



MINISTRY OF SUPPLY

DIRECTORATE OF
ROYAL ORDNANCE FACTORIES
EXPLOSIVES

INDUSTRIAL EXPERIMENTATION

FOURTH EDITION

LONDON: HIS MAJESTY'S STATIONERY OFFICE
1949

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by K. A. BROWNLEE, M.A.

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(Reprinted 1950)

“We see that the theory of probabilities is at bottom only commonsense reduced to calculation : it makes us appreciate with exactitude what reasonable minds feel by a sort of instinct.”

P. S. LAPLACE

“Mathematics may be compared to a mill of exquisite workmanship which grinds you stuff of any degree of fineness, but nevertheless what you get out depends on what you put in—and as the grandest mill in the world will not extract wheat flour from peaspods so pages of formulae will not get a definite result out of loose data.”

T. H. HUXLEY

“We have usually no knowledge that any one factor will exert its effects independently of all others that can be varied, or that its effects are particularly simply related to variations in these other factors. . . .

“ . . . If the investigator, in these circumstances, confines his attention to any single factor, we may infer either that he is the unfortunate victim of a doctrinaire theory as to how experimentation should proceed, or that the time, material or equipment at his disposal is too limited to allow him to give attention to more than one narrow aspect of his problem.”

R. A. FISHER

FOREWORD

The present Monograph is based on an earlier Memorandum produced by the Directorate of Ordnance Factories (Explosives) for the use, primarily, of those concerned with pilot plant and plant scale experiments on chemical manufacturing processes in the Royal Ordnance Factories (Explosives). Much work of this type was being carried out and it had become evident that it was desirable for the results of such experiments to be subjected to critical tests of significance. A convenient account of the straightforward tests of significance, written from the point of view of the individual who has to apply them in practice without necessarily a full knowledge of their theoretical background, was not readily available, and an attempt was therefore made to prepare one.

It was evident that to apply tests of significance conveniently and economically the experiments had to be planned in appropriate forms. It is considered that the methods outlined should be as much a standard tool of the industrial experimenter as a chemical balance is of the laboratory experimenter. In carrying out an industrial experiment the choice is not between using a statistical design with the application of the appropriate tests of significance or the ordinary methods: the choice is between correct or incorrect methods. Even the simplest experiment requires an estimate of the significance of its results.

The practice sometimes followed of consulting the statistician only after the experiment is completed and asking him "what he can make of the results" cannot be too strongly condemned. It is essential to have the experiment in a form suitable for analysis and in general this can only be attained by designing the experiment in consultation with the statistician, or with due regard to the statistical principles involved.

The present Monograph, therefore, is intended to be a guide to both the planning and the interpretation of experiments on the industrial scale, and it is hoped that the methods described will become part of the everyday technique to those who carry out such experiments.

R. C. BOWDEN,
Director of Ordnance Factories (Explosives)
Ministry of Supply

January, 1946

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INDEX

FROM THE PREFACE TO THE FIRST EDITION

The present monograph is intended to provide for those who carry out investigational work a guide to modern statistical methods, both the use of tests of significance to attain reliability in deductions from experimental data and the use of statistical design to attain the maximum precision with the minimum expenditure.

The subject is treated entirely from the practical point of view. Theory is at a minimum and the only mathematics involved is simple arithmetic. Each statistical method discussed is illustrated with practical examples worked out in detail to show what exactly is involved in its use. Some of the more advanced techniques may appear rather formidable, but it is strongly recommended that they should be approached through the use of the simpler methods. A little practice in these will lead to a familiarity and confidence with which it will be realized that the advanced techniques are merely comparatively easy extensions of the simpler.

The theoretical background on which the author has drawn is of course that largely developed by Professor R. A. Fisher, expounded in his "Statistical Methods for Research Workers" and "The Design of Experiments."

Indebtedness is expressed to Prof. R. A. Fisher and Dr. Frank A. Yates for permission to reprint Tables III-VI, IX, XII and XXIII from their book "Statistical Tables for Biological, Agricultural, and Medical Research" (Oliver & Boyd, Edinburgh and London), and to the British Standards Institution for permission to reprint certain factors of Quality Control Charts from B.S. 600R, "Quality Control Charts."

The examples used here are all the results of experimental and investigational work in the Royal Ordnance Factories (Explosives), and the author's thanks are due to Mr. A. C. H. Cairns, Mr. A. H. Woodhead, Mr. D. Newman and others whose collaboration has been invaluable in the development of the ideas and outlook presented here. The author is also indebted to Dr. R. C. Bowden, Director of Ordnance Factories (Explosives), for his active interest and encouragement throughout this work.

January, 1946.

K. A. B.

PREFACE TO THE SECOND EDITION

The principal additions to this Edition are a substantial enlargement of Chapter I and two new chapters, Chapter XIII on balanced incomplete blocks and Chapter XIV on confounding. Further additions are the components of variance for unequal column size in Chapter VII (d), the exact formula for the residual variance about a regression line in Chapter IX (h), the Doolittle method of computation in multiple regression in Chapter X (d), and the partitioning of sums of squares in Chapter XII (c).

Minor additions have also been made at many points.

November, 1946

K. A. B.

PREFACE TO THE THIRD EDITION

Chapter I has been extended by sections on Youden Squares and on Randomisation. In Chapter IX (g) the comparison of regression lines is discussed at greater length. Chapter XII (g) has a numerical example of a Latin Square. The methods of computation for Youden Squares and some examples of them are given in Chapter XIII (e). Chapter XIV is extended by discussions of confounding with factors at three levels (Section (g)) and four levels (Section (h)). Tables of the angular transformation (VI) and of random numbers (VII) are added to the Appendix. A number of minor additions have also been made.

October, 1947

K. A. B.

PREFACE TO THE FOURTH EDITION

The principal addition to this Edition is Chapter XVI on the fractional replication of factorial experiments.

Of the other modifications the most important is the expansion and rearrangement of the earlier material on Latin Squares into a new Chapter, Chapter XIII. Others are the calculation of the percentage of the normal curve lying outside any particular limit (Chapter II (j)), Bartlett's test for variances with unequal numbers of degrees of freedom (Chapter IV (c)), proofs of the simple equations for simple and multiple regression (Chapter IX (d) and Chapter X (b) respectively), the treatment of missing values in factorial experiments (Chapter XII (k)), the possible designs for balanced incomplete blocks (Chapter XIV (c)) and their efficiency (Chapter XIV (d)).

A Bibliography and an Index have been added.

November, 1948

K. A. B.

APPENDIX

TABLE I

Table of t

Degrees of Freedom	t				
	0.10	0.05	0.02	0.01	0.001
1	6.31	12.71	31.82	63.66	636.62
2	2.92	4.30	6.97	9.93	31.60
3	2.35	3.18	4.54	5.84	12.94
4	2.13	2.78	3.75	4.60	8.61
5	2.02	2.57	3.37	4.03	6.86
6	1.94	2.45	3.14	3.71	5.96
7	1.90	2.37	3.00	3.50	5.41
8	1.86	2.31	2.90	3.36	5.04
9	1.83	2.26	2.82	3.25	4.78
10	1.81	2.23	2.76	3.17	4.59
11	1.80	2.20	2.72	3.11	4.44
12	1.78	2.18	2.68	3.06	4.32
13	1.77	2.16	2.65	3.01	4.22
14	1.76	2.15	2.62	2.98	4.14
15	1.75	2.13	2.60	2.95	4.07
16	1.75	2.12	2.58	2.92	4.02
17	1.74	2.11	2.57	2.90	3.97
18	1.73	2.10	2.55	2.88	3.92
19	1.73	2.09	2.54	2.86	3.88
20	1.73	2.09	2.53	2.85	3.85
21	1.72	2.08	2.52	2.83	3.82
22	1.72	2.07	2.51	2.82	3.79
23	1.71	2.07	2.50	2.81	3.77
24	1.71	2.06	2.49	2.80	3.75
25	1.71	2.06	2.48	2.79	3.73
26	1.71	2.06	2.48	2.78	3.71
27	1.70	2.05	2.47	2.77	3.69
28	1.70	2.05	2.47	2.76	3.67
29	1.70	2.04	2.46	2.76	3.66
30	1.70	2.04	2.46	2.75	3.65
40	1.68	2.02	2.42	2.70	3.55
60	1.67	2.00	2.39	2.66	3.46
120	1.66	1.98	2.36	2.62	3.37
∞	1.65	1.96	2.33	2.58	3.29

Abridged from Table III of "Statistical Tables for Biological, Agricultural and Medical Research." (R. A. Fisher and F. Yates : Oliver and Boyd).

TABLE II
Table of χ^2

Degrees of Freedom										
	0.99	0.98	0.95	0.90	0.50	0.10	0.05	0.02	0.01	0.001
1	0.000	0.001	0.004	0.015	0.455	2.71	3.84	5.41	6.64	10.83
2	0.020	0.040	0.103	0.211	1.386	4.61	5.99	7.82	9.21	13.82
3	0.115	0.185	0.352	0.584	2.366	6.25	7.82	9.84	11.34	16.27
4	0.297	0.429	0.711	1.064	3.357	7.78	9.49	11.67	13.28	18.47
5	0.554	0.752	1.145	1.610	4.351	9.24	11.07	13.39	15.09	20.52
6	0.872	1.134	1.635	2.204	5.35	10.65	12.59	15.03	16.81	22.46
7	1.239	1.564	2.167	2.833	6.35	12.02	14.07	16.62	18.48	24.32
8	1.646	2.032	2.733	3.490	7.34	13.36	15.51	18.17	20.09	26.13
9	2.088	2.532	3.325	4.168	8.34	14.68	16.92	19.68	21.67	27.88
10	2.558	3.059	3.940	4.865	9.34	15.99	18.31	21.16	23.21	29.59
11	3.05	3.61	4.57	5.58	10.34	17.28	19.68	22.62	24.73	31.26
12	3.57	4.18	5.23	6.30	11.34	18.55	21.03	24.05	26.22	32.91
13	4.11	4.76	5.89	7.04	12.34	19.81	22.36	25.47	27.69	34.53
14	4.66	5.37	6.57	7.79	13.34	21.06	23.69	26.87	29.14	36.12
15	5.23	5.99	7.26	8.55	14.34	22.31	25.00	28.26	30.58	37.70
16	5.81	6.61	7.96	9.31	15.34	23.54	26.30	39.63	32.00	39.25
17	6.41	7.26	8.67	10.09	16.34	24.77	27.59	31.00	33.41	40.79
18	7.02	7.91	9.39	10.87	17.34	25.99	28.87	32.35	34.81	42.31
19	7.63	8.57	10.12	11.65	18.34	27.20	30.14	33.69	36.19	43.82
20	8.26	9.24	10.85	12.44	19.34	28.41	31.41	35.02	37.57	45.32
21	8.90	9.91	11.59	13.34	20.34	29.61	32.67	36.34	38.93	46.80
22	9.54	10.60	12.34	14.04	21.34	30.81	33.92	37.66	40.29	48.27
23	10.20	11.29	13.09	14.85	22.34	32.01	35.17	38.97	41.64	49.73
24	10.86	11.99	13.85	15.66	23.34	33.20	36.42	40.27	42.98	51.18
25	11.52	12.70	14.61	16.47	24.34	34.38	37.65	41.57	44.31	52.62
26	12.20	13.41	15.38	17.29	25.34	35.56	38.89	42.86	45.64	54.05
27	12.88	14.12	16.15	18.11	26.34	36.74	40.11	44.14	46.96	55.48
28	13.56	14.85	16.93	18.94	27.34	37.92	41.34	45.42	48.28	56.89
29	14.26	15.57	17.71	19.77	28.34	39.09	42.56	46.69	49.59	58.30
30	14.95	16.31	18.49	20.60	29.34	40.26	43.77	47.96	50.89	59.70

Abridged from Table IV of "Statistical Tables for Biological, Agricultural and Medical Research." (R. A. Fisher and F. Yates: Oliver and Boyd).

TABLE III
Tables of Variance Ratio (i)

$N_2 \backslash N_1$	0.20 Significance Level								
	1	2	3	4	5	6	12	24	∞
1	9.5	12.0	13.1	13.7	14.0	14.3	14.9	15.2	15.6
2	3.6	4.0	4.2	4.2	4.3	4.3	4.4	4.4	4.5
3	2.7	2.9	2.9	3.0	3.0	3.0	3.0	3.0	3.0
4	2.4	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.4
5	2.2	2.3	2.3	2.2	2.2	2.2	2.2	2.2	2.1
6	2.1	2.1	2.1	2.1	2.1	2.1	2.0	2.0	2.0
7	2.0	2.0	2.0	2.0	2.0	2.0	1.9	1.9	1.8
8	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.7
9	1.9	1.9	1.9	1.9	1.9	1.8	1.8	1.7	1.7
10	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.6
11	1.9	1.9	1.8	1.8	1.8	1.8	1.7	1.6	1.6
12	1.8	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.5
13	1.8	1.8	1.8	1.8	1.7	1.7	1.6	1.6	1.5
14	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.5
15	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.5	1.5
16	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.5	1.4
17	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.5	1.4
18	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4
19	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4
20	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4
22	1.8	1.7	1.7	1.6	1.6	1.6	1.5	1.4	1.4
24	1.7	1.7	1.7	1.6	1.6	1.6	1.5	1.4	1.3
26	1.7	1.7	1.7	1.6	1.6	1.6	1.5	1.4	1.3
28	1.7	1.7	1.7	1.6	1.6	1.6	1.5	1.4	1.3
30	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	1.3
40	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.4	1.2
60	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.3	1.2
120	1.7	1.6	1.6	1.5	1.5	1.5	1.4	1.3	1.1
∞	1.6	1.6	1.6	1.5	1.5	1.4	1.3	1.2	1.0

Abridged from Table V of "Statistical Tables for Biological, Agricultural and Medical Research." (R. A. Fisher and F. Yates: Oliver and Boyd).

Table of Variance Ratio (ii)

N ₂ \ N ₁	0.05 Significance Level								
	1	2	3	4	5	6	12	24	∞
1	164.4	199.5	215.7	224.6	230.2	234.0	234.9	249.0	254.3
2	18.5	19.2	19.2	19.3	19.3	19.3	19.4	19.5	19.5
3	10.1	9.6	9.3	9.1	9.0	8.9	8.7	8.6	8.5
4	7.7	6.9	6.6	6.4	6.3	6.2	5.9	5.8	5.6
5	6.6	5.8	5.4	5.2	5.1	5.0	4.7	4.5	4.4
6	6.0	5.1	4.8	4.5	4.4	4.3	4.0	3.8	3.7
7	5.6	4.7	4.4	4.1	4.0	3.9	3.6	3.4	3.2
8	5.3	4.5	4.1	3.8	3.7	3.6	3.3	3.1	2.9
9	5.1	4.3	3.9	3.6	3.5	3.4	3.1	2.9	2.7
10	5.0	4.1	3.7	3.5	3.3	3.2	2.9	2.7	2.5
11	4.8	4.0	3.6	3.4	3.2	3.1	2.8	2.6	2.4
12	4.8	3.9	3.5	3.3	3.1	3.0	2.7	2.5	2.3
13	4.7	3.8	3.4	3.2	3.0	2.9	2.6	2.4	2.2
14	4.6	3.7	3.3	3.1	3.0	2.9	2.5	2.3	2.1
15	4.5	3.7	3.3	3.1	2.9	2.8	2.5	2.3	2.1
16	4.5	3.6	3.2	3.0	2.9	2.7	2.4	2.2	2.0
17	4.5	3.6	3.2	3.0	2.8	2.7	2.4	2.2	2.0
18	4.4	3.6	3.2	2.9	2.8	2.7	2.3	2.1	1.9
19	4.4	3.5	3.1	2.9	2.7	2.6	2.3	2.1	1.9
20	4.4	3.5	3.1	2.9	2.7	2.6	2.3	2.1	1.8
22	4.3	3.4	3.1	2.8	2.7	2.6	2.2	2.0	1.8
24	4.3	3.4	3.0	2.8	2.6	2.5	2.2	2.0	1.7
26	4.2	3.4	3.0	2.7	2.6	2.5	2.2	2.0	1.7
28	4.2	3.3	3.0	2.7	2.6	2.4	2.1	1.9	1.7
30	4.2	3.3	2.9	2.7	2.5	2.4	2.1	1.9	1.6
40	4.1	3.2	2.9	2.6	2.5	2.3	2.0	1.8	1.5
60	4.0	3.2	2.8	2.5	2.4	2.3	1.9	1.7	1.4
120	3.9	3.1	2.7	2.5	2.3	2.2	1.8	1.6	1.3
∞	3.8	3.0	2.6	2.4	2.2	2.1	1.8	1.5	1.0

Abridged from Table V of "Statistical Tables for Biological, Agricultural, and Medical Research." (R. A. Fisher and F. Yates : Oliver and Boyd).

Tables of Variance Ratio (iii)

$N_2 \backslash N_1$	0.01 Significance Level									
	1	2	3	4	5	6	8	12	24	∞
1	4052	4999	5403	5625	5764	5859	5981	6106	6234	6366
2	98.5	99.0	99.2	99.3	99.3	99.4	99.3	99.4	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.5	27.1	26.6	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	14.8	14.4	13.9	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.3	9.9	9.5	9.0
6	13.7	10.9	9.8	9.2	8.8	8.5	8.1	7.7	7.3	6.9
7	12.3	9.6	8.5	7.9	7.5	7.2	6.8	6.5	6.1	5.7
8	11.3	8.7	7.6	7.0	6.6	6.4	6.0	5.7	5.3	4.9
9	10.6	8.0	7.0	6.4	6.1	5.8	5.5	5.1	4.7	4.3
10	10.0	7.6	6.6	6.0	5.6	5.4	5.1	4.7	4.3	3.9
11	9.7	7.2	6.2	5.7	5.3	5.1	4.7	4.4	4.0	3.6
12	9.3	6.9	6.0	5.4	5.1	4.8	4.5	4.2	3.8	3.4
13	9.1	6.7	5.7	5.2	4.9	4.6	4.3	4.0	3.6	3.2
14	8.9	6.5	5.6	5.0	4.7	4.5	4.1	3.8	3.4	3.0
15	8.7	6.4	5.4	4.9	4.6	4.3	4.0	3.7	3.3	2.9
16	8.5	6.2	5.3	4.8	4.4	4.2	3.9	3.6	3.2	2.8
17	8.4	6.1	5.2	4.7	4.3	4.1	3.8	3.5	3.1	2.7
18	8.3	6.0	5.1	4.6	4.3	4.0	3.7	3.4	3.0	2.6
19	8.2	5.9	5.0	4.5	4.2	3.9	3.6	3.3	2.9	2.5
20	8.1	5.9	4.9	4.4	4.1	3.9	3.6	3.2	2.9	2.4
22	7.9	5.7	4.8	4.3	4.0	3.8	3.5	3.1	2.8	2.3
24	7.8	5.6	4.7	4.2	3.9	3.7	3.3	3.0	2.7	2.2
26	7.7	5.5	4.6	4.1	3.8	3.6	3.3	3.0	2.6	2.1
28	7.6	5.5	4.6	4.1	3.8	3.5	3.2	2.9	2.5	2.1
30	7.6	5.4	4.5	4.0	3.7	3.5	3.2	2.8	2.5	2.0
40	7.3	5.2	4.3	3.8	3.5	3.3	3.0	2.7	2.3	1.8
60	7.1	5.0	4.1	3.7	3.3	3.1	2.8	2.5	2.1	1.6
120	6.9	4.8	4.0	3.5	3.2	3.0	2.7	2.3	2.0	1.4
∞	6.6	4.6	3.8	3.3	3.0	2.8	2.5	2.2	1.8	1.0

Abridged from Table V of "Statistical Tables for Biological, Agricultural and Medical Research." (R. A. Fisher and F. Yates : Oliver and Boyd).

Table of Variance Ratio (iv)

N ₂	0.001 Significance Level										
	1	2	3	4	5	6	8	12	24	∞	
1		varying from 400,000 to 800,000									
2	998	999	999	999	999	999	999	999	999	999	
3	167	148	141	137	135	133	131	128	126	123	
4	74.1	61.3	56.2	53.4	51.7	50.5	49.0	47.4	45.8	44.1	
5	47.0	36.6	33.2	31.1	29.8	28.8	27.6	26.4	25.1	23.8	
6	35.5	27.0	23.7	21.9	20.8	20.0	19.0	18.0	16.9	15.8	
7	29.2	21.7	18.8	17.2	16.2	15.5	14.6	13.7	12.7	11.7	
8	25.4	18.5	15.8	14.4	13.5	12.9	12.0	11.2	10.3	9.3	
9	22.9	16.4	13.9	12.6	11.7	11.1	10.4	9.6	8.7	7.8	
10	21.0	14.9	12.6	11.3	10.5	9.9	9.2	8.5	7.6	6.8	
11	19.7	13.8	11.6	10.4	9.6	9.1	8.3	7.6	6.9	6.0	
12	18.6	13.0	10.8	9.6	8.9	8.4	7.7	7.0	6.3	5.4	
13	17.8	12.3	10.2	9.1	8.4	7.9	7.2	6.5	5.8	5.0	
14	17.1	11.8	9.7	8.6	7.9	7.4	6.8	6.1	5.4	4.6	
15	16.6	11.3	9.3	8.3	7.6	7.1	6.5	5.8	5.1	4.3	
16	16.1	11.0	9.0	7.9	7.3	6.8	6.2	5.6	4.9	4.1	
17	15.7	10.7	8.7	7.7	7.0	6.6	6.0	5.3	4.6	3.9	
18	15.4	10.4	8.5	7.5	6.8	6.4	5.8	5.1	4.5	3.7	
19	15.1	10.2	8.3	7.3	6.6	6.2	5.6	5.0	4.3	3.5	
20	14.8	10.0	8.1	7.1	6.5	6.0	5.4	4.8	4.2	3.4	
22	14.4	9.6	7.8	6.8	6.2	5.8	5.2	4.6	3.9	3.2	
24	14.0	9.3	7.6	6.6	6.0	5.6	5.0	4.4	3.7	3.0	
26	13.7	9.1	7.4	6.4	5.8	5.4	4.8	4.2	3.6	2.8	
28	13.5	8.9	7.2	6.3	5.7	5.2	4.7	4.1	3.5	2.7	
30	13.3	8.8	7.1	6.1	5.5	5.1	4.6	4.0	3.4	2.6	
40	12.6	8.2	6.6	5.7	5.1	4.7	4.2	3.6	3.0	2.2	
60	12.0	7.8	6.2	5.3	4.8	4.4	3.9	3.3	2.7	1.9	
120	11.4	7.3	5.8	5.0	4.4	4.0	3.5	3.0	2.4	1.6	
∞	10.8	6.9	5.4	5.6	4.1	3.7	3.3	2.7	2.1	1.0	

Abridged from Table V of "Statistical Tables for Biological, Agricultural and Medical Research." (R. A. Fisher and F. Yates : Oliver and Boyd).

TABLE IV
Table of the Correlation Coefficient

Degrees of Freedom	r				
	0.10	0.05	0.02	0.01	0.001
1	.988	.997	.999	1.000	1.000
2	.900	.950	.980	.990	.999
3	.805	.878	.934	.959	.992
4	.729	.811	.882	.917	.974
5	.669	.754	.833	.874	.951
6	.621	.707	.789	.834	.925
7	.582	.666	.750	.798	.898
8	.549	.632	.716	.765	.872
9	.521	.602	.685	.735	.847
10	.497	.576	.658	.708	.823
11	.476	.553	.634	.684	.801
12	.457	.532	.612	.661	.780
13	.441	.514	.592	.641	.760
14	.426	.497	.574	.623	.742
15	.412	.482	.558	.606	.725
16	.400	.468	.543	.590	.708
17	.389	.456	.528	.575	.693
18	.378	.444	.516	.561	.679
19	.369	.433	.503	.549	.665
20	.360	.423	.492	.537	.652
25	.323	.381	.445	.487	.597
30	.296	.349	.409	.449	.554
35	.275	.325	.381	.418	.519
40	.257	.304	.358	.393	.490
45	.243	.287	.338	.372	.465
50	.231	.273	.322	.354	.443
60	.211	.250	.295	.325	.408
70	.195	.232	.274	.302	.380
80	.183	.217	.256	.283	.357
90	.173	.205	.242	.267	.337
100	.164	.195	.230	.254	.321

Abridged from Table VI of "Statistical Tables for Biological, Agricultural and Medical Research." (R. A. Fisher and F. Yates : Oliver and Boyd).

TABLE V
Factors for Control Charts

Number in Sample	A' 0.025	A' 0.001	D' 0.975	D' 0.999	d _n
2	1.229	1.937	2.81	4.12	1.13
3	0.668	1.054	2.17	2.98	1.69
4	0.476	0.750	1.93	2.57	2.06
5	0.377	0.594	1.81	2.34	2.33
6	0.316	0.498	1.72	2.21	2.53
7	0.274	0.432	1.66	2.11	2.70
8	0.244	0.384	1.62	2.04	2.85
9	0.220	0.347	1.58	1.99	2.97
10	0.202	0.317	1.56	1.93	3.08

Abridged from B. S. 600R "Quality Control Charts". (B. P. Dudding and W. J. Jennett. British Standards Institution.)

TABLE VI
The Angular Transformation of Percentages to Degrees

P%	0	1	2	3	4	5	6	7	8	9
0	0	5.7	8.1	10.0	11.5	12.9	14.2	15.3	16.4	17.5
10	18.4	19.4	20.3	21.1	22.0	22.8	23.6	24.4	25.1	25.8
20	26.6	27.3	28.0	28.7	29.3	30.0	30.7	31.3	31.9	32.6
30	33.2	33.8	34.4	35.1	35.7	36.3	36.9	37.5	38.1	38.6
40	39.2	39.8	40.6	41.0	41.6	42.1	42.7	43.3	43.9	44.4
50	45.0	45.6	46.1	46.7	47.3	47.9	48.4	49.0	49.6	50.2
60	50.8	51.4	51.9	52.5	53.1	53.7	54.3	54.9	55.6	56.2
70	56.8	57.4	58.1	58.7	59.3	60.0	60.7	61.3	62.0	62.7
80	63.4	64.2	64.9	65.6	66.4	67.2	68.0	68.9	69.7	70.6
90	71.6	72.5	73.6	74.7	75.8	77.1	78.5	80.0	81.0	84.3

From Table XII of "Statistical Tables for Biological, Agricultural and Medical Research" (R. A. Fisher and F. Yates ; Oliver and Boyd).

TABLE VII
Abbreviated Table of Probits

%	0	1	2	3	4	5	6	7	8	9
50	5.000	5.025	5.050	5.075	5.100	5.126	5.151	5.176	5.202	5.227
60	5.253	5.279	5.305	5.332	5.358	5.385	5.412	5.440	5.468	5.496
70	5.524	5.553	5.583	5.613	5.643	5.674	5.706	5.739	5.772	5.806
80	5.842	5.878	5.915	5.954	5.995	6.036	6.080	6.126	6.175	6.227
90	6.282	6.341	6.405	6.476	6.555	6.645	6.751	6.881	7.054	7.326

Abridged from Table IX of "Statistical Tables for Biological, Agricultural and Medical Research." (R. A. Fisher and F. Yates : Oliver and Boyd).

TABLE VIII
Random Numbers (I)

03 47 43 73 86	36 96 47 36 61	46 98 63 71 62	33 26 16 80 45	60 11 14 10 95
97 74 24 67 62	42 81 14 57 20	42 53 32 37 32	27 07 36 07 51	24 51 79 89 73
16 76 62 27 66	56 50 26 71 07	32 90 79 78 53	13 55 38 58 59	88 97 54 14 10
12 56 85 99 26	96 96 68 27 31	05 03 72 93 15	57 12 10 14 21	88 26 49 81 76
55 59 56 35 64	38 54 82 46 22	31 62 43 09 90	06 18 44 32 53	23 83 01 30 30
16 22 77 94 39	49 54 43 54 82	17 37 93 23 78	87 35 20 96 43	84 26 34 91 64
84 42 17 53 31	57 24 55 06 88	77 04 74 47 67	21 76 33 50 25	83 92 12 06 76
63 01 63 78 59	16 95 55 67 19	98 10 50 71 75	12 86 73 58 07	44 39 52 38 79
33 21 12 34 29	78 64 56 07 82	52 42 07 44 38	15 51 00 13 42	99 66 02 79 54
57 60 86 32 44	09 47 27 96 54	49 17 46 09 62	90 52 84 77 27	08 02 73 43 28
18 18 07 92 46	44 17 16 58 09	79 83 86 19 62	06 76 50 03 10	55 23 64 05 05
26 62 38 97 75	84 16 07 44 99	83 11 46 32 24	20 14 85 88 45	10 93 72 88 71
23 42 40 64 74	82 97 77 77 81	07 45 32 14 08	32 98 94 07 72	93 85 79 10 75
52 36 28 19 95	50 92 26 11 97	00 56 76 31 38	80 22 02 53 53	86 60 42 04 53
37 85 94 35 12	83 39 50 08 30	42 34 07 96 88	54 42 06 87 98	35 85 29 48 39
70 29 17 12 13	40 33 20 38 26	13 89 51 03 74	17 76 37 13 04	07 74 21 19 30
56 62 18 37 35	96 83 50 87 75	97 12 25 93 47	70 33 24 03 54	97 77 46 44 80
99 49 57 22 77	88 42 95 45 72	16 64 36 16 00	04 43 18 66 79	94 77 24 21 90
16 08 15 04 72	33 27 14 34 90	45 59 34 68 49	12 72 07 34 45	99 27 72 95 14
31 16 93 32 43	50 27 89 87 19	20 15 37 00 49	52 85 66 60 44	38 68 88 11 80
68 34 30 13 70	55 74 30 77 40	44 22 78 84 26	04 33 46 09 52	68 07 97 06 57
74 57 25 65 76	59 29 97 68 60	71 91 38 67 54	13 58 18 24 76	15 54 55 95 91
27 42 37 86 53	48 55 90 65 72	96 57 69 36 10	96 46 92 42 45	97 60 49 04 92
00 39 68 29 61	66 37 32 20 30	77 84 57 03 29	10 45 65 04 26	11 04 96 67 24
29 94 98 94 24	68 49 69 10 82	53 75 91 93 30	34 25 20 57 27	40 48 73 51 92
16 90 82 66 59	83 62 64 11 12	67 19 00 71 74	60 47 21 29 68	02 02 37 03 31
11 27 94 75 06	06 09 19 74 66	02 94 37 34 02	76 70 90 30 86	38 45 94 30 38
35 24 10 16 20	33 32 51 26 38	79 78 45 04 91	16 92 53 56 16	02 75 50 95 98
38 23 16 86 38	42 38 97 01 50	87 75 66 81 41	40 01 74 91 62	48 51 84 08 32
31 96 25 91 47	96 44 33 49 13	34 86 82 53 91	00 52 43 48 85	27 55 26 89 62
66 67 40 67 14	64 05 71 95 86	11 05 65 09 68	76 83 20 37 90	57 16 00 11 66
14 90 84 45 11	75 73 88 05 90	52 27 41 14 86	22 98 12 22 08	07 52 74 95 80
68 05 51 18 00	33 96 02 75 19	07 60 62 93 55	59 33 82 43 90	49 37 38 44 59
20 46 78 73 90	97 51 40 14 02	04 02 33 31 08	39 54 16 49 36	47 95 93 13 30
64 19 58 97 79	15 06 15 93 20	01 90 10 75 06	40 78 78 89 62	02 67 74 17 33
05 26 93 70 60	22 35 85 15 13	92 03 51 59 77	59 56 78 06 83	52 91 05 70 74
07 97 10 88 23	09 98 42 99 64	61 71 62 99 15	06 51 29 16 93	58 05 77 09 51
68 71 86 85 85	54 87 66 47 54	73 32 08 11 12	44 95 92 63 16	29 56 24 29 48
26 99 61 65 53	58 37 78 80 70	42 10 50 67 42	32 17 55 85 74	94 44 67 16 94
14 65 52 68 75	87 59 36 22 41	26 78 63 06 55	13 08 27 01 50	15 29 39 39 43
17 53 77 58 71	71 41 61 50 72	12 41 94 96 26	44 95 27 36 99	02 96 74 30 83
90 26 59 21 19	23 52 23 33 12	96 93 02 18 39	07 02 18 36 07	25 99 32 70 23
41 23 52 55 99	31 04 49 69 96	10 47 48 45 88	13 41 43 89 20	97 17 14 49 17
60 20 50 81 69	31 99 73 68 68	35 81 33 03 76	24 30 12 48 60	18 99 10 72 34
91 25 38 05 90	94 58 28 41 36	45 37 59 03 09	90 35 57 29 12	82 62 54 65 60
34 50 57 74 37	98 80 33 00 91	09 77 93 19 82	74 94 80 04 04	45 07 31 66 49
85 22 04 39 43	73 81 53 94 79	33 62 46 86 28	08 31 54 46 31	53 94 13 38 47
09 79 13 77 48	73 82 97 22 21	05 03 27 24 83	72 89 44 05 60	35 80 39 94 88
88 75 80 18 14	22 95 75 42 49	39 32 82 22 49	02 48 07 70 37	16 04 61 67 87
90 96 23 70 00	39 00 03 06 90	55 85 78 38 36	94 37 30 69 32	90 89 00 76 33

Abridged from Table XXXIII of "Statistical Tables for Biological, Agricultural, and Medical Research" (R. A. Fisher and F. Yates : Oliver and Boyd).

TABLE VIII
Random Numbers (II)

53 74 23 99 67	61 32 28 69 84	94 62 67 86 24	98 33 41 19 95	47 53 53 38 09
63 38 06 86 54	99 00 65 26 94	02 82 90 23 07	79 62 67 80 60	75 91 12 81 19
35 30 58 21 46	06 72 17 10 94	25 21 31 75 96	49 28 24 00 49	55 65 79 78 07
63 43 36 82 69	65 51 18 37 88	61 38 44 12 45	32 92 85 88 65	54 34 81 85 35
98 25 37 55 26	01 91 82 81 46	74 71 12 94 97	24 02 71 37 07	03 92 18 66 75
02 63 21 17 69	71 50 80 89 56	38 15 70 11 48	43 40 45 86 98	00 83 26 91 03
64 55 22 21 82	48 22 28 06 00	61 54 13 43 91	82 78 12 23 29	06 66 24 12 27
85 07 26 13 89	01 10 07 82 04	59 63 69 36 03	69 11 15 83 80	13 29 54 19 28
58 54 16 24 15	51 54 44 82 00	62 61 65 04 69	38 18 65 18 97	85 72 13 49 21
34 85 27 84 87	61 48 64 56 26	90 18 48 13 26	37 70 15 42 57	65 65 80 39 07
03 92 18 27 46	57 99 16 96 56	30 33 72 85 22	84 64 38 56 98	99 01 30 98 64
62 95 30 27 59	37 75 41 66 48	86 97 80 61 45	23 53 04 01 63	45 76 08 64 27
08 45 93 15 22	60 21 75 46 91	98 77 27 85 42	28 88 61 08 84	69 62 03 42 73
07 08 55 18 40	45 44 75 13 90	24 94 96 61 02	57 55 66 83 15	73 42 37 11 61
01 85 89 95 66	51 10 19 34 88	15 84 97 19 75	12 76 39 43 78	64 63 91 08 25
72 84 71 14 35	19 11 58 49 26	50 11 17 17 76	86 31 57 20 18	95 60 78 46 75
88 78 28 16 84	13 52 53 94 53	75 45 69 30 96	73 89 65 70 31	99 17 43 48 76
45 17 75 65 57	28 40 19 72 12	25 12 74 75 67	60 40 60 81 19	24 62 01 61 16
96 76 28 12 54	22 01 11 94 25	71 96 16 16 88	68 64 36 74 45	19 59 50 88 92
43 31 67 72 30	24 02 92 08 63	38 32 36 66 02	69 36 38 25 39	48 03 45 15 22
50 44 66 44 21	66 06 58 05 62	68 15 54 35 02	42 35 48 96 32	14 52 41 52 48
22 66 22 15 86	26 63 75 41 99	58 42 36 72 24	58 37 52 18 51	03 37 18 39 11
96 24 40 14 51	23 22 30 88 57	95 67 47 29 83	94 69 40 06 07	18 16 36 78 86
31 73 91 61 19	60 20 72 93 48	98 57 07 23 69	65 95 39 69 58	56 80 30 19 44
78 60 73 99 84	43 89 94 36 45	56 69 47 07 41	90 22 91 07 12	78 35 34 08 72
84 37 90 61 56	70 10 23 98 05	85 11 34 76 60	76 48 45 34 60	01 64 18 39 96
36 67 10 08 23	98 93 35 08 86	99 29 76 29 81	33 34 91 58 93	63 14 52 32 52
07 28 59 07 48	89 64 58 89 75	83 85 62 27 89	30 14 78 56 27	86 63 59 80 02
10 15 83 87 60	79 24 31 66 56	21 48 24 06 93	91 98 94 05 49	01 47 59 38 00
55 19 68 97 65	03 73 52 16 56	00 53 55 90 27	33 42 29 38 87	22 13 88 83 34
53 81 29 13 39	35 01 20 71 34	62 33 74 82 14	53 73 19 09 03	56 54 29 56 93
51 86 32 68 92	33 98 74 66 99	40 14 71 94 58	45 94 19 38 81	14 44 99 81 07
35 91 70 29 13	80 03 54 07 27	96 94 78 32 66	50 95 52 74 33	13 80 55 62 54
37 71 67 95 13	20 02 44 95 94	64 85 04 05 72	01 32 90 76 14	53 89 74 60 41
93 66 13 83 27	92 79 64 64 72	28 54 96 53 84	48 14 52 98 94	56 07 93 89 30
02 96 08 45 65	13 05 00 41 84	93 07 54 72 59	21 45 57 09 77	19 48 56 27 44
49 83 43 48 35	82 88 33 69 96	72 36 04 19 76	47 45 15 18 60	82 11 08 95 97
84 60 71 62 46	40 80 81 30 37	34 39 23 05 38	25 15 35 71 30	88 12 57 21 77
18 17 30 88 71	44 91 14 88 47	89 23 30 63 15	56 34 20 47 89	99 82 93 24 98
79 69 10 61 78	71 32 76 95 62	87 00 22 58 40	92 54 01 75 25	43 11 71 99 31
75 93 36 57 83	56 20 14 82 11	74 21 97 90 65	96 42 68 63 86	74 54 13 26 94
38 30 92 29 03	06 28 81 39 38	62 25 06 84 63	61 29 08 93 67	04 32 92 08 09
51 29 50 10 34	31 57 75 95 80	51 97 02 74 77	76 15 48 49 44	18 55 63 77 09
21 31 38 86 24	37 79 81 53 74	73 24 16 10 33	52 83 90 94 76	70 47 14 54 36
29 01 23 87 88	58 02 39 37 67	42 10 14 20 92	16 55 23 42 45	54 96 09 11 06
95 33 95 22 00	18 74 72 00 18	38 79 58 69 32	81 76 80 26 92	82 80 84 25 39
90 84 60 79 80	24 36 59 87 38	82 07 53 89 35	96 35 23 79 18	05 98 90 07 35
46 40 62 98 82	54 79 20 56 95	15 74 80 08 32	16 46 70 50 80	67 72 16 42 79
20 31 89 03 43	38 46 82 68 72	32 14 82 99 70	80 60 47 18 97	63 49 30 21 30
71 59 73 05 50	08 22 23 71 77	91 01 93 20 49	82 96 59 26 94	66 39 67 98 60

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