

# **The Theory of Inbreeding**

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By

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## APPENDIX A

### SPECIES BEARING ONE OFFSPRING AT A BIRTH

#### 27. Model Mating Systems

In many species, and those among the slowest breeders, one offspring only is to be expected at a birth. The use of successive sib-matings is then particularly time-consuming, for, whichever may be the sex of the first-born, a second mating will certainly be required, and this will produce an offspring of the right sex in only half the trials. The frequency of a delay of 1, 2, 3, ... birth intervals after the first birth is given by the geometric series

$$\begin{array}{ccccccc} 1 & 2 & 3 & \dots & \dots & \dots & \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \dots & \dots & \dots & \end{array}$$

from which it is easily seen that the average delay, beyond the time needed for the first birth, will be two birth intervals.

The alternative of using parent-offspring matings, using each animal twice only, which, as has been seen, is equally efficacious in securing homozygosis, is manifestly preferable for such species, since young of the right sex will be supplied by the first birth in half the cases, by the second birth in quarter, and so on. Such a system therefore saves time, as compared with sib-mating, at the rate of exactly one birth interval in each generation.

In cases, however, in which the first, or the first two births produce offspring of the wrong sex, it may well be asked whether use cannot be made of these to advance the progress of the line more expeditiously than by waiting passively for the appearance of an

offspring of the desired sex. Several alternative possibilities present themselves, some of which are discussed below as examples of the manner in which such operational policies may be investigated.

MODEL C.—If the first offspring is always used with the parent of opposite sex, the generation matrix is

$$\begin{array}{cccc}
 & & aa \times ab & ab \times ab \\
 & & u_0 & v_0 \\
 u_1 & \cdot & \cdot & \cdot & \frac{1}{2} & \frac{1}{2} \\
 v_1 & \cdot & \cdot & \cdot & \frac{1}{4} & \frac{1}{2}
 \end{array}$$

The equation for  $\lambda$  is

$$8\lambda^2 - 8\lambda + 1 = 0.$$

The dominant root

$$\begin{aligned}
 \lambda &= .85355,33906 = \frac{1}{4}(2 + \sqrt{2}) \\
 -\log_e \lambda &= .15835,88971 \\
 -1/\log_e \lambda &= 6.31476992.
 \end{aligned}$$

If  $g$  is the ratio of the minimal generation time to the time interval between births, the time taken by this method to attain a standard degree of inbreeding may be compared with that required by other methods:—

- (C) First offspring method . . . . .  $(6.31477)g$
- (B) Alternate parent-offspring method . . . . .  $(4.71842)(g+1)$
- (A) Sib-matings . . . . .  $(4.71842)(g+2)$ .

It is obvious that some parent-offspring method is quicker than the sib-method with animals bearing only one at a birth; also, if  $g$  is less than 2.9557, the use of the first offspring is quicker than the policy of waiting for one of the sex wanted.

The comparatively high value of the root, .85355, shows that the use of the same animal in more than two generations is somewhat disadvantageous. Reiterated use is also avoidable, for a possible policy would be to use the same parent no more than three times,

and to continue every third mating until an offspring of suitable sex appears. Although, as will appear, this method can be improved on, it is in many circumstances better than those discussed above. The method of calculating its speed is also of some interest.

In this case (model *D*) we must distinguish six possible types of mating, in three of which the parent is used for the second time, and in three for the third time. Writing the genotype of the parent first, these are :—

$ab \times aa$	$aa \times ab$	$ab \times ab$
$u$	$v$	$w$

using the parent for the second time, and  $u'$ ,  $v'$ ,  $w'$  when used for the third time. The generation matrix is then found to be :—

TABLE 55

	$u_0/4$	$v_0/4$	$w_0/4$	$u'_0/2$	$v'_0/2$	$w'_0/2$
$u_1$	$-4\lambda$	1	1	.	1	1
$v_1$	1	$-4\lambda$	.	1	.	.
$w_1$	.	1	$1-4\lambda$	.	1	1
$u'_1$	1	.	1	$-2\lambda$	.	.
$v'_1$	.	1	.	.	$-2\lambda$	.
$w'_1$	1	.	1	.	.	$-2\lambda$

Two of the roots for  $\lambda$  are zero, and the remainder are roots of the quartic equation

$$32\lambda^4 - 8\lambda^3 - 10\lambda^2 - 3\lambda - 1 = 0.$$

The dominant root is

$$\begin{aligned} \lambda &= .82358,93624 \\ -\log_e \lambda &= .19408,32199 \\ -1/\log_e \lambda &= 5.14253. \end{aligned}$$

Since the third use of a parent only occurs when on the second use the offspring is of the wrong sex, only one third of the matings will be of this kind, after which an offspring of the right sex must be awaited.



The time needed for a standard amount of inbreeding is therefore

$$(5.14253)(g + \frac{1}{3})$$

and this is less than the value for alternate parent-offspring matings, if  $g$  is less than 6.91, a value exceeded by man, but I think not by other animals. It is also less than the value for the method using always the first offspring when  $g$  exceeds 1.48, which also must generally be the case.

This method is not, however, to be recommended in practice since, for example, when the parent is a male, he might have a second offspring by his daughter, before the first by his granddaughter. If this turns out to be a male, either a mating to his mother or to his sister will become possible. If the granddaughter bears a son, he with his mother will carry on the line; but if a daughter, she could be mated to any brother of her mother.

When a male is mated repeatedly with a daughter and a granddaughter with a view to using a male born to one of the matings with his mother, the order of preference among the males born will influence both the retardation of the matings relative to the minimal generation time, and the proportion of cases in which one animal is used three times in succession. Only an exact calculation can show the simultaneous effect of both influences.

Let us suppose that when a male is bred to a daughter and granddaughter the order of preference is as follows :—

Offspring of Daughter.	Offspring of Granddaughter.
1	
2	
4	3
6	5
.	7
.	.
.	.
.	.

while a female is only mated to a grandson if the first two born to her son are both male. Then the fraction of males used in a third generation is

$$\frac{1}{8} \left( 1 + \frac{1}{4} + \frac{1}{4^2} + \dots \right) = \frac{1}{6}$$

while that of females is

$$\frac{1}{8} \left( 1 + \frac{1}{2} + \frac{1}{2^2} + \dots \right) = \frac{1}{4}$$

In all, therefore,

$$\frac{1}{2} \left( \frac{1}{6} + \frac{1}{4} \right) = \frac{5}{24}$$

is the frequency of carrying on the line by third matings. Further, for males the average delay is

$$\left( \frac{1}{4} + \frac{1}{32} \right) \left\{ 1 + 2 \left( \frac{1}{4} \right) + 3 \left( \frac{1}{4} \right)^2 + \dots \right\} = \frac{1}{2}$$

and for females

$$\frac{1}{4} + \frac{1}{16} \left\{ 1 + 2 \left( \frac{1}{2} \right) + 3 \left( \frac{1}{2} \right)^2 + \dots \right\} = \frac{1}{2}$$

The situation is then that of twelve second matings by males, two are carried on by a third generation using the first born, and ten as second matings of the mother, with a total delay of  $5\frac{1}{3}$  birth intervals; the two third generation matings being continued as second matings to the female with a total delay of  $\frac{2}{3}$  of a birth interval. While of twelve second matings of females, three are carried on by third generation matings to the first born, and nine as second generation matings of the father, with a total delay of three birth intervals; the three third generation matings being continued as second matings to the male, with a total delay of three birth intervals. Thus the total delay is twelve birth intervals in twenty-nine generations, or the average time of a generation is  $g + \frac{12}{29}$ .

The generation matrix depends only on the propor-

tion of third matings used, irrespective of the time elapsed ; multiplied by 48 it is :—

TABLE 56

-48λ	19	19	.	24	24
19	-48λ	.	24	.	.
.	19	19-48λ	.	24	24
5	.	5	-48λ	.	.
.	5	.	.	-48λ	.
5	.	5	.	.	-48λ

From this the equation for λ is found to be

$$4608\lambda^4 - 1824\lambda^3 - 1202\lambda^2 - 285\lambda - 25 = 0$$

of which the dominant root is

$$\begin{aligned} \lambda &= \cdot 81743,52344 \\ - \log_e \lambda &= \cdot 20158,36035 \\ -1/\log_e \lambda &= 4\cdot 960720925. \end{aligned}$$

The time taken on this system (model *E*) to make one unit of progress is

$$4\cdot 96072(g + \frac{19}{35});$$

this is generally, but only slightly, shorter than the time needed in the first model case involving third matings. By using, however, a different schedule of preference (model *F*)

	1		1
	3		3
Males	5	2	2
	.	4	4
	.	6	5
	.	.	.
	.	.	.

the delay is reduced to  $\frac{17}{65}$  birth intervals, at the expense of using an increased proportion of third matings. This, of course, increases λ, which is now the largest root of

$$\begin{aligned} 18432\lambda^4 - 5952\lambda^3 - 5186\lambda^2 - 1581\lambda - 289 &= 0 \\ \lambda &= \cdot 82111,28303 \\ - \log_e \lambda &= \cdot 19709,47486 \\ -1/\log_e \lambda &= 5\cdot 073701898. \end{aligned}$$

Table 57 shows that in the range of  $g$  considered the speed of this method is 31 to 80 per cent. higher than that of sib-matings. In all special cases, other methods will suggest themselves; it is in order that these may be scrupulously examined before adoption that the full examples set out above have been presented.

TABLE 57

*Table of Time, in Birth Intervals, required to Make Unit Advance in Inbreeding, Using Different Methods*

$g =$	1.6	2.0	2.4	2.8	3.2	3.6	4.0	
Sib-mating ( $A$ ) . . .	16.986	18.874	20.761	22.648	24.536	26.423	28.311	$4.71842 (g+2)$
Alternate parents ( $B$ ) . . .	12.268	14.155	16.043	17.930	19.817	21.705	23.592	$4.71842 (g+1)$
First born ( $C$ ) . . .	10.104	12.630	15.155	17.681	20.207	22.733	25.259	$6.31477 g$
Model $D$ . . . . .	9.942	11.999	14.056	16.113	18.170	20.227	22.284	$5.14253 (g+\frac{1}{3})$
Model $F$ . . . . .	9.445	11.474	13.504	15.533	17.563	19.592	21.622	$5.07370 (g+\frac{1}{6})$

### 28. Time Criterion for Choice of Mating

An objection to any schedule of preference based on birth order is that it takes no account of the actual deviations from average times, which though they cannot be foreseen are known to the experimenter who makes the choice. To take proper account of the circumstances we should need a rule of the form, "The offspring of a third mating is to be preferred to an offspring of the preceding second mating if born not more than  $x$  days later."

Since third matings will not, to judge by the results of the preceding section, need to be very numerous, we may, as an approximation, treat the third matings as exceptional irregularities in a series of alternate

parent-child matings. For such a series the matrix :—

TABLE 58

			$u_0/2$	$v_0/2$	$w_0/2$
$u_1$	.	.	$-2\lambda$	1	1
$v_1$	.	.	1	$-2\lambda$	.
$w_1$	.	.	.	1	$1-2\lambda$

has a dominant root

$$\lambda = \epsilon = \frac{1}{4}(\sqrt{5} + 1),$$

with frequencies

$$\frac{f(u)}{2\epsilon} = f(v) = (2\epsilon - 1)f(w) = \frac{f(w)}{2\epsilon}$$

and complexities

$$c(u) = \frac{c(v)}{2\epsilon} = (2\epsilon - 1)c(w) = \frac{c(w)}{2\epsilon}.$$

The expected effect of a third mating is given by the matrix :—

TABLE 59

			$u_0$	$v_0$	$w_0$
$u_1$	.	.	$\frac{1}{2}$	.	$\frac{1}{2}$
$v_1$	.	.	.	$\frac{1}{2}$	.
$w_1$	.	.	$\frac{1}{2}$	.	$\frac{1}{2}$

so that the reduction in complexity due to a third mating is

$$\frac{1}{2}\{f(u)+f(w)\}\{c(u)+c(w)\} + \frac{1}{2}f(v)c(v) \\ \div f(u)c(u) + f(v)c(v) + f(w)c(w),$$

which may be reduced to

$$\frac{4\epsilon + 3}{4\epsilon + 4} = \frac{1}{10}(7 + 2\epsilon) = .86180,33989.$$

The negative natural logarithm is .14872,81098 or 70.176 per cent. of that of a generation of continuous alternate-parent mating. This suggests that when third generation matings are sufficiently rare they will be preferred if the offspring of the right sex born from

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the third generation mating is not younger than that born to the second generation mating by more than 70 per cent. of the observed average generation length. If, for example, this average were found to be 890 days, we should prefer the offspring of the third mating to that of the second if born not more than 623 days later. In this way allowance is made for the actual ages of the animals available.

## APPENDIX B

### THE EFFICACY OF SELF-STERILITY MECHANISMS AMONG HERMAPHRODITES IN DIMINISHING UNIONS BETWEEN NEAR RELATIONS

The three chief methods of avoiding self-fertilisation in hermaphrodite plants are distylism, tristylism and a system of self-sterility allelomorphs.

All of these are effective in preventing self-fertilisation, but unequally effective in lowering the frequency of matings between near relatives. With distyly, all progenies resemble the general population in giving the two phenotypes in equal numbers; consequently neither sib-matings nor parent-offspring matings are in any degree diminished by this mechanism. The same applies to dioecious organisms.

In two cases the inheritance of the polymorphic variation of a tristyllic species has now been elucidated. The genetic analysis reveals the number of genotypes giving each form of flower, and subject to equal viability and fertility of the different legitimate crosses, the frequencies of the different genotypes of an open-pollinated population, the frequencies of the different legitimate matings, and the frequencies of the genotypes produced by each. On the basis of such an analysis we can therefore infer, subject to the simplifying conditions imposed, the effects of the system on the frequencies of matings between individuals of chosen degrees of kinship.

In *Oxalis valdiviensis* (7) there is disomic inheritance involving two Mendelian factors, rather closely linked. There are seven genotypes produced by legitimate pollination of which one is Long-styled, two are Mid-styled and four Short-styled. The frequencies in an

open-pollinated population in genetic equilibrium are shown, with their genetic symbols, in the following table.

TABLE 60

Symbol.	Frequency.	Style Type.
$ms/ms$ . . .	1	<i>L</i>
$Ms/ms$ . . .	$4\sqrt{3}-6$	<i>M</i>
$Ms/Ms$ . . .	$7-4\sqrt{3}$	<i>M</i>
$mS/ms$ . . .	$4-2\sqrt{3}$	<i>S</i>
$MS/ms$ . . .	$3\sqrt{3}-5$	<i>S</i>
$mS/Ms$ . . .	$3\sqrt{3}-5$	<i>S</i>
$MS/Ms$ . . .	$7-4\sqrt{3}$	<i>S</i>

For each style type the frequencies given above add to unity; they are therefore referable to a total population of three plants.

Legitimate matings among these seven genotypes, without distinguishing reciprocal matings, which are equivalent, number fourteen. Of these two are between Long and Mid, four between Long and Short, and eight between Mid and Short. The frequencies of the different matings will be three times the product of the frequencies of the two genotypes involved, or one third of the product of the values given above. The frequencies expressed in reference to a total of three matings, are as shown in Table 61.

The proportion of parent-offspring matings which would be illegitimate is half the sum of the frequencies in the offspring of the parental phenotypes; for sib-matings it is the sum of the squares of the three phenotypic frequencies. Parallel matings of double heterozygotes in coupling and repulsion which are of equal frequency may conveniently be averaged. It will be noticed that linkage makes no difference to the general frequency of parent-offspring matings, but that it does somewhat increase the frequency of illegitimate matings among sibs.



TABLE 61

Symbol.	Frequency.	Offspring.			Proportion Incompatible.	
		<i>L</i>	<i>M</i>	<i>S</i>	Parent-Offspring.	Sibs.
<i>Long and Mid</i>						
<i>ms/ms = Ms/ms</i>	$4\sqrt{3}-6$	$\frac{1}{2}$	$\frac{1}{2}$	...	$\frac{1}{2}$	$\frac{1}{2}$
<i>ms/ms = Ms/Ms</i>	$7-4\sqrt{3}$	...	1	...	$\frac{1}{2}$	1
<i>Long and Short</i>						
<i>ms/ms = mS/ms</i>	$4-2\sqrt{3}$	$\frac{1}{2}$	...	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
<i>ms/ms = MS/ms</i>	$3\sqrt{3}-5$	$\frac{1}{2}q$	$\frac{1}{2}p$	$\frac{1}{2}$	$\frac{3}{8}$	$(1-pq)/2$
<i>ms/ms = Ms/mS</i>	$3\sqrt{3}-5$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}$	$\frac{3}{8}$	$(1-pq)/2$
<i>ms/ms = MS/Ms</i>	$7-4\sqrt{3}$	...	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$
<i>Mid and Short</i>						
<i>Ms/ms = mS/ms</i>	$28\sqrt{3}-48$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$
<i>Ms/ms = MS/ms</i>	$66-38\sqrt{3}$	$\frac{1}{2}q$	$\frac{1}{2}(1+p)$	$\frac{1}{2}$	$\frac{7}{16}$	$(7-pq)/16$
<i>Ms/ms = mS/Ms</i>	$66-38\sqrt{3}$	$\frac{1}{2}p$	$\frac{1}{2}(1+q)$	$\frac{1}{2}$	$\frac{7}{16}$	$(7-pq)/16$
<i>Ms/ms = MS/Ms</i>	$52\sqrt{3}-90$	...	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
<i>Ms/Ms = mS/ms</i>	$52-30\sqrt{3}$	...	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
<i>Ms/Ms = MS/ms</i>	$41\sqrt{3}-71$	...	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
<i>Ms/Ms = mS/Ms</i>	$41\sqrt{3}-71$	...	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
<i>Ms/Ms = MS/Ms</i>	$97-56\sqrt{3}$	...	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

If for all fourteen types of mating the proportion of illegitimate parent-offspring matings is multiplied by the frequency and added, the general proportion of illegitimate matings is found to be

$$(6\sqrt{3}-5)/12.$$

The proportion legitimate is therefore

$$(17-6\sqrt{3})/12,$$

and, since with unrelated plants this proportion is two-thirds, the effect of the genetic system is to reduce the proportion of parent-offspring matings in the ratio

$$(17-6\sqrt{3})/8$$

or

$$82.596 \text{ per cent.}$$

For sib-matings we have a corresponding reduction in the ratio

$$\{(1+3\sqrt{3})+(13-7\sqrt{3})pq\}/8$$

or, as a percentage,

$$77.452+(10.946)pq.$$

For sib-matings, therefore, the proportionate reduction ranges from 77.452 per cent. for absolute linkage ( $pq = 0$ ) to 80.588 for very loose linkage, or independence ( $pq = \frac{1}{4}$ ). For *Oxalis valdiviensis* the recombination fraction has been found to be about 7 per cent., and  $pq$  must be about  $\frac{1}{15}$ .

Compared with a distylic species, therefore, the tristily in *O. valdiviensis* avoids about one in six parent-offspring matings, and a little more than one in five sib-matings. The latter effect is slightly enhanced by the close linkage of the two genes.

A second case of tristily is supplied by *Lythrum salicaria*, (8) in which the loci for Mid- and Short-style are unlinked. Inheritance is, however, tetrasomic, and double reduction has been shown to occur at both loci. At the locus for Mid the frequency of double reduction has been determined to be about 8 per cent., and, using this value, the frequencies of fifteen genotypes in a cross-pollinated population, with equal viability and

fertility, have been calculated to be as follows, for a population of 1000 plants.

TABLE 62

		Not Short, $s_4$ .	Short, $Ss_2$ and $S_2s_2$ .
Long	$m_4$	333·3333	179·6645
Mid	$Mm_3$	275·9408	114·7611
Mid	$M_2m_3$	52·1439	33·6590
Mid	$M_3m$	4·9246	4·9246
Mid	$M_4$	0·3241	0·3241

Owing to double reduction at the Short locus a certain fraction of each class of Short genotype will be duplex for the Short gene. In equilibrium this fraction is

$$3a/(4-a),$$

where  $a$  is the frequency of double reduction at the Short locus. The frequency  $a$  has been very roughly determined at about  $2\frac{1}{2}$  per cent., so that about one in fifty-three of each class of Short will be duplex. This proportion does not affect the frequency of legitimate matings of parent and offspring, but would have some influence on the frequency for whole sibs.

The frequency of legitimate matings for parent and offspring may be expeditiously calculated from the frequency of the different style types from Long, Mid or Short seed-parents.

TABLE 63

*Style Types of Legitimate Offspring from Different Seed-Parents*

Seed-Parent.	Long.	Mid.	Short.
Long . . . .	429·96	370·04	250·00
Mid . . . .	322·47	427·53	250·00
Short . . . .	247·57	252·43	500·00

Whence it appears that the proportions of legitimate matings with offspring are :—

TABLE 64

Seed-Parent.	Per cent.
Long . . . . .	57·004
Mid . . . . .	57·247
Short . . . . .	50·000

Thus, on the average, 54.750 of parent-offspring crosses are legitimate, or 81.175 per cent. of the proportion in the general population.

The protection afforded against matings of parent with offspring in *L. salicaria* is thus very nearly equal to, though a trifle greater than in *O. valdiviensis*.

The effect of a series of self-sterility allelomorphs is less determinate owing to its dependence on the frequencies with which these occur. If all are rare, as seems often to be the case, then almost all matings will be of the type

$$s_1s_2 = s_3s_4,$$

and all genotypes of offspring will be cross-fertile with both parents. Since there are four intra-sterile types of offspring appearing in equal numbers, compatible sib-matings will be reduced to 75 per cent., or rather more than in *O. valdiviensis*. This form of self-sterility is, therefore, an efficient one for annual plants, though not so efficient as tristylly for perennials.

There will, however, be a proportion of matings of the type

$$s_1s_2 = s_1s_3,$$

giving only two genotypes, and having therefore 50 per cent. protection against sib-matings. In these crosses, moreover, half the offspring will be incompatible with the pollen parent, so that there will be only 75 per cent. compatibility in parent-offspring crosses. The large number of self-sterility alleles usually found will, however, make such matings comparatively rare.

In effect, then, a series of self-sterility alleles provides the greatest protection against homozygosity for annual plants, in which sib-mating is of importance, while tristylly may be more effectual in perennials. Tristylly also is not upset if the powerful additional protection of polysomic inheritance is superimposed on the self-sterility mechanism, as in *Lythrum salicaria*.

## REFERENCES

- (7) R. A. FISHER and V. C. MARTIN (1948). Genetics of style-length in *Oxalis*. *Nature*, clxii, 533.
- (8) R. A. FISHER and V. E. MARTIN (1947). Spontaneous occurrence in *Lythrum salicaria* of plants duplex for the short-style gene. clx, 541.

## APPENDIX C

### THE FUNCTION OF INBREEDING IN ANIMAL AND PLANT IMPROVEMENT

Great practical success has attended the production of improved varieties, most conspicuously in maize, by the following cycle of operations :—

- (a) Choice of foundation stock.
- (b) Inbreeding to produce a homozygous, or nearly homozygous, line.
- (c) Crossing chosen lines ; the first generation from a pair of lines may be further crossed either to a third line, or to another first cross from different parent lines. Further crosses may be made using independent inbred material. The result of such crosses is used for production, and not as a self-perpetuating variety ; it is produced anew as required from the inbred lines maintained permanently as parent stock.

In interpreting the working of such a cycle of operations it is important to observe that, starting with a cross-bred population containing numerous genotypes with definite frequencies, the whole cycle will merely reproduce these same genotypes with the same probabilities provided that :—

- (a) The foundation individuals are chosen at random.
- (b) Inbreeding is carried on without selection.
- (c) Inbred lines are crossed at random.

Any improvement actually effected must therefore be ascribed to selection at one or more of these three stages.

Most animal and plant material of value to mankind owes its value to the selection in past generations of the

visible good qualities of individuals. Carried out over long periods, and by innumerable individual breeders, each careful to preserve his best stock, this is undoubtedly an effective method of improvement; it is indeed the necessary foundation for all improvement. Since, however, domesticated varieties do not very rapidly change their average performance, the dramatic improvement effected by inbreeding cannot be ascribed to the single act of the selection of the individuals used to found the line. Its success may, of course, owe much to the choice of good stock from which these individuals are selected, but improvement above the level already attained by this stock is often far greater than careful selection within it, for a single generation, could possibly effect. We cannot ascribe the success of the programme merely to the fact that the foundation individuals are not chosen at random.

It would be quite impossible to carry out an inbreeding programme without selection, for the numerous recessive defects which inbreeding uncovers, will by lethality or severe debility exercise a selection quite out of the experimenter's control. There are, however, grave difficulties of a quantitative nature, in the way of accepting the view that the success of inbreeding programmes is due simply to the purification of the stock by the elimination of a fraction of the recessive heritable defects.

In a cross-bred population carrying numerous recessive factors, the effect of each of which, when homozygous, is to lower somewhat the physiological efficiency, the general performance measured by yield of seed will be to some extent, but not greatly, depressed by the chance occurrence of such homozygotes. Suppose such a recessive gene to have a frequency  $p$ ; in random crossing the frequency of the homozygotes will be  $p^2$ . If these have their yield lowered by the fraction  $k$ , the

average loss due to this fraction will be  $kp^2$ , and the total loss of crop from this cause will be

$$S(kp^2)$$

the summation being taken over all such recessives.

If no conscious selection is exercised in such a self-perpetuating cross-bred population each frequency  $p$  will come to equilibrium with the mutation rate, by which the recessive defect is produced. If the chance of survival is equated to the yield, as is reasonable with grain crops, the rate of elimination of the genes is also measured by  $kp^2$ , so that the sum of these quantities for all factors must be equal to the total of the mutation rates by which they are supported.

Since the great majority of mutants are known to be deleterious, we might now equate the depression of yield in the cross-bred crop to the total mutation rate to which it is subject, with, however, a few reservations.

(i) Recessives causing complete lethality will contribute to the total mutation rate, but can scarcely affect the performance of the crop as grown, for this will be sown with excess seed, and the lethals will not appear at harvest; the same is true of severely handicapped types, which will have been crowded out, yielding their space to more efficient competitors. (ii) Improved crops may now be regarded as working near to a "ceiling," or physical limit of production. As such a ceiling is approached each factor in the genotype has less and less effect upon the yield; it is probable therefore that many frequencies are not now in equilibrium with their mutation rates; since mutation may not have had time to increase these frequencies up to the new level of equilibrium. For both these reasons it would appear that the total elimination of deleterious recessives would make less difference to the yield of cross-bred commercial crops than the total mutation rate would



suggest. Perhaps no more than a 1 per cent. improvement could be looked for from this cause. Differences of the order of 20 per cent. remain to be explained.

Factors in which selection favours the heterozygote over both homozygotes will establish a stable polymorphism in which a considerable fraction of the population will be below the optimum. Such factors, if frequent, might explain a great advantage in some first-cross hybrids, but scarcely in later crosses, unless a multiplicity of alleles, all deleterious when homozygous, were assumed.

It should be noted that the phase of inbreeding is not favourable to the exercise of deliberate selection, partly because such selection often can only be applied at the expense of retarding the inbreeding process, partly because the effects of factors on which selection might usefully be exercised is masked by the segregation of a mass of deleterious recessives of no consequence to the final product.

At the third stage at which selection is exercised the conditions seem to be more favourable. We are not now confined to the physiologically deleterious factors, which by selection have become recessive. The difference between two different crosses may depend on any of the factors available for evolutionary adaptation. It may be noted first that "hybrid corn" has been an immense success in that species, in which thousands of inbred lines have been produced. In species in which so far only a few inbred lines have become available, success has not been conspicuous.

When many homozygous lines are available the conditions for effective selection seem to be at their best. (i) We are selecting the actual genotype to be used for production, not merely an ancestor of it. (ii) Owing to the reliability of breeding performance achieved by inbreeding, lots of all sizes will be available for testing.

Many crosses get no further than a first inspection, but promising crosses may be tested in quantity with all the precision which modern experimental design makes possible. (iii) Any special advantage, limited perhaps by locality or by industrial use, remains a permanent property of the hybrid, which will reappear whenever it is made up. It can later be produced in the quantity appropriate to the special role it is to play. Every careful determination of quality is a permanent contribution to the optimal utilisation of the material.

The practical moral of these facts is obvious. As the basis of future livestock and plant improvement there is required not a single inbred line, or a few only, but a deliberately planned multiplicity. The price paid for reliability of breeding behaviour is the impoverishment of the genic content, due to the elimination of many genes. There need be no such impoverishment if many inbred lines are created simultaneously.

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