

ACTUARIAL STATISTICS

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COORDINATING EDITOR:
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VOLUME II CONSTRUCTION OF MORTALITY, & OTHER TABLES

by
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CONTENTS

	PAGE
<i>Introduction</i>	xiii
<i>Editor's Preface</i>	xv
<i>Authors' Preface</i>	xvi

CHAPTER I ✓

PRELIMINARY SURVEY

Introduction	1
Objects of a mortality investigation	2
Nature of the data required	3
Sources of the data	4
Choice of data	5
Homogeneity of data	6
Errors due to insufficient data	6
Summary of the process	7

CHAPTER II ✓

CALCULATION OF THE RATE OF MORTALITY. FROM FIRST PRINCIPLES

Definition of the rate of mortality	9
Application of the definition	10
Fractional periods of exposure	11
Assumption underlying the method of dealing with fractional exposures	13
Exposed to risk at age x	13
Approximations to lengths of fractional periods of exposure	14
Withdrawals and new entrants	16
Errors of approximation	17
Alternative definition of q_x	17
The life table	18

✓ CHAPTER III

EXPOSED TO RISK FORMULAE

	PAGE
Connection between exposed to risk at successive ages . . .	19
Exposed to risk when there are no fractional periods of exposure	19
Underlying theory of exposed to risk formulae	20
Fractional periods of exposure	22
Test of formula	25
Alternative demonstration of the treatment of fractional exposures	27

CHAPTER IV ✓

EXPOSED TO RISK FORMULAE.
AGE GROUPING BY DATE

Approximations to age	30
Method I—Age nearest birthday	32
Method I—Exposed to risk formula	33
Method I—Analysis of exposed to risk	34
Method II—Age nearest birthday on a fixed date other than the starting date of the experience	35
Method III—Age next (or last) birthday	36
Method III—Practical application	37
Method IV—Age nearest birthday at entry and calendar years of entry and exit	38
Method V—Age next birthday at entry and calendar years of entry and exit	39
Methods IV and V—Analysis of exposed to risk	40

CHAPTER V

✓ EXPOSED TO RISK FORMULAE.
AGE GROUPING BY EVENT

Method I—Policy anniversaries as limiting dates of experience	43
Method II—Fixed limiting dates	44
Alternative method for fractional exposures	47

✓ CHAPTER VI

PRACTICAL APPLICATION OF THE METHODS

Summary of operations	49
Collection of data	50
Mortality experience cards	52
Calculation of b_x , n_x , w_x etc.	54

CHAPTER VII

EXAMPLES OF THE CONSTRUCTION OF EXPOSED TO RISK FORMULAE

	PAGE
Examples	56
Examination questions	67

CHAPTER VIII ✓

ERRORS OF APPROXIMATION

Uniform distribution of deaths	69
Treatment of fractional exposures	71
Constancy of k , h and s	73
Use of exact age $x+l$	74
Entry and exit in the same year of age	75

CHAPTER IX ✓

THE CENTRAL RATE OF MORTALITY

Definition of the central rate of mortality	79
Exposed to risk formulae	79
Errors introduced by central exposed to risk formulae	80
Alternative formula for E_x^c	81
Alternative formula for E_x^c : fractional exposures	82
Distribution of deaths: alternative assumption	83
Calendar year method of calculating m_x	84

CHAPTER X ✓

THE CENSUS METHOD

Theory of the census method	87
Practical application of the method	88
Comparison with the modified calendar year method	90
Investigation of underlying assumptions	90
Application of the census method to life office data	92
Alternative demonstration of the census method	93
Experience covering more than one calendar year	95
Relation between deaths and exposed to risk	96
Distribution of deaths over the year of age	97

✓CHAPTER XI

CONSTRUCTION OF SELECT TABLES

	PAGE
Temporary initial selection	100
Select tables: duration of selection	101
Select rates of mortality: exposed to risk	103
✕Alternative method of dealing with fractional exposures	105
Ultimate data	106
Mechanical processes of sorting and grouping data	108
Financial effect of using select tables	110
Other types of temporary initial selection	111
Exact-age method	112
Rates of withdrawal	113

✓CHAPTER XII

USE OF CENTRAL RATES OF MORTALITY FOR SELECT TABLES. APPLICATION OF THE CENSUS METHOD

Definition of $m_{[x]+t}$	117
Formulae for $E_{[x]+t}^o$	117
✓Census method	118
Practical application of the census method	118
Investigation of underlying assumptions	119
Effect of variations in the distribution of entry dates over the calendar year on the function ${}^oP_{x+t}$	121
Experience covering more than one calendar year	121
✕Modification of the census method	124
Error at duration 0	125
Modified policy-year method	125

✓CHAPTER XIII

THE GENERAL THEORY OF SELECTION

Definition of selection	131
Class selection	132
Class selection: the problem of heterogeneity	133
Time selection	135
Spurious selection: definition and examples	136
✕Indications of spurious selection	138
Causes of spurious selection	139
✕Multiple selection	143
Methods of investigating selection	145

CHAPTER XIV

THE CONSTRUCTION OF MORTALITY TABLES
FROM LIFE OFFICE RECORDS

	PAGE
The selection of lives assured	148
Requirements of a mortality table for office use	149
Accuracy of the data	150
Unit of investigation	152
Investigations based on policies	152
Investigations based on sums assured	155
Investigations based on sums at risk	156
Effect of withdrawals	157

Omitted
CHAPTER XV

HISTORY OF MORTALITY TABLES OF
ASSURED LIVES

Northampton Table	158
Carlisle Table	159
Morgan's Equitable Table	159
Seventeen Offices' Table	160
Institute of Actuaries Tables (H^M and H^F)	160
H^M Experience: select tables	161
British Offices' Life Tables (1863-93)	162
The Continuous Investigation—A 1924-29 Table	168

CHAPTER XVI

FORECASTING RATES OF MORTALITY.
GENERAL THEORY OF PROJECTION

General definition of the rate of mortality	173
Forecasting rates of mortality	173
Generation mortality	174
Tables of generation mortality	175
General theory of projection	177
Projection by curve fitting	178
Statistical approach to the theory of projection	179

CONTENTS
Original
 CHAPTER XVII

MORTALITY TABLES OF ANNUITANTS

	PAGE
Collection and tabulation of data	183
Variation of q_x with year of birth	184
Office Annuitants' Experience 1900-20: method of projection .	186
Office Annuitants' Experience 1900-20: suggested explanation of special features	188
Government Annuitants' Experience 1900-20: method of projection	192
Continuous Investigation	193
Late intimation of deaths	193

✓ CHAPTER XVIII

COMPARISON OF MORTALITY TABLES

Principal objects of comparison	194
Comparison of rates of mortality	194
Comparison of expectations of life	195
Comparison of actual and expected deaths	196
Mortality index figures	198
Comparison of mortality indices	202
<i>✓</i> Special indices devised for particular cases	206

✓ CHAPTER XIX

DOUBLE- AND MULTIPLE-DECREMENT TABLES

Double-decrement tables	207
Types of double-decrement tables	208
Dependent rates of decrement	209
Relations between dependent and independent rates of decrement	211
Distribution of deaths and withdrawals	213
Central rates of decrement	215
Marriage and mortality table	216
Total and permanent disability table	219
<i>✓</i> Multiple-decrement tables	219
Service tables	220
<i>✓</i> Graded service tables	222

✓ CHAPTER XX

✓ CONSTRUCTION OF NATIONAL LIFE TABLES

	PAGE
Method of constructing tables	232
Estimates of the mean population	233
Rates of mortality at individual ages	236
Errors in census data	237
Exposed to risk at young ages	238
Possible sources of error	240
<i>✓</i> Population statistics: some general problems	245

Quitted
CHAPTER XXI

NATIONAL LIFE TABLES.
SECTIONAL INVESTIGATIONS

Types of sectional investigation	249
Marital status	249
Density of population	251
Geographical district	251
Occupation	251
Occupation: mortality indices	252
Occupation: errors	254
Occupation: female lives	255
Social class or economic status	255
<i>✓</i> Effect of the different factors on rates of mortality	256
Use of control populations	256
<i>✓</i> Conclusions drawn from the 1931 investigation	257

CHAPTER XXII

SICKNESS RATES

Definition of sickness	259
Essential difference between sickness rates and rates of decrement	259
Force of sickness and rates of sickness	260
Calculation of sickness rates	262
Waiting period for new entrants: treatment of lapses	262
Exposed to risk formulae	263
Periods of sickness pay	266

	PAGE
Off period	266
Subdivision of sickness rates according to period of pay . . .	267
Collection and tabulation of data	268
Factors influencing sickness rates	271
Advantage of central sickness rates	273
Proportion sick in a year	273
Comparison of actual and expected sickness	274
Selection	274
× Special types of sickness benefit	275
× Standard tables	278
Analysis of sickness rates into short periods	279

INTRODUCTION TO VOL. II OF THE PART III TEXT-BOOK

THE issue of the present volume by Messrs J. L. Anderson, B.A., F.I.A., and J. B. Dow, M.A., F.F.A., completes the Text-Book for the Part III Examinations of the Institute and Faculty; the first volume, that on Statistics and Graduation by Mr Herbert Tetley, M.A., F.I.A., was published in 1946.

The services of all these authors, and of the co-ordinating editor, Mr H. Freeman, M.A., F.I.A., were acknowledged by our respective predecessors in office in their Introduction to Mr Tetley's volume; but we should like to add our tribute of gratitude for their unsparing labours, which have so greatly eased the task of present and future students for Part III. The present volume will, we believe, assist those students in obtaining a clear grasp of main principles, and will prove to be a valuable addition to actuarial literature.

A. H. ROWELL

President, The Institute of Actuaries

COLIN S. PENN

President, The Faculty of Actuaries

June 1947

EDITOR'S PREFACE

THE subject under the heading of the second volume of Actuarial Statistics is one which often causes difficulty to the student. One of the reasons for this is probably that, up to the present, the official reading has been diffuse. The need for an up-to-date text-book on the compilation and interpretation of mortality and similar tables has long been evident, and this book should satisfy that need.

Editing the book has been a most interesting task. Even when the authors of a text-book know their subject as well as do the authors of this one, they do not always express themselves logically and lucidly. In the present book the reader cannot fail to be struck with the clearness with which the successive arguments for the adoption of the formulae for the exposed to risk have been set out. The authors are at pains to stress that formulae are not to be learnt by rote but are to be evolved from first principles and devote a considerable portion of the book to the processes for solving the many different types of problem that beset the student in his first approach to the theory and practice of the construction of mortality tables.

H. F.

AUTHORS' PREFACE

THIS text-book has been written with special regard to the requirements of the student who is encountering for the first time the problems involved in the construction of mortality tables. We have therefore dealt at considerable length with the subject of Exposed to Risk Formulae which is a frequent cause of confusion at the first approach. The underlying theory has been built up from first principles and has then been applied to demonstrate the simplest type of formula. More complicated formulae have been developed by reference to the simpler types.

While some readers, with previous experience of the subject, may prefer a more general approach, we feel strongly that the average student should begin with the simplest cases and proceed by easy stages to the more complicated without any attempt to formulate a general method.

We have not thought it necessary to include any examples to be worked by students, but a number of representative examples have been included in the text to illustrate particular points.

We have had access to, and have made full use of, the notes provided for students by the Actuarial Tuition Service and we gratefully acknowledge our debt to their authors. We should also like to record our thanks to a number of our professional colleagues who have helped with advice and criticism, and to Mr H. Freeman who, as co-ordinating editor, has greatly eased the work of preparing the book for the press.

J. L. A.

J. B. D.