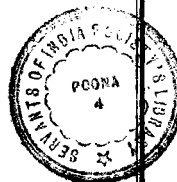


PUBLIC AIDS TO TRANSPORTATION

VOLUME III

PUBLIC AIDS TO TRANSPORTATION



VOLUME III PUBLIC AIDS TO TRANSPORTATION BY WATER

**Part I
WATERWAY IMPROVEMENTS AND RELATED AIDS**

**Part II
GOVERNMENT TRANSPORTATION OPERATIONS ON INLAND WATERWAYS
AN ANALYSIS OF THE INLAND WATERWAYS CORPORATION**

**Part III
THE PANAMA CANAL**

Published after expiration of Emergency Railroad Transportation Act, 1923

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Further discussion of the public aids received by carriers by water and of the other subjects treated in this volume, and a comparative analysis of the aids received by the several forms of transportation, are found in volume I. A foreword by the former Federal Coordinator of Transportation to this entire series also is found in that volume.

PREFATORY NOTE

This volume is in three parts. Part I, *Waterway Improvements and Related Aids*, develops the costs incurred in the provision and maintenance of waterway facilities, including publicly owned terminals, appraises the economic soundness of the expenditures made, and discusses the "waterway problem" in various of its other phases. It was submitted in tentative form to many organized groups and individuals known to have a substantial interest in these subjects. It likewise was sent to various Government departments for comment and criticism.

Part II, *Government Transportation Operations on Inland Waterways.—An Analysis of the Inland Waterways Corporation*, consists principally of an appraisal of the financial results of the Federal Barge Line operations, but also traces the antecedent operations of the United States Railroad Administration and the War Department. It was sent for comment to the official in charge of the Corporation.

Part III, *The Panama Canal*, analyzes the results achieved in the operation of our only toll-bearing Federal waterway facility. It was reviewed by the officials in charge.

The suggestions and criticisms received from these several private and Government sources were carefully considered in the final revision of the reports. The former Federal Coordinator of Transportation has elsewhere (vol. I) expressed his appreciation for the assistance thus afforded and in particular for that given by other Government departments.

I wish to make grateful acknowledgment of the cooperation given by members of the staff of the Section of Research of the Federal Coordinator of Transportation and by others. Mr. Henry C. Wilson bore much of the burden of assembling and analyzing the costs of waterway improvements presented in chapter IV of part I and materially aided on certain of the other chapters. Mr. James O. Riley has rendered invaluable assistance in connection with the final revision of this part and particularly as to chapters III-VI. Mr. Ronald A. Shadburne and Mr. Carl J. Baer also require special mention. Mr. Shadburne aided substantially on

various parts of the report, particularly those relating to the history of water transportation on the Mississippi River system, to barge and rail rates, and to the costs of barge transportation. Mr. Baer, a former vice president of an important common carrier barge line whose active and enthusiastic interest in inland waterway problems is well known, was engaged to make a detailed study of operators' costs. (See p. 55.) The results of his work were used in preparing chapter V, but with various modifications and amplifications, made after he left the Coordinator's service, for which he is not to be held responsible. Others who have contributed in various ways are Messrs. W. L. Fulton, Andrew L. Malone, and James W. O'Brien. Statistical assistance was given by a number of individuals, including Messrs. Rolston Lyon and John S. Andrews.

Messrs. R. M. Stockman and E. J. DeMuth, examiners loaned by the Bureau of Accounts of the Interstate Commerce Commission, require special mention in connection with part II. They analyzed accounting and statistical records of the Inland Waterways Corporation and its predecessors to the extent that these records were available in Washington, and Mr. Stockman brought forward an accounting report which was used to a large extent in the preparation of this part. Mr. DeMuth contributed to various sections of that report and particularly to those relating to terminal costs. Mr. R. A. Shadburne also requires mention. Mr. Herron Adams rendered valuable statistical assistance.

Acknowledgment also is made of the services rendered by Messrs. Stockman and DeMuth in connection with the accounting analysis presented in part III, *The Panama Canal*.

The Bureau of Valuation, Interstate Commerce Commission, kindly furnished data on service lives of waterway facilities for each of the three reports.

Each of the persons named gave much uncompensated overtime to the work, for which real appreciation is expressed.

CHARLES S. MORGAN,
Director, Coordinator's Section of Research.

PUBLIC AIDS TO TRANSPORTATION BY WATER

PART I

WATERWAY IMPROVEMENTS AND RELATED AIDS

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The highway companies represented in this table handled in the calendar year 1935, 2,619,786 tons of freight in line-haul operations and 1,973,513 tons in local operations. They carried 107,727,485 passengers in line-haul operations. Assets of the 128 companies amounted to \$89,508,108.

No figures were presented in this study to show the use of motor vehicles by class II and III railroads, or by electric railroads, nor were figures included to show the number of motor carriers which carried railroad freight under contract but in which the railroads had no financial interest. The Railway Express Agency, Inc., and Southeastern Express Co., subsidiaries of rail carriers, also were not covered in the Commission's tabulation. On January 1, 1937, the former owned 10,258 vehicles and the latter 483, mostly operated on city streets.¹

Statement No. 35-A of Statistics of Railways in the United States for 1936 shows that 130 operating steam railways giving pick-up and delivery service paid out \$10,209,206 in 1936 to operators of motor trucks or horse-drawn vehicles through contract arrangements.²

To bring this review as near to date as possible, reference may be made to an unofficial source which indicates that in 1938 railroads made use of a total of 63,781 trucks. Of this total, 9,111 were reported as being used in terminal transfer service, 5,890 in intercity service, and 48,780 in store-door delivery service.³

The only ready means of indicating the relative importance of motor-vehicle operations in which the railroads are interested is on the basis of the number of vehicles used. It appears from the 1938 figures given above that freight vehicles in which the railroads are interested constitute between 1 and 2 percent of all for-hire trucks operated in intercity service and 15-20 percent of those operated locally.

No figures have been found which would fairly reflect operation of passenger motor vehicles in rail service after 1936,⁴ but it is believed the number of such vehicles would now run to about 14 percent of the total common carrier busses in intercity operation.

Since May 1, 1936, acquisitions of and investments in motor-carrier lines by railroads, railroad subsidiaries and by motor carriers in which railroads have a substantial stock interest have been relatively small and would not materially change the investment figure shown above. An examination of finance cases before the Interstate Commerce Commission involving rail participation in highway operations (36 cases involving 25 railroad subsidiaries) indicates that as of January 1, 1939, the investment of railroad companies had not

increased by more than \$1,000,000 over what it was 3 years earlier.

Space does not permit a more detailed description of the motor operations of the railroads; that is, of the relative use they make of motor vehicles in substitution for, in supplementation of, or in competition with rail service. The aforementioned reports on Coordination of Motor Transportation and Regulation of Transportation Agencies define these terms and classify the uses, and current railroad literature provides much information of this kind, including some which emphasizes the beneficial effects on rail service and costs of coordinated rail-motor service.⁵

Mention has been made at earlier points (pp. 1 and 15) of the importance of motor transportation, both urban and rural, as a feeder of traffic to the rails.⁷ This function has diminished, relatively, with the growth of intercity motor transport service. In the beginning, the railroads did much to bring about the improvement of roads as feeders.

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¹ See especially the monthly Motor Transport Section of Railway Age.

² See, in this connection, California Highway Transportation Survey, 1934, pp. 9-10.

¹ The latter company has subsequently been dissolved.

² It is reasonable to assume that a very small percentage of the total disbursement was for service rendered by horse-drawn vehicles.

³ Automobile Facts and Figures, 1938.

⁴ As an example of railroad participation in the operation of motor carrier companies, see M.C.-F-33, Greyhound Corporation—Change in Terms of Common Stock, 1 M. C. C. 7; and No. M.C.-F-110, Greyhound Motors, 1934, 1 M. C. C. 342.

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APPENDIX A

SOURCES AND EXPLANATION OF CERTAIN ITEMS OF HIGHWAY EXPENDITURES AND REVENUES USED IN TEXT OF REPORT

Construction expenditures.—The Bureau of Public Roads tables giving annual construction expenditures under the supervision of the State highway departments vary in the treatment of administration and engineering expenses. The computations shown in table A-1 are for the purpose of making the data more nearly comparable for the period under investigation.

Refunds for construction work performed for others.—The Bureau of Public Roads tables of State highway receipts include an item of miscellaneous revenue or miscellaneous income. Analysis of the returns filed with the Bureau by State authorities discloses many miscellaneous receipts, such as those from sale of material and cement sacks, railroad participation in the

cost of highway work, cash discounts on material purchases, equipment and garage rents, etc., which should act to reduce the cost of construction or maintenance. Information necessary to make such an apportionment is lacking, but as a large part of the reported items are of the nature of credits to construction, the entire amounts of such items are treated in this report as construction credits. Details reported by certain States for the years 1925 to 1932 and 1933 form the basis of estimates of the total amounts of such credits for those years and for the years 1921 to 1924 and 1933 to 1937, for which no details were reported. (See table A-2.)

TABLE A-1.—Adjustment of construction expenditures on State highway systems to include pro rata share of administration and engineering expenses, 1921-37

Year	Construction expenditures as reported in various Bureau of Public Roads sources ¹ (1)	Miscellaneous expenses (when as reported with administration and engineering excluded) (2)	Percent miscellaneous expenses of construction expenditures (3)	Miscellaneous expenses, including administration and engineering (4)	Computed miscellaneous expenses, excluding administration and engineering (1/2 average of 3) (5)	Administration and engineering (4-5) (6)	Total construction and maintenance expenditures (7)	Percent construction of total (1+7) (8)	Administration and engineering pro rata share to construction (6×8) (9)	Adjusted construction expenditures (1+9) (10)
1921	\$284,302,460			\$16,417,306	\$1,884,925	\$14,532,383	\$340,135,619	81.430	\$11,833,719	\$296,136,170
1922	287,461,018			19,075,406	1,904,868	18,070,538	302,801,509	79.234	14,318,011	301,779,059
1923	379,992,788			24,407,190	1,854,353	24,557,777	355,321,948	78.800	19,346,012	390,335,800
1924	383,335,096			31,163,081	2,834,886	28,328,195	407,142,253	78.465	22,470,409	404,806,105
1925	389,327,384			35,618,083	2,881,241	35,031,842	408,630,944	78.544	27,530,213	416,007,507
1926	458,174,816			38,260,111	3,861,458	34,398,653	491,791,929	73.227	29,531,419	521,705,055
1927	494,317,317			45,823,267	2,573,981	45,143,328	543,000,676	74.481	34,949,261	488,856,578
1928	535,043,138	\$4,011,620	0.00746							538,043,138
1929	537,400,626	5,834,358	.00991							537,400,626
1930	713,117,048	2,227,459	.00312							715,117,045
1931	730,954,833	4,318,553	.00590							730,954,833
1932	551,445,859	4,416,117	.00801							551,445,859
		Average 1928-32	.00683							
1933	\$446,841,108	4,623,181	.01038							
		Average 1928-33	.00710							514,809,489
1933	\$45,153,000			\$3,715,000		\$46,868,000	\$91,213,000	66.133	\$3,136,821	\$94,349,821
1934	463,050,000	(²)		24,450,000	2,227,726	21,172,274	484,222,000	78.288	16,161,004	470,211,004
1935	387,445,000	(²)		26,300,000	2,837,860	23,462,140	407,780,000	70.394	18,727,131	374,172,131
1936	534,887,000	(²)		33,638,000	3,783,498	29,854,502	706,870,000	75.337	22,502,644	685,396,644
1937	437,906,000			41,164,000	3,266,949	37,897,051	640,902,000	71.441	27,078,590	494,946,590

¹ Construction expenditures for 1921 to 1937 are disbursements on State highways, taken from Bureau of Public Roads table F-2, revised.
² In 1933, a change in reporting from a fiscal to a calendar year basis occurred. The lower figure represents the amount necessary to carry the data through Dec. 31, 1933.
³ Exclusive of amounts for secondary roads under State control or urban extensions of State systems, given separately in Bureau of Public Roads table SF-2, beginning with 1934.

⁴ Beginning with 1934, the SF-3 series of tables of the Bureau of Public Roads qualify the data given for administrative, engineering and miscellaneous expense by the statement that the treatment of such items "is not uniform, as some States charge all or a large portion of overhead costs to construction and maintenance."

AIDS TO MOTOR VEHICLE TRANSPORTATION

TABLE A-2.—Analysis of "miscellaneous revenues," State highway departments, 1921-37¹

(All amounts in thousands)

(a) DISTRIBUTION IN YEARS FOR WHICH INFORMATION IS AVAILABLE IN REPORTS OF BUREAU OF PUBLIC ROADS

Item	1925	1926	1927	1928	1929	1930	1931 ²	1932 ³
1. Fines, fees, tolls, permits, etc.....	\$511	\$388	\$239	\$1,732	\$181	\$1,790	\$7,407	\$8,091
2. Amounts indicated as refunds for work performed and other amounts representing a reduction of cost of highways, such as railroad participation, sale of material, cash discounts, equipment, and garage rentals, etc.....	7,514	5,490	2,550	2,951	3,728	4,004	7,193	7,721
3. Interest received on deposits.....	471	854	1,109	264	1,440	2,248	1,803	1,269
4. Miscellaneous receipts, transfers, etc., not identified with users or construction.....	835	2,748	5,808	2,600	1,380	4,538	1,586	4,581
Total.....	9,331	9,445	9,706	7,447	6,729	12,573	17,989	31,682
Percent item 2 of total.....	80.52	57.50	26.27	39.63	55.40	31.84	39.80	22.12

(b) DETERMINATION OF ITEM (2) BY YEARS ON BASIS OF AMOUNTS REPORTED PLUS A PORTION OF AMOUNTS NOT SEPARATELY REPORTED, ON BASIS OF PROPORTIONS FOUND IN (a)

Year	Reported	Total not detailed	Proportion assign- able to item 2		Total item 2	Year	Reported	Total not detailed	Proportion assign- able to item 2		Total item 2	
			Percent	Amount					Percent	Amount		
1921.....		\$7,227	45.64	\$3,303	\$3,303	1921.....		\$7,193	59.80		\$7,193	
1922.....		8,236	45.64	3,759	3,759	1922.....		7,721	52.12		7,721	
1923.....		15,498	45.64	7,074	7,074	1923.....		\$9,153	45.64	\$4,177	4,177	
1924.....		13,360	45.64	6,093	6,093	1924.....		9,561	45.64	4,359	4,359	
1925.....		7,514	80.52	4,936	12,120	1925.....		8,370	45.64	3,820	3,820	
1926.....		5,490	3,779	57.80	2,185	7,645	1926.....		9,153	45.64	4,177	4,177
1927.....		2,550	2,816	26.27	730	3,289	1927.....		4,135	45.64	1,887	1,887
1928.....		2,951	3,185	39.03	2,947	4,698	1928.....					
1929.....		3,728	4,098	55.40	2,769	6,497	1929.....					
1930.....		4,004	4,611	31.84	1,436	5,440	Total.....	41,121	111,673		52,431	93,553

¹ See p. 67.² Tolls received by the State of New Jersey in 1931 (\$5,961,348) and 1932 (\$6,868,140) from the Port of New York Authority and the Delaware River Joint Commission are disregarded in the apportionment of amounts not separately reported.³ Item 2 as adjusted, 1925 to 1932, \$33,999,000; total miscellaneous income, 1925 to 1932, \$129,286,000; percent item 2 to total miscellaneous income, 45.64.⁴ Excludes \$5,883,603 received by Pennsylvania from Federal unemployment relief fund. For 1933 and subsequent years the items included in miscellaneous revenue are not exactly the same as for prior years. However, for present purposes the same relationship of refunds to the total is assumed.

APPENDIX B

THE RATIONAL DESIGN OF PORTLAND CEMENT CONCRETE PAVEMENTS¹

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Reference has been made at various points in chapters V and VI to questions which enter into the rational design of portland cement concrete pavements. This appendix presents a more detailed analysis of this matter than could appropriately be given there.

¹ By Warner Tufts.

² The word "rational" as used here means not merely "reasonable" (the ordinary sense), but more precisely "attained by reason" (in a theoretical sense). A rational equation is deduced from general laws of nature, and applies with equal certainty to the range of observations as well as beyond it. It is distinguished from an empirical equation, which may be a satisfactory expression of the observations, but which may not express any law, and which cannot be depended upon beyond the observed limits. For discussion, see O. P. Steinmetz, *Engineering Mathematics*, p. 216.

EARLY STEPS TOWARD A RATIONAL DESIGN²

At the close of the World War concrete roads in the United States lacked most of the elements of modern design. They were narrow and high crowned, constructed of poorly controlled materials, and their design reflected little of the theoretical knowledge of the structural properties of concrete slab which was available at the time.³

Application of corner formula to road design.—It was not until 1919, and after pressure measurements had been taken under the Camp Humphreys (Virginia) concrete road, that A. T. Goldbeck, then engineer of tests of the Bureau of Public Roads, outlined the first steps toward a rational design of portland cement concrete road slab.⁴ His formulas were based on conditions of maximum severity. They assumed a monolithic slab spanning the entire width of the road surface, and therefore do not apply to interior slab design where a longitudinal joint has since become standard. They are noteworthy, however, in that they offered the first applications of accepted formulas in mechanics to the design of road-surface slab. One of these, referred to as the corner formula still is used as a working basis of practical design.⁵

The application of this formula to the corner of a concrete pavement slab assumed (1) no subgrade support between the point of loading and the fulcrum or axis of rotation, (2) loading at a point, (3) loading at the extreme tip of the corner, and (4) an axis of rotation forming an angle of 45° with the two sides of the corner. These assumptions permitted the expression that the corner thickness in design is equal to the square root of three times the load divided by the safe working stress of the concrete; i. e., $T = \sqrt{\frac{3W}{S}}$.

³ A historical résumé may be found in H. M. Westergaard and W. A. Slater, *Moments and Stresses in Slabs*, Proceedings, American Concrete Institute, vol. 17, 1921, p. 415, et. seq.

⁴ A. T. Goldbeck, *Thickness of Concrete Slabs, Public Roads*, April 1919, p. 34.
⁵ This equation, based on well-established laws of the mechanics of loaded beams, and attributed to Barré de Saint-Venant (1856), expresses the relation between the unit flexural stress, the moment, and the properties of the cross-section, in any beam of uniform thickness, thus: $M/S = Tc$. In it, M is the moment of force applied at the point of loading and is the product of the load multiplied by the distance from the point to the axis of rotation; S is the flexural or bending stress developed in the fibers of the surface of the beam; I is the moment of inertia of the cross-section at the axis of rotation and is the product of the area of that cross-section multiplied by the square of the thickness divided by 12; and c is the distance between the interior neutral axis (at which compressive stresses on the concave side and tensile stresses on the convex side reach zero) and the surface fibers of the beam, or in a beam of uniform thickness, c is equal to one-half of the thickness.

Not one of these assumptions was, however, quite true.⁶ In addition, the formula depended entirely on a determinable value for safe working stress, which prior to 1923 was pretty much of a speculation. Nevertheless, the values of thickness resulting from this equation very closely approximated the edge thicknesses found to be adequate in actual experience, and, therefore, despite its known theoretical weaknesses, it increased in acceptance as a working rule.

Development of thickened edge as an element in design theory.—As early as 1909, thickened edges in the form of inverted or blind curbs were used in Wayne County, Mich.,⁷ and thickened edge pavement somewhat similar in design followed in California, in Cuyohoga County, Ohio, and in Chautauqua County, N. Y.⁸ This practice was directly contrary to the popular thickened center design of the period, however, and was employed only in special cases where additional support was required for vehicles getting on and off the pavement or where an especially insecure subgrade prevailed, and the presence of such an inverted curb might retard the seepage of water from the shoulders.

It was after the war, however, that general notice was taken of the fact that transverse failure proceeded inward from the edges. It was at this time, too, that engineers were being called upon to look to some source other than width limitation for the economies necessitated by extensive construction. Minimum requirements in width had increased considerably. The saving in concrete that would result from a diminished interior thickness came forward as a solution. Maricopa County, in Arizona, was utilizing a 9"-6"-9" design on 18-foot widths and an 8"-5"-8" on some of the 16-foot roadway in a 309-mile paving program on which construction was started in 1920.⁹

The second step in the direction of a rational design of concrete road slab involved the question of interior thickness, and appears to have been first expounded by Clifford Older, chief highway engineer of Illinois.¹⁰ He observed that the greatest stresses in the interior would occur at the corners formed by the intersection

of joints and cracks, which are frequently present even before the opening of the road to traffic loads. He assumed the application of the usual corner formula, but his contribution was the added assumption that the maximum condition would be represented by the support of the load over two adjacent corners. Interior thickness of slab would thus be equal to the square root of one and a half times the wheel load divided by the safe working stress of the concrete, i. e., $T = \sqrt{\frac{1.5W}{S}}$, and would be equal to approximately 71 percent of the edge thickness.

This interior thickness formula has all the theoretical shortcomings of the corner formula from which it is derived, but, as in the case of the corner formula, it has proved its practical utility through the many miles of acceptable highway surface constructed in accordance with it.¹¹

Design engineers were nevertheless aware at the time, not only of the possible weaknesses in the assumptions underlying these formulas, but also of the fact that these formulas anticipated the worst conditions and did not allow for the many intermediate ones. Nor did the formulas give consideration to the varied developments in the use of reinforcing metal. They were based on static wheel loads, and it was thought by some that the impact produced by fast moving vehicles would add materially to the force of the static weight. They assumed a closely ascertainable value for safe working stress, whereas it was known that the practices and conditions of the time caused wide variations even with a specified mix.

Engineers wanted practical proof of the qualities of any design. They wanted to know the relative merits of the various designs in actual performance. They knew that concrete road slab 6 inches thick constructed in 1893 in Bellefontaine, Ohio, was still giving service. They were aware that slab of a uniform 4-inch thickness, of a mix having a very low modulus of rupture, constituted almost all of the California concrete construction during 1912 and 1913, and that two-thirds of it was still giving good service in 1920.¹² It was already being recognized that cracking of concrete slab did not mean it had failed as a surface. But at what point in the life of the pavement, and under what conditions, practical failure would actually take place was entirely unknown.

In response, therefore, to the practical need of the highway engineer, everywhere confronted with an unprecedentedly large construction program, two intensive field tests were undertaken almost simult-

⁶ Though no subgrade support may be present at the outer corner under extreme considerations entering into design, the absence of such support will not be likely to terminate at a definite straight line, beyond which a complete support may be expected. On the contrary, it is likely that the transition from complete nonsupport to a maximum support will be gradual, thus necessitating some modification in the beam theory. Nor does the loading ever occur at a single point represented by the extreme corner, for, however close to the corner it may occur in actual circumstance, it still will be distributed over an appreciable tire contact area. And though this area would itself tend to change the assumed relation between the distance from the loading center to the axis of rotation and the linear dimension along that axis, that relation is further disturbed by the practical absence of such a well-defined axis. The theoretical rupture from overloading would form a straight line at angles of 45° with the sides of the corner, and would correspond with the axis of rotation. As a matter of fact, such a break would be extremely uncommon, because corner breaks are not always symmetrical, and when they are they usually occur in the form of an arc of a circle whose center is along the diagonal bisecting the corner angle.

⁷ Frank R. Rogers, *The Concrete Roads of Wayne County, Mich.*, Engineering News, Oct. 9, 1918, p. 730.

⁸ *Concrete Roads With Thickened Edges*, Concrete, October 1923, pp. 156-157.

⁹ L. W. Teller and E. G. Sutherland, *The Structural Design of Concrete Pavements*, Public Roads, December 1925, pp. 201-202, contains a brief outline of these historical developments.

¹⁰ Public Roads, February 1921, p. 9.

¹¹ This formula for interior thickness and the corner formula for edge thickness, with an allowance for impact, were recommended by T. E. Agg and B. A. Moyer in *General Engineering Handbook*, C. E. O'Rourke, ed., as recently as 1922.

¹² At this time the Bureau of Public Roads investigated the physical condition of the California Highway System and reported that, "No definite correlation could be found between the total of vehicles and the condition of the concrete pavement . . ." It did find a correlation, however, between defective pavement and poor subgrade support. See *Report of a Study of the California Highway System by the U. S. Bureau of Public Roads to the California Highway Commission, etc.*, 1920.

taneously. In these tests actual road pavement of various designs was to be subjected to actual truck traffic in such degree that destruction could take place and some practical evaluation made of the elements entering into such destruction.

The Pittsburg, Calif., experimental road.—The desire of the Columbia Steel Co. of California to determine the efficiency of a special reinforcing steel which it was manufacturing led it, in February 1921, to invite the cooperation of all who would be interested in undertaking an intensive highway research project involving an experimental road containing sections of various designs. The intention was to secure information on the relative merits of enough different types of pavement, including types embodying the ideas of the Columbia Steel Co., to enable engineers to design highways with greater certainty of results. It was considered vital that the designs and construction methods should represent, to the fullest possible extent, the ideas of engineers actually engaged in highway construction.

Thirteen designs were selected from those submitted, including three by the Columbia Steel Co.¹³ Of these, eight were of uniform design thicknesses of 5 and 6 inches, and reinforced with steel; four were of the inverted curb type, two of which involved additional inverted curbs beneath the interior portion and one of which contained reinforcing, while still another was of the Maricopa County type, i. e., plain concrete with thickened edges tapering to the interior thickness at 2 feet from the edge.¹⁴ These 13 designs were embodied in a continuous track constructed on a specially compacted adobe clay fill, and were subjected to truck traffic for 7½ hours a day from November 9, 1921, to August 21, 1922, equivalent to a total of 7,365,100 tons, or approximately 12 years of miscellaneous truck traffic of 1,000 trucks per day. The applied loads varied over a wide range.

The records of the various tests revealed much that was valuable to the highway engineer, as will be indicated later in this résumé; but the outstanding contribution to the problem of road slab design was the demonstration of the economic superiority of the Maricopa type, thickened edge pavement slab.

The Bates, Ill., experimental road tests.—At the end of March 1922, while the Pittsburg tests were but half completed, test traffic was started on the Bates experimental road in Illinois, under the auspices of the highways division of the department of public works and buildings of that State.¹⁵

¹³ Fully described in Lloyd Aldrich and John B. Leonard's Report of Highway Research at Pittsburg, Calif., 1921 and 1922, California State Printing Office, Sacramento, 1923.

¹⁴ Actual thickness, averaged from a series of measurements, showed a plus variation of as much as 25 percent over design thickness.

¹⁵ Reported in Test Road News Bulletins, Nos. 1-7, inclusive, 1922, and in Bulletin No. 21, Bates Experimental Road or Highway Research in Illinois, by Clifford Olds, 1924. Special tests on fatigue in concrete conducted preliminarily to the Bates road tests are reported in detail in Engineering Report No. 34-1, Fatigue of Concrete, 1924. All of the above documents were issued by the division of highways, department of public works and buildings, State of Illinois.

This road, consisting originally of 63 sections of various high-type pavements classified in 6 major groups, one of which was portland cement concrete, did not include the thickened edge principle until traffic breaks in the pavements had revealed the importance of strengthened edges. Accordingly, in the fall of 1922, additional sections embodying this design were constructed and the tests were resumed in the spring of 1923.

Like the Pittsburg tests, these sections were subjected to continuous truck traffic until the point of destruction was reached; but, unlike the Pittsburg tests, the loads were applied in definite increments and in a definite number of applications of each successive increment. These loadings may be tabulated as follows:

Wheel loads used and number of round trips for each loading—Bates experimental road, 1922

Increment No.	Load, in pounds on each rear wheel	Load, in pounds on each front wheel	Gross load	Net carried or live load in pounds	Number of round trips	
					Day	Night
1.....	2,600	2,200	6,500		1,000	
2.....	4,800	2,100	11,300		2,167	1,033
3.....	4,800	2,000	13,200		2,500	1,000
4.....	2,600	1,900	14,800	3,500	2,000	1,000
5.....	5,600	1,800	16,000	3,300	2,000	1,000
6.....	4,000	1,933	19,800	3,300	4,000	3,000

1922, ORIGINAL SECTIONS (EXCLUDING SEC. 83)

1.....	4,800	1,800	12,600	1,300	2,000	1,000
2.....	4,000	1,600	15,200	3,900	2,000	1,000
3.....	4,800	1,800	16,200	4,900	2,500	1,500
4.....	4,000	1,600	19,200	7,900	2,000	1,000
5.....	9,000	1,700	21,400	10,100	2,000	1,000
6.....	10,000	2,000	24,000	12,700	2,000	1,000
7.....	11,000	2,600	27,300	15,000	2,000	1,000
8.....	12,000	2,600	29,200	17,000	2,000	1,000
9.....	13,000	2,600	31,000	18,700	2,000	1,000

1922, NEW SECTIONS (AND INCLUDING SEC. 83)

1.....	6,600	1,800	16,600	5,800	2,000	1,000
2.....	6,750	1,700	20,800	6,600	2,000	1,000
3.....	2,600	1,600	19,200	7,900	7,000	3,000
4.....	10,000	2,000	24,000	12,700	2,000	1,000
5.....	11,000	2,800	27,300	15,000	2,000	1,000
6.....	12,000	2,600	29,200	17,000	2,000	1,000
7.....	13,000	2,600	31,000	18,700	2,000	1,000

The Bates road tests were planned with two general forms of research in mind. The traffic tests were but one of these and were designed to furnish empirical data supplementary to the other tests, which took the form of special investigations bearing on important factors involved in the rational design of pavement surfaces.

Outstanding among these special investigations were the fatigue tests of concrete. Though the resulting data are not readily susceptible to complete analysis, they pointed very strongly to the following conclusions: (1) Stresses which are small in relation to the stress sufficient to cause rupture appear to have a strengthening influence on the slab;¹⁶ (2) stresses equal to 50

¹⁶ This phenomenon is characteristic of crystalline solids, and is referred to in the case of metals as work hardening. Further discussion of it will be found in Appendix C, The Dynamics of Loaded Materials.

percent or less of the rupture stress may be applied indefinitely without injury to the slab; and (3) stresses greater than 50 or 55 percent of the modulus of rupture cause fatigue with a rapidity which increases in an undetermined way with their relative magnitude.

The Pittsburg tests had revealed that continued applications of heavy loads would ultimately produce flexural fatigue, but the relative magnitude of such fatigue-producing loads had not been determined. The Bates road tests thus contributed a fundamentally important element to a rational solution by establishing the safe working stress of concrete at approximately 50 percent of the modulus of rupture.¹⁷

The traffic tests were conducted in the hope that, among other things, they would bring out, roughly at least, the relation between wheel loads and thickness for each type of pavement used. When the 1922 truck runs were completed, an attempt at analysis of the data indicated, according to the report, that the corner formula "* * *" when used in connection with the design of rigid pavement slabs of fairly constant cross section and having no special provision for edge strengthening, may be used with considerable confidence.¹⁸

It has been popularly assumed that these Bates road tests definitely established the validity of that formula for use in rigid pavement design. Several qualifying factors, however, inadequately stressed or altogether neglected in the report and generally overlooked in consideration of it, should be pointed out.

(1) Careful examination of the text of the various reports, the diagrammatic presentation of cracks developed under the several wheel load increments, and the points used in the analysis as indicative of the relation between slab failure and loading, reveals many ambiguities which can be accounted for in no other way save that recorded load failures were determined by observation and were classified on the basis of judgment as "critical", "incipient", "decisive", "progressive", or merely "failures." Failures were further distinguished from breaks, which were disregarded unless they involved broken sections so small that subsequent applications of the same wheel load depressed the section and led to progressive failure due to impact caused by the depression. It is known that cracks may occur in concrete, and particularly in a covered concrete base, sometime before they can be observed or before their effects (in the instance of a base) can be noted on the surface. From the point of view of a formula of design it is the inception of the crack that marks the critical point. For this reason values of wheel load may be said to be overstated in the analysis.

(2) From the 1922 loadings shown in the foregoing table, it may be observed that the second loading

(3,500 pounds wheel load) was preceded by 1,000 applications of a wheel load of 2,500 pounds, equivalent to 71.4 percent of the second wheel load; the third increment by 1,000 applications of a load equal to 55.6 percent and 3,200 applications of a load equal to 77.8 percent; and that this practice prevailed throughout the loadings with increasing importance, until the sixth and last increment was preceded by 1,000 applications of 31.2 percent, 3,300 of 43.8 percent, 3,000 of 56.3 percent, 3,000 of 68.8 percent, and 3,000 of 81.3 percent. Moreover, each rear wheel loading was accompanied by front wheel loads, which in the lowest limit equaled 90 percent of the loading used in the analysis. Thus, the breaks assumed to be caused by even the 2,500-pound wheel load were in reality caused by the 2,500-pound wheel load acting together with a 2,250-pound wheel load, if the conclusions regarding fatigue are to be considered even remotely reliable. For this reason, values of wheel load used in the analysis are materially understated.

(3) The report as presented overlooked the fact that in all rigid types (except the plain concrete and monolithic brick), the bituminous surfacing that was present, the transference of the loads at joints, and the use of bar or mesh reinforcement contributed strength or deferred the observation of cracks. The effect was to reduce the importance of thickness and to minimize the importance of wheel loads, and particularly of the smaller wheel loads.

(4) And, though other unconsidered elements were present which have already been referred to in discussion of the weakness of the corner formula itself, one important additional qualification should be made. The loads that broke the pavement slabs were moving loads, while the loads considered in the analysis were static loads.

So long as the actual stresses caused by the moving wheel loads were not measured, a correlation between the static loads of the same wheels and the destruction caused by them in movement could be assumed to include impact or dynamic effect; but two conditions were present in the tests to offset the validity of the procedure in this particular instance. In the first place, relatively far greater impacts were present with the heavier loads because the same vehicles were used throughout the tests, and the final loadings were applied under springs considerably overloaded, and through solid tires practically worn out, whereas the springs were greatly underloaded in initial runs and the tires were new.¹⁹ In the second place, the final loadings were applied on pavement already rutted and cracked, thereby accelerating the effects of the moving wheels, whereas the initial loadings were applied on new pavement.

¹⁷ Fatigue tests conducted under the direction of W. K. Hatt at Purdue University, 1922, and reported by him in the Proceedings of the Fourth Annual Meeting of the Highway Research Board, 1924, materially substantiated the findings of these Illinois tests.

¹⁹ The importance of these factors is shown in Appendix D, Dynamic Wheel Loads and Tire Contact Area.

In making these criticisms, it is by no means intended to overlook the importance of this pioneering work from the vantage point of a decade and a half of accumulated thought about and experience in road design. Historically, the Bates road tests were of profound importance and influence. They conclusively established the advantage of the Maricopa design for plain concrete; they illustrated the advantages of edge reinforcement and the transference of loads across joints; they indicated a ratio of approximately 2 to 3 between interior and edge thickness; and they provided a factor of safety to be used in connection with the corner formula. Through the seeming substantiation they gave to this formula, they led to its general acceptance and thus standardized rigid surface design within limits which are still acceptable as close approximations.

The Pittsburg and Bates road tests may be definitely credited with the modern portland cement concrete highway. The period since their completion has been one of fine modification and adjustment through wide and intensive research, which has been largely influenced by the forward impetus given by the tests made on these two experimental roads.

THE WESTERGAARD ANALYSIS OF STRESSES IN CONCRETE ROAD SLABS

The properties of concrete slab.—Of the several properties of concrete to be considered in the design of road slabs, its resistance to bending—its beam action—is of primary importance. The compressive strength of concrete or its resistance to crushing is so very much higher than stresses from any loads likely to be imposed on a road surface that it may be ignored in surface slab design.¹⁰ The tensile strength of concrete, on the other hand, is very low but may be disregarded in considering the effect of traffic. The tensile stresses produced by wheel loads are likely to be small in comparison to such stresses caused by the expansion and contraction of the slab under the influence of temperature and moisture. The low tensile strength and the high degree of friction between the slab and the subgrade, however, are responsible for the cracks that will inevitably occur in the pavement unless joints are provided to relieve the tension produced by such expansion

or contraction. The closer the spacing of these joints, the less likelihood there is of the formation of such cracks. It was the 5-foot joint interval of the Bellefontaine-Ohio pavement that enabled it to withstand over 40 years of service without the development of these natural cracks, and similarly it is the joint interval of the concrete sidewalk that enables it to withstand weather changes without the development of tensile ruptures. This tendency toward tensile rupture is also reduced to some extent as the thickness is increased.

As indicated, the value of concrete slab is in its beam action; i. e., its capacity for spreading loads over wide areas of subgrade and thus increasing the load capacity of a uniform but soft subgrade, or for bridging the soft spots in a nonuniform one. If a subgrade were perfectly stable, this characteristic of concrete would not be essential, and if at the same time concrete were used purely as a surface finish to provide smoothness and resist abrasion, it possibly could be laid with the thickness and slab areas of sidewalks. Subgrades, however, are not stable, particularly in the presence of moisture and frost, and frequently they lack uniformity.

In addition to compressive and tensile strength, a third type of strength, therefore, is necessarily present in concrete in the form of resistance to rupture in bending. It is, of course, closely related in character to tensile strength, because in a given beam the modulus of rupture is determined by the rupture stress of the outer surface fibers in tension. But since the rupture point is beyond the elastic limit of the material, the modulus of rupture in flexure is higher than the modulus of rupture in tension, and must be determined by test. In the type of concrete used in road construction this modulus will vary between 500 and 750 pounds per square inch, at 28 days after pouring.

The bending of a beam produces compressive stresses in the fibers of its concave surface and tensile stresses in those of its convex surface. For this reason, still another element to be considered in analyzing the resistance of concrete to bending is the relative magnitude or modulus of its elasticity. This modulus is the ratio of the longitudinal stress, in tension or compression, to the accompanying strain or deformation.

Deformation, however, is not in one direction only, for as one dimension changes under load another dimension changes in an opposite manner. Stretched wire becomes thinner, or a compressed cylinder expands laterally. When, therefore, Hooke's law¹¹ is applied to deformation for the determination of stresses, one unit deformation must be modified by the other. The two being considered proportional to each other, the ratio of the unit lateral deformation to the unit deformation in the direction of the application of the force is a constant and is known as Poisson's ratio, the value of which

¹⁰ The following table is presented by Herbert H. Scofield in *General Engineering Handbook*, C. E. O'Rourke, ed., 1933, and is taken from test data of the Research Laboratory of the Portland Cement Association.

Strength of plain concrete, pounds per square inch		
Compressive	Flexural	Tensile
1,000	280	110
2,000	375	200
3,000	485	278
4,000	590	360
5,000	678	400
6,000	765	490
7,000	835	520
8,000	930	580
9,000	1,010	630

¹¹ This law states that within the elastic limits of a material, stress is proportional to deformation or strain.

in the case of concrete used in road construction may be taken as 0.15.²¹

When a load is applied to an elastic beam, the resulting stresses are calculable, within the limits of the assumed conditions, according to ordinary postulates of theoretical mechanics. But when beam action takes place in all directions, as with an elastic slab, and when that slab is in turn supported throughout its length and width by a subgrade also elastic in its reaction, conditions are present which call for special theoretical consideration. The variability of such conditions must also be considered, as, for example, in the matter of sub-grade support, position of load, and area of load application.

Westergaard load stress equations.—In 1926 Harald M. Westergaard presented the results of such a theoretical consideration, upon the basis of which a series of tests was conducted by the United States Bureau of Public Roads. These tests provided data which pointed to supplementary amplifications of the formula, published in December 1933.²²

The assumptions underlying the original analysis were that the slab would act as a homogeneous, elastically isotropic solid in equilibrium and that the reactions of the subgrade would be vertical only and would be proportional to the deflections of the slab. Three conditions of loading were examined as being representative of limits: (1) A wheel load applied close to a rectangular corner, tending to produce a corner break, with the critical stress at the top of the slab; (2) a wheel load applied at the interior of the slab surface, with the critical stress occurring at the bottom of the slab; and (3) a wheel load applied at the edge, but away from a corner, and again, with the critical stress occurring at the bottom of the slab. The first of these, being an intermediate condition, does not need to be considered in practical design, since the equation would result in smaller values for corner thickness than for edge thickness.

The assumption that the subgrade reaction would be proportional to slab deflection was found, through the tests, not to be wholly valid in the case of interior loading. Actually there was a closer concentration of subgrade reactions around the load than of deflections. Accordingly, a correction was introduced in the formula for critical stresses developed by a load applied to the interior of the slab.

The following two equations, for critical stresses produced, are the result of this further analysis by Wester-

gaard, pertinent to the practical design of concrete road slab:

$$S_e = 2.117(1 + 0.54\sigma) \frac{P}{h^3} \left[\log\left(\frac{h}{b}\right) + \frac{1}{3} \log\left(\frac{E}{K}\right) - 0.2666 \right]^{22}$$

$$S_i = 1.1(1 + \sigma) \frac{P}{h^3} \left[\log\left(\frac{h}{b}\right) + \frac{1}{3} \log\left(\frac{E}{K}\right) - 0.089 \right. \\ \left. - 13.64 \left(\frac{l}{L}\right)^2 Z \right]^{24}$$

wherein S_e and S_i represent edge and interior stresses, respectively, and

P =load on slab.

h =slab thickness.

σ =Poisson's ratio.

a =radius of area of load distribution.

$b = \sqrt{1.6a^2 + h^2} - 0.675h$ when a is less than $1.724h$, and $b = a$ when a is greater than $1.724h$.

E =modulus of elasticity of concrete in slab.

k =modulus of subgrade reaction=load per square inch of area per inch of deflection, expressed in pounds per cubic inch.

$l = \sqrt{\frac{Eh^3}{12(1-\sigma^2)k}}$ =measure of stiffness of slab relative to stiffness of subgrade.

$K = kl = \sqrt{\frac{Eh^3k^3}{12(1-\sigma^2)}}$ =coefficient of subgrade stiffness.

L =maximum value of horizontal radial distance of the resultant of the load P from the point of application.

Z =reduction ratio of the maximum deflection.

Values of L and Z are to be estimated for the particular circumstances according to a procedure outlined, and are to be based on particular test results. A plausible value of L may be $5l$. Z is positive, but less than 0.39. Test experience has suggested 0.2 as a possible norm.

Logarithms are to the base 10.

The multiplicity of terms involved, confusing though it is to the general reader, may be taken as indicative of the inclusiveness of the equations. They are not susceptible to direct conversion into equations for thickness h . In the generalized type formula, $S = m(c-g)$, say, to which they both conform, the coefficient m contains $\frac{P}{h^3}$ and if h did not recur in the term $(c-g)$ it could be said that the thickness was directly proportional to the square root of the load, as in the case of the beam formula, i. e., that $h = \sqrt{\frac{P}{S}(c-g)}$; but h does appear in the term $(c-g)$, not only as $\log h$ but also as an element in $\log b$ and $\log K$, and it cannot therefore be exactly isolated by mere algebraic manipulation. Moreover, the fact that it does so reappear establishes

²¹ F. E. Richart and N. H. Boy, Digest of Test Data on Poisson's Ratio for Concrete, Proceedings, American Society for Testing Materials, vol. 30, part I, 1930, pp. 601-607. Tests made in 1907 by A. N. Talbot cover a range which approximates the reported limits for the entire digest, i. e., from 0.10 to 0.30, the larger values occurring close to the ultimate load.

²² The original theory is presented in Stresses in Concrete Pavement Computed by Theoretical Analysis, by H. M. Westergaard, Public Roads, April 1926. The supplementary analysis appears in Analytical Tools for Judging Results of Structural Tests of Concrete Pavements, Public Roads, December 1933. The tests referred to are discussed in the Structural Design of Concrete Pavements, by L. W. Teller and E. C. Sutherland, Public Roads, October 1935.

²³ Equation (30) Public Roads, December 1933, p. 195.

²⁴ Equation (37) same, p. 198.

that under the theory the thickness will not vary exactly as the square root of the load.

The element of the load, however, is a constant in the coefficient and does not appear elsewhere in the formula. Therefore, under fixed conditions other than load, stress is always proportional to load, and the ratio of load to stress, under the given conditions, will be a constant. If, then, various values of this ratio are derived for any given set of conditions, but for a range of different thicknesses and radii of contact areas, a graph may be made which shows the thickness required

specification calls merely for the minimum. Similarly, the thickness resulting from the application of the design theory will serve but as a working minimum. And the equations themselves, comprehensive though they appear, leave out of consideration, as Westergaard points out, causes of change in the volume of the slab, as temperature and moisture, the effects of the gradation in thickness between the edge and the interior, and the results of local soft or hard spots in the subgrade. They also ignore the horizontal components of the reaction of the subgrade and the dynamic

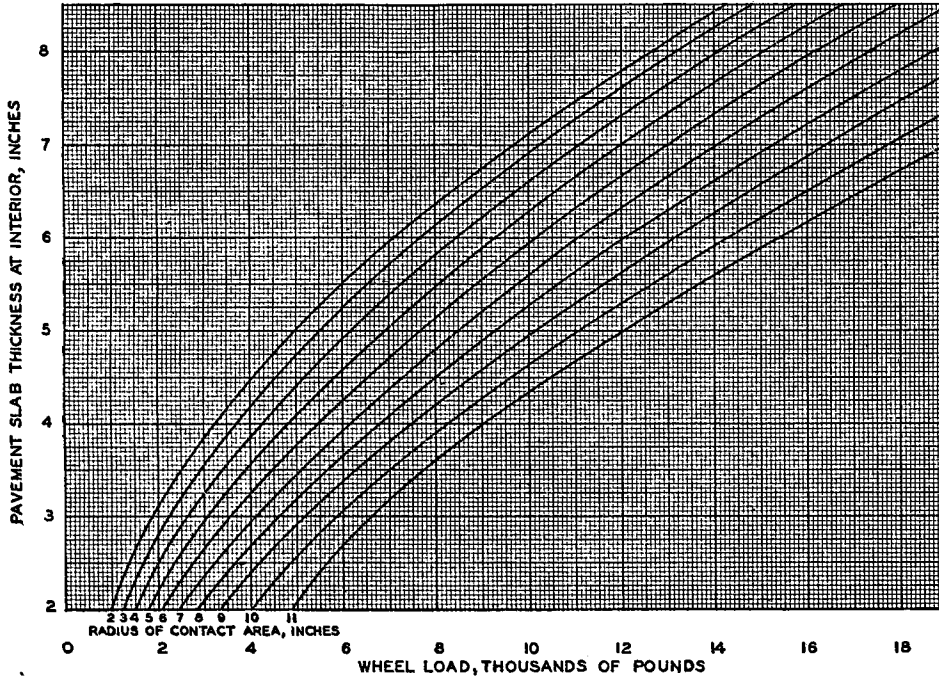


FIGURE B-1.—Relation between wheel-load and pavement slab thickness at interior for different values of contact area radius, with $S=300$ pounds per square inch, $k=100$ pounds per cubic inch, $R=3,000,000$ pounds per square inch, $\nu=0.15$, $l/L=0.05$, and $Z=0.2$.

for a particular wheel load and with a particular contact area. This procedure in its many possible modifications is accepted practice for the determination of slab thickness according to the Westergaard equations. It is resorted to here in figures B-1 and B-2 and tables B-1 and B-2.

The inclusion in an equation for road slab design of such small variables as Poisson's ratio, for example, does not mean that the last word in a rational design has been attained or that it is expected that construction practice will exactly conform to design specifications. When a modulus of rupture is specified for a particular design, it is known that it will vary widely, and the

effects due to the inertia of the pavement and the subgrade. Moreover, the design theory, however sound its basis, depends for effective practical application on the accuracy with which its elements can be measured. Progress in the immediate future would seem to lie in the improvement of this technique of measurement—an improvement which is gradually taking place in the laboratory.

Gradation in thickness from edge to interior of slab.—Were the relation between slab thickness for edge and interior loadings independent of the manner in which the transition between the thickened edge and the interior is achieved in practical construction, the Westergaard

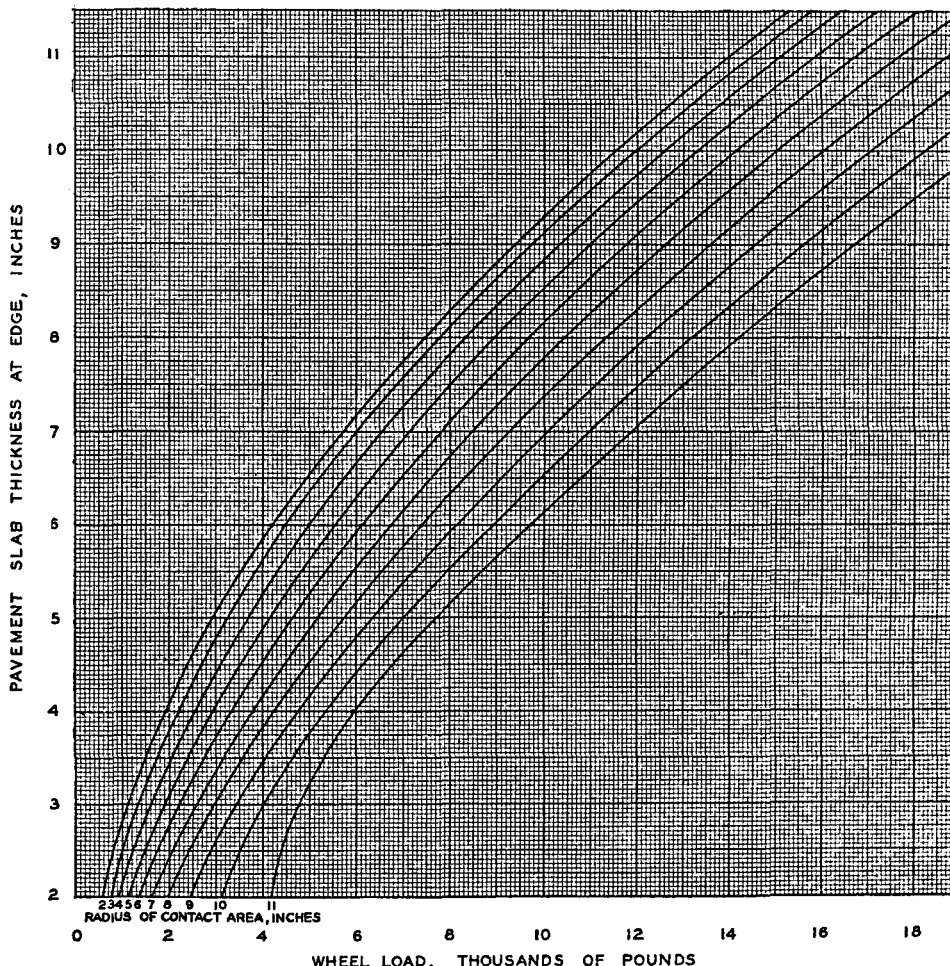


FIGURE B-2.—Relation between wheel-load and pavement slab thickness at edge for different values of contact area radius, with $S=300$ pounds per square inch, $k=100$ pounds per cubic inch, $E=3,000,000$ pounds per square inch, $\nu=0.15$, $4/L=0.02$, and $Z=0.02$.

equations indicate that it would vary somewhat with the thickness/contact-radius ratio, h/a , and with the modulus of subgrade reaction, k . The relation, however, appears to be modified by the transition design.

In tests by the Bureau of Public Roads, on a subgrade having a modulus of subgrade reaction, k , of approximately 225 pounds per cubic inch, and with loads applied through a bearing block having a radius of 4 inches, the most satisfactory transition design of several tested appeared to be that in which the transition between the edge and interior thickness was made at a constant rate in the outer 2 or 3 feet of the pave-

ment slab, the thickness thereafter remaining uniform. With such a design, and under the conditions of the tests, the Bureau concluded that "the thickness of the extreme edge must be about one and two-thirds times that of the interior of the slab if the section is to be completely balanced for stresses caused by load."²⁵

The effects of multiple-wheel loadings.—The discus-

²⁵ L. W. Teller and Earl C. Sutherland, *The Structural Design of Concrete Pavements*, pt. 3, Public Roads, December 1935, p. 211. The modulus of elasticity of the concrete, E , was 5,000,000 pounds per square inch and Poisson's ratio, ν , was taken as 0.15. With these values the Westergaard equations point to an edge thickness slightly greater than one and one-third times that of the interior for a balance in stress.

sion thus far has related solely to the effects produced by a single wheel load. In the case of two vehicles passing, and that of the six-wheel truck, loads are applied to the pavement at points in close proximity to each other, and the question arises as to the possible superposition of the resulting stresses.

Tests conducted by the Bureau of Public Roads in 1925 led to the conclusion that "In the case of the six-wheel vehicles the maximum tension produced in the pavement seems to be a function of the wheel load and not of the axle spacing."²⁶ Westergaard's conclusion as to this problem, on the basis of theoretical analysis in 1926 was " * * * that the main part of the state of stresses at a given point is due to a wheel load right over the point."²⁷ Again in 1935 the Bureau of Public Roads reported the results of further tests and again concluded, " * * * the data indicate that if several loads are applied simultaneously, as would occur in the case of a vehicle on the pavement, the critical stress under each load will not be increased by the presence of the other loads, providing the distance between the loads is approximately 3 to 4 feet or more. Thus it is to be expected that four- and six-wheel vehicles of the usual types will not have wheel loads so closely spaced that the effect of adjacent wheel loads on stresses need be considered."²⁸

Stresses due to volume changes.—The Westergaard load stress equations include no allowance for the influence of contraction or expansion stresses, resulting from temperature or moisture changes, on the total stresses to be considered in pavement design. The knowledge that such changes have an appreciable influence has raised a question as to the minimum practical thickness for concrete pavement slab.²⁹ An analysis of these stresses as supplementary elements in design is presented by Westergaard in Public Roads, May 1927. He introduces the discussion with the following statement:

The fact that cracks sometimes develop in a new pavement before any load is put on it shows the importance of the stresses which ordinarily are considered to be secondary—stresses caused

²⁶ L. W. Teller, *The Six-Wheel Truck and the Pavement*, Public Roads, October 1925, p. 184.

²⁷ H. M. Westergaard, *Stresses in Concrete Pavements Computed by Theoretical Analysis*, Public Roads, April 1926, p. 35.

²⁸ L. W. Teller and E. C. Sutherland, *The Structural Design of Concrete Pavement*, pt. 2, Public Roads, December 1935, p. 213.

²⁹ In the Report of Highway Research at Pittsburg, Calif., 1921-22, vertical temperature deflections in the edges of the pavement slab, occurring in a wavelike manner, are noted as a cause of transverse cracks. It is stated: "This is attributable to two causes which may in some degree act independently but under most conditions are jointly present. One of these conditions is the unequal expansion of the extreme fibers of the concrete because of their difference in temperature. The other is the existence of an axial stress caused by the longitudinal expansion of the entire section and the frictional resistance of the subgrade at the bearing points * * *. The stress in the concrete will be a maximum under a combined action of the axial and flexural forces. The rate in which these two forces act cannot be stated but in many cases the flexural forces may be suspected of being the predominant factor." Clifford Older, in his report on the *State Experimental Road* (Bulletin No. 21, Illinois Department of Public Works and Buildings, January 1924), presents a striking proof of temperature deflection, by comparative graphs of simultaneous air temperature readings and measured pavement slab deflections over a considerable period. These graphs also illustrate the relative constancy of the temperature at the bottom surface of concrete pavement as compared to marked variations at the top surface.

by changes of temperature, by setting of the concrete, and by changes of the moisture content. In a piece of concrete free to expand and contract the setting will produce a shrinkage equal to that caused by a certain drop in temperature, and the absorption of moisture will produce a swelling equal to that produced by a certain rise in temperature. In each case the essential feature is a tendency to change of volume. Thus, the study of stresses due to variations of temperature will supply information also about the other secondary stresses. With this condition in view, the discussion is limited to the case of variations of temperature.

The stresses caused by variations of temperature should be considered from two points of view. One has reference to the early life of the pavement, before the pavement has obtained its full strength and before it has been opened to traffic. The other has reference to the later life, after the pavement has gained strength and after it has been opened to traffic. According to the first point of view, the stresses caused by variations of temperature are considered as an independent cause of cracks and are treated by themselves. According to the other point of view the temperature stresses are to be combined with the stresses caused by the loads. This combination in most cases is a simple matter of addition.

Whatever particular effect is studied, three places on the panel of pavement should be examined—the corner, the interior area, and the edges.

Two type cases of temperature variation are studied. One involves the slower changes, such as are caused by seasonal differences in temperature, etc., assuming uniformity in temperature throughout the slab at any given moment. The analysis is simple. The uniform change is found to be unimportant as a contributory cause of corner breaks, and, though it results in stresses of considerable magnitude in the interior and at the edge, which in a continuous slab would have to be added to those caused by a load, it need not affect design in terms of thickness. These stresses, which may be relieved naturally by the plastic flow of the concrete, are in practice relieved by appropriate spacing of longitudinal and transverse joints.

In 1916, Charles Upham was introducing joints in his pavement construction in Delaware, spaced in conformity with the observed average spacing of cracks occurring naturally in similar surfaces of the period. In 1925, Goldbeck presented a rational method of joint spacing, based on theoretical analysis of the several factors of slab dimensions and weight, the coefficient of subgrade friction, the allowable tensions in the concrete and the reinforcing steel, and the moduli of elasticity of the concrete and steel.³⁰

The other type case in Westergaard's temperature stress analysis involves unequal temperature changes within the slab, caused by quick changes, such as a change from a cool night to a hot day, and vice versa. This results in curling stresses which may combine with wheel load stresses and require consideration in design.

The magnitude of the temperature stresses occurring in a road slab, i. e., a slab long in relation to its width,

³⁰ A. T. Goldbeck, *The Interrelation of Longitudinal Steel and Transverse Cracks in Concrete Roads*, Public Roads, August 1925. A correlative analysis, *The Mechanics of Progressive Cracking in Concrete Pavements*, is presented by H. M. Westergaard in Public Roads, June 1928.

may be computed by the aid of the following equations:

For lateral stresses:

$$S_{12} = S_0 \left[1 - \frac{2 \cos \lambda \cosh \lambda}{\sin 2\lambda + \sinh 2\lambda} \left((\tan \lambda + \tanh \lambda) \right. \right.$$

$$\left. \left. \cos \frac{y}{l\sqrt{2}} \cosh \frac{y}{l\sqrt{2}} + (\tan \lambda - \tanh \lambda) \sin \frac{y}{l\sqrt{2}} \sinh \frac{y}{l\sqrt{2}} \right) \right]^{21}$$

in which

$$S_0 = \frac{Ee_t t}{2(1-\sigma)}$$

$$\lambda = \frac{b}{l\sqrt{8}}$$

E = modulus of elasticity of concrete in slab.

e_t = coefficient of temperature expansion.

t = difference in degrees Fahrenheit in temperature at top and bottom of slab.

σ = Poisson's ratio for the concrete.

b = width of slab.

y = distance from longitudinal axis to point of stress.²²

$$l = \sqrt{\frac{Eh^3}{12(1-\sigma^2)k}} = \text{radius of relative stiffness.}$$

h = thickness of slab.

k = modulus of subgrade reaction in pounds per cubic inch.

For longitudinal stresses:

$$S_{12} = S_0 + \sigma(S_{12} - S_0)^{23}$$

Computations follow for the stresses due to a 20° F. difference in temperature²⁴ varying uniformly between the top and bottom of a 10-foot road slab. It is assumed that $E=3,000,000$ pounds per square inch, $e_t=0.000006$ inches per inch of length per degree Fahrenheit, $k=100$ pounds per cubic inch, and $\sigma=0.15$.

²¹ Equation (20), Analysis of Stresses in Concrete Roads Caused by Variations in Temperature, by H. M. Westergaard, Public Roads, May 1927.

²² Maximum S_{12} is developed at $y=0$, zero S_{12} is developed at $y=\frac{b}{2}$.

²³ Equation (21), Analysis of Stresses in Concrete Roads Caused by Variations in Temperature, by H. M. Westergaard, Public Roads, May 1927. This notation is identical with that for the equation for F_{12} .

²⁴ See the following:

Daily temperature data, in degree F., from United States Weather Bureau Compilations

Place	Period of record years	Average daily maximum	Average daily minimum	Average daily range
San Francisco.....	59	62.4	50.2	12.2
New Orleans.....	59	76.9	62.0	14.9
St. Louis.....	58	61.6	47.0	14.6
Chicago.....	63	58.6	45.3	13.3
New York City.....	45	59.6	45.0	14.6

These figures provide the average daily range and not the extremes; 20° F. was therefore assumed to be satisfactory for purposes of illustration. It is further justified by actual measurements in test slabs. See L. W. Teller and E. O. Sutherland, The Structural Design of Concrete Pavements, pt. 2, table 1, Public Roads, November 1935.

Values of temperature stress to be added to interior load stress

Slab thickness (inches)	Lateral interior stress (pounds per square inch)	Longitudinal interior stress (pounds per square inch)
1 2	228	214
4	180	207
6	117	198
8	86	190
10	68	183
12	54	183

¹ In practical design of present-day concrete road slab, a 2-inch thickness never occurs.

At the edge of the slab the values of temperature stress to be added to load stresses do not vary with thickness. Here, lateral stresses are of course zero. Longitudinal stresses vary with conditions other than thickness, and for those assumed in the table above they are a constant 180 pounds per square inch.

In the case of the interior of the slab, computations should be made for both lateral and longitudinal temperature stresses and those having the higher value, in either instance, should be added to load stresses.

The temperature stresses may in instances be considerably greater than load stresses.²⁵ Thus, Bates road records indicate differences of temperature of a much wider range than the 20° F. used above. When volume changes due to sudden moisture changes accompany these temperature differences, the resulting stresses, insofar as they are additive, may importantly reduce the load capacity of pavements. Moreover, the constant correction at the edge, occurring with a variable correction at the interior, results in a distortion at certain points in the normal design relation between edge and center thickness.

These complications are due primarily to two factors, which in no way impair the validity of the theory under the assumptions used. (1) The Westergaard analysis assumes a uniform gradation in temperature between the top and the bottom of the slab. The Bates road thermograph readings show that this is an exceptional condition. (2) Load stress design requires a gradation of thickness increasing from interior to edge. The temperature stress equations, however, assume a uniform thickness.

A series of tests were conducted by the United States Bureau of Public Roads between 1930 and 1935, to serve as the basis of a general investigation of the problem of concrete pavement design. A part of this study dealt with the effects of temperature and moisture conditions on the size, shape, and load-carrying ability of pavement slabs. According to the report,²⁶

²⁵ Compare Table 3, Stresses in Concrete Pavements Computed by Theoretical Analysis, by H. M. Westergaard, Public Roads, April 1926.

²⁶ The general report, The Structural Design of Concrete Pavements, by L. W. Teller and E. O. Sutherland, is in four parts, as follows: Pt. 1. A Description of the Investigation; Pt. 2. Observed Effects of Variations in Temperature and Moisture on the Size, Shape, and Stress Resistance of Concrete Pavement Slabs; Pt. 3. A Study of Concrete Pavement Cross Sections; Pt. 4. A Study of the Structural Action of Several Types of Transverse and Longitudinal Joint Designs. These parts appeared in Public Roads, for October, November, and December 1935, and September 1936, respectively.

it was planned " * * * to provide information, not heretofore available, on the complex relations created by temperature and moisture variations; and the practical significance of these relations with respect to the design of the pavement slab as a load-carrying structure." 37

The report concluded:

It is definitely indicated by all of the data * * * that, except in the immediate vicinity of the slab edge, the pavement should be of uniform thickness * * * that the edge thickness need not extend more than approximately 2 feet from the edge * * * that the desired balance of stresses is accomplished to a satisfactory degree by a cross section in which the increase in slab thickness is developed at a uniform rate as the edge is approached. 38

And finally:

Since a balanced cross section should in all cases be designed on the basis of combined load and warping stresses, it is obvious that economy demands that the stresses resulting from warping be limited to low values. The most practical way of doing this is by constructing short pavement slabs. In short slabs the cross section may be designed on the basis of load alone. 39

Historically, however, just as slab thickness has been added to compensate for the unknowns in subgrade performance, and just as a safety factor allowance is made for unknowns in materials, so metal reinforcement has been used as a safeguard against the undetermined elements in volume change.

Steel reinforcement.—Early tests of reinforcing in pavements, which include the Pittsburg and Bates road tests and those conducted at about the same time by the Bureau of Public Roads, 40 indicated some retarding effects of the reinforcing metal on the formation of visible cracks, and a very marked capacity to hold the cracked portions together and thus prevent the progressive destruction that follows the development of such cracks in plain concrete. This action depends, not so much on the supporting powers of the metal itself, which would involve its shearing strength, as on the binding effect due to its elasticity and tensile strength, which tends to hold broken sections sufficiently close together to secure by the friction between the rough faces of the fracture a high degree of mutual support under moving loads. The use of reinforcing metal in some form is almost universal in contemporary concrete surface design where fractures are likely to occur from uneven subgrade performance or unequal volume change.

In a summary of the status of the problem of reinforcement, presented before the Highway Research Board it is stated:

When first deposited on the subgrade in a plastic state, the concrete comprising both pavements and pavement bases may shrink due to loss of moisture, and may lose volume due to solu-

tion of cement constituents. After hardening, it may swell due to moisture absorption, may shrink due to moisture loss, may expand due to rise in temperature, may contract due to drop in temperature, may warp due to the temperature or moisture content in the top of the slabs differing from that in the bottom of the slabs, may distort due to nonuniform subgrade support, and may deflect due to traffic loads. 41

In the report it is further explained how steel reinforcement reduces these changes and the stresses caused by them:

During the shrinkage of plastic concrete it serves to reduce the extent of transverse and longitudinal movement of plastic concrete; to distribute uniformly throughout the length and breadth of slabs that movement which does occur; and to reduce, according to indications, the vertical permeability of slabs. In this manner it serves to reduce the amount of cracking of plastic concrete and tends to increase the uniformity of the concrete existing at different locations in the slab.

During the shrinkage and swell of rigid concrete it reduces the extent of swell due to wetting and of shrinkage due to drying the concrete and thus serves to reduce the amount of cracking due to the alternate wetting and drying of concrete slabs.

During the contraction and expansion of concrete it serves to hold fractured slabs together and thus decreases the widths of the cracks due to the contraction of concrete. This benefit may be utilized with a knowledge of the subgrade friction to predetermine the length of slab which can be maintained free of contraction cracks.

During the warp it reduces the magnitude of movement due to swell, shrinkage, and flow of concrete, and therefore, when properly placed in the slab would serve to reduce the warping produced by these phenomena.

During the bending of slabs due to nonuniform subgrade support it allows greater stretch of the concrete before the appearance of visible cracks, and, within limits, prevents the separation of cracked slab fragments. These benefits utilized with intelligent crack control permit the determination of the length of slabs which can be maintained free of cracks due to nonuniform subgrade support.

During the bending of slabs due to load it tends to reduce the magnitude of impacts due to wheel loads because reduction in faulting produces smoother pavements and tends to increase very appreciably the modulus of rupture of slabs laid on wet subgrades.

The report deals with the functions of steel reinforcement. However, it is supplementary to a previous investigation of the Highway Research Board, which deals with the economic value of reinforcement. 42 The first three conclusions of the latter report are as follows:

1. The amount of cracking and subsequent disintegrating is a function of time; thus the rate of cracking is a measure of the life of the pavement.
2. The data show that steel reinforcement reduced the rate of cracking and thus increased the life of the pavement. This applies to both concrete pavements and other pavements laid upon a concrete base.
3. Crack reduction is more economically accomplished by the use of steel reinforcement than by additional thickness of concrete.

³⁷ C. A. Hogentogler, E. A. Willis, and F. A. Roberson, *Functions of Steel Reinforcement in Concrete Pavements and Pavement Bases*, Proceedings, Eleventh Annual Meeting, Highway Research Board, 1931.

³⁸ C. A. Hogentogler, *Report of Investigation of the Economic Value of Reinforcement in Concrete Roads*, Proceedings, Fifth Annual Meeting, Highway Research Board, 1926, pt. II.

³⁹ *The Structural Design of Concrete Pavements*, op. cit., pt. 1, p. 144.

⁴⁰ Same, pt. I, p. 520.

⁴¹ Same, pt. I, p. 201.

⁴² L. W. Talbot, *Impact Tests on Concrete Pavement Slabs*, Public Roads, April 1924.

Nonuniformity of subgrade.—Nonuniformity of subgrade may result from irregular settlement or compaction under lanes of travel and to readjustment of the subsoil to the new environmental conditions created by the addition of an impervious surface, which alters the normal capillarity and evaporation. In the case of certain types of subsoil, nonuniformity may also be due to the frost of severe winters and the thaw of ensuing springs.

To regard these conditions as elements to be considered in slab design would imply that they were general. The resulting design theory, contemplating maximum conditions, would consequently regard a road surface as a series of spans and cantilevers and would therefore revert to beam or corner formulas. But these conditions are not universal, and present opinion is inclined to consider them as a problem of subgrade improvement and distinct from the problem of slab design.

When uneven spots occur in soft subgrades, their effect on concrete slab is relatively slight, because soft subgrades are capable of adjusting themselves in the direction of uniformity under a rigid slab; a most valuable quality of concrete is its ability to carry traffic on such subgrades where other surfaces would fail. Where subgrades are relatively rigid, however, soft spots or deformations are more severe on concrete pavement,⁴³ and, if difficult to correct, may require either added thickness or steel reinforcement, or a form of construction by stages wherein the subgrades are compacted by traffic before being surfaced with concrete.

MOTOR-VEHICLE IMPACTS AS A FACTOR IN DESIGN

It is shown in the preceding discussion that the wheel load and its area of application are elementary considerations in the balanced design of concrete pavement slab. But these considerations have always been regarded in their static aspects alone. This is so because the complex phenomenon of the load exerted by a moving wheel, and the more difficult problem of the inertia of the slab and subgrade, have not been sufficiently understood to warrant generalization.

Sufficient evidence exists, however, to permit useful estimates of the forces that are normally developed by moving wheel loads and the tire contact areas produced by these forces at the instant of impact.⁴⁴ And on the basis of the plausibility of two strongly indicated hypotheses, these may be employed in lieu of static wheel loads and static contact areas in design formulas.

One of these hypotheses is that impact blows result in increased stresses. The other is that the increased stresses are of the same general magnitude as stresses caused under equivalent static forces and areas of application.

A phenomenon of momentary loading of elastic materials permits a body to be loaded considerably beyond its static yield-point if only the loading is of sufficiently brief duration. This has frequently led to the notion that impact blows may be ignored because they automatically raise the yield point. A. E. H. Love, however, limits the time factor to the order of one-thousandth of a second, or less,⁴⁵ which is considerably less than the measured duration of motor vehicle impacts.⁴⁶ But the tests on which Love bases his limitation relate to dense materials such as metals and the duration factor, quite likely an inverse function of density, might well be much greater in the case of concrete.

In the summer of 1923 the Bureau of Public Roads completed a series of impact tests on a large number of concrete slabs.⁴⁷ Among the striking results of the investigation were the exceptionally high unit fiber tensile deformations recorded. The average deformation of road slab of a particular mix indicated a unit fiber stress in tension at elastic failure of 1,210 pounds per square inch. Such a modulus of rupture was recognized as being at least twice as great as the static modulus for the same concrete. The concrete was actually strained to a greater degree under impact without rupture than was possible under static loads.

In the Bates road tests a practical coincidence was found between observed slab deflections due to static and impact applications of wheel load. And the report concluded, "that the impact factor is not of sufficient importance to justify any great precautions in design."⁴⁸

In the Pittsburg tests, pavement deflections were measured under moving as well as static loads. It is stated in the report:

While the difference in deflections between moving and static conditions are small and varying in amount they very consistently point to the conclusion that the moving produces less deflection in the pavement than does the static or quiescent load.

From calculation of the stresses produced, however, the following conclusion was reached:

The results of this analysis indicate that for two conditions of loading the one which produces the greater deflections does not necessarily cause maximum moments in the pavements. This question arises in a comparison of the distortions caused by the static and moving loads. Under the latter loads the deflections are uniformly less than under static loads, yet the stresses caused by the moving loads are, from the calculations, generally greater. This result is due to the greater angular movements in the slab caused by the moving load and which was found in 48 cases out of 67 examined. These relative angular movements are probably due to the action of the subgrade, for with the two classes of conditions of loading on a slab supported only at its

⁴³ A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (4th ed.), p. 117.

⁴⁴ In the Proceedings of the Seventh Annual Meeting, Highway Research Board, 1927, James A. Buchanan and E. B. Smith reported this duration for truck impacts to be between 0.02 and 0.08 second.

⁴⁵ L. W. Teller, *Impact Tests on Concrete Pavement Slabs*, Public Roads, April 1924.

⁴⁶ Clifford Older, *Bulletin No. 21 Bates Experimental Road or Highway Research in Illinois, 1924*, p. 18.

⁴⁷ Indicated by H. M. Westergaard in *Mechanics of Progressive Cracking in Concrete Pavements*, Public Roads, June 1929, pp. 70-71.

⁴⁸ Appendix D, *Dynamic Wheel Loads and Tire Contact Areas*.

ends, the higher stresses would not be expected to accompany the smaller deflections. The element of time may well be suspected of influencing these results.⁴⁹

According to Stephen Timoshenko, "The action of a suddenly applied force is not transmitted at once to all parts of the body. At the beginning the remote portions of the body remain undisturbed, and deformations produced by the force are propagated through the body in the form of elastic waves."⁵⁰ The velocity of transmission being a function of density, pavement and subgrade reactions would involve an added time lag, further concentrating the stresses in the immediate zone of force application, and contributing to the angularity of slab movement observed.

Having noted the discrepancy between the modulus of rupture under impact and the static modulus, the Bureau of Public Roads, in 1925, in cooperation with Johns Hopkins University, undertook a special investigation of this phenomenon.⁵¹ The series of tests indicated, in the words of the report:

* * * that plain concrete beams subjected to static load have a considerably lower modulus of rupture than exactly similar beams subjected to successive impacts of increasing magnitude, caused by dropping a weight upon the beam from progressively increasing heights, when the weight is dropped but once from each height.

Under these conditions, which are similar to those prevailing in the slab impact tests of the Bureau, in which the weight was dropped only five times from each height, the beams resisted without failure impact forces considerably in excess of the static load which caused failure. In other words, they were capable of greater elastic deformation under impacts applied as described than under ordinary static load applications.

When, on the other hand, the beams were subjected to repeated applications of impact of the same magnitude, the greatest impact they would successfully withstand was that which pro-

⁴⁹ Measurement of deflections was taken in observation tunnels beneath the road with the aid of instruments connected to steel rods embedded in the surface slab. The sensitivity of the instrument was such that deflections caused by a man walking on the slab surface above the tunnel were recorded on the dial. Report of Highway Research at Pittsburg, Calif., 1921 and 1922, by Lloyd Aldrich and John B. Leonard, p. 137.

⁵⁰ Theory of Elasticity, p. 261.
⁵¹ J. T. Thompson, Static and Impact Strains in Concrete, Public Roads, July 1925.

TABLE B-1.—*Ratios of load in pounds to stress in pounds per square inch,¹ computed from Westergaard equation for interior pavement slab loading for different values of h, k, and a.²*
[E=3,000,000 pounds per square inch; $\mu=0.15$; $l/L=0.2$; $Z=0.2$]

Thickness of slab (inches) h	Modulus of subgrade reaction (pounds per cubic inch) k	Values of $\frac{P}{S}$									
		a=2 inches	a=3 inches	a=4 inches	a=5 inches	a=6 inches	a=7 inches	a=8 inches	a=9 inches	a=10 inches	a=11 inches
2	80	3.0488	3.7513	4.4888	5.1640	5.8312	6.7831	7.7459	8.9574	10.1874	11.7165
	100	3.2835	4.1190	4.9675	5.8576	6.9091	8.0673	9.4667	11.2202	13.3914	16.0404
	300	3.5623	4.5697	5.6324	6.9470	8.5408	10.4509	12.7099	15.3750	18.4901	22.0911
4	80	10.3620	11.6288	13.1341	14.7833	16.4763	18.2794	19.9501	21.7377	23.5327	25.3148
	100	10.9634	12.4022	14.2465	16.1714	18.2648	20.5102	22.6340	24.9122	27.3579	30.0605
	300	11.7293	13.4933	15.6773	17.9941	20.4918	23.3611	26.1659	29.2558	32.7395	36.6584
6	80	32.3717	36.0190	40.1239	44.5855	49.3777	54.4604	59.8928	65.7191	71.8804	78.3148
	100	33.9640	38.0429	42.6322	47.7719	53.2965	59.1628	65.4275	72.1401	79.2408	86.6795
	300	35.8454	40.5103	45.7107	51.4965	57.8093	64.5009	71.6299	79.1491	87.1104	95.4657
8	80	30.8207	41.3106	48.8336	57.3941	66.9938	77.6757	89.4801	102.4491	116.5328	131.6819
	100	42.3364	44.0256	46.9105	50.2831	54.0786	58.2668	62.8101	67.7619	73.0608	78.6592
	300	43.1850	47.1106	52.4334	58.3245	64.6941	71.5452	78.8195	86.4699	94.4459	102.7028
10	80	61.5953	65.7877	70.6297	75.9734	81.7737	87.9858	94.5641	101.4639	108.7318	116.3119
	100	66.4379	67.9071	71.1339	74.9794	79.3210	84.0930	89.2488	94.7416	100.5208	106.5399
	300	68.7738	74.0058	79.3076	84.7494	90.3074	96.0497	102.0439	108.3425	114.8917	121.6428
12	80	88.8332	91.5738	94.6497	98.0237	101.6833	105.5838	109.6709	113.9909	118.5818	123.3828
	100	94.3430	97.4731	100.9605	104.7640	108.8497	113.1734	117.6825	122.3330	127.1704	132.2408
	300	101.1947	104.1968	107.4989	111.0633	114.9346	119.0668	123.4057	127.9958	132.7804	137.8001

¹ These values, multiplied by the slab working stress of the concrete, provide the pavement supportability in terms of wheel load; or divided into the wheel load, they yield the slab stress produced.

duced a tensile stress equal to about 55 percent of the static modulus of rupture, indicating that the behavior of the concrete under impact is very similar to its behavior under static loads, as evidenced by static fatigue tests.

Since road slab design is concerned with repeated impact loads, it is clear, from the finding of this series of tests, that the possibility of an increased yield point under impact must be ignored in design.

The question thus reverting to the relative production of stresses, one may conclude from theoretical considerations and from the findings of the Pittsburg studies, that the greater force of the moving wheel load produces greater stresses than the static wheel load. Moreover, the factors entering into the production of impact, i. e., wheel load, unsprung load, and speed, would also tend to concentrate deflections and increase the stresses under moving loads; while the factor of tire deflection, which reduces impact, would tend to decrease the force concentrations, and the dynamic stresses would thus probably be proportional to dynamic load.

But in the dynamic situation there is still another factor to be considered. The principal stresses in a road slab produced by static wheel loads are bending stresses at the bottom of the slab beneath the center of the load. Certain subordinate and reciprocal stresses occur at the top of the slab at some distance from the load, due to upward pressure from the subgrade and to the compression of the surface fibers immediately beneath the load. When a wheel load is set in motion over the slab, these stresses are reversed and alternating stresses result, which may be quite as productive of failure and hence as important in design as the simple bending stresses thus far considered.⁵²

For these reasons, values of force under impact appear to be more plausible criteria of design than mere static forces.

⁵² The Pittsburg tests indicated "a wave motion in the pavement ahead of the advancing load. The dial indicated this action by registering negative readings or showed waverings of the needles in the negative direction. In a few instances this tendency was perceptible throughout the width of the pavement. * * *

AIDS TO MOTOR VEHICLE TRANSPORTATION

TABLE B-2.—Ratios of load in pounds to stress in pounds per square inch,¹ computed from Westergaard equation for edge pavement slab loading for different values of *h*, *k*, and *a*.²[*E*=3,000,000 pounds per square inch; *ν*=0.15]

Thickness of slab (inches) <i>h</i>	Modulus of subgrade reaction (pounds per cubic inch) <i>k</i>	Values of $\frac{P}{S}$									
		<i>a</i> =2 inches	<i>a</i> =3 inches	<i>a</i> =4 inches	<i>a</i> =5 inches	<i>a</i> =6 inches	<i>a</i> =7 inches	<i>a</i> =8 inches	<i>a</i> =9 inches	<i>a</i> =10 inches	<i>a</i> =11 inches
2	50	1.8023	2.2571	2.7281	3.2141	3.7322	4.2851	4.8454	5.4088	5.9828	6.5678
	100	1.9538	2.4096	2.8910	3.3790	3.8988	4.4521	5.0210	5.5958	6.1768	6.7648
	200	2.1908	2.6515	3.1357	3.6240	4.1388	4.6801	5.2488	5.8258	6.4118	7.0068
4	50	0.6250	0.8590	1.1150	1.3930	1.6940	2.0180	2.3650	2.7350	3.1280	3.5430
	100	0.4425	0.6404	0.8535	1.0820	1.3260	1.5850	1.8590	2.1480	2.4520	2.7710
	200	0.3228	0.4633	0.6274	0.8157	1.0270	1.2610	1.5180	1.7980	2.0990	2.4210
6	50	19.0081	14.0895	10.4000	7.8879	5.8867	4.3218	3.2168	2.3654	1.7600	1.3110
	100	13.8707	10.1070	7.4634	5.5711	4.1311	3.0297	2.2377	1.6328	1.2077	0.8977
	200	14.8570	10.2844	7.6732	5.7421	4.2910	3.1894	2.3974	1.7924	1.3674	1.0574
8	50	23.2710	17.1930	12.7701	9.5993	7.0925	5.2001	3.8101	2.8101	2.0901	1.5601
	100	24.8254	18.3774	13.6888	10.3118	7.7189	5.7828	4.2828	3.1828	2.3628	1.7428
	200	26.9335	19.8162	14.9191	11.4158	8.5242	6.3785	4.7070	3.4970	2.5970	1.9470
10	50	35.9573	27.3218	20.0854	14.8545	11.0221	8.1453	6.0165	4.4362	3.2962	2.4562
	100	38.3406	29.8833	22.1904	16.4322	12.1228	9.0131	6.6411	4.9154	3.6454	2.7454
	200	41.0526	32.8296	24.5181	18.4916	13.7400	10.3070	7.6270	5.6370	4.1870	3.1370
12	50	52.1351	39.5754	29.4785	22.1918	16.5823	12.3983	9.1373	6.7131	4.9619	3.6419
	100	55.8008	42.2405	31.4182	23.7918	17.8584	13.3584	9.9873	7.2773	5.3523	3.9423
	200	59.5632	45.1492	33.9569	25.6010	19.5640	14.6849	10.9289	8.0552	5.8641	4.3441

¹ These values, multiplied by the safe working stress of the concrete, provide the pavement supportability in terms of wheel load; or divided into wheel load, they yield the slab stress produced.

² *a*=radius of tire contact area.

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APPENDIX C

THE DYNAMICS OF LOADED MATERIALS AS RELATED TO EARTH ROADS, SUBGRADES, AND FLEXIBLE PAVEMENTS¹

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¹ By Warner Tufts. The problems presented in the allocation of the costs of Sec-
tion pavements, earth roads, and subgrades are discussed at pp. 92-98 and 117-118,
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NOTATIONS

- A = area.
 a = radius of circular area.
 c = coefficient of cohesion.
 c_1, c_2 = constant coefficients or parameters.
 E = Young's modulus of elasticity (see p. 228).
 E' = complete elliptic integral of the second kind with the modulus $k' = \sqrt{1 - r_1^2/r_2^2}$ shown by figure C-22, page 222.
 $E(k, \phi) = E(k, \theta)$, elliptic integrals of the second kind with $k = r_1/r_2$ shown by figure C-22, page 222.
 $F(k, \phi) = F(k, \theta)$, elliptic integrals of the first kind with $k = r_1/r_2$ shown by figure C-22, page 222.
 f = coefficient of friction.
 h = height of earth column between given horizontal planes.
 K' = complete elliptic integral of the first kind with the modulus $k' = \sqrt{1 - r_1^2/r_2^2}$ shown by figure C-22, page 222.
 k = the ratio r_1/r_2 shown by figure C-22, page 222, used as a modulus in elliptic integrals
 k' = the complimentary modulus $\sqrt{1 - r_1^2/r_2^2}$ shown by figure C-22, page 222, in the complete elliptic integrals.
 M_+, M_0 = arbitrary grouping of recurring terms in stress equations, defined on page 222.
 m, n = constant coefficients or parameters.
 P = total applied load, concentrated at a theoretical point or distributed over a finite area.
 P_r = the perimeter of a circular area.
 p = pressure intensity.
 Q = total shearing stress required to cause displacement along a shear plane.
 q = unit shearing stress required to cause displacement along a shear plane.
 q_1 = a point within a semi-infinite mass determined by the distances z and ρ as shown by figure C-22, page 222.
 R = radial distance in any direction below a horizontal boundary plane from a point on the plane.
 \widehat{RR} = a stress in the direction of R with R normal to the plane across which the stress acts.
 r_1, r_2 = dimensions described in figure C-22, page 222.
 S = maximum stress-difference, i. e., $T_1 - T_2$ (see p. 225).
 T_1, T_2 = the greatest and least principal stresses respectively (see p. 225).
 u_e = vertical displacement of the horizontal plane boundary at the edge of a uniformly loaded circular area.
 u_c = vertical displacement of the horizontal plane boundary at the center of a uniformly loaded circular area.
 V = Newtonian potential of surface distribution.
 z = depth below a loaded horizontal boundary plane.
 \widehat{zz} = vertical stress components.
 Θ = (Theta) an angle shown by figure C-22, page 222.
 κ = (Kappa) compression modulus (see p. 228).
 λ = (Lambda) elastic modulus defined by E and σ (see p. 228).
 μ = (Mu) modulus of rigidity (see p. 228).
 ρ = (Rho) radial direction in cylindrical coordinates.
 σ = (Sigma) Poisson's ratio of lateral to longitudinal strain (see p. 228).
 Φ = (Phi) angle of internal friction, angle of repose, or slope of slip planes in granular media.

NOTATIONS—Continued

ϕ =(Phi) an angle shown by figure C-22, page 222.
 χ =(Chi) Boussinesq's logarithmic three-dimensional potential.
 ω =(Omega) tangential direction in cylindrical coordinates, i. e., in the same plane with ρ but at right angles to ρ .
 $\widehat{\rho\rho}$ =radial stress components in cylindrical coordinates.
 $\widehat{\rho z}, \widehat{\omega\rho}, \widehat{\omega z}$ =components of shear in cylindrical coordinates.
 $\widehat{\omega\omega}$ =tangential stress components in cylindrical coordinates.

In all of the above stress components the first letter under the crescent signifies the direction of the normal to the plane across which or along which the stress acts, while the second denotes the direction of the stress itself.

→ The sign of approaching equality or coincidence as a limit.
 $>$ =is greater than.
 $<$ =is less than.
 \leq =is less than or equal to, i. e., is not greater than.

NOTE.—Minor nonrecurring notations such as r , b , I , and d are sufficiently defined where used.

WHEEL LOADS AND THE DESIGN OF EARTH ROADS, SUBGRADES, AND FLEXIBLE PAVEMENTS

Throughout the literature on road design, published since the motor vehicle became a dominant consideration, the design of surfacing, or wearing course, appears to be based almost wholly on the expected volume of traffic and the availability of materials. It is not until a choice has to be made in the matter of thickness of concrete slab that the anticipated wheel load is given explicit consideration in design. It is true that in the days of the steel tire and narrow wheel rim, and even in the early days of the solid-tired truck, sporadic efforts were made to classify roads and subgrade soils according to their unit load-bearing power, but no results of consequence were obtained.

This failure to utilize the wheel load as a factor in rationalized flexible pavement design may, in a sense, be attributed to the great variety in local earths and paving materials and to the necessity for wheel loads of all existing magnitudes to travel on all road types. On the light traffic route the problem was distinctly one to be met by the practical experience and ingenuity of the local engineer. It was not until traffic reached a considerable volume, with its accompanying weight of public demand and its accompanying economic justification of road improvement, that a more standardized surface could be adopted. Differences in design, at least so far as flexible surfaces were concerned, resulted not from any important differences in the size of vehicles constituting a cross section of traffic, but rather from the differences in the materials economically justifiable for road construction purposes in the light of existing or anticipated volume of traffic.

Failure to arrive at a basis of design which would consider wheel loads may be attributed much more positively, however, to a lack of definite knowledge concerning the relation of wheel loads of varying magnitude to the stability of flexible pavements on earth subgrades.

The need in the present investigation for allocating the pavement and subgrade improvement costs of this type of road construction, as well as the costs of earth roads, to motor vehicle groups having widely different wheel-load characteristics, has given rise to the research which is reported in the following pages.

The theory that is developed is offered, not as a basis of design, because far too much experiment remains to be made in the specific physical characteristics of the materials commonly employed; but it is offered as an indication of the relative requirements of different motor vehicle groups in the stability that is provided. It is put forward as the logical consequence of a vast body of data and supporting hypothesis carefully evaluated with reference to the particular problem.

Earth roads.—Lowest in the scale of roads having a nonrigid wearing course is the plain earth road. Its surface is in fact nothing more than the natural soil. Improvement does not extend beyond the simplest requirements of grading and drainage. The group comprises approximately three-fourths of all rural road mileage in the United States. E. W. James suggests roads of this type as suitable initial construction for average daily traffic up to 100 vehicles.² It is, however, unlikely that much of this large mileage ever attains a daily traffic of 100 vehicles.³ Economy justifies no more than a minimum of improvement.

A study of the reactions of the nonsurfaced road to traffic in terms of individual wheel loads rather than volume will appropriately introduce the more complex reactions of the other and higher types of nonrigid surfaced construction.

It is obvious that the wheel load is not a direct consideration in the design of the plain earth road, if the term "design" may be used at all in this connection.

² E. W. James, *Highway Construction, Administration and Finance*, reprinted from *Ingenieria Internacional*, by the Highway Education Board, Washington, D. C.

³ An average daily traffic of 100 vehicles per day for the 2,500,000 miles of this type of road would imply almost 85 billion annual vehicle-miles, or more than 3,000 miles of operation on this road type per year for every motor vehicle registered. One-sixth of this figure might be a high average.

It is known, too, that under certain circumstances such roads are suitable for any ordinary magnitude of wheel load, while under other conditions they may serve adequately for light traffic, but with the concentrated passage of an unusual volume of traffic or of heavy wheel loads they may be seriously damaged. In the case of the earth road, then, consideration may be directed from wheel loads as elements in design to wheel loads as elements in destruction.

The problem of earth stability has received the attention of engineers and physicists for 200 years. Until recently the efforts toward a solution followed almost without exception the known laws of mechanics. These laws assumed homogeneity, i. e., the smallest element taken from the material has the same physical properties as the entire body of material; they assumed isotropy, i. e., the elastic properties are the same in all directions through the material; and they assumed perfect elasticity under the action of external force, i. e., (1) when the load increases the deformation increases in the same ratio, (2) when the load diminishes, the deformation diminishes in the same ratio, and (3) when the load is removed, all deformation vanishes.

The Pittsburg experimental road investigation had found that "an elastic action of the subgrade was very clear and distinct in all soil observations."⁴ This finding was substantiated in the Bates road tests,⁴ and by other experimental undertakings.⁶ Nevertheless, the wide departures from theory that were very commonly encountered, and the empirical modifications of mechanical laws that were usually necessary in order to make them fit the case of earth and granular materials, have brought forward quite another type of approach in recent years.

This newer attack takes the material into the laboratory, measures its characteristics and examines its constituents, endeavoring thereby to develop a "soil science" through which the action of an earth material may be predicted on the basis of its constituents, or its stability increased through their modification. Much progress has been made,⁸ but the results thus

far are almost wholly of a qualitative sort, and may not be employed in a quantitative generalization of behavior. Many of the facts, however, lead to an understanding of the causes of that behavior, and are therefore of importance to the approach employed later in this study.

In 1927, Charles Terzaghi outlined certain principles of soil classification⁹ based on compressibility and permeability, to which are related such characteristics as lateral flow, volume change, slaking, etc. He stated: "We want to describe our soils not by more or less abstract figures like the Atterburg limits, the dye absorption number, the moisture equivalent, etc.—but rather by data with a direct, simple, and well-known bearing on the behavior of the subgrade."

In 1929, together with C. A. Hogentogler, he presented a classification of all subgrade types in eight uniform subgrade groups and three nonuniform subgrade groups.¹⁰ The groups are numbered in the order of their relative stability under wheel loads and in the presence of moisture. For each group is listed the typical earth of a particular region which is most characteristic of the group. The report, moreover, discusses the earth constituents and qualities which determine its stability. Certain indicated treatments by which the supporting power may be increased are also discussed.

These and further researches by the Bureau of Public Roads were brought together in 1931.¹¹ A summary is given of the physical properties of earths and their effect on subgrade performance, and a discussion given of the soil constants which serve to identify the constituents of the various earths.

The conclusions in the introductory summary are:

- (a) That the subgrade instead of the pavement really supports the wheel load.
- (b) That the manner in which the subgrade supports the wheel load depends upon its reaction to both load and climatic changes.
- (c) That these reactions depend upon five basic physical characteristics of soil, to wit, cohesion, internal friction, compressibility, elasticity, and capillarity.
- (d) That these physical characteristics control such important performances of subgrade as shrinkage, expansion, frost heave, the settlement of fills, sliding in cuts, and lateral flow of soft undersoils.
- (e) That these physical characteristics are furnished by soil constituents easily identified in the laboratory.
- (f) And that subgrades may be arranged in definite groups according to the characteristics of the soil constituents.

The last two statements (e and f) are definite indications that subgrade control is emerging from the level

¹ Lloyd Aldrich and John B. Leonard, Report of Highway Research at Pittsburg, Calif., 1921 and 1922, p. 91. See also pp. 128, here.

² Clifford Oldar, Bulletin No. 21, department of public works and buildings, division of highways, State of Illinois, January 1924, pp. 25-29. See also p. 185, herein.

³ George Paswell, Transmission of Pressure Through Solids and Soils, Transactions American Society of Civil Engineers, vol. 55, 1922.

⁴ The term is purposely enclosed in quotation marks. In communications to the Federal Coordinator of Transportation both D. F. Kryzine and John H. Griffith quite logically take exception to it. Kryzine suggests its inadequacy "since it refers to an existing agricultural science." He prefers "engineering soil science" or "soil mechanics." From Griffith's viewpoint, "The term 'soil' has already been pre-empted by bacteriologists and agriculturists." Their physics of soils relates to questions of fertility and suitability for farming. Their approach is from organic rather than inorganic chemistry. He believes, "the term 'earth' is more appropriate for the uses of geologists, mineralogists, petrologists, physicists, ceramists and engineers."

⁵ An historical outline of the development, from 1891 to 1928, is presented in Soil Road Surfaces, by C. A. Hogentogler, Public Roads, February 1928. Both theory and recent practical applications are illustrated in Studies on Sediments, by Tetsu Watanabe, Geotechnical Committee, Government Railways of Japan, Bulletin No. 4, 1925, pp. 11-61. A recent textbook dealing comprehensively with the subject is Engineering Properties of Soil, by C. A. Hogentogler, et al.

⁹ Charles Terzaghi, Principles of Final Soil Classification, Public Roads, May 1927.

¹⁰ C. A. Hogentogler and Charles Terzaghi, Interrelationship of Load, Road, and Subgrade, Public Roads, May 1929.

¹¹ C. A. Hogentogler, A. N. Wintermeyer, and E. A. Wills, Subgrade Soil Constants, Their Significance, and Their Application in Practice, Public Roads, June 1931, and July 1931.

of trial and error. In an understanding, however, of the destructive characteristics of various wheel loads on a nonsurfaced road, the second and third of these statements are of primary importance. In the light of the discussion on which they are based it follows that the manner in which the subgrade supports the wheel load depends upon (1) the cohesion, internal friction, compressibility, and elasticity of the earth material, and (2) the manner in which these characteristics are affected by weather, which depends in turn on the porosity and capillarity of the subgrade earth.

Cohesion is provided, among other things, by moisture and by the natural stickiness of colloids between the particles; and friction by the sharpness, angularity, and hardness of the particles themselves. Moist clay has cohesion, but is deficient in friction. Dry sand has friction, but is deficient in cohesion. If friction is dominant, enormous bearing power may be attained by loading or confining the surface surrounding the area of load application, as with a pavement. If cohesion is dominant, this practice contributes little.

Compression by a load represents a decrease in porosity, i. e., in the ratio between the volume of the voids in the earth mass and the volume of the total mass. The rate of compressibility depends, of course, on whether the voids contain air or water, but in either case it is dependent on permeability. Permeability may be reduced by consolidation, under traffic or by rolling, and thus the rate of compressibility may be reduced.

Elasticity represents the tendency of a compressed earth to return to its former shape. It tends to prevent both permanent consolidation and permanent reduction of permeability.

The earth of any nonsurfaced road will have these four characteristics in varying degrees, depending on the consolidation that has taken place under traffic and on such constituents as gravel, coarse sand, fine sand, silt, cohesive clay, gluey colloids, mica flakes, diatoms, peat, etc., and certain chemicals. Therefore, it is these constituents which determine the load supportability of an earth mass.

The remaining factor, capillarity, controls the reaction of the subgrade or earth road to weather. Its force varies inversely with the size of the pores between the particles and must therefore be dependent on the soil constituents. Capillarity in a subgrade may be defined as "The ability of soils to transmit moisture in a finely divided state in all directions in spite of the direction in which gravity acts, and the force of gravity."¹² Capillarity draws moisture into the subgrade and retains it. Expansion results with the increased moisture content to an extent which depends on the initial degree of consolidation and moisture content.

Evaporation, acting together with capillary force, causes shrinkage to a degree which depends on the volume of moisture content before evaporation and on the shrinkage limit. At sufficiently low temperature, the moisture in the large capillary pores freezes, forming ice particles. The moisture in the smaller capillaries, however, does not freeze and these continue to supply moisture to the large capillaries, thus contributing to the further increase of the amount of frozen moisture in the soil mass. The resulting ice layers cause "heaves" in the surface, the magnitude of which equals the combined thickness of the layers. The size of the latter depends on the force of capillarity, the area of pore space, and the frictional resistance.¹³ These weather factors affect the earth layers of a given subgrade or nonsurfaced road in much the same way as changes in soil constituents, and they will therefore disturb the relative supportability offered to wheel loads in a similar manner.

There remains one additional factor to be noted in the behavior of earths under the influence of weather. Sufficient cohesion between the particles can prevent loss of stability due to moisture absorption, if the earth is not manipulated. This suggests one answer to the important question as to why roads which are adequate for light traffic may quickly fail under the concentrated effect of a few heavy loads. This condition would occur mainly with earths whose resistance to shear under critical moisture conditions is dependent almost wholly on cohesion. The manipulation that results in breakdown will be related to frequency of load application and therefore to traffic volume.¹⁴

It will be observed in figure C-1 that the solid curves indicate a compressibility in the earth constituents which is a function of unit pressure. This compressibility is also a function of time in very much the same way.¹⁵ It will also be observed that the broken lines represent expansion or elastic recovery upon removal of the load. This likewise is a function of time.

The presence of the element of time, or lag, in earth reaction is another factor which introduces traffic volume into the problem. Compression and the formation of ruts and the manipulation of clay soils in the presence of moisture have a cumulative effect when wheel load quickly follows wheel load. But earths are more plastic and viscous than elastic, and the permanent deformations that occur are equally cumulative under traffic.

¹² Capillarity is but one of several factors influencing frost heaves. A broad discussion of various causes is to be found in C. A. Hoegentogler, et al., *Engineering Properties of Soil*, pp. 135-157.

¹³ It also depends on the relation between deflection and pressure intensity under various areas of pressure application, as discussed later. D. F. Krynine suggests another cause. "The upper crust of an earth mass, if duly compressed by a given load, represents an integral part of the whole mass only for that or smaller loads. For greater loads it may again become an agglomeration of small individual bodies which, however, can be consolidated by gradual increases in the transition to the larger load."

¹⁴ Charles Terzaghi, *Principles of Final Soil Classification*, Public Roads, May 1927, p. 42.

¹⁵ See footnote 11 on p. 199.

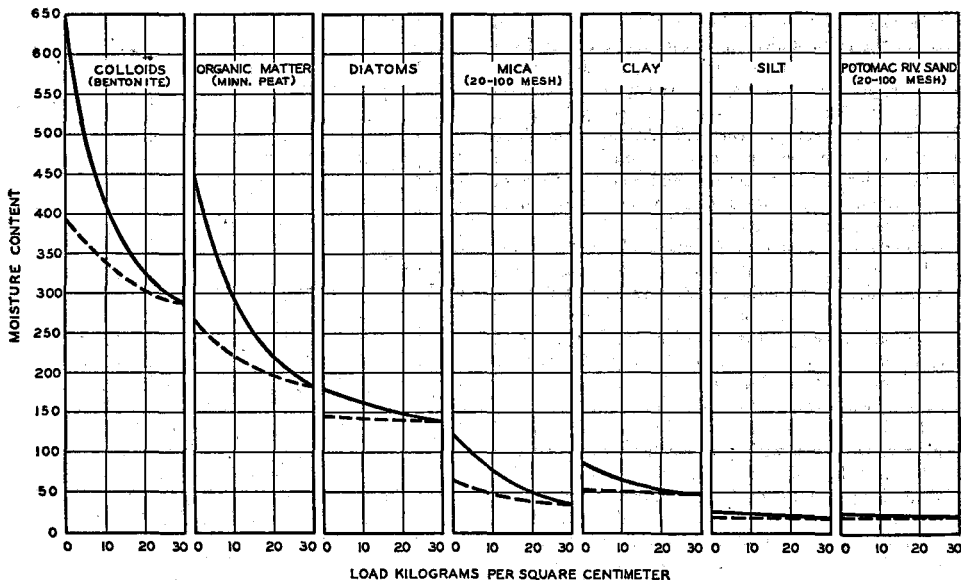


FIGURE C-1.—Compression and expansion curves for representative soil constituents. (From Public Roads, June 1931.)

All flexible surfaced roads are subject to the basic weaknesses of the nonsurfaced road, mitigated only by the degree to which definite measures have been taken to improve subgrade stability by soil correction, drainage, and protection from moisture by water-shedding surfaces, and modified, of course, by the resistance offered by the surface itself. The present status of soil science permits the diagnosis of these weaknesses, and provides relative means for their correction.¹⁸ But the science is still qualitative, and the absolute supportability to be derived from a given corrective is not known; neither is the specific resistance that may be anticipated from different earths to different wheel loads applied over different tire contact areas.

The problem of the relative destruction of earth roads by different wheel loads, and the associated problem of the design of nonrigid pavements for earth subgrades, resolves into a question of the relative resistance of different materials, whether elastic or plastic or viscous, to different intensities of stress under static and dynamic loadings over different areas of pressure application.

Investigations of the transmission of pressure through earths.—According to Jacob Feld,¹⁷ Couplet gave "a complete theoretical analysis of soil physics" in 1727.

¹⁸ C. A. Hogentogler, *Soil Road Surfaces*, Public Roads, February 1936; Highway Research Board, National Research Council, *Progress Reports of Project Committee on Stabilized Soil Road Surfaces*, 1935 and 1938; C. A. Hogentogler and E. A. Willis, *Stabilized Soil Roads*, Public Roads, May 1936.

¹⁷ Feld on the Science of Foundations, *Transactions, American Society of Civil Engineers*, vol. 93, 1929, p. 314.

And whatever the influence of the Newtonian conception of material bodies, as made up of small parts possessed of finite sizes and definite shapes acting on each other by means of central forces,¹⁹ the next century and a quarter was to witness a joint theoretical development of the mechanics of elastic solids and of pulverulent or granular materials. The practical need for experimental verification in the case of earths was apparently not felt until much later.¹⁹

In 1908 an exhaustive series of tests was started, at the Engineering Experiment Station of the University of Iowa under the direction of Anson Marston, to determine the loads transmitted to culverts and underground conduits.²⁰ Tests for the same purpose had been made earlier,²¹ and have been conducted since,²² but do not compare in magnitude or scope with Marston's undertakings, which are still in progress and on which a score of reports have already been issued.

Marston's investigations revealed a close relation between the actual vertical pressure transmitted to any

¹⁹ A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (1927), p. 6.

²⁰ D. F. Krynine, in *Krynine on Soil Mechanics Research*, *Transactions, American Society of Civil Engineers*, vol. 58, 1925, p. 274, gives 1870 as an early year in the investigation of the transmission of pressure through soils. See also footnote 25, below.

²¹ For a summary report to 1926, see Anson Marston, *The Theory of External Loads on Closed Conduits*, Bulletin No. 95, Iowa Engineering Experiment Station, 1926.

²² *Journal of Association of Engineering Societies*, vol. 19, December 1927.

²³ *Proceedings, American Railway Engineering Association*, vol. 27, 1926, p. 794, and vol. 28, 1928, p. 827. Also G. M. Brauns, *Earth Pressures on Culvert Pipes*, Public Roads, January 1929, p. 222.

point at any depth considered and the theoretical vertical stress at the point indicated by the elastic theory for a concentrated loading at the surface.²³ They also verified, within approximate limits, the theory of horizontal pressures in a granular material advanced by W. J. M. Rankine a half century earlier.²⁴ Attention in these reports, however, was concentrated on the problem of the underground structure, and another type of investigation going forward at the time was looked to as more pertinent to the road builder.

Among the earliest experiments made to determine the distribution of vertical stresses on horizontal planes at various depths, were those of Steiner-Kick at Prague, in 1879.²⁵ In 1910 the Joint Committee on Stresses in Railroad Track of the American Society of Civil Engineers and the American Railway Engineering Association commenced a series of tests with granular materials, with similar objectives, under the direction of Marvin L. Enger at the University of Illinois.²⁶ Similar tests with a variety of earth types were made by Roy B. Fehr and C. R. Thomas under the direction of J. A. Moyer at Pennsylvania State College in 1914 and 1915.²⁷ And in 1916, A. T. Goldbeck, of the United States Bureau of Public Roads, utilizing a pneumatic pressure gage of his own design which gave very accurate results, measured pressure distribution in sand (figs. C-2 and C-3).²⁸

The significant disclosure common to these tests was that pressure is transmitted through these granular and plastic materials in a conelike manner, concentrated somewhat at a distance beneath the center of the bearing block and diminishing in intensity in some way both vertically and laterally as the distance from the bearing

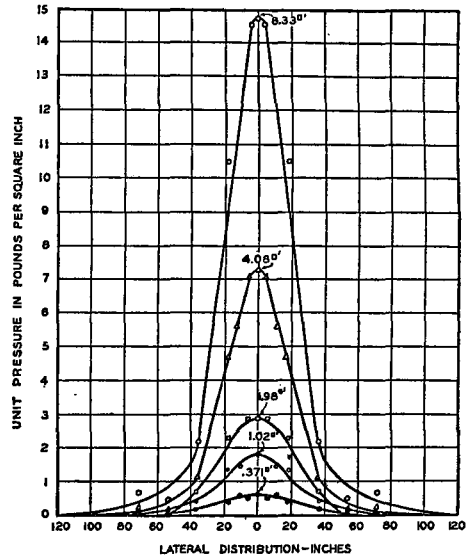


FIGURE C-2.—Pressure distribution in an earth mass at a constant depth from a 4,000 pound per square foot loading on various bearing areas. (From Public Roads, January 1928.)

block increased. In 1922, George Paaswell pointed out that the distribution of pressure as experimentally determined "seems to follow almost the identical law of transmission, as determined from the mathematical theory of elasticity." And, reviewing the tests mentioned herein, added: "The agreement in stress distribution between material of so varied a character as that covered in the various reports mentioned, as well as the elastic solids discussed in this paper, seem to indicate that stress distribution under a surface loading follows a simple general law of distribution, regardless of the character of the material."²⁹

Paaswell also cited the ordinary "rule of thumb" of the time which assumed "a uniform distribution of pressure along any horizontal plane below the loaded surface, increasing in spread as the distance below the surface increases and confined, roughly, within planes making an angle of 45° with the vertical." Actually, he found a 1:2 slope was required to enclose the limiting zone of influence, but after averaging the pressure intensity on various planes he concluded that if the effects of unequal pressure distribution are ignored, "it is seen that a 1:1 slope gives a reasonable distribution of pressure."

The "law" of the inverse square.—According to the distribution of pressure from a point loading, as derived from the mathematical theory of elasticity,³⁰ the maxi-

²³ M. G. Spangler, Clyde Mason, and Robley Winfrey, Experimental Determination of Static and Impact Loads Transmitted to Culverts, Iowa Engineering Experiment Station, Bulletin No. 79, 1926, p. 6. The special case of the equation for this distribution is:

$$\frac{P}{2\pi} = \text{vertical pressure at the point} = \frac{3P}{2\pi} \frac{s^2}{r^3}$$

where P is the concentrated load at the surface, s the depth below the surface, and r the distance from the concentrated load to the point considered. The particular case of this equation was given by Joseph Boussinesq in *Comptes Rendus* * * * de l'Académie des Sciences, Paris, Tome 82, July-December 1881, p. 704, Eq. (8). Discussion of it may be found in John Perry, *Applied Mechanics*, (1907) p. 380; A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (1927), p. 192; Stephen Timoshenko, *Theory of Elasticity*, p. 331; and John H. Griffith, *Dynamics of Earth and Other Macroscopic Matter*, Iowa Engineering Experiment Station, Bulletin No. 117, pp. 66 and 67, and 66 and 67. Griffith's exposition of a more general case is discussed later.

²⁴ That the ratio of active horizontal pressure at any point in the fill to the vertical pressure which causes the active horizontal pressure = $\tan^2\left(45^\circ - \frac{\phi}{2}\right)$, in which ϕ is the angle of internal friction in the fill material. See Amos Mason and A. O. Anderson, *Theory of Loads on Pipes and Ditches*, Iowa Engineering Experiment Station, Bulletin No. 81, 1913, p. 32; also W. J. M. Rankine, *On the Stability of Loose Earth*, *Philosophical Transactions, The Royal Society of London*, vol. 147, 1857, pp. 9-37.

²⁵ A. E. Cummings, *Distribution of Stresses Under a Foundation*, Proceedings, American Society of Civil Engineers, August 1925, p. 628. Cummings gives a variety of examples, with the resulting experimental data.

²⁶ Marvin L. Enger, *The Distribution of Pressure by Granular Materials*, Engineering Record, vol. 78, No. 4, January 1922, pp. 106-108; George Paaswell, *Transmission of Pressure Through Solids and Soils*, Transactions, American Society of Civil Engineers, vol. 53, 1922, p. 1563; also Enger on the Science of Foundations, Transactions, American Society of Civil Engineers, vol. 53, 1923, p. 871.

²⁷ J. A. Moyer, *Distribution of Vertical Soil Pressure*, Engineering Record, vol. 68, No. 22, May 25, 1914, p. 808, and vol. 71, No. 11, March 25, 1916, p. 330.

²⁸ A. T. Goldbeck, *Distribution of Pressure Through Earth Fills*, Proceedings, American Society for Testing Materials, 1917, pt. 2, pp. 640-661.

²⁹ George Paaswell, *Transmission of Pressure Through Solids and Soils*, Transactions, American Society of Civil Engineers, vol. 53, 1922, pp. 1563-1577.

³⁰ See equation in footnote 23, above.

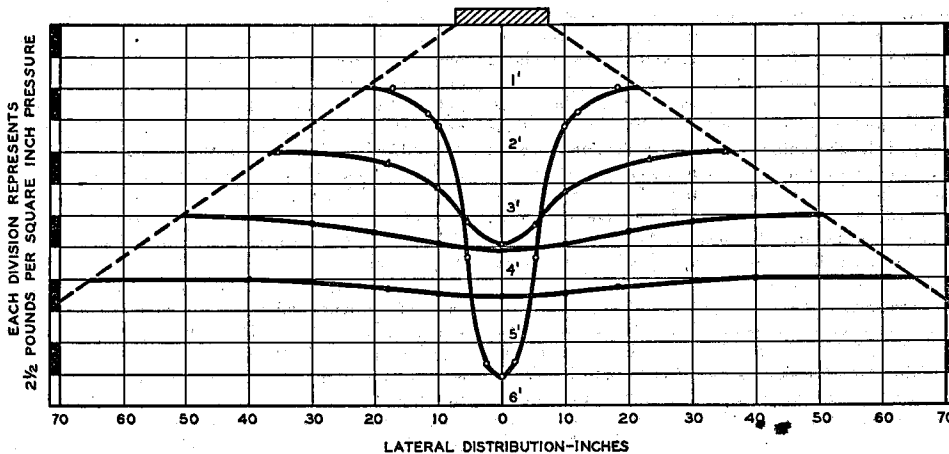


FIGURE C-2.—Pressure distribution in an earth mass at various depths from a 4,000 pound loading on a bearing area of 1 square foot. (From Public Roads, January 1926.)

imum vertical stress occurs on the axis of loading and is inversely proportional in intensity to the square of the depth.⁵¹ When loads are applied through finite bearing areas, as in the case of Marston's experiments, the validity of the law extends only where depths are large in relation to the radius of the bearing area. D. P. Krynine says: "At a certain depth, about twice the diameter of the loaded area, any structure acts practically as a concentrated load."⁵² A. E. Cummings, who considers various distributions of load over the bearing area, also states that, "The character of the pressure distribution at the surface is especially important near the surface but loses its importance as the depth increases."⁵³

Nevertheless, highway engineers looked at wheel loads as point loadings, and disregarding the failure of the concept in the critical region close to the surface, i. e., within 14 to 34 inches for motor-vehicle-tire contact areas, accepted as a practical law that the maximum vertical pressure on different horizontal planes would be inversely proportional to the square of their distance from the point of loading at the surface. From this proportionality it would follow that the depth at which given maximum vertical stresses would occur would vary in direct proportion to the square root of the applied load, and the pavement thickness re-

quired to reduce stresses to a desired limit would vary similarly.

The "law", however, is not valid for distributed loadings applied over motor-vehicle-tire contact areas at depths comparable to pavement and base course thickness. The mathematical theory for such loadings uniformly applied (p. 222 and fig. C-23), shows the vertical stresses immediately beneath the loaded surface to be equal to the applied pressure intensity instead of infinite as with the point loading, and it shows the vertical stresses on the axis of loading to be in the ratio 100:44:13 at depths of 1, 2, and 4 times the radius of a given area, while the inverse squares of these depths are in the ratio of 100:25:6, and finally, it shows these stresses to be proportional to pressure intensity rather than to total load.

But quite aside from such disagreement between mathematical theory and the law of the inverse square, and as a matter of practical design, the maximum vertical stresses, whatever relation they may bear to depth and applied load and dimensions of bearing area, are of no significance unless the capacity of the loaded material to resist failure is understood. And such resistance is based not only on the vertical stresses produced but also on the intensity of the horizontal and shearing stresses resulting from the surface loading. Marston's tests were unique in that they included the problem of lateral thrust as well as the pressure transmitted vertically.

The inverse square theory has persisted in the minds of engineers because circumstances have not required its use.⁵⁴ Maximum wheel loads have been

⁵¹ It is of interest that the Newtonian law of the inverse square applies in this purely theoretical case of a point loading. John H. Griffith, in discussing the more general case in nature, quotes J. H. Poincaré as follows: " . . . the law according to which force varies as function of the distance is perhaps not the law of Newton, but it is an analogous law; in place of the exponent -3, we have probably a different exponent, and it is from this change of exponent that arises all the diversity of physical phenomena . . ." (John H. Griffith, *The Pressure Under Substructures, Engineering and Contracting*, vol. 43, March 1929, p. 117.)

⁵² Krynine on Distribution of Stresses Under a Foundation, Proceedings, American Society of Civil Engineers, pt. 1, October 1924, p. 1293.

⁵³ A. E. Cummings, Distribution of Stresses Under a Foundation, Proceedings, American Society of Civil Engineers, August 1924, p. 821.

⁵⁴ Modifications of this theory, as well as other formulas offered from time to time as suitable for flexible pavement design, are discussed by A. C. Benkelman in *Present Knowledge of the Design of Flexible Pavements*, Public Roads, January 1933.

the criteria in flexible pavement design and these have been accommodated in matters of pavement thickness by the judgment of the engineer and the accumulated experience of trial and error.

The problem of earth foundations under buildings, towers, and similar engineering structures contains many of the elements of the road problem and, though failure of the earth foundation under such structures cannot be tolerated, the question of supportability is here reduced to one of allowable relative settlement rather than failure. Studies of the settlement produced under different conditions of loading have therefore superseded stress distribution studies in foundation research.

The case of the building foundation is sometimes complicated importantly by long time compression resulting from the slow escape of water, but if this one factor is eliminated,³⁵ as it may be through laboratory tests, the settlement can itself serve to indicate the nature of the various stress distributions,³⁶ and the problem becomes the familiar though little understood one of the semi-infinite body loaded over a small area of its bounding plane.

The relation of pressure intensity and area of load application to the surface deformation of loaded materials.—In 1828, following the development of the general equations of elastic equilibrium by Navier in 1821, and their publication in 1827,³⁷ G. Lamé and B. P. E. Clapeyron reported the first solution to the problem of a purely normal³⁸ load applied to an elastic body bounded by an unlimited plane.³⁹ Their solutions, arrived at by means of Fourier's series and expressed as quadruple integrals, were of little use for practical calculations. In 1902, Horace Lamb, utilizing the method of series, expressed the components of displacement and of stress in terms of single integrals involving Bessel's functions.⁴⁰ Again, in 1916, Bessel's functions were employed by Kwan-ichi Terazawa in an extended paper on the problem.⁴¹ The general method, however, is applicable to areas having a circular boundary only, and has so far been developed only in cases where pressure is distributed symmetrically about the center.

In 1885, Joseph Boussinesq published a formal general solution applicable to any form of boundary of the

loaded area and any distribution of the pressure over the area.⁴² This solution, known as the potential method, derives its name through its use of the derivatives of a certain function called by Boussinesq, "le potentiel logarithmique à trois variables", and defined as a certain double integral taken over the loaded area. The difficulty of evaluating the integral has been the major obstacle in the development of the formal solution in special cases. The solution also utilizes derivatives of another function which is the Newtonian potential of a surface distribution and which is likewise defined as a double integral.

Recent applications of the potential method to the cases of the rectangular and the circular bearing area were made in 1924 by S. D. Carothers,⁴³ and much more comprehensively, in 1929, by A. E. H. Love.⁴⁴ Love's equations are employed in calculations made later in this report.

From the application of Boussinesq's solution to the case of a uniformly loaded circular area, the deflection u_c at the edge is seen to be $2/\pi$ or 0.6366 times the maximum deflection, at the center, u_0 , and may be expressed

$$u_c = \frac{4(1-\sigma^2)}{E} pa$$

in which σ is Poisson's ratio, p is pressure per unit of loaded area, a is radius of loaded area, and E is the modulus of elasticity.⁴⁵

According to this solution the surface deflection, in the case of an elastic solid, is in direct proportion to the applied pressure and also varies directly with the radius of the loaded area, which of course implies a similar relation to the diameter, the perimeter, and the square root of the area.

In 1923, A. T. Goldbeck and M. J. Bussard conducted tests to determine the influence of area on the penetration of loaded circular bearing blocks into various sand-clay mixtures. Areas tested ranged from 1 square inch to 9 square feet. Penetrations up to 0.1 inch were recorded. The report concluded: "When a given unit of load is supplied to a soil over various areas, the depth of penetration is directly proportional to the square root of the area over which the load is applied."⁴⁶

The load-deflection diagrams published in the report (fig. C-4) indicate, moreover, that the penetration of a

³⁵ The mechanics of the compression of water from earths are given by Charles Terzaghi in *Principles of Final Soil Classification*, Public Roads, May 1927, and also by C. A. Hogentogler, et al., in *Engineering Properties of Soil*, pp. 177-203. In the following sections of the present text it will be assumed that though the extrusion of a fluid matrix may be a factor in road foundation design it need not be considered in determining the relation between pressure intensity and stress differences resulting from the application of motor vehicle wheel loads to road surfaces.

³⁶ Simultaneous equations for different deflections are solved for the elastic moduli. These may then be employed in the equations for stress at points in the loaded body.

³⁷ *Mémoires de l'Académie des Sciences*, Paris, tome 7, 1827.

³⁸ In engineering usage the word "normal" means "perpendicular to a plane."

³⁹ G. Lamé and B. P. E. Clapeyron, *Mémoire sur l'équilibre intérieur des corps solides homogènes*, *Oeuvres*, Journal, vol. 7, 1831, p. 400.

⁴⁰ Horace Lamb, *On Boussinesq's Problem*, *Proceedings*, London Mathematical Society, vol. 24, 1902, p. 276.

⁴¹ Kwan-ichi Terazawa, *On the Elastic Equilibrium of a Semi-Infinite Solid Under Given Boundary Conditions*, *Journal of the College of Science, Imperial University, Tokyo*, vol. 37, article 7, December 1916.

⁴² Joseph Boussinesq, *Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques*, Paris, 1885. An account of the theory is given in Todhunter and Pearson, *History of the Theory of Elasticity*, vol. 2, pt. 2, 1885, pp. 337-377.

⁴³ S. D. Carothers, *Test Loads on Foundations as Affected by Scale of Tested Area*, *Proceedings, International Mathematical Congress*, Toronto, 1924, vol. 2, pp. 237-249.

⁴⁴ A. E. H. Love, *The Stress Produced in a Semi-Infinite Solid by Pressure on Part of the Boundary*, *Philosophical Transactions of the Royal Society of London*, vol. 228 (series A), November 1926. This was the year of Boussinesq's death.

⁴⁵ Stephen Timoshenko, *Theory of Elasticity*, given as Equation (202).

⁴⁶ A. T. Goldbeck and M. J. Bussard, *The Supporting Value of Soil as Influenced by Bearing Area*, *Public Roads*, January 1928. The same finding was reported by M. Wolterbeek to the *Ministère des Travaux Publics des Pays Bas*, in 1923, which report was abstracted by A. B. Bijla, Chief of Highways and Bridges in Belgium, in *Le Génie Civil*, May 26, 1923, p. 490.

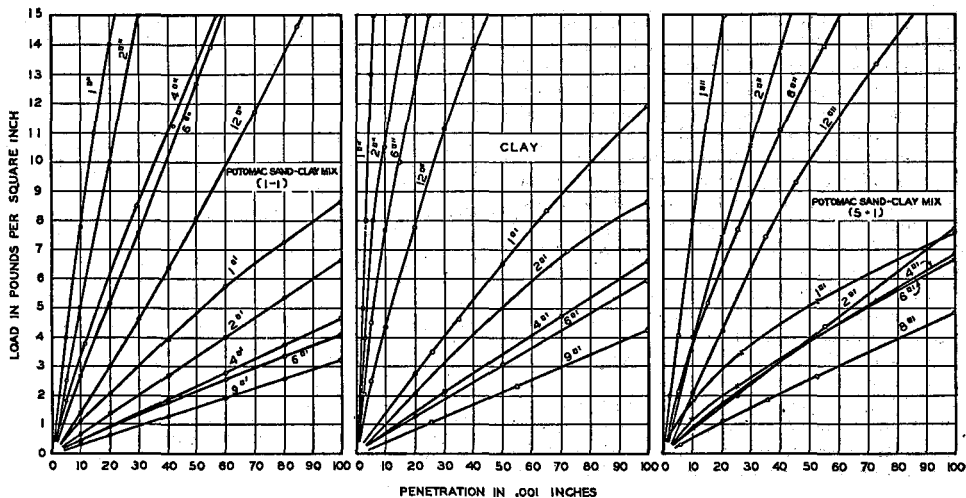


FIGURE C-4.—Bearing value curves for earths. (From Public Roads, January 1926.)

given area is, within the limits of the tests, essentially proportional to the unit pressure and that this is particularly true with respect to the lighter loads and smaller penetrations. Harald M. Westergaard's assumption of a constant modulus of subgrade reaction under concrete pavement in pounds per square inch per inch of deflection for a given uniform subgrade is in accord, though Westergaard stated that it might vary considerably, depending upon the area over which the pressure is applied.⁴⁷

In 1931, John H. Griffith reported the results of a cooperative investigation into the properties of earths by the United States Bureau of Public Roads and the Iowa Engineering Experiment Station. The relations between load and deflection⁴⁸ were calculated, after certain elastic properties had been measured, in accordance with the mathematical theory of elasticity. The resulting values of deflection compared favorably with observed values. The maximum penetrations were 0.1 inch, as in the case of the Goldbeck-Bussard tests. Again the deflections were proportional to applied pressure as well as to the radius of the bearing area.

Evidence from tests as well as competent theoretical analysis could be reviewed at length in support of the foregoing proportionality between penetration in soils and the square root of the bearing area and the unit load. On the other hand, a similar quantity of evidence contradicts these relations, and equally competent authority refuses to apply assumptions that relate to

homogeneous and elastically isotropic solids to the analysis of occurrence in materials which in their natural state are never strictly homogeneous or strictly isotropic, which are more nearly plastic than elastic, and which frequently react as fluids rather than solids.

In 1927, Terzaghi cited two series of soil tests the results of which conform to the square root of the area relation of the elastic theory. He cited another, made on hard pan, which reveals a much greater disparity between the settlement of two areas than would be measured by the difference in their square roots; and also one, made on clean sand with occasional layers of sand clay and yellow clay, which shows a much closer similarity of settlement than the theory could allow.⁴⁹ He said: "Suppose a foundation company sincerely tries to obtain some reliable basis for drawing conclusions from its loading tests. The engineers of the firm make their observations year by year, accumulate the data in their files, and finally, when they try to correlate the data, they face grotesque contradictions of the type quoted."

He then accounted for the contradictions cited by restating a conclusion which he had published in 1925,⁵⁰ i. e., "The relation between the diameter of the loaded area and the settlement produced by a given unit load depends essentially on the cohesion (actual shearing strength) of the soil. For soils with great cohesion, the settlement produced by a given unit load increases in direct proportion with the diameter of the loaded area. On the other hand, for perfectly cohesionless soils the size of the area has very little effect."

⁴⁷ H. M. Westergaard, *Stresses in Concrete Pavements Computed by Theoretical Analysis*, Public Roads, April 1926, p. 24.

⁴⁸ John H. Griffith, *Physical Properties of Earths*, Iowa Engineering Experiment Station, Bulletin 101, Equation 6; also John H. Griffith, *Dynamics of Earth and Other Macroscopic Matter*, Iowa Engineering Experiment Station, Bulletin 117, Equation 112.

⁴⁹ Charles Terzaghi, *The Science of Foundations—Its Present and Future*, Transactions, American Society of Civil Engineers, vol. 36, 1926, pp. 274-276.

⁵⁰ Charles Terzaghi, *Erdbaumechanik*, Vienna, 1925.

In the discussion of Terzaghi's paper, Clement C. Williams and William S. Housel independently advanced a concept⁵¹ which satisfied Terzaghi's conclusion as to the influence of cohesion, but which goes somewhat farther, and which lately has gained considerable acceptance among foundation and highway engineers.⁵² This concept attributes settlement to shear along the perimeter of the bearing block and to compression beneath its area and may be expressed for a given deflection: $P = P_1 m + An$, wherein P represents the total load imposed, P_1 and A are the perimeter and area, respectively, and m and n are coefficients representing resistance to shear along the perimeter and resistance to compression beneath the area.

Referring to the elements m and n , Williams said: "These supporting factors * * * vary with the character of the soil (mineral and granulometric composition, compactness, water content, etc.) and with the depth to which the load penetrates the soil. The two terms are indeterminate, their relative values depending on which represents the more rigid element in the total support".⁵³

Authorities were speculating at this time as to the effect of the shape of the loaded area.⁵⁴ In 1924, Dr. F. Kögler, at Freiberg, Germany, had conducted tests with bearing blocks of various sizes and shapes and related his results to the area/perimeter ratio.⁵⁵ Housel, utilizing an equivalent though slightly different expression of the Williams formula, illustrated from test results and theoretical considerations that the different effects of square and round plates are comprehended by the equation,

$$p = \frac{P_1 m}{A} + n,$$

in which p is the unit pressure required to produce a given settlement not too near the ultimate, and therefore the supporting capacity at that settlement.

Stephen Timoshenko gives the solutions for various rectangular shapes and circles in a generalized equation

⁵¹ Williams on the Science of Foundations, and Housel on the Science of Foundations, Transactions, American Society of Civil Engineers, vol. 93, 1929, p. 309 and p. 326, respectively.

⁵² For example, Roland Vokac, Studies in the Proportioning of Low Cost Bituminous Mixtures of Dense Graded Aggregate Type, Proceedings, Tenth Annual Asphalt Paving Conference, 1932, p. 337. Edwin J. Beugler, Soil Tests to Determine Bearing Capacity, The Military Engineer, July-August 1933, pp. 326-377. Bernard E. Gray, The Design and Construction of Bituminous Pavements, Part I, mimeo., The Asphalt Institute, 1934, p. 9. H. J. Summers, Determining Bearing Power in Earth Foundations, Engineering News Record, April 19, 1934.

⁵³ In 1934, Williams said: "With respect to the first question (What unit load may be placed on a given soil), several factors affect the result, involving both the footing surface and the soil characteristics. The writer has pointed out that the area and shape of the footing constitute one of these factors." He indicates that the earlier form of equation was intended to provide for just that. His elaboration, however, does not alter the form, other than to make it a function of m . (Williams on Earths and Foundations, Proceedings, American Society of Civil Engineers, vol. 60, No. 6, August 1934, p. 854, Equation (70) p. 855.)

⁵⁴ In the discussion of Terzaghi's paper (footnote 49, above), E. P. Goodrich, p. 304; Jacob Feld, p. 321; W. S. Housel, p. 322; Lazarus White, p. 343; E. D. N. Simham, p. 347; and the author, p. 391, all make reference to the influence of the shape of the bearing area on settlement.

⁵⁵ Kögler on Soil Mechanics Research, Transactions, American Society of Civil Engineers, vol. 98, 1933, p. 299. This is the reciprocal of the perimeter/area ratio. It is applicable to sand, and Kögler's use of it may be a result of his experience with that material.

for average deflection, u_a , caused by a uniformly distributed load, P , in the case of an elastic solid,⁵⁶

$$u_a = c_1 \frac{(1 - \sigma^2)}{E} \cdot \frac{P}{A}$$

with the coefficient c_1 depending on the shape of the area. Since P is the unit pressure multiplied by the area, this equation, when appropriately transformed, shows unit pressure (supporting capacity) to be inversely related to the square root of the area. The perimeter/area ratio may also be generalized as a coefficient times the reciprocal of the square root of the area, and the relatives determined from the case of the elastic solid and from the Williams-Housel concept do not differ materially where cohesion and friction are high in relation to compressive resistance, i. e., where n is relatively small.

In discussing Williams' formula, Terzaghi points out that it is not a direct derivation from the theoretical considerations which were offered as a background and that it could be accepted "merely as an empirical equation".⁵⁷ Housel carried the concept much farther than Williams, with very interesting results,⁵⁸ but he repeatedly recognized limitations, and said, "It is apparent that the method of using the straight-line bearing capacity equation is limited in its ability to account for various observations which have been made * * * In addition, this method is limited by the necessity for assuming some constant settlement and analyzing the data for only that particular case. It is obvious that a more general study of soil action is necessary for a better understanding of the relation between the several factors entering into bearing capacity. That the more general relation must include settlement as one of the variables is quite evident * * *".⁵⁹

Housel utilizes Williams' diagram⁶⁰ for the derivation of a relation between settlement, size of bearing area (square) and load, and also adopts Williams' mathematics to the point where⁶¹

$$u_a = \frac{Ph}{I(b^2 + br^2)}$$

in which u_a is the settlement of the bearing block, P is the total load applied, h is the total depth of soil compressed, I is the modulus of incompressibility,⁶² b is the side of the bearing block, and r is the tangent of the angle of load distribution.

⁵⁶ Stephen Timoshenko, Theory of Elasticity, p. 238, Equation (207) and table.

⁵⁷ Terzaghi on the Science of Foundations, Transactions, American Society of Civil Engineers, vol. 63, 1929, p. 288.

⁵⁸ W. S. Housel, A Practical Method for the Selection of Foundations, p. 62.

⁵⁹ Same, p. 62.

⁶⁰ Same, p. 62.

⁶¹ Same, equation (4) p. 63.

⁶² In place of I , Williams uses C , which he terms the modulus of compressibility. Housel states that his quantity I is "analogous to the modulus of elasticity in more rigid substances", and it is definitely so by his definition, i. e., "load per unit of area divided by the settlement per unit of depth."

This expression leads to the following equation for p , the pressure per unit of area required to produce a given settlement u_a ,

$$p = u_a I \left[\frac{b^2}{b^2 h} + \frac{b r}{b^2} \right]$$

Bearing in mind that the area, b^2 , considered in the basic analysis was square, with a perimeter of $4b$, the equation may be further reduced to terms of area A and perimeter P_r , with the following result:

$$p = u_a I \left[\frac{1}{h} + \frac{P_r r}{A} \right]$$

which of course may be generalized into

$$p = \frac{P_r}{A} m + n.$$

Housel's basic equation for u_a may also be reduced to terms of area A and perimeter P_r , for a circular area, with the result that

$$p = u_a I \left[\frac{1}{h} + \frac{P_r r \sqrt{A}}{A} \right],$$

and a more general form, applicable to squares and circles alike is achieved, because in the case of the square the term r/A may be multiplied by any b/b without altering its value and the result will equal $r\sqrt{A}/P_r$. This term, multiplied by $u_a I$ is Housel's m , the coefficient of P_r/A .

The general equation for p , the pressure required to produce a given deflection u_a , may be further reduced to

$$p = \frac{u_a I r}{\sqrt{A}} + \frac{u_a I}{h}$$

in which $u_a I r/\sqrt{A}$ is equal to $P_r m/A$ and $u_a I/h$ is equal to n .

From test results Housel finds that in the lower range of applied pressures in the case of a clay soil, the material exhibits properties of an elastic solid and the value of the second term, i. e., n , in the equation is negligible. With n equal to or approximating zero, the resulting equation is closely allied to that for the elastic solid, which may be reduced to.

$$p = \frac{u_a E}{c_1 (1 - \sigma^2) \sqrt{A}}$$

In terms of the perimeter/area ratio, this may be rewritten

$$p = \frac{P_r}{A} c_2 u_a \frac{E}{(1 - \sigma^2)}$$

wherein $c_2 = \sqrt{A}/c_1 P_r$. Here the curve of p as a function of P_r/A may be considered a logarithmic curve in which the exponent of P_r/A is unity.⁴² The curve of

p as a function of A is also a logarithmic curve in which the exponent of A is minus $\frac{1}{2}$.

These relations hold for Housel's equation in the limiting case when $n=0$, but Housel's equation differs from that of the elastic solid by the absence of any adjusting coefficient c_1 or c_2 to provide for the variation in the shape of the bearing area, from which it appears that either in the limiting case of the elastic solid, the equation is not applicable or that the perimeter/area ratio is an inadequate provision for the factor of bearing area shape. Terzaghi's experience with earths is that "the influence of the shape of the area may be very different, depending on whether the bearing capacity of the material is essentially due to cohesion or to internal friction."⁴⁴ If, as is thus indicated, the P_r/A ratio is not wholly adequate as between solids and earths and as between different earths, one must conclude that in its present stage of development the equation is insufficiently generalized to permit broad interpretation, however convenient a tool it may be in the handling of specific test results.

Housel finds that under increasing pressure his parameters m and n increase in a particular manner with each type of clay earth investigated and he interprets this change in parameters as a change in state of the body under test and, considered in their full range, from initial applications of pressure to pressures beyond the bearing capacity limit, they represent a transition from the solid to the liquid state.⁴⁵

In reviewing theoretical developments one repeatedly returns to the notion that, whatever the nature of the ultimate solution of the problem of deflection in loaded earths, the underlying theory will relate closely to that of the elastic solid. The historical development of physical laws has ever been from the particular to the general. The algebraic results of Boussinesq's general potential theory are dependent for their wide practical usefulness on the constants resulting from the geometrical nature of stress distributions and upon the constancy of the elastic moduli. When a more general solution is made available it will logically comprehend the field of constant moduli merely as a limiting case in the continuity.

The modulus of elasticity, E , so familiar in the mechanics of elastic solids, is perceptibly a constant only in some solid materials and its constant characteristics are definitely limited in all materials. Permanent deformations set in early or late in the deflection-load curve, depending on the properties of the material and rate of loading, and in the case of plastic or ductile "solids" the state of linear elasticity may be so brief in relation to the curvilinear, under normal loads, that Hooke's law must be discarded in an analysis of total reactions. This appears to be the case with earths.

⁴² R. D. N. Simham, of Madras, India, in discussion of Terzaghi's paper (p. 267 et seq., footnote 41) considers at length the various factors influencing settlement as it progresses, and suggests such a logarithmic relation. In connection with the suggestion that the exponent is unity, let it be remembered that Housel's tests were conducted with clay soils.

⁴⁴ Terzaghi on the Science of Foundations, Transactions, American Society of Civil Engineers, vol. 91, 1926, p. 361.

⁴⁵ W. S. Housel, A Practical Method for the Selection of Foundations, pp. 101-112.

MECHANICAL PROPERTIES OF MATERIALS

Elasticity, plasticity, and fluidity.—When an external force producing deformation in a body is removed and the body resumes its former shape, it is said to be elastic. When the force exceeds a certain value and the deformation does not disappear upon its removal, the body is said to be plastic, and its state under a force just equal to this critical value is termed the limit of perfect elasticity. If a constant force causes a deformation which increases continuously with time, the body is said to be viscous. If such a continuous deformation occurs only when the constant force exceeds a certain value the body is a solid, however soft it may be. If no critical value exists for the force causing continuous deformation the body is a fluid, however hard it may be.⁶⁵

These definitions classify materials as solids or fluids. The former have properties of elasticity, plasticity, and viscosity,⁶⁷ in the order given, while the latter have viscosity but lack elasticity and plasticity. The classifications, however, are clear only if the forces of gravity and the stresses intrinsic to the material are alone involved, and if normal atmospheric temperatures are considered. Similarly, the laws of solids or of fluids will apply only as the material remains within the classification and in a state anticipated in the assumptions on which the law is based.

Under sufficiently high velocities, or if confined, water reacts as a solid, and over a sufficient duration of time a glacier flows under the action of gravity. Appropriate temperature changes glass from an elastic solid to a viscous liquid. High pressure causes fluids to assume properties of solids and solids to behave as liquids. Molecular water reacts as a solid. Meanwhile, the ideal "continuous" solid assumed as the basis of all laws of mechanics is non-existent, and theories concerning it would fail were it not for the smallness of the particles in relation to the volume of the mass to which the laws

are applied, and for the random nature of their orientation.⁶⁶

Time, temperature, pressure, and particle dimensions are thus the criteria of classification. They are the relatives which indicate differences between materials of degree only. When their full range is considered, differences in kind vanish to the point of insignificance.

Thus it is that the deflection-pressure (strain-stress) curves for various materials regarded as solids closely resemble each other as to form.⁶⁹ They originate in a regime of elastic deformation, some part of which is a line of proportionality.⁷⁰ They pass through a transition phase in which elastic deformation combines with viscous flow to form a plastic state. They terminate in a regime where viscous flow is controlling.⁷¹

The magnitude of pressure or of deflection at which these stages appear is entirely dependent on the characteristics of the particular material and does not appear to be readily subject to generalization. The limits, however, are plain.

During the initial period of linear elastic deformation, even though in a given material it may be imperceptibly brief, the laws of linear elasticity are applicable and the surface deflection produced by a uniformly applied pressure over a given area varies directly with the pressure intensity, i. e., according to a coefficient times the pressure to the first power. For different areas the deflection varies directly as their square roots.

During the final period of viscous flow, and beyond a critical load intensity, the deflection is continuous with time, and varies therefore according to a coefficient times the pressure to a power which approaches infinity, i. e., the curve during this regime is essentially a straight line vertical to the base and parallel to the deflection axis.⁷² The particular position occupied by this section of the curve with reference to the base, i. e.,

⁶⁵These definitions are in accord with those expounded by James Clerk Maxwell in his *Theory of Heat* (4th ed.), pp. 295-298.

⁶⁶In the light of these definitions "plasticity" may be considered as a transition state between the elastic regime, where time is not a factor of primary consideration, and the viscous state, where time is controlling. The student of rheology, however, will take exception to this terminology, preferring to consider the flow that occurs continuously with time, beyond the elastic and plastic stages, as "plastic flow", and reserving the term "viscous flow" for that deformation, continuous with time, which requires no fixed minimum load for its occurrence. (See Eugene C. Bingham, *Plasticity and Elasticity*, Journal of the Franklin Institute, vol. 197, No. 1, January 1924, pp. 99-115; *First Report on Viscosity and Plasticity*, Committee for the Study of Viscosity, Academy of Sciences, Amsterdam, 1935, pp. 5-16; and Proposed Revised Tentative Definitions of the Terms Consistency and Plasticity, American Society for Testing Materials, Designation E 24-T, issued 1934, revised 1937.) This would preclude the use of the term "viscosity" in connection with solids.

⁶⁷Within useful limits these definitions appear quite satisfactory, but, strictly speaking, so long as surface tension exists in the fluid (as it does in liquids) the system may be subjected to some small stress without continuous deformation. The rod floating on water is an example. Similarly, the assumption that elastic deformation is independent of time is in the strictest sense inaccurate, if very small values of time are considered, because whenever a system in nature changes from one configuration to another it can do so only with the passage of time.

⁶⁸Lord Kelvin (Sir William Thomson) in his *Mathematical and Physical Papers*, vol. 1, p. 291, refers to the relative quality of homogeneity. A village of houses may be a homogeneous mass from the point of view of dynamical behavior. For further discussion see Stephen Timoshenko, *Theory of Elasticity*, p. 1; Eugene C. Bingham, *Fluidity and Plasticity*, pp. 215-217, 229-231; A. Nédal, *Plasticity*, Chs. I-IV, in: John H. Griffin, *Dynamics of Earth and Other Macroscopic Matter*, Bulletin No. 117, loc. cit., pp. 29 and 117; Richard V. Southwell, *Strength of Materials*, Encyclopaedia Britannica (14th ed.), vol. 16, pp. 61-57; and Gustav Tammann, *The States of Aggregation*, pp. 1-5.

⁶⁹Indefinitely extended solids bounded by one plane surface are being considered, so that sudden reversals, as in the bending of a bar or stretching of a wire, are irrelevant.

⁷⁰Even though it be infinitesimal.

⁷¹R. N. Traxler suggests the use of "permanent deformation" in place of "viscous flow" and D. P. Kryzine offers the term "reversible" for deformations which are elastic and "irreversible" for those which are plastic. (Communications to the Federal Coordinator of Transportation.) These suggestions afford valuable clarification but their use would leave the distinction incomplete. "Viscosity is a general term," says Love, "for all those properties of matter in virtue of which the resistance which a body offers to any change, depends upon the rate at which the change is effected." (A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity*, (1927) p. 117.)

⁷²This kind of a plot considers deflection to be a dependent function of pressure and therein differs from the usual stress-strain diagram. In the viscous regime the rate of deflection varies directly with the critical pressure, but increases in deflection no longer depend on increases in pressure.

the pressure at which the viscous state occurs, for different areas, depends altogether on the range of pressure through which the particular material has sustained the loaded area during the elastic and plastic periods. Otherwise, according to the conditions which characterize viscosity of solids, as soon as the critical unit loading is reached by each deflection curve it would become perpendicular to the base and curves for different areas would coincide thereafter.

Shear.—The degree of deflection to which a given material will sustain a loaded area during the elastic and plastic stages depends on its resistance to internal structural deformation, the mechanism of which is complex and surrounded with controversy.

Much of the diversity of view may be attributed to a failure to harmonize into a common working theory the reactions of microscopic structural elements with those of a macroscopic or visible nature. Such a harmonization need not be minutely precise. The essential correctness of a mechanical theory lies in its effectiveness, even though its fundamental assumptions in an exact sense may be known to be false, its empirical determinations merely approximations, and its mathematics only relatively precise.

Griffith, more persistently and clearly than any other recent writer, has led the way to this kind of synthesis through established principles of mathematical physics.⁷⁵ He states: "It is assumed as a fundamental hypothesis in formulating physical laws that like fields, acting on like mechanical systems, occasion like dynamical behaviors. In the language of the philosopher Spencer, "The entire process of things displayed in the aggregate of the visible universe is analogous to the entire process of things as displayed in the smallest aggregates."'

There is much reason to agree with Griffith, and, as has already been pointed out, to consider the element of particle dimension as affecting "degree" rather than "kind." But Griffith's equations contain coefficients which must be experimentally determined. This, of course, is true of the most firmly established laws of mechanics when applied to a given structural material. The real difficulty, however, in the case of materials passing quickly over the elastic and into the viscous stage lies in the fact that these coefficients are variables, not only as between different materials, but also as between different states of stress in the same material, and the determination of moduli for all values of stress is essentially synonymous with the determination of strain for all values of stress.⁷⁴

⁷⁴ John H. Griffith, *The Pressure Under Substructures*, Engineering and Contracting, March 1930, p. 118; John H. Griffith, *Physical Properties of Earths*, Iowa Engineering Experiment Station, Bulletin No. 101, 1931; John H. Griffith, *Dynamics of Earth and Other Macroscopic Matter*, loc. cit., Bulletin No. 117, 1934.

⁷⁵ This may be said generally of the more important contributions which have recently been made. The determination of angles of friction, stress-differences, and coefficients of compressibility or rigidity are well nigh hopeless unless a full range of observations is taken.

Internal structural deformation under stress consists of (1) deformation of the particles with respect to their original shape, (2) compression, readjustment, or extrusion of the boundary matrix or dispersion medium that may surround the particles, and (3) relative displacement of the particles with respect to each other. These occur separately or together, and the degree to which and the order in which each occurs varies with different materials and importantly influences their elastic and plastic properties.

A particle in the sense above is wholly a relative thing. In a strictly amorphous material, the molecular arrangement is entirely orderless,⁷⁶ and the term "particle" in relation to it can have no significance except as it refers to the individual molecule. A crystalline material, on the other hand, has a definitely ordered molecular structure in which the particle becomes perceptible under the microscope and consists of a minute element of the crystal grain bounded by planes of weakness in the molecular lattice. According to Hogentogler and Terzaghi, clay, silt, and sand may be distinguished by the size of their particles, which in the case of clay are less than one two-hundredth of a millimeter, and in the case of sand are greater than one-twentieth of a millimeter in diameter.⁷⁶ In reference to a coarse gravel, the term may apply to a sizeable stone. Particles may thus be defined as those component elements of a body having the greatest relative independence and having certain mechanical properties in common which influence the reactions of the body as a whole.

The boundary matrix must be similarly regarded as a relative element in the mass structure of the material. In absolute characteristics it may differ significantly between different materials. In a dense and highly crystalline substance such as steel the intimate affinity between the boundaries of the particles in the crystal grains and between these grains themselves may minimize the presence of a perceptible matrix, and the unordered arrangement of molecules in an amorphous material such as glass may quite preclude it. Yet it appears that the study of changing states under a sufficient range of stress is akin to the study of a changing material, and that in both the amorphous and the crystalline arrangements of molecules critical conditions of strain introduce a varied structure in

⁷⁶ This is by definition only. H. M. Dawson refers to a "microcrystalline" structure in many "apparently" amorphous substances, and he prefers to consider others as supercooled viscous liquids (*Physical Chemistry, Encyclopaedia Britannica* (14th ed.), vol. 5, p. 365). A. Nédal (*Plasticity*, p. 284) refers to researches of S. E. Sheppard, R. C. Houck, M. Rabier, and G. W. Stewart indicating a "regional orientation of molecules" in liquids, Eugene C. Bingham (*Fluidity and Plasticity*, pp. 95-97) discusses "crystalline liquids," and Gustav Tammann (*The States of Aggregation*) devotes a chapter to "liquid crystals."

⁷⁷ As adopted by the United States Bureau of Public Roads. See C. A. Hogentogler and Charles Terzaghi, *Interrelationship of Load, Road, and Subgrade*, Public Roads, May 1929, p. 46; also accepted by Bernard A. Koen in *The Physical Properties of the Soil*, p. 205, though he presents a table of different values "now in use in Great Britain," Bernard A. Koen, *The Physical Properties of the Soil, Encyclopaedia Britannica* (14th ed.), vol. 20, p. 222.

which elements having certain characteristics are dispersed among an element having quite different characteristics.

Henry S. Prichard conceived of the flow by which deformation in steel and wrought iron is accomplished "as consisting of the sliding relatively to each other of perfectly elastic solid particles separated by thin highly viscous mobile or liquid films."⁷⁷ Photomicrographic studies by J. A. Ewing and Walter Rosenhain had substantiated that the particles comprising the crystal grain were deformed with respect to their original shape during the period of elastic deformation and actually slipped along well defined cleavage planes during plastic deformation.⁷⁸ Various authorities have since witnessed the same phenomenon with the aid of the microscope.⁷⁹ The theory of the liquid films separating the crystal particles was first conceived and slowly developed through photomicrographic studies made over an extended period of years (1900-1915) by George T. Beilby. The investigations revealed that polish on the surface of crystalline solids produced by rubbing is the result of surface flow of dislodged molecules which solidify into an amorphous film upon removal of the force. From them he developed the reasonable hypothesis that a similar liquefaction and flow occur during slipping between the surfaces of the crystal particles.⁸⁰ And he referred to the resulting amorphous solid material as "a rigid matrix to the deformed grains which are enclosed within it."

Zay Jeffries, who accepts the amorphous theory merely as a partial explanation of strain- or work-hardening because of the quantity of amorphous cement that would have to be developed by the extremely small movement between the individual particles of the crystal grain, says, nevertheless, "At the grain boundaries the conditions are different. The shapes of the original grains must change in a manner depending on the change in shape of the piece of metal taken as a whole. No matter what the direction of extension, the grain boundary surface is increased by cold work. Each slip intersecting a grain boundary produces one "lapped-over" fragment, the surface atoms of which may be utilized to increase amorphous metal at the grain boundary. As deformation becomes severe, the grain

boundary layer of amorphous metal should become thicker by the same process."⁸¹

B. P. Haigh states: "In any body of ordinary metal, some parts are crystalline and other parts are amorphous; in particular, the crystal boundaries are amorphous, and amorphous layers surround particles of slag and other constituents."⁸²

The concept of a matrix in a crystalline solid is introduced in a slightly different way by Wilder D. Bancroft, who, incidentally, makes the rather too sweeping statement that, "The weight of evidence seems to indicate that the alleged presence of an undercooled liquid in metals and alloys is a myth." He points out, "Had the proponents of the hypothesis been willing to be satisfied with arguing that at a point of contact of two crystals the fields of force that determine the arrangement of the molecules in the space lattice and in the crystals extend beyond the individual crystal, so that the outer layer or layers of molecules are directioned, not by a single field of force, but by the resultant of two (or more) such fields—resulting in surface conditions, surface energy, and surface forces different from the conditions of energy and forces in the body of the crystal—little objection could be raised to the theory."⁸³ Boundary forces in the crystalline aggregation as well as the boundary forces in the molecular aggregation may thus, for practical purposes, also be considered as a matrix.

In a purely amorphous material⁸⁴ on the other hand, the behavior will be quite different. The absence of crystalline slip planes as well as absence of an ordered molecular structure affords no definite limit of deformation before liquefaction occurs. External stress will tend to change the positions of the random spaced molecules and will vary in its intensity of effect on each, and, in the words of Frederick A. Lindemann, "What might be called local liquefaction occurs in innumerable minute regions dispersed at random through the body. It becomes soft and viscous and changes gradually by imperceptible degrees into a liquid."⁸⁵

In a plastic clay the matrix already exists and through its mixture of air, water, organic substances, absorbed salts, colloids, etc., the solid particles are dispersed. In a dry sand or gravel the dispersing medium may consist entirely of air.

Deformation of the particles with respect to their original shape occurs more readily in an elastic solid than either deformation of the matrix or the displace-

⁷⁷ Henry S. Prichard, *Effects of Straining Structural Steel and Wrought Iron*, Transactions, American Society of Civil Engineers, vol. 39, 1916, p. 1501.

⁷⁸ J. A. Ewing, *Strength of Materials*, Encyclopaedia Britannica (11th ed.), vol. 25, p. 1916; Richard V. Southwell, *Strength of Materials*, Encyclopaedia Britannica (14th ed.), vol. 16, p. 65; and Henry S. Prichard, *Effects of Straining Structural Steel and Wrought Iron*, Transactions, American Society of Civil Engineers, vol. 39, 1916, pp. 1505-1522.

⁷⁹ A. Nádai, *Plasticity*, pp. 30-34, 284; Richard V. Southwell, *Strength of Materials*, op. cit., p. 65.

⁸⁰ George T. Beilby, *Aggregation and Flow of Solids*, chs. V-VIII, inc.; Richard V. Southwell, *Strength of Materials*, op. cit., p. 65. The term "reasonable" is used advisedly and not with disregard of the opposition that may be attributed largely to the more comprehensive reaches of the "amorphous theory" as developed by Beilby, G. D. Bengough, and Rosenhain. (See G. D. Bengough, *A Study of the Properties of Alloys at High Temperature*, Journal, Institute of Metals, No. 1, 1913, pp. 128-190.)

⁸¹ Zay Jeffries and Robert S. Archer, *The Science of Metals*, p. 83.

⁸² B. P. Haigh, *The Theory of Rupture in Fatigue*, Proceedings, First International Congress for Applied Mechanics, Delft, 1924, p. 231.

⁸³ Wilder D. Bancroft, *Applied Colloid Chemistry* (3d ed.), p. 453.

⁸⁴ Solid materials in quantities available for engineering purposes are never purely crystalline or purely amorphous.

⁸⁵ Frederick A. Lindemann, *Theory of (the) Solid State*, Encyclopaedia Britannica (14th ed.), vol. 20, p. 947.

ment of the particles with respect to each other.⁵⁶ In the case of earths, the reverse of this is generally true.⁵⁷ In either case, if the action is insufficient to break down the molecular attraction or other intrinsic pressure, the work that has been done is transformed into potential energy of strain and the body resumes its former shape upon removal of the load.⁵⁸

Deformation of the matrix in earths will depend on the permeability of the material, which is a function of the relative distribution of the particles by size, the smallness of their average size, their angularity and shape, and of their state of consolidation.⁵⁹ It follows the laws of fluid friction, but need not be permanent in the presence of capillarity or an abundance of scale-like particles in the material, and need not be accompanied by a reorientation of the particles with respect to each other. Deformation of the matrix in a dense material such as steel also follows the laws of fluid friction, but is an accompaniment to some permanent displacement of the particles relative to each other.

In all solid materials this permanent displacement must take place before the material can be said to be plastic. It is known as shear, and the degree to which it occurs depends on two sources of resistance termed cohesion and friction.

The resistance to shear offered by cohesion.—The molecular theory of solids holds that, "The characteristic of having shape reduces to the fact that the molecules are held in position by the forces they exert upon one another", that a maximum force of attraction between molecules occurs at distances comparable to their diameters, and that this force diminishes with rapidity as the distances between the molecules increase. This attraction, which is probably electro-magnetic in origin even in fine-grained earths, is referred to as cohesion. It is effective in the solid state in both the crystalline and the amorphous aggregations. The liquid and gaseous states are a result of an increase in the kinetic energy of the molecules which enables them to partially

(in the liquid) or completely (in the ideal gas) overcome the forces of mutual attraction.⁶⁰

In the case of a crystalline metal a special condition is present. Rosenhain states: "The forces which are at work in producing cohesion between adjacent crystals must, however, be of a somewhat different character, for it is obvious that the regular arrangement of molecules in oriented layers cannot be carried on through a crystal boundary, while it appears that the actual cohesion between adjacent crystals is stronger than that between different layers of the same crystals."⁶¹ If during the formation of the crystal grains the boundary molecules form incomplete space lattices and are more or less randomly distributed as a result, an explanation may be present in the finding of Beilby that in most if not all ductile metals the amorphous state is the hard state, and the crystalline the soft,⁶² yet, from the fact that the crystal grain has a definite boundary which need not, in an unstressed state, take the form of an amorphous matrix, still another explanation may be deducted.

According to Lindemann, "The mutual attraction of the molecules gives rise, of course, on the surface of a solid to an uncompensated inward attraction which is known in liquids as surface tension." Alfred W. Porter says, regarding the surfaces of materials: "There is, therefore, a thin, indefinitely bounded, layer near the surface which is in a different condition from the main

⁵⁶ Frederick A. Lindemann, *Theory of the Solid State* Encyclopaedia Britannica (14th ed.), vol. 20, pp. 946-947; James H. Jeans, *Kinetic Theory of Matter*, Encyclopaedia Britannica (14th ed.), vol. 18, p. 886; and H. M. Dawson, *Physical Chemistry*, Encyclopaedia Britannica (14th ed.), vol. 6, p. 388. In the articles cited Lindemann and Dawson speak of the "molecules" in describing the solid, the liquid, and the gaseous states, as do many other recent writers; yet Jeans states, "Modern research on the crystalline structure of solids has made it clear that in a solid the atoms form independent units, and that the best motion is not a motion of complete molecules but of complete atoms only. In a sense, there are no molecules in a solid." He adds, "In a mass of common salt the atoms of sodium alternate with atoms of chlorine, but a given atom of sodium is no more closely associated with the atom of chlorine on its right than that on its left." He also says that, "The same general conclusion emerges from a study of the specific heats of solids." Jeans reverts to "molecules" in the case of a liquid and gas. Dawson, who is careful only in the case of his "crystalline" solid to use the term "atoms or molecules" or "atoms (molecules)", discussed the case of common salt, which is strongly polar (and therefore crystalline), in the light of X-ray studies by M. Laue, W. H. and W. L. Bragg, and others. He says: "The material structural elements are sodium and chlorine ions, and molecules of sodium chloride in the ordinary sense are nonexistent. Substances of non-polar character give X-ray spectra, on the other hand, which clearly show that the lattice points are associated with molecular units." Lindemann refers to peculiarities of metallic crystals and suggests the assumption of an electron space-lattice. He summarizes by using the terms "atom" and "molecule" as though they were interchangeable. He clarifies his paper by so doing. That is also an objective here.

⁵⁷ Walter Rosenhain, *An Introduction to the Study of Physical Metallurgy*, pp. 257-258; James A. Ewing, *Strength of Materials*, Encyclopaedia Britannica (11th ed.), vol. 25, p. 1018.

⁵⁸ George T. Beilby, *Aggregation and Flow of Solids*, p. 145.

⁵⁹ Extensive discussion of the progressive action of strain in metals is presented by Henry S. Friehard, in *The Effects of Straining Structural Steel and Wrought Iron*, Transactions, American Society of Civil Engineers, vol. 80, 1918, pp. 1429-1442. The particular characteristics mentioned above is cited in a quotation from Walter Rosenhain, p. 1521. See also Richard V. Southwell, on *Strength of Materials*, op. cit., p. 64.

⁶⁰ Glennon Gilboy, *Soil Mechanics Research*, Transactions, American Society of Civil Engineers, vol. 86, 1933, p. 226.

⁶¹ Eugene C. Bingham, *Fluidity and Plasticity*, on "Elastic Deformation, Plastic, Viscous, and Turbulent Flow", p. 4; and Stephen Timoshenko and Glennon H. MacCullough, *Elements of Strength of Materials*, on "Elasticity", p. 2.

⁶² R. N. Traxler and L. A. H. Baum, *Permeability of Compacted Powders*, *Physica*, vol. 7, January 1936, p. 2.

body of the substances. Many of the properties will be different in this surface layer from elsewhere * * *. The most prominent peculiarity is that it possesses more potential energy than the rest of the body per unit of volume."⁸³

The action of the "uncompensated inward attraction" or resultant surface force is twofold. Its first effect is toward a reduction of surface area. The second law of thermodynamics requires that free energy tend toward a minimum, and since the free surface energy of a liquid is the product of its surface tension and the area of its surface, and since the surface tension at a given temperature is constant, the tendency can only be toward a reduction of area.⁸⁴ If the mass in its liquid state is large and if gravity is opposed, the forces of gravity will predominate, but if the mass is small or if gravity is not opposed the normal form assumed will be spherical. Familiar instances are the raindrop, the drop of lead, and the mercury globule. Beilby was able to obtain films of gold, silver, and platinum so thin that the forces of surface tension were enabled to draw them into new forms of aggregation even though the materials were still in the solid state, and he concluded, "At the surfaces of solids and liquids alike these potential forces are always present, and ready, as it were, to take advantage of any increase of the mobility of the molecules to impress their characteristic forms of aggregation upon them."⁸⁵

The second effect of the "inward attraction" is referred to by Maxwell as "cohesion" if it acts between "neighboring parts of the same substance" and as "adhesion" if it acts "between portions of matter of different kinds." Porter refers to both manifestations of the attraction as "surface action", and quotes Newton to the effect that, "The parts of all homogeneal hard bodies, which fully touch one another, stick together very strongly."⁸⁶ This is now known to be true of substances of the same kind or of different kinds as well as of different states. Terzaghi says: "That any two bodies if brought into contact stick to each other and that the total force is a function of the total points of contact."⁸⁷ And Eugene C. Bingham gives as an example the familiar case of glass, two surfaces of which "ordinarily touch each other at very few points and they do not adhere strongly, but when the two surfaces are ground to an optical surface and cleaned, it is difficult to separate the two surfaces without tearing them after they have been brought together."⁸⁸ In this illustration the term "cleaned" may be only relative and the strong cohesion may be the result of adsorption of a thin layer of air or moisture

on the glass surfaces; but, even so, it would imply an adhesion between the adsorbed air or moisture and the glass. Beilby conducted experiments in which he was careful to remove most of such influences between the contact surfaces of glass and minute globules of metals (including polarization of the globules) and found that the forces were sufficient between the resulting small points of contact to permit the suspension of globules up to a certain size from the glass surface.⁸⁹

This second effect of the uncompensated surface forces is, like molecular cohesion, dependent on distance. According to Maxwell, "These forces are quite insensible between two portions of matter separated by any distance which we can measure. It is only when the distances become exceedingly small that these forces become perceptible."¹ It appears therefore that this effect varies with compression or compaction of a material as these affect closeness of contact and total area of contact. The intimate three-dimensional mosaic formed by the irregular crystal grains in a metal is thus a direct source of strength, and the actual total surface area of the grains, which is a function of their smallness, is also such a source. Similarly, compaction, or reduction of grain size, or grading so as to reduce void space,² in a granular medium, materially adds to its resistance to deformation.

In the case of earths, molecular cohesion offers resistance to deformation and displacement as does the surface cohesion and adhesion between particle and particle. But when moisture is present in minute films between the particles, a joint effect from the adhesion between the particles and the moisture and from the surface tension in the moisture becomes an added and very important source of strength.

This joint effect is commonly known as capillary action, and may be effectively illustrated by the familiar rise of a column of water in a hair-size tube, because it is readily understood that the force acting to raise the water is also acting on the walls of the tube to draw them together. The illustration, however, leads to the misconception that this action depends solely on the presence of tubes or pores, and it does not clarify the fact that a minute particle of water adhering to two particles of soil also results in capillary action and that the term "capillary" refers only to the distance between the two surfaces with which the moisture is in contact.

Hogentogler states that, "Very fine vapor films have an adhesive power so great that they cannot be removed from glass by heating at a temperature up to 500° C."³ Maxwell presents an equation for the surface forces in a liquid film drawing two plates together, and says: "The force thus produced by the introduction of a

⁸³ Alfred W. Porter, *Surface Tension*, Encyclopaedia Britannica (14th ed.), vol. 21, pp. 595, 597.

⁸⁴ William Thomas, *Adsorption*, Encyclopaedia Britannica (14th ed.), vol. 1, p. 183.

⁸⁵ George T. Beilby, *Aggregation and Flow of Solids*, p. 26.

⁸⁶ Alfred W. Porter, *Surface Tension*, op. cit., p. 562.

⁸⁷ Charles Terzaghi, *Principles of Final Soil Classification*, Public Roads, May 1927, p. 38.

⁸⁸ Eugene C. Bingham, *Fluidity and Plasticity*, p. 261.

⁸⁹ George T. Beilby, *Aggregation and Flow of Solids*, pp. 71-78.

¹ James Clerk Maxwell and Lord Rayleigh, *Capillary Action*, Encyclopaedia Britannica (11th ed.), vol. 5, p. 251.

² R. N. Traxler, L. A. H. Baum, and C. U. Pittman, *Experimental Determination of Void Content of Close-Packed Mineral Powders*, Industrial and Engineering Chemistry, vol. 5, May 15, 1933, p. 165.

³ C. A. Hogentogler, *Soil Road Surfaces*, Public Roads, February 1935, p. 276.

drop of water between two plates is enormous, and is often sufficient to press certain parts of the plates together so powerfully as to bruise them or break them."⁴

Capillary force is inversely proportional to the radius in the case of a tube, inversely proportional to the distance of separation in the case of plates, and inversely proportional to the radii of the spheres in the case of globular particles in contact.⁵

Terzaghi distinguishes between the intrinsic pressure in soils resulting from capillary forces, which he calls "apparent cohesion" and that resulting from molecular attraction between the particles which he calls "true cohesion," but the distinction itself is more apparent than true.⁶ Bernard Keen points out that, "With very fine soil particles high stress values are found experimentally, but these are not due entirely to the moisture film. The organic material acts as a binding or cementing agent, and physical forces associated with colloidal material also operate." He cites tests in which the crushing strength of briquettes made with clay colloids and sand was found to be 19 times the strength of portland cement briquettes having the same percentage of sand.⁷ And Terzaghi refers to water retained in very fine soil fissures (smaller than 0.0001 millimeter) which reacts as a solid and does not evaporate even if heated to the boiling point of normal water.⁸

It has been pointed out that a maximum force of attraction between molecules occurs at distances comparable to their diameters and that this diminishes with rapidity as the distances increase. It may now be stated that the attraction also diminishes with increasing rapidity as the distances decrease below the zone of maximum attraction, very quickly reaching a boundary below which actual repulsion increases at this increasing rate. This boundary of zero attraction and zero repulsion marks the zone across which the molecules oscillate in a sort of dynamic equilibrium between the two forces.⁹

The mechanics of these opposing forces between molecules may perhaps best be conceived if it is assumed that two molecules are brought into such an equilibrium (the solid state) with respect to each other. The net force between their points of nearest approach is zero, because their mutual repulsions balance their mutual attractions. If they are moved apart, the attractions will increase up to a critical point in the manner stated

and then decrease very rapidly. If they are forced together the repulsions will increase at an increasing rate indefinitely. At other points on their surfaces the same forces will be present but will have a net attractive influence. If a mass of such molecules is considered, it is clear that a molecule in the interior is acted upon equally from all sides, while adjacent molecules at the surface are definitely held only to the mass and to each other.

The force intensity that holds the surface molecules to each other is clearly a constant, for like molecules, and will be the same however large the mass of the aggregation may be. In a sense, it is a measure of the work that would be required to separate these surface molecules from each other. If the system were complete and independent, i. e., if it did not involve the presence of other molecules within the attractive range of the surface, it might be assumed that the surface molecules would have no net attraction for each other, because any closer approach would be met by repulsion, and that the resulting equilibrium might offer a resistance to change called surface tension, but that in this surface tension there could be no contractile properties. Strictly, and in effect, that is not the case.

The oscillations of the interior molecules are symmetrical, because their inward and outward swings are opposed by forces which exactly balance. This is not true of the surface molecules which on their "inward incursions are repelled by their neighbours, but on their outward swings are limited by the attraction of these same neighbouring molecules."¹⁰ Lindemann adds, "* * * it is this very attraction which gives rise to the internal pressure in solids." It does so because the repulsive force is greater than the attractive force for a given molecular movement normal to the surface, which would result in the surface molecules escaping from the aggregation were they not held to each other. And whether one looks on it as an outward pressure from the inside or an inward pressure from the outside does not matter. The holding together of the surface molecules involves their movement away from each other up to the point of equilibrium of these surface force peculiarities and they are constantly ready to return toward each other upon removal of these peculiarities.

The system, however, is not complete, and the free molecular surfaces will attract all of the molecules from the material adjoining or surrounding the aggregation that they can. These attracted molecules will, if they are in a fluid state, crowd each other around the surface, exerting another compressive or contractile influence. But it is obvious that this external influence will detract from the internal one by its own value and will not alter the surface tension constant. In very thin filaments of materials ordinarily classifiable

⁴ James Clerk Maxwell and Lord Rayleigh, *Capillary Action*, op. cit., p. 263.
⁵ The analogy of globular soil particles is employed by W. B. Haines and R. A. Fisher (Bernard A. Keen, *The Physical Properties of the Soil*, pp. 109-146) in contrast to the capillary tube analogy as employed by Terzaghi (Charles Terzaghi, *Principles of Final Soil Classification*, Public Roads, May 1927, pp. 30-41).

⁶ Charles Terzaghi, *Erdbaumechanik*, p. 94. This is also discussed in his extensive paper, *Principles of Soil Mechanics*, eight parts, *Engineering News-Record*, November 6 to December 31, inclusive, 1925. He states the interesting fact, however, that "the true cohesion is very small compared with the apparent cohesion" (*Engineering News-Record*, November 4, 1925, p. 743).

⁷ Bernard A. Keen, *The Physical Properties of the Soil*, p. 134.

⁸ Charles Terzaghi, *Principles of Final Soil Classification*, op. cit., p. 51.

⁹ See footnote 50, p. 211.

¹⁰ Frederick A. Lindemann, *Theory of (the) Solid State*, *Encyclopaedia Britannica* (14th ed.), vol. 20, p. 608.

as viscous liquids, this external influence may be presumed to be dominating, and thus account for the unusual tensile strengths observed in the laboratory.

The attractions and affinities usually referred to as cohesion, adhesion, surface tension, interfacial tension, capillary action, and adsorption are all manifestations of these residual attractive forces in the surface of a solid or fluid after its molecules have attained positions of relative equilibrium with respect to their own intermolecular forces of attraction and repulsion under the energy conditions induced by temperature, hydrostatic pressure,¹¹ and electrical or chemical action. They are not readily classifiable separately, because within the range of these controlling influences they are closely related. Their common behavior is a resistance to shear through resistance to any increase in intermolecular distances. They are the forces of cohesion.

In a perfectly homogeneous solid the intrinsic pressure of cohesion is present before the application of external load and upon its application will resist sliding between adjacent molecules along the planes of shear insofar as this sliding represents separation of the molecules beyond the distances of their natural equilibrium. As shearing stresses, which may be considered proportional to the applied load, are increased the resulting molecular separations are met by an increasing resistance up to the point of their maximum mutual attraction, beyond which the resistance very rapidly declines, and, were there no other form of resistance, permanent separation would occur.

The total cohesive resistance at which this permanent separation of molecules occurs, obviously depends on the number of molecules affected, and therefore on area of the rupture surface. Its intensity, however, is a constant for any solid material in a given state. As applied loads increase the shearing stress along the planes of shear, they also increase the opposing stresses perpendicular to those planes, thereby causing closer compaction of the molecules along their respective surfaces. In the perfectly homogeneous solid considered, this movement would result in added intermolecular repulsion exactly equivalent to the added force, and the net effect on the forces of attraction would be zero. If, however, the material is an agglomeration of hard particles dispersed in a softer matrix, or if it is a composite of solids, liquids, and gases having different surface tensions, as in the case of a soil; or if there are minute voids too great for these surface forces to span, the compaction resulting from the increased force may increase the cohesive resistance to shear.

In a perfectly homogeneous material this cohesive resistance to shear is, except for lateral effects, identical with the tensile strength. This has repeatedly been pointed out,¹² largely under the influence of A. A.

Griffith, who presented a theory and experimental results leading to the conclusion that "rupture stress is comparable to the intrinsic pressure of the solid."¹³

The equilibrium of the oscillating molecules (or atoms) in the solid state, as hypothesized by the kinetic theory,¹⁴ is a relative equilibrium which attains actuality only at the uncommonly low temperature of 273° below zero on the Centigrade scale. Temperature above this provides the molecules (or atoms) with the heat energy which sets them in vibration with respect to each other, though the amplitude of individual oscillations is controlled by chance. In the solid state the average excursions occur within the zone of maximum attraction, but do not permit the molecules to escape from their average positions with respect to each other. If separation is induced in this state by outside forces, the energy will be given off as heat, will be retained as potential energy of strain to be used up in the elastic return, or will go into the formation of new surface and exist as surface energy.¹⁵

As heat is added the average magnitude of the excursions from the point of equilibrium increases until some of the molecules encounter each other so closely (collision) that the excessive repelling force causes them to rebound beyond the point where their own attraction prevails, and they assume new positions with respect to the aggregation, around which they oscillate as before. The plastic state is thus introduced. When the energy of motion of the molecules is so great that there is constant rearrangement of position among them the substance is a perfect liquid, even though none of the molecules have escaped the attraction of the aggregation. Finally, when the temperature is sufficiently high, and their heat energy enables them all to escape the attraction of the group and describe free paths in space, until checked by encounters with similarly wandering molecules or other obstacles, the substance becomes a gas.

The ability of molecules to change position thus distinguishes the fluid from the solid,¹⁶ and is a measure of the "fluidity" of a material. Its converse, "viscosity", is a measure of the resistance to changing place. A true solid therefore has infinite viscosity and zero fluidity (so long as it remains a solid). Pitch will flow from a small crack in a barrel, but if struck with a hammer will fracture like glass. Under the latter

¹² A. A. Griffith, *The Theory of Rupture*, Proceedings, First International Congress of Applied Mechanics, Delft, 1924, p. 947.

¹³ James H. Jeans, *The Kinetic Theory of Matter*, Encyclopaedia Britannica (14th ed.), v. 18, pp. 385-390, or other references in footnote 90, p. 211.

¹⁴ A. A. Griffith reports (*Theory of Rupture*, Proceedings, First International Congress for Applied Mechanics, Delft, 1924, p. 62) rupture strain energy of silica rods so great that if it were converted into heat the rod would become red hot. He states: "It is of interest therefore to inquire what becomes of this energy when fracture occurs. It is not converted into heat; indeed, measurements have shown that the defects of elasticity of the strong material is so inappreciably small that such an operation would take several minutes, at least. What actually happens is that the energy is used up almost entirely in forming new surfaces, that is, surfaces of fracture in the material."

¹⁵ The "flow" of wheat, or coal, or sand through pipes and orifices is made possible through an analogous changing of position by particles.

¹¹ Such uniform pressure from all directions does not give rise to shearing forces and needs no discussion here.

¹² A. Nádai, *Elasticità*, p. 18; Stephen Timoshenko, *Theory of Elasticity*, p. 144; Frederick A. Lindemann, *Theory of (the) Solid State*, op. cit., p. 947.

condition, the molecules have no time to change position. Time is a controlling factor, therefore, in viscous flow. So also must be the number of molecules involved in the translation in a given unit of time, and consequently compact concentrations of molecules will have a high viscosity. In this respect hydrostatic pressure and temperature also are important influences.¹⁷

The oscillation of the molecules brings on this fluid or viscous state through the resulting encounters with each other's repellant forces, which increase at a much faster rate for a given excursion from equilibrium than do their forces of attraction. They do not destroy cohesion merely because they render it ineffective, but from a mechanical point of view it results in the same thing. So that it appears reasonable to say that if effective cohesion is gradually broken down, a solid will pass progressively through the plastic into the liquid state, and that the reverse of this will occur in the case of increases in cohesion. It appears equally reasonable to assume that if localized repellant encounters are mechanically induced through applied force, in a substance already homogeneous, the effective cohesion will also be broken down.¹⁸

It has been pointed out that the element of cohesion in shear is the increasing resistance of molecules to separation up to a point beyond which the resistance very rapidly declines. As shearing stresses are increased by an increasing applied load, the imperceptibly small movements that introduce effective cohesion must also be met by the opposing forces of molecular repulsion, because each unit of molecular separation is substituted by a unit of molecular approach. Meanwhile, though the resistance due to attraction is increasing, the resistance due to repulsion is increasing at a much faster rate.¹⁹ The growing excess in repulsion may very quickly result in a separation of the two planes of shear by an amount just sufficient to compensate for this excess and thus release this energy in the form of surface energy in the new surfaces. On the other hand, it may result in heat. Or both occurrences may take place together. In any case effective cohesion is reduced.

If shear took place at once throughout a single plane, the process of rupture just described, i. e., that of surface formation, would result abruptly, while the heat effects would be a function of time. A load applied to an indefinitely large mass of a material could scarcely result in the former case, unless it were applied suddenly. A bullet against a heavy slab of glass or

marble will cause such a fracture. But ordinarily, shear will take place gradually, forming countless minute fissures²⁰ known as Lüders' lines, at different planes of weakness. From the progressive nature of these it is obvious that the failure at one point of weakness will result in the transference of the stress to the remaining stronger points, and as these in turn fail the resistance will be supplied by more and more remote sources of strength. This process will go on till the accompanying accumulation of minute fractures at the plane of maximum shear finally permits sliding to occur. In the case of metals, the generated heat might definitely influence this behavior, but in the case of earths the effects of heat need not be considered, because the distributed separation of the particles is analogous to liquefaction in its effects and hence analogous to a break-down in cohesion.

In view of the above characteristics of cohesion, it is clear that the maximum value of cohesion, i. e., the value beyond which permanent deformations arise, is a constant, except in the initial stages of stress in an unconsolidated material.

Another force is present, however, to supplement the resistance of cohesion in shear. It is friction. But instead of being related to the forces of attraction, it is related to the forces of repulsion.

The resistance to shear offered by friction.—The resistance of friction, as a force opposing rupture, is assumed to differ from cohesion in that its action is a mechanical one and dependent on minute surface irregularities which tend to obstruct sliding between two solid surfaces in contact with each other under pressure.

In the case of a crystalline solid, if the grain boundaries consist of irregularly spaced projections of geometrical crystal particles imperfectly interlocking with similar projections from adjoining grains, as Jeffries suggests,²¹ then any tendency for grain rotation necessary to bring the randomly oriented slip planes into alinement under stress, would be opposed by the mechanically interlocking particles. Again, according to Bancroft, "A steel containing 0.90 percent carbon in the form of small, round particles of cementite having an average diameter on the order of 0.001 inch has a Brinell hardness of about 160, as compared with a hardness of about 80 for the ferrite matrix when free from carbon. * * * The ferrite matrix in the steel * * * is soft because of the presence of many planes of weakness. The cementite particles scattered through this matrix harden it by interfering with slip along these planes."²²

In both cases the interference is a mechanical one and constitutes friction.²³ In both cases, too, it is quite

¹⁷ The behavior of materials in the plastic and fluid regimes affords a strong temptation to elaborate this necessary background. See A. Nédal, *Plasticity*, and Eugene C. Bingham, *Fluidity and Plasticity*. Recent outstanding references are, R. Howink, *Elasticity, Plasticity, and Structure of Matter*; G. W. Sestl Blair, *An Introduction to Industrial Rheology*; and First Report of Committee on Viscosity and Plasticity of the Academy of Sciences at Amsterdam, 1934.

¹⁸ True hydrostatic pressure results in an opposite effect, but if a metal compressed in a cylinder be subjected to great pressure and a small aperture made in the cylinder wall, the metal will extrude as a viscous fluid.

¹⁹ An interesting excursion at this point would be to investigate the influence of this molecular attraction-repulsion curve on the phenomena of elastic recovery and hysteresis, and perhaps, also, what is termed "elastic after-effect."

²⁰ A. Nédal, *Plasticity*, pp. 87, 112, 118, etc.

²¹ See Jeffries and Robert S. Archer, *The Science of Metals*, 2d. ed., p. 78.

²² Wilder D. Bancroft, *Applied Colloid Chemistry*, p. 448.

²³ Unless the resistance to movement is sufficiently strong to overcome resistance to plastic flow, and local failure takes place with its components of both friction and cohesion.

plain that this resistance will be in addition to the resistance from cohesion, and in view of the nature of cohesion it is difficult to conceive of two solid particles which touch without some cohesion, however imperceptible it may be. It is equally difficult to conceive of two solid particles which touch without some frictional resistance to movement as long as there is a cohesive pressure drawing them together.

In a strict sense, however, as has been shown, two particles not only do not touch but manifest a violent resistance to touching. This resistance could not be effective in the complete absence of pressure perpendicular to the sliding planes. If the only pressure were cohesion, the resistance could not come into existence until effective cohesion had been overcome. Perfect materials, in sufficiently thin rods, in tension would not have this resistance, because pressure would be absent. But whatever may be the relation between normal pressure and the reduction of molecular distances that result, equilibrium demands that at any given reduction the force causing it can be neither more nor less than the opposing resistance. Theoretically, pure friction is therefore directly proportional to the unit pressure normal to the plane of sliding.

The resistance offered by friction to shear is called static friction. When separation of the particles has taken place and sliding continues, the resistance is referred to as kinetic friction. And if localized liquefaction occurs, or if the particles are sufficiently disturbed so that they change position readily, the resistance is termed fluid friction. Without going into the maze of indefinite and contradictory distinctions appearing in the textbooks, which nevertheless have historic and practical usefulness, and serve to classify behaviors, the following definition appears suitable and sufficient for both cohesion and friction:

Cohesion and friction are the resistances offered by a substance to mechanical disturbance of its molecular or atomic equilibrium or degree of freedom, when unaccompanied by appropriate changes in energy. (1) Cohesion is the resistance to molecular or atomic separations and has a constant maximum value with each substance and state. (2) Friction is the resistance to molecular or atomic contacts and therefore has no apparent maximum, but in the state of equilibrium (the solid) varies directly with the intensity of normal force and in the state of freedom (the fluid) varies directly with the velocity of force application.²⁴

²⁴ Up to the point of turbulent flow, which probably reintroduces cohesion. L. Prandtl (in *Applied Hydro- and Aero-mechanics*, by C. G. Tietjens, p. 4, and in *Fundamentals of Hydro- and Aero-mechanics*, by the same author, pp. 107-109) refers to viscosity of fluids and proceeds on the assumption that "The motion of a fluid is governed essentially by inertia and friction." The two terms appear to be unnecessary. In the interior of a fluid the molecular inertia is a change in the degree of freedom may be referred to as friction, while on its boundary or surface the molecular inertia is a change in equilibrium due to fluid-solid or gas-liquid contacts may be called cohesion. If velocities are sufficient to break down cohesion at the boundary, friction in the form of freedom-inertia may also be present there.

Charles Augustin de Coulomb, in 1780, formulated a law for the ultimate resistance to shear in the case of soil particles which may be presented as:

$$Q = Pf + cA$$

in which Q is the total stress required to cause displacement along any given plane, P , the total applied load normal to that plane, f the coefficient of friction, c the coefficient of cohesion, and A the area over which P is applied.²⁵

Since static friction is directly proportional to area and to unit pressure the equation may be rewritten:

$$q = pf + c$$

in which q is the resistance per unit of area at the shear plane, and in which p is the unit pressure normal to that plane. The relation thus expressed may be presented graphically by a straight line having a positive intercept on the Y -axis.

In the resulting shear diagram, cohesion is a constant throughout the range of pressure. The fact that cohesion may conceivably increase under pressure in a loose or granular material where the compaction results in a reduced porosity, an elimination of air, a reduction in the size of the moisture films, colloid particles, and capillaries, or an increase in the surfaces of contact, need not affect the validity of the law, in which a continuous homogeneity was assumed. But even if homogeneity had not been assumed, this fact is similar to the fact that resistance to separation increases with load, while the cohesion in Coulomb's equation represents the maximum resistance before failure. The situation is clarified by the relation of loading in each instance. In Coulomb's equation the pressure is taken as normal to the plane of shear. In the case of compaction the pressure is compressive pressure that acts only imperceptibly on planes of shear because these are scarcely defined in the body of the material until compaction has occurred, provided the load is increased slowly enough. If the compaction involves the elimination of water or other fluid matrix or if it involves an accompanying readjustment of particles, the material will pass, during loading, from the fluid to the solid state in its behavior. If the material is highly impermeable, or if the load is applied rapidly enough, and the water or fluid matrix is not eliminated, the effective cohesion will constitute the initial resistance to shear because the fluid friction of the matrix will tend to prevent static friction between the solid particles; and in either circumstance velocity of load application will be controlling.

²⁵ William Cain, *Cohesion in Earth*, Transactions, American Society of Civil Engineers, vol. 80, 1916, p. 1316; also, Charles Terzaghi, *The Mechanics of Shear Failures on Clay Slopes*, Public Roads, December 1926, p. 179.

An action similar in effect will occur with dry granular materials. The mobility of the particles will decrease under increasing load or will not have time to be effective under high velocities of load application, and the reaction of the material under slowly applied loads will pass from the fluid to the solid type of behavior.

When such actions are encountered in shearing resistance tests the verification of Coulomb's law is difficult. D. S. Berry made shear tests on chilled lead shot, Ottawa sand (characterized by rounded edges), and sharp concrete sand (fig. C-5). The materials were sensibly dry, and at zero normal load the shearing resistance curves appeared to originate at the zero value. The chilled shot indicated a direct proportionality of resistance to load. The Ottawa sand showed a greater resistance but the proportionality diminished with load. The sharp sand offered still greater resistance, but showed a much greater change in the proportionality. Both sands approached a condition of strength, part of which was apparently constant and independent of pressure and therefore comparable to cohesion in its effect.

Berry said,²⁶ "The important influence of mechanical arrangement and particle shape on shearing resistance which is apparent from these data indicates that some revision of the conventional conception of friction as a source of internal resistance is necessary. It is not clear how the following observations can be described by existing conceptions:

(1) In most cases for compacted materials the ratio of tangential components to normal components of stress, generally referred to as the tangent of the angle of internal friction, is greater than unity, resulting in so-called angles of friction as high as 70°. The minimum value for this angle, obtained for chilled lead shot, was 43°.

(2) There are also noted large variations in this angle with changes in degree of compaction.

(3) As transverse shearing force is applied, there is motion normal to the shearing plane as well as tangential movement, indicating a stage of volume expansion previous to failure which introduces static reactions between individual particles not accounted for in existing theory.

(4) There is variation in the so-called coefficient of internal friction, noted in the lower range of normal load in the various tests as static resistance due to interlocking, which is not directly proportional to normal pressure applied.²⁷

It is almost necessary that these manifestations be present. Osborne Reynolds, who, in 1885, discovered the phenomenon of dilatancy, i. e., the expansion of granular masses under load, accounted for it by pointing out the tendency of a loose granular material in equilibrium to approximate a condition of closest packing, and that any change to another arrangement without distorting the individual grains can take place only by

²⁶ D. S. Berry, Stability of Granular Materials, Proceedings, American Society for Testing Materials, pt. 2, 1924, p. 304, and fig. 14.

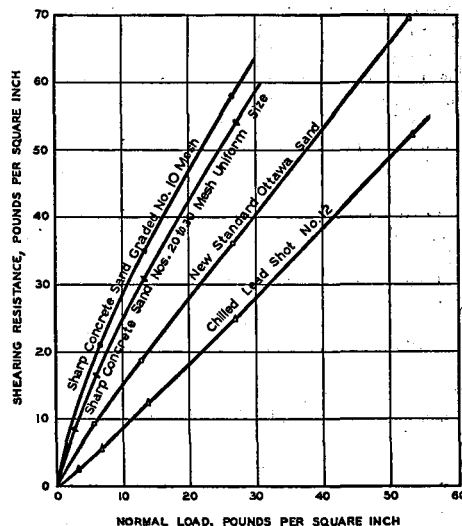


FIGURE C-5.—Effect of type of particle on shearing resistance. (From Proceedings, American Society for Testing Materials, Part 2, 1925.)

increasing the average distance of the grains and thus increasing the volume.²⁷

The condition of closest packing represents a condition of minimum energy and therefore is analogous to molecular equilibrium, any disturbance of which is opposed by cohesion. In the case of finite solid particles, the actual cohesion under zero load may be imperceptible because the contact points are relatively few and because they are otherwise separated by finite air films. As load is increased, the contact points are increased, and retained air may be squeezed out to molecular films, and it is conceivable that some increasing cohesion may thus be brought into play. This view has been held by William Cain, Terzaghi, Krynine, and others.²⁸

It is more obvious, however, that in addition to the ordinary shearing resistance furnished by the sliding friction of the particles, which is proportional to normal

²⁷ Osborne Reynolds, On the Dilatancy of Media Composed of Rigid Particles in Contact, Philosophical Magazine, vol. 20, 1885, p. 469. This is discussed at some length by John H. Griffith in Progress Report: Special Committee . . . On the Bearing Value of Soils, Proceedings, American Society of Civil Engineers, vol. 46, 1922, p. 629, and also by Arthur Casagrande in Characteristics of Cohesionless Soils Journal, Boston Society of Civil Engineers, January 1924, pp. 16-22.

²⁸ William Cain, Cohesion in Earth, op. cit., vol. 80, 1914, p. 1212. Terzaghi on The Solution of Foundations, Transactions, American Society of Civil Engineers, vol. 32, 1922, pp. 380, 383, and especially Charles Terzaghi, The Mechanics of Shear Failures on Clay Slopes, Public Roads, December 1923, p. 180. Krynine on Soil Mechanics Research, Transactions, American Society of Civil Engineers, vol. 66, 1922, p. 370, fig. 26. Increasing cohesion in clay is demonstrated by C. A. Hogentogler in Engineering Properties of Soil, p. 221.

pressure, there arises a resistance of the particles to rolling or rotation which is a function of their angularity and sharpness, and which is also affected by pressure. The pressure effects, however, depart from proportionality, because as pressure increases the amount of rotation is reduced and the movement more and more becomes one of sliding.²⁹

This additional resistance in a granular material may result in so-called coefficients of internal friction which are greater than unity, and it is responsible for coefficients which decrease with increasing pressure.³⁰

The nature of internal friction of solids requires that the tangent of the angle of friction, i. e., the coefficient, be no greater than unity, because internal friction cannot offer a resistance in the solid state greater than the static pressure normal to the plane of sliding, which produces it, unless that pressure is assisted by the attractive force of the molecules, which is cohesion. In a homogeneous solid this limitation is independent of velocity of load application. At the lower limit it cannot be zero, because that would imply a zero resistance to molecular displacements, which is the fluid state.

In the fluid state, static loading cannot produce an internal resistance to molecular movement, in a semi-infinite body. The coefficient of friction may be measured according to terms employed in the case of solid friction, i. e., the coefficient is equal to the unit resistance divided by the normal pressure. Since the resistance is zero, it will be zero. It may also be measured in terms employed in dynamic fluid loading, i. e., the coefficient is equal to the unit resistance divided by the velocity of loading. Again the resistance is zero and the coefficient is zero.³¹

Dynamic loading of a fluid changes unordered molecular motion into ordered motion, through the net momentum imparted to the molecules in a direction parallel to the axis of loading. The action is "collisional," "translational," and "diffusional,"³² whether or not consideration is given to boundary surfaces, as the walls of a tube, the surfaces of a lubricated bearing, or the wing surface of an aeroplane. The resistance constitutes friction, and is directly proportional to the velocity of load application, for any given unbounded fluid up to velocity limits which are thought to be comparable to the speed of light.³³ With boundary conditions present it is a constant up to a critical point, depending on several factors associated with the

conditions and with the fluid.³⁴ The coefficient of this proportionality will vary, with different fluids, upward from zero. At the upper limit, however, it cannot be infinity, because, as has already been suggested, viscosity (which is measured by this coefficient) is infinite only in the perfect solid.

In view of the fact that materials pass gradually from the solid to the liquid state,³⁵ or from a liquid-like behavior to the behavior of a solid,³⁶ it is of some significance that the lower limit of the friction coefficient in the solid state characterizes the fluid state, and that the upper limit of the friction coefficient in the fluid state characterizes the solid state.

Where the two states or behaviors coexist, as they frequently do in earths, the internal resistance will combine these diverse characteristics, following a successive behavior corresponding to the elements in the composite of the material which offer the remaining least resistance as each resistance is overcome. In a semi-infinite body such as an earth road the pressure distribution downward will have lateral effects at increasing depths as loads are increased and these lateral reactions may be greater or less than those encountered by the distributions from a lighter load.

However, the magnitude of pressures required for some of these transitions to take place appreciably in earths may be quite beyond ordinary wheel loads, and the critical depths in connection with road surfaces may rarely extend beyond the range of 7 to 17 inches.³⁷ For these reasons one need scarcely consider the extremes that have been outlined except as they distinctly present the limiting effects of pressure intensity on materials having various mechanical properties, and open the way to an examination of the limiting effects of different areas of load application.

The limiting effects of area as determined by the mechanical properties of a material.—The vertical compression of a prism of material results in lateral expansion. If the body is indefinite in extent and bounded only by a horizontal plane on a small section of which is imposed a downward pressure, a lateral thrust results within the body emanating from the elements beneath the loaded area.³⁸ The controlling influence of load is its downward and outward thrust, and in the resistance of the material to this particular action lies the clue to the area effects.

²⁹ Fluidity and Plasticity, *idem*, section on "Transition from Linear to Turbulent Flow", pp. 35-49.

³⁰ Under increasing stress. During these transitions, deformations in the solid state will disappear upon removal of load, those of the solid-liquid phase will only partly recover, and those in liquid regime will remain. With respect to the liquid stage, this would be nonsense were the normal state of the molecules not that of the solid and the one into which the molecules immediately freeze upon load removal.

³¹ Under increasing velocity of load application.

³² Arthur Casagrande states in an unpublished manuscript loaned to the Federal Coordinator of Transportation: "From experiments it is known that the zone of lateral movement beneath a loaded area goes to a depth about equal to the width of a loaded area."

³³ This explains why static friction of fluids is not considered in the usual references. See, however, L. Prandtl, in *Fundamentals of Hydro- and Aero-mechanics*, by O. G. Tietjens, p. 13.

³⁴ Eugene O. Bingham uses these terms in relation to flow along boundary surfaces, in *Fluidity and Plasticity*, pp. 147-162.

³⁵ Mass is assumed to increase at such velocities.

³⁶ Interesting photographic evidence of this is presented by M. L. Enger, *Transactions, American Society of Civil Engineers*, vol. 53, 1922, p. 1261, fig. 3, and by Roland Vokac, *Proceedings, Tenth Annual Asphalt Paving Conference*, 1923, p. 334, 335.

Terzaghi has approached the problem of load penetration by assuming, (a) materials without friction, and (b) materials without cohesion.³⁹ He concludes that in the former case the resistance of the material to initial deflections, i. e., the required pressure, will vary inversely as the diameter, while for ultimate deflections near the bearing capacity limit the resistance will be independent of the size of the loaded area. In the case of the cohesionless material the initial deflections require a pressure independent of size of area, while at ultimate deflections the resistance is directly proportional to the diameter.⁴⁰

Just how he arrives at these conclusions theoretically is not explained except by the statement that, if the assumptions which he outlines are granted, " * * * they represent a simple consequence of the laws of similitude." And though he admits that the assumptions are valid only within limits, it is of some significance that in the light of test results (figs. C-6 to C-21) his conclusions are essentially verified. They may, however, be arrived at directly from a consideration of the mechanical properties of the two groups of materials.

A view now widely accepted among authorities is perhaps best expressed by Kögler, who states: "The settlement of a loaded area is due to two factors: First, to the compression (elastic and nonelastic), of the soil; and, second, to the displacement of the soil sidewise and upward."⁴¹

According to Kögler both elastic and nonelastic compression are proportional to the diameter. With respect to elastic deformation this relation is of course true, as has already been shown. In both cases of compression the total movement, whether molecular or a result of compaction of perceptible particles, is proportional to the force and dependent on the volume of material affected, and, according to the geometrical necessities of pressure distribution, like pressures on different areas of a uniform material will act on prisms, or bulbs, which are similar. The initial depth of material affected being related to the diameters, and the reductions in volume being similar, the reductions in depth will be proportional to the diameters. This also means that at any given deflection the pressures are related inversely as the square roots of the areas. And, to the extent that compression takes place before shear, this will be the case with all materials.

In the case of settlement resulting from particle displacements alone, on the other hand, the particle rotations beneath a loaded area encounter a resistance to lateral movement which is proportional to the number

of particles involved in the movement in a given direction outward from the center of a loaded area. This number is a direct function of pressure intensity and the radius of the loaded area. And pressures causing like deflections under different areas are thus, in this case, related directly as the square root of the areas.

Deformation in a semi-infinite mass, loaded over a small area of its boundary plane, can occur only under these two characteristic internal actions of the material. They may act separately and result in the limiting area effects just described; or they may act jointly and result in intermediate area effects similar to those described by Terzaghi.

Under ultimate deflections in the case of compression, particle rotation accompanies shear, and a semifluid condition exists wherein the resistance approximates independence of area. Under the initial deflections in the case of particle displacement the resistance of the surface particles around the perimeter is negligible,⁴² as in the fluid state, and again the resistance is independent of area.

From these conclusions it may be deduced that at any given deflection the resistance of the material will vary according to the general law

$$p = mA^n$$

in which p is the resistance per unit of area, m a coefficient depending on the deflection and on the material, A the area of load application, and n an exponent depending on the dominance of cohesion or friction as a factor of support.

The exponent n would, moreover, range between minus five-tenths for the elastic materials and for the early deflections in compressible materials to plus five-tenths for the ultimate deflections in materials with friction but without cohesion.

A study of deflection, however, is not an investigation of failure, or the safe supportability offered by different types of material, even though it shows an interesting relationship in their respective dynamical behaviors. Yet, it is significant that at one end of the range is the case of the elastic solid, for in its presence in the continuum lies a clue to the relative behavior of various materials acting independently or jointly.

FIGURES C-4 TO C-8, INCLUSIVE

The Goldbeck-Bussard data (tables C-3A to C-3C, inclusive, p. 247) were read directly from enlargements of the three pressure-penetration diagrams appearing in the article, The Supporting Value of Soil as Influenced by Bearing Area, by A. T. Goldbeck and M. J. Bussard, Public Roads, January 1925.

A decrease in the influence of area may be noted as between the typical elastic curve and the earths tested. A distinct decrease in the influence of area is evident as between the clay and the 5 to 1 sand-clay mix.

³⁹ Loading the surface area surrounding the perimeter thus lends important supportability to granular materials.

³⁹ Terzaghi on the Science of Foundation, Transactions, American Society of Civil Engineers, vol. 63, 1926, pp. 330-333.

⁴⁰ Terzaghi appears to be the only authority who has given such relative limits of the diverse reaction of these different materials. Definite mention of the nature of the diversity may also be credited, however, to Lieut. Col. F. S. Benson, United States Corps of Engineers, Engineering News-Record, March 18, 1924, pp. 345-346, and to D. S. Berry, Proceedings, American Society for Testing Materials, pt. 2, 1924, p. 804.

⁴¹ Kögler on Soil Mechanics Research, Transactions, American Society of Civil Engineers, vol. 66, 1923, p. 208.

Though the 1 to 1 sand-clay mix might be expected to indicate some of the characteristic influence of sand, according to Clement C. Williams (Proceedings, American Society of Civil Engineers, vol. 61, No. 8, October 1935, p. 1252), "A relatively small clay content will convert sand to a plastic mass. * * * with fine sand of fairly uniform grain, 20 percent clay will make the mass essentially plastic."

The small penetrations recorded obscure the diminishing influence of area at successive depths.

FIGURES C-4 TO C-11, INCLUSIVE

The United States Bureau of Standards data (tables C-3D to C-3F, inclusive, p. 247) have been taken from the results of tests made in the laboratories of the Bureau for the use of the Coordinator. In these tests deflections were measured for various loadings. From the resulting pressure-penetration curves readings were taken for given deflections. In order to obtain a satisfactory range of readings, two points in table C-4, indicated by parentheses, were extrapolated. Their probable error is negligible.

Clay 1 contained 79 percent, by weight, of a mixture of refractory clays and 21 percent water; clay 2 of 87 percent yellow surface clay and 13 percent water; and clay 3 contained 45 percent yellow surface clay, 45 percent common sand, and 10 percent water.

Clay 1 departs somewhat from the behavior of clay and approaches that of sand. Absence of the cohesive action of natural colloids may be partly responsible.

Clay 2 is stiffer than clay 3, of which it is a component, and though clay 3 contains 45 percent sand its behavior is in an opposite direction to that expected, i. e., there is relatively less resistance under the larger areas. The materials were hand trowelled and then compacted under a 64-pound load, applied through a rectangular plate of approximately 10 square inches, before testing. The resulting pressure intensity of 6.4 pounds per square inch could not compact the deeper strata reached by the greater stress intensities of the larger areas. The sand ratio was insufficient for the introduction of appreciable friction.

FIGURES C-12 TO C-15, INCLUSIVE

A most interesting array of data (tables C-3G to C-3J, inclusive, p. 247) is that developed in the laboratory by D. S. Berry, of the University of Michigan, and the Research and Testing Division of the Michigan State Highway Department, under the direction of William S. Housel. Data showing the resistance of sand were taken from curves in his article on Stability of Granular Materials (Proceedings, American Society for Testing Materials, pt. 2, 1935, figs. 10 and 11). Data relating to lead shot and graded aggregate were taken from curves furnished the Coordinator by Housel.

The materials respond, in the measured range of deflection, almost exactly as expected. Under increasing

load, resistance increases beneath the loaded area but not in the surrounding material. The lateral and upward displacement of this material surrounding the larger pressure zone, as compared to the smaller, occurs under yield pressures comparable to the square roots of the areas. The settlement that takes place is a result of rotation and readjustment and sliding of the particles. During the small settlements, in which rotation and reorientation is taking place freely, the material acts as a fluid, in which stage area has no influence on resistance under static loading.

FIGURES C-16 TO C-19, INCLUSIVE

The data for figure 16 (table C-3K, p. 247) were taken from figure 26, page 44, of A Practical Basis for the Selection of Foundation, by William S. Housel, University of Michigan Engineering Research Bulletin No. 13. The data for figures C-17 and C-18 (tables C-3L and C-3M, p. 248) were furnished the Coordinator by Housel. The Stiff Blue Clay contains sufficient sand and pebbles to control its behavior. At early deflections its action approaches that of the clay matrix, while at later ones it resembles that of a granular material.

The Terzaghi data table (C-3N, p. 248) providing figure 19 are readings from figure 1, page 273, The Science of Foundations, Transactions, American Society of Civil Engineers, volume 93, 1929. Items in parentheses are extrapolations.

All of these tests were made in the field. They are interesting for their general agreement with laboratory results.

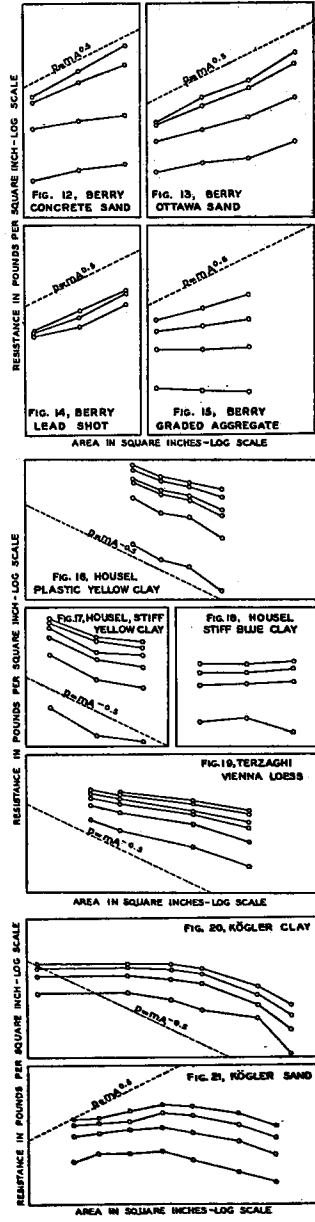
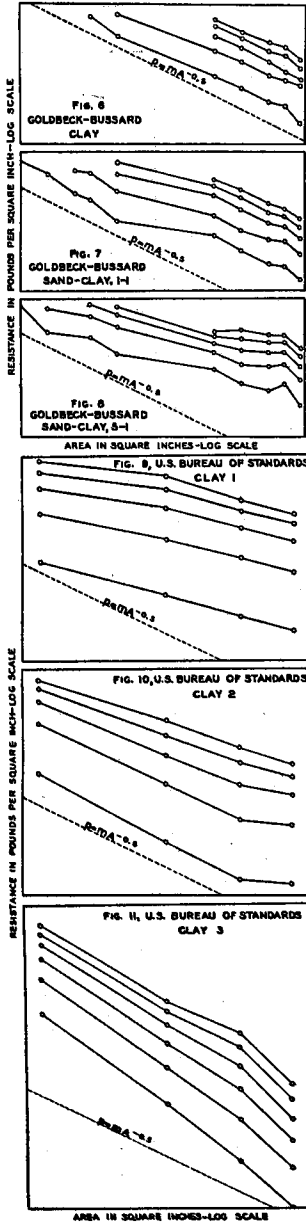
FIGURES C-20 AND C-21

These figures, based on data from the two diagrams, page 300, of Kögler on Soil Mechanics Research, Transactions, American Society of Civil Engineers, volume 98, 1933, are notable for their departure from the usual relation between pressure and areas of different sizes. They are results of tests performed in the Soils Laboratory of the Bergakademie, in Freiberg, Germany (tables C-3O and C-3P, p. 248).

For the smaller areas the action of each material is somewhat in conformity with expectations, while for the larger areas the behavior of both the sand and the clay approaches that of an elastic solid. The implication is that in both cases the pressure distribution of the larger areas reaches a layer of material somewhat more deformable than that at the surface.

STRESS DISTRIBUTIONS IN THE ELASTIC SOLID AND THE BEHAVIOR OF LOADED MATERIALS

The uniformly loaded circular area.—In 1929, following the potential method of Boussinesq, A. E. H. Love carried through the solution of the problem of the stresses produced in a semi-infinite solid by pressure



FIGURES 6 to 21, Inc.—The effects of bearing area on the resistance of clay and granular materials at increasing penetrations.

applied on a part of its boundary plane, in several important cases.⁴³

Of these, the one of the uniformly loaded circular area appears most nearly to represent the conditions of the pressure-deflection tests already considered (figs. C-6 to C-21). These tests were made with flat circular bearing blocks; and though in the instances of small diameter the actual pressure distribution may have been greatest at the circular boundary, as in the case of a rigid die, the relative penetration of different bearing blocks is not thereby affected. Again, the objective of this study is to relate the verified and theoretical behavior of materials in general to the problem of roads, and since the case of the uniformly loaded circular area may be conjectured most nearly to represent the manner of loading under pneumatic tires,⁴⁴ it has a special significance.

Using cylindrical coordinates, Love develops the basic differential equations for deflection and stress in terms of certain derivatives of the Newtonian potential of surface distribution, V , with respect to the depth, z , below the boundary plane and the horizontal distance, ρ , from the central vertical axis, and in terms of certain derivatives of Boussinesq's three-dimensional logarithmic potential, χ , with respect to ρ . He also employs certain elastic moduli⁴⁵ in which may be substituted the familiar Poisson's ratio, σ , and the equations become:

$$\begin{aligned}\widehat{\rho\rho} &= \frac{1}{2\pi} \left[2\sigma \frac{\partial V}{\partial z} - (1-2\sigma) \frac{\partial^2 \chi}{\partial \rho^2} - z \frac{\partial^2 V}{\partial \rho^2} \right] \\ \widehat{\omega\omega} &= \frac{1}{2\pi} \left[2\sigma \frac{\partial V}{\partial z} - (1-2\sigma) \frac{1}{\rho} \frac{\partial \chi}{\partial \rho} - \frac{z}{\rho} \frac{\partial V}{\partial \rho} \right] \\ \widehat{z\bar{z}} &= \frac{1}{2\pi} \left[\frac{\partial V}{\partial z} - z \frac{\partial^2 V}{\partial z^2} \right] \\ \widehat{\rho z} &= \frac{1}{2\pi} \left[-z \frac{\partial^2 V}{\partial \rho \partial z} \right], \widehat{\omega\rho} = 0, \widehat{\omega z} = 0.\end{aligned}$$

In these, $\widehat{\rho\rho}$ and $\widehat{\omega\omega}$ are, respectively, the radial and tangential components of stress, while $\widehat{z\bar{z}}$ is the vertical component. Shearing stresses are denoted by $\widehat{\rho z}$, $\widehat{\omega\rho}$ and $\widehat{\omega z}$.⁴⁶

Love then evaluates the several derivatives in terms of the simple geometrical dimensions shown in figure C-22, and the elliptic integrals⁴⁷ $F(k, \phi)$, $E(k, \phi)$, $F(k, \theta)$ and $E(k, \theta)$, in which $k = r_1/r_2$, and also the complete elliptic

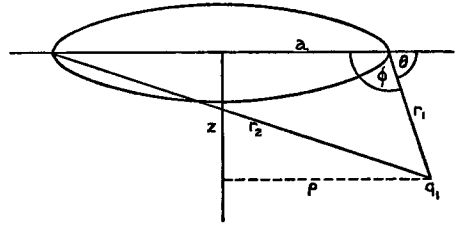


FIGURE C-22.—Geometrical dimensions employed in calculation of stresses beneath a uniformly loaded circular area.

integrals K' and E' , in which the modulus $k' = \sqrt{1-k^2}$. If one substitutes M_* , say, for $K'E(k, \phi) - (K' - E')F(k, \phi)$ and M_* , say, for $K'E(k, \theta) - (K' - E')F(k, \theta)$, the results may be given as follows:

When ρ is less than a ,

$$\begin{aligned}\frac{1}{\rho} \frac{\partial \chi}{\partial \rho} &= p \left[\pi + \frac{2r_2}{\rho^2} E' - \frac{z(2a^2 + 2\rho^2 + z^2)}{r_2 \rho^2} K' + \frac{a^2 + \rho^2}{\rho^2} M_* \right] \\ \frac{\partial^2 \chi}{\partial \rho^2} &= p \left[\pi - \frac{2r_2}{\rho^2} E' + \frac{z(2a^2 + z^2)}{r_2 \rho^2} K' - \frac{a^2 + \rho^2}{\rho^2} M_* \right] \\ \frac{\partial V}{\partial z} &= 2p \left[\frac{z}{r_2} K' + M_* - \pi \right]\end{aligned}$$

When ρ is greater than a ,

$$\begin{aligned}\frac{1}{\rho} \frac{\partial \chi}{\partial \rho} &= p \left[\frac{a^2 + 2r_2}{\rho^2} E' - \frac{z(2a^2 + 2\rho^2 + z^2)}{r_2 \rho^2} K' - \frac{a^2 - \rho^2}{\rho^2} M_* \right] \\ \frac{\partial^2 \chi}{\partial \rho^2} &= p \left[-\frac{a^2 - 2r_2}{\rho^2} E' + \frac{z(2a^2 + z^2)}{r_2 \rho^2} K' + \frac{a^2 + \rho^2}{\rho^2} M_* \right] \\ \frac{\partial V}{\partial z} &= 2p \left[\frac{z}{r_2} K' - M_* \right]\end{aligned}$$

The remaining derivatives with their coefficients may be expressed as follows:

$$\begin{aligned}\frac{z}{\rho} \frac{\partial V}{\partial \rho} &= \frac{2r_2}{\rho^2} p \left[2E' - \frac{r_2^2 + r_1^2}{r_2^2} K' \right] \\ z \frac{\partial^2 V}{\partial \rho^2} &= p \left[\frac{z(\rho^2 + z^2 + a^2)(\rho^2 - z^2 - a^2)}{r_1^2 r_2 \rho^2} E' \right. \\ &\quad \left. - \frac{2z}{r_2} K' + \frac{2r_2}{\rho^2} \left(\frac{r_2^2 + r_1^2}{r_2^2} K' - E' \right) \right] \\ z \frac{\partial^2 V}{\partial \rho \partial z} &= \frac{z^2}{r_2 \rho} p \left[\frac{r_2^2 + r_1^2}{r_1^2} E' - 2K' \right] \\ z \frac{\partial^2 V}{\partial z^2} &= p \left[\frac{2z}{r_2} K' - \left(\frac{2r_2}{\rho^2} + z \frac{(\rho^2 + z^2 + a^2)(\rho^2 - z^2 - a^2)}{r_1^2 r_2 \rho^2} \right) E' \right]\end{aligned}$$

In all of these equations the quantity, p , represents the intensity of the uniform pressure.

Thus, by using ordinary tables of elliptic integrals⁴⁸ one may, without interpolation, compute the stresses occurring at certain fixed points along the circular arc described by the point, q_1 , as the angle, ϕ , increases from 0° to 180° .

For the purposes of this investigation, however, it is more convenient to know the stresses occurring at different distances, ρ , on planes parallel to the boundary surface at various depths, z , as may be developed from table C-4 and figures C-23 to C-28 herewith. Com-

⁴³ A. E. H. Love, *The Stress Produced in a Semi-Infinite Elastic Solid by Pressure on Part of the Boundary*, Philosophical Transactions of the Royal Society of London, vol. 228 (series A), November 1926, pp. 377-420.

⁴⁴ Road abstracts: Department of Scientific and Industrial Research, Great Britain, vol. 3, No. 12, March 1937, reviewing experiments conducted by H. Martin in Germany, states: "For pneumatic tyres the pressure distribution is found to be sensibly uniform."

⁴⁵ $\frac{\lambda}{\lambda + 2\mu}$ and $\frac{\mu}{\lambda + 2\mu} = 1 - 2\nu$, see p. 228 below.

⁴⁶ The notation is Love's following Pearson, following Castiglione, Maxwell, Boussinesq, Saint-Venant, Cauchy, and Corollis. Also see Isaac Todhunter and Karl Pearson, *A History of the Theory of Elasticity*, vol. 1, pp. 882-883 and p. 322; and A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (1927), p. 614. The first letter of the symbol represents the direction of the normal to the plane under consideration, while the second represents the direction of the stress component itself.

⁴⁷ An excellent chapter on elliptic integrals appears in *Higher Mathematics for Engineers and Physicists*, by Ivan and Elizabeth Sokolnikoff.

⁴⁸ For example, see B. O. Peirce, *A Short Table of Integrals*, 3d ed., pp. 121-123.

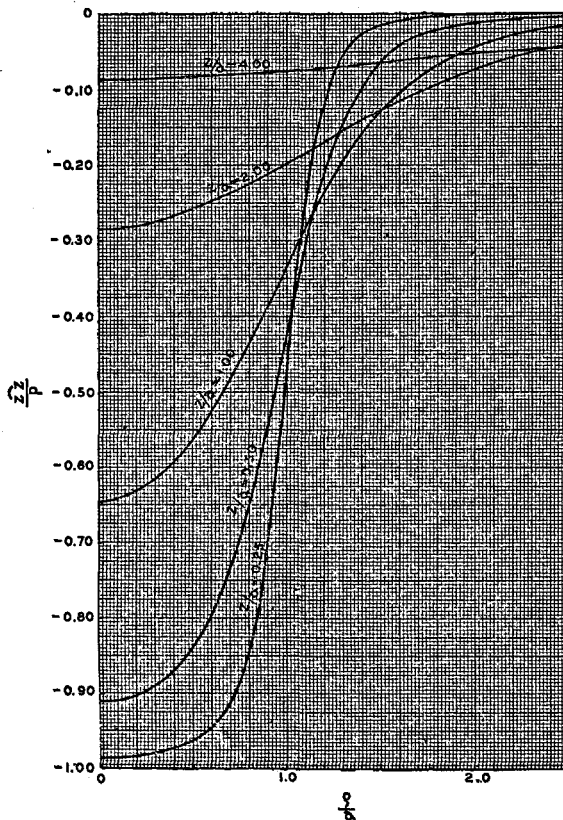


FIGURE C-23.—The Stress Component σ_z at Various Relative Depths $\frac{z}{a}$ and Relative Lateral Distances $\frac{\rho}{a}$ from the Vertical Axis in a Semi-infinite Isotropic Solid, Expressed as a Decimal Part of a Uniform Pressure Applied over a Circular Area at the Boundary Plane.

putation thus involved the use of tables of elliptic integrals calculated for small intervals of $\sin^{-1}k$. Tables in Legendre's classical work⁴⁹ provide numerical values for intervals of 1° . These were employed with Legendre's own equation for interpolation.⁵⁰ Accuracy to the eighth decimal was sought in this interpolation procedure because sixth place accuracy in all numerical values employed in the calculation of the derivatives was desired.

Along the vertical axis, the stress components may be found by evaluating the limits to which the relevant derivatives of V and χ tend as k approaches unity, but it is simpler, as Love points out to observe that when $\rho=0$,

$$V=2\pi p[(a^2+z^2)^{1/2}-z]$$

and that in this limiting case:

$$\frac{\partial^2 V}{\partial \rho^2} = \frac{1}{\rho} \frac{\partial V}{\partial \rho} = -\frac{1}{2} \frac{\partial^2 V}{\partial z^2} = \pi p \left[-\frac{a^2}{(a^2+z^2)^{3/2}} \right]$$

and that also in this limiting case:

$$\frac{\partial^2 \chi}{\partial \rho^2} = \frac{1}{\rho} \frac{\partial \chi}{\partial \rho} = -\frac{1}{2} \frac{\partial V}{\partial z} = \pi p \left[1 - \frac{z}{(a^2+z^2)^{1/2}} \right]$$

Love also provides the equations for the components of vertical and lateral displacement. For vertical displacement of the pressed area the results are identical with those already discussed (p. 206) in connection with the elastic solid and need no further mention here.

Stress distribution and the stability of isotropic materials.—A knowledge of stress distribution in the case of the elastic solid has led to various speculations as to

⁴⁹ Adrien M. Legendre, *Traité des Fonctions Elliptiques*, etc., Tome Second (1826), table IX.

⁵⁰ Same, p. 302. Two values in the table were found to be in error. These were corrected by the same method.

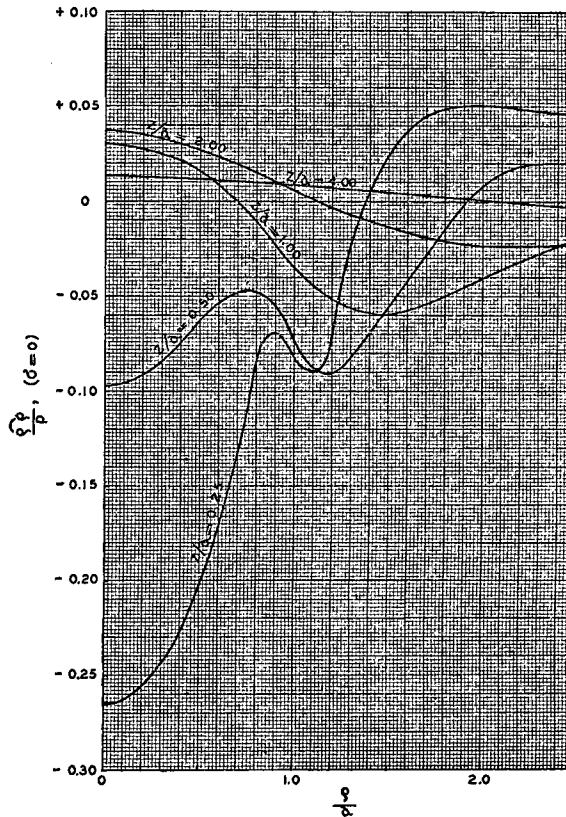


FIGURE O-24.—The Stress Component, $\hat{\sigma}_p$, at Various Relative Depths $\frac{z}{a}$ and Relative Lateral Distances $\frac{r}{a}$ from the Vertical Axis in a Semi-Infinite Isotropic Solid, Expressed as a Decimal Part of a Uniform Pressure Applied over a Circular Area at the Boundary Plane. $\sigma=0$.

the criteria of rupture.⁵¹ These have limited, (a) the maximum extension, (b) the maximum tension, (c) the maximum stress, (d) the maximum stress-difference, (e) the maximum shear, and, in the case of granular materials, (f) the critical ratio between the principal stresses.⁵²

Carothers⁵³ considers each of these from the point of view of their applicability to the case of earths and very logically concludes that in this instance the conditions of stability must (a) preclude tension everywhere, and

(b) require that the maximum stress-difference at any point shall be below a certain limit. This also is the conclusion arrived at by Love⁵⁴ in dealing with the same problem.

Love says: "For a discussion of strength or weakness the most important quantities are the stress-difference and the tensile stress. The stress-difference is the excess of the algebraically greatest principal stress at a point above the algebraically smallest principal stress at the same point. It is generally conceded that failure is likely to occur if at any point the stress-difference exceeds some limit depending on the material."⁵⁵

He points out that the stress component $\hat{\omega}$ is always a principal stress, that the axial plane through the point,

⁵¹ These are discussed by A. E. H. Love in *A Treatise on the Mathematical Theory of Elasticity* (1927), p. 121; Richard V. Southwell, *The Strength of Materials*, Encyclopaedia Britannica (14th ed.), vol. 18, p. 68; A. Nádai, *Plasticity*, p. 56; and Stephen Timoshenko and Gleason H. MacCullough, *Elements of Strength of Materials*, p. 219. A somewhat extended paper (together with discussion by various writers) is by Joseph Marin, *Failure Theories of Materials Subjected to Combined Stresses*, Transactions, American Society of Civil Engineers, No. 101, 1936, pp. 1168-1194.

⁵² Rankine's ratio, p. 236.

⁵³ S. D. Carothers, *Test Loads on Foundations as Affected by Scale of Tested Area*, Proceedings, International Mathematical Congress, Toronto, 1924, vol. 2, p. 829.

⁵⁴ A. E. H. Love, *The Stress Produced in a Semi-Infinite Solid by Pressure on Part of the Boundary*, Philosophical Transactions of the Royal Society of London, vol. 228 (series A), November 1926, p. 417.

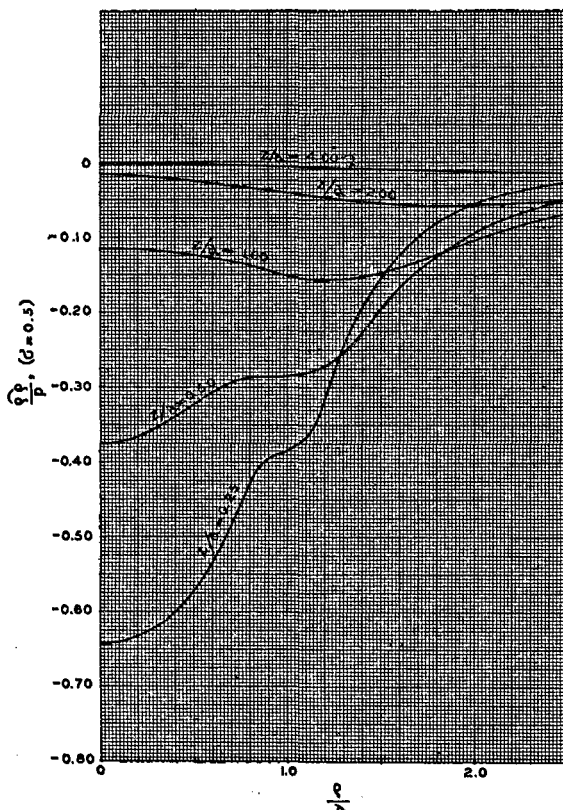


FIGURE C-22.—The Stress Component, $\hat{\sigma}_z$, at Various Relative Depths $\frac{z}{a}$ and Relative Lateral Distances $\frac{r}{a}$ from the Vertical Axis in a Semi-Infinite Isotropic Solid, Expressed as a Decimal Part of a Uniform Pressure Applied over a Circular Area at the Boundary Plane. $\nu=0.5$.

q_1 (fig. C-22), is always a principal plane of this stress at q_1 and that the remaining principal planes of stress at q_1 are at right angles to this plane. His formulas for the remaining principal stresses T_1 and T_2 , are:²²

$$T_1 = \frac{1}{2}(\hat{\rho}\hat{\rho} + \hat{z}\hat{z} + S)$$

$$T_2 = \frac{1}{2}(\hat{\rho}\hat{\rho} + \hat{z}\hat{z} - S)$$

$$S = T_1 - T_2 = [(\hat{\rho}\hat{\rho} - \hat{z}\hat{z})^2 + 4\hat{\rho}\hat{z}^2]^{\frac{1}{2}}$$

with the condition that the stress difference S is always positive. Thus, the maximum stress difference is either S , or it may be $T_1 - \infty$. Computed values of S/\hat{p} are here given in table C-17.

From these equations and computations a significant and conclusive observation may at once be made, with respect to the relative effect of different areas on the condition of rupture or plasticity in an isotropic solid. The stress-difference, at any given relative depth z/a , is proportional to pressure. The only effect of area is an alteration of the depth at which the stress-difference occurs, but the magnitude with which it occurs and not the position of its occurrence determines the yield point. Therefore, in the case of the homogeneous isotropic solid, to which the mathematical theory is rigorously applicable, failure is independent of any area effects and is directly dependent on pressure intensity.

Thus, as earth materials embody the characteristics premised in the elastic theory they will deflect under load in proportion to pressure intensity and in proportion to the radius, or the square root, of the loaded area.

²² Identical results are given by Stephen Timoshenko in *Theory of Elasticity*, p. 19, Eq. 14, and by A. Nöel in *Plasticity*, p. 48, Eq. 21.

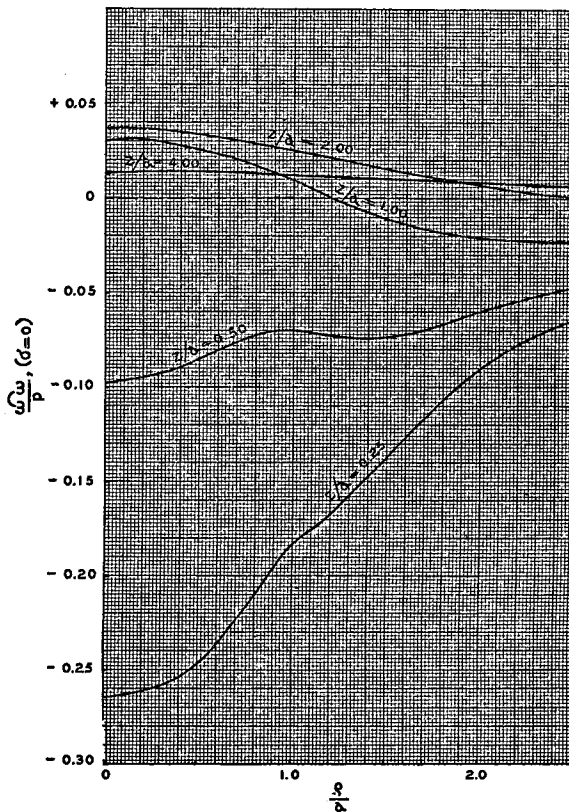


FIGURE C-20.—The Stress Component, σ_{zz} , at Various Relative Depths $\frac{z}{a}$ and Relative Lateral Distances $\frac{r}{a}$ from the Vertical Axis in a Semi-infinite Isotropic Solid, Expressed as a Decimal Part of a Uniform Pressure Applied over a Circular Area at the Boundary Plane. $\sigma = 0$.

The proportionality will persist under increasing pressure intensity until the yield point is attained. At this point, though the pressure intensities and stress-differences under different areas are identical, the deflections under the larger areas are greater than under the smaller. The material will not undergo a sudden transition to the viscous stage because the failure that occurs at this point, though permanent, is local in character and will spread only gradually under increasing applications of pressure. In order, then, to produce the same deflection under a smaller area as exists under a larger, at this point, additional pressure will be required. Thus it is that strain-stress curves for plastic earths do not coincide under usual test conditions even after noticeable failure is taking place. Such coincidence cannot take place until a nearly fluid action is reached. This accounts for the fact that the

pressure area curves for clays (figs. C-6 to C-20) merely approach the point where the pressure required to produce a given deflection under the different areas are independent of area, i. e., where pressure varies as the area to the zero power.

From an examination of the deflection characteristics of loaded earths as witnessed by test results, and from a consideration of the limiting mechanical properties of all materials, it was deduced that all materials would sustain loads according to the relative expression $p = mA^n$, where p and A are pressure and area, respectively, and m and n are variable parameters with the latter ranging between -0.5 and $+0.5$.

In the case of cohesive earths, it was deduced that the exponent, n , would be -0.5 under initial loadings and approach zero as failure progressed. This deduction has been harmonized with mathematical theory;

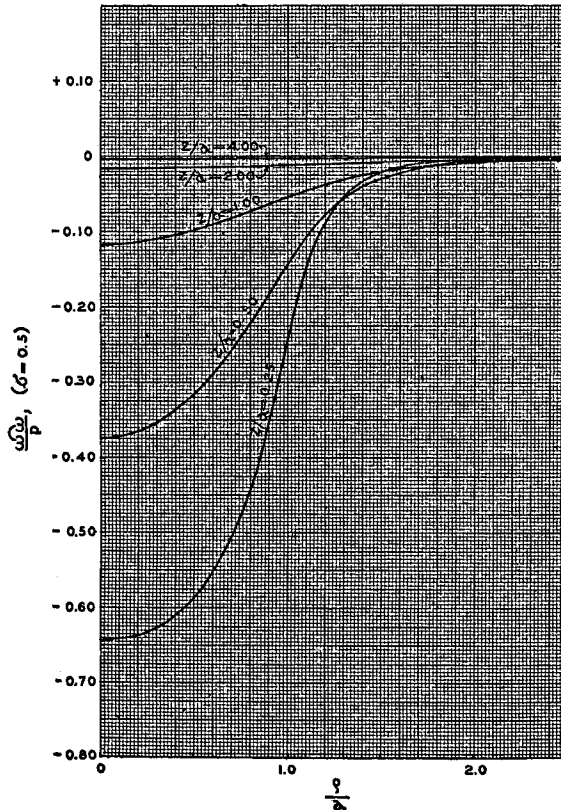


FIGURE C-37.—The Stress Component, $\sigma_{\theta\theta}$, at Various Relative Depths $\frac{z}{a}$, and Relative Lateral Distances $\frac{r}{a}$ from the Vertical Axis in a Semi-Infinite Isotropic Solid, Expressed as a Decimal Part of a Uniform Pressure Applied over a Circular Area at the Boundary Plane. $\nu = 0.5$.

but while the deduction does not permit determination of the point at which permanent yield impends, and is therefore of slight consequence as an aid to rational design, the mathematical theory shows that if the pressure-area relation at any given deflection in a purely cohesive material is other than according to the inverse square root of the area, the elastic limit of the material has been exceeded and yield has taken place. Moreover, the fact that large deformations under large areas and small deformations under small ones, in proportion to the square roots of the areas, exist at the failure point, eliminates the consideration of fixed deformations as criteria in flexible pavement failure or design.

The conclusion that pressure intensity as a cause of failure in the isotropic solid is independent of area effects immediately raises the question as to the validity

of considering area of load application as a factor in the design of rigid pavement when the design theory assumes that both the pavement slab and the subgrade possess elastically isotropic properties.⁵⁸ The answer, of course, is simple. The action of the slab is that of a beam, and quite different from that of an indefinitely extended solid. In the slab the critical stresses are the tensile stresses in the surface fibers at the top or bottom of the slab, depending on the position of the load with respect to the slab boundary. These stresses are dependent on bending moments which in turn are influenced by area of load application. The surface fibers will rupture from tension before shearing forces can be of consequence. In the indefinitely extended solid, shearing forces are of major consequence.

⁵⁸ See appendix B, I. a., the Rational Design of Portland Cement Concrete Pavements, p. 164.

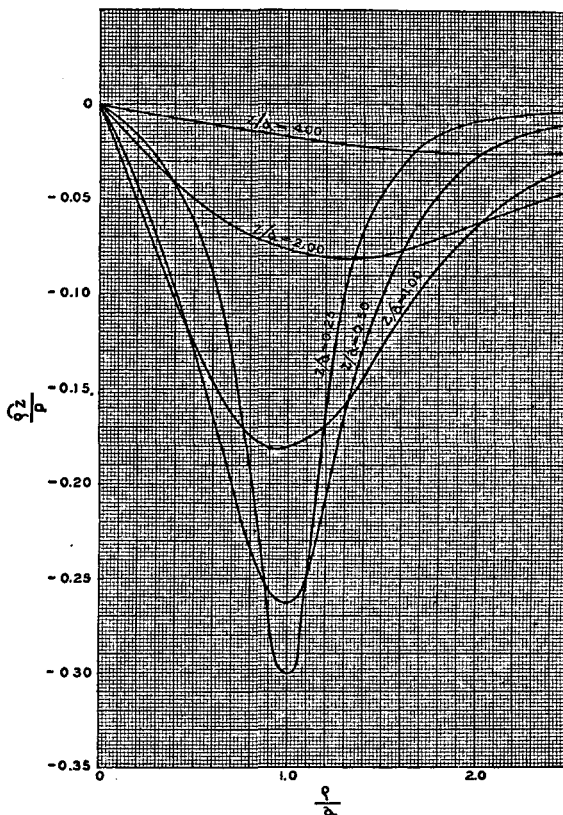


FIGURE C-28.—The Stress Component, $\hat{\sigma}_z$, at Various Relative Depths $\frac{z}{a}$ and Relative Lateral Distances $\frac{\rho}{a}$ from the Vertical Axis in a Semi-Infinite Isotropic Solid, Expressed as a Decimal Part of a Uniform Pressure Applied over a Circular Area at the Boundary Plane.

It remains then to examine, in the light of the mathematical theory, the case of materials which, at early deflections, offer a resistance which is independent of area effects, and which at ultimate deflections (progressive failure) offer a resistance which varies directly with the square root of the area of load application.

Stress distribution and the stability of aeolotropic materials.—One of the underlying assumptions in the solution of the problem of stress distribution in a semi-infinite elastic body, loaded over a small area of its plane boundary, is that the body is elastically isotropic; that its elastic properties are the same in all directions. When this is not true, the body is said to be elastically aeolotropic, i. e., anisotropic or heterotropic.⁵⁷

In accordance with the assumption of elastic isotropy, it must follow that the coefficients by which the elastic properties are measured, such as λ and μ (Lamé's constants), or the more familiar E and σ , are equally applicable to any small element of the body, wherever taken, and equally constant with respect to that element. These moduli are readily available in texts on elasticity⁵⁸ but are here taken, together with their definitions, from Griffith.⁵⁹ They are:

$$E = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)} \quad \text{Young's extension modulus, or ratio of tensile stress to tensile strain.}$$

$$\sigma = \frac{\lambda}{2(\lambda + \mu)} \quad \text{Poisson's ratio of lateral to longitudinal strain.}$$

⁵⁷ Stephen Timoshenko, *Theory of Elasticity*, p. 1, and Max Planck, *Introduction to Theoretical Physics*, vol. 2, p. 65. D. P. Kryzine suggests this usage of "elastically isotropic" as against the more general "elastic and isotropic."

⁵⁸ See, for example, A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (1927), pp. 103-4.

⁵⁹ John H. Griffith, *Dynamics of Earth and Other Macroscopic Matter*, Iowa Engineering Experiment Station Bulletin No. 117, p. 30.

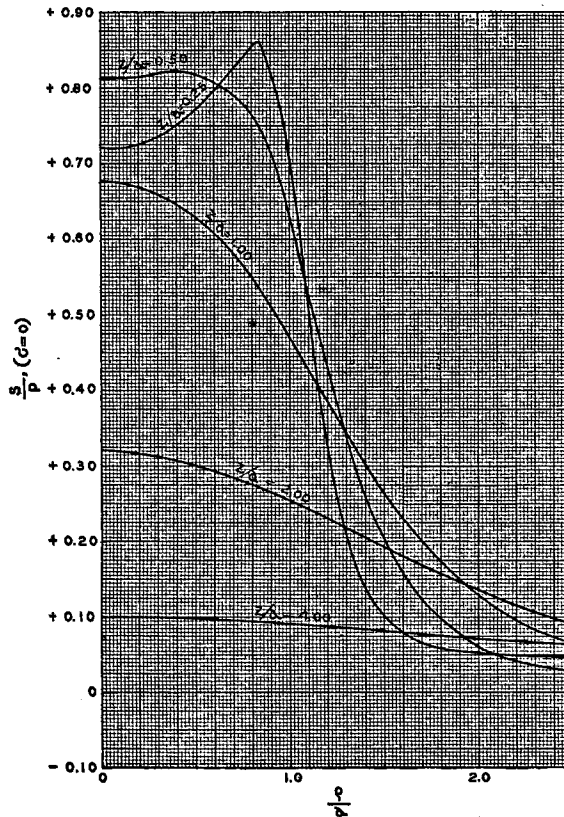


FIGURE C-24.—The Stress Difference, S , at Various Relative Depths $\frac{z}{b}$ and Relative Lateral Distances $\frac{\rho}{b}$ from the Vertical Axis in a Semi-Infinite Isotropic Solid, Expressed as a Decimal Part of a Uniform Pressure Applied over a Circular Area at the Boundary Plane. $\sigma=0$.

$\mu = \frac{E}{2(1+\sigma)}$ Rigidity, or ratio of shearing stress to angular strain.

$\kappa = \frac{E}{3(1-2\sigma)} = \lambda + \frac{2}{3}\mu$ Compression modulus, or ratio of hydrostatic pressure to volumetric strain

$\lambda = \frac{E\sigma}{(1+\sigma)(1-2\sigma)}$ Defined by these equations.

Since the modulus of elasticity, E , is a measure of the stress required to produce a given deformation, it will vary importantly with different materials, and if the stresses are known it is an essential factor in the determination of strains or deformations. On the other hand, if the deformations are known, a knowledge of the relative deformations along different axes of a small element is all that is required for a determination

of the stresses, so long as the actual deformations are proportional to stress. These relative deformations or movements are measured with the aid of Poisson's ratio and therefore this ratio alone appears as the differentiating modulus in the determination of stresses in elastically isotropic materials, however different the materials may be.

It may be observed, moreover, that the modulus appears only in the equations for the horizontal stress components, $\hat{\rho}\hat{\rho}$ and $\hat{\omega}\hat{\omega}$, and is absent in the equations for axial and shearing stresses, $\hat{z}\hat{z}$ and $\hat{\rho}\hat{z}$. This holds for point loadings⁶⁰ as well as for symmetrically distributed loadings. Attention was called by Boussinesq to this absence of elastic moduli in the final equations for stresses normal to planes parallel to the

⁶⁰ See sources mentioned in footnote 51, p. 220.

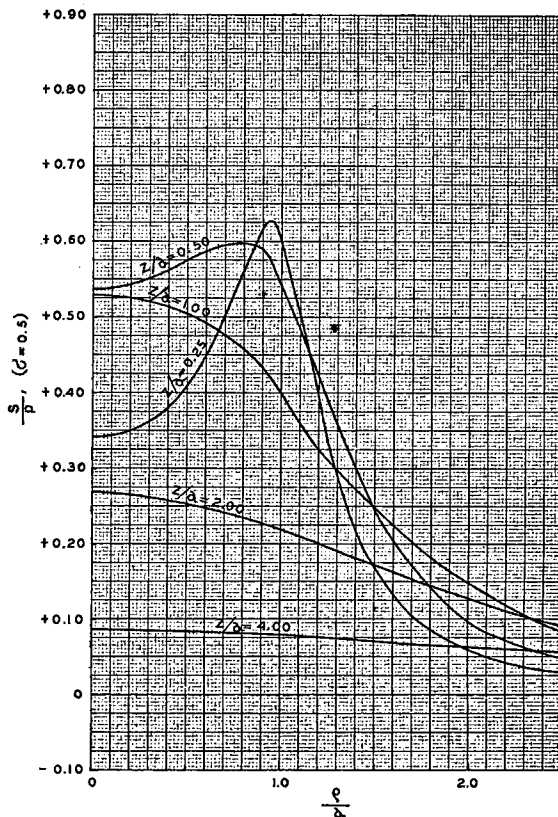


FIGURE C-30.—The Stress Difference, S , at Various Relative Depths $\frac{z}{a}$ and Relative Lateral Distances $\frac{\rho}{a}$ from the Vertical Axis in a Semi-infinite Isotropic Solid, Expressed as a Decimal Part of a Uniform Pressure Applied over a Circular Area at the Boundary Plane. $\nu=0.5$.

boundary, and he observed that such would be the case in all isotropic solids.⁶¹

Boussinesq also called to attention that "Throughout the whole surface of a sphere touching the surface of the given (boundary) plane at the point of application of the normal force (point loading) * * *, the stress, which an elementary plane parallel to the surface of the

body is subjected to, is a constant."⁶² The equation for these stresses, already given,⁶³ may be modified to show the radial stresses emanating from the point, thus:

$$RR = \frac{-3P}{2\pi} \frac{z^3}{R^4}$$

where P is the applied point loading and R is the radius vector drawn from the point, in any plane through the z axis. From this equation it may be noted that the stresses RR , in any direction of R whatever, are independent of elastic moduli so long as the stress trans-

⁶¹ Joseph Boussinesq, Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques, * * * pp. 105-106; also Isaac Todhunter and Karl Pearson, A History of the Theory of Elasticity, vol. 2, part 2, p. 249. The moduli enter in the derivation of the equations but vanish by cancellation. See above references and Stephen Timoshenko, Theory of Elasticity, pp. 7-11 and 228-231, and S. D. Carothers, Test Loads on Foundations as Affected by Scale of Bearing Area, Proceedings, International Mathematical Congress, Toronto, 1924, vol. 2, p. 543, eq. 49.

⁶² See Boussinesq and Todhunter-Pearson sources, footnote 61. Translation quoted is by Pearson; parentheses are added.

⁶³ See footnote 25, p. 202.

mitting properties are the same in all directions; in other words, so long as the material is perfectly isotropic.

If the elastic properties are not the same in all directions, it is easy to see that the values of $\hat{E}\hat{R}$ will be different along different directions or slopes of R from those which the equation indicates, and that therefore Boussinesq's spheres of constant stress, \hat{z}_z , on planes parallel to the boundary will be distorted by an amount which will depend on the magnitude of the difference in the elastic properties along the several directions of R . Stresses in an aeotropic body may, therefore, not be calculated by an unmodified application of the equations of the isotropic solid.

In 1856, Rankine presented " * * * certain principles which are common to every case of the internal equilibrium of a solid mass in what manner soever constituted," and then proceeded "to combine those principles with the peculiar principle of the stability of an incoherent granular mass."⁶⁴ And in his famous memoir on pulverulence, of 1876, Boussinesq makes a direct transition from the consideration of the limit of elastic stability in isotropic mediums to an analysis of the equilibrium of a cohesionless pulverulent mass.⁶⁵ The step is accomplished by the substitution of a certain constant, which may be considered as related to the angle of internal friction, for the elastic moduli in the case of the isotropic solid.⁶⁶ Agreement with Rankine results.

According to Karl Pearson, "Boussinesq bases his theory of pulverulence on the hypothesis that masses of pulverulent material stand midway between solid and fluid bodies, and act like fluids when not subjected to pressure, but when subjected to pressure gain an elasticity of form as well as of bulk, and act like solids."⁶⁷

In 1899, J. H. Michell investigated the problem of the semi-infinite solid (1) when two, elastically different, isotropic solids are soldered together over the plane, and (2) when there is free slipping over the plane.⁶⁸ A year later he analyzed the problem of stress distribution in an aeotropic elastic solid, having an axis of elastic symmetry.

In the later paper,⁶⁹ he considers the surface depression due to a normal pressure concentrated at a point. His equations contain interrelated constants which in the case of the isotropic solid would be the elastic moduli λ , μ and $\lambda+2\mu$. When these are substituted, the equation is identical with the Boussinesq results. He con-

cludes: "It appears, therefore, that the law of depression is the same as for an isotropic solid."

He also considers stresses across planes parallel to the boundary. In order to pass to the case of an isotropic solid, he introduces two variables which vanish in the limiting case, in which he again obtains Boussinesq's results.

These important solutions to the problems cited are historically significant in their disclosure of a continuity between the isotropic case and cases known to be imperfectly isotropic or definitely aeotropic, even though the solutions do not comprehend the problem of area effects on stress distributions in a semi-infinite body.

Griffith has more recently provided such a continuity for all of the principal stress components resulting from a point loading,⁷⁰ by considering the derivation of the stress equations in the isotropic case and introducing a variable parameter which satisfies test results for other types of matter. Krynine⁷¹ shows this parameter to be equal to two plus the reciprocal of the coefficient of pressure at rest, which is usually given as the ratio of lateral to vertical pressure for matter uniformly stressed under its own weight. Carothers, following earlier authorities, gives this coefficient as $\lambda/(\lambda+2\mu)$ ⁷² while Griffith also gives the equivalent $\sigma/(1-\sigma)$, in terms of Poisson's ratio.⁷³

Utilizing Griffith's parameter, which may be called n , Krynine adapts it to the equation for stress along the vertical axis under a uniformly loaded circular area to the following effect:

$$\hat{z}_{z,r=\sigma} = p \left[\left(\frac{z}{(a^2 + z^2)^{1/2}} \right)^n - 1 \right]$$

in which

$$n = 2 + \frac{1-\sigma}{\sigma} = \frac{1+\sigma}{\sigma}$$

Bearing in mind that the distribution of the stresses, \hat{z}_z , on planes parallel to the boundary is the same for all isotropic bodies and independent of elastic moduli, and that in such cases $n=3$, one may conclude that in the elastically isotropic case the coefficient of pressure at rest, i. e., the ratio between the principal pressures, is unity.

Such a coefficient implies a hydrostatic condition under the action of gravity, an initial Poisson's ratio of 0.5 in any isotropic body, and incompressibility. These seeming anomalies are removed by the fact that

⁶⁴ W. J. M. Rankine, On the Stability of Loose Earth, Philosophical Transactions, The Royal Society of London, vol. 147, p. 17.

⁶⁵ Joseph Boussinesq, Essai théorique sur l'équilibre d'élasticité des masses pulvérulentes * * * Mémoires Couronnés, Académie Royale de Belgique, vol. 49, 1876, pp. 83-85.

⁶⁶ Same, p. 85.

⁶⁷ Isaac Todhunter and Karl Pearson, A History of the Theory of Elasticity, vol. 2, pt. 2, p. 218.

⁶⁸ J. H. Michell, The Transmission of Stress Across a Plane of Discontinuity, and the Potential Solutions for a Plane Boundary, Proceedings, London Mathematical Society, vol. 21, 1900, p. 152.

⁶⁹ J. H. Michell, The Stress in an Aeotropic Elastic Solid with an Infinite Plane Boundary, Proceedings, London Mathematical Society, vol. 23, 1901, p. 267.

⁷⁰ John H. Griffith, The Pressure Under Substructures, Engineering and Contracting, March 1933, and Griffith on Earths and Foundations, Proceedings, American Society of Civil Engineers, November 1933, p. 1461 et seq.

⁷¹ Krynine on Distribution of Stresses Under a Foundation, Proceedings, American Society of Civil Engineers, pt. 1, October 1933, p. 1255.

⁷² S. D. Carothers, Test Loads on Foundations as Affected by Scale of Tested Area, Proceedings, International Mathematical Congress, Toronto, 1924, vol. 2, p. 336.

⁷³ John H. Griffith, Dynamics of Earth and Other Macroscopic Matter, op. cit., pp. 19 and 49-51.

Poisson's ratio, σ , leads to a measure of volume change⁷⁴ under load, and a total absence of volume change (which corresponds to $\sigma=0.5$) as well as of initial stress, from the action of gravity or other body forces, is assumed as the starting point for load stress calculations.⁷⁵ A value of $n=3$ simply signifies, therefore, absence of initial stress in the body.

Love says, "The initial state of a body may be too far removed from the unstressed state to permit of the stress and strain being calculated by the principle of superposition * * *" and adds, "A body in equilibrium under the mutual gravitation of its parts is in a state of stress, and when the body is large the stress may be enormous."⁷⁶

In examining the stress distribution in foundation earths in the light of the elastic theory, Carothers⁷⁷ suggests a consideration of the existing initial stresses in the ground as necessary to the evaluation of the total stress differences under the applied pressure. He utilizes the weight of the earth and the coefficient of pressure at rest in their calculation, as does Griffith.⁷⁸

He cites three different cases where these initial stresses in the unloaded material will differ widely: (1) The case of an earth formed by gradual accretion in horizontal layers. The pressure in this case would be greater on horizontal planes than on vertical planes; (2) the case of an earth formed from a semi-fluid impalpable mass which has solidified gradually from the top downward. The pressure in this case would be of a hydrostatic nature and equal in all directions; (3) the case of a layer of earth now denuded of strata with which it had once been overlaid. Here the natural earth pressure might conceivably correspond with that of slate formations and be greater on vertical planes than on horizontal ones.

Though all three of these limiting cases might well be considered in the problem of building foundations, the last of these would be of slight if other than theoretical interest in earth road and subgrade problems. And since the second conforms to the elastically isotropic case already discussed, it remains to examine the case wherein the pressure on horizontal planes, owing to gradual accretion or the influence of gravity, is greater than on vertical planes. This case, too, would include that of earth roads and subgrades, rolled or compacted under the action of traffic and of superimposed base courses and flexible pavements.

The problem appears to be three-fold, involving (1) different elastically isotropic strata superimposed in

well-defined layers, (2) superimposed isotropic and aeotropic layers, and (3) unstratified material with negligible cohesion, such as a loose gravel or sand.

The first case obviously is the one investigated by Michell, whose solution involves only an adjustment of the elastic moduli. It will be considered later in connection with an examination of the second type of stratified material. It becomes of immediate importance to consider the case of the loose granular medium in the light of the concept of isotropy.

Krynine recently made a valuable contribution to the thought on the subject when he said:⁷⁹

Bodies termed "isotropic" in reality may possess quite different properties, depending on their degree of isotropy. If the material in a body possesses exactly the same properties throughout, including the stress-transmitting capacity, any point of any line drawn within that body will be a part of an isotropic "continuum", and such a state of isotropy may be termed "monotonous." On the other hand, if the material consists of units of particles interspersed with voids or some other material, with different stress-transmitting capacities, the body may possess "statistical" isotropy. Owing to the presence of a great number of particles pressed against each other, stresses within such a body may be generated so as to simulate a "continuum," and this similarity is the greater, the greater the degree of isotropy approaches the "monotony" referred to; for instance, if two straight lines, 1 inch long, drawn within a sand and a clay mass, pierce 100 and 100,000 soil particles, respectively, it is evident that the clay mass is much more properly termed "monotonously" isotropic than the sand mass. Furthermore, attracted moisture, which is almost solid in the case of a moist clay mass, approaches more the stress-transmitting capacity of the soil particles than water in the voids of a sand mass. Hence, formulas of the theory of elasticity, the Boussinesq formula among them, developed for the case of an isotropic elastic "continuum" should be applied to clays rather than sands. It does not follow, however, that clays are elastic, and sand inelastic. Each sand particle is an elastic body, as demonstrated by the rebound after releasing a load from a sand mass; but the reason why the Boussinesq formula cannot be applied unconditionally to sand, lies in their lack of monotonous isotropy. Since statistical isotropy requires the presence of a great number of particles, it does not exist at all where this condition is not fulfilled. Hence, an earth mass, especially a natural sand mass, is not isotropic close to its boundary.

The statistical concept is not without historical antecedent in the development of the elastic theory. S. D. Poisson, in 1828, whose hypotheses and mathematical formulations lead to the strictest sense of isotropy (see Planck's definition, p. 228 above), assumed that an "irregular action of the molecules in the immediate neighborhood of the one considered may be neglected in comparison with the total action of those more remote, which is regular."⁸⁰ His results agreed with those of his contemporary, A. L. Cauchy, who assumed a perfectly symmetrical distribution of molecules and hence a regular action throughout.⁸¹ It remained for

⁷⁴ Stephen Timoshenko and Gleason H. MacCullough, *Elements of Strength of Materials*, p. 44. The unit volume change is $\epsilon(1-2\nu)$, where ϵ is the unit strain, sufficiently small to permit neglect of its squares and cubes.

⁷⁵ Stephen Timoshenko, *Theory of Elasticity*, p. 4. A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity*, (1927), pp. 75-77 and 100-111.

⁷⁶ Sams, p. 100.

⁷⁷ S. D. Carothers, *Test Loads on Foundations as Affected by Scale of Tested Area*, Proceedings, International Mathematical Congress, Toronto, 1924, vol. 3, pp. 530-531.

⁷⁸ John E. Griffith, *Dynamics of Earth and Other Macroscopic Matter*, loc. cit., Bulletin 117, pp. 40-44.

⁷⁹ Krynine on Distribution of Stresses Under a Foundation, Proceedings, American Society of Civil Engineers, pt. 1, October 1935, pp. 1263-1264.

⁸⁰ S. D. Poisson, *Mémoire sur l'équilibre et le mouvement des corps élastiques*, Mémoires de l'Académie des Sciences, Paris, tome 8, 1828, pp. 393-406.

⁸¹ A. L. Cauchy, *De la pression ou tension dans un système de points matériels* This article occupies pp. 219-226 of Cauchy's *Exercices de Mathématique*, 1828.

R. Clausius, two decades later, to show in the light of experiment that either assumption was perfectly legitimate since the required value of the elastic constants is not that of a particular point, but the mean value for an indefinite number of points.⁸³

Both Poisson and Cauchy were sustained by Clausius and later by Saint-Venant in their conclusion that in elastically isotropic bodies $\lambda = \mu$, from which it follows that $\sigma = \frac{1}{2}$.⁸³ Observed experimental departures later came to be associated with aeolotropy, and the absence of materials which could with assurance be supposed perfectly isotropic with respect to elastic properties led investigators to experiment with materials, such as crystals, with known kinds of aeolotropy.⁸⁴ And though Love states that "The best modern experiments support the conclusion that Poisson's ratio can differ sensibly from the value $\frac{1}{2}$ in materials which may without cavil be treated as isotropic and homogeneous,"⁸⁵ the validity of Love's conclusion rests on statistical isotropy.

Griffith says:

Isotropic matter in the light of the present theory is regarded a singular structure in which the organisation breaks down by means of, for example, a chaotic arrangement of the individual homogeneous component structures, which occurs for a crystalline structure in contradistinction to a crystal structure * * *. By the theory of probability, if displacement or strain measurements are taken along any direction, the properties should be about the same owing to the compensation.⁸⁶

The same thought is stated by Timoshenko in discussing the case of steel:

The material is very far from being homogeneous, but experience shows that solutions of the theory of elasticity based on the assumptions of homogeneity and isotropy can be applied to steel structures with very great accuracy. The explanation of this is that the crystals are very small; usually there are millions of them in one cubic inch of steel. While the elastic properties of a single crystal may be very different in different directions, the crystals are ordinarily distributed at random, and the elastic properties of larger pieces of metal represent averages of properties of the crystals. So long as the geometrical dimensions defining the form of a body are large in comparison with the dimensions of a single crystal the assumption of homogeneity can be used with great accuracy, and if the crystals are orientated at random the material can be treated as isotropic.⁸⁷

In the light of these statements by Krynine, Love, Griffith, and Timoshenko, one may conclude that where favored directions of stress transmission are distributed at random through small elements of the body, the body is effectually isotropic, and that where favored directions exist for the body as a whole it is aeolotropic.

⁸³ Fergendoff's Annalen, vol. 78, 1848, pp. 46-67.

⁸⁴ Navier, Lamé, Clapeyron and others of the time assumed $\lambda = \mu$ in solid bodies in general.

⁸⁵ This is discussed by A. E. H. Love in *A Treatise on the Mathematical Theory of Elasticity*, (1927), p. 10, and at some length by Isaac Todhunter and Esai Pearson in *A History of the Theory of Elasticity*, vol. 1, pp. 497-505 and 753.

⁸⁶ Same, p. 14.

⁸⁷ John H. Griffith, *Dynamics of Earth and Other Macroscopic Matter*, loc. cit., Bulletin No. 117, p. 24.

⁸⁸ Stephen Timoshenko, *Theory of Elasticity*, p. 1.

Thus, Michell's "aeolotropic elastic solid having an axis of elastic symmetry" possessed what Love and others refer to as transverse or cylindrical isotropy.

In the case of a granular mass, such as sand, assumed to be lacking in cohesion and therefore in tensile resistance, it is obvious that the stress transmitting qualities in the vertical direction are quite different from those in a lateral direction if planes near the surface are considered, and that if the particles are small enough the properties will be essentially the same along any line drawn to the axis and rotated about it without change in the angle. The mass may thus be said to be cylindrically or transversely isotropic. Such symmetry could not be supposed, however, if a mass of the same dimensions were composed of large, irregular boulders. And this is consistent with Krynine's discussion of statistical isotropy in earth masses.

From Michell's findings relative to cylindrical isotropy, one might readily suppose that appropriate substitution in the equations of elasticity for the constant moduli would be sufficient for the determination of stress distribution in an aeolotropic body such as a sand mass. But Krynine says, "Probably, at a certain depth within the earth all matter obeys the Boussinesq law with a concentration factor, $n = 3$, and the difference between the stress distribution in sand and clay is merely a surface phenomenon,"⁸⁸ and moduli, variable with depth, implied in Krynine's statement, are supported by test results.⁸⁹ And in harmony with the famous "Principle of Saint-Venant"⁹⁰ and with the theory of local perturbations as enunciated by his most noteworthy pupil, Boussinesq, wherein the manner of load distribution over a pressed area and the effects of the area itself as opposed to a point loading are of importance near the surface but decrease in importance with distance from the surface, one may suppose that departures from the action of a true elastically isotropic solid are also of significance near the surface only.

To offset purely local effects of loading or boundary conditions in the application of the elastic theory it is

⁸⁸ Krynine on Distribution of Stresses Under a Foundation, Proceedings, American Society of Civil Engineers, pt. 1, October 1934, p. 1294.

⁸⁹ Since the deflection, u_0 , at the boundary of a pressed circular area is given by the equation

$$u_0 = 4 \frac{(1-\nu^2)pe}{E}$$

where p is the applied pressure intensity and e is the radius of the bearing area, it follows that

$$\frac{(1-\nu^2)}{E} = \frac{u_0}{4pe}$$

and this equation may be employed in connection with tables 1 to 15 to show the variation.

⁹⁰ This principle, enunciated in 1855 by Barré de Saint-Venant states that if the forces acting on a small part of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, the redistribution of loading produces changes in the stresses locally but has a negligible effect on the stresses at distances which are large in comparison with the linear dimensions of the loaded portion of the surface. See A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity*, (1927), p. 21 and 131; and Stephen Timoshenko, *Theory of Elasticity*, p. 31.

correct to make local modifying assumptions⁹¹ and to pass from the case of a given elastic solid to that of another one simply introduces an appropriate change in Poisson's ratio. But from table C-17 and figures C-29 and C-30 one may observe that this modulus may itself be regarded as a local modifier because it vanishes in importance at distances which are large in comparison with the linear dimensions of the loaded area. The introduction of a modulus variable with depth to accommodate the case of a material having decreasing anisotropy as distance from the surface increases, appears to be, therefore, a wholly logical procedure.

Since the isotropic solid has a coefficient of pressure at rest of unity, a material having a very low degree of superficial isotropy, such as dry sand, has, near the surface at least, a coefficient which is close to zero. Under small pressure applications the lateral resistance is negligible, tensile resistance is absent, and the movement that takes place is caused by the rotational action of the particles under purely vertical pressure. In this stage, like pressures over different areas result in like movement. Settlement is essentially independent of area.

As rotational resistance increases under the developed friction the effect is similar to the introduction of an appreciable lateral pressure, because the arching action of the particles transfers a part of the load to the material in the region where $\rho > a$. This transfer is known as the "bin effect" because it closely resembles the transfer of a part of the weight of the contents of a storage bin to the walls of the bin.⁹²

The introduced lateral resistance, however, does not bear a constant ratio to vertical pressure because it is directly dependent on the depth considered. The result is comparable to a Poisson's ratio which increases with the depth to which failure extends⁹³ and hence with the area of load application.

If a fixed maximum stress-difference is attained under a given pressure and area at a given depth, the same stress-difference is not attained by a larger area at a depth proportional to its radius, as in the monotonously isotropic material. Nor, in fact, is it attained at all under the same pressure, because an increase in Poisson's ratio is accompanied by a decrease in the maximum stress-difference that can occur.

⁹¹ A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (1927) p. 189. Particular cases of stress indeterminacy are referred to by A. E. H. Love in *The Stresses Produced in a Semi-infinite Solid by Pressure on Part of the Boundary*, *Philosophical Transactions of the Royal Society of London*, vol. 228 (series A), November 1926, pp. 401, 402; as well as by Kwan-ichi Tazawa in *On the Elastic Equilibrium of a Semi-infinite Solid Under Given Boundary Conditions*, *Journal of the College of Science, Imperial University, Tokyo*, vol. 37, article 7, December 7, 1916, p. 15.

⁹² Charles Terzaghi, *Arching in Sand*, *Engineering News-Record*, May 14, 1936, p. 692.

⁹³ L. M. Newmark suggests the empirical transformations of the depth value, z , to, z_1 , say, and the substitution of the adjusted values in the elastic equations. See Newmark on *Distribution of Stresses Under a Foundation*, *Proceedings, American Society of Civil Engineers*, January, 1936, p. 137. Though practical, this approach cannot be as rational as the adjustment of the dimensionless Poisson's number, ν , for various depths.

One may consider the maximum stress-difference along the vertical axis. This can be expressed, according to the mathematical theory for an isotropic solid, as

$$S_{\max.} = p \left[\frac{(1-2\sigma)}{2} + \frac{2}{9} (1+\sigma)(2+2\sigma)^{3/2} \right], \quad \rho=0$$

and it occurs at a depth

$$\frac{z}{a} = \left[\frac{2(1+\sigma)}{7-2\sigma} \right]^{1/2}$$

The following table will illustrate the relation between the pressure required to produce a given stress-difference on the vertical axis and the depth-radius ratio:

σ	$\frac{S_{\max.}}{p} (\rho=0)$	$\frac{z}{a}$	Relative depth	Relative pressure to produce given $S_{\max.}$
0	0.8143	0.8345	1.0000	1.0000
.1	.7626	.8688	1.0442	1.0678
.2	.7131	.9050	1.1282	1.1419
.3	.6658	.9374	1.1925	1.2230
.4	.6206	.9720	1.2572	1.3121
.5	.5774	.7071	1.3229	1.4108

In the case of a cohesionless granular mass, uniform as to distribution of particle size and shape, variation in Poisson's ratio will be related to the influence of gravity and hence this modulus may be assumed to vary directly with depth. From the foregoing table it may be observed that for practical purposes Poisson's ratio, the maximum stress-difference, and the depth at which it occurs, all bear an essentially linear relation to each other. It follows, therefore, that whatever the rate of variation in Poisson's ratio with depth these relations will not be disturbed and that the required pressure intensity on different areas to produce a given maximum stress-difference along the vertical axis will vary almost directly with their radii.

Actually, the required pressure for the larger areas is slightly greater than the direct ratio of their radii, but a compensation occurs in the zone close to the bounding edge of the bearing area. Here the question of area does not enter. The maximum stress-difference occurs at $\phi = 70^\circ 55' 39''$, with $k \rightarrow 0$, $\rho \rightarrow a$, and may be determined from the equation

$$S_{\max.} = p \left[\frac{(1-2\sigma)^3}{4} - 0.3932\sigma + 0.5586 \right]^{1/2}$$

and it is found, under any value of σ , to be greater than the stress-difference on the vertical axis. Like the latter, however, it varies directly with pressure intensity.

Another critical zone lies on the surface boundary of the material immediately outside the pressed area. Here

$$\frac{T_1}{p} = \frac{\omega\omega}{p} = \frac{(1-2\sigma)}{2} = \frac{S}{p}$$

and, therefore, in this zone

$$\frac{T_1 - \omega\omega}{p} = 1 - 2\sigma$$

and this particular stress-difference is greater than the stress-difference $\frac{S}{p}$ in the zone considered. It, too, is independent of area of load application, but is dependent on pressure intensity. Computed values of these stress differences occurring near the edge of the loaded area follow:

	$\sigma=0$	$\sigma=0.1$	$\sigma=0.2$	$\sigma=0.3$	$\sigma=0.4$	$\sigma=0.5$
$\frac{S_{max}}{p}, (k \rightarrow 0)$	0.8992	0.8241	0.7549	0.6838	0.6418	0.6017
$\frac{T_1 - \omega\omega}{p}, (\sigma \rightarrow 0)$	1.0	0.8	0.6	0.4	0.2	0

In the case of a granular material it is only necessary to consider such values of these stress-differences as are associated with small values of Poisson's ratio.

The magnitude of S indicates that failure will progress from this zone toward the axis, and under the assumption of a uniform pressure this would always be the case.

At the instant of local yielding, however, in this zone, in a loose material,⁶⁶ it is quite likely that the assumption of uniform pressure is, for the instant at least, not valid; in which case $T_1 - \omega\omega$ becomes the critical stress-difference. Moreover, it is conceivable that a material such as a dry sand might sustain a larger stress-difference under the edge of the bearing area, due to its arching action, until movement had first taken place as a result of its yielding to a smaller stress-difference just outside this area.

On the other hand, a reduced support just inside the edge of the bearing area might momentarily place the entire load on a smaller area of support, thereby increasing the pressure intensity. The part of the load to be redistributed, being proportional to the perimeters for different areas, would have a relatively greater effect for the smaller areas. If, for example, a load, resulting in a pressure of one unit when applied over an area with a radius of one unit, were thus redistributed over an area having a radius of 0.9 unit, the new pressure intensity would be 1.2346 units. At the same time, a load resulting in a pressure intensity of 1.4103 units when applied over an area with a radius of 1.3229 units, if reapplied over an area having a radius of 1.2229 units would result in a new pressure intensity of 1.6503 units. The ratio of 1 to 1.4103 of the original

pressures is thus changed to 1 to 1.3367, and brought closer to the ratio of the original radii of 1 to 1.3229.⁶⁶

Whatever the nature of the actual occurrence, it is clear that in the case of stress-differences near the boundary a relatively minor influence, independent of area, acts in conjunction with an influence which increases at a rate slightly faster than the square root of the area. In the other case an inverse modifier enters. But in either case the modification indicates an approximately direct relation between the yield point and the square root of the loaded area, for a superficially aeolotropic body, weak in cohesion and subjected to a given uniform pressure intensity, applied over a circular area. And this relation, suggested by mathematical theory, and confirmed by a consideration of the mechanical properties of granular materials, is further substantiated by test results (figs. C-12 to C-15, inclusive).

Rankine's criterion of failure in a granular medium.—

In presenting his "mathematical theory of that kind of stability, which, in a mass composed of separate grains, arises wholly from the mutual friction of those grains, and not from any adhesion amongst them", Rankine assumed his theory applicable to earths and he believed that the angle of internal friction was identical with the angle of repose.

His fundamental theorem stated that, "At each point in a mass of earth, the ratio of the difference of the greatest and least pressures to their sum cannot exceed the sine of the angle of repose."⁶⁷ Stated in equation form this is, in our own notation,

$$\frac{T_1 - T_2}{T_1 + T_2} \leq \sin \phi$$

which, of course, may also be put

$$\frac{T_2}{T_1} \leq \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left[45^\circ - \frac{\phi}{2} \right]$$

In discussing the characteristic differences between a cohesive frictionless material and a cohesionless granular material, Terzaghi states that, in the former, "failure occurs as soon as the difference between the greatest and least principal stress exceeds the compressive strength of the material," while in the latter, "failure occurs as soon as the ratio between the greatest and least principal stress exceeds a definite limiting value," and he adds that "This condition represents the most distinguishing feature" between the two cases.⁶⁷

⁶⁶ See preceding table of stress-differences along the vertical axis, i. e., $\frac{S_{max}}{p}, (\nu=0)$.

⁶⁷ W. J. M. Rankine, On the Stability of Loose Earth, *Philosophical Transactions The Royal Society of London*, vol. 147, 1857, pp. 17-18.

⁶⁸ Terzaghi on the Science of Foundations, *Transactions, The American Society of Civil Engineers*, vol. 63, 1923, p. 381. See also, Leo Jürgenson, The Application of Theories of Elasticity and Plasticity to Foundation Problems, *Journal, Boston Society of Civil Engineers*, vol. 21, No. 3, July 1924, p. 320.

⁶⁶ In discussing the local yielding around the edge of a rigid die, in the case of an isotropic solid, Timoshenko says: "In actual cases we shall have yielding of material along the boundary. This yielding, however, is of local character and does not substantially affect the distribution of pressure at points some distance from the boundary of the die." (Stephen Timoshenko, *Theory of Elasticity*, p. 336.) See footnote 90, p. 333.

The limiting condition of stability as described by Rankine has frequently led to the misconception of an increasing ratio, T_2/T_1 , under applied pressure, which approaches a fixed maximum determined by the obliquity of the friction angle. A similar misinterpretation may easily be made of Terzaghi's statement.

In granular as well as cohesive mediums the principal stresses T_1 and T_2 are each proportional to applied pressure at all points in the mass, and therefore at any given point they maintain a constant ratio to each other within the limits of the stability of the medium. Within these limits it is obvious that, under increasing loads, though the ratio between the pressures may be a constant at any selected point, the difference between them will be a constantly increasing quantity.

In Terzaghi's "frictionless" material the "compressive strength" is the resistance to shear afforded by cohesion and is therefore a constant and independent of load increments. And because this constant is analytically equal to one-half the principal stress-difference, $T_1 - T_2$, it is convenient to utilize this difference as the criterion of rupture in materials where cohesion affords the significant strength.⁸⁸ It is of interest, however, to note that Joseph Marin, after correlating many theories of rupture, in materials possessing cohesive strength, with test results concluded that "the precisely correct theory of failure" was probably a combination of a number of those theories, depending on the ratio of the principal stresses.⁸⁹ This ratio, of course, at any point is dependent on Poisson's ratio for the material.

In Terzaghi's "cohesionless" material the "definite limiting value" is the resistance to shear afforded by friction. The compressive strength of the grains is only partially brought into play because rotation and sliding of the grains takes place before their compressive limit is exceeded. But the resistance to friction, though theoretically dependent on the pressure normal to the planes of sliding, and hence to the applied pressure, is at the same time determined by the magnitude of the normal pressure that those planes can support, which in turn is determined by the resistance afforded laterally by the weight and internal friction of the surrounding mass. Housel has effectively stated this in another way. Referring to the internal stability of granular materials, he says,

* * * if frictional resistance is the controlling factor, granular material would have infinite bearing capacity up to the crushing strength of the particles themselves. * * * The failure universally observed is due to lateral displacement.

He also says,

The stability of the soil arch cannot be disturbed except by applying a vertical pressure which exceeds the critical value as

⁸⁸ Stephen Timoshenko, *Theory of Elasticity*, pp. 15-20; A. Nádai, *Plasticity*, pp. 44-48; Hamilton Gray, *Stress Distribution in Elastic Solids*, Proceedings, International Conference on Soil Mechanics, Cambridge, Mass., 1936, vol. 2, p. 159.

⁸⁹ Joseph Marin, *Failure Theories in Materials Subjected to Combined Stresses*, American Society of Civil Engineers, No. 101, 1934, p. 1177.

controlled by the horizontal thrust available as support from an adjacent element * * *¹

In such materials, therefore, the really limiting condition of stability is the lateral support limiting the magnitude of T_1 , and though at this limit it is demonstrated that the ratio T_2/T_1 is related to the angle of repose, or so-called internal friction,² or to the inclination of the slip planes to the vertical axis,³ it can also be stated that at this limit the difference between the greatest and least principal stresses will attain a fixed maximum which not only is determined by the angle ϕ but also by their sum, or that

$$T_1 - T_2 = S = (T_1 + T_2) \sin \phi$$

Under increasing pressure up to this point the ratio T_2/T_1 remains a constant. Beyond this point an increase in T_1 cannot be sustained and any increase in T_2 with its resulting increase in the ratio T_2/T_1 will signify failure. Up to this point, however, the difference $T_1 - T_2$ has been constantly increasing with pressure, and therefore there is reason to adopt, with Carothers,⁴ a limited stress ratio, in a comparative study such as this.

The superimposition of isotropic and aeotropic materials.—Michell's mathematical investigation of the transmission of force across planes of discontinuity in axially isotropic materials has already been referred to. Point loading was assumed. In the one case slipping was permitted between the layers; in the other it was not. Surface deflections were found to follow the law of deflections of the continuous elastic solid. Pressure transmission followed a systematic departure from that of the continuous solid, but the departure was accounted for by the assumption of variable moduli. The difference was one of degree only.

In 1935, M. A. Biot gave the results of a similar mathematical investigation.⁵ Again a point loading was assumed, and again the body was considered as elastic and axially isotropic. Slippage and absence of slippage between planes of discontinuity were again important factors. The report states:

We here investigate the effect on such a pressure distribution,⁶ taken at a depth h , of the presence at that depth of a slippery

¹ William S. Housel, *Internal Stability of Granular Materials*, Proceedings, American Society for Testing Materials, vol. 36, 1936, pt. 2.

² Reynolds says, "It is a very difficult question to say exactly what part friction plays; for although we may perhaps still assume without error

$$\frac{T_2}{T_1} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

where ϕ is the angle of repose, we cannot assume that $\tan \phi$ has any relation to the actual friction between the particles." (Osborne Reynolds, *On the Dilatancy of Media Composed of Rigid Particles in Contact*, Scientific Papers (1903), p. 203. See footnote 24, p. 202, above.)

³ A. Nádai, *The Phenomenon of Slip in Plastic Materials*, Proceedings, American Society for Testing Materials, vol. 31, pt. 2, 1931, pp. 24-28.

⁴ S. D. Carothers, *Test Loads on Foundations as Affected by Scale of Tested Area*, Proceedings, International Mathematical Congress, Toronto, 1924, vol. 2, p. 530.

⁵ M. A. Biot, *Effect of Certain Discontinuities on the Pressure Distribution in a Loaded Soil*, *Physics*, vol. 6, December 1935, pp. 267-276.

⁶ The Boussinesq distribution, which serves as Biot's case (e).

rigid bed (case b); of a perfectly rough rigid bed (case c); and of a perfectly flexible but inextensible thin layer embedded in the material (case d).

The results afford a useful complement to Griffith's theory of variable parameters, previously discussed, in that they permit the determination of numerical values of the coefficient to be applied to $-3P/2\pi z^2$ in the several cases.⁷

Carothers carries the investigation to uniform pressures applied over finite bearing areas.⁸ In addition to the case of the indefinitely extended body, he also considers, "when at a depth h below the surface there is a horizontal layer of frictionless material," and "when at a depth h below the surface there is a horizontal surface on which the value of friction is not restricted."

In view of the conclusion of the project committee on stabilized soil road surfaces, of the Highway Research Board, that "Earth structures lose stability mainly as a result of soil being squeezed out from beneath the loaded area",⁹ the question of slippage or nonlippage appears to have some significance in the general theory.

Carothers, however, investigates building foundation areas and certain results are, therefore, unrelated to the road problem. Considering a layer of foundation earth 60 feet in depth, he says:

Working out the stress differences for various cases it will be seen that for quite narrow strips the maximum stress difference in the central plane occurs near the surface whereas in the case of much wider strips the maximum stress difference occurs at deeper levels; when the width of the strip becomes 120 feet or twice the thickness of the earth the maximum stress difference in the central plane occurs at the earth foundation level and the whole thickness of the foundation is subjected at every level to the full maximum at some point, whereas with the narrower strips only a portion of the thickness is subjected to this maximum.

Such a situation is unrelated to the road problem, owing to the far smaller areas of pressure application, except in the case of improper design where a thin, moist clay layer beneath the surface overlies a rigid rock foundation or in the case of a solidly frozen subgrade underlying a stratum of thawed-out clay. But these occurrences, together with the occasional instances where a subgrade or substratum flows from beneath the surface layer, though troublesome to the road builder, are not sufficiently general to warrant more than passing mention in the present study. They are problems in stabilization through drainage and reduction in capillarity.

As stated earlier, Carothers finds it necessary to take into account the original stress-differences in the ground. These will be a function of the weight of the earth material. Due to symmetry, under similar areas a given stress-difference from a given applied pressure intensity will occur at depths which are in proportion to

lateral dimensions of the areas and hence at greater depths for larger areas. But the weight of earth at these greater depths will be greater, and hence the stress-difference, after correction for initial stresses, may also be greater. He thus concludes that for the case where the stress on horizontal planes is greater than on vertical planes, and where such a correction must therefore be made, "The narrower the strip the greater the load which can be carried per foot super on any given foundation."

Earths weigh in the neighborhood of 100 pounds per cubic foot, or say 0.06 pound per cubic inch. Maximum stress differences under tire contact areas occur at depths ranging between limits of 2 to 6 inches. Stress-differences diminish with great rapidity at depths below these maxima. If Poisson's ratio were zero, the 0.4 pound maximum per square inch resulting from this source would be negligible compared to the 40 to 60 pounds per square inch stress-difference resulting at the same points from wheel loads on the surface.

Carothers, moreover, assumes that the initial stress difference due to gravity is continuous throughout the 60-foot depth and from the building foundation viewpoint his assumption appears wholly logical. But it precludes consideration of the earth mass as an indefinitely extended body.

In the case of the indefinitely extended isotropic body he is, of course, in agreement with the mathematical theory as presented herein, and since the instance where initial pressures on horizontal and vertical planes are equal requires no modification of that theory he appropriately concludes that in this case "All widths of strip should carry the same intensity of loading." He arrives at exactly the same conclusion for the case of the material in which the pressure on vertical planes is greater than on horizontal planes in the initial state, because under this circumstance "The maximum stress difference occurs at the edge of the strip." In both cases he qualifies his conclusion with, "provided the ground does not vary with depth."

In view of his assumptions, this qualification seems unnecessary, but since variation of the material with depth is likely to be a frequent occurrence in the road problem it is of some interest to examine its effects on the relative supportability under various areas of load application, both in the case where the variation is gradual to the depth considered and where it is abrupt, as in the case of a pavement and subgrade combination.

The cases to be considered are those of, (1) the rigid pavement, (2) the flexible pavement, and (3) the unpaved road. They involve, in their limiting aspects, an underlying earth mass (a) dominantly cohesive or (b) dominantly granular.

The rigid pavement, its base course and subgrade.—The supportability afforded by the Portland cement concrete pavement under the assumption of an elastic reaction from the subgrade is fully discussed elsewhere

⁷ $-3P/2\pi z^2$ is the vertical pressure in the continuous elastic solid at any depth, z , along the axis, resulting from a point loading, P .

⁸ S. D. Carothers, "Tire Loads on Foundations as Affected by Scale of Tired Area, Proceedings, International Mathematical Congress, Toronto, 1924, vol. 2, pp. 537-549.

⁹ Highway Research Board (first) Progress Report of Project Committee on Stabilized Soil Road Surfaces, 1934, p. 8.

(appendix B). An hypothesis of interest, however, is that whatever may be the characteristic action of the subgrade earth in its natural state, the elastic action of the pavement, together with its beam strength, enters in some degree into the resistance of the subgrade to deformation under load and thus strengthens the validity of the assumption. This action applies, as well, to bituminous or other high-type pavements having concrete bases.

In all these cases the normal action of the subgrade material would appear to be restricted by the weight and rigidity of the pavement from which it, in a sense, would borrow density, superficial isotropy, and elasticity. In some cases even a quasi-hydrostatic condition might exist wherein shearing stresses in the subgrade are reduced to a minimum.

The condition should hold true, though perhaps in different degree, both for interior and edge loadings on the pavement. In the former case the vertical deflections in the subgrade will cause lateral and upward movement of the surrounding subgrade material, unless it is highly permeable and unconsolidated and unless it is in a fluid state and unrestricted at the pavement edge. The upward moving material will receive a downward thrust upon recovery of the deflected slab until it again, in turn, has recovered. In the case of edge loading, the vertical deflection of the pavement will result in the compression of the subgrade at an angle toward the interior, due to the curvature of the crown and the slope of the under-surface of the thickened pavement edge. The lateral movement will thus be downward at an angle in the direction away from the pavement and upward at an angle toward it.

But whatever the soundness of the hypothesis, the fact remains that the subgrades unsuited to flexible types of construction frequently render satisfactory service beneath rigid pavement. And this is largely a result of the elimination of local shear in the subgrade material through the beam action of the slab. In the case of a soft clay, shear is prevented until the clay mass begins to flow from beneath the pavement, while with a dry sand the usual susceptibility of surface particles to individual action is obviated by the restraining load of the pavement and the mass acts as a continuous solid.

In the Bates road tests, in which the subgrade consisted of a "brown silt loam" with a moisture content of approximately 28 percent, it was observed that, "Even at midday, when the edges of the slab were warped downward the maximum amount, the bottom surface of the edge never appeared to be in contact with the subgrade except when deflected by the passing of a heavy wheel load."¹⁰ One may infer that the statement applies to the concrete sections of uniform thickness and not necessarily to the later

test sections having thickened edges, but whether this is so or not, the cause lies in an outward crowding of the subgrade material at the edge.

Such permanent outward crowding may be regarded as subgrade failure in extreme cases, and the condition of its occurrence will be a critical stress difference in the subgrade material. The relation to pressure intensity and area of load application may be examined in the light of a practical example.

According to the Westergaard theory (appendix B, p. 185), and assuming for the concrete slab a modulus of elasticity of 3,000,000 pounds per square inch, a Poisson's ratio of 0.15, a modulus of subgrade reaction of 100 pounds per cubic inch, and a uniform pressure intensity over the loaded area of 50 pounds per square inch, the following edge thicknesses are indicated:

Radius of contact area, inches.....	2	4	6	8
Pavement slab thickness at edge, inches.....	2.2	4.0	5.7	7.4

From the equations already provided it may be deduced that in each instance the maximum stress difference on the axis of loading occurs within the pavement slab and that therefore all other maxima occur therein. The question then arises as to the relative importance of the maximum stress differences that are present in the subgrade material immediately beneath the pavement. These, of course, occur on the axis, and the equation, which also may be deduced from the preceding formulas, is:

$$S_{z=0} = p \left[\frac{(1-2\sigma)}{2} + (1+\sigma) \frac{z}{(a^2+z^2)^{1/2}} - \frac{3}{2} \frac{z^3}{(a^2+z^2)^{3/2}} \right]$$

The required numerical values, for the points just below the pavement slabs are:

Radius of contact area, inches.....	2	4	6	8
Stress-differences in subgrade, lbs./in. ²	29.0	31.7	32.6	33.1

For practical purposes, it may be concluded that where the design of the slab is rational, the tendency toward subgrade failure is essentially equalized under different loads and contact areas, and that therefore where subgrade stability must be increased through subgrade improvement under concrete slab, it is required in virtually equal degree by like pressure intensities regardless of the magnitude of the total load or the area of pressure application.

Strictly, however, there is a slightly greater stress difference beneath the larger area than under the smaller. In a sand subgrade this greater stress difference would be compensated by the restraining action of the pavement weight, which, of course, would be greater for the thicker slabs and hence the larger areas. But in the case of a clay soil, which may be considered isotropic, the stress differences at the subgrade may be equalized simply by increasing the thickness of the slab, because in such a material the depths at which a

¹⁰ Clifford Older, Department of Public Works and Buildings, Division of Highways, State of Illinois, Bulletin No. 21, p. 49.

given stress difference will occur are proportional to the radii of the bearing areas.

In actual practice such equalization is not sought, because pavements are designed for given subgrade and given maximum load conditions. Rarely is the question of area introduced, while the problem of a finely differentiated design for various loads and areas of load application has never confronted the road builder.

The engineer has, however, considered the load distributing characteristics of granular ballast materials, and it may appear plausible that the necessary equalization could be obtained by the addition of a base course sufficiently stable to withstand the greater stress differences occurring under the pavements designed for the larger areas.

The Pittsburg tests found broken-stone ballast to be a contributory cause of failure,¹¹ and one hears the argument advanced that such base courses are of little use where added support is needed because the lack of tensile resistance in the granular mass soon permits it to be swallowed up by the soft subgrade.

On the other hand, the addition of a granular base course to the subgrade before paving with concrete slab appears to be recognized practice in certain States, and load distribution is, but one of many reasons advanced for its use. The permeable nature of a granular base course permits drainage at the subgrade surface and thereby minimizes the detrimental effects of spring thaws when the frozen earth layer beneath prevents the usual run-off. It allows for a certain ventilation which permits a more uniform drying and consolidation of the subgrade. And during frost heaves or in the case of a rocky and nonuniform subgrade it provides a more uniformly distributed subgrade reaction to pavement deflections. It may also permit a readier longitudinal movement of the pavement during expansion and contraction, and through the insulation afforded, it may permit the slab to be more uniformly affected by atmospheric temperature changes.

Except for the consideration that it may serve as a distributor of load, it is readily perceived that some constant thickness is required for a given subgrade condition, and that this thickness will not vary with pavement design.

In order, however, to eliminate the differential in stress differences occurring at the subgrade surface in the four cases shown above, the following thicknesses would have to be added, if stress distributing benefits are assumed:

Radius of contact area, inches.....	2	4	6	8
Base course material, inches.....	0	0.4	0.9	1.4

If the constant thickness to be added may be taken as 6 inches,¹² the stress differences occurring at the sub-

grade surface, and the total base course material required to equalize them, would be as follows:

Radius of contact area, inches..	2	4	6	8
Stress differences in subgrade, lbs./in. ²	4.6	10.9	15.8	19.4
Base course material, inches....	6	12.4	18.9	25.4

The assumption of such stress distribution involves, at the same time, the assumption of isotropy. In an indefinitely extended mass of the granular material, and under the confinement of the surface particles by the weight and friction of the slab, the surface peculiarities of the granular material would no doubt be minimized and the body would assume appreciable isotropy. But when such a material, lacking tensile resistance, is interposed between two isotropic bodies one of which is subject to failure under small stress differences, it may not be assumed that the material will take on the resistance qualities of the other body, however great they may be. It will in fact accommodate scarcely greater stress differences than the softer material. The lateral movement of the subgrade, under load, will result in corresponding lateral movement of the granular particles at the subgrade, and such movement in a material incapable of resisting tension is accompanied by particle separations, which of course is failure. It is true that arching occurs in the granular particles, but it is equally true that the arch is no stronger than its footing.

It may therefore be concluded that base courses may promote uniformity of subgrade performance, and where the subgrade can originally sustain the stress differences transmitted through the pavement they may actually improve its supportability, and that therefore their effect is of essentially equal benefit to any rationally designed pavement slab. But at the same time it must be concluded that their effect is negligible in the improvement of subgrade supportability through stress distribution under rigid pavements.¹³

Subgrade supportability under rigid pavements may be increased through drainage and consolidation, and any other means whereby the friction and cohesion of the material is uniformly increased, but these things merely result in raising the failure point and they do not alter the relative independence of area for the attainment of this point under properly designed slab thicknesses.

If the subgrade affords poor support, as in the case of a soft clay, total supportability may be increased by appropriate increases in slab thickness. Conversely, if the subgrade affords a high degree of support, as in the case of a confined sand, relatively thin pavement slab

¹¹ In an unpublished manuscript loaned to the Coordinator, Arthur Casagrande states: "A base course of usual thickness does not help to increase the load distribution by any appreciable amount beneath a rigid pavement. It is only warranted for the purpose of reducing the detrimental effects of frost heaving and nonuniform subgrade conditions. Outside of these cases, a base course beneath a concrete pavement has no value and must be considered a wasted expenditure."

¹² Lloyd Aldrich and John B. Leonard, Report of Highway Research at Pittsburg, Calif., 1931 and 1932, pp. 145-146.

¹³ Six inches is assumed for purposes of estimate, by Charles H. Breed, in Report Upon Cost of Roads Required for Heavy Motor Vehicles, etc., p. 32.

may suffice. But these design features are provided for in the Westergaard theory.

In the case of the rigid pavement, its base course and subgrade, the relation between load, area of application, and the yield point of the structure may be summarized thus: (1) With respect to the slab, the yield point is related to pressure intensity and area of application as provided in the Westergaard theory. (2) With respect to the subgrade, and base course materials that may be present, the yield point under properly designed slab may be regarded as directly related to pressure intensity and independent of area.

The flexible pavement, its base course and subgrade.—It has been shown that, under the assumption of a uniform distribution of pressure, the greatest stress difference occurs at the surface of the body near the edge of the loaded area where $k \rightarrow 0$. This stress difference is unassociated with physical dimensions but is inversely dependent on the value of Poisson's ratio. Other and lesser maxima occur over a basin shaped surface until the smallest maximum stress difference is reached at the bottom of this surface in the neighborhood of the vertical axis. Failure in a uniform material as well as in a material lacking superficial density is therefore likely to originate at the surface of the loaded body.

If the load distribution is not uniform, but tends to zero at the boundary of the loaded area, the stress difference maxima are distributed somewhat differently near the surface and in a manner corresponding closely to the distribution at a plane $z > a$, for the uniformly applied pressure. The greatest difference in this case would occur at the vertical axis as provided by the formula already given for S when $\rho = 0$. In this case, however, $T_1 - \omega \bar{\omega}$ would not be altered, and a danger zone would therefore still exist at the surface, though it would occur just outside the boundary of the pressed area.¹⁴

The supportability of a road foundation may therefore be enhanced by merely providing a layer of material at the surface possessing sufficient stability to withstand these critical stress differences. Such a layer is here referred to as a pavement, even though the degree of stability in some of the surfacing materials employed in practice is insufficient to permit the resulting roadways to be commonly considered as paved. Sand-clay and gravel roads, and even the earth road compacted at the surface by traffic, are in this sense strengthened by a pavement, albeit the strengthening may be relatively meager and its durability dependent almost wholly on the weather.

The fundamental consideration in design is that permanent deformations in the structure are to be avoided. In the case of the paved road the structure consists of both pavement and subgrade, and together they are a

statically indeterminate system in which the action of the one is influenced by that of the other. Thus, in the case of a rigid pavement the high modulus of elasticity of the slab produces a tendency in the plastic subgrade to recover from deformation. The relatively large permanent deformations to which the subgrade might otherwise be susceptible are eliminated, however, by the beam action of the concrete slab, and the frequency of load application is thereby prevented from being a considerable factor.

In the case of a flexible pavement, on the other hand, appreciable elasticity in the slab is usually lacking and its beam action is negligible. The avoidance of plastic yield in the structure is therefore dependent on the elastic qualities in the subgrade, and may be achieved through the control of stress differences occurring in the subgrade surface.

In the presence of a subgrade consisting of a cohesive but relatively frictionless clay of such supporting power that none of the maxima of stress differences resulting within anticipated loads and a given range of areas would cause plastic deformations, it may well be that a surfacing coat would be required for purposes of waterproofing and the reduction of tractive resistance. In this case the thickness of the required coating would be independent of area and also independent of pressure intensity, and, as in the case of the elastic solid, pressure intensities at the yield point would be independent of area.

Again, the supporting power may be such that one need consider only the stress differences at the point where $k \rightarrow 0$, or those at a point just outside the boundary of the loaded area. In either consideration, the critical stress difference occurs at the surface and is therefore independent of area but directly dependent on pressure intensity. The thickness of the resistant pavement required to eliminate this stress difference from the subgrade would thus be uniform for different areas under like pressure intensities. In practice, however, this case would scarcely be of importance because of the small percentage difference that exists between $S_{\max.}, (\rho = 0)$ and $S_{\max.}, (k \rightarrow 0)$. When $\sigma = 0$ the former is $0.8143p$ and the latter $0.8992p$, and when $\sigma = 0.5$ they are $0.5774p$ and $0.6017p$, respectively.

If, on the other hand, a clay subgrade were not sufficiently stable to resist any of the maxima of the stress-differences, supportability could be gained by the addition of pavement thicknesses. In this case the limited stress-difference permissible at the subgrade surface could be attained under pavement thicknesses proportional to the radii of the bearing areas and in some way variable with applied pressure intensities. The maxima of stress-differences vary, however, under different areas and different pressure intensities, with depth. Moreover, under a given area and applied pressure the stress-differences diminish rapidly with depth after attaining these maxima. Therefore it is

¹⁴ A. E. H. Love, The Stress Produced in a Semi-infinite Solid by Pressure on Part of the Boundary, Philosophical Transactions of the Royal Society of London, vol. 228 (series A), November 1926, p. 420.

only necessary to have successive depths of the pavement sufficiently resistant to sustain the successive stress-differences. Thus the question of base courses beneath the wearing course of a flexible pavement is introduced as a matter of economy.

These base courses may range from the stabilized sand-clay or gravel layer of a traffic-bound surfacing, to a fairly high type premixed clay concrete¹⁶ or bituminous concrete, but, whatever their composition, they must combine cohesive resistance with the frictional resistance of a hard aggregate if resistance to stress-differences is to be realized.

On a sandy subgrade quite another condition is present. Here the weakness is primarily a surface characteristic and if adequate cohesion can be introduced near the surface, or if the surface particles are confined beneath a cohesive mat, the normal tendency for failure to take place first under the smaller areas is retarded, and the composite assumes properties of the isotropic solid wherein failure is uninfluenced by area.

Thus, the design of flexible pavements is unaffected by area of load application where a clay subgrade is sufficiently cohesive to resist the stress-differences resulting from the applied pressure or where the crushing strength of the particles in a granular subgrade is not exceeded. Design is also independent of area where cohesion and friction combined to provide the necessary resistance.

Only in the case where a weakly cohesive subgrade, lacking in particle strength, requires the reduction of stress-differences to a given value at the subgrade surface does the influence of area appear, because such reduction can be accomplished by the superposition of flexible pavement thicknesses which, so far as area influence is concerned, are proportional to the radii of the areas of load application.

But whether area effects are to be taken into consideration or not may not be determined from the natural state of the subgrade. It will depend on the state which the subgrade assumes or in which it can be maintained after the pavement has been added.

From the point of view of the supportability to be gained through pavement thickness, natural earths which can sustain a stress-difference of 50 pounds per square inch need no pavement so far as the heaviest motor vehicle wheel loads are concerned, and if they cannot sustain a stress-difference of approximately 30 pounds per square inch they must be paved in order to accommodate even the private car, if supportability is to be gained through paving.

Supportability may be gained, however, through the reduction of capillarity or the elimination of surface

¹⁶ The term is of recent origin. The material represents a controlled mix of graded aggregate, sand, clay, and calcium chloride or sodium chloride. Recent discussions of this and other practical base course developments appear in Proceedings, 1928 Convention, American Road Builders Association, in which the following papers are given: Thomas E. Morson, Stabilization and Mixed-in-Place Construction; F. V. Rengel, Bituminous Soil and Base Course Stabilization; A. C. Tilley, Soil Stabilization Through the Use of Bituminous Materials; and F. C. Lang, Stabilization of Soil and Gravel Roads.

moisture or both, and the latter may often be materially aided by the mere addition of a waterproof wearing course. The necessary supportability in soft earths may therefore frequently be gained by such waterproofing without recourse to the stress diminishing properties of pavement thickness.

With porous earths having a high degree of capillarity, the question of moisture, particularly in the presence of frost, is an important one, and the addition of a waterproof blanket may have little if any effect as a solution.¹⁸ But in such cases the addition of pavement thicknesses is scarcely a better solution from an economic viewpoint. If, say, a sustainable stress difference of 10 pounds per square inch is granted, a 10-inch flexible pavement would scarcely support the lightest loads experienced and a 25-inch pavement would be inadequate for the heaviest; while, if a fluid condition of the subgrade is considered, flexible pavement thickness can be discarded entirely as a solution to the problem of supportability. But these things can be better understood by an examination of the effects of pressure intensity on pavement thickness requirements.

Thus far the joint effects of area and pressure intensity on stresses and stress-differences have been examined as well as the influence of area on the depths at which the maxima of stress-differences occur. It has, moreover, been observed that surface deflections and stress-differences at the surface resulting from a uniformly applied normal pressure are directly related to pressure intensity and that the magnitude of stresses and stress-differences at any given depth are also so related.

It now becomes of interest to examine the effects of pressure intensity on the depth at which a given stress difference occurs. It is sufficient to investigate merely the values of $S_{max-\rho}$, when $\rho=0$, and the stress differences occurring at greater depths on the vertical axis.

The general equation for S when $\rho=0$ is given on page 238 and computed values may be found in table C-4. From this equation, or the computed values, it is readily seen that no simple relation, either arithmetic or logarithmic, exists between pressure intensity and the depth at which a given stress difference is attained under different areas of load application or at which different stress differences are attained under the same area.

Since computations may be made, however, for different values of z/a and the results obtained in terms of S/p , as is done in table C-4, it is necessary only to multiply z/a values by any selected a value and to multiply S/p values by any selected p value and to plot the obtained values of S as dependent variables of z . From the resulting curves, readings may be

¹⁸ Frank T. Sheets of the Portland Cement Association calls attention "to the fact that the placing of a water-proof blanket will, in many cases, aggravate conditions" by cutting off surface evaporation, increasing the moisture content from capillarity, and accentuating instability (communication to the Federal Coordinator of Transportation).

taken for the depth at which a given stress difference will occur under the selected pressure-area combinations.

It is of practical importance to select typical wheel pressures and tire contact areas. A Poisson's ratio of 0.25 may be employed for illustrative purposes, and as probably representative of earths within their elastic stage. The following table will serve.

TABLE C-1.—Typical motor vehicle wheel loads and tire contact areas

Group	Vehicle type	Average wheel load (pounds) ¹	Average contact radius, a (inches)	Average pressure, p (pounds per square inch)
1	Passenger cars	1,999	3.87	47.3
2	Taxicabs	2,675	3.85	59.0
3	Busses, 7 passengers and less	3,130	4.01	61.9
4	Trucks, 1½ tons and less	4,922	4.62	78.2
5	Combinations, ¼ to 2 tons	4,922	3.92	78.2
6	School busses	4,984	4.84	73.7
7	Farm trucks	5,520	4.84	76.0
8	Busses, 8 to 20 passengers	6,893	6.09	46.0
9	Trucks, 1½ to 3 tons	8,647	6.67	61.9
10	Combinations, 3 to 5 tons	8,647	6.67	61.9
11	Busses, over 20 passengers	9,430	7.73	50.3
12	Trucks, 3 to 5 tons	11,418	8.72	57.8
13	Trucks, 5 tons	13,933	8.72	58.2
14	Combinations, 5 tons	13,933	8.72	58.2
15	Trucks, over 5 tons	15,864	9.72	58.0
16	Combinations, over 5 tons	15,864	8.72	58.0

¹ Because of requirements elsewhere, the values given represent impact conditions. The difference between this group and group 11 would scarcely appear on the suggested graph.

The resulting stress-difference values in pounds per square inch and the values of depth in inches are given in the following table, and are plotted in figure C-31.

TABLE C-2.—Stress-differences, S, in pounds per square inch at various depths, z, in inches, under typical wheel loads

[$\nu=0.25, \rho=0$]

z/a	S/p	Group 1		Group 2		Group 3		Group 4		Group 5		Group 6	
		a=3.87 p=47.3		a=3.83 p=59.0		a=4.01 p=61.9		a=4.62 p=78.2		a=4.84 p=73.7		a=4.84 p=76.0	
		S	z	S	z	S	z	S	z	S	z	S	z
0.6202	0.6892	32.00	2.28	33.07	2.38	42.66	2.49	50.45	2.87	50.70	2.88	51.69	3.00
0.75	0.6760	31.97	2.75	39.21	2.87	51.84	3.01	49.48	3.47	49.83	3.48	50.70	3.63
1.00	0.6636	28.55	3.67	35.01	3.83	37.98	4.01	44.18	4.62	44.49	4.64	45.27	4.84
1.25	0.6519	24.21	4.49	29.09	4.79	31.69	5.01	37.47	5.78	37.73	5.80	38.39	6.08
1.50	0.6407	20.15	5.13	24.71	5.75	26.37	6.22	31.18	6.93	31.40	6.95	32.03	7.26
1.75	0.6355	16.72	6.42	20.50	6.70	21.88	7.22	25.88	8.09	26.05	8.12	26.51	8.47
2.00	0.6247	13.94	7.34	17.09	7.66	18.24	8.02	21.57	9.24	21.72	9.26	22.10	9.68
2.50	0.6100	9.53	9.13	11.98	10.88	13.03	11.37	11.55	13.18	13.41	13.61	15.72	13.00
3.00	0.1551	5.94	11.01	9.00	11.98	9.60	12.63	10.55	14.11	14.31	14.51	16.88	13.62
4.00	0.0831	4.40	14.98	5.40	15.32	5.70	16.04	6.81	18.48	6.88	18.56	6.98	19.36
5.00	0.0614	2.90	18.85	3.86	19.16	3.80	20.03	4.49	22.10	4.53	22.20	4.61	24.20
7.00	0.0322	1.62	25.69	1.87	30.81	1.99	28.07	2.36	32.94	2.37	32.48	2.42	33.88

z/a	S/p	Group 7		Group 8		Group 9		Group 10		Group 11			
		a=6.09 p=49.0		a=6.67 p=61.9		a=7.78 p=60.3		a=7.93 p=57.8		a=8.73 p=58.2		a=8.72 p=58.0	
		S	z	S	z	S	z	S	z	S	z	S	z
0.6202	0.6892	33.77	4.15	42.66	4.14	34.87	4.70	39.84	4.92	40.11	5.41	39.97	5.41
0.75	0.6760	33.12	5.02	41.84	5.00	34.00	5.60	35.07	5.95	39.34	6.55	39.21	6.54
1.00	0.6636	29.58	6.06	37.30	6.07	30.36	7.78	34.89	7.93	35.13	8.73	35.01	8.72
1.25	0.6519	25.86	6.81	34.23	7.78	26.89	9.01	29.70	10.01	29.10	10.99	29.10	10.99
1.50	0.6407	20.87	10.04	26.37	10.01	21.43	11.60	24.92	11.90	24.71	13.04	24.71	13.04
1.75	0.6355	17.32	11.71	21.88	11.97	17.76	13.63	20.43	13.88	20.57	15.28	20.50	15.28
2.00	0.6247	14.44	13.38	18.24	13.34	14.92	15.46	17.03	15.86	17.15	17.48	17.09	17.44
2.50	0.6100	10.29	16.73	13.00	16.80	10.86	18.33	12.14	19.83	12.29	21.83	12.18	21.80
3.00	0.1551	7.40	20.07	9.60	20.01	7.80	23.19	8.96	23.70	9.03	26.19	9.00	26.19
4.00	0.0931	4.56	26.76	5.76	26.98	4.68	30.92	5.38	31.72	5.42	34.92	5.40	34.88
5.00	0.0614	3.31	33.45	3.80	33.25	3.09	38.65	3.55	39.65	3.57	43.65	3.56	43.60
7.00	0.0322	1.88	45.83	1.99	44.69	1.62	54.11	1.88	53.01	1.87	61.11	1.87	61.04

¹ Maximum value.

From figure C-31 it is seen at once that the maximum stress difference on the vertical axis produced by passenger cars (group 1) is 32.6 pounds per square inch, and that it occurs at a depth of 2.28 inches. If the subgrade can just support the stress-difference produced, this group will then require a pavement of 2.28 inches in thickness and the heaviest group will require a pavement of 9.70 to 9.75 inches in thickness. If, on the other hand, it is desired to limit the stress-difference at the subgrade to approximately half of the sustainable value or, say, 15 pounds per square inch, then the passenger car group will require a pavement 7 inches thick and the heaviest group a thickness of 19.3 inches. Thicknesses for the several groups under the two conditions follow:

Group	Wheel load, pounds	Thickness at S=32.6 lbs./in. ²	Index	Thickness at S=15 lbs./in. ²	Index
1	1,999	2.3	100.0	7.0	100.0
2	2,675	4.3	167.0	8.4	120.0
3	3,130	4.9	213.0	9.2	131.4
4	4,922	6.7	291.3	11.7	167.1
5	4,984	6.8	291.7	11.8	168.6
6	4,520	7.1	308.7	12.5	178.6
7	6,893	6.4	234.8	13.0	184.7
8	8,647	8.1	352.2	15.3	218.0
9	9,430	6.7	291.3	15.4	220.0
10	11,418	8.8	382.6	17.3	247.1
11	13,933	9.7	421.7	19.8	275.7
12	15,864	9.8	426.1	19.9	274.3

From figure C-31 it is also seen that the greatest stress-differences are caused by small trucks and combinations, school busses, and farm trucks. These stress-differences occur at a depth of approximately 3 inches. They are between 50 and 52 pounds per square inch, and remain higher than those caused by any other vehicle group until a depth of 5½ to 6 inches is reached. It may be said, therefore, that where flexible rural road pavements up to approximately 6 inches in thickness are adequate for this group of vehicles, they are suitable construction for all heavier groups.

If for safety's sake, however, the stress-differences caused by farm trucks, say, are to be reduced at the subgrade surface by 50 percent, or to 26 pounds per square inch, as may be done by an 8½ inch pavement, limited use of the structure may still be permitted the heaviest group because the maximum stress-difference produced at the subgrade surface will still be 31 percent less than the sustainable value.

More important than these considerations, however, to a rational design of flexible roads, is the fact—clearly shown by figure C-31—that required pavement thicknesses increase in greater and greater degree as sustainable stress-differences diminish by fixed amounts and that for very low values of subgrade supportability pavement thicknesses become prohibitive. One may conclude, therefore, that beyond moderate thicknesses economic design lies in the direction of subgrade improvement through drainage and the reduction of

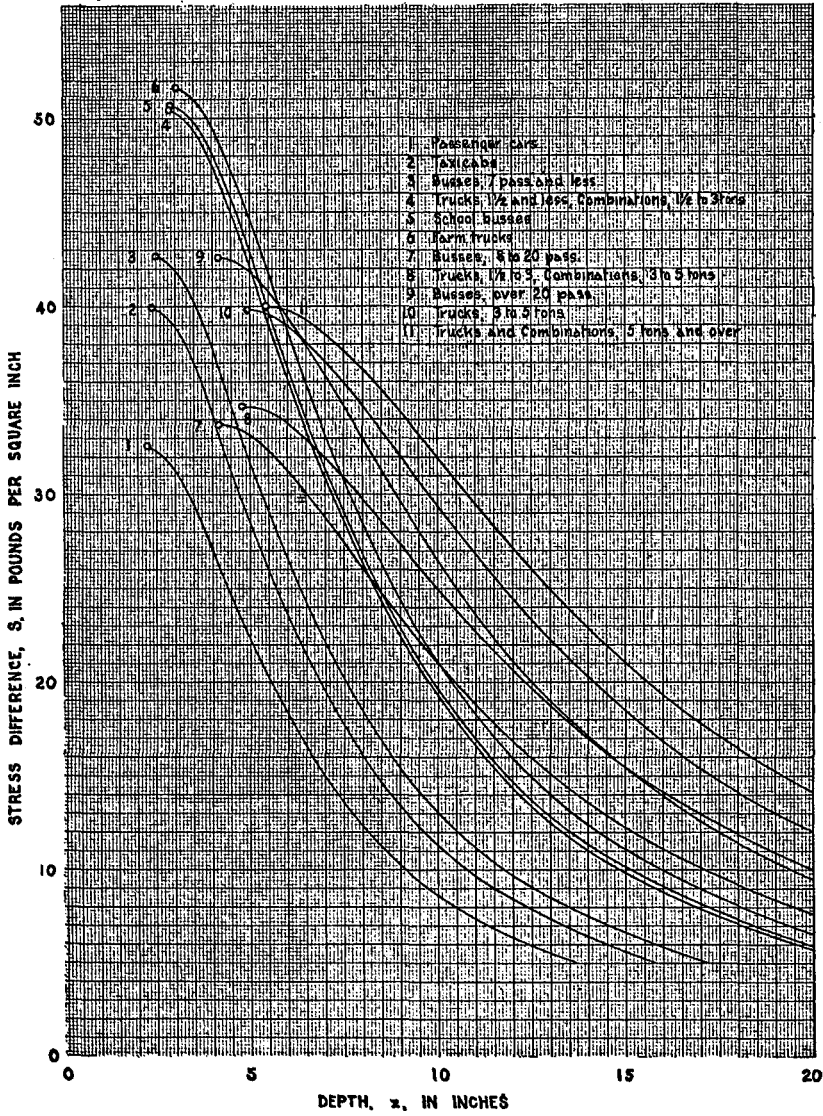


FIGURE C-11.—The Stress Difference, S, at Various Depths along the Vertical Axis of Loading under Typical Motor Vehicle Wheel Loads and The Contact Area, $\mu=0.35$.

capillarity rather than in the direction of increased pavement thickness.¹⁷

The earth road.—It was suggested at the outset that the plain earth road, lacking in design, be regarded in terms of its susceptibility to failure under different loads applied over different areas. Under the assumption of uniformity, the occurrence of failure has been observed to be independent of area or inversely related to the square root of the area according to whether the strength of the earth material is derived mainly from cohesion or from friction.

These relations are strictly qualitative, however, and they apply only so long as the material retains the stipulated characteristics. As the relative importance of cohesion and friction is altered by moisture or the action of traffic these relations may also be altered. Cohesion in a clay mass may be broken down by moisture, while in a sandy earth it may be increased by moisture, and frequently the effects of moisture may be expedited under the action of traffic loads. When cohesion is increased in the sandy material its action then conforms more closely to that of the isotropic solid; when cohesion is reduced in plastic earths, on the other hand, a point may even be reached where the material assumes a fluid action.

Thus, the failure of the plain earth road is directly related to pressure intensity. Where the material is predominantly clay, failure is independent of area of load application, but where the material is predominantly sand or gravel it is inversely related to the square roots of the areas and will therefore occur first under the smaller of the two different areas.

Effects of dynamic loading.—It would appear from a consideration of the mechanical properties of materials that the effects of area of load application in the solid state are uninfluenced by the rate of load application, so long as the stresses produced do not exceed the elastic strength of the material. This qualifying clause is, in fact, not necessary, because when the elastic strength is exceeded the material will pass out of the solid state momentarily during the load application. And though stresses may be concentrated in the vicinity of the load when it is applied suddenly,¹⁸ and thereby deter failure to a certain extent, it is reasonable to assume that the contours of stress distribution under different areas would remain similar.

¹⁷ It is of some significance that in a communication to the Federal Coordinator of Transportation, the Asphalt Institute stated: "We are firmly convinced as to the coming of a completely different conception of highway design and that instead of designing the pavement to carry the load, as well as to resist abrasion and moisture, the load supporting function will be taken care of almost in its entirety by the subgrade itself after appropriate treatment. The function of surfacing will be purely to take wear and tear and to provide waterproofing. Every indication in the behavior of properly designed and treated subgrades points to the correctness of this view and the economics of the situation will certainly dictate its acceptance by the designing engineer."

¹⁸ Appendix B, p. 163. It has been suggested earlier that, in the case of the crystalline solid, deformation produces sliding and hence liquefaction between the crystal particles. Stress concentrations may thus be a result of momentary resistance afforded by the fluid friction under suddenly applied forces.

In the fluid state shear occurs under the slightest pressure, but since movement changes the unordered molecular motion into ordered motion around the edges of the loaded area, frictional resistance is set up under suddenly applied forces which varies directly with the velocity of force application. Moreover, the resistance varies with the number of molecules so affected, and therefore with the radii of the bearing area.

Solid particles, lacking in cohesion, act as a fluid under dynamic loading just as they do under initial deflections caused by static loads. The resistance to shear varies directly with the square root of the bearing area and directly with the velocity of loading. It is not by chance that the salt beds of Bonneville and the sands of Daytona afford such excellent courses for automobile speed records.

Plastic materials represent a combined state. Under dynamic loading, those of their elements which are viscous offer greater resistance to the larger areas, at a given pressure, than to the smaller. They also offer greater resistance to rapidly moving loads than to slowly moving ones. The bituminous pavement beneath a taxicab stand may be ruined before the roadway accommodating much heavier moving vehicles shows appreciable surface deformation.¹⁹ On the other hand, those of the elements in plastic materials which are solid offer a resistance which is independent of area at the yield point, and if the bituminous binder is present in sufficiently thin layers between the particles of the hard aggregate, the aggregate itself may come into play in the total resistance, as it may even under static loading.

Instead of referring specifically to fluid or solid behavior, one may describe the reactions under dynamic loading in a different way. If the resistance to plastic yielding is afforded primarily by cohesion, it is independent of area and independent of velocity of loading. If, on the other hand, the resistance is afforded primarily by friction, it will be directly proportional to the square root of the area and directly proportional to the velocity of load application.

Thus, were moving loads alone to be considered in flexible pavement design, or with regard to their destructiveness of unpaved roads, one could make a favorable allowance for the larger areas, wherever viscous action would be presumed. The pavement, however, is called upon to support various degrees of velocity in load application which range down to zero and therefore static loadings may not be neglected.

It is not to be overlooked, on the other hand, that stress concentrations under suddenly applied forces must occur in viscous as well as solid bodies, and that whatever the effects of area may be on the resistance

¹⁹ Such a relative effect is mentioned by E. F. Kelley in a Symposium on Research Features of Flexible-Type Bituminous Roads, Proceedings, Highway Research Board, 1933, pt. 2, p. 12.

to failure, the proportionality of pressure intensity to the stress differences causing failure is constant under all circumstances. If tire contact areas increased in direct proportion to motor vehicle impact loads on the road surface, dynamic loads might well be neglected in area consideration, but they do not increase in direct proportion. Elsewhere it is shown that areas increase according to the two-thirds power of impact load (appendix D, equation 6), and therefore pressure intensities increase under impact load and these changed intensities together with the changed areas over which they occur are the important consideration.²⁰

Abrasion and raveling.—With the sand-clay and gravel surfaces, some of the treated types, and the water-bound macadam roads, there is involved a factor which is not generally of importance with the higher types of flexible pavement or with the nonsurfaced road. It is the displacement of the wearing course by traffic. It occurs in the case of sand-clay surfacing but is unimportant because the loss of material is largely a result of its penetration into the subgrade which it thereby improves. Because of its susceptibility to surface dislodgment, the water-bound macadam pavement is not well suited to rubber-tired traffic in appreciable volume. Surface dislodgment or bunching also occurs among some treated types of the lower class, and notably in the case of the gravel road in the form of rhythmic corrugations. Frequently these dislodgments result in actual loss of wearing course.

Where the pavement is sufficiently cohesive to withstand the tractive pull of the driving wheel, surface failure in the pavement may result from stress-differences caused by loads not contemplated in the design, a weakened subgrade or frost heaves. Ordinarily, however, the design theory must regard the tractive pull in the light of its influence on the stress-differences that would result from a purely normal application of load. Some analysis may be made, however, as to the relative effects of different areas and pressure intensities.

In the first place, the phenomenon involves separation of particles, and hence tangential shear, at the upper surface of the pavement. Resistance to it will therefore be directly related to pressure intensity. In the second place, stress-differences at the surface being independent of area, and the occurrence of shear in cohesive materials as well as superficially in materials weak in cohesion also being independent of area, one might anticipate

²⁰ One of the fundamental assumptions in the calculation of the stress distributions already discussed was that the uniform pressure on the boundary plane of the semi-infinite solid was normal to that plane. Moreover, in appendix III the observed values of impact represent vertical impact normal to the road surface. In practical cases it would seem necessary to consider the tangential forces exerted by the moving vehicle and the angular resultants below the road surface before a proper analysis of the effects of dynamic wheel loadings could be made. It appears safe to assume, however, (1) that tangential effects, as between different wheel loads, would be proportional to vertical effects, and (2) that the resulting angularity of stress distribution would merely modify the stress contours without changing the similarity of the stress diagrams. The relative effects of different wheel loads would thus remain unchanged.

independence of area in this case as well, provided only that the resistance to stress-differences is a constant.

The discussion covering viscous and granular materials, however, indicates that with these materials the resistance is not a constant but varies with the velocity of load application and with the relative number of molecules or particles brought into play. In an indefinitely extended body the number of particles would vary with the perimeter of the loaded area, and therefore with its radius and its square root. In the case of the surface, as now considered, the number of particles is directed related to the area, but the resistance to movement is the resistance offered by each particle and does not depend on any cumulative effect on the particles surrounding the loaded prism as in the case of internal shear. Therefore, in the present case the resistance per unit of area is a constant and independent of area. And this is identical with the phenomenon of sliding friction between solids.

It appears, therefore, that displacement of the wearing course, due to abrasion, is a direct function of the unit tractive pull exerted at the road surface by the driving wheel, and that the extent of its occurrence is proportional to the distance traveled, the number of driving wheels, and the diameter of the contact area. It also appears that raveling, which involves progressive dislodgment of surface particles where superficial failure has already taken place, is a function of tractive pull in the same way.

If a more detailed analysis were possible, the mechanics of surface abrasion and raveling might be revealed to be somewhat more complex. The peculiar harmonic action of sprung load and driving wheel, and the tendency of light wheel loads to leave the road should perhaps also be considered. But whatever the net result of such factors might be, surface dislodgments are of limited importance in maintenance compared to permanent surface deformations when the total effect on all flexible-pavement maintenance is considered. The finer discrepancies may thus be neglected.

SUMMARY

Unconfined materials are subject to but two types of deformation under static load.

The first of these is generally termed "elastic" deformation. It results from small changes in the distances between the particles normally associated with their equilibrium under the influence of mutual attraction and of gravity. It is common to all solids and is absent in all fluids, however soft the solids may be or however hard the fluids. The principal resistance to it is cohesion. Its principal characteristics are its independence of time and its disappearance upon removal of the load.

The second type of deformation is commonly known as viscous flow. It results from a change in the relative

position of the particles normally associated with their degree of freedom under the influence of mutual repulsion and of gravity. It is common to all fluids but is absent in solids. The principal resistance to it is friction. Its principal characteristics are its proportionality to time and its persistence upon removal of the load.

Materials utilized for structural purposes may exhibit, successively under increasing load, properties of solids, properties of a combined state, and, finally, properties of fluids. Thus deformations may pass from the elastic, through the plastic, into the viscous stages. For this reason it is necessary in structural design to eliminate stress intensities which cause deformations beyond the elastic stage.

The limits of elastic deformation may be determined only by laboratory test with the particular material considered. The deformations occurring during the elastic stage and the stress distributions accompanying them have been, however, a subject of intensive mathematical investigation. Parallel experimental investigation has proved the resulting theory to be rigorously applicable where the fundamental assumptions of homogeneity and isotropy are statistically tenable.

Many criteria for the mathematical determination of the elastic limit, after certain moduli are known, have been proposed during the development of the science. A generalization of the most valid elements in these is that the elastic limit is reached when the maximum stress difference in the body attains a critical value, depending on the material.

When an indefinitely extended, isotropic, elastic body is loaded on part of a boundary plane, and the load is distributed uniformly over a circular area, the mathematical theory shows the magnitude of the maxima of stress-differences to be directly related to pressure intensity but independent of the area of load application. A consideration of the mechanical properties of fine-grained and cohesive materials indicates permanent deformation to take place under similar circumstances in such a clay earth. Laboratory and field tests with a wide variety of earths show this to be the general case for clays.

The mathematical theory of elasticity is applicable to phenomena other than stress distribution in an isotropic solid. The requisite is an adjustment in the moduli or parameters employed in connection with the fundamental differential equations. The pertinent modulus in the equations of stress distribution in an isotropic solid is a function of the ratio of lateral to vertical movement of a loaded element, i. e., Poisson's ratio. If then a Poisson's ratio of zero, or some finite but small

value, be employed in connection with dry, cohesionless sands, in computations of the stress-differences produced near the surface, where continuous isotropy may not be assumed, and if the value be increased for successive depths where continuous isotropy is approached, it is found that the maxima of stress-differences under different areas are directly related as pressure intensity, and for practical purposes inversely related as the square roots of the areas. This conclusion conforms with theoretical considerations of the mechanical properties of weakly cohesive granular materials, and it is further supported by experimental results with sandy earths.

Therefore, in an indefinitely extended body of earth, as in the case of an earth road, the relative contribution of different loaded areas to failure is directly related to pressure intensity. In the case of clay soils, it is independent of area, while in the case of predominantly sandy soils it is in inverse proportion to the square root of the area.

Design in earth roads considers merely the question of drainage. It enters importantly, however, where the thickness of flexible pavement is involved. Here it is indicated that the thickness of pavement over a weak clay subgrade varies as the square root of the area of load application modified by the pressure intensity in a manner implicit in the equation for stress differences along the vertical axis

$$S\rho_{-o} = p \left[\frac{(1-2\sigma)}{2} + (1+\sigma) \frac{z}{(a^2+z^2)^{1/2}} - \frac{3}{2} \frac{z^2}{(a^2+z^2)^{3/2}} \right],$$

but that in this case the use of pavement thickness as a means of reducing stress differences at the subgrade surface is justified only within very moderate limits. In the case of a sufficiently stable clay subgrade, or in the case of a sand subgrade weak in cohesion, the thickness of the resistant surface layer may be independent of area, and need be only just sufficient to provide a waterproof blanket appropriately smooth and cohesive to resist the lateral movement of the subgrade and the surface abrasion of the driving wheel.

Unit surface abrasion, moreover, under different areas is indicated to be independent of area but directly dependent on the unit tractive pull exerted at the road surface.

Dynamic loading does not alter the relative effect of areas of different sizes on the critical stress differences within the body of the material. Motor vehicle impact phenomena, however, disturb the static relation between pressure intensity and tire contact area under different wheel loads, and the resulting greater pressure intensities therefore require the consideration of impact loads and areas in dealing with flexible pavement design or the relative destruction of earth roads.

TABLES

TABLE C-3.—The effects of bearing area on the resistance of clay and granular materials

C-3A. GOLDBECK-BUSSARD DATA—CLAY

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations				
	0.02 inch	0.04 inch	0.06 inch	0.08 inch	0.10 inch
6	12.00				
12	7.76	18.81			
144	5.77	5.28	7.72	10.00	11.52
288	3.08	4.04	5.80	7.44	8.68
576	1.46	2.81	4.10	5.56	6.67
864	1.31	2.52	3.97	4.80	6.33
1,506	.84	1.71	2.60	3.46	4.33

C-3B. GOLDBECK-BUSSARD DATA—SAND-CLAY (1 TO 1)

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations				
	0.02 inch	0.04 inch	0.06 inch	0.08 inch	0.10 inch
1	14.00				
2	10.00				
4	6.11	11.00			
6	4.76	10.50			
12	3.00	6.40	6.92	13.69	
144	2.07	3.94	5.71	7.36	8.63
288	1.40	2.65	4.00	5.39	6.69
576	.89	1.89	2.83	3.74	4.65
864	.98	1.78	2.59	3.36	4.11
1,506	.67	1.28	1.92	2.54	3.20

C-3C. GOLDBECK-BUSSARD DATA—SAND-CLAY (5 TO 1)

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations				
	0.02 inch	0.04 inch	0.06 inch	0.08 inch	0.10 inch
1	14.78				
2	7.40	18.90			
4	6.43	11.11	16.11		
12	4.24	6.42	11.62	14.29	
144	2.89	4.40	5.87	6.51	7.58
288	1.98	3.24	4.93	5.31	7.75
576	1.62	3.04	4.34	5.63	6.88
864	1.53	3.21	4.38	5.59	6.59
1,506	1.11	2.18	3.03	3.62	4.38

C-3D. UNITED STATES BUREAU OF STANDARDS DATA—CLAY 1

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations				
	0.04 inch	0.08 inch	0.12 inch	0.16 inch	0.20 inch
0.12	30.50	33.00	70.00	(94.00)	(95.00)
0.49	21.00	39.00	55.50	63.50	79.50
1.10	16.50	31.50	44.00	53.50	60.00
1.96	14.00	27.00	38.00	45.00	51.50

C-3E. UNITED STATES BUREAU OF STANDARDS DATA—CLAY 2

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations				
	0.01 inch	0.02 inch	0.03 inch	0.04 inch	0.05 inch
0.12	133	244	314	364	404
0.49	64	123	170	213	253
1.10	49	83	123	166	196
1.96	40	77	108	133	158

TABLE C-3.—The effects of bearing area on the resistance of clay and granular materials—Continued

C-3F. UNITED STATES BUREAU OF STANDARDS DATA—CLAY 3

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations					
	0.04 inch	0.06 inch	0.08 inch	0.10 inch	0.12 inch	0.14 inch
0.12	140	209	264	313	356	394
0.49	50	76	100	123	145	163
1.10	26	42	59	77	95	112
1.96	15	24	33	42	53	63

C-3G. BERRY DATA—CONCRETE SAND (SHARP)

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations			
	0.02 inch	0.04 inch	0.08 inch	0.16 inch
4	17.8	39.0	59.6	64.6
8	20.9	44.4	80.5	95.7
16	23.8	48.2	104.1	128.6

C-3H. BERRY DATA—OTTAWA SAND (SMOOTH)

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations			
	0.008 inch	0.016 inch	0.032 inch	0.064 inch
4	6.0	9.7	12.5	12.9
8	7.9	11.7	15.6	16.2
16	7.6	14.0	22.0	24.6
32	9.7	19.0	32.0	36.2

C-3I. BERRY DATA—LEAD SHOT (EXTRA FINISH NO. 12)

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations		
	0.005 inch	0.010 inch	0.020 inch
4	23.8	24.0	25.5
8	27.3	31.8	35.0
16	38.8	45.2	47.4

C-3J. BERRY DATA—GRADED AGGREGATE (3/4-INCH MAXIMUM)

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations			
	0.02 inch	0.04 inch	0.06 inch	0.10 inch
4	43	76	101	129
8	40	76	109	143
16	39	75	120	177

C-3K. HOUSEL DATA—PLASTIC YELLOW CLAY

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations					
	0.1 inch	0.4 inch	0.8 inch	1 inch	2 inches	3.5 inches
144	20.8	66.3	97.9	107.4	134.8	152.5
288	12.7	45.1	72.9	80.9	101.7	114.9
576	11.4	40.2	63.2	70.7	88.4	98.9
1,506	6.2	28.9	42.4	46.1	61.1	68.2

TABLE C-3.—The effects of bearing area on the resistance of clay and granular materials—Continued

C-3L. HOUSEL DATA—STIFF YELLOW CLAY

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations					
	0.4 inch	1.0 inch	1.6 inches	2.2 inches	2.8 inches	3.4 inches
144.....	45.1	102.8	134.0	154.0	168.8	179.2
288.....	29.9	70.1	96.8	112.5	126.4	134.7
676.....	27.1	61.8	88.4	102.1	114.6	126.0

C-3M. HOUSEL DATA—STIFF BLUE CLAY (CONTAINING SAND AND PEBBLES)

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations			
	0.3 inch	1.5 inches	2.7 inches	3.9 inches
144.....	72.9	127.8	155.6	179.2
288.....	77.8	130.6	156.2	178.5
676.....	62.5	138.1	168.3	188.1

C-3N. TERZAGHI DATA—VIENNA LOESS (68 PERCENT SAND)

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations					
	0.2 inch	0.4 inch	0.6 inch	0.8 inch	1 inch	1.2 inches
7.6.....	33.5	48.0	(58.0)	(68.0)	(72.0)	(88.0)
15.2.....	(26.0)	40.0	50.0	(67.0)	(83.0)	(88.0)
96.8.....	(16.5)	28.7	38.2	44.0	49.0	54.2
888.8.....	(10.0)	(19.0)	(26.8)	33.0	38.4	42.6

TABLE C-3.—The effects of bearing area on the resistance of clay and granular materials—Continued

C-3O. KÜGLER DATA—CLAY

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations			
	0.1 inch	0.2 inch	0.3 inch	0.4 inch
1.3.....	28.0	40.2	(40.2)	(58.0)
12.2.....	26.0	40.2	(40.2)	(58.0)
35.2.....	21.7	36.4	(40.8)	(58.0)
76.1.....	18.8	33.2	42.2	(48.0)
304.3.....	(13.0)	19.2	24.6	29.6
694.4.....	(5.0)	(10.2)	14.7	16.0

C-3P. KÜGLER DATA—SAND

Bearing area (square inches)	Resistance in pounds per square inch at various penetrations			
	0.04 inch	0.08 inch	0.12 inch	0.16 inch
7.8.....	3.0	5.8	7.7	9.0
14.7.....	2.7	6.8	8.1	9.3
31.2.....	3.7	6.9	9.4	11.1
70.1.....	3.9	7.2	10.4	12.0
149.1.....	2.8	6.8	9.8	12.2
467.8.....	2.3	5.0	7.6	10.2
1,168.6.....	1.8	3.6	5.5	7.4

TABLE C-4.—Stresses in a semi-infinite isotropic solid from a load uniformly distributed over a small circular area of its boundary plan

$\frac{r}{a}$	$\frac{\sigma_z}{p}$			$\frac{\sigma_{\theta\theta}}{p}$			$\frac{\tau_{rz}}{p}$	$\frac{\tau_{\theta z}}{p}$	$\frac{S}{p}$		
	$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$	$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$			$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$
$\frac{z}{a}=0.25$											
0	-0.2646	-0.4540	-0.6433	-0.2646	-0.4540	-0.6433	-0.9857	0	0.7211	0.5318	0.3424
0.0670	-0.2647	-0.4535	-0.6428	-0.2633	-0.4536	-0.6419	-0.9855	-0.0054	0.7209	0.5318	0.3428
0.1311	-0.2648	-0.4534	-0.6427	-0.2619	-0.4516	-0.6394	-0.9851	-0.0104	0.7194	0.5319	0.3432
0.670	-0.1828	-0.3849	-0.5471	-0.2401	-0.4027	-0.5652	-0.9542	-0.0781	0.7871	0.6097	0.4361
0.75	-0.1111	-0.2845	-0.4080	-0.2173	-0.3469	-0.4765	-0.8838	-0.1697	0.8361	0.6702	0.5322
0.884	-0.0608	-0.2333	-0.3070	-0.1978	-0.2920	-0.3952	-0.7943	-0.2593	0.8144	0.7057	0.6080
1	-0.0222	-0.2335	-0.3849	-0.1832	-0.2131	-0.2430	-0.4590	-0.2988	0.7959	0.6390	0.6023
1.1166	-0.0892	-0.2252	-0.3813	-0.1735	-0.1653	-0.1371	-0.2696	-0.2312	0.4776	0.4628	0.4870
1.25	-0.0457	-0.1835	-0.2813	-0.1630	-0.1149	-0.0877	-0.0757	-0.1297	0.2609	0.2745	0.3322
1.4330	+0.0116	-0.0842	-0.1800	-0.1438	-0.0859	-0.0284	-0.0211	-0.0575	0.1195	0.1311	0.1691
1.6869	+0.0444	-0.0286	-0.1015	-0.1172	-0.0638	-0.0105	-0.0056	-0.0227	0.0675	0.0509	0.1061
2.4178	+0.0490	+0.0079	-0.0301	-0.0608	-0.0341	-0.0014	-0.0005	-0.0037	0.0471	0.0113	0.0306
$\frac{z}{a}=0.50$											
0	-0.0975	-0.2357	-0.3759	-0.0975	-0.2357	-0.3759	-0.9106	0	0.8150	0.6749	0.5367
0.1340	-0.0961	-0.2327	-0.3703	-0.0981	-0.2329	-0.3698	-0.9095	-0.0299	0.8136	0.6764	0.5395
0.2659	-0.0848	-0.2208	-0.3587	-0.0959	-0.2246	-0.3569	-0.9113	-0.0584	0.8170	0.6852	0.5504
0.5	-0.0623	-0.1938	-0.3249	-0.0851	-0.2009	-0.3168	-0.8396	-0.1287	0.8188	0.6951	0.5754
0.7668	-0.0474	-0.1675	-0.2875	-0.0736	-0.1832	-0.2828	-0.7775	-0.2259	0.7553	0.6813	0.5968
1	-0.0284	-0.0754	-0.1809	-0.0701	-0.1054	-0.1406	-0.4175	-0.2921	0.6259	0.6781	0.6408
1.1820	-0.014	-0.0366	-0.1036	-0.0726	-0.0769	-0.0811	-0.2254	-0.2142	0.4481	0.5203	0.5118
1.5	-0.0059	-0.0211	-0.0602	-0.0730	-0.0510	-0.0283	-0.0504	-0.1019	0.2038	0.2140	0.2448
1.7141	-0.0287	-0.0848	-0.1430	-0.0655	-0.0420	-0.0146	-0.0268	-0.0890	0.1181	0.1316	0.1657
2.0722	+0.0138	-0.0315	-0.0848	-0.0593	-0.0324	-0.0085	-0.0385	-0.0290	0.0565	0.0886	0.0974
2.8600	+0.0176	-0.0075	-0.0324	-0.0394	-0.0202	-0.0013	-0.0013	-0.0094	0.0227	0.0142	0.0337
3.8356	+0.0176	+0.0035	-0.0106	-0.0237	-0.0105	+0.0026	+0.0026	+0.0019	0.0205	0.0039	0.0137

APPENDIX

TABLE C-4.—Stresses in a semi-infinite isotropic solid from a load uniformly distributed over a small circular area of its boundary plane—Continued

$\frac{r}{a}$	$\frac{z}{a}$			$\frac{z}{a}$			$\frac{z}{a}$	$\frac{z}{a}$	$\frac{z}{a}$		
	$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$	$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$			$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$
$\frac{z}{a} = 0.75$											
0	0.000	-0.1080	-0.2080	-0.0080	-0.1080	-0.2080	-0.7840	0	0.7780	0.6760	0.6760
0.1029	-0.0091	-0.1085	-0.2079	-0.0095	-0.1080	-0.2066	-0.7803	0.0294	0.7754	0.6748	0.6763
0.3707	-0.0083	-0.1081	-0.2099	-0.0111	-0.1071	-0.2058	-0.7854	0.0445	0.7585	0.6660	0.6740
0.6700	-0.0100	-0.1026	-0.1951	-0.0173	-0.0940	-0.1920	-0.8231	0.0601	0.7284	0.6447	0.6682
1.0000	-0.0287	-0.1138	-0.1889	-0.0318	-0.0940	-0.1920	-0.8231	0.0811	0.6506	0.5884	0.6326
1.2010	-0.0440	-0.1216	-0.2087	-0.0496	-0.1010	-0.2084	-0.8745	0.1011	0.6467	0.5648	0.6152
1.4330	-0.0730	-0.1412	-0.2094	-0.0747	-0.1098	-0.2046	-0.9308	0.1211	0.6208	0.5393	0.5859
1.7000	-0.1040	-0.1604	-0.1860	-0.1018	-0.1209	-0.2009	-0.9911	0.1412	0.5872	0.5225	0.5627
2.0111	-0.1492	-0.1956	-0.1419	-0.1390	-0.1339	-0.1939	-1.0612	0.1612	0.5468	0.5100	0.5397
2.3707	-0.2222	-0.2692	-0.1081	-0.1778	-0.1222	-0.1666	-1.1448	0.1812	0.5068	0.5097	0.5118
2.7800	-0.3300	-0.3800	-0.0581	-0.2175	-0.1022	-0.1397	-1.2333	0.2012	0.4668	0.5177	0.4818
3.2300	-0.4800	-0.4800	-0.0106	-0.2212	-0.1008	-0.1300	-1.3277	0.2212	0.4268	0.5110	0.4605
3.7300	-0.6800	-0.6800	-0.0000	-0.2100	-0.1000	-0.1300	-1.4277	0.2412	0.3868	0.5098	0.4405
$\frac{z}{a} = 1.00$											
0	0.0303	-0.0429	-0.1161	0.0303	-0.0429	-0.1161	-0.6464	0	0.6768	0.6036	0.6303
0.1800	0.0270	-0.0449	-0.1174	0.0313	-0.0410	-0.1132	-0.6378	0.0424	0.6708	0.5990	0.6273
0.4228	0.0200	-0.0503	-0.1205	0.0274	-0.0373	-0.1019	-0.6264	0.0811	0.6438	0.5781	0.6136
0.6930	0.0147	-0.0520	-0.1227	0.0244	-0.0317	-0.0858	-0.6103	0.1211	0.5981	0.5417	0.6000
0.9125	-0.0247	-0.0850	-0.1453	0.0127	-0.0242	-0.0612	-0.5773	0.1612	0.6059	0.4658	0.6305
1.1793	-0.0499	-0.1031	-0.1683	0.0013	-0.0191	-0.0396	-0.5401	0.2012	0.5718	0.3948	0.6122
1.4983	-0.0894	-0.1046	-0.1499	-0.0101	-0.0163	-0.0225	-0.5033	0.2412	0.5325	0.2790	0.6054
1.8591	-0.1438	-0.0944	-0.1208	-0.0148	-0.0133	-0.0106	-0.4666	0.2812	0.4912	0.1627	0.5923
2.2811	-0.2223	-0.0478	-0.0733	-0.0230	-0.0132	-0.0034	-0.4300	0.3212	0.4508	0.0608	0.6083
2.7442	-0.3300	-0.0223	-0.0397	-0.0296	-0.0108	-0.0011	-0.4030	0.3612	0.4077	0.0327	0.6444
3.2300	-0.4800	-0.0048	-0.0181	-0.0139	-0.0087	-0.0001	-0.3766	0.4012	0.3677	0.0088	0.6137
$\frac{z}{a} = 1.25$											
0	0.0428	-0.0120	-0.0688	0.0428	-0.0120	-0.0688	-0.6239	0	0.5887	0.6119	0.4671
0.1247	0.0399	-0.0143	-0.0556	0.0443	-0.0104	-0.0650	-0.6197	0.0276	0.5823	0.6083	0.4648
0.4171	0.0305	-0.0224	-0.0752	0.0398	-0.0104	-0.0616	-0.6174	0.0678	0.5728	0.6017	0.4580
0.6511	0.0123	-0.0378	-0.0878	0.0328	-0.0087	-0.0499	-0.6100	0.1093	0.4921	0.4999	0.4094
0.9006	-0.0089	-0.0550	-0.1017	0.0248	-0.0074	-0.0396	-0.5922	0.1461	0.4340	0.4013	0.3708
1.1094	-0.0271	-0.0700	-0.1100	0.0184	-0.0097	-0.0298	-0.5654	0.1803	0.3699	0.3453	0.3248
1.4550	-0.0440	-0.0812	-0.1179	0.0036	-0.0069	-0.0174	-0.5479	0.2204	0.2657	0.2337	0.2466
1.8793	-0.0435	-0.0781	-0.1027	-0.0074	-0.0080	-0.0085	-0.5296	0.2612	0.1644	0.1623	0.1656
2.4007	-0.0281	-0.0716	-0.0601	-0.0142	-0.0086	-0.0030	-0.5123	0.3012	0.0788	0.0822	0.0909
3.0451	-0.0112	-0.0267	-0.0422	-0.0150	-0.0081	-0.0011	-0.4950	0.3412	0.0385	0.0400	0.0496
3.7300	-0.0003	-0.0089	-0.0181	-0.0120	-0.0061	-0.0002	-0.4781	0.3812	0.0110	0.0130	0.0196
$\frac{z}{a} = 1.50$											
0	0.0440	+0.0090	-0.0399	0.0440	0.0090	-0.0399	-0.4240	0	0.4680	0.4260	0.3940
0.1340	0.0416	-0.0000	-0.0417	0.0449	0.0031	-0.0837	-0.4203	0.0295	0.4642	0.4229	0.3818
0.4540	0.0310	-0.0094	-0.0499	0.0398	0.0022	-0.0335	-0.3833	0.0746	0.4404	0.4025	0.3633
0.7945	0.0127	-0.0235	-0.0637	0.0332	0.0019	-0.0294	-0.3440	0.1080	0.3679	0.3651	0.3357
1.1000	-0.0095	-0.0422	-0.0777	0.0251	0.0011	-0.0256	-0.3062	0.1491	0.3400	0.3202	0.2976
1.4645	-0.0233	-0.0556	-0.0879	0.0166	-0.0000	-0.0167	-0.2692	0.1885	0.2860	0.2855	0.2640
1.8460	-0.0338	-0.0628	-0.0914	0.0083	-0.0015	-0.0114	-0.2329	0.2269	0.2250	0.2144	0.2073
2.2400	-0.0398	-0.0616	-0.0866	0.0007	-0.0032	-0.0079	-0.2000	0.2638	0.1824	0.1878	0.1877
2.6500	-0.0318	-0.0527	-0.0736	-0.0054	-0.0047	-0.0040	-0.1635	0.3051	0.1108	0.1108	0.1143
3.0700	-0.0115	-0.0350	-0.0548	-0.0065	-0.0057	-0.0019	-0.1272	0.3444	0.0644	0.0640	0.0738
3.5000	-0.0046	-0.0218	-0.0359	-0.0077	-0.0037	-0.0007	-0.0908	0.3812	0.0140	0.0140	0.0198
4.0000	-0.0042	-0.0138	-0.0227	-0.0101	-0.0052	-0.0003	-0.0532	0.4170	0.0085	0.0196	0.0196
$\frac{z}{a} = 1.75$											
0	0.0410	0.0080	-0.0240	0.0410	0.0080	-0.0240	-0.3455	0	0.3605	0.3335	0.3206
0.1840	0.0387	0.0061	-0.0296	0.0400	0.0084	-0.0241	-0.3406	0.0293	0.3626	0.3502	0.3170
0.3311	0.0370	-0.0045	-0.0381	0.0384	0.0074	-0.0218	-0.3058	0.0663	0.3553	0.3299	0.3006
0.4869	0.0307	-0.0200	-0.0496	0.0306	0.0061	-0.0177	-0.2332	0.1002	0.3188	0.2848	0.2719
1.1531	-0.0077	-0.0350	-0.0628	0.0221	0.0044	-0.0134	-0.1947	0.1376	0.2704	0.2626	0.2602
1.4639	-0.0213	-0.0438	-0.0704	0.0141	0.0028	-0.0095	-0.1586	0.1721	0.2184	0.2082	0.1963
1.8100	-0.0264	-0.0500	-0.0717	0.0066	0.0002	-0.0062	-0.1201	0.2099	0.1657	0.1689	0.1649
2.2064	-0.0285	-0.0469	-0.0638	-0.0003	0.0018	-0.0036	-0.0828	0.2468	0.1160	0.1167	0.1143
2.6500	-0.0235	-0.0373	-0.0529	-0.0049	-0.0034	-0.0015	-0.0459	0.2837	0.0681	0.0732	0.0786
3.0900	-0.0139	-0.0240	-0.0351	-0.0078	-0.0042	-0.0007	-0.0102	0.3207	0.0375	0.0399	0.0440
3.7300	-0.0036	-0.0108	-0.0180	-0.0076	-0.0039	-0.0008	-0.0027	0.3577	0.0138	0.0140	0.0140

TABLE C-4.—Stresses in a semi-infinite isotropic solid from a load uniformly distributed over a small circular area of its boundary plane—
Continued

$\frac{z}{a}$	$\left(\frac{\partial \sigma}{\partial z}\right)$			$\left(\frac{\partial \tau}{\partial z}\right)$			$\left(\frac{\partial \sigma}{\partial r}\right)$	$\left(\frac{\partial \tau}{\partial r}\right)$	$\frac{S}{p}$		
	$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$	$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$			$\sigma=0$	$\sigma=0.25$	$\sigma=0.5$
$\frac{z}{a}=2.00$											
0	0.0367	0.0103	-0.0161	0.0267	0.0103	-0.0161	-0.2845	0	0.3211	0.2947	0.2683
.2721	.0331	.0071	-.0189	.0263	.0105	-.0153	-.2787	-.0285	.3150	.2894	.2640
.6473	.0213	-.0037	-.0267	.0216	.0091	-.0135	-.2430	-.0606	.2807	.2632	.2461
1.5227	-.0093	-.0306	-.0310	.0180	.0050	-.0360	-.1459	-.0802	.2107	.1975	.1859
1.7270	-.0195	-.0386	-.0575	.0108	.0027	-.0384	-.0969	-.0721	.1940	.1806	.1702
2.1547	-.0284	-.0401	-.0565	.0045	.0005	-.0383	-.0612	-.0571	.1301	.1181	.1143
2.6782	-.0217	-.0352	-.0488	-.0011	-.0014	-.0318	-.0329	-.0393	.0794	.0766	.0692
3.3832	-.0181	-.0298	-.0361	-.0029	-.0029	-.0261	-.0229	-.0229	.0483	.0465	.0425
4.4641	-.0088	-.0142	-.0215	-.0061	-.0032	-.0203	-.0047	-.0100	.0201	.0222	.0261
$\frac{z}{a}=2.50$											
0	0.0283	0.0104	-0.0075	0.0283	0.0104	-0.0075	-0.1096	0	0.2278	0.2100	0.1921
.8301	.0254	.0078	-.0098	.0278	.0104	-.0070	-.1065	-.0212	.2230	.2057	.1886
.7813	.0189	-.0010	-.0179	.0241	.0090	-.0092	-.1081	-.0442	.2041	.1890	.1742
1.8187	-.0034	-.0128	-.0280	.0189	.0071	-.0048	-.1327	-.0583	.1754	.1635	.1524
1.6699	-.0076	-.0218	-.0360	.0131	.0048	-.0053	-.0958	-.0537	.1420	.1337	.1284
2.1658	-.0148	-.0270	-.0394	.0078	.0026	-.0023	-.0921	-.0481	.1024	.1024	.0988
2.7005	-.0164	-.0289	-.0374	.0025	.0006	-.0013	-.0357	-.0359	.0748	.0728	.0718
3.5	-.0186	-.0220	-.0303	-.0013	-.0010	-.0035	-.0174	-.0227	.0456	.0457	.0472
4.6704	-.0081	-.0142	-.0204	-.0035	-.0019	-.0002	-.0066	-.0128	.0284	.0246	.0271
$\frac{z}{a}=3.00$											
0	0.0218	0.0089	-0.0039	0.0218	0.0089	-0.0039	-0.1462	0	0.1680	0.1551	0.1428
.7375	.0148	-.0035	-.0098	.0199	.0083	-.0032	-.1300	-.0281	.1554	.1440	.1327
1.2825	.0066	-.0030	-.0175	.0159	.0066	-.0027	-.1046	-.0392	.1363	.1261	.1172
1.8038	-.0038	-.0141	-.0245	.0113	.0047	-.0020	-.0761	-.0416	.1101	.1036	.0975
2.3889	-.0096	-.0180	-.0282	.0077	.0027	-.0013	-.0497	-.0366	.0838	.0793	.0762
3.5173	-.0115	-.0184	-.0254	.0010	.0002	-.0005	-.0203	-.0226	.0498	.0480	.0482
4.6763	-.0083	-.0137	-.0190	-.0018	-.0009	-.0002	-.0088	-.0128	.0267	.0261	.0276
$\frac{z}{a}=4.00$											
0	0.0136	0.0061	-0.0013	0.0136	0.0061	-0.0013	-0.0860	0	0.1005	0.0931	0.0860
.2947	.0121	.0040	-.0022	.0141	.0066	-.0009	-.0856	-.0058	.0984	.0912	.0840
1.0088	-.0088	-.0015	-.0056	.0124	.0038	-.0009	-.0761	-.0176	.0913	.0825	.0789
1.7053	.0028	-.0038	-.0106	.0096	.0044	-.0008	-.0664	-.0239	.0790	.0710	.0663
2.4559	-.0029	-.0031	-.0148	.0068	.0030	-.0006	-.0463	-.0243	.0643	.0661	.0580
3.8094	-.0061	-.0112	-.0164	.0036	.0016	-.0004	-.0365	-.0205	.0504	.0477	.0452
4.8664	-.0071	-.0113	-.0166	.0010	.0003	-.0006	-.0208	-.0144	.0348	.0329	.0305
$\frac{z}{a}=5.00$											
0	0.0081	0.0043	-0.0006	0.0091	0.0043	-0.0006	-0.0371	0	0.0688	0.0614	0.0606
1	.0067	.0029	-.0026	.0088	.0045	-.0002	-.0328	-.0069	.0622	.0578	.0554
1.8816	-.0028	-.0016	-.0060	.0070	.0034	-.0003	-.0410	-.0151	.0532	.0495	.0462
2.8198	-.0011	-.0051	-.0091	.0049	.0023	-.0002	-.0333	-.0102	.0471	.0443	.0417
4.6010	-.0045	-.0076	-.0108	.0019	.0009	-.0001	-.0241	-.0121	.0311	.0292	.0275
$\frac{z}{a}=7.00$											
0	0.0049	0.0024	-0.0002	0.0049	0.0024	-0.0002	-0.0299	0	0.0347	0.0322	0.0297
1	.0038	.0014	-.0010	.0049	.0025	+.0000	-.0288	-.0039	.0331	.0306	.0284
1.6124	.0031	.0015	-.0017	.0048	.0023	+.0000	-.0245	-.0059	.0300	.0275	.0257
2.8756	.0006	-.0016	-.0038	.0036	.0017	-.0001	-.0235	-.0082	.0268	.0275	.0268
8.5478	-.0004	-.0023	-.0047	.0029	.0014	-.0000	-.0224	-.0085	.0278	.0262	.0246
4.2641	-.0011	-.0031	-.0061	.0023	.0011	-.0000	-.0210	-.0063	.0277	.0244	.0229

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APPENDIX D

DYNAMIC WHEEL LOADS AND TIRE CONTACT AREAS ON VARIOUS ROAD SURFACES ¹

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THE INTERRELATED EFFECTS OF LOAD, SPEED, TIRES, AND ROAD ROUGHNESS ON MOTOR VEHICLE IMPACTS

The maximum wheel load of the vehicle and the tire contact area through which it is applied to the road surface are fundamental considerations in a rational pavement design. They are, likewise, important factors in determining the stability that may be expected from a given subgrade or unpaved earth road.²

¹ By Warner Tufts.

² See appendix C, above.

³ Thomas R. Agg and Ralph A. Moyer in the recent General Engineering Handbook (O. E. O'Rourke, editor, McGraw-Hill, 1923), p. 331, suggest impact factors of 1.6 for solid rubber tires and 1.3 for pneumatic tires in connection with the "corner formula" as modified by Clifford Older (appendix B, p. 182). Since this formula results in greater slab thickness than the more exact Westergaard equations, these impact allowances are actually greater than the values stated.

Because moving loads are more severe on concrete slab than static ones, some authorities have recommended the use of constant impact allowances in slab design formulas.³ But fixed impact factors, however suitable where upper limits of anticipated wheel loads are to be used as design criteria, are not accurate for use in dealing, as in the present study, with a whole range of vehicle groups having radically different load, speed, tire, spring, and unsprung load characteristics. Neither are they applicable to different degrees of surface smoothness.

It is important, therefore, to examine the joint effect of the various elements causing impact, and also to consider the relation between the increased dynamic loads and the resulting increase in tire contact areas.

The principal measurements of road impacts are given in two investigations of the United States Bureau of Public Roads.⁴ These measurements relate to the average of the maximum impact forces developed under given load, speed, vehicle, and tire conditions at a frequency of 100 per mile. The average of these 100 worst impacts may be assumed to be a fair measure of the particular surface condition. From these and related source data, the necessary equations are developed and herein given to permit statistical representation within moderate limits of variation.

In table D-4 are given impact values resulting from the application of these equations to three road surface conditions representing excellent, intermediate, and poor surfaces as found in practical operating conditions. The best of these corresponds to the general run of the best concrete and high-type surfacing, the intermediate to the average road surface, and the worst to conditions such as are sometimes encountered and beyond which reconstruction or extensive repairs are needed. The computations have been based on a single speed of 45 miles per hour for pneumatic tires. This speed is the maximum recommended for trucks and busses in the proposed uniform code of the American Association of State Highway Officials. It comprises the general maximum of impacts reached within practical traveling speeds. Solid tire reactions have been computed for a speed of 10 miles per hour. This speed is the maximum recommended for these tires in the code.

⁴ Presented in *The Interrelated Effects of Load, Speed, Tires, and Road Roughness on Motor Truck Impact*, by James A. Buchanan, Public Roads, September 1930, and in *Road Impact Produced by a Heavy Motor Bus*, by the same author, Public Roads, November 1932.

Road roughness versus speed.—A consideration of the theory and of the available data indicates that, at present, calibrated values for road roughness may not be derived independently of speed. A given small obstacle produces reactions which vary through a certain range of speed conditions. Other roughnesses react in a similar individual manner. Obstructions which are important at one speed may be quite ineffective at higher speeds; but at these higher speeds other obstruc-

Calibrated values of roughness, R_c , and S_c , in indicator units per mile¹

Test road	R_c	S_c	R	R	R	R	R	R	R	R
			5	10	15	20	25	30	35	
			m. p. h.	m. p. h.	m. p. h.	m. p. h.	m. p. h.	m. p. h.	m. p. h.	m. p. h.
H.....	117	34	31	69	80	94	107	115	117	
D.....	151	30	67	117	141	147	149	151	151	
A.....	293	66	50	96	141	180	212	239	252	
C.....	395	45	134	228	283	325	352	373	388	
E.....	780	23	604	717	755	778	780	780	780	

¹ Values of R_c and S_c were derived by graphical extrapolation from various renderings of the data in curve form. Values of R are identical with the roughness indicator units employed. They are not comparable to R values elsewhere used save as they are divided by 211.7; i. e., the value of road A at 25 m. p. h., to which all values of R elsewhere employed are relative.

These five roads are insufficient to permit complete generalization as to the interrelation of these characteristics, though within the five instances the following equation⁴ may be employed as an expression of the relation between the fixed maximum roughness, R_c , and the critical speed, S_c , at which it is attained:

$$S_c = \frac{R_c^2}{(0.012R_c + 3.095)^4} \quad (1)$$

The roughness measure would most likely vary with the design and mounting of the calibration instrument. The roughness calibrations are, however, relatives covering a sufficient range of road surface conditions to permit selection of intermediate road roughness values.⁷

This discussion is directed toward a determination of the constancies and differences found in the data rather than toward an evaluation of their causes. However,

⁴ Here and elsewhere in the derivation of equations, because the data seemed to lend themselves most readily to this type of analysis, values were converted to a form in which they would approach straight lines either on quadrille or on logarithmic plots. The method of least squares was applied to determine the constants. Independent variables were then successively eliminated.

⁷ "The test road surfaces were calibrated with the relative roughness indicator (See. An Instrument for the Measurement of Relative Road Roughness, Public Roads, vol. 7, No. 7, September 1930) being developed by the Bureau. This was carefully done by varying speeds, and the maximum and minimum roughness factors obtained at a car speed of 30 miles per hour were approximately 800 to 100 units per mile. The instrument was installed between the front axle and chassis frame of a large sedan automobile fully equipped with rebound snubbers and 6.50 by 20 balloon tires inflated to a pressure of 35 pounds.

"Since the vehicle on which the device is mounted is an integral part of the apparatus, the data obtained with one vehicle are not directly comparable with those obtained with another. However, as the same vehicle was used to obtain data on each of a series of roads it is believed that the respective factors obtained indicate the relative roughness with reasonable accuracy.

"In making the selection of road surfaces for these tests it was necessary to give consideration to three conditions. First, they should include the extremes of surface condition from very smooth to very rough; second, they should furnish a sufficient number of types so that the data would cover the range between extremes as well as possible; and, third, the pavements selected should be of such character that they would not change in condition during the tests. It would have been very difficult to fulfill all these conditions perfectly. It is believed, however, that the sections selected represent a very wide range in roughness and that all of them maintained their characteristics during the tests, but with the pavements which were available in the vicinity of Washington it was not possible to cover the range in roughness in as uniform a manner as might be desired. Referring to fig. 5, it will be noted that there is a gap between the curves for roads C and E. However, it is thought that the majority of pavements carrying any considerable amount of traffic will be found to have surfaces as smooth or smoother than road C, so that the deficiency just mentioned is not so important as might appear from a cursory inspection of the surface roughness data contained in this figure."—Public Roads, September 1930.

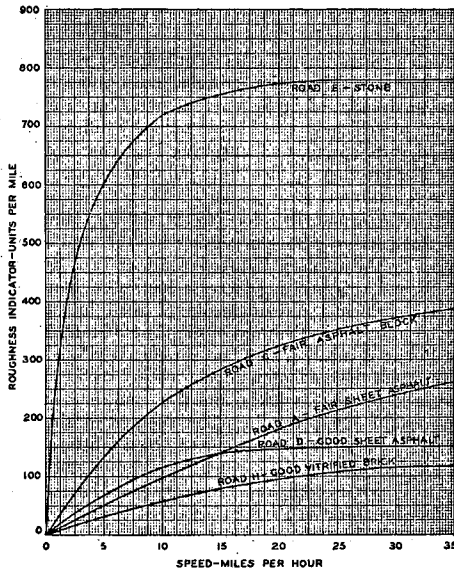


FIGURE D-1.—Calibration of roughness of test road sections with relative roughness indicator. (From Public Roads, September 1930.)

tions, unimportant before, may produce considerable effect. For the present, it seems that any measure of roughness, R , in a formula for impact must be a function of both roughness and speed.

Test data are given below on five roads representing a wide range of roughness conditions.⁵ From these data it is to be observed that for each road there is a characteristic relation between roughness and speed, as well as a definite maximum roughness, R_c , attained at a somewhat less definite critical speed, S_c . (See accompanying table and fig. D-1.)

⁵ Here, as elsewhere, unless otherwise stated, the analysis is based on the data or the graphs published in The Interrelated Effects of Load, Speed, Time, and Road Roughness on Motor Truck Impact, above. The specific reference is to fig. 5, p. 144, of that report.

the fact that a maximum in roughness is reached at all may in some way be related to a source of difficulty in the analysis that was not quite overcome for the speeds below 15 miles per hour. A curious vibration phenomenon appears to be present which suggests an interplay of several periodic forces, the effect of which is most manifest at very low speeds.

Changes in the installation of the instrument used in measuring this interplay may yield different values of the maximum roughness or of critical speeds, but the usefulness of the equation is not thereby impaired. Motor vehicles, motor vehicle springs, and rubber tires are highly standardized and are closely adapted to each other for the maximum of smoothness and economy in operation.

Road roughness, R , in the calibration units used, may be interpreted in terms of R_s and S_s by the following equations:

$$R = \frac{S^2}{b(S+a)_{2.3}} \quad (1a)$$

$$b = \frac{0.671 \sqrt{\frac{1}{R_s} - 0.00116}}{S_s} \quad (1b)$$

$$a = \frac{5.59}{bR_s} - 17.52 \quad (1c)$$

By substituting from equation (1) for S_s , R would then be expressed in terms of S and R_s alone. This equation is modified in calculations of impact in this study by the modulus, 0.47, required to adjust all R values into relatives, using the roughness value of road A at 25 miles per hour as 100.

This roughness equation fits the data reasonably well for all speeds up to 50 miles an hour, except that in the case of road E a marked departure occurs at speeds below 15 miles an hour. This departure, however, is unimportant for the purposes of this investigation.

Since roughness comprehends an element of speed, it was thought that the correction of impact values for roughness by the roughness formula in the process of successive elimination of variables, would yield impact values independent of speed. This was not the case. Values of impact still revealed a speed influence after the correction. The inference is that the reactions of the particular roughness calibration apparatus to speed are not similar to those of a moving vehicle.

Impact.—The impact force delivered to a road surface may be represented as a percentage of the wheel load. It is more convenient, however, to express the impact force as the static wheel load plus a dynamic increment.³

³ The Interrelated Effects of Load, Speed, Tires, and Road Roughness on Motor Truck Impact, above, p. 144.

In Public Roads, September 1930,³ there are presented 342 values of impact force determined by tests conducted with a range of combinations of load, speed, tires, and road roughness conditions. These are tabulated herein in terms of 1,000-pound units as F , in table D-1. They serve as the basis for this investigation and from them, together with data contained in tables D-2 and D-3, the various equations presented below were derived.

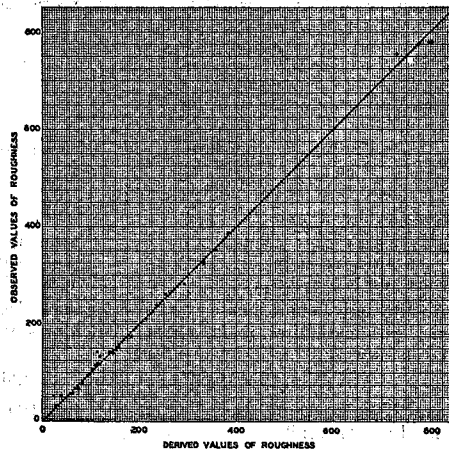


FIGURE D-2.—Agreement of derived and observed road roughness values.

Static wheel loads were subtracted from these values of F_s and the remaining dynamic increments were analyzed for their relation to the other variables involved. A satisfactory expression for total impact in terms of these variables is given in the equation:

$$F = W + \frac{cRW_1(S^{0.4} - 1.75)}{D} \quad (2)$$

where F = impact force of right rear wheel in pounds.
 W = static wheel load in pounds, at capacity load.
 R = measure of relative road roughness (road A at 25 m. p. h. = 100).
 W_1 = unsprung load, right rear wheel, in pounds.
 S = speed in miles per hour.
 D = tire deflection under recommended static wheel load, at recommended pressure, in inches.

The coefficient c varies with the ratio of actual to capacity load, having the value:

$$c = 0.0101 + 0.0037 \frac{L}{L_c} \quad (2a)$$

where L = actual vehicle loading in pounds.
 L_c = vehicle capacity load in pounds.

³ Same, Figs. 8 to 12, inclusive.

Manufacturing practice, moreover, affords the following typical relations between unsprung load and wheel load (see table D-2):

Passenger cars.....	$W_1 = 0.212 W$	} (2b)
Buses.....	$W_1 = .183 W$	
6-wheel trucks ¹⁰	$W_1 = .177 W$	
Taxis and trucks below 3¼ tons capacity.....	$W_1 = .150 W$	
Trucks above 3¼ tons capacity.....	$W_1 = .125 W$	

Manufacturing practice also affords the typical relations between tire deflection, wheel load, and tire pressure (see table D-3) given by the equation:

$$D = W \left[\frac{0.0259}{P} - n \right] + 0.22 \quad (2c)$$

where D = tire deflection in inches.

W = wheel load in pounds.

P = tire inflation pressure in pounds per square inch.

n = a constant depending on tire type.

Typical values of n are:

Truck and bus high-pressure tires.....	$n = 0.00005$
Truck and bus balloon tires.....	$n = .00010$
Passenger car balloon tires.....	$n = .00015$

An alternative approximate expression for the deflection of pneumatic tires, useful where inflation pressure is not known, is:

$$D = M \sqrt{W} \quad (2d)$$

where M is constant depending on the tire type. Typical values are:¹¹

Truck and bus high-pressure tires.....	$M = 0.0167$
Truck and bus balloon tires.....	$M = .0228$
Passenger car balloon tires.....	$M = .0267$

Discussion of computed and observed impact force values.—It will be recalled that the formula for road roughness is applicable for speeds up to 50 miles an hour. The data for road E fit the equation rather poorly at the lower speeds, but this is an old stone road of extraordinary roughness.¹² Hence in fitting the impact formula the data for road E were ignored wherever their inclusion would have altered the conclusions materially.

There actions of new and worn solid tires on road H, a brick pavement, were greatly amplified by the joints between the bricks.¹³ Solid tire values for this road were also similarly omitted. The particular new cushion tires on the wheel loads of 2,500 and 5,000 pounds, and the particular new solid tires on all wheel loads had tire deflection characteristics which deviated from the trends established for the pneumatics, the heavier new cushions, the worn solids, and other cushion and solid tire deflection data. These unusual influences were likewise eliminated.

¹⁰ See discussion of unsprung load at p. 267 below.

¹¹ Manufacturers' data on solid tire deflections at recommended static loads were not secured. Tests data indicate the rough value, $M = 0.007$. This value was used in table 1.

¹² See footnote 7, p. 254.

¹³ Public Roads, September 1930, p. 151.

These eliminations are justified by the need for deriving a formula applicable to balloon tires and other developments since these 1926 truck tests. This objective is of greater practical importance than the derivation of a mere formula of best fit for the entire body of data.

The remaining 147 data values have been given the major influence in determining the equation (2) for impact. In figure D-3 these data values are plotted

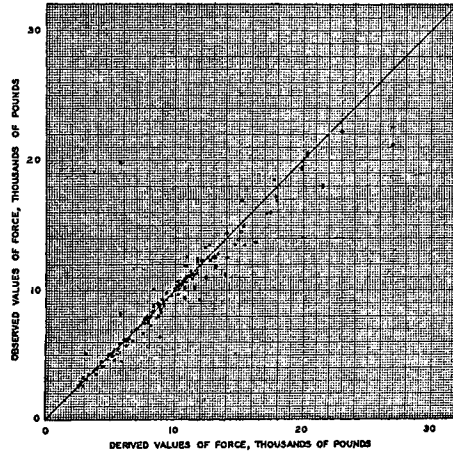


FIGURE D-3.—Agreement of significant values of impact force derived and observed.

against the corresponding values computed from the formula. In figure D-4 a similar comparison is shown between the observed points and the corresponding

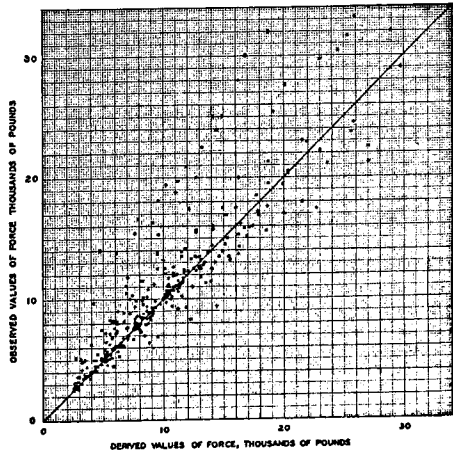


FIGURE D-4.—Agreement of all values of impact force derived and observed. Major deviations occur in cases not applicable.

formula values for all the original 342 data values. All major deviations in the latter figure represent observations for road E or for solid tires on road H.

Equation applied to balloon tires.—The formula was applied for check purposes to more recent data for bus impacts with balloon tires on concrete surfaces.¹⁴ The resulting values of roughness placed the smoothest concrete road on which the tests were run—the Mount Vernon Boulevard—about 45 percent below those for road H (fig. D-1). For the roughest concrete surface the values were well above those for road D but below those for road A. These results appear to agree reasonably well with actual roughness measurements of such roads.

The importance of the several elements.—It will be observed from equation (2) that impact reactions do not vary directly with wheel load. Impact will always be greater than wheel load, but as wheel load increases impact will increase at a diminishing rate.

The coefficient c , depending on the load/capacity ratio, influences the dynamic increment directly, though within relatively narrow limits. An increase from a 50 percent underload to a 50 percent overload—a 200 percent increase in the ratio—causes but a 31 percent increase in the coefficient.

Influences of road roughness and unsprung load on the dynamic increment are in direct proportion. Any change in either will result in a corresponding change in the dynamic increment.

Influences of speed, according to the relation ($S^{0.4}-1.75$), are somewhat greater proportionately at low speeds than at high.

Finally, the influence of tire deflection on the dynamic increment is in inverse proportion. For a change in D there is a corresponding inverse change in the increment.

Unsprung load.—Unsprung load, or that part of the wheel load below the springs, bears a fairly definite relation to total wheel load. The relation is determined by certain standards of travel smoothness and varies with vehicle type (table D-2). It does not appear that the ratio of unsprung to total load is related to the impact increment, though equation (2) suggests a fairly definite relation for the actual magnitude of the unsprung load.

In table D-4 the calculations for the six-wheel truck are subject to uncertainty. The two instances presented in table D-2 may not warrant generalization. But if these six-wheel truck unsprung loads are representative, then the six-wheel truck will result in slightly larger values of impact than its four-wheeled companion, though the excess will be but 4 to 8 percent throughout the load ranges. It is doubtful whether any other factor enters to change this possible small excess, and therefore it does not appear that this truck needs to be

considered separately from all trucks of its wheel-load range.¹⁵

Tire deflection characteristics.—Tire deflection characteristics differ radically under two sets of conditions. In the preceding discussion consideration is given to that deflection or "give" of a tire which corresponds to the recommended pressure and static load. It is seen by equation (2d) that deflection varies according to the square root of the recommended load. However, this relation changes when a tire, inflated to a given pressure, is subjected to increasing loads. Such dynamic deflection changes are illustrated in various issues of Public Roads dealing with the subject.¹⁶

The deflection of a given tire under different loads varies according to the curvilinear relation, $D=aW^n$, wherein n is a fraction. Solids, cushions, and the carcasses of pneumatics follow the same form of curve, having approximately the same values of n . However, for pneumatic tires, as soon as the load becomes sufficient to be supported by the air within the carcass, the exponent becomes approximately 1.0, so that the curve has the linear relation, $D=aW+b$. Thus, pneumatic tires, beyond extremely light loads, have a constant increase of deflection with load, instead of the decreasing increase of other tires.

Theoretically, this difference in trend should show marked differences in impact reaction between pneumatic and solid or cushion tires for increasing loads, since the same increases in load result in greater deflections, at the instant of impact, in the pneumatic than in the solid tire. But such differences are not apparent in observed impact data.

Inquiry was made of leading tire manufacturers as to the causes of uniformity in deflection qualities. It appears that a tire is designed and manufactured with definite objectives as to its cost, life, and traveling smoothness. It is then tested for loading, air pressure, and deflection qualities. Beyond an optimum deflection, there is too much internal friction within the carcass; above an optimum pressure there is too much rigidity; overloading creates destructive stresses. A balanced adjustment is sought. The net result seems to be that the allowable load and recommended pressure are adapted to the particular allowable deflection so as to yield the greatest ultimate economy. It is similarity in manufacturing technique, therefore, that results in the general uniformity of deflection characteristics (table D-3). This uniformity is further promoted by the

¹⁴ It is of interest to note the statement of L. W. Tuller before the Seventh Annual Meeting of the Highway Research Board as an established fact: "That, with a six-wheel vehicle of a type having equal load distribution on the four rear wheels and rear axle spacings between the limits of 3 feet and 10 feet, the maximum stress produced in the pavement is a function of the wheel load and not of the axle spacing." See Proceedings, p. 173. All standard American 6-wheel trucks are of this type.

¹⁵ For example, James A. Buchanan, *Motor Truck Impact as Affected by Rubber Tread Thickness of Tires*, Public Roads, September 1930, fig. 3 and table 3; The Interrelated Effects of Load, Speed, Tires, and Road Roughness on Motor Truck Impact, Public Roads, September, 1930 table 3 and fig. 4; Impact Reactions Developed by a Modern Motor Bus, Public Roads, April 1931, figs. 4, 14, and 15; Road Impact Produced by a Heavy Motor Bus, Public Roads, November 1932, table 1 and fig. 2.

¹⁶ James A. Buchanan, *Road Impact Produced by a Heavy Motor Bus*, Public Roads, November 1932.

definite standards developed and subscribed to by all leading manufacturers through the Tire and Rim Association, Inc.

These uniform tire characteristics do not change importantly from year to year for a given tire type—solid, high pressure, or balloon. Practically all new trucks sold in 1933 in the United States were sold with balloon tires as original equipment. Buses and private cars on the highway, with negligible exceptions, were equipped with balloon tires. These things were not true in 1926. And yet 1926 high-pressure tires had the same deflection formula values (equations (2c) and (2d)) as the 1933 high-pressure tires. The deflections for 1929-30 truck and bus balloon tires also were approximately the same as for the tires of 1933.

TIRE CONTACT AREAS

Single tires.—The tire contact area data in table D-3 are the actual contact areas of static wheel loads under recommended single-tire loading practice. These data permit the following expressions of contact area as a function of wheel load for the different pneumatic-tire types:

For balloon tires with a load rating of less than 1,400 pounds, the static contact area, in square inches, would be:

$$\text{Static area, in square inches,} = 0.004825 (\text{wheel load, pounds})^{1.25} \quad (3)$$

For balloon tires with a load rating of 1,400 pounds or more, the static contact area, in square inches, would be:

$$\text{Static area, in square inches} = 0.001189 (\text{wheel load, pounds})^{1.25} + 18.68 \quad (4)$$

For high-pressure pneumatics:

$$\text{Static area, in square inches} = 0.0375 (\text{wheel load, pounds})^{0.875} \quad (5)$$

When the load is increased on these tires, as under impact, a change in the area/load relation occurs (table 5), which may be expressed as a factor to be applied to the above-mentioned static load areas:

$$\text{Dynamic area} = 1.070 (\text{static area}) \left[\frac{\text{impact force}}{\text{static wheel load}} \right]^{0.687} \quad (6)$$

Dual tires.—Computations for dual tire contact areas apply to the gross contact area. This is the sum

of the areas of the two tires plus the area of the space between the two contact impressions. Such data were unavailable. A brief analysis of limits will suffice.

Standard specifications for the center-to-center spacing of dual tires of given cross-section and standard cross-section specifications for given tire loadings are developed by the Tire and Rim Association, and adopted by manufacturers. A certain range of allowance is made. For purposes here, it is well to know the minimum and maximum conditions. Minimum allowance figures for high-pressure tires and maximum allowance figures for balloons were taken from the association's 1933 handbook. They follow:

Wheel load	High pressure		Balloon	
	Cross section	Dual spacing	Cross section	Dual spacing
	<i>Inches</i>	<i>Inches</i>	<i>Inches</i>	<i>Inches</i>
2,000	4.00	† 6.50	5.50	7.25
3,000	5.00	7.75	6.50	8.25
4,000	5.00	7.75	7.50	10.00
5,000	8.00	9.00	8.00	11.50
8,000	8.00	11.50	9.75	12.75
12,000	10.00	12.75	12.50	† 15.25

† Estimated.

† Dual high pressure tires are almost always used in preference to balloons for very heavy wheel loads.

Dual tires for a given wheel load consists of two tires with a load rating of just half the wheel load. The individual contact areas of each of the two tires must, therefore, be computed on the basis of half the wheel load. The shape of these individual areas is somewhat oblong, rounded at the ends and bulging slightly at the sides. If they are treated as rectangles with semicircular ends, the error is small. If the greatest length of this area is $1\frac{1}{2}$ times its width,¹⁷ it is equal to 1.32 times the square root of the single tire area, and the gross contact area of the dual tire will be equal to this length times the center-to-center spacing plus the single tire contact area. If, on the other hand, the length of the individual tire area is as much as two times its width (a somewhat unlikely extreme), then it is equal to 1.50 times the square root of the single tire area and the gross area is again available as above.

Dynamic contact areas of dual tires require the calculation of dual tire impacts. These are considerably

¹⁷ Inspection of such contact area diagrams as were available, and consultation with manufacturers, indicated this to be a fair relation for standard loading. It would not change greatly under increased load, due to the expansion taking place in all directions.

more severe than impacts for like wheel loads on single tires because of the reduced deflection of the small tires in the dual combination (equations (2c) and (2d)). If a difference existed in the weight of the dual wheel with its tires, another cause of impact difference would be present (equation (2b)); however, dual tires for a given load weigh less, by an average constant of 21 pounds, than single tires (table D-6), and the dual wheel is considerably smaller in diameter than the single wheel. No regular differential in unsprung weight appears. Wheel weights are shown in table D-7.

From computed dynamic loadings and areas for individual tires, the minimum and maximum areas for high-pressure and balloon duals are developed as outlined above. They are presented in the following table for the larger vehicle types and wheel loads considered earlier in making impact calculations (table D-4).

Computed dual tire contact areas under impact

[Square inches]

	Static wheel load (pounds)	High pressure duals				Balloon duals	
		Minimum		Maximum		Minimum	Maximum
Light truck.....	2,000	76.3	82.5	110.1	118.6		
Bus.....	2,000	80.6	87.0	115.6	124.3		
Light truck.....	3,000	103.4	111.8	127.5	131.4		
Truck.....	4,000	114.6	123.6	143.3	153.6		
6-wheel truck.....	4,000	123.9	133.4	153.6	163.5		
Bus.....	5,000	151.5	163.6	180.7	196.4		
Truck.....	8,000	208.8	226.3	222.4	241.7		
6-wheel truck.....	8,000	221.1	240.3	233.9	253.9		
Bus.....	8,000	223.8	242.1	235.0	255.1		
Truck.....	12,000	275.4	288.0	312.1	339.3		
6-wheel truck.....	12,000	288.7	312.0	326.1	354.1		

1 Dual high pressure tires are almost always used in preference to balloons for very heavy wheel loads.

TABLE D-1.—Derived values of impact force (F_d) and observed values (F_o)

[In thousands of pounds]

Wheel load (pounds)	Speed (m. p. h.)	High pressure pneumatic tires										New cushion tires																
		Road H		Road D		Road A		Road C		Road E		Road H		Road D		Road A		Road C		Road E								
		F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o							
3,500.....	5	2.5	2.7	2.6	2.6	2.5	2.5	2.6	2.7	2.9	2.7	2.7	2.6	2.8	2.6	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7					
	10	2.9	3.2	3.2	3.1	3.1	3.1	3.0	3.0	3.0	2.9	2.9	2.8	2.7	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8					
	15	3.3	3.6	3.6	3.5	3.5	3.5	3.4	3.4	3.4	3.3	3.3	3.2	3.1	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2				
	20	3.7	4.0	4.0	3.9	3.9	3.9	3.8	3.8	3.8	3.7	3.7	3.6	3.5	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6				
	25	4.0	4.4	4.4	4.3	4.3	4.3	4.2	4.2	4.2	4.1	4.1	4.0	3.9	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0				
	30	4.4	4.8	4.8	4.7	4.7	4.7	4.6	4.6	4.6	4.5	4.5	4.4	4.3	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4				
5,000.....	5	4.0	4.1	4.1	4.1	4.0	4.0	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1				
	10	4.3	4.6	4.6	4.5	4.5	4.5	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4				
	15	4.7	5.0	5.0	4.9	4.9	4.9	4.8	4.8	4.8	4.7	4.7	4.7	4.6	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7				
	20	5.0	5.3	5.3	5.2	5.2	5.2	5.1	5.1	5.1	5.0	5.0	5.0	4.9	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0				
	25	5.4	5.8	5.8	5.7	5.7	5.7	5.6	5.6	5.6	5.5	5.5	5.5	5.4	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5				
	30	5.8	6.2	6.2	6.1	6.1	6.1	6.0	6.0	6.0	5.9	5.9	5.9	5.8	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9				
7,500.....	5	7.5	7.6	7.6	7.5	7.5	7.5	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4	7.4				
	10	7.9	8.2	8.2	8.1	8.1	8.1	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0				
	15	8.3	8.6	8.6	8.5	8.5	8.5	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4				
	20	8.6	9.0	9.0	8.9	8.9	8.9	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8			
	25	9.0	9.3	9.3	9.2	9.2	9.2	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1			
	30	9.4	9.7	9.7	9.6	9.6	9.6	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5			
10,000.....	5	10.0	10.1	10.1	10.1	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0			
	10	10.3	10.4	10.4	10.4	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3			
	15	10.7	10.8	10.8	10.8	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7	10.7			
	20	11.0	11.1	11.1	11.1	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0			
	25	11.4	11.5	11.5	11.5	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4	11.4			
	30	11.8	11.9	11.9	11.9	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8			
New solid tires	Speed (m. p. h.)	Road H	Road D	Road A	Road C	Road E	Road H	Road D	Road A	Road C	Road E	Road H	Road D	Road A	Road C	Road E	Road H	Road D	Road A	Road C	Road E	Road H	Road D	Road A	Road C	Road E		
		F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	F_d	F_o	
2,500.....	5	2.6	2.1	2.7	2.8	2.7	2.7	2.6	2.6	2.4	2.7	2.7	2.9	2.6	2.7	2.9	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1		
	10	3.2	4.3	3.9	4.0	3.9	3.9	3.3	3.3	3.0	3.3	3.3	4.2	3.7	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8		
	15	4.0	6.2	5.1	5.8	5.1	5.8	5.0	5.0	4.5	5.0	5.0	6.1	4.8	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0		
	20	4.8	6.8	6.1	6.8	6.1	6.8	6.0	6.0	5.5	6.0	6.0	7.1	5.8	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0		
	25	5.6	8.3	6.9	10.6	6.9	11.1	13.0	13.7	12.8	13.6	13.6	14.6	12.1	13.1	13.1	13.1	13.1	13.1	13.1	13.1	13.1	13.1	13.1	13.1	13.1	13.1	
	30	6.2	8.0	
5,000.....	5	6.1	5.3	5.3	5.3	5.3	5.0	5.3	5.7	6.1	7.2	5.3	7.3	6.5	6.1	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2		
	10	6.7	7.3	6.4	6.4	6.4	6.1	7.8	7.5	12.7	15.8	7.0	11.8	8.8	8.1	7.3	12.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3			
	15	6.7	8.9	7.6	8.4	7.6	8.1	10.6	10.0	10.1	25.4	9.1	16.3	11.9	8.3	12.4	11.0	19.9	19.9	19.9	19.9	19.9	19.9	19.9	19.9			
	20	7.4	11.3	6.1	8.0	6.1	8.9	10.6	10.0	11.5	22.0	7.9	18.5	10.8	12.6	13.4	12.6	21.5	21.5	21.5	21.5	21.5	21.5	21.5	21.5			
	25	8.1	11.6	8.6	10.6	8.6	11.1	13.0	13.7	12.8	24.8	12.1	20.9	12.1	14.6	17.1	18.9	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0			
	30	8.8		
7,500.....	5	7.6	8.0	7.7	7.8	7.7	7.7	7.8	7.8	8.6	10.2	7.7	9.6	8.0	7.9	8.0	8.3	10.1	10.2	10.2	10.2	10.2	10.2	10.2	10.2			
	10	8.3	9.6	9.1	9.3	9.1	9.9	10.5	11.0	16.0	17.8	8.6	13.3	11.5	12.9	10.7	15.3	15.9	15.9	15.9	15.9	15.9	15.9	15.9	15.9			
	15	9.2	11.2	10.4	11.3	10.4	11.1	13.7	13.8	22.0	23																	

AIDS TO MOTOR VEHICLE TRANSPORTATION

TABLE D-2.—Motor vehicle wheel load data ¹

Vehicle gross weight	Wheel load	Ratio to gross weight	Unsprung wheel load	Ratio to wheel load
PASSENGER CARS				
2,375	672.5	0.281	142.5	0.212
3,005	810.0	.270	175.5	.217
3,265	872.5	.267	172.0	.197
3,325	906.0	.273	179.0	.198
3,590	937.5	.261	193.5	.199
3,910	1,025.0	.262	201.5	.197
4,360	1,140.0	.262	244.0	.214
4,765	1,257.5	.263	255.5	.227
4,865	1,350.0	.274	258.0	.217
5,235	1,360.0	.260	307.5	.226
6,435	1,367.5	.207	307.5	.220
6,700	1,467.5	.224	320.0	.218
6,340	1,610.0	.264	345.0	.214
LIGHT TRUCKS AND COMMERCIAL CARS				
3,800	1,125.0	0.296	117.0	0.157
4,100	1,322.5	.323	150.5	.136
7,500	2,800.0	.374	429.0	.153
7,500	2,850.0	.380	429.0	.151
7,000	3,000.0	.390	461.0	.154
7,000	3,050.0	.380	461.0	.151
8,300	3,165.0	.382	484.0	.153
8,300	3,215.0	.388	484.0	.151
TAXICABS				
5,130	1,515.0	0.298	237.5	0.157
TRUCKS (4-WHEEL)				
8,800	2,962.5	0.349	450.0	0.152
10,500	3,775.0	.360	530.0	.140
15,000	4,970.0	.330	762.5	.153
10,000	6,527.5	.396	882.5	.140
10,000	7,247.5	.382	900.0	.124
22,000	8,945.0	.398	1,037.5	.120
22,000	8,975.0	.395	1,100.0	.127
25,000	9,927.5	.397	1,282.5	.129
30,000	10,692.5	.356	1,325.0	.121
34,000	12,262.5	.359	1,362.5	.127
40,000	15,072.5	.377	1,926.0	.126
TRUCKS (6-WHEEL)				
40,000	7,907.5	0.198	1,412.5	0.179
50,000	9,941.5	.199	1,738.5	.174
BUSSES				
9,490	2,300.0	0.243	405.5	0.196
10,850	2,755.0	.254	502.5	.182
10,050	6,157.5	.320	984.0	.158
21,630	6,810.0	.317	1,261.0	.188
21,980	7,146.0	.325	1,122.5	.157
26,050	8,386.0	.322	1,585.0	.189

¹Manufacturers' data. Average ratios of unsprung wheel load to wheel load: Passenger cars=0.213, buses=0.183, 6-wheel trucks=0.177, light trucks (approximately below 3½ tons capacity) and taxicabs=0.150, other trucks=0.125.

TABLE D-3.—Tire data ¹

Size (inches)	Load (pounds)	Inflation (pounds per square inch)	Deflection (inches)	Contact area (square inches)
PASSENGER BALLOON				
4.40-21	760	32	0.73
4.40-21	760	32	18.9
4.50-21	800	32	.78
4.50-21	800	32	15.8
4.75-19	840	32	22.6
4.75-19	855	32	.85
5.25-17	885	32	.92
5.00-16	895	32	23.8
4.50-21	900	36	.76
4.75-19	945	36	.72
5.25-21	1,000	32	26.7
5.50-18	1,000	32	26.2
5.00-16	1,005	36	.79
5.25-18	1,040	36	.79
5.50-17	1,075	36	.81
5.25-21	1,120	36	.80
6.00-21	1,190	32	1.01
6.00-20	1,190	32	31.1
6.00-21	1,190	32	35.5
6.00-18	1,205	3690
6.50-20	1,220	32	1.09
6.00-21	1,230	36	.98
6.50-18	1,275	36	1.00
6.50-20	1,430	36	31.4
6.50-20	1,600	36	1.06
7.00-18	1,625	36	1.05
7.00-21	1,700	36	1.12
7.00-21	1,700	36	34.7
7.50-18	1,710	36	1.02
TRUCK AND BUS HIGH-PRESSURE				
30 x 6	1,700	76	.70	25.3
30 x 6	1,700	76	.71
30 x 6	1,750	76	.76
32 x 6	1,950	78	.78
32 x 6	2,200	80	.81	31.4
32 x 6	2,200	80	.85
36 x 6	2,600	80	.90
34 x 7	2,800	85	.87	35.7
34 x 7	2,800	85	.97
36 x 8	3,000	90	.90	45.1
36 x 8	3,600	90	1.11
38 x 9	4,500	95	1.12	56.8
38 x 9	4,500	95	1.00	55.8
40 x 10	5,500	100	1.50	72.8
40 x 10	5,500	100	1.50
44 x 10	5,500	100	1.11
TRUCK AND BUS BALLOON				
5.50-20	1,225	40	0.80
6.00-20	1,400	40	.87	28.8
6.00-20	1,450	45	.90
6.50-20	1,650	50	1.01
7.00-20	1,900	55	.90
7.00-20	1,900	55	1.04	33.8
7.50-20	2,100	55	.93
7.50-20	2,100	55	1.10	35.9
8.25-20	2,350	60	1.01	46.8
8.25-20	2,550	60	1.20	40.4
9.00-20	3,200	65	1.12	49.2
9.00-20	3,250	65	1.18	47.9
9.75-20	3,300	70	1.16	54.8
10.50-20	4,700	75	1.30	64.9
11.25-20	5,490	80	1.28
12.00-20	6,250	85	1.45
12.75-20	7,200	90	1.40
13.50-20	8,200	95	1.52

¹These tires represent popular models of leading manufacturers, from whom data were secured.

TABLE D-4.—Computed motor vehicle impact forces ¹

Type of vehicle	Static wheel load (pounds)	Solid tires at 10 m. p. h.			High-pressure tires at 45 m. p. h.			Balloon tires at 45 m. p. h.		
		Excellent road surface	Intermediate road surface	Poor road surface	Excellent road surface	Intermediate road surface	Poor road surface	Excellent road surface	Intermediate road surface	Poor road surface
Passenger car	500	813	1,818	1,817
Light truck	1,000	1,100	1,510	2,037	1,500	2,300	3,108	1,367	2,545	2,545
Passenger car	1,000	1,443	2,168	2,995
Do	1,500	2,043	2,918	3,784
Light truck	2,000	2,142	2,788	3,465	2,708	3,551	4,981	2,618	3,356	4,194
Bus	3,000	2,173	2,895	3,789	2,893	4,238	6,037	2,622	3,654	4,694
Light truck	2,000	3,173	3,898	4,798	3,897	5,267	6,531	3,635	4,663	5,674
Truck	4,000	4,167	4,956	6,761	4,534	6,131	7,518	4,958	6,088	6,574
6-wheel truck	4,000	4,236	5,224	6,447	5,181	7,069	8,975	4,853	6,262	7,643
Bus	5,000	5,273	6,415	7,828	6,366	8,572	10,753	6,000	7,615	9,212
Truck	8,000	8,296	9,222	10,443	8,179	11,065	12,989	8,884	10,299	11,639
6-wheel truck	8,000	8,334	9,731	11,460	8,670	12,308	15,036	9,223	11,190	13,153
Bus	8,000	8,346	9,790	11,578	8,726	12,516	15,274	9,265	11,808	13,328
Truck	12,000	12,289	13,407	14,992	13,444	15,778	18,966	13,638	14,767	16,132
6-wheel truck	12,000	12,410	14,120	16,238	13,939	17,073	20,171	13,498	15,918	18,311

¹Derived from equation (2), p. 255.

TABLE D-4.—Computed motor vehicle impact forces—Continued

	10 m. p. h.	15 m. p. h.	20 m. p. h.	25 m. p. h.	30 m. p. h.	35 m. p. h.	40 m. p. h.	45 m. p. h.
Excellent road surface.....	14	22	28	38	37	40	42	45
Intermediate road surface.....	72	94	103	109	113	115	117	118
Poor road surface.....	145	170	182	188	190	191	191	191

¹ See pp. 258.

TABLE D-5.—Tire contact areas under successive load increments
(Figures in parentheses are recommended loads and corresponding static areas)

Wheel loads (pounds)	Contact area (square inches)	Tire type	Source of data
(8,000)	(92)	Balloon.....	Public Roads, November 1932, p. 137.
16,000	162	do.....	
24,000	211	do.....	
(5,000)	(78.85)	do.....	Public Roads, April 1931, p. 27.
12,000	133.33	do.....	
18,000	178.22	do.....	
(8,000)	(48.39)	do.....	Public Roads, April 1931, p. 31.
4,700	85.28	do.....	
6,000	78.92	do.....	
9,000	101.12	do.....	Public Roads, April 1931, p. 31.
9,400	104.44	do.....	
(4,700)	(60.25)	do.....	
9,400	106.82	do.....	Public Roads, April 1931, p. 31.
14,100	141.90	do.....	
(3,000)	(34.88)	High-pressure.....	
4,700	48.96	do.....	Public Roads, April 1931, p. 31.
5,000	58.25	do.....	
9,300	78.37	do.....	
9,400	78.33	do.....	

¹ There is no indication of any consistent difference in the reaction of high-pressure and balloon tires to successive increments in loading, despite the wide range of increments beyond recommended loads.

TABLE D-6.—Truck and bus tire weight data for 1933¹

Wheel load (pounds)	Tire weight (pounds)	Wheel load (pounds)	Tire weight (pounds)	Wheel load (pounds)	Tire weight (pounds)
HIGH-PRESSURE					
1,600	39.2	2,200	62.3	3,600	138.0
1,700	44.1	2,200	68.4	4,000	133.5
1,700	40.7	2,200	119.5	4,500	181.8
1,700	50.8	2,800	100.1	4,500	181.4
1,700	46.1	2,800	85.2	5,000	176.9
1,850	54.0	2,200	88.0	5,000	226.0
2,200	72.5	3,600	125.4	5,500	228.1
2,200	78.6	3,600	139.6	6,000	205.9
BALLOON					
1,400	35.4	2,100	61.6	3,250	127.5
1,400	39.4	2,550	117.6	3,900	150.6
1,400	38.1	2,550	106.7	3,900	122.9
1,900	56.9	2,550	94.1	3,900	124.8
1,900	66.2	2,550	83.5	4,700	150.9
1,900	55.1	3,250	100.4	4,700	148.2
2,100	65.3	3,250	101.3	6,025	198.1
2,100	64.3	3,250	128.8	6,250	197.1

¹ Furnished by manufacturers.

TABLE D-7.—Truck and bus rear wheel weight data¹

Wheel load (pounds)	Wheel weight (pounds)	Wheel load (pounds)	Wheel weight (pounds)
SINGLE WHEELS FOR HIGH-PRESSURE TIRES			
1,700	39.4	2,200	61.1
1,900	43.6	2,200	64.6
1,350	44.9	2,200	71.1
2,200	60.1	3,200	83.2

¹ Furnished by manufacturers.

TABLE D-7.—Truck and bus rear wheel weight data—Continued

Wheel load (pounds)	Wheel weights (pounds)	Wheel load (pounds)	Wheel weight (pounds)
SINGLE WHEELS FOR HIGH-PRESSURE TIRES—Continued			
4,500	91.3	5,500	129.0
5,500	116.0	6,000	148.0
5,500	118.0		
SINGLE WHEELS FOR BALLOON TIRES			
1,650	42.3	3,250	90.2
1,900	48.1	3,900	88.9
1,900	51.1	4,700	110.1
2,100	51.1	4,700	113.6
2,100	55.6	4,700	118.0
2,500	63.9	5,450	117.9
2,550	70.9	5,450	110.9
2,550	71.2	5,450	127.9
DUAL WHEELS FOR HIGH-PRESSURE TIRES			
3,200	86.8	4,400	132.2
3,200	87.8	5,600	131.2
2,400	79.2	5,600	145.2
3,400	88.2	5,600	158.2
3,900	95.0	6,400	205.4
3,900	88.0	7,200	187.6
3,900	103.5	7,200	183.1
4,400	100.2	7,200	204.7
4,400	104.2	8,000	221.9
4,400	127.7		
DUAL WHEELS FOR BALLOON TIRES			
2,450	62.0	5,100	127.6
2,800	61.0	5,100	141.8
2,800	68.0	5,100	137.8
3,200	64.6	5,500	150.4
3,200	67.6	5,500	162.4
3,800	98.2	5,500	204.4
3,800	104.2	7,000	217.4
3,800	126.2	7,800	189.8
4,200	104.2	7,800	203.8
4,200	113.7	8,400	210.0
4,200	141.2		

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APPENDIX E

DISTRIBUTION OF TRAVEL OF GROUPS OF MOTOR VEHICLES, BY ROAD SYSTEMS¹

Until recently, data as to the distribution by highway systems of the travel of all vehicles have been available for only a few States. The highway planning surveys are producing much material along this line, all of which is not available at the present time. However, data for 17 States, largely from this source, indicate the following percentage distribution of total travel on the various highway and street systems in 1936:²

	Percent
Primary rural highways and their transcity connections.....	56.4
Secondary highways and local rural roads.....	13.4
City streets.....	30.2
Total.....	100.0

As indicated, the percentage for primary roads includes mileage traveled on transcity connections. These connections in 1936 aggregated 20,000 miles, or in the neighborhood of 10 percent of all city-street mileage. For the purpose of this report, the costs of such streets, when reported as State highway expenditures and separated from other expenditures, have been transferred to the costs of city streets.³ It is therefore necessary to include the travel on transcity connections with that on other city streets. This adjustment is reflected in the estimated percentages indicated in the following paragraphs.

In estimating the travel characteristics of the several vehicle groups, recourse was had to preliminary underlying road use data for individual States collected in the course of the planning surveys. Relations between percentages of total travel on the different systems, on the one hand, and population density and percentages of urban population, on the other, were established and used in estimating probable national characteristics. The material used in estimating the distribution of total passenger car and total truck traffic by highway systems reflects experience in about a third of all the States; it constitutes, therefore, quite a large sample, although the geographic distribution of such States is not entirely adequate. However, for the other vehicle

classifications (school busses, common carrier busses, etc., all of minor importance), information is either completely lacking or very meager. For the groups of larger trucks the sample is considerably smaller than it is for the entire group, for the reason that in some States the mileage of such vehicles is negligible or such vehicles are completely absent. Therefore, in some cases the distributions are based almost entirely upon estimates, while in others the supporting material is substantial. On the whole, it is believed that the percentages given will be found to be reasonable.

Farm-owned cars and trucks have been segregated and their respective travel characteristics estimated. Substantial data are available indicating such characteristics for vehicles owned by residents of unincorporated areas. However, residents of unincorporated areas are not all engaged in farming,⁴ and vehicles used by those engaged in other pursuits have quite different travel characteristics from those of farm-owned vehicles. The nonfarming group living in unincorporated areas accounts for a substantial part of the total travel of rural residents, and a greater percentage of their total travel is on city streets. In using travel by residents of unincorporated areas as a measure of the use of farm-owned vehicles, it has been assumed that the use of city streets by such vehicles is only one-half as great as that indicated by the travel characteristics of all residents of unincorporated areas, and that the difference is made up by the greater use of secondary and local roads.

The base percentages to be used for each of the several groups of vehicles, without distinction between

¹ Thus, of a total rural population in 1930 of \$3,830,325, the Bureau of the Census classified 23,023,710, or 44 percent, as rural nonfarm population. It is to be noted that "rural," as used by the census, includes both incorporated and unincorporated places of less than 2,500 population. The population of such places amounted to 9,185,453 in 1930. In a survey of 11 Western States made during 1929 and 1930, a classification of trucks in traffic on Federal-aid roads by status of ownership was made, with the following results:

Percentage of trucks from farms.....	26.3
Percentage of trucks from villages (owners living in rural communities of less than 2,500 population).....	37.0
Percentage of trucks from cities (all communities of more than 2,500 population).....	46.7
Total.....	100.0

(Report of a Survey of Traffic on the Federal-aid Highway Systems of 11 Western States, 1930).

Surveys of traffic in other parts of the country indicate a smaller percentage of farm trucks on State highways, the range being from about 11 to 19 percent of the total truck traffic.

⁴ Of the population classified by the census as rural nonfarm, 14,479,257 (23,023,710 minus 9,185,453) live in other than incorporated towns or villages. A substantial part of this population lives in small communities which are similar in all respects to incorporated towns or villages except that they are not incorporated. An unknown part of the 37.0 percent of all trucks set forth above is owned by such residents.

¹ The data herein set out relate for the most part to the year 1936. Comparable data are not available for 1932. It has been necessary, therefore, to assume that no significant changes in the relative distribution of the travel of any given vehicle group on the various highway and street systems have occurred in the intervening period. Significant changes have occurred in the distribution of traffic as between different vehicle types on any given highway or street system. Illustrative of this change is the increasing number of trucks relative to passenger cars. Such changes are discussed later in connection with the actual distributions.

² Robert H. Faddock, and Roe F. Rogers, Preliminary Results of Road Use Studies Public Roads, May 1933.

³ See p. 162, above. Reports of the Bureau of Public Roads have separated these expenditures since 1933. It is understood that prior to 1933 expenditures by State highway departments on city streets were relatively minor.

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for-hire and private truck operations, are given for 1932 in table E-1, below:

TABLE E-1.—Estimated percentage distribution of travel of various vehicle types by road and street systems

Class of motor vehicle	Percent of total travel on—			
	Primary highways	Secondary and local roads	Streets	Total
Passenger cars.....	43	13	44	100.0
Farm.....	44	45	11	100.0
Other.....	43.0	10.8	46.2	100.0
Taxicabs.....	8	2	90	100.0
School busses.....	35	60	5	100.0
Contract busses.....	45	5	50	100.0
Common carrier busses.....	3	3	95	100.0
Trucks—all.....	46.5	14.5	39	100.0
Farm trucks.....	54	37	9	100.0
Other trucks.....				
1½ tons and less.....	{ 36.7	{ 17.0	{ 42.3	100.0
Over 1½ and less than 3.....	{ (35.0)	{ (9.5)	{ (55.5)	100.0
3 and less than 5.....	{ 50.5	{ 3.5	{ 44.0	100.0
5 tons.....	{ 55.0	{ 4.0	{ 41.0	100.0
Over 5 tons.....	{ 62.0	{ 2.5	{ 35.5	100.0
Total (all vehicles).....	{ 43.0	{ 13.1	{ 43.9	100.0

¹ Based upon the estimated travel characteristics of vehicles owned by residents of unincorporated areas and assuming that the relative use of city streets by strictly farm-owned cars is only 50 percent of that for the group as a whole and, further, that such difference is made up for by increased use of secondary and local roads.

² Determined by deduction of travel of farm-owned cars from all passenger car travel.

³ Trucks owned in all incorporated areas. The reflection of travel of trucks owned by nonfarm rural residents has been assumed in a subsequent adjustment to relate solely to the 1½-ton and less group. This assumption is substantially accurate for the reason that the greater part of the use of trucks by residents of unincorporated areas is confined to that group (see p. 127). Also, it is probable that the characteristics of the relatively small travel of nonfarm, rural-owned trucks of greater capacity do not differ greatly from those of urban-owned trucks.

⁴ Distribution determined from estimated total truck travel and the difference therefrom after deductions of all other groups estimated in the manner indicated.

⁵ Trucks owned in incorporated areas only (see footnote above).

⁶ Derived from total after distribution of individual items.

From the foregoing it is seen that the travel characteristics of the various vehicle groups vary widely and that, while there is little difference between those of passenger cars and trucks as a whole, the individual truck groups show marked differences. As would be expected, a substantial part of the total travel of farm-owned vehicles is on secondary and local roads.

As noted, the travel characteristics of one group, the 1½-ton and less truck, have been determined by sub-

tracting the travel of the other truck groups from the total truck travel on each of the systems. Also, the percentage distribution of the total travel of all vehicles on the respective systems has been determined after the allocation of individual items.

The next step is to distribute to the various road systems the mileages estimated at pages 129-131 and set forth in column 4 of table 46.⁵ See table E-2, below. The result is a distribution of 43 percent of all travel in 1932 to State primary roads, 13.1 percent to secondary and local roads, and 43.9 percent to streets, including transcity connections. Published information indicating what the distribution was at an earlier date is very meager. However, considering the many variables present, the percentages indicated appear reasonable when compared with the results of these earlier studies.

As set forth above,⁶ the percentage distribution of the travel of any particular vehicle group between the several systems has been assumed to have been the same in 1932 as it was in 1936. While there may have been slight changes in the use made of various vehicle types, so far as relative use of the several systems is concerned, nothing in the way of concrete evidence is available to indicate what such changes were, except in the case of the 1½-ton-truck group.⁷

⁵ Percentages given in table E-1 times column 4, table 46.

⁶ Footnote 1, p. 263.

⁷ The 1½-ton vehicle, as it is recognized today, was not as important a factor in 1932 as in 1936. In recent years the single-unit truck of this rated capacity has supplanted trucks of much greater rated capacity which were in demand in 1932 and earlier years. Also, the tractor of the 1½-ton type is used in combinations which at times reach quite large rated capacities. It is understood that the road use studies of the highway planning surveys classify combinations of this nature by the rated capacity of the tractor and that therefore the use of highways and streets by such combinations is included with that indicated for the 1½-ton and less group. However, such units are included herein with the heavier groups. (See p. 129.) Therefore, owing to the general use of 1½-ton trucks for loads much greater than their rated capacity, their use for long hauls has increased, with a larger percentage of their total mileage on rural roads. However, since the travel of the 1½-ton and less group on each of the road systems has been determined by the subtraction, as previously noted, it is believed that these changes have been accounted for.

TABLE E-2.—Distribution of estimated travel by vehicle types to road and street systems

Class of motor vehicle	State primary			Secondary and local			Streets			Total		
	Annual mileage (000)	Percentage of total travel for vehicle type	Percentage of total travel on system	Annual mileage (000)	Percentage of total travel for vehicle type	Percentage of total travel on system	Annual mileage (000)	Percentage of total travel for vehicle type	Percentage of total travel on system	Annual mileage (000)	Percentage of total travel for vehicle type	Percentage of total travel on system
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Passenger cars and similar types.....	82,400,003	43	82.07	18,965,117	13	81.18	63,851,166	44	82.26	145,116,286	100.0	82.05
Taxicabs and other for-hire basic types of passenger vehicles.....	212,749	8	.28	53,187	2	.23	2,393,424	90	3.08	2,659,360	100.0	1.80
School busses.....	179,193	35	.23	307,188	6	1.32	25,599	5	.03	511,980	100.0	.29
Contract carrier busses.....	19,980	45	.03	2,190	5	.01	21,200	50	.03	42,400	100.0	.02
Common carrier busses.....	405,166	42	.51	33,238	3	.14	610,456	55	.79	1,109,820	100.0	.63
All trucks.....	12,756,705	46.5	16.73	3,977,807	14.5	17.12	10,696,172	39	13.75	27,433,775	100.0	15.51
Farm trucks.....	1,350,000	64	1.78	825,000	37	3.98	228,000	9	.29	2,403,000	100.0	1.41
Other:												
1½ tons and less.....	5,706,634	39.7	7.51	2,874,969	17.9	11.09	6,075,121	42.5	7.82	14,356,725	100.0	8.12
Over 1½ and less than 3.....	2,800,542	20.5	3.69	305,010	5.5	1.31	2,440,077	44.0	3.14	5,546,629	100.0	3.14
3 tons and less than 5.....	1,728,290	55.0	2.27	128,667	4.0	.54	1,268,294	41.0	1.66	3,144,181	100.0	1.77
5 tons.....	395,493	62.0	.48	14,777	2.5	.06	376,829	35.5	.37	891,988	100.0	.33
Over 5 tons.....	804,866	62.0	1.06	32,454	2.5	.14	460,851	35.5	.59	1,296,171	100.0	.73
Grand total.....	76,053,896	43.0	100.0	23,238,807	13.1	100.0	77,601,017	43.9	100.0	176,878,721	100.0	100.0

¹ This classification reflects that observed in the collection of data as to payments made by the various groups of vehicles and does not necessarily signify the true status of the operations.

² Determined from total truck mileage minus all other groups.

³ Determined from the total of all items.

On the other hand, owing to changes in the makeup of total vehicles registered, some differences should exist, 1932 compared with 1936, in the ratio of the travel of a particular vehicle group to the total travel on any highway or street system. Thus in 1930, for every truck registered there were 6.615 passenger cars; in 1932, 6.468; in 1934, 6.297; and in 1936, 6.014. Necessarily, such changes in numbers will also produce changes in the relative importance of each group in traffic.⁸

The average mileages used in table 46 result in 83.5 percent of the total traffic on all road systems being assigned to passenger cars, including taxicabs and other for-hire cars. Small variations from this figure are found in the mileages as distributed to systems in table E-2, notably 82.3 percent for State primary roads, 81.4 percent for secondary and local roads, and 85.4 percent for streets. According to a number of preliminary planning survey reports, the 1936 percentages would be slightly lower in each instance, reflecting the increasing importance of truck travel to total travel. Truck and bus traffic respectively account for 15.5 and 0.9 percent of the total 1932 traffic. The traffic of trucks, as in the case of passenger cars, varies in importance on the different systems, accounting for 16.8, 17.1, and 13.8 on the State primary, secondary and local and street systems, respectively. Bus traffic, including school and contract busses, accounts for 0.9, 1.7⁹ and 0.8 percent, respectively, for the three systems. While no exact comparison can be made with earlier surveys in various States, owing to differences in the dates at which made and systems covered and also to differences peculiar to the individual States, it would appear that the foregoing percentages fall within the limits previously found.

The travel of farm-owned trucks has been segregated in tables 46 and E-2 and, while no distinction has been

made in the passenger car mileage distributed in tables 46 and E-2, the estimated percentage distribution of the travel of farm passenger cars is presented in table E-1. Segregating farm-owned passenger car traffic, it is found that the 1932 travel of passenger vehicles so owned constitutes 6.2 percent of the passenger car traffic on all roads and streets and 6.4, 21.7, and 1.5 percent of the total on State primary roads, secondary and local roads, and streets, respectively. Farm trucks represent a somewhat higher percentage of total truck traffic, constituting 9.1 percent of truck travel on all systems and 10.6, 23.3, and 2.1 percent of that on the State primary, secondary and local, and street systems, respectively. Of the travel of all vehicles, farm-owned vehicles account for 6.6 percent on all systems and 7.1, 21.7, and 1.6 on State primary roads, secondary and local roads, and streets, respectively. Situs of ownership studies have been made in a number of published surveys and a segregation made of vehicle ownership into three classifications: Farm, village, and city. The first two classifications represent population classified as rural by the Bureau of the Census, i. e., persons living in places of 2,500 population or less. If the farm classification employed in these surveys excludes persons living in rural areas (exclusive of incorporated or unincorporated villages less than 2,500 population) not engaged in farming, then it would appear that the travel of vehicles designated herein as farm vehicles is somewhat understated.¹⁰

In the present analysis trucks accounted for 15.5 percent of the traffic on all systems in 1932, and 16.8, 17.1, and 13.8 percent on the State primary, secondary and local, and street systems. These ratios are all lower than the simple averages of the relations found in available preliminary highway planning survey reports (from individual States) for 1936. This increase between 1932 and 1936 agrees with the trend indicated above.

Truck traffic is made up principally of that of 1½-ton-and-less group, which constitutes 61.4 percent of total

⁸ The relative decrease in passenger car and the complementary increase of truck traffic may be demonstrated by comparison of the results of certain early road surveys with the preliminary results of the highway planning surveys in the same States. The survey made of eleven western States in 1929-30 (Report of a Survey of Traffic on the Federal-aid Highway Systems of Eleven Western States, 1930) indicated for 7 States that passenger cars accounted for 83.4 percent of the total traffic on Federal-aid roads. In 1936 traffic of passenger vehicles registered in the same States accounted for only 78.3 percent of all traffic on the State systems. While in some States little or no change has taken place over a period of years, it is generally true that a small increase in the ratio of truck traffic to all traffic has occurred between 1932 and 1936 (see pp. 161-162).

NOTE.—While the comparison for the 7 States is between Federal-aid roads in 1929-30 and State primary highways in 1936, it is unlikely that any large difference in a distribution of this kind would exist between the traffic on the two. Also, the 1936 data represent in each case the traffic within the State of vehicles registered in the same State. However, in only those cases where foreign traffic is distributed in a manner different from that of local traffic would any difference arise. The Florida traffic survey made in 1933-34 (Public Roads, June 1935) indicated that of all traffic on the State highway system 78.3, 31.0, and 0.6 percent represented passenger cars, trucks, and busses, respectively. Traffic of only those vehicles registered within the State accounted for 80.3 percent of the total traffic on that system and indicated only slight differences in their distribution, being 76.1, 32.8, and 1.0, respectively, for the same 3 items.

⁹ This relatively high percentage for secondary and local roads arises from the inclusion of school busses, with their large travel on such road systems.

¹⁰ See also footnote 4, p. 263.

A summary of travel of farm-owned vehicles, as found in published surveys made by the Bureau of Public Roads in conjunction with State highway bodies in the several States, is given in the following tabulation:

State	Road system covered	Date of survey	Percent of traffic of the vehicle type indicated represented by farm vehicles		
			Passenger cars	Trucks	All vehicles
Pennsylvania.....	Rural roads.....	1923-24	7.1		
Ohio.....	do.....	1925	12.4		15.5
New Hampshire.....	do.....	1926	6.1		11.4
Vermont.....	do.....	1926	10.1		18.9
Eleven western States.....	Federal-aid.....	1929-30	15.7		26.3
Indiana.....	State.....	1933-33			18.4
Do.....	County.....	1933-33			48.7
Connecticut.....	State.....	1933-34		8.1	
Arkansas.....	do.....	1934-35		17.4	

truck traffic on all systems.¹¹ This relationship varies with the different systems, being 55.3, 88.0, and 59.9 percent in the case of State primary, secondary and local, and street systems, respectively. An average of the results of preliminary road-use studies for 1936 in a group of States indicates that about 85 percent of all truck mileage is that of vehicles of 1½ tons or less rated capacity. For the State primary, secondary and local, and street systems these percentages are 80.6, 93.3, and 87.2 percent, respectively. However, as previously stated, it is understood that the mileage of tractors of 1½ tons rated capacity, although these tractors are used in conjunction with semitrailers of much greater capacity, has been included with that of this latter group. On the other hand, preliminary data from loadometer station reports in 10 States show that about 82 percent of all single-unit trucks appearing on primary roads are of 1½-ton capacity or less. If all tractor semitrailer and other combinations which were observed at these stations are included as being greater than 1½ tons, then the 1½-ton-and-less group constitutes approximately 70 percent of all trucks on primary routes in these States. While some part of the difference between the 55.3 percent used herein and the 70-80 percent indicated above for 1936 may arise from the increasing popularity of the 1½-ton truck, comparison with published surveys made at other times would not support this view. Complementary to this seeming understatement of the mileage of the 1½-ton-and-less group is an apparent overstatement of that of the heavier groups, shown by the distribution of total truck travel given in tables E-2 and E-3, as compared with averages found in certain preliminary road-use surveys for 1936.

Group	Percent of total truck travel	
	As used herein	Average of certain preliminary road use surveys
1½ tons and less.....	61.4	85.4
Over 1½ and less than 3.....	20.2	10.0
3 and less than 5.....	11.5	2.7
5 tons.....	2.2	1.0
Over 5 tons.....	4.7	.9
Total.....	100.0	100.0

In the present report, trucks greater than 1½ tons capacity accounted for 38.6 percent of all truck traffic in 1932, while an average of the results of preliminary

¹¹ This percentage includes all farm-owned truck traffic with the 1½-ton-and-less group. There is a slight error in such a procedure, since some farm-owned trucks are of greater capacity. However, substantially all strictly farm trucks would be included in this group. As illustrative of the low average capacity of such trucks, the average gross weight of all farm trucks in Vermont in 1926 was only 4,440 pounds as compared with an average gross weight of city-owned trucks of 7,200 pounds. (The average gross weight of all 1½-ton-and-less units was 5,140 pounds.) In Ohio in 1926 the average net load per farm-owned truck was but 1,530 pounds compared with 2,720 pounds for the average city-owned truck. (The average net cargo for 1½-ton trucks was 700 pounds and for 1-1½-ton trucks, 1,930.) The New Hampshire survey of 1925 indicated a gross weight of farm-owned trucks of 4,200 pounds as against 8,000 pounds for city-owned vehicles. (Average gross weight of all trucks 1½ tons and less, 4,940 pounds.)

road use surveys indicates such use to have been only 14.6 percent of the total in 1936. The explanation set forth above of the difference in the treatment of 1½-ton tractors is, of course, acting here in a complementary capacity. Also, in both instances the displacement of heavier units by the 1½-ton unit as now recognized is an important factor. With increasing use of the 1½-ton truck for long hauls, its importance in total traffic would be increased. In addition, the road use studies are confined, so far as segregation by capacity groups is concerned, to traffic within the State of vehicles owned therein. It would seem, therefore, with the generally much greater average annual mileage of the heavier units and consequently greater cross-boundary travel, that the omission of foreign traffic would tend to overstate the use of the 1½-ton truck. It is not known to what extent the foregoing considerations would diminish the differences indicated. However, in view of the data examined from earlier published surveys and loadometer station observations, it would appear that some overstatement of the relative importance of the heavier truck groups and a corresponding understatement of that of the 1½-ton-and-less group exists in the distributions used herein.

Attention is next turned to the distribution to road and street systems of truck traffic by capacity groups, separated as between single units and combinations and as between private and for-hire operations. Table 46 (column 4) gives the estimated total mileage of all groups, which may be summarized as follows:

Group	Total truck travel (100 miles)	Percent
Single units:		
Private.....	120,528,115	82.3
For-hire.....	4,405,804	17.7
Total.....	24,933,919	100.0
Combinations:		
Private.....	1,914,643	78.6
For-hire.....	585,213	23.4
Total.....	2,499,856	100.0
Total, private.....	22,442,758	81.8
Total, for-hire.....	4,991,017	18.2
Grand total.....	27,433,775	100.0

¹ Includes farm trucks. Although a considerable number of trailers are used in conjunction with farm trucks, they are as a rule quite small and for purposes of this comparison have been included as single units (see also pp. 128-29).

Private operations predominate in both the single unit and combination classifications, accounting for more than three-fourths of the total travel in each instance.¹² Data are not available which would indicate the reasonableness of this figure for traffic on all road and street systems. However, highway planning survey data obtained at loadometer stations located on State primary roads are given presently. Hereinafter, total truck travel is distributed 81.8 percent to private and 18.2 percent to for-hire operations. Studies of the

¹² This statement, in terms of trucks, does not indicate the relative tonnage carried in private and for-hire operations. Also, the classification made may not in all respects conform to the legal status of the operations.

distribution of all truck traffic, intercity and local, between private and for-hire operations are practically nonexistent. However, previous studies, largely confined to rural roads, are in reasonable agreement with the distribution found above insofar as the data for comparable road systems are concerned.

Also, little information is available that indicates the differing relative uses of the various road and street systems by private and for-hire trucks of various capacities. Data from road use studies for 1936 in several States, not particularly representative, indicate that the relative use of secondary and local roads, and streets, by for-hire trucks is about one-half that of private trucks and that their use of primary roads is greater than that of private trucks. This relationship does not hold true of the for-hire 1½-ton and less group, particularly for 1932, owing to the considerable use of this type of truck for local hauling on streets. Therefore, for the purposes of the allocations which follow it has been estimated that the mileage of the 1½-ton for-hire group is distributed 35 percent on primary roads, 9 percent on secondary and local roads, and 56 percent on streets. Substantial differences would be expected in the distributions to road and street systems of the travel of single units, on the one hand, and combinations on the other. Under ordinary circumstances, there would be little economic justification for the use of large for-hire combinations in general hauling on city streets. Likewise, except in the case of highly integrated operations,¹⁸ little need would exist for the use of combinations for general city work in private operations. It has therefore been assumed that a much smaller proportion of the mileage of combinations is on streets than that of single units, and further that the relative use of streets by for-hire combinations is less than that of private combinations. For-hire combinations of less than 5 tons have been assumed to travel 83 percent of their total mileage on primary roads, 1 percent on secondary and local roads, and 16 percent on streets. The mileage of for-hire combinations greater than 5 tons has been assumed to be divided 85 percent, 1 percent, and 14 percent, respectively. These percentages are based largely upon the estimated distribution of the travel of the heavier single units found in the following paragraphs. The travel of private combinations has been estimated upon the basis of a differential applied to all truck travel in the various capacity groups, as explained below.

Tables E-1 and E-2 indicate generally increasing use of primary roads and decreasing use of secondary and local roads and streets as vehicle capacity increases. It was pointed out above that use of secondary and local roads, and streets is substantially less for for-hire

trucks than for private trucks. The following differentials have been applied to the estimated distribution of all truck travel on these two systems in an attempt to approximate for-hire travel thereon:

Relative travel on secondary and local roads, and streets traveled by all trucks, estimated to be traveled by for-hire trucks	
Capacity group:	Percent
Over 1½ tons and less than 3	50
3 tons and less than 5	45
5 tons and over	40

Thus, for the entire group of over 1½ tons and less than 3, the travel on secondary and local roads, and streets is estimated to be 5.5 and 44.0 percent (table E-2), respectively. Applying the differential indicated for this group to the estimated travel of all trucks, the travel of for-hire trucks of this capacity becomes 2.7 and 22 percent, respectively. The travel on the primary road system of all trucks of this capacity is estimated to be 50.5 percent. Increasing this amount by 2.7 plus 22 (the 50 percent deducted from the other two systems), the estimated travel of all for-hire trucks of this capacity on the primary road system becomes 75.3 percent.

The travel of the other for-hire groups has been determined in a similar manner. Using the total travel of each for-hire group from column 4 of table 46 and the distribution of each group's travel estimated as above, the distribution of the travel of private trucks may be determined by subtraction. Thus, for the group of over 1½ and less than 3 tons capacity the total travel is distributed to the various street systems as given in table E-2. Table E-3 gives this distribution designated as "grand total." The total travel for the same capacity for-hire group is taken from col. 4 of table 46, page 131 (shown also in column 8 of table E-3), and distributed to the several systems by use of the percentages determined above; i. e., 75, 3 and 22 percent to the primary, secondary and local, and street systems, respectively. Deduction of the amounts so determined from the "grand total" for the group, gives the distribution of the total private truck travel indicated in columns 2, 4 and 6 of table E-3. Travel of for-hire combinations is distributed on the basis previously indicated and the travel of single for-hire units determined by subtraction. The remaining step is to allocate to systems the travel of private single units and combinations. It was pointed out above that under certain conditions substantial use of streets is made by private combinations. However, it is believed that the greater part of the total mileage of such combinations is performed on primary roads. It is believed that as rated capacity increases, private combinations more nearly approach the travel characteristics of for-hire combinations. However, their relative use of city streets will always be greater than that of for-hire combinations. Using the estimated travel of all vehicles, it has been assumed that private com-

¹⁸ Such as chain stores delivering large quantities of merchandise from central warehouses to individual stores, etc. In such cases, while one unit/trailer is being used for delivery purposes, another is being loaded at the central distributing point, thereby assuring almost continuous operation of the power unit. Similar instances are found in the delivery of goods by large department stores, industrial plants, etc.

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binations travel on secondary and local roads, and streets only 75 percent as much as all vehicles in the group of over 1½ and less than 3 tons capacity. For the other three groups, 3 tons and less than 5, 5 tons, and over 5 tons, slightly smaller percentages, or 72.5, 70,

and 70 percent, respectively, have been used. The distribution of the travel of private single units has been computed by subtraction, as in the case of for-hire single units. These computations are all given in table E-3 for the various capacity groups.

TABLE E-3.—Distribution of truck travel to road systems, segregated as to that of single units and combinations, and private and for-hire operations, 1932¹

Vehicle type (1)	Primary roads (000 miles) (2)	Percent of total travel of vehicle type (3)	Secondary and local roads (000 miles) (4)	Percent of total travel of vehicle type (5)	Streets (000 miles) (6)	Percent of total travel of vehicle type (7)	All roads and streets ² (000 miles) (8)	Percent of total travel of vehicle type (9)
1½ and less:								
Private.....	4,808,837	40.8	2,342,744	19.9	4,630,186	39.3	11,776,448	100.0
For-hire.....	808,097	35.0	222,228	9.0	1,444,886	56.0	2,580,277	100.0
Total.....	5,706,834	39.7	2,574,969	17.9	6,075,121	42.3	14,356,725	100.0
Over 1½ and less than 3:								
Private, single.....	1,127,035	38.2	219,096	7.0	1,793,082	56.7	3,109,163	100.0
Private, combinations.....	807,675	63.0	51,281	4.0	423,068	33.0	1,282,024	100.0
Total.....	1,934,710	44.1	270,377	6.2	2,186,100	49.8	4,391,187	100.0
For-hire, single.....	708,084	73.4	32,765	3.4	222,830	23.2	968,649	100.0
For-hire, combinations.....	158,868	63.0	1,378	1.0	30,347	16.0	187,733	100.0
Total.....	866,952	76.0	34,143	3.0	253,177	22.0	1,184,442	100.0
Grand total.....	2,800,542	50.5	305,010	5.5	2,440,077	44.0	5,545,629	100.0
3 and less than 5:								
Private, single.....	769,217	40.8	101,404	5.4	1,014,102	53.8	1,884,723	100.0
Private, combinations.....	242,141	67.0	10,842	3.0	108,422	20.0	361,405	100.0
Total.....	1,011,358	45.0	112,246	5.0	1,122,524	50.0	2,246,128	100.0
For-hire, single.....	545,004	79.1	11,371	1.7	132,645	19.2	689,019	100.0
For-hire, combinations.....	171,858	83.0	2,070	1.0	33,126	18.0	207,054	100.0
Total.....	716,862	80.0	13,441	1.5	165,770	18.5	896,053	100.0
Grand total.....	1,728,200	55.0	125,687	4.0	1,288,294	41.0	3,142,181	100.0
5 tons:								
Private, single.....	191,672	51.6	11,564	3.1	167,882	45.2	371,118	100.0
Private, combinations.....	74,022	73.0	5,028	2.0	25,340	25.0	101,400	100.0
Total.....	265,694	56.2	13,592	2.9	193,222	40.9	472,518	100.0
For-hire, single.....	42,106	85.0	405	1.0	5,935	14.0	49,337	100.0
For-hire, combinations.....	58,662	85.0	690	1.0	9,662	14.0	69,014	100.0
Total.....	100,768	85.0	1,185	1.0	15,597	14.0	118,651	100.0
Grand total.....	366,463	62.0	14,777	2.5	208,819	35.5	591,069	100.0
Over 5 tons:								
Private, single.....	476,492	53.6	26,641	3.0	394,660	43.4	898,863	100.0
Private, combinations.....	123,964	73.0	3,396	2.0	42,454	25.0	169,814	100.0
Total.....	600,456	56.7	30,037	2.9	427,014	40.4	1,068,677	100.0
For-hire, single.....	102,274	85.0	1,203	1.0	19,546	14.0	120,222	100.0
For-hire, combinations.....	103,166	85.0	1,214	1.0	16,992	14.0	121,372	100.0
Total.....	205,440	85.0	2,417	1.0	36,538	14.0	241,694	100.0
Grand total.....	806,866	62.0	52,454	2.5	463,552	35.5	1,289,171	100.0
All trucks:								
Total, private.....	9,964,726	44.4	3,693,096	16.5	8,794,036	39.1	22,442,738	100.0
Total, for-hire.....	2,791,979	55.9	293,901	5.7	1,915,136	38.4	4,991,017	100.0
Total.....	12,756,705	48.5	3,977,897	14.5	10,699,172	39.0	27,433,755	100.0

¹ Totals in column 9 are from column 4 of table 46.
² Distribution of total travel of group from table E-2.

³ Includes farm trucks not included in the individual items listed.

The distribution of all truck traffic results in varying percentages of private and for-hire operations on each of the systems, as follows:

System	Percent of total truck travel		
	Private	For-hire	Total
Primary.....	78.1	21.9	100.0
Secondary and local.....	82.9	7.1	100.0
Streets.....	82.1	17.9	100.0
Total.....	81.8	18.2	100.0

Relatively, for-hire operations are greatest on the primary system and least on the secondary and local system. On the basis of available data it would appear that the estimated distribution on primary roads is reasonable. This distribution also agrees closely with averages of preliminary loadometer station data for 1936 for 15 States, which show 78.3 percent private and 21.7 percent for-hire operations. Data are not available which would indicate the reasonableness of the distributions on the secondary and local road and street systems.

The distribution of for-hire and private operations as between single units and combinations found herein is as follows for primary roads:

Type	Percent of total truck travel on primary roads		
	Private	For-hire	Total
Single-unit trucks.....	79.1	20.9	100.0
Combinations.....	71.8	28.2	100.0
Total.....	78.1	21.9	100.0

The loadometer station data referred to above check the distribution of single-unit trucks and the total closely. On the other hand, there is a marked difference in the distribution of combinations, loadometer station data indicating that 38.8 percent of the total combinations observed were private. Two explanations of this seeming difference are possible, the first that such combinations have increased to a greater extent in recent years with a more rapid increase in the for-hire field than in the private field, and the second that in many

instances private use of combinations is a part-time use not compensated for in the data used herein.

The distribution of all truck traffic between road and street systems, subdivided between private and for-hire operations, as computed in table E-3, gives the following:

System	Private ¹	For-hire	Total
Primary.....	44.4	55.6	100.0
Secondary and local.....	16.5	83.5	100.0
Streets.....	88.1	11.9	100.0
Total.....	100.0	100.0	100.0

¹ Including farm trucks.

For-hire carriers show a greater part of their total mileage on primary roads and a smaller part on secondary and local roads than private trucks. Little difference is found in their relative use of streets. The large use of city streets by for-hire trucks results from the assumption that the 1½-ton and less group travels the bulk of its mileage on streets.

The allocation of the travel of contract and common carrier busses by seating capacity is based entirely on estimate. Nothing in the way of published information or available unpublished information would indicate the travel of the various capacity types. It seems reasonable, so far as common carrier busses are concerned, that the smaller the unit the greater its use in cases where space demands are small, such as intercommunity service between small places or between cities and outlying suburban developments. Of course, these small units are sometimes used in shuttle service solely within cities or as the principal means of mass transportation in small places. However, it is believed that, with increasing capacity, a greater part of the total mileage is performed on city streets, owing to the large mileage traveled by city mass transportation units. Such an adjustment is reflected in the percentages used in table E-3. Contract carrier busses are assumed to travel 50 percent of their total mileage on rural roads.

The estimated distribution of the travel of all vehicle types, from the foregoing analysis, is given in table 46, page 131.

APPENDIX F

DETAILS OF ALLOCATION OF COSTS TO GROUPS OF MOTOR VEHICLES

Tables F-1 to F-6, herewith, set out in detail the steps taken in allocating the several cost elements to groups of motor vehicles. The costs per vehicle so derived are summarized in table 50, page 139. An explanation will enable the reader to follow the processes involved.

Table F-1 (columns 3-7) develops the index of costs of rigid pavements assignable to the several vehicle groups. Columns 3 and 4 give the interior and edge thicknesses required by each group. The former is derived from the Westergaard equations and the latter on the basis of the ratio, approximately 1.4, of edge to interior thickness. Column 5 gives in square inches the cross section area of the pavement slab (WT). Column 6, by use of the formula developed at pages 89-91, converts the width and thickness requirements into a cost per mile of road. This cost per mile, converted to 100 at the maximum requirement, gives the index presented in column 7.

This index is reduced in table F-2 to increments of cost required by the several vehicle groups (column 3) and applied (columns 4-7) to the assignable annual costs arising from rigid pavement on each of the road and street systems. The total so allocated is \$138,271,000, as set out at p. 139. The increments of cost of rigid pavement on primary roads so derived are then allocated to vehicle groups according to principles adopted for use in the present report; i. e., all vehicles share in the costs for which they are directly responsible and all costs of groups with lesser requirements. The requirements of passenger cars, taxicabs, and the groups of smallest common and contract carrier busses are the same and represent the least requirement of any of the groups shown. Therefore, all vehicles which use primary roads share in their costs and they share in none of the costs of other groups. Thus, the cost increment of \$69,194,535 (column 4) is reduced to a vehicle-mile cost of 0.09100 cent through use of the entire traffic on primary roads (76,033,895,000 vehicle-miles, columns 8 and 11). On the other hand, the group with the greatest requirements¹ shares in the costs assignable to every other vehicle group, while no other vehicles share any of the costs assigned to it. Thus, the costs assignable to this group, \$226,868, are reduced to a vehicle-mile charge of 0.06191 cent (column 12) by dividing by 366,462,000 vehicle-miles (columns 8 and 11). However, this charge represents only that arising

from this group's special requirements; to arrive at the total cost per vehicle-mile it is necessary to add the charges of every other group with which it has shared. These amounts, severally shown in column 12, range on a cumulative basis (column 13) from the above-mentioned 0.09100 cent per vehicle-mile for the basic vehicle group to 1.15509 cents for the group with the greatest requirements. The costs given in column 13 are applied to the travel of the individual vehicle types (column 8), to produce the total costs chargeable against the group given in column 14. The costs are then reduced to the per-vehicle charge given in column 15 on the basis of the number of vehicles shown in column 2 of table F-1.

The same process is observed in the case of secondary and local roads and city streets.

The total cost per vehicle arising from rigid pavements on all roads and streets is given in column 26 of table F-2. These amounts range from \$2.30 in the case of the farm truck to \$354.67 in the case of the for-hire combination of over 5 tons. Generally, the greater part of the total cost arises from primary rural roads. However, in those instances in which the major part of the travel of any vehicle type is performed on city streets, the costs arising from such streets is a substantial part of the total. Thus, of a total charge of \$7.10 in the case of taxicabs and other for-hire passenger car types, \$5.56, or 78 percent, represents city street costs. This charge is almost 6 times the charge for passenger cars on city streets. The explanation lies in the fact, first, that the average annual mileage of taxicabs is about 2.9 times that of passenger cars and, second, that only 44 percent of the total mileage of the latter, as compared with 90 percent of the mileage of the former, is on city streets. The same condition holds true of busses and other vehicle types, a relatively large part of the use of which is on city streets. At the other extreme are those vehicle types which perform most of their mileage on primary roads. In the case of the heaviest for-hire combination, 91 percent of the total costs of rigid pavement arises from use of the primary road systems.

Table F-1 (columns 8 to 10) develops the method used in allocating the costs of flexible pavement and grading.² Impact pressure intensities, expressed in pounds per

¹ Private and for-hire trucks and combinations of 5 tons rated capacity.

² This discussion refers to the unmodified method of allocating the costs of these elements. In the final analysis, a modified method was adopted. See pp. 120-122.

square inch of tire contact area, (column 8), are multiplied by the width of pavement required by each vehicle group (column 11) and the resultant amounts (column 9) converted into an index of required cost with the maximum requirement as 100 (column 10).

In Table F-3 this index of cost is converted into increments of cost required by each vehicle group and applied to the costs of flexible pavements and grading by systems (columns 2-7). These costs aggregate \$227,704,000. The detailed steps in the assignment of these increments to vehicle groups are similar to those described under rigid pavements. It is to be noted, however, that the relative cost requirements of the several vehicle groups are considerably different from those found in the case of rigid pavements. Thus, the farm truck is here found to have the highest cost requirement, while for rigid pavement costs the requirements generally increased with weight (table F-1).

The total costs per vehicle (column 26) range from \$5.41 in the case of passenger cars to \$112.05 in the case of the for-hire combinations of over 1½ and less than 3 tons capacity. The latter vehicle type has the highest costs, owing to the high-pressure intensities of the wheel loads and tire type assigned and its relatively high mileage. As in the case of rigid pavement costs, the greater part of the costs assigned to taxicabs arises from the use of city streets. The charge against taxicabs for city streets is higher than that found for the heaviest common carrier bus. It must, however, be noted that the charge to any vehicle group is an average charge and that this group of busses covers both over-the-road and strictly city types. If the latter were segregated, the costs allocated for use of city streets, both relative to total assigned costs and in actual magnitude, would be greatly increased. The charge against the school bus for use of secondary and local roads represents approximately two-thirds of the total costs allocated in this table to this type of vehicle. This result is explained by the relatively great use made of this type of road and by the high pressure intensity of the assumed tire type.

Table F-4 sets out the allocation of the costs of structures. Increments of the cost of this cost element have been assigned to the several vehicle groups on the basis of width requirements alone (see column 11 of table F-1). The assignment of the aggregate cost, \$78,930,000, has been accomplished by the same means as those used for the two preceding elements of cost. The total costs per vehicle given in column 26 range from \$2.97 in the case of the passenger car to \$23.12 in the case of the heaviest for-hire combination. Since there are only two requirement groups (those with and those without additional width requirements), the costs vary generally as the mileages of the different vehicle types on the individual road or street system.

The total costs per vehicle arising from all capital cost elements are given in summary form in table 50, at page 139. The costs there given range from \$12.24 in the case of the passenger car to \$454.47 in the case of the heaviest for-hire combination. Owing to the large per-vehicle costs that are represented by the costs of rigid pavements on primary rural roads, the general trend of costs is to increase with weight and mileage. That is, the larger the vehicle the greater the annual mileage and the greater the relative use of primary rural roads (see table 46 and pp. 263-270.)

Table F-5 presents the three indices for the allocation of maintenance costs. The first of these indices (columns 3-5) is vehicle-miles. It is used in the allocation of those maintenance costs which are not associated with vehicle characteristics. Such costs constitute 71.6 percent of the total. The resulting per-vehicle costs are presented in columns 3, 5 and 7 of table F-6 for the respective road systems. Since the percent of total traffic on any given system arising from a particular group is the basis of such allocation, the per-vehicle costs there given vary directly with use.

Slightly less than one-fourth (21.3 percent) of the total maintenance costs are allocated on the basis of responsibility for wear and tear arising from vertical loadings. The index for the allocation of these costs is derived from a combination of pressure intensities and total vehicle-miles (column 8 of table F-5). The resulting per-vehicle costs are given in column 9 of table F-6. Use of pressure intensities and vehicle-miles results in a gently increasing charge against the heavier and more traveled vehicles. The costs range from \$1.39 in the case of the farm truck to \$14.96 in the case of the heaviest for-hire vehicle.

The remaining 7.1 percent of the maintenance costs are allocated on the basis of responsibility for costs arising from horizontal wear and tear. The index for this allocation is developed in columns 9-15 of table F-5 and represents a combination of tractive pull, diameter of tire contact area, and traffic on the secondary and local road system. The last of these factors is used for the reason that this type of maintenance cost is largely confined to the lower type of road and street. The first two factors, tractive pull and the diameter of the tire contact area, increase with increasing weight and size of vehicle. However, since the use of secondary and local roads generally declines with increase in the weight and size of vehicle, the net result is a cost per vehicle which, with a few exceptions, does not vary greatly in amount, one vehicle type compared with another. The principal exception is the school bus, which, owing to its large use of secondary and local roads, is assigned the highest per-vehicle charge (\$11.18).

Total per-vehicle costs arising from maintenance are given in column 12 of table F-6. They range from

\$6.84 for the farm truck to \$63.69 for the heaviest for-hire combination. Vehicle mileage is the most important element in the allocation of these costs. These two vehicle groups have the smallest and the largest annual mileage, respectively.

All per-vehicle costs, both capital and maintenance, are summarized in table 50 at page 139. The range is from \$24.28 in the case of the passenger car to \$518.16 in the case of the heaviest for-hire vehicle. The net result of the allocation of all cost elements is a charge which generally increases with vehicle size and weight. The heavy charges arising from the costs of rigid pavements, which increase sharply with weight and mileage traveled, bulk large in the final result.

The reader's attention is called to the explanation of the modified method of allocating the costs of flexible pavement and grading used in the final stages of the report and of the results thus obtained. This explanation is found at pages 150-152.

Also, attention may be called to the treatment of the "basic vehicle" concept in the chapters to which this

appendix relates. The discussion in the early chapters of section B employs a meaning that has been given this term by others, which generally includes the light truck groups as well as passenger cars. Such meaning served the purpose of those chapters. In the later chapters the term is used in a somewhat flexible sense, depending on the cost element to be allocated. In most cases it signifies passenger cars (including in some cases taxicabs and small busses of passenger-car design). Practical considerations dictated this difference of treatment. The computations required in producing the results given in chaps. XII and XIII have been exceedingly complex and would have been considerably more complex if further refinement had been undertaken. A test check has shown that the differences in the final allocations to groups of vehicles would not have been more than a few percentage points in the more important instances and that they would generally have been much less.

TABLE F-1.—Development of indexes for allocation of construction costs chargeable to motor-vehicle users, 1932

Class of motor vehicle (1)	Number registered 1932 ¹ (2)	Index of relative cost of requirements in—								
		Rigid pavement					Flexible pavement			Structures
		Required concrete slab thickness, interior ² (inches) (3)	Required concrete slab thickness, edge ² (inches) (4)	Required width times average thickness ² (5)	\$6,885 plus \$11.04 WT (dollars) (6)	Index of cost of required width and thickness (maximum = 100) (7)	Pressure intensity under impact (pounds per inch) (8)	Pressure intensity multiplied by required width (9)	Index of required cost (maximum = 100) (10)	Index of required pavement width (11)
Passenger cars.....	20,730,898	4.00	5.66	1,000.8	17,384	70.5462	47.3	0.66.0	60.0685	95.2381
Taxicabs and other for-hire cars.....	124,908	4.00	5.66	1,000.8	17,384	70.5462	68.0	1,163.0	78.5538	95.2381
School busses.....	61,198	4.15	5.97	1,068.2	18,896	74.8660	73.7	1,547.7	68.8666	100.0000
Contract busses (seats):										
7 and less.....	2,790	4.00	5.66	1,000.8	17,384	70.5462	61.9	1,238.0	78.6039	95.2381
8 to 20.....	716	4.43	6.26	1,159.2	19,138	77.6439	49.0	1,028.0	65.3333	100.0000
Over 20.....	734	5.15	7.26	1,848.2	21,219	86.1091	50.3	1,056.3	67.0687	100.0000
Common-carrier busses (seats):										
7 and less.....	3,810	4.00	5.66	1,000.8	17,384	70.5462	61.9	1,238.0	78.6039	95.2381
8 to 20.....	6,796	4.43	6.26	1,159.2	19,138	77.6439	49.0	1,028.0	65.3333	100.0000
Over 20.....	28,878	5.15	7.26	1,848.2	21,219	86.1091	50.3	1,056.3	67.0687	100.0000
Farm trucks.....	1,000,900	4.41	6.24	1,164.2	19,077	77.4166	78.0	1,676.0	100.0000	100.0000
Private trucks (nonfarm) (tons):										
1½ and less.....	1,384,850	4.55	6.45	1,194.5	19,522	79.2222	73.2	1,837.2	97.6000	100.0000
Over 1½ and less than 3, single vehicles.....	262,216	4.18	7.33	1,365.8	21,308	86.4500	61.9	1,296.9	82.5333	100.0000
Over 1½ and less than 3, combinations.....	75,229	4.56	6.45	1,194.5	19,522	79.2222	73.2	1,837.2	97.6000	100.0000
3 and less than 6, single vehicles.....	105,640	4.79	8.10	1,617.0	23,083	93.6734	57.6	1,218.8	77.0687	100.0000
3 and less than 6, combinations.....	19,446	4.18	7.33	1,365.8	21,308	86.4500	61.9	1,296.9	82.5333	100.0000
6 and less than 7, single vehicles.....	28,651	4.33	8.05	1,653.2	24,642	100.0000	65.3	1,222.2	77.6000	100.0000
6 and less than 7, combinations.....	5,066	4.33	8.05	1,653.2	24,642	100.0000	65.3	1,222.2	77.6000	100.0000
Over 6 and single vehicles.....	89,840	6.31	8.92	1,653.1	24,585	99.7887	68.0	1,218.0	77.3333	100.0000
Over 6 and combinations.....	6,903	6.31	8.92	1,653.1	24,585	99.7887	68.0	1,218.0	77.3333	100.0000
For-hire trucks (tons):										
1½ and less.....	140,103	4.55	6.45	1,194.5	19,522	79.2222	73.2	1,837.2	97.6000	100.0000
Over 1½ and less than 3, single vehicles.....	45,433	4.18	7.33	1,365.8	21,308	86.4500	61.9	1,296.9	82.5333	100.0000
Over 1½ and less than 3, combinations.....	7,801	4.56	6.45	1,194.5	19,522	79.2222	73.2	1,837.2	97.6000	100.0000
3 and less than 6, single vehicles.....	27,308	4.79	8.10	1,617.0	23,083	93.6734	57.6	1,218.8	77.0687	100.0000
3 and less than 6, combinations.....	7,876	4.18	7.33	1,365.8	21,308	86.4500	61.9	1,296.9	82.5333	100.0000
6 and single vehicles.....	1,760	4.33	8.05	1,653.2	24,642	100.0000	65.3	1,222.2	77.6000	100.0000
6 and combinations.....	3,452	4.33	8.05	1,653.2	24,642	100.0000	65.3	1,222.2	77.6000	100.0000
Over 6 and single vehicles.....	3,776	6.31	8.92	1,653.1	24,585	99.7887	68.0	1,218.0	77.3333	100.0000
Over 6 and combinations.....	3,458	6.31	8.92	1,653.1	24,585	99.7887	68.0	1,218.0	77.3333	100.0000
Total.....	34,130,000									

¹ See pp. 126-131.

² Slab thickness taken according to Westergaard equations, with $Z=3,000,000$ lbs. per in.², $e=0.15$, $f=0.2$, $Z=0.2$, $k=100$ lbs. per in.², and $S=300$ lbs. per in.²

³ Edge thickness assumed to diminish to interior thickness at 2 feet from edge.

⁴ Passenger cars, taxicabs, and busses of 7 seats or 7 seats or less assumed to require 20-foot pavement width; all other vehicles assumed to require 21 feet.

AIDS TO MOTOR VEHICLE TRANSPORTATION

TABLE F-2.—Allocation of cost of rigid pavement to classes of motor vehicles, 1932¹

Class of motor vehicle	Index of required cost (maximum=100) (from column 7 of table F-1)	Increments in index (percent) (from column 2)	Increments of cost assignable to vehicle types by systems ²				Estimated annual mileage on ³		
			State primary highways (3) x \$98,084,000	Secondary and local roads (3) x \$4,187,000	City streets (3) x \$34,000,000	Total \$138,271,000 (4)+(5)+(6)	Primary (000)	Secondary and local (000)	City streets (000)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Passenger cars.....	70.5482	70.5482	\$60,194,535	\$4,284,693	\$22,985,708	\$97,564,936	62,400,003	18,865,117	63,851,166
Taxicabs.....							212,749	33,187	2,393,424
Contract buses, 7-passenger or less.....							12,556	1,396	13,950
Common carrier buses, 7-passenger or less.....							56,902	6,323	7,025
School buses.....	74.3660	3.8228	3,740,555	286,517	1,299,752	5,286,824	170,193	307,188	25,599
Farm trucks.....	77.4168	3.9478	2,959,208	188,555	1,036,184	4,213,947	1,850,000	925,000	225,000
Contract buses, 8 to 20 passengers.....	77.6439	2.273	222,945	14,053	77,282	814,290	3,222	358	3,580
Common carrier buses, 8 to 20 passengers.....							100,930	11,217	91,723
Private trucks (nonfarm), 1½ tons and less.....	79.2225	1.5786	1,548,884	97,668	536,734	2,182,746	4,803,537	2,242,744	4,030,166
Private combinations, over 1½ and less than 3 tons.....									
For-hire trucks, 1½ tons and less.....							807,875	61,281	422,068
For-hire combinations, over 1½ and less than 3 tons.....							903,097	232,225	1,444,955
Contract buses, over 20 passengers.....	86.1061	6.8896	6,764,653	426,074	2,341,444	9,522,171	165,898	1,878	30,047
Common carrier buses, over 20 passengers.....							3,303	18,867	3,870
Private trucks, over 1½ and less than 3 tons.....	86.4500	3.4609	334,368	21,001	116,005	471,366	808,214	15,738	511,636
Private combinations, 3 and less than 5 tons.....							1,127,056	219,008	1,763,032
For-hire trucks, over 1½ and less than 3 tons.....							242,141	10,842	108,422
For-hire combinations, 3 and less than 5 tons.....							709,964	22,756	222,930
Private trucks, 3 and less than 5 tons.....	93.6734	7.2234	7,084,000	448,013	2,485,956	9,987,868	171,538	3,398	33,135
For-hire trucks, 3 and less than 5 tons.....							769,217	101,404	1,014,102
Private trucks, over 5 tons.....	96.7697	6.0653	4,978,514	377,116	2,073,402	8,428,032	645,904	11,371	132,645
For-hire trucks, over 5 tons.....							476,463	23,941	384,560
Private combinations, over 5 tons.....							129,904	3,968	42,474
For-hire trucks, over 5 tons.....							102,274	1,203	16,845
For-hire combinations, over 5 tons.....							103,196	1,214	19,992
Private combinations, 5 tons.....	100.0000	29.18	298,888	14,811	78,642	318,341	191,972	11,564	167,823
For-hire trucks, 5 tons.....							74,022	2,028	26,350
For-hire combinations, 5 tons.....							42,106	495	6,935
For-hire combinations, 5 tons.....							68,462	690	9,658
Total.....		100.000	98,084,000	6,187,000	34,000,000	138,271,000	76,033,895	23,238,807	77,601,017

Class of motor vehicle	Cost per vehicle on—									
	Primary roads					Secondary roads				
	Cumulated vehicle-miles (000) (from column 8)	Cost per vehicle-mile (cents) (4+11)	Cumulated cost per vehicle-mile (cents) (from column 12)	Costs chargeable against group (13X6)	Cost per vehicle (15)	Cumulated vehicle-miles (000) (from column 9)	Cost per vehicle-mile (cents) (5+16)	Cumulated cost per vehicle-mile (cents) (from column 17)	Costs chargeable against group (18X9)	Cost per vehicle (20)
(1)	(11)	(12)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	
Passenger cars.....	76,033,895	0.09100	0.09100	\$56,788,624	\$2.74	23,238,807	0.01878	0.01878	\$3,543,320	\$0.17
Taxicabs.....	13,833,892		0.09100	193,002	1.46	4,373,690		0.01878	9,889	.08
Contract buses, 7-passenger or less.....	13,408,588		0.09100	11,423	4.09	4,330,303		0.01878	202	.02
Common carrier buses, 7-passenger or less.....	0.09100			51,781	18.43	4,319,108		0.01878	1,187	.43
School buses.....	13,851,686	0.2908	1.1908	213,383	4.17	4,312,765	0.0548	0.2926	74,524	1.46
Farm trucks.....	13,172,463	0.2269	1.4177	1,613,835	1.91	4,906,597	0.0471	0.2897	267,973	.27
Contract buses, 8 to 20 passengers.....	11,923,493	0.0180	1.4366	4,629	6.47	3,880,587	0.0046	0.02943	105	.15
Common carrier buses, 8 to 20 passengers.....	11,919,271		1.4366	145,025	21.83	3,860,239		0.02943	3,301	.49
Private trucks (non-farm), 1½ tons and less.....	11,718,321	0.01321	1.5687	7,686,308	5.44	3,969,022	0.0618	0.03261	763,969	.55
Private combinations, over 1½ and less than 3 tons.....	6,914,784		1.5687	1,267,000	16.82	726,278		0.03261	16,723	.22
For-hire trucks, 1½ tons and less.....	6,107,109		1.5687	1,416,688	10.11	674,997		0.03261	75,726	.64
For-hire combinations, over 1½ and less than 3 tons.....	5,204,012		1.5687	244,510	31.84	442,772		0.03261	612	.08
Contract buses, over 20 passengers.....	5,048,144	0.13380	2.9067	6,601	13.08	440,894	0.0664	1.2826	474	.65
Common carrier buses, over 20 passengers.....	5,044,841		2.9067	890,178	37.33	440,327		1.2826	20,367	.55
Private trucks, over 1½ and less than 3 tons.....	4,736,527	0.0776	2.9773	3,358,201	11.89	426,789	0.0497	1.3422	204,071	1.04
Private combinations, 3 and less than 5 tons.....	3,806,492		2.9773	720,926	37.08	205,673		1.3422	14,662	.78
For-hire trucks, over 1½ and less than 3 tons.....	3,367,351		2.9773	2,118,778	46.60	194,831		1.3422	48,904	.97
For-hire combinations, 3 and less than 5 tons.....	2,457,397		2.9773	611,613	64.97	169,076		1.3422	5,778	.32
Private trucks, 3 and less than 5 tons.....	2,485,549	26.605	6.8278	4,482,843	42.44	100,000	0.07031	413.53	419,336	3.97
For-hire trucks, 3 and less than 5 tons.....	1,716,332		6.8278	2,176,174	116.33	88,602		413.53	47,022	1.72
Private trucks, over 5 tons.....	1,171,328	0.1040	1.09318	5,197,653	132.05	47,231	0.7645	1.21198	322,894	8.20
Private combinations, over 5 tons.....	696,396		1.09318	1,353,160	196.31	26,690		1.21198	41,159	5.96
For-hire trucks, over 5 tons.....	671,902		1.09318	1,116,039	296.00	17,194		1.21198	14,580	3.89
For-hire combinations, over 5 tons.....	490,628		1.09318	1,127,790	223.38	15,991		1.21198	14,713	4.22
Private trucks, 5 tons.....	368,462	0.6191	1.15509	2,213,894	118.71	14,777	0.06885	1.30883	151,433	8.12
Private combinations, 5 tons.....	174,790		1.15509	855,021	167.78	3,213		1.30883	26,543	5.21
For-hire trucks, 5 tons.....	100,768		1.15509	486,392	276.34	1,185		1.30883	6,479	6.68
For-hire combinations, 5 tons.....	58,602		1.15509	677,699	276.25	690		1.30883	9,051	9.38
Total.....				98,084,000					6,187,000	

¹ See p. 132.
² For basic amounts allocated, see p. 139.
³ From table 46.

APPENDIX

TABLE F-2.—Allocation of cost of rigid pavement to classes of motor vehicles, 1932—Continued

Class of motor vehicle (1)	Cost per vehicle on—					Total (column 15+20+22) (23)
	City streets					
	Cumulated vehicle-miles (000) (from column 10)	Cost per vehicle-mile (cents) (6+21)	Cumulated cost per vehicle-mile (cents) (from column 22)	Costs chargeable against group (23×10)	Cost per vehicle	
(21)	(22)	(23)	(24)	(25)	(26)	
Passenger cars.....	77,001,017	0.03091	0.03091	\$19,735,967	\$0.95	\$3.86
Taxicabs.....	18,749,851	0.03091	736,807	5.58	7.10
Contract buses, 7-passenger or less.....	11,356,427	0.03091	4,319	1.55	5.73
Common carrier buses, 7-passenger or less.....	11,942,477	0.03091	2,171	.77	19.03
School buses.....	11,335,432	.01147	.04238	10,849	.21	5.84
Farm trucks.....	11,309,853	.00915	.05154	115,866	.13	2.30
Contract buses, 8 to 20 passengers.....	11,084,833	.00670	.05224	6,830	2.61	9.23
Common carrier buses, 8 to 20 passengers.....	11,081,27305224	47,942	7.05	28.87
Private trucks (non-farm), 1½ tons and less.....	10,889,500	.00488	.03712	2,644,751	1.91	7.90
Private combinations, over 1½ and less than 3 tons.....	6,349,33403712	241,668	3.21	20.23
For-hire trucks, 1½ tons and less.....	5,938,39603712	325,338	5.89	16.54
For-hire combinations, over 1½ and less than 3 tons.....	4,491,31103712	17,168	2.20	33.82
Contract buses, over 20 passengers.....	4,023	4.43	19.21
Common carrier buses, over 20 passengers.....	4,457,594	.02248	.05980	560,777	23.49	61.87
Private trucks, over 1½ and less than 3 tons.....	5,945,336	.00294	.11254	1,934,118	7.03	19.09
Private combinations, 3 and less than 5 tons.....	2,182,06411254	122,018	6.28	44.11
For-hire trucks, over 1½ and less than 3 tons.....	2,174,48211254	232,011	8.54	33.01
For-hire combinations, 3 and less than 5 tons.....	1,850,55211254	37,270	4.73	70.05
For-hire trucks, 3 and less than 5 tons.....	1,817,437	.13618	.24787	2,811,636	25.77	70.18
For-hire combinations, over 3 tons.....	303,32524787	328,022	13.03	130.08
Private trucks, over 5 tons.....	670,680	.80900	.65667	2,140,730	54.39	194.94
Private combinations, over 5 tons.....	236,12065667	236,829	34.24	236.51
For-hire trucks, over 5 tons.....	243,36665667	65,771	24.83	324.78
For-hire combinations, over 5 tons.....	226,62165667	64,389	28.41	354.87
Private trucks, 5 tons.....	300,829	.03748	.69415	997,471	53.48	180.81
Private combinations, 5 tons.....	41,94769415	160,517	20.59	203.85
For-hire trucks, 1½ tons and less.....	16,89769415	41,204	22.41	303.44
For-hire combinations, 5 tons.....	9,66369415	57,407	23.41	303.44
Total.....	84,000,000

TABLE F-3.—Allocation of costs of flexible pavement and grading to classes of motor vehicles, 1932¹

Class of motor vehicle (1)	Index of re- quired cost (maximum = 100) (from column 10 of table F-1) (2)	Increments in index (per- cent) (from column 2) (3)	Increments of cost assignable to vehicle types by sys- tems ²				Estimated annual mileage on— ³		
			State pri- mary roads (3) × \$111,- 678,000 (4)	Secondary and local roads (3) × \$38,484,000 (5)	City streets (3) × \$62,547,- 000 (6)	Total (\$227,704,000 (4)+(5)+(6) (7)	Primary (000) (8)	Secondary and local (000) (9)	City streets (000) (10)
			(4)	(5)	(6)	(7)	(8)	(9)	(10)
Passenger cars.....	60.0635	60.0635	\$67,074,712	\$32,124,302	\$37,567,917	\$136,766,991	63,400,008	18,865,117	63,851,166
Contract buses, 8 to 20 passengers.....	65.8333	5.2698	4,824,944	2,818,800	3,296,102	11,960,546	3,222	3,222	3,893
Common carrier buses, 8 to 20 passengers.....	100,350	11,217	91,773
Contract buses, over 20 passengers.....	67.0667	1.7834	1,938,740	627,062	1,084,190	3,647,022	3,303	867	3,670
Common carrier buses, over 20 passengers.....	808,314	16,768	511,658
Taxicabs.....	78.6508	8.5941	7,352,663	3,521,445	4,119,157	14,992,259	212,749	53,187	2,369,424
Private trucks, 3 and less than 5 tons.....	77.0667	8.4169	3,814,638	1,826,950	3,136,543	9,778,141	789,817	101,404	1,014,109
For-hire trucks, 3 and less than 5 tons.....	645,004	11,371	122,645
Private trucks, over 5 tons.....	77.8334	.2697	267,839	143,842	166,818	607,267	475,462	28,641	364,560
Private combinations, over 5 tons.....	123,364	3,398	42,454
For-hire trucks, over 5 tons.....	102,274	1,203	16,846
Private combinations, over 3 tons.....	108,166	1,214	18,992
Private trucks, 5 tons.....	77.6661	.2667	297,853	142,843	168,813	607,267	161,373	11,564	167,863
Private combinations, 4 tons.....	74,022	2,028	26,350
For-hire trucks, 5 tons.....	42,106	495	6,833
For-hire combinations, 5 tons.....	85,662	960	9,663
Contract buses, 7-passenger or less.....	78.6663	1.0033	1,130,368	636,851	637,471	2,364,528	1,303	1,303	1,840
Common carrier buses, 7-passenger or less.....	56,902	6,323	7,022
Private trucks, over 1½ and less than 3 tons.....	82.5334	3.9501	4,368,961	2,101,976	3,468,160	8,943,995	1,127,635	219,096	1,793,032
For-hire trucks, over 1½ and less than 3 tons.....	708,964	32,755	223,320
Private combinations, 3 and less than 5 tons.....	242,141	10,642	108,422
For-hire combinations, 3 and less than 5 tons.....	171,836	2,070	33,122
Private trucks (non-farm), 1½ tons and less.....	67.8331	15.0667	14,828,436	8,068,274	8,423,789	36,307,479	4,863,337	2,942,744	4,630,166
For-hire trucks, 1½ tons and less.....	903,697	232,225	1,444,866
Private combinations, over 1½ and less than 3 tons.....	807,675	51,261	428,068
For-hire combinations, over 1½ and less than 3 tons.....	168,968	1,878	30,017
School buses.....	68.2667	.6663	744,413	358,524	416,933	1,517,874	179,198	307,188	25,650
Farm trucks.....	100.0000	1.7333	1,935,626	937,030	1,054,127	3,945,783	1,340,010	625,000	226,000
Total.....	100.0000	111,673,000	53,484,000	63,647,000	237,704,000	76,083,966	23,283,807	77,001,017

¹ See pp. 129-133.
² For basic amounts allocated, see pp. 129-130.
³ From table 46.

AIDS TO MOTOR VEHICLE TRANSPORTATION

TABLE F-3.—Allocation of costs of flexible pavement and grading to classes of motor vehicles, 1932—Continued

Class of motor vehicle (1)	Cost per vehicle on—									
	Primary roads					Secondary and local roads				
	Cumulated vehicle-miles (000) (from column 8)	Cost per vehicle- mile (cents) (4+11)	Cumulated cost per vehi- cle-mile (cents) (from column 12)	Costs charge- able against group (8 X 13)	Cost per vehicle	Cumulated vehicle- miles (000) (from col- umn 9)	Cost per vehicle- mile (cents) (5 + 16)	Cumulated cost per mile (cents) (from col- umn 17)	Costs chargeable against group (8 X 18)	Cost per vehicle
Passenger cars.....	76,033,895	0.08922	0.08922	\$58,045,930	\$2.86	23,238,807	0.13824	0.13824	\$26,078,173	\$1.26
Contract busses, 8 to 20 passengers.....	12,632,892	.04316	.13138	4,233	5.91	4,373,600	.06444	.20268	22,726	1.01
Common carrier busses, 8 to 20 passengers.....	13,830,67013138	132,628	19.51	4,373,33220298	22,726	3.34
Contract busses, over 20 passengers.....	13,529,720	.01431	.14569	4,812	6.86	4,362,116	.02125	.22393	822	1.12
Common carrier busses, over 20 passengers.....	13,529,41714569	44,663	18.81	4,351,74822393	35,287	1.48
Taxis.....	15,118,108	.05563	.20132	429,306	3.2208103	102,199	1.22
Private trucks, 3 and less than 5 tons.....	13,005,354	.02933	.23985	1,774,199	16.79	4,292,803	.04256	.24732	252,399	3.34
For-hire trucks, 3 and less than 5 tons.....	12,236,13723985	1,287,022	48.04	4,191,39924752	38,516	1.46
Private trucks, over 5 tons.....	11,691,138	.00255	.23220	1,108,777	28.17	4,183,028	.00341	.25093	93,401	2.88
Private combinations, over 5 tons.....	11,215,67123220	289,084	41.88	4,153,38725093	11,918	1.78
For-hire trucks, over 5 tons.....	11,091,70723220	238,503	63.16	4,149,99125093	4,222	1.12
For-hire combinations, over 5 tons.....	10,989,43323220	240,583	68.97	4,148,78825093	4,299	1.52
Private trucks, 5 tons.....	10,836,287	.00274	.23594	452,231	34.25	4,147,574	.00344	.25437	40,379	2.30
Private combinations, 5 tons.....	10,694,59523594	174,848	34.27	4,139,01025437	7,187	1.41
For-hire trucks, 5 tons.....	10,620,57323594	98,245	68.45	4,133,88225437	1,764	1.00
For-hire combinations, 5 tons.....	10,678,46723594	138,407	68.45	4,133,48725437	2,448	1.00
Contract busses, 7 passengers or less.....	10,519,895	.01065	.24659	30,959	11.10	4,132,797	.01268	.26735	5,128	1.84
Common carrier busses, 7 passengers or less.....	10,507,85024659	140,315	49.93	4,131,40226735	26,228	3.27
Private trucks, over 1½ and less than 3 tons.....	10,459,348	.04206	.28859	3,252,310	11.52	4,125,079	.00996	.41831	916,503	2.85
Common carrier busses, over 1½ and less than 3 tons.....	9,323,31328859	2,048,885	45.08	3,905,98341831	137,017	8.01
Private combinations, 3 and less than 5 tons.....	8,613,34928859	698,704	35.94	3,873,22841831	46,353	2.33
For-hire combinations, 3 and less than 5 tons.....	8,371,30828859	493,807	62.37	3,862,33641831	8,659	1.10
Private trucks (nonfarm), 1½ tons and less.....	8,590,370	.20520	.49379	23,718,885	17.1226875	.62706	14,600,410	18.21
For-hire trucks, 1½ tons and less.....	3,395,83349379	4,450,402	31.83	1,517,57262706	1,456,190	10.89
Private combinations, over 1½ and less than 3 tons.....	2,462,73849379	3,968,218	52.94	1,285,34762706	321,863	4.27
For-hire combinations, over 1½ and less than 3 tons.....	1,985,99149379	799,561	98.05	1,234,99562706	11,778	1.50
School busses.....	1,529,159	.04888	.94947	97,028	18.59	1,223,18865598	9,012,128	36.25
Farm trucks.....	1,260,000	.14338	.68685	9,258,975	9.36	925,000	.10022	.73621	6,994,048	6.39
Total.....	111,678,000	53,484,000

Class of motor vehicle (1)	Cost per vehicle on—					Total (col- umn 16 + 20 + 25)
	City streets					
	Cumulated vehicle-miles (000) (from column 10)	Cost per vehicle-mile (cents) (6+21)	Cumulated cost per vehicle-mile (cents) (from column 22)	Costs charge- able against group (23 X 10)	Cost per vehicle	
Passenger cars.....	77,001,017	0.04841	0.04841	\$30,911,280	\$1.49	\$5.41
Contract busses, 8 to 20 passengers.....	13,746,271	.02397	.07238	2,861	2.62	10.54
Common carrier busses, 8 to 20 passengers.....	13,634,498	.00794	.07238	66,425	0.77	32.63
Contract busses, over 20 passengers.....	12,650,62806032	2,645	4.02	11.70
Common carrier busses, over 20 passengers.....	13,159,170	.05134	11196	410,964	17.71	37.60
Taxis.....	10,745,745	.01868	13184	2,672,497	29.10	24.54
Private trucks, 3 and less than 5 tons.....	9,731,644	.00174	13238	1,333,650	12.53	32.78
Private trucks, over 5 tons.....	9,598,969	13238	174,481	6.39	53.88
Private combinations, over 5 tons.....	9,214,430	13238	512,542	13.02	43.57
For-hire trucks, over 5 tons.....	9,171,985	13238	64,638	8.39	51.81
For-hire combinations, over 5 tons.....	9,155,149	13238	22,451	8.96	70.23
Private trucks, 5 tons.....	8,138,148	.00188	13511	22,947	6.49	78.68
Private combinations, 5 tons.....	8,970,296	13511	226,925	12.18	38.03
For-hire trucks, 5 tons.....	8,944,916	13511	84,260	6.72	42.40
For-hire combinations, 5 tons.....	8,929,319	.00703	14214	9,870	5.32	62.77
Contract busses, 7 passengers or less.....	8,914,999	14214	13,034	8.22	62.77
Common carrier busses, 7 passengers or less.....	8,907,344	.00700	14214	19,829	7.11	20.03
Private trucks, over 1½ and less than 3 tons.....	7,144,312	16974	9,985	8.55	61.76
Private combinations, 3 and less than 5 tons.....	6,920,382	16974	2,992,571	10.69	26.37
For-hire combinations, 3 and less than 5 tons.....	6,911,960	16974	184,036	9.46	47.73
Private trucks (non farm), 1½ tons and less.....	6,778,836	.13902	20876	56,226	7.14	71.21
For-hire trucks, 1½ tons and less.....	2,148,969	20876	14,204,101	10.82	28.06
Private combinations, over 1½ and less than 3 tons.....	703,714	20876	4,461,443	31.83	74.05
For-hire combinations, over 1½ and less than 3 tons.....	290,646	20876	1,306,268	17.34	74.88
School busses.....	250,599	.16038	47614	62,773	11.89	112.05
Farm trucks.....	225,000	.48183	95967	121,531	2.33	60.73
Total.....	2,183,183	2.15	18.40

TABLE F-4.—Allocation of costs of structures to classes of motor vehicles, 1952¹

Class of motor vehicle (1)	Index of required pavement width (maximum=100) (from column 11 of table F-1) (2)	Increment in index (percent) (from column 2) (3)	Increments of cost assignable to vehicle types by systems ²			Total (\$78,930,000) (7)	Estimated annual mileage on—		
			Primary (\$46,068,000) (4)	Secondary and local roads (\$16,824,000) (5)	City streets (\$17,038,000) (6)		Primary (000) (8)	Secondary and local roads (000) (9)	City streets (000) (10)
Passenger cars.....	95.2381	95.2381	\$42,922,000	\$16,028,000	\$16,227,000	\$75,172,000	62,400,003	18,805,117	63,851,166
Taxis and other for-hire cars.....							212,749	53,187	2,803,424
Contract buses, 7 and less seats.....							12,556	1,306	13,862
Common-carrier buses, 7 and less seats.....							65,902	6,323	7,026
School buses.....							179,193	307,188	26,599
Contract buses (seats):									
5 to 20.....							3,222	858	3,880
Over 20.....							3,303	387	3,670
Common carrier buses (seats):									
5 to 20.....							100,950	11,217	91,773
Over 20.....	308,314	15,758	611,608						
Farm trucks.....	1,360,000	926,000	236,000						
Private trucks (nonfarm) (tons):									
1/4 and less.....	4,803,537	2,342,744	4,630,166						
Over 1/4 and less than 3/4:									
Single.....	1,127,035	219,086	1,788,033						
Combination.....	307,076	51,281	458,058						
3/4 and less than 3/2:									
Single.....	796,217	101,404	1,014,109						
Combination.....	242,141	10,843	108,423						
3/2 and less than 2:									
Single.....	191,672	11,664	167,869						
Combinations.....	74,022	3,028	26,850						
Over 2:									
Single.....	475,462	26,641	354,560						
Combination.....	123,954	3,306	42,454						
For-hire trucks (tons):									
1/4 and less.....	908,097	232,226	1,444,956						
Over 1/4 and less than 3/4:									
Single.....	709,994	32,755	223,930						
Combination.....	156,895	1,878	30,947						
3/4 and less than 3/2:									
Single.....	545,004	11,871	132,645						
Combination.....	171,836	3,070	33,125						
3/2 and less than 2:									
Single.....	42,108	493	6,925						
Combinations.....	58,663	690	9,663						
Over 2:									
Single.....	102,274	1,302	16,845						
Combination.....	103,165	1,214	16,999						
Total.....	100.0000	100.0000	46,068,000	16,824,000	17,038,000	78,930,000	78,033,985	23,238,907	77,601,017

Class of motor vehicle (1)	Cost per vehicle on—									
	Primary roads					Secondary and local roads				
	Cumulated vehicle-miles (from column 8) (000) (11)	Cost per vehicle-mile (4+11) (cents) (12)	Cumulated cost per vehicle-mile (from column 12) (cents) (13)	Cost chargeable against group (6×13) (dollars) (14)	Cost per vehicle (15)	Cumulated vehicle-miles (from column 9) (000) (16)	Cost per vehicle-mile (5+16) (cents) (17)	Cumulated cost per vehicle-mile (from column 17) (cents) (18)	Cost chargeable against group (6×18) (dollars) (19)	Cost per vehicle (cents) (20)
Passenger cars.....	75,033,896	0.05645	0.05645	\$1,226,051	\$1.70	23,238,907	0.06896	0.06896	12,007,457	0.53
Taxis and other for-hire cars.....			0.06645	120,097	0.91			0.06896	26,672	0.28
Contract buses, 7 and less seats.....			0.05645	7,067	2.34			0.06896	953	0.84
Common-carrier buses, 7 and less seats.....			0.05645	32,121	11.43			0.06896	4,300	1.55
School buses.....			0.07353	130,961	2.54			0.07353	368,851	3.25
Contract buses (seats):										
5 to 20.....			0.07353	2,337	2.26			0.07353	312	0.44
Over 20.....			0.07353	2,304	2.26			0.07353	321	0.44
Common carrier buses (seats):										
5 to 20.....			0.07353	73,209	10.77			0.07353	9,817	1.44
Over 20.....	0.07353	223,690	6.36	0.07353	12,791	0.95				
Farm trucks.....	0.07353	979,030	0.98	0.07353	806,600	0.81				
Private trucks (nonfarm) (tons):										
1/4 and less.....	0.07353	2,483,226	2.83	0.07353	2,050,370	1.48				
Over 1/4 and less than 3/4:										
Single.....	0.07353	817,326	2.90	0.07353	191,728	0.68				
Combination.....	0.07353	484,728	7.76	0.07353	44,581	0.60				
3/4 and less than 3/2:										
Single.....	0.07353	657,656	5.28	0.07353	56,749	0.84				
Combination.....	0.07353	173,601	3.03	0.07353	6,489	0.49				
3/2 and less than 2:										
Single.....	0.07353	139,001	7.45	0.07353	10,121	0.84				
Combinations.....	0.07353	53,651	10.83	0.07353	1,775	0.26				
Over 2:										
Single.....	0.07353	344,805	2.76	0.07353	23,316	0.80				
Combination.....	0.07353	80,590	13.03	0.07353	2,973	0.43				
For-hire trucks (tons):										
1/4 and less.....	0.07353	654,026	4.67	0.07353	308,263	1.45				
Over 1/4 and less than 3/4:										
Single.....	0.07353	514,698	11.33	0.07353	26,687	0.68				
Combination.....	0.07353	113,026	14.49	0.07353	1,644	0.21				
3/4 and less than 3/2:										
Single.....	0.07353	395,337	14.49	0.07353	9,902	0.26				
Combination.....	0.07353	124,617	14.23	0.07353	1,813	0.23				
3/2 and less than 2:										
Single.....	0.07353	80,538	17.34	0.07353	632	0.26				
Combinations.....	0.07353	43,643	17.26	0.07353	604	0.26				
Over 2:										
Single.....	0.07353	74,169	19.04	0.07353	1,028	0.26				
Combination.....	0.07353	74,816	21.45	0.07353	1,028	0.26				
Total.....										
			44,068,000					16,824,000		

¹ See pp. 119-120.

² For basic amounts allocated, see pp. 119-120.

³ From table 65.

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TABLE F-4.—Allocation of costs of structures to classes of motor vehicles, 1958—Continued

Class of motor vehicle (1)	Cost per vehicle on—					Total (dollars) (26)
	City streets					
	Cumulated vehicle-miles (from column 10) (000) (21)	Cost per vehicle-mile (6+21) (cents) (22)	Cumulated cost per vehicle-mile (from column 22) (cents) (23)	Cost charge- able against group (10×23) (dollars) (24)	Cost per vehicle (dollars) (25)	
Passenger cars.....			0.02001	18,358,419	0.64	9.07
Taxis and other for-hire cars.....			.02001	500,465	2.76	4.95
Contract busses, 7 and less seats.....	77,801,017	0.02001	.02001	2,917	1.04	8.92
Common carrier busses, 7 and less seats.....			.02001	1,469	.62	12.50
School busses.....			.02806	7,183	.14	7.93
Contract busses (seats):						
8 to 20.....			.02806	1,005	1.40	5.10
Over 20.....			.02806	1,060	1.40	5.10
Common carrier busses (seats):						
8 to 20.....			.02806	25,792	3.79	16.00
Over 20.....			.02806	143,371	6.01	13.95
Farm trucks.....			.02806	83,135	.06	1.85
Private trucks (nonfarm) (tons):						
1½ and less.....			.02806	1,299,226	.94	4.94
Over 1½ and less than 3:						
Single.....			.02806	494,707	1.75	5.33
Combination.....			.02806	113,713	1.58	9.98
3 and less than 6:						
Single.....			.02806	284,557	2.69	8.81
Combination.....			.02806	30,423	1.66	11.08
5, single vehicle.....	11,338,423	.00715	.02806	47,108	2.53	10.52
5, combinations.....			.02806	7,113	1.40	12.28
Over 5:						
Single.....			.02806	107,908	2.74	12.09
Combination.....			.02806	11,913	1.73	15.18
For-hire trucks (tons):						
1½ and less.....			.02806	405,454	2.89	9.01
Over 1½ and less than 3:						
Single.....			.02806	62,535	1.38	13.34
Combination.....			.02806	8,431	1.09	15.79
3 and less than 6:						
Single.....			.02806	37,220	1.36	16.20
Combination.....			.02806	9,295	1.18	17.23
5, single.....			.02806	1,946	1.11	18.70
5, combinations.....			.02806	2,711	1.11	18.71
Over 5:						
Single.....			.02806	4,727	1.26	21.17
Combination.....			.02806	4,708	1.57	23.12
Total.....				17,038,000		

TABLE F-5.—Development of indexes for allocation of maintenance costs chargeable to motor vehicles, 1952

Class of motor vehicle	Number registered, 1952	Indexes for allocation on basis of traffic volume (Table 4f, column 6, 8 and 10)			Index for allocation on basis of pressure intensity		
		Index of traffic on primary rural roads (percent of total)	Index of traffic on secondary and local rural roads (percent of total)	Index of traffic on city streets (percent of total)	Pressure intensity per square inch of tire contact area	Product of pressure intensity and total annual mileage (000,000)	Index of allocation (percent of product total)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Passenger cars.....	20,730,898	82.0687	81.7794	82.2614	47.3	4,894,000	78.2008
Taxis.....	152,988	.2708	.2289	2.0843	58.0	154,243	1.7125
School buses.....	151,198	.2857	1.3210	.0380	73.7	37,733	4.189
Contract buses (seats):							
7 and less.....	2,700	.0165	.0050	.0180	61.9	1,727	.0192
8 to 20.....	716	.0042	.0015	.0045	49.0	351	.0039
Over 20.....	734	.0043	.0016	.0047	50.3	368	.0041
Common carrier buses (seats):							
7 and less.....	2,810	.0748	.0772	.0901	61.9	4,348	.0483
8 to 20.....	6,798	.1228	.0483	1.183	49.0	9,998	1.110
Over 20.....	28,878	.4056	.0678	.6588	50.3	42,007	4.667
Farm trucks.....	1,009,000	1.7768	8.6894	2.890	73.0	187,000	2.0813
Private trucks (nonfarm) (tons):							
1/4 and less.....	1,384,000	6.3176	10.0811	5.9666	73.2	863,086	9.8710
Over 1/4 and less than 3/4:							
Single vehicles.....	282,315	1.4822	.9428	2.2710	61.9	192,457	2.1868
Combinations.....	76,330	1.0623	.2207	.5453	73.2	93,844	1.0419
3/4 and less than 1:							
Single vehicles.....	104,640	1.0117	.4364	1.3088	57.5	108,937	1.2095
Combinations.....	10,445	.3185	.0487	.1897	61.9	22,371	.2484
1, single vehicles.....	18,851	.3221	.0488	.2163	58.2	21,599	.2398
Combinations.....	8,996	.0974	.0387	.0327	58.2	4,901	.0546
Over 1:							
Single vehicles.....	36,360	.6283	.1146	.4956	58.0	61,426	.6710
Combinations.....	6,903	.1630	.0140	.0547	58.0	9,549	1.064
For-hire trucks (tons):							
1/4 and less.....	140,108	1.1878	.9088	1.8620	73.2	184,876	2.0970
Over 1/4 and less than 3/4:							
Single vehicles.....	45,453	.9337	1.400	.2886	61.9	69,366	.7648
Combinations.....	7,801	.2080	.0061	.0387	73.2	18,740	.2096
3/4 and less than 1:							
Single vehicles.....	27,303	.7168	.0489	.1708	57.5	89,825	.9822
Combinations.....	7,876	.2260	.0089	.0427	61.9	15,615	.1723
1, single vehicles.....	1,700	.0554	.0021	.0089	58.2	3,883	.0430
Combinations.....	2,452	.0772	.0030	.0126	58.2	4,017	.0446
Over 1:							
Single vehicles.....	3,776	.1345	.0032	.0217	58.0	6,979	.0775
Combinations.....	3,488	.1367	.0032	.0219	58.0	7,040	.0782
Total.....	24,150,000	100.0000	100.0000	100.0000		9,006,788	100.0000

Class of motor vehicle	Index for allocation on basis of abrasion and wear						
	Rolling resistance (pounds)	Air resistance (pounds)	Total tractive pull at road surface (pounds per square inch)	Diameter of tire contact area (inches)	Traffic on secondary and local rural roads (000 annual vehicle miles)	Product of tractive pull, diameter of contact area and traffic volume (000,000)	Index of allocation (percent of product total)
(1)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Passenger cars.....	42.5	104.9	147.4	7.34	18,865,117	26,410,547	64.0106
Taxis.....	63.8	104.9	173.7	7.65	63,187	70,767	.2219
School buses.....	133.0	162.7	287.7	9.28	307,198	820,149	3.6721
Contract buses (seats):							
7 and less.....	76.0	104.9	179.9	8.03	1,296	2,013	.0063
8 to 20.....	150.0	212.5	362.5	13.40	338	1,739	.0055
Over 20.....	333.1	238.3	468.4	15.46	367	2,601	.0083
Common carrier buses (seats):							
7 and less.....	76.0	104.9	179.9	8.03	6,323	9,123	.0286
8 to 20.....	160.0	212.5	362.5	13.40	18,717	54,457	1.770
Over 20.....	333.1	238.3	468.4	15.46	11,678	111,678	3.622
Farm trucks.....	131.3	135.9	287.2	9.66	925,000	2,302,978	7.3226
Private trucks (nonfarm) (tons):							
1/4 and less.....	105.8	140.4	248.9	9.26	2,342,744	4,854,488	15.7369
Over 1/4 and less than 3/4:							
Single vehicles.....	151.1	136.4	267.5	13.34	319,086	840,178	2.8349
Combinations.....	306.1	133.3	336.3	9.26	61,261	161,121	.5653
3/4 and less than 1:							
Single vehicles.....	231.3	121.3	343.1	11.96	101,404	667,880	1.7810
Combinations.....	356.3	126.3	386.1	13.34	10,843	65,843	.1751
1, single vehicles.....	304.6	110.6	418.3	17.46	11,464	83,323	.2229
Combinations.....	304.6	122.1	426.7	17.46	2,028	16,109	.0474
Over 1:							
Single vehicles.....	415.3	97.1	512.4	17.44	26,641	283,071	.7466
Combinations.....	417.6	96.1	614.6	17.44	4,866	26,460	.1143
For-hire trucks (tons):							
1/4 and less.....	106.5	140.4	248.9	9.26	232,236	438,789	1.6683
Over 1/4 and less than 3/4:							
Single vehicles.....	151.1	136.4	267.5	13.34	34,736	126,674	.3940
Combinations.....	306.1	133.3	336.3	9.26	1,678	4,901	.0166
3/4 and less than 1:							
Single vehicles.....	231.3	121.3	343.1	11.96	11,271	63,990	.1997
Combinations.....	356.3	126.3	386.1	13.34	3,070	10,623	.0334
1, single vehicles.....	304.6	110.6	418.3	17.46	666	3,623	.1118
Combinations.....	304.6	122.1	426.7	17.46	660	5,141	.0161
Over 1:							
Single vehicles.....	415.3	97.1	512.4	17.44	1,208	10,739	.0337
Combinations.....	417.6	96.1	614.6	17.44	1,214	12,620	.0409
Total.....					28,238,007	31,694,237	100.0000

¹ Includes 37,196 tax-exempt school buses. See footnote 2, table 4f.

AIDS TO MOTOR VEHICLE TRANSPORTATION

TABLE F-6.—Allocation of maintenance costs to classes of motor vehicles, 1952¹

Class of motor vehicle (1)	Allocation of \$224,933,000 on basis of traffic volume						Allocation of \$66,734,000 on basis of pressure intensity		Allocation of \$22,243,000 on basis of abrasion and ravelling		Total (column 4+5+7+9+11) per vehicle (12)
	\$104,548,000 State highway maintenance		\$44,100,000 county and local road maintenance		\$76,305,000 city street maintenance		Per class (\$66,734,000 X index in column 8, table F-5)	Per vehicle	Per class (\$22,243,000 X index in column 10 of table F-5)	Per vehicle	
	Per class (\$104,548,000 X index in column 3 of table F-5)	Per vehicle	Per class (\$44,100,000 X index in column 4 of table F-5)	Per vehicle	Per class (\$76,305,000 X index in column 5 of table F-5)	Per vehicle					
(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		
Passenger cars.....	\$85,801,185	\$4.14	\$35,800,114	\$1.73	\$62,784,822	\$3.03	\$50,857,714	\$2.45	\$14,237,578	\$0.69	\$12.04
Taxicabs.....	292,525	2.20	100,945	1.76	2,353,475	17.70	1,142,820	8.59	49,357	.37	20.02
School buses.....	245,430	4.61	582,958	11.39	25,181	.49	279,549	8.48	572,112	11.18	33.33
Contract buses (seats):											
7 and less.....	17,250	6.18	2,648	.95	13,735	4.92	12,813	4.50	1,401	.50	17.14
8 to 20.....	4,391	6.13	862	.92	3,510	4.90	2,803	2.64	1,223	1.71	17.30
Over 20.....	4,495	6.13	708	.96	3,558	4.99	2,750	3.73	1,534	2.49	15.30
Common carrier buses (seats):											
7 and less.....	78,202	27.83	11,995	4.27	6,944	2.47	32,233	11.47	6,361	2.20	48.30
8 to 20.....	338,840	30.42	21,800	3.13	90,299	13.25	74,075	10.90	35,013	5.59	53.22
Over 20.....	423,849	17.75	25,900	1.25	603,079	21.07	311,448	13.04	77,895	3.26	56.37
Farm trucks.....	1,856,260	1.86	1,755,356	1.78	221,208	.22	1,389,268	1.39	1,606,502	1.61	6.84
Private trucks (nonfarm) (tons):											
1½ and less.....	6,694,924	4.77	4,445,765	3.21	4,552,814	3.20	6,387,111	4.61	3,721,210	2.69	15.57
Over 1½ and less than 3:											
Single vehicles.....	1,649,610	5.49	415,775	1.47	1,733,573	6.14	1,425,972	5.03	885,082	2.08	20.22
Combinations.....	1,110,613	14.74	97,329	1.29	416,015	5.62	695,301	9.23	112,394	1.49	32.27
3 and less than 5:											
Single vehicles.....	1,037,712	10.01	102,452	1.82	997,154	0.44	807,148	7.64	396,148	3.75	32.05
Combinations.....	532,865	17.12	20,595	1.06	105,598	3.48	165,707	8.52	35,947	2.00	34.15
5, single vehicles.....	293,896	16.07	21,932	1.15	165,948	3.35	190,023	3.53	35,477	3.14	35.82
5, combinations.....	101,830	19.98	3,837	.75	24,952	4.90	43,711	8.58	10,548	2.07	26.28
Over 5:											
Single vehicles.....	653,739	16.61	50,539	1.28	378,188	0.61	381,051	0.68	160,095	4.22	41.40
Combinations.....	170,413	24.69	6,439	.93	41,736	6.05	75,007	10.58	25,424	3.88	45.93
For-hire trucks (tons):											
1½ and less.....	1,241,821	8.86	440,691	3.15	1,420,709	10.14	1,299,419	9.99	368,856	2.63	34.77
Over 1½ and less than 3:											
Single vehicles.....	976,165	21.48	62,137	1.37	220,216	4.84	443,314	9.75	67,637	1.93	39.37
Combinations.....	214,322	37.47	3,572	.45	20,530	3.79	101,836	13.05	4,115	.53	45.30
3 and less than 5:											
Single vehicles.....	749,400	27.45	21,555	.70	130,405	4.73	205,088	10.81	44,419	1.63	45.45
Combinations.....	235,278	30.00	3,925	.50	32,522	4.14	94,992	12.06	7,429	.94	47.84
5, single vehicles.....	57,920	33.01	328	.53	5,701	3.85	21,356	12.13	2,613	1.45	50.85
5, combinations.....	50,711	32.62	1,323	.64	9,438	3.89	20,763	12.14	3,581	1.46	50.95
Over 5:											
Single vehicles.....	140,617	37.24	2,293	.61	16,133	4.39	61,719	13.70	7,495	1.69	37.93
Combinations.....	141,873	44.67	2,263	.66	16,711	4.79	62,188	14.95	6,007	2.61	33.69
Total.....	104,548,000		44,100,000		76,305,000		66,734,000		22,243,000		

¹ See pp. 140-141.

APPENDIX G

ANALYSIS OF RECENT STUDIES OF HIGHWAY COSTS AND MOTOR-VEHICLE USER PAYMENTS

A number of studies of the highway costs of particular States and comparisons with the payments made by motor-vehicle users have appeared in recent years. All cannot be mentioned. The Pennsylvania Motor Revenue Study, made by the Bureau of Public Roads, as an illustration of method, and the two reports of the Highway Cost Commission of the State of Washington (1935 and 1937), have been referred to earlier. Others include a report by William D. Ennis, of Stevens Institute of Technology, on Motor Vehicle Taxation in New Jersey, with Special Reference to Equity of Present Taxes of Heavy Vehicles;¹ brief studies by John S. Worley, of the University of Michigan;² a Study of the Motor Transportation Act and Fees and Taxes Paid by the Road Users for the Highway Facilities Provided, prepared by an interim committee appointed by the Governor of Oregon (January 1937); a Study of Missouri Highway and Street Costs Chargeable to Motor Vehicles, by H. M. Warbitzky, of the staff of the State highway commission (1937); and a Study of Highway Costs and Motor Vehicle Taxation, by V. L. Glover and others, of the division of highways of the department of public works and buildings of the State of Illinois (1938).³

Space limitations do not permit of further comment on the Pennsylvania and Washington studies or of consideration of the reports of Professors Ennis and Worley. It is possible, however, to consider briefly the methods used and results obtained in the three studies last named and at a later point to comment more extensively on the report on national conditions made to the Association of American Railroads by Messrs. C. B. Breed, Clifford Older, and W. S. Downs.⁴ Brief mention will also be made of certain engineering studies of highway costs and their allocation which have been offered over a period of years to the Highway Research Board of the National Research Council.⁵ Comments on these studies in the light of

the reasoning and facts employed in the present report will sharpen and clarify the issues present in this highly controversial field.⁶

Oregon, Missouri, and Illinois Studies

The Missouri and Illinois studies were prepared in defense of fees and regulations imposed on certain equipment of interstate common carriers of property, by some of whom injunctions against enforcement of the charges and regulations had been obtained. The Federal courts in each instance sustained the State.⁷ However, it is reasonable to assume that the reports in question did not necessarily or in all respects reflect the views of the respective States on the subject of motor-vehicle taxation.⁸ There has been a tendency to accord the reports more weight than is justified by the circumstances under which they were prepared.

In making comparisons of the methods and results of the three State studies with those of the present report, sight must not be lost of the fact that the differences encountered may be explained in part by differences between conditions in the three States and national conditions. It also is to be borne in mind that considerations which the present report finds have no place in the determination of public aid may enter into investigations for purposes of taxation and regulation, provided they are supported by legislative standards and can find support in the courts.

The Missouri and Illinois studies employ the type of "annual cost" analysis which has been used in the present report. The Oregon study, however, uses annual "highway requirements," defined as "those funds necessary to discharge debt service, administer, extend, reconstruct, and maintain highways * * *."⁹ It thus includes only interest actually paid, and brings in amortization through retirements of highway debt

¹ It may be noted that certain of the more recent studies appear to have been financed, directly or indirectly, by the present report as circulated in preliminary form for criticism and that the "railroad study" was prepared, at least in part, to meet the conclusions of this report, from which, in its unreviewed form, it drew certain material. See preliminary note, p. iii.

² See *Breakers Freight Lines, Inc., et al., v. Public Service Commission of Missouri*, 28 F. Supp. 895, June 15, 1939, and *Breakers Freight Lines, Inc., et al., v. Edward J. Hughes, Secretary of State of Illinois*, 26 Fed. Supp. 908. The United States Supreme Court, on appeal by the State of Missouri, held that it was without jurisdiction since the appellant had received a favorable decision in the lower court.

³ In at least the Missouri proceeding, the court found it unnecessary, in fact, to consider the State's cost study.

⁴ Report, p. 8 for details of amounts involved, see pp. 54-57. This method has also been followed in the very recent report on Highway Costs and Motor Vehicle Taxation, made by Messrs. G. C. Dillman, D. P. Locklin, G. L. Wilson, and J. S. Worley to the Illinois Highway Users Conference, Inc. (released May 4, 1939).

¹ Published in January 1933; indicated to have been made to New Jersey Taxpayers Association at the request of Associated Railroads of New Jersey.

² *Automotive Vehicle Fees and Taxes in Michigan*, published by the National Highway Users Conference in April 1933, and *Motor Truck Transportation in Michigan, 1934*, a study financed by the Fruehauf Trailer Co.

³ A special Royal Commission on Transportation, Provinces of Ontario, rendered a report in 1938 which, among other things, discussed motor-vehicle tax problems at length.

⁴ *Highway Costs. A Study of Highway Costs and Motor Vehicle Payments in the United States, January 1939*. See pp. iii, 144-149, above.

⁵ Mention may also be made of *A Study in Highway Economics*, by Wilfred Owen. This brief study was published in 1934 under the auspices of the Harvard Phi Beta Kappa Society.

and replacements of facilities. A duplication of charges could thereby result.

Two of the reports make direct use of the so-called "public utility" concept and such use may be inferred in the case of the third.¹⁰ The present report recognizes that such a concept, properly used, has some validity in approaching the problem of assigning costs. However, it emphasizes the differences in treatment required when consideration is given to the relations of highways and streets to all who use or benefit by them.

The differences between the two approaches take on two very practical forms.¹¹ The Missouri report includes an interest charge on the value of right-of-way underlying road and street improvements, as does the Illinois study in one of its alternative statements of costs.¹² The former admits that "right-of-way occupied by streets was compensated for in the original sale price of abutting property" and that, upon abandonment, the lands would revert to the abutting owners, but finds justification for setting up an interest charge on the "tremendous value [that] is represented in the hundreds of thousands of acres occupied by highways and streets" in the fact that these lands are "used as a foundation for the roadway of a public transportation system operating in competition with private transportation systems." A capital value of nearly \$34,000,000, derived from the values of adjacent lands, is thus set up against motor-vehicle users. This value is

¹⁰ "The motor transportation system of the State is a great public corporation . . . Each highway user thus becomes a stockholder in the corporation. He purchases stock by paying motor vehicle fees and gasoline taxes, and receives direct dividends in the decreased operating costs of his motor vehicle, besides many other indirect benefits." (Oregon report, p. 8.)

¹¹ "A modern highway system is a public utility, comparable both in importance and magnitude of investment to a privately owned railway system. It is perhaps more a matter of public interest than railways, waterways, telephone systems, or power and light plants . . . the sovereignty . . . supplies facilities which are an actual monopoly and which are therefore subject to governmental regulations. . . . A publicly owned utility must know its annual cost, determined in the same manner as for a privately owned public utility, in order that it may determine as accurately as possible the proper annual charges. . . ." (Illinois report, pp. 3-4.)

¹² Application of the public utility concept to highways appears to derive from studies made by Dean Anson Marston of Iowa State College. (See, especially, *The Engineering Valuation of Highway Systems, a Problem in Public Utilities Valuation*, in Proceedings, 13th annual meeting, Highway Research Board, pp. 43-57. He also appeared in support of the State in the *Missouri case*.) Dean Marston regards highways as in all essential respects public utility enterprises, with all the implications in the way of finding fair value and a reasonable rate of return which that term connotes in private utility undertakings. The original cost, cost of reproduction new and less depreciation (estimated at "spot" and "parid" prices), going concern value, working capital, and all the other paraphernalia of valuation technique are brought into his study. He would, however, limit the value of lands to lands owned by the State, excluding easements, which constitute much the larger part of the total.

The procedure suggested by Dean Marston would not only entail large expense, particularly if the States imposed on themselves a degree of accuracy commonly required of public utilities, but also raises such difficult questions as who would pay for highways if their reproduction cost fell below their original cost or why, if reproduction cost were to rise above original cost, the user should pay on the higher figure. No State would give up revenue under the first of these circumstances, and probably no court would sustain fees (rates) under the second. The theory has an appeal for some, but as a practical matter the States would do well to confine themselves to recorded costs.

¹³ Further comment on the public utility concept appears at pp. 261-267.

¹⁴ The value of right-of-way was not considered in the so-called "basic cost study" in the Illinois report.

not amortized; "it is a pure gratuity; the principal is never recovered; only the interest on the principal is charged in this study."¹³

The Illinois study employs substantially the same reasoning and sets up a land value of \$584,000,000 on which motor-vehicle users are to pay a return. It adds: "The purpose of demanding a return upon the capital invested in right-of-way is to compensate the State for money permanently invested in land used for the special purpose of transportation. Without such a return, the State would have to furnish such right-of-way free of charge."¹⁴ That the State has made an "investment" in right-of-way, except where it has acquired lands by purchase, as for new motor roads or widening of existing ones, is not supportable. Most of the lands used for highway purposes have been dedicated to the State for public use without direct compensation.

Also, the two reports employ an erroneous premise as to what privately owned public utilities may do under similar circumstances. It is a well-recognized principle of valuation and rate-making that property owned by the public and used by a public utility may not enter the rate base and become a charge upon rate payers.¹⁵ Utilities may include in their investment account or rate base only the costs, of relatively minor importance, incurred in acquiring the right of use.^{15a} It is doubtful, therefore, whether the courts, given a case in which this matter was a specific issue, would sustain a schedule of road-use charges premised on a value of right-of-way dissociated from costs of acquisition. Removal of this item would change the results of the two studies very materially.

The other matter referred to above is the item of taxes. The present report does not include taxes in computing public aid, for reasons indicated at page 52; it does, however, show what such a charge would amount to and thus permits those who wish to do so to

¹³ Report, pp. 33-41.

¹⁴ Report, appendix C, p. 6.

¹⁵ It may be urged that there is an inconsistency here with the setting up, as public aid, of a value for railroad occupancy of public domain for which no payment was made (see vol. II, pp. 48-70), in that no similar value has been assigned to the use of public domain in motor-vehicle operations, particularly those of commercial users. On close examination, however, no difference in treatment will be found. The value of \$118,000,000 set up in volume II for railroad use of the public domain is, as shown by the method in which it was derived, a measure or estimate of the damages accruing to property owners and municipalities without recompense. The use made of streets by railroads and that made by users of automotive equipment cannot be equated to a common basis. The use of streets (or highways) by commercial trucks, so far as usage alone is concerned, is not different from use by owners of privately operated trucks or of passenger cars. Any differential cost requirements as between different classes of motor-vehicle users have been taken care of in the method used herein. Any additional cost requirements necessitated by railroad and other public utility use are reflected in the percentages of total annual cost assigned to other than motor-vehicle users. The value set up in the railroad report should not be confused with such annual cost assignments. It represents an estimate of the value to the railroads, measured by costs incurred by others, of a use completely foreign to the intended purposes of streets and highways, compensation for which in the form of damage payments was overlooked by individuals or communities desiring the rail facilities.

^{15a} They may also, of course, charge operating expenses with paving and similar costs incurred in the use of public facilities.

argue the equities of the terms under which motor transportation competes with other forms of transportation.

Both the Missouri and Illinois studies set up such a charge on unamortized construction costs⁶ and for reasons similar to those used in justifying an interest charge. The former report adds, however: "Whether or not the State, as a matter of policy, should actually include such an item or should make a charge against the private passenger automobile, or other classes of highway users, is not considered in this study. If the legislature elects to eliminate such items from those which it considers in fixing a use charge for the highways, it is not the province of this study to advise or to criticize * * *. This observation also applies to the item of interest as heretofore discussed."⁷

That an effort to collect a tax charge of this kind from the motoring public would be most vigorously resisted cannot be doubted. The question is, however, one of policy and ultimately for the courts to pass on. If an attempt were made to set up such a charge against only commercial users of the highways, it would appear that a basis would be laid for a strong charge of discrimination. In any event, it is not believed likely that any State would make an issue of a tax charge on publicly owned facilities used preponderantly for other than ordinary commercial purposes. The Missouri report casts doubts on the wisdom of such an attempt.

The next important difference between the reports under review and the present study is found in the proportions in which the costs of roads and streets are assigned to motor-vehicle users as a class. It will be recalled that in this study the following percentages have been used: State highways, 1932, 80 percent; 1933-37, 83 percent; county and local roads, 1932, 31.5 percent; 1933-37, 34 percent; city streets, 1932, 28.5 percent; 1933-37, 30 percent.

The Oregon study uses 100 percent for the primary and secondary roads, but states that these matters are "of grave import and affect the allocation of the motorist's dollar on the various classes of roads in the State." It says: "There are many benefits that the general public and the adjacent property owners derive from highways. For instance, the benefits which accrue to urban centers and rural communities due to their being connected by arterial highways are material. Likewise, the benefits in rural districts for roads that merely serve the land mainly accrue to the adjacent owners of that land." This reasoning comports with that used in the present study and casts serious doubts on the validity of the conclusions reached in the Oregon report respecting the failure of heavy vehicles to bear

their share of the costs. The report leaves the matter, however, for further consideration upon completion of the Highway Planning Survey.⁸

The Missouri report concludes that 90 percent of the annual cost of State highways, 66 percent of that of county and local roads, and 50 percent of that of city streets are assignable to motor-vehicle users as a class. These ratios it regards as minima. The subject receives only brief discussion, however, and the treatment presumably reflects the approach of an engineer. "There is," it is said in discussing State highways, "the benefit to pedestrian traffic, to horse-drawn traffic, to utility companies which, by our statutes, are permitted to occupy portions of the right-of-way and indirectly pay for the privilege, to publicly owned vehicles exempt from tax, and to adjoining property. Social, spiritual, and economic benefits may accrue to the public in general, the expense of which might not properly be charged to the motor-vehicle user alone."⁹ This language agrees in part with that used in the present study, though a meaningless contrast is drawn between "social and community" and "economic and commercial" uses. The important question is the values assigned.

These values are derived by comparing the cost of an "average local road" before the motor era with the cost of roads now in use. The present 14,458 miles of State highways would have cost, it is said, \$2,000 per mile, or a total of \$26,916,000. This amount is 9.6 percent of the actual cost of constructing the State highways now in use. Ten percent is used in the subsequent calculations. In the case of county and local roads, the "annual per capita expenditure since the advent of the motor vehicle has been three times the per capita expenditure prior to that time." Two-thirds of the cost of the present county and local road improvements is therefore assigned to motor-vehicle users. Similar reasoning applied to city-street expenditures yields a ratio of 50 percent.¹⁰

The Illinois report reasons along the same lines to substantially the same conclusions. It concedes that State and county roads confer benefits on adjoining property and society in general, that township roads "are maintained primarily for the local land owner and communities," and that city streets "serve a purpose similar to that of rural roads," though the greater degree of their improvement is said to justify a larger allocation of costs to motor-vehicle users. Much is said about the feeder aspect of local roads and city

⁶ Report, pp. 7, 12. An earlier Oregon report, which there has not been opportunity to analyze, reaches quite different conclusions. In *An Analysis of Highway Tax Structures in Oregon*, the Oregon Highway Commission found that only 33.4 percent of the annual cost of main trunk highways and 10.8 percent of the cost of intermediate (secondary) highways and load-service roads should be charged to motor-vehicle users as a class. For city streets it used 18.6 percent. (Technical Bulletin No. 10, 1936, pp. 113-113.)

⁷ Report, p. 18.

⁸ Report, pp. 17-21. It is to be noted that the estimated cost of State highways given above apparently should be \$29,916,000.

⁴ See also p. 264, below.

⁵ Report, p. 30. The reference to interest leaves the reader somewhat uncertain as to what the report recommends on that subject.

streets, and mention is made of the right which any motor-vehicle user has to take his vehicle over any available road.¹¹ Here, again, facts which seem to be of controlling importance are recited and then dismissed without adequate consideration of what they mean.

The allocation in the case of State highways in the two studies does not differ materially from that made in the present study. The other allocations differ very substantially and err through failure to give sufficient consideration to the land-service and community uses made of highways and streets through relying too heavily on deductions drawn from an historical record of expenditures without consideration of its economic and social connotations. The reasoning employed in the present report comprehends more phases of this question of motor-vehicle user responsibility and employs more extensive statistical evidence. The public-utility concept adopted by the writers of the two reports appears to have failed them in this instance, as it, along with the emphasis given the need for treating all roads and streets as a unit, appears to require assignment of 100 percent responsibility to motor-vehicle users.

Possibly the most significant consideration is whether Missouri and Illinois are prepared to raise by motor-vehicle taxation and devote to county and local roads and city streets motor-vehicle taxes commensurate with the responsibility assigned the motor-vehicle users and to reduce real estate and other taxes correspondingly. This practical phase of the matter has disconcerting possibilities.

While there are many other points which could be discussed if space permitted, such as the propriety of the States' assessing charges on investments made by the Federal Government,¹² only three further matters can be discussed in any detail. These are, first, the service lives used in computing depreciation or amortization; second, the differential costs assigned to the larger vehicles; and third, the use of the "ton-mile" basis¹³ in allocating differential costs.

¹¹ Report, pp. 15-17.

¹² Missouri Report, p. 23, and Illinois Report, p. 78.

¹³ The Illinois Report uses both an "increment" and "ton-mile" basis and finds that they give somewhat similar results. The Missouri Report uses the "ton-mile" basis only and refers to the general similarity of results found in the Illinois Report.

In connection with the assignment of differential costs to heavier vehicles, it is of interest to note the opinion expressed by Missouri State highway engineers in response to the questionnaire circulated by the Washington State Highway Cost Commission (Report, 1935, pp. 100-161). This opinion indicated that if only basic vehicles were to be considered in building roads, no reduction in costs arising from width requirements could be made, that concrete pavement costs associated with thickness could be reduced by only 8 to 10 percent, that no reduction could be made in excavation costs, and that a 20 percent reduction could be made in the costs of structures. It was also stated that a reduction in the costs of flexible pavements of 40 to 70 percent could be made if bitumens were used instead of portland cement concrete pavements. Reductions in the cost of maintenance were stated as "unknown." On the basis of these opinions and the distribution of Missouri road costs by elements, it is difficult to understand how the "ton-mile" basis applied to all costs could be used as a means of determining differential cost assignments.

No response was made by the State of Illinois to the aforementioned questionnaire.

Both studies present considerable detail as to the distribution of costs among the various elements of highways and streets and as to the average lives of these elements. However, only in the case of grading on State highways is salvage mentioned; the Illinois Report considers 10 percent of the costs assignable to grading to be salvaged. The precise manner in which allowance is made for this factor is, however, difficult to understand.¹⁴ Both reports arrive at weighted lives for all elements on each of the three systems (primary roads, secondary-local roads, and city streets), and these lives are in turn converted into annual rates for ease of computation. Beyond this step, it is difficult to determine just what the authors of the respective reports had in mind. The annual rates (derived on a straight-line basis) are applied to remaining unamortized construction costs at the beginning of each year in determining the annual amortization charge. That is, an annual rate which, applied to 100 percent of the construction costs to be amortized, would completely remove such costs at the end of the assumed average life, has been applied as a constant percentage to a declining capital sum. The weighted average lives and the annual amortization rates used in the reports are shown at page 285 in juxtaposition with the effective average lives and annual rates as determined from the last year presented in the computations in the respective reports.

The longer the period from the base year the greater the effective lives would become. Theoretically, no expenditure would ever be completely eliminated.¹⁵ Use of this method results in an understatement of the yearly amortization charge. However, since the costs would remain on the books indefinitely, taxes, interest, and amortization charges would be computed for a

¹⁴ In the computation of the weighted average service life shown in table 6, p. 72 of the Illinois Report, no allowance has been made for salvage. Also, in the computation of the "total annual fixed costs of primary road systems," the total construction cost to be amortized is taken to be 100 percent, except as noted below.

¹⁵ As an example of the effect of such a method extended over a long period of time, an initial expenditure of \$100 may be followed through for 100 years, using lives as determined and the method actually used in each report. It will be seen that, after the number of years indicated, the following amounts still remain as unamortized in the investment accounts:

Remaining unamortized cost at end of indicated year following the date of investment of \$100 on the systems shown

End of year	Illinois study			Missouri study		
	Primary	Secondary-local	City streets	Primary	Secondary-local	City streets
10th.....	\$72.04	\$71.62	\$65.02	\$76.61	\$69.24	\$69.24
20th.....	60.02	49.42	40.31	58.96	48.08	48.08
30th.....	54.78	34.11	26.00	43.37	30.50	30.50
40th.....	34.12	23.55	15.49	31.48	20.34	20.34
50th.....	11.64	11.21	5.94	17.40	9.00	9.00
100th.....	2.71	2.55	.88	5.32	1.76	1.76

System	As stated in reports			
	Weighted average life (years)		Annual amortisation rate (percent)	
	Missouri	Illinois	Missouri	Illinois
Primary.....	34.3	27.6	2.92	3.58
Secondary-local.....	25.0	21.4	4.00	4.67
City streets.....	25.0	27.3	4.00	3.64

System	Effective amounts as computed from the last year given in computation of annual amortisation charge ¹			
	Weighted average life (years)		Annual amortisation rate (percent)	
	Missouri	Illinois	Missouri	Illinois
Primary.....	39.7	35.8	2.52	2.79
Secondary-local.....	35.8	33.3	2.72	3.00
City streets.....	35.6	32.3	2.78	3.10

¹ Thus, in the case of the Missouri report the cumulated construction costs assignable to motor-vehicle users and the annual amortisation charges as given in the report for the years indicated, are as follows:

System	Cumulated construction costs	
	Amount	Year through which cumulated
Primary.....	\$245,307,904	1934
Secondary-local.....	165,607,914	1934
City streets.....	90,828,089	1934

System	Annual amortisation charge	
	Amount	Year
Primary.....	\$6,178,830	1935
Secondary-local.....	4,838,139	1935
City streets.....	2,486,629	1935

For the Illinois report the corresponding data are as follows:

System	Cumulated construction costs	
	Amount	Year through which cumulated
Primary.....	\$390,305,828	1935
Secondary-local.....	214,324,139	1935
City streets.....	207,928,081	1935

System	Annual amortisation charge	
	Amount	Year
Primary.....	\$10,830,526	1935
Secondary-local.....	10,830,329	1935
City streets.....	7,968,245	1931

TABLE G-1.—Comparison of results obtained in railroad report with those given in the present report with respect to total annual costs of highways and streets, costs assignable to motor-vehicle users as a class, payments made by such users, and net over- or underpayments

(Note: Amounts given below in parentheses include taxes, which have not been recognized as a proper element in the determination of cost or public aid. Such amounts are shown for comparative purposes only)

[All amounts in thousands]

Period	State highways		County and local roads		City streets		Total, all roads and streets	
	Railroad report	Present report ¹	Railroad report	Present report ¹	Railroad report	Present report ¹	Railroad report	Present report ¹
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A. AMORTIZATION								
1921-32.....	\$1,530,767	\$227,855	\$3,228,028	\$1,057,042	\$2,294,028	\$1,553,478	\$6,600,174	\$2,527,276
1933-37.....	1,174,618	634,094	963,716	634,433	578,700	694,660	2,017,170	2,615,095
Total.....	2,705,385	1,611,949	4,191,744	1,691,475	2,872,728	2,248,138	8,617,344	5,142,371

See footnotes at end of table.
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considerable period beyond the lives stated to have been used in the report. The net result of the understatement of amortization charges and the overstatement of interest and tax charges is an understatement of total annual fixed costs.

Discussion of the second and third points mentioned above is found at pages 297-305, below, and in chap. XII.

Report on Highway Costs, by Breed, Older, and Downs

Readers of this report, hereinafter referred to as the "railroad report," who have followed the text of the present study will have no difficulty in recognizing both the similarities and the differences. The similarities have been accounted for (see footnote 6, page 281). The differences remain for consideration. The major points of comparison, so far as the over-all picture is concerned, are indicated in table G-1, herewith.

Total annual costs of \$31,120,214,000 are found in the railroad report for the period 1921-37; the corresponding costs developed in the present report aggregate \$25,519,089,000, a difference of \$5,601,125,000, or 22 percent.

Taxes.—A large part of this difference is explained by the inclusion in the railroad report of taxes on the investment in roadway facilities. This item amounts to \$2,202,448,000. For reasons indicated at page 52, above, taxes should not be included in the calculation of public aid. They may, however, appropriately be considered in comparing the costs of an agency which pays taxes on all its facilities for the support of the general functions of Government with those of an agency which uses tax-free public facilities. Taxes in the amount of \$1,950,225,000 are shown in table G-1, but are enclosed in parentheses to indicate the special treatment given them. The difference between this amount and that given in the railroad report comes about mainly from the higher unamortized costs which the latter uses, as explained below. The rate used is 1.30 percent, or only a little more than that of 1.25 percent used in the present study. (See page 52, above.) If taxes were included in the present report for other than reference purposes, the costs plus taxes shown in the two studies for the period 1921-37 would differ by \$3,650,900,000, or 14 percent.

AIDS TO MOTOR VEHICLE TRANSPORTATION

TABLE G-1.—Comparison of results obtained in railroad report with those given in the present report with respect to total annual costs of highways and streets, costs assignable to motor-vehicle users as a class, payments made by such users, and net over- or underpayments—Continued

Period (1)	[All amounts in thousands]							
	State highways		County and local roads		City streets		Total, all roads and streets	
	Railroad report	Present report ¹	Railroad report	Present report ¹	Railroad report	Present report ¹	Railroad report	Present report
(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
B. INTEREST								
1921-32.....	\$1,154,056	\$1,210,566	\$2,018,491	\$901,266	\$1,275,533	\$1,648,143	\$4,448,390	\$2,750,975
1933-37.....	1,175,675	1,202,564	872,233	659,038	864,612	969,840	2,012,520	2,067,476
Total.....	2,329,731	2,413,130	2,890,724	1,560,304	2,140,145	2,618,022	7,960,900	6,717,451
C. TAXES								
1921-32.....	\$368,007	\$(266,049)	\$588,122	\$(290,353)	\$390,253	\$(484,748)	\$1,328,382	\$(1,001,150)
1933-37.....	369,618	(383,106)	261,978	(183,073)	204,470	(292,896)	876,066	(869,075)
Total.....	712,625	(739,155)	850,100	(473,426)	594,723	(777,644)	2,204,448	(1,870,225)
D. MAINTENANCE AND OPERATIONS EXPENSES								
1921-32.....	\$1,861,221	\$1,861,221	\$3,149,647	\$3,103,848	\$3,170,407	\$3,064,881	\$8,200,475	\$8,293,750
1933-37.....	1,316,445	1,981,477	958,037	1,200,000	1,261,473	1,768,000	3,430,545	4,019,477
Total.....	3,078,666	3,842,698	4,107,684	4,303,848	4,431,880	4,832,881	11,631,020	12,313,227
E. TOTAL ANNUAL COST								
1921-32.....	\$4,749,071	\$3,999,642	\$8,998,345	\$5,061,956	\$7,130,730	\$7,466,502	\$20,876,148	\$16,527,100
1933-37.....	3,933,169	4,235,691	3,041,554	2,354,815	3,209,345	3,435,759	10,244,068	11,818,260
Total.....	8,682,240	8,235,333	12,039,899	7,416,771	10,340,075	10,902,261	31,120,216	28,345,360
F. ANNUAL COSTS ASSIGNABLE TO MOTOR-VEHICLE USERS								
1921-32.....	4,269,378	3,199,718	8,074,413	1,217,230	3,444,603	1,877,728	16,788,349	8,019,701
Percent of total annual costs.....	89.90	(80.0)	89.75	24.0	48.81	25.1	75.63	48.4
1933-37.....	3,602,759	2,447,461	2,765,428	878,785	1,573,142	1,037,621	7,991,329	6,538,807
Percent of total annual costs.....	90.6	(58.0)	90.6	37.4	48.25	24.0	77.0	46.0
Total.....	7,872,137	5,647,179	10,839,841	2,096,015	5,017,745	2,915,349	24,779,678	14,558,508
Percent of total annual cost.....	90.2	(68.1)	90.0	27.4	48.25	26.6	76.1	51.1
G. MOTOR-VEHICLE USER PAYMENTS²								
1921-32.....							4,031,395	4,069,838
1933-37.....							4,840,086	4,474,931
Total.....							10,881,481	8,544,769
H. UNDER- OR OVERPAYMENT BY MOTOR-VEHICLE USERS³ (= EXCESS OF PAYMENTS OVER COSTS)								
1921-32.....							9,756,954	(1,049,877)
1933-37.....							3,041,243	(5,283,807)
Total.....							12,798,197	(6,333,684)

¹ In the period 1933-37, costs incurred by State highway departments for urban extensions of State systems or secondary roads under the jurisdictions of such departments have been included herein with those of city streets and county and local roads (see p. 158). The effect of such treatment is of relatively minor importance in the comparisons presented for items A to C, but should be considered when judging the differences on individual systems as to items D to F.

² Excludes relatively small amounts reported in Bureau of Public Roads tables as costs incurred for urban extensions of State systems and secondary roads under the jurisdiction of State highway departments (see also pp. 157-158).

³ The sum of the individual items is \$1,038,000 less than the amount given. This difference results from the inclusion in column 14 of table 13 of amounts different from those computed in table 17.

⁴ Estimated sales value of surplus war materials assignable to motor-vehicle users of \$23,000,000 (see p. 80) not included.

⁵ The sum of the individual items is \$7,818,000 less than the amount given. This difference results from several minor errors in the computation of annual costs for the carry-down investment as of Dec. 31, 1920.

⁶ Includes the estimated sales value of surplus war materials set forth in footnote 4.

⁷ Percentages shown have been derived prior to the inclusion of estimated sales value of surplus war materials set forth in footnote 4 and 5.

⁸ Costs assignable to motor-vehicle users were reported in the railroad report as a single item for all highways and roads. The amount given here has been segregated by systems on the same basis as used in that report for the period 1921-32.

⁹ Motor-vehicle user payments segregated by systems are given in the railroad report and the present report; however, such a comparison would merely illustrate the results obtained from different bases of assignment to systems and therefore is not presented here.

¹⁰ Exclusive of "legal" diversions (see pp. 54-55, 80-81, and 180).

¹¹ The amounts given are unadjusted differences. For the effect of adjustment to eliminate the effect of overpayments in any given year see pp. 80-81.

Interest.—Interest charges differ by \$643,000,000. The rates used in the present report were carried into the railroad report. The latter applies these rates to unamortized costs which in the aggregate are higher than those used herein.

Investments in road and street facilities and amortization.—The principal difference between the two reports is found in the amounts set up for amortization or depreciation: \$9,907,993,000 in the railroad report and \$5,852,411,000 in the present study. To explain this difference, consideration must be given to the investment figures used in the two reports and to the treatment of service lives and salvage values.

(a) **Investments in facilities handed down in 1920.**—The investments used are substantially the same, except with respect to the amounts computed as having been invested in road and street facilities at the opening of the period covered by the two reports. Here a very substantial difference is found, as shown below.

Report and item	Estimated investment in roads and streets as of Dec. 31, 1920 (in thousands)			
	State highways ¹	County and local roads	City streets	All roads and streets
Railroad report:				
Estimated original cost.....	\$1,263,013	\$7,715,472	\$2,410,838	\$11,391,970
Depreciated cost to Dec. 31, 1920.....	690,307	3,559,756	1,470,757	4,961,550
Percent depreciated of original cost.....	50	46	61.0	
Present report:				
Estimated original cost.....	\$790,703	\$464,793	\$4,096,137	\$5,351,633
Depreciated cost to Dec. 31, 1920.....	\$443,373	\$300,528	\$2,142,845	\$2,846,646
Percent depreciated of original cost.....	56.7	64.6	52.5	

¹ As subsequently defined.
² Represents estimated construction expenditures, plus interest during construction, for the period 1904 to 1920, amortized on the basis of a 20-year life.
³ Excludes 72 percent of the cost of unimproved earth and partly graded roads, amounting to \$66,334,000 in terms of original cost. See p. 59.
⁴ Excludes 72 percent of the cost of sand clay and lower type roads, amounting to \$34,633,000 in terms of original cost. See p. 59.
⁵ See pp. 71-72. The amount given is the estimated original cost of streets as of July 1, 1913, plus construction expenditures made subsequent thereto. Owing to the fact that some of the facilities in existence in 1913 would no longer have been in existence in 1920, the original cost of facilities in existence on Dec. 31, 1920, would have been somewhat less. For this reason, the percent depreciated of original cost is not given. The difference between the estimated original cost used in the railroad report and that used herein would therefore be less than is indicated by the figures given above.

The estimates in the railroad report of the original cost of State highways and county and local roads in existence in 1920,¹⁶ after omitting differences arising from the exclusion of lower type roads,¹⁷ exceed those used in the present report by \$403,917,000 and \$6,187,996,000, respectively. On the other hand, the carry-over costs of the city streets of this period found in the present report are somewhat higher. The exact difference cannot be stated.¹⁸

The difference of \$403,917,000 in the case of State highways may be broken down as follows (in thousands):

Road type	Railroad report	Present report	Difference
Surfaced roads.....	\$981,435	\$703,351	\$278,084
Non-surfaced roads.....	370,576	154,745	215,831
Total.....	1,351,011	858,096	492,917

¹⁶ State highway systems as such were not laid out until 1921; reference above is, of course, to the roads in existence in 1920 which went into such systems.
¹⁷ See footnotes 3 and 4 of tabulation above.
¹⁸ See footnotes 3 and 4 of tabulation above.

In both reports the estimated cost of the various types of roads is based upon average costs of Federal-aid roads constructed during the period 1917-24 and in both instances reductions have been made to reflect lower price levels and lower types of construction.¹⁹ However, the railroad report has made insufficient allowance for these factors. Its deduction of only 25 percent from road-building costs which were at an all-time peak for the past three decades is entirely insufficient to compensate for the lower price levels of earlier years and leaves nothing to reflect the lower standards of construction of State highways constructed prior to 1921 than of those built from 1917 to 1924. Thus, the annual report of the Bureau of Public Roads for 1920, page 2, commented on the high construction costs prevailing in 1920 and made the following comparisons of the costs of some of the principal road types:

Road type	1917 ¹	1920
Earth.....	\$3,100	\$4,100
Sand clay.....	2,400	4,606
Gravel.....	4,836	7,250
Concrete.....	21,165	40,000
Brick.....	35,000	53,000

¹ These costs may also be compared with those prevailing some years prior to 1917, presented at pp. 18-19 herein.
² Although not stated, this designation undoubtedly refers to graded and drained earth roads.

Contributing to the above-mentioned major error is the overstatement in the railroad report of the mileage of portland cement concrete roads in existence on December 31, 1920. This excess arises from estimating road mileages on a basis which is not as comprehensive as that used in the present report (see p. 57).

Another important source of difference is found in the amortization charges set up for facilities in existence on December 31, 1920. This difference results from the elimination from the investment used in the present report of the cost of unimproved earth²⁰ and partly graded roads (with the exception of 28 percent estimated to represent the cost of structures and certain minor items pertaining to these roads which are not replaced by yearly maintenance work). Inasmuch as the only classifications of work recognized in the case of such roads are grading and structures (aside from the aforementioned minor items), any work not representing the latter classification must be the former. Therefore, any work which provided mere passability, such as blading or dragging, and which is generally repeated yearly, would be capitalized in the

¹⁹ See pp. 57-58, above.
²⁰ Unimproved roads have been defined for purposes of the highway planning surveys as "a road composed wholly or with minor local exceptions, of the natural ground of the region traversed, which may or may not have been bladed, which does not conform in respect to alignment, grade and drainage, at least to the definition of a 'graded and drained earth road,' and on which the only work that has been done by public authority is that required to maintain a condition of bare passability for horse-drawn or mule vehicles." (Bureau of Public Roads, Planning Survey Memorandum No. 305, Nov. 23, 1920.) [Italics supplied.] It is to be noted that this definition reflects a "modern" idea of what constitutes an unimproved road. In earlier days, around 1920, such roads may have had a higher classification.

railroad report. This procedure results in charging amortization on "grading" and at the same time in the setting up of a yearly charge for maintenance work, though such work would be substantially a repetition of the first cost. The costs of such roads have been capitalized in the railroad report at the unreasonably high figure of \$2,000 per mile.²¹ As these costs represent a fairly large item in the total, a significant overstatement results.²²

As stated above, the original cost of county and local roads in existence on December 31, 1920, presented in the railroad report is \$6,187,996,000 in excess of that used herein. No detailed explanation is given of how the estimated original cost of \$7,719,472,000 shown in the railroad report was derived. It is stated that an estimated average cost of \$2,973 per mile was used and that for other than earth roads a one-third reduction of the 1917-24 costs of Federal-aid roads would give costs which are representative of standards of construction of such roads. On these bases, an average cost of about \$2,000 per mile apparently was applied to about 2,300,000 miles of earth roads, of which about 2,100,000 miles represented unimproved earth and partially graded and drained roads. When consideration is given to the generally much lower standards of construction of county and local roads of the period prior to 1920 and to the fact that many of the more important county roads had been transferred to the State systems, the average cost per mile used is far too high. Many miles of county and local roads were little more than wagon tracks at that time, and in some cases probably existed on paper only.²³ It is doubtful whether any capital costs have ever been incurred on a substantial part of this system. The only expenditures incurred were those required to maintain the road in a passable condition and in many cases even this much was not done.

As in the case of State highways, only 23 percent of the cost of unimproved and partially graded roads has been included in the investment shown in the present report as carried into the period beginning with 1921. In addition, and for the same reasons, a similar percentage of the costs of sand clay and top soil, and improved earth roads has been included in the costs to be amortized over subsequent years. Although such roads are of a higher type than unimproved roads, it is felt that the allowance of 23 percent used herein is adequate to reflect the costs of improvements which were not repeated by yearly maintenance work. The entire

estimated original cost of these latter types of road has been included herein in the case of the State system for the reasons that grading quantities were much heavier on these more important roads and that in many instances these roads were only a step toward a higher type of improvement.

The difference between the two reports in the 1920 investment in city streets is somewhat less than would appear from a comparison of the figures presented at page 287. The explanation is given in footnote 5 of the tabulation on that page. The costs used in the railroad report cover only the period 1904 to 1920, whereas those used herein are intended to cover a longer period and to include all expenditures for facilities which were useful in 1920.

(b) *Investment, 1921-37.*—As noted previously, the investment figures for the period 1921-32 and 1933-37 are generally the same in the two reports, except that in certain instances those used herein include some items which are not covered in the railroad report. The amounts used are compared below (in millions):

Report and period	State highways ¹	County and local roads	City streets	Total
Railroad report:				
1921-32	\$5,552	\$3,244	\$4,832	\$13,628
1933-37	2,642	654	811	4,107
Total	8,194	4,198	5,643	18,035
Present report:				
1921-32	\$5,643	\$3,282	4,317	\$13,242
1933-37 ²	2,481	1,091	907	4,569
Total	8,124	4,373	5,224	17,811

¹ Amounts given represent net construction expenditures; i. e., after refunds for construction work performed for others.

² Includes 75 percent of the expenditures by the Federal Government on roads in public domain (see pp. 66-67).

³ In the period 1933-37, the construction costs reported by the Bureau of Public Roads for urban extensions of State highways and for secondary roads under State control are included herein with city streets and county and local roads, respectively.

Considering the magnitude of the foregoing figures and the many estimates which were resorted to in the computation of certain of them, the data as used in the two reports are in reasonable agreement. The only difference of importance is in the amounts used for city streets in the period 1921-32. Those used herein are taken from a Department of Commerce publication, *Construction Activity in the United States, 1915-32*, and, upon check, were found to be reasonably reliable. (See p. 72.)

(c) *Salvage values.*—Neither report recognizes salvage in the case of roads in existence prior to 1921. In the subsequent periods the railroad report allows 30 percent for salvage in the case of intermediate type pavements on all road and street systems, but it makes no allowance in the case of other pavement types. For use herein, salvage values of 25 percent and 40 percent were adopted for intermediate and high type pavements, respectively.

Some engineers regard salvage as a "myth" and others are influenced in their opinion as to what it amounts to

²¹ See p. 287.

²² Such roads represented approximately 80 percent of the total State highway mileage in existence on December 31, 1920, but accounted for only 13.5 percent of the total estimated cost of all State highways shown in the railroad report. In the case of county and local roads, however, failure to exclude capital costs for roads of this type introduces a major error.

²³ Thus, in the mileage inventory of 1921 and 1922 (U. S. Department of Agriculture Bulletin No. 1379, *Rural Highway Mileage, Income, and Expenditures, 1921 and 1922*, p. 44) the mileage of county roads given for Oklahoma, is stated to include "about 26,000 miles of section lines declared public highways by law but not generally used."

by their own particular experience. Little published information of a national character is available. However, such information will ultimately become available as a part of the highway planning surveys. In deriving the estimates used herein, considerable use was made of opinions developed from preliminary results of this work.³⁴ For this reason, it is believed that a much broader basis of estimate was available in the preparation of this report than that used by the authors of the railroad report.

(d) *Service lives.*—Similar lives were used in each report in amortizing the carryover investment in State highways. Somewhat different lives were used in the case of county and local roads and city streets carried down in 1920, as shown below:

Element	Railroad report		Present report	
	County and local roads	City streets	County and local roads	City streets
Surfacing.....	11	20	10	25
Grading.....	30		30	
Structures.....	15		25	

On the basis of the distribution of the original cost among the various elements of county and local roads set out at page 68, herein, an average age of 19.0 years would be obtained for the lives used in the railroad report, or about one-fifth less than that used in the present report. The life of 20 years used for city streets is about 43 percent less.

A longer life than that used for State highways for grading (protected by a wearing course) on county and local roads is justified when the uses of such roads and their lesser susceptibility to obsolescence are considered. Although physical depreciation is greater on the type of structure found on county and local roads than on structures provided on State highways, the smaller influence of obsolescence justifies the life used of 25 years.

The life of 20 years for city streets used in the railroad report is not sufficient to cover surfacing, much less the more permanent items of curbing, gutters, grading, and structures. Furthermore, the relative costs of these items were greater in the period prior to 1921 than they have been in later years.

The assumptions as to the useful life remaining in the various elements of the carry-over investment also differ. In each case, except that of city streets, the railroad report assumes a 50-percent condition as of December 31, 1920. With the exception of surfacing and structures on county and local roads, higher percentages have been assumed in the present report.³⁵ Taken by itself, this difference has no effect upon the yearly amortization charge, but the higher percentage used herein spreads a large part of the estimated original

cost of such facilities over a greater number of years during the period beginning with 1921.

The service lives used in the two reports for road and street improvements made between 1921 and 1937 are given below:

Service lives used for road and street improvements during the period 1921-37, in the railroad report and in the present report

Element	State highways		County and local roads		City streets		
	Railroad report	Present report		Railroad report	Present report	Railroad report	Present report
		1921-32	1933-37				
Surfacing:							
High-type.....	20	24	26	20	25	20	25
Intermediate type.....	10	12	12	10	12		
Excavation.....	50	55	75	50	65		
Structures.....	20	20	25	20	25		
Average.....	29	40.9	51.6	33.4	27.0	29	38
Percentage of costs amortized.....	95.9	79.2	88.2	93.1	89.2	95.9	77

¹ Includes bituminous macadam.
² The lives given are those assigned to bituminous macadam, waterbound macadam (surface treated), and gravel roads, reading from top to bottom, respectively. (See p. 70.)
³ Grading under nonsurfaced roads, including those surfaced with sand-clay or topsoil, has been, in effect, treated as a maintenance item. (See p. 70.)
 It is to be noted that the life used for grading is the same as that used for State highways in the period 1921-32. For reasons indicated in footnote 4 of table 59, no change was made in the lives of the various elements on county and local roads for the later period. However, as pointed out at p. 70, longer lives could be justified.
⁴ Cost less salvage value.

The weighted average lives shown above differ quite widely as between the two reports. The principal differences lie in the lives assigned grading and structures and in the treatment of grading on county and local roads, previously discussed (see above). Differences in lives assigned surfacing are on the whole relatively minor, but those used in the railroad report are generally somewhat lower than those considered reasonable for use in the present report.

The railroad report used a weighted average life for city streets which is 24 percent less than the 38 years used herein. Obsolescence in the case of the general run of city streets is of much less importance than in the case of State highways. Street realignment is for obvious reasons practically impossible, and with the exception of possible express highways, street widening, which also is restricted, is the only remedy for traffic congestion. On the other hand, long-lived elements, such as grading and structures, have constituted a smaller portion of the construction costs of city streets than they do of the costs of State highways in recent years. All factors considered, the life of 29 years used in the railroad report is far too low.

It may be noted, in this connection, that the railroad report apparently has given very little weight to the effects of improved planning of highway improvements or to the higher types of construction of more recent years, as the lives assigned to grading and structures provided in the years 1921-37 are substantially the same as those used for improvements in existence on

³⁴ See p. 62.
³⁵ See pp. 58 and 68.

December 31, 1920.²⁶ Obsolescence and limiting economic life are the sole factors entering into the determination of the life of grading.²⁷ In the case of structures on State highways, obsolescence is also the principal factor where such structures are of more or less permanent construction. If grading and structures which were designed for speeds and traffic volumes far below those which influenced design between 1921 and 1937, can be assigned lives of 50 and 25 years, respectively, much longer lives should be used in this later period.²⁸

As indicated at pp. 61-63, considerable preliminary material gathered in connection with the highway planning surveys was used in the final preparation of the present report. Although the greater part of this material related to the lives of surface types, some general opinions as to other elements were obtained. It is felt that the lives used in the present report are reasonable.

(c) *Summary.*—The total difference of slightly more than \$4,000,000,000 in the amortization charges given in the two reports results from marked overstatement by the authors of the railroad report of the early investment in rural roads, particularly county and local roads, from the failure to recognize salvage on certain types of road surfacing, and from the use of service lives for the various highway and street elements which are generally too short.

Maintenance and operation expense.—The costs found in the present report for the period 1921-37 exceed those given in the railroad report by \$1,309,207,000. Of this difference, \$710,183,000 arises from the inclusion herein of the estimated cost of city traffic police for the period 1921-37, and of State highway patrols for the period 1933-37. These items were recognized in the railroad report as properly includible but were assumed to be largely offset by certain revenue payments which were not included (see p. 51 of that report). This assumption is not justified. The greater part of the remaining difference arises from the manner in which maintenance and operation expense has been estimated for cities of less than 30,000 population (see pp. 44 and 46 of the railroad report and pp. 72-73 herein), and from the treatment of relief expenditures in the period 1933-37 (see p. 101 of the railroad report and pp. 155-158 herein).

²⁶ Grading on State highways and county and local roads has been assigned the same life (50 years) and the life assigned structures on each of the systems has been increased 5 years over that used for facilities in existence on December 31, 1920. The life assigned structures on county and local roads is in reasonable agreement with that used herein.

²⁷ It may be noted that in valuation proceedings before the Interstate Commerce Commission, the railroads strenuously urged that their grading has an indefinite life. Obsolescence is relied upon to justify the 50-year life used in the railroad report. (See p. 28.)

²⁸ State highways only.

Annual costs assignable to motor-vehicle users.—Large differences are to be noted in the percentages of responsibility for road and street costs used in the two reports.²⁹ These differences are indicated in section F of table G-1 and are summarized below. They are discussed further at pages 291-297.

	Percentage of total annual costs assigned to motor-vehicle users as a class, 1921-37			
	State highways	County and local roads	City streets	Total
Railroad report.....	90.2	90.0	48.25	76.1
Present report.....	81.3	27.4	23.9	40.6

¹ Exclusive of taxes.

These differences, together with the overstatement of annual costs previously discussed, produce costs assignable to motor-vehicle users which are \$13,321,110,000³⁰ in excess of those found herein. This excess is distributed by road systems as follows: State highways, \$2,184,968,000, county and local roads, \$8,733,786,000, and city streets, \$2,402,356,000.

Motor-vehicle user payments.—The total motor-vehicle user payments found in the railroad report are \$337,212,000 in excess of those used in the present report. Certain omissions are stated to have been made in the railroad report because of lack of definite information; however, certain deductions made in the present report cause the payments found herein to be smaller. The principal items not included in the railroad report but which have been considered in the present report and

²⁹ It may be useful in this connection to have in summary form the percentages of motor-vehicle user responsibility for road and streets found in a number of reports. Use has been made in part of table 1 of the report of Messrs. Dillman and other referred to at p. 281, above.

Class of highway	Present report, national basis, 1937 ¹	Duncan, national basis, 1935 ²	Breed, Older, Downs, national basis, 1938 ³	Ennis, New Jersey, 1935 ⁴	Missouri, 1936 ⁵	Illinois, 1937 ⁶	Oregon, 1936 ⁷
Main trunk.....	83	82	90.2	85	90	90	85.6
Intermediate.....	54	53	90.0	86	96	90	10.9
Local service roads.....	34	82	90.0	85	96	90	10.9
City streets.....	30	25	48.3	51	80	50	18.5

¹ P. 159, above.

² Economist for Association of American Railroads, in Highway Competition, p. 6.

³ Report on Highway Costs, p. IX.

⁴ Report mentioned at p. 281, above.

⁵ Weitzel report, pp. 20-21 (see p. 281, above).

⁶ Glover report, pp. 11-13 (see p. 281, above).

⁷ Oregon Highway Commission, An Analysis of Highway Tax Structures in Oregon (Technical Bull. No. 10, pp. 112-113).

There is reasonably close agreement in case of the percentages shown for State highways; marked differences exist in the case of the lesser rural roads; four of the reports agree on 80 percent or thereabouts in the case of city streets, the percentage used in the present report falling about midway between the extremes.

³⁰ Prior to inclusion of \$25,000,000, the estimated sales value of surplus war materials, or to other minor adjustments. (See p. 80, herein.)

the page or table references to the present report are as follows:

Item	User pay-ments	Deduction from user pay-ments
Miscellaneous State taxes (tables 28 and 61).....	\$162,180,000
Estimated county and local, and municipal govern-ment taxes (table 28 and 61).....	202,267,000
Estimated personal property taxes included in regis-tration fees (p. 53 and tables 28 and 61).....	\$188,888,000
"Legal" diversions (pp. 54-55, 81 and 160).....	240,437,000
Costs of administration of State gasoline and registra-tion fee collections (1923-27 only).....	142,642,000
Total.....	364,437,000	669,964,000
Net difference.....	315,527,000

¹ In the railroad report, costs of administration were deducted from user payments in the earlier period but not in the later.

In addition, several relatively minor sources of revenue, not considered as motor-vehicle user revenue, were excluded from the present report.²¹

Under- or over-payment by motor-vehicle users.—The railroad report finds a net underpayment by motor-vehicle users of \$12,798,197,000 for the entire period 1921-37, as compared with the net overpayment of \$160,701,000²² found herein. If taxes be included, the overpayment as found in the present report becomes an underpayment of \$760,443,000. The amounts first given represent a difference of \$12,958,898,000; if taxes were to be included in the present report, the two reports would differ by \$12,037,754,000.

These differences result from a combination of all of the items previously discussed, but principally from a marked overstatement in the railroad report of the responsibility of motor-vehicle users for road and street costs, use of costs far too high for rural roads in existence on December 31, 1920, and the inclusion of taxes as an element of highway and street cost. Also contributing to the overstatement, but to a less important degree, are the use of service lives which are too short and failure to recognize salvage on certain types of road surfaces.

Assignment of responsibility for road and street costs.—The assignment of responsibility for road and street costs between motor-vehicle users and others and the inclusion of the interest and tax charges mentioned earlier are made to rest, in the railroad report, on the so-called "public utility concept." The reasoning employed may be summed up as follows:

A public utility is an enterprise which "functions as an agency of the State to perform a service to citizens of the State. Such an enterprise sells its services to the general public in order to obtain the revenue for its support. Highways and streets fall clearly within this definition of a public utility," and have always been of this character. The early toll roads, the railroads, and the tramway and electric streetcar have been proto-

types. Each provided its own facilities and users were charged for their support rather than the taxpayers. "When the motor vehicle appeared it was only another form of streetcar possessing all the commercial possibilities of the electric car" but differing only with respect to the character of the ownership. The motor vehicle needed "tracks." Cities had already provided such tracks, "largely for commercial purposes," but none that were adequate were available in rural areas. Free use of streets prevented the establishment of toll roads, as did the small number of vehicles and "a strong element of public opinion * * * opposed to the inconvenience of such a method." Initially, reliance had to be placed on general taxes, but soon "public policy demanded that highway financing follow the principles of the public utility concept." Special motor-vehicle taxes, "in effect, a form of toll," were resorted to and "as the number of vehicles increased and their commercial aspects became more clearly recognized, the charges were increased * * *. Taxpayers became more and more reluctant to furnish public money for highway improvements, which were so clearly an economic or commercial enterprise." (Report, pp. 1-2. Expressions italicized will be especially commented on later.)

There are, however, "other reasons of an economic nature why the highways should be managed and financed as a public utility enterprise." (a) " * * * the capital invested * * * serves to produce wealth. This wealth benefits all the people but it does not distribute itself uniformly among the citizens or in proportions equitable with the payment of general taxes. It is distributed approximately in proportion to highway use or services rendered * * *. Consequently, the cost * * * should be paid for in proportion to highway service." (b) "Another economic reason" is that, unless the public utility concept is applied, the several forms of transportation will not be on an equal footing. The result will be detrimental to the general welfare. (P. 3.)

"The public utility concept as applied to highways is widely accepted." (Pp. 3-4, 10.)

Several "principles" flow from the public utility concept: (1) To avoid subsidy, " * * * the costs shall be paid for entirely by users"; (2) " * * * the costs shall be distributed among the users * * * on an equitable basis through the rates charged"; (3) " * * * the enterprise is a monopoly within its own field of service"; (4) " * * * the type and character of the facilities must be adjusted to the kinds of use which are economically justified"; that is, the facilities must be limited to those which can be supported by the rates which can be borne by the users; and (5) "the public utility shall be considered as a system in which all parts work together cooperatively for common service * * * in union there is strength. (P. 4.)"

Physically, all highways and streets in a State belong to one system. The motor-vehicle user recognizes no

²¹ These amounts account for the greater part of the difference between \$314,527,000 (difference between \$669,964,000 and \$354,437,000 above) and the amount set forth in the text above of \$337,213,000, and consist principally of other receipts in connection with the motor-fuel tax, such as distributors' and dealers' license, inspection fees, etc.

²² The amounts given in the present report are not adjusted to eliminate the effect of overpayments by motor-vehicle users. (See pp. 80-81 and 159-160.)

political boundaries and is not concerned about the class of highway he uses. "He expects to use all the roads when he so desires and *hopes to find them all serviceable.*" All roads and streets are dependent on one another; "*city streets represent terminal facilities for the trunk highways and the secondary rural roads are branch lines or feeders * * **" Railroad trunk lines are similarly dependent on terminal facilities and feeder lines. A State is a separate legal entity and it alone must solve the problem of financing the highways. Political subdivisions mean nothing in legal contemplation; "*all of these road revenues, except Federal aid, were determined by acts of the State legislature.*" All roads in a State are in reality State roads, all should be treated as a single public utility, and all revenues collected from motor-vehicle users must be set against the costs of all parts of this public utility. Branch lines and terminals can not be expected to be separately self-supporting, "nor is the volume of traffic over a road, much less the origin of traffic, any criterion for the method of financing such facilities." Not only are "the so-called local highways * * * important 'feeder' or 'terminal facilities' but their ready-to-serve value to the motor vehicle is very real * * *. *The motor vehicle has removed the local use concept from these highways * * *. Distinction between classes of roads and even city streets and rural roads is fast disappearing. It is recognized that all classes of roads are essential for the full service of the highway and street facilities.*" (Pp. 4-7.)

"Highway improvements on all and *any part* of the public utility are made *solely for transportation purposes*" and must be justified by the "transport benefits" to be derived from not only the particular improvements but all other road and street facilities in the State. *Such benefits are measured by what the traffic will pay.* However, despite the fact that more will be spent on some parts than on others, the cost, distributed on "some equitable unit such as a vehicle mile or a ton-mile" will be "much higher" on the less-used roads. However, the total cost divided by total traffic will give a unit cost which can be allocated among all uses. "It follows, therefore, that if the highway management and finance are on a sound basis, the traffic on *any part* of the public utility, paying rates equitably proportioned, will meet its full responsibility for all the highway improvements on all parts." (Pp. 7-8.)

The farmer should pay no land taxes for road purposes; *he pays sufficiently through special motor-vehicle taxes.* Land may benefit by road improvements, but "justification for the improvements must depend upon the savings in transportation cost and not upon land production or increased land values * * *" (P. 8. See also below.)

"Of course, when a highway is improved for motor-vehicle traffic, the *other uses* represented by animal or foot traffic *may* continue and the *general public*, other

than motor-vehicle owners, *may thus profit by the improvement.*" However, they may suffer inconvenience, as witness dangers inherent in mixed traffic and the burden put on those who place conduits under streets. "Light and air requirements within the cities have nothing to do with street improvements. They demand only space * * * an open field * * * or the open right-of-way." Paving in cities contributes to health, but "the motor-vehicle *uses the pavements*, requires all the facilities, and city dwellers receive no greater benefits from the paving than motor vehicles receive from the storm sewerage or drainage system for which they do not pay." (Pp. 8-9.)

"National defense is no reason for highway improvements any more than it is a reason for building railroads, telegraph, telephone, electric, or any other public-utility service. In case of necessity all the national resources will be used for defense." (P. 9.)

Public vehicles (school busses, ambulances, police cars, fire trucks, etc.) "are instruments of social use and as such their use of the highways justifies a charge against general taxes for the support of highway facilities." But such taxes should be merely proportionate to the number of such vehicles in relation to the total of all motor vehicles. "It is only a question of public policy whether the license fees and gasoline taxes for such vehicles are to be paid directly into the public treasury or the amount credited to the highway transport industry." Postal vehicles, usually privately owned, pay motor-vehicle taxes in the same way as any other like vehicles and, if these taxes are passed on to the Government, the latter is paying as any user is. (P. 9.)

"The question of special benefits to the owners of private property by reason of close proximity to highway improvements admits of no consideration in the allocation of highway costs for rural roads." The improvements are made to meet traffic requirements and not to benefit land. The same is true of railroads. Private property is as often damaged as benefited by the close proximity of trunk highways. The benefits or damage do not affect the productivity of the land. If land values are increased, "it may be assumed that the extra use of the highways by vehicles engaged in transporting the products will pay their fair share of the highway costs." If highway costs are charged against the land, then the costs of railroads and other public utilities should be similarly charged. (Pp. 9-10.)

Use of general taxes in the early days of motor vehicles and the continued use of such taxes for certain road purposes reflected a policy which "was not in accord with public utility finance and accounting. *The problem is now to place highway financing on a public utility accounting basis.*" The change in conditions was gradual. "Those uses of the highways which existed before the motor vehicle appeared gradually merged into the economic aspects of the motor vehicle

and finally disappeared in the public utility concept, just as the old town pump disappeared when the modern water supply came into existence as a public utility." Facilities provided "primarily for the benefit of motor vehicles" during this period of transition in many instances "constitute real values in the present highway utility." All should be charged against motor-vehicle users except "the old rights-of-way, only, [which] constitute the government's real responsibility with respect to the costs of the present highway utility." These rights-of-way may be used by any public utility enterprise (water, gas, motor-vehicle, etc.) and should be charged against such users according to the costs of the improvements which they respectively require. "They also furnish access to land, light, and air; and may, therefore, be classed as a common social requirement." However, "all highway improvements such as grading, drainage structures, bridges, and surface improvements are essentially a motor-vehicle requirement [and] the motor vehicle should pay all the costs of such facilities." (Pp. 12-13.)

Question remains as to when motor-vehicle responsibility began. The so-called Salter Report (1932) is cited as having declared that the responsibility of motor-vehicle users for the aforementioned "legacy from the past" was cancelled by the "community uses" then made of the roads and streets of England and that all future road expenditures should be borne by the motor vehicle. The report of the special Royal Commission on Transportation of the Province of Ontario, Canada, rendered in 1938, is also cited as reflecting the public utility concept and as having used the year 1918 as the point at which motor-vehicle responsibility begins. This report recommends that \$3 per capita be set up as a charge against general taxes for the so-called "social necessity value of roads and streets." (Pp. 13-14.)

The railroad report adopts the "basic period plan." This plan "is to determine government expenditures for highway improvements on some equitable basis prior to the appearance of the automobile which is to be compared with expenditures for improvements thereafter made to accommodate it. If a period or a representative year can be found which fairly marks the beginning of this transition, the expenditures of that time can be used as fairly measuring the money value of those traditional uses of the highway before the motor vehicle became the predominant factor. Certainly, all the costs beyond this amount are chargeable to the motor vehicle." (P. 14.)

The improvements brought down from the pre-motor period outweigh "any so-called traditional use that is not directly a motor-vehicle charge." As these improvements have decreased in value through depletion and obsolescence, "so have the so-called traditional uses decreased due to the predominance of the motor vehicle. Consequently, there should come a time when the motor vehicle should properly be charged with all

highway and street costs, excluding rights-of-way and sidewalks." However, owing to lack of data, it has not been possible to assign 100-percent responsibility even in 1937. The resulting "error benefits the motor vehicle at the expense of the taxpayer, because it leaves the highways still a subsidized public utility." This "major concession to those who contend that there are some highway costs which do not clearly benefit the motor vehicle" is made because "there is no way to isolate these costs if they do exist; nor is there any way to prove that all the so-called traditional uses which included important commercial uses, have been merged in the public utility aspects of the present highway system." (Pp. 14-15.)

Highway expenditures in several States and probably in the United States were for many years "quite uniform in proportion to population." The per-capita expenditure "did not increase appreciably for many years prior to the appearance of the motor vehicle. This per-capita expenditure before the motor-vehicle era may well be taken as the measure of so-called traditional uses, inasmuch as it represents the amount which the people were willing to spend for highway facilities when there was no motor-vehicle influence." It is "futile to speculate as to what would be the highway costs today and who would pay for them if the motor vehicle had never existed. * * * The relation between the average annual per capita highway expenditures in a period before the motor-vehicle era and that in a period of motor-vehicle operation may be assumed to represent the *maximum* proportion of the annual highway cost for the later period that can be charged to the so-called social or community requirements." (Italics in original.) The year 1904 satisfies the conditions stated and is used as the basic year. It is the first year for which data as to highway expenditures in the United States as a whole are available. There were only 55,000 motor vehicles in that year, but the increase thereafter was rapid. "The real significance, however, is not in the rapid rise of highway expenditures or in the rise of motor-vehicle registrations, but in the fact that men began to think of road and street improvements in terms of the motor vehicle, when this influence became a factor in the amount of money expended for highways and streets. * * * There is abundant evidence to prove that the motor vehicle had stimulated interest in road improvement before 1904 and its effect on road expenditures after that date was very marked. * * * The expenditures for 1904 are, therefore, considered a generous measure of the monetary value of traditional uses and are accepted * * * as representing the per-capita expenditure which the people were willing to make for highway and street facilities without motor vehicle influence." For rural roads this per-capita figure was 97 cents or, rounded off, \$1. " * * * this method gives the highway users a free right-of-way, appraised in 1904 at

\$341,899,306, which increased in value with settlement and development of the country." (Pp. 15-21.)

Street expenditures of cities of 30,000 and greater population were \$5.32 per capita in 1904, or much more than the 97 cents found above. This difference "leads to the conclusion that the higher degree of improvement of city streets which existed in 1904 offered facilities which were satisfactory for the use of the multiplying motor vehicles in the course of succeeding years. It was not until the highly intensified use of these facilities by motor vehicles and the increase of their unit weight within the period just preceding and following the World War had either greatly damaged them or rendered them inadequate and unsatisfactory that a rapid increase in per-capita expenditures occurred. For this reason a fair and reasonable conclusion is that street facilities provided by a relatively stable per-capita expenditure in the pre-motor vehicle days were far beyond those defined here as community uses. * * * No doubt pre-automobile standard of street facilities extending into the period under consideration furnished services to motor-vehicle users for which they should pay a part of the cost. There is no feasible method, however, to reduce the per-capita expenditure in 1904 to a figure more truly representative of the cost of facilities that community use alone would require. * * * Rather than assume an arbitrary division until better data are available, it is decided for this report to charge against the taxpayers all the per capita expenditures in 1904 as their obligation." (Pp. 21, 23-24.)

No adjustment need be made for variations in price levels in the period subsequent to 1904 as such variations "could have had little or no effect on the amount of money which was expended for highways. The best measure of the willingness of the public to provide highway facilities is the actual number of dollars appropriated or expended rather than the amount of work that was done." Also, "improvements made in the equipment used and in the art of roadbuilding, including better design, would tend to balance any increase in costs of labor or material." (Pp. 25-26.)

This somewhat lengthy exposition of the reasoning of the three engineers has been necessary for a clear understanding of the issues presented.

It will help, at the outset, to have a clear idea of how far the public utility concept advanced in this report has been accepted. The report states (p. vi) that it has been "approved by many authorities * * * and sustained by the courts as applicable to highway finance and to charges to users for highway service." The authorities cited (pp. 3-4) include Mr. Roy W. Crum, Director of the Highway Research Board, and Dean-Emeritus Anston Marston, of Iowa State College. It is true that these men have advocated this concept, but it is also true that it has gained little or no accept-

ance among highway engineers.³³ Other authorities cited (p. 10) are W. S. Downs, himself an author of the railroad report; W. D. Ennis, whose report, as noted above (but not in the railroad report) was written for the Associated Railroads of New Jersey; the Werbitzky and Glover reports, written under the special conditions described at page 281; the decision in *Brashears v. Hughes*, which was based on a record that admittedly was weak with respect to the evidence presented by the affected motor carriers and which therefore was not capable of appeal; the companion Missouri decision, which did not involve use of the underlying Werbitzky study; the Salter report, which, while directed primarily at bringing about an equalization of competitive conditions between rail and road transport and to that end seeking to impose costs on road users comparable to those borne by railroads, did in effect make a very substantial allowance for "community uses," and which, by reason, among other things, of numerous protests occasioned by the failure of the committee to consult with road users other than motor-freight carriers and by the nature of the reasoning and conclusions, was not adopted by Parliament in revising motor-vehicle tax rates; and finally the report of the Royal Commission on Transportation of the Province of Ontario, which openly and in effect rejected the public utility concept.³⁴

Studies which have not employed the public utility concept are not cited in the railroad report.

Therefore, the statement (p. 10) that "whenever studies have been made to allocate highway costs, the public utility concept has appeared and its principles have been followed," is misleading and erroneous.

There appears in this connection one of several inconsistencies or confusions of statement which make it difficult to appraise the report. In one place the writers state that soon after the appearance of the motor vehi-

³³ Comment on Dean Marston's view appears at p. 232, above.

³⁴ "It was urged on behalf of the Railway Association of Canada that the highway system of the Province should be considered and treated as a public utility would be treated if privately operated and financed upon the usual basis of public-utility accounting. This would involve charging the publicly-owned utility with taxes, with interest on its outstanding investment cost to date and with such annual amounts as would retire the various items of investment cost during the estimated lifetime of the works constructed with such investment of investment. On the other hand, it was urged on behalf of the Automotive Transport Association of Ontario that the provision of highways is a fundamental obligation of government, and that the users of the highways should not be directly charged with any loans which the Government had to assume in order to provide the highways.

"The Commission does not accept either of these viewpoints. It does, however, accept the principle that there is an obligation on the public as a whole to supply itself with a basic system of roads and that such obligation is a local necessity for the social needs of restricted areas. Accordingly, as is pointed out in chapter XIII, the cost of satisfying these wholly local needs should be assumed by the local community." (Report, p. 198.)

"Whatever the annual cost of the roads of the Province may be, it is evident that some share of it should be borne by the public as a whole. Every citizen, whether he personally makes use of motor vehicles or not, has an interest in the provision and maintenance of highways, which is associated with the comfort and convenience of living and the general social well-being of the community. The combined interest of all the members of the community in this regard is here called the 'social-necessity value.'" (Report, p. 212.)

There follows an itemization of the "benefits to communal life afforded by public roads, altogether apart from the right to operate motor vehicles on them." Local support of rural roads and city streets on the basis of \$3 per capita, was suggested.

Neither this report nor the Salter report included taxes on the highway investment with road costs.

cle public policy demanded "that highway financing follow the principles of the public utility concept." However, a few pages on the report indicates that the concept has not been used in the framing of State highway plans and policies. "The problem," it is said, "*is now to place highway financing on a public utility accounting basis.*" (Italics supplied.) No way of reconciling these statements is apparent, and yet the first of the two is necessary if the conclusions as to subsidy are to be sustained. It is true that highway-user taxes came to bear a large part of the cost of the highways, but these taxes were trivial in amount in 1912, 8 years after 1904, the year from which the calculations in the report are dated, and did not begin to yield large revenues until after the War. Also, for many years these taxes were used exclusively on the State system and they are now so used in large part. County and local roads and city streets continued to be financed wholly by general taxation and special assessments until recently and they are still largely so financed.⁸⁵ Taxpayers as such may have been reluctant, as the engineers state, to pay taxes for road and street purposes, but the big job of building the main highways had to be disposed of before motor-vehicle taxes could be used in these other ways. The city resident and the farmer, surveying the street or road before his house, most certainly never conceived of it as "clearly an economic or commercial enterprise."

The best test of the foregoing remarks and in fact of the major conclusion of the railroad report is to ask whether it would have been possible to have raised nearly \$10,000,000,000 of additional revenue (the subsidy figure given in the report) from motor-vehicle users in the period 1921-32. The answer clearly is "no." In other words, roads and streets of the kind provided in this period could not have been afforded if their major purpose had been that of serving motor-vehicle operations and if motor-vehicle users had had to pay a much larger bill. The functions of these facilities were broader, as was their support.⁸⁶ While the report states, and properly, that changes in the methods of road financing were gradual, it fails to reflect this trend in the conclusions drawn as to the responsibility for road costs over the period in question. As application of the railroads' public utility concept therefore would have been self-defeating, the concept itself lacks the validity ascribed to it.

Also difficult to understand is the statement that "those uses of the highways which existed before the

⁸⁵ See pp. 77-78.

⁸⁶ The railroad report itself testifies to this fact when it states that toll roads were not established after motor vehicles began to come into general use because the number of such vehicles was too small to support the facilities. (Report, p. 2.) Mention is also made of the early toll roads of this country as an indication of the validity of the public utility concept, but the fact is overlooked that for the most part these roads were not financially successful and that even before the advent of the railroad many had been turned over to the States or local communities for support from general tax revenues.

motor vehicle appeared gradually merged into the economic aspects of the motor vehicle and finally disappeared in the public-utility concept." It presumably means that horse-drawn vehicles were gradually displaced by motor vehicles and that, owing to the greater mobility of the latter, the uses made of roads and streets were so broadened as to create a new conception of responsibility for costs. So interpreted, the statement is partially true. If, however, it signifies that all such uses became economic in character, it is in error. While the word "economic" is not defined and may suggest a variety of things, it can be inferred to refer to transportation uses and more specifically to transportation that is competitive with other agencies of transportation (railroads or street railways). Clearly, however, a vast amount of the use of motor-vehicles has been created by the availability of such vehicles or is merely a substitute for local movements which had to occur by horse-drawn vehicles if they were to take place at all. It is therefore erroneous in a high degree to assert that county and local roads are merely feeders of and city streets terminal facilities for the trunk highways. They do have these functions, just as they served the railroads when horse-drawn vehicles were in use, but substantially half of all the mileage of motor vehicles is made in local operations which in very large part are not competitive with any preexisting or presently existing form of transportation. The highway planning surveys have abundantly shown that such is the case.⁸⁷ Therefore, the effort made in the railroad report to build up the conception that all roads and streets, from the busy main highway or street down to the *cul de sac* or the dirt road used by one or two farm families, are a unit must undergo serious revision. The detailed analysis of road and street uses presented in chapter III, above, need not be repeated here, even in summary form. It, however, reflects the realities of road and street use and the degrees to which it is possible to identify such use with the land served. There has been no fundamental change in this respect since the motor vehicle displaced the horse. Roads and streets are generally better than they were then, but there still are tens of thousands of miles of country roads which no motor-vehicle user would drive over unless he were identified in some way with the farms or other property which lie alongside. Mere physical interconnection signifies little so far as the great body of motor-vehicle users is concerned. If, then, as the railroad report urges, the distinction between classes of roads and streets is breaking down (and such is the case to only a minor degree), it is because of continued rejection of the public utility principle as conceived in that report. The end result would be to place on motor-vehicle users as a class the

⁸⁷ The average length of trips made by trucks which travel outside of city limits is about 16 miles.

burden of providing all the highways and byways of the nation. If such an attempt were made, motor-vehicle taxes would either have to rise to prohibitive heights or a vast mileage of roads and streets would have to be abandoned. Nothing is gained by discussing a problem of this kind in impractical terms. It requires no powers of prediction to state that the State and its subdivisions will not look upon their facilities as something whose services are for sale under a schedule of rates. Such procedure would carry the benefit basis of taxation to an extreme that no one else has had the temerity to discuss. Streets and roads are too closely knit in with the everyday life of the people to be pulled apart from other socially-provided facilities, though certain streets and roads, viz, those which are essentially a general transportation medium, may be and in fact have such distinctive characteristics as to be treated in large part on such a basis.

It should be clear, from various of the foregoing remarks, that the test for road and street improvements laid down in the railroad report is far too narrow a one. The report states that such improvements are made solely for transportation purposes and their justification is found only in transportation benefits. These benefits are, it is said, to be measured by savings in motor-vehicle transportation costs and all roads and streets are to be thrown together in computing such costs and savings. The answer to this reasoning is found in chapter III. Road facilities confer benefits not associated with motor-vehicle users as such, and society may devote funds to road purposes where the savings in transportation charges are negligible in proportion to the benefits expected to follow from the expenditures, as, for example, by the removal of isolation or sectional backwardness. There may be "subsidy" in such latter cases but it is not "subsidy" to motor-vehicle users as such. Thus, recent Federal road expenditures in less-favored areas are not and should not be financed by motor-vehicle users, except to the extent that they use the facilities. (See p. 27, above.) Considered alone, the nonmotor-vehicle uses of road and street facilities are of larger proportions than the railroad engineers appear willing to concede and they are not to be minimized by considerations of inconvenience, any more than the responsibility of the motor-vehicle user is to be lessened by the mere fact of his sharing a facility with pedestrians, bicyclists, or telephone poles. Furthermore, these uses which do not arise out of motor-vehicle operations are not going to vanish to a point where they can be considered compensated for by a "free" right-of-way. They are in every way as vital as motor-vehicle use and would remain if such use were to cease. They do not, to be sure, require as high a degree of improvement as that to which roads and streets have been carried under volume traffic, a fact which has been allowed for in the present report, but they cannot be submerged and ultimately eliminated without causing all business and

social intercourse to cease. Streets and highways are to be supported 100 percent by their users, as the railroad report states, but the users comprise all of society and their relations to the State are far broader than those which arise out of motor-vehicle operation. These facts seem so self-evident as not to require serious explanation.

Confusion exists also as to the treatment which the railroads intended to give the facilities handed down from the period before motor vehicles became numerous. At one place it is asserted that only the cost of the bare right-of-way so brought down is to be taken out of the motor-vehicle user's bill; elsewhere it appears that the cost of all improvements so brought down are so to be treated and that, as these wear out, there is left only the old right-of-way as a residue of "government" responsibility. The point is not important quantitatively, but it leads the writers of the report to the conclusion that, in not charging motor vehicles with 100 percent of the responsibility for roads and streets in 1937 (the last year covered in the report), a major concession is made to those who do not think all the costs should be borne by the motor vehicle. The amount so conceded is, nevertheless, labelled "subsidy." Here there is failure to let the reasoning used lead to its own conclusion. If a share of the cost should be borne by others, it should be removed from the picture entirely.

All of the foregoing remarks have laid a basis for appraising the reasoning and statistical computations from which the specific percentages of motor-vehicle responsibility presented in the report are derived. The procedure there used is to find out what was being spent on roads before the motor vehicle made its appearance, to consider this amount the measure of "so-called traditional uses" of roads, to convert it to a per capita basis, to find the percentage rise in per capita expenditures thereafter, and to consider such rise a measure of the road costs that are attributable to motor-vehicle users.

Needless to say, this procedure largely discards the reasoning hitherto employed in the railroad report. On this basis, motor-vehicle users never would assume complete responsibility for road costs.

The present report employs the same reasoning in part. However, it does not find that 1904 is a good year from which to start. There were only 55,000 motor vehicles at that time and their use was largely confined to streets. The "good roads" movement was not even then in full swing. As late as 1912, there were only 312,000 motor vehicles, few of which were owned on farms or made much use of rural roads. Special taxes collected from their owners aggregated the insignificant sum of \$1,000,000. Roads were still being designed principally for horse-drawn vehicles, as witness the destruction which was subsequently wrought by faster moving, rubber-tired motor vehicles. Possibly aware of the weakness of their initial year (which in fact was selected because of the availability of data), the authors of the railroad report state that more

significant than the rise in per capita expenditures after 1904 was the fact that men began to think in terms of motor vehicles. This new means of transportation, it is said, had in fact stimulated interest in roadbuilding before 1904. Therefore, it is stated further, it is generous to set up even 1904 expenditures as the measure of the "traditional uses."

There is no denying that cross-country runs, tests of the endurance of both car and driver, created an interest in better roads around the turn of the century. However, farmers had very few cars and substantially no trucks in 1904 and were no keener about paying for roads for the "city motorist" than they had been before about making the trips of city bicyclists more pleasant.³⁷ The real test was what city owners were being asked to pay for rural roads and the answer is that they paid substantially nothing in 1904, or as late as 1912. It is also very important to note that, when city money began to flow in volume into rural road channels, it was the main highways that got all the money for many years. To forget this fact and assign like responsibility for all rural roads is distinctly not in accord with what was actually transpiring.

It is necessary, if use is to be made of the "basic year" method, to start with a much later year than 1904. The year 1915 is used in the present report.³⁸ Even this year is not entirely satisfactory, as there is every reason to believe that per-capita road costs would have gone on increasing without motor-vehicle influence.³⁹

Application of the reasoning employed in assigning responsibility for rural roads led to serious difficulties when the authors came to city streets, for here an expenditure of \$5.32 in 1904 stubbornly refused to advance for 10 to 15 years. In this dilemma, it is reasoned that the expenditures which had been made on streets before 1904 provided improvements which "were far beyond those defined here as community uses." The figure of \$5.32 should, it is said, be cut back to give what the cities should have spent had they been mindful of what their real community needs were. However, lacking a statistical basis for such a reduction, the 1904 expenditures are used as a measure of the so-called traditional uses.

Comment is hardly necessary. Suffice it to say that improvements of city streets were far from adequate in 1904 and that, without motor-vehicle influence, they would have increased steadily as health and other community standards rose.⁴⁰

There are other points, such as the failure to adjust per capita costs for changes in price levels, which could be mentioned if space permitted. Already, however, more consideration has been given the railroad report than it appears to merit.

³⁷ See p. 7, above.

³⁸ It is to be noted, however, that other factors are there considered in reaching the conclusion presented.

³⁹ See pp. 25-27 above.

⁴⁰ See p. 28, above.

In using the public-utility concept the railroad engineers were seeking to place on users of roads and streets the same kinds of costs that are incurred in the private provision of right-of-way facilities. This objective has not been discussed on its merits, as it goes to questions of tax policy rather than to those of costs and public aid with which this report is concerned. It may be noted, however, that consideration has been given to the public-utility concept in chapters III and IV, above, mainly as a means of orientation as to the nature of road costs and uses. The interest charge and the percentage of motor-vehicle responsibility for State highways which the railroad report adduces from this concept accord substantially with the treatment given these items in the present report. Divergence becomes marked in the treatment of taxes and of responsibility for the costs of county and local roads and city streets. This divergence results from the failure of the railroad report to reflect the realities of road and street use and to give sufficient weight to trends appearing in the period after 1921. The result is overemphasis of motor-vehicle use of streets and roads, of the unity of all streets and roads, and of distant as compared with short-haul uses of these facilities. The statistical demonstration of the extent of motor-vehicle responsibility does not follow out the adopted public-utility concept consistently and is in itself open to serious criticism.

Other aspects of the railroad report now require consideration.

Comparison of methods used in the present report and in the railroad report in allocating costs to groups of motor vehicles.—Tables G-2 and G-3 present, the latter in the form of multiples, the general methods followed in the present report⁴¹ and in the railroad report in allocating the assignable annual costs of all highways and streets for 1932. Also presented in table G-3 are multiples derived from Glover's use of the increment method.⁴² A comparison cannot be presented for each of the detailed elements, owing to differences in the methods used in the several reports. Thus, in the present report construction costs of flexible pavements and excavation are allocated in a single step, while in the railroad report the costs represented by the latter element are allocated with those of structures.⁴³ Also, in the present report costs arising from the additional width requirements of vehicles other than basic vehicles

⁴¹ The multiples given in the present report are on two bases: (1) that in which the costs of flexible pavements and grading are allocated to the various vehicle groups on the basis of "pressure intensity-vehicle miles" (see pp. 48-50), and (2) that in which the costs of these elements are allocated on the modified basis finally adopted. (See table G-3 and pp. 150-151). The former is designated in tables G-2 and G-3 as "unmodified" and the latter as "modified."

⁴² See p. 281, above. Since his data refer to one State, peculiar conditions therein are to some extent reflected in the costs assigned to the respective vehicle groups. Influence of the conditions prevailing in Illinois is diminished somewhat by the use by Glover of certain national averages which he assumed to be representative of Illinois; e. g., distribution of vehicles by types. However, the comparison is of interest and the results are generally comparable with the national results presented in the railroad and the present report.

⁴³ However, the modified method finally adopted in the present report allocates the costs of flexible pavement and grading separately.

have been treated with the elements affected; i. e., additional pavement widths with pavement costs, etc., whereas the railroad report disposes of all additional requirements as one item. The comparison is therefore confined to the two major cost elements (construction, maintenance and operation) and to total costs. Owing to the large differences in the costs assigned in the respective reports to motor-vehicle users as a class in 1932,⁴⁴ a direct comparison of costs would indicate to a large extent differences arising from the basic amounts to be allocated. Therefore, the comparison is presented on a relative basis only. The multiples are based on costs assigned to the basic vehicle as 1.

An over-all comparison of the costs allocated to the several vehicle groups is presented in columns 2-5 of table G-3. In the case of annual costs represented by construction, the largest charge assigned to any vehicle in the present report (over 5-ton combinations) is 33.3 times (37.1 times as unmodified) the cost assigned the basic vehicle, while in the railroad report the over 5-ton for-hire combination is charged 85 times the amount assigned the basic vehicle. Glover made no distinction, in assigning costs, between private and for-hire vehicles. The maximum assignment of costs in that report is to the group of largest trucks designated X (those greater than 5 tons rated capacity) and amounts to 33.3 times the cost assigned to the basic vehicle. Maintenance and operations in each of the reports varied between smaller limits. Thus, in the present report the maximum charge is 5.3 times (over 5-ton for-hire combinations) that against the basic vehicle (column 6), whereas the maximum multiple found in the railroad report is 36.6 for the same type vehicle (column 7). Similarly, in the Glover report the group of largest vehicles is assigned costs approximately 15.2 times those set up against the basic vehicle (column 8).

On the basis of total annual costs; i. e., the sum of the foregoing construction costs and maintenance and operations expense, the maximum charge in each instance is to the group of largest for-hire trucks and amounts to the following multiples of basic vehicle charges: 20.8 (21.3 unmodified), 65.0, and 28.1 for the present report, the railroad report, and the Glover report, respectively (columns 9-12).

The multiples given above depend not only on the method used in allocating the individual elements of cost but also on the number of vehicles or vehicle-miles entering into the allocation, and the relative importance in the total assignable costs of each individual element (rigid pavement, etc.). Differences in method have been illustrated in table G-2 and are discussed in more detail in the following pages. Little difference exists between the railroad and present report as to the number and mileage of vehicles or as to the relative distribution of costs of all road and street systems

between elements. However, owing to the fact that it is confined to a single State, substantial differences exist between the Glover report and the present report. Even if all other variables were the same, these differences would exert considerable influence on the multiples of over-all costs.

The following tabulation indicates the distribution of total assignable costs to the principal cost elements used in each of the reports:

Percentage of assignable annual costs arising from the principal elements of road and street costs as used in the present report, the railroad report, and the Glover report

Report	Capital costs arising from—				Maintenance costs	Total
	Pavement		Grading	Structures		
	Rigid	Flexible				
Present.....	18.2	12.8	17.2	10.4	41.4	100.0
Railroad.....	18.4	11.7	21.6	11.5	36.7	100.0
Glover.....	31.1	2.8	23.3	16.2	26.8	100.0

The extensive mileage of rigid pavements in Illinois accounts for the larger percentage of the costs that is ascribed to this element in the Glover report.

The results of the various methods used in each report in the allocation to vehicle groups of the costs of the several elements⁴⁵ are indicated in the following comparison of the percentage of the total charge per vehicle arising from each of the elements.

Percent of total costs per vehicle as found in the present report, the railroad report, and the Glover report for selected vehicle types arising from the several elements of cost

Vehicle type and report	Capital costs				Maintenance	Total
	Pavement		Grading (including earth roads)	Structures		
	Rigid	Flexible				
Passenger car:						
Present report:						
Modified.....	14.7	11.2	17.0	11.8	46.8	100.0
Unmodified.....	18.9	9.5	12.8	12.2	49.6	100.0
Railroad report.....	8.8	4.6	20.0	15.4	41.2	100.0
Glover report.....	22.4	1.2	23.1	19.4	28.9	100.0
Heavyest vehicle:						
Present report:						
Modified.....	55.0	12.3	16.8	4.2	11.7	100.0
Unmodified.....	68.4	5.9	8.9	4.5	12.8	100.0
Railroad report.....	42.2	26.8	4.0	2.8	22.3	100.0
Glover report.....	68.7	8.6	4.1	2.9	15.7	100.0
Farm truck:						
Present report:						
Modified.....	11.1	18.6	28.1	9.0	38.2	100.0
Unmodified.....	7.8	24.9	37.7	6.8	22.8	100.0
Railroad report.....	23.7	16.0	13.2	7.0	38.1	100.0
Glover report (not given).....						

¹ See discussion under "flexible pavements," p. 203.

² Adjusted to include costs arising from additional width requirements.

The effect of the Glover distribution of basic costs is apparent in the percentages here given; i. e., costs arising from rigid pavement are much higher for the basic vehicle, etc. However, the greater part of the difference between the railroad report and the present report arises from differences in method of allocation to vehicle types.

⁴⁵ With allowance, of course, for the fact that in the Glover report the relative distribution of costs is somewhat different from that found in the present report and the railroad report.

⁴⁴ See table G-1.

APPENDIX

TABLE G-2.—Comparison of methods of allocating costs to groups of motor vehicles used in the railroad report with those used in the present report, including a comparison with the increment method used in Glover report (see p. 281)

Type of cost and road or street system	Highway or street element	Governing factor in cost allocation and principal vehicle or other characteristics considered	Method employed in—			
			Railroad report	Page reference	Present report	Page reference
Capital costs: Rural roads.....	Pavement: Rigid.	Thickness: Wheel loads.....	<p>Thickness requirements (used for ratio of thickness only) of various vehicles derived from Older's corner formula modified for impact, and the resulting thickness adjusted for cost on the basis of: Cost of rigid pavement—\$3,100+-\$3,526 where t=thickness and \$2,526 the constant cost per mile. (It is to be noted that the adjustment for width requirements of heavier vehicles is made in a later map for all highway elements (table 22d). The formula given apparently applies only to an 18-foot width for pavement.) The cost index for the required thickness, translated into increments, has been used in assigning to each vehicle group the costs for which it is considered responsible.</p> <p>[Glover in his "increment" method has followed a procedure similar to that above, except that the cost formula is based on Illinois experience, which gives the following:</p> <p>Cost of rigid pavement—\$3,700+-\$2,300,</p>	60	<p>Thickness requirements of various vehicles derived from Westergaard formula, modified by the use of 4 inches as the minimum practical interior thickness (the thickness obtained from the formula is 2.6 inches for the basic vehicle) and the resulting thickness adjusted for cost on the basis of: Cost of rigid pavement—\$11.64 WT+-\$6,335 where W=road width, T=thickness, and \$6,335 the constant cost per mile. The costs for which each vehicle group is responsible have been derived from the cost requirements of each group based on the computed cost index.</p>	81
		Width: Vehicle widths, speeds, operating characteristics, relative volume of vehicles of different sizes and weights, etc.	<p>A width of 18 feet on improved roads is assumed as adequate for the basic vehicle; similar freedom for heavier vehicles requires a 20-foot pavement. Although the added costs due to wider requirements are not set forth separately, 13 percent of the entire capital cost of improved rural roads is assigned to vehicles wider than the basic vehicle, and this percentage is stated to be "based primarily on the factor of additional width." This amount covers, in addition to pavements, additional traffic lanes, widening on grades, increased grading yardage, grade reduction, and increased cost of structures, all assignable to vehicles wider than the basic vehicle.</p> <p>[Glover (increment method), using Illinois experience, finds the following percentage of annual cost assignable to other than basic vehicles for rural roads as a result of additional width requirements, etc.: Pavement, 132 percent; grading, 9.19 percent; and structures, 5.19 percent. (Note: The additional requirements on secondary and local roads for grading and structures are considered to apply only to surfaced roads; i. e., to 44.17 percent of the total costs, applicable to secondary and local roads for these elements.) The weighted percentage applied to all capital costs of rural roads is 4.30 percent or, excluding costs assignable to grading and structures on unsurfaced rural secondary and local roads, 5.30 percent.]</p>	72	<p>A width of 20 feet is taken as satisfying requirements of the basic vehicle and 21 feet for all wider vehicles. On a percentage basis this amounts to an assignment of 4.76 percent of all pavement costs to wider vehicles. This amount also covers the cost of maintenance roads assignable to wider vehicles.</p> <p>(It is to be noted that all trucks, from the smallest to the largest, above in the additional width requirements. In the railroad report only those trucks of 3 tons or greater rated capacity are assumed to require additional width. Glover has assumed additional width requirements for all trucks except the lightest group of 3,000 pounds or less gross weight.</p>	106
		Average cost per unit.....	<p>The costs assignable to the various groups are determined on the basis of vehicle-miles; i. e., the total mileage of all vehicles within any given group above the basic vehicle has been used in determining an average cost per vehicle-mile of that group and the average cost of each lighter group. The costs assigned to all vehicles wider than the basic vehicle are prorated to each group in proportion to the total mileage of each group. The costs per vehicle-mile so obtained are in each case expanded to a per-vehicle basis.</p> <p>[Glover (increment method) has determined average costs per vehicle group on the basis of number of vehicles only.]</p> <p>No distinction has been made as to variations in traffic on the various road and street systems in either report.</p>	80	<p>The costs assigned to the various groups are reduced to a per-vehicle basis through use of vehicle-miles; however, consideration is given to the radically different traffic distribution on each of the road and street systems in the derivation of unit costs.</p>	100-105

TABLE G-2.—Comparison of methods of allocating costs to groups of motor vehicles used in the railroad report with those used in the present report, including a comparison with the increment method used in Glover report (see p. 281)—Continued

Type of cost and road or street system	Highway or street element	Governing factor in cost allocation and principal vehicle or other characteristics considered	Method employed in—			
			Railroad report	Page reference	Present report	Page reference
Capital costs—Con. Rural roads.....	Pavement: Flexible (except earth).	Thickness: Wheel load (railroad report). Pressure intensity, impact wheel loads, and vehicle-miles (present report).	Relative thickness requirements of various vehicle groups derived in a manner similar to that used for rigid pavements, except that ratios of the square roots of the wheel loads are set forth instead of theoretical thickness. The resulting relative thicknesses are translated into a cost index on the assumption that 90 percent of the costs of flexible pavement vary with thickness and the remaining 10 percent are constant. [Glover has followed the same method, except that no adjustment has been made for constant costs.]	68	In the final determination of per-vehicle costs arising from flexible pavements (and grading), the method set forth in the body of the report (chap. VII; see (a) below) has been modified to allow for estimated conditions under which such elements were actually constructed. (a) <i>Responsibility (unmodified) basis.</i> —The relative cost requirement of each vehicle group has been determined on the basis of dynamic pressure intensities, modified by width requirements (same as under rigid pavement). The index so obtained has been assumed to be equal to the index of required costs, no adjustment having been made for constant or variable costs (see p. 69).	93
Earth roads.....	All: Damage.	Width. Average cost per unit.	See rigid pavement.	72	Same as rigid pavement.	105
Grading.....	All: Use made of, and additional requirements of various vehicle groups.	Same as earth roads. The additional cost assigned to vehicles with greater requirements than those of the basic vehicle are as set forth under rigid pavements—width—given above. No allowance is made for any specific element, such as grading, etc., however, as stated above, the principal factor considered is width. The over-all allowance for such factors is 12 percent. Average costs per vehicle are derived on the basis of vehicle-miles. [Glover: Same as above, except that width requirements specifically affecting grading are set forth separately. See rigid pavement—width.]	Same as earth roads. The additional cost assigned to vehicles with greater requirements than those of the basic vehicle are as set forth under rigid pavements—width—given above. No allowance is made for any specific element, such as grading, etc., however, as stated above, the principal factor considered is width. The over-all allowance for such factors is 12 percent. Average costs per vehicle are derived on the basis of vehicle-miles. [Glover: Same as above, except that width requirements specifically affecting grading are set forth separately. See rigid pavement—width.]	68	Same as given above for earth roads. Average costs per vehicle determined as outlined under rigid pavements.	117
Structures.....	Width and weight: Vehicle weights, widths, speeds and other operating characteristics.	Same as grading above.	Same as grading above.	71	Of the total costs, 95.2381 percent has been assigned to all vehicles and the remaining 4.7619 percent to vehicles with requirements greater than those of the basic vehicle. The requirements of basic vehicles and others have been assumed to be the same as pavement width requirements. No adjustment has been made for constant costs.	119
City streets, all.....	All: Same as rural roads.	Same as for rural roads, except that no assignment of costs to other than basic vehicles for additional width requirements has been made. [Glover—same.]	Same as for rural roads, except that no assignment of costs to other than basic vehicles for additional width requirements has been made. [Glover—same.]	68	Same as for rural roads.	126

APPENDIX

TABLE G-2.—Comparison of methods of allocating costs to groups of motor vehicles used in the railroad report with those used in the present report, including a comparison with the increment method used in Glover report (see p. 251)—Continued

Type of cost and road or street system	Highway or street element.	Governing factor in cost allocation and principal vehicle or other characteristics considered	Method employed in—			
			Railroad report	Page reference	Present report	Page reference
Maintenance costs: Rural roads, all		Responsibility of various groups for maintenance costs, climatic and subsoil conditions, condition of capital and maintenance items, traffic volume, age of pavement, inclusion of costs not relating to the traveled roadway, etc.	Of the total maintenance costs, 75 percent are assumed to represent items arising from use of the traveled way, and the remaining 25 percent items not related thereto. The former are assigned to vehicle groups on the basis of the proportionate part of the total gross ton-miles performed by vehicles in that group and the latter on the basis of vehicle-miles similarly distributed to groups. In no case has the method used shared the costs assignable to any given group with those of any other group, as in the case of certain capital cost items. [Glover: The assignment of costs is based on the use of ton-miles and vehicle-miles, each applied to a percentage of maintenance costs as follows: Percentage of total maintenance costs allocated by:	57	Of the total maintenance and operations costs other than those estimated as arising from traffic police control, street cleaning, snow and ice removal, street lighting, and such other service items, two-thirds have been treated as necessitated by natural causes, approximately one-fourth as arising from traffic wear attributable to vertical loading, and the remaining one-ninth as arising from traffic wear resulting from abrasion and revealing. The first group, consisting of all service items, and two-thirds of the costs remaining after exclusion of such items, have been assigned to vehicle groups on the basis of each group's contribution to traffic volume. Such amounts are assigned by road systems; i. e. State highways, county and local roads, and city streets. The second group, consisting of costs attributable to vertical loading, has been assigned to vehicle groups on the basis of pressure intensities and the traffic volume of vehicles within that given group. The last-mentioned group, representing costs assignable to horizontal wear from abrasion and revealing, has been assigned to vehicle groups, based on the relative distribution of an index representative of each group's contribution to the total wear computed from the product of total tractive pull at road surface, diameter of the contact area and traffic volume on county and local roads. In no case has the method used shared the costs assignable to any given group with those of any other group, as in the case of capital costs.	120-126
City streets, all	do	do	The total costs of maintenance have been assigned to vehicle groups in equal proportion on the basis of each group's contribution to the total ton-mileage and total vehicle-mileage.	57	Same as for rural roads, except as the costs treated above on the basis of traffic volume are considered separately for city streets.	

TABLE G-3.—Costs assigned per vehicle, by vehicle groups, expressed as multiples of base vehicle costs, segregated as to costs arising from construction and maintenance work, as given in the railroad report, the Glover report, and the present report

Class of motor vehicle (1)	Construction costs				Maintenance and operations expense			Total costs			
	Present report		Railroad report	Glover report ²	Present report	Railroad report	Glover report ²	Present report		Railroad report	Glover report ²
	Unmodified ¹	Modified ²	(4)	(5)	(6)	(7)	(8)	Unmodified ¹	Modified ²	(11)	(12)
Passenger cars	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Taxis	3.0	2.2	2.9	2.5	2.5	4.2	1.0	2.7	2.3	4.0	1.0
School buses	6.1	2.5	2.7	2.8	2.8	3.5	1.0	4.4	2.3	2.9	2.9
Contract buses (seats):											
7 and less	2.4	1.7	2.0	1.4	2.1	1.4	1.0	1.9	1.6	2.1	2.1
8 to 20	2.0	2.0	2.5	1.4	4.0	1.4	1.7	1.7	1.7	2.3	2.3
Over 20	2.9	2.0	11.2	1.6	5.4	1.6	2.3	2.3	2.3	2.3	2.3
Common carrier buses (seats):											
7 and less	7.8	5.3	5.1	4.0	5.4	5.4	1.0	5.9	4.8	5.2	5.2
8 to 20	6.3	6.2	22.4	4.4	11.7	4.4	5.4	5.4	5.4	18.0	18.0
Over 20	5.4	6.2	39.3	4.7	19.4	4.7	7.1	7.1	7.1	31.1	31.1
Trucks:											
Private trucks:											
Farm trucks:											
1 1/4 and less	1.8	1.0	.8	.6	.7	.7	1.0	1.3	.8	.9	.9
Other private:											
Over 1 1/4 and less than 2:	4.1	2.4	2.3	1.5	2.2	2.2	1.0	2.9	2.0	2.3	2.3
Single	4.1	2.3	4.9	1.7	2.8	2.8	A=1.2	2.9	2.6	4.4	A=1.2
Combinations	5.5	6.0	4.7	2.7	7.7	7.7	B=1.0	5.5	4.0	5.0	B=1.0
3 and less than 2:											
Single	9.1	2.2	26.3	2.7	8.9	8.9	C=1.1	5.9	5.7	19.1	C=1.1
Combinations	8.4	5.0	16.2	2.8	10.2	10.2	D=4.0	5.6	4.9	12.7	D=4.0
4, single vehicles	12.7	17.4	22.5	3.2	12.7	12.7	E=2.5	10.9	10.9	30.9	E=2.5
4, combinations	21.0	19.2	48.5	2.0	12.5	12.5	F=6.1	12.1	11.8	33.5	F=6.1
Over 4:											
Single	20.4	19.2	64.9	2.4	19.2	19.2	G=7.7	12.0	12.0	49.9	G=7.7
Combinations	24.3	22.9	60.0	2.8	25.8	25.8	H=12.2	14.4	14.2	44.9	H=12.2
For hire:											
1 1/4 and less	2.1	4.7	5.1	2.9	4.7	4.7	1.0	5.5	2.9	4.9	4.9
Over 1 1/4 and less than 2:											
Single	10.0	7.9	9.4	2.9	7.3	7.3	1.0	6.7	5.8	8.5	8.5
Combinations	12.2	7.5	6.6	2.8	10.9	10.9	1.0	8.5	6.6	8.4	8.4
3 and less than 2:											
Single	16.4	14.4	37.1	2.8	12.6	12.6	1.0	16.1	9.5	27.6	27.6
Combinations	12.9	10.2	22.9	4.0	14.4	14.4	1.0	8.5	7.3	19.4	19.4
4, single	21.4	22.2	65.5	4.2	18.0	18.0	1.0	17.5	17.4	47.5	47.5
4, combinations	21.4	22.6	68.6	4.2	18.9	18.9	1.0	18.0	17.4	47.5	47.5
Over 4:											
Single	26.0	21.0	77.7	4.3	27.2	27.2	1.0	18.5	18.0	58.9	58.9
Combinations	27.1	22.2	84.2	4.2	26.6	26.6	1.0	21.2	20.8	64.9	64.9

¹ The multiples given represent those obtained from per-vehicle costs which reflect an allocation of flexible pavement and grading costs solely on the "presence-intensity-vehicle-mile" or responsibility basis. In the final analysis (pp. 150-155), consideration was given to other factors. See table G-3 under "flexible pavement and grading."

² Multiples derived from using the modified method of allocating costs of flexible pavement and grading. See pp. 150-154, above, and also footnote 1.

³ Glover has classified trucks by gross weights. The grouping given represents the approximate manufacturer's rated capacity derived from the relationship given in his report. The alphabetical designations used by Glover represent the following gross weights: A=3,500 pounds; B=3,000-3,500 pounds; C=2,000-3,000 pounds; D=10,000-12,000 pounds; E=12,000-14,000 pounds; F=14,000-16,000 pounds; G=16,000-20,000; H=20,000-24,000 pounds; and I=24,000-30,000 pounds. All trucks have been shown above opposite private trucks, since no other breakdown was given.

Many individual points of difference are indicated in the comparisons presented in table G-2, which in varying degrees contribute to the markedly different results presented above and in table G-3. The more important of these differences have been discussed at pages 147-149, and are considered in more detail in the pages following. However, before discussing these matters, attention needs to be directed to one factor which, though common to practically all of the steps taken in making an allocation to vehicle groups, is in some respect used in a different manner in each of the three reports.

The present report and the railroad report, after the assignment of increments of cost (see table G-2) to vehicle groups, reduce these increments to a per-vehicle charge on the basis of vehicle-miles or appearances in traffic. On the other hand, Glover's increment basis uses solely the number of vehicles. In each report the unit of measurement (either vehicle-miles or number of vehicles) is used in a manner which makes each group share in costs considered to be specifically assignable to it and in all costs of groups with lesser requirements. It would appear that the number of appearances in traffic, measuring as it does the extent of use of a facility, is the more appropriate basis. There is, however, one rather important difference between the railroad report and the present report in the use of vehicle-miles. The former considers the costs and travel of all systems together, while the latter treats such items by systems. The differences in the results thus obtained reflect the importance of the costs of a given system and the distribution of its traffic between vehicle groups. The cost per vehicle of rigid pavement assigned on the basis of use is approximately the same computed by either method in the case of passenger cars. However, on the basis used herein the for-hire 5-ton combination is assigned rigid pavement costs amounting to approximately \$303, with which may be compared costs of about \$260 that would result if all systems are considered as a unit.

While different measures of vehicle use (vehicle-miles or ton-miles) have been used in allocating other capital or maintenance costs, the costs and allocation factors in the railroad and Glover reports have been treated without regard to system, whereas, with minor exceptions,⁴⁵ all costs in the present report have been allocated by systems. Owing to the fact that there are substantial differences in the relative use of the several systems by each vehicle type, it is believed that the basis of allocation used herein, which reflects the variations in the cost and the use made of the respective systems by vehicle type, gives more reasonable results.

(a) *Rigid pavements.*—A substantial part of the total charge against heavier vehicles represents rigid

pavement costs. Thus, of a total charge of \$545 (\$518 unmodified) against for-hire combinations of greater than 5 tons capacity found in the present report, approximately \$355, or 65 percent, is ascribable to this factor.⁴⁷ In the railroad and Glover reports such charges against the heaviest vehicles, exclusive of additional width requirements on rigid pavement, account for 37 and 68 percent, respectively, of the total. Adjusted to include the costs arising from additional width requirements, these percentages become 42 and 69, respectively.

Comparison of the methods used in the present report and the railroad report in the allocation of the various elements of cost has been presented in table 56 on a basis similar to that used in table G-3. The greatest charge against any vehicle group found in the present report occurs in the case of over 5-ton for-hire combinations and amounts to 91.9 times the cost assigned the basic vehicle. Exclusive of additional width requirements, the railroad report finds charges for the heaviest for-hire combinations (\$1,019.03) which are 272 times those found for the basic vehicle. Adjusted to include the additional width requirements, to secure comparability with the multiple given above for the present report, this figure becomes 310. The charge against the heaviest vehicle group as found by Glover, similarly adjusted, is approximately 86 times that against the basic vehicle.

The Glover report, with the exception of two important steps, follows generally the same method as the railroad report. The two steps are Glover's assumption of a minimum thickness of rigid pavement necessitated by the basic vehicle above the computed theoretical thickness,⁴⁸ and the determination of per-vehicle costs, as previously noted, on the basis of number of vehicles only.

The failure of the railroad report to recognize a practical minimum thickness produces a relative thickness requirement for the heaviest vehicle 3.16 times that of the basic vehicle. This relative thickness requirement compares with a ratio of 1.58 in the present report and of 2 in the Glover report. Previous statements by the authors of the railroad report, as set forth at page 84 herein, indicate that these engineers would not consider practical thicknesses as small as those used in their allocation. As indicative of the impracticability of the minimum thickness of 2.6 inches adopted for the basic vehicle, their qualifying statement that "the thicknesses * * * were determined solely for the purpose of reaching a ratio of thickness which is translated into a ratio of cost," is illumi-

⁴⁷ This amount reflects the additional requirements of vehicles larger than the basic vehicle. The railroad and Glover reports treat such requirements separately.

⁴⁸ Glover determined that a theoretical center thickness of 2 inches was required by the basic vehicle and 6 inches for the heaviest group of vehicles. However, his actual center thickness was obtained by adding a constant thickness of 2 inches to all theoretical thicknesses, giving 4 inches as that necessitated by the basic vehicle and 8 inches for the heaviest group. An explanation of the use of this constant thickness of 2 inches is not given.

⁴⁵ In the case of a relatively minor proportion of maintenance costs allocated on the basis of responsibility for wear arising from vertical loadings. (See table F-6, columns 8 and 9.)

nating. If, as stated by engineers of the Bureau of Public Roads (see p. 84, above), a 4-inch center thickness represents the thinnest pavement which would be constructed by them under practical conditions and then under only the most favorable circumstances, the ratio of thickness developed in the railroad report would mean that pavements in excess of 12 inches would be necessary to maintain such a ratio for the heaviest vehicles. The mere fact that the results are to be used for a "ratio of cost" would not diminish this practical effect.

The assumption in the railroad report, through use of the "corner formula," that the pavement thickness requirements of various vehicles are in proportion to the square roots of their respective wheel loads assumes loading at a point and completely disregards tire contact areas and unit pressure intensities, thereby resulting in an overstatement of the maximum thickness required by heavier vehicles. Thus, the fact that the heavy wheel load of a large vehicle is distributed over a much greater tire area than that of the basic vehicle is not considered. The Westergaard formula used in the present report, in addition to compensating for variations in unit pressure intensities, encompasses other factors not covered by the "corner formula." A complete discussion of both formulas appears in chapter VI and appendix B.

The thicknesses as found in the various reports have been translated into a cost index. Owing to adjustment for constant costs, the resulting multiples of aggregate pavement cost requirements would be somewhat less than the multiples of thickness indicated above.

(b) *Flexible pavements.*—Flexible pavement costs account for a much smaller part of the total costs assigned to various vehicle groups in the railroad and Glover reports than do rigid pavement costs. On the other hand, owing to the use in the present report of pressure intensities as the basis of allocation (unmodified method),⁴⁸ the upward trend of charges with increasing size and wheel load, found in the case of rigid pavements, is diminished considerably. In several instances, notably farm trucks and other small units with high pressure intensities, the charge for flexible pavements constitutes a substantial part of the total. Thus, on the unmodified basis given in the present report, the charge against the heaviest for-hire combination constitutes only 5.9 percent of the total cost of \$518, whereas, in the case of the farm truck it amounts to approximately 24.9 percent of the total.⁴⁹ As both the railroad report and the Glover report have used the square root of the wheel load as the basis of allocation, the heaviest charges fall upon the heaviest vehicle.

⁴⁸ See table 30 and pp. 92-93.

⁴⁹ The allocation in the present report on the unmodified basis includes earth roads and grading with flexible pavements. However, the percentages given here exclude such amounts.

In the railroad report 26.8 percent of the total cost assigned the heaviest for-hire combination arises from flexible pavement costs. The comparable percentage as found by Glover for the heaviest group is 8.6 percent.⁵¹ Naturally, the small percentage of flexible surface roads in Illinois, as compared with the national percentage, is of controlling importance in this instance.

On a relative basis, the railroad report charges the heaviest for-hire vehicle, in the case of flexible pavements, 308⁵² times the amount found for the basic vehicle and Glover 194 times. On the unmodified basis given in the present report, the maximum assignment is to the for-hire combination of over 1½ and less than 3 tons and represents 19.4 times the charge against the basic vehicle. The charge against the heaviest for-hire combination is 13.3 times that to the basic vehicle. In fact, if vehicle-miles were not used in the allocation, the heaviest charges would generally be against the smallest trucks, since their pressure intensities are as a rule much greater than those of the larger trucks.

The modification of the method of allocating costs of flexible pavement finally adopted (see table 57 and pp. 150-155) changes substantially the distribution of costs presented above. The adopted costs increase with weight and mileage, and the maximum charge against the heaviest for-hire combination is 22.8 times that against the basic vehicle. The costs assigned to the farm truck by this method are only 1.3 times the charge to the basic vehicle, as compared with a similar multiple of 3.2 obtained on the unmodified basis.

Fundamentally, the railroad report follows the same method that is used in the allocation of rigid pavement costs. With the exception of the adjustments made to establish a minimum thickness for rigid pavements and to compensate for constant costs, Glover also has followed a similar method.⁵³ The principal difference between the railroad and Glover reports is that the former uses vehicle-miles in its allocation, whereas the latter uses number of vehicles only.

The adjustment for constant costs made in the allocation of rigid pavement costs in the three reports has been directly made in the case of flexible pavements only in the railroad report. No adjustment for constant costs has been made in the present report, for the reason that the method of allocation used is largely independent of this factor.⁵⁴

While the costs of flexible pavements are not the most important element to be allocated in a highway cost study, their allocation represents one of the most difficult problems. However, it is believed that, judged solely from the viewpoint of theoretical responsibility

⁵¹ Includes adjustment for additional width requirements, which, as previously noted, have been handled in each of the reports through a separate adjustment.

⁵² See preceding footnote. Also see table 26, p. 147.

⁵³ These adjustments substantially account for the differences in the relative charge against the heaviest vehicles in the case of rigid and flexible pavements; i. e., 30 times for rigid pavement and 194 times for flexible pavement.

⁵⁴ See pp. 92-93.

for costs (see p. 150), the present method gives more reasonable results than any heretofore presented and in particular that designated as the square root of the wheel load. This view is substantiated by the Bureau of Public Roads. (See p. 148.)

On the other hand, it has been recognized, in use of the final modified method, that the square root of the wheel load has exerted some influence on the design of roads as they were constructed during the period under consideration (1921-32) and particularly in the earlier part. However, on the basis of data presented at pages 150-152, a substantial modification of the method used in the railroad and Glover reports has been adopted for use herein. There appears to be no supportable evidence that the costs of flexible pavements were increased 1.5 times to accommodate vehicles heavier than passenger cars, as is claimed in the railroad report.⁵⁷ The portion of the costs of flexible pavements (one-third) allocated on the "modified square root of the wheel load" basis adopted is assigned 70 percent to passenger cars and 30 percent to other vehicles, the latter amount being distributed to the various vehicle groups in proportion to the square root of their impact wheel loads. On the basis of available information, it is believed that this method provides a more reasonable approach than that used in the railroad or Glover report.

(c) *Grading (including earth roads).*—Grading costs, including those of earth roads, have been allocated in the present report (unmodified basis) with flexible pavement costs and in the railroad and Glover reports with those for structures. The modified basis used herein in the allocation of grading treats this item separately. In the comparisons which follow, using data provided by the unmodified method, the element of grading has been segregated in each case.

In the present report (modified basis), the maximum charge for grading has been assigned to the for-hire over 1½- and less than 3-ton combination and represents 23.7 percent of the total costs assignable to that vehicle type. The charge to the heaviest for-hire vehicle constitutes only 6.8 percent of its total assigned costs. The method employed in the railroad and Glover reports in the allocation of grading costs is on the basis of vehicle-miles. In the former report, the greatest assignment is to the largest common-carrier bus and represents 10.5 percent⁵⁸ of the total cost assigned to that type vehicle. The total charge to the heaviest truck combination is slightly less, but on a percentage basis is much smaller. It is of interest to note that the charge to these two vehicle types for additional width requirements is greater than their share of the basic

road costs.⁵⁷ Glover's maximum assignment of costs is to the heaviest group of trucks and constitutes 4.1 percent of the total charge to that group.

The comparison of maximum charges with those assigned the basic vehicle produces relative charges of 8.3, 11.5, and 4.1 for the present report, the railroad report and the Glover report, respectively.⁵⁸ As indicated above, these maximum charges apply to over 1½- and less than 3-ton for-hire combination, largest common-carrier bus,⁵⁸ and heaviest truck groups, respectively.

The unmodified method used in the present report ("pressure-intensity vehicle-miles") results in relative maximum charges which are midway between those found in the other two reports.

On the unmodified basis, all grading costs are allocated solely according to the "pressure intensity-vehicle-mile" method. Owing to the fact that pressure intensities of the smaller trucks and busses are generally greater than those of the heavier vehicles, the use of this factor in combination with vehicle-miles, though it generally produces higher charges for the heavier vehicles, has eliminated any large variations between light and heavy trucks. However, the heaviest charge by this unmodified basis is against the for-hire combination of over 1½ and less than 3 tons capacity and amounts to 32.6 percent of the total costs assigned to this group. The costs so assigned for grading are 21.7 times those of the basic vehicles. The charge against the heaviest for-hire vehicle amounts to 8.9 percent of the total assignable costs by this method and is 14.8 times the charge to the basic vehicle.

While the range in maximum costs for grading is higher in the present report than in the other two reports, it is believed that the latter, through use of vehicle-miles as the sole basis of allocation, have failed to allow for differential cost requirements of the several vehicle groups, particularly those arising from unprotected surfaces.

(d) *Structures.*—Costs of structures in the railroad and Glover reports are allocated on the same basis as grading; i. e., in proportion to the total vehicle mileage of any group and the number of vehicles in that group. In the present report, 95.238 percent of the cost of structures has been assigned to all vehicles and the remainder to those with additional width requirements. This cost element, when allocated to the several groups, generally produces smaller charges than those arising from any other element and represents in the case of

⁵⁷ Includes costs arising from additional width requirements.

⁵⁸ Glover finds that such costs were increased 2 times to accommodate these same vehicles. Allowing for additional requirements as to width, etc., the increase in costs assumed for other than passenger cars becomes 1.9 and 2.1 for the railroad and Glover reports, respectively.

⁵⁸ Thus, the largest common-carrier bus is charged \$61.23 for grading on the basic road and \$77.21 for additional width requirements. Likewise, the heaviest truck combination is charged \$61.19 and \$76.78, respectively.

⁵⁸ The charge to the heaviest truck combination in the railroad report is only slightly below that to the largest common-carrier bus.

the heaviest group only 4.2 percent of the total assigned costs. Likewise, the charge against the heaviest groups in the railroad and Glover reports accounts for 2.8 and 2.9 percent, respectively, of the total costs. On a basis relative to the charge against the basic vehicle, multiples of 7.8, 11.8, and 4.2 are found for the heaviest vehicles in the present report, the railroad report, and the Glover report, respectively.

(e) *Maintenance and operations expense.*—This expense group, as is indicated in table G-2, is allocated in each of the reports by a combination of several methods. The railroad and Glover reports allocate these costs on the vehicle-mile basis and on the ton-mile basis. The present report has used three methods; viz, vehicle-miles, pressure intensity-vehicle-miles, and a basis, intended to take care of abrasion and raveling,³⁹ which represents a combination of several factors.

In each of the reports those maintenance and operations costs not related to the traveled way or to traffic wear and tear have been allocated on the basis of each group's contribution to the total vehicle mileage. Of the total maintenance and operations expense, 71.6, 30.1, and 44.6 percent have been so allocated in the present report, the railroad report and the Glover report, respectively. Except for minor differences resulting from slightly different annual mileages and numbers of vehicles, the maximum multiples of basic vehicle charges as found in each of the reports are in substantial agreement. Thus, the maximum multiple of the basic vehicle charges is 5.2 for the heaviest for-hire combination in the present report, 5.0 for the same unit in the railroad report⁴⁰ and 3.3 for the heaviest group in the Glover report.

The remaining maintenance and operations expense has been allocated in a manner which in each report is assumed to measure each vehicle's contribution to wear and tear. The railroad and Glover reports have adopted gross ton-miles as such a measure. As indicated in table G-2, wear from vertical loadings, and abrasion and raveling have been handled separately in the present report. The result of the assignment of expenses arising from wear and tear places a maximum charge against the heaviest vehicle which is 56 times the charge against the basic vehicle in the case of the railroad report and 27.3 times in the case of the Glover report.⁴¹ The combination method used in the present report results in a maximum assignment to the heaviest

for-hire group which is 5.6 times that to the basic vehicle.⁴²

The railroad and Glover reports consider too large a part of the total costs of maintenance to arise from wear and tear associated with vehicle characteristics. Also, it is believed that there is little basis for the assumption that the responsibility for such costs varies according to gross ton mileage.

REPORT ON HIGHWAY TAXATION, FINANCE, AND ADMINISTRATION BY NATIONAL HIGHWAY USERS CONFERENCE

A few comments may be made, by way of contrast with the foregoing, on a report on Highway Taxation, Finance, and Administration, prepared by representatives of member associations of the National Highway Users Conference and released by the conference in December 1938, shortly before the appearance of the Breed-Older-Downs report. The two reports serve to emphasize how far apart the railroad and highway groups, as thus represented, are on essential points.

The highway group advances a fivefold classification of roads and streets.⁴³ The several classes and the methods of support recommended for each are as follows:

1. Main trunk interstate highways: Those which connect the principal centers of population and on which "the traffic is predominantly general," to be constructed and maintained jointly by the Federal Government and motor-vehicle users.
2. Main trunk State highways: Those which connect the principal population centers in each State (including connecting streets) "on which traffic is predominantly general but does not include the major portion of interstate movement," to be supported mainly by motor-vehicle users but, because of their effect on the general welfare, entitled to Federal support.
3. Intermediate highways: Those which "constitute the principal feeders (including farm-to-market roads) to the main trunk highway systems * * * and to railroads and waterways, on which the traffic is predominantly intrastate in character * * * and which, to the extent they serve agriculture and the

³⁹ This relative charge is the weighted result of the allocation of 21.3 percent of the total maintenance and operation expenses on the basis of pressure intensity-vehicle-miles, and 7.1 percent by an index derived from the product of tractive pull at the road surface, diameter of the tire contact area, and traffic on secondary and local roads. The former factor assigns the heaviest charge (6.1 times that to the basic vehicle) to the heaviest for-hire combination. The latter method charges the heaviest private combination 5.3 times the basic vehicle charge and the school bus 16.3 times. Owing to the fact that traffic on secondary and local roads is a factor and that the travel of heavier trucks is relatively limited on such roads, small passenger-vehicle charges for such vehicles result. The result of weighting the two methods of allocation gives the multiple indicated in the text above.

⁴⁰ Apart from roads and streets "lying within areas held or owned by the United States Government."

³⁹ See table F-4.

⁴⁰ The largest common-carrier bus in the railroad report is charged substantially the same amount as the heaviest for-hire combination and in the present report a slightly smaller amount.

⁴¹ The much lower relative charge in the Glover report results from the smaller average annual mileage and load assumed for the heaviest group.

general welfare of the State, may be regarded as roads of general motor use," to be supported by motor-vehicle users to the extent of such general motor use, by the Federal Government to the extent they affect the general welfare, and by general revenue of counties or comparable local subdivisions to the extent they contribute to local welfare.

4. Land service roads: Those which connect with roads of the foregoing three classes, "the principal service of which is that of giving access to the land traversed or penetrated by them, and on which the traffic is predominantly a movement originating at or destined to properties immediately served by them," to be supported exclusively from the revenues of the counties and comparable subdivisions other than special motor vehicle imposts.

5. City streets: (a) Arterial highways, which carry through traffic and connect with roads of classes 1 and 2, to be supported to the extent of their general use by motor-vehicle users and the Federal Government and otherwise from the general revenues of the city; and (b) all other streets, predominantly local in their use, to be supported from local revenues other than special motor vehicle imposts.

Traffic surveys intended to show the nature and extent of the use are the only means suggested for the placing of a given road or street in its proper class. The criteria given are very general in character, in contrast with the definiteness that characterizes the conclusions of the railroad report and also those of the present study. Thus, there is no indication of the extent of the assumed Federal responsibility for all except land-service roads and streets, though on the other hand there is a clearly expressed declaration that the Government should cease collecting special excise taxes on motor vehicles, tires, accessories or motor fuel.⁴⁴ Federal sharing of the cost of maintaining all roads and streets, except those noted, would be a distinct innovation, except as relief expenditures may have shown that there is justification for permanently breaking down the established doctrine that maintenance is strictly a State and local matter. Failure to provide for any use of general State revenues also is objectionable, in view, among other things, of the merit of State-aid under some circumstances. The emphasis on the "predominantly general" character of the traffic on roads of classes 1 and 2 is also questionable in view of the findings of the highway planning surveys as to the preponderantly local character of motor-vehicle operations.

⁴⁴ Elimination of these excises is justified on the ground of the Government's obligation to carry the mail and provide for the national defense. These obligations appear, however, to have been invoked too lightly. (See pp. 22-23, 27 and 54-55, above.)

It is of interest to recall that bills were introduced in Congress and passed the House in the period just before the Federal Aid Act was passed in 1916, which would have caused Federal aid to have taken the form of payments for the privilege of using the roads in delivering the mail. Payments were to be on a mileage basis and to vary with the type of road. Back of this proposal was the desire to protect States' rights and local autonomy, with emphasis on the type of local road that was believed to be of most interest to the farmer. (A. F. MacDonald, Federal Aid—A Study of the American Subsidy System, pp. 87 ff.)

Also to be noted is the emphasis given to State planning. For this purpose, all roads and streets in a State are to be treated as an organic whole and from such a planned highway program "there should be developed an annual budget of funds needed for highways." The practical difficulties which would be encountered in dovetailing Federal, State, and local budgetary provisions for roads and streets are much greater than the report appears to have assumed.

The cost elements recognized differ from those used in the railroad report and also from those used in the present study. Taxes are excluded, and properly, but interest is limited to that on actually outstanding debt incurred for highways of general use, and no provision is made for amortization except as highway debt has been incurred and is written down from year to year. The budget is to include the first cost of new highways and of improvements. Such a provision presumably would preclude further borrowing for highways and streets and make future construction or improvement work largely dependent on what residue of funds remains after the necessary costs of existing roads have been provided for.

It is further stated that the annual budget "should not exceed the economic return from resulting road service," that new roads "must in every instance be predicated on traffic needs," and that the program of development of a State "must be consistent with and measured by the taxpayer's ability to pay." Measurement of the "economic return," if that term can be given definite meaning, is most difficult; traffic needs alone cannot determine whether a given road should be provided; and the "taxpayer's ability to pay" is somewhat indeterminate, particularly when Federal contributions are considered. If the language quoted is intended merely to recommend that conservatism in road expenditures is desirable, no serious objection can be taken to it.

Special motor imposts, it is said, should be used solely for highway purposes; no diversion would be permitted. However, as indicated at pages 54-55, there are conditions under which diversion is both appropriate and desirable.

All special motor imposts should "be reasonable and equitable in amount and incidence;" they should "tend to encourage the use of motor vehicles" and "should be determined in harmony with basic technical and economic analyses as the science of highway transport advances." These general precepts are coupled with statements as to the forms the taxes should take: A motor fuel tax in all cases and a registration fee, the latter "graduated according to weight or horsepower" in the case of passenger cars, "based on mileage operated and graduated according to seating capacity" in the case of busses, and "graduated so that it will increase more than directly with weight" in the case of trucks. A weight-mile basis is recommended, therefore, only in

the case of busses and no specific indication is given of the extent to which the truck fee should "increase more than directly with weight."

All in all, the report is so very general that it does not contribute greatly to the solution of the problems it discusses.

Comments could also be offered on the studies of methods of determining road costs which were made a number of years ago by members of the committee on transportation economics of the Highway Research Board, a division of the National Research Council.⁶⁴

⁶⁴ See Proceedings, Highway Research Board, 8th meeting, 1929, pp. 850-68; 10th meeting, 1930, pp. 829-69; 12th meeting, 1932, pp. 28-65; and 13th meeting, 1933, particularly pp. 70-115. The name of the committee varied in this period. Dean T. R. Agg, of Iowa State College, was chairman in each of the years indicated until 1932, when Professor R. L. Morrison, of the University of Michigan, began acting in that capacity. A brief summary of the several reports is given by W. L. Crum, Director of the Board, in *Engineering News-Record*, June 26, July 6 and 13, 1932.

These reports are of particular interest because of the attention given to the problem of determining the costs of individual roads or stretches of road. That the methods recommended in the papers given were not always accepted by members of the Board is indicated by the discussion that followed the offering of the reports. Points of criticism, suggested by the analyses developed in the present report, include the use of present value of right-of-way easements in determining what was designated "cost of construction" and the failure to consider any deduction from costs to reflect responsibilities other than those of motor-vehicle users as such. As these and other points of difference can be readily discerned by those who are familiar with the reports, no further discussion appears necessary here.

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