

# **THE THEORY OF WAGES**



MACMILLAN AND CO., LIMITED  
LONDON · BOMBAY · CALCUTTA · MADRAS  
MELBOURNE

THE MACMILLAN COMPANY  
NEW YORK · BOSTON · CHICAGO  
DALLAS · ATLANTA · SAN FRANCISCO

THE MACMILLAN COMPANY  
OF CANADA, LIMITED  
TORONTO

# THE THEORY OF WAGES

BY

J. R. HICKS, M.A., B.LITT.

LECTURER IN ECONOMICS, LONDON SCHOOL OF ECONOMICS

MACMILLAN AND CO., LIMITED  
ST. MARTIN'S STREET, LONDON

1932

**COPYRIGHT**

**PRINTED IN GREAT BRITAIN**

## PREFACE

THE task which is attempted in this book is a restatement of the theory of wages in a form which shall be reasonably abreast of modern economic knowledge. It is thus an undertaking which seems to need little apology. Periodical reconsiderations of each of the main departments of economic theory are an important part of the duty of economists; since, for one thing, one field is often illuminated by advances which have been made in others, and for another, the events of contemporary history make it necessary to examine possibilities, of which earlier writers may have been aware, but which they naturally regarded as not worthy of special attention. Such a reconsideration of wage theory seems long overdue. For the most recent comprehensive statements of a positive theory of wages in English—of anything more than an elementary character—are now thirty or forty years old. We have to go back for them to Marshall's *Principles* and Clark's *Distribution of Wealth*. Since that time important work on the subject has indeed been done, but it is nearly all special studies; even Professor Pigou's treatment of Labour, in the *Economics of Welfare*, ought probably so to be reckoned for our purposes. Of these works much use has been made in the following pages: to them

this book owes a great debt; but they have not removed the need for some undertaking like the present.

The historical fact which dominates the wage-history of the present century—both in Britain and in other countries—is the growth of Trade Union power and the development of State Regulation of Wages. This fact, which is due to a complex of causes, and which could not have been wholly foreseen by economists thirty years ago, alters very considerably the range of problems with which we have to deal. It might even appear at first sight as if it ought to change the whole structure of our theory—that we ought to treat the regulation of wages as the normal case, and take its consideration first. But this course does not prove satisfactory. The same forces which determine wages in a free market are still present under regulation; they only work rather differently. It is therefore best for us to begin in the traditional manner with the determination of wages under competition; though at a later stage we must examine regulation in more detail than the traditional theories do.

By proceeding in this way, we secure the great advantage of being able to build directly upon familiar doctrines; and we naturally start with a consideration of that principle which was regarded by the economists of Marshall's generation as the basis of their theory of wages—the principle of Marginal Productivity. The validity and the importance of this principle we shall see no reason to question; but its very importance has one awkward consequence. For we shall get into endless difficulties if we allow any obscurity about so

essential a principle to persist; and it is unfortunately the case that its original propounders did leave it—or at least its application—in some obscurity. We are therefore faced at the start with the hard task of trying to make clear something which Marshall and J. B. Clark did not make altogether clear; and we cannot hope to do this if we shirk difficulties. The reader must therefore be asked to follow Chapter I. with attention and some patience; but he may be assured that relatively smooth waters lie beyond.

One very important aspect of the theory of wages it has unfortunately been necessary to leave undiscussed—the relation of wages to general industrial fluctuations or trade cycles. In this branch of economics recent years have certainly seen striking advances; it does seem probable that in a few years' time we shall possess the main lines of an established theory of fluctuations; but that time is not yet. Thus to discuss trade fluctuations from any angle is hazardous, since nothing useful can be said unless one is prepared to take sides on the critical issues. And most of these lie altogether outside the theory of wages, although they have a direct bearing upon it.

Thus I must confine myself here to stating a personal opinion. It is my own belief that some parts of this book—particularly the last chapters—have considerable relevance to the theory of fluctuations, although they are not stated with that particular reference. But I shall make no attempt to defend this view at present.

I have to acknowledge a great debt of gratitude for

the help I have received in the preparation of this book. I work in an atmosphere which is very conducive to the making of such studies as the present, and I know what I owe to it. Professor Lionel Robbins, Professor Arnold Plant, and Dr. F. C. Benham, of the London School of Economics, and also Professor W. H. Hutt of the University of Cape Town, have all read the whole, or large parts, of my manuscript, and made most valuable suggestions—which I fear I have not always accepted. I have also to acknowledge the valuable criticisms which, at more than one stage in the development of my ideas, I have received from Mr. D. H. Robertson; and the generous assistance of Professor F. A. Hayek, in connection with those difficult points where the present enquiry begins to abut on the theory of Capital.



# CONTENTS

## PART I.—THE FREE MARKET

### CHAPTER I. MARGINAL PRODUCTIVITY AND THE DEMAND FOR LABOUR

	PAGE
1. The problems of wages in the Free Market are classified into problems of (a) demand, (b) supply, and (c) their interaction . . . . .	1
Methods are proposed whereby consideration of (b) . . . . .	2
and (c) may be postponed . . . . .	4
The conception of equilibrium . . . . .	6
2. The conditions of equilibrium: (a) Equalisation of wages . . . . .	7
(b) Marginal Productivity . . . . .	8
The conventional proof of the marginal productivity principle . . . . .	8
Another way of looking at it: output and method of production . . . . .	10
Marshall's "net productivity" principle . . . . .	12
Variation of methods leads to marginal productivity . . . . .	14
3. The real labour market is not in equilibrium . . . . .	18
and the adjustment of wages is not instantaneous . . . . .	19
Demand for labour becomes more elastic when time is allowed . . . . .	20

### CHAPTER II. CONTINUITY AND INDIVIDUAL DIFFERENCE

Current objections to marginal productivity . . . . .	23
1. Exact equality of wages and marginal products is apparently prevented by the indivisibility of the human unit . . . . .	24
but this indeterminateness is greatly reduced by the individual differences of employers . . . . .	27
2. The individual differences of labourers . . . . .	27
give us a new set of conditions of equilibrium . . . . .	29
The apparent "range of indeterminateness" . . . . .	32
is negligible save for exceptional men . . . . .	33

	PAGE
3. A man's efficiency depends partly on the efficiency of his employer . . . . .	34
The "Gospel of High Wages" . . . . .	36
4. Exact adjustment of wages to individual differences is impossible . . . . .	37
but there are devices for securing a rough approximation . . . . .	38
The standard rate, promotion, piece rates . . . . .	38

### CHAPTER III. UNEMPLOYMENT

Effect of unemployment on wages depends on the cause of the unemployment . . . . .	42
1. "Normal" unemployment . . . . .	42
The standard rate makes it difficult for subnormal men to secure regular employment . . . . .	43
The "unemployable" . . . . .	44
The unemployed through variation in activity between firms . . . . .	45
The possibility of short-period exploitation . . . . .	46
2. Casual unemployment . . . . .	47
very unlikely to affect wages . . . . .	48
3. With seasonal fluctuations . . . . .	49
and other kinds which are foreseen . . . . .	51
the effect on wages is the result of conscious policy . . . . .	51
Wage policy of employers in depression a question of circulating capital . . . . .	53
affected by foresight . . . . .	55
"Good" and "bad" employers . . . . .	55
The rigidity of wages . . . . .	57

### CHAPTER IV. THE WORKING OF COMPETITION

1. Dynamic problems—the cost of change . . . . .	58
and the fact of foresight . . . . .	59
2. The theory of "bargaining advantage" . . . . .	61
needs to be corrected for foresight in the case of repeated contracts . . . . .	62
Casual and "regular" trades . . . . .	63
In a casual market with unspecialised employers . . . . .	64
bargaining advantage is on the side of labour . . . . .	65
3. Great competitiveness in an ordinary casual market . . . . .	66
4. "Regularity" makes for rigidity of wage rates . . . . .	69
How wages rise in a regular trade . . . . .	71

## CONTENTS

xi

	PAGE
5. Local differences in wages . . . . .	74
are partly due to the slow transmission of wage changes . . . . .	76
Occupational differences in wages . . . . .	76
also subject to slow adjustment by movement of juveniles . . . . .	77
and to some extent by movement of adults . . . . .	78
"Fair" wages . . . . .	80
6. Exploitation of labour . . . . .	81
diminished by mobility and foresight . . . . .	83
Exploitation by monopolist employer improbable . . . . .	84
State employment . . . . .	85
7. The wage system . . . . .	86

### CHAPTER V. INDIVIDUAL SUPPLY OF LABOUR

1. Variations in quantity of labour supplied per head . . . . .	89
"Quantity of labour" must be defined objectively . . . . .	90
and this can only be done by making reference to prices . . . . .	91
Limited validity of this definition . . . . .	91
2. Reactions of wages on efficiency through (a) ability, (b) willingness to work . . . . .	93
Reactions through ability complicate transferences of labour from declining trades . . . . .	95
but are generally very favourable to economic progress . . . . .	95
"Diminishing returns" of efficiency . . . . .	96
3. A change in wages may affect willingness to work in either direction . . . . .	97
The possibility of increased effort due to reduced wages . . . . .	99
may have serious consequences in trade depressions . . . . .	101
4. Variation of hours. The "output optimum" . . . . .	103
Possibility of competition failing to reduce excessive hours . . . . .	106
The "conditions of labour" . . . . .	110

### CHAPTER VI. DISTRIBUTION AND ECONOMIC PROGRESS

1. Absolute and relative shares of the social income . . . . .	112
Limitations of the following analysis . . . . .	113
2. The effects of an increased supply of one factor of production can be stated in three propositions . . . . .	114
can be stated in three propositions . . . . .	115
✓The "elasticity of substitution" . . . . .	117
Methods by which substitution can take place . . . . .	120
3. Inventions must increase the social dividend . . . . .	121
Labour-saving and capital-saving inventions . . . . .	121

	PAGE
Professor Pigou's definition . . . . .	122
slightly modified . . . . .	123
Another classification: autonomous and induced inventions . . . . .	125
Two kinds of induced inventions . . . . .	125
4. Two extreme cases: invention—(a) lethargic, (b) very active . . . . .	127
Possibility but improbability of inventions reducing real income of labour . . . . .	128
Much greater probability of labour's relative share being reduced . . . . .	130
5. Historical application . . . . .	130
Tendency towards a fall in the elasticity of substitution . . . . .	132
6. Application to the theory of money wages . . . . .	133
A possible dilemma . . . . .	134

## PART II.—THE REGULATION OF WAGES

### CHAPTER VII. THE THEORY OF INDUSTRIAL DISPUTES

1. Origin of labour combination . . . . .	136
in customs and rights . . . . .	137
and the idea of fair wages . . . . .	138
2. The threat of a strike sets before employers a choice . . . . .	140
which is considered . . . . .	141
and illustrated diagrammatically . . . . .	143
The dangers of bluff . . . . .	144
increased by presumption in favour of negotiation as a more promising method . . . . .	145
Conciliation . . . . .	147
Arbitration . . . . .	148
Compulsory Arbitration . . . . .	151
3. The conditions which favour Union success . . . . .	152
expressed partly in Union's resistance curve . . . . .	153
and partly in employer's concession curve . . . . .	154
Good trade favours the Union . . . . .	155
and inelasticity of demand for the product . . . . .	156
The possibility of substitution . . . . .	157

### CHAPTER VIII. THE GROWTH OF TRADE UNION POWER

Economic issues in Trade Union history . . . . .	159
1. Limited power of local clubs . . . . .	160
due to blacklegs and competition of outside firms . . . . .	161

## CONTENTS

xiii

2.	Extension of the area of organisation . . . . .	163
	brings gains through the "piecemeal" method . . . . .	163
	The "common rule" . . . . .	164
	and limitation of entry to the trade . . . . .	165
3.	Rise of employers' associations . . . . .	166
	follows the establishment of standards . . . . .	167
	Central employers' associations function as reserves . . . . .	169
4.	Rationalisation of wages after the war . . . . .	171
	greatly increases Union strength . . . . .	172
	"Sheltered" and "unsheltered" trades . . . . .	173
5.	Union influence on wages still more evident after the return to gold . . . . .	175
	The effect of unemployment insurance . . . . .	177

### CHAPTER IX. WAGE-REGULATION AND UNEMPLOYMENT

1.	The deduction from marginal productivity an inadequate de- scription . . . . .	179
	Partial control . . . . .	180
	and complete control of wages throughout a whole community	180
2.	Regulation of wages in a single firm . . . . .	181
	Regulation throughout an industry . . . . .	183
	The apparent lack of connection between wages and unemploy- ment . . . . .	184
3.	Regulation throughout a stationary closed community . . . . .	185
	Shifting of capital between industries . . . . .	187
	And change of methods . . . . .	188
	Distribution of unemployment between industries . . . . .	189
4.	Apparent conflict with observed fact . . . . .	190
	explained by the destruction of capital . . . . .	192
	Ways in which this takes place . . . . .	193
	and its consequences on unemployment . . . . .	194
	The situation in a progressive community . . . . .	195
	Unemployment benefit . . . . .	195

### CHAPTER X. FURTHER CONSEQUENCES OF WAGE-REGULATION

1.	Raising of wages without capital loss finally results in a deter- minate amount of unemployment . . . . .	198
2.	If there is loss of capital, number unemployed will be greater . . . . .	199
	but equilibrium will also be harder to reach . . . . .	199
	Reasons for this . . . . .	200
	False rationalisation . . . . .	203

	PAGE
3. Beneficence of inventions . . . . .	203
diminished by the high wages . . . . .	204
4. The accumulation of capital in a progressive economy . . . . .	205
may outweigh the tendencies to decline . . . . .	206
Changes in population . . . . .	207
5. Effects of high wages on efficiency of labour . . . . .	207
have to be large . . . . .	208
and rapid, if much is to be hoped from them . . . . .	209
6. The regulation of money wages not thoroughly considered . . . . .	211
7. Regulation of wages within a national area . . . . .	213
has effects more serious than in a closed community . . . . .	213
Ultimate difficulty of maintaining Gold Standard . . . . .	215

#### CHAPTER XI. HOURS AND CONDITIONS

1. Trade Union pressure easily reduces hours below the output optimum . . . . .	217
2. Reductions beyond the output optimum: (a) in the single firm . . . . .	218
(b) in a whole industry . . . . .	219
Possibility of shifting burden in this case . . . . .	220
(c) in a closed community. Real and money wages . . . . .	221
3. Regulation of hours in a single country . . . . .	224
International regulation of hours . . . . .	225
4. Regulation of the conditions of labour . . . . .	227
5. Conclusion . . . . .	229

#### APPENDIX

(i.) The <i>Co-ordination of the Laws of Distribution</i> . . . . .	233
(ii.) Increasing returns . . . . .	239
(iii.) The elasticity of derived demand . . . . .	241
(iv.) The distribution of the national dividend . . . . .	246

## APPENDIX

THE principal object of this appendix is the construction of a mathematical proof of the conclusions about absolute and relative shares in the Social Dividend put forward in Chapter VI; but since the chief value of such a mathematical proof must lie in the disclosure of the exact assumptions and the precise limitations under which the propositions are true, it is convenient to begin with a consideration of certain problems whose connection with these propositions may appear at first sight a little remote.

*Euler's Theorem &c.*

### ✓ (i.) THE CO-ORDINATION OF THE LAWS OF DISTRIBUTION

Ever since the early days of the marginal productivity theory in the eighteen-nineties, the mathematical application of the theory has been greatly hampered by the difficulty which was raised by P. H. Wicksteed, in his essay, "The Co-ordination of the Laws of Distribution" (1894). (If each factor is paid according to its marginal product, is the total product exhausted, or is there a surplus or deficit?) Clearly (it is most consonant with the conditions of equilibrium that each factor should be remunerated according to its marginal product, including the factor which "employs" the others, and takes the surplus for its share. But will there be enough residue to pay the employing factor its marginal product?)

The solution which Wicksteed himself offered to his own problem is unsatisfactory, as, indeed, he admitted on subsequent occasions.<sup>1</sup> But it is not true, as most English and American economists seem still to imagine, that the problem remained unsolved. Within a few months of the publication of

<sup>1</sup> *Common Sense of Political Economy*, p. 373. The argument in the text of the *Common Sense*, while perfectly valid, does not meet the mathematical difficulty. See also Robbins, "The Economic Works of Philip Wicksteed" (*Economica*, November, 1930).

Wicksteed's Essay, Léon Walras put forward a solution which is altogether free from the objections to which Wicksteed's own solution is liable.<sup>1</sup> But, unfortunately, Walras expressed himself in so crabbed and obscure a manner that it is doubtful if he conveyed his point to anyone who did not possess some further assistance. Anyone who knows the answer can see that Walras has got it; but anyone who does not must find it almost impossible to get it from Walras.

A perfectly intelligible solution did, however, appear a few years later in the *Vorlesungen* of Knut Wicksell.<sup>2</sup> With Wicksell's aid it is not difficult to clear up this matter; after which we shall be in a position to proceed with our principal enquiry.

The first thing on which we have to be clear, if we want to see our way towards a solution of this question, is that we are concerned solely with the internal coherence of the conditions of economic equilibrium. Our problem is purely one of the conditions of equilibrium, and therefore it is extremely unwise to complicate our discussions with the consideration of phenomena which only arise in the real world because the economic system is not in equilibrium; and among these fall the greater part of the activities of enterprise and management. If we persist in thinking of the factor which receives the residue as the "entrepreneur", we shall get into endless difficulties; but fortunately, without any serious departure from reality, we can think of our typical firm as a Joint Stock Company, and suppose the residue to fall to the capitalist as capitalist, management (so far as management is required) being hired like labour of other grades. Or, alternatively, we can follow Wicksell's example, and suppose the landlord or the labourer to take the residue, hiring other factors.

Once we adopt this assumption, the most ordinary non-mathematical analysis shows that every factor must get its marginal product. For every *hired* factor must get its marginal

<sup>1</sup> "Note sur la réfutation de la Théorie anglaise du fermage de M. Wicksteed." This was republished as an appendix to the third edition of Walras' *Eléments* (1896). It is omitted in subsequent editions.

<sup>2</sup> Vol. i., pp. 186-191.



product, since otherwise the demand for it would expand or contract; and every *unhired* factor (which is "acting as entrepreneur") must get its marginal product, since if it got less, its owners would prefer to hire it out; and if it got more, some would be transferred from the hired to the unhired class.

This is a perfectly satisfactory line of argument, and it is evidently reasoning of this kind which has generally persuaded non-mathematical economists (for example, J. B. Clark and his followers) that the "adding-up" difficulty is a delusion. And we shall see that they are right.

The trouble is that the alternative mathematical line of approach did not appear to lead to the same conclusion.

Let  $x$  = the amount of product, and  $a, b, c, \dots$  the quantities of factors required to make that product  $x$ . In order that the marginal productivity law should be fulfilled, the share of the product which goes to the factor  $a$  must be  $a \frac{\partial x}{\partial a}$ , and similarly for the other factors. If the product is to be exactly divided among the factors, leaving no residue, positive or negative, then

$$x = a \frac{\partial x}{\partial a} + b \frac{\partial x}{\partial b} + \dots$$

Wicksteed's explanation was based upon the well-known mathematical proposition, due to Euler, that if  $x$  is a homogeneous function of the first degree in  $a, b, c, \dots$  so that it can be written

$$x = a f\left(\frac{b}{a}, \frac{c}{a}, \dots\right)$$

this relation

$$x = a \frac{\partial x}{\partial a} + b \frac{\partial x}{\partial b} + \dots$$

will always be satisfied.

It was this that drew the scathing remark of Edgeworth: "There is a magnificence in this generalisation which recalls the youth of philosophy. Justice is a perfect cube, said the ancient sage; and rational conduct is a homogeneous function, adds the modern savant."<sup>1</sup>

<sup>1</sup> "Theory of Distribution," in *Papers*, vol. i., p. 31.

But when it is expressed in economic language, the Wicksteed-Euler proposition appears much less ridiculous than it seems to have appeared to Edgeworth. It means simply that there will be no residue, positive or negative, if the commodity in question is produced under conditions of "constant returns"—using that ill-treated expression in yet another unfamiliar, but nevertheless highly convenient, sense. The production function will have the requisite form if a proportional increase in *all* the quantities of factors employed will increase the quantity of product in the same proportion in which the factors were increased; that is to say, if the amounts of factors required per unit of product (the "coefficients of production") are independent of the amount of product.

Put in this way, the condition appears much less startling; yet it is doubtful if it can be considered to be generally satisfied. So long as all the factors are increased in the same proportion, the general condition of diminishing returns—the disproportionate increase of some factors—is absent. But the condition of increasing returns—economies of specialisation and co-operation due to size—may be present. It does seem possible that "increasing returns" (used here in a special sense, but one that has many of the implications of the ordinary meaning) may come in to upset the marginal productivity theory, as they are inclined to upset, unless we are very careful, so many economic generalisations.

We may now turn to the solution of Walras and Wicksell.

We are concerned here solely with one part of the general equilibrium system, the conditions that a particular firm should be in equilibrium. We assume perfect competition, both in the market where the firm sells its products, and in the market where it buys its factors. Thus, so far as the action of this particular firm is concerned, we can assume all the prices with which it deals to be given; for the influence of its individual action on prices, whether of product or of factors, will be negligible. In order that the firm should be in equilibrium, two conditions have to be satisfied: (1) the unit cost of production of

its product must be a minimum; (2) that unit cost must equal the selling price of the product. The first condition must be fulfilled, since otherwise the owners of that factor which is "acting as entrepreneur" could increase their profits by a change in methods. The second condition must be fulfilled, since otherwise the owners of that factor would be receiving a return either higher or lower than was being earned by similar services elsewhere in the market, and someone would therefore have an incentive to act differently. In order to minimise its costs of production, the firm can vary indefinitely the quantities of factors which it uses, and therefore, of course, the quantity of product it turns out. The production function (the relation between the quantities of factors and the quantity of product) is naturally given by technical considerations.<sup>1</sup> The coefficients of production do not only have to be chosen so that the unit cost of production for a given output is a minimum; the output has also to be chosen so that the unit cost of production is a minimum.

We have then

$$x = f(a, b, c, \dots) \text{ (production function).}$$

$$\text{Total cost of production} = ap_a + bp_b + \dots$$

where  $p_a, p_b$  are the prices of the factors.

$$\text{Cost of production per unit} = \pi_x = \frac{1}{x} (ap_a + bp_b + \dots) \text{---(1)}$$

$\pi_x = p_x$ , i.e. cost of production = selling price.

In order that  $\pi_x$  should be a minimum

$$\frac{\partial \pi_x}{\partial a}, \frac{\partial \pi_x}{\partial b}, \dots \text{ must all} = 0.$$

$$\begin{aligned} \text{Now } \frac{\partial \pi_x}{\partial a} &= \frac{\partial}{\partial a} \left\{ \frac{1}{x} (ap_a + bp_b + \dots) \right\} \\ &= \frac{1}{x} p_a - \frac{1}{x^2} \frac{\partial x}{\partial a} (ap_a + bp_b + \dots) \end{aligned}$$

<sup>1</sup> Once we grant the universality of substitution, as we have seen cause to do, as a result of the discussions of Chapter I., the existence of a production function follows necessarily.

$$\begin{aligned}
 &= \frac{1}{x} p_a - \frac{1}{x^2} \frac{\partial x}{\partial a} \cdot x \pi_x \\
 &= \frac{1}{x} \left( p_a - \pi_x \frac{\partial x}{\partial a} \right).
 \end{aligned}$$

Then, since  $\frac{\partial \pi_x}{\partial a} = 0$ ,  $p_a = \pi_x \frac{\partial x}{\partial a} = p_x \frac{\partial x}{\partial a}$ , and similarly for the other factors.

This is the marginal productivity law, and by substituting in (1) we have

$$x = a \frac{\partial x}{\partial a} + b \frac{\partial x}{\partial b} + \dots$$

proved independently of any assumption about "constant returns".

The explanation which lies behind this proof lies in the essential hypothesis that each firm is producing at that scale of output which makes its unit cost a minimum. If, as before, we assume that the prices of the factors are constant, and if we assume further that the proportions in which the factors are employed remain unchanged as output varies, we can construct a (very specialised) cost curve for the firm, giving the cost per unit of producing various outputs. Wicksteed thought he had proved that it was a necessary condition for the truth of the marginal productivity theory that this curve should be a horizontal straight line. Walras and Wicksell showed that it was only necessary that the curve should have a minimum point, and that in equilibrium output must be at that point.

Now it is clear that in the neighbourhood of the minimum point, where the tangent to the curve must be horizontal, the curve will approximate very closely to the straight line. It is not surprising that, at this point, Wicksteed's condition should be satisfied. Where Wicksteed went wrong was in his assumption that he could argue from the shape of the curve at one particular point to the general shape of the curve.

Wicksteed's difficulty can therefore be overcome by substituting for his untenable condition of "constant returns" the condition of "minimum cost" which appears, on the surface

at least, more in keeping with the fundamental assumptions on which it is reasonable to base an equilibrium theory. But, as Mr. Sraffa has pointed out,<sup>1</sup> the condition of minimum cost is not without its difficulties. We are excluded from the assumption of diminishing returns in the usual sense; but if we assume no tendency to diminishing returns—that a simultaneous increase in all the factors in the same proportion will never increase the product less than proportionately—then either competitive equilibrium is impossible (which will be the case if increasing returns go on indefinitely) or alternatively the distribution of output among the different firms in an industry will be altogether indeterminate (if increasing returns give way to constant returns). Neither of these conclusions is welcome; but if we are to avoid them, we are driven to assume that “technical diseconomies” will, after a certain point, induce diminishing returns. There can be little question that in fact there is generally a limit to the extent to which any firm can grow under given conditions, independently of the limitation of the market. But a doubt must remain how far the limitations which we do find in experience have not been assumed away on the level of abstraction on which we are now working.

Further consideration of this point would lead us too far into the more arid regions of higher general theory; its relevance to the theory of distribution is remote.

#### (ii.) INCREASING RETURNS

The marginal product which measures the actual return which a factor of production must get in a state of equilibrium, is the addition which is made to the product of a firm when a small unit is added to the supply of the factor available to that firm, when the organisation of the firm is adjusted to the new supply (so that it is used in the most economical way), but when the rest of the organisation of industry, including the general system of prices, remains unchanged. Now there is no

<sup>1</sup> “The Laws of Returns under Competitive Conditions” (*Econ. Jour.*, 1926).

reason why this increment should be the same as the increment of production which would accrue if the additional unit were made available to the whole of industry, and the whole organisation of industry, including the general price-system, were adjusted to the new supply.)

If all the firms were operating in accordance with Wicksteed's law, under conditions of "constant cost"; and if we leave out of account the fact that the allocation of the increase in resources to one firm only would mean an uneconomic distribution of production; then there can be no question that these two "marginal products" would be equal. But in fact an increase in the supply of one factor generally involves a complicated redistribution of production between firms and between industries, and in consequence of these changes it is quite likely that the marginal product of a factor in the second sense will be greater than the marginal product in the first sense. The division of labour progresses as the supply of the factors increases, and the advantages of the division of labour are gained as much, or more, through an increase in specialisation between firms and between industries, as through an increase in the size of firms.<sup>1</sup>)

Thus we have to distinguish between the "private" marginal product, which does, in equilibrium, equal the wage of labour; and the "social" marginal product, which results from an increase in the supply of labour, when we suppose that increase to have worked out its full effect. And in general it is safe to assume that the latter will exceed the former.)

This divergence has awkward consequences for the application of the general marginal productivity theory. (If we can assume "constant returns" and a consequent equality of "social" and "private" marginal products, it is possible to deduce certain not uninteresting results about the effect of increases in the factors on the distribution of the product. But in so far as we have to allow for increasing returns, these re-

<sup>1</sup> Cf. Allyn Young, "Increasing Returns and Economic Progress" (*Econ. Jour.*, 1928); Shove, "Varying Costs and Marginal Net Products" (*Econ. Jour.*, 1928).

sults are surrounded by a margin of doubt. Yet it does not seem probable that the divergence would be very great)

Nevertheless, the reader is asked to bear in mind that the exact conclusions of the following pages depend for their strict validity upon the assumption of "constant returns" in the Wicksteed-Wicksell sense; and thus upon the identity of "private" and "social" marginal products.<sup>1</sup>

### (iii.) THE ELASTICITY OF DERIVED DEMAND

In examining the effects on Distribution of changes in the supply of the factors of production, it is convenient to begin with the special case of a change in the supply of a factor which is specialised to some particular purpose, and can only be used in one industry. The problem which is then raised within that industry is then simply a problem of the elasticity of derived demand—the problem which was studied by Marshall in his well-known example of plasterers' wages. Marshall gave four rules for the things on which the elasticity of derived demand depends; and in their discussions of this matter, economists have generally been content to use Marshall's rules, without making them the subject of any further investigation. These rules are an excellent example of the convenience of the elasticity concept, in enabling essentially mathematical notions to be used in formally non-mathematical arguments. But such procedure, although convenient, is dangerous; it will enable us to proceed more securely, if, instead of merely accepting Marshall's conclusions, we examine their mathematical foundation.

Marshall himself no doubt derived his rules from mathematics; Note XV. in the mathematical appendix to the *Prin-*

<sup>1</sup> Of the two rules about absolute and relative shares in the Dividend put forward in Chapter VI. and to whose consideration this discussion is ultimately leading, it seems extremely improbable that the rule about absolute shares could possibly be affected by increasing returns. The rule about relative shares, on the other hand, almost certainly must be affected to some extent, although it is unlikely that the difference would be very serious unless it could be shown that an increase in one particular factor would be much more likely to call forth a strong development of those tendencies making for increasing returns than an increase in the other.

*ciples* is enough to assure us of that. But he does not there give the full mathematical derivation; (he confines himself to a simplified case, that in which the proportions of factors employed (the "coefficients of production") remain constant.) A more extended enquiry, he assures us, would lead to "substantially the same results." But we may as well see for ourselves.

The four rules (in Professor Pigou's more convenient formulation) are:

I. "The demand for anything is likely to be more elastic, the more readily substitutes for that thing can be obtained."

II. "The demand for anything is likely to be less elastic, the less important is the part played by the cost of that thing in the total cost of some other thing, in the production of which it is employed."

III. "The demand for anything is likely to be more elastic, the more elastic is the supply of co-operant agents of production."

IV. "The demand for anything is likely to be more elastic, the more elastic is the demand for any further thing which it contributes to produce."<sup>1</sup>

We may now proceed to our mathematical enquiry.

A product is being made by the co-operation of two factors,  $a$  and  $b$ , which are remunerated according to the value of their marginal products. Let  $x$  be the quantity of product ( $x$  is thus a function of  $a$  and  $b$ ),  $p_x$  its price;  $p_a$  and  $p_b$  the prices of the factors  $a$  and  $b$  respectively. If  $\eta$  is the elasticity of demand for the product, and  $e$  the elasticity of supply of  $b$ , how is  $\lambda$ , the elasticity of demand for  $a$ , determined?

We have  $p_a = p_x \frac{\partial x}{\partial a}$ ,  $p_b = p_x \frac{\partial x}{\partial b}$  (marginal products).

$$\text{Also} \quad \eta = - \frac{p_x}{x} \frac{dx}{dp_x}, \quad e = \frac{p_b}{b} \frac{db}{dp_b}, \quad \lambda = - \frac{p_a}{a} \frac{da}{dp_a}$$

<sup>1</sup> Marshall, *Principles*, bk. v., ch. vi.; Pigou, *Economics of Welfare*, bk. iv., ch. v.



Since the total expenditure of the firm equals total receipts,

$$p_x x = p_a a + p_b b.$$

This can also be written

$$x = a \frac{\partial x}{\partial a} + b \frac{\partial x}{\partial b}.$$

Since we are assuming "constant returns" we can treat this last equation as an identity, and differentiate it partially with respect to  $b$ ,

$$\frac{\partial x}{\partial b} = a \frac{\partial^2 x}{\partial a \partial b} + b \frac{\partial^2 x}{\partial b^2} + \frac{\partial x}{\partial b}.$$

$$\therefore b \frac{\partial^2 x}{\partial b^2} = -a \frac{\partial^2 x}{\partial a \partial b} \dots \dots \dots (1).$$

Further, the total differential of  $x$ ,

$$dx = \frac{\partial x}{\partial a} da + \frac{\partial x}{\partial b} db$$

$$\therefore p_x dx = p_a da + p_b db \dots \dots \dots (2).$$

Since the condition of equality of receipts and expenditure must still be satisfied after we have made our small change in  $a$ ,

$$p_x dx + x dp_x = p_a da + a dp_a + p_b db + b dp_b.$$

But from (2) this becomes

$$x dp_x = a dp_a + b dp_b.$$

And by the elasticity formulæ,

$$\frac{p_x dx}{x} = \frac{p_a da}{a} - \frac{p_b db}{b} \dots \dots \dots (3).$$

Now the change in  $b$ , which results from the change in  $a$  as independent variable,

$$= db = \frac{be}{p_b} dp_b = \frac{be}{p_b} d \left( p_x \frac{\partial x}{\partial b} \right).$$

By expansion and application of (1), this becomes

$$db = \frac{be}{p_b} \left\{ - \frac{p_b dx}{x \eta} + p_x \frac{\partial^2 x}{\partial a \partial b} \left( da - \frac{a}{b} db \right) \right\}.$$

Now write  $\sigma = \frac{p_a p_b}{p_x^2 x \frac{\partial^2 x}{\partial a \partial b}}$  and  $\kappa = \frac{p_a a}{p_x x}$ , and simplify.

$$\text{Then } \frac{p_x dx}{\eta} = \frac{p_a da}{\sigma} - \frac{p_b db}{1 - \kappa} \left( \frac{1}{e} + \frac{\kappa}{\sigma} \right) \dots \dots (4).$$

Eliminating  $dx$ ,  $da$ ,  $db$  between (2), (3) and (4), we get

$$\frac{\lambda - \sigma}{\eta - \lambda} = \frac{\kappa}{1 - \kappa} \frac{e + \sigma}{e + \eta}$$

$$\text{or } \lambda = \frac{\sigma(\eta + e) + \kappa e(\eta - \sigma)}{\eta + e - \kappa(\eta - \sigma)}.$$

This gives us a value for the elasticity of demand for  $a$ , in terms of  $\eta$ ,  $e$ ,  $\kappa$ , and  $\sigma$ .<sup>1</sup>

These are in fact the four Marshallian variables.  $\kappa$ ,  $e$ ,  $\eta$  correspond to the rules (II), (III), and (IV) quoted above.  $\sigma$  is a suitable measure for (I); it is the "elasticity of substitution".

Its principal component,  $\frac{\partial^2 x}{\partial a \partial b}$ , gives the rate of change of the marginal product of one factor for a change in the other factor.

If  $\frac{\partial^2 x}{\partial a \partial b}$  is infinite,  $\sigma = 0$ , and there is no substitution possible at all; the coefficients of production are strictly proportional. If  $\frac{\partial^2 x}{\partial a \partial b} = 0$ ,  $\sigma$  is infinite, the factors are perfectly rival or their

use is indifferent. If we had a third factor, or more, then  $\frac{\partial^2 x}{\partial a \partial b}$  might be negative, and the factors would be rival in the more ordinary sense of the term; an increase in one would diminish the marginal product of the other. But with only two factors, and under the assumption that there can be no "diminishing returns" to all the factors together, this is impossible.

But although  $\frac{\partial^2 x}{\partial a \partial b}$  is thus to some extent a test of the amount of substitution possible, it is not a suitable measure of

<sup>1</sup> When  $\sigma=0$ , this reduces to Marshall's formula (*Principles, Mathematical Appendix, Note XV.*).

the "elasticity of substitution". For its magnitude depends on the units in which  $x$ ,  $a$ , and  $b$  are measured. Just as we have to multiply  $\frac{dx}{dp}$  by  $\frac{p}{x}$  in order to get the *elasticity* of demand, so we must multiply  $\frac{\partial^2 x}{\partial a \partial b}$  by a further factor in order to get the elasticity of substitution.  $\frac{p_x^2 x}{p_a p_b}$  is a suitable multiplier. But I have taken the reciprocal of this expression, in order to have a measure increasing with the facility of substitution.

$$\text{Since } \frac{p_a p_b}{p_x^2 x \frac{\partial^2 x}{\partial a \partial b}} = \frac{\frac{\partial x}{\partial a} \frac{\partial x}{\partial b}}{x \frac{\partial^2 x}{\partial a \partial b}}, \sigma \text{ could also have been written}$$

in this latter form.

So far we have only shown that the elasticity of derived demand depends upon Marshall's four variables. We have still to examine how it moves with the four variables—i.e., to test the rules.

Taking the formula for  $\lambda$ , and differentiating it partially by each in turn of the four variables on which it depends, we get:

- (1)  $\frac{\partial \lambda}{\partial \sigma} = (1 - \kappa) \times \text{a square.}$
- (2)  $\frac{\partial \lambda}{\partial \kappa} = (\eta - \sigma)(\eta + e)(e + \sigma) \times \text{a square.}$
- (3)  $\frac{\partial \lambda}{\partial e} = \kappa(1 - \kappa) \times \text{a square.}$
- (4)  $\frac{\partial \lambda}{\partial \eta} = \kappa \times \text{a square.}$

The first, third, and fourth of these expressions are always positive. The first, third, and fourth rules are universally true. But the second rule is not universally true. Even if we concern ourselves only with cases where  $e$  is positive ( $\eta$  and  $\sigma$  must be positive) the second rule is only true so long as  $\eta > \sigma$ ; so long as the elasticity of demand for the final product is greater than

the elasticity of substitution. Of course, in the usual cases taken for illustration of this rule, the condition for its validity is fulfilled. It is supposed that the demand for the product is fairly elastic, while substitution is difficult. But if technical change is easy, while the product has an inelastic demand, the rule works the other way. For example, a factor may find it easier to benefit itself by a restriction in supply if it plays a large part in the process of production than if it plays a small part. *It is "important to be unimportant" only when the consumer can substitute more easily than the entrepreneur.* Further even if  $\eta > \sigma$ , but if the difference is small, the importance of this second rule will be negligible.

#### (iv.) THE DISTRIBUTION OF THE NATIONAL DIVIDEND

The last part of our enquiry—the application of these results to the wider problem discussed in Chapter VI.—now presents little difficulty. We are now concerned no longer with the money demand for a factor of production engaged in the making of a particular product, but with the real demand for a general group of factors of the traditional kind "labour" or "capital". To this we can still apply our formula, but in a considerably simplified form. Since the total product of a closed community does not need to be sold outside that community, we can write  $p_x = 1$ , and  $\eta = \text{infinity}$ . The elasticity of demand for one of these groups of factors is therefore given by the following formula, derived from the formula of the last section:

$$\lambda = \frac{\sigma + \kappa e}{1 - \kappa}$$

From this formula<sup>1</sup> the second and third of the rules given above in Chapter VI. can be directly derived.

<sup>1</sup> It may be interesting to illustrate the significance of this formula by an arithmetical example. If we suppose  $\sigma=1$ , the elasticity of supply of the factors to be zero, and the dividend to be divided between labour and capital in the proportions of 75 per cent. to 25 per cent., then the elasticity of demand for labour (measured in terms of real goods) will be 4; and the elasticity of demand for capital  $1\frac{1}{3}$ .

For

$$\frac{d}{da} (bp_b) = \frac{p_a(1+e)}{\lambda}$$

$$a \frac{d}{da} \left( \frac{ap_a}{x} \right) = \frac{\kappa(\sigma-1)}{\lambda}$$

The rules are therefore valid so long as  $\lambda$  is positive; that is to say, in practically every conceivable case. (It was shown above on p. 98, footnote, that  $e$  may always be taken to be greater than  $-1$ ).

It only remains for us now to make a few remarks on the reason which led Dr. Dalton<sup>1</sup> to arrive at a conclusion so different from that which is evidently to be derived from the last of the above formulæ. Dr. Dalton constructed a formula giving a test for the conditions under which an increase in  $a$  would increase its relative share. In our notation, his formula is  $\lambda > \frac{1}{1-\kappa}$ . It is evident that this formula is correct, so long as  $e$  can be neglected. He then proceeded to apply to this formula estimates for the elasticities of demand for labour and capital—estimates derived from Marshall's rules, but not from any formula. He thus naturally overlooked the precise way in which  $\lambda$  increases with  $\kappa$ . The larger  $\kappa$  is, the higher is the obstacle that has to be jumped before a factor can increase its relative share; but since the jumper increases in strength at exactly the same rate, the obstacle is irrelevant. The condition for increased relative share depends on  $\sigma$ , and on  $\sigma$  alone.

<sup>1</sup> See above, p. 119.

X:952  
62  
9906

no guarantee that those men whose efficiency he has improved will stay with him. The terms he is offering to his employees are better than those offered by his rivals; at least, they are better to a man in ordinary circumstances. But a man's relative valuation of income and leisure may change; and if he is faced with misfortune (for example, an illness in his family) it often does change. Although under normal circumstances he may prefer the shorter hours to a rise in wages, he may not always prefer them. If he is in difficulties the temptation to go elsewhere, to work longer hours, but to offer his improved efficiency as a claim to higher wages than are generally being paid, may be irresistible. The first employer must then replace him with another man, whose efficiency has to be worked up; and instead of reaping his expected profits, he is faced with another period of loss.

In spite of all these difficulties, it must not be assumed that a purely competitive system is powerless to reduce the hours of labour, so as to give the labourer some of the fruits of industrial progress in the form of increased leisure. Even the darkest days of the Industrial Revolution had their Robert Owen; and there can be little doubt that since that time the number of employers who are highly competent and adventurous and at the same time sympathetic to the needs of labour, has been on the increase. They can be relied upon to do something to mitigate excessive hours; and their success must induce others to follow their example. However, the struggle is not an easy one. It does seem probable that there are occasions when interference to reduce hours may secure to large numbers of workmen an increase in leisure at the cost of a fall in wages; which, nevertheless, seems to most