

IMPERIAL BUREAU OF SOIL SCIENCE

TECHNICAL COMMUNICATION No. 35

THE DESIGN AND ANALYSIS OF FACTORIAL EXPERIMENTS

by

F. YATES, M.A.

(Rothamsted Experimental Station)



Price, 5/-

Imperial Bureau of Soil Science, Harpenden, England)

1937

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