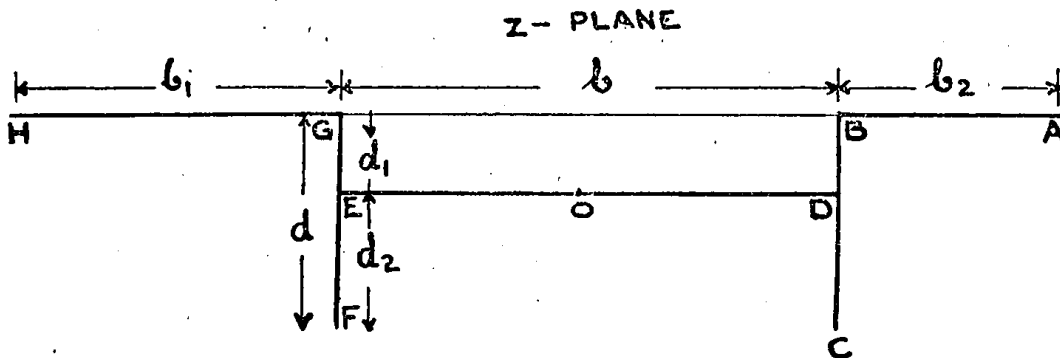


Figure 7A.1

Let $ABCDEFGH$ be the profile of a structure consisting of upstream and downstream aprons HG and BA , two equal vertical sheet piles GF and BC , and the depressed horizontal floor ED . Let the length of each sheet pile be $d_1 + d_2$ and let the depression equal d_1 , the length of the portion depressed being b . Let b_1 and b_2 be the lengths of the aprons.

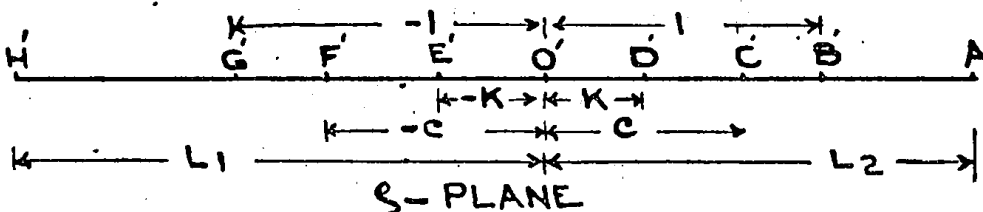


Figure 7A.2

Let us carry the profile over to ζ -plane so that the portion $BCDEFG$, which is symmetrical on the z -plane is carried over into the ζ -axis $B'C'D'E'F'G'$ with the points disposed symmetrically about the origin, O' . The latter will obviously correspond to the middle point O of ED , the origin chosen for the z -plane.

If the values of ζ are ± 1 for B and G , $\pm c$ for C and F , and $\pm k$ for D and E , the necessary transformation takes the form :-

$$Z = A \int \frac{\zeta^2 - c^2}{\sqrt{(\zeta^2 - k^2)(\zeta^2 - 1)}} \cdot d\zeta + B \quad \dots \quad (7A.1)$$

Substitute

$$\zeta = k \operatorname{Sn}(u, k) \text{ and let } 1 - c^2 = c'^2.*$$

Then

$$Z = A \int (dn^2 u - c'^2) du + B = A [E(u) - c'^2 u] + B \quad \dots \quad (7A.2)$$

* The theory given here is a summary of two forthcoming publications of the Irrigation Research Institute, Vol. II, No. 20 and Vol. II, No. 22, viz., "Notes on Khosla's Principle of Independent Variables, I—Mutual Interference of equal piles at ends of a floor" and "Notes on Khosla's Principle of Independent Variables, II—Pressure distribution under a floor with equal end piles and a varying intermediate pile."

The corresponding values of z , ζ , and u are given in the following table :—

Point ..	A	B	C	D	E	F	G	H
Value of z ..	$b_2 + \frac{b}{2} - id$	$\frac{b}{2} - id_1$	$\frac{b}{2} + id_2$	$\frac{b}{2}$	$-\frac{b}{2}$	$-\frac{b}{2} + id_2$	$-\frac{b}{2} - id_1$	$-b_1 - \frac{b}{2} - id_1$
.. .. ζ ..	L_2	1	c	k	$-k$	$-c$	-1	$-L_1$
.. .. u ..	$\rho_2 + iK'$	$K + iK'$	$K + \mu i$	K	$-K$	$\mu i - K$	$iK' - K$	$-(\rho_1 + iK')$

Substituting the corresponding pair of values of Z and U in (7A.2) we get, taking complementary points

$$b/2 = A(E - c'^2K) + B \quad \dots \quad \text{point } D$$

$$-\frac{b}{2} = A(c'^2K - E) + B \quad \dots \quad \text{point } E$$

Hence we have $B = 0, A = \frac{b}{2(E - c'^2K)} \quad \dots \quad (7A.3)$

Next $\frac{b}{2} - id_1 = A[E(K + iK') - c'^2(K + iK')] \dots \text{point } B$
 $= A[E + i(K' - E) - c'^2(K + iK')]$,

Equating imaginaries, $d_1 = A(E' - c'^2K)$ or $\frac{d_1}{b} = \frac{E' - c'^2K'}{2(E - c'^2K)} \quad \dots \quad (7A.4)$

Also $b/2 + id_2 = A[E(K + \mu i) - c'^2(K + \mu i)] \dots \text{point } C$
 $= A[E + E(\mu i) - k^2 \operatorname{Sn}(K + \mu i) \operatorname{Sn}(\mu i) - c'^2(K + \mu i)]$

Or $\frac{id_2}{A} = E(\mu i) - k^2 \operatorname{Sn}(K + \mu i) \operatorname{Sn}(\mu i) - ic'^2\mu$.

But $E(\mu i) = i\mu + \frac{\operatorname{Sn}(\mu i)}{\operatorname{Sn}(K - \mu i)} - iE(\mu, k')$.

$$\therefore \frac{id_2}{A} = ic^2\mu - iE(\mu, k') - \frac{\operatorname{Sn}(\mu i) \operatorname{dn}^2(K + \mu i)}{\operatorname{Sn}(K + \mu i)}$$

Again $\operatorname{Sn}(K + \mu i) = c/k, \therefore \operatorname{cn}(K + \mu i) = \frac{\sqrt{k^2 - c^2}}{k}$

and $\operatorname{dn}(K + \mu i) = \sqrt{1 - c^2} = c'$

And $\operatorname{Sn}(\mu i) = \operatorname{Sn}(K + \mu i - K) = \frac{\operatorname{cn}(K + \mu i)}{\operatorname{dn}(K + \mu i)} - \frac{i\sqrt{c^2 - k^2}}{c'k}$.

Or $\frac{d_2}{A} = c^2\mu - E(\mu, k') + \frac{\sqrt{c^2 - k^2} \cdot c'^2}{c'k} \cdot \frac{k}{c}$
 $= \mu c^2 - E(\mu, k') + \frac{\sqrt{c^2 - k^2} \cdot c'}{c}$

Or $\frac{d_2}{b} = \frac{\mu c^2 - E(\mu, k') + \sqrt{1 - k^2/c^2} \cdot c'}{2(E - c'^2K)} \dots \dots \dots (7A.5)$

For A , we get $b_2 + b/2 - id = A[E(\rho_2 + iK') - c'^2(iK + \rho_2)]$

or $b_2 + b/2 = A \left[E(\rho_2) + \frac{\operatorname{Sn} \rho_2 \operatorname{cn} \rho_2}{\operatorname{dn} \rho_2} - c'^2 \rho_2 \right]$

$$\therefore 2b_2/b + 1 = \frac{E(\rho_2) + \frac{\operatorname{Sn} \rho_2 \operatorname{cn} \rho_2}{\operatorname{dn} \rho_2} - c'^2 \rho_2}{E - c'^2K} \quad \dots \quad (7A.6)$$

Similarly $\frac{2b_1}{b} + 1 = \frac{E(\rho_1) - \frac{\operatorname{Sn} \rho_1 \operatorname{cn} \rho_1}{\operatorname{dn} \rho_1} - c'^2 \rho_1}{E - c'^2K} \quad \dots \quad (7A.6a)$

Where $kSn(\rho_2 + iK') = L_2$ or $Sn\rho_2 = \frac{1}{L_2}$, $Sn\rho_1 = \frac{1}{L_1}$.*

The last four equations determine ρ_1 , ρ_2 , c , and k from known values of the four ratios b_1/b , b_2/b , d_1/b , and d_2/b .

Again $\zeta = \lambda \cosh \omega - \lambda_1 \dots \dots \dots (7.23)$

and for the stream line along the face of the pile, $u=0$.

$$\therefore v = \cos^{-1} \frac{\zeta + \lambda_1}{\lambda}, \text{ where } \lambda = \frac{L_1 + L_2}{2}, \lambda_1 = \frac{L_1 - L_2}{2}$$

$$\therefore P_{B,G} = \frac{H}{\pi} \cos^{-1} \frac{\lambda_1 \pm 1}{\lambda}; P_{C,F} = \frac{H}{\pi} \cos^{-1} \frac{\lambda_1 \pm c}{\lambda}; P_{D,E} = \frac{H}{\pi} \cos^{-1} \frac{\lambda_1 \pm k}{\lambda}.$$

If we make the aprons vanish, $L_1 = L_2 = 1$, $\lambda = 1$, $\lambda_1 = 0$,

and $P_{C,F} = \frac{H}{\pi} \cos^{-1}(\pm c)$, $P_{D,E} = \frac{H}{\pi} \cos^{-1}(\pm k) \dots \dots (7A.7)$

Case I. Depressed floor without aprons.

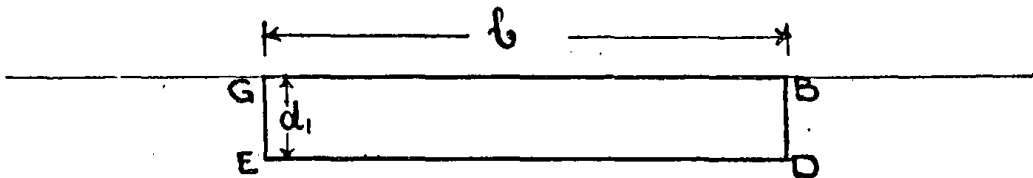


Figure 7A.3

Here d_2 vanishes, leaving a depressed floor of depth d_1 and breadth b ; obviously c equals k . Equation (7A.4) now gives

$$\frac{2d_1}{b} = \frac{E' - k^2K'}{E - k^2K} \dots \dots \dots (7A.8)$$

while from (7A.2) and (7A.3) we get

$$Z = \frac{b}{2} \frac{E(u) - k'^2 u}{E - k'^2 K}, \text{ where } u = Sn^{-1}\left(\frac{\zeta}{k}\right) \dots \dots \dots (7A.9)$$

The pressure at any point under the floor is given by

$$\frac{H}{\pi} \cos^{-1}(\zeta) = \frac{H}{\pi} \cos^{-1}(kSn u) \dots \dots \dots (7A.10)$$

where the distance x of the point from the centre is

$$x = \frac{b}{2} \frac{E(u) - k'^2 u}{E - k'^2 K} \dots \dots \dots (7A.10a)$$

In particular we have for the point D , the pressure as

$$P_D = \frac{H}{\pi} \cos^{-1}(k) \dots \dots \dots (7A.11)$$

Putting $\frac{\pi P_D}{H} = \nu_D = \theta$,

we have $k = \cos \theta$, and (7A.8) then gives

$$\frac{2}{\alpha} = \frac{E\left(\frac{\pi}{2} - \theta\right) - \cos^2 \theta \cdot K\left(\frac{\pi}{2} - \theta\right)}{E(\theta) - \sin^2 \theta \cdot K}, \text{ where } \alpha = b/d \dots \dots \dots (7A.12)$$

If $2/c = \alpha$, and we have another floor for which $2/\alpha_1 = 1/c$ and $k = \cos \theta_1$, then from (7A.12)

$$\frac{E\left(\frac{\pi}{2} - \theta\right) - \cos^2 \theta \cdot K\left(\frac{\pi}{2} - \theta\right)}{E(\theta) - \sin^2 \theta \cdot K} = \frac{E(\theta_1) - K \cdot \sin^2 \theta_1}{E\left(\frac{\pi}{2} - \theta_1\right) - \cos^2 \theta_1 \cdot K\left(\frac{\pi}{2} - \theta_1\right)}$$

which gives $\frac{\pi}{2} - \theta = \theta_1$ or $\theta + \theta_1 = \frac{\pi}{2}$.

*For definitions of the functions Sn , cn , dn and E , K , E' , K' , and for explanations of the other steps, reference may be made to the relevant Chapter in Whittaker and Watson's Modern Analysis or to any other book on elliptic functions.

If
$$\phi_\alpha = \frac{P_D}{H} \times 100,$$

then as
$$\frac{P_D}{H} = \frac{\theta}{\pi}, \quad \theta = \frac{\pi}{100} \cdot \phi_\alpha, \text{ also } \alpha\alpha_1 = 4.$$

This gives
$$\phi_\alpha + \phi_{\alpha_1} = \phi_\alpha + \phi_{\alpha/\alpha} = 50 \quad \dots \dots \dots (7A.13)$$

In particular, if $\alpha=2$, i. e. if the floor is twice as long as it is deep $\phi_\alpha = \phi_{\alpha/\alpha} = 25 \dots (7A.14)$

In this case only a quarter of the head remains at D .

Equation (7A.13) can be utilized in finding the value of ϕ_α for values of α less than 2, if the value of ϕ_α for values of α above 2 are known.

Thus $\phi_1 + \phi_{\frac{1}{4}} = 50, \phi_{\frac{1}{2}} + \phi_8 = 50$; and so on.

The exit gradient at the toe is given by

$$\begin{aligned} G_{\text{EXIT}} &= \frac{H}{\pi} \cdot \frac{dv}{d\zeta} \cdot \frac{d\zeta}{dz} \quad \left(\text{as } \frac{dz}{ds} \text{ equals unity at exit} \right), \\ &= \frac{H}{\pi A} \cdot \left[\frac{1}{\sqrt{\zeta^2 - 1}} \cdot \frac{\sqrt{\zeta^2 - 1}}{\sqrt{\zeta^2 - k^2}} \right]_{\zeta=1} \\ &= \frac{H}{\pi A k'} = \frac{H}{\pi} \cdot \frac{2(E - k'^2 K)}{b k'} \\ &= \frac{H}{\pi d} \cdot \frac{E' - k'^2 K'}{k'} \quad \dots \dots \dots (7A.15) \end{aligned}$$

The graph showing the theoretical values of ϕ_D has already been incorporated by Mr. Khosla in Plate No. VII. 6. The agreement with experimental values has also been noticed by him in Chapter IV. (See Table IV-3 on page 26.)

Case II. Equal sheet piles at heel and toe without aprons.

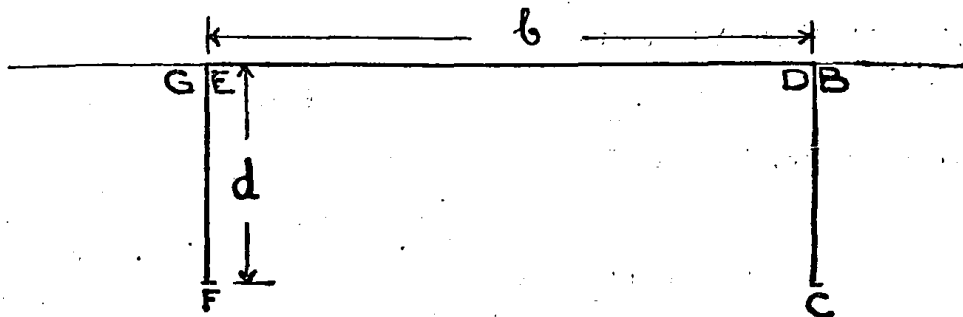


Figure 7A.4

Here d_1 vanishes and Equation (7A.4) gives
$$c^2 = E'/K' \quad \dots \dots \dots (7A.16)$$

Hence
$$E - c'^2 K = K[E/K + E'/K' - 1] = \frac{\pi}{2K'} \quad \dots \dots \dots (7A.16a)$$

Substituting in (7A.5) we get

$$\begin{aligned} \frac{d_2}{b} &= \frac{\mu E'/K' - E(\mu, k') + \sqrt{1 - \frac{k'^2 K^2}{E}} \cdot \sqrt{1 - \frac{E'}{K'}} \cdot K'}{\pi} \\ &= \frac{K'}{\pi} \cdot \left[\frac{(E' - k'^2 K')(K' - E')}{E' K'} - Z_n(\mu, k') \right] \quad \dots \dots \dots (7A.17) \end{aligned}$$

Where Z_n is Jacobi's Zeta-function, and μ is given by

$$dn(\mu, k') = \frac{k}{c} = k \cdot \frac{1}{\sqrt{1 - \frac{K'}{E'}}}$$

The potential function at any point is given by $\cos^{-1}(kSnu)$, where u is given by

$$\frac{2z}{b} = \frac{E(u) - c'^2u}{E - c'^2K}$$

$$= \frac{Zn(u) - \frac{\pi u}{2KK'}}{K \cdot \frac{\pi}{2KK'}}$$

$$\therefore z = \frac{bK'}{\pi} \cdot Zn(u) - \frac{bu}{2K} \dots \dots \dots (7A.19)$$

In particular the percentage pressures at C and D are given by $\frac{100}{\pi} \cos^{-1}(c)$ and $\frac{100}{\pi} \cos^{-1}(k)$.

Choosing any k , we can get c and μ from (7A.16) and (7A.18), and then derive the value of d_2/b from (7A.17). Plotting the values of $\cos^{-1}(k)$ and $\cos^{-1}(c)$ against those of d_2/b , we get the curves shown in Plate VII.A.1 which show the values of ϕ_C and ϕ_D (called ϕ_D and ϕ_B in Plate VII.A.1) for various values of $\alpha = \frac{b}{a}$.

The values obtained from the theoretical curves for certain values of α are compared in the following table with those obtained from electrical experiments and from Khosla's Method of Independent Variables¹ :-

TABLE VII.A.1

α	ϕ_F		ϕ_B			ϕ_D			ϕ_C	
	Theory	(2) Expt.	(1) Khosla	Theory	(2) Expt.	(1) Khosla	Theory	(2) Expt.	Theory	(2) Expt.
3	71.6	71.7	58.5	58.6	58.2	41.5	41.4	42.1	28.4	29.2
4	73.9	73.1	61.9	62.1	61.3	38.1	37.9	40.0	26.1	26.2
6	77.3	76.6	67.0	67.1	66.3	33.0	32.9	34.1	22.7	24.0
8	79.7	78.7	70.6	70.8	69.7	29.4	29.2	30.7	20.3	21.7
12	82.8	81.7	75.3	75.4	74.0	24.7	24.6	26.4	17.2	18.9
24	87.4	85.7	82.1	82.2	79.7	17.9	17.8	20.3	12.6	15.2

(1) derived from Equation 10.1 (Chapter X, page 115).
 (2) Table Case VIII(a) 1 (Chapter IX, page 107).

The comparatively large differences at higher values of α with the experimental values obtained from the electrical method appear to be due to a very slight error in measurement of length of the small sheet pile used. For $\alpha=24$, the sheet pile was $\frac{1}{2}$ " long and an increase of 1/10th of its length would make the theory agree with experiment.

It has been noticed that when $\alpha > 2$ the values of ϕ_D and ϕ_C are very nearly in the ratio of $\sqrt{2} : 1$ or

$$\phi_C = 0.7\phi_D \dots \dots \dots (7A.20)$$

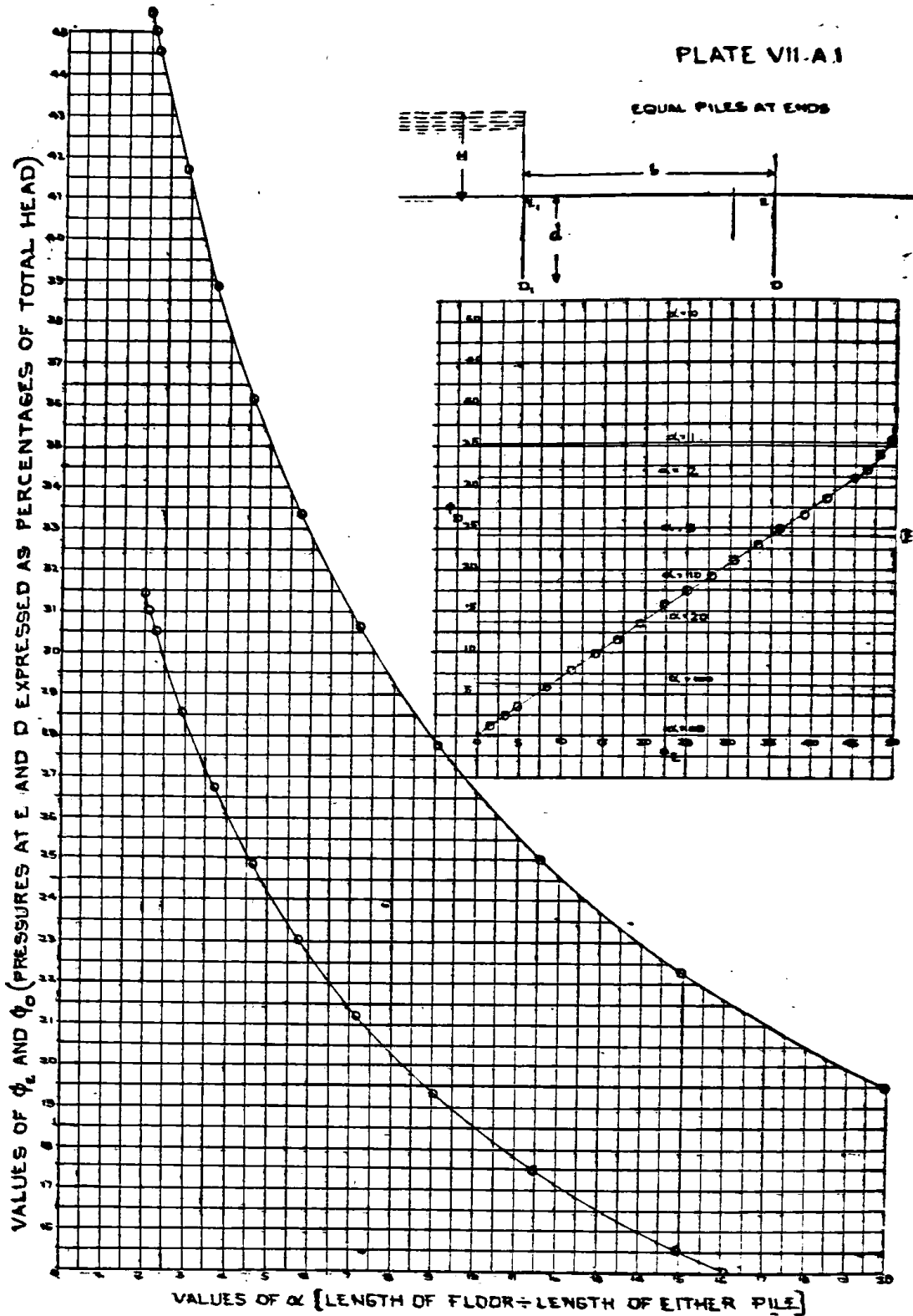
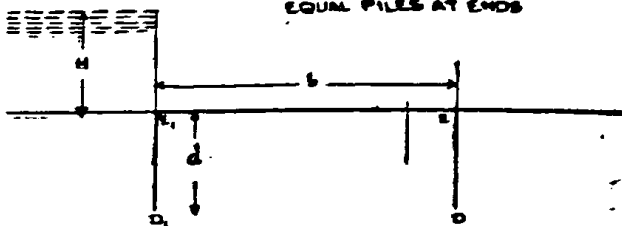
This means that the side of the pile next the floor takes only about 30 per cent. of the total pressure cut off by the pile.

Mr. Khosla's Method of Independent Variables¹ receives a very striking confirmation of its rationality from the results of this particular case in that his method gives results differing from the theoretical by 0.2 per cent. or less.

¹ See Page 120

PLATE VII-A.1

EQUAL PILES AT ENDS



CHAPTER VIII

Exit Gradients as Related to Weir Design

Weir failure from seepage flow can occur by :—

- (a) Undermining of the sub-soil.
- (b) Uplift due to pressure under the floor being in excess of the weight on the floor.

The failure by undermining is the most common, so that a knowledge of its causes and of the measures to prevent it, is of the utmost importance both for design of new works and for ascertaining the safety of existing ones.

CAVITY UNDER KHANKI WEIR (PIER 5)



This cavity took 900 c.ft. of cement and sand to fill in.

Note the 19 feet length of bamboo inside the cavity.

The undermining of the sub-soil starts from the tail end of the work. It begins at the surface due to the residual force of seepage water at this end being in excess of the restraining forces of the sub-soil which tend to hold the latter in position. Once the surface is disturbed, the dislocation of sub-soil particles works further down and, if progressive, leads to the formation of cavities* below the floor into which the latter may collapse. According to the commonly accepted ideas this undermining is supposed to result from what is known as "piping", that is, the erosion of sub-soil by the high velocities of flow of water through it, when such velocities exceed a certain limit. But as will be shortly explained this conception of undermining is incomplete. Water has a certain residual force at each point along its flow through the sub-soil which acts in the direction of flow and is proportional to the pressure gradient at that point. At the tail end this force is obviously upwards and will tend to lift up the soil particles if it is more than the submerged weight of the latter. The frictional resistance, cohesion, *etc.*, of the adjacent soil will have to be considered in certain cases. Once the surface particles are disturbed, the resistance against upward pressure of water will be further reduced, tending to progressive disruption of the sub-soil. The flow gathers into a series of pipes in the latter and dislocation of particles is accelerated. The sub-soil is thus progressively undermined. Soil erosion can also occur through natural pipes or faults in the sub-soil.

This conception of undermining by "Flotation" was put forward by Professor Charles Terzaghi¹ in 1925 and independently arrived at by F. F. Haigh² in 1930.

As water flows through the sandy sub-soil, it is well known that the velocity of flow at any point in the medium is given by Darcy's Law :—

$$v = K \frac{h}{l}$$

*Cavity under Pier 5, Khanki Weir. Photograph reproduced from Punjab Engineering Congress paper No. 195 of 1936, 'Reconstruction of the Khanki Weir' by Rai Bahadur A. N. Khosla.

¹ CH. TERZAGHI Erdbau Mechanik.

² F. F. HAIGH Paper No. 138 of Punjab Engineering Congress, 1930, by A. N. Khosla—
Discussion by

This water exerts a force (F) on the sandy medium along its line of flow. Besides this force, the particles of the medium are subject to two other forces :—

- (1) the force of gravity, or weight (W) which acts downwards,
- and (2) the force of buoyancy (B) which acts opposite to gravity.

These two latter forces can be combined into one and if

w = Weight of unit volume of water,

ρ = Specific gravity of sand particles,

e = Pore space in unit volume,

then the weight of soil particles per unit volume = $w(1 - e) \rho = W$

The weight of displaced water = $w(1 - e) = B$

Hence the resultant of the forces (1) and (2), that is the weight of the soil particles less buoyancy

$$= w(1 - e) (\rho - 1) \quad (8.1)$$

This may be called the submerged weight W_s , which will obviously always act downwards as $\rho > 1$

The next step is to determine the value of F,

Let us assume a cylinder of soil along one of the stream lines.

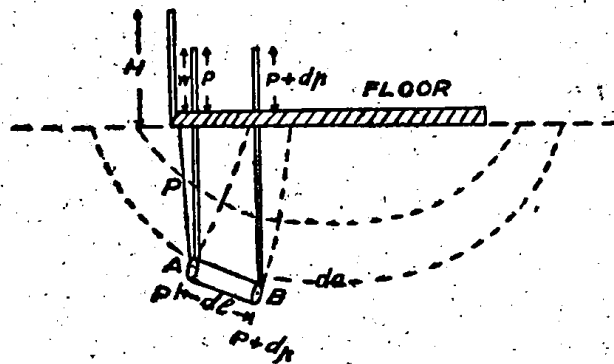


Figure 8.1

Let da = sectional area of cylinder,
and dl = length of cylinder.

The Pressure at face A = $p da$ and acts along the stream line from left to right.

The Pressure at face B = $(p + dp) da$ and acts along the stream line from right to left.

Neglecting the curvature of the stream line we get the force acting on the cylinder along the stream line as

$$p da - (p + dp) da = - dp da$$

The force per unit volume of the cylinder

$$= - \frac{dp da}{dl da} = - \frac{dp}{dl} \quad (8.2)$$

Hence $F = - \frac{dp}{dl}$ = Pressure gradient at that point. So that, the action of water on any point in the sandy medium through which it percolates, can be given by a force F, which, per unit volume, is equal to the gradient of pressure at that point and which acts in the direction of flow.

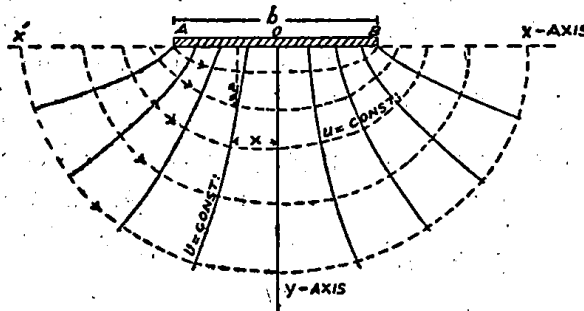


Figure 8.2

Thus the force acting on any particle in the sub-soil consists of :—

- (a) a force $F = - \frac{dp}{dl}$ acting in the direction of flow,

and (b) the submerged weight $W_s = w(1 - e) (\rho - 1)$ acting downwards.

For stability of the particles, the resultant R of F and W_s must have no upward component.

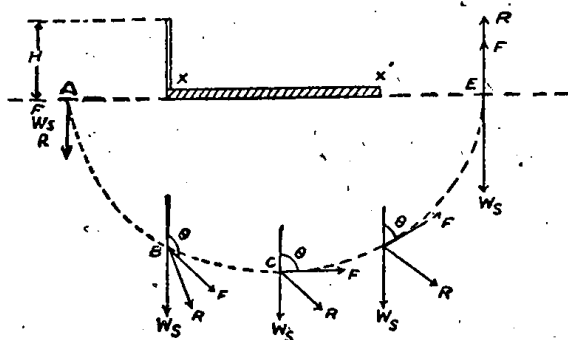


Figure 8.3

Now the vertical component of R is given by

$$R_v = F \cos\theta - W_s$$

From Figure 8.3 R_v in the region A to C must necessarily be downwards as F has no upward vertical component. From C onwards, F has an upward vertical component, which goes on increasing rapidly as the stream line approaches E the exit end. At E, F is vertically upwards; at X' it is infinite, beyond which point it rapidly falls off as shown in Figure 7.10 (Chapter VII). Thus, for some length beyond X' at the surface, where F acts vertically upwards, and W_s vertically downwards, the resultant $R_v = F - W_s$ will be vertically upwards, so that, the soil particles in this region will be lifted up and be in a state of unstable equilibrium. This region of unstable equilibrium will extend to the limit where $R=0$ or $F=W_s$. At this point the soil particles will be just stable and F acting upwards will have its critical value which will be just resisted by the submerged weight of the particle acting downwards. The slightest increase in this value of F will lead to instability. The soil particles will start to be lifted up or "float" so to speak. The gradient of pressure at which this occurs has been aptly called by Terzaghi the "flotation gradient." Haigh calls it the bursting gradient. One of the authors¹ called it the critical gradient. This is equal to the critical value of the force F_c and is given by,

$$\begin{aligned} \text{Flotation Gradient} &= F_{\text{critical}} = W_s \\ &= w(1 - \epsilon)(\rho - 1) \end{aligned}$$

If the pore space = 40 per cent, and $\rho = 2.65$ and $w = 1$

$$\begin{aligned} G_f &= (1 - 0.4)(2.65 - 1) \\ &= 0.6 \times 1.65 = 0.99, \text{ say unity.} \end{aligned}$$

Thus for the class of soil mentioned above, the Flotation Gradient, $G_f = \text{unity or } 1/1$. This flotation gradient will vary with the pore space of the sub-soil and the density of the soil particles. The density of the soil particles, though generally in the neighbourhood of 2.6 to 2.7 for the Punjab sands, may vary in extreme cases from 1.8 to 2.8. Similarly the pore space which for Punjab sands ranges between 37 and 42 per cent. may be as low as 20 per cent. and as high as 45 per cent. Working on these limits the extreme values of G_f may range from 0.44 to 1.44.

Table for G_f .

Pore Space	Densities					
	2.8	2.6	2.4	2.2	2.0	1.8
0.20	1.44	1.28	1.12	0.96	0.80	0.64
0.25	1.35	1.20	1.05	0.90	0.75	0.60
0.30	1.26	1.12	0.98	0.84	0.70	0.56
0.35	1.17	1.04	0.91	0.78	0.65	0.52
0.40	1.08	0.96	0.84	0.72	0.60	0.48
0.45	0.99	0.88	0.77	0.66	0.55	0.44

(1) A. N. KHOSLA . . . Paper No. 138, Punjab Engineering Congress, 1930.

Terzaghi in his experiments used sands of 2.65 specific gravity and found the following values for G_f

Pore Space. = e	G_f observed.	G_f Computed $1.65 \times (1 - e)$
0.43	0.95	0.94
0.355	1.11	1.06
0.430	1.13	0.94
	to	
	1.16	
0.410	0.96	0.97
0.440	1.05	0.92
0.350	1.08	1.07

Dr. Vaidhianathan¹ carried out some very careful experiments in 1935, with Punjab sands. The technique and results of these are of great value.

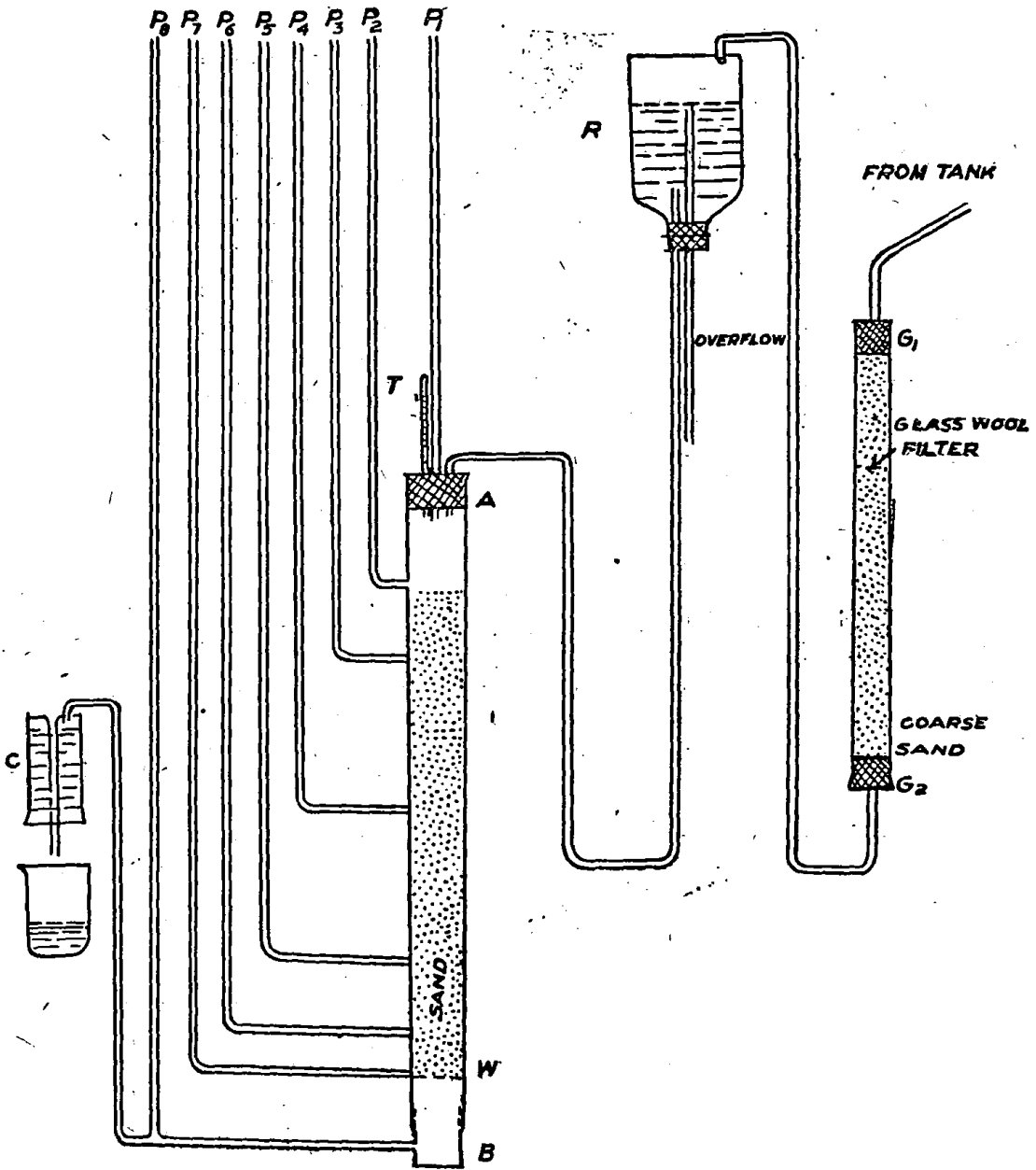


FIGURE 8.4

(1) V. I. VAIDHIANATHAN . . . Memoir of the Punjab Irrigation Research Institute, Vol.V, No. 5, 1935.

Figure 8.4 shows the apparatus used by him. The description of the experiments is best given in his own words:—

“The method of carrying out the experiment will be clear from the following description and diagrams. The experimental tube A B (Figure 8.4) was 85 cms. long with an internal diameter of 4.2 cms. The length of the sand column was 63.0 cms. The positions of the manometers P_1 , P_2 , etc., are as shown in the figure. The manometers were fitted with wire-gauze to prevent the sand passing into them. R is a constant level reservoir which can be adjusted to any required height. Water which has been filtered is supplied to the reservoir R from a special tank.

W is a wire-gauze on which the sand column rests. The meshes of the gauze are too small to allow the sand particles to pass through. The water that flows out is collected through the over-flow at C. At the end A, the tube is not completely filled with sand. T is a thermometer to record the temperature of the water.

The sand to be examined is first washed and particles below 0.04 mm., are removed by sedimentation. The cohesive forces due to fine particles are thus eliminated. The size distribution, pore space and density are then determined. The sand is packed uniformly under water in the experimental tube AB by tapping and stirring.

It is of the greatest importance that *the water used should be free from fine suspended matter* and air bubbles. When the experiments were begun, untreated tap water was being used for the experiments. The manometer tubes P_2 , P_3 , etc., showed a progressive fall with the lapse of time while the head remained constant. The sources of error were then investigated. It was found that, when a very small quantity of fine clay was introduced into the reservoir, R, a quick drop of pressure occurred in the manometer tubes. It was then thought that traces of fine particles in the tap water might be causing trouble by forming a filter skin and choking the passage. Tap water also contains small air bubbles. These small bubbles used to get lodged in the sand in the tube and thus choke the passages for the water. When the tap water was allowed to stand in a special reservoir for two days before the experiment and afterwards carefully filtered through glass wool and coarse sand as shown in G_1 , G_2 , the pressures became steady with time.

This point has been specially stressed here because of its importance to model work carried out in laboratories. When the pressures are observed by tubes inserted in sand under models as is usually done, the results are likely to be vitiated if due attention is not paid to this source of error.

EXPERIMENTS WITH WATER FLOWING DOWNWARDS

The first set of observations was taken with water flowing downwards in the sand column. To begin with, the reservoir R was fixed in a low position and a hydraulic gradient of nearly 0.5 was obtained. When the flow had continued for half an hour, water was collected through the over-flow C for a convenient interval and the weight of the water was determined. This observation was repeated. The reservoir R was then raised to obtain a higher hydraulic gradient and after a flow had been maintained for half an hour the weight of the over-flowing water during a definite interval was again determined. Observations were thus taken for the hydraulic gradients of 0.5, 0.75, 1.0, 1.25 and 1.5, as nearly as possible. The experiments were generally repeated twice, the gradients first increasing and then decreasing.

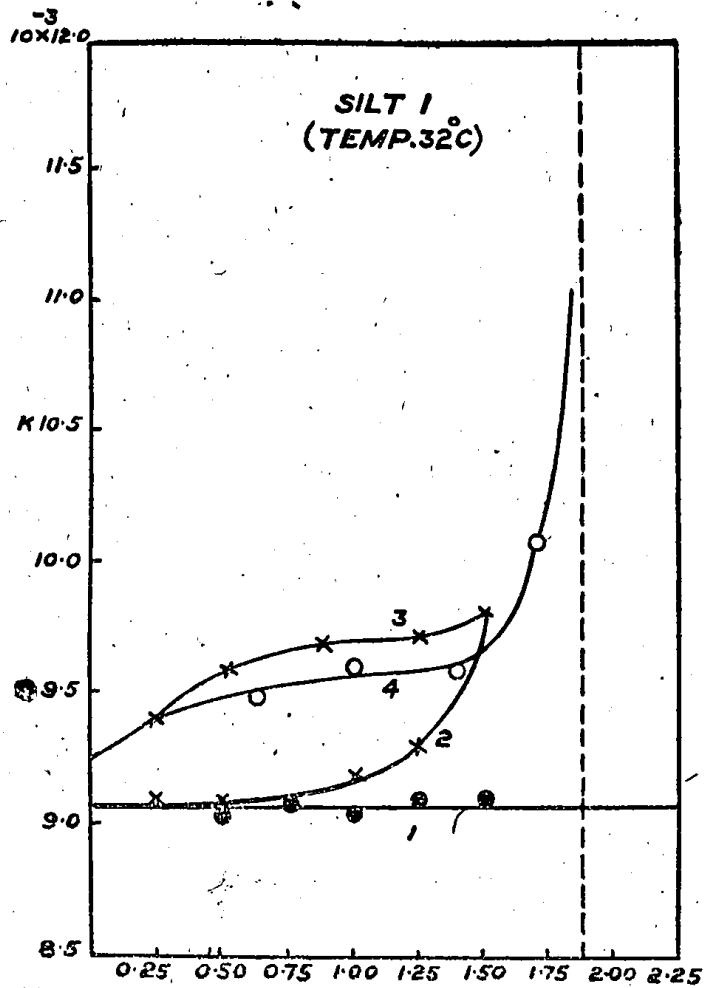


Figure 8.5

HYDRAULIC GRADIENTS (i)

EXPERIMENTS WITH WATER FLOWING UPWARDS

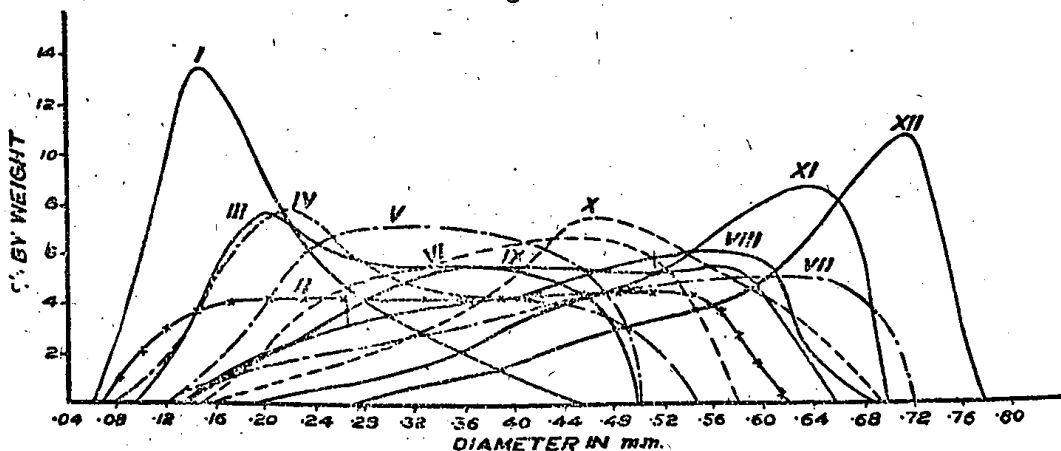
The second and the subsequent sets of observations were taken with water flowing upwards. In these sets of observations water was entering at B (Figure 8.4) and coming out at the end A.

In the second set water was allowed to flow upwards, the pressure gradients being slowly increased beginning from 0.25 to about 1.5. This part of the experiment is shown in Curve 2, Figure 8.5. When a gradient of 1.5 was reached the third set of observations was begun.

In the third set the pressure gradients were decreased, otherwise the observations were taken in the same way as in the second set, Curve 3, Figure 8.5 is thus traced.

In the fourth set the pressure gradients were again gradually increased as in the second set, but this time the increase of the pressure gradient was continued until the sand particles were removed along with the out-flowing water (Curve 4, Figure 8.5). After re-packing the sand, the experiment was repeated.

Figure 8.6



SIZE DISTRIBUTION CURVES

Vaidhianathan took sands of different orders of packing and wide size distributions (Figure 8.6). These results are tabulated below:—

Specimen No. of sand	Specific Gravity of sand	Pore Space	Transmission constant	FLOTATION GRADIENT	
				Calculated	Observed
I	2.68	41.3	0.0167	0.99	1.00
II	2.69	38.5	0.0414	1.04	0.85
III	2.69	40.3	0.0422	1.01	1.00
IV	2.64	39.9	0.0434	0.99	1.00
V	2.65	41.5	0.0778	0.97	0.95
VI	2.68	41.0	0.0854	0.99	0.95
VII	2.67	38.1	0.0945	1.03	1.00
VIII	2.69	38.2	0.0948	1.03	1.25
IX	2.70	38.8	0.1023	1.04	1.25
X	2.69	39.2	0.1521	1.04	1.00
XI	2.68	39.0	0.2219	1.02	1.00
XII	2.69	38.4	0.3002	1.04	0.95

In downward flow, the transmission constant or the discharge remains absolutely steady irrespective of the pressure gradient (Curve No. 1, Figure 8.5). In upward flow, it remains constant to about a gradient of 1 : 1; beyond that it increases rapidly (Curve 2, Figure 8.5). The gradient is then lowered but the transmission constant remains much above its previous value. Compare Curves 2 and 3. When the gradient is again increased, the transmission constant increases first slowly and then rapidly till at a gradient of about 1.7/1 the soil definitely blows off and the soil medium finally breaks up.

These curves have great practical significance. If the sub-soil at the tail end of a work has once been subjected to a gradient more than the critical the whole soil mass gets disturbed as a result of flotation and does not settle back to its normal packing even though the gradient is reduced. Thus once the stage or flotation is crossed, the mischief is done.

While Vaidhianathan's tabular results show close agreement between observed and calculated values of G_f , his curves lead one to believe that the values at which floating actually occurred, might be somewhat lower than the computed values. The wide range of size distribution, however, does not seem to have any material effect.

The tests for flotation gradient have been extended to lead shot by Mochman and Dougherty¹ who found that lead shot was lifted under gradient of 5.88 to 1 against a calculated value of 6.0 to 1 as obtained from the relation $G_f = (1 - \epsilon) (\rho - 1)$ where ϵ and ρ were the pore space and specific gravity respectively of lead shot.

Another experimental confirmation of the final break up of soil is available in Harbans Lal's experiments with a depressed floor. With a floor length of 12' and depression $\frac{1}{2}$ ' the soil particles started definitely working away at a head of 9.33 inches. This gives length of depression = $12/\frac{1}{2} = 24$.

Pressure at the bottom corner of the floor $\frac{1}{2}$ ' below floor level is 9 per cent. of 9.33 or 0.84'. Exit gradient for this = $\frac{0.84 \times 0.84}{0.5} = 1.4$ or 1.4 in 1, (see case VII, p. 40.) This is not greatly different from 1.7/1 obtained by Vaidhianathan.

The True Significance of Exit Gradients

Before proceeding to a consideration of a factor of safety it will be as well to mention here in clear terms that in any ordinary structure, the critical value of 1 : 1 or nearly so, is almost impossible of occurrence if other factors do not intervene. For instance, with a floor

¹ MOCHMAN AND DOUGHERTY. Harza's paper on "Uplift and Seepage under Dams." Paper 1920 Am. Soc., C. E.—Discussion by.

length of 100 ft., and head 20 ft. the exit gradient is $1/3$ for a 2 ft. depression and $2/3$ for a one foot depression of floor with no sheet piles. Or in other words, if the exit gradients were considered purely from the academic point of view, failures of any normal structure should be well nigh impossible.

But we must always keep in mind the various other factors which create conditions suitable for the development of critical values of exit gradients. A scour hole occurs which extends down to the bottom of the toe wall. The exit gradient at that place will necessarily be at or above the critical value and the force due to this in conjunction with the earth pressure will do the damage. The exit gradient at the tail end can be increased many times by the occurrence of local surges or waves which in action resemble the working of an intermittent pump. At one time the pressure goes up say 2 ft. above the mean water level. The next moment it falls two feet below that level. Thus in a short time the tail soil is subjected to a differential head of 4 ft. This will increase the otherwise harmless force of water due to exit gradient to one that will exceed the critical so that soil particles will start moving. The movement of soil of the bed and side slopes and the appearance of springs when intermittent pumping is done in a drain, are phenomena of every day occurrence.

Another factor is the sudden ponding up upstream of the weir by closing off flow downstream. In this case the spring level at the downstream end does not drop as fast as the water level so that a considerable differential head may be set up at the tail end resulting in excessive exit gradients till such time as the time lag lasts.

Thus, scour, wave action, sudden application or reduction of head, and high spring level, will have a very important bearing on the actual exit gradients that may occur at the tail end of any structure.

Factor of Safety

It will thus be clear that it is not enough to design a structure so that the exit gradient at the tail end does not exceed the critical value G_c . Theoretically, such a structure should be safe and could be built at small cost. But as has been pointed out earlier, the slightest increase in this value, from whatever source, will upset the stability of the sub-soil at the exit end. This indicates the need for the application of a generous factor of safety in designs. In the sub-soil there are quite a large number of uncertain factors. The foundation soil may not be quite homogeneous even though definite stratification may be absent. The packing and pore space may differ in places. There may be local intrusion of clay beds which deflect flow or there may be zones of very porous material, which may induce concentration of flow from all round. Fresh deposits in a formerly deep river channel may have entirely different soil formation from that of the sub-soil on either side. There may be faults and fissures in the sub-soil formation. Where continuous clay beds or bands occur, this problem will require special treatment, but apart from that, the uncertainties in soil formation are generally of sufficient importance to upset results obtained from theoretical considerations. With the most extensive borings it will not be possible in a work of any magnitude to determine exactly all the minor deviations from the normal homogeneous formation accepted for purposes of design.

All these deviations and uncertainties in nature must be allowed for if safe structures are to be designed. A factor of safety has, thus, to be applied to the critical value of the exit gradient to obtain a safe value for such gradient in much the same way as one is applied to the crushing strength of concrete to get the safe working stress in compression or to the elastic limit of steel to get safe tensile stresses in the latter.

This factor of safety will also cover cases where, due to retrogression of bed levels or flood scour, the upper part of the piles at the downstream end is exposed. The exit gradient will, in this case, increase rapidly as the depth exposed increases.

In the table on page 109 it has been shown that in the extreme case the value of G_c may range between 1.44 and 0.44. With ordinary sands 0.9 to 1.1 are more usual.

A factor of safety must take note of the class of material. The specific weight and pore space have been considered. The angle of repose at which the class of sub-soil will stand should also be considered. No work has so far been done to correlate the critical gradient and the angle of repose for soils. This will be a useful study. Till further information is available it is proposed to apply the following factors of safety to critical values of exit gradients—

Shingle	4 to 5
Coarse Sand	5 to 6
Fine Sand	6 to 7

These will not apply to clays which are more or less impervious.

In all cases, it must be remembered that the damage can occur from excess exit gradients due to seepage flow alone or in conjunction with those imposed by pulsations and suction of surface flow.

Determination of Exit Gradients

The full significance of exit gradients in relation to stability of structures against undermining has been clearly set forth above. The exit gradients for standard forms of structures have been theoretically worked out in Chapter VII. The relevant formulae are reproduced below along with reference to the plates in which they are plotted graphically.

Case	Standard Form	Formula
------	---------------	---------

I.

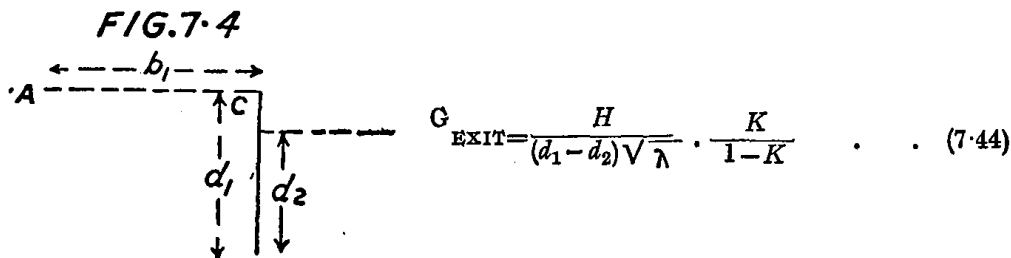


Table VII/3 and Plate VII/3c.

II.

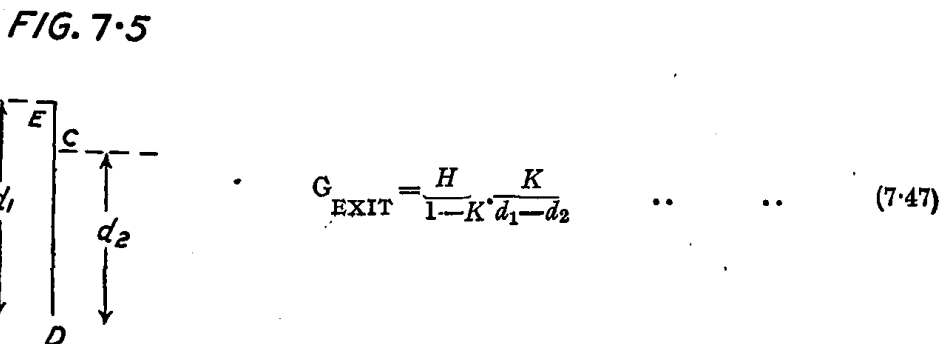
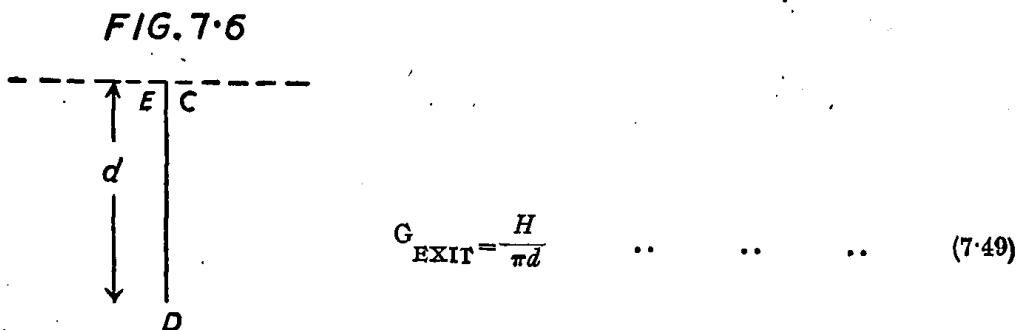


Table VII/1 and Plate VII/4.

III.



V.

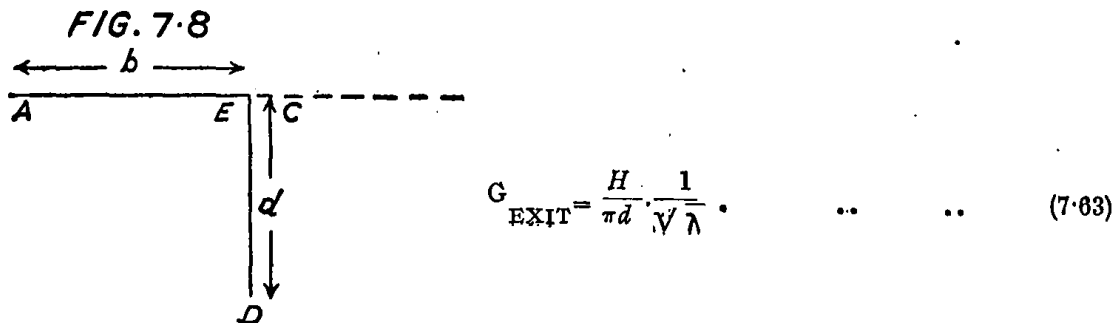
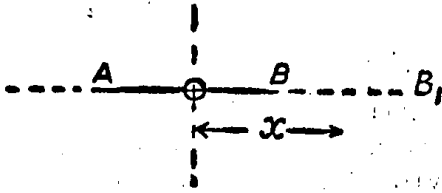
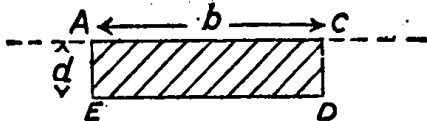


Table VII/5 and Plate VII/5.

Case	Standard Form	Formula
VI.	FIG. 7.9-7.10	$G_{\text{EXIT}} = \text{infinity} \dots \dots \dots (7.64)$
		$G_{\text{EXIT}} \text{ along } B B_1 = \frac{2H}{\pi b} \frac{1}{\sqrt{\left(\frac{2x}{b}\right)^2 - 1}} \quad (7.68)$
VII.	FIG. 7.11	$G_{\text{EXIT}} = \frac{0.84 P_{D1}}{d} \dots \dots \dots (7.71)$
		

The above formulae furnish solutions for all cases met with in practice.

From the above cases it will be noticed that a floor of the type of case VI, with exit gradient at $B = \text{infinity}$, is unstable. Undermining must start at B. But it should also be noticed (Figure 7.10 and Table 7.2) that the *infinite gradient rapidly falls off and soon gets to a safe figure depending on the ratio $\frac{H}{b}$* . But all the same, a certain length is subjected to gradient in excess of the critical, and trouble must necessarily start in this length.

This case is never constructed in actual practice, case VII is more usual. But if the soil at the exit end is scoured out to depth d , it will degenerate very nearly into case VI and unstable conditions will set in.

If lowering of the downstream bed occurs due to flood scour or retrogression of bed levels, case V will get back to case I and case III to II.

The factor of safety mentioned above, therefore, covers not only the unfavourable sub-soil formations, but also the possible occurrence of local tail erosion or scour. This factor of safety will permit the structure to hold on till such time as the scour and other damage can be made good.

Of the six cases mentioned above, the only form which is unstable is the one in which $d=0$ at the tail end, viz., case (VI). This points to the absolute necessity of having some depth of piles or vertical cut-off at the tail end.

The next in order of instability is form No. VII. If d is small in comparison with H the gradient at exit may exceed the critical value and undermining may set in.

In all cases, except VI, d occurs as denominator so that the greater the value of d in relation to H , the less the exit gradient and the more stable the work. This is the fundamental principle to be kept in mind for designs. d cannot, however, be made too great; as will be explained in Chapter IX, the uplift pressures under the work increase as d increases, and the floor has to be thickened in consequence. The proper depth (d) for any given length of floor will be that which gives a safe value for the exit gradients and at the same time a minimum thickness for the floor, or that which ensures absolute safety at minimum cost. This aspect will be more fully dealt with in Chapter X.

CHAPTER IX

Standard Forms

In Chapter IV it has been shown that the distribution of pressures under the various structures dealt with in that section, as derived mathematically from the potential law, are identical with those obtained from the hydraulic or from the electric model.

In Chapter V, it has been shown that the results obtained on models agree with those on the prototype in the field. It has further been shown that any departure from the normal results on the prototype can be exactly reproduced on one or the other model by simulating on the latter, the conditions of silt and temperature under which observations relating to the particular results were made.

It may, thus, be taken as established that both for purposes of design and for subsequent field analysis of data, the pressure distribution under any structure as derived mathematically or as given by the hydraulic or electric model will exactly represent such distribution under the full size field structure. This applies to soils which are homogeneous or very nearly so. In cases of non-homogeneous soils, the results derived from mathematically or models can, at best, be approximate, as it is almost impossible to ascertain in detail all the elements of non-homogeneity under any big structure and, therefore, the simulation of such conditions on the models cannot in all cases be done with any degree of reliance.

In a number of complicated cases, mathematical solutions may be either impossible or too involved, and the facilities for model testing may be absent. For the solution of such cases, extensive experiments have been carried out on specially planned standard forms so that some simple empirical combinations of one or more of these will give the various complicated forms.

The various standard forms are given in the order in which their mathematical solutions have been worked out, and not necessarily in the order of experiments.

In this chapter experimental data are given generally for such cases for which the theoretical values have not been or cannot be worked out. In other cases the representative pressure contours, derived experimentally, have been shown and references made to the mathematical equations of Chapter VII.

The comparison of the theoretical and experimental results for the various forms has already been made in Chapter IV.

The experimental data, regarding distribution of pressures under the standard forms, from which the diagrams of this chapter have been prepared, were obtained by Vaidhianathan and Gurdas Ram. Use has been made of all data, published (1,2,3) or un-published.

The photographs for stream lines have been supplied by Harbans Lal from the *Hydraulic Model*.

The general case. Stepped floor with pile line at the step or fall.

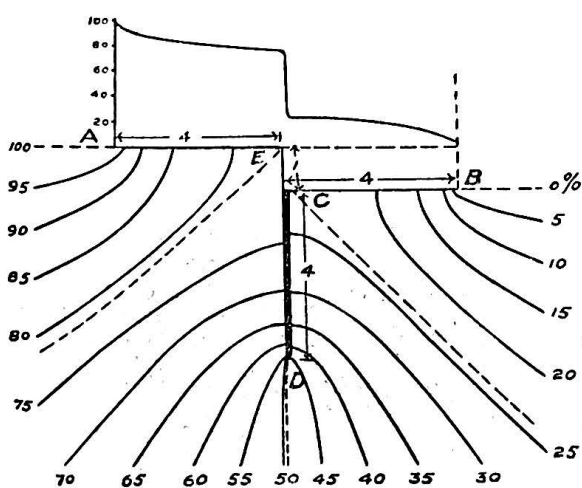


Figure 9-1

Length of U. S. Floor	:	4
Length of D. S. Floor	4
Length of Central Sheet Pile from the D. S. Floor	4	4
U. S. Floor Higher than the D. S. Floor by	1

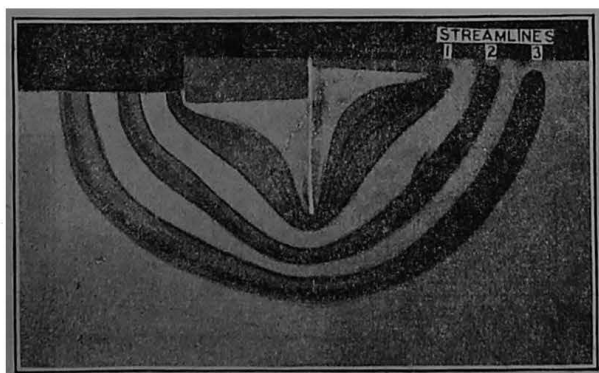


PHOTO 9-1

- (1) Uplift Pressure on Weirs. A Floor with a Line of Sheet Piles, by V. I. Vaidhianathan, Gurdas Ram and E. McKenzie Taylor. (Proceedings of the Indian Academy of Science, Vol. II, No. 6, December, 1935.)
- (2) The Design of Falls with reference to Uplift Pressure, by Gurdas Ram, M.Sc., and V. I. Vaidhianathan, D. Sc., F. Inst. P. (Proceedings of the Indian Academy of Science, Vol. III, No. 4, April, 1936.)
- (3) Uplift Pressure and Design of Weirs with two Sheet Piles, by Gurdas Ram and V. I. Vaidhianathan (Proceedings of the Indian Academy of Science, Vol. IV, No. 2, August, 1936.)

This is the general case worked out in Chapter VII from which all the other cases have been derived. The theoretical solutions are given in Equations 7.25, 7.36, 7.37 and 7.38.

This case covers the various types of weirs and falls. Seventeen different cases of this were tried in model form.

The following table gives the experimental and theoretical values of ϕ_E , ϕ_D and ϕ_C .

TABLE

$\delta = \frac{d_2}{d_1 - d_2}$	$\delta_1 = \frac{b_1}{d_1 - d_2}$	$\delta_2 = \frac{b_2}{d_1 - d_2}$	ϕ_E		ϕ_D		ϕ_C	
			Exp.	Theor.	Exp.	Theor.	Exp.	Theor.
4	4	0.0	77.0	76.3	43.4	43.8	0.0	0.0
4	4	1.0	77.1	76.5	44.0	44.3	6.8	6.8
4	4	2.0	77.4	76.9	45.6	45.6	12.4	13.7
4	4	3.0	78.2	77.1	47.1	47.3	18.7	19.4
4	4	4.0	78.6	78.1	48.9	49.2	24.2	24.6
4	4	5.0	79.3	78.8	50.5	51.2	27.9	28.9
4	4	6.0	79.9	79.5	53.3	53.1	31.9	32.7
4	4	7.0	80.7	80.0	55.4	54.8	37.1	36.0
4	4	8.0	81.7	80.8	56.5	56.4	39.4	38.9
0.0	7	7	56.7	56.8	53.7	53.9	53.7	53.9
0.5	7	7	58.4		52.0		48.2	
1.0	7	7	60.2	59.8	51.4	51.7	45.6	46.6
1.5	7	7	62.1		51.0		43.3	
2.0	7	7	63.2	63.2	50.6	50.9	40.4	41.6
3.0	7	7	66.9	66.6	50.3	50.4	36.2	37.2
4.0	7	7	69.7	69.6	49.8	50.0	32.3	33.4
5.0	7	7	72.6	72.4	49.5	49.8	29.2	30.0

CASE 1

*Floor with Sheet Pile at Downstream End, with Step.
Scour at Tail End.*

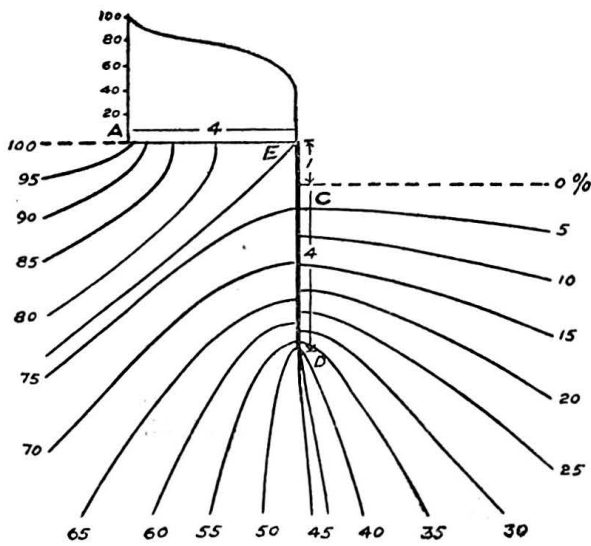


Figure 9.2

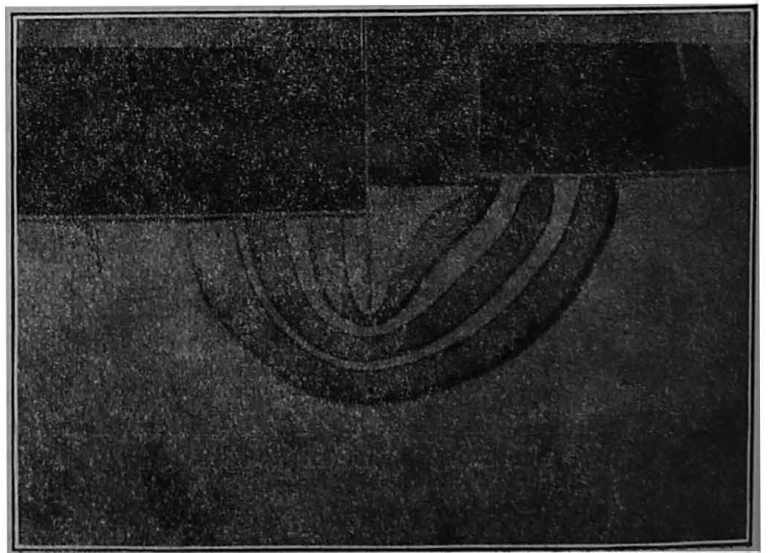


PHOTO 9.2

The mathematical solution for this case is given in Equations (7.39) and (7.40). The exit gradient at C is given by Equation (7.44).

This case occurs where settlement of the downstream apron has taken place due to scour or retrogression of levels. The abutments of an aqueduct where the stream bed between the abutments is protected by loose pitching only, is another instance.

The exit gradients in this case are of special importance in determining the factor of safety against flotation at the toe of a weir.

CASE II

Single Sheet Pile, no aprons. Unequal fill on two sides.

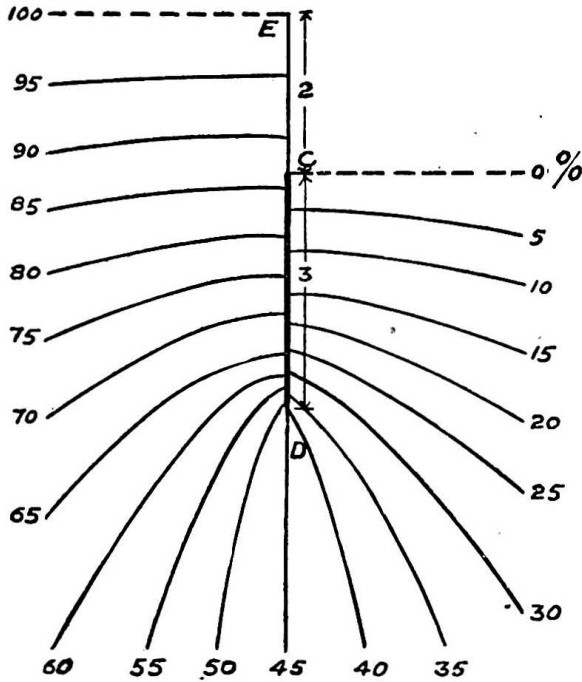


Figure 9.3

SINGLE SHEET PILE.
Length=6
Drop = 2

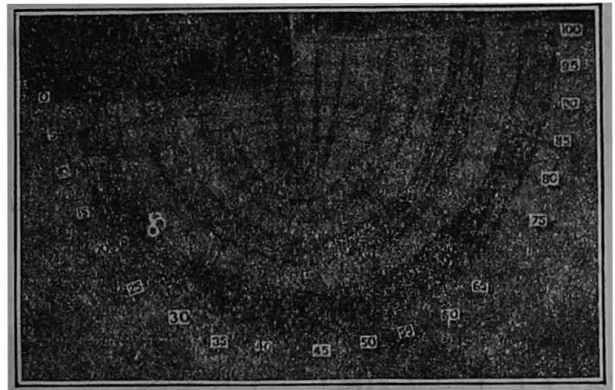


PHOTO 9.3

The mathematical solution of this case is given by Equations 7.46 and 7.47.

Abutments of bridges and aqueducts, cofferdams with pumping and excavation within, and ordinary wells during sinking are instances of this case. It will also determine the stability of toe walls should erosion or scour occur close to them.

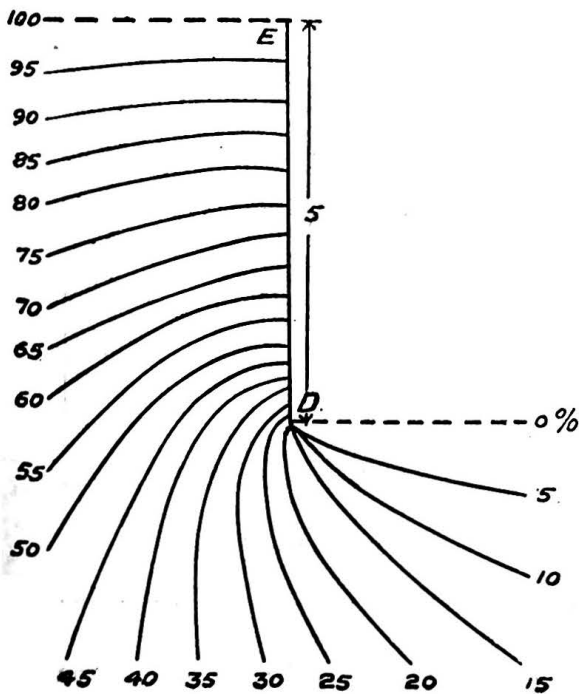


Figure 9.4

STREAM LINES
and
Pressure Contours.
Length of pile=6
Drop=5

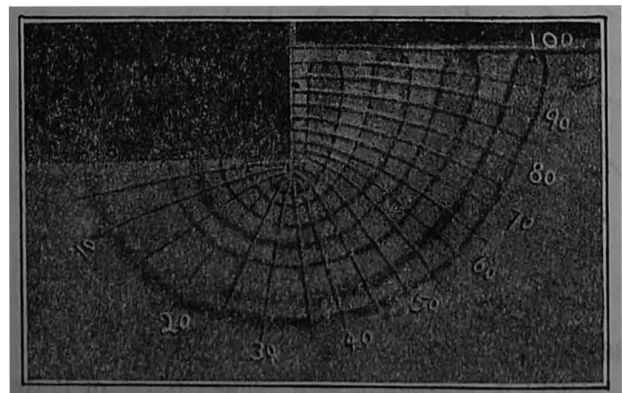


PHOTO 9.4

CASE III

Single Sheet Pile. No Aprons. Equal Fill.

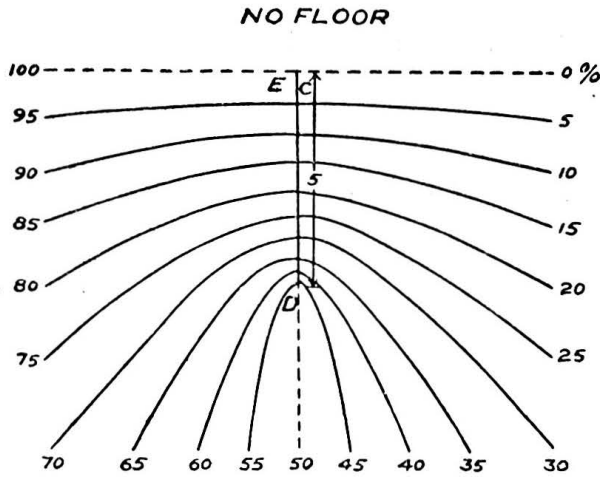


Figure 9-5

STREAM LINES
and
Pressure Contours.

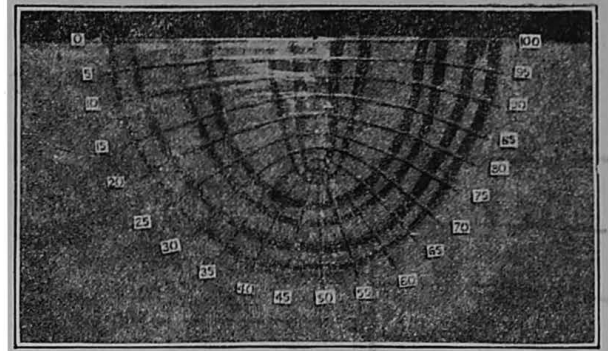


PHOTO 9-5

The theoretical solution for this case is given by Equations 7-48 and 7-49. Cofferdams are an instance of this case.

CASE IV

Floor with Sheet Pile not at End.

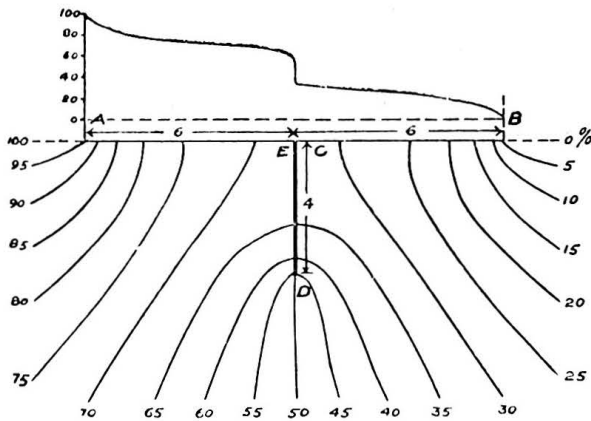


Figure 9-6

Floor 12
Central Sheet Pile 4

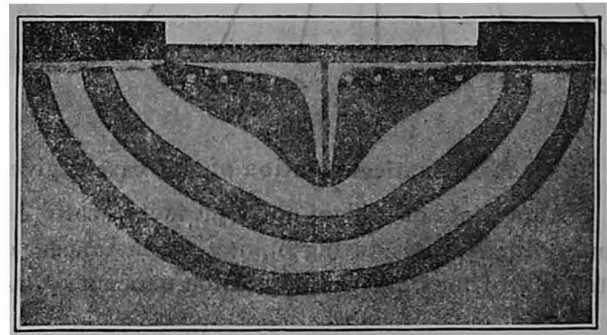


PHOTO 9-6

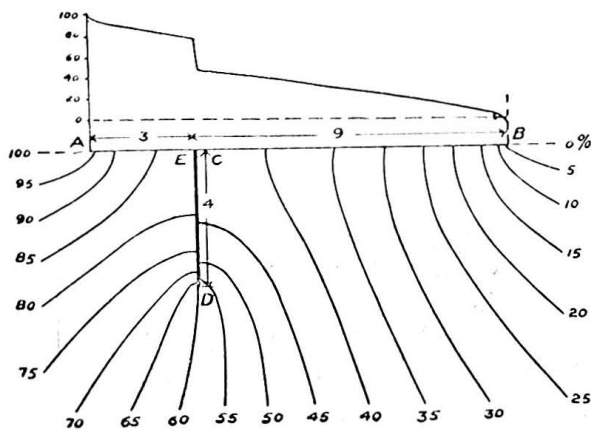


Figure 9-6 (a)

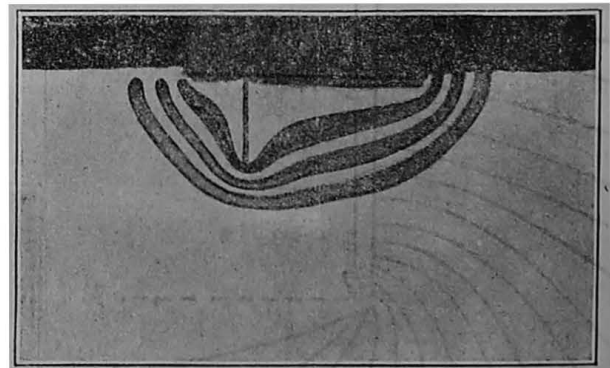


PHOTO 9-6(a)

The mathematical solutions for this case are given by Equations 7-59, 7-60, 7-36, 7-37 and 7-38.

This is a common case. The curves of Plate VII/6 give graphically the pressures at the points E, C and D. These curves will be frequently used for design. The allowance for the thickness of floor can be made by a combination of this with case VII of depressed floor.

CASE V

Floor with Pile at End

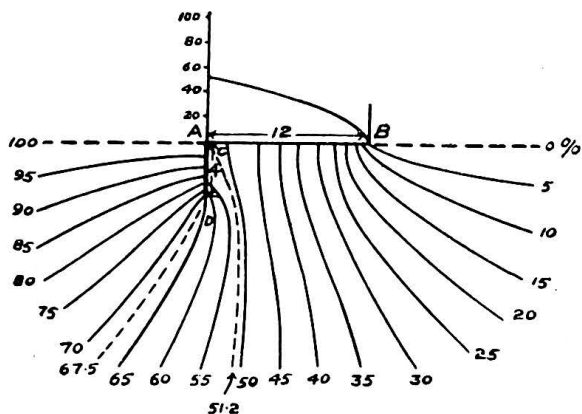


Figure 9-7

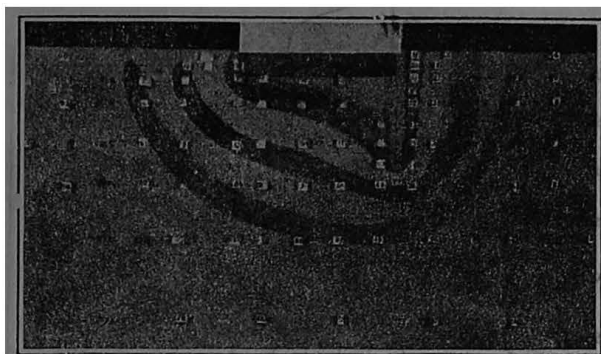


PHOTO 9-7

This is another common form for designs. The mathematical solutions are given by Equations 7-61, 7-62, and 7-63. The graphical solutions are given by curves of Plate VII/6. Floor thickness can be allowed for as in the previous case.

The exit gradients for this case are given on Plate VII/5.

CASE VI

Simple Floor. - No pile lines.

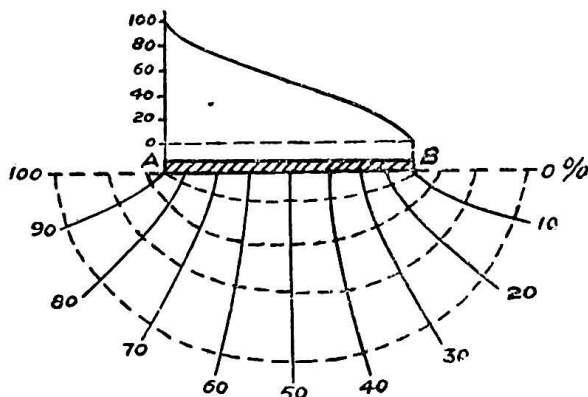


Figure 9-8

The solutions for this case are given by Equations 7-7, 7-64 and 7-68.

This case is more or less of theoretical interest. This, however, is the fundamental form of which all other forms are modifications. This will be rarely referred to for purposes of design except if it be to demonstrate unstable forms of structures.

CASE VII

Depressed Floor. No sheet pile.

Impervious Floor Length 12

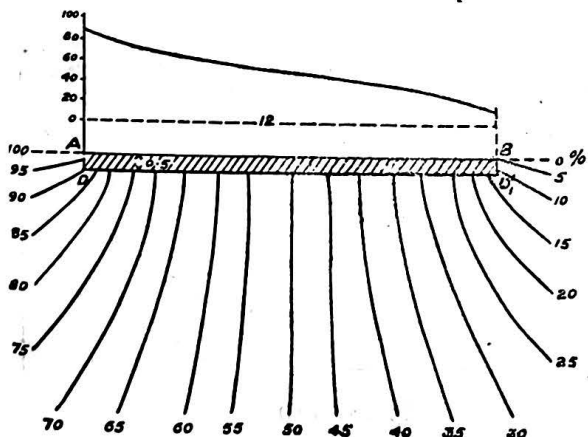


Figure 9-9

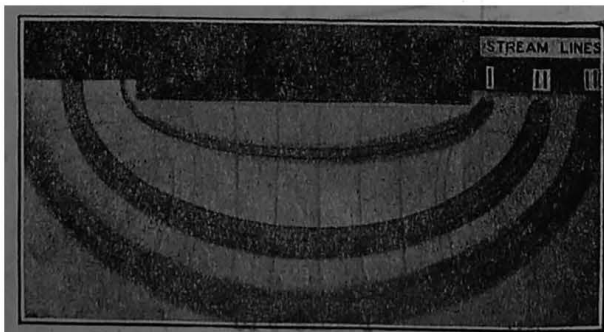


PHOTO 9-9

CASE VII—contd.

Depressed Floor. No Sheet pile.

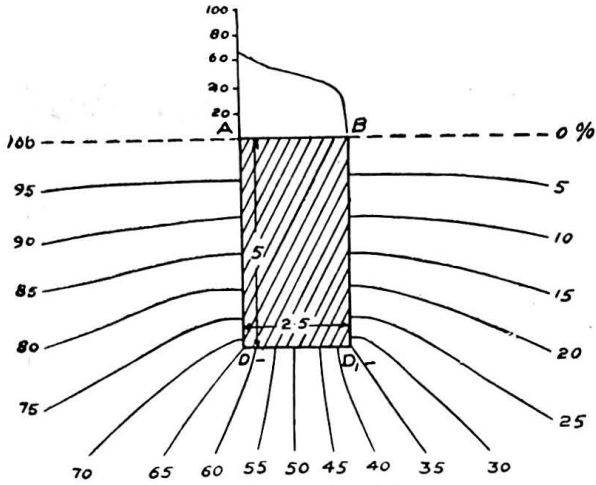


Figure 9-6 (a)

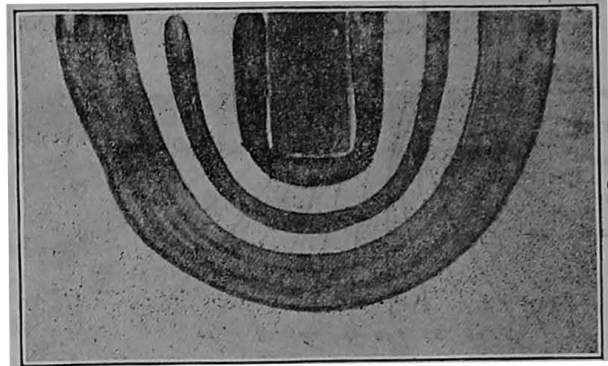


PHOTO 9-9 (a)

In practice this case is not common, but it occurs in combination with all other cases. It represents the thickness of the floor. The other application of this case is for wells with considerable base width.

It will be noticed in Plate VII/6 that the pressure at points D and D' are greater than those at the bottom of sheet piles of the same depth, so that the pressures under the floor in this case are higher than if the same length were bounded by sheet piles of depth equal to the depression, the base length being the same in both cases.

The solutions for this case are given by Equations 7-70 and 7-71. The curve of Plate VII/6 for this case is derived from experiments as well as from theory. The values are almost identical in both cases with those of Table IV. 3.

CASE VIII.

Floor and Multiple Sheet Piles

Thus far only such cases have been dealt with for which complete mathematical solutions are available. The experimental work done on these forms served merely to give an experimental confirmation of the mathematical formulae. Where calculations are laborious experimental results furnish a simple guide.

But in actual design, the bulk of cases are either mathematically indeterminate or too involved, so that theoretical solutions are out of the question. These cases consist of combinations of floor with sheet piles. The greatest merit of experimental work lies in the solutions of such cases. From any analysis of the experimental results for these cases it has been possible to derive simple rules (See Chapter X) which enable pressures under complicated forms to be determined with a fair degree of accuracy.

(a) FLOOR WITH A SHEET PILE AT EITHER END

(1) End Piles Equal.*

Length of U. S. Sheet Pile	$\frac{3}{4}$
Length of D. S. Sheet Pile	$\frac{3}{4}$
Length of Floor	12

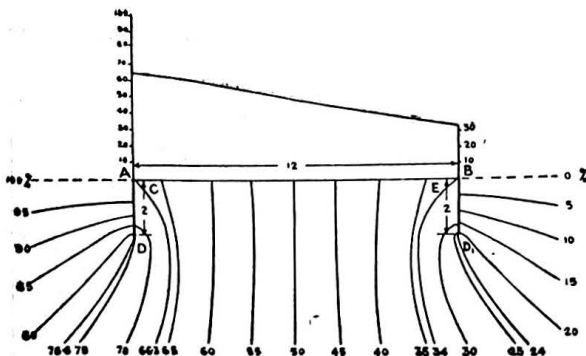


Figure 9-10

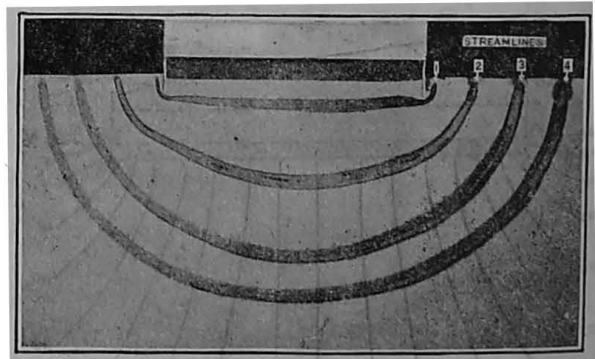


PHOTO 9-10

* A solution to this case has been derived by Malhotra, and compared with experimental results. See Chapter VII, Appendix pp. 88-89.

The pile lengths ranged from 0.5 to 6 and values of $\alpha = \frac{\text{floor length}}{\text{depth of pile}}$ from 24 to 2. The following table gives the values at the corner points D, C, E and D₁ with b=12.

d	$\frac{1}{2}$	1	1 $\frac{1}{2}$	2	3	4
d ₁	$\frac{1}{2}$	1	1 $\frac{1}{2}$	2	3	4
D	85.7	81.7	78.7	76.6	73.2	71.7
C	79.7	74.0	69.7	66.3	61.3	58.2
E	20.3	26.4	30.7	34.1	40.0	42.1
D ₁	15.2	18.9	21.7	24.0	26.2	29.2

(2) Piles Unequal.

Length of U. S. Sheet Pile	3
Length of D. S. Sheet Pile	1
Length of Floor	12

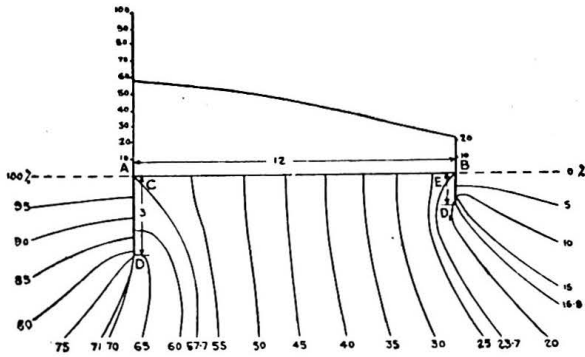


Figure 9-11

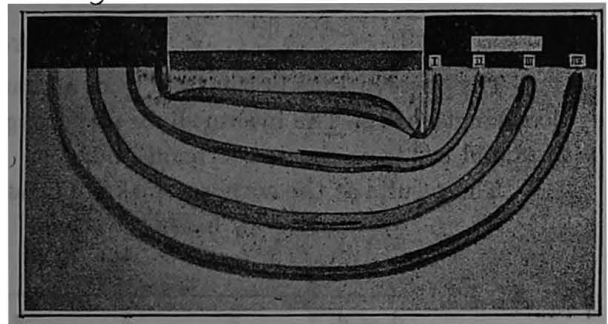


PHOTO 9-11

See Plate IX. 2. Figure 1

Here again the piles ranged from 0.5 to 6 and values of α from 24 to 2. The experimental values for the corner points are given in the table.

b=12

d	2	3	3	3	3	3	3
d ₁	1	1	2	3	4	5	6
D	75.0	70.0	71.9	73.2	74.4	75.8	77.0
C	64.5	57.7	59.1	61.3	63.4	65.2	67.3
E	25.1	23.7	32.22	40.0	44.2	48.7	52.9
D ₁	18.1	16.8	22.0	26.2	29.7	32.9	35.0

(b) FLOOR WITH PILES AT ENDS AND MIDDLE.

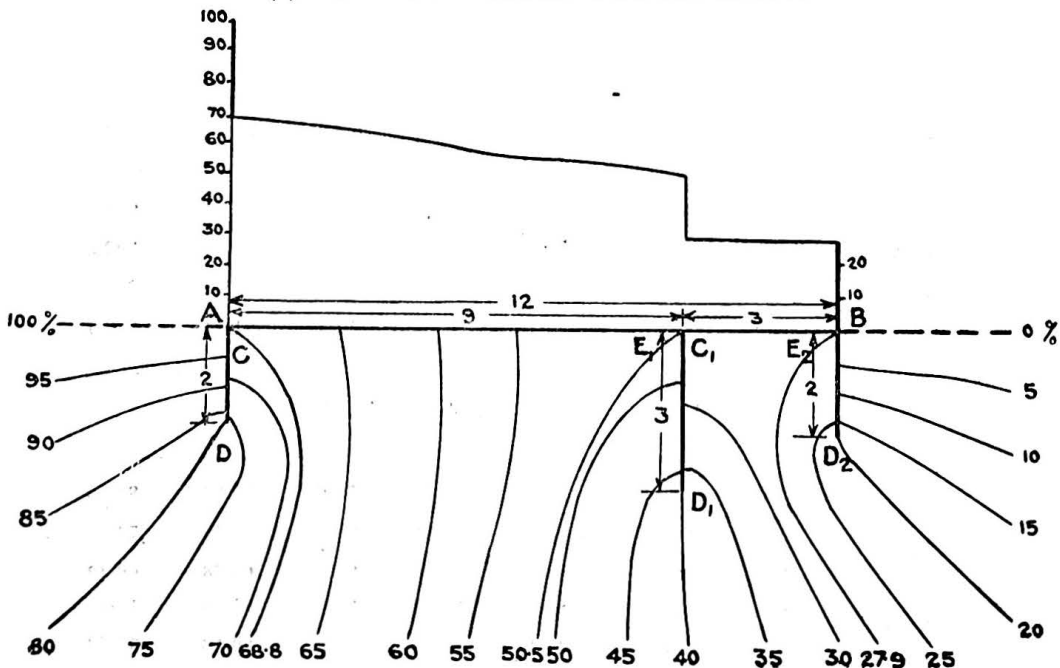


Figure 9-12

See Plate IX. 2. Figures 2 & 3,

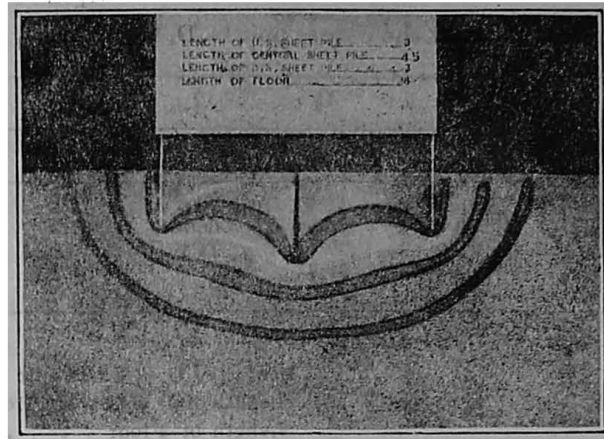
FLOOR WITH PILES AT ENDS AND MIDDLE—*contd.*

PHOTO 9·12

The observations in this case were all carried out with the end piles 2 units long and floor length of 12. The intermediate piles ranged from 0·5 to 5 and were located at $b/8$, $b/4$, $3b/8$ and $b/2$ from the downstream end.

The results at the corner points are tabulated below:—

$$b_1/b = \frac{3}{4}.$$

$$b_1=9, b=12.$$

$d = d_2$	2	2	2	2	2	2	2
d_1	0.5	1.0	1.5	2.0	3.0	4.0	5.0
Φ at D	76.4	76.92	77.45	77.88	78.45	79.02	80.26
Φ at c	66.5	66.50	66.62	67.18	68.79	69.72	71.67
Φ at E_1	42.3	43.21	45.0	46.92	50.5	54.30	58.59
Φ at D_1	40.0	39.75	40.0	40.0	39.12	41.66	41.99
Φ at C_1	38.3	36.12	34.8	32.58	28.92	26.10	23.33
Φ at E_2	33.25	32.10	31.82	30.34	27.88	25.00	22.66
Φ at D_2	23.54	23.05	23.08	22.69	20.98	20.58	17.24

b =length of floor.

b_1 =distance of intermediate pile from upstream end.

$$b_1/b = \frac{5}{8}.$$

$$b_1=7.5, b=12.$$

Φ at D	77.5	77.17	77.28	77.20	77.92	79.24	80.00
Φ at c	66.8	66.75	67.06	67.52	68.70	70.48	72.13
Φ at E_1	47.03	48.78	50.30	52.28	55.40	59.29	62.60
Φ at D_1	44.70	45.00	45.00	44.73	45.72	45.52	45.61
Φ at C_1	42.83	41.07	39.48	37.72	35.10	31.66	28.39
Φ at E_2	33.24	33.09	32.13	31.68	29.53	27.89	25.62
Φ at D_2	23.15	23.16	23.40	22.90	22.27	21.41	18.99

$$b_1/b = 1 : \frac{1}{2}.$$

$$b_1=6, b=12.$$

Φ at D	77.27	77.29	77.59	77.94	78.6	79.23	80.40
Φ at c	66.52	67.32	67.32	67.90	69.3	71.43	72.97
Φ at E_1	52.30	54.06	55.83	57.66	60.93	63.88	66.42
Φ at D_1	50.0	50.00	50.00	50.00	50.00	50.00	50.00
Φ at C_1	47.79	46.27	44.14	42.93	39.3	36.56	33.89
Φ at E_2	33.67	33.30	32.80	32.60	30.92	29.07	27.48
Φ at D_2	22.98	22.90	23.50	23.08	22.53	21.27	20.00

$d=d_2$	$b_1/b = \frac{7}{8}$			$b_1=10.5, b=12$			
	2	2	2	2	2	2	2
d_1	0.5	1.0	1.5	2.0	3.0	4.0	5.0
Φ at D	77.0	76.96	77.12	77.26	78.00	78.75	80.20
Φ at C	66.78	66.68	66.55	67.00	67.90	70.00	71.42
Φ at E_1	32.27	38.20	40.00	41.53	45.00	49.89	54.17
Φ at D_1	35.96	34.70	34.03	33.90	34.25	36.17	38.19
Φ at C_1	34.00	31.80	30.18	28.90	23.95	20.65	17.68
Φ at E_2	33.00	31.68	30.00	28.32	23.98	20.52	17.40
Φ at D_2	23.68	23.13	22.84	22.50	20.00	17.82	15.98

(c) FLOOR WITH A PAIR OF PILES NOT AT THE ENDS

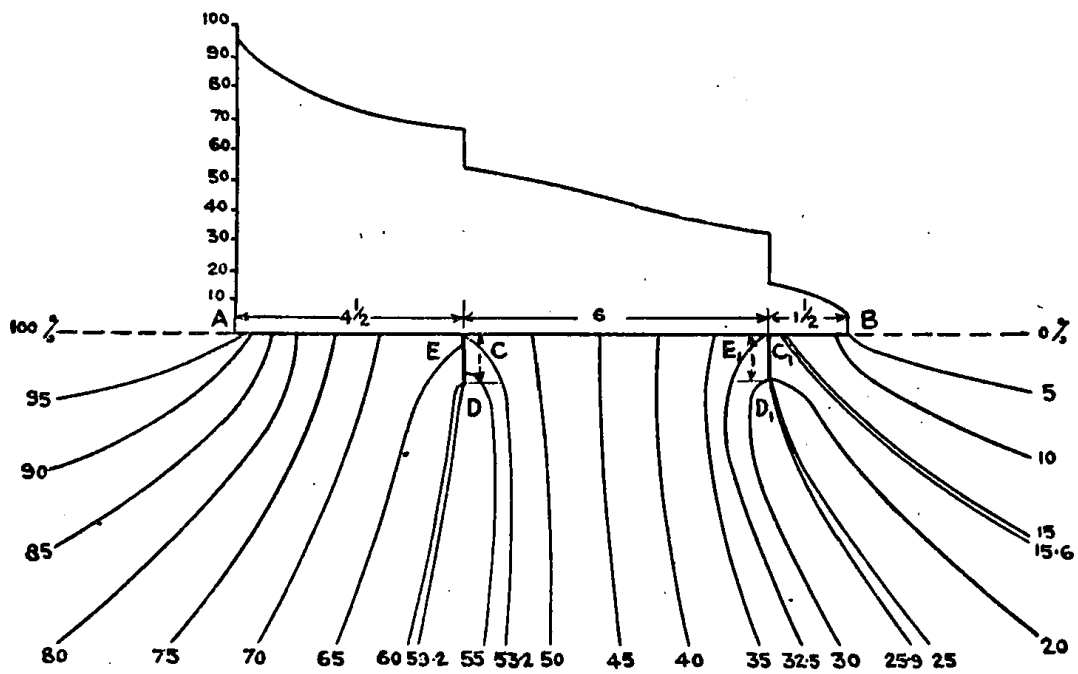


Figure 9.13

See Plate IX. 1. Figure 2

This case consisted of a pair of piles each $b/12$ long and $b/2$ apart which were moved up and down along the floor. The results at the corner points are tabulated below:—

$b=12$ and distance between piles= $b/2$						
$d=d_1=$	1	1	1	1	1	1
$b_r=$	6	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	3
E	58.8	60.7		65.2	67.9	
D	53.4	55.0	56.8	59.2	61.6	
C	47.9	49.2	51.0	53.2	55.5	
E_r	27.3	28.2	30.0	32.5	35.2	40.0
D_r	19.9	21.3	23.4	25.9	28.4	
C_r	00.0	6.3	11.5	15.6	19.6	

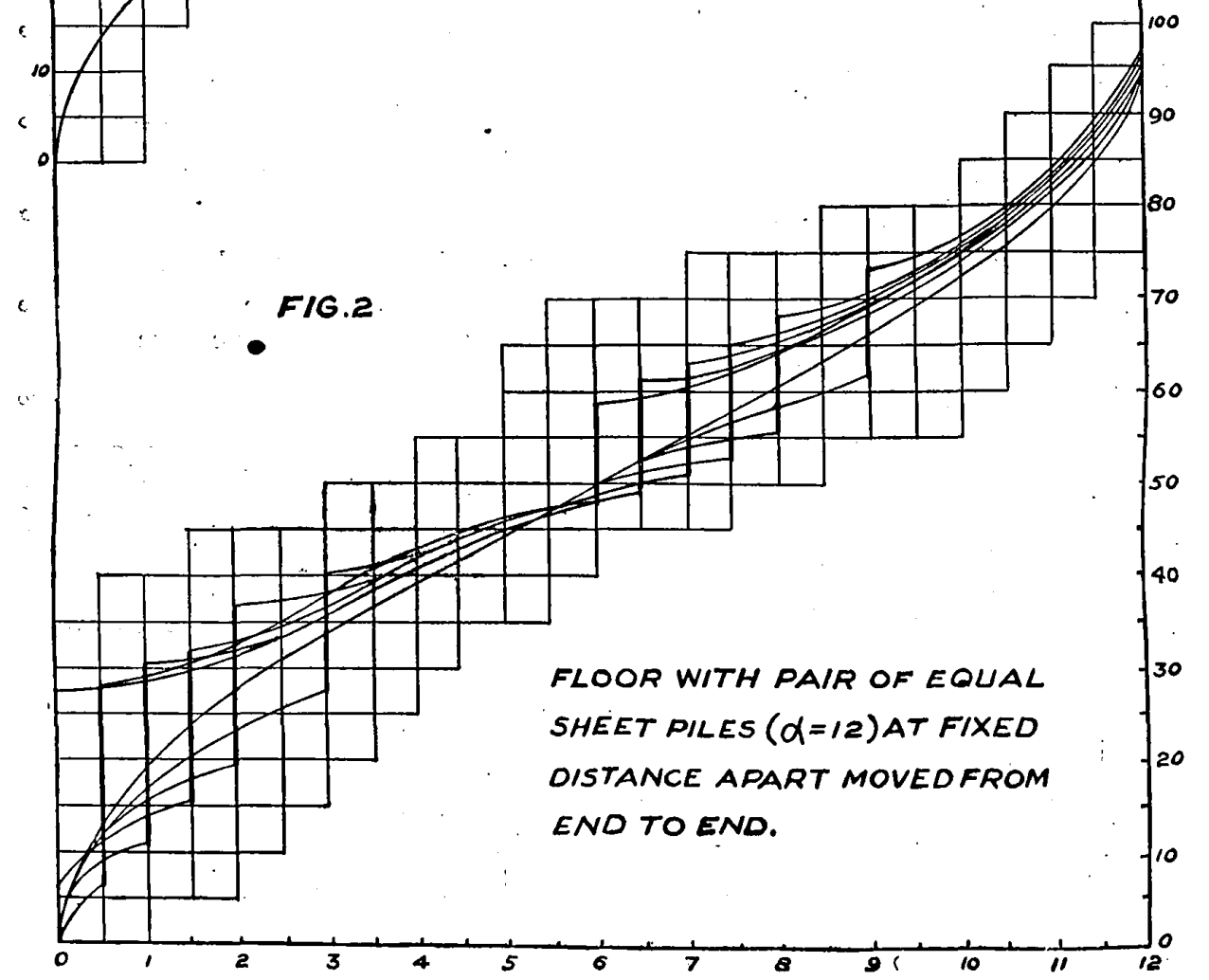
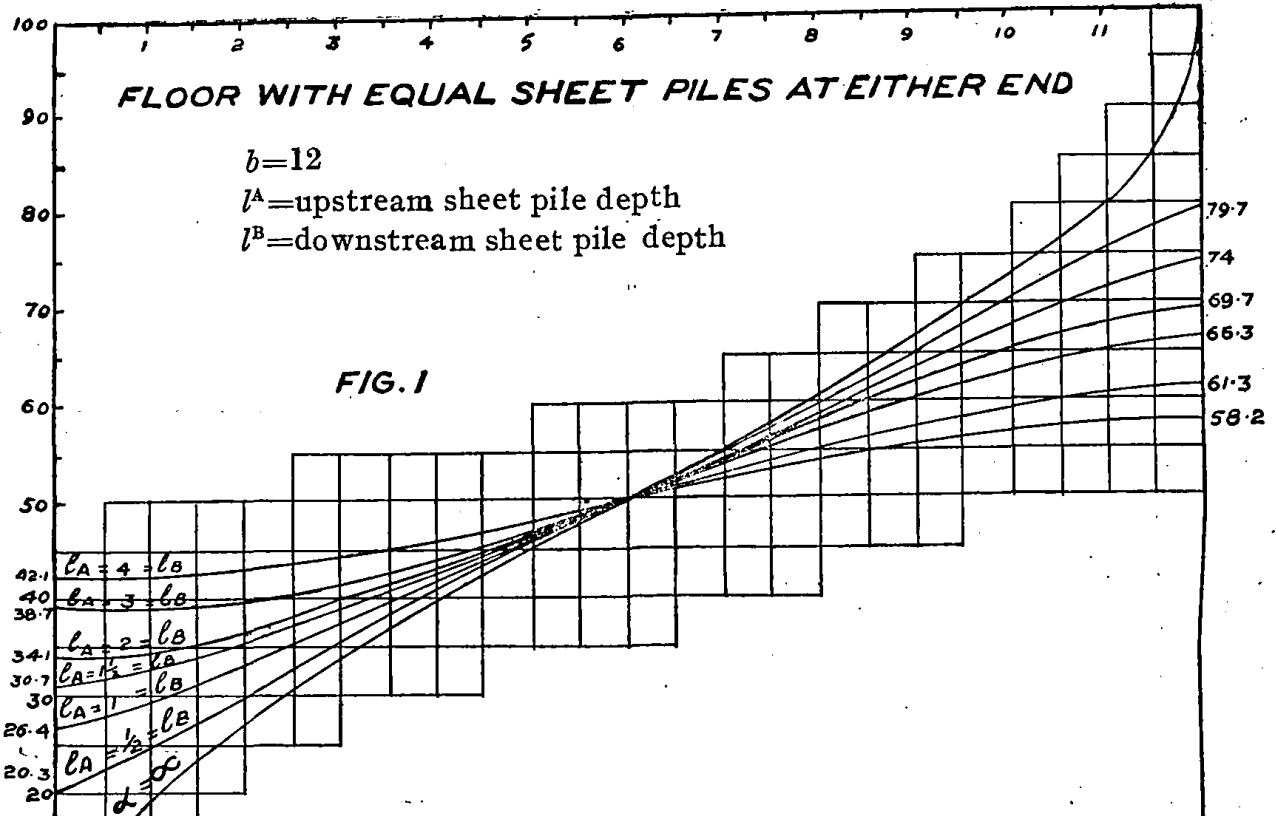
b_1 =distance of pile from upstream end.

The hydraulic gradient diagrams of Plate IX, 1 and 2 show graphically the mutual influence of piles in various positions. One general feature which is noticeable in each case is that the insertion of a pile anywhere results in heading up or increase of pressure upstream

and a draw-down or reduction downstream, the increase or reduction being maximum at the pile decreasing rapidly further out. The main influence is confined to a radius equal to the depth of the pile. At twice that radius the influence becomes negligible. The hydraulic gradient line between any two corners is very nearly a straight line; in any case, the departure from a straight line is small and generally negligible in designs.

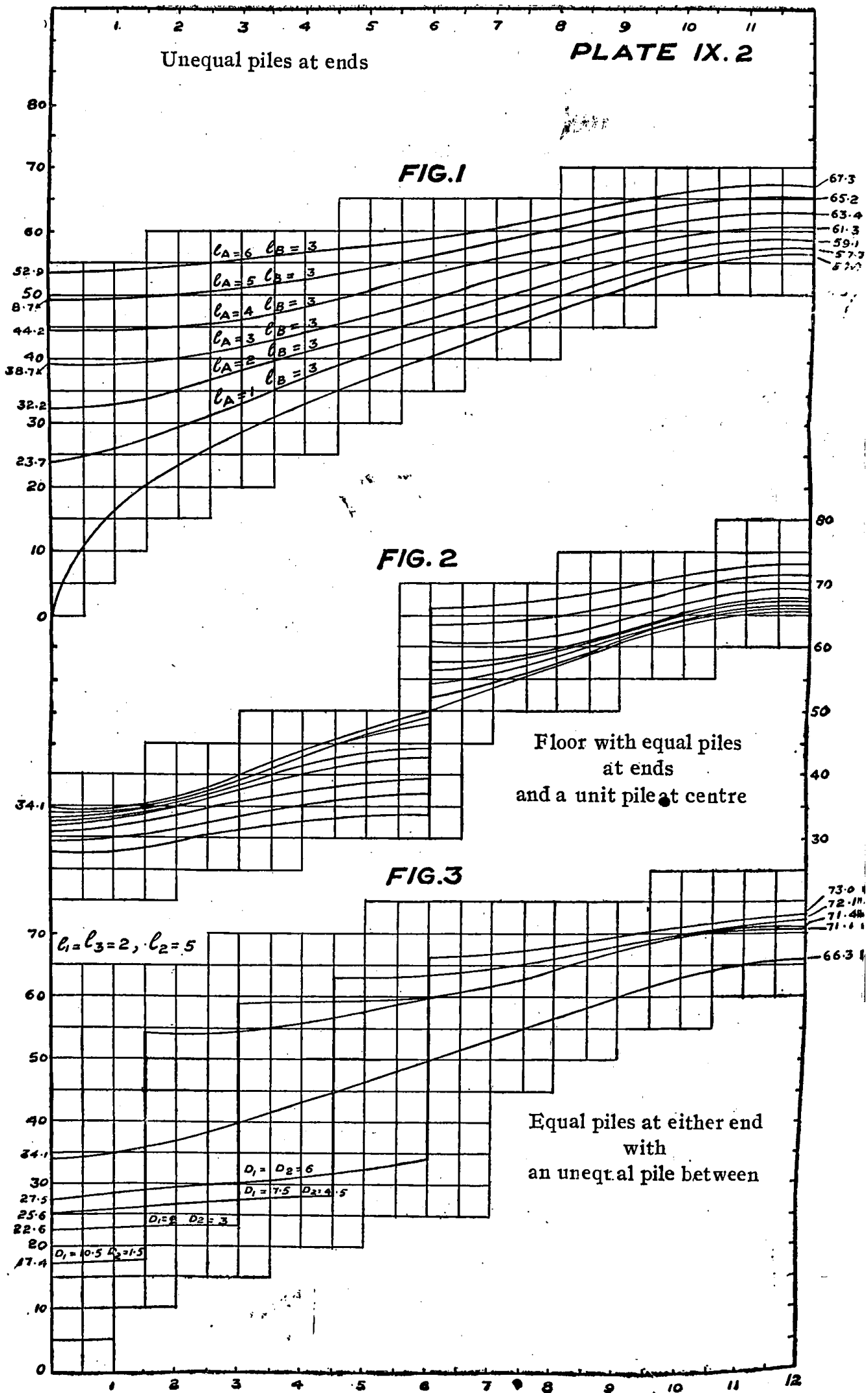
The mathematical solutions of Chapter VII and the experimental results given in this chapter have made it possible to frame empirical rules which will give the pressures at the various key points correct within about 2½ per cent. These rules have been derived and their application to designs are explained in the next chapter.

PLATE IX.1



Unequal piles at ends

PLATE IX.2



CHAPTER X

Working Rules and General Principles of Design

The fundamentals of sub-soil hydraulics and their applications to the design of structures resting on saturated soils have been fully dealt with in the previous chapters. This chapter will deal with the methods of design of new structures and of analysing the existing ones with a view to ascertaining their stability.

Water in its flow through the sub-soil under a weir or other hydraulic structures, loses head gradually along its passage till at the exit the head is reduced to zero. Thus, at each point under the weir floor, the water has a certain residual pressure, and a certain pressure gradient, *i.e.*, the rate at which this head is being dissipated.

As has been shown before (Chapter VIII), the pressure gradient at any point represents the force which the sub-soil water exerts in the direction of flow. It follows that there are two forces which weirs have to withstand, firstly, the residual pressure which will tend to lift up the weir floor if the weight on the latter is less than the upward pressure of water at that point, and secondly, the pressure gradient or the force of water acting along the direction of flow. This latter is of no moment except at the tail end where the water emerges from the sub-soil. If at this end the upward force of water is in excess of the restraining force of the sub-soil, *viz.*, weight, internal friction, *etc.*, the surface soil will be lifted up followed by progressive disruption of that further down. This may result in undermining of the foundation soil and ultimate failure of the structure.

The two essentials to be considered in weir design, therefore, are :—

- (a) Residual or uplift pressure on the weir floor, and
- (b) Exit gradients.

These two essentials are inter-connected. For any given foundation profile of a weir in a given class of soil, there will be a definite distribution of pressure and a definite exit gradient. To safeguard against undermining, the exit gradient must not be allowed to exceed a certain safe limit, generally $1/5$ to $1/7$. The uplift pressures must be kept as low as possible, consistent with safety at the exit, so as to keep the floor thickness at a minimum. The aim of this investigation is to determine designs which will ensure absolute safety with utmost economy.

Khosla's Method of Independent Variables¹ for the Determination of Uplift Pressures and Exit Gradients

Before attempting to proceed with the discussion of actual designs, it is necessary to know the pressure distribution for any given foundation profile.

Until recently the profession was wedded to the so-called Bligh's Creep Theory. In Chapter I, this theory has been shown to be incorrect. The very experiments of Col. Clibborn, which are supposed to form the basis of this theory, go to disprove it. The field researches at the Upper Chenab Siphons (1926 to 1929)^{2,3} showed how the pressure gradient lines derived from this theory were entirely different from those given by actual observations at site. Since then intensive field and laboratory researches have been carried out in the Punjab with a view to discovering a simple, rational and correct basis for design, and thus to fill the blank created by the rejection of the Bligh Theory.

A new method—the method of independent variables¹—has now been evolved, which in conjunction with Plate VII-6 of Chapter VII, enables uplift pressures, under any complicated weir sections to be determined with almost mathematical accuracy. (See page 104 for equal sheet piles.)

In Chapter VII, we have complete solutions of :—

- (a) A stepped horizontal floor with a pile line at the step but no depression of the former in the sub-soil.

(1) For mathematical justification of this method, see Punjab Irrigation Research Publications, Vol. II No. 20 "Notes on Khosla's Principle of Independent Variables I—Mutual interference of equal piles at ends of a floor" and Vol. II No. 22 "Notes on Khosla's Principle of Independent Variables II—Pressure Distribution under a floor with equal end piles and a varying intermediate pile", by J. K. Malhotra and E. McKenzie Taylor.

(2) "Hydraulic Gradients in Sub-soil Water Flow" by A. N. Khosla, Paper No. 138, Punjab Engineering Congress (1930).

(3) "Stability of Weirs and Canal Works" by A. N. Khosla, Paper No. 142, Punjab Engineering Congress (1930).

- (b) A horizontal floor (without step) with a pile line anywhere along it but no depression.
- (c) A horizontal floor with depression in the sub-soil but no pile line.¹
- (d) A horizontal floor with equal pile lines at either end but no depression of the former¹.
- (e) A horizontal floor with no piles and no depression.
- (f) A sloping floor with no piles and no depression.²
- (g) A vertical pile line with equal or unequal fills on the two sides or no fill at all on the downstream side.

These are the basic or elementary forms for which complete mathematical solutions are available. Solutions will become too involved or indeterminate if two or more of these elementary forms are grouped in a weir section. In general, the usual weir sections rarely conform to any one single elementary form. They consist of a combination of almost all the forms mentioned above. They have the floor in one or more steps, one, two or more pile lines of equal or unequal lengths, and varying thicknesses of the floor depressed into the sub-soil. Any method, to be of universal application, must aim at a complete solution of any such complex combination of the elementary forms. *The method of independent variables furnishes such a complete solution.*

In this method, a complex weir section is split up into its elementary standard forms,—the *entire* length of the floor with any one of the pile lines, *etc.*, making up one such form. Each elementary form is then treated as independent of the others. The pressures at the key points are then read off the curves of Plate VII/6. These key points are the junction points of the floor and the pile line of that particular elementary form, the bottom point of that pile line and the bottom corners in the case of a depressed floor.

The exit gradients can be obtained from Plate VII/5.

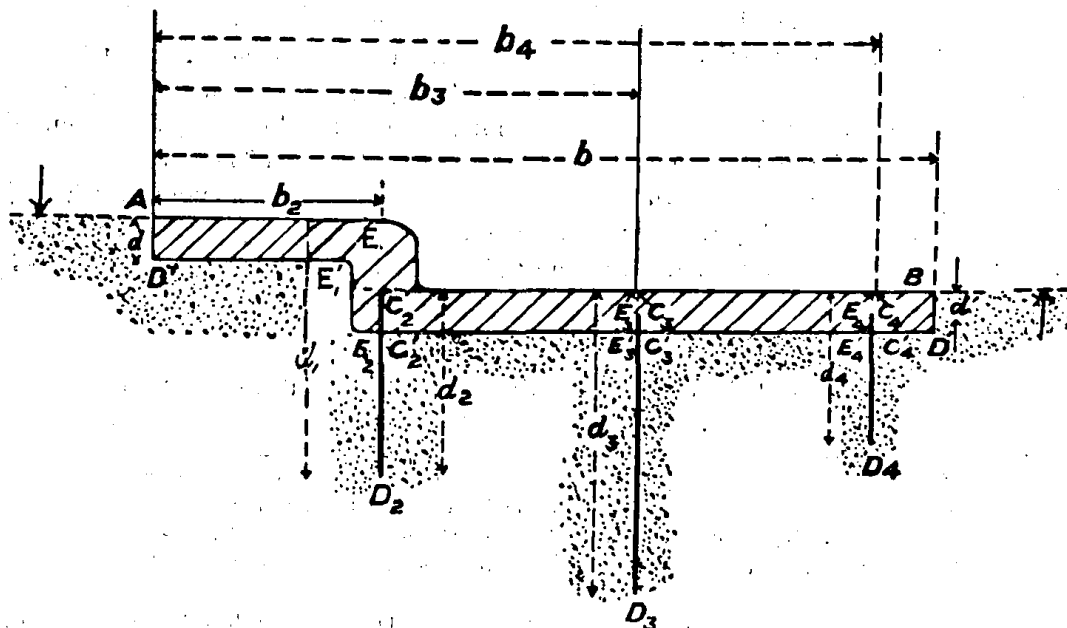
The readings at the junction points are then corrected for :—

- (a) the mutual interference of piles,
- (b) the floor thicknesses,
- (c) the slope of the floor.

For simplicity of working the pressure curves for the essential forms, *viz.*, a floor with pile line not at the end or at the end and for a depressed floor, are all grouped in one single plate—Plate VII/6. Great accuracy will be possible in some cases, with other relevant curves of Chapter VII, but the difference will rarely exceed 1 to 2 per cent, which may be ignored for all practical purposes. These other curves, however, form an excellent check. They have helped in establishing the validity of the method of independent variables.

For an illustration take Figure 10-1

FIG. 10-1



(1) Punjab Irrigation Research Institute Memoirs (under preparation) by J. K. Malhotra, M.A.

(2) "Uplift Pressures under a sloping floor" by J. K. Malhotra, Punjab Irrigation Research Publication, Vol. II, No. 18. See Plate X-1.

This complex section will split up into the following elementary forms, illustrated in Figures 10.2 to 10.5.

FIG. 10.2

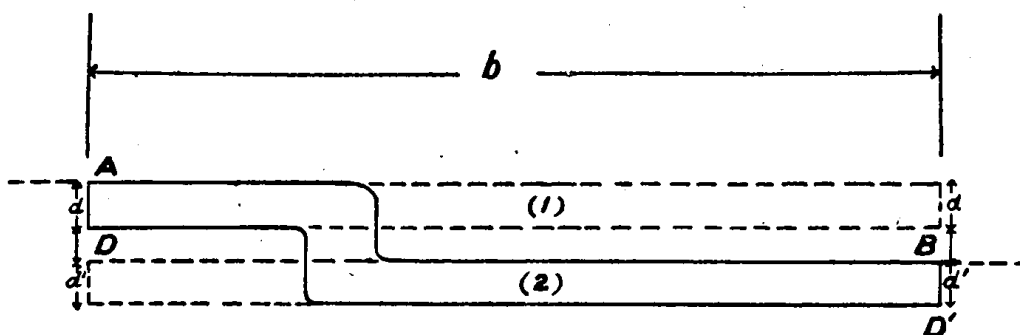


Figure 10.2 resolves itself into two cases of depressed floor and no sheet piles.

Form (1).

$$1/\alpha = d/b$$

Read $\varphi_{D'}$ from curves of $\varphi_{D'}$ (depressed floor)—right-hand diagram and top-most curve (Plate VII/6).

Form (2).

$$1/\alpha = d'/b.$$

Similarly read $\varphi_{D'}$ for this value of α .

FIG. 10.3

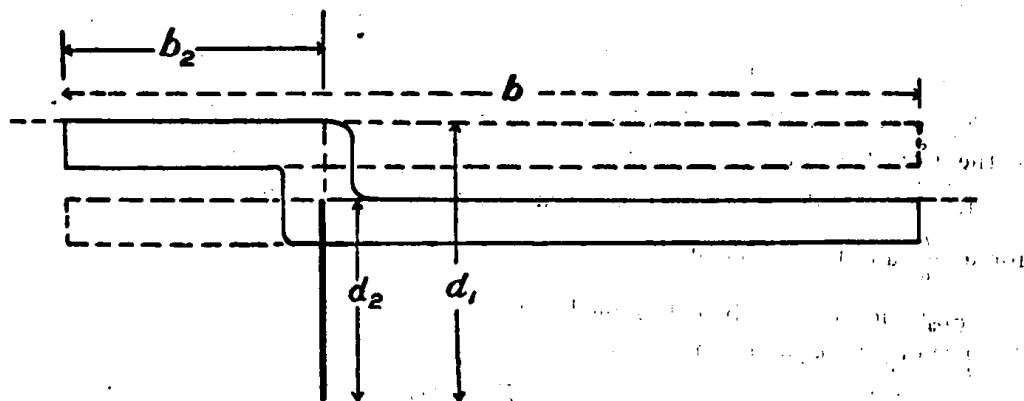


Figure 10.3 represents a plain floor with a pile of depth d_1 or d_2 at distance b_2 from the upstream end.

The base ratio b_2/b remains unchanged but α has two values.

$$\alpha = \frac{b}{d_1} \text{ and } \alpha = \frac{b}{d_2}$$

Read the values of φ_{D_2} for the two cases from the right-hand bottom curves of Plate VII/6. Where the fall is small compared with the depth of piles the two values of φ_{D_2} will not differ very much. Otherwise the value of φ_{D_2} obtained for $\alpha = b/d_1$ should be used for the part of the floor on the left and that for $\alpha = \frac{b}{d_2}$ for that on the right of the pile line.

$$\left. \begin{array}{l} \text{Read } \varphi_{E_1} \text{ for } \alpha = \frac{b}{d_1} \\ \varphi_{C_2} \text{ for } \alpha = \frac{b}{d_2} \end{array} \right\} \text{Curves of Plate VII/6.}$$

φ_{E_1} and φ_{C_2} represent values at points on the floor d_1 and d_2 above the bottom of the pile respectively.

To obtain $\varphi E_1'$ and $\varphi C_2'$ under the floor, assume a proportional drop along the vertical $E_1 D_2$ and $D_2 C_2$. Then

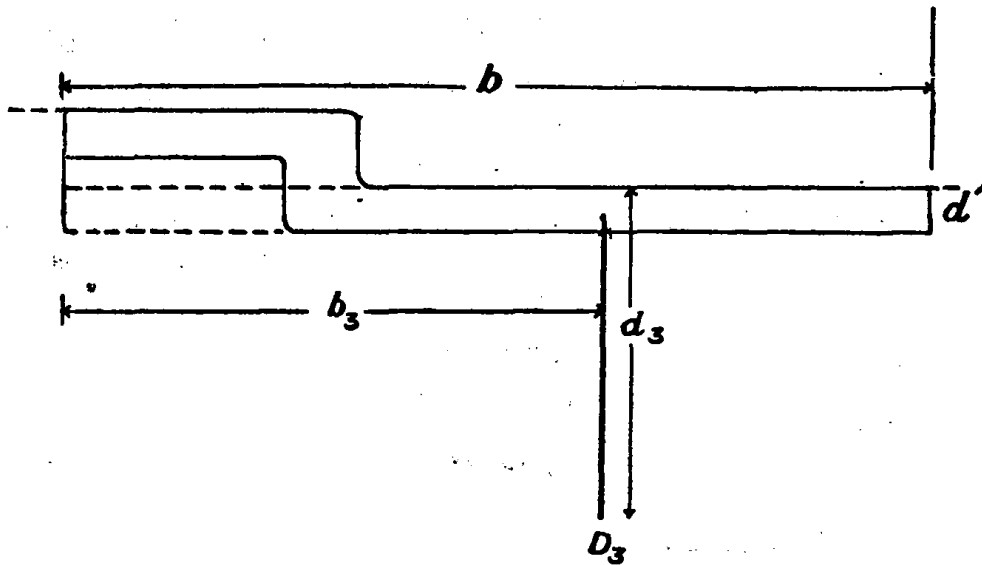
$$\varphi E_1' = \varphi E_1 - \frac{d}{d_1}(\varphi E_1 - \varphi D_2) \text{ for } \alpha = \frac{b}{d_1}$$

$$\varphi C_2' = \varphi C_2 + \frac{d'}{d_2}(\varphi D_2 - \varphi C_2) \text{ for } \alpha = \frac{b}{d_2}$$

$\varphi C_2'$ will be further corrected for the interference of the pile line. This will be explained later.

Form (3).

FIG. 10.4



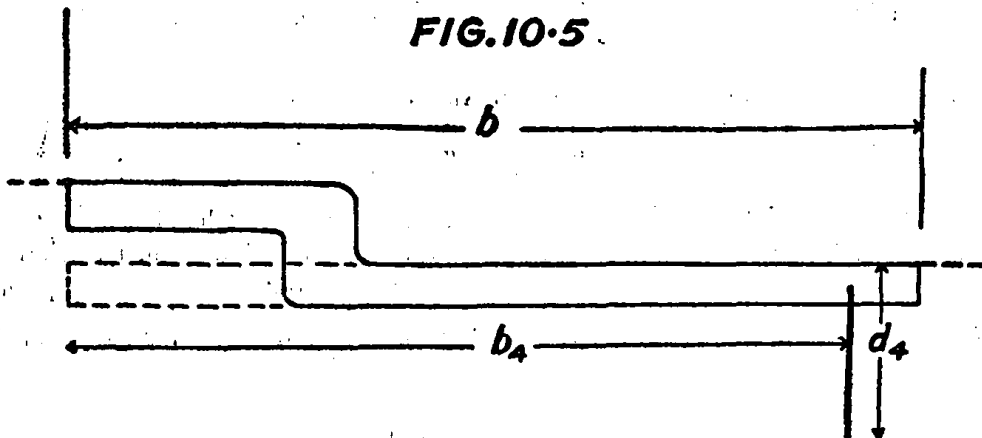
This case is similar to the above except that the floor of length b is assumed horizontal at the downstream level and all pile lines except d_3 are considered non-existent.

Read φE_3 and φC_3 from the φC curves, and φD_3 from the φD curves (Plate VII/6) for $\alpha = \frac{b}{d_3}$ and base ratio = b_3/b .

$\varphi E_3'$ and $\varphi C_3'$ can be determined as for case (b). These have to be corrected for interference of piles d_2 and d_4 respectively.

Form (4).

FIG. 10.5



This case is similar to (3) except that all pile lines except d_4 are assumed as non-existent.

Read φE_4 and φC_4 from the φC curves, and φD_4 from the φD curves, for $\alpha = b/d_4$ and base ratio = b_4/b .

Determine $\varphi E_4'$ and $\varphi C_4'$ as before. $\varphi E_4'$ will have to be corrected for interference of pile d_3 .

Mutual Interference of Piles.

The mutual interference of piles is given by a simple formula :—

$$C = 19 \sqrt{\frac{D}{b'}} \cdot \frac{d+D}{b} \quad \dots \quad (10.1)$$

where C = the correction to be applied as percentage of head,

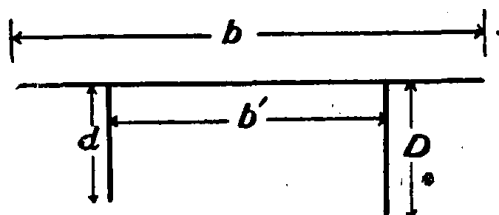
b' = distance between the two piles,

D = the depth of pile whose influence has to be determined on the neighbouring pile of depth (d),

d = depth of pile on which the effect of pile (D) is sought to be determined, and

b = total floor length.

FIG. 10.6



This correction (C) is additive for points in the rear or back-water and subtractive for points forward in the direction of flow. This equation gives results within about $2\frac{1}{2}$ per cent of those obtained by experiments and almost exactly those obtained from theory by Malhotra for the case of two equal piles at either end (see Table VII/A/1 of Appendix to Chapter VII).

But this equation does not apply to the effect of an outer pile on an intermediate pile if the latter is equal to or smaller than the former and is at a distance less than twice the length of the outer pile. Subject to these limitations¹ the equation 10.1 can be applied to find the influence of outer piles on intermediate ones and *vice versa* irrespective of their depth or spacing.

The mutual influence of piles is local as already stated in Chapter IX. It mainly extends to a distance equal to the depth of the pile beyond which it gradually falls off till the residual effect at twice that distance is negligible in most cases. Where the spacing is close and the depth of piles great, the residual effect may be considerable. The above equation gives this influence for all cases. It takes note of the depth of either pile, of the distance between the two and the floor length.

Malhotra² has offered a mathematical justification for the formula of mutual interference and for the method of independent variables for the case of two equal piles at head and toe of a floor.

The mutual interference of pile lines in the above case (Figure 10.1) will be as below :—

$$\text{at } C_2'; C = + 19 \sqrt{\frac{d_3 - d'}{b_3 - b_2}} \cdot \frac{(d_2 - d') + (d_3 - d')}{b}$$

$$\text{where } d_2 - d' = C_2' D_2$$

$$\text{and } d_3 - d' = E_3' D_3.$$

Where the floor is level, this correction may as well be applied at C_2 by taking the equation

$$C = + 19 \sqrt{\frac{d_3}{b_3 - b_2}} \cdot \frac{d_2 + d_3}{b}$$

and then applying the correction for floor thickness to obtain pressure at C_2 . The former method is, however, more suitable.

(1) When such limitations exist, the solution can be found from the Appendix to this Chapter (p.147), which forms a summary of the Punjab Irrigation Research Publication, Vol. II, No. 22. "Notes on Khosla's Principle of Independent Variables II—Pressure Distribution under a floor with equal end piles and a varying intermediate pile" by J. K. Malhotra and E. McKenzie Taylor.

(2) Memoir of the Punjab Irrigation Research Institute, Vol. II, No. 20, by J. K. Malhotra, and E. McKenzie Taylor. "Notes on Khosla's Principle of Independent Variables I—Mutual Interference of equal piles at ends of a floor." See also page 89 of this Publication.

Similarly, the correction for the point E_3' for interference of pile d_2 will be

$$C = -19 \sqrt{\frac{d_2 - d'}{b_3 - b_2} \cdot \frac{(d_2 - d') + (d_3 - d')}{b}}$$

The correction at C_3' will be

$$C = +19 \sqrt{\frac{d_4 - d'}{b_4 - b_3} \cdot \frac{(d_3 - d') + (d_4 - d')}{b}}$$

$$\text{and at } E_4', C = -19 \sqrt{\frac{d_3 - d'}{b_4 - b_3} \cdot \frac{(d_3 - d') + (d_4 - d')}{b}}$$

where $d_3 - d' = C_3' D_3$ and $d_4 - d' = E_4' D_4'$

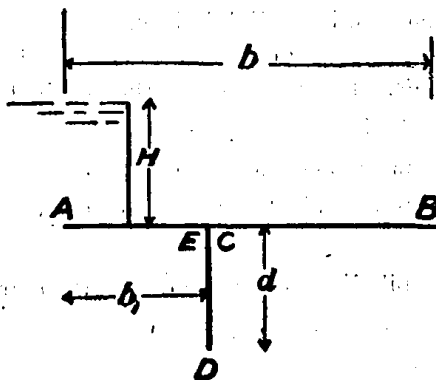
Method of using Curves of Plate VII/6.

Curves for φC and φE .

It has been explained in Chapter IV that the stream lines and pressure lines will remain unaltered under a work whether the head is applied from the upstream or the downstream end. The only change will be in the values to be assigned to the equi-pressure lines.

If $\varphi C = \frac{P_c}{H} \times 100$, be the pressure percentage at any point, with the flow reversed the pressure percentage at this point will become $100 - \varphi C = (1 - \frac{P_c}{H})100$. In other words an equi-pressure line which indicated 5 per cent with normal flow will indicate $(100 - 5) = 95$ per cent with reversed flow. This property of reversibility simplifies the work of plotting curves. For example, assume the flow as in Figure 10.7 from A to B. The values of φC can be directly obtained from the φC curves of Plate VII/6, for values of $\alpha = \frac{b}{d}$ and base ratio $\frac{b_1}{b}$

FIG. 10.7



To obtain the value of φE , assume the flow to be reversed. The rear point C will now become the facing point. φE will read the same as φC with a base ratio of $\frac{b - b_1}{b}$. But the head in this case is negative. Hence φE for base ratio $\frac{b_1}{b} = 100 - \varphi C$ for base ratio $\frac{b - b_1}{b}$.

The curves of φD are exactly symmetrical with respect to base ratio $\frac{b_1}{b} = 0.5$. Hence to save space only one-half of them have been plotted. For the base ratios 0.5 to 1, φD can be read direct. For those less than 0.5, φD can be determined from those curves by making use of the principle of reversibility. Thus for base ratio of $\frac{b_1}{b} = 0.4$, φD should be read at base ratio $1 - \frac{b_1}{b} = 0.6$ and the reading subtracted from 100. The same principle of reversibility helps in obtaining values of φC and φD for

the case where the pile line is at the downstream end and values of $\phi D'$, for the depressed floor. (See curves at top right-hand of Plate VII/6.)

The above instructions have been noted on Plate VII/6.

Tables X/1, X/2 and X/3 show the comparative values of ϕ at the key points :—

- (a) as read off the curves of plate VII/6,
- (b) as corrected by Equation 10.1,
- (c) as obtained from theory by Malhotra for cases of equal piles at either end,
- (d) as obtained from the electric model by Vaidhianathan.

The agreement between (b) and (d) is within very close limits except in cases which come under exceptions mentioned above in dealing with the mutual interference of pile lines.

For the cases of equal pile lines at either end which have been theoretically worked out, the differences between the corrected and theoretical values are no more than 0.2 to 0.3 per cent whereas the differences between the theoretical and experimental values vary up to 2.5 per cent. This shows that the corrected values, being in closer agreement with the theoretical ones, are more reliable than the experimental values.

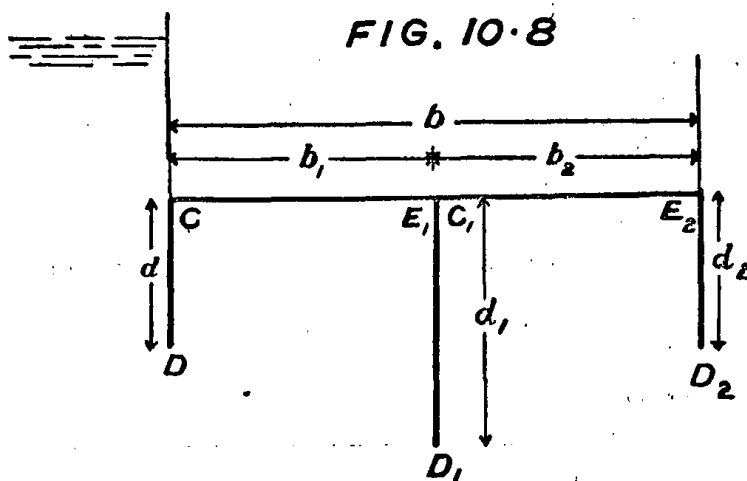


TABLE X/1.

Equal piles at ends, no pile in the middle. Case VIII/a/1 of Chapter IX.

$$b=12. \quad d_1=0. \quad d=d_2.$$

$$d=d_2=\frac{1}{2}.$$

	Curve (a)	Corrected (b)	Theoretical (c)	Experimental (d)
ϕC	81.8	82.1	82.2	79.7
ϕE_2	18.2	17.9	17.8	20.3
$d=d_2=1.$				
ϕC	74.4	75.3	75.4	74.0
ϕE_2	25.6	24.7	24.6	26.4
$d=d_2=1\frac{1}{2}$				
ϕC	68.9	70.6	70.8	69.7
ϕE_2	31.1	29.4	29.2	30.7

$d = d_2 = 2$

	Curve (a)	Corrected (b)	Theoretical (c)	Experimental (d)
φC	64.4	67.0	67.1	66.3
φE_2	35.6	33.0	32.9	34.1

$d = d_2 = 3$

φC	57.0	61.9	62.1	61.3
φE_2	43.0	38.1	37.9	40.0

$d = d_2 = 4$

φC	51.2	58.5	58.6	58.2
φE_2	48.8	41.5	41.4	42.1

TABLE X/2

Unequal piles at ends, no pile in middle. Case VIII/a/2 of Chapter IX.

$b = 12, d_1 = 0$

$d_2 = 2, d = 1$

$d_2 = 3, d = 1$

	Curve (a)	Corrected (b)	Experimental (d)	Curve (a)	Corrected (b)	Experimental (d)
φC	64.4	65.8	64.5	57.0	58.8	57.7
φE_2	25.6	23.7	25.1	25.6	22.4	23.7
$d_2 = 3, d = 2$				$d_2 = 3, d = 3$		
φC	57.0	60.2	59.1	57.0	61.8	61.3
φE_2	35.6	31.6	32.2	43.0	38.3	40.0
$d_2 = 3, d = 4$				$d_2 = 3, d = 5$		
φC	57.0	63.4	63.4	57.0	65.2	65.2
φE_2	48.8	43.3	44.2	53.8	47.5	48.7
$d_2 = 3, d = 6$						
φC	57.0	67.1	67.3
φE_2	57.6	50.5	52.9

Table X/3

Two equal piles at either end, a variable middle pile with varying base ratios. Case VIII (b) of Chapter IX

$b=12. \quad d=d_2=2$

$d_1=0.5$ $b_1/b_2=6/6$				$d_1=1$ $b_1/b_2=6/6$		
	Curve (a)	Corrected (b)	Experimental (d)	Curve (a)	Corrected (b)	Experimental (d)
φC . . .	64.4	65.5	66.5	64.4	66.3	67.3
φE_1 . . .	52.4	50.1	52.3	55.5	52.8	54.1
φC_1 . . .	47.5	49.8	47.8	44.7	47.4	46.3
φE_2 . . .	35.6	34.5	33.7	35.6	33.7	33.3
$d_1=1.5$ $b_1/b_2=6/6$				$d_1=2$ $b_1/b_2=6/6$		
φC . . .	64.4	67.2	67.3	64.4	68.1	67.9
φE_1 . . .	57.8	54.6	55.8	60.1	56.5	57.7
φC_1 . . .	42.3	45.5	44.1	39.7	43.4	42.5
φE_2 . . .	35.6	32.8	32.8	35.6	32.0	32.6
$d_1=3$ $b_1/b_2=6/6$				$d_1=4$ $b_1/b_2=6/6$		
φC . . .	64.4	70.0	69.3	64.4	72.2	71.4
φE_1 . . .	64.8	60.2	60.9	68.5	63.0	63.9
φC_1 . . .	35.0	39.6	39.3	31.0	36.5	36.6
φE_2 . . .	35.6	30.0	30.9	35.6	27.9	29.1
$d_1=5$ $b_1/b_2=6/6$				$d_1=0.5$ $b_1/b_2=7.5/4.5$		
φC . . .	61.4	74.5	73.0	64.4	65.4	66.8
φE_1 . . .	71.6	65.2	66.4	44.6	42.6	47.0
φC_1 . . .	27.9	34.3	33.9	39.4	42.0	42.8
φE_2 . . .	35.6	25.5	27.5	35.6	34.3	33.2
$d_1=1$ $b_1/b_2=7.5/4.5$				$d_1=1.5$ $b_1/b_2=7.5/4.5$		
φC . . .	64.4	66.1	68.8	64.4	66.9	67.1
φE_1 . . .	47.4	45.0	48.8	50.0	47.1	50.3
φC_1 . . .	36.4	39.6	41.1	34.0	37.7	39.5
φE_2 . . .	35.6	33.4	33.1	35.6	32.4	32.1
$d_1=2$ $b_1/b_2=7.5/4.5$				$d_1=3$ $b_1/b_2=7.5/4.5$		
φC . . .	64.4	67.7	67.5	64.4	69.4	68.7
φE_1 . . .	58.0	49.7	52.3	58.0	53.9	55.4
φC_1 . . .	31.8	36.0	37.7	27.6	32.0	35.1
φE_2 . . .	35.6	31.4	31.7	35.6	29.1	29.5

Table X/3—Contd.

$d_1=4$ $b_1/b_2=7.5/4.5$				$d_1=5$ $b_1/b_2=7.5/4.5$		
	Curve. (a)	Corrected. (b)	Experimental. (d)	Curve. (a)	Corrected. (b)	Experimental. (d)
φC	64.4	71.3	70.5	64.4	73.0	72.1
φE_1	62.3	57.4	59.3	66.5	60.8	62.6
φC_1	21.0	30.3	31.7	21.6	29.0	28.4
φE_2	35.6	26.6	27.9	35.6	24.0	25.6
$d_1=0.5$ $b_1/b_2=9/3$				$d_1=1$ $b_1/b_2=9/3$		
φC	64.4	65.3	66.5	64.4	66.0	66.5
φE_1	36.3	34.4	42.3	39.5	37.3	43.2
φC_1	30.2	33.4	38.3	27.3	31.2	36.1
φE_2	35.6	34.0	33.3	35.6	32.9	32.1
$d_1=1.5$ $b_1/b_2=9/3$				$d_1=2$ $b_1/b_2=9/3$		
φC	64.4	66.6	66.6	64.4	67.3	67.2
φE_1	42.8	40.2	45.0	46.2	44.0	46.9
φC_1	25.0	29.5	34.8	22.9	28.9	32.6
φE_2	35.6	31.7	31.8	35.6	30.4	30.3
$d_1=3$ $b_1/b_2=9/3$				$d_1=4$ $b_1/b_2=9/3$		
φC	64.4	68.9	68.8	64.4	70.7	69.7
φE_1	51.7	47.9	50.5	56.8	52.3	54.3
φC_1	19.2	25.7	28.9	16.5	24.3	26.1
φE_2	35.6	27.7	27.9	35.6	24.7	25.0
$d_1=5$ $b_1/b_2=9/3$				$d_1=0.5$ $b_1/b_2=10.5/1.5$		
φC	64.4	72.6	71.7	64.4	65.3	66.8
φE_1	61.4	56.2	58.6	27.0	25.3	32.3
φC_1	14.8	23.9	23.3	18.8	23.4	33.0
φE_2	35.6	21.3	22.7	35.6	33.4	34.0
$d_1=1$ $b_1/b_2=10.5/1.5$				$d_1=1.5$ $b_1/b_2=10.5/1.5$		
φC	64.4	65.9	66.7	64.4	66.5	66.7
φE_1	31.5	29.4	38.2	35.8	33.0	40.0
φC_1	16.3	21.8	31.7	14.0	20.4	30.0
φE_2	35.6	31.8	31.8	35.6	30.1	30.2

Table X/3—Contd.

$d_1=2$ $b_1/b_2=10.5/1.5$				$d_1=3$ $b_1/b_2=10.5/1.5$		
	Curve (a)	Corrected (b)	Experimental (d)	Curve (a)	Corrected (b)	Experimental (d)
φC	64.4	67.2	67.0	64.4	68.6	67.9
φE_1	39.8	37.0	41.5	46.5	43.1	45.0
φC_1	12.4	19.7	28.3	10.2	19.3	23.7
φE_2	35.6	28.3	28.9	35.6	24.4	24.0
$d_1=4$ $b_1/b_2=10.5/1.5$				$d_1=5$ $b_1/b_2=10.5/1.5$		
φC	64.4	70.2	70.0	64.4	72.0	71.4
φE_1	52.3	48.2	49.9	57.0	52.2	54.2
φC_1	8.4	19.4	20.5	7.2	20.0	17.4
φE_2	35.6	20.1	20.7	35.6	15.4	17.7

Table X/4 gives the comparative values of φ at the corner points for a stepped floor. See general case of Chapter IX. The agreement in this case is as satisfactory as in the previous cases.

Table X/4

δ	δ_1	δ_2	φE		φD			φC	
			Experimental	As computed	Experimental	Computed from U/S	Computed from D/S	Experimental	As computed
$\frac{d_2}{d_1-d_2}$	$\frac{b_1}{d_1-d_2}$	$\frac{b_2}{d_1-d_2}$							
4.0	4.0	0.0	77.0	77.0	43.4	45.8	44.3	0.0	0.0
4.0	4.0	1.0	77.1	77.2	44.0	46.3	45.0	6.8	7.1
4.0	4.0	2.0	77.4	77.5	45.6	47.0	46.0	12.9	13.8
4.0	4.0	3.0	78.2	78.0	47.1	48.3	48.0	18.7	20.0
4.0	4.0	4.0	78.6	78.5	48.9	50.0	50.0	24.2	24.9
4.0	4.0	5.0	79.3	79.2	50.5	51.5	52.1	27.9	29.9
4.0	4.0	6.0	79.9	79.7	53.3	53.3	53.8	31.9	33.1
4.0	4.0	7.0	80.7	80.0	55.4	54.7	55.6	37.1	36.8
4.0	4.0	8.0	81.7	80.5	56.5	56.0	57.1	39.4	39.3
0.0	7.0	7.0	56.7	54.6	53.7	50.0	50.0	53.7	53.7
0.5	7.0	7.0	58.4	56.8	52.0	50.0	50.0	48.2	47.8
1.0	7.0	7.0	60.2	58.9	51.4	50.0	50.0	45.6	45.4
1.5	7.9	7.0	62.1	61.0	51.0	50.0	50.0	43.3	43.2
2.0	7.0	7.0	63.2	63.0	50.6	50.0	50.0	40.4	41.2
3.0	7.0	7.0	66.9	67.0	50.3	50.0	50.0	36.2	37.0
4.0	7.0	7.0	69.7	70.0	49.8	50.0	50.5	32.3	33.0
5.0	7.0	7.0	72.6	73.0	49.5	50.0	50.0	29.2	30.0

Sloping Floors

Both theory and experiments have, so far, been confined to an investigation of horizontal floors and vertical sheet piles, as these are the elements of most common occurrence. A theoretical investigation of the sloping floors or glacis has been made by Malhotra¹. The results are plotted in Plate X.I. The general conclusion from these curves is that the pressure percentages under a floor sloping down or sloping up in the direction of flow are respectively greater or less than those under a horizontal floor for the same base ratios.

In the method of independent variables this can be allowed for by the application of a suitable percentage correction for the sloping glacis, the correction being *plus* for the down and *minus* for the up slopes following the direction of flow.

For a sloping floor the lengths of pile D in the equation 10.1 giving the mutual interference of piles, will be the length below the level at pile (d) at which the interference is sought to be calculated. The corrections for a sloping floor will be best explained by an example.

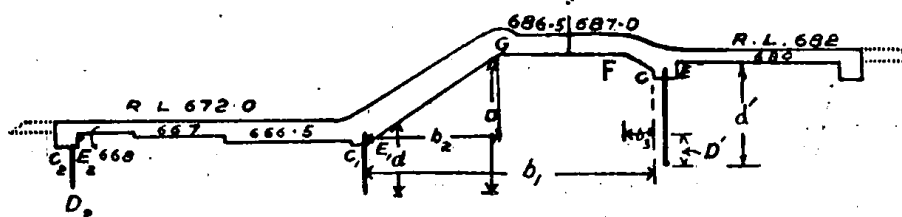


Figure 10.9.

Correction for up slope C F.

Up slope of underside of floor at $C=1:1$.From Fig. 3 Plate X.I for $1:1$ slope correction = 11.2%.Length of slope = b_3 (along horizontal).Spacing of piles = b_1 .Correction at C & F = $-b_3/b_1 \times 11.2\%$.Correction for down slope $G E_1 \frac{1}{2}$ to 1.

Similarly correction at E_1 & G = $+b_3/b_1 \times 6.5\%$ as the correction for $\frac{1}{2}:1$ slope is 6.5% (Fig. 3 Pl. X.1).

The influence of pile d' on pile d is given by

$$c = -19 \sqrt{\frac{D'}{b_1} \frac{D'+d}{b}}$$

and that of pile d on pile d' by

$$c = +19 \sqrt{\frac{D}{b_1} \frac{D+d'}{b}}$$

The Method of Independent Variables in relation to Exit Gradients.

The exit gradients can be simply determined from Plate VII.5 even though the level of the pervious bed downstream of the D. S. pile line may be below or above the level of the impervious floor.

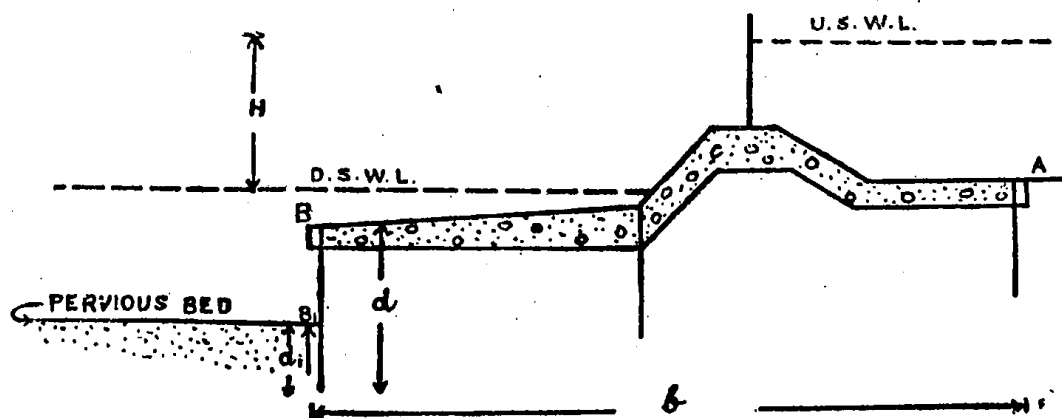


Figure 10.9 (a).

(1) Punjab Irrigation Research Institute Memoir, Vol. II, No. 18. "Uplift pressures under a sloping floor" by J. K. Malhotra,

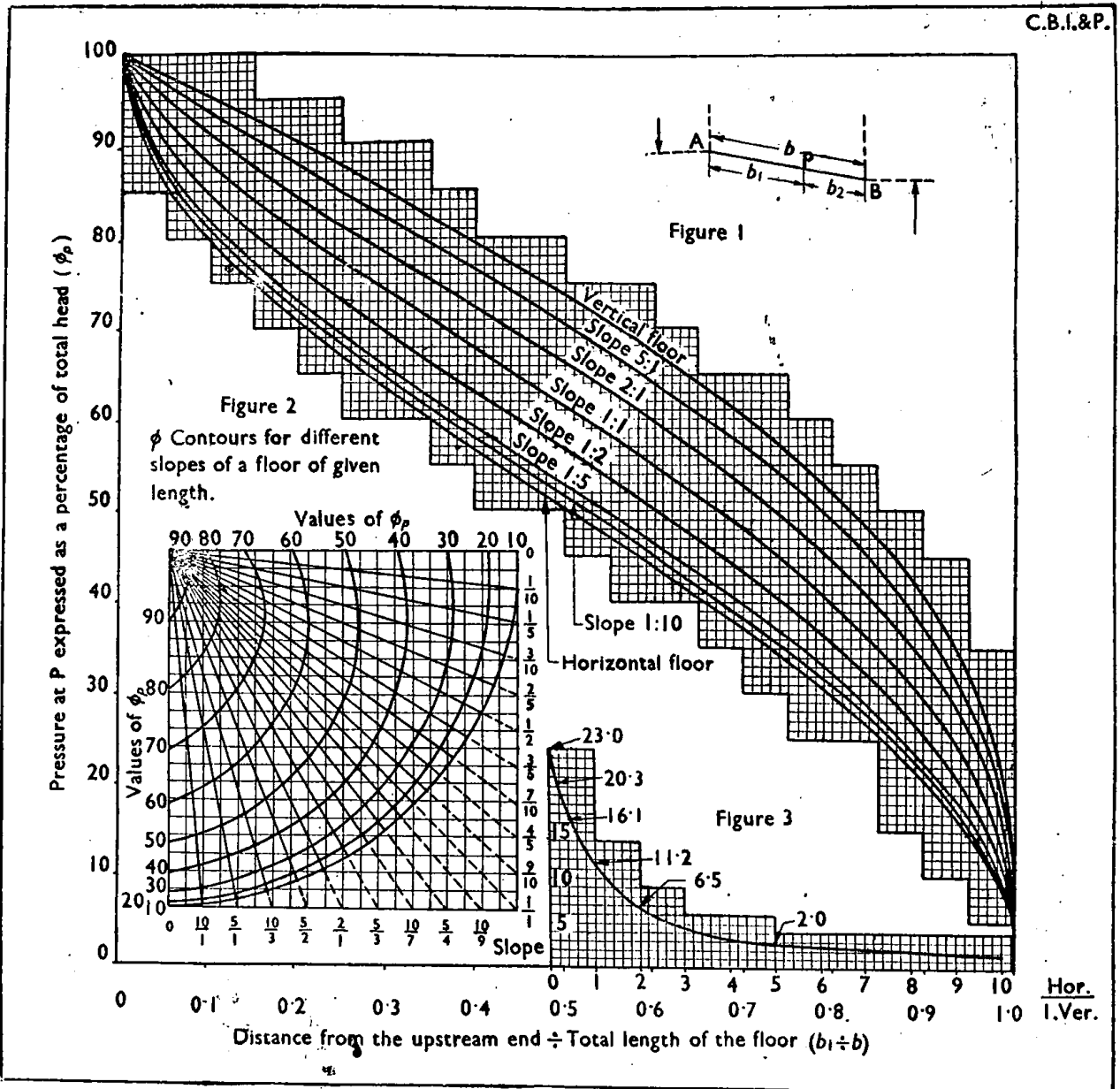


PLATE X.1 ; Showing Pressure Distribution under floors with different slopes

Figure 10.9 (a) shows a typical section of a weir floor. Assume that the downstream bed has scoured down to a depth $(d-d_1)$. To find the exit gradient at B_1 , assume the downstream pile line to be of depth d_1 instead of d , and the floor length b as usual. Then $\alpha = \frac{b}{d_1}$.

Read value of $\frac{1}{\pi\sqrt{\lambda}}$ for this value of α . The exit gradient at B_1 will then be :—

$$G_E \text{ at } B_1 = \frac{H}{d_1} \cdot \frac{1}{\pi\sqrt{\lambda}} \text{ for } \alpha = \frac{b}{d_1}$$

This exit gradient can be more accurately obtained from Plates VI/3C and VII/4 but the difference is small and for all practical purposes, the simpler curves of Plate VII/5 will do.

The method of independent variables, thus offers a complete and accurate solution both in respect of uplift pressures and exit gradients for any complicated foundation profile. The pressure under the floor at the key points of any component standard form explained on page 130, can be simply read off the curves of Plate VII/6 and corrected for the interference of the adjoining sheet piles or sloping floors from Equation 10-1 and Plate X/1 respectively. The exit gradient can be obtained from Plate VII/5.

Thus between Plates VII/5, VII/6 and X/1 and Equation 10-1, it is now possible to determine, with almost mathematical accuracy, the uplift pressure under and exit gradients at the end of the floor of any complicated weir section.

The Application of the New Method to investigate pressures under some actual weirs.

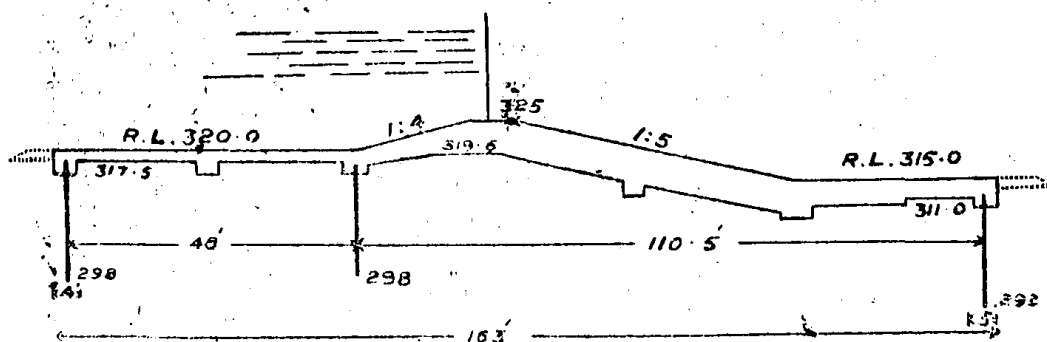
The following typical weir sections will be investigated:—

- (a) Panjnad Weir.
- (b) Khauki Weir—Bay 8,
 - Case I.—Upstream apron pervious.
 - Case II.—Upstream apron impervious.
- (c) Deg Escape Head.
- (d) Lloyd Barrage.
- (e) Proposed Kalabagh Weir (Indus River).

The actual and experimental values of uplift pressure under *a, b, c* and *d* have been compared in Chapter VI.

The values of uplifts have also been calculated for a large number of other cases including several cases of the proposed Trimmu Weir. The experimental values have in all cases been obtained by the Electric Method by Vaidhianathan and Gurdas Ram. The agreement in all these cases is as good as in the cases worked out in detail in the following pages.

FIG. 10.10
PANJNAD WEIR



Length of floor = $b = 163'$.

Upstream pile line.

$$d = 320 - 298 = 22'$$

$$1/\alpha = 22/163 = 0.135$$

$$\phi_D = 77.7\%$$

$$\phi_C = 68\%$$

$$\phi_D - \phi_C = 9.7\%$$

$$\phi_C \text{ corrected} = 68 + 1.1 + 2.9 = 72.0\%$$

$$\phi_C \text{ correction for depth} = \frac{320 - 317.5}{32} \times 9.7$$

$$= +1.1\%$$

Interference of 2nd pile line

$$D = d = 317.5 - 298 = 19.5'$$

$$C = 19 \sqrt{19.5/48} \times \frac{19.5 + 19.5}{163} = +2.9\%$$

Intermediate pile line

$d = 320 - 298 = 22'$, $b/d = 163/22 = 7.4 = \alpha$
 $b_1 = 50$, $b_1/b = 50/163 = 0.307$.
 $\varphi E_1 = 71.4\%$.
 $\varphi D_1 = 61.4\%$.
 $\varphi C_1 = 53.2\%$.
 $\varphi E_1 - \varphi D_1 = 10\%$.
 $\varphi D_1 - \varphi C_1 = 8.2\%$.
 φE_1 corrected $= 71.4 - 1.1 - 2.9 = 67.4\%$.
 φC_1 corrected $= 53.2 + 0.9 + 2.5 - 0.24 = 56.4\%$.

φE_1 correction for depth $= \frac{320 - 317.5}{22} \times 10$
 $= -1.1\%$.

φC_1 correction for depth $= 2.5/22 \times 8.5$
 $= +0.9\%$.

Interference of upstream pile line on φE_1
 $= -2.9\%$, as the two pile lines are equal and placed at the same level.

Interference of downstream pile on φC_1
 $D = 317.5 - 292 = 25.5'$
 $d = 317.5 - 298 = 19.5'$

$C = 19 \sqrt{25.5/110.5} \times \frac{25.5 + 19.5}{163} = +2.5\%$.

φC_1 correction for slope.

slope 1 : 4, correction $= -3.3\%$.

(Plate X.I. Figure 3.)

Length of slope 8' and spacing between piles $= 110.5'$.

Correction $= 8/110.5 \times 3.3 = -0.24\%$.

Downstream pile line

$d = 315 - 292 = 23'$.
 $1/\alpha = d/b = 23/163 = 0.143$.
 $\varphi D_2 = 23\%$.
 $\varphi E_2 = 33\%$.
 $\varphi E_2 - \varphi D_2 = 10\%$.
 φE_2 corrected $= 33 - 1.7 - 1.3 = 30\%$.

φE_2 correction for depth $= \frac{315 - 311}{23} \times 10$
 $= -1.7\%$.

Interference of intermediate pile line.

$D = 311 - 298 = 13'$

$d = 311 - 292 = 21'$

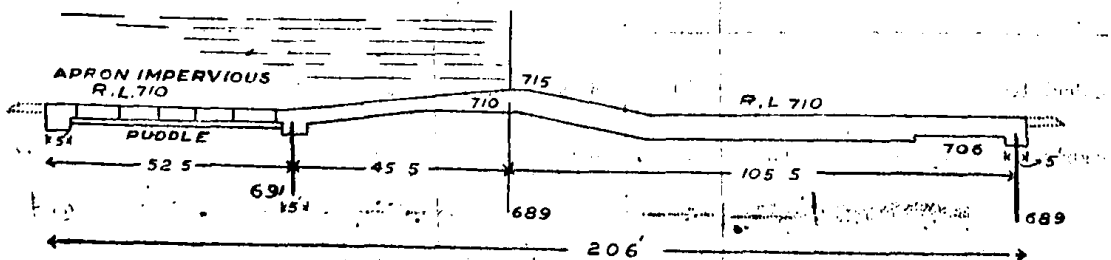
$C = 19 \sqrt{13/110.5} \times \frac{13 + 21}{163} = -1.3\%$.

Comparative Table

	U. S. Pile		Middle Pile.			D. S. Pile.	
	φD	φC	φE_1	φD_1	φC_1	φE_2	φD_2
Calculated	77.7	72.0	67.4	61.4	56.4	30.0	23.0
Experimental	75.8	70.4	64.0	58.7	54.2	30.8	23.0
Difference	1.9	1.6	3.4*	2.7	2.2	0.8	0.0

* This relatively large difference is due to the actual pressure point in the experiment being at R.L. 315 instead of 317.5. This will reduce the value of φD_1 by 1.2%, so that the actual difference is no more than 2.4%.

FIG. 10.11.
KHANKI WEIR



The upstream apron, in this case, though not part of the permanent concrete floor, is nevertheless impervious as the blocks rest on stone bedded in puddle. The design of this weir is discussed later in the chapter.

Bottom of upstream toe wall
 Depression = $d = 710 - 705 = 5'$.
 $1/\alpha = d/b = 5/206 = .024$.
 $\phi_{D1} = 94\%$.

Upstream pile line.

$d = 710 - 691 = 19'$.
 $\alpha = 206/19 = 10.8$.
 $b_1/b = 52.5/206 = 0.255$.
 $\phi_E = 73\%$.
 $\phi_D = 65.6\%$.
 $\phi_C = 59.1\%$.
 $\phi_E - \phi_D = 7.4\%$.
 $\phi_D - \phi_C = 6.5\%$.
 ϕ_E corrected = $73 - 2 = 71\%$.
 ϕ_C corrected = $59.1 + 1.7 + 1.6 = 62.4\%$.

ϕ_E correction for depth at R. L. 705.

$$= \frac{710 - 705}{19} \times 7.4 = -2\%$$

ϕ_C correction for depth = $5/19 \times 6.5 = +1.7\%$.

Influence of middle pile on ϕ_C .

$$D = 705 - 689 = 16'$$

$$d = 705 - 691 = 14'$$

$$C = 19 \sqrt{16/45.5} \times \frac{16 + 14}{206} = +1.6\%$$

Middle pile line.

$d = 710 - 689 = 21'$.
 $\alpha = 206/21 = 9.8$.
 $b_1/b = 98/206 = 0.475$.
 $\phi_{E1} = 58.3\%$.
 $\phi_{D1} = 51.5\%$.
 $\phi_{C1} = 45.9\%$.
 ϕ_{E1} corrected = $58.3 - 2.5 = 55.8\%$.
 ϕ_{C1} corrected = $45.9 + 1.7 = 47.6\%$.

No correction for depth necessary in this case as top of pile is taken at R. L. 710.

Influence of upstream pile on ϕ_{E1} .

$$D = 710 - 691 = 19'$$

$$d = 710 - 689 = 21'$$

$$C = 19 \sqrt{19/45.5} \times \frac{19 + 21}{206} = -2.5\%$$

Influence of D. S. pile on ϕ_{C1} .

$$D = 710 - 689 = 21' = d.$$

$$C = 19 \sqrt{\frac{21}{105.5}} \times \frac{21 + 21}{206} = +1.7\%$$

Downstream pile line.

$d = 710 - 689 = 21'$.
 $1/\alpha = 21/206 = 0.1$.
 $\phi_{E2} = 28\%$.
 $\phi_{D2} = 20\%$.
 $\phi_{E2} - \phi_{D2} = 8\%$.
 ϕ_{E2} corrected = $28 - 1.5 - 1.1 = 25.4\%$.

ϕ_{E2} correction for depth at R. L. 706 = $4/21 \times 8 = -1.5\%$.

Influence of middle pile on ϕ_{E2} .

$$D = 706 - 689 = 17'$$

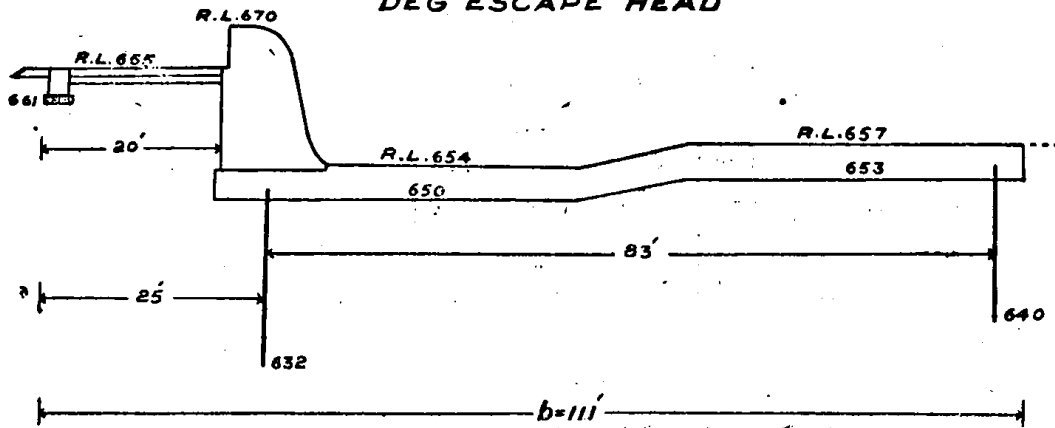
$$d = 706 - 689 = 17'$$

$$C = 19 \sqrt{17/105.5} \times \frac{17 + 17}{206} = -1.1\%$$

Comparative Table

	Toe	U. S. Pile		Middle Pile			D. S. Pile		
	ϕ_D	ϕ_E	ϕ_D	ϕ_C	ϕ_{E1}	ϕ_{D1}	ϕ_{C1}	ϕ_{E2}	ϕ_{D2}
Calculated	94.0	71.0	65.6	62.4	55.8	51.5	47.6	25.4	20.0
Experimental	93.4	72.5	67.4	62.7	58.6	53.0	47.4	25.6	20.0
Difference	0.6	1.5	1.8	0.3	2.8	1.5	0.2	0.2	0.0

FIG. 10-12.
DEG ESCAPE HEAD



Bottom of upstream toe wall.

$$d = 665 - 661 = 4'$$

$$1/\alpha = d/b = 4/111 = 0.036$$

$$\varphi_{D'} = 92\%$$

Junction of upstream apron with drop wall.

$$d = 665 - 632 = 33'$$

$$b_1 = 20$$

$$\alpha = 111/33 = 3.36$$

$$b_1/b = 20/111 = 0.18$$

$$\varphi_{E'} = 87.0\%$$

$$\varphi_D = 64.5\%$$

$$\varphi_{E'} - \varphi_D = 22.5\%$$

$$\varphi_E \text{ corrected} = 87.0 - 1.4 = 85.6\%$$

$$\varphi_E \text{ correction for depth} = 2/33 \times 22.5 = -1.4\%$$

Upstream pile line.

$$d = 657 - 632 = 25'$$

$$b_1 = 25'$$

$$\alpha = 111/25 = 4.44$$

$$b_1/b = 25/111 = 0.225$$

$$\varphi_E = 81.7\%$$

$$\varphi_D = 64.5\%$$

$$\varphi_C = 51.5\%$$

$$\varphi_E - \varphi_D = 17.2\%$$

$$\varphi_D - \varphi_C = 13.0\%$$

$$\varphi_E \text{ corrected} = 81.7 - 4.8 = 76.9\%$$

$$\varphi_C \text{ corrected} = 51.5 + 3.6 + 1.7 = 56.8\%$$

$$\varphi_E \text{ correction for depth} = \frac{657 - 650}{25} \times 17.2 = -4.8\%$$

$$\varphi_C \text{ correction for depth} = 7/25 \times 13 = +3.6\%$$

Interference of downstream pile on φ_C .

$$D = 650 - 640 = 10'$$

$$d = 650 - 632 = 18'$$

$$C = +19 \sqrt{10/83} \times \frac{10+18}{111} = +1.7\%$$

Downstream pile line.

$$d = 657 - 640 = 17'$$

$$b_1 = 108$$

$$\alpha = 111/17 = 6.5$$

$$b_1/b = 108/111 = 0.973$$

$$\varphi_{E_1} = 34.8\%$$

$$\varphi_{D_1} = 24.3\%$$

$$\varphi_{C_1} = 3.0\%$$

$$\varphi_{E_1} - \varphi_{D_1} = 10.5\%$$

$$\varphi_{D_1} - \varphi_{C_1} = 21.3\%$$

$$\varphi_{C_1} \text{ corrected} = 3.0 + 5 = 8\%$$

$$\varphi_{E_1} \text{ corrected} = 34.8 - 2.5 - 2.8 = 29.5\%$$

$$\varphi_{E_1} \text{ correction for depth} = 4/17 \times 10.5 = -2.5\%$$

$$\varphi_{C_1} \text{ correction for depth} = 4/17 \times 21.3 = +5\%$$

Influence of upstream pile on φ_{E_1} .

$$D = 653 - 632 = 21'$$

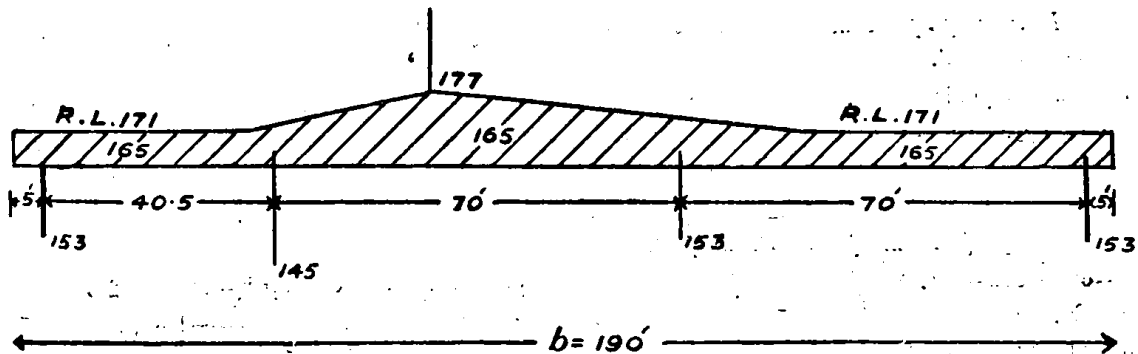
$$d = 653 - 640 = 13'$$

$$C = 19 \sqrt{21/83} \times \frac{21+13}{111} = -2.8\%$$

Comparative Table.

	U/S Toe.	U. S. Pile.					D. S. Pile.		
	$\phi D'$	$\phi E'$	ϕD	ϕE	ϕD	ϕC	ϕE_1	ϕD_1	ϕC_1
Calculated	92.0	85.6	64.5	76.9	64.5	56.8	29.5	24.3	8.0
Experimental	93.0	83.9	64.9	78.4	64.9	55.9	32.6	23.6	8.2
Difference	1.0	1.7	0.4	1.5	0.4	0.9	3.1	0.7	1.8

FIG. 10.13
LLOYD BARRAGE



For upstream and downstream toes of floor.

$$d=6'.$$

$$1/\alpha=6/190=0.032.$$

$$\phi D'=8\% \text{ and } 92\%.$$

Upstream pile line.

$$d=171-153=18'.$$

$$\alpha=190/18=10.6.$$

$$b_1=5', b_1/b=5/190=0.026.$$

$$\phi E=96.3\%.$$

$$\phi D=80.0\%.$$

$$\phi C=72.2\%.$$

$$\phi E-\phi D=16.3\%.$$

$$\phi D-\phi C=7.8\%.$$

$$\phi E \text{ corrected}=96.3-5.4=90.9\%.$$

$$\phi C \text{ corrected}=72.2+2.6+2.3=77.1\%.$$

$$\phi E \text{ correction for thickness}=6/18 \times 16.3 \\ =-5.4\%.$$

$$\phi C \text{ correction for thickness}=6/18 \times 7.8 \\ =+2.6\%.$$

Influence of second pile on ϕC .

$$D=165-145=20'$$

$$d=165-153=12'$$

$$C=19\sqrt{20/40.5} \times \frac{20+12}{190}=+2.3\%.$$

Second pile line.

$$a=171-145=26'.$$

$$b_1=45.5'.$$

$$\alpha=190/26=7.3.$$

$$b_1/b=0.24.$$

$$\phi E_1=77.0\%.$$

$$\phi D_1=65.5\%.$$

$$\phi C_1=57.3\%.$$

$$\phi E_1-\phi D_1=11.5\%.$$

$$\phi D_1-\phi C_1=8.5\%.$$

$$\phi E_1 \text{ corrected}=77.0-2.7-1.8=72.5\%.$$

$$\phi C_1 \text{ corrected}=57.0+2+1.3=60.3\%.$$

$$\phi E_1 \text{ correction for depth}=6/26 \times 11.5 \\ =-2.7\%.$$

$$\phi C_1 \text{ correction for depth}=6/26 \times 8.5=+2\%.$$

Influence of 1st pile on ϕE_1

$$D=165-153=12'$$

$$d=165-145=20'$$

$$C=19\sqrt{12/40.5} \times \frac{12+20}{190}=-1.8\%.$$

Influence of 3rd pile on ϕC_1

$$D=12', d=20'$$

$$C=19\sqrt{12/70} \times \frac{12+20}{190}=+1.3\%.$$

Third pile line.

$d=171-153=18.$
 $b_1=115.5, \alpha=190/18=10.6.$
 $b_1/b=115.5/190=0.61.$
 $\varphi_{E_2}=49.2\%.$
 $\varphi_{D_2}=43.2\%.$
 $\varphi_{C_2}=36.7\%.$
 $\varphi_{E_2}-\varphi_{D_2}=6.0\%.$
 $\varphi_{D_2}-\varphi_{C_2}=6.5\%.$
 $\varphi_{E_2} \text{ corrected}=49.2-2-1.7=45.5\%.$
 $\varphi_{C_2} \text{ corrected}=36.7+2.2+1=39.9\%.$

φ_{E_2} correction for depth= $6/18 \times 6 = -2\%.$
 φ_{C_2} correction for depth= $6/18 \times 6.5 = +2.2\%.$
 Effect of 2nd pile on $\varphi_{E_2}.$
 $D=165-145=20.$
 $d=165-153=12'.$
 $C=19\sqrt{20/70} \times \frac{12+20}{190} = -1.7\%.$
 Effect of 4th pile on $\varphi_{C_2}.$
 $D=12, d=12.$
 $C=19\sqrt{12/70} \times \frac{12+12}{190} = +1\%.$

Fourth pile line.

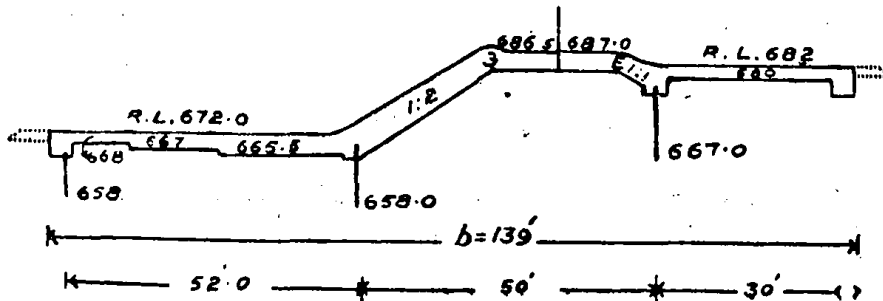
$\varphi_{C_3}=100-\varphi_E=100-90.9=9.1\%.$
 $\varphi_{D_3}=100-\varphi_D=100-80=20\%.$
 $\varphi_{E_3}=100-\varphi_C.$
 Corrected for depth only= $100-74.8$
 $=25.2\%.$
 Corrected for pile effect, $\varphi_{E_3}=25.2-1$
 $=24.2\%.$

Effect of 3rd pile line on $\varphi_{E_2}.$
 $=19\sqrt{12/70} \times \frac{12+12}{190} = -1\%.$

Comparative Table

	Toe		1st Pile Line			2nd Pile Line			3rd Pile Line			4th Pile Line		
	$\varphi_{D'}$		φ_E	φ_D	φ_C	φ_{E_1}	φ_{D_1}	φ_{C_1}	φ_{D_2}	φ_{E_2}	φ_{C_2}	φ_{E_3}	φ_{D_3}	φ_{C_3}
Calculated.	92.0	8.0	90.9	80.0	77.1	72.5	65.5	60.3	45.5	43.2	39.9	24.2	20.0	9.1
Experimental.	94.0	6.4	91.6	81.9	76.1	70.8	64.2	58.3	46.4	43.1	39.4	24.5	19.3	8.3
Difference.	2.0	1.6	0.7	1.9	1.0	1.7	1.3	2.0	0.9	0.1	0.5	0.3	0.7	0.8

FIG. 10-14
KALA BAGH WEIR
 (PROPOSED)



$b=139'.$

Upstream toe of floor,

$d=682-677=5'.$
 $1/\alpha=5/139=0.036.$
 $\varphi_{D'}=92.0\%.$

First pile line.

$$d = 682 - 667 = 15'$$

$$b_1 = 34'$$

$$\alpha = 139/15 = 9.26$$

$$b_1/b = 34/139 = 0.244$$

$$\varphi_E = 74.7\%$$

$$\varphi_D = 66.3\%$$

$$\varphi_C = 59.0\%$$

$$\varphi_E - \varphi_D = 8.4\%$$

$$\varphi_D - \varphi_C = 7.3\%$$

$$\varphi_E \text{ corrected} = 74.7 - 1.1 = 73.6\%$$

$$\varphi_C \text{ corrected} = 59.0 + 1.5 + 3 - 0.9 = 62.6\%$$

$$\varphi_E \text{ correction for depth} = 2/15 \times 8.4 = -1.1\%$$

$$\varphi_C \text{ correction for depth} = 3/15 \times 7.3 = +1.5\%$$

Influence of 2nd pile on φ_C .

$$D = 679 - 658 = 21'$$

$$d = 679 - 667 = 12'$$

$$C = 19\sqrt{21/50} \times \frac{12+12}{139} = +3\%$$

φ_C correction for slope.

Slope 1 in 1.3 upwards.

Correction = -9.2%

Proportional length = 5/50 = 1/10.

φ_C correction = 9.2, 10 = -0.9%.

Second pile line.

$$d = 672 - 658 = 14'$$

$$b_1 = 84$$

$$\alpha = 139/14 = 9.9$$

$$b_1/b = 84/139 = 0.6$$

$$\varphi_{E_1} = 50.4\%$$

$$\varphi_{D_1} = 44\%$$

$$\varphi_{C_1} = 37\%$$

$$\varphi_{E_1} - \varphi_{D_1} = 6.4\%$$

$$\varphi_{D_1} - \varphi_{C_1} = 7\%$$

$$\varphi_{E_1} \text{ corrected} = 50.4 - 2.7 + 3.4 = 51.2\%$$

$$\varphi_{C_1} \text{ corrected} = 37 + 3 + 0.7 = 40.7\%$$

$$\varphi_{E_1} \text{ correction for depth} = \frac{672-666}{14} \times 6.4 = -2.7\%$$

$$\varphi_{C_1} \text{ correction for depth} = 6/14 \times 7 = +3\%$$

Effect of 1st pile on φ_{E_1} = nil, as the bottom of the latter is at R. L. 667 and φ_{E_1} relates to 666.

φ_{E_1} correction for slope.

Slope = 1/1.4 upwards; correction = 9%.

Proportion of length = 19/50.

$$\varphi_{E_1} \text{ correction for slope} = \frac{9 \times 19}{50} = 3.4\%$$

Effect of 3rd pile on φ_{C_1} .

$$D = 666 - 658 = 8'$$

$$d = 666 - 658 = 8'$$

$$C = 19\sqrt{8/52} \times \frac{8+8}{139} = +0.7\%$$

Third or D/S pile line.

$$d = 672 - 658 = 14'$$

$$\alpha = 14/139 = 0.1$$

$$\varphi_{E_1} = 28.3$$

$$\varphi_{D_2} = 19.6$$

$$\varphi_E - \varphi_{D_2} = 8.6\%$$

$$\varphi_{E_2} \text{ corrected} = 28.3 - 3.6 - 0.7 = 24.0\%$$

$$\varphi_{E_2} \text{ correction for depth} = \frac{672-666}{14} \times 8.6 = -3.6\%$$

$$\varphi_{E_2} \text{ correction for 2nd pile line} = -0.7\%$$

Comparative Table

	Toe	1st Pile			2nd pile			3rd Pile	
		φ_D'	φ_E	φ_D	φ_C	φ_{E_1}	φ_{D_1}	φ_{C_1}	φ_{E_2}
Calculated	92.0	73.6	66.3	62.6	51.2	44.0	40.7	24.0	19.6
Experimental	91.8	74.6	67.7	62.4	52.6	47.0	43.3	27.0	21.3
Difference	0.2	1.0	1.4	0.2	1.4	3.0	2.6	3.0	1.7

The above illustrations demonstrate the soundness of the method of independent variables in determining uplift pressures under weirs.

Design of New Works

As stated before there are two main factors for consideration in design of weirs :—

- (a) Uplift Pressures, and
- (b) Exit Gradients.

The uplift pressures will be provided against by a suitable thickness of floor at different points and the exit gradients by a suitable depth of pile line at the downstream end of the floor.

The *depth of the downstream pile line* will be determined from two considerations :—

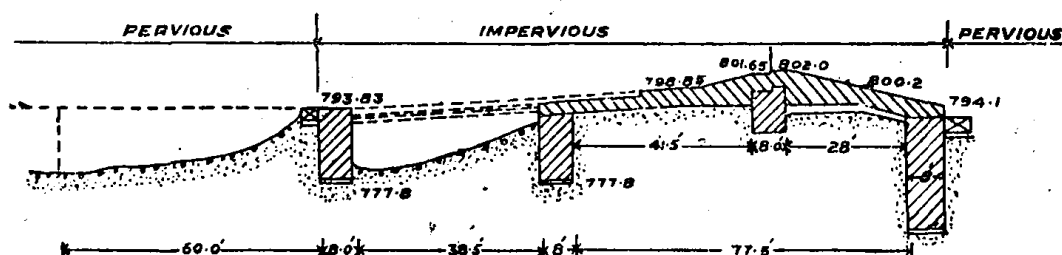
- (a) that with a suitable length of floor, it gives a safe exit gradient for a given head (Chapter VIII), and
- (b) that its bottom is nearly at or below the level of normal flood scour for that section of the work for which the depth of piles is being determined.

There are occasions when the flexible apron downstream of the downstream pile line and upstream of the upstream pile line may be washed away, and the scour may extend very close to the piles. If the bottom of these piles is at a high level, the subsoil behind the piles may run out by simple earth pressure. This can happen at both the upstream and the downstream piles irrespective of any consideration of the exit gradients.

The failure (in 1895) of Bay¹ of the Khanki Weir¹ occurred from upstream scour. The extensive damage to the Rasul Weir Bays 2, 3, 4 and 5² (August 1929) occurred mostly from the same cause. The damage to the Deoha Barrage (September 1929) occurred presumably from a combination of scour upstream and undermining downstream³. The failure of the Islam Weir⁴ (September 1929) occurred by the retrogression of the downstream river bed and flood scour at the downstream end which accelerated the undermining of the subsoil under the downstream floor. This floor ends in a shallow curtain wall and has no deep sheet piling.

A typical case of dangerous flood scour is shown in Figure 10-15. This occurred during the record flood of 1928 in the Chenab River.

FIG. 10-15
MERALA WEIR SEPT: (1928)



The scour of September 1934 at the Panjad Weir (Bay 44) which extended right up to the downstream line of wells was reproduced on a model and is shown by the Photograph 10-1. If this scour had been more accentuated, it might have caused damage to the line of wells and perhaps to the inverted filter.

The magnitude of the scour upstream or downstream is obviously as important as that of the exit gradient.

Depth of Scour.

The depth of scour for any discharge per foot run may be worked out from Kennedy's silt formula.

$$V_0 = 0.84 d^{0.64}$$

which applies to Punjab conditions. It can be modified to suit other grades of bed sands.

(1) History of Khanki Headworks, Punjab Irrigation Branch Publication.

(2) "Repairs and Remodelling of Rasul Weir (1929-1931)" by T. A. W. Foy, P. E. C. Paper No. 155 (1932).

(3) "Design and Construction of the Sarda Canal" by Sir Bernard Darley, Kt., C. I. E., Paper No. 4814, Institute of Civil Engineers, London.

(4) Report by the Islam Enquiry Committee into the Failure of the Islam Weir, Punjab Irrigation Branch Publication (1930).

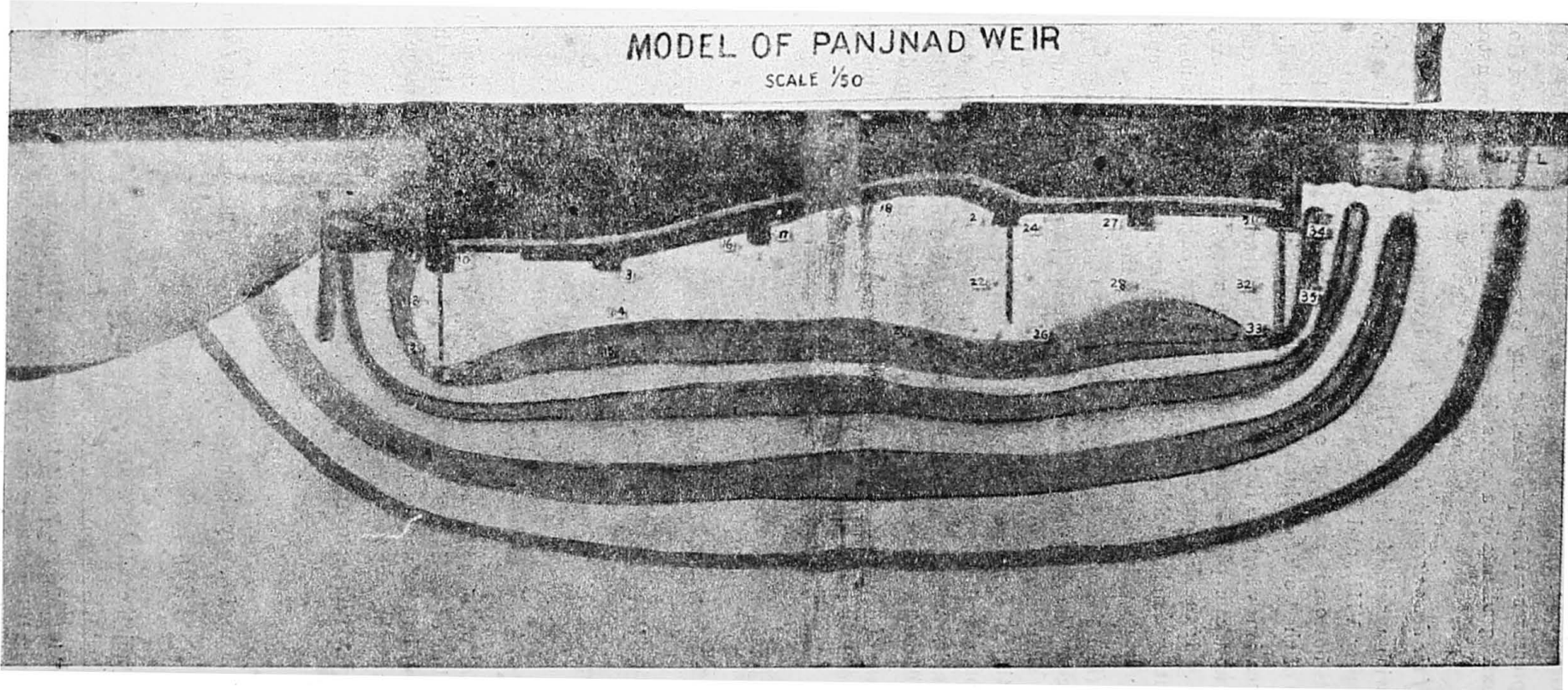
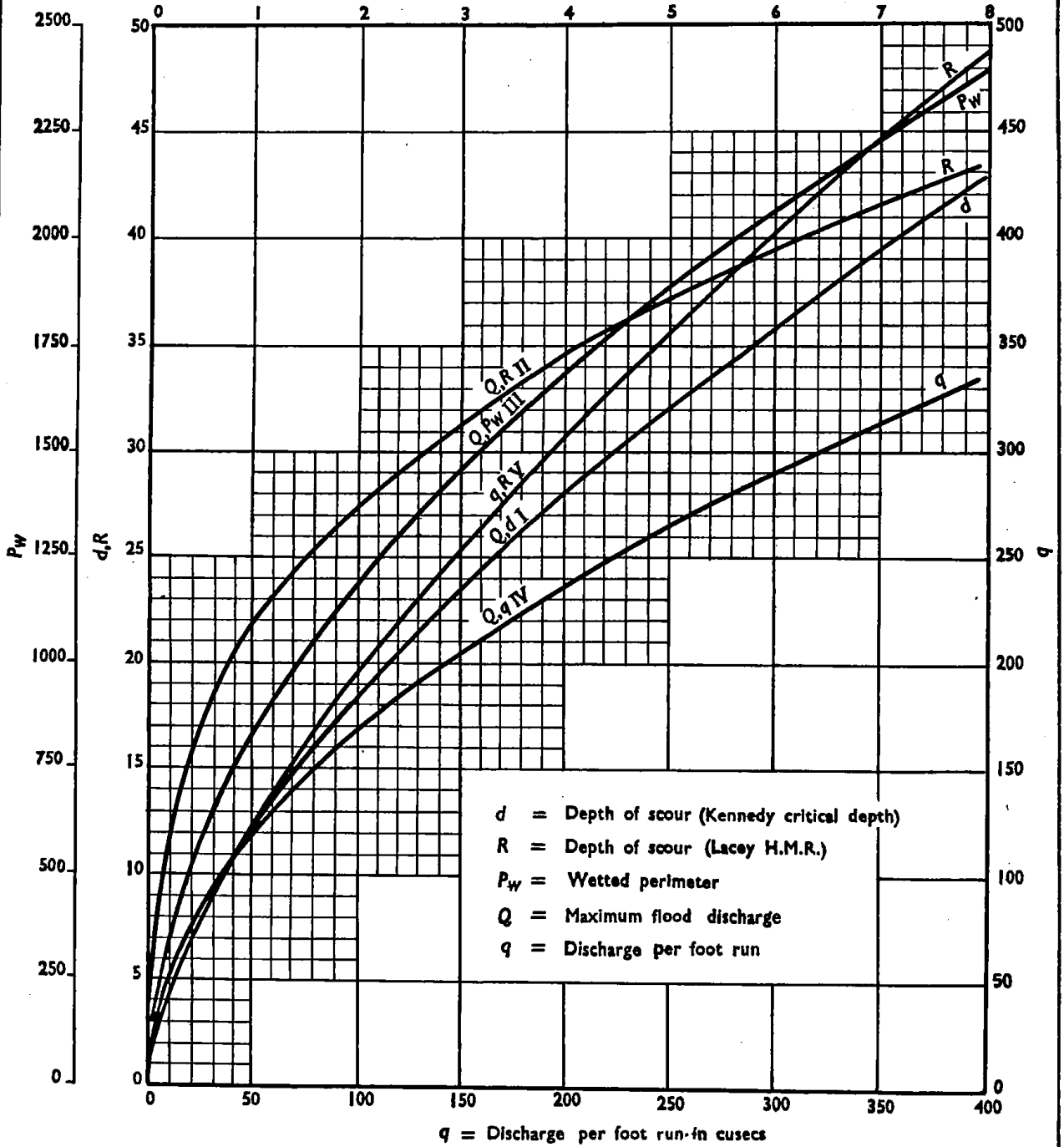


Photo 10-1 : Showing scour downstream of Panjnad Weir Bay 44. (September 1934)

Q = Maximum flood discharge in lakh cusecs



Let q = discharge per foot run,
 then $q = V_o d = 0.84 d^{1.64}$
 or $d = 1.11 q^{0.61}$ (10.2)

The critical depth d , which is also the depth to which scour will take place in normal flow, can be read off directly from Plate X. 2, Curve I.

Another method of determining depth of scour is that given by Lacey¹

$$V_o = \left(\frac{Qf^2}{3.8} \right)^{\frac{1}{5}}$$

and $R = 0.7305 V_o^2 f$
 where V_o = Critical velocity,
 R = Hydraulic mean depth,
 Q = Maximum flood discharge,
 f = Silt factor.

Hence—

$$R = 0.7305 V_o^2 f$$

$$= 0.7305 \left(\frac{Q}{3.8f} \right)^{\frac{2}{5}} \dots \dots \dots (10.3)$$

Plate X. 2, Curve II.

R in this formula is the depth of scour corresponding to d in the Kennedy formula. The values of R can be read off the curve in Plate X.2, Curve II, for values of $f=1$. This relationship, however, is not directly applicable to the design of weirs.

For any given flood discharge Q , the minimum stable waterway is given by

$$P_w = 2.67 \sqrt{Q} \dots \dots \dots (10.4)$$

Plate X. 2, Curve III.

The discharge per foot run is given by :—

$$q = \frac{Q}{P_w} = \frac{Q}{2.67 \sqrt{Q}} = \frac{\sqrt{Q}}{2.67} \dots \dots \dots (10.5)$$

which is a unique value of discharge per foot run for a given maximum flood discharge through the minimum stable waterway.

Plate X. 2, Curve IV.

In a weir where the water-way differs materially from the minimum stable water-way given by the Equation (10.4), the depth of scour obtained by the Lacey formula will apply only to that section of the weir whose discharge per foot run equals that given by Equation (10.5). This depth of scour will also occur along the guide-banks should the whole river hug a particular bank and run in a width equal to the minimum stable water-way.

Combining Equations (10.3) and (10.5) we get a direct relationship between the discharge per foot run and R the depth of scour.

From Equation (10.4)

$$Q = (2.67q)^2$$

Substituting in Equation (10.3) we get

$$R = 0.7305 \frac{(2.67q)^{\frac{2}{5}}}{(3.8f)^{\frac{1}{5}}} = 0.9 \left(\frac{q^2}{f} \right)^{\frac{1}{5}} \dots \dots \dots (10.6)$$

The graphs of this equation are plotted in Plate X.3 for different values of (f). Curve V of Plate X.2 is the plot of this equation for $f=1$.

Kennedy's formula gives the relation—

$$d = 1.11 q^{0.61} \text{ (for standard sands only, with } f=1 \text{)} \dots \dots \dots (10.2)$$

(Plate X.2, Curve I.)

(1) Lacey on Stable Channels in Alluvium, Paper No. 4716(1933), Institute of Civil Engineers, London, page 279.

Lacey's formula gives somewhat higher values than Kennedy's and may be accepted as being on the safer side.

The curves of R for different values of f are given in Plate X.3.

In a weir with undersluices, the discharges per foot run are different in the weir and undersluice bays. The depth of scour for which the work has to be designed will, therefore, differ for the two cases. These depths obviously cannot be got from the Lacey formula (Equation 10.3). The formula (10.6) will be the correct one to use with due allowance for the value of f . Where there are no undersluices, that is, the crest is at one uniform level throughout the length of the weir, the formula (10.3) will still not apply as the maximum discharge per foot run is, generally speaking, much less than that required for minimum stable water-way. In this case, too, the formula (10.6) will apply.

According to Spring¹, scours are classed as :—

- (a) normal, caused by straight unobstructed currents,
- (b) abnormal, caused by deflected, but nevertheless onward flowing currents, and
- (c) extraordinary, caused by swirls set up in non-forward moving water.

Class—

- (a) ranging between 30 and 40 ft.
- (b) ranging between 60 and 100 ft.
- (c) ranging between 80 and 120 ft.

With good design, the extraordinary swirls should never occur but the normal and abnormal scours will have to be allowed for in the sections of the work where they are likely to occur.

Lacey has given a clearer definition of the classes of scour which can be suitably adopted for designs.

Let R = depth of scour in a channel with a level bed, as for instance the downstream apron of a weir.

Class A.—Straight reach	1.25 R .	(Lacey gives 1.27.)
Class B.—Moderate bend	1.50 R .	
Class C.—Severe bend	1.75 R .	
Class D.—Right-angled bend	2.0 R .	

Class A is likely to occur anywhere just below the loose aprons.

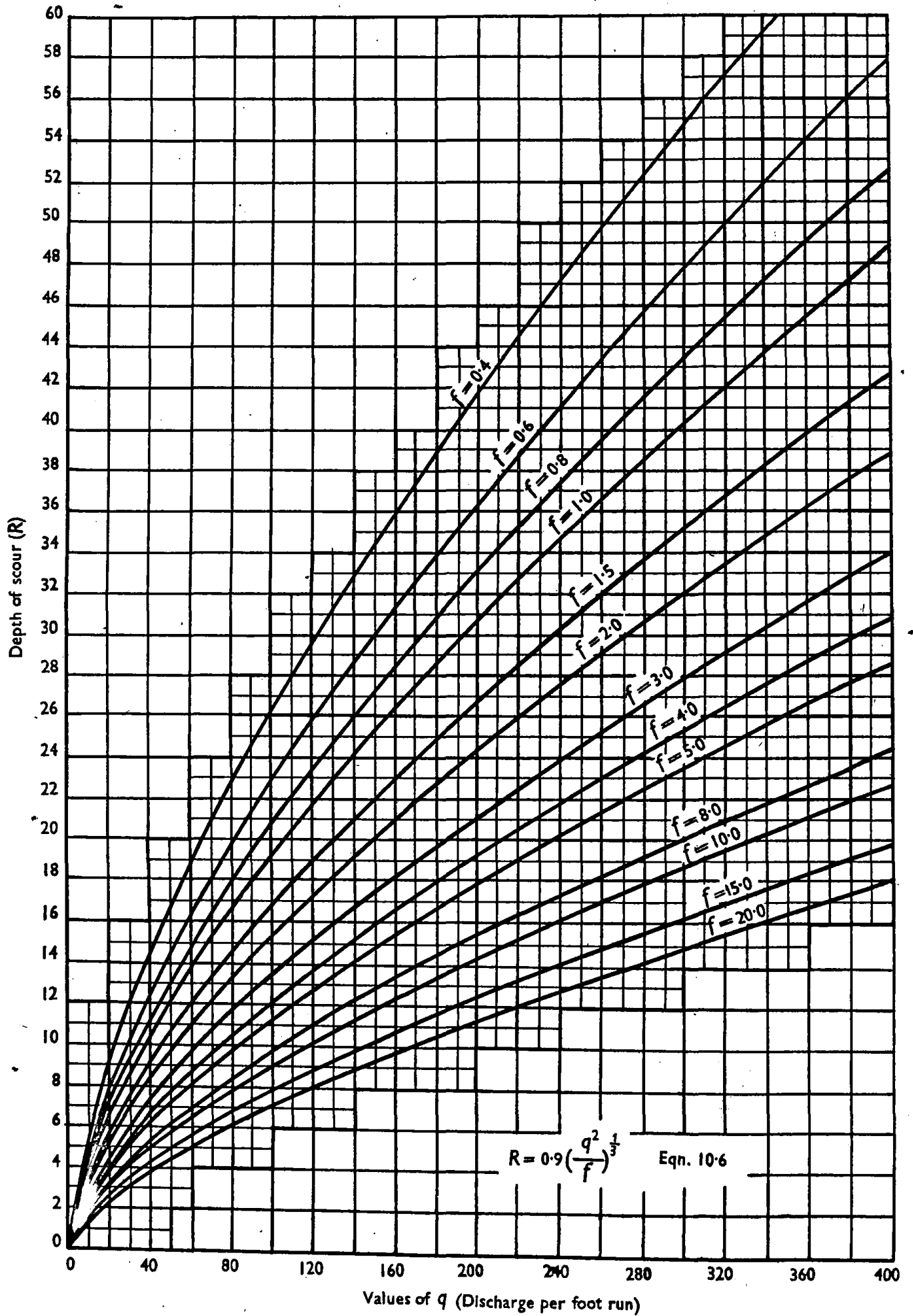
Class B is likely to occur anywhere along the aprons of guide banks in the straight reach and Classes C and D at and below the noses of guide banks, or at the loose weir aprons should heavy swirls set in for any reason.

For the design of sheet piles, it will be enough to take the piles to a depth of scour equal to R . For the design of loose aprons, upstream and downstream of the weir, the depth of scour should be taken between 1.25 R and 2.0 R to suit local concentrations of flow. This will be discussed in greater detail in the next chapter. These scour depths will be reckoned below maximum flood level.

At Khanki, during the dismantling of Bay 8 for conversion into undersluices, the original blocks and stones of the loose downstream apron were met with as low as R . L. 704 to 703 and of the upstream apron between R . Ls. 705 and 706. In 1895 when 100 ft. of the weir crest subsided in Bay 1, scour depth to R . L. 701 was discovered upstream of the crest. To this depth were the wells and wooden piles upstream of the crest sunk and driven in 1896 and 1897. The maximum flood level upstream is 738.0 and downstream 734.6.

Thus, scour upstream = 32 to 33 ft., and scour downstream = 30.6 to 31.6 ft.

(1) "River Training and Control", by F. J. Spring, Technical Publication 153, Government of India, Chapter V, Para. 7 (1903).



The discharge per foot run of weir is 145 cusecs, which gives $R=25'$ (Equation 10.5).

$$\begin{aligned}\text{Normal Scour} &= 1.25 R \text{ for } f=1. \\ &= 25 \times 1.25. \\ &= 31.3 \text{ ft.}\end{aligned}$$

This fits in with the observed depths of scour.

The new undersluices have been designed for a discharge per foot run of 387 cusecs. For this $R=48'$ nearly (for $f=1$).

$R, L.$ of likely scour hole below the apron $= 734.6 - 48 = 686.6$.

Further down, the scour may be $48 \times 1.25 = 60$ ft. The downstream sheet piles in the weir bays go down to about $R, L. 700$ except where the presence of clay beds necessitated otherwise. In Bay 8, the bottom of these piles is at about $R, L. 689$.

The scour hole in Bay 4 of Merala Weir (see Figure 10.15) during the record flood of 1928 went down to $R, L. 776.5$ or $811.3 - 776.5 = 34.8$ ft. below maximum flood level downstream. The discharge per foot run was about 160 cusecs giving $R=26.5$ and $1.25 R=33.1$ ft. It will be noticed that the downstream wells go down to $R, L. 777.8$ (top of well curb).

These scour holes occur not only during floods under normal flow conditions, but also at lower stages if there is parallel flow along the weir either upstream or downstream, or if there are swirls.

Depth of Sheet Piling

Having decided on the depth of scour below maximum flood level, the next step is to ascertain the level of the downstream floor, such that the standing wave under the worst conditions, does not form further downstream than the toe of the glacis. This latter aspect is dealt with in the next chapter. The depth of the pile line will then be the depth below the top of the downstream floor to the bottom of the hole due to normal scour. This is a general indication but slight departures are permissible to suit individual cases.

Piles at the Upstream and Downstream Ends Necessary

It should be borne in mind that scour holes can occur both upstream and downstream so that a pile line is required at the upstream end of the floor just as much as at the downstream end in order to prevent failure by slipping of the subsoil into the scour holes by simple earth pressure.

In the case of a scour hole at the upstream end, the flow of seepage water will tend to keep the soil in position as its force will oppose slipping due to earth pressure. At the downstream end this force will assist and accelerate slipping due to earth pressure as the two forces will act together. This restraint against undermining at the upstream end and acceleration of it at the downstream end will increase in direct proportion to the applied head. Failure at the upstream end will generally occur at low heads when the restraint exercised by seepage flow against slipping is reduced and failures downstream will similarly occur at high heads when the acceleration imposed by the seepage is increased.

The depth of the upstream pile line will be governed by the depth of scour only and of the downstream one by the depth of scour and exit gradient.

Under normal conditions the bottom of the scour upstream will be higher than the bottom of the scour downstream by the amount of afflux or difference between upstream and downstream levels in maximum flood. The sheet piles at the upstream end can, therefore, be higher than those at the downstream end by that amount.

The necessity for piles at the upstream and downstream ends is obvious. They are to protect the work against undermining of the foundation soil. The pressures under the downstream floor increase as the depth of the downstream pile line increases. The upstream pile line has little effect in reducing these pressures as the spacing of these two is generally much more than the range of influence of either. It must, therefore, be clearly understood

that the upstream end pile has little influence on the uplift pressures under the downstream floor. Looked at from that point of view, the upstream pile should not be necessary. It will effect a reduction of pressures under the upstream floor. Such reduction is of no consequence in the case of weirs where the load of water on top of the upstream floor is much in excess of the uplift pressure. In the case of high dams the upstream pile line with consequent reduction of pressure at the upstream end, is of very great importance as it might effect considerable economies in the mass of the structure.

Intermediate Piling

The intermediate sheet pile lines are required neither to prevent undermining of the floor at the upstream nor at the downstream end; nor do they materially alter the pressure distribution to give less uplift pressures under the downstream floor. But they act as important secondary lines of defence, so that even if the *pacca* (impervious) floor is damaged at either the upstream or the downstream end by failure of the end piles under abnormal scour, the rest of the floor and the superstructure will be saved from collapse by the intermediate piles. The uplift pressures will increase but the undermining will be arrested as the exit gradients will still be below the critical value. The structure will thus be saved from actual failure which on a major work would be disastrous. A very apt demonstration of the value of a pile line or a line of wells under the crest was furnished by the Deoha Barrage¹. The floor collapsed at one point upstream and blew up downstream. The damage occurred in four bays and extended to another three. "It was fortunate that the main structure was founded on deep wells and thus remained undamaged."

Opinions differ as to whether there should be one or more intermediate piles. The best value will be obtained with one deep pile line directly under the crest. This has been done in the central and right undersluices of the Khanki Weir. In that case if both the upstream and downstream floors fail, but there is some length left upstream and downstream, the essential part of the structure, *viz.*, the gates and gearing, will be saved. A deep pile line by itself can stand a large head. For a 20 ft. head and 30 ft. depth of piling, with no upstream or downstream floor, the exit gradient will be $GE=1/4.5$ which is more than four times the critical.

The next best alternative is to have one intermediate pile between the upstream end and the crest, and another between the downstream end and the crest, located under the upstream and downstream toes of the glacis. The Sukkur Barrage is an instance of this combination. A similar combination is contemplated at the proposed Trimmu Weir.

The recommendation (f), page 8 of the Islam Enquiry Committee Report², is against carrying a line of piles vertically below the crest of a weir as it was considered that such a pile line might lead to unequal settlement and cracks. This recommendation appears to be open to serious question. With a deep pile line under the heavy gates and gearing, the floor concrete reinforced against temperature and the base pressure judiciously spread, the chances of unequal settlement or of cracks should be remote.

Wells

Interlocked steel sheet piles should in all cases be preferred to wells, as the gaps between the individual wells can rarely be blocked effectively all the way from top to bottom. On the upstream side their use will be less objectionable as their function is to restrain the earth against slippage into scour holes, although, if the gaps are not properly blocked, some slippage can occur. On the downstream side, the use of wells is definitely risky as undermining of the sub soil can set in through any of the gaps if they remain open at any place along the depth. Here the earth pressure and the force of subsoil water act together. By themselves wells are more stable than sheet piles. A suitable place for wells is just downstream of the inverted filter as at the Panjnad Weir.

(1) "Design and Construction of the Sarda Canal" by Sir Bernard Darley, Kt., C.I.E., Paper No. 4814, Institute of Civil Engineers, London.

(2) Report by the Islam Enquiry Committee into the Failure of the Islam Weir, Punjab Irrigation Branch Publication (1930)

Examples of designs.

The process of design will be best explained by an example. The newly constructed right undersluices of the Khanki Weir¹ (Figure 10.11) afford a good example of how to proceed with a new design and how to analyse a given one.

$$H = \text{total head} = 19 \text{ ft.}$$

$$H. F. L. = 738.0 \text{ upstream.}$$

$$734.6 \text{ downstream.}$$

$$\text{Afflux} = 3.4 \text{ ft.}$$

$$q = \text{discharge per foot run} = 387 \text{ cusecs.}$$

Level of upstream and downstream horizontal floor = 710.0. This was arrived at from a consideration of the standing wave forming not lower than the toe of the glacis. R = depth of scour = 48' (Plate X.2). Bottom of scour hole downstream = 734.6 - 48 = 686.6. Bottom of scour hole upstream = 738 - 48 = 690.0. The bottom level of the downstream piles is actually at 689.0 and of the upstream piles at 691.0.

$$d = \text{depth of downstream pile line below floor}$$

$$= 710 - 689 = 21 \text{ ft.}$$

$$d_1 = \text{depth of upstream pile line below floor}$$

$$= 710 - 691 = 19 \text{ ft.}$$

$$GE = \text{Exit gradient} = \frac{H}{\pi d \sqrt{\lambda}}$$

$$\frac{H}{d} = 19/21.$$

If factor of safety is 6 so that $GE = 1/6$

$$\frac{1}{\pi \sqrt{\lambda}} = \frac{21 \times GE}{19} = \frac{21 \times 1/6}{19} = 0.18.$$

$$\text{From Plate VII.5, for a value of } \frac{1}{\pi \sqrt{\lambda}} = 0.18, \alpha = 5$$

Thus for a 21 ft. pile line, the floor length to give an exit gradient of $1/6$ need not have been more than $5 \times 21 = 105$ ft.

The actual floor length = 156 ft.

This floor length was put in to give a long enough passage for the standing wave to dissipate its destructive energy before getting on to the pervious apron.

The actual exit gradient in this case works out as below :—

$$\alpha = 156/21 = 7.4.$$

$$\frac{1}{\pi \sqrt{\lambda}} = 0.16 \text{ (from Curve, Plate VII/5)}$$

$$GE = \frac{H}{d} \times \frac{1}{\pi \sqrt{\lambda}} = \frac{19 \times 0.16}{21} = 1/7.$$

Thus the floor length is suitable.

To determine the uplift pressures,

Upstream pile line.

$$b = 156.$$

$$1/\alpha = 19/156 = 0.122.$$

$$\varphi_D = 78.8\%.$$

$$\varphi_C = 69.0\%$$

$$\varphi_D - \varphi_C = 9.8\%$$

$$\varphi_C \text{ corrected } 69.0 + 2.6 + 2.4 = 74\%.$$

φ_C correction for depth at R, L. 705

$$= \frac{710 - 705}{19} \times 9.8 = +2.6\%.$$

Effect of middle pile on $\varphi_C = 19 \sqrt{16/45.5} \times$

$$\frac{16 + 14}{156} = +2.4\%.$$

(1) "Reconstruction of the Khanki Weir", by A. N. Khosla, Punjab Engineering Congress Paper No. 195 (1930)

Middle pile line.

$$d = 710 - 689 = 21 \text{ ft.}$$

$$\alpha = 156/21 = 7.4.$$

$$b_1/b = 48/156 = .308.$$

$$\varphi_{E_1} = 71.2.$$

$$\varphi_{D_1} = 61.7:$$

$$\varphi_{C_1} = 53.2\%:$$

$$\varphi_{E_1} - \varphi_{D_1} = 9.5\%:$$

$$\varphi_{D_1} - \varphi_{C_1} = 8.5\%:$$

$$\varphi_{E_1} \text{ corrected} = 71.2 - 3.3 = 67.9\%:$$

$$\varphi_{C_1} \text{ corrected} = 53.2 + 2.2 = 55.4\%:$$

No correction for depth is necessary in this case.

Effect of upstream pile on φ_{E_1} .

$$D = 710 - 691 = 19'.$$

$$d = 710 - 689 = 21'.$$

$$C = 19 \sqrt{19/45.5} \times \frac{19+21}{156} = -3.3\%.$$

Effect of downstream pile on φ_{C_1} .

$$D = 710 - 689 = 21'.$$

$$d = 710 - 689 = 21'.$$

$$C = 19 \sqrt{21/105.5} \times \frac{21+21}{156} = +2.2\%.$$

Downstream pile line:

$$d = 710 - 689 = 21.$$

$$1/\alpha = 21/156 = 0.134.$$

$$\varphi_{E_2} = 32.5\%:$$

$$\varphi_{D_2} = 22.2\%$$

$$\varphi_{E_2} - \varphi_{D_2} = 10.3\%.$$

$$\varphi_{E_2} \text{ corrected} = 32.5 - 2 - 1.5 = 29.0\%$$

$$\varphi_{E_2} \text{ correction for depth} = 4/21 \times 10.3 = -2\%.$$

Effect of middle pile line.

$$D = 17', d = 17'.$$

$$C = 19 \sqrt{17/105.5} \times \frac{17+17}{156} = -1.5\%.$$

Comparative Table.

	U. S. Pile		Middle Pile			D. S. Pile	
	φ_D	φ_C	φ_{E_1}	φ_{D_1}	φ_{C_1}	φ_{E_2}	φ_{D_2}
Calculated	78.8	74.0	67.9	61.7	55.4	29.0	22.2
Experimental	81.6	73.0	67.9	61.3	53.7	29.6	22.2
Difference	2.8	1.0	0.0	0.4	1.7	0.6	0.0

For a head of 19' the absolute uplift pressures work out as below :—

$$P_C = \frac{19 \times 74}{100} = 14.1'.$$

$$P_{E_1} = \frac{19 \times 67.9}{100} = 12.9'.$$

$$P_{C_1} = \frac{19 \times 55.4}{100} = 10.6'.$$

$$P_{E_2} = \frac{19 \times 29}{100} = 5.5'.$$

The floor thickness below the middle pile line can now be calculated, taking the submerged weight of stone masonry or concrete as 1.4 times that of water. The pressures along the floor between the middle and downstream pile lines can be obtained by assuming a linear drop from 10.6 to 5.5 ft. along the length between the two pile lines.

Floor to act as one mass

The designed thickness of floor obtained as above, will be adequate so long as the floor acts as one mass for the entire thickness. If this thickness is in layers and they separate, the full pressure can be transmitted through cracks under any one of the layers. The latter not being individually strong against the total uplift pressure will be lifted and damage will result. It is, therefore, essential to ensure that unless the floor is laid *en-masse* in one layer, the various layers are tied to each other by steel reinforcement or otherwise so that they become inseparable.

Damage on account of separation of layers occurred at Narora¹&² (1898) at Merala³ 1928 and 1929, at Rasul⁴, Islam⁵, and Ferozepur in 1929.

Of the four weirs on the Sutlej Valley Scheme, the Ferozepur, Suleimanki and Islam Weirs had the floor concrete laid in layers. The Panjnad Weir had it done in one mass. In all except the latter weir, there has been greater or less separation of layers. At Suleimanki the trouble arose during construction and was effectively set right. At Islam the trouble was acute and widespread and was one of the contributory causes of its failure in 1929. This point has been dealt with at length by Khosla in his paper on the reconstruction of the Khanki Weir⁶, and will be noticed in the next chapter.

The Inverted Filter

It is usual and, indeed, necessary to provide a protection, of graded ballast and stone overlaid with heavy blocks for the bed soil immediately downstream of the impervious floor. The idea is to prevent erosion of this soil by the action of waves and undermining by seepage flow. This graded filter permits free movement of water but prevents the movement of sand. It consists of a layer of fine *bajri* or *kankar* resting directly on the bed soil, followed by ballast of increasing grades and finally spawls and boulders. This filter bed of layers of increasing porosity, from bottom up, is overlaid and protected by a series of heavy blocks (with open joints) against dislocation by the action of surface flow. An alternative form of protective covering for the filter bed is a slab of concrete or masonry perforated by a series of 1½" G. I. pipes which are bedded into the filter bed. This latter method is generally less satisfactory.

The use of the inverted filter is at present restricted to the tail end of the work but it can, with advantage, be extended in a less expensive form to the upstream end and to the aprons and slopes of guide banks which suffer damage by suction of the subsoil due to wave action.

The idea of the inverted filter originated with Mr. Beresford in 1899⁷ and was used for the first time at the Rasul Weir.

Pressure Relief or Drainage

The idea of the inverted filter can be carried a step further and applied at suitable points under the impervious floor to reduce uplift pressures, and thus economise in first cost. In this connection it must be clearly understood and remembered that the effect of isolated obstructions is purely local and has a limited radius of action. Any system of drainage to be effective will have to be very elaborate and extensive. For weirs on shingle beds, drainage can be used with a considerable degree of success but for those resting on fine sand drainage under the impervious floor is likely to be full of grave risks. There are two schools of thought those favouring drainage⁸ and those against it⁹. The conditions necessary for permanent pressure relief have been given by Haigh⁸ as:—

- (a) the pressure relief should be horizontal and should extend to the whole area to be relieved,
- (b) the gradient in the underlying material should be safe,
- (c) the filter and its outlet should be protected from any danger of choking from downstream."

(¹) Engineering Conference at Simla. (1913), Paper No. 7, by C. H. Hutton, Chief Engineer.

(²) "Design of Irrigation Works" by W. G. Bligh, pages 185-193.

(³) "Stability of Weirs and Canal Works" by A. N. Khosla. Paper 142, P. E. C. (1930)

(⁴) Repairs and Remodelling of Rasul Weir (1929-1931)" by T. A. W. Foy, P. E. C. Paper No. 155 (1932).

(⁵) Report by the Islam Enquiry Committee into the Failure of the Islam Weir Punjab Irrigation Branch Publication (1930).

(⁶) "Reconstruction of the Khanki Weir" by A. N. Khosla, Punjab Engineering Congress Paper No. 195 (1936).

(⁷) "Experiments on the passage of water through sand." Technical Paper No. 97, Government of India, page 31.

(⁸) "Design of Weirs on Sand Foundations" by F. F. Haigh, I.S.E., M.I.C.E., Paper No. 182, P. E. C. (1935) page 100.

(⁹) "Uplift and Seepage under Dams on Sand" by L. F. Harza, M.Am. Soc.C.E. Paper 1920, Am.Soc.C.E. (1935).

According to Haigh, "the best form of filter is a layer of ballast on a layer of sand, the permeability of which is 10 times that of the sand foundation, *i.e.*, average grain size is about 3 times as large. The purpose of the sand layer is to provide for any locally excessive gradients which might result from the placing of the ballast directly on the sub-foundation and might cause the latter to penetrate the ballasts. The ballast merely acts as a French drain to prevent the escape of sand with the seepage flow. Only one exit should be provided. More than one orifice permits flow from outside through the filter with consequent silting. The orifice should be arranged to discharge vertically downwards at the site of lowest pressure and might be provided with a silt trap in the thickness of the floor."

While there is no question that pressure relief by drainage will lead to considerable economy by reducing the uplift pressures and therefore the floor thickness, its introduction in the design of major weirs cannot be recommended in the present state of our knowledge. If allowance is made in design for reduced uplift pressures due to drainage and if the drainage system gets somehow fully or partially choked, the weir floor will have to withstand pressures much in excess of those for which it has been designed and may give way. If, on the other hand, some fault occurs in the filter and sand movement starts, the work may fail by undermining. The use of pressure relief would be perfectly legitimate downstream of the impervious floor.

Design of Flanks

As a general safe guide, the foundations of abutments and flank walls between any pair of pile lines should go down to the level of the bottom of those pile lines. If these latter are at different levels, the flank foundations can step down from one end to the other to suit these levels. This, in effect, means a complete boxing in of the foundations of the entire weir by means of pile lines of suitable depth upstream, downstream, and on the two flanks. With such an arrangement of levels for the flank foundations, the uplifts under the weir due to flank flow will not be more than those due to direct flow under the weir. The Panjad Annexe and the reconstructed Bays 4 and 8 of the Khanki Weir are examples of completely boxed in foundations.

Plate X'4 shows the pressures at either side of the flanks at the corresponding pressure points. The pressures due to flank flow, by the time they are transmitted to the weir floor adjoining the flanks, will be no more than those from direct flow under the floor. The location of pressure pipes of the flank of Panjad Weir is given in Plate V'2 by reference to which the results plotted in Figure 1, Plate X'4, can be correctly interpreted.

The foundations of the upstream and downstream return walls of the flank should similarly go down to the levels of the bottom of the upstream and downstream pile lines respectively. These should project into the bank by a length equal to twice the depth of the corresponding pile line below the floor level. Thus if the upstream pile line goes 20 ft. below the upstream floor level, the upstream return should have its foundation 20 ft. below that level and should be $2 \times 20 = 40$ ft. in length. These projections have nothing to do with the uplifts under the floor. They are to prevent outflanking due to seepage flow which, if not given suitable length and properly directed, may undermine the work from the flanks.

The fill behind the flanks will be saturated up to the level of the pressure gradient line at each point. These walls will have to be designed to withstand earth pressure due to the dry soil above the pressure gradient line and saturated soil below that line. Opinions differ as to the exact method of determining the horizontal pressure on retaining walls due to saturated fills, and experimental evidence on the subject is meagre and inconclusive.

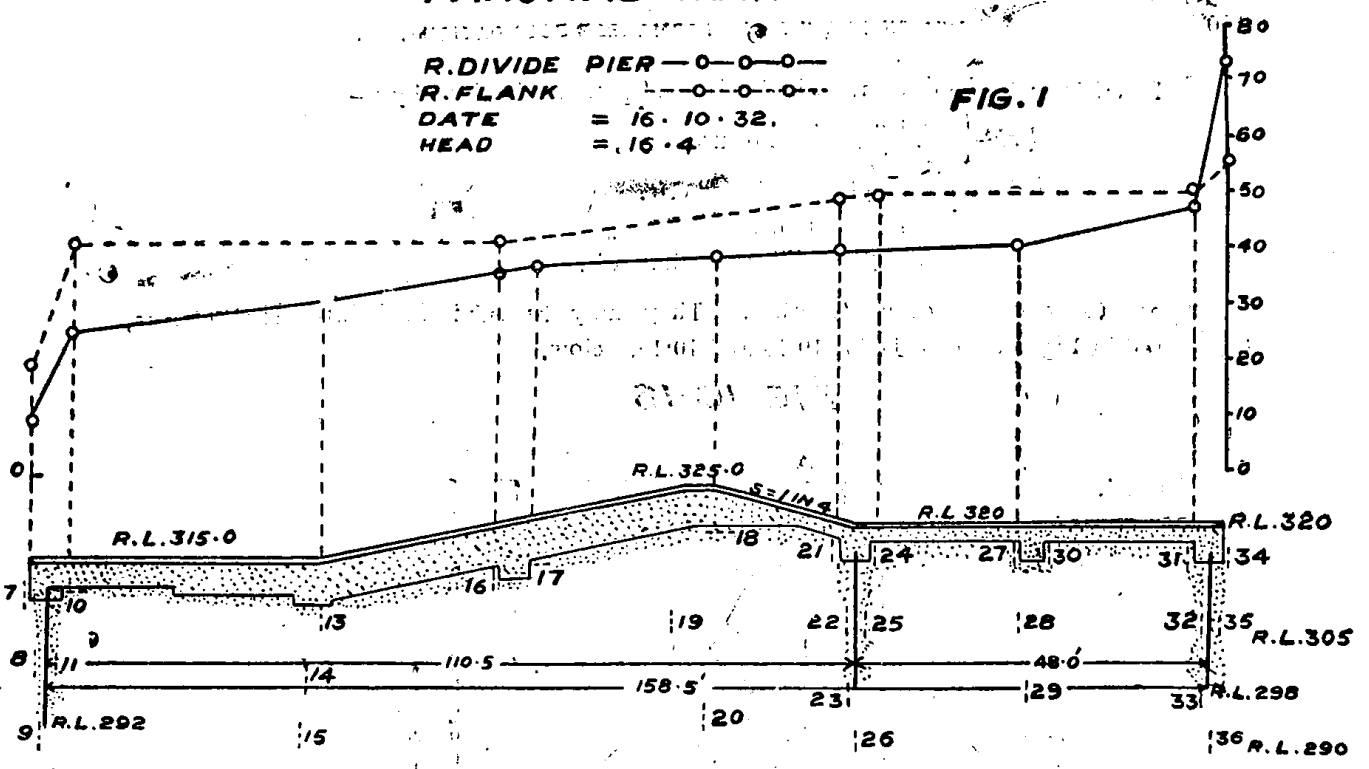
In designing the abutments and wing walls of hydraulic structures, three distinct types of pressures are met with. They are:—

- (a) Pressure due to dry soil.
- (b) Pressure due to saturated soil for the entire or part of the depth.
- (c) Hydraulic pressure acting in conjunction with saturated soil pressure, the former being proportional to the depth of water in case of no downward flow or somewhat less than this due to gradual loss of head in case there is downward flow.

PANJNAD WEIR PLATE X-4

R. DIVIDE PIER —○—○—○—
 R. FLANK —○—○—○—○—
 DATE = 16.10.32.
 HEAD = 16.4

FIG. 1



KHANKI WEIR

R. DIVIDE PIER —○—○—○—
 R. FLANK —○—○—○—○—
 DATE = 19.2.36
 HEAD = 10.55

FIG. 2

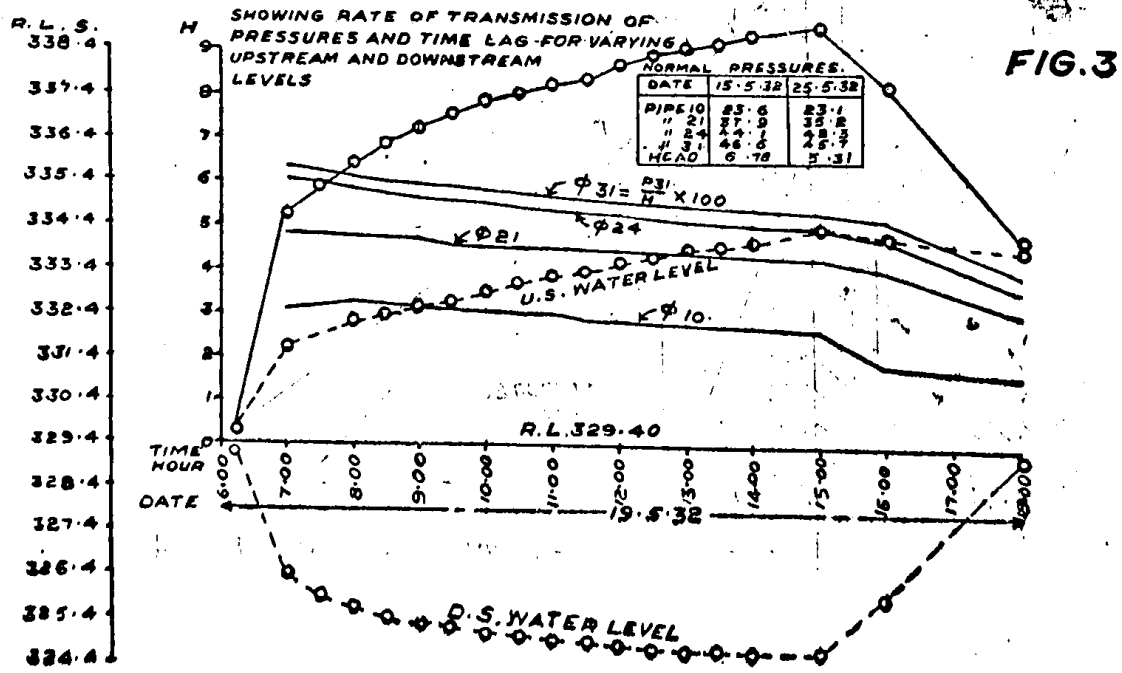
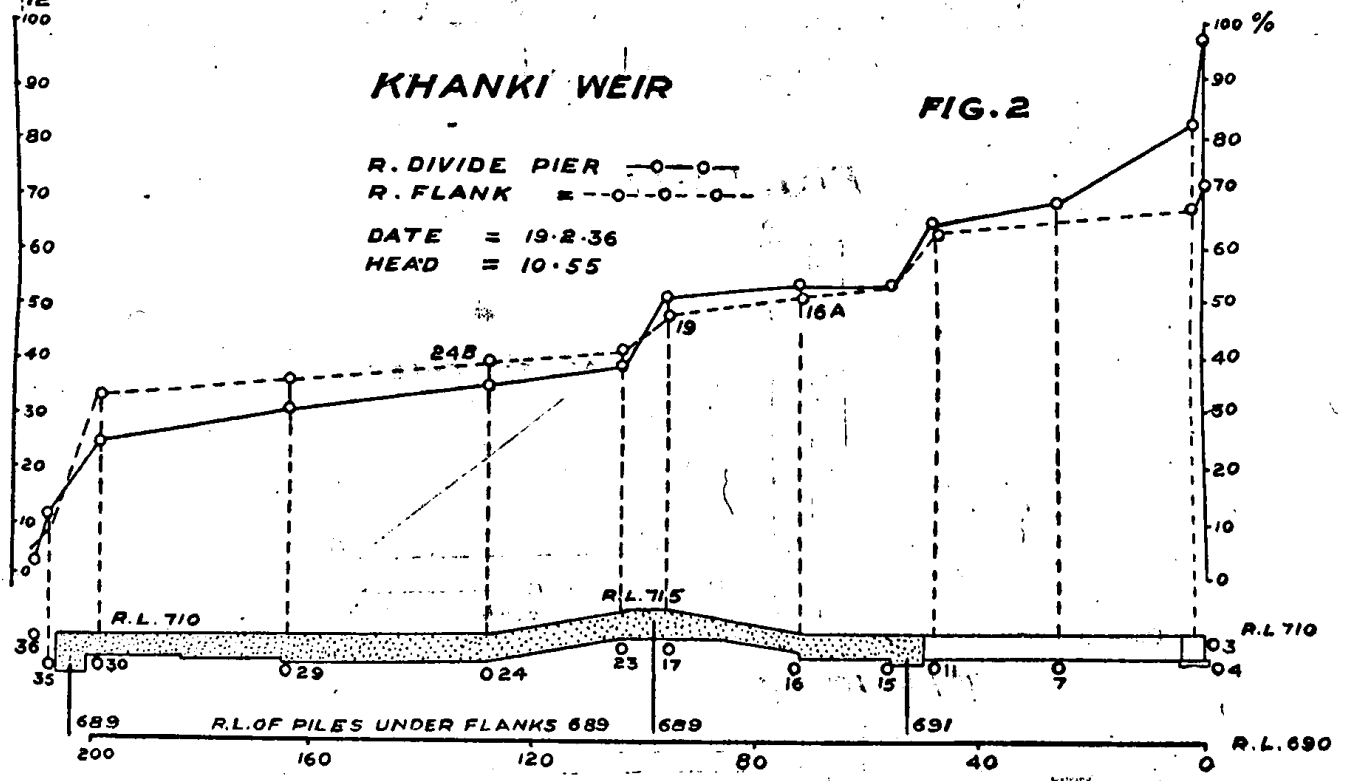


FIG. 3

Type (a) is easy of solution. The horizontal pressure is given by :-

$$\frac{1}{2}wh^2 \frac{1 - \sin \phi}{1 + \sin \phi}, \quad \text{no surcharge,}$$

where w = weight of soil per c.ft.

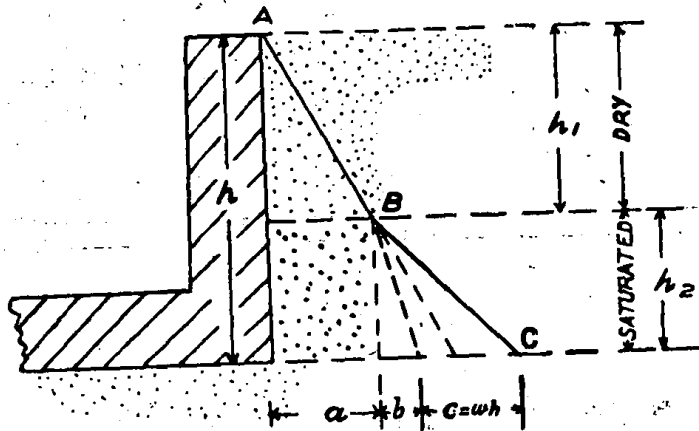
h = height of fill.

ϕ = angle of repose of that soil.

Types (b) and (c) occur together. They may be subdivided into four classes, illustrated in Figures 10-16, 10-17, 10-18 and 10-19, below.

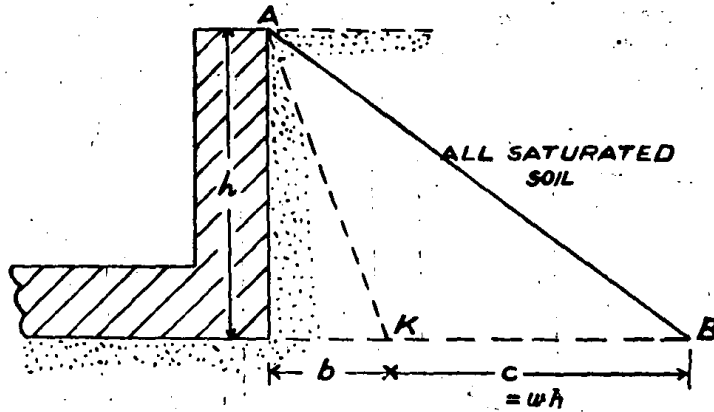
FIG. 10-16

Case. 1.



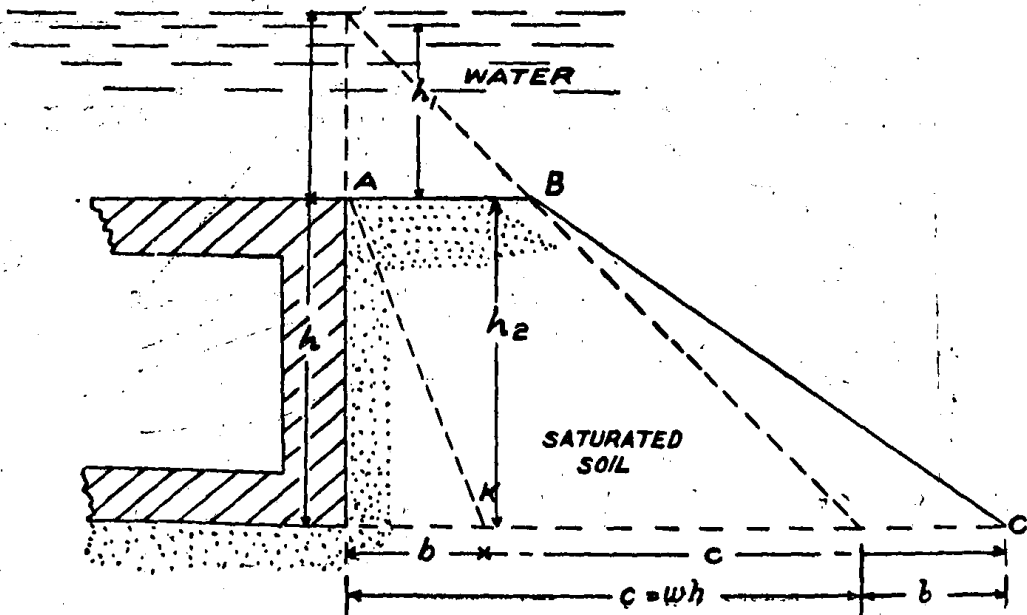
Case. 2.

FIG. 10-17.



Case. 3.

FIG. 10-18.



Cases 1, 2 and 3 relate to abutments and wings of weirs and other structures having an impervious floor between the abutments.

Case 1.—The pressure diagram in this case is given by the line ABC . (Figure 10-16) At B , the pressure is :—

$$a = w_1 h_1 \left(\frac{1 - \sin \varphi}{1 + \sin \varphi} \right)$$

where w_1 = weight of dry soil per c. ft.

The pressure at the base is given by $a + b + c$.

$$\text{Here } b = w_2 h_2 \left(\frac{1 - \sin \varphi_1}{1 + \sin \varphi_1} \right)$$

= pressure due to saturated soil

where w_2 = submerged weight of soil,

$$= (2.65 - 1) (1 - 40/100) \times 62.5$$

$$= 62.5 \text{ lbs. (approx.) for a soil of specific gravity 2.65 and pore space 40\%.}$$

φ_1 = angle of repose of saturated soil under water ;

and $c = w h_2$, where w = weight of 1 c. ft. of water = 62.5 lbs.

Case 2.—The pressure line is given by AB . (Figure 10-17)

The pressure at base = $b + c$

where b and c have the same significance as in Case 1, *viz.*

$$b = w_2 h \left(\frac{1 - \sin \varphi_1}{1 + \sin \varphi_1} \right)$$

$c = w h$ = full hydrostatic pressure.

Case 3.—The pressure line is given by BC . (Figure 10-18)

Pressure at $A = w h_1$ (hydrostatic pressure)

Pressure at

$$\text{base} = b + c$$

$$\text{where } b = w_2 h_2 \left(\frac{1 - \sin \varphi_1}{1 + \sin \varphi_1} \right)$$

and $c = w(h_1 + h_2)$ = full hydrostatic pressure.

Case 4 relates to aqueducts with pervious floor between the abutments and to cofferdams and open wells, with inside levels of soil and water below those outside. These latter cases are common in under-water foundations, dredging and pumping.

In cases 1 to 3 there is little downward flow. It is mostly paralld to the walls. In case 4, there is downward flow so that the intensity of hydraulic pressure at any point will be less than the actual hydrostatic pressure at that point. The actual pressure at any point can be determined as in the case of a single sheet pile with unequal fill on the two sides (Case 2 & II, Chapters VII and IX).

Opinions differ as to the value of the angle of repose of saturated sands below water. H. G. Husted¹ accepts φ_1 the angle of repose under water, to be greater than φ the angle of repose of dry material.

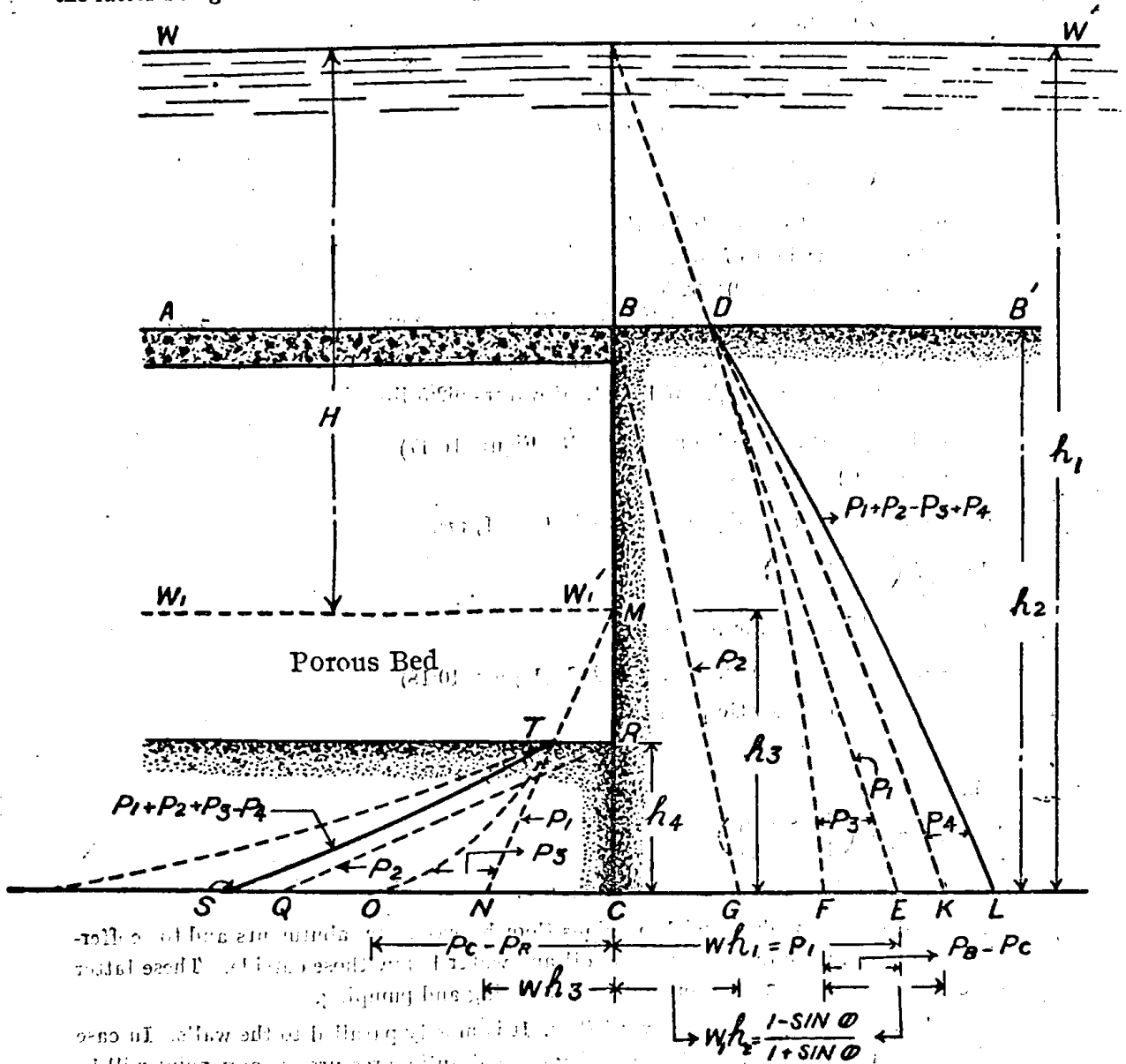
According to the latest test carried out in Europe² and America,³ the value of φ_1 is either equal to or a trifle less than φ . According to the exhaustive tests of Parsons³, the value of $w_2 \left(\frac{1 - \sin \varphi_1}{1 + \sin \varphi_1} \right)$ is 25% of $w_1 \left(\frac{1 - \sin \varphi}{1 + \sin \varphi} \right)$ in case of sands and 43% in case of fine gravels, or in other words; the total pressure on the retaining wall or sheet piling will be the full hydrostatic pressure *plus* 25% of dry earth pressure in case of sands and 43% of that pressure in case of fine gravels. Jenkin's² tests, however, show that the pressure on the retaining wall will be made up of the full hydrostatic pressure *plus* half the dry earth pressure—the latter being the pressure due to submerged weight of sand which is nearly half that of dry sand (Specific gravity of sand taken as 2.65 and pore space 40%).

(1) Engineering News Record, Vol. 81, p. 441.

(2) "The pressure on Retaining walls" by C. F. Jenkin, Paper No. 4867 (1032-34), Institute of Civil Engineers, London.

(3) "Some Soil Pressure Tests" by H. De B. Parsons, Paper No. 1881, Am. Soc. C. E. (1035), pages 12 and 30.

For purposes of designs, it would be quite reasonable to take the pressure as made up of the full hydrostatic pressure plus 25% of the pressure due to dry earth. But for extra caution, it may be taken as the full hydrostatic pressure plus 50% of the dry earth pressure, the latter being the same as the earth pressure due to the submerged weight of the soil.



Case 4.—(Figure 10-19)

The distribution of pressures and the stresses set up in the wall or sheet pile in this case are somewhat complicated but can be determined with fair approximation.

In Figure 10-19 AB is the roof of an aqueduct and BC the side wall or a line of interlocked steel sheet piles. BB' is the porous sand surface, WW' is the free water level above the aqueduct and w₁w₁' the water level below. The difference between the two levels=H.

The forces on the right of BC are:—

1. Hydrostatic pressure (P₁)=wh

where w=weight of water=62.5 lbs.

h=depth of the particular point below ww'.

This pressure is given by the horizontal intercepts between DE and BC.

2. Active Earth pressure (P₂)=w₂ h ((1-sin φ) / (1+sin φ))

where w₂=submerged weight of soil

=62.5 lbs. per c. ft. as explained above.

φ=Angle of repose of dry soil or of that under water.

h=depth of the point below AB.

This pressure is given by the intercept between BC and BG.

3. Since the water is at different levels above and below the aqueduct the pressure at any point due to water will be less than the static by the amount of head lost by water in flowing from BB' to that point. If P_B = the pressure at B (above $W_1 W_1'$) and P_C = pressure at C above the same datum, the loss in transit from B to C = $P_B - P_C$. On the diagram this loss is represented by

$$FE = (P_B - P_C)w$$

The hydraulic pressure line will now be represented by the curved line DF . This force due to this lost head may be called (P_3).

The resultant pressure line due to these three forces is represented by the curved line DK

$$\text{where } FK = w_2 h_2 \frac{1 - \sin \phi}{1 + \sin \phi} = CG$$

4. In addition to the forces due to earth and water pressures there is yet another force (P_4) which the water exerts in the direction of flow and which is proportional to the pressure gradient at each point (See Figure 8.1, Chap. VIII). As the flow is all downwards this force (P_4) will act downwards and will on the average, equal $0.8 \left(\frac{P_B - P_C}{h_2} \right) w$. lbs. per sq. ft. or

$$(P_4) = 50 \left(\frac{P_B - P_C}{h_2} \right) \text{ at each point.}$$

In this case $P_B = H$

This force varies from point to point being minimum at B and maximum at C . This force will produce additional horizontal thrust in the same way as a surcharge and can be provided for by adding P_4 to the weight w_2 of the soil which will now become $w_2 + P_4$ per c. ft.

The curved line DL represents the combined effect of all the above four sets of forces, viz. :-

1. Water pressure (P_1)
2. Active earth pressure (P_2)
3. Loss of head in flow through subsoil (P_3)
4. Force of flowing water along stream lines (P_4)

The centre of action of each force can be worked out, but for all practical purposes it will be enough to assume the forces P_2 , P_3 and P_4 to act at the centroid of the earth pressure. The centre of action of P_1 (hydrostatic pressure) can be easily worked out.

Similarly, the forces on the downstream side are :-

1. Water pressure (P_1') = wh' .
where h' is the depth of any part below $W_1 W_1'$.
This is given by line MN .

2. Passive Earth pressure (P_2')
 $= w_2 h' \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)$

These pressures are given by the line RQ .

3. Force (P_3') consequent on the water being in flow through the subsoil from Q to R .

If P_C = Pressure at C and $w h_3$ the hydrostatic pressure of water, then at point C $P_3' = P_C - w h_3$

The combined pressures due to 1 and 3 are given by the curved line OT .

4. The force (P_4') due to upward flow of water through the subsoil, will in this case, act upward, thus reducing the submerged weight of the soil w_2 by P_4' lbs. The net weight of the soil at the downstream end will thus be $(w_2 - P_4')$ per cubic foot. The value of this force will be on the average

$$(P_4') = 0.8 \cdot \frac{(P_C - P_R)}{h_4} \times w = 50 \cdot \frac{P_C - P_R}{h_4} = 50 \cdot \frac{P_C}{h_4}$$

as in this case $P_R = 0$

The reduction in the weight of the soil in this case, can as a rough rule, be allowed for by taking the effective submerged weight of soil as $\frac{2}{3} w_2$, so that the passive earth pressure becomes $\frac{2}{3} w_2 \frac{1 + \sin \phi}{1 - \sin \phi}$.

The curved line TS represents the collective effect of the above four forces.

5. There is yet another force which tends to disturb the stability of the fill at the toe and that is the upward heave by earth pressure of the fill on the right hand. This is given by $w_2 h_2 \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2$. This if desired can be allowed for in the same way as the pressure of water due to its flow through subsoil. It is, however, recommended that this may be ignored in calculations. The passive earth pressure as given by $w_2 h_1 \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)$ is shown by experiments¹ to be very much on the low side. It works out in actual practice to very nearly twice that value.

The pressures P_B, P_C etc., at any point can be worked out by the method of independent variables explained in the earlier pages.

Strength of Sheet piles at the upstream and downstream ends of a weir floor

As in the above case of abutments of an aqueduct, the sheet piles upstream and downstream of a weir floor where the soil in front has been scoured or eroded (thus leaving the tops of the piles exposed) will be subjected to the same four forces— P_1, P_2, P_3 and P_4 .

Upstream piles

On the upstream face the water pressure will be given by $P_1 - P_3$ and the earth pressure by $P_2 + P_4$.

On the downstream face the water pressure will be given by $P_1' + P_3'$ and earth pressure by $P_2' - P_4'$.

On the upstream face of the upstream piles due to downward flow of water the hydraulic pressure will be less than the static and the weight of soil will be greater than the submerged weight due to the downward force of water acting in the direction of flow. On the downstream face of the upstream piles, the water pressure will be more than the static but the weight of the soil will be reduced due to the upward force of flowing water. The two sets of conditions tend toward stability.

Downstream piles

The forces on their upstream face will be $P_1 - P_3$ and $P_2 + P_4$.

Those on the downstream face will be $P_1' + P_3'$ and $P_2' - P_4'$.

The stability of the piles may now be calculated. The factors to be provided against are:—

1. Spewing or forward movement of the soil in front of the sheet piles where scour has taken place.
2. Failure of the pile line as a beam in the unsupported length between the bottom of floor and earth fill (at the scoured end) or as cantilever at its junction with the floor.

3. Failure of the floor slab at its junction with the sheet piles due to cantilever action, Figure 10·19 (a) represents a downstream end pile line (depth d_1) with the soil in front scoured to a depth (d_2) leaving a depth of bedding= $d_1 - d_2 = d_3$.

The forces acting on the two sides of the pile line are shown in Figure 10·19 (a). They have the same significance as explained before. The static water pressures P_1 and P_1' being the same on the two sides will cancel out.

Taking moments of the forces about E,

$$MR = \text{Moments on the right} = P_3 x_1 + (P_2 + P_4) x_2$$

$$ML = \text{Moments on the left} = (P_2' + P_3' - P_4') x_3$$

If $MR > ML$ the toe of the pile will have a tendency to move out but will be restrained by the resistance moment of the pile acting as a cantilever at E. The pile must be strong enough to resist this moment otherwise it will fail.

If Z = section modulus of the pile

and f = safe tensile stress

then, for stability

$$MR - ML \text{ must be less than } fZ$$

$$\text{or } Z \text{ must be greater than } \frac{MR - ML}{f}$$

For steel sheet piles, $f = 16,000$ lbs. per square inch.

(1) "Design of Bulkheads" by Paul Baumann, Paper No. 1908, American Society of Civil Engineers (1936).

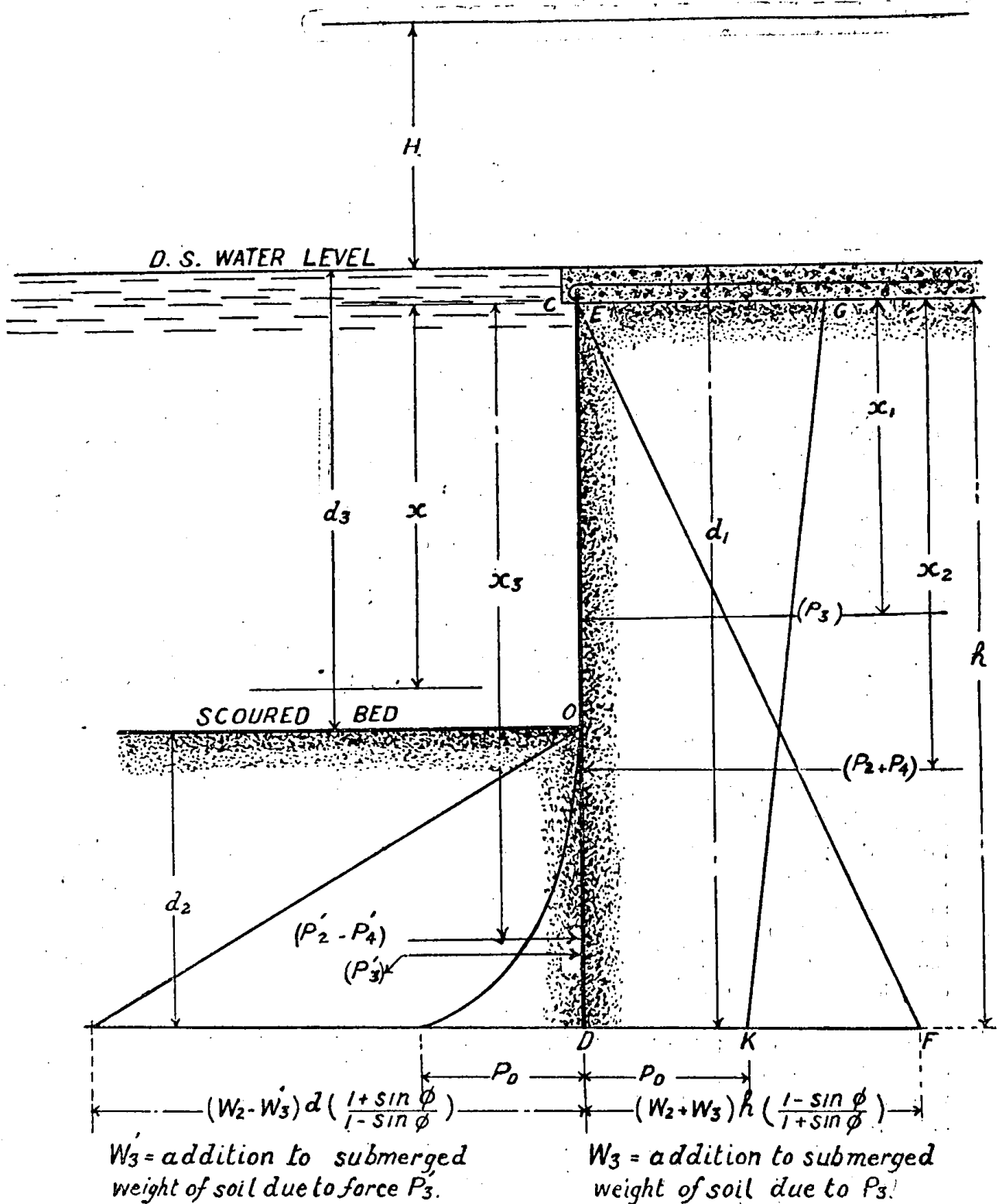


Figure 10-19(a)

If M_R is equal to or less than M_L , the maximum bending moment in the pile will occur at some point x below the bottom of the floor slab. The value of x can be determined by taking moments about the point of maximum B. M. and differentiating for x . The value of maximum B. M. can then be determined and the section of the sheet piles tested for strength.

The floor slab to which the steel sheet pile is tied should also be tested for shear and tension. Suitable reinforcement should be provided at the bottom and top so that the slab is at least as strong as the sheet pile at the junction.

The value of ϕ in case of sand may be taken as 35°

$$\text{or } \frac{1 - \sin \phi}{1 + \sin \phi} = 0.271$$

$$\text{and } \frac{1 + \sin \phi}{1 - \sin \phi} = 3.52$$

Magnitude of Uplift Pressure for Design of Floor Thickness

Opinions differ as to whether the effective pressure at any point under the floor is the full indicated pressure or only a fraction of it. One school of thought considers that as the water and soil cannot be in contact with the underside of the floor at one and the same time hence the full indicated pressure will act only on that part of the surface which is not in contact with the soil. The other school contends that water will exert full indicated pressure on the entire area directly or through the soil grains as the case may be. The problem is similar to that of retaining walls with saturated fills and as in that case, the experimental evidence is meagre and inconclusive. But the general indication²³ ²⁴ is in favour of the assumption that *the full indicated pressure acts on the entire surface.*

A safe policy appears to be to design the floor thickness for the total indicated pressure at the various points acting on the entire base area. This assumption appears to be the only sound one, as local settlements of the subsoil are not unlikely under certain parts of the floor where the entire area will be in direct contact with the water.

The floor should be designed as a purely gravity section at each point. The tension of the floor concrete or masonry will be an additional safeguard and afford a useful factor of safety which, in a well-designed structure, should never be needed. Cases of sudden application of head due to an unstable standing wave do occur but these should be foreseen and fully provided for in design. Cases are, however, on record in which a floor has been saved from blowing up or collapse due to its action as a beam.

The floor may as well be designed as a reinforced concrete slab held down by the weight of piers. The surface reinforcement for temperature will also partially take the tension due to uplift pressures. Where these pressures are high requiring great thickness of floor and consequently excessive pumping and expensive excavation, the reinforced concrete design may be actually cheaper and more convenient in construction. But unless the difference in cost is pronounced or constructional difficulties are great, the gravity design with temperature reinforcement on the surface is much to be preferred.

Great economies may be possible in a reinforced concrete design by treating the floor as a "flexible" raft, so that the maximum pressure intensities occur under and joining the piers thus reducing, considerably, the bending moments in the middle of the slab which would occur if the distribution were uniform along the entire span between the piers. The load of the piers has to be considered in conjunction with the uplift pressures. At and downstream of the crest where the piers are made heavy to carry the load of the gates, superstructure and the road bridge, the pressure intensities under the floor slab will be governed more by the weight of the piers and the superimposed loads than by uplift pressures. But the design must take into account both factors together and provide for the worst conditions at each section.

Time Lag

Cases have come to light where the floor thickness was much less than that required to withstand the uplift pressures as indicated by pressure pipes. In some cases like the Narora Weir failure occurred, but in others like the Merala Weir, the floor stood. The latter can be due to:—

- (a) The full hydrostatic pressure not acting on the entire area, but on a fraction of it.
- (b) Beam action in the floor slab in which the tension of the masonry comes into play and withstands the movement due to uplift in excess of that balanced by the gravity section.
- (c) Time lag between the application of the excessive head and its effect at the section of the floor under consideration.

Items (a) and (b) have been dealt with above. The beam action is a considerable factor depending on the material used and the workmanship.

The phenomenon of time lag has already been discussed in Chapter VI. The effect of any application of head is felt throughout the structure almost instantaneously or within a very short space of time, so that this factor is of no material assistance in saving an otherwise weak floor. No conclusive experimental data exist on this subject in relation to weirs. Some observations were made at the Panjnad Weir in 1932 to investigate the time lag in the

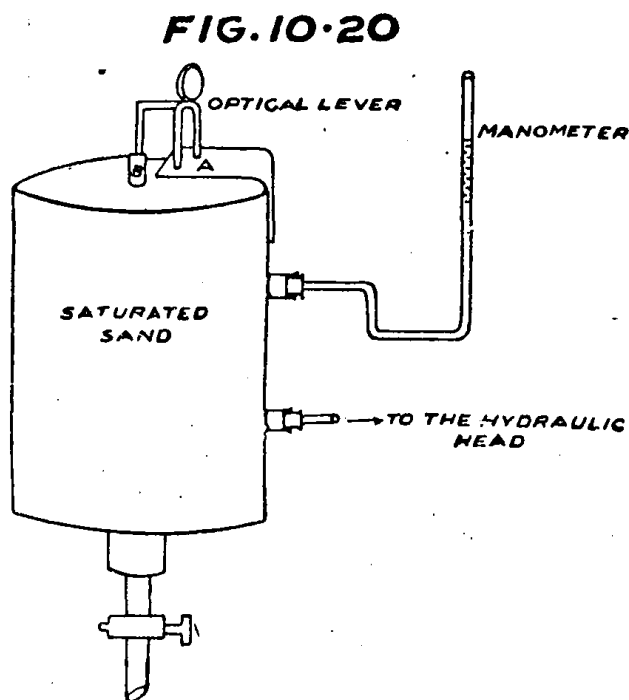
(23) "Hydrostatic Uplift in Pervious Soils." By H. De B. Parsons, M. Am. Soc. C. E., Paper No. 1713 of American Soc. of Civil Engineers (1929).

(24) "Simple Tests Determine Hydrostatic Uplift", by Karl Terzaghi, Engineering News Record, Jun^o 18, 1936, pp. 872-76.

response of pressure pipes to an application or withdrawal of head. The results have been plotted in Plate X-4, Figure 3.

Experiments were recently carried out by Dr. Vaidhianathan with an apparatus shown in Figure 10-20. The reservoir about 2" in diameter was filled with sand saturated with water. The leak-tight cover of the reservoir was a thin brass plate which simulated the impervious masonry of the weir. One arm B of an optical lever rested on this plate and the two other arms on a brass strip A which was independent of the movement of B. Any microscopic displacement of B would obviously cause a tilt of the mirror which was capable of measurement by a beam of light reflected from the mirror of the optical lever. When the pressure head was changed the optical lever responded instantaneously whereas the manometer showed considerable time lag due to its comparatively large bore. This experiment showed that the response of the subsoil to change of pressure was instantaneous although the pressure pipes might take some time to reflect it.

While evidence on the point is inconclusive, it will be safe and fairly correct to ignore the time lag as of no consequence in influencing the stability of a weir floor under sudden application of excessive heads.



APPENDIX TO CHAPTER X

An empirical formula for the pressure percentages at the joins of a variable intermediate pile to a floor, which is also provided with equal end-piles.*

This formula has been evolved to supplement the correction formulated by Khosla for the mutual interference of piles in such cases as gave results departing widely from his "corrected" values. It will apply so long as the distance between the inner pile and either one of the outer piles is not less than one-eighth of the floor length. (See page 133).

For end-piles each equal to $\frac{1}{8}$ of the floor length, with the inner pile under the down-stream half of the floor, the formula giving the percentage pressures at the points E_1 and C_1 is—

$$\varphi_{E_1}, C_1 = 50 - 40 \frac{x}{b} \pm \frac{42D}{b} \quad (10-A. 1)$$

where x is the distance of the top of the inner pile from the centre, D is the length of the pile and b is the floor length.

This formula fits the experimental data with a maximum difference of about 1.5 per cent. Below are shown some values: experimental, Khosla's, and those given by this formula:—

x/b	D/b	φ_{E_1}			φ_{C_1}		
		Expt	(10-A. 1)	Khosla	Expt	(10-A. 1)	Khosla
1/8	1/24	47.0	46.8	42.6	42.8	43.2	42.0
1/8	1/12	48.8	48.5	45.0	41.1	41.5	39.6
1/8	1/8	50.3	50.2	47.1	39.5	39.8	37.7
1/4	1/24	42.3	41.8	34.4	38.3	38.2	33.4
1/4	1/12	43.2	43.5	37.3	36.1	36.5	31.2
1/4	1/8	45.0	45.2	40.2	34.8	34.8	29.5
3/8	1/24	37.3	36.8	25.3	34.0	33.2	23.4
3/8	1/12	38.2	38.5	29.4	31.8	31.5	21.8
3/8	1/8	40.0	40.2	33.0	30.2	29.8	20.4

The basis of this formula is the assumption that the inner pile is a disturbing factor in the distribution set up by the equal end-piles, and that the pressure cut off by a pile is proportional to its length. The distribution values with no inner pile for the case of equal end-piles each $\frac{1}{8}$ of the floor length could be very nearly taken as $50 - 40 \frac{x}{b}$, and it only remained to apply the correction for the inner pile.

For end-piles not equal to $\frac{1}{8}$ of the floor length, but not more than $\frac{1}{4}$ th of it, the following modified formula may be found useful.

$$\varphi_{E_1, C_1} = 50 - 63 (1 - 2d/b) (x/b \mp D/b) \quad (10-A. 2)$$

where d is the depth of either one of the outer piles.

*By J. K. Malhotra, Officiating Mathematical Officer, Punjab Irrigation Research Institute, Lahore.

CHAPTER XI

Design of Weirs in relation to Surface Flow

In Chapter X the principles governing the design of weirs in relation to the flow of water through the permeable sub-soil have been discussed. The principles governing these designs in relation to the flow of water over the surface, will be dealt with in this Chapter.

With the extensive and comprehensive field and laboratory researches in the Punjab, it has been possible to reach a stage of finality in the treatment of the subject with regard to sub-surface flow for which the controlling factors in designs can now be determined with a fair degree of accuracy, and correct designs can be worked out with the help of simple rules. But the subject of surface flow suffers from a partial lack of such researches. The material discussed in this Chapter represents the accumulated experience in the field supplemented in certain instances by a limited amount of experimental investigation. The recommendations made as a result of the study of the material are tentative, but probably represent the best practice at the present time. These recommendations will no doubt have to be modified somewhat when the subjects discussed have been more thoroughly investigated both in the field and in the laboratory.

For the present it will be desirable to deal only with those aspects of the subject which have a direct and intimate bearing on the general problem and a discussion of which will lead to an elucidation of the fundamentals of design. The questions of the selection of site, flood investigation, pond level and silt exclusion though of vital importance in the location and design of headworks, fall outside the scope of this work which essentially deals with the factors governing the designs and stability of such works and the determination of the safest and most economical sections for them. In subsurface flow, these governing factors have been shown to be the exit gradients and uplift pressures which depend on the total head and the foundation profile of the work. In surface flow there is the additional factor of the dynamic impact, and the designs in this respect will depend on (1) the afflux, (2) the changes in the regime of the river caused by the construction of a weir, (3) the discharge per foot run, (4) flood scour, and (5) the position and depth of the trough of the standing wave.

AFFLUX

By afflux is meant the rise in maximum flood level of the river upstream of the weir as a result of its construction. As will be shown later on, this afflux though confined in the beginning, to a short length of the river above the weir, extends gradually very far up till the final slope of the river upstream of the weir is much the same as it was before the construction of the weir. In the design of weirs founded on alluvial sands, the afflux is limited to between 3 and 4 feet—more commonly, three feet. In very steep reaches of the rivers with boulders or rock bed, the afflux may safely be considerably higher.

The amount of afflux will determine the top levels of guide banks and their lengths, and the top levels and sections of a flood protection *bunds*. It will govern the dynamic action, as the greater the afflux or fall of levels from upstream to downstream, the greater will be the action. It will also control the depth and location of the standing wave.

By providing a high afflux, the width of weir can be narrowed but the cost of training works will go up and the risk of failure by outflanking will increase. The discharge per foot run, the depth of scour and, therefore, the section of the loose protections upstream and downstream, as well as the depths of piles at either end will also increase.

Table XI-I

The following table shows the afflux at the various weirs in the Punjab :—

Name of Weir.	Maximum Flood Levels.		Afflux.	Maximum flood Discharge.
	U/S.	D/S.		
Rupar	877.0	872.5	4.5	315,500
Ferozepur	651.5	648.5	3.0	450,000
Suleimanki	572.0	569.0	3.0	325,000
Islam	457.0	455.0	2.0	275,000
Panjnad	341.5	338.5	3.0	700,000
Mersala	814.1	811.1	3.0	718,000
Khanki	738.0	735.0	3.0	750,000
Rasul	724.2	723.5	0.7	875,000
Kalabagh (proposed)	699.0	694.0	5.0	1,200,000 Absolute maximum.
			3.0	900,000 Possible maximum.

THE EFFECT OF CONSTRUCTION OF WEIRS ON THE REGIME OF A RIVER

In the initial stages, the effects of the construction of a weir and the withdrawals of water by the off-taking canals are :—

- The slope of the river upstream of the weir flattens due to the ponding up of supplies.
- An increase in tortuosity, as a result of ponding up, as the bulk of the silt charge of the river water deposits in the pond, leading to the formation of irregular shoals.
- A progressive degradation or retrogression of bed levels downstream, due to the picking up of bed silt by the relatively silt-free water escaping over the weir.
- These effects continue for the first few years after which as a result of continuous silting up of the pond and increasing tortuosity, the bed levels upstream will tend to rise as the bed levels at the weir are fixed. This can be explained by the fact that the water will need greater head to overcome the increased distance over which it has to travel. An increase in tortuosity will necessarily enhance the rise of levels due to weir afflux with the result that this rise will be felt higher up the river than would otherwise be the case. As a result of this progressive rise of bed levels, there will be a tendency on the part of the river to regain its original slope.
- A stage will be reached when the upstream pond will absorb no further silt burden. Owing to the off-taking canals withdrawing comparatively silt-free water, the excess of silt will go downstream of the weir, while the amount of water passing over the weir will be below the normal due to canal withdrawals. The river below the weir will, thus, have to carry an excessive silt charge with a lower discharge. This will result in progressive silting up downstream, an increase in tortuosity and, therefore, a recovery of bed levels downstream.

The final bed levels downstream may actually be higher than those existing before the weir was constructed. This re-adjustment may take 20 to 30 years. Below Khanki, this rise is of the order of 2 feet and is still continuing.

In passing, it may be mentioned that the bed silt of rivers gets finer and the surface slope flatter as the sea is approached. This progressive reduction in the grade of silt is not due to the silt being worn down by attrition but due to the silt selective behaviour of flow. As the rivers emerge from the hills, the coarser particles are dropped and the finer alluvium is picked up. This process extends down the river constantly and continuously.

The regime of any one river of an alluvial system will alter in sympathy with any changes in any other river of that system due to the withdrawal of water by canals. A lowering of levels in any alluvial river leads to a retrogression of levels proceeding upstream in a river of similar character which joins it.

EFFECT OF REGIME CHANGES ON WEIR DESIGN

The changes in the regime of a river caused by the construction of a weir and the withdrawals by the off-taking canals, have an important bearing on design. There has been serious trouble in the past due to a lack of proper appreciation of these effects.

(a) Retrogression of Levels

In the first few years after the construction of a weir, the retrogression of bed levels downstream is rapid and progressive. In the case of the Punjab weirs this has ranged between 4 and 7 feet. This lowering of the bed levels in the early stages if not duly allowed for in design may result in a failure like that of the Islam Weir in 1929. This retrogression may undermine the stability of a work by an increase in the exit gradient beyond the safe limits. It will increase the destructive action of the standing wave as with the increased fall and decreased depth of downstream water due to the lowering of the water levels at that end, the wave will tend to travel down to the block and loose protection area.

As a result of the retrogression in bed levels, while the low water levels have been found to drop from 4 to 7 feet, the maximum flood levels have not been known to have dropped by more than 1 to 1.5 ft.

(b) Restoration of the original slope upstream of the weir

In course of time, the weir upstream will regain its original slope which implies that the effect of afflux due to the construction of the weir will not be confined in magnitude and length to the usually accepted distance of double the length of the pond. It will travel very far up and will be felt in full all that distance. In other words, the entire bed of the river will ultimately rise uniformly throughout the zone of protection and training works. As, however, the full effect will not be felt until after a lapse of many years, the free-board may, in the first instance, be reckoned above the H. F. L. as determined by the back-water curves.

(c) Recovery of downstream bed levels

The process of recovery of downstream bed levels after the initial retrogression is slow but steady. It may take 20 to 30 years but the bed levels may rise higher than those before the construction of the weir. At Khanki the rise above the original bed level has been of the order of 2 feet. A rise in the downstream bed levels may lead to loss of control of the river in respect of silt regulation, making it necessary to raise the weir crest. Khanki and Merala are instances of such raising. The Khanki Weir crest was raised by two feet in 1910-11 in Bays 4, 5 and 6 and in 1917 in the remaining bays. It was again raised by a further 2 feet in 1920-22. Merala was raised by 2 feet in 1925-26 in order to obtain control of the river and to improve silt conditions in the Main Line. These facts point to the necessity of designing for a pond level sufficiently above the full supply level in the canal so as to leave ample margin even after the rise in the downstream bed has taken place.

The subject of changes in regime of rivers has been dealt with exhaustively by Messrs. Nicholson and Trench in their Report on the Bhakra Dam Storage.¹

DISCHARGE PER FOOT RUN AND WATER-WAY

In the Punjab weirs founded on the alluvial sands, the safe rule has been to provide sufficient water-way to pass the maximum flood discharge with an afflux not exceeding 3 feet. In shingle beds a bigger afflux may be permissible.

A likely figure to adopt for width of water-way would be the minimum stable width for the maximum flood discharge given by the Lacey formula

$$P_w = 2.67 Q^{\frac{1}{3}}$$

where P_w = minimum stable width in feet,

and Q = maximum flood discharge in cusecs.

But the river discharges vary through a very considerable range from a few hundred cusecs in winter to a few hundred thousand cusecs in the monsoon. The minimum stable width that will suit one set of conditions will not suit another. There are two schools of thought in this respect. One school advocates a width of the weir nearly equal to the minimum stable width required for the maximum flood discharge. The argument advanced in favour of this is that with a restricted width of water-way the shoaling upstream can be mostly eliminated and a nearly straight and stable approach to the weir secured. But as has been explained above under regime changes, there must be shoaling and an increase in tortuosity as a result of the ponding up. The Sutlej Valley weirs are examples of restricted width of water-way. The shoals persist at all of them and in no case a straight stable channel has been secured and maintained.

(1) Report of the Committee of Superintending Engineers on the probable effects of the Bhakra Dam on inundation canals on the Indus (1931). Printed at the C. M. G. Press, Lahore. 2 G

The second school advocates, what may be called, a loose weir, with a relatively greater width and shallower depth of water over the crest. In this case, the discharge per foot run over the weir will be relatively smaller and as the dynamic action of water and the depth of scour are governed by the discharge per foot run, the length and thickness of the impervious and pervious floors can be made smaller. With an increase in the discharge per foot run, the depth over the crest and, therefore, the cost of gates will increase, so that what is saved on account of reduced width may be used up by the heavier section of the floor and loose protection and by the additional quantity of steelwork. The possibility of damage due to local concentration of flow will, obviously, be greater with a higher discharge per foot run.

In using the P, Q formula in connection with determining the water-way, it must be remembered that this formula applies to regime channels whose bed and slope profile must conform to an ellipse. The construction of a weir with a level crest alters the regime conditions and the only formula that can apply in this case will be the one based on a discharge per foot run as against the total discharge.

The necessary water-way may be provided at one level over a barrage having a uniform crest level or partly at a low level through the undersluices and partly at a higher level over the weir crest. In the latter case, the relative capacity of the undersluices and their depth below the weir crest will need careful consideration.

Table XI-2

Showing Water-way, Discharge per foot run, etc., on Various Weirs

RIVER	SUTLEJ				JHELUM	CHENAB				INDUS	
	Rupar	Ferozepore	Suleimanki	Islam	Rasul	Merala	Khanki	Trimmu (Proposed)	Panjnad	Sukkur Barrage	
Upstream H. F. L.	877.00	651.50	572.00	547.00	724.00	814.10	738.00	493.50	341.50	201.00	
Crest Levels	Weir	866.50	633.50	560.00	441.00	711.50	802.00	727.00	477.50	325.00	177.00
	Undersluices	857.00	633.50	552.00	435.50	701.00	792.00	715.00	472.00	325.00	176.00
Width between abutments (B)	2663	1956	2223	1621	4400	4475	4414	3022	3400	4725	
Length of clear water-way.	Weir	2202	1200	1440	921	4000	4000	3000	2220	2820	3240
	Undersluices	240	540	480	480	240	240	960	420	240	720
	TOTAL	2532	1740	1920	1401	4240	4240	3960	2640	3060	3960
Designed maximum flood discharge (Q)	315,500	450,000	325,000	275,000	875,000	718,010	750,000	645,000	700,000	1,500,000	
Max. discharging capacity.	Weir	251,60	310,000	210,000	218,000	771,000	647,000	435,000	502,000	640,500	1,230,000
	Undersluices	63,000	140,000	115,000	170,000	104,000	80,000	372,000	143,000	59,500	270,000
	TOTAL	315,500	450,000	350,000	388,000	875,000	736,000	807,000	645,000	700,000	1,500,000
Minimum stable width = $P_w = 2.67\sqrt{Q}$	1500	1784	1520	1400	2501	2263	2312	2140	2230	3260	
Ratio $\frac{B}{P_w}$	1.7	1.10	1.47	1.15	1.76	1.98	1.91	1.41	1.56	1.45	
Discharge per foot run	Weir	110	148	146	208	193	160	145	226	248	380
	Between piers										
	Undersluices	266	250	240	342	433	357	387	340	248	380
	Outside piers										
	Weir	108	133	126	186	165	157	142	203	222	340
	Undersluices	213	237	205	203	370	235	300	305	222	340
Depth of scour below H. F. L. (R) for $f=1$.	Weir	20.7	84	23.6	29.5	26.8	26	24.4	31.0	32	43.6
	Undersluices	32	94	31.3	30.7	40.3	33.6	41.0	40.5	32	43.6

UNDERSLUICES

The main functions of the undersluices are to provide and maintain a definite and deep approach channel to the canal head regulators and to control the entry of silt into the canal by providing a deep and relatively still-water pond in front of the head regulators. The crest

level of the undersluices, their sluicing capacity and their distribution along the weir will be governed mainly by the above two considerations. The Khanki Weir has three sets of undersluices, viz., 12 spans of 20 feet on the left adjoining the canal head regulator, 18 spans of 20 feet in the middle of the weir (Bay 4) and 18 spans of 20 feet on the extreme right. The latter two—the middle and the right—were added during the reconstruction of 1933-35.¹ The probable low water discharge of the river at the time of diversion over the completed weir, which will have to be passed through the undersluices with as little heading up as possible should be taken into consideration in determining their capacity. The floor of the undersluices should be placed as low as possible subject to the limit imposed by a consideration of the cost of foundations and floor, and the difficulties of pumping during construction. A low floor level in the undersluices will be of very great help and may mean a large saving in cost should heavy repairs become subsequently necessary in any part of the weir, as it will be possible to pass winter freshets at a level several feet lower than would otherwise be the case, thus greatly reducing the cost and risk involved in pumping.

As stated before, the river bed upstream will rise due to the afflux during floods, the ponding up of supplies during the low water period and the withdrawals of comparatively silt-free water into the canal. The depression of the floor and the sluicing capacity of the undersluices will not altogether arrest this rise.

The discharge per foot run through the undersluices will be obviously high necessitating a considerably thicker section of floor and heavier protections upstream and downstream.

WEIR

The level of the crest of the weir will be determined by the permissible afflux during the maximum flood, the discharge per foot run and the pond level. The latter can be maintained by a permanent masonry crest with its top at pond level as at Rasul, or one at a lower level supplemented by falling shutters as at Khanki and Mera'a, or counterbalanced gates as at Suleimanki and Islam.

The experience at Rasul indicates that a permanently raised crest is likely to result in a loss of control in training the river. Unless there are strong reasons to the contrary in any particular case, this type would appear to be unsuitable. The choice between a weir with shutters and one with counterbalanced gates is largely a matter of cost and convenience of working. A shuttered weir will generally be cheaper but will lack the speed and effective control possible with a gated weir. With the latter type, a roadway can be constructed across the river at small cost. Where cost permits and means of communications across the river are desirable, a gate controlled weir will naturally suggest itself. Modern practice favours this course.

BARRAGE

A barrage is a gate controlled weir right across the river with the crest at one uniform level. Ferozepur and Panjnad Weirs are examples of this. The rise of bed levels upstream, the retrogression of levels downstream and subsequent recovery will take place irrespective of whether it is a barrage or a weir with undersluices, but the latter will afford better control of the approach channel to the canal regulators and better facilities for inspection and repairs of the weir floor, more particularly the upstream part. Where silt trouble is likely to be acute a barrage will possibly be less effective in silt exclusion.

Ferozepur and Panjnad Weirs are of the barrage type. The so-called undersluices are just a part of the barrage separated from the rest by a divide wall to form a pocket in front of the regulators.

Islam weir was originally constructed as a barrage but a set of depressed undersluices were added during reconstruction after the 1929 failure.

Depth of Scour and Section of Aprons

The method of determining flood scour and the various classes of scour have been dealt with in Chapter X. The depth of scour is given by

$$R = 0.9 (q^2/f)^{1/3} \quad \dots \dots \dots \quad (\text{Eqn. 10.6})$$

where R = depth of scour,

q = discharge per foot run, at ends of impervious floor (outside the piers),

and f = Lacey's silt factor.

(1) "Reconstruction of the Khanki Weir" by R. B. A. N. Khosla, Paper No. 106, Punjab Engineering Congress, 1936, 2 G 2

The Lacey formula has been accepted in preference to that of Kennedy as the former is hydrodynamically more rational and takes note of different grades of bed material.

The values of R for the different values of q and f can be obtained from the Plates X.2 and X. 3. The values of f can be obtained from Lacey's formula connecting Q , slope and f . The total discharge and slope at that discharge for any site can be observed and therefore f can be determined.

The relationship is given by

$$S = \frac{1}{1788} \times \frac{f^{5/3}}{Q^{1/6}}$$

where S = slope,

f = silt factor,

and Q = total discharge.

This formula can be re-written in terms of q the discharge per foot run. for

$$P_w = \frac{8}{9} \sqrt[3]{Q}$$

$$Q = P_w \times q = \frac{8}{9} \sqrt[3]{Q} \times q. \text{ (This applies where width is large compared to depth).}$$

Or $\sqrt[3]{Q} = \frac{9}{8} q$, so that

$$S = \frac{1}{1788} \times \frac{1}{(\frac{9}{8})^{1/3}} \cdot \left(\frac{f^5}{q}\right)^{1/3} = \frac{1}{2480} \left(\frac{f^5}{q}\right)^{1/3}$$

The relation of the depth of flood scour and the depths of sheet piling at the upstream and downstream ends of the pacca floor, has been discussed in the last chapter. The lengths and depths of pervious protections to the pacca floor will now be considered in relation to flood scour.

According to Spring¹ the quantity of stone in the aprons should be sufficient to afford approximately 3 feet cover over a slope of 2 : 1 below the level at which the apron is originally laid to the bottom of the deepest scour that is likely to occur at the particular locality.

WEIR AND GUIDE BANK APRONS

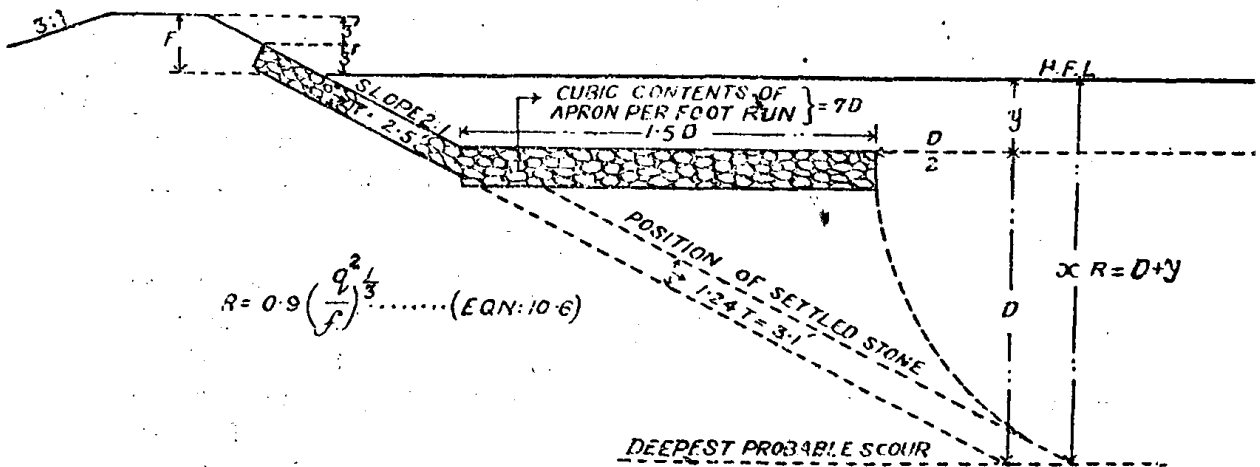


Figure 11.0

If T be the thickness of stone on the slope (see Figure 11.0), the depth of covering over the slope made by the falling apron due to scour should, according to Spring¹, be $1.24 T$. In the Punjab weirs, T is generally 2.5' so that $1.24 T = 3.1'$.

The following Table² gives the thickness of stone pitching necessary to protect sand surfaces for various grades of sands and slopes of rivers.

Fall per mile in inches	3	9	12	18	24
Sand Classification.	Thickness of Stone Pitching in inches.							
Very Coarse	16	19	22	25	28
Coarse	22	25	28	31	34
Medium	28	31	34	37	40
Fine	34	37	40	43	46
Very fine	40	43	46	49	52

(1) Technical Paper No. 153. Government of India (1903). "River Training and Control on the Guide Bank System", by F. J. E. Spring, C, I, E, (Plate XX),

(2) Page 31 of above,

"By the hand packing and careful gradation of the stone, that is by keeping the smaller stuff, such as quarry refuse or even bricks next the sand and the large 80 to 160 lbs. stone for the outside, these thicknesses may perhaps be safely reduced by 6" to 9" all round." According to Spring¹, the design of apron should be based on the unreduced thickness of the slope stone and that any reduction due to graded packing should be confined to the slope stone only. The gradation of stone will be as desirable in the aprons as it is on the slopes, as by this means, the suction of sand from between the interstices of stone, by the high velocity jets, will be reduced to a minimum.

Now, if D be the depth of scour below the level at which the apron stone is laid, the length of sand face to be covered on a slope of 2 : 1 will be $\sqrt{5} D = 2.23 D$.

The necessary quantity of stone per foot run will be

$$3.1 \times 2.236 D = 6.93 D, \text{ say } 7 D \quad (11.0).$$

Before attempting to determine values D , it will be desirable to decide the class of scour that is likely to be met with at different places along the weir and guide banks. The following values are offered as a provisional guide which may have to be modified subsequently as a result of more exact knowledge.

Depth of scour for Design of Aprons

Locality	Range	Mean
1. Upstream of Pacca floor	1.25 R to 1.75 R	1.5 R
2. Downstream of Pacca floor	1.75 R to 2.25 R	2.0 R
3. Noses of Guide Banks	2.00 R to 2.5 R	2.25 R
4. Transition from Nose to straight	1.25 R to 1.75 R	1.5 R
5. Straight reaches of Guide Banks	1.0 R to 1.5 R	1.25 R

The value of R is calculated on the normal discharge per foot run at the section concerned, without allowing for any concentration of flow. If the usual 20% is allowed for concentration the above coefficients will need to be reduced correspondingly.

The values of D can now be determined from

$$D = xR - (\text{High flood level} - \text{floor level}) = xR - Y$$

where x is the multiplier in the above table, and R is given by equation (10.6).

At the upstream end a further protection of blocks over loose stone should be given so as to protect the soil adjoining the upstream end of the *pacca* floor. The length of this protection should be nearly equal to D and the thickness the same as that of the stone apron at this end. It is generally 4 feet in the major Punjab weirs and consists of 2 feet deep concrete blocks over 2 feet of graded stone.

At the downstream end, there should be an area of inverted filter of length equal to $1.5D$ to $2D$ and depth equal to that of the downstream stone apron. This should be made up of deep blocks (3 to 4 feet over 2 feet of graded filter bed made up of fine *bajri* laid over the sand of the bed. Wide shallow blocks are apt to be carried away by the current of water. Deep blocks get wedged in and resist dislocation. This has been discussed in the Islam Enquiry Committee Report² and in a paper on the reconstruction of the Khanki Weir.³ The interstices between the blocks are filled up with *bajri*.

The total quantity of material per foot run in the pervious protections upstream and downstream, comprising the loose stone, blocks, and inverted filter sections, may suitably be equal to $10D$ as against $7D$ for the loose stone protection only.

The upstream block protection and the downstream filter area are meant to be immovable. They are flexible and are supposed to adjust themselves to slight subsidence but they are not intended to fall in the same way as the loose aprons. Whenever these protections are damaged they should be made good at once. Their existence, intact, will be a definite safeguard against any damage to *pacca* floor.

The following Table gives the discharges per foot run over the various weirs and the protections as provided and as calculated by the above method.

(1) Technical Paper No. 153, Government of India (1903). "River Training and Control on the Guide Bank System" by F. J. E. Spring, C. I. E. (Plate XX). Page 49.

(2) Report of the Islam Enquiry Committee into the Failure of Islam Weir in September 1929.

(3) "Reconstruction of the Khanki Weir," by R. B. A. N. Khosla, Paper No. 195 of Punjab Engineering Congress (1930). Page 9.

Table XI₃

Analysing stone in aprons in relation to discharge per foot run

RIVER	SUTLEJ				JHELUM	CHENAB				INDUS		
	Name of weir	Rupar	Ferozepore	Sulel-manki	Islam	Rasul	Merala	Khanki	Trimmu (Proposed)	Panjudad	Sukkur Barrage	
H. F. L.	U/S	877.00	651.50	672.00	457.00	724.20	814.10	738.00	493.50	341.50	201.00	
	D/S	872.50	649.50	569.00	455.00	723.50	810.00	735.00	490.50	338.50	198.00	
Depth of scour (R) below H. F. for f=1.	Weir	20.7	34.0	23.6	29.5	26.8	26.0	24.4	31.0	32.0	43.6	
	Under-slucices.	32.0	34.0	31.3	39.7	46.3	38.6	41.0	40.5	32.0	43.6	
Weir floor level	U/S	857.00	630.50	552.00	438.00	702.00	792.10	722.00	470.00	320.00	171.00	
	y	20.0	21.0	20.0	19.0	22.2	22.0	16	23.5	21.5	30	
	D/S	857.00	626.50	549.00	434.00	701.29	793.83 789.83	715.00	470.00	316.00	171.00	
	y	15.5	22.0	20.0	21.0	22.3	16.17 20.17	20.05	20.5	222.5	27	
Undersluice floor level..	U/S	857.00	630.50	552.00	438.00	701.00	792.10	710.00	463.00	320.00	175.00	
	y	20.1	21.0	20.0	19.0	23.2	22.0	23.0	25.5	21.5	26.0	
	D/S	857.00	626.50	549.00	434.00	701.00	792.10	710.00	464.00	315.00	175.05	
	y	15.5	22.0	20.0	21.0	22.5	17.9	25.5	26.5	23.5	22.95	
Quantity of stone per foot run in aprons = 7 D.	Weir	U/S	120	92	45	72	120*	160	120	200	154	380
		D/S	..	192	120	93.6	150	200	300	300	192	670
	Under-slucices.	U/S	620	92	Paved	72	120	760	136	250	154	..
		D/S	620	192	200	117	140	215	480	360	425	950
Depth of scour (D) which these quantities of stone will effectively cover.	Weir	U/S	17.1	13.1	6.4	10.3	17.1	23.0	17.1	23.6	22.0	54.3
		D/S	..	27.4	17.1	13.4	21.4	23.6	42.9	43.0	27.4	96.0
	Under-slucices.	U/S	88.6	13.1	..	10.3	17.1	108.6	19.4	36.0	22.0	..
		D/S	88.6	27.4	28.6	16.7	20.0	30.7	63.6	51.4	60.7	135.7
$X = \frac{D+y}{R}$..	Weir	U/S	1.79	1.0	1.12	0.99	1.47	1.73	1.36	1.68	1.36	1.93
		D/S	0.74	1.45	1.57	1.17	1.62	1.70 1.88	2.60	2.05	1.56	2.32
	Under-slucices	U/S	3.40	1.00	0.64	0.74	0.87	3.39	1.15	1.52	1.36	0.6
		D/S	3.24	1.45	1.55	0.96	0.92	1.26	2.23	1.90	2.03	3.64
Blocks and stone protection = 10D.	Weir	U/S	120	178	45	149	120	230	324	200	242	380
		D/S	..	469	260	368	230	320	474	405	453	1162
	Under-slucices.	U/S	620	178	..	140	120	823	336	460	242	..
		D/S	620	469	444	391	400	515	720	730	693	1565

Table XI/3—contd.

Analysing stone in aprons in relation to discharge per foot run—contd.

RIVER		SUTLEJ				JHELUM	CHENAB				INDUS	
Name of weir		Rupar	Feroze- pur.	Sulei- manki	I lam	Rasul	raia	Khanki	Trimmu (Proposed)	Panjad	Sukkur Barrage	
Depth of scour (D) which these quanti- ties of blocks and stone will effectively cover.	Weir	U/S	12	17.8	4.5	14.9	12	28	32.4	20	24.2	88.0
		D/S	..	46.9	26	36.8	23	32	47.4	40.5	45.3	116.2
	Under- sluices.	U/S	62	17.8	..	14	12	82.3	33.6	46	24.2	..
		D/S	62	46.9	44.4	39.1	40	51.5	72.0	78	59.3	156.5
$X = \frac{D+y}{R}$	Weir	U/S	1.55	1.14	1.04	1.15	1.27	1.92	2.0	1.4	1.43	1.56
		D/S	0.75	2.03	1.99	1.06	1.70	$\frac{1.85}{2.07}$	2.77	1.94	2.12	3.28
	Under- sluices.	U/S	2.63	1.14	0.64	0.82	0.76	2.7	1.5	1.76	1.43	0.6
		D/S	2.40	2.03	2.06	1.52	1.35	1.80	2.36	2.58	2.59	4.11

Downstream of the inverted filter area, the Islam Enquiry Committee¹ recommended a line of wells 20 feet deep. This recommendation was given effect to in the design of the Panjad Weir Annexe and deserves consideration in designing any new major weir. The recommendation (g) of the Committee reads as follows:—

- “A line of piles or wells should always be provided at the end of the permeable floor. This is the most vulnerable point in the work and failure to hold it places the whole weir in danger.”

In considering the discharge per foot run and the relative depth of scour, it must be remembered that what may be suitable for a bridge on deep well foundations will, in general, not be suitable for a weir with comparatively shallow foundations. A heavy concentration of flow may pass harmlessly by the deep wells of the bridge but it may work havoc on a weir. In a bridge, the narrowing can be done to very nearly the minimum stable width, but such narrowing in the case of a weir will expose it to serious risks.

Length, Shape and Free-board of Guide Banks

A detailed discussion of these will be outside the scope of this work but the following quotations from Spring² and the Islam Enquiry Committee Report³ are given for ready reference as they seem to embody modern practice.

According to Spring²—

- “(i) The choice between parallelism, convergence and divergence must be dictated by the condition of the bed of the river, for construction purposes, during the working season. For it is better, if possible, not to be obliged to lay the apron in deep water.
- (ii) But, if practicable, it is better that the guide banks should approach each other near their upper ends, before their upper curves begin. The amount of contraction may be, say, anything up to double the combined thicknesses of the bridge piers below low water.”

NOTE.—The guide banks at the Suleimanki Weir are curved inwards, as they run upstream, to form “bottleneck, the width between the noses being 1,600 feet compared with 2,223 feet on the weir. The object was to induce central flow and prevent the formation of shoals within the guide banks an object which has not been attained.⁴

- (1) Page 8 of the Islam Enquiry Committee Report.
 (2) Chapter VIII, para. 12, p. 41.
 (3) Recommendation (b), p. 9.
 (4) Completion Report of Sutlej Valley Project : 1935, page 42.

- (iii) The length of the upstream part of the guide banks may be made equal to, or say up to a tenth longer than the bridge. But due attention should be paid to the possibility of the river bending round above one guide bank into the still-water area at the back of it, and eroding the main approach bank. In specially wide *Khadirs* this may involve the use of very long guide banks.
- (iv) The length of the downstream part of the guide bank may be a tenth to a fifth of the length of the bridge, according to the judgment that may be formed as to the activity of the swirl or disturbance likely to be caused by the splaying out of the water on leaving the bridge. For the swirl—if there is one—must be kept far enough away not to endanger the approach banks.
- (v) The radius of the curve of the downstream end of the guide bank may be such as the main line trains can run on, say 300 to 600 feet because it is convenient to take the stone service line by this route.
- (vi) The radius of the upstream curved part of the guide bank may be anything from 500 to 1,000 feet according to the estimate that may be formed of the probable velocity of the current past it. The curve should be carried well round to the back, fully 120° to 140°."

On account of the afflux caused by the construction of a wier, the length of guide banks will have to be somewhat greater than that recommended under (iii).

Recommendation (b) of the Islam Committee¹ runs as follows :—

"We consider that curved guide banks, as adopted in the Panjnad design, are, provided that a sufficient length of straight stem is given, for preferable to the straight banks constructed at Ferozepur, Suleimanki and Islam. When the river approaches the work at an angle, as it almost invariably does, a straight guide bank on the side from which the river comes acts as a spur, and, besides inducing heavy action round its own nose, forces the current across to the opposite bank; and not only is the guide bank on that side attacked, but a large shoal is formed on the near side, which extends downstream almost to the weir, contracting the channel, which thus scours excessively. Curved banks not only guide the water more smoothly to the weir, but are likely to prove far less expensive in maintenance than the straight type."

The following Table gives the lengths of guide banks, the distance between them and quantity of stone, *etc.*, at the various Punjab weirs, and at the Lloyed Barrage (Sukkur).

Table XI/4

Name of Headworks	Panjnad	Islam	Sulei- manki	Feroze- pore	Merala	Sukkur		
V/S H. F. L.	341.50	457.00	572.00	651.50	814.1	201		
R. L. of spoin (at nose and at shank)	392.5	448.00	558.00	638.75	805.00	186		
Normal distance of nose from C. L. of weir	Upstream. Downstream	3000	3600	3500	2700	3500	Left. 13350	Right. 5950
		600	830	1125	600	1575	1625	1625
Length along guide banks from C. L. of weir	Upstream . . Downstream	3700	3600	3650	2800	3700	14850	6050
		650	880	1125	650	1575	1800	1800
Width between guide banks	3606.6	1717	1600	2200	4475	5000 (approx.)		
Designed max. flood discharge (Q)	700,000	275,000	325,000	450,000	718,000	1,500,000		
Discharge per foot run (q)	194	160	203	205	160	300		
Depth of scour below H. F. L. (R) for $f=1$	30.5	26.5	31.0	31.0	26.5	40.3		
(ν)	12	14	14	17.75	9.1	15		
Quantity of stone per foot run = 7D	At nose. At shank*	300	240	240	400	455	225	
		113	90	75	90	275	225	
(D)	At nose At shank	43	84.3	34.3	57.1	65	32.1	
		16.1	13	10.7	13	39.3	32.1	
$X = \frac{D+Y}{R}$	At nose At shank*	1.8	1.82	1.56	2.41	2.8	1.16	
		0.92	1.02	0.90	0.99	1.82	1.16	

The quantities of stone at the nose of most guide banks have been subsequently found inadequate. More or less serious damage occurred at these and much larger quantities had to be added. The value of X at the nose should not be less than 2.0 and at shanks not less than 1.25.

(1) Recommendation (b), p. 9 of the Report of the Islam Enquiry Committee into failure of Islam Weir.

A free-board of six feet is allowed at the guide banks and the portions of the marginal bunds in the neighbourhood of the weir. Beyond that a free-board of 3 feet is considered sufficient. As explained earlier, this free-board will ultimately have to be above the highest flood levels at each site raised by the amount of afflux over the weir.

THE STANDING WAVE

As stated before, and as will be explained later, the thickness and length of the downstream floor depend also on the position and height of the standing wave. The latter, in turn, depends on the discharge per foot run, the afflux, the shape of the glacis and the level of the downstream floor.

A detailed discussion of the standing wave or hydraulic jump is contained in the Central Board of Irrigation Publication No. 7¹ and relevant extracts are reproduced below. This brief exposition will help in visualising the exact significance of this phenomenon in weir design.

“When a shallow stream moving with a high velocity, strikes water of sufficient depth there is commonly produced a striking phenomenon which has been appropriately called the Hydraulic Jump (or the standing wave). It consists of an abrupt rise in the surface in the region of impact between the rapidly moving stream and the more slowly moving wall of water, accompanied by a great tumbling of the commingling water, and the production of a white foamy condition throughout the moving mass. Under suitable conditions this standing wave remains steadily in one position. The surface at the beginning of the abrupt rise is constantly falling against the on-coming stream moving at high velocity, and farther along in the jump, masses of water are continually boiling to the surface from greater depths. So much foam is produced that some time must elapse before it can all rise to the surface and the water again become clear. This phenomenon is constantly illustrated in the surf of the sea-shore.

The Momentum Formula

The theory of the standing-wave may be concisely stated as follows :—

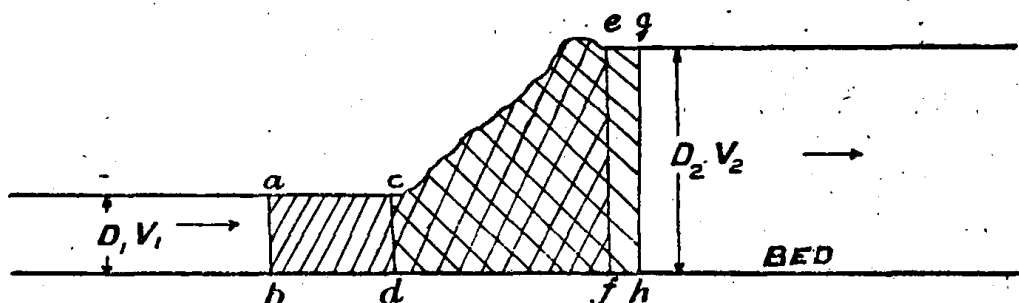


Figure 11.1

Let $abfe$ (Figure 11.1) represent a mass of water moving through a standing wave. In a short interval of time it is supposed to move to the position $cdhg$. The standing wave has the following characteristics :—

- The water entering at ab has a nearly uniform steady high velocity and is transparent.
- The water leaving at gh has a fairly uniform but relatively low velocity and is transparent.
- Between c and e the surface rises rapidly and is much disturbed by spraying and spattering. Much of the surface water appears to be moving down the slope. The whole mass is so milky as to suggest the presence of much internal impact. The milky condition of the water reduces its specific gravity and accordingly the surface at the top of the rise is above the normal level, but as soon as all the air bubbles reach the surface, so that the water is again transparent, the surface becomes smooth and level.

The moving mass of water loses much momentum in passing from the position $abfe$ to the position $cdhg$. According to Newton's Second Law of Motion, the rate of loss of momentum must be equal to the unbalanced force acting on the moving mass to retard its motion.

Against the face ab is the static pressure of the water acting towards the right. Opposed to this force are the static pressure acting against the face ef and the surface friction along the bottom bf . The latter is small and may be neglected.

(1) "The Standing Wave or Hydraulic Jump." C. B. I. Publication No. 7 by A.M.R. Montagu, I. S. R. M. I. C. E., etc.

Other assumptions made in the following analysis are as follows :—

- (a) the "jump" is assumed to take place instantaneously,
- (b) stream line flow immediately before and after the wave,
- (c) friction is neglected,
- (d) parallel side constraints,
- (e) horizontal bed.

In Figure 11-2 Let q be the discharge in cusecs per unit width of the stream,

D_1 be the depth of stream entering the standing-wave,

V_1 be the velocity of the same stream,

P_1 be the total pressure over the vertical section of unit width,

D_2 be the depth of stream leaving the standing-wave,

V_2 be the velocity of the same stream,

P_2 be the total pressure over the vertical section of unit width.

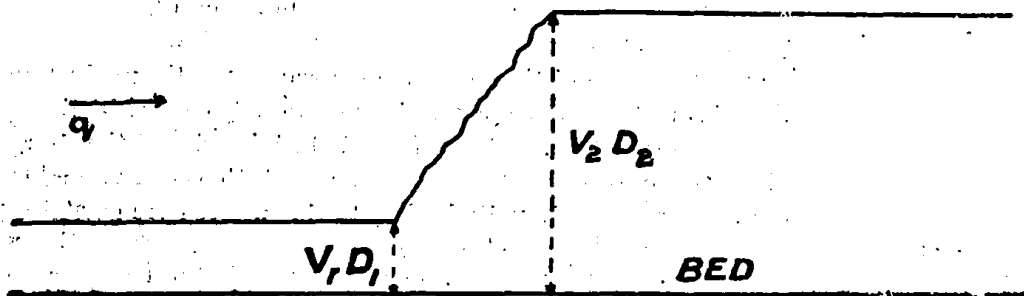


Figure 11-2

Then $q = V_1 \cdot D_1 = V_2 \cdot D_2$

Mass per second = $\frac{W}{g} q$

and change of momentum per second = $\frac{W}{g} q (V_1 - V_2)$

change of pressure = $\frac{1}{2} W (D_2^2 - D_1^2)$

$\therefore D_2^2 - D_1^2 = \frac{2}{g} q (V_1 - V_2)$ (11-1)

This is the fundamental momentum equation for the standing-wave.

Substituting q/D for V

$D_2^2 - D_1^2 = \frac{2}{g} q (q/D_1 - q/D_2)$

$\therefore D_2 + D_1 = \frac{2}{g} q^2 \cdot \frac{1}{D_1 \cdot D_2}$

or $D_1 \cdot D_2 (D_1 + D_2) = \frac{2}{g} q^2$ (11-2)

By substituting the product $V D$ for q , the equation (2) can be thrown into other useful forms, thus :—

$D_1 \cdot D_2 (D_1 + D_2) = \frac{2}{g} \cdot V_1^2 \cdot D_1^2$

$D_2^2 + D_1 \cdot D_2 = \frac{2}{g} \cdot V_1^2 \cdot D_1$

$\therefore D_2^2 + D_1 \cdot D_2 + \frac{1}{4} D_1^2 = \frac{2}{g} \cdot V_1^2 \cdot D_1 + \frac{1}{4} D_1^2$

$\therefore D_2 + \frac{1}{2} D_1 = \pm \sqrt{\frac{2}{g} \cdot V_1^2 \cdot D_1 + \frac{1}{4} D_1^2}$

$\therefore D_2 = -\frac{1}{2} D_1 \pm \sqrt{\frac{2}{g} \cdot V_1^2 \cdot D_1 + \frac{1}{4} D_1^2}$ (11-3)

or, by eliminating V_1

$= -\frac{1}{2} D_1 \pm \sqrt{\frac{2 \cdot q^2}{g \cdot D_1} + \frac{1}{4} D_1^2}$ (11-3-A)

Alternatively

$$D_1 \cdot D_2 (D_1 + D_2) = \frac{2}{g} \cdot V_2^2 \cdot D_2^2$$

whence

$$D_1 = -\frac{1}{2}D_2 \pm \sqrt{\frac{2}{g} \cdot V_2^2 \cdot D_2 + \frac{1}{4}D_2^2} \quad \dots \quad (11.4)$$

$$= -\frac{1}{2}D_2 \pm \sqrt{\frac{2}{g} \cdot \frac{q^2}{D_2} + \frac{1}{4}D_2^2} \quad \dots \quad (11.4-A)$$

Equations (11.3-A) and (11.4-A) can of course also be obtained direct from Equation 11.2.

Clearly, only the *plus* value of the expression below the root sign can be used, as D_1 and D_2 must always be positive.

The applications of the above equations to a standing-wave on a sloping glacis, involves not only the assumptions (a) to (d) on page 180 but also neglect of the component of gravity parallel to the bed (Figure 11.3)

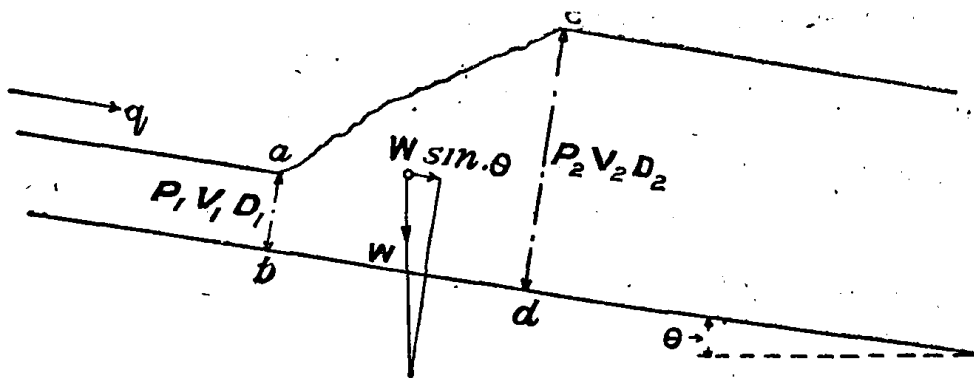


Figure 11.3 Standing wave on Inclined Glacis

In this connection, Bakhmeteff¹ (pp. 233) remarks :—

“The advantage of considering the jump in a canal with a horizontal bottom, lies specifically in the fact that the effect of gravity is eliminated. In case the bottom were not horizontal, as in Figure 11.3, one would have to add to the forces contributing to the change of the momentum, the gravity component $W \sin \theta$ where W is the weight of the liquid body $abcd$. That would require knowing the length and the form of the jump, unless, as done by many authors in the past, the effect of the weight component is simply neglected. Such an approximation, however, is scarcely permissible and, as observations have shown, leads to serious incongruencies”.

For a full treatment of the standing-wave in all its aspects, the advanced student is referred to this Author.¹ Nevertheless, provided that the slope of the glacis is not too pronounced, neglect of the gravity component is justified on the grounds that the error so introduced is practically cancelled by the error involved in the other assumptions. Model tests made *ad hoc*, show that the simplified treatment rendered possible by the neglect of the gravity component on a sloping glacis, produces results which are well within the limits of accuracy required by the practical engineer.

For purposes of this note, therefore, this approximation is accepted and the simple equations derived for the case of a standing-wave on horizontal bed are applied to the case of the gently sloping glacis.

(1) “Hydraulics of Open Channels” by Boris A. Bakhmeteff, M. Am. Soc. C. E. (1932).

The loss of energy in the standing-wave can be computed thus :—

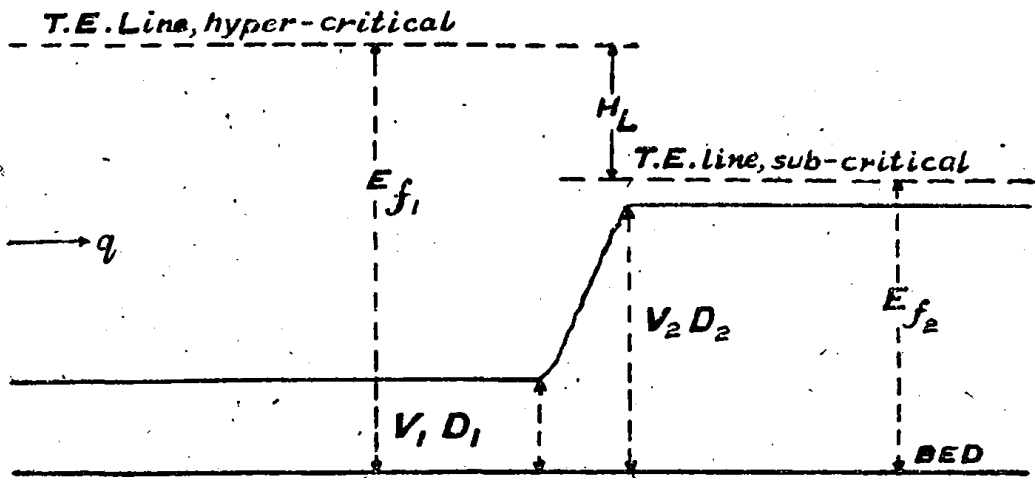


Figure 11.4

Let H_L be the loss of energy,
 E_{f1} be the energy of flow above the wave,
 E_{f2} be the energy of flow below the wave.

Then

$$\begin{aligned} H_L &= E_{f1} - E_{f2} \\ &= \left(D_1 + \frac{V_1^2}{2g} \right) - \left(D_2 + \frac{V_2^2}{2g} \right) \\ &= D_1 - D_2 + \frac{1}{2g} (V_1^2 - V_2^2) \\ &= D_1 - D_2 + \frac{q^2}{2g} \left(\frac{1}{D_1^3} - \frac{1}{D_2^3} \right) \end{aligned} \quad (11.5)$$

This is the fundamental equation for loss of energy in the standing-wave.

Either D_1 or D_2 can be eliminated by the substitution of the values given by Equations 11.4-A) or (11.3-A).

Rewriting equation (11.5)

$$H_L = D_1 + \frac{q^2}{2g} \cdot \frac{1}{D_1^3} - D_2 - \frac{q^2}{2g} \cdot \frac{1}{D_2^3}$$

Substitute the values given in Equation (11.2) for the quantity $q^2/2g$, then

$$H_L = D_1 - D_2 + \frac{D_1 \cdot D_2 (D_1 + D_2)}{4D_1^3} - \frac{D_1 \cdot D_2 (D_1 + D_2)}{4D_2^3}$$

which, upon reduction, simplifies to :—

$$= \frac{(D_2 - D_1)^3}{4D_1 \cdot D_2} \quad (11.6)$$

It is now necessary to examine the relationship of D_1 and D_2 to D_c the critical depth for the discharge q .

By definition, water is flowing at critical depth and velocity when the energy of flow is a minimum.

From any text-book on Hydro-dynamics,

$$q = V_c \cdot D_c$$

$$\text{and } \frac{V_c^3}{2g} = \frac{1}{3} D_c$$

$$\therefore V_c = \sqrt{g \cdot D_c}$$

$$\text{and } q = \sqrt{g \cdot D_c^3}$$

$$\text{or } D_c = 3\sqrt{q^2/g}$$

From equation (11.2)

$$\begin{aligned} D_1 \cdot D_2 (D_1 + D_2) &= \frac{2}{g} q^2 \\ &= 2 D_c^3 \end{aligned}$$

Divide through by D_c^3 and the above equation can be written

$$\frac{D_1}{D_c} \cdot \frac{D_2}{D_c} \left(\frac{D_1}{D_c} + \frac{D_2}{D_c} \right) = 2.$$

Let $x = D_1/D_c$

and $y = D_2/D_c$

$$\text{Then } xy(x+y) = 2 \quad \dots \dots \dots (11.7)$$

which is a *dimensionless* equation.

The relationship of HL to D_c is found as follows :—

Put $z = HL/D_c$

and divide equation (6) through by D_c . then :—

$$z = \frac{(y-x)^3}{4xy} \quad \dots \dots \dots (11.8)$$

This particular analysis is due to E. S. Crump who utilised it to determine the position of the standing-wave, when HL and q are known."

STANDING-WAVE AND DESIGN OF DOWNSTREAM FLOOR

In connection with the standing-wave the following facts which have an intimate bearing on designs, should be carefully noted and remembered. These are :—

- (a) On a level floor with low friction, the position of the standing-wave is unstable. For a slight change in the depth or velocity, the position of the standing-wave will vary through a wide range. The position of the jump on a horizontal and smooth floor cannot, therefore, be closely predicted.
- (b) On a sloping glacis, the position of the standing-wave is definite and can be closely predicted.
- (c) The length of the jump is approximately five times the height. While the beginning of the jump is fairly definite, its lower end is indefinite. In estimating the length, the lower end is taken as the place where the water surface becomes and remains sensibly level, The position of this is variable and difficult to locate.

The exact significance of the above three characteristics of the standing-wave will be best explained by a reference to Plate XI.3, Figures 1 and 2.

Figure 2 (Plate XI. 3) shows the water profiles over the weir with varying discharges. Profile 1 represents conditions with the max- flood discharge and the original downstream bed levels. Curve 2 represents the profile of the water surface for the maximum flood discharge with a foot or so lowering of downstream flood level due to retrogression of river bed. Curves 3 and 4 show the water profiles with smaller discharges per foot run. In the case of Curve 4, the standing-wave forms nearly at the toe of the glacis.

Curve 5 shows a hypothetical case where owing to heavy retrogression of bed levels, the standing-wave forms on the permeable floor.

The dotted Curves 1, 2, 3 and 4 represent the pressure gradient lines under the conditions of flow of the full curves 1, 2, 3 and 4 respectively.

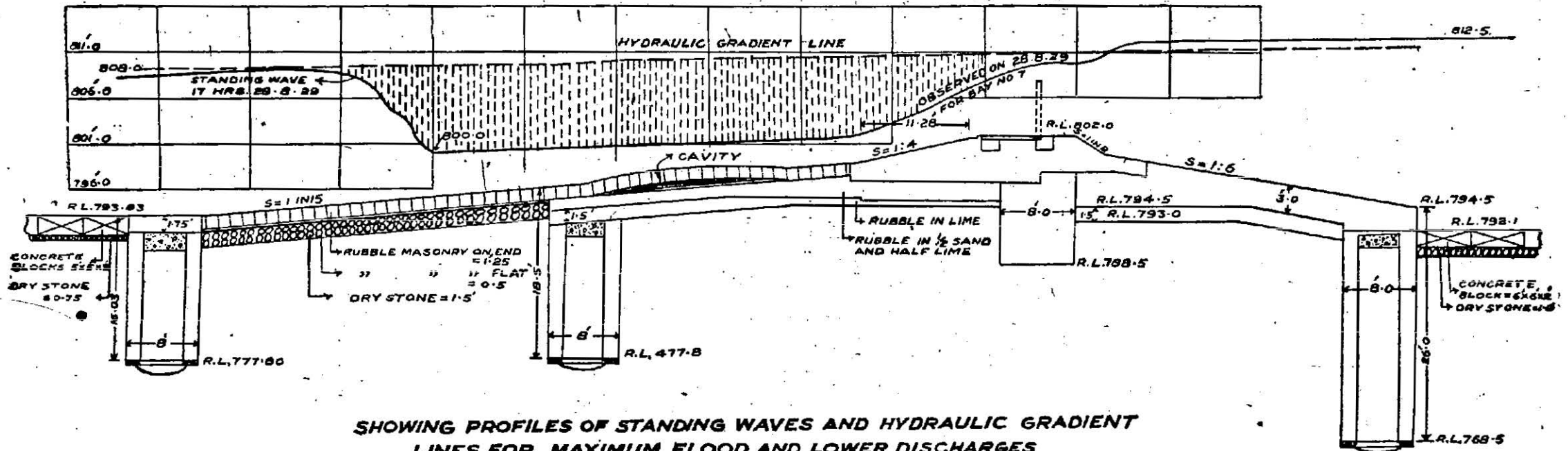
The dotted Curve 6 is the pressure gradient line for the maximum head when there is no flow downstream and the pond is fully up.

The uplift pressures, at any point of the weir, due to sub-surface flow are given by the vertical intercepts between the pressure gradient lines and the respective downstream water levels or, the weir floor level where the latter is higher than the downstream water level. The maximum uplifts due to this cause will occur when the pond is fully up and there is no water on the downstream floor.

But the standing-wave superimposes a new set of conditions. For instance, during the maximum flood when the total head over the weir is no more than 3 feet, the uplift pressures at any point of the weir due to sub-surface flow cannot exceed 3 feet. But the standing-waves have troughs of 9.4' and 11.2' which represent the unbalanced heads or uplift pressures at these troughs. The floor thickness will have to be designed to withstand these pressures of 9.4 to 11.2 ft. Again, the standing-wave is not stationary. It moves downstream with the retrogression of levels and reduction of discharge per foot run. The depths of trough in all cases are about 10 ft. so that in the region of the standing-wave the floor will have to be to withstand these uplifts, with due allowance for spread.

SHOWING UPLIFT DUE TO UNBALANCED HEAD AT TROUGH OF STANDING WAVE
(MERALA WEIR AUGUST 1929)

PLATE XI-3
FIG. 1



SHOWING PROFILES OF STANDING WAVES AND HYDRAULIC GRADIENT LINES FOR MAXIMUM FLOOD AND LOWER DISCHARGES PROPOSED TRIMMU WEIR (ALTERNATIVE SECTION)

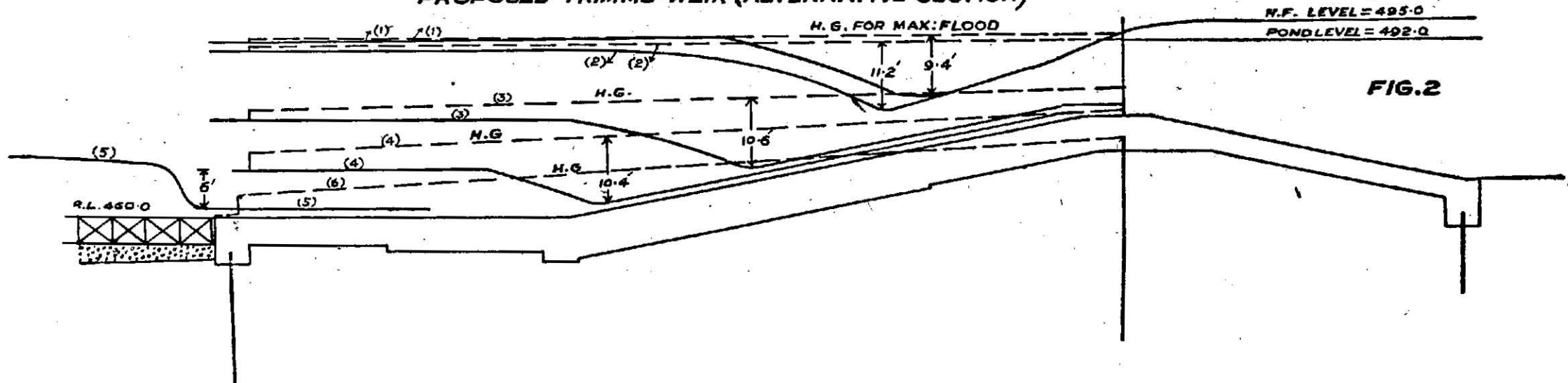
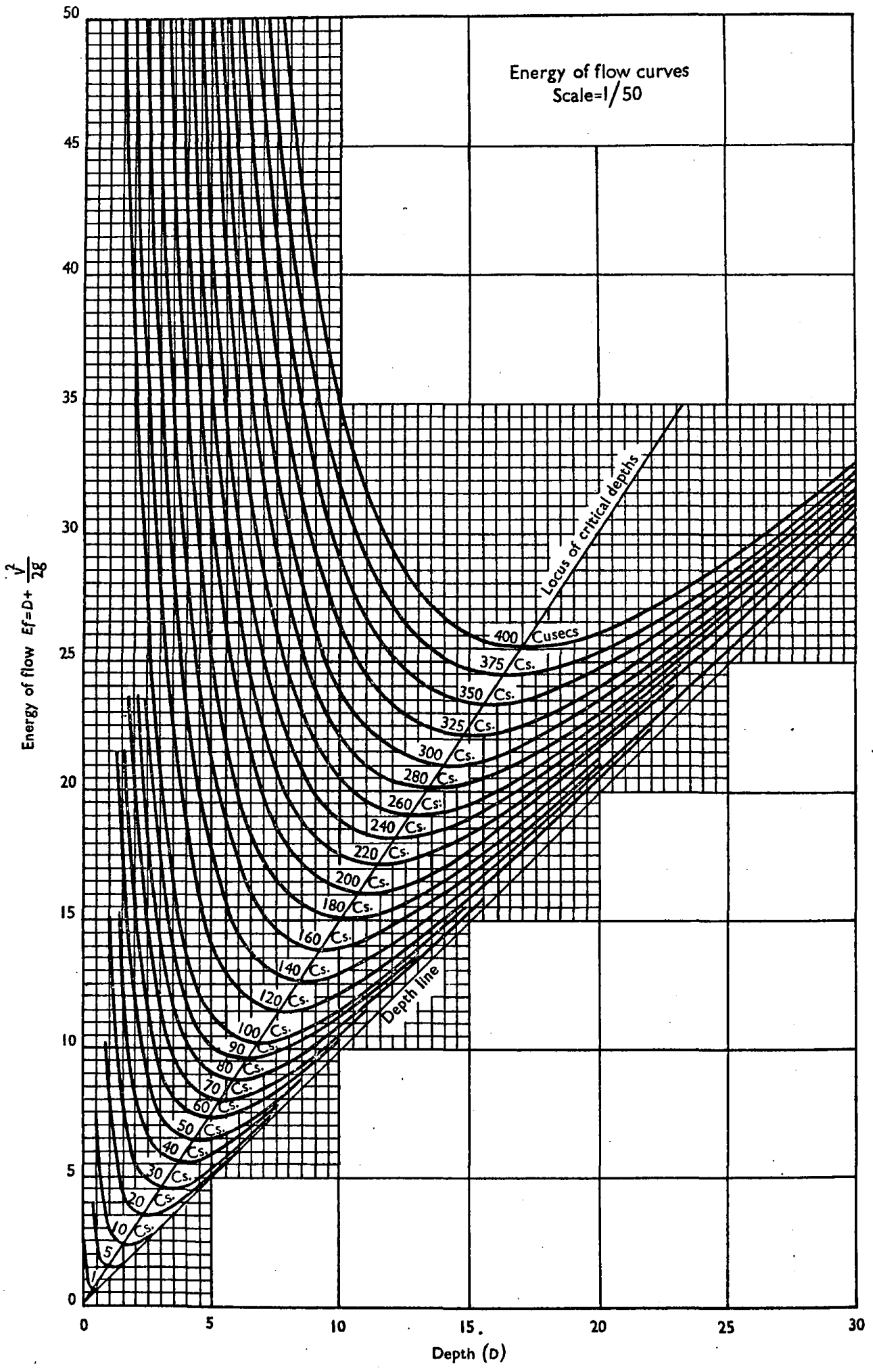


FIG. 2



Thus, the final thickness of the section at each point will have to be designed for the uplift due to sub-surface flow or the standing-wave, whichever is higher. The thickness of the sloping floor could be very small if only the seepage flow were considered. But due to the standing-wave it has to be as big as, if not at places bigger than, that of the horizontal floor.

Next consider the case of Curve 5, the standing-wave forming on the permeable apron. The unbalanced head in this case may be about 6 feet. This head will suck out sand from beneath the loose stone and, to a smaller extent, from below the filter bed and will soon lead to undermining of the floor with consequent risk to the structure. It has been stated above that the position of the standing-wave forming on a smooth horizontal floor is unstable. It can form on the *pacca* horizontal floor or can travel down to the pervious floor downstream with its attendant dangers mentioned above. For a structure to be safe, the standing-wave should, therefore, be confined to the sloping glacis and not permitted to form on the horizontal. A most essential condition for a safe design, therefore, is that the standing-wave should not form lower down than the toe of the glacis.

Thickness of floor as determined by the Standing-wave

In determining the thickness of the floor for the trough of the hydraulic jump, it should be remembered that :—

- (a) The actual position of the standing-wave is somewhat upstream of the calculated position by a distance approximately equal to $(D_2 - D_1)$. (See Figure 11.5.)¹ This is a provisional figure which experimental evidence may confirm or modify,

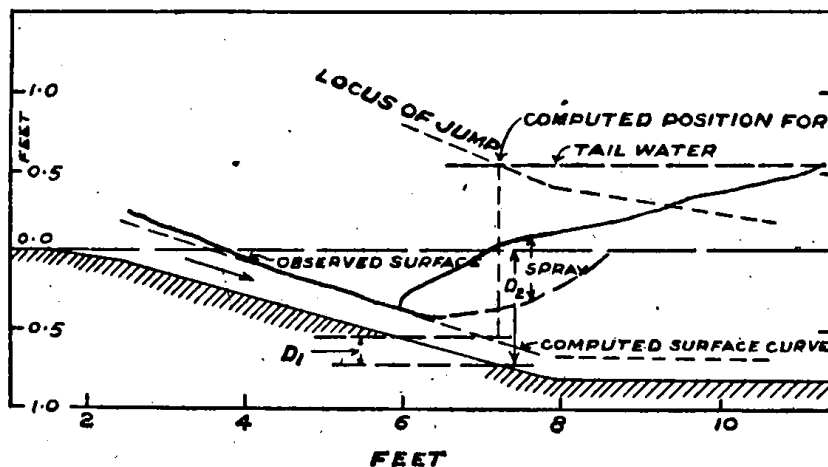


DIAGRAM SHOWING OBSERVED AND COMPUTED POSITIONS OF THE HYDRAULIC JUMP

Figure 11.5

- (b) Due to the forward movement of the upstream end of the jump, the actual depth of trough will be somewhat smaller than calculated.
- (c) The uplift due to the maximum depth at the trough will operate only at the deepest point of the trough. It will become smaller on either side. Since the floor slab has always got some beam action, it may be designed in sections for the average uplift over each section. This point is of great importance as the standing-wave can form all over the glacis in different stages of supply and if no allowance is made for spread, the entire length will have to be designed for the very maximum uplifts—a course which would be wasteful.
- (d) At the toe of the glacis and some distance above and below, the jet of water flowing down the glacis will exert a downward force on the floor equal to the rate of change of momentum in the vertical direction. At the toe this force is considerable and may be as much as two thirds the thickness of jet. It becomes smaller at points removed upstream of the toe. It can be easily calculated for the straight glacis as well as for a curved one. In the latter case acceleration equals V^2/R where V is the velocity of the jet and R the radius of curvature of the glacis near the horizontal floor.

Due to this downward force the thickness of the floor at the toe and some distance above and below can be correspondingly reduced.

(1) "Theory of the Hydraulic Jump and Backwater Curves" by Sherman M. Woodward, U. S. A. (1917), p. 87

Length of Horizontal Floor as determined by the Standing-wave

It has been laid down above that for a safe design the standing-wave should on no account form lower down than the toe of the glacis. It has also been stated that the main disturbance of the standing-wave dies out at a distance from the point of formation equal to five times the jump or $5(D_2 - D_1)$. In order, therefore, that the filter area and the stone protection be immune from the main turbulence of the standing-wave, the length of the horizontal floor should be nearly $5(D_2 - D_1)$, where

D_1 = depth of water just upstream of the point of formation of the wave,
and D_2 = depth of water just downstream of the point of formation of the wave.

THE GLACIS

The design of the glacis, its slope and shape, do not seem to have received much attention so far. The older weirs were generally designed on a slope of 1/15 so as to follow the Bligh gradient line of 1 : 15 and thus require a comparatively small thickness of floor to withstand the small uplift pressures resulting with such a design. No consideration was given to the standing-wave. From actual observations at site and from calculations it has, however, been found that the flatter the glacis, the more intense the wave and the greater the range of the trough, requiring heavy thicknesses along a much greater length of the floor. (Plate XI.3, Figure 1). The steeper the floor, the less the range and depth of the trough of the wave and therefore, the shorter the length and thickness of the glacis. (Plate XI.3, Figure 2).

Slopes of between 1 : 3 and 1 : 5 for the glacis are considered to be the most suitable both for the maximum dissipation of energy and economy.

Montagu¹ has dealt with the design of the glacis profile from a new aspect. According to him, the whole theory of a standing-wave postulates a horizontal velocity and it is during the change of this horizontal velocity from the hyper-critical to the sub-critical stage that the destruction of surplus energy takes place.

The fact that all water moving down a glacis has a vertical component of velocity seems to have been lost sight of so far. The high speed jet which so often persists below a standing wave is the outcome of this vertical component of velocity, the energy of which is unaffected by the occurrence of the standing-wave. He suggests that the profile of the glacis should be so designed that the maximum horizontal acceleration is imparted to the stream at all stages of discharge.

Bose¹ has, similarly, worked out a form of glacis which will give the flowing stream the maximum horizontal acceleration.

The profiles as given by Montagu and Bose are being tested in the Punjab Irrigation Research Institute and it is as yet premature to pronounce on their merits.

Level of Downstream Floor as determined by the Standing-wave.

An essential condition for safety is that the standing-wave shall not form lower down than the toe of the glacis. For a maximum flood, the downstream level, the afflux and the discharge per foot run are known. The loss of head LH will be appreciably the same as the afflux. Where necessary, correction for velocity of approach upstream and velocity of recess downstream, may be allowed.

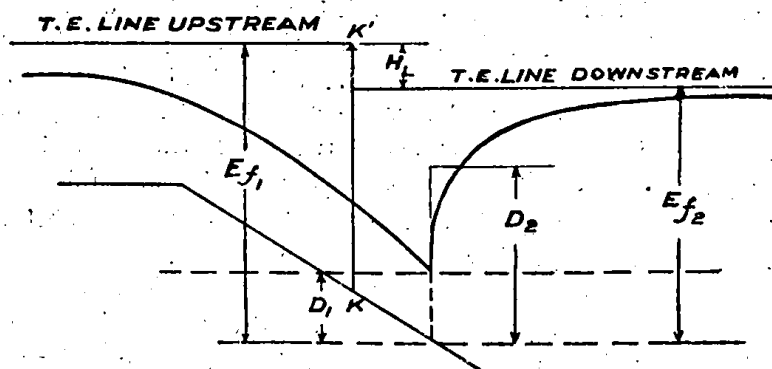
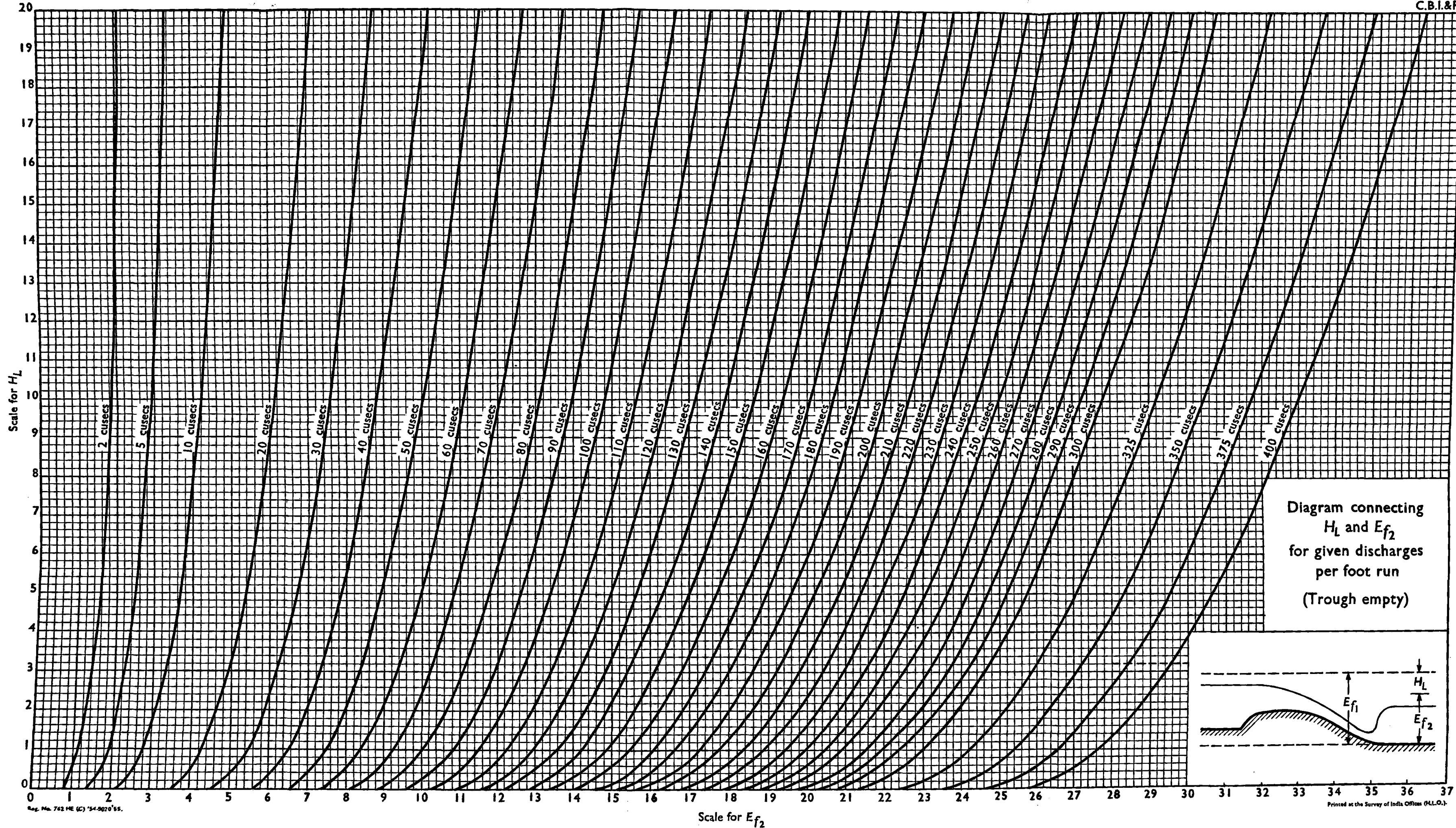


Figure 11.6

$$\text{Now } LH = E_{f1} - E_{f2} \quad (11.5)$$

where E_{f1} = Energy of flow just upstream of the point of formation of the wave,
and E_{f2} = Energy of flow just downstream of that point.

(1) "Irrigation Canal Falls," Appendix VI.B, Central Board of Irrigation Publication, No. 10 (1935).



The profile of water flowing over the weir can be easily worked out from Montagu's¹ hydraulic diagrams which have been extended by Kanwar Sain and are reproduced as Plate XI. I. For instance, at a point K on the glacis the total energy of flow is KK' , *i.e.* the vertical intercept between the total energy line upstream and the point on the glacis. Given the discharge per foot run (q) and the total energy at the point (KK'), the depth of the jet at that point can be read off directly from these curves. Thus for $E_{f1}=20$ and discharge = 240 *cs*, the depth D_1 (Figure 11-6) at hyper-critical flow will be = 9 feet and that at sub-critical flow (D_2) = 16.8 ft. Upstream of the wave there is hyper-critical flow. Downstream of it, there is sub-critical flow. It should be remembered in this connection that the total energy for any point upstream of the standing-wave should be taken below the upstream T. E. Line and that for points downstream of the standing-wave, below the downstream T. E. Line.

Now, given the discharge per foot run and L_H , the afflux, the energy of flow (E_{f2}) below the standing-wave can be directly read off the curves of Plate XI. 2. These curves have been prepared by Kanwar Sain, I. S. E., and are an extension of Blench's curves².

Thus for an afflux of 3 feet and discharge per foot run of 300 *cusecs* we get from Plate XI.2, $E_{f2}=24.64$. From Plate XI. 1, for discharge per foot run $q=300$ and $E_f=24.64$, we get $D_2=216$.

The level of the downstream floor for these conditions will be—
 =Downstream max. flood length— D_2
 =Downstream max. flood level—21.6

The downstream level so obtained should be tested for other conditions of flow when the discharge per foot run is lower and the effect of retrogression of down-stream bed levels of the river fully operative. It should be noticed in this connection that while in low supplies the water level may go down by as much as 4 to 7 feet due to retrogression of the downstream river bed, the maximum flood levels will not be materially altered, a reduction of about a foot or so being the maximum that might be expected.

The discharge per foot run over the downstream floor should be calculated on the width of the floor at the point under consideration, inclusive or exclusive of the width of piers according as the wave forms downstream of or between the piers. Another point to consider in the designs is the velocity with which the water passes over the loose stone apron. The one-man stone commonly used in the aprons will ordinarily not be shifted with velocities as high as 18' per second³, but it would be best to limit these velocities to about 10 feet per second.

Having determined the length of the horizontal floor and its level, and having got the crest level, the slope of the glacis and the working head, we can complete the design with reference to sub-surface flow. One or more trials may be necessary to work out the most economical design.

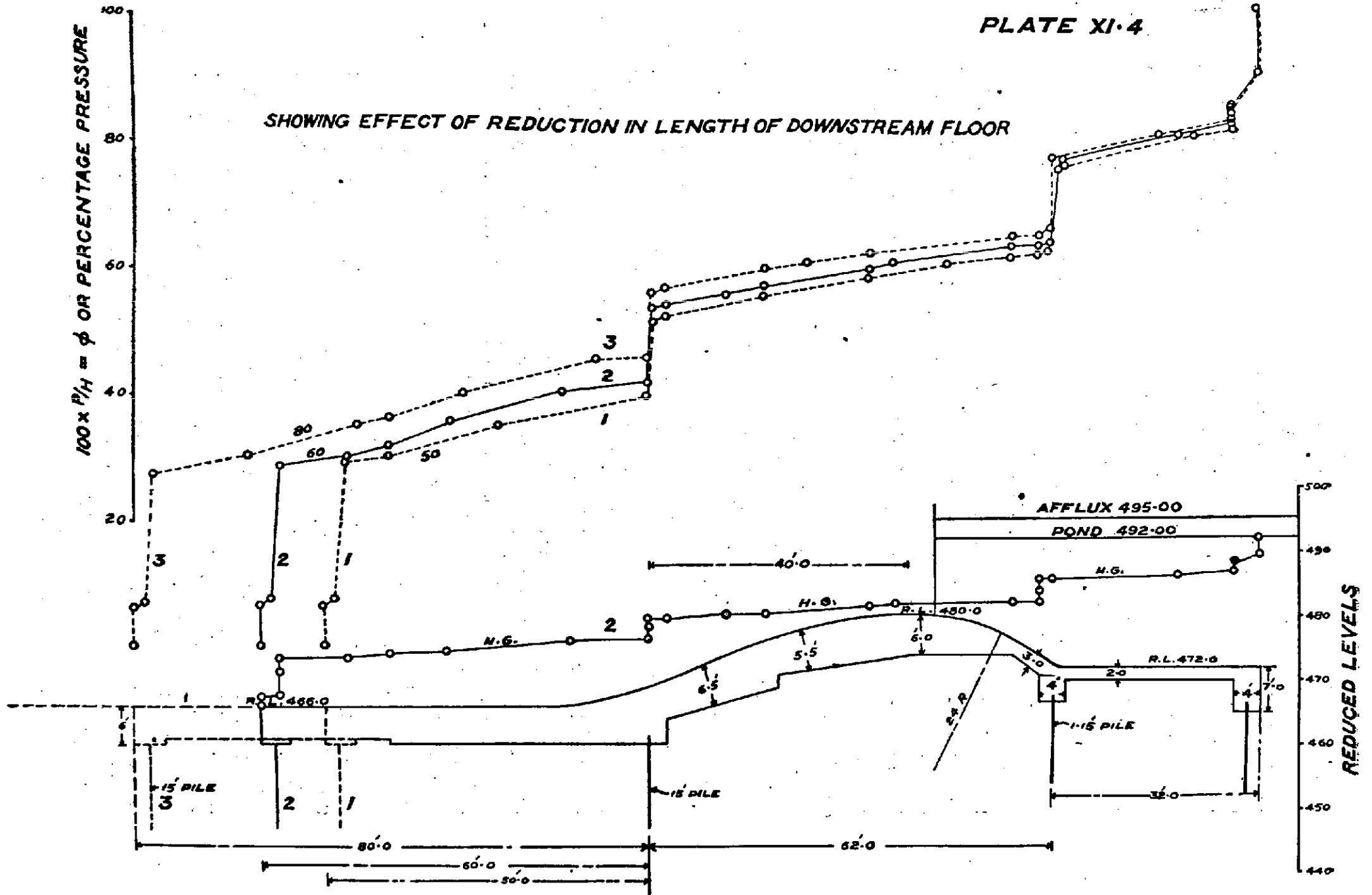
The effect of the reduction of the length of the downstream horizontal floor, other factors remaining unchanged, is best illustrated by Plate XI.4 (proposed Trimmu Weir). A reduction in length reduces uplift pressures and increases the exit gradients. If the exit gradient is fixed, the length of the horizontal floor will be determined by the ratio $\alpha=b/d$ required to give that value of exit gradient for the given head and depth of piling (d). The length so obtained will have to be increased if it is found to be insufficient for the requirement of the standing-wave.

(1) "Hydraulic Diagrams," by A. M. R. Montagu, M.L.C.E., &c., C.B.I. Publication No. 4.

(2) C.B.I. Publication No. 7, "Hydraulic Jump," Plate V (1934).

(3) Page 43. "River Training and Control" by F. J. E. Spring (1903).

SHOWING EFFECT OF REDUCTION IN LENGTH OF DOWNSTREAM FLOOR



Known :—

1. Maximum flood discharge.
2. Maximum flood level.
3. Minimum water level.
4. River bed level at Weir site.

To decide :—

1. Afflux. (P. 169).
2. Pond Level. This will be determined from a consideration of the full supply levels in the off-taking canals and of winter (low discharge period) storage, if any.
3. Discharge per foot run (pp. 171-172)
4. Length of water-way (pp. 171-172)
5. Type of Weir, *viz.*, Barrage or weir and Undersluices (pp. 172-173)
6. Crest levels (pp. 172-173)

To determine :—

1. Total length of impervious floor, and depth and location of sheet piles (pp. 142-156)
2. Exit gradient at the end of the impervious floor (pp. 140-142)
3. Depth of sheet piles in relation to exit gradient and flood scour (pp. 152-154)
4. Level and length of horizontal part of the downstream impervious floor (pp. 186-187)
5. Thickness of downstream impervious floor:—
 - (a) With referenece to uplift pressures (pp. 142-156 & 166)
 - (b) With referenece to standing-wave (p. 185)
6. Design of flanks (pp. 158-165)
7. Length and thickness of upstream and downstream aprons (pp. 173-177)
8. Inverted filter (p. 157)
9. Length, shape and free board of guide banks (pp. 177-179)

The details of designs of weir section with reference to flow of water through the subsoil have been illustrated by examples in Chapter X. The following will give a general outline of the procedure for a complete design in respect of subsoil and surface flow :—

The first step is to fix the total length of the impervious floor and the depths of the vertical cut offs, namely, the upstream and the downstream end piles and the intermediate piles if any, with reference to uplift pressures, exit gradient, earth pressures and flood scour. The detail of designs for *exit gradient and uplift pressures due to subsoil flow* have been fully illustrated on pages 155, 156 & 166. The stability of end piles against earth pressure and scour have been dealt with on pages 164-165. The design of piles with reference to flood scour has been dealt with on pages 153-154.

Next refer to Figure 11-6. The downstream Total Energy Line is the maximum flood level at the weir site before the construction of the weir, plus the head due to velocity of approach. The afflux is fixed as stated on page 169. This is generally 3 feet. The upstream Total Energy Line will therefore be the downstream Total Energy Line plus this afflux. Now, having decided discharge per foot run, q (pages 171-172), the crest level can be worked out as below :—

From the weir formula

$$q = CK^{3/2}$$

Where $C = 3.09$ (theoretical value)

and

$$K = H + H_v \text{ or the total head including velocity head.}$$

The crest level can now be obtained by subtracting K from the upstream Total Energy Line.

Refer again to Figure 11-6. The crest level having been determined, the upstream Total Energy Line will remain fixed for a given value of q . The downstream Total Energy Line will however have to be somewhat lowered, for purposes of designs, to allow for a possible drop in the maximum flood level after the construction of the Weir due to retrogression of the downstream river-bed levels. (See page 171.) A lowering of approximately 1 ft. will be sufficient in most cases. Thus while the upstream Total Energy Line will remain unaffected by a retrogression in bed levels, the downstream Total Energy Line will be depressed by approximately one foot, so that the difference between the two will become H_L (Loss of head) = afflux + 1 foot = 4 feet, where afflux is assumed 3 feet.

Now, the upstream and the downstream Total Energy Lines have been fixed for the final design.

Let us next consider a point on the glacis where it is decided that the standing wave should form. In Figure 11.6, E_{f1} is the depth of this point below the upstream Total Energy Line and E_{f2} is its depth below the downstream Total Energy Line depressed by 1 ft. as explained above). If this point is to be the toe of the glacis, the level of the downstream floor will be the same as the level of this point. Now we know the discharge per foot run q , and the loss of head H_L . The value of E_{f2} can then be directly read off the curves of Plate XI/2 against a value of $H_L = \text{afflux} + 1$ (=4ft. in general). The downstream floor level, which will fulfil the essential condition for safety that the standing waves does not form lower down than the toe of the glacis, can now be found out by subtracting this value of E_{f2} from the downstream Total Energy Line.

The next step is to find the depth of the standing wave, as this will determine the length of the horizontal floor (page 186), and the thickness of this part of the floor and the glacis (page 185). Now E_{f1} —the depth of the point of formation of the standing wave below the upstream Total Energy Line is known, being $=E_{f2} + H_L$. The depths D_1 and D_2 (Figure 11.6) for that given value of q can be directly obtained from Plate XI/1, D_1 and D_2 being the hypercritical and subcritical depths of water upstream and downstream, respectively, of the point of formation of the standing wave. Now, the height of the standing wave is given by $D_2 - D_1$. The length of the horizontal floor will therefore be $5(D_2 - D_1)$. (See page 186.)

The next step is to determine the floor thicknesses with reference to the standing wave page 185). For this it is necessary to work out the complete profile of the standing wave and the hydraulic gradient due to the particular difference in the upstream and downstream levels for the given value of q . The intercept between these two lines will give the unbalanced head for which the floor has to be designed. (See Plate XI-3.) The profile of the standing wave can be determined as explained on page 187. The thicknesses so obtained should now be tested for the maximum uplift pressures which will occur with the maximum pond level upstream and no escape downstream under conditions of maximum retrogression of bed levels. The determination of these latter pressures has been dealt with in Chapter X (pages 142-156, 166.)

The semi-pervious floor and the loose aprons upstream and downstream can now be designed as explained on pages 157, 173-177.

The design flanks can similarly be worked out as explained on pages 158-165.

This completes the design of the weir section and its protections. The training works can be designed as explained on pages 177-178.

It should be carefully noted that if the discharge per foot run q is the normal figure obtained by dividing the total discharge in a particular section of the weir by the length of crest in that section, the details of design worked out above will have to be suitably modified to allow for a possible concentration of flow in any particular section. On any major weir it is usual to allow an additional 20 per cent. for such concentration. A simple way of dealing with this concentration is to take the active discharge per foot run as $q + 20 \text{ per cent.} = 5/4 q$. (See page 175. The details of design as worked out for this increased figure can then be taken as definitely suitable and adequate.

ENERGY DISSIPATION AND PREVENTION OF DANGEROUS SCOUR DOWNSTREAM OF APRONS.

The length of the downstream horizontal floor can be reduced with a corresponding saving in cost, if the range of turbulence of the standing wave can be restricted and the normal steady level of the downstream water obtained in a distance less than five times the jump from the toe of the glacis. A number of devices for restricting the range of turbulence have been tried with varying degrees of success in preventing dangerous bed erosion downstream of the aprons.

Before proceeding to a discussion of the devices for the dissipation of energy, it will be desirable to refer briefly to the various factors leading to bed erosion downstream of weirs.

The bed erosion below weirs is due to the formation of vortices in the effluent water. These vortices¹ may be horizontal or vertical. The horizontal vortices may be positive or negative according as they cause accretion or erosion of the bed sand at the toe of the apron.

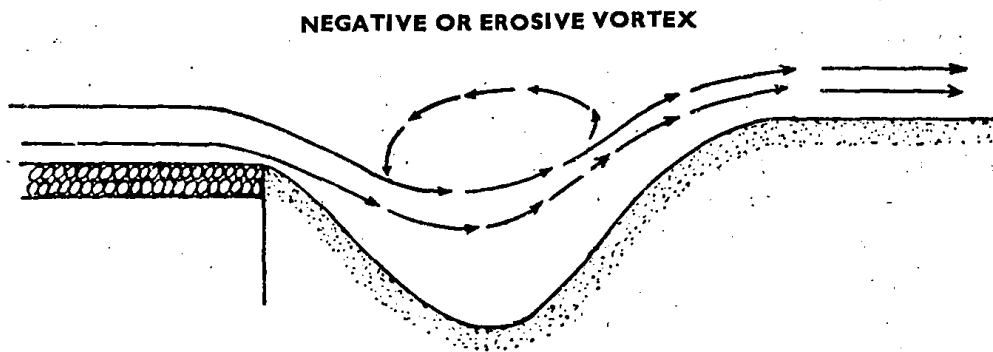


Figure 11.7

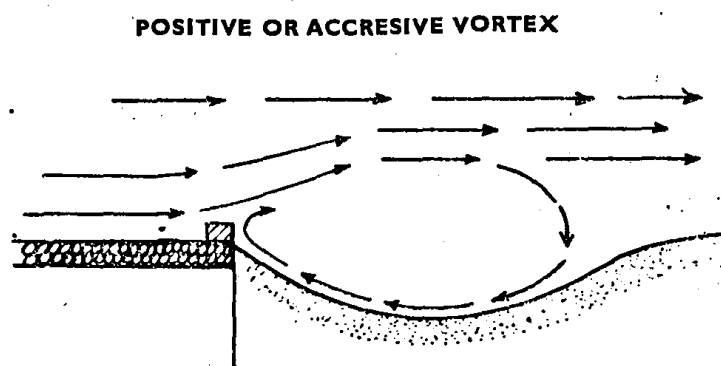


Figure 11.8

In the case of positive vortices, the currents along the bed tend to move the eroded material back towards the toe of the apron thus preventing dangerous scour holes occurring in the adjoining bed of the channel. In this case the lines of maximum velocity of the effluent water at the toe have an upward trend.

In the negative vortices, the eroded material of the bed is carried forward along the stream and scour progressively increases. The bed adjoining the apron becomes unstable and deep scour holes form. These conditions are dangerous to the stability of the work. The lines of maximum velocity of the effluent water at the end of the apron have in this case a downward trend.

Vertical vortices have a boring action. The high velocity jets dig out the bed material which is thrown up and carried forward with the current. The scour holes due to vertical vortices or "swirls" can go very deep indeed and if unchecked can be very dangerous. These vortices may be caused by (a) oblique flow, *i.e.*, the flow over the weir being at an angle instead of normal, due to an oblique approach of the river; (b) unequal discharge per foot run passing through adjoining bays, due to unequal openings of the gates in case of a level crest or to a difference in crest levels—as in a weir bay adjoining an undersluice bay.

These vortices are met with at the noses of guide banks and divide groynes and at every change of level in floor if not separated by a suitable length of divide wall. The failures of the upstream nose of the river guide bank at Suleimanki in 1925 and of the Rasul².

(1) "The Causes and Prevention of Bed Erosion, with special reference to the Protection of Structures controlling Rivers and Canals", by A. D. D. Butcher, C.B.E., and J. D. Atkinson, M.I.C.E., Paper No. 4885 of The Institute of Civil Engineers, (London).

(2) "Repairs and Remodelling of Rasul Weir (1929-31)", page 127, Paper No. 155, Punjab Engineering Congress, 1932.

Weir in 1929 were due to cause (a). Instances of damage caused by running very unequal discharges per foot run in adjoining bays not separated by divide walls, can be found at many of the Punjab weirs.

To prevent bed erosion, it will therefore be necessary to prevent the occurrence of the vertical and negative horizontal vortices. The former can be provided against by :—

- (a) Constructing suitable lengths of divide walls at every appreciable change in crest levels.
- (b) Maintaining as far as possible a uniform discharge per foot run over the crest except in special cases which should be dealt with on their merits.

The negative horizontal vortices can be prevented by a number of devices each one of which is designed to give an upward bend to the high velocity jets of the effluent water. These devices—generally known as energy-dissipating devices—may take any of the following forms and may be located somewhere at the tail end of the floor depending upon the local characteristics.

- (a) A continuous dwarf wall, such as,
 1. a baffle, biff or lip wall, or
 2. Rehbock's dentated sills.
- (b) Arrows.
- (c) Raised blocks.
- (d) A roughened or toothed floor.
- (e) A combination of two or more of the above forms.

The baffle wall has been discussed at length by Inglis¹. Figure 11.9 shows a baffle acting effectively.

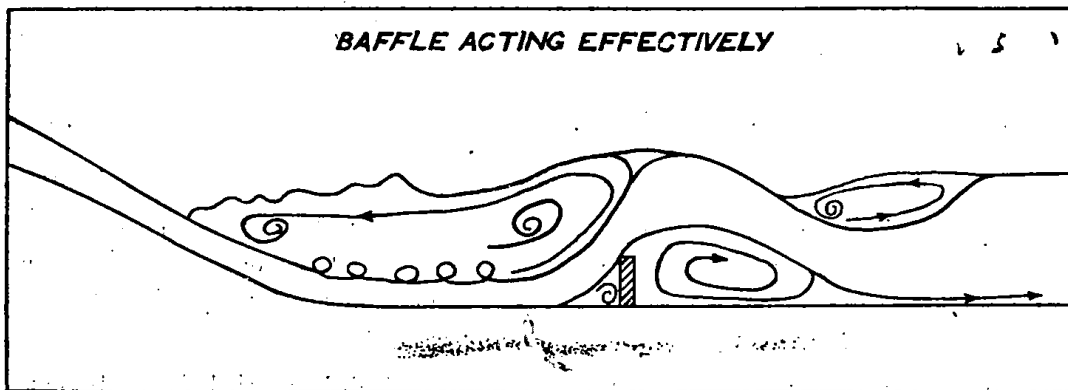


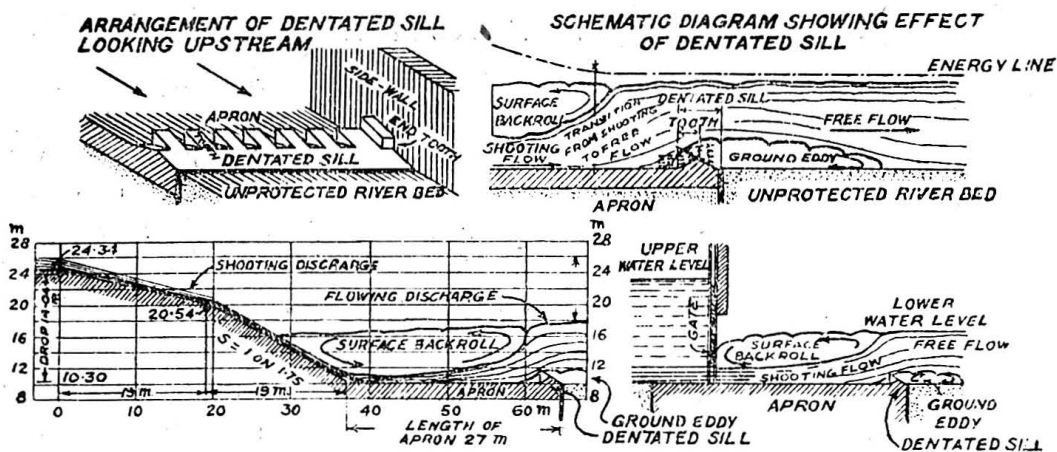
Figure 11.9

The dentated sill is best described by a quotation from Rehbock² :—

"It is necessary to deflect the largest velocities to the surface and to diminish the bottom velocities. After many experiments with more than twenty devices, the author (Dr. Theodor Rehbock) succeeded in finding in the "dentated sill" the desired solution. The dentated sill is a flat roof-like sill on which are placed teeth (dentates) with their vertical faces upstream and with gently sloping faces on the downstream side. By means of such dentated sills, the height of which need be only one-tenth of the height of the overfall, the bottom velocities are materially diminished and the depth of scour of the bed decreased to a fraction of its former amount. Figure 11-10 shows the arrangement and mode of operation of the dentated sill. The stream of water is lifted from the bed of the stream by the sill and is prevented by the filaments of water flowing between the teeth from dropping suddenly to the bottom. The water filaments directed upwards increase the surface velocities while the bottom velocities are diminished even beyond the roller, by the ground roller formed below the dentated sill."

(1) "The Dissipation of Energy below Falls" by C. C. Inglis, M.I.C.E., and D. Joglekar. Bombay Technical Paper No. 44 (1933).

(2) "Hydraulic Laboratory Practice," by John R. Freeman, (1929) page 185, published by the American Society of Mechanical Engineers.



LONGITUDINAL SECTION THROUGH FLOOD-DISCHARGE STRUCTURE AT THE FRIEDLAND POWER STATION

ARRANGEMENT OF DENTATED SILL BELOW SLUICE

CONFIGURATION OF THE RIVER BED-LEVEL USING AN APRON WITHOUT DENTATED SILL

CONFIGURATION OF THE RIVER BED-LEVEL USING AN APRON WITH DENTATED SILL.

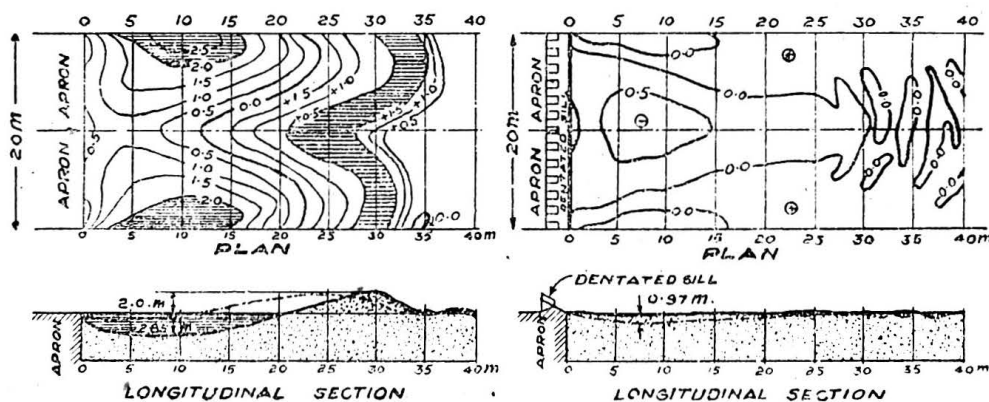


Figure 11.10

The arrows and raised blocks have been used below Khanki Weir (Reconstructed¹) with considerable success. They break up the high velocity bottom jets and direct them one against the other. They also deflect them upward so that only the positive vortices can form. They are illustrated in Photograph No. 11.1.



Photo 11.1

Showing arrows and blocks downstream of Bay 8 of Khanki Weir

(1) Pages 117 and 133. Punjab Engineering Congress, Paper No. 162, "Reconstruction of Khanki Weir" by A.N. Khosla (1936).

Failures of Dams and their Lessons

It would be highly desirable to add a chapter analysing the main features of design of the various existing weirs and dams including cases of failures and their causes. An analysis of this nature based on the general principles of design laid down in this publication would be of immense value as it would draw attention to features which have led to waste or instability and those that have resulted in economy and added stability. It would also show how vast sums of money could have been saved in first cost and many a disaster averted by a knowledge of the fundamentals outlined in these chapters.

The Balloki Weir is a typical example of a wasteful design. The floor length is about 300 feet for a head of about 14 feet. The proposed Trimmu Weir, which in design had the advantage of the above knowledge, is to have no more than about 160 feet length of floor for a total head of 26 feet.

The failures of the Khanki Weir in 1895 and of the Narora Weir in 1898 paved the way for a thorough investigation of the factors governing the design of hydraulic works resting on permeable foundations. The failures of the Islam Weir (Punjab) and of the Deoha Barrage (U. P.) in 1929 and damage to the Anderson Weir (Bengal) in 1935, established the necessity for piles at the upstream and downstream ends of the impervious floors. The existence of deep wells under the crest of the Deoha Barrage which saved the gates and gearing from collapse showed that a deep line of piles or wells under the gates were an important second line of defence. The failure of the Rasul Weir in 1929 and the damage to the Merala Weir in 1928 pointed to the necessity for providing against bed scour and the unbalanced head at the trough of standing wave during floods. These, along with the failure at Islam and damage at Ferozepur, showed that, for safety, the entire thickness of the impervious floor must act as one mass as the separation of layers might result in disaster. The Islam failure also called attention to the dangerous consequences of a retrogression of downstream bed levels if not adequately provided against in the original design.

As, however, the material available for such an analysis is limited, a detailed discussion must necessarily be left out for the present. But it is hoped that engineers connected with the design and maintenance of works of this nature will analyse their structures in the light of the fundamentals set forth above and furnish the results to the Central Board of Irrigation, so that if of sufficient value, they can be incorporated in the next edition of this publication.

Need for further Research

As has been stated earlier, the experimental work on the surface flow in relation to weir design has been meagre and of a fragmentary nature, and at best qualitative. Future research in this direction must aim at obtaining quantitative results. The nucleus for this already exists. Co-ordinated effort is required to bring together what has already been done and to cover new ground on a carefully thought-out plan, so as to make the investigation complete and comprehensive. Field and laboratory work must, as in the case of sub-surface flow, proceed in close collaboration. The model work in this case will be somewhat more difficult, as the scale factor which is linear in the case of sub surface flow will assume different proportions in different aspects of surface flow.

Large economies will be possible in first construction and in subsequent maintenance as a result of precise knowledge and understanding of the various factors governing design. The small sums spent in research in the field and on models will be more than repaid in one single new major work. We have paid heavily in the past for our ignorance.

List of Publications by the Central Board of Irrigation

Publication No.	Title	Date	Number in stock on 5th March 1938.
1	Digest of Technical Notes, 1931 (Reprint) A summary of views of Chief Engineers on the following subjects:— I. Waterlogging and Reclamation. II. Effect of deforestation and afforestation on river floods. III. Design of works on sand foundations. IV. Fluming of canal works. V. Regulation at canal headworks. VI. Design of canal falls.	24th October 1931	36
2	Digest of Technical Notes, 1932 A summary of views of Chief engineers on the following subjects:— I. Waterlogging, II. Design of works on sand foundations. III. Design of canal falls. IV. Staunching of canals.	15th October 1932.	..
3	Catalogue of Books in the Library of the Central Board of Irrigation, Simla. First Supplement to the Catalogue of Books Second Supplement to the Catalogue of Books.	2nd January 1934. 2nd January 1935. 2nd January 1936. 191
4	Hydraulic Diagrams: Energy-of-Flow, Pressure-plus-Momentum Diagrams for the simple graphic solution of problems involving a change of section in a stream of water.	31st July 1934	24
5	Glossary of Technical and Vernacular Terms in connection with Irrigation in India together with Standard Notations.	15th October 1934.	..
6	Fluming: A series of examples illustrating the use of Hydraulic Diagrams—Central Board of Irrigation Publication No. 4.	1st November 1934.	26
7	Standing Wave or Hydraulic Jump	1st December 1934.	28
8	Observation and Record of Pressures below Works on Permeable Foundations.	15th December 1934.	..
9	Stabilization of Soils by the use of Bitumen Emulsion	1st February 1935.	..
10	Irrigation Canal Falls.	15th February 1935.	29
11	Annual Report (Technical) of the work of the Central Board of Irrigation, India, 1934-1935.	1st May 1936	166
12	Design of Weirs on Permeable Foundations	1st September 1936.	..
13	Annual Report (Administrative) of the work of the Central Board of Irrigation, India, 1935-36.	1st October 1936.	32
14	Annual Report (Technical) of the work of the Central Board of Irrigation, India, 1935-36.	1st April 1937.	223
15	Annual Report (Administrative) of the work of the Central Board of Irrigation, India, 1936-37.	15th September 1937.	31
16	Annual Report (Technical) of the work of the Central Board of Irrigation, India, 1936-37.	April, 1938	..

List of Sub-committees provided by the Central Board of Irrigation

Serial No.	Name	Date
1	Sub-committee convened to examine the Nyaunggyat Dam Project, Burma	12th December 1927.
2	Sub-committee convened to enquire into the causes of floods in Orissa and possible remedies.	22nd August 1928.
3	Sub-committee convened to report on the practicability of the proposal of the Engineer-in-Chief, Cauvery Mettur Project to utilize cement concrete in place of cyclopean masonry in surki mortar in the construction of the Mettur Dam.	11th April 1929.
4	Sub-committee convened to examine the causes of failure of the Islam Weir, Sutlej Valley Project, on the 19th September 1929.	10th December 1929.
5	Sub-committee convened to report, on the organization of the Irrigation Department, Bengal and the practicability of separating the Department and forming a separate Board of Waterways.	13th March 1930.
6	Sub-committee convened to enquire into the desirability of remodelling the headworks and canal systems of the Son Canals in Bihar and Orissa.	16th January 1931.
7	Sub-committee convened by the Government of India to report upon the Quetta Drainage Scheme, Baluchistan.	24th February 1934.
8	Sub-committee convened by the Government of India to report on the conversion of the Paharpur Canal (North-West Frontier Province) into a perennial canal.	15th September 1934.
9	Sub-committee convened by the Government of India to report on Distribution of the Waters of the Indus and its Tributaries.	1934-35.
10	Sub-committee convened to suggest protective measures against damage caused by floods in 1935 to "Anderson Weir" of the Damodar Canal, Bengal.	1935-36.
11	Sub-committee convened to examine the Haveli (Punjab) Project, 1935 .	10th August 1936
12	Sub-committee convened by the Government of India, to examine the Khairpur State Lloyd Barrage Project, 1935-36.	13th December 1937.
13	Sub-committee convened by the Government of India, to examine the Spin Karez Dam Project in Baluchistan.	27th January 1938.

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