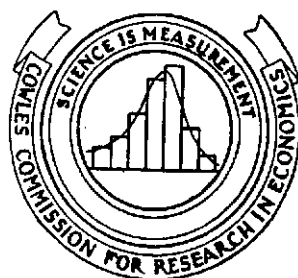


THE ANALYSIS OF ECONOMIC TIME SERIES

By
HAROLD T. DAVIS



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This work surveys the problem of analyzing economic time series beginning with historical origins and ending with a critical appraisal of what has been accomplished by modern statistical methods. Numerous examples illuminate the text. Many tables and 169 charts illustrate the contents of the book.

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PREFACE

The object of the present volume is to set forth in some detail the present status of the problem of analyzing and interpreting that very extensive set of data known as economic time series. This perplexing problem has engaged the attention of economists and statisticians for many years, but the extraordinary intensity with which it has been attacked during the past decade attests the importance which it has for modern economic development.

Since its beginning the laboratory of the Cowles Commission for Research in Economics has had as a major interest the investigation of the nature and action of stock price series. In the course of this investigation a number of interesting but difficult problems were encountered concerning the nature of economic time series in general, and the relation of these series to the basic postulates of economic theory in particular. To most of these questions only partial answers were discovered in the literature and in many cases these answers were not accompanied by careful statistical analyses. Therefore, it seemed to the author that a systematic treatise on the nature of economic series might fill a present need.

To one who works with statistical data it soon becomes apparent that the conclusions derived at the end of a process of analysis are intimately related to the postulates which underlie the tools employed in the investigation. The employment of a linear trend for the reference of residuals, or the graduation of a series of production data by means of the logistic curve, implies economic assumptions which must be carefully defined and subjected to realistic criticism. That is to say, conclusions mathematically derived are no better than the postulates upon which they rest. Hence it has seemed necessary to make a careful re-examination of the various mathematical devices which have been used in the study of economic data in order to appraise their weakness and their strength, and to define the range of their validity.

There is a perpetual fascination in economic time series, derived not only from their immense importance in the lives of all of us, but also from their statistical nature. Differing from the series encountered in the experiments of physical science, every economic time series possesses a large random element. But the series themselves are not random, in spite of some popular belief to the contrary, nor are

they sufficiently regular to satisfy most mathematical postulates. Hence, in many instances, the analysis must proceed from a description of the differences between random series and series that are not random. Correlations take the place of functions and serial relationships replace the more familiar functional equations of the exact sciences.

In the course of preparing so extensive a manuscript the author has become indebted to many people. Foremost among these is Mr. Alfred Cowles, president of the Cowles Commission, who for nearly a decade has liberally supported a scientific laboratory devoted to the investigation of problems in economic theory and economic statistics. His personal interest in these investigations and his own scientific contributions to the subject have been a source of inspiration and satisfaction to the author.

From Mr. Dickson H. Leavens, managing editor of *Econometrica* and research associate of the Cowles Commission, the author has received services too numerous to mention. Mr. Leavens assumed full editorial supervision of the manuscript and the planning of the charts is to be credited entirely to him.

During the preparation of the book the author received many suggestions from Dr. C. F. Roos, former research director of the Cowles Commission, and from Professor T. O. Yntema, the present research director. Their broad knowledge of economic problems was placed generously at his disposal.

A special debt of thanks is also due Professor Gerhard Tintner of Iowa State College, who read the entire proof carefully and offered many valuable suggestions. His exceptionally wide acquaintance with economic and statistical literature, especially that of European schools, has made his criticism of great value.

To Mr. Herbert E. Jones, research associate of the Cowles Commission, the author is indebted for a number of essential contributions to the book. Mr. Jones undertook a thorough investigation of problems relating to the theory and application of serial correlation. In particular, he studied the properties of random series and then applied his analysis to the problem of determining the nature of the structural elements in economic time series. Much of the material in Chapters 3 and 4 is derived from his studies.

Throughout the long and arduous calculations presented at many places in the book the laboratory staff of the Cowles Commission has played an indispensable role. The brunt of this work has been assumed by Mr. Forrest Danson, research associate of the Cowles Commission and director of the computing laboratory. The author is especially

indebted to him. In this phase of the work numerous computations were made by Miss Emma Manning, Miss Anne M. Lescisin, Mr. Edward Morris, and Mrs. Martha Belschner Swanson. Miss Kathryn Withers did the arduous work of inking and lettering the charts and Miss Mary Jo Lawley helped in preparing the manuscript for the printer.

To the great experience of Professor Irving Fisher in monetary theory and to the statistical studies of Mr. Carl Snyder on economic trends and the theory of prices the author owes a special debt. From conversations with Professor Ragnar Frisch of Oslo, Norway, and from his writings, more particularly his studies of harmonic analysis, confluence analysis, and the dynamics of cycles, the author has derived many valuable suggestions. Professor J. W. Angell of Columbia University very kindly supplied the author with monetary data which would otherwise have been inaccessible to him.

The author would also like to acknowledge his appreciation of the critical advice received from Dr. John Smith, research associate of the Cowles Commission, who has brought to bear upon the analysis a broad knowledge of statistical sampling. His criticism has been particularly valuable in connection with some of the material in Chapter 5. From other colleagues in the research staff of the Cowles Commission many helpful suggestions have been received. Professor Francis McIntyre, Dr. Abraham Wald, Dr. Edward N. Chapman, and the late Mr. W. F. C. Nelson all brought unique experience to bear upon certain aspects of the problem.

During the preparation of the book a series of conferences on economic problems was held in Colorado Springs under the auspices of the Cowles Commission. Some 200 lectures were given at these conferences and the author received many valuable suggestions both from the lectures and from informal conferences with the speakers. The effects of this unusual experience will be noted in many parts of the book.

The appraisal of the author's debt would not be complete without mention also of the help received in two other statistical laboratories, one at Indiana University and the other at Northwestern University. In the operation of these laboratories the author has been particularly indebted to Dean Fernandus Payne of Indiana University and to Professor E. J. Moulton of Northwestern University, both of whom have taken a personal interest in the work. In both laboratories many of the author's students contributed generously of their time. Colleagues in the departments of both physics and astronomy gave generously of their information at various stages of the writing of the manuscript.

Finally, but not least, the author must acknowledge his debt to the Principia Press and to its editor, Professor J. R. Kantor, who has extended in every way his cordial co-operation. The manuscript has been put into type and printed by the Dentan Printing Company of Colorado Springs, who have met all the unusual requests incidental to the production of a mathematical and statistical treatise with unflinching cheerfulness.

From these acknowledgments it will be apparent that the present work is in many respects a collaborative effort. Such virtues as the work may have are to be shared by those who have been mentioned here; unfortunately, the responsibility for the errors must be assumed only by the author himself.

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Evanston, Illinois
November, 1941

TABLE OF CONTENTS

CHAPTER 1

HISTORY OF THE PROBLEM

1.	Time Series	1
2.	Astronomical Time Series	1
3.	Economic Time Series	4
4.	Types of Time Series	7
5.	Economic Crises and Their Significance	9
6.	The Problem of Trends	15
7.	The Evidence for Cycles	24
8.	Harmonic Analysis	28
9.	The Advantages and Limitations of Harmonic Analysis	33
10.	The Erratic-Shock Theory of Economic Time Series	36
11.	Historical Summary of Application of Harmonic Analysis	43
12.	The Theory of Business Cycles	44
13.	Mathematical Attempts to Account for Cycles	49
14.	Historical Summary of the Theory of Serial Correlation	52
15.	The Analysis of Random Series	56
16.	The Present Status of the Problem	59

CHAPTER 2

THE TECHNIQUE OF HARMONIC ANALYSIS

1.	Harmonic Analysis	60
2.	Fourier Series	61
3.	The Theorems of Bessel and Parseval and Their Significance	64
4.	The Technique of Harmonic Analysis	67
5.	A Mathematical Example	71
6.	The Effect of a Linear Trend in Harmonic Analysis	75
7.	Applications to Economic Time Series	77
8.	Other Methods of Harmonic Analysis	83
9.	The Exact Determination of the Period	87
10.	Orthogonal Functions	89
11.	Minimizing by the Method of Least Squares	95
12.	Relationship to the Theory of Multiple Correlation	97

CHAPTER 3

SERIAL CORRELATION ANALYSIS

1.	Introduction	102
2.	Examples of Lag Correlation	104
3.	Inverse Serial Correlation	111
4.	The Lag-Correlation Function for a Harmonic Sum	119
5.	The Lag-Correlation Function and Its Harmonic Analysis for Statistical Data	122
6.	Some Examples Useful in the Analysis of Economic Time Series— Continuous Spectra	123
7.	Yule's Theory of Random Variation	129
8.	Lag Correlation and Its Relation to Supply and Demand Curves	135

THE ANALYSIS OF ECONOMIC TIME SERIES

CHAPTER 4

THE THEORY OF RANDOM SERIES

1.	Definitions and Examples	142
2.	Goutereau's Constant	144
3.	Yule's Theory	149
4.	Generalization of Yule's Theory of the Differences of Random Series	151
5.	Accumulated Random Series	155
6.	Random Series Smoothed by a Moving Average	159
7.	The Theory of Sequences and Reversals	164
8.	An Application to Stock-Market Action	170

CHAPTER 5

THE DEGREES OF FREEDOM IN ECONOMIC TIME SERIES

1.	Preliminary Definitions	175
2.	Economic Time Series as a Problem in Inverse Probability	180
3.	Significance Tests and the Problem of Degrees of Freedom	183
4.	Schuster's Significance Test in Harmonic Analysis	186
5.	Walker's Significance Test in Harmonic Analysis	188
6.	R. A. Fisher's Test of Significance	189
7.	Factor Analysis	192
8.	The Method of Elementary Energies	198
9.	Examples Illustrating the Method of Elementary Energies	205

CHAPTER 6

THE ANALYSIS OF TRENDS

1.	Introduction	208
2.	Types of Trend	209
3.	Technical Discussion of the Linear Trend	210
4.	Extension of the Foregoing Theory to Polynomial Trends	217
5.	Formulas for the Correlation of Residuals from Polynomial Trends	222
6.	Example of the Reduction of Series to their Random Elements	227
7.	Economic Significance of the Example	234
8.	Seasonal Variation	237
9.	The Variate Difference Method and Its Application	240
10.	The Logistic Trend	247
11.	The Growth of Population	255
12.	The Growth of Production	265
13.	Frequency Distribution of Time Series	271
14.	The Stability of Trends	274
15.	General Critique of the Economic Significance of Trends	275

CHAPTER 7

PERIODOGRAM ANALYSIS

1.	Introduction	276
2.	The Constructed Sine-Cosine Series	278
3.	Random Series	280
4.	Random Series Smoothed with a Moving Average	282
5.	Cumulated Random Series (Smoothed)	285
6.	The Cowles Commission All Stocks Index (1880-1896)	286
7.	The Dow-Jones Industrial Averages (1897-1913)	288
8.	The Dow-Jones Industrial Averages (1914-1924)	290
9.	The Dow-Jones Industrial Averages (1925-1934)	292

TABLE OF CONTENTS

10.	Rail Stock Prices (Monthly, 1831-1855) - - - - -	298
11.	Rail Stock Prices (Monthly, 1856-1880) - - - - -	294
12.	Rail Stock Prices (Annually, 1831-1930) - - - - -	296
13.	Rail Stock Prices (at Four-Month Intervals, 1831-1930) - - - - -	298
14.	American Industrial Activity (Annually, 1831-1930) - - - - -	299
15.	American Wholesale Prices (Annually, 1831-1930) - - - - -	304
16.	Sauerbeck's Index of English Wholesale Prices (1818-1913) - - - - -	305
17.	Commercial-Paper Rates (Annually, 1831-1930) - - - - -	307
18.	Rail Bond Prices (Annually, 1831-1930) - - - - -	309
19.	Business Failures in the United States (1867-1932) - - - - -	310
20.	Wheat Prices in Western Europe (1500-1869) - - - - -	312
21.	Sunspot Numbers (1750-1900) - - - - -	317
22.	Galvanometer Series - - - - -	319
23.	Stock Price Indexes (Daily, 1927) - - - - -	320
24.	Stock Price Indexes (Daily, 1930) - - - - -	321
25.	Stock Price Indexes (Weekly, 1922-1927) - - - - -	323
26.	Stock Price Indexes (Weekly, 1929-1935) - - - - -	324
27.	General Summary - - - - -	325

CHAPTER 8

THE EVIDENCE AND EXPLANATION OF CYCLES

1.	The Enumeration of Theories - - - - -	327
2.	The Maximizing of Profits as an Example of How Cycles May Be Expected to Arise - - - - -	333
3.	The Macrodynamic Theory of Cycles - - - - -	339
4.	The Interest Theory of Cycles - - - - -	348
5.	The Building Cycle and Its Influence - - - - -	351
6.	Other Cycle Theories—Statistical Hysteresis - - - - -	355
7.	The Random-Shock Theory of Cycles—The Galvanometer Experiment - - - - -	366
8.	The Perturbation Theory of Cycles - - - - -	370
9.	The Resonance Theory of Crises - - - - -	378
10.	Generalization of the Perturbation Theory - - - - -	380
11.	Conclusions - - - - -	384

CHAPTER 9

THE NATURE OF WEALTH AND INCOME

1.	The Nature of the Problem - - - - -	386
2.	The Nature of Wealth - - - - -	388
3.	The Nature of Income - - - - -	390
4.	The Distribution of Incomes—Pareto's Law - - - - -	394
5.	Income Data - - - - -	397
6.	The Pareto Distribution - - - - -	400
7.	The Statistical Verification of Pareto's Distribution - - - - -	402
8.	Formulas for the General Distribution Function - - - - -	404
9.	A New General Distribution Function - - - - -	408
10.	Theoretical Derivation of the Distribution Function - - - - -	412
11.	Statistical Verification - - - - -	413
12.	Relationships Between the Distribution of Income and Total Real Income - - - - -	418
13.	Curves of Concentration - - - - -	424
14.	The Generalized Law of Inequality - - - - -	426
15.	Application of the Pareto Distribution of Income to Political Events - - - - -	434

TABLE OF CONTENTS

16.	Conclusions - - - - -	440
	Selected Bibliography - - - - -	440
CHAPTER 10		
THE DYNAMICS OF TRENDS FROM THE POINT OF VIEW OF THE EQUATION OF EXCHANGE		
1.	Historical Introduction - - - - -	444
2.	The Variables in the Equation of Exchange - - - - -	447
3.	Circulating Money (M) - - - - -	448
4.	Circulating Deposits (M') - - - - -	451
5.	The Velocity of Circulating Deposits (V') - - - - -	454
6.	The Velocity of Circulating Money (V) - - - - -	461
7.	Price (P) - - - - -	463
8.	Trade (T) - - - - -	473
9.	Snyder's Theory of Price - - - - -	476
10.	The Statistical Test of Snyder's Theory - - - - -	478
11.	Modifications of Snyder's Theory - - - - -	481
12.	Gold and Silver and Their Relationship to the Level of Prices - - - - -	484
13.	The Index of Trade from Spanish Treasure Data - - - - -	498
14.	Conclusion - - - - -	496
	Selected Bibliography - - - - -	497
CHAPTER 11		
FORECASTING ECONOMIC TIME SERIES		
1.	Introduction - - - - -	499
2.	The Present Status of Forecasting Stock Prices - - - - -	501
3.	The Standard Error of Forecast - - - - -	504
4.	The Standard Error of Forecast for Linear Trends - - - - -	517
5.	The Standard Error of Forecast for Harmonic Sums - - - - -	522
6.	Forecasting Logistic Trends - - - - -	524
7.	The Standard Error of Forecast for Polynomial Trends - - - - -	529
8.	Moving-Periodogram Analysis - - - - -	530
9.	The Method of Probable-Error Bands - - - - -	534
10.	The Dow Theory of Forecasting - - - - -	538
11.	General Conclusions - - - - -	544
CHAPTER 12		
INTERPRETATION AND CRITIQUE		
1.	Introduction - - - - -	546
2.	Probability Evidence for the Forty-Month Cycle - - - - -	546
3.	Probability Evidence for the Ten- and Twenty-Year Cycles - - - - -	550
4.	The Fifty-Year War Cycle - - - - -	552
5.	The Interaction of World Crises - - - - -	557
6.	The Theory of Jevons and Its Present Status - - - - -	561
7.	Economic Time Series and the Interpretation of History - - - - -	571
8.	General Summary - - - - -	576
—		
	Table 1. Values of the Walker Probability Function, P_w - - - - -	585
	Table 2. Values of the Fisher Probability Function, P_f - - - - -	6'
	Index of Names - - - - -	6
	Index of Subjects - - - - -	6..