A Model of Market Clearing Exchange Rates

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Abstract

This paper formulates a model of exchange rate determination that describes the market processes by which the foreign exchange markets are cleared and international receipts of countries are brought into equality with their international payments. It is shown that corresponding to any configuration of intercountry payments there exists a unique positive set of exchange rates that simultaneously clear the markets for all currencies. The model is capable of being explicitly solved for the actual world economy provided the balance of payments data which are routinely collected by central banks and reported to the IMF are arranged by their countrywise origins and destinations.

This paper proposes a model in which currency exchange rates are determined by the simultaneous operation of two activities of foreign exchange dealers

1. the matching of demands and supplies of currencies by intermediating between buyers and sellers of currencies
2. arbitraging across foreign exchange quotations in different local financial centers

The buyers and sellers in (1) include both speculators and non-speculators. However, the inclusion of speculative transactions on foreign exchange dealers' own accounts has been discussed separately in Section 5 below. Other issues that are germane to the subject, such as the management of exchange rate targets, balance of payments targets and, the consequences of multiple exchange rates have also been explored within the context of the basic model.

I The Intercountry Payments Matrix

Consider a world of freely floating exchange rates. Suppose there are Z countries A, B, … Z each having its own national currency. Let $P_{ij}(i, j = A...Z, i \neq j)$ denote the total payments (on all accounts, visible, invisible and autonomous and accommodating capital accounts) made by country i to country j in its own currency. Obviously, $P_{ij} = 0$ for $i = j$ since no country makes international payments to itself. It will be supposed that if $P_{ij} > 0$ for some i, then $P_{ki} > 0$ for at least one k i.e. no country will make international payments without having some international receipts. We shall also suppose that there is at least one country that has international economic relations with all countries directly or indirectly, i.e. the world economy does not break up into independent sub-worlds.

The matrix of intercountry payments may then be written out as follows

Table 1: The Intercountry Payments Matrix

	A	B	C	…	Z	Total
A	0	P_{AB}	P_{AC}	…	P_{AZ}	P_A
B	P_{BA}	0	P_{BC}	…	P_{BZ}	P_B
C	P_{CA}	P_{CB}	0	…	P_{CZ}	P_C
:	:	:	:	:	:	:
:	:	:	:	:	:	:
Z	P_{ZA}	P_{ZB}	P_{ZC}	…	0	P_Z

The row sums of the matrix are the total payments made by the countries to the rest of the world and may be said to be the total supplies of the currencies that are offered for sale in the foreign exchange market. If the matrix is read columnwise, each column shows the payments that a country will receive from the rest of the world and may be said to constitute the demand for its currency since all these receipts represent the supplies of the various currencies that are offered in exchange for the currency of the recipient country. It is the job of the foreign exchange market to ensure that these multilateral demands and supplies are cleared.[1]

Let $E_{ij}(i \neq j)$ denote the spot exchange rates showing the units of currency i that exchange for a unit of currency j. $E_{ij} = 1$ for $i = j$ by definition. Foreign exchange markets are cleared when exchange rates are such as to equate the total supplies of the currencies to the total demands for them, i.e. the total payments P_i are equal to the total receipts of each country $R_i = \sum_j P_{ij} E_{ji} = P_i$. We thus obtain Z balance of payments equations,

$$
\begin{aligned}
&0 + P_{BA}E_{AB} + P_{CA}E_{AC} + ... + P_{ZA}E_{AZ} = P_A \\
&P_{AB}E_{BA} + 0 + P_{CB}E_{BC} + ... + P_{ZB}E_{BZ} = P_B \\
&P_{AC}E_{CA} + P_{BC}E_{CB} + 0 + ... + P_{ZC}E_{CZ} = P_C \qquad \ldots (1) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&P_{AZ}E_{ZA} + P_{BZ}E_{ZB} + P_{CZ}E_{ZC} + ... + 0 = P_Z
\end{aligned}
$$

Observe in (1) that the left-hand sides represent the total receipts of each country in its home currency and these are equated to the country's total payments. There are Z equations in (1) of which any one is redundant in view of the fact that if payments and receipts of any $Z-1$ countries balance so do those of the Z^{th}. Thus there are only $Z-1$ independent equations. As against these there are $Z(Z-1)$ unknown exchange rates $E_{ij}(i, j = A....Z, i \neq j)$. So it appears that we are $Z(Z-1) - (Z-1) = (Z-1)^2$ equations short of the required number.

It is well-known however, that the arbitrage operations of foreign exchange dealers will establish some relations between the unknown exchange rates. Specifically, arbitrage activities will ensure that bilateral exchange rates will be reciprocals of one another and that direct quotes will be equal of indirect quotes. The exchange neutrality conditions are,

$$E_{ij}E_{ji}=1 \qquad \forall i,j \;\; i\neq j \qquad \ldots (2a)$$

$$E_{ij}E_{jk}E_{ki}=1 \qquad \forall i,j,k \;\; i\neq j\neq k \qquad \ldots (2b)$$

Equations 2(a) are brought about by 2-currency arbitrage and equations 2(b) by three-currency arbitrage. (2a) eliminate between them $Z(Z-1)/2$ exchange rates and (2b) eliminate $(Z-1)(Z-2)/2$ exchange rates, i.e. they supply exactly the $(Z-1)^2$ equations that are required.[2]

Before demonstrating the explicit solution for the exchange rates however, it may be worthwhile to make a small detour to bring out the role of the equations in (2) in terms of an alternative derivation. The equations in (1) are in the own currencies of the countries. Suppose we require that these equations balance not only in their own currencies but also in terms of any one of the currencies, say the currency of country A.

We may then express all the equations in (1) in currency A by keeping the first equation as it is, multiplying the second equation by E_{AB}, the third by E_{AC}.... and so on to obtain,

$$\begin{aligned}
&0+P_{BA}E_{AB}+P_{CA}E_{AC}+\ldots+P_{ZA}E_{AZ}=P_A\\
&P_{AB}E_{BA}E_{BA}+0+P_{CB}E_{AB}E_{AC}+\ldots+P_{ZB}E_{AB}E_{BZ}=P_BE_{AB}\\
&P_{AC}E_{AC}E_{CA}+P_{BC}E_{AC}E_{CB}+0+\ldots+P_{ZC}E_{AC}E_{CZ}=P_CE_{AC} \qquad \ldots (3)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&P_{AZ}E_{AZ}E_{ZA}+P_{BZ}E_{AZ}E_{BZ}+P_{CZ}E_{AZ}E_{CZ}+\ldots+0=P_ZE_{AZ}
\end{aligned}$$

Now observe that the elements in the first column of (3) are the payments made by country A in the currency of country A. Thus

$$P_{AB}E_{AB}E_{BA}+P_{AC}E_{AC}E_{CA}+\ldots+P_{AZ}E_{AZ}E_{ZA}=P_A$$

But by definition

$$P_{AB}+P_{AC}+\ldots+P_{AZ}=P_A$$

For both to hold, $E_{AB}E_{BA}=1, E_{AC}E_{CA}=1,\ldots,E_{AZ}E_{ZA}=1$ which give $Z-1$ two currency neutrality conditions

Likewise, compare the remaining $Z-1$ column sums. Thus, for the second column

$$P_{BA}E_{AB}+P_{BC}E_{AC}E_{CB}+\ldots+P_{BZ}E_{AZ}E_{ZB}=P_BE_{AB}$$

Since both sides are payments by country B in terms of currency A. But by definition,

$$P_{BA}+P_{BC}+\ldots+P_{BZ}=P_B$$

so that for both to hold, $E_{AC}E_{CB}=E_{AB},\ldots,E_{AZ}E_{ZB}=E_{AB}$ which gives $Z-2$ three currency neutrality conditions. The $Z-1$ column comparisons will therefore yield $(Z-1)(Z-2)$ three currency neutrality conditions. We therefore obtain a total of $(Z-1)+(Z-1)(Z-2)=(Z-1)^2$ independent equations to eliminate

$(Z-1)^2$ exchange rates. And we have *Z-1* independent balance of payments equations to determine $Z-1$ exchange rates.

The purpose of this detour was only to show that the neutrality conditions which in reality are brought about by currency arbitrage invariably determine a configuration of exchange rates such that the balance of payments of all countries balance in terms of *all* the currencies.[3]

II Solution of the Exchange Rates

Using the neutrality conditions, the equations in (3) can all be expressed in terms of some one currency A;

$$0 + P_{BA}E_{AB} + P_{CA}E_{AC} + ... + P_{ZA}E_{AZ} = P_A$$
$$P_{AB} + 0 + P_{CB}E_{AC} + ... + P_{ZB}E_{AZ} = P_B E_{AB}$$
$$P_{AC} + P_{BC}E_{AB} + 0 + ... + P_{ZC}E_{AZ} = P_C E_{AC}$$
$$.....$$
$$P_{AZ} + P_{BZ}E_{AB} + P_{CZ}E_{AC} + ... + 0 = P_Z E_{AZ}$$

In vector-matrix notation the system is,

$$\begin{bmatrix} P_A & -P_{BA} & -P_{CA} & ... & -P_{ZA} \\ -P_{AB} & P_B & -P_{CB} & ... & -P_{ZB} \\ -P_{AC} & -P_{BC} & P_C & ... & -P_{ZC} \\ \mathrm{M} & \mathrm{M} & & \mathrm{M} & \\ -P_{AZ} & -P_{BZ} & -P_{CZ} & ... & P_Z \end{bmatrix} \begin{bmatrix} 1 \\ E_{AB} \\ E_{AC} \\ \mathrm{M} \\ E_{AZ} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathrm{M} \\ 0 \end{bmatrix} \quad \ldots (4)$$

Equation (4) is a system of homogenous equations. Since all the column sums of the matrix add up to zero its determinant vanishes and a unique solution for the exchange rates can be obtained up to a scalar multiple. Of course since $E_{AA} = 1$ by definition a unique solution for $Z-1$ exchange rates $E_{AB}...E_{AZ}$ exists. To explicitly solve for the exchange rates, eliminate any one equation from (4), say the first equation for country A, and write the remaining $Z-1$ equations as a system of non-homogenous equations,

$$\begin{bmatrix} P_B & -P_{CB} & ... & -P_{ZB} \\ -P_{BC} & P_C & ... & -P_{ZC} \\ \mathrm{M} & \mathrm{M} & & \mathrm{M} \\ -P_{BZ} & -P_{CZ} & ... & P_Z \end{bmatrix} \begin{bmatrix} E_{AB} \\ E_{AC} \\ \mathrm{M} \\ E_{AZ} \end{bmatrix} = \begin{bmatrix} P_{AB} \\ P_{AC} \\ \mathrm{M} \\ P_{AZ} \end{bmatrix} \quad \ldots (5)$$

Note that since the receipts of country A have been eliminated and by assumption there is at least one country making payments to it, the column sums of the matrix on the left hand side of (5) are non-negative with at least one column-sum being strictly positive. Therefore the inverse exists and consists only of positive elements. It follows that the exchange rate solution obtained must be strictly positive because the vector on the right hand side is strictly non-negative [4]

Some properties of the exchange rate solution are at once apparent. If (5) is solved by Cramer's rule

$$E_{AJ} = \frac{\det P_j}{\det P} \qquad j = B,...Z$$

where P_j is the matrix obtained by replacing the j^{th} column by the vector on the right hand side and P is the $Z-1 x Z-1$ matrix in (5). Now suppose that the vector of A's payments to the rest of the world were to increase by a uniform factor $(1+k_A)$ then $\det(P_j)$ increases by that factor and so do all the E_{Aj} showing a proportionate depreciation in the currency of country A against all currencies. If the payments of any country other than A were to rise by a factor $(1+k_j)$ there would be no change in the exchange rates $E_{Ai}(i \neq j)$ but E_{Aj} would decrease by the factor $(1+k_j)$, i.e. currency A would appreciate against currency j. Of course currency j would depreciate against all other currencies by the factor $(1+k_j)$.

It is unlikely that in reality intercountry payments (on all accounts, current as well as capital) would exhibit such stylized trends except in rare cases such as a country undergoing unusually rapid inflation. Usually, one would expect to observe substantial changes in the composition of intercountry payments and therefore changes in exchange rates relative to one another.

III Forward Exchange Rates

The determination of forward exchange rates is straightforward since covered interest arbitrage would ensure that,

$$F_{ij}^t = E_{ij} \frac{(1+r_{it})^t}{(1+r_{jt})^t} \quad i \neq j \qquad \ldots (6)$$

where t stands for the maturity of the forward contract and r_{it}, r_{jt} are the spot interest rates for maturity t in countries i and j.

While the above represents the final relationship determining the forward exchange rates, the actual process of multilateral clearing of spot and forward markets may be quite complicated. This is because at any given time there will simultaneously prevail a spot demand-supply position as depicted by the spot intercountry payments matrix and several forward intercountry payments matrices for the various maturities of the ultimate buyers and sellers which reflect their foreign exchange risk management policies. If the spot and forward exchange rates are solved for these individual markets they would obviously show very substantial violations of the interest parity condition. The fact that such violations are not observed in practice only means that arbitrageurs rapidly set into motion intercountry payments on their own account to make spot-forward, forward-forward and covered interest arbitrage transactions to restore the parities.

IV The Speculative Force

We have up to this point ignored the third activity of foreign exchange dealers which has an obvious bearing on exchange rate movements viz. speculative buying and

selling of currencies. It has been documented that inter-dealer transactions constitute more than 90 per cent of the daily turnover in the markets for important convertible currencies.[5] And although foreign exchange dealers are known to be definitely cautious about keeping open positions for too long it is likely that a significant proportion of inter-dealer transactions will be of a speculative nature especially due to the advantage of leverage. To include speculative transactions into the model of exchange rates, it would be necessary to add the periodic changes in the values of the net open long positions of the foreign exchange dealers to the usual intercountry' payments on the visible, invisible and capital accounts which represent dealers currency trades on their clients' behalf. Presumably such data may be obtained from dealers' associations that play a self-regulatory role and/or from central banks.

V Other Exchange Rate Regimes

In a fixed exchange rate system the parity rates are fixed exogenously at say E^*_{Aj}, so that the system of exchange rates (4) will be used to solve for the balance of payments deficits / surpluses for Z-1 countries. For moderate deviations of the parity rates from the market-clearing rates the simple rule is that countries that have pegged their exchange rates at appreciated levels would experience deficits against those that have fixed them at depreciated levels. Of course the sum of the surpluses and deficits add up exactly to zero.

If countries pursue balance of payments targets B^*_i measured in terms of currency A $(i = A,...Z)$ equation in (5) can be used to find the corresponding exchange rate target E^*_{Aj},

$$\begin{bmatrix} P_B & -P_{CB} & \dots & -P_{ZB} \\ -P_B & P_{BC} & \dots & -P_{ZC} \\ \text{M} & \text{M} & & \text{M} \\ -P_{BZ} & -P_{CZ} & \dots & P_Z \end{bmatrix} \begin{bmatrix} E^*_{AB} \\ E^*_{AC} \\ \text{M} \\ E^*_{AZ} \end{bmatrix} = \begin{bmatrix} P_{AB} + B^*_B \\ P_{AC} + B^*_C \\ \text{M} \\ P_{AZ} + B^*_Z \end{bmatrix} \qquad \dots (7)$$

where B^*_i represent deficits if positive and surpluses if negative. The balance of payments deficit / surplus of A is simply the sum of B_i. A has no control over it.[6]

If a country were to follow a dual exchange rate system with a fixed exchange rate for some intercountry transactions and a market determined rate for others, that country's row of payments must be separated into parts, one portion P^c_{Aj} will be converted at the controlled rate E^c_{Aj} and the remaining payments P^f_{Aj} at the free market rate. For example if country A follows a dual exchange rate system, then the system to solve the market clearing rates in the presence of the controlled rate is,

$$\begin{bmatrix} P_B - P_{AB}^C E_{BA}^C & -P_{CB} & -P_{DB} & .... & P_{ZB} \\ -P_{BC} & P_C - P_{AC}^C E_{CA}^C & -P_{DC} & .... & P_{ZC} \\ -P_{BD} & -P_{CD} & P_D - P_{AD}^C E_{DA}^C & .... & P_{ZD} \\ ..... & & & & \\ -P_{BZ} & -P_{CZ} & -P_{DZ} & ..... & P_Z - P_{AZ}^C E_{ZA}^C \end{bmatrix} \begin{bmatrix} E_{AB} \\ E_{AC} \\ E_{AD} \\ M \\ E_{AZ} \end{bmatrix} = \begin{bmatrix} P_{AB}^f \\ P_{AC}^f \\ M \\ P_{AZ}^f \end{bmatrix} \quad \ldots (8)$$

A similar procedure applies if country *i* is using a controlled exchange rate to convert a part of its receipts in which case it is that country's column in the payments matrix that must be separated into the controlled and free market components. It is evident that if the country subsidies its priority sectors by fixing an appreciated controlled rate for some of its payments and/or a depreciated controlled rate for some its receipts the subsidy that it thereby gives to its priority sectors is made good by a depreciation of its free market rate that translates into a 'tax' on the other sectors.

VI Numerical Example

Some of the propositions discussed above are illustrated in the following numerical example. Consider a world of 7 countries with the following intercountry payments matrix.

Table 2: International Payments Matrix: A 7x7 Example

	A	B	C	D	E	F	G	Total
A	0	100	75	20	400	500	10	1105
B	5	0	5	2	4	4	10	30
C	1000	2000	0	1550	5000	10000	12000	31550
D	200	300	150	0	10000	250	300	11200
E	250	250	500	1000	0	300	200	2500
F	7000	10000	3000	8000	20000	0	25000	73000
G	500	6000	200	300	15000	10000	0	32000

The market clearing exchange rates in the currency of country A are $E_{AB} = 51.052$, $E_{AC} = 0.045$, $E_{AD} = 0.215$, $E_{AE} = 1.921$, $E_{AF} = 0.034$, $E_{AG} = 0.074$

Table 3 gives all the cross exchange rates where the elements below the main diagonal are reciprocals of the corresponding elements above it.

Table 3: Market Clearing Exchange Rates

	A	B	C	D	E	F	G
A	1.0000	51.0523	0.0457	0.2155	1.9211	0.0347	0.0746
B	0.0195	1.0000	0.0009	0.0042	0.0376	0.0006	0.0014
C	21.8738	1116.71	1.0000	4.7156	42.022	0.7603	1.6316
D	4.6385	236.808	0.2120	1.0000	8.9111	0.1612	0.3460
E	0.5205	26.5743	0.0237	0.1122	1.0000	0.0181	0.0388
F	28.7677	1468.66	1.3151	6.2018	52.2662	1.0000	2.1459
G	13.4056	684.391	0.6128	2.8900	25.7538	0.4659	1.0000

Every country's overall receipts and payments will be in balance at these exchange rates but they may have surpluses and deficits with individual countries e.g. country A has the following net position with countries $B,...Z$; +155.26, -29.28, 23.116, 80.27, -256.67, 27.29 which sum to zero.

If payments by country A to other countries increase uniformly by 10%, i.e. the first row rises to (0, 110, 82.50, 22, 440, 550, 11) with total payments of 1215.30, the exchange rates change to 56.157, 0.050, 0.237, 2.113, 0.038, 0.082 showing a depreciation of 10% but all other exchange rates are unaffected. If receipts of country A (the first column) were to rise by 10% the exchange rates change to 46.047, 0.041, 0.195, 1.745, 0.032, 0.067 showing an appreciation in A's currency by 10%. But in this case the other exchange rates do not remain unchanged because the composition of intercountry payments have undergone a change. For example in the former case $E_{DB} = 236.088$ while in the latter it is $E_{DB} = 235.1269$

If countries adopt a fixed exchange rate system and fix the rates (2.5, 2.5, 0.5, 3.5, 30, 1) the equations solve for the balance of payment surpluses/deficits. These, measured in A's currency, are 10152.5, – 466087.5, 371450.0, 45439.0, -158964.0, -268120.0, 466130.0 and sum up to zero.

Observe in the above that countries that have fixed parities that stand at appreciated levels with respect to the market clearing rates show deficits and vice versa. But this rule applies only for moderate deviations of parities from the market clearing rates.

The exchange rate system can be used in the reverse. For instance if the countries choose to target the sizes of their surpluses/deficits, the system can be used to find the exchange rates that these countries must target. Thus suppose countries B,…,G set their deficit / surplus targets at 100, 150, - 200, -300, 250, 120 in the currency of country A. Then the corresponding exchange rate targets are 41.548, 0.018, 0.222, 1.887, 0.027, 0.059. At these exchange rates country A has –120 deficit which equals the sum of the surpluses/deficits of the 6 countries.

Finally consider how the equations are applied to determine the market clearing exchange rates in a regime of dual/multiple exchange rates. Suppose that country A decides that (to take a simple case) one-half of its payments to other countries will be converted at controlled exchange rates. Suppose the exchange rates of currency A in the freely floating regime and the controlled exchange rates are,

Table 4: Exchange Rates of Country A under Free Market and Dual Regimes

	Free Market Rate	Controlled Rate
E_{AB}	51.052	40.00
E_{AC}	0.045	0.03
E_{AD}	0.2150	0.1
E_{AE}	1.921	1.2
E_{AF}	0.034	0.02
E_{AG}	0.074	0.03

Then the market clearing exchange rates in the presence of the controlled rates obtained by solving equation (8) are as follows, $E_{AB} = 145.5113$, $E_{AC} = 0.1310$, $E_{AD} = 0.6225$, $E_{AE} = 5.5331$, $E_{AF} = 0.1009$, $E_{AG} = 0.2152$

Observe by comparing these to the free market exchange rates of A that they show a depreciation in all cases. It is due to the fact that the controlled exchange rates have been fixed at an appreciated level to subsidise 'priority' payments and the subsidy must be made up by a corresponding tax on 'non-priority' payments.

Less obviously, the effects of A's adopting a dual exchange rate policy are not simply confined to a cross-subsidisation between categories of foreign exchange payers, it results in a change in all exchange rates, e.g. $E_{DB} = 233.753, E_{FE} = 54.784$ as compared to $E_{DB} = 236.8088$ and $E_{FE} = 52.2662$ in the free market conditions.

VII Exclusion/Inclusion of Countries

To get an idea of the sensitivity of the solution of the exchange rates to the exclusion of countries, say due to unavailability of their intercountry payments data, consider the following. Suppose that the data of country A which has direct economic relations with other countries and accounts for 6.81 per cent of total world payments worth 16220.088 in the currency of A were not available. Then a comparison of the exchange rate solutions under the scenarios of availability and non-availability reveals the following:

Table 5: Effect of Excluding Country A

	With A	Without A	Discrepancy (%)
E_{BC}	0.0008954	0.0008137	10.04
E_{BD}	0.0042220	0.0038906	8.53
E_{BE}	0.0376300	0.0346780	8.51
E_{BF}	0.0006800	0.0005570	22.16
E_{BG}	0.0014610	0.0013270	10.09

As is to be expected the discrepancies are large. This is true of other cross-rates as well. For example exchange rate E_{CF} which is $E_{CF} = 0.7603305$ if A is not excluded changes to $E_{CF} = 0.684896$ if A is excluded, a discrepancy of 11.01 per cent.

However, the discrepancies are much smaller if small countries were excluded. For example suppose that country G which makes payment of 200 and 300 to countries C and D and receives 100 and 100 from them and has no economic relations with other countries were excluded. The exchange rates with inclusion of G and without inclusion are as follows:

Table 6: Effect of Excluding Country G

	With G	Without G	Discrepancy (%)
E_{AB}	56.274788	56.283371	- 0.0152
E_{AC}	0.0685951	0.06877180	- 0.2575
E_{AD}	0.2055489	0.20510502	+ 0.2164
E_{AE}	1.6702998	1.66900970	+ 0.0773
E_{AF}	0.0409066	0.04093390	- 0.0665

Small as the discrepancies are the largest impact is observed in the case of exchange rates with respect to currencies C and D with whom country G has economic relations.

VIII Concluding Remarks

The model of exchange rates presented in this paper may be said to be a first approximation of the old 'balance of payments theory of exchange rates' [Keynes (1930), Robinson (1937)]. It is no more than that. A complete theory of exchange rates must explain why the intercountry payments matrix comes to be what it is and how the individual components of intercountry payments will evolve over different economic episodes in response to the different economic forces that operate in the individual countries.

Despite its limited scope and purpose, the model by itself is versatile enough to explain the workings of the multicountry foreign exchange market under a variety of exchange rate regimes like the fixed and managed floating regimes and administered multiple rate regimes. As such it may prove useful for resolving problems of the multi-country co-ordination of BOP targets and exchange rate policies. Most of the data that are required by the model are being compiled and reported by the central banks of all the member countries to the IMF. It will only be necessary to compile this data into an intercountry payments matrix in the own currencies of the member countries. Data on the net open position of forex dealers will of course need to be separately elicited.

Endnotes

1. Kenen (1994) has called these exchange rates "market clearing exchange rates" as opposed to "equilibrium exchange rates". This is an unusual way of employing the terminology considering that the term equilibrium is generally used to describe situations in which markets are cleared. Kenen makes this distinction on subtler grounds. Any set of forex market clearing exchange rates may not be an equilibrium set of exchange rates because other markets, e.g. commodity/asset markets may not be cleared at every market clearing set of exchange rates. Only a set of market clearing exchange rates which also clears commodity and asset markets would qualify to be called equilibrium exchange rates.

Kenen proposed a two currency model [*see* Kenen (1994) p. 319] to determine the market clearing exchange rate which is a special case of the model presented in this paper.

Supply of yen = (Demand for dollars) x Yen per dollar

where the supply of yen which are the payments made by Japan to USA also represent the demand for dollars. Krugman (1980) presented a 3-country 3-currency example which is identical to the formulation in this paper but used it as a framework to identify efficient structures for the exchange mechanism and to bring out the importance of vehicle currencies. Krugman's model is as follows. Let α, β, χ represent currencies of countries A, B, and C. Then the balance of payments equations are

$$P_{BA} + P_{CA} = E_{\alpha\beta} P_{AB} + E_{\alpha\chi} P_{AC}$$
$$P_{AB} + P_{CB} = E_{\beta\alpha} P_{BA} + E_{\beta\chi} P_{BC}$$
$$P_{AC} + P_{BC} = E_{x\alpha} E_{CA} + E_{x\beta} E_{CB}$$

where the items on left hand side represent receipts of the respective countries in terms of their own currencies and the items on right hand side are the payments made by them in foreign currencies converted into their own currencies by means of the exchange rates.

2. Every textbook on foreign exchange markets explains the neutrality conditions e.g. Levi (1996), Riehl & Rodriguez (1973). The fact that equations 2(a) and 2(b) form a linearly independent set of equations was first proved by Chacholiades (1971) as a theorem, "if two-currency and three-currency arbitrage is not profitable then m-currency arbitrage (m >3) is not profitable either".

3. The neutrality conditions can be derived from other criteria as well. Consider this criterion, "the world income must be the same irrespective of the currency in which it is measured". Let national incomes in domestic currencies be $Y_A, Y_B, ... Y_Z$. Then the world income in various currencies may be expressed as,

$$\begin{aligned} W_A &= Y_A + E_{AB} Y_B + E_{AC} Y_C + ... + E_{AZ} Y_Z \\ W_B &= E_{BA} Y_A + Y_B + E_{BC} Y_C + ... + E_{BZ} Y_Z \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ W_Z &= E_{ZA} Y_A + E_{ZB} Y_B + E_{ZC} Y_C + ... + Y_Z \end{aligned} \quad \ldots (9)$$

Of course

$$W_A = E_{AB} W_B = E_{AC} W_C + ... + E_{AZ} W_Z$$

Multiply W_B ..., W_z by E_{AB}, E_{AC} ..., E_{AZ} to express them in the currency of country A

$$W_A = (E_{AB} E_{BA}) Y_A + E_{AB} Y_B + (E_{AB} E_{BC}) Y_C + ... + (E_{AB} E_{BZ}) Y_Z$$
$$W_A = (E_{AC} E_{CA}) Y_A + (E_{AC} E_{CB}) Y_B + E_{AC} Y_C + ... + (E_{AC} E_{CZ}) Y_Z$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$W_A = (E_{AZ} E_{ZA}) Y_A + (E_{AZ} E_{ZB}) Y_B + (E_{A2} E_{2C}) Y_C + ... + E_{AZ} Y_Z$$

and compare each of these $(Z-1)$ equations to the first equation in 9. From the first comparison

$$E_{AB} E_{BA} = 1 \quad E_{AB} E_{BC} = E_{AC} ..... E_{AB} E_{BZ} = E_{AZ}$$

From the second we obtain

$$E_{AC} E_{CA} = 1 \quad E_{AC} E_{CB} = E_{AB} ..... E_{AC} E_{CZ} = E_{AZ}$$

And so on. From the last comparison

$$E_{AZ} E_{ZA} = 1 \quad E_{AZ} E_{ZB} = E_{AB} .....$$

These give a total of $(Z-1)$ two currency and $(Z-1)(Z-2)$ three-currency neutrality conditions, i.e. $(Z-1)^2$ in all.

4. Consider equation 5

$$
\begin{aligned}
&P_B E_{AB} - P_{CB} E_{AC} \ldots - P_{ZB} E_{AZ} = P_{AB} \\
&- P_{BC} E_{AB} - P_C E_{AC} \ldots - P_{ZC} E_{AZ} = P_{AC} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&- P_{BZ} E_{AB} - P_{CZ} E_{AC} \ldots - P_Z E_{AZ} = P_{AZ}
\end{aligned}
$$

Divide the respective equations by $P_B \ldots . P_Z$ to get,

$$
\begin{bmatrix}
1 & -p_{CB} & \ldots . & -p_{ZB} \\
-p_{BC} & 1 & \ldots . & -p_{ZC} \\
\mathrm{M} & \mathrm{M} & & \mathrm{M} \\
-p_{BZ} & -p_{CZ} & \ldots . & 1
\end{bmatrix}
\begin{bmatrix}
E_{AB} \\ E_{AC} \\ \mathrm{M} \\ E_{AZ}
\end{bmatrix}
=
\begin{bmatrix}
p_{AB} \\ p_{AC} \\ \mathrm{M} \\ p_{AZ}
\end{bmatrix}
$$

where the off-diagonal elements are shares p_{ij} of receipts/payments of countries from / to other countries. It is clear that the row sums are all non-negative with at least one being strictly positive in view of the fact that country A must be making positive payments to at least one country and country A's row has been excluded. Likewise the column sums must be non-negative with at least one being strictly positive because country A must be receiving payment from at least one country but country A's column has been excluded. Thus the matrix fulfils Solow's (1952) sufficiency condition for a positive inverse. And since the vector on the right hand side is non-negative, the solution of exchange rates must be positive.

5. Wolgast M (1997) quoted in Manfred (1998)

6. Only $(Z-1)$ countries can pursue independent balance of payments targets. McKinnon (1979) gives an analysis of currencies that serve as numeraire and also *vehicle* currencies.

References

Chacholiades (1971), The Sufficiency of Three Point Arbitrage to Insure Consistent Cross Rates of Exchange, *The Southern Economic Journal,* Vol. 38, No. 1, pp. 86-88.

Kenen, Peter B. (1994), *The International Economy,* 3rd Edition, Cambridge University Press, Cambridge.

Keynes, J.M. (1930), *Treatise on Money*, Macmillan and Co., London.

Krugman, Paul (1980), Vehicle Currencies and the Structure of International Exchange, *Journal of Money, Credit and Banking,* Vol. 12, pp. 513-526.

Levi, Maurice (1996), *International Finance*, 3rd International Edition, McGraw-Hill, Singapore.

Manfred, Willams (1998), "Comment" in Wagner H. (Ed.), *Current Issues in Monetary Theory*, Physica-Verlag, Heidelberg.

McKinnon, R. (1979), *Money in International Exchange,* Oxford University Press, New York.

Riehl, Heinz and Rita M. Rodriguez (1973), *Foreign Exchange Markets,* McGraw Hill, New York.

Robinson, Joan (1937), The Foreign Exchanges in *Essays in the Theory of Employment*, pp. 134-155, Macmillan and Co., London

Solow, Robert M. (1952), On the Structure of Linear Models, *Econometrica*, 20, January, pp. 29-46.