

# Fisher's Equation: Some Methodological Doubts

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1. The idea of a 'real' rate of interest, as opposed to the nominal or money rate of interest, the rate at which real world transactions take place, was introduced by Irving Fisher in his 1930 classic, *The Theory of Interest*. The object of this distinction was to unveil the interest phenomenon of its monetary shroud and to investigate the reality which was supposed to lie beneath – a reality in which thriftiness (time preference in Fisher's words) and productivity govern the lending and borrowing of real capital to establish a real rate of interest. Once the real rate is established in this way then, all else remaining the same, monetary forces may be allowed to affect only the nominal rate.<sup>1</sup> This conclusion appeared to be consistent with the doctrine of neutrality of money as applied to financial markets and squared well with Fisher's quantity theory of money.

In keeping with this logic, Fisher first defined the real rate of interest as,

$$r \equiv \frac{1+R}{1+p} - 1 = R - p - rp \approx R - p \quad \dots (1)$$

with R being the nominal rate and p the rate of inflation. Next, having decided that the 'real' object r was determined outside the monetary world, he chose to rewrite the identity (1) in the form

$$R \equiv r + p \quad \dots (2)$$

This manner of writing it is intended to convey the idea that the 'dependent variable' is the nominal rate which, in some sense, must be 'explained' by the real rate and the inflation rate; that the latter are the building blocks that make R what it is seen to be. Of course the fact remains that (2) is as much an identity as (1) is. To convert it into a behavioural equation Fisher took the next step – of supposing that r would be determined by real forces and that only the *expected* rate of inflation can have any influence on the nominal interest rate. He thus obtained the equation

$$R = r + p^e \quad \dots (3)$$

which has now become one of the most widely used equations in economics during the last fifty years.<sup>2</sup> In Fisher's words, "The influence of changes in the purchasing power of money on the money rate of interest will be different according to whether or not that change is foreseen" [Fisher 1930, p. 45].

2. After decades of empirical inquiry, this equation has become a foundation stone of modern macroeconomic, monetary and financial theories. It is today a sacred cow of not only the science of economics but also of the art of central banking. I wish to

show that despite its established status, Fisher's equation stands on rather shaky foundations and I leave it to the readers to judge whether I should be hanged, or only flogged, for this heresy.

Consider equation (1) which defines an unobservable real interest rate in terms of observable variables, the nominal interest rate and the rate of inflation. This equation has been transformed into equation (3) making the observed nominal interest rate the sum of two unobservable variables, the real interest rate and the expected inflation rate. Then equation (3) is read in the following way: given the real rate as determined by real forces that are given in the long-run, changes in the expected inflation rate are simply transmitted into changes in the nominal interest rate. The suspicious thing about this is that the transmission mechanism is omitted; specifically, the manner in which the expected rate of inflation causes a change in the decisions of market participants and their transactions and thus causes a change in the nominal interest rate is nowhere articulated. But surely that is what Fisher means in his statement quoted earlier. Specifically, it is by representing the inflation rate in terms of expectations that the identity (1) is converted into the equation (3).

Thus, consider the consequence of allowing expectations to affect the transactions of market participants. Suppose people expect inflation to occur at a rate  $p^e$  in future. They have an amount of money  $M$  which they can invest in a deposit/bond that pays the nominal interest rate  $R$  (no matter whether it contains  $p^e$  or not) or, alternatively, to buy goods at their current prices (in the same weightage as they appear in the price index) and sell them later. If they do not have access to  $M$ , they simply borrow money and buy commodities if  $R < p^e$  or sell commodities and invest at  $R$  if  $R > p^e$ . This by itself will cause a change in the going market prices of commodities and bonds. The terminal wealth for the two investments  $M(1+R)$  and  $M(1+p^e)$  are equalised in equilibrium, so that

$$R = p^e \quad \dots (4)$$

and the real interest rate, which in this context shows the net (arbitrage) rate of profit, becomes zero. In other words, Fisher's equation (3) does not obtain if foreseen inflation is allowed to affect portfolio decisions. Incidentally, it may be recalled that these were the precise grounds on which Keynes (1936, p. 142) took objection to Fisher's idea, "... If it [inflation] is foreseen, the prices of existing goods will be forthwith so adjusted that the advantages of holding money and of holding goods are again equalized ...".

3. Considering the important role played by Fisher's equation in modern macroeconomic theory, it may be subjected to another test. Consider a static economy. Suppose it undergoes continuing inflation in the money supply, wage rate and prices for a period sufficiently long so as to make it the expected inflation rate. Then the nominal values of the national income for this economy from any one year to the next are;

$$Y_1 = w_1L + RK_1 \quad \dots 5(a)$$

$$Y_0 = w_0L + rK_0 \quad \dots 5(b)$$

If the capital stock is regarded as the value of produced means of production,

$$\frac{Y_1}{Y_0} = \frac{w_1}{w_0} = \frac{K_1}{K_0} = 1 + p$$

which on substitution in 5(a) and 5(b) results in,

$$R = r \quad \dots (6)$$

Equation (3) is not obtained. Even if we suppose that K represents real capital whose size like that of labour or of real output does not change from year to year equation (5) stands replaced by

$$Y_1 = w_1L + RK$$

$$Y_0 = w_0L + rK$$

Then substituting  $\frac{Y_1}{Y_0} = \frac{w_1}{w_0} = (1 + p)$  gives

$$R = r(1 + p) \quad \dots (7)$$

i.e., even if the impact of inflation were to be coaxed to pass through exclusively into the nominal interest rate the resulting equation is not Fisher's equation!

If, to make one more attempt to find a suitable macroeconomic role for Fisher's equation, we were to substitute  $(1 + r)(1 + p) - 1$  in 5(a), the result is,

$$p = 0$$

which clearly contradicts the initial assumption that the economy is undergoing inflation. Nor does substituting the approximation  $r + p$  for R help because it simply results in  $r = 1$ , which is nonsensical.

Another way to make the point is to write the nominal national income of the economy undergoing continuous uniform inflation for any given year as

$$Y_N = WL + [(1+r)(1+p) - 1] K$$

where W is the nominal wage rate. To obtain the real income deflate both sides by the price factor so that

$$\begin{aligned} \frac{Y_N}{1+p} &= \left( \frac{W}{1+p} \right) L + (1+r)K - \frac{K}{1+p} \\ Y_R &= wL + rK + K \left( 1 - \frac{1}{1+p} \right) \\ &= wL + rK + \left( \frac{p}{1+p} \right) K \end{aligned}$$

The last term shows an error in the measurement of the real national income. The inevitable conclusion is that Fisher's equation is incompatible with the national income identity!

4. Econometric studies of the inflation-interest relationship has become an industry in its own right alongside such subjects as testing the CAPM, testing for market efficiency or testing for the purchasing power parity theory. Countless papers have been written on this subject and although their conclusions go this way and that, as is only to be expected when the data handled differs across countries, economic conditions, time periods and definitions of the variables, the consensus seems to be that Fisher's equation passes the econometric tests of validity. Since a review of that literature is beyond the scope of this paper, and more frankly, beyond the endowment of my econometric capabilities, I shall not attempt it. I shall confine my comment to one simple test of Fisher's equation, a test whose multivariate generalizations are found in the literature.

Fisher's equation (3) is usually cast into the corresponding econometric form

$$R_t = a + bp_t^e + u \quad \dots (8)$$

where  $R_t$  is a suitably chosen nominal rate,  $p_t^e$  is extracted from a suitably chosen price index by using one of many expectations models and the random term  $u_t$  is supposed to have the customary properties. Then, if  $a$ , the estimate of the real rate  $r$  is found to be significantly positive and  $b$  is found to be significantly close to 1, the underlying equation (3) is regarded as valid. But what this procedure ignores is that equation (3) itself is a numerical approximation with  $rp \approx 0$  which is made purely for conversational convenience; the exact equation must include the multiplicative term and ought to be read as,

$$R = r + (1+r)p^e \quad \dots 9(a)$$

with the econometric form

$$R_t = a + (1+a)p_t^e + u_t \quad \dots 9(b)$$

Seen against the backdrop of equation 9(a) the findings that  $a > 0$  which implies  $r > 0$  and  $b = 1$  which implies  $r = 0$  are actually contradictory! And there is no conceivable way of identifying whether the regression of  $R_t$  on  $p_t^e$  is one that pertains to the mathematical form (3) or 9(a). I conclude that even when the data are best suited to

test Fisher's equation, it is impossible to say from the results that Fisher equation is verified.

5. The equation may be approached from yet another viewpoint which permits a more detailed examination of the effects of inflation. Consider a firm that uses a stock of working capital  $K$ , flow inputs of  $C$  and labour worth  $W$  for its annual production which generates a sales revenue  $S$ . If  $r$  is the rate of interest, its annual activity is given by

$$rK + C + W = S$$

If  $K=100$ ,  $C=50$ ,  $W=50$  and  $r=20$  per cent and the firm is fully leveraged, its profit and loss account is as follows:

Sales Revenue	120
Less Cost of Goods Sold	100
Less Interest	20
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Profit	0
	----

The firm can continue its operations from year to year simply by renewing the loan. (Note that we are assuming implicitly that 20 per cent is the normal rate of profit so the zero profit should be considered as zero supernormal profit)

Next, suppose that there is inflation at a 10 per cent rate which in the first stage is confined to the output sold by the firm. The profit and loss account will show

Sales Revenue	132
Less Cost of Goods Sold	100
Less Interest	20
	----
Profit	12
	----

The excess profit of 12 that the owners begin to enjoy is directly attributable to the (opportunity) losses suffered by the input suppliers and workers which is worth 10 and those of lenders which is worth two. There are two noteworthy points. Firstly, the rate of operating profit (interest plus profit)  $R$  can be expressed as,

$$R = (1+r)(1+p) - 1$$

$$\text{i.e., } 0.32 = (1.2)(1.1) - 1$$

Keynes (1936, p. 142) had made a similar observation when criticizing Fisher, "The mistake lies in supposing that it is the rate of interest on which changes in the value of money will directly react, instead of the marginal efficiency of a given stock of capital." The rate  $R=0.32$  is not the nominal rate of interest, it is the nominal rate of operating *profit*, which has an obvious similarity with the marginal efficiency of capital.

Secondly, if lenders were to actually charge the rate of interest of 32 per cent in accordance with Fisher's formula they would entirely recover what they had lost to the owners;

Sales Revenue	132
Less Cost of Goods Sold	100
Less Interest	32
	----
Excess Profit	0
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This looks okay at first glance but begins to create difficulties when inflation is considered to be economy wide so that input prices and wages too inflate by 10 per cent. Then the position of the firm is as follows;

Sales Revenue	132
Less Cost of Goods Sold	110
Less Interest	32
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Profit	-10
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And this leads to a further worsening when inflation touches the values of stocks themselves whose requirements would increase to 110 with the result that the quantum of financing increases to 110. The picture is,

Sales Revenue	132
Less Cost of Goods Sold	110
Less Interest	32.2
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Profit	-13.2
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In other words, if nominal interest rates were to change according to Fisher's formula otherwise viable firms in a zero inflationary condition are driven into insolvency in a condition of inflation at a uniform rate. Not a very convincing conclusion.

The more likely scenarios would be as follows. When, in the first bout, inflation causes excess profits to appear, two forces will be activated. Firstly, inflation will begin to spread through the system of production considering that the inflated sales revenues of some firms are costs of production for other firms, and so on. This will cause the costs of production of all firms to rise which, in time, will begin to inflate the values of the stocks and fixed capital items  $K$  as they come up for replenishment and replacement. These effects will work towards reducing the excess profits. Secondly, the divergence between the rate of profit and the rate of interest will cause the financial portfolios to be reshuffled away from deposits and bonds and towards equities. The result will either be an outflow of deposits or a selling pressure in the bond market. If it is the former, banks will raise the interest rate offered on deposits to bid them back. They are able to do this by raising the interest rate on loans which entrepreneurs do not mind paying due to the increased profitability. This causes interest rates to rise. If it is the latter bond prices will drop and bond yields will rise. These effects explain why Fisher's equation may be found to hold good in the short-run. In time, however, after inflation has spread throughout the system of production and excess profits experienced in the early stages move southwards, interest rates too follow suit and in the new equilibrium the profit and loss account will look as follows:

Sales Revenue	132
Less Cost of Goods Sold	110
Less Interest	22
	----
Profit	0
	----

The interest payment of 22 on an inflated capital of 110 once again is a 20 per cent rate; the nominal and real rate are equal to one another and back at their original level. Needless to say, continuing inflation at disuniform rates from year to year or at disuniform rates across classes of commodities will keep both the production system and the financial system in a disturbed state but that is outside the scope of a discussion of Fisher's equation.

6. Friedman (1956) was perhaps the first economist to make Fisher's equation a foundational building block of macroeconomic models. But the version that Friedman advances [Friedman (1956), equation 9] of Fisher's equation is:

$$r_b = r_e + \frac{1}{p} \frac{dp}{dt}$$

where  $r_b$  is the rate of interest on bonds and  $r_e$ , Fisher's real rate, is made equivalent to the rate of return on, of all things, *equity*. But this is exactly the opposite of our finding in the example of the previous section viz. that it is the rate of operating profit (including that on equities) which increases with inflation at the prevailing nominal rate of interest. Moreover, the return on equity is the sole non-contractual 'price' in an economy; all other prices including the rate of interest are contractually fixed and vary by a revision in the terms of the contract as the economic forces governing their demands and supplies vary over time. The return on equity can become *negative*, sometimes for sustained periods of time. Surely, it cannot stand for Fisher's real rate, which must be strictly positive on account of its being determined by the real forces of the marginal productivity of capital and a time preference for current over deferred consumption. So I think Friedman's version of Fisher's equation is simply a misrepresentation. It is intuitively far more comfortable to suppose that a variable rate of inflation affects, in the first place, a variable non-contractual return on equity with the contracted rate being temporarily fixed. To my knowledge Friedman's version of Fisher's equation has never been empirically tested. One does not need to be a prophet to foretell that if it were tested with short-term rates, the results would either be rejected or inconclusive and if it were tested with long-term rates, the equation would be simply rejected.<sup>3</sup>

7. Having said all this does not absolve me from the task of answering two important questions. Firstly, why does short-term interest rate behaviour seem to adhere closely to Fisher's equation when it does so? [For example, *see* Mishkin 2009] Secondly, why do market players, particularly money and bond market traders, swear by Fisher's equation in their day-to-day work? To my mind the answer to both questions is one and the same. Central bankers use the interest rate instrument to achieve their short-term inflation targets. If that is so, bond traders must use something like Fisher's equation to anticipate central bank actions on interest rates in response to inflation rates and maximize their profits by selling securities if an interest rate increase (consequent upon a rising the inflation rate above the target) is anticipated and buying

them if an interest rate decrease (the actual inflation rate falling below the target) seems imminent. Anticipations of interest rate increases cause bonds to be sold and their yields to rise and *vice versa*. If the very same bond yield data is mapped against the inflation rate, lo and behold, it must give a good fit. To conclude, if Fisher's equation is found to work it could only be because of the general impression that somebody as powerful as a Central Bank is using it!

## Notes

1. Fisher's terminology, now almost universally adopted in the economics literature, is most misleading. Both the adjectives 'real' and 'nominal' are used to convey the opposites of their conventional meanings. Thus, the adjective nominal which usually means "unreal", "unimportant" or "insignificant" has been attached to the actual rate of interest, but 'actual' conventionally means "real". On the other hand, the adjective 'real' has been attached to the "unobservable" rate and no grounds are given for it to be even conceivably observable so that, in conventional terms, it has a flavour of unreality. Even if it is understood that Fisher used the term real rate of interest to rhyme with real capital, I suspect that the transfer of epithet has somewhat prejudiced matters in such a way that only an inquiry into 'real' interest rates is regarded as a *real* = true = genuine inquiry into interest rate theory.

2. Similar definitions can be set up for other contractually determined prices as well. For instance, suppose a wage (or rent) contract to receive an amount of money wage  $W$ . If inflation is expected to take place the expected remuneration received in real terms is

$$w = \frac{W}{1 + p^e} \approx W(1 - p^e)$$

which can be cast into an appropriate econometric form,

$$W = A + B [p^e (1 + p^e)] + u$$

and tested. It is surprising that these Fisher-like equations for other prices have been entirely ignored in the macroeconomic literature even though a whole school of macroeconomic thought viz. rational expectations theory, relies on their empirical validity.

3. Prescott and Mehra (1985) have shown that for the US economy over long time-series the return on equities always exceeds the rate of interest. The equity premium puzzle has been subsequently found to be a feature of several financial markets.

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